Chapter 11

Probability

Unit Structure

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11.0 Objectives:

After going through this unit, you will able to:

- Know the basic concept of probability.
- Probability axioms.
- Conditional probability and its examples.
- Independent events and multiplication theorem of probability.
- Baye's formula of probability.
- Expected value of probability.

11.1 Introduction:

Some time in daily life certain things come to mind like "I will be success today', I will complete this work in hour, I will be selected for job and so on. There are many possible results for these things but we are happy when we get required result. Probability theory deals with experiments whose outcome is not predictable with certainty. Probability is very useful concept. These days many field in computer science such as machine learning, computational linguistics, cryptography, computer vision, robotics other also like science, engineering, medicine and management.

Probability is mathematical calculation to calculate the chance of occurrence of particular happing, we need some basic concept on random experiment , sample space, and events.

11.2 Basic concept of probability:

Random experiment: When experiment can be repeated any number of times under the similar conditions but we get different results on same experiment, also result is not predictable such experiment is called random experiment. For. e.g. A coin is tossed, A die is rolled and so on.

Outcomes: The result which we get from random experiment is called outcomes of random experiment.

Sample space: The set of all possible outcomes of random experiment is called sample space. The set of sample space is denoted by S and number of elements of sample space can be written as$n(S)$. For e.g. A die is rolled, we get $= \{1, 2, 3, 4, 5, 6\}$, $n(S) = 6$.

Events: Any subset of the sample space is called an event. Or a set of sample point which satisfies the required condition is called an events. Number of elements in event set is denoted by $n(E)$. For example in the experiment of throwing of a dia. The sample space is

S = {1, 2, 3, 4, 5, 6 } each of the following can be event i) A: even number i.e. A = { 2, 4, 6} ii) B: multiple of 3 i.e. B = { 3, 6} iii) C: prime numbers i.e. C = { 2, 3, 5}.

Types of events:

Impossible event: An event which does not occurred in random experiment is called impossible event. It is denoted by $\emptyset$ set. i.e. $n(\emptyset) = 0$. For example getting number 7 when die is rolled. The probability measure assigned to impossible event is Zero.

Equally likely events: when all events get equal chance of occurrences is called equally likely events. For e.g. Events of occurrence of head or tail in tossing a coin are equally likely events.

Certain event: An event which contains all sample space elements is called certain events. i.e. $n(A) = n(S)$.

Mutually exclusive events: Two events A and B of sample space S, it does not have any common elements are called mutually exclusive events. In the experiment of throwing of a die A: number less than 2 , B: multiple of 3. There fore $n(A \cap B) = 0$

Exhaustive events: Two events A and B of sample space S, elements of event A and B occurred together are called exhaustive events. For e.g. In a thrown of fair die occurrence of even number and occurrence of odd number are exhaustive events. There fore $n(A \cup B) = 1$.

Complement event: Let S be sample space and A be any event than complement of A is denoted by $\overline{A}$ is set of elements from sample space S, which does not belong to A. For e.g. if a die is thrown, S = {1, 2, 3, 4, 5, 6} and A: odd numbers, A = {1, 3, 5}, then $\overline{A} = \{2, 4, 6\}$.

Probability: For any random experiment, sample space S with required chance of happing event E than the probability of event E is define as

$$P(E) = \frac{n(E)}{n(S)}$$

Basic properties of probability:

1) The probability of an event E lies between 0 and 1. i.e. $0 \leq P(E) \leq 1$.
2) The probability of impossible event is zero. i.e. $P(\emptyset) = 0$.
3) The probability of certain event is unity. i.e. $P(E) = 1$.
4) If A and B are exhaustive events than probability of $P(A \cup B) = 1$.
5) If A and B are mutually exclusive events than probability of $P(A \cap B) = 0$.
6) If A be any event of sample space than probability of complement of A is given by $P(A) + P(\overline{A}) = 1 \Rightarrow \therefore P(\overline{A}) = 1 - P(A)$.

11.3 Probability Axioms:

Let S be a sample space. A probability function P from the set of all event in S to the set of real numbers satisfies the following three axioms for all events A and B in S.

i) $P(A) \geq 0$.
ii) $P(\emptyset) = 0$ and $P(S) = 1$.
iii) If A and B are two disjoint sets i.e. $A \cap B = \emptyset$) than the probability of the union of A and B is $P(A \cup B) = P(A) + P(B)$.

Theorem: Prove that for every event A of sample space S, $0 \leq P(A) \leq 1$.

