$$\begin{bmatrix} K & 2 & 2 & 2 \\ 2 & K & 2 & 2 \\ 2 & 2 & K & 2 \\ 2 & 2 & 2 & K \end{bmatrix} \quad R_i \longleftrightarrow \frac{R_i}{2} \quad R_4 \longleftrightarrow R_1$$

$$\begin{bmatrix} 1 & 1 & 1 & \frac{K}{2} \\ 1 & \frac{K}{2} & 1 & 1 \\ 1 & 1 & \frac{K}{2} & 1 \\ \frac{K}{2} & 1 & 1 & 1 \end{bmatrix} \quad \begin{matrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \\ R_4 \to R_4 - \frac{K}{2}R_1 \end{matrix}$$

$$\begin{bmatrix} 1 & 1 & 1 & \frac{K}{2} \\ 0 & \frac{K}{2}-1 & 0 & 1-\frac{K}{2} \\ 0 & 0 & -1+\frac{K}{2} & 1-\frac{K}{2} \\ 0 & 1-\frac{K}{2} & 1-\frac{K}{2} & 1-\frac{K^2}{4} \end{bmatrix} \quad R_i \longrightarrow \frac{R_i}{\frac{K}{2}-1}$$

$i = 2, 3, 4$

$K \neq 2$ otherwise rank of given matrix will be 1. ——— ①

$$\begin{bmatrix} 1 & 1 & 1 & \frac{K}{2} \\ 0 & 1 & 0 & -1 \\ 0 & 0 & +1 & -1 \\ 0 & -1 & -1 & -(1+\frac{K}{2}) \end{bmatrix} \quad R_4 \longrightarrow R_4 + R_2 + R_3$$

$$\begin{bmatrix} 1 & 1 & 1 & \frac{K}{2} \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & -(3+\frac{K}{2}) \end{bmatrix}$$

So, for rank $= 3$

$$-(3+\tfrac{K}{2}) = 0$$

$$\Rightarrow K = -6$$

——— ②

We have deducted one mark for following:

1. No explanation : why $K \neq 2$?

2. dividing by $K+\alpha$ (for any α) without giving reason why $K \neq -\alpha$?

We have given Zero mark for following:

1. $\text{Rank}(A) = 3 \Rightarrow \det(A) = 0$

2. $\text{Rank}(A) = 3 \Rightarrow AX = 0$ has a nontrivial sol^n.

3. $\text{Rank}(A) = 3 \Rightarrow$ Last row will be a l.c. of preceeding rows.

Although many have found correct answer $(K = -6)$.