EE 101
Electrical Sciences

Department of Electronics & Electrical Engineering

Syllabus

- Fundamental Laws of Electrical Engineering
  - *Coulomb's law*
  - *Ohm's law*
  - *Kirchoff's law*
  - *Ampere's law*
  - *Faraday's law of electromagnetic induction*
- The circuit elements
  - Ideal independent current and voltage sources
  - Energy and power
  - Resistance, inductance and capacitance
- Network Theory
  - Series and parallel combination of circuit elements
  - Network analysis by mesh currents and node pair voltages
  - Thevenin's theorem, Norton's theorem
  - Network reduction by star delta transformation

Syllabus

- Circuit differential equations
  - The differential operator
  - General formulation of circuit differential equations
  - The forced solutions, and natural response
- Circuit Dynamics and Forced Responses
  - First order circuits (R-L, R-C circuits)
  - The impulse response
  - Second order circuits (R-L-C circuits)
  - Step response of second order circuits
  - Response of first order and second order circuits to sinusoidal inputs
- Sinusoidal Steady State Response of Circuits
  - Average and effective values of a periodic function
  - Instantaneous and average power, power factor
  - Phasor representation of sinusoids
  - Application of network theorems to complex impedances and balanced three phase circuits

Syllabus

- Magnetic Theory and circuits
  - Definition of Magnetic quantities
  - Theory of magnetism
  - The magnetic circuits
  - Hysteresis and eddy current losses in magnetic materials
- Electromechanical energy conversion
  - Basic analysis of electromagnetic torque
  - Analysis of induced voltage
  - Construction features of electric machines
- Transformers
  - Theory of operation and phasor diagrams
  - Equivalent circuit and parameters
- Three phase induction machines
  - Theory of operation and phasor diagrams
  - Equivalent circuit and parameters
- Introduction to synchronous machines and DC machines

Books

1. Vincent Del Toro, ‘Electrical Engineering Fundamentals’, Second Edition, PHI Learning Private Limited, New Delhi, 2010.
2. Bhag S. Guru, Huseyin R. Hiziroglu, “Electric Machinery and Transformers”, 3rd Edition, Oxford University Press, 2001.
3. G B Shrestha, M H Haque, “AC Circuits and Machines”, Pearson Prentice Hall, 2006

Lectures 1-4

Fundamental Laws of Electrical Engineering
DC Circuits

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Introduction

- There are two broad branches of classical electrical engineering
  - Electric circuit theory
  - Electromechanical energy conversion

- Both these branches are based on of experimentally established fundamental laws. The electric circuit theory mainly consists of :
  - Coulomb Law(1785),
  - Ohm's Law (1827),
  - Faraday's law (1831), and
  - Kirchhoff's law (1857)

- The electromechanical energy conversion can be treated and analyzed by applying just two of the fundamental laws, namely Ampere's Law (1825) and Faraday's Law of Induction (1831).

Coulomb’s Law

- Fig.1 shows two point *positive* charges of value Q_1 and Q_2 separated by a distance of *r* meters.
- Charles A. Coulomb showed experimentally that a force of repulsion exists between Q_1 and Q_2 and is given by

$$F = \frac{Q_1 Q_2}{4\pi\varepsilon r^2} \ [\mathrm{N}] \qquad (1)$$

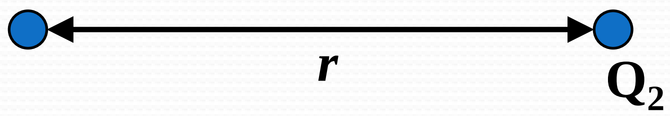


Figure 1

where, Q_1 and Q_2 are in coulombs

r is the separation between the charges in metres

4π is a proportionality constant, and

ε is the permittivity

- The Coulomb’s law is also frequently written as

$$F = \mathcal{E}\, Q_2 \ [\mathrm{N}] \qquad (2)$$

And, $\mathcal{E} = \dfrac{Q_1}{4\pi\varepsilon r^2}$ [N/C] is known as electric field intensity.

Permittivity

- In equation (1) all the quantities can be known through measurements with the exception of the proportionality factor ε.
- The factor ε is a property of the medium in which the experiment is performed and this factor is known as *permittivity*. The permittivity of a substance describes how easy it is to set up an electric field in it.
- When the experiment is performed in a vacuum, the value of the permittivity is: $\varepsilon_0 = 8.854 \times 10^{-12}$ [F/m]
- Repeating the Coulomb's experiment in oil for the same values of Q_1 and Q_2 and *r*, it is found that the resulting force is only about half as much as for air.
- The permittivity of oil is greater than that of air. A convenient way of expressing this difference is to introduce a quantity called relative permittivity defined as: $\varepsilon_r = \varepsilon / \varepsilon_0$, so that

$$F = \frac{Q_1 Q_2}{4\pi\varepsilon_0\varepsilon_r r^2} \text{ [N]}$$

Potential Difference

- In Fig. 2 the charge Q_1 is assumed to be located at some fixed point in space. The charge Q_2 is assumed initially to be on a horizontal path from Q_1 and is infinitely far away. In this condition the force between Q_1 and Q_2 is zero.
- As Q_2 is moved in the direction of Q_1, greater and greater forces are required to accomplish this.

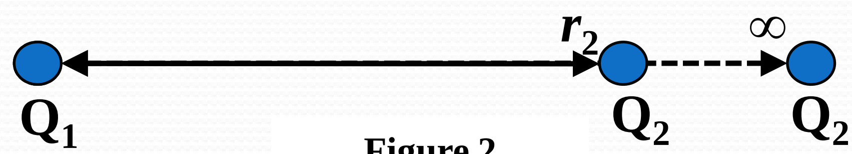


Figure 2

- Since the force exerted on Q_2 acts along a distance which measures from infinity up to r_2, it follows that work is being done on Q_2.
- The work done on Q_2 is in the form of *potential energy* and is recovered in the form of *kinetic energy* immediately upon release of the force which keeps Q_2 fixed at a position r_2 meters from Q_1.
- The work done on Q_2 to bring it from infinity to r_2 can also be expressed in terms of the *electric field intensity* associated with charge Q_1 (eqs. 1 and 2).
- In moving Q_2 from infinity to r_2, work must be done against this electric field.

Potential Difference

- A quantity, known as *work per unit charge* also known as *voltage* is defined as work done per coulomb on moving Q_2 from infinity to r_2.
- To obtain the *voltage* it is necessary to perform an integration of the electric field intensity (eq. 2) with respect to the distance.
- Mathematically voltage is defined as:

$$\text{Voltage} = \frac{\text{work}}{\text{charge}}, \quad \text{or} \quad V_2 = \frac{W_2}{Q_2}$$

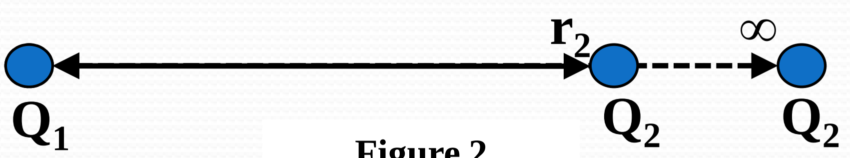


Figure 2

$$\text{ie,} \;\; V_2 = -\int_{\infty}^{r_2} E dr = \frac{Q_1}{4\pi\varepsilon r_2} \;\; [\text{V}] \qquad (3)$$

- The minus sign is arbitrarily inserted to indicate that work is done on the charge against the action of the electric field produced by Q_1.

Potential Difference

- To determine the *potential difference* between any two point charges, another charge Q_3 is considered and this charge is moved from infinity to r_3.
- From Fig. 3 it can be seen that r_3 is smaller than r_2, hence more work per unit charge must be done to place a charge at point 3.
- Mathematically voltage or potential of Q_3 is defined as:

$$V_3 = \frac{Q_1}{4\pi\varepsilon r_3} \text{ [V]} \qquad (4)$$

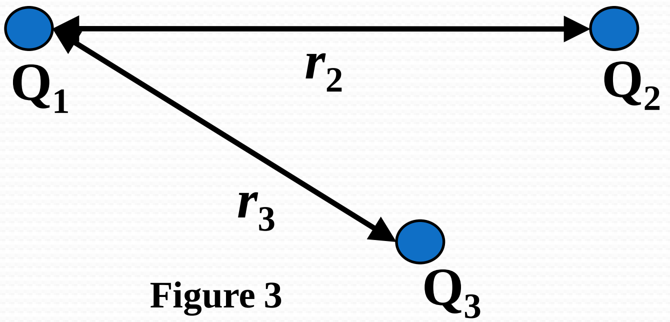


Figure 3

- Since V_3 is greater than V_2, there exists a potential difference between these two points.
- When a unit charge moves from a point of higher potential (point 3) to one of lower potential (point 2) it gives away energy.