Proof: $S = A \cup \overline{A}$, $\emptyset = A \cap \overline{A}$.

$$\therefore 1 = P(S) = P(A \cup \overline{A}) = P(A) + P(\overline{A})$$

$$\therefore 1 = P(A) + P(\overline{A})$$

$\Rightarrow P(A) = 1 - P(\overline{A})$ or $P(\overline{A}) = 1 - P(A)$.

If $P(A) \geq 0$. than $P(\overline{A}) \leq 1$.

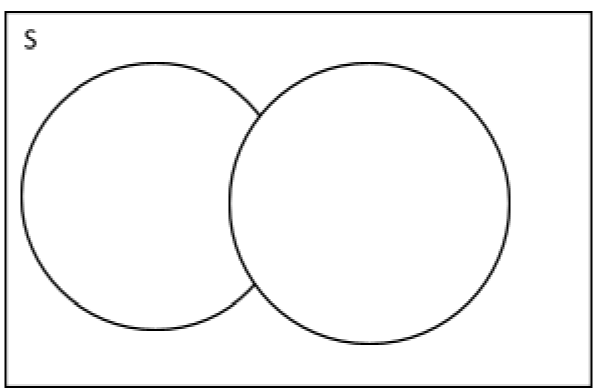
$\therefore$ for every event A; $0 \leq P(A) \leq 1$.

11.3.1 Addition theorem of probability:

Theorem: If A and B are two events of sample space S, then probability of union of A and B is given by $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

Proof: A and B are two events of sample space S.

B



Now from diagram probability of union of two events A and B is given by,

$$P(A \cup B) = P(A \cap \overline{B}) + P(A \cap B) + P(B \cap \overline{A})$$

But $P(A \cap \overline{B}) = P(A) - P(A \cap B)$ and $P(B \cap \overline{A}) = P(B) - P(A \cap B)$.

$\therefore P(A \cup B) = P(A) - P(A \cap B) + P(A \cap B) + P(B) - P(A \cap B)$

$$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B).$$

Note: The above theorem can be extended to three events A, B and C as shown below:

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$

Example 1: A bag contains 4 black and 6 white balls; two balls are selected at random. Find the probability that balls are i) both are different colors. ii) both are of same colors.

Solution: Total number of balls in bag = 4 blacks + 6 white = 10 balls

To select two balls at random, we get

$n(S) = C(10, 2) = 45.$

i) A be the event to select both are different colors.

$$\therefore n(A) = C(4, 1) \times C(6, 1) = 4 \times 6 = 24.$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{24}{45} = 0.53.$$

ii) To select both are same colors.

Let Abe the event to select both are black balls

$$n(A) = C(4, 2) = 6$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{6}{45}$$

Let B be the event to select both are white balls.

$$n(B) = C(6, 2) = 15$$

$P(B) = \frac{n(B)}{n(S)} = \frac{15}{45}$.

A and B are disjoint event.

$\therefore$ The required probability is

$P(A \cup B) = P(A) + P(B) = \frac{6}{45} + \frac{15}{45} = \frac{21}{45} = 0.467.$

Example 2: From 40 tickets marked from 1 to 40, one ticket is drawn at random. Find the probability that it is marked with a multiple of 3 or 4.

Solution: From 40 tickets marked with 1 to 40, one ticket is drawn at random

$n(S) = C(40, 1) = 40$

it is marked with a multiple of 3 or 4, we need to select in two parts.

Let A be the event to select multiple of 3,

i.e. A = { 3,6,9,….,39}

$$n(A) = C(13, 1) = 13$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{13}{40}$$

Let B be the event to select multiple of 4.

i.e. B = {4,8,12, …., 40}

$$n(B) = C(10, 1) = 10$$

$$P(B) = \frac{n(B)}{n(s)} = \frac{10}{40}.$$

Here A and B are not disjoint.

$A \cap B$ be the event to select multiple of 3 and 4.

i.e. $A \cap B$ = {12,24,36}

$$n(A \cap B) = C(3, 1) = 3$$

$$P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{3}{40}$$

∴ The required probability is

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{13}{40} + \frac{10}{40} - \frac{3}{40} = \frac{20}{40} = 0.5.$$

Example 3: If the probability is 0.45 that a program development job; 0.8 that a networking job applicant has a graduate degree and 0.35 that applied for both. Find the probability that applied for atleast one of jobs. If number of graduate are 500 then how many are not applied for jobs?

Solution: Let Probability of program development job= $P(A) = 0.45$.

