DEPARTMENT OF MATHEMATICS
Indian Institute of Technology Guwahati
MA101: Mathematics I, July - November, 2014
Tutorial Sheet: LA - 1

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1. Supply two examples each and explain their geometrical meaning.
   (a) Two linear equations in two variables with exactly one solution.
   (b) Two linear equations in two variables with infinitely many solutions.
   (c) Two linear equations in two variables with no solutions.
   (d) Three linear equations in two variables with exactly one solution.
   (e) Three linear equations in two variables with no solutions.

2. For what values of $\lambda \in \mathbb{R}$, the following system of equations has (i) no solution, (ii) a unique solution, and (iii) infinitely many solutions?

   $$(5-\lambda)x + 4y + 2z = 4, \quad 4x + (5-\lambda)y + 2z = 4, \quad 2x + 2y + (2-\lambda)z = 2.$$

   Also, find the solutions whenever they exist.

3. Prove that the interchange of two rows of a matrix can be accomplished by a finite sequence of elementary row operations of the other two types.

4. Let A be an $n \times n$ matrix. If the system $A^2\mathbf{x} = \mathbf{0}$ has a non-trivial solution then show that the system $A\mathbf{x} = \mathbf{0}$ also has a non-trivial solution.

5. Let A and B be two 2×3 matrices that are in reduced row echelon form. If the systems $A\mathbf{x} = \mathbf{0}$ and $B\mathbf{x} = \mathbf{0}$ have the same solution set then show that $A = B$.

6. Prove or disprove: There exist two solutions $\mathbf{x}_1$ and $\mathbf{x}_2$ of some consistent non-homogeneous system $A\mathbf{x} = \mathbf{b}$ such that $\mathbf{x}_1 + \mathbf{x}_2$ is also a solution of $A\mathbf{x} = \mathbf{b}$.

7. Prove or disprove: If two matrices of the same order have the same rank then they must be row equivalent.
