start

3/12/2022

Unit II

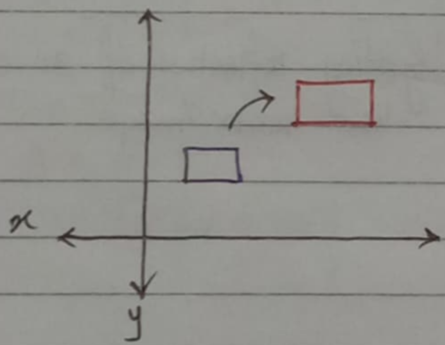
Chapter I : Transformation.

The process of changing position, orientation, size of an object by using mathematical operations.

2D → x , y
3D → x , y , z

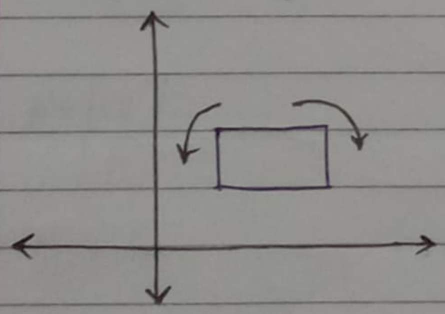
- **Translation:**

Change in position of object.



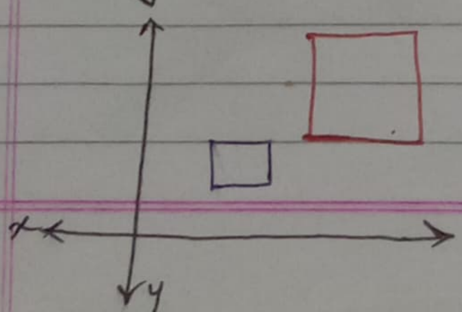
- **Rotation.**

Rotating the object in clockwise and anti-clockwise direction.



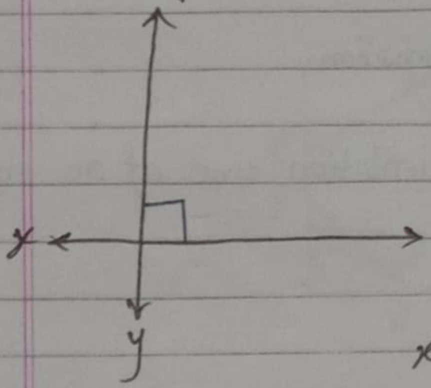
- **Scaling**

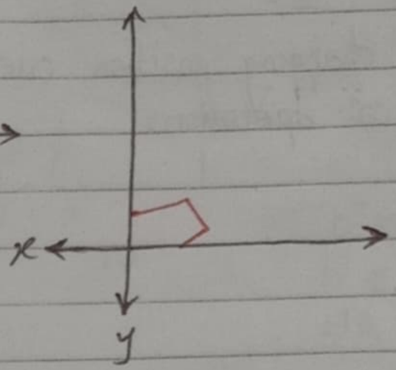
Change in size of object.



- ## Shearing

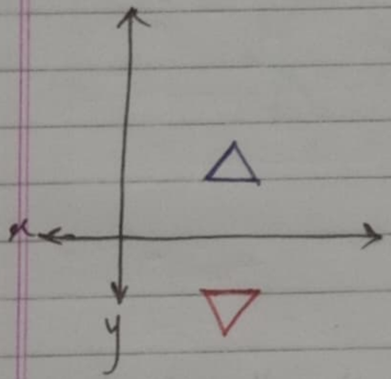
Change in shape of object





- ## Reflection

Getting reflection of an object like mirror image.



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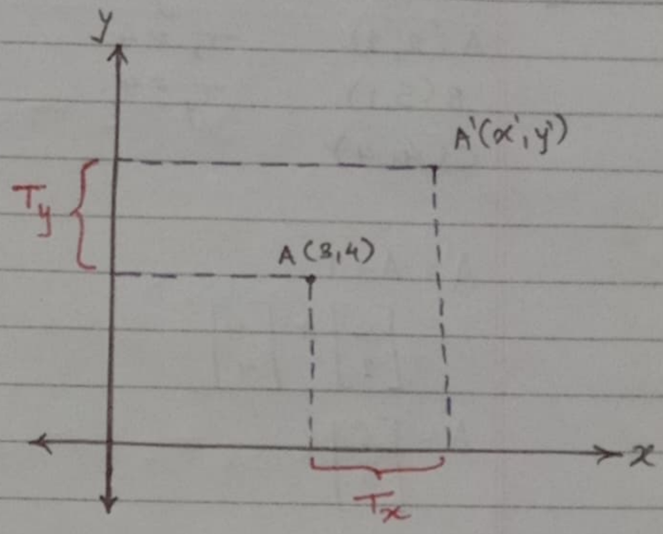
1. Translation

$A(3,4)$ $T_x = 4$
$A'(x', y')$ $T_y = 6$
$T_x = 4$
$T_y = 6$ $x = 3, y = 4$
$x' = 7, y' = 10$



$x + T_x = x'$
$y + T_y = y'$

$A' = A + T$

$A = \begin{bmatrix} x \\ y \end{bmatrix}$ $A' = \begin{bmatrix} x' \\ y' \end{bmatrix}$ $T = \begin{bmatrix} T_x \\ T_y \end{bmatrix}$