Probability of networking job = $P(B) = 0.8$.

Probability of both jobs = $P(A \cap B) = 0.35$.

Probability of atleast one i.e. to find $P(A \cup B)$.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cup B) = 0.45 + 0.8 - 0.35 = 0.9$$

Now there are 500 application, first to find probability that not applied for job.

$$P(\overline{A \cup B}) = 1 - P(A \cup B) = 1 - 0.9 = 0.1$$

Number of graduate not applied for job = $0.1 \times 500 = 50$.

Check your Progress:

1. A card is drawn from pack of 52 cards at random. Find the probability that it is a face card or a diamond card.
2. If $P(A) = \frac{3}{8}$ and $(B) = \frac{5}{8}$, $P(A \cup B) = \frac{7}{8}$ than find i) $P(\overline{A \cup B})$ ii) $P(A \cap B)$.
3. In a class of 60 students, 50 passed in computers, 40 passed in mathematics and 35 passed in both. What is the probability that a student selected at random has i) Passed in atleast one subject, ii) failed in both the subjects, iii) passed in only one subject.

11.4 Conditional Probability:

In many case we have the occurrence of an event A and are required to find out the probability of occurrence an event B which depend on event A this kind of problem is called conditional probability problems.

Definition: Let A and B be two events. The conditional probability of event B, if an event A has occurred is defined by the relation,

$P(A) = \frac{P(B \cap A)}{P(A)}$ if and only if $P(A) > 0$.

In case when $P(A) = 0$, $P(B|A)$ is not define because $P(B \cap A) = 0$ and $P(A) = \frac{0}{0}$ which is an indeterminate quantity.

Similarly, Let A and B be two events. The conditional probability of event A, if an event B has occurred is defined by the relation,

$P(B) = \frac{P(A \cap B)}{P(B)}$ If and only if $P(B) > 0$.

Example 4: A pair of fair dice is rolled. What is the probability that the sum of upper most face is 6, given that both of the numbers are odd?

Solution: A pair of fair dice is rolled, therefore $n(S) = 36$.

A to select both are odd number, i.e. A = {(1,1), (1,3), (1,5), (3,1), (3,3), (3,5), (5,1),(5,3), (5,5)}.

$$P(A) = \frac{n(A)}{n(S)} = \frac{9}{36}$$

B is event that the sum is 6, i.e. B = { ((1,5),(2,4), (3,3),(4,2), (5,1)}.

$$P(B) = \frac{n(B)}{n(S)} = \frac{5}{36}$$

$A \cap B$ = { (1,5), (3,3), (5,1)}

$P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{3}{36}$

By the definition of conditional probability,

$P(A) = \frac{P(A \cap B)}{P(A)} = \frac{\frac{3}{36}}{\frac{9}{36}} = \frac{1}{3}$.

Example 5: If A and B are two events of sample space S, such that $P(A) = 0.85$, $P(B) = 0.7$ and $P(A \cup B) = 0.95$. Find i) $P(A \cap B)$, ii) $P(A|B)$, iii) $P(A)$.

Solution: Given that $P(A) = 0.85$, $P(B) = 0.7$ and $P(A \cup B) = 0.95$.

i) By Addition theorem,

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$0.95 = 0.85 + 0.7 - P(A \cap B)$

$P(A \cap B) = 1.55 - 0.95 = 0.6.$

ii) By the definition of conditional probability ,

$P(B) = \frac{P(A \cap B)}{P(B)} = \frac{0.6}{0.7} = 0.857.$

iii) $P(A) = \frac{P(A \cap B)}{P(A)} = \frac{0.6}{0.85} = 0.706$

Example 6: An urn *A* contains 4 Red and 5 Green balls. Another urn *B* contains 5 Red and 6 Green balls. A ball is transferred from the urn *A* to the urn *B*, then a ball is drawn from urn *B*. find the probability that it is Red.

Solution: Here there are two cases of transferring a ball from urn A to B.

Case I: When Red ball is transferred from urn A to B.

There for probability of Red ball from urn A is $P\left(R_A\right) = \frac{4}{9}$

After transfer of red ball, urn B contains 6 Red and 6 Green balls.

Now probability of red ball from urn B = $P\left(R_A\right) \times P\left(R_A\right) = \frac{6}{12} \times \frac{4}{9} = \frac{24}{108}$.

Case II: When Green ball is transferred from urn A to B.

There for probability of Green ball from urn A is $P\left(G_A\right) = \frac{5}{9}$

After transfer of red ball, urn B contains 5 Red and 7 Green balls.

