STATISTICAL RETHINKING 2026
A03 SOLUTIONS

I asked for a prior predictive simulation of height as a function of age. What makes this exercise tricky is the strongly non-linear relationship between these variables. People get taller for a couple of decades, and then they slowly shrink into old age. So by asking you to develop priors for the relationship, I'm asking you to try to model it. I'm not looking for a specific solution - there are lots of ways to do okay in this task. I'll show you my first attempt.

When thinking about priors, it's useful to find some anchor points. In this case, a person at ago zero (birth) should have height (length) around 50cm. That's just a human average. And a person at conception should have height (length) of about zero centimeters. On the other end, terminal average human height is globally around 155cm or 160cm. It varies a lot by population and sex. But these are priors, so let's go easy on ourselves.

We are looking for a ballistic increase in height from birth to about age 20. Then a flat (or slightly declining) trend for the rest of life. How can we get a function of age to do that? There are several options. You could just use two lines: one for birth to 20, and the other from the 20 and above. Or you could use a kind of biological growth model with a terminal height, if you are familiar with those models.

Here is a simple implementation of the two-lines approach. This is called a piecewise regression model. The idea is to have an equation for ages below 20:

$$\mathrm{E}(H_i|A_i < 20) = \alpha + \beta A_i$$

where α is the expected height H_i at age zero (birth) and β is the slope for the first 20 years. Then we need a second equation:

$$\mathrm{E}(H_i|A_i \geq 20) = \alpha + \beta 20$$

This is the horizontal line for the expected height at age 20 (end of growth).

Now we want priors on α and β. I said height at birth should be about 50cm. So $\alpha \sim \text{Normal}(50, 5)$ should do. For β, we want expected height at age 20 to be around 150cm. So we need another 100cm of height. This requires:

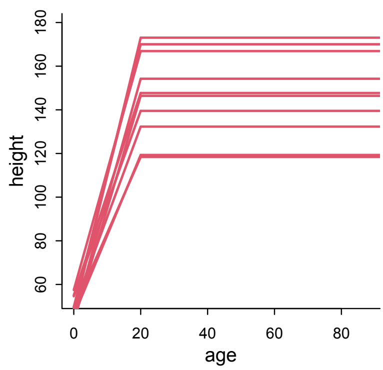
$$\alpha + \beta 20 = 150$$

If $\alpha = 50$, then we can solve for $\beta = 5$. So maybe a prior like $\beta \sim \text{Normal}(5, 1)$.

Let's set up the data and then simulate some priors.

```
# sim from priors
n <- 10
a <- rnorm(n,50,5)
b <- rnorm(n,5,1)
plot( NULL , xlim=range(d$age) , ylim=range(d$height) ,
```

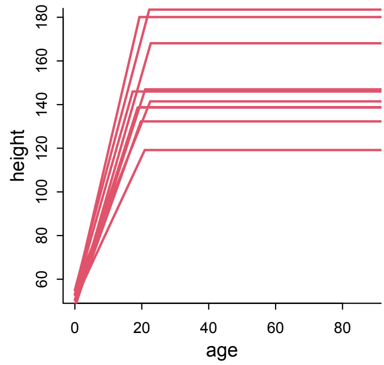
```
    xlab="age" , ylab="height" )
# draw two lines for each draw from priors
for ( i in 1:n ) {
    lines( c(0,20) , c( a[i] + b[i]*0 , a[i] + b[i]*20 ) , lwd=2, col=2 )
    lines( c(20,100) , c( a[i] + b[i]*20 , a[i] + b[i]*20 ) , lwd=2, col=2 )
}
```



Those are very loose at the adult ages, but this is a prior that at least covers the real data and allows for some extreme surprises, if the data demand.

We could expand this model to allow the age of stable height to float as well - we just need a parameter for the 20 in all the code above, and a prior on it. Let x be the age of terminal growth. Then for example with a prior $x \sim \text{Normal}(20, 2)$:

```
# sim from priors
n <- 10
a <- rnorm(n,50,5)
b <- rnorm(n,5,1)
x <- rnorm(n,20,2)
plot( NULL , xlim=range(d$age) , ylim=range(d$height) ,
    xlab="age" , ylab="height" )
# draw two lines for each draw from priors
for ( i in 1:n ) {
    lines( c(0,x[i]) , c( a[i] + b[i]*0 , a[i] + b[i]*x[i] ) ,
        lwd=2, col=2 )
    lines( c(x[i],100) , c( a[i] + b[i]*x[i] , a[i] + b[i]*x[i] ) ,
        lwd=2, col=2 )
}
```



Another approach would be some biological growth model. The most basic approach is to first write down a differential equation for the rate of change in height. We want the rate of growth to be greatest at birth and to decline towards the plateau. Human growth is more complicated than that, but let's keep this simple as an example. So as an example, we could say that the rate of change in height (length) is given by:

$$\frac{dH}{dA} = \beta(H_{\text{max}} - H)$$

where β is a rate and H_{max} is adult maximum height. If we solve this equation for an explicit solution as a function of A, we get:

$$H(A) = H_{\text{max}}\big(1 - \exp(-\beta(A - A_0))\big)$$

where A_0 is the age at which $H = 0$. So we have three variables: H_{max}, β, and A_0. We have a prior already for H_{max}: It should be around 150cm. And A_0 should be around conception, so like $-9/12 = -3/4$.

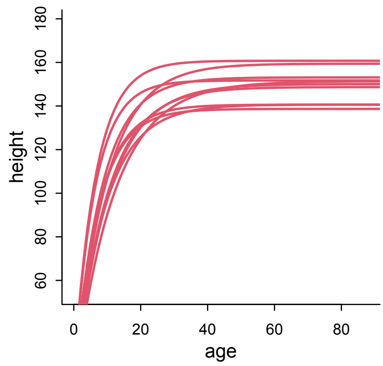
A prior for β is harder. But we can figure this out with a special trick known as "algebra". The term $\exp(-\beta(A - A_0))$ goes to zero in the limit as A increases. So let's solve for the value of β that at $A = 20$ leaves a proportion x of growth:

$$x = \exp(-\beta(A - A_0)) \implies \beta = \frac{\log(1/x)}{A - A_0}$$

Plugging in $A = 20$ and $A_0 = 3/4$ and $x = 0.1$, we get $\beta \approx 0.12$. So let's center our prior on that and see how it works out.

```
# sim from priors
n <- 10
Hmax <- rnorm(n,150,10)
b <- rnorm(n, 0.12 ,0.02)
plot( NULL , xlim=range(d$age) , ylim=range(d$height) ,
    xlab="age" , ylab="height" )
# draw two lines for each draw from priors
for ( i in 1:n ) {
    curve( Hmax[i]*( 1 - exp(-b[i]*(x+3/4)) ) , add=TRUE ,
```

```
        lwd=2 , col=2 , from=-1 , to=100 )
}
```



I might widen both priors just to be conservative. But these are not bad.