Ohm's Law

- On the basis of experiments, Ohm showed that current flow in a circuit composed of a battery and conductors can be expressed as

$$I = \frac{A}{\rho}\frac{\mathrm{d}v}{\mathrm{d}l} \quad [\mathrm{A}] \qquad (5)$$

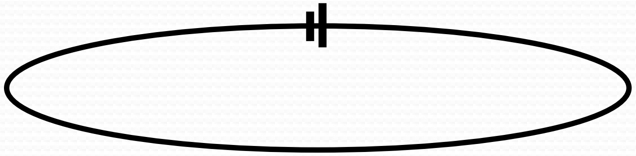


Figure 4

where, I is the current,

A and ρ are the cross section and the resistivity of the conductor, and

$\mathrm{d}v / \mathrm{d}l$ is the change of voltage along the length.

- In case of a uniform conductor, the current can be expressed as

$$I = \frac{A}{\rho}\frac{\mathrm{V}}{\mathrm{L}} = \frac{\mathrm{V}}{\rho \mathrm{L}/A} \quad [\mathrm{A}] \qquad (6)$$

where, V and L are total voltage and the length of the conductor.

- Ohm's law states that the current in a wire is proportional to the voltage across its ends, which can be expressed as:

$$I = V / R \qquad \text{or} \qquad V = IR \qquad \text{where,} \qquad R = \rho L / A \qquad (7)$$

where, R is called the resistance of the conductor.

Ohm's Law

- During Ohm's time, experimental verification of his law was achieved exclusively by using direct current sources, namely the voltaic cells.
- Further experimentation has shown that Ohm's law is also valid when the potential difference across a linear resistor is time varying. In this case Ohm's law is expressed as

$$v = iR \qquad (8)$$

- The lower case letters are used to indicate the non constant natures of the quantities v and i.

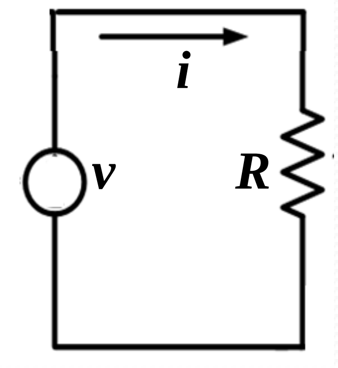


Figure 5

Example

- A negative point charge Q_1 of magnitude 2×10^{-12} C is placed in vacuum.
    a. Find the value of the electric field at a point 1 m from the charge.
    b. Compute the work done in bringing an electron from a point very far removed from Q_1 to a point 0.5 m away
    c. What force is acting on the electron in part (b)?

Solution

a. **0.018 V/m**

b. **0.0576×10^{-19} Joules**

c. **115.2×10^{-22}** N

Kirchhoff's Laws

- Gustav Robert Kirchhoff published the first systematic formulation of the principles governing the behavior of the electric circuits.
- His work is embodied in two laws:
  - The current law
  - The voltage law
- Kirchhoff's current law states that the sum of the current entering or leaving a junction at any instant is equal to zero. A junction point is that place in a circuit where two or more circuit elements are joined together (Fig.6).

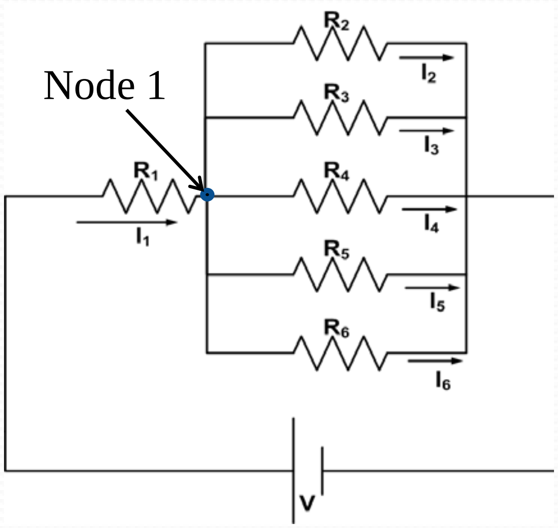


Figure 6

- Mathematically the law is expressed as:

$$\sum_{j=1}^{k} I_j = 0 \qquad (9)$$

where, $j =$ number of circuit elements connected to a node

Kirchoff's Current Law (KCL)

- Applying the Kirchoff's current law to node 1 in Fig.6 gives

$$I_1 - I_2 - I_3 - I_4 - I_5 - I_6 = 0 \qquad (10)$$

- The Kirchhoff's law is the restatement of the *principle of conservation of charge*.
- It may be noted that if the current entering a node is taken as +ve, then the current going out of the node will be –ve, and vice versa.

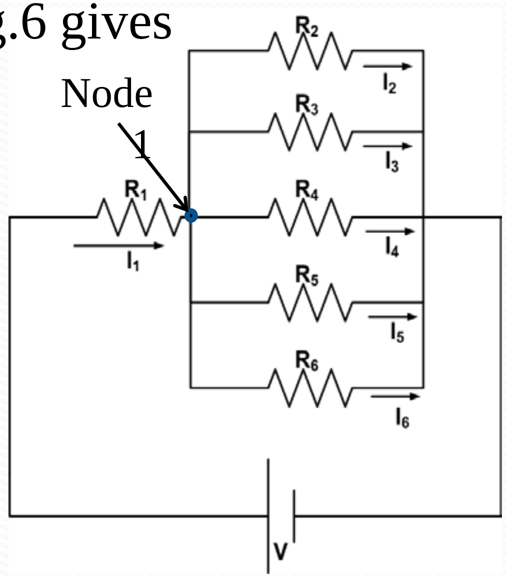


Figure. 6: Kirchhoff's current law

- For two resistances in parallel, KCL can be used to show that:
  - Current division in parallel resistances:

$$I_1 = \frac{R_2}{R_1 + R_2} I, \qquad I_2 = \frac{R_1}{R_1 + R_2} I \qquad (11)$$

  - Parallel combination of resistances:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}, \quad \text{or} \quad R_{eq} = \frac{R_1 R_2}{R_1 + R_2} \qquad (12)$$

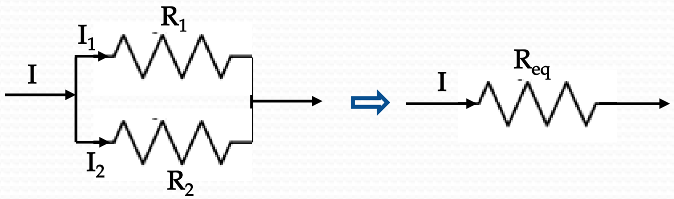


Figure. 7: Parallel Resistances

Kirchoff’s Voltage Law (KVL)

- Kirchoff’s voltage law states that *at any instant of time, the sum of the voltages in a closed circuit is zero*.
- Application of Kirchhoff’s voltage law to the circuit shown inn Figure.7 gives:

$$V_1 + V_2 + V_3 - E = 0 \qquad (13)$$

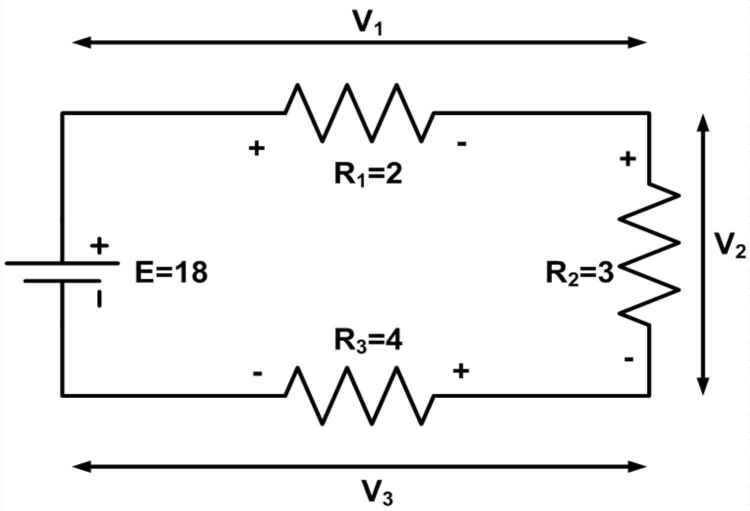


Figure.8: Kirchhoff's voltage law

- It should be noted that the polarity of various voltages should be properly taken care of.
- For two resistances in series, KVL can be used to show that:
  - Voltage division in series resistances:

$$V_1 = \frac{R_1}{R_1 + R_2} V, \quad V_2 = \frac{R_{12}}{R_1 + R_2} V \qquad (14)$$