Now probability of red ball from urn B = $P\left(G_A\right) \times P\left(G_A\right) = \frac{5}{12} \times \frac{5}{9} = \frac{25}{108}$.

Therefore required probability $= \frac{24}{108} + \frac{25}{108} = \frac{49}{108} = 0.4537.$

Check your progress:

1. A family has two children. What is the probability that both are boys, given at least one is boy?
2. Two dice are rolled. What is the condition probability that the sum of the numbers on the dice exceeds 8, given that the first shows 4?
3. Consider a medical test that screens for a COVID-19 in 10 people in 1000. Suppose that the false positive rate is 4% and the false negative rate is 1%. Then 99% of the time a person who has the condition tests positive for it, and 96% of the time a person who does not have the condition tests negative for it. a) What is the probability that a randomly chosen person who tests positive for the COVID-19 actually has the disease? b) What is the probability that a randomly chosen person who tests negative for the COVID-19 does not indeed have the disease?

11.5 Independent events:

Independent events: Two events are said to be independent if the occurrence of one of them does not affect and is not affected by the occurrence or non-occurrence of other.

i.e. $P\left(\frac{B}{A}\right) = P(B)$ or $P\left(\frac{A}{B}\right) = P(A)$.

Multiplication theorem of probability: If A and B are any two events associated with an experiment, then the probability of simultaneous occurrence of events A and B is given by

$$P(A \cap B) = P(A)\, P\left(\frac{B}{A}\right)$$

Where $P\left(\frac{B}{A}\right)$ denotes the conditional probability of event B given that event A has already occurred.

OR

$$P(A \cap B) = P(B)\, P\left(\frac{A}{B}\right)$$

Where $P\left(\frac{A}{B}\right)$ denotes the conditional probability of event A given that event B has already occurred.

11.5.1 For Independent events multiplication theorem:

If A and B are independent events then multiplication theorem can be written as,

$$P(A \cap B) = P(A)\, P(B)$$

Proof. Multiplication theorem can be given by,

If A and B are any two events associated with an experiment, then the probability of simultaneous occurrence of events A and B is given by

$$P(A \cap B) = P(A)\, P\left(\frac{B}{A}\right)$$

By definition of independent events, $P\left(\frac{B}{A}\right) = P(B)$ or $P\left(\frac{A}{B}\right) = P(A)$.

$$\therefore P(A \cap B) = P(A)\, P(B) .$$

Note:

1) If A and B are independent event then, $\overline{A}$ and $\overline{B}$ are independent event.

2) If A and B are independent event then, $\overline{A}$ and B are independent event.

3) If A and B are independent event then, A and $\overline{B}$ are independent event.

Example 7: Manish and Mandar are trying to make Software for company. Probability that Manish can be success is $\frac{1}{5}$ and Mandar can be success is $\frac{3}{5}$, both are doing independently. Find the probability that i) both are success. ii) Atleast one will get success. iii) None of them will success. iv) Only Mandar will success but Manish will not success.

Solution: Let probability that Manish will success is $P(A) = \frac{1}{5} = 0.2$.

Therefore probability that Manish will not success is $P(\overline{A}) = 1 - P(A) = 1 - 0.2 = 0.8$.

Probability that Mandar will success is $P(B) = \frac{3}{5} = 0.6$.

Therefore probability that Mandar will not success is $P(\overline{B}) = 1 - P(B) = 1 - 0.6 = 0.4$.

i) Both are success i.e. $P(A \cap B)$.
$P(A \cap B) = P(A) \times P(B) = 0.2 \times 0.6 = 0.12$ $\because$ A and B are independent events.

ii) Atleast one will get success. i.e. $P(A \cup B)$
By addition theorem,
$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.2 + 0.6 - 0.12 = 0.68$.

iii) None of them will success. $P(\overline{A \cup B})$ or $P(\overline{A} \cap \overline{B})$
[By DeMorgan's law both are same]

$P(\overline{A \cup B}) = 1 - P(A \cup B) = 1 - 0.68 = 0.32$.

Or

If A and B are independent than $\overline{A}$ and $\overline{B}$ are also independent.

$$P(\overline{A} \cap \overline{B}) = P(\overline{A}) \times P(\overline{B}) = 0.8 \times 0.4 = 0.32.$$

iv) Only Mandar will success but Manish will not success. i.e. $P(\overline{A} \cap B)$.

$$P(\overline{A} \cap B) = P(\overline{A}) \times P(B) = 0.8 \times 0.6 = 0.48$$

Example 8: 50 coding done by two students A and B, both are trying independently. Number of correct coding by student A is 35 and student B is 40. Find the probability of only one of them will do correct coding.