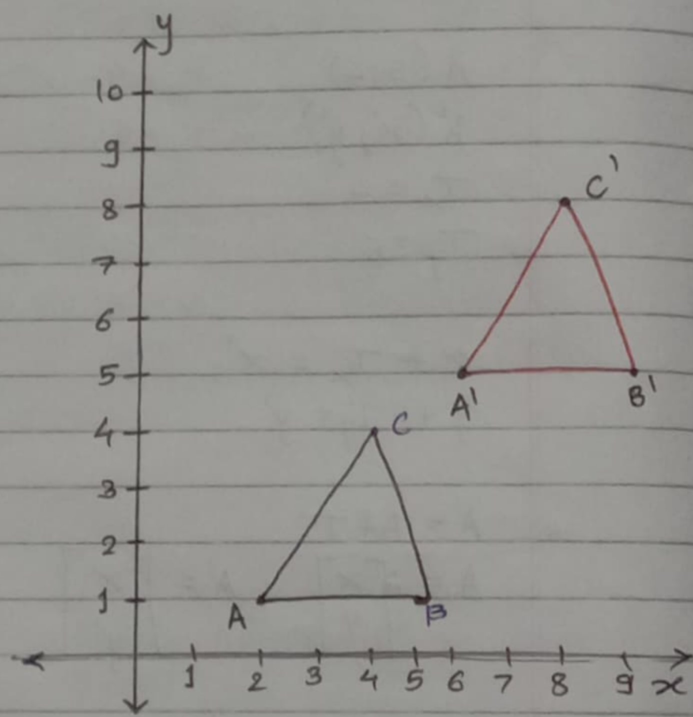
$$A' = A + T$$
$$= \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} T_x \\ T_y \end{bmatrix}$$
$$= \begin{bmatrix} 3 \\ 4 \end{bmatrix} + \begin{bmatrix} 4 \\ 6 \end{bmatrix}$$
$$A' = \begin{bmatrix} 7 \\ 10 \end{bmatrix}$$

Q.1] Translate ΔABC by 4 units in x direction & 4 units in y direction

$A(2,1)$ $\quad T_x = 4$

$B(5,1)$ $\quad T_y = 4$

$C(4,4)$



$A' = A + T$

$= \begin{bmatrix} 2 \\ 1 \end{bmatrix} + \begin{bmatrix} 4 \\ 4 \end{bmatrix}$

$A' = \begin{bmatrix} 6 \\ 5 \end{bmatrix}$

$B' = B + T$

$= \begin{bmatrix} 5 \\ 1 \end{bmatrix} + \begin{bmatrix} 4 \\ 4 \end{bmatrix}$

$B' = \begin{bmatrix} 9 \\ 5 \end{bmatrix}$

$C' = C + T$

$= \begin{bmatrix} 4 \\ 4 \end{bmatrix} + \begin{bmatrix} 4 \\ 4 \end{bmatrix}$

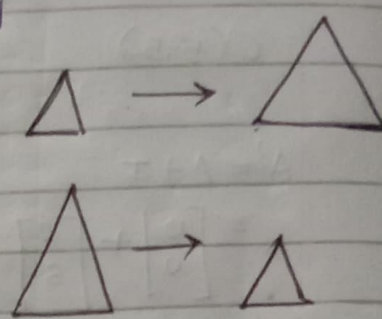
$C' = \begin{bmatrix} 8 \\ 8 \end{bmatrix}$

2. **Scaling (change in size)**

S_x & S_y

If S_x & $S_y > 1 \rightarrow$ small to big

S_x & $S_y < 1 \rightarrow$ big to small



S_x & $S_y = 1 \rightarrow$ No scaling

$X' = x \cdot S_x$

$Y' = y \cdot S_y$

$\cancel{Z' = z \cdot S_z}$

$A = [x \quad y]_{1\times 2}$ $\qquad S = \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix}_{2\times 2}$

$A' = A \cdot S$

$$\boxed{A' = [S_x \cdot x \quad y \cdot S_y]_{1\times 2}}$$

Q.1] Scale ΔABC by 3 units to x direction & 2 units to y direction.

A (2,1) $\quad S_x = 3$

B (5,1) $\quad S_y = 2$

C (4,4)

Solution: $A' = A \cdot S$

$= [2, 1]_{1\times 2} \cdot \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}_{2\times 2}$