  - Series combination of resistances:

$$R_{eq} = R_1 + R_2 \qquad (15)$$

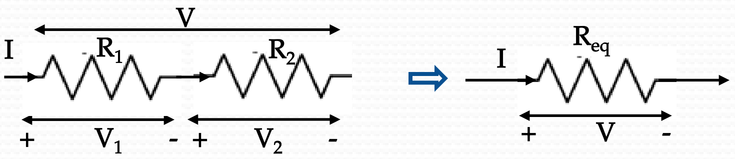


Figure. 9: Series Resistances

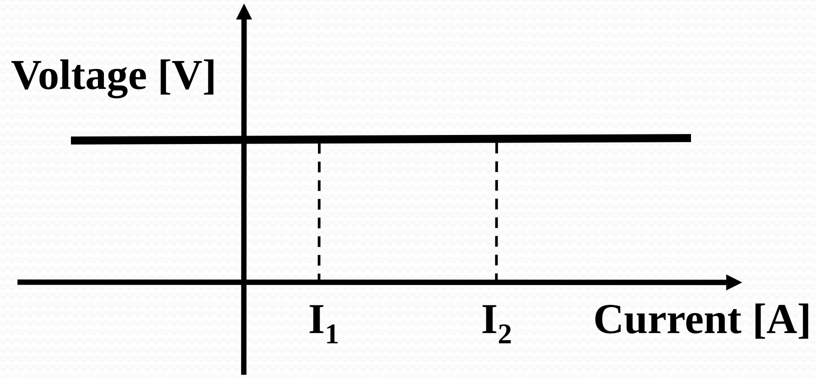
Ideal Independent Voltage Source

- For an independent voltage source, the magnitude of the voltage is not dependent on either the magnitude or direction of the current.
- The characteristics of an independent voltage source is shown in Fig. 8. A notable point about the ideal voltage source is its zero internal resistance. This is readily demonstrated by invoking Ohm's law in incremental form:

$$R = \frac{\Delta V}{\Delta I}$$

From Fig. 8,

$$R = \frac{\Delta V}{I_2 - I_1} = 0$$



Figfure. 8: Characteristics of ideal voltage source

- *It may be noted that an ideal voltage source is capable of supplying any amount of power, which is not true with practical voltage sources.

Ideal Independent Current Source

- The ideal independent current source is a two terminal element which supplies its specified current to the circuit it is connected to, independent of the value and direction of the voltage appearing across its terminals.
- The voltage vs. current characteristics of an ideal current source are shown In Fig. 9.

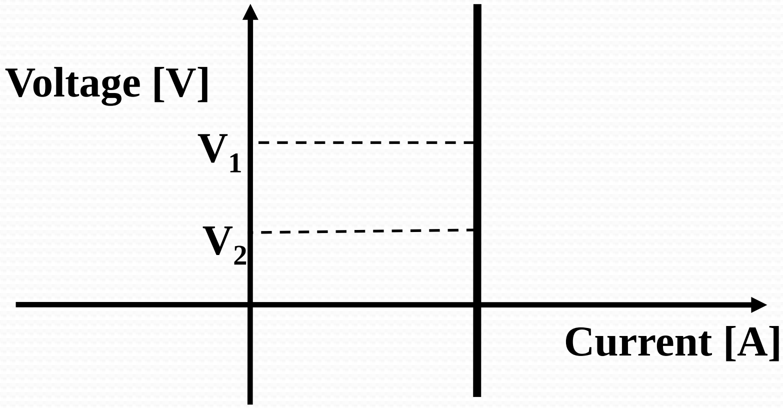


Figure. 9: Characteristics of ideal current source

- It may also be noted that an ideal current source is capable of supplying any amount of power, which is not true with practical current sources.

Ideal Dependent Sources

- In case of ideal *dependent* or *controlled* source, the source quantity is determined by a voltage or current existing at some other location in the system or circuit being analyzed.
- The *dependent* sources appear in the equivalent electrical models for many electronic devices such as transistors, operational amplifiers and integrated circuits.
- The dependent sources are represented as diamonds to distinguish them from independent sources (Fig. 10).

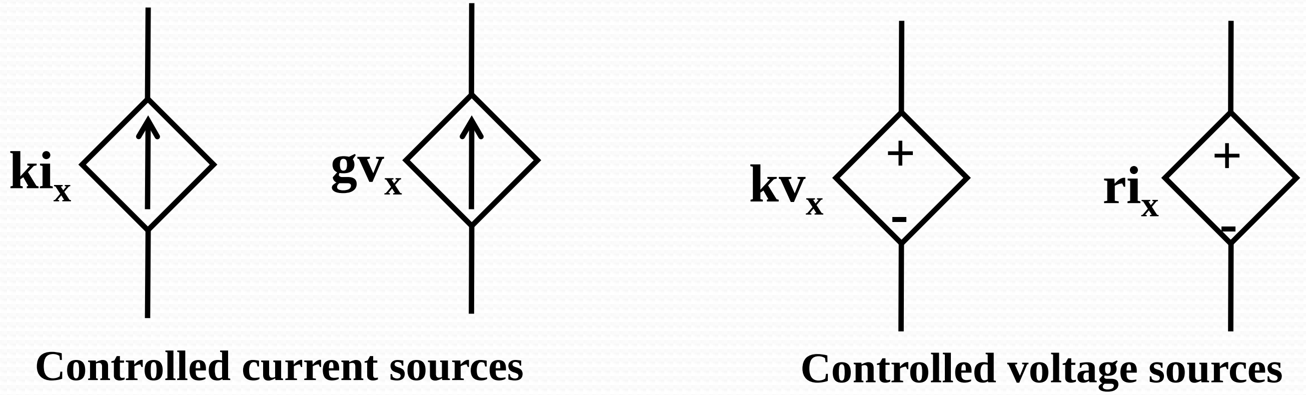


Figure 10: Controlled or dependent sources

Example 1

- Determine v_x in the circuit shown in Fig. 11(a)

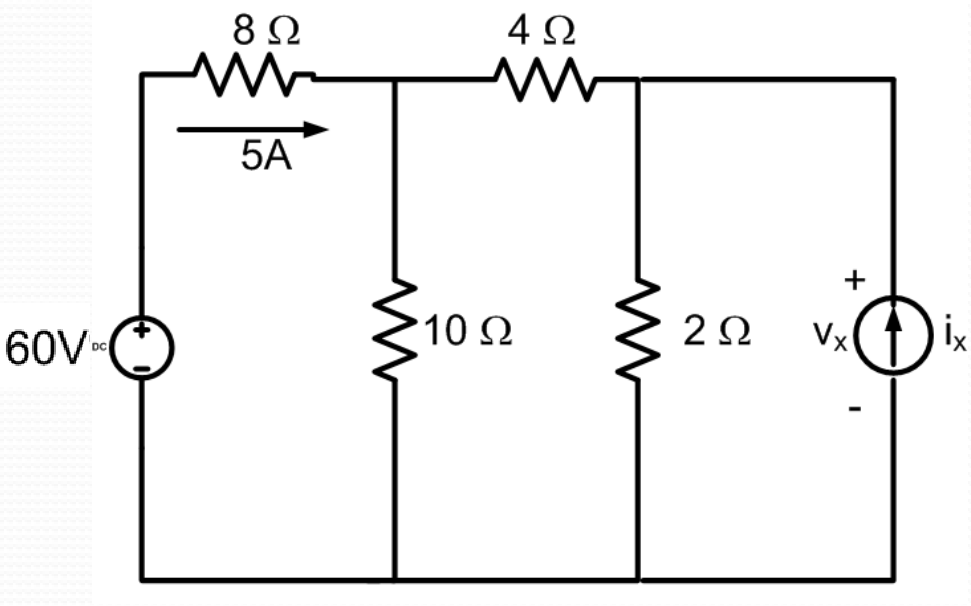


Fig.11a: Circuit for the example 1

Solution

- First the given network is properly labeled (Fig.11b)