Solution: Let probability of student A get correct coding is $P(A) = \frac{35}{50} = 0.7$

Probability of student A get wrong coding is $P(\overline{A}) = 1 - 0.7 = 0.3$

Probability of student B get correct coding is $P(B) = \frac{40}{50} = 0.8$

Probability of student B get wrong coding is $P(\overline{B}) = 1 - 0.8 = 0.2$.

The probability of only one of them will do correct coding.

i.e. A will correct than B will not or B will correct than A will not.

$$P(A\cap\overline{B}) + P(B\cap\overline{A}) = P(A)\times P(\overline{B}) + P(B)\times P(\overline{A}).$$

$$= 0.7\times0.2 + 0.8\times0.3 = 0.14 + 0.24 = 0.38$$

Example 9: Given that $P(A) = \frac{3}{7}$, $P(B) = \frac{2}{7}$, if A and B are independent events than find i) $P(A\cap B)$, ii) $P(\overline{B})$, iii) $P(A\cup B)$, iv) $P(\overline{A} \cap \overline{B})$.

Solution: Given that $P(A) = \frac{3}{7}$, $P(B) = \frac{2}{7}$.

i) A and B are independent events,

$$\therefore P(A\cap B) = P(A)\times P(B) = \frac{3}{7} \times \frac{2}{7} = \frac{6}{49} = 0.122$$

ii)$P(\overline{B}) = 1 - P(B) = 1 - \frac{2}{7} = \frac{5}{7} = 0.714.$

iii)By addition theorem,

$$P(A\cup B) = P(A) + P(B) - P(A\cap B) = \frac{3}{7} + \frac{2}{7} - \frac{6}{49} = \frac{29}{49} = 0.592.$$

iv) $P(\overline{A} \cap \overline{B}) = P(\overline{A\cup B}) = 1 - P(A\cup B) = 1 - 0.592 = 0.408.$

Check your progress:

1. If $P(A) = \frac{2}{5}$, $P(B) = \frac{1}{3}$ and if A and B are independent events, find $(i)\ P(A \cap B)$, $(ii)\ P(A\cup B)$, $(iii) P(\overline{A} \cap \overline{B})$.
2. The probability that A , B and C can solve the same problem independently are $\frac{1}{3}, \frac{2}{5}\ and\ \frac{3}{4}$ respectively. Find the probability that i) the problem remain unsolved, ii) the problem is solved, iii) only one of them solve the problem.
3. The probability that Ram can shoot a target is $\frac{2}{5}$ and probability of Laxman can shoot at the same target is $\frac{4}{5}$. A and B shot independently. Find the probability that (i) the target is not shot at all, (ii) the target is shot by at least one of them. (iii) the target shot by only one of them. iv) target shot by both.

11.6 Bayes formula:

In 1763, Thomas Bayes put forward a theory of revising the prior probabilities of mutually exclusive and exhaustive events whenever new information is received. These new probabilities are called as posterior probabilities. The generalized formula of bayes theorem is given below:

Suppose $A_1, A_2, \ldots\ldots, A_k$ are k mutually exclusive events defined in B (a collection of events) each being a subset of the sample space S such that $\cup_{i=1}^{k} A_i = S$ and $P(A_i) > 0,\ \forall\, i = 1, 2,, \ldots k$.

For Some arbitrary event B, which is associated with A_i such that $P(B) > 0$, we can find out the probabilities $P(A_1),\ P(A_2), \ldots\ldots, P(A_k)$.

In Baye's approach we want to find the posterior probability of an event A_i given that B has occurred. i.e. $P(B)$.

By definition of conditional probability, $P(B) = \frac{P(A_i \cap B)}{P(B)}$

$\because B \in S$ such that $B \cap S = B$.

$$B = B \cap (A_1 \cup A_2 \cup \ldots. \cup A_k)$$

$\cup_{i=1}^{k} A_i = S$ and A_i's are disjoint.

i.e. $B = (B \cap E_1) \cup (B \cap E_2) \cup \ldots. \cup (B \cap E_k)$

$$\therefore P(B) = \sum_{i=1}^{k} P(B \cap A_i)$$

$P(B \cap A_i) = P(B) \times P(B) \Rightarrow P(B) = \frac{P(B \cap A_i)}{P(B)}$

But $P(B \cap A_i) = P(A_i)P(A_i)$ and $P(B) = \sum_{i=1}^{k} P(A_i)P(A_i)$.