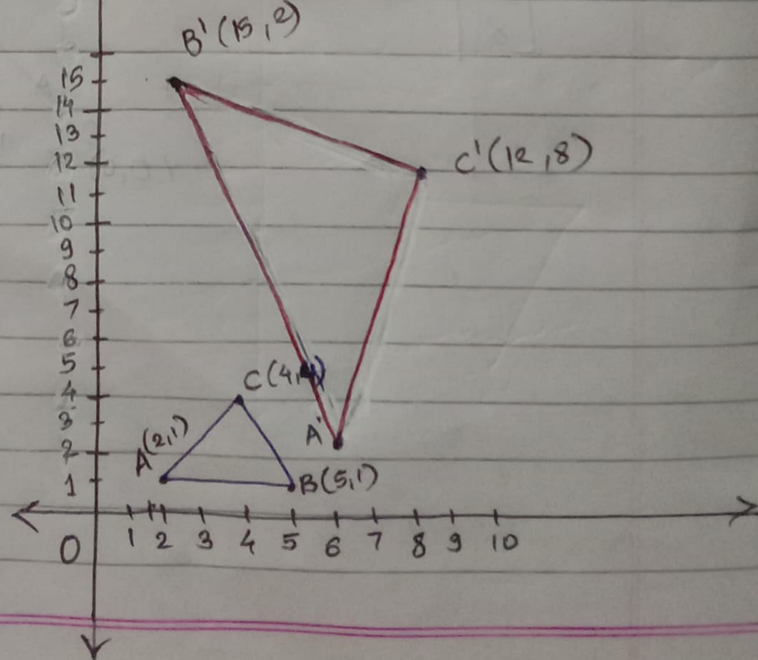
$A' = [6 \quad 2]$

$B' = [5 \quad 1] \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$

$B' = [15 \quad 2]$

$C' = [4 \quad 4] \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$

$C' = [12 \quad 8]$



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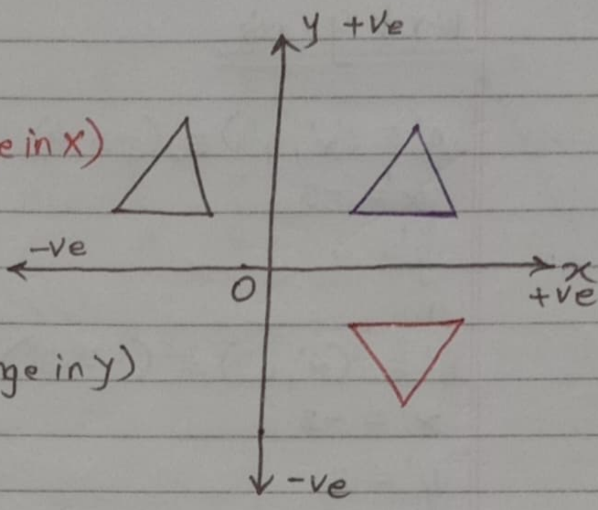
3. Reflection

Reflection on X-axis (No change in X)

$x' = x$

$y' = -y$



Reflection on y-axis (No change in y)

$x' = -x$

$y' = y$

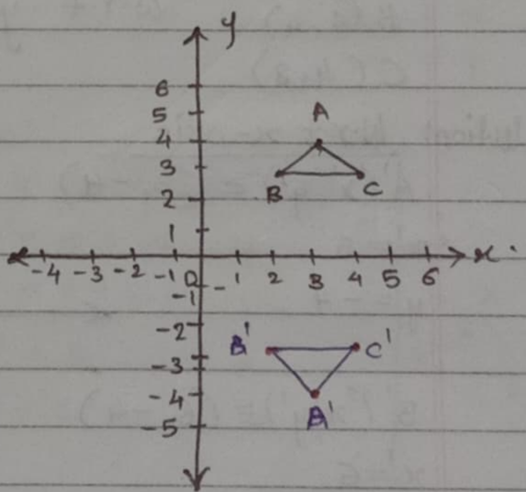
Que: 1] A (3, 4) w.r.t x-axis
B (2, 3) w.r.t y-axis.
C (4, 3)