$$v_8 = 5 \times 8 = 40V$$

- Using KVL in loop I,

$$-60 + v_8 + v_{10} = 0$$

$$\therefore \quad v_{10} = 20 \;\; \text{V}$$

And, $i_{10} = v_{10} / 10 = 2 \;\; \text{A}$

- Using KCL at node X

$$i_4 = 5 - i_{10} = 5 - \frac{v_{10}}{10} = 3 \;\; \text{A}$$

$$\Rightarrow \quad v_4 = 4 \times 3 = 12$$

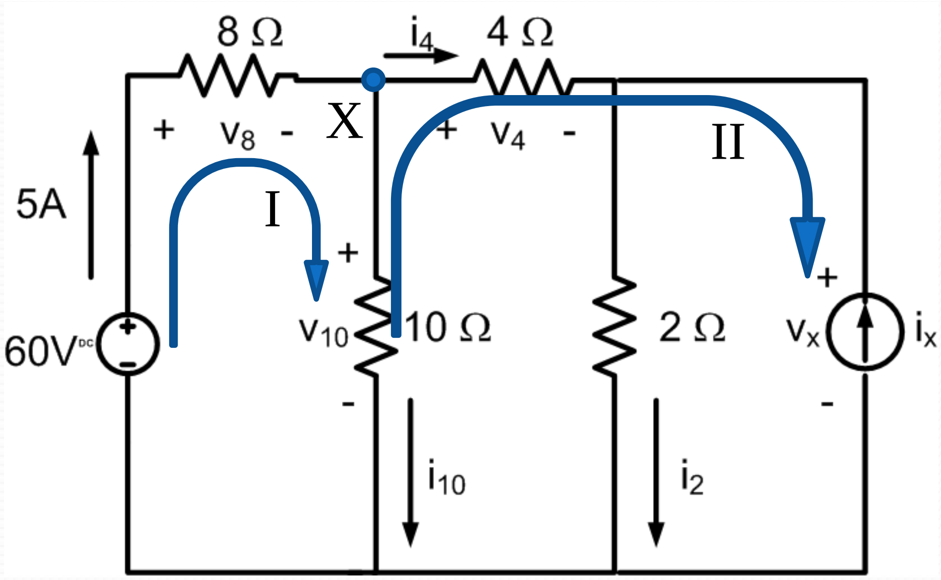


Fig.11(b): Labeled circuit

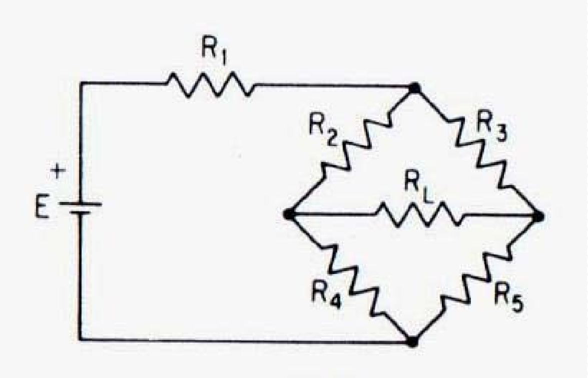
- Using KVL in loop II,

$$-v_{10} + v_4 + v_x = 0$$

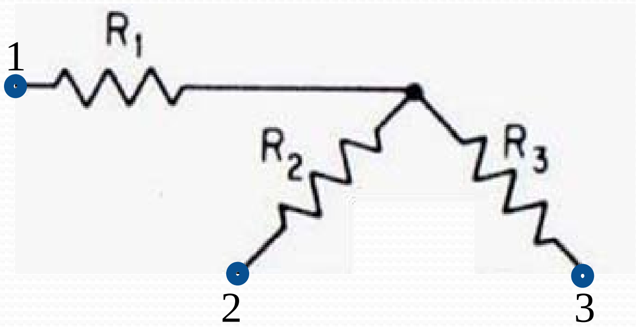
$$\therefore v_x = v_{10} - v_4 = 20 - 12 = 8 \;\; \text{V}$$

Star and Delta Networks

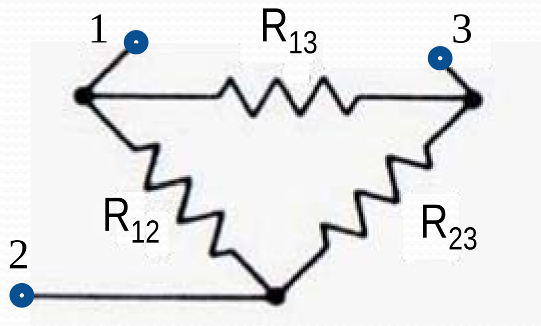
- Consider the circuit shown, which is not easily amenable to series parallel circuit reduction techniques.



- A 'Y' or Star network is a three-terminal three-branch network with a common point as shown.
  - The resistance connected between any terminal i and the common point in the Y network is denoted R_i.

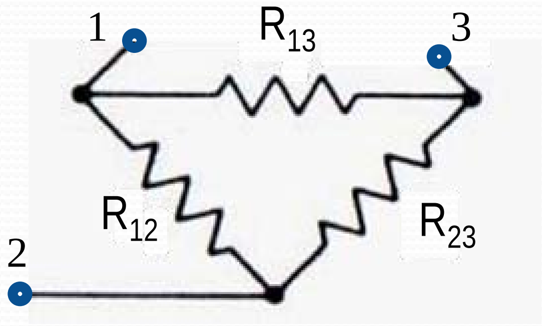


- A Δ or Delta network is a three-terminal three-branch network as shown.
  - The resistance connected between any two terminals i and j in the Δ network is denoted R_{ij}.

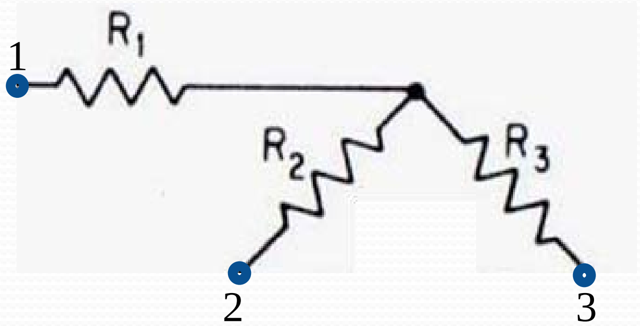


- Star and Delta networks are commonly found in many circuits and are basic configurations in 3-phase circuits.

Delta Star Transformation



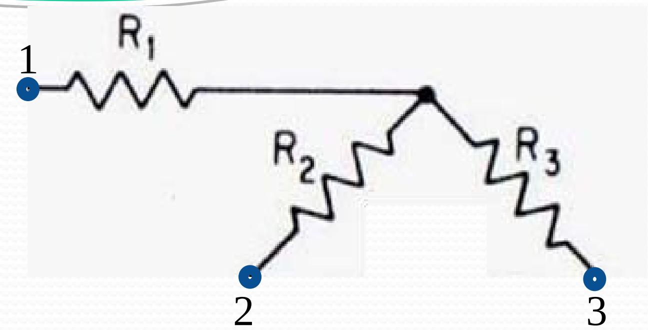
- Every Δ (Delta) network has an equivalent Y (Star) network and vice versa.
- If the two circuits shown are equivalent, then their impedances should match under all circumstances.



  - If terminal 1 is open, then, $R_2 + R_3 = \dfrac{(R_{12} + R_{13})R_{23}}{R_{12} + R_{13} + R_{23}}$
  - If terminal 2 is open, $R_1 + R_3 = \dfrac{(R_{12} + R_{23})R_{13}}{R_{12} + R_{13} + R_{23}}$
  - And, if terminal 3 open, $R_1 + R_2 = \dfrac{(R_{13} + R_{23})R_{12}}{R_{12} + R_{13} + R_{23}}$
  - Adding all three, $R_1 + R_2 + R_3 = \dfrac{(R_{12}R_{13} + R_{21}R_{23} + R_{31}R_{32})}{R_{12} + R_{13} + R_{23}}$
  - Then subtracting each from the total:

$$R_1 = \frac{R_{12}R_{13}}{R_{12} + R_{13} + R_{23}}, \qquad R_2 = \frac{R_{21}R_{23}}{R_{12} + R_{13} + R_{23}}, \qquad R_3 = \frac{R_{31}R_{32}}{R_{12} + R_{13} + R_{23}}$$

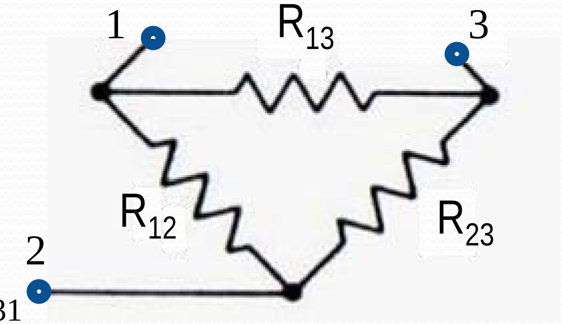
Star Delta Transformation



- Every Y (Star) network has an equivalent Δ (Delta) network.
- Using Δ to Y conversion relations:

$$R_1 = \frac{R_{12}R_{13}}{R_{12}+R_{13}+R_{23}} \qquad R_2 = \frac{R_{21}R_{23}}{R_{12}+R_{13}+R_{23}},$$

$$R_3 = \frac{R_{31}R_{32}}{R_{12}+R_{13}+R_{23}}$$



We get:

$$R_1R_2 + R_2R_3 + R_3R_1 = \frac{R_{12}R_{23}R_{31}(R_{12}+R_{13}+R_{23})}{(R_{12}+R_{13}+R_{23})^2} = \frac{R_{12}R_{23}R_{31}}{(R_{12}+R_{13}+R_{23})}$$

Taking the ratios:

$$R_{23} = R_2 + R_3 + \frac{R_2R_3}{R_1}, \qquad R_{31} = R_3 + R_1 + \frac{R_3R_1}{R_2}, \qquad R_{12} = R_1 + R_2 + \frac{R_1R_2}{R_3}$$

- It should be noted that above conversion equations are equally valid for AC circuits with complex impedances

Star Delta Transformation - Example

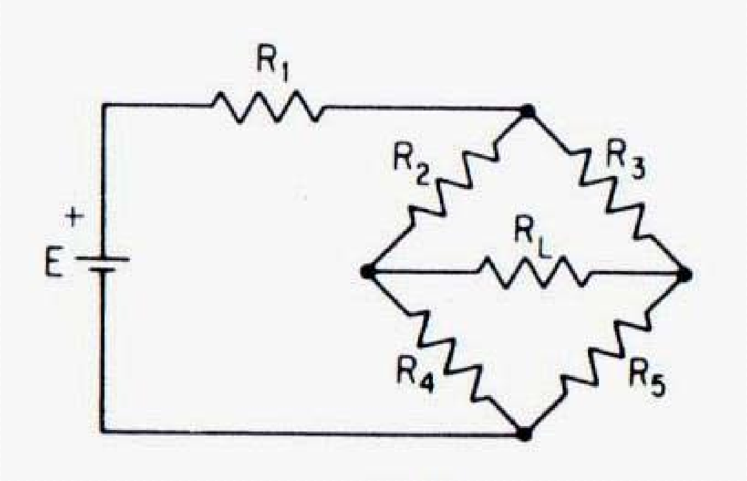
Example:

Find the current supplied by E, when

$E = 10$ V,

$R_1 = 3\ \Omega$, $R_2 = 10\ \Omega$, $R_3 = 5\ \Omega$,

$R_L = 10\ \Omega$, $R_4 = 2\ \Omega$, and $R_5 = 4\ \Omega$.