Therefore we get,

$P(B) = \frac{P(A_i)P(A_i)}{\sum_{i=1}^{k} P(A_i)P(A_i).}$ this known as Baye's formula.

Example 10: There are three bags, first bag contains 2 white, 2 black, 2 red balls; second bag 3 white, 2 black, 1 red balls and third bag 1 white 2 black, 3 red balls. Two balls are drawn from a bag chosen at random. These are found to be one white and I black. Find the probability that the balls so drawn came from the third bag.

Solution: Let B_1 be the first bag, B_2 be the second bag and B_3 be the third bag.

A denotes the two ball are white and black.

First select the bag from any three bags,))

i.e. $P(B_1) = P(B_2) = P(B_3) = \frac{1}{3}$.

Probability of white and black ball from first bag:

$P(B_1) = \frac{C(2,1)\times C(2,1)}{C(6,2)} = \frac{4}{15}$.

Probability of white and black ball from second bag:

$$P(B_2) = \frac{C(3,1)\times C(2,1)}{C(6,2)} = \frac{6}{15}.$$

Probability of white and black ball from third bag:

$$P(B_3) = \frac{C(1,1)\times C(2,1)}{C(6,2)} = \frac{2}{15}$$

By Baye's theorem,

$$P(A) = \frac{P(B_3)P(A|B_3)}{P(B_1)P(B_1)+P(B_2)P(B_2)+P(B_3)P(A|B_3)}$$

$$= \frac{\frac{1}{3}\times\frac{2}{15}}{\frac{1}{3}\times\frac{4}{15}+\frac{1}{3}\times\frac{6}{15}+\frac{1}{3}\times\frac{2}{15}} = \frac{\frac{2}{45}}{\frac{12}{45}} = \frac{1}{6}.$$

Example 11: A company has two factories F_1 and F_2 that produce the same chip, each producing 55% and 45% of the total production. The probability of a defective chip at F_1 and F_2 is 0.07 and 0.03 respectively. Suppose someone shows us a defective chip. What is the probability that this chip comes from factory F_1?

Solution: Let F_i denote the event that the chip is produced by factory. A denote the event that chip is defective.

Given that $P(F_1) = 0.55$, $P(F_2) = 0.45$, $P(F_1) = 0.07$, $P(F_2) = 0.03$.

By Bayes' formula,

$$P(A) = \frac{P(F_1)P(A|F_1)}{P(F_1)P(F_1)+P(F_2)P(A|F_2)} = \frac{0.55\times0.07}{0.55\times0.07+0.45\times0.03} = \frac{0.0385}{0.052} = 0.74.$$

11.7 Expected Value:

In order to understand the behavior of a random variable, we may want to look at its average value. For probability we need to find Average is called expected value of random variable X. for that first we have to learn some basic concept of random variable.

Random Variable: A probability measurable real valued functions, say X, defined over the sample space of a random experiment with respective probability is called a random variable.

Types of random variables: There are two type of random variable.

Discrete Random Variable: A random variable is said to be discrete random variable if it takes finite or countably infinite number of values. Thus discrete random variable takes only isolated values.

Continuous Random variable: A random variable is continuous if its set of possible values consists of an entire interval on the number line.

Probability Distribution of a random variable: All possible values of the random variable, along with its corresponding probabilities, so that $\sum_{i=1}^{n} P_i = 1$, is called a probability distribution of a random variable.

The probability function always follow the following properties:

i) $P(x_i) \geq 0$ for all value of i.

ii) $\sum_{i=1}^{n} P_i = 1$.

The set of values x_i with their probability P_i constitute a discrete probability distribution of the discrete variable X.

For e.g. Three coins are tossed, the probability distribution of the discrete variable X is getting head.

X= x_i	0	1	2	3

$P(x_i)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

Expectation of a random variable (Mean) :

All the probability information of a random variable is contained in probability mass function for random variable, it is often useful to consider various numerical characteristics of that random variable. One such number is the expectation of a random variable.

If random variable X takes values $x_1,\ x_2,\ \ldots.. x_n$ with corresponding probabilities $P_1, P_{2,} \ldots\ldots. P_n$ respectively, then expectation of random variable X is

$$E(X) = \sum_{i=1}^{n} p_i x_i \quad \text{where} \sum_{i=1}^{n} P_i = 1$$

Example 12: In Vijay sales every day sale of number of laptops with his past experience the probability per day are given below:

No. of laptop	0	1	2	3	4	5
Probability	0.05	0.15	0.25	0.2	0.15	0.2

Find his expected number of laptops can be sale?