Solution: w.r.t x-axis

$A' (x', y') \equiv (3, -4)$

$x' = 3$

$y' = -4$



$B' (x', y') \equiv (2, -3)$

$x' = 2$

$y' = -3$

$C' (x', y') \equiv (4, -3)$

$x' = 4$

$y' = -3$

<u>W.r.t y-axis.</u>

$A' \equiv (x', y') \equiv (-3, 4)$

$x' = -3$

$y' = 4$

$B' \equiv (x', y') \equiv (-2, 3)$

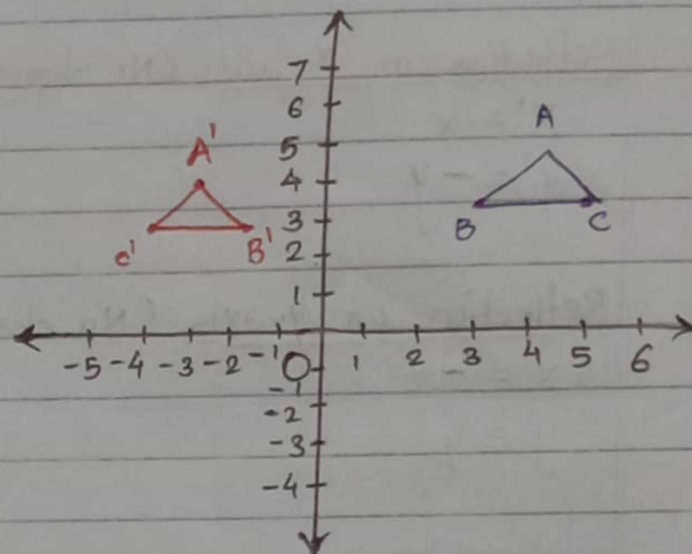
$x' = -2$

$y' = 3$

$C' \equiv (x', y') \equiv (-4, 3)$

$x' = -4$

$y' = 3$



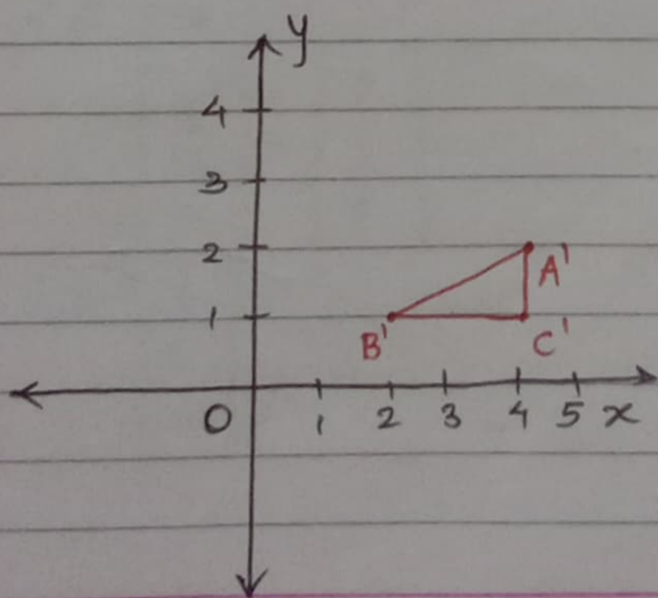
4. **Shearing**

In x-axis

$$x' = x + sh_x \cdot y$$
$$y' = y$$

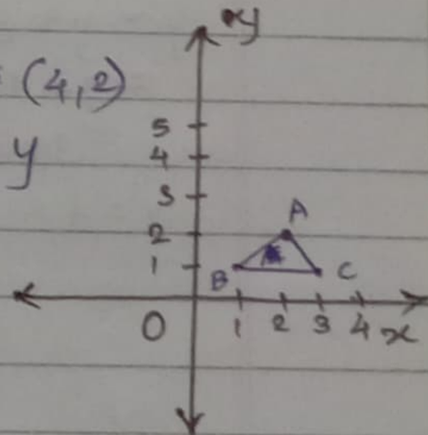
In y-axis

$$x' = x$$
$$y' = y + sh_y \cdot x$$



Que: 1] $sh_x = 1$ A (2,2)
$sh_y = 1$ B (1,1)
C (3,1)
w.r.t x &
w.r.t y

Solution:



$A' \equiv (x', y') \equiv (4,2)$
$x' = x + sh_x \cdot y$
$= 2 + 1 \cdot 2$
$x' = 4$
$y' = 2$

$B' \equiv (x', y') \equiv (2,1)$
$x' = x + sh_x \cdot y$
$= 1 + 1 \cdot 1$
$x' = 2 \quad y' = 1$
$C' \equiv (x', y') \equiv (4,1)$
$x' = x + sh_x \cdot y$
$= 3 + 1 \cdot 1$
$x' = 3 + 1$
$= 4 \quad y' = 1$

W.r.t y-axis.

$A' \equiv (x', y') \equiv (2, 4)$

$x' = x = 2$

$y' = y + sh_y \cdot x$

$= 2 + 1 \cdot 2$

$y' = 4$

$B' \equiv (x', y') \equiv (1, 2)$

$x' = x = 1$

$y' = y + sh_y \cdot x$

$= 1 + 1 \cdot 1$

$= 1 + 1$

$y' = 2$

$C' \equiv (x', y') \equiv (3, 4)$

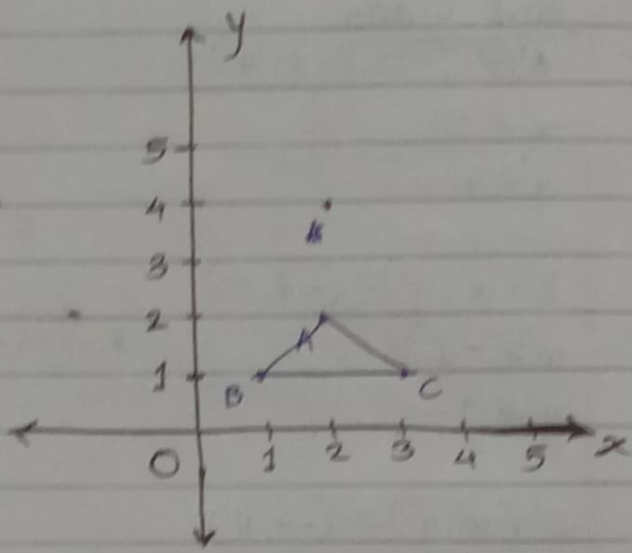
$x' = x = 3$

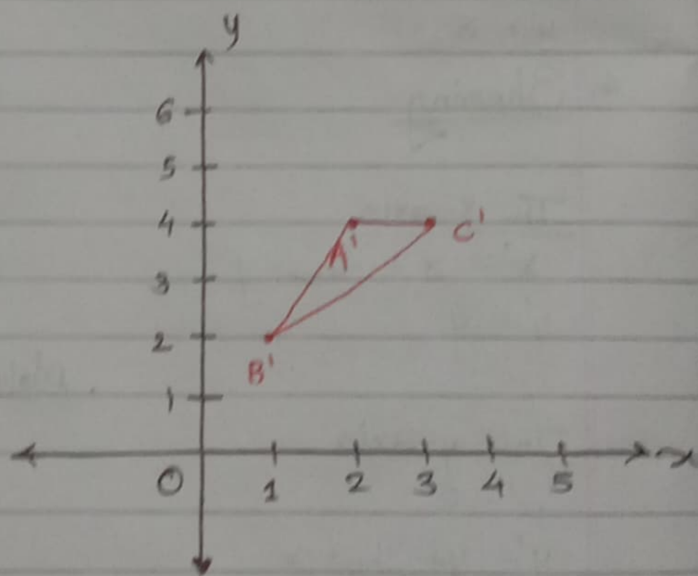
$y' = y + sh_y \cdot x$

$= 1 + 1 \cdot 3$

$= 1 + 3$

$y' = 4$





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3d Transformation (X, Y, Z)

1. Translation (3D)

$$X' = X + T_x$$
$$Y' = Y + T_y$$
$$Z' = Z + T_z$$

Que: 1] 3D object with co-ordinates A(0,3,1), B(3,3,2), C(3,0,0), D(0,0,0). Apply translation.