Ans. 1.25 A

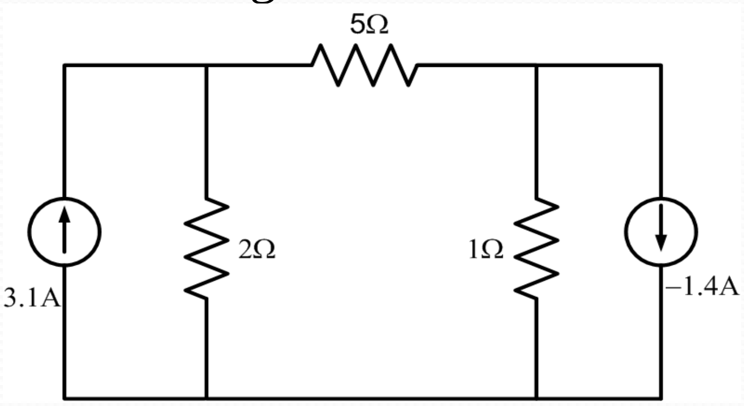
Network Techniques and Theorems

There are several systematic techniques and useful theorems which are benefitially employed to solve network problems.

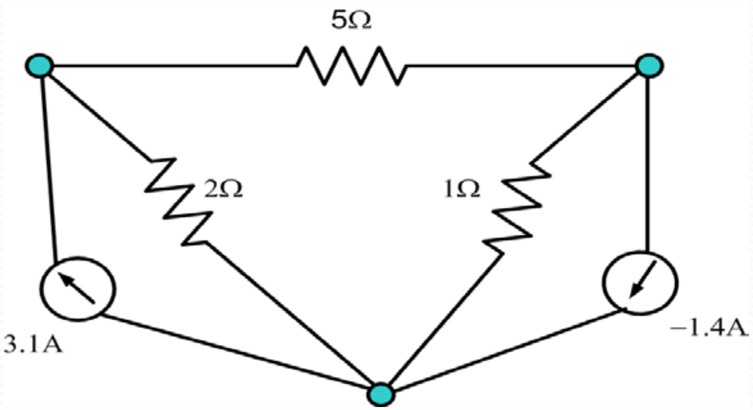
- Nodal Analysis
- Mesh Analysis
- Superposition Theorem
- Thevenin's Theorem
- Norton's Theorem

Nodal Analysis

- To illustrate the basic mechanics of the technique, consider the three node circuit shown in Fig.a. In this network there are three nodes.
- A three node circuit should have two unknown voltages and two equations; a 10-node circuit will have nine unknown voltages and nine equations; an N-node circuit will need (N-1) voltages and (N-1) equations.
- In the Fig. b, it is shown that the network has three unique nodes. One node is designated as a *reference node* (Fig.c) and it is the negative terminal of the nodal voltages.



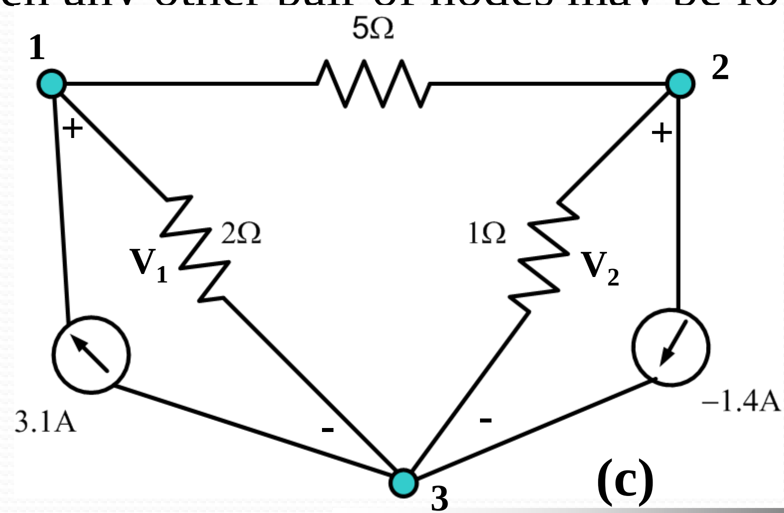
(a)



(b)

Nodal Analysis

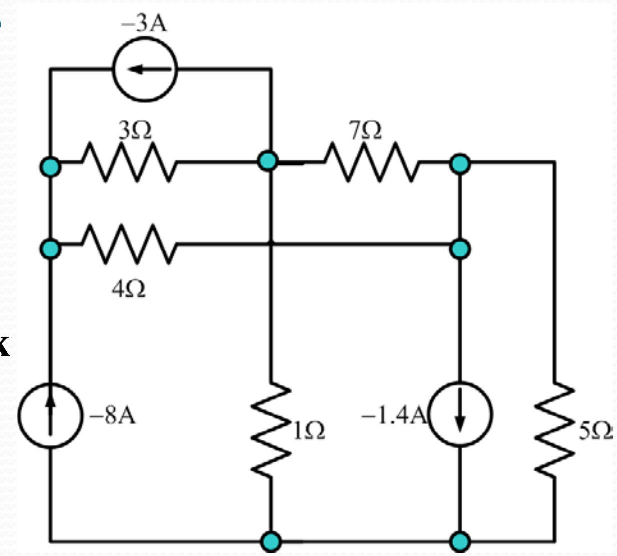
- In the Fig. b, one node is designated as a *reference node* (also in Fig.c) and it is the negative terminal of the nodal voltages.
- As a general rule, a node having maximum number of branches is chosen as the reference node. By doing so the number of equations defining the network is reduced.
- In Fig. c, the voltage of node 1 relative to the reference node is defined as V_1 and V_2 is defined as the voltage of node 2 with respect to the reference node. The voltage between any other pair of nodes may be found in terms of V_1 and V_2.



(c)

Nodal Analysis – General Procedure

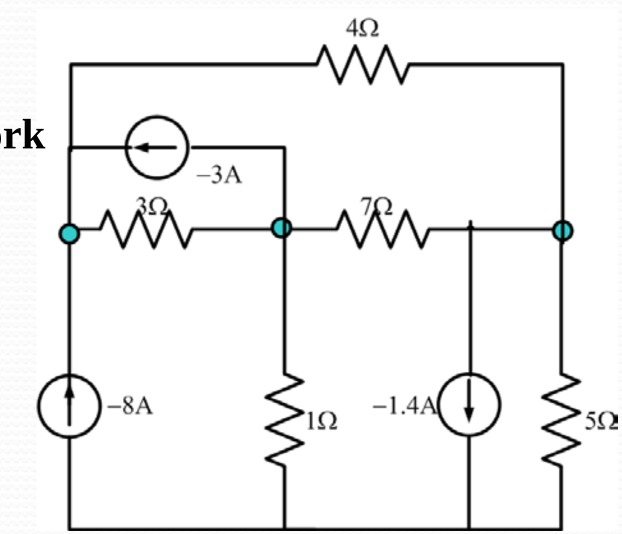
- The node analysis is a general method and can always be applied to any electrical network. Consider the circuit shown in the Fig.



A 4 node network

- Identify the goal of the problem: There are four nodes in this network. The bottom most node is selected as the reference node and all the other three nodes are labeled as shown in Fig.

Redrawn network



- Collect the information: There are three unknown voltages V_1, V_2 and V_3. All current sources and resistors have designated values which are marked on the schematic.