Solution: Let X be the random variable that denote number of laptop sale per day.

To calculate expected value, $E(X) = \sum_{i=1}^{n} p_i x_i$

$$E(X) = (0\times0.05) + (1\times0.15) + (2\times0.25) + (3\times0.2) + (4\times0.15) + (5\times0.2)$$

$$E(X) = 2.85 \sim 3$$

Therefore expected number of laptops sale per day is 3.

Example 13: A random variable X has probability mass function as follow:

X= x_i	-1	0	1	2	3
P(x_i)	K	0.2	0.3	2k	2k

Find the value of k, and expected value.

Solution: A random variable X has probability mass function,

$\sum_{i=1}^{n} P_i = 1.$

$\Rightarrow$ k + 0.2 + 0.3 + 2k +2k = 1

$\Rightarrow$5k = 0.5

$\Rightarrow$ k = 0.1

Therefore the probability distribution of random variable X is

X= x_i	-1	0	1	2	3
P(x_i)	0.1	0.2	0.3	0.2	0.2

To calculate expected value, $E(X) = \sum_{i=1}^{n} p_i x_i$

$E(X) = (-1\times 0.1) + (0\times 0.2) + (1\times 0.3) + (2\times 0.2) + (3\times 0.2) = 1.2$.

Example 15: A box contains 5 white and 7 black balls. A person draws 3 balls at random. He gets Rs. 50 for every white ball and losses Rs. 10 every black ball. Find the expectation of him.

Solution: Total number of balls in box = 5 white + 7 black = 12 balls.

To select 3 balls at random, $n(s) = C(12, 3) = \frac{12\times 11\times 10}{3\times 2\times 1} = 220.$

Let A be the event getting white ball.

A takes value of 0, 1, 2 and 3 white ball.

Case I : no white ball. i.e. A = 0,

$$P(A = 0) = \frac{C(7,3)}{220} = \frac{35}{220}$$

Case II: one white ball i.e. A = 1,

$$P(A = 1) = \frac{C(5,1)\times C(7,2)}{220} = \frac{105}{220}$$

Case III: two white balls i.e. A = 2,

$$P(A = 2) = \frac{C(5,2)\times C(7,1)}{220} = \frac{70}{220}$$

Case IV: three white balls i.e. A = 3,

$$P(A = 3) = \frac{C(5,3)}{220} = \frac{10}{220}$$

Now let X be amount he get from the game.

Therefore the probability distribution of X is as follows:

$X = x_i$	-30	30	90	150
$P(x_i)$	$\frac{35}{220}$	$\frac{105}{220}$	$\frac{70}{220}$	$\frac{10}{220}$

To calculate expected value, $E(X) = \sum_{i=1}^{n} p_i x_i$

$E(X) = \left(-\ 30\times\frac{35}{220}\right) + \left(30\times\frac{105}{220}\right) + \left(90\times\frac{70}{220}\right) + \left(150\times\frac{10}{220}\right)$ = Rs. 45.

11.8 Let us sum up:

In this chapter we have learn:

- Basic concept of probability like random experiment, outcomes, sample space, events and types of events.
- Probability Axioms and its basic properties.
- Addition theorem of probability for disjoint events.
- Condition Probability for dependent events.
- Independent events.
- For Independent events multiplication theorem.
- Baye's formula and its application.
- Expected Value for discrete random probability distribution.

11.9 Unit end Exercises:

1. A card is drawn at random from well shuffled pack of card find the probability that it is red or king card.

2. There are 30 tickets bearing numbers from 1 to 15 in a bag. One ticket is drawn from the bag at random. Find the probability that the ticket bears a number, which is even, or a multiple of 3.

3. In a group of 200 persons, 100 like sweet food items, 120 like salty food items and 50 like both. A person is selected at random find the probability that the person (i). Like sweet food items but not salty food items (ii). Likes neither.

4. A bag contains 7 white balls & 5 red balls. One ball is drawn from bag and it is replaced after noting its color. In the second draw again one ball is drawn and its color is noted. The probability of the event that both the balls drawn are of different colors.

5. the probability of A winning a race is $\frac{1}{3}$ & that B wins a race is $\frac{3}{5}$. Find the probability that (a). either of the two wins a race. b), no one wins the race.