$T_x = 1,\ T_y = 1,\ T_z = 2$

Solution:

$A' = A + T$

$= \begin{bmatrix} 0 \\ 3 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$

$A' = \begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix}$

$B' = B + T$

$= \begin{bmatrix} 3 \\ 3 \\ 2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$

$B' = \begin{bmatrix} 4 \\ 4 \\ 4 \end{bmatrix}$

$C' = C + T$

$= \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$

$C' = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$

$D' = D + T$

$= \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$

$D' = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$

2. Scaling (3D)

$x' = x \cdot S_x$
$y' = y \cdot S_y$
$z' = z \cdot S_z$

Que: I] 3D object with coordinates $A(0,3,3)$, $B(3,3,6)$, $C(3,0,1)$, $D(0,0,0)$ Apply Scaling $S_x = 2$, $S_y = 3$, $S_z = 3$

Answer: $\cancel{A' = A \cdot S_x}$
$\cancel{= [0, 3 \ 3]}$ [

$A' = (0, 9, 9)$
$x' = 0 \times 2 = 0$
$Y' = 3 \times 3 = 9$
$z' = 3 \times 3 = 9$

$B'(6, 9, 18)$
$X' = 3 \times 2 = 6$
$Y' = 3 \times 3 = 9$
$Z' = 6 \times 3 = 18$

$C'(6, 0, 3)$
$x' = 3 \times 2 = 6$
$y' = 0 \times 3 = 0$
$z' = 1 \times 3 = 3$

$D'(0, 0, 0)$
$x' = 0 \times 2 = 0$
$Y' = 0 \times 3 = 0$
$z' = 0 \times 3 = 0$

3. Reflection (3D)

W.r.t. XY Plane	W.r.t YZ Plane	W.r.t ZX Plane.
$x' = x$ $y' = y$ $z' = -z$	$x' = -x$ $y' = y$ $z' = z$	$x' = x$ $y' = -y$ $z' = z$

Que: 1] 3D triangle with co-ordinates
A(3,4,1)
B(6,4,2)
C(5,6,3)
Apply reflection on XY plane.

Solution:

$A'(3,4,-1)$	$B' = (6,4,-2)$	$C'(5,6,-3)$
$x' = 3$	$x' = 6$	$x' = 5$
$y' = 4$	$y' = 4$	$y' = 6$
$z' = -1$	$z' = -2$	$z' = -3$

4. Shearing (3D)

x-axis	y-axis	z-axis.
$x' = x$ $y' = y + sh_y \cdot x$ $z' = z + sh_z \cdot x$	$x' = x + sh_x \cdot y$ $y' = y$ $z' = z + sh_z \cdot y$	$x' = x + sh_x \cdot z$ $y' = y + sh_y \cdot z$ $z' = z$

Que 1] 3D triangle with points $A(0,0,0)$,
$B(1,1,2)$
$C(1,1,3)$

Shear $sh_x = 2$
$sh_y = 2$
$sh_z = 3$

Solution: X-axis ⇒

$A'(0,0,0)$
$x' = x = 0$
$y' = 0 + 2\cdot 0 = 0$
$z' = 0 + 3\cdot 0 = 0$

$B'(1,3,5)$
$x' = 1$
$y' = 1 + 2\cdot 1 = 3$
$z' = 2 + 3\cdot 1 = 5$

$C'(1,3,6)$
$x' = 1$
$y' = 1 + 2\cdot 1 = 3$
$z' = 3 + 3\cdot 1 = \cancel{3}6$

Y-axis ⇒

$A'(0,0,0)$
$x' = 0 + 2\cdot 0 = 0$
$y' = 0$
$z' = 0 + 3\cdot 0 = 0$

$B'(3,1,5)$
$x' = 1 + 2\cdot 1 = 3$
$y' = 1$
$z' = 2 + 3\cdot 1 = 5$

$C'(3,1,6)$
$x' = 1 + 2\cdot 1 = 3$
$y' = 1$
$z' = 3 + 3\cdot 1 = 6$

Z-axis ⇒

$A'(0,0,0)$
$x' = 0 + 2\cdot 0 = 0$
$y' = 0 + 2\cdot 0 = 0$
$z' = 0$

$B'(5,5,2)$
$x' = 1 + 2\cdot 2 = 5$
$y' = 1 + 2\cdot 2 = 5$
$z' = 2$

$C'(7,7,3)$
$x' = 1 + 2\cdot 3 = 7$
$y' = 1 + 2\cdot 3 = 7$
$z' = 3$

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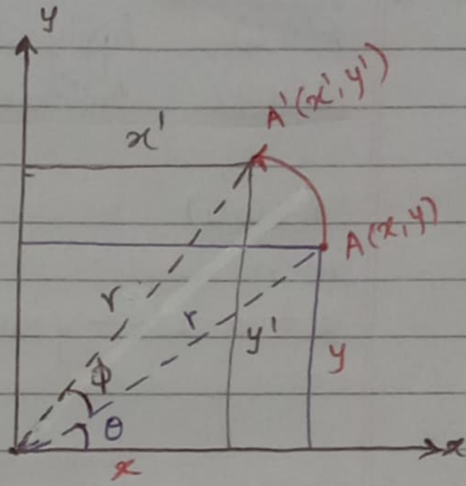
2D Rotation.