(1)

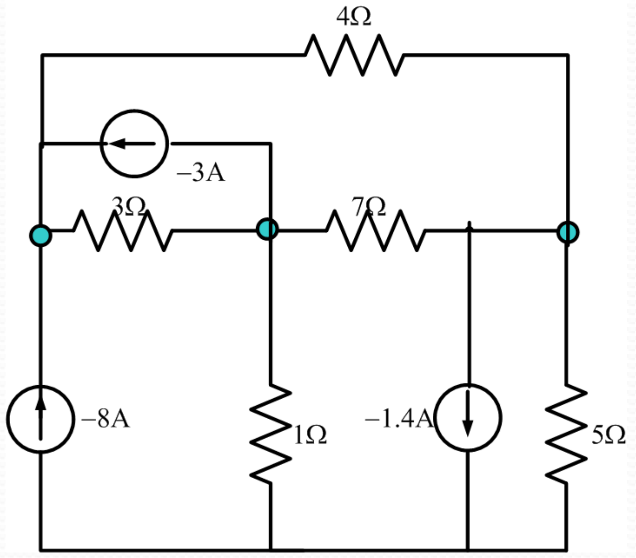
Nodal Analysis – General Procedure

- **Devise a plan**: This problem is well suited for nodal analysis and three independent KCL equations may be written.
- ***Construct an appropriate set of equations***:

$$\frac{V_1 - V_2}{3} + \frac{V_1 - V_3}{4} = -8 - 3 \qquad \text{at node 1}$$

$$\frac{V_2 - V_1}{3} + V_2 + \frac{V_2 - V_3}{7} = 3 \qquad \text{at node 2}$$

$$\frac{V_3}{5} + \frac{V_3 - V_2}{7} + \frac{V_3 - V_1}{4} = 1.4 \qquad \text{at node 3}$$



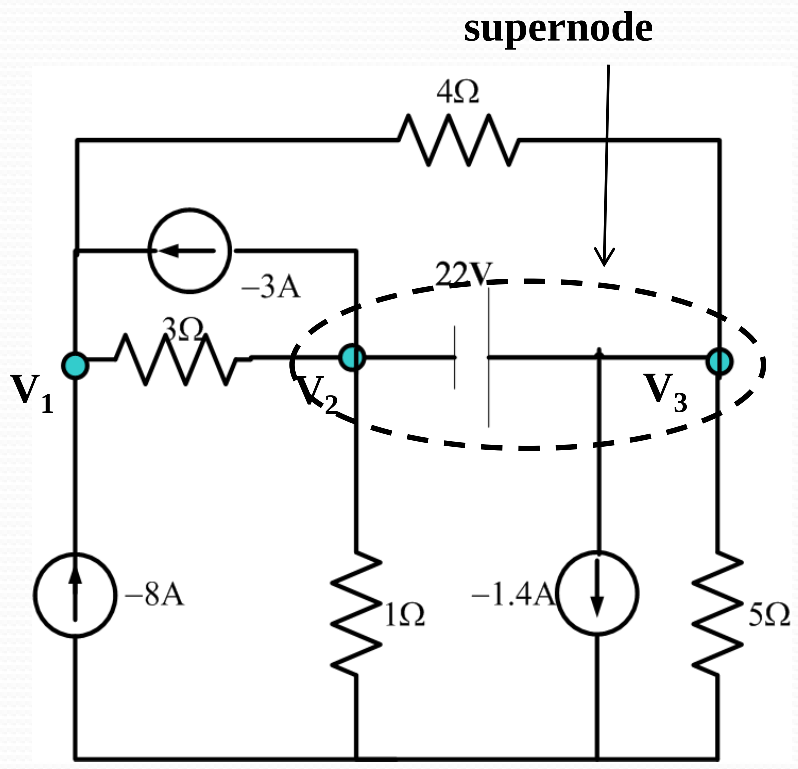
- Solve for voltages.

The Supernode

- Consider the network shown in the fig. If KCL is applied as in the previous example then we will run into difficulty at nodes 2 and 3 because the current in the branch 23 is unknown.

- To overcome the dilemma, node 2, node 3 together with the voltage source is considered a ***supernode*** and KCL is then applied.

  The super node is shown as a region enclosed by the broken lines.



The Supernode

The super node is shown as a region enclosed by the broken lines.

- The system of equations are:

at node 1

$$\frac{V_1 - V_2}{3} + \frac{V_1 - V_3}{4} = -8 - 3$$

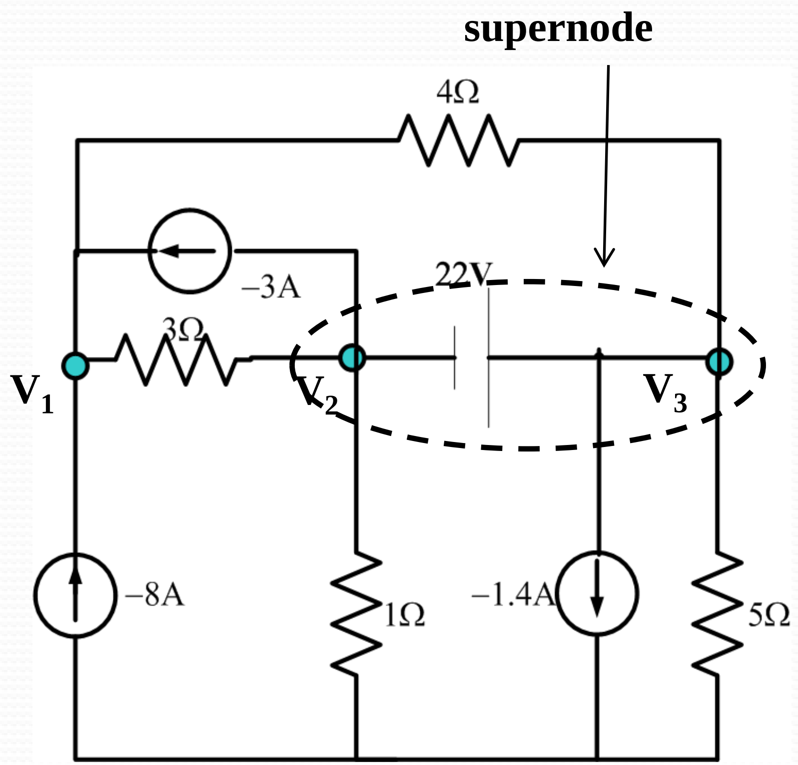
at supernode

$$\frac{V_2 - V_1}{3} + \frac{V_3 - V_1}{4} + \frac{V_3}{5} + V_2 = 3 + 1.4$$

the third equation is

$$V_2 - V_3 = -22$$

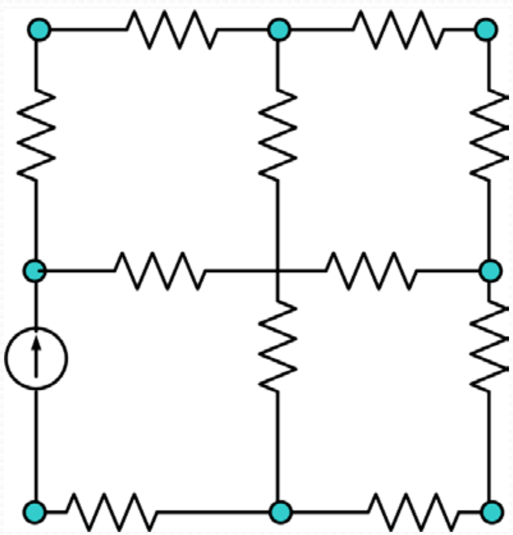
- Then solve for the three unknown voltages.



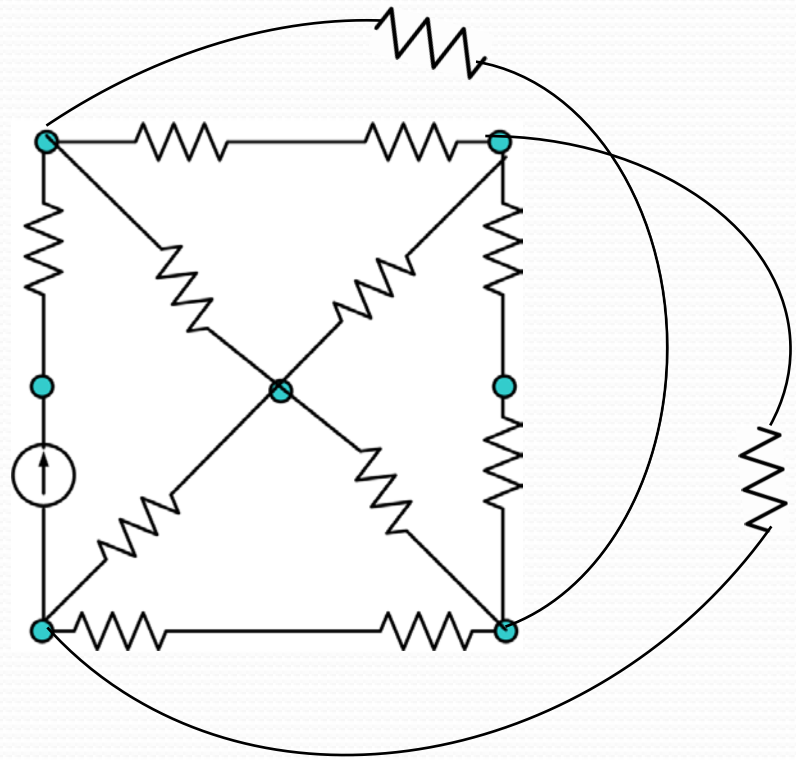
Mesh Analysis

- The node analysis is a general method and can always be applied to any electrical network.
- Another technique to analyze the networks is *mesh analysis*. *This technique is not general and can be applied to only planar networks*.
- If it is possible to draw the diagram of the network on a plane surface in such a way that no branch passes over or under any other branch, then the network is said to be a *planar network.*
- If a network is planar, mesh analysis can be used to accomplish the analysis. This concept involves the concept of a *mesh current*.