6. Three machines A, B & C manufacture respectively 0.3, 0.5 & 0.2 of the total production. The percentage of defective items produced by A, B & C is 4 , 3 & 2 percent respectively. for an item chosen at random , what is the probability it is defective.

7. An urn A contains 3 white & 5 black balls. Another urn B contains 5 white & 7 black balls. A ball is transferred from the urn A to the urn B, then a ball is drawn from urn B. find the probability that it is white.

8. A husband & wife appear in an interview for two vacancies in the same post. The probability of husband selection is $\frac{1}{7}$ & that of wife's selection is $\frac{1}{5}$. What is the probability that, a). both of them will be selected. b). only one of them will be selected. c). none of them will be selected?

9. A problem statistics is given to 3 students A,B & C whose chances of solving if are $\frac{1}{2}$, $\frac{3}{4}$ & $\frac{1}{4}$ respectively. What is the probability that the problem will be solved?

10. A bag contains 8 white & 6 red balls. Find the probability of drawing 2 balls of the same color.

11. Find the probability of drawing an ace or a spade or both from a deck of cards?

12. A can hit a target 3 times in a 5 shots, B 2 times in 5 shots & C 3 times in a 4 shots. they fire a volley. What is the probability that a).2 shots hit? b). at least 2 shots hit?

13. A purse contains 2 silver & 4 cooper coins & a second purse contains 4 silver & 4 cooper coins. If a coin is selected at random from one of the two purses, what is the probability that it is a silver coin?

14. The contain of a three urns are : 1 white, 2 red, 3 green balls; 2 white, 1 red, 1 green balls & 4 white, 5 red, 3 green balls. Two balls are drawn from an urn chosen at random. This are found to be 1 white & 1 green. Find the probability that the balls so drawn come from the second urn.

15. Three machines A,B & C produced identical items. Of there respective output 2%, 4% & 5% of items are faulty. On a certain day A has produced 30% of the total output, B has produced 25% & C the remainder. An item selected at random is found to be faulty. What are the chances that it was produced by the machine with the highest output?

16. A person speaks truth 3 times out of 7. When a die is thrown, he says that the result is a 1. What is the probability that it is actually a 1?

17. There are three radio stations A, B and C which can be received in a city of 1000 families. The following information is available on the basis of a survey:

(a). 1200 families listen to radio station A
(b). 1100 families listen to radio station B.
(c). 800 families listen to radio station C.
(d). 865 families listen to radio station A & B.
(e). 450 families listen to radio station A & C.
(f). 400 families listen to radio station B & C.
(g). 100 families listen to radio station A, B & C.
The probability that a family selected at random listens at least to one radio station.

18. The probability distribution of a random variable x is as follows.

X	1	3	5	7	9
P(x)	K	2k	3k	3k	K

Find value of (i). K (ii). E(x)

19. A player tossed 3 coins. He wins Rs. 200 if all 3 coins show tail, Rs. 100 if 2 coins show tail, Rs. 50 if one tail appears and loses Rs. 40 if no tail appears. Find his mathematical expectation.

20. The probability distribution of daily demand of cell phones in a mobile gallery is given below. Find the expected mean .

Demand	5	10	15	20
Probability	0.4	0.22	0.28	0.10

21. If $P(A) = \frac{4}{15}$, $P(B) = \frac{7}{15}$ and if A and B are independent events, find
$(i)\ P(A \cap B)$, $(ii)\ P(A \cup B)$, $(iii) P(\overline{A} \cap \overline{B})$.

22. If $P(A) = \frac{5}{9}$, $P(\overline{B}) = \frac{2}{9}$ and if A and B are independent events, find
$(i)\ P(A \cap B)$, $(ii)\ P(A \cup B)$, $(iii) P(\overline{A} \cap \overline{B})$.

23. If $P(A) = 0.65$, $P(B) = 0.75$ and $P(A \cap B) = 0.45$, where A and B are events of sample space S, find $(i) P(A|B)$, $(ii)\ P(A \cup B)$, $(iii) P(\overline{A} \cap \overline{B})$.

24. A box containing 5 red and 3 black balls, 3 balls are drawn at random from box. Find the expected number of red balls drawn.

25. Two fair dice are rolled. X denotes the sum of the numbers appearing on the uppermost faces of the dice. Find the expected value.

11.10 List of References:

- Discrete Mathematics with Applications by Sussana S. Epp
- Discrete Mathematics and its Applications by Kenneth H. Rosen
- Discrete Mathematical Structures by B Kolman, RC Busby, S Ross
- Discrete structures by Liu