$$\boxed{x' = x\cdot\cos\theta - y\cdot\sin\theta}$$
$$\boxed{y' = x\cdot\sin\theta + y\cdot\cos\theta}$$



(Triangle in blue)

$\cos\theta = \dfrac{x}{r} \Rightarrow \quad x = r\cdot\cos\theta$

$\sin\theta = \dfrac{y}{r} \Rightarrow \quad y = r\cdot\sin\theta$

(Triangle in black)

$\cos(\theta+\phi) = \dfrac{x'}{r} \Rightarrow x' = r\cdot\cos(\theta+\phi)$

$\sin(\theta+\phi) = \dfrac{y'}{r} \Rightarrow y' = r\cdot\sin(\theta+\phi)$

$\cos(\theta+\phi) = \cos\theta\cdot\cos\phi - \sin\theta\cdot\sin\phi$

$\sin(\theta+\phi) = \sin\theta\cdot\cos\phi + \cos\theta\cdot\sin\phi$

$x' = r\cdot\cos(\theta+\phi)$

$\therefore x' = r[\cos\theta\cdot\cos\phi - \sin\theta\cdot\sin\phi]$

$\therefore x' = \underbrace{r\cos\theta}_{x}\cdot\cos\phi - \underbrace{r\sin\theta}_{y}\cdot\sin\phi$

$y' = r\cdot\sin(\theta+\phi)$

$\therefore y' = r[\sin\theta\cdot\cos\phi + \cos\theta\cdot\sin\phi]$

$\therefore y' = \underbrace{r\sin\theta}_{y}\cdot\cos\phi + \underbrace{r\cos\theta}_{x}\cdot\sin\phi$

$$\boxed{x' = x\cdot\cos\theta - y\cdot\sin\theta}$$

$$\boxed{y' = x\cdot\sin\theta + y\cos\theta}$$

Que: 1] $l(AB)$
$A(0,0)$ Apply 30° Rotation
$B(4,4)$ Anticlockwise.

$A(0,0)$

A'

$x' = x \cdot \cos\theta - y \sin\theta = 0 \cdot \cos 30^\circ - 0 \cdot \sin 30^\circ$

$= 0 \cdot \frac{\sqrt{3}}{2} - 0 \cdot \frac{1}{2}$

$x' = 0$

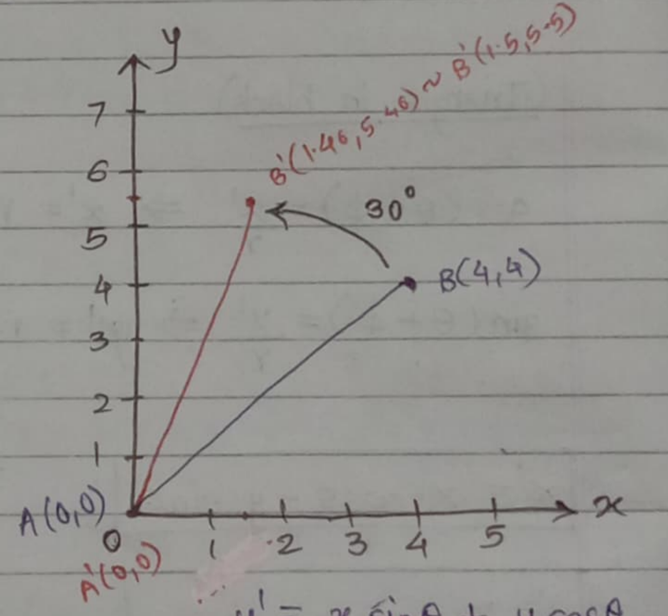
$y' = x \cdot \sin\theta - y \cdot \cos\theta$

$= 0 \cdot \sin 30^\circ - 0 \cdot \cos 90^\circ$

$= 0 \cdot \frac{1}{2} - 0 \cdot \frac{\sqrt{3}}{2}$

$y' = 0$

$\boxed{A'(0,0)}$



B'

$x' = x \cos\theta - y \sin\theta$

$= 4 \cdot \cos 30^\circ - 4 \cdot \sin 30^\circ$

$= 4 \cdot \frac{\sqrt{3}}{2} - 4 \cdot \frac{1}{2}$

$= 2(\sqrt{3} - 1)$

$= 2(1.73 - 1) = 2(0.73)$

$x' = 1.46$

$y' = x \sin\theta + y \cos\theta$

$= 4 \cdot \sin 30^\circ + 4 \cdot \cos 30^\circ$

$= 4 \cdot \left(\frac{1}{2}\right) + 4 \cdot \left(\frac{\sqrt{3}}{2}\right)$

$= 2(1 + \sqrt{3})$

$= 2(1 + 1.73) = 2(2.73)$

$= 5.46.$

$\boxed{B'(1.46, 5.46)}$

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Que: 2] $A(0,0)$ Rotate by $90°$
$B(1,0)$ Anticlockwise.
$C(1,1)$

Solution: A'

$x' = x \cdot \cos\theta - y \cdot \sin\theta$
$\cancel{y} = 0 \cdot \cos 90° - 0 \cdot \sin 90°$
$= 0 \cdot 0 - 0 \cdot 1 = 0$ $x' = 0$