Planar and Non Planar Networks



a. Planar network



b. Non-planar network

Fig: Planar and non planar networks

Mesh Analysis

- Consider the two mesh circuit shown in the Fig. We define a *mesh current* as a current that flows only around the perimeter of a mesh.
- The left hand mesh has a current i_1 flowing in a clockwise direction and a mesh current of i_2 is established in the remaining mesh.
- The system of equations for the network are as follows:

$$6i_1 + 3(i_1 - i_2) = 42 \quad \text{for mesh 1}$$
$$3(i_2 - i_1) + 4i_2 = 10 \quad \text{for mesh 2}$$

- Solving the above two equations, the mesh currents i_1 and i_2 are obtained as

$$i_1 = 6 \ \text{A}, \ \text{and} \quad i_2 = 4 \ \text{A}$$

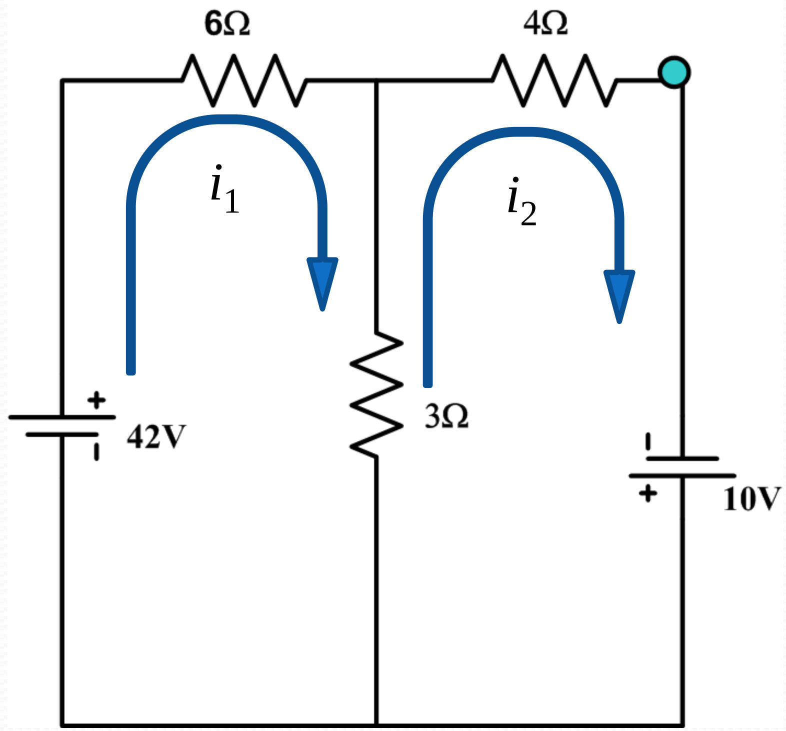


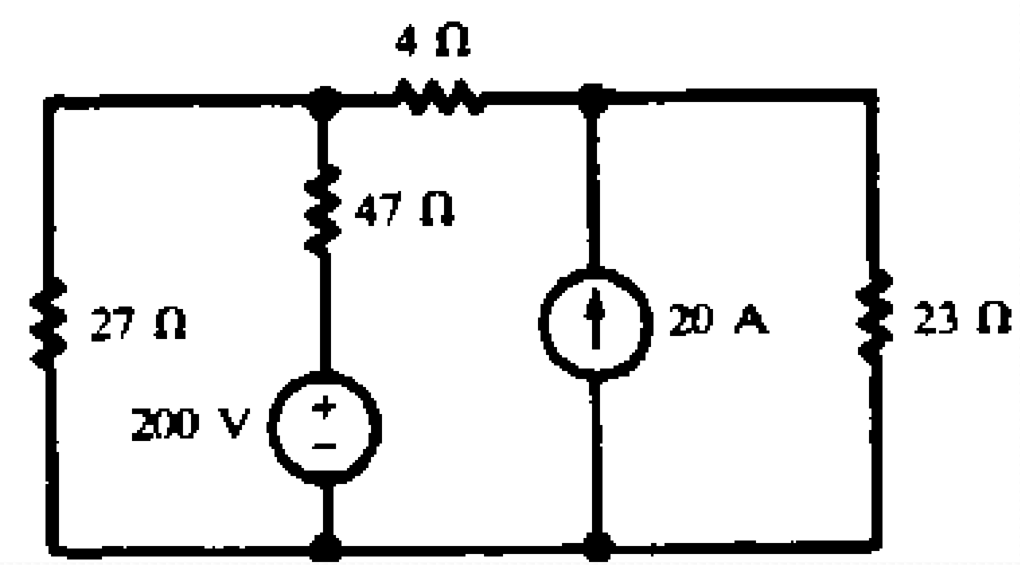
Fig.5: Mesh Analysis

Superposition Theorem

- According to superposition theorem, in a linear network containing two or more sources, the response in any element is equal to the algebraic sum of the responses caused by individual sources acting alone while the other sources are non-operative.
- This theorem is useful for analyzing networks that have large number of independent sources as it makes it possible to consider the effects of each source separately.
- Superposition theorem is applicable to any linear circuit having time varying or time invariant elements.
- The limitations of superposition theorem are
  - It is not applicable to the networks consisting of non linear elements like transistors, diodes, etc.
  - It is not applicable to the networks consisting of any dependent sources

Superposition Theorem - Example

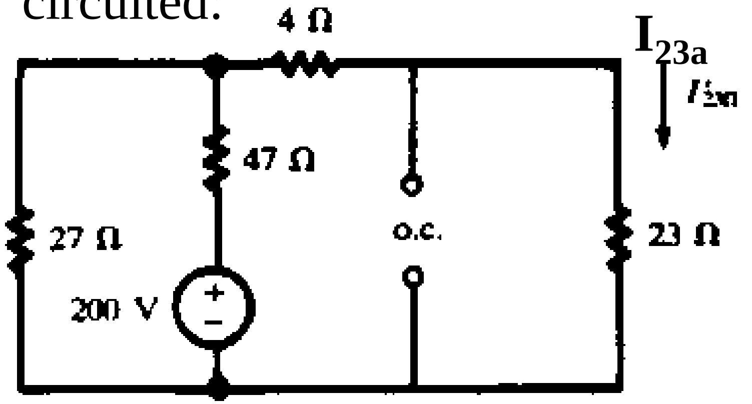
Compute the current in the 23 Ohm resistor using superposition theorem.



- The circuit contains 2 independent sources

Superposition Theorem - Solution

- With 200V source acting alone and the 20A current open circuited:

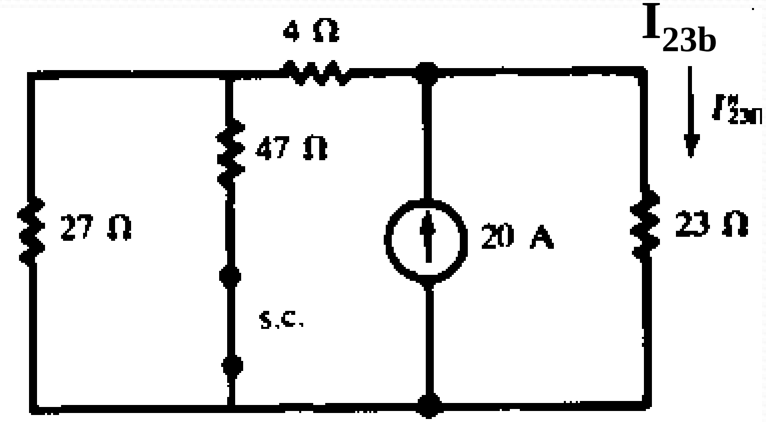


$$R_{eq} = 47 + \frac{27 \times (4+23)}{54} = 60.5\ \Omega$$

$$I_{total} = \frac{200}{60.5} = 3.31\ \text{A}$$

$$I_{23a} = \left(\frac{27}{54}\right) \times 3.31 = 1.65\ \text{A}$$

- With 20A source acting alone and the 200V source short circuited:



$$R_{eq} = 4 + \frac{27 \times 47}{74} = 21.15\ \Omega$$

$$I_{23b} = \left(\frac{21.15}{21.15+23}\right) \times 20 = 9.58\ \text{A}$$

Curent through the 23 Ω is: $I_{23} = I_{23a} + I_{23b} = 11.23\ \text{A}$

Thevenin’s Theorem

- Thevenin’s theorem states that any two-terminal linear network having a number of sources and resistances can be replaced by a simple equivalent circuit consisting of a single voltage source in series with a resistance.
- The value of the voltage source is equal to the open circuit voltage across the two terminals of the network.
- The resistance is equal to the equivalent resistance measured between the terminals with all the energy sources replaced by their internal resistances.

