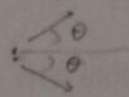
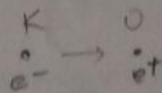


Q3. (a) At what speed should a meter stick move if its length as observed by a stationary observer to shrink to 0.5m? [2]

(b) A rod of proper length 60 cm is travelling along its length in the positive x -direction with speed $0.6c$ as seen from a stationary frame S . In the same frame S , a particle is moving in the direction opposite to the rod with a speed $0.6c$. c is the speed of light in vacuum. In the frame of the particle, (i) what is the speed of the rod, (ii) what is the length of the rod and (iii) how much time the particle would take to cross the rod? (iv) In the S frame, what is the time required for the particle to cross the rod? [2+1+1+2=6]

$$E = K + m_0 c^2$$



$$- \frac{m_0 c^2}{h}$$

Q4. An electron e^- with kinetic energy K makes a head-on collision with a positron e^+ at rest. Electron and positron have the same rest mass m_0 . In the collision, the two particles annihilate each other and are replaced by two photons. The emitted photons travel making equal angles θ with the electron's direction of motion. Determine (i) the energy E_{ph} , (ii) momentum p_{ph} and (iii) angle of emission θ of each photon in terms of K , m_0 and c , the speed of light in vacuum. (iv) If $K = 1$ MeV and rest mass energy of electron (or positron) is 0.5 MeV, what is the value of θ ? [3+1+3+1=8]

Q5. The n -th eigenstate of a particle of mass m in a one dimensional infinite square well potential extending from $x = 0$ to $x = a$ is $\psi_n(x) = \sqrt{2/a} \sin(n\pi x/a)$ with energy eigenvalue $E_n = n^2 \pi^2 \hbar^2 / (2ma^2)$, $n = 1, 2, 3, \dots$. The initial ($t = 0$) wavefunction of the particle is given by

$$\Psi(x, 0) = A[2\psi_1(x) + 3\psi_2(x)]$$

where A is a constant. (a) Normalize $\Psi(x, 0)$. (b) What is the wave function $\Psi(x, t)$ at an arbitrary time t ? (c) Find the lowest possible value of time $T (\neq 0)$ for which $\Psi(x, 0) = \Psi(x, T)$. (d) What is the average energy of the particle? [4x2=8]

$$T = \frac{ma^2}{3\pi\hbar}$$

Q6. The n -th eigenstate $\psi_n(x)$ of a particle of mass m in a potential $V(x) = m\omega^2 x^2 / 2$ can be obtained from the $(n+1)$ -th eigenstate following $\psi_n(x) = A_n a_- \psi_{n+1}(x)$ where A_n is a constant and $a_- = (i\hat{p} + m\omega x) / \sqrt{2\hbar m\omega}$. (a) Given the second excited state

$$\psi_2(x) = \frac{1}{\sqrt{2}} \left(\frac{m\omega}{\pi\hbar} \right)^{1/4} \left(\frac{2m\omega x^2}{\hbar} - 1 \right) e^{-m\omega x^2 / (2\hbar)},$$

determine the first excited state $\psi_1(x)$. (b) Normalize $\psi_1(x)$. (c) Calculate average potential energy $\langle V \rangle$ of the particle in the state $\psi_1(x)$. (d) If the energy of the n -th eigenstate is $E_n = \left(n + \frac{1}{2}\right) \hbar\omega$, $n = 0, 1, 2, \dots$, find the average kinetic energy $\langle T \rangle$ of the particle in the state $\psi_1(x)$, without explicit calculation of $\langle p^2 \rangle$. [3+2+2+1=8]

Useful integrals: $\int_0^\infty x^2 e^{-\alpha x^2} dx = \frac{1}{4\alpha} \sqrt{\frac{\pi}{\alpha}}$ and $\int_0^\infty x^4 e^{-\alpha x^2} dx = \frac{3}{8\alpha^2} \sqrt{\frac{\pi}{\alpha}}$

$$B_1 \phi_1 - B_2 \phi_2$$