$y' = x \cdot \sin\theta + y \cdot \cos\theta$
$= 0 \cdot 1 + 0 \cdot 0$
$y' = 0$

$A'(0,0)$

B'
$x' = x \cos\theta - y \cdot \sin\theta$
$= 1 \cdot 0 - 0 \cdot 1$
$x' = 0$

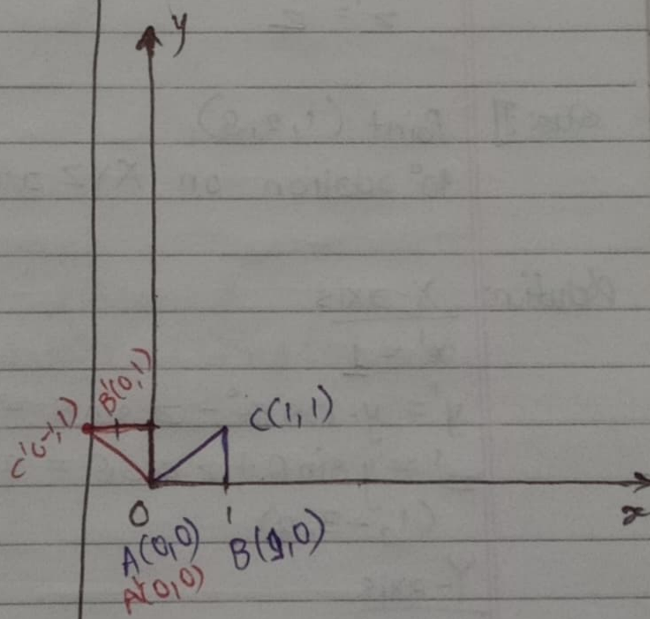
$y' = x \sin\theta + y \cos\theta$
$= 1 \cdot 1 + 0 \cdot 0.$
$y' = 1$

$B'(0,1)$

C'
$x' = x \cos\theta - y \sin\theta$
$= 1 \cdot 0 - 1 \cdot 1$
$= 1 - 1$
$x' = -1$

$y' = x \sin\theta + y \cos\theta$
$= 1 \cdot 1 + 1 \cdot 0$
$y' = 1$

$C'(-1,1)$



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3D Rotation

X-axis

$x' = x$
$y' = y \cdot \cos\theta - z \sin\theta$
$z' = y \sin\theta + z \cos\theta$

Y-axis

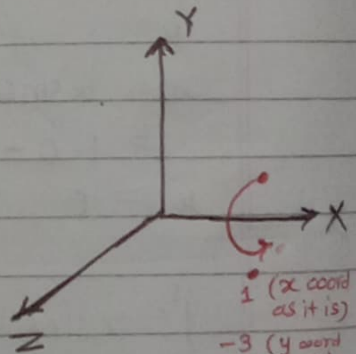
$x' = z \cdot \sin\theta + x \cdot \cos\theta$
$y' = y$
$z' = y \cdot \cos\theta - x \cdot \sin\theta$

Z-axis

$x' = x \cdot \cos\theta - y \cdot \sin\theta$
$y' = x \cdot \sin\theta + y \cdot \cos\theta$
$z' = z$

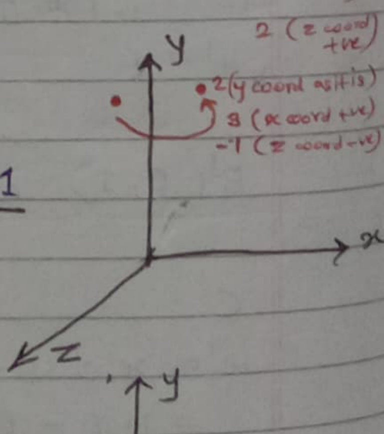
Que: I] Point (1, 2, 3)
90° rotation on XYZ axis.

Solution: **X-axis**



$x' = \underline{1}$
$y' = y \cdot \cos 90^\circ - z \sin\theta = 2(0) - 3(1) = \underline{-3}$
$z' = y \sin\theta + z \cos\theta = 2(1) + 3(0) = \underline{2}$
$(1, -3, 2)$

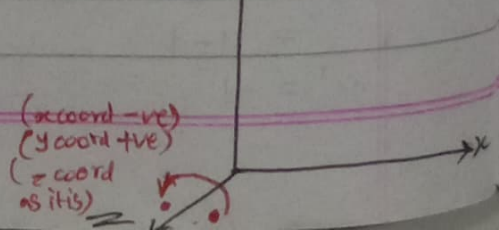
Y-axis



$x' = z \sin\theta + x \cdot \cos\theta = 3(1) + 1(0) = \underline{3}$
$y' = \underline{2}$
$z' = y \cos\theta - x \sin\theta = 2(0) - 1(1) = \underline{-1}$
$(3, 2, -1)$

Z-axis

$x' = x \cos\theta - y \sin\theta = 1(0) - 2(1) = \underline{-2}$
$y' = x \sin\theta + y \cos\theta = 1(1) + 2(0) = \underline{1}$
$z' = \underline{3}$
$(-2, 1, 3)$



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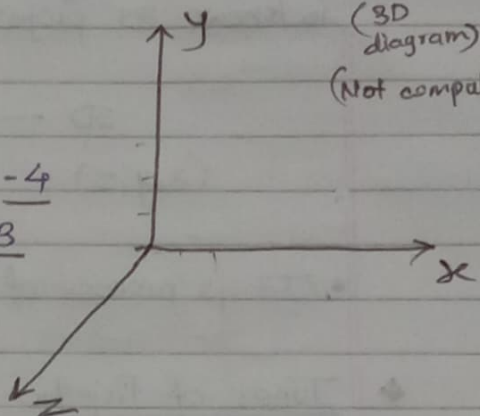
Que:2] (2, 3, 4) 90° rotation.