Procedure to Obtain Thevenin's Equivalent Circuit

The steps involved in determining the Thevenin's equivalent circuit are:

- Temporarily remove the load resistance where the current is required
- Find the open circuit voltage V_{OC} across the two terminals from where the load resistance has been removed. This is known the Thevenin's voltage V_{TH}.
- Calculate the resistance of the whole network as seen from these two terminals, after all voltage sources are replaced by short circuit and all current sources are replaced by open circuit leaving internal resistance (if any). This is called Thevenin's resistance, R_{TH}.
- Replace the entire network by a single Thevenin's voltage source V_{TH} and resistance R_{TH}.
- Connect the resistance (R_L), across which the current value is desired,, back to its terminals from where it was previously removed.
- Finally, calculate the current flowing through R_L using the following expression:

$$I_L = \frac{V_{TH}}{R_{TH} + R_L}$$

Norton’s Theorem

- Norton’s theorem states that any two-terminal linear network with current sources, voltage sources and resistances can be replaced by an equivalent circuit consisting of a current source in parallel with a resistance
- The value of the current source is equal to the short circuit current between the two terminals of the network and the resistance is equivalent resistance measured between the terminals of the network with all the energy sources replace by their internal resistance.

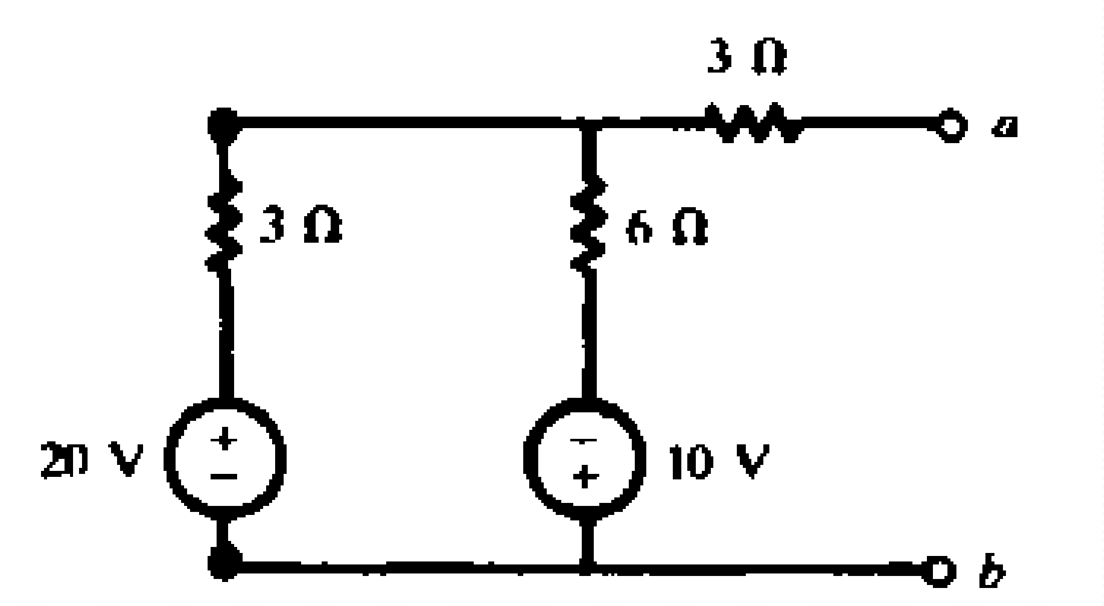
Procedure to Obtain Norton's Equivalent Circuit

The steps involved in obtaining the Norton's equivalent circuit are:

- Temporarily remove the load resistance across the two terminals and short circuit these terminals.
- Calculate the short circuit current I_N.
- Calculate the resistance of the whole network as seen at these two terminals, after all voltage sources are replaced by short circuit and current sources are replaced by open circuit leaving internal resistances (if any). This is Norton's resistance R_N.
- Replace the entire network by a single Norton's current source whose short circuit current is I_N and parallel with Norton's resistance R_N.
- Connect the load resistance (R_L) back to its terminals from where it was previously removed.
- Finally calculate the current flowing through R_L.

Thevenin's and Norton's Equivalent Circuits

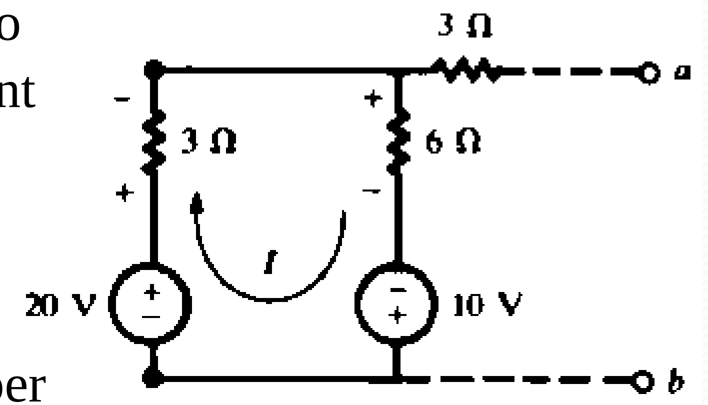
- Obtain the Thevenin's and Norton's Equivalent Circuit for the network:



Thevenin's Equivalent Circuit

- With the terminals *a* and *b* open, the two voltage sources drive a clockwise current through the 3 Ω and 6 Ω resistors

$$I = \frac{20+10}{3+6} = \frac{30}{9}\ \text{A}$$



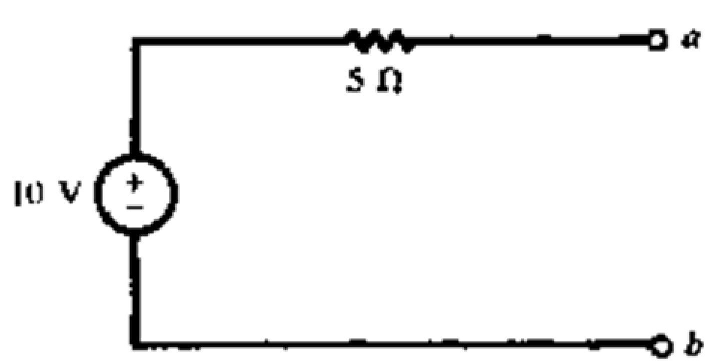
- Since no current passes through the upper right 3Ohm resistor, the Thevenin's voltage can be taken from either active branch:

$$V_{TH} = 20 - \left(\frac{30}{9}\right) \times 3 = 10\ \text{V}$$

- The equivalent resistance as seen from terminals *a* and *b* is

$$R_{eq} = 3 + \frac{3 \times 6}{9} = 5\ \Omega$$

- The Thevenin's equivalent circuits is:

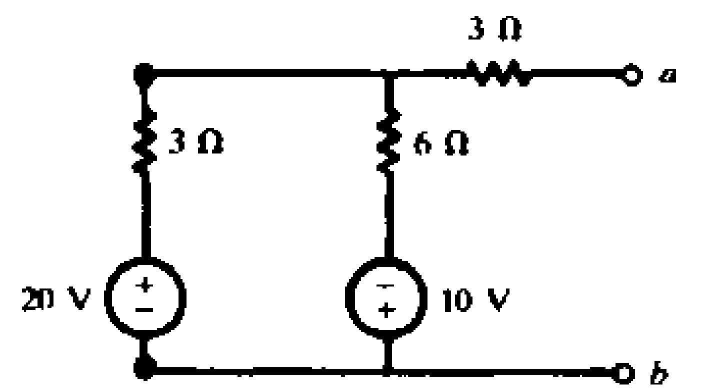


Thevenin's eq. circuit

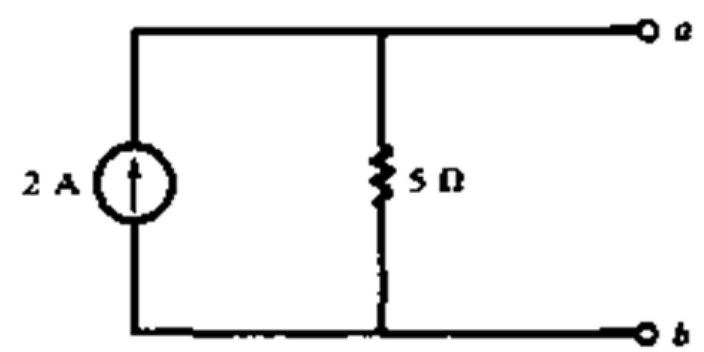
Norton's Equivalent Circuit

- When a short circuit is applied to the terminals, current I_{sc} results from the two sources. By superposition theorem, this current is given by

$$I_{sc} = \left(\frac{6}{6+3}\right)\left[\frac{20}{3+3\times 6/9}\right] - \left(\frac{3}{3+3}\right)\left[\frac{10}{6+3\times 3/6}\right] = 2\ \text{A}$$



- The Norton's equivalent circuit is:



Norton's eq. circuit