Solution: X-axis



(3D diagram)
(Not compulsory)

$x' = \underline{2}$

$y' = y\cos\theta - z\sin\theta = 3(0) - 4(1) = \underline{-4}$

$z' = y\sin\theta + z\cos\theta = 3(1) + 4(0) = \underline{3}$

$(2, -4, 3)$

Y-axis

$x' = z\sin\theta + x\cos\theta = 4(1) + 2(0) = 4$

$y' = 3$

$z' = y\cos\theta - x\sin\theta = 3(0) - 2(1) = -2$

$(4, 3, -2)$

Z-axis

$x' = x\cos\theta - y\sin\theta = 2(0) - 3(1) = -3$

$y' = x\sin\theta + y\cos\theta = 2(1) + 3(0) = 2(1) + 3(0) = 2$

$z' = 4$

Que: 2] Explain Window to viewport transformation.

Definitions:

- World Coordinate System:

    This is object space or the space in which the application model is defined.

- World Window (or clipping):

    This is the rectangle in the world defining the region that is to be displayed.

- Viewport:

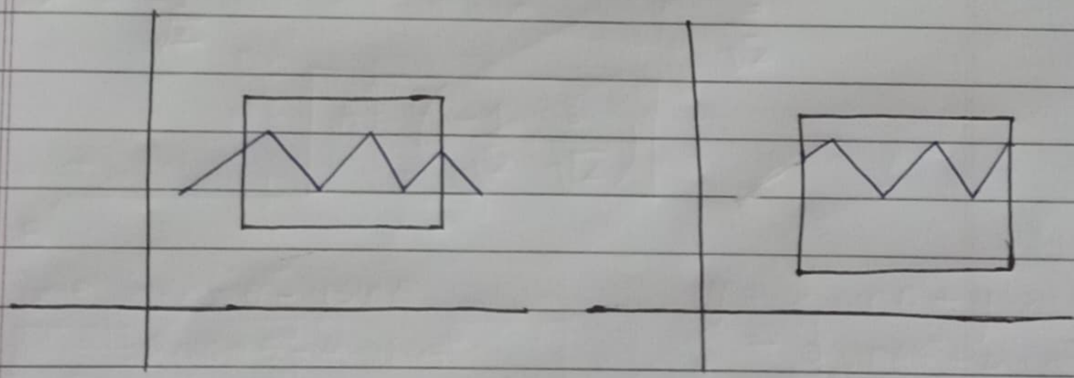
    The rectangular portion of the interface window that defines where the image will actually appear (usually the entire interface window but in some cases modified to be a portion of the interface window).

- **Viewing Transformation:**

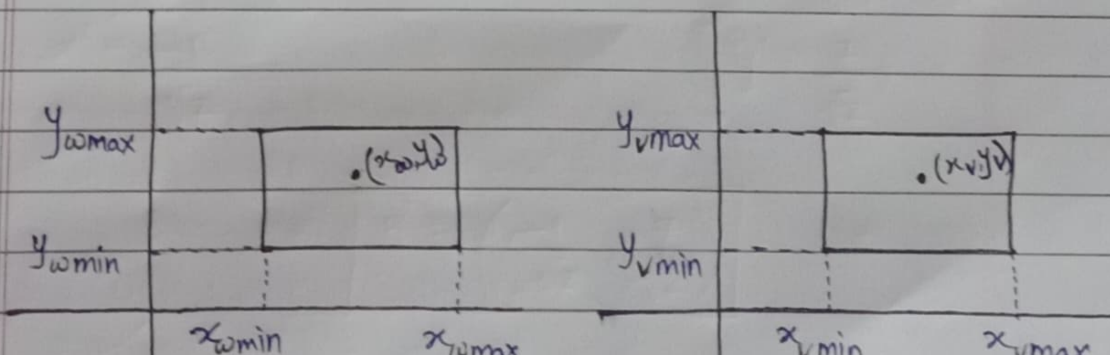
The process of mapping a world window in World Coordinates to the Viewport

Window - to - Viewport mapping

Window-to-viewport mapping is the process of mapping or transforming a two dimensional, world-coordinate scene to device coordinates. In particular, objects inside the world or clipping window are mapped to the viewport. The viewport is displayed in the interface window on the screen. In other words, the clipping window is used to select the part of the scene that is to be displayed. The viewport then positions the scene on the output device.



Window　　　　　　Viewport



$$\frac{x_v - x_{vmin}}{x_{vmax} - x_{vmin}} = \frac{x_w - x_{wmin}}{x_{wmax} - x_{wmin}}$$

$$x_v = \frac{(x_w - x_{wmin}) \cdot (x_{vmax} - x_{vmin})}{x_{wmax} - x_{wmin}} + x_{vmin}$$

$$\frac{y_v - y_{vmin}}{y_{vmax} - y_{vmin}} = \frac{y_w - y_{wmin}}{y_{wmax} - y_{wmin}}$$

$$y_v = \frac{(y_w - y_{wmin}) \cdot (y_{vmax} - y_{vmin})}{y_{wmax} - y_{wmin}} + y_{vmin}$$