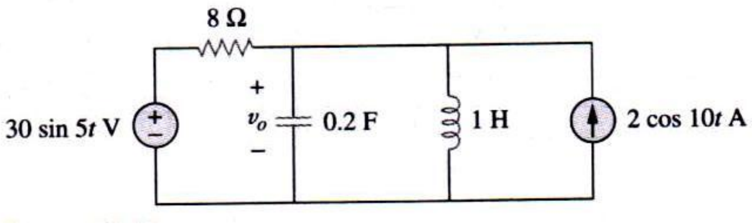
EE 101 Solutions to Tutorial Problems 6

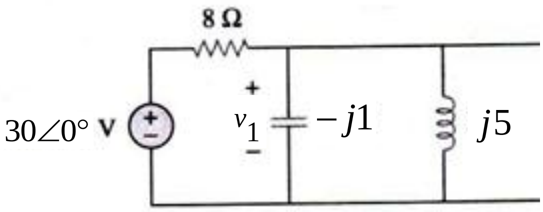
(18 September 2014)

Solution to Q1

1. Since the two sources have two different frequencies, superposition will have to be applied to find the voltage across the capacitor due to each source separately; and add them together.



- When considering the voltage source only, the current source will be open circuit, and the frequency domain circuit will be as shown below.



$X_L = j5\times1 = j5 \qquad X_C = -j/(5\times0.2) = -j1$

Equivalent impedance

= 8 + (-j1 // j5) = 8 + (5/j4) = 8 – j 1.25 = 8.1∠-8.8

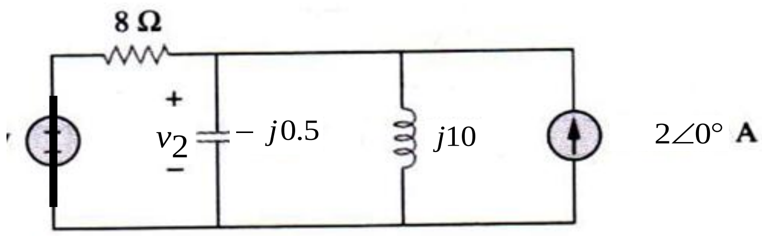
Current in the circuit

$$=\frac{30\angle0^\circ}{8.1\angle-8.88^\circ}=3.7\angle8.88^\circ$$

Therefore, $V_1 = 30\angle0^\circ - 8\times3.7\angle8.88^\circ = 30-29.2-j4.57 = 4.64\angle-80.1^\circ$

$v_1 = 4.64\sin(5t-80.1^\circ)$

- When considering current source, the voltage source will be short circuited, and the frequency domain circuit will be



$X_L = j10\times1 = j10$

$X_C = -j/(10\times0.2) = -j0.5$

Equivalent impedance

= 8//*j*10//*j*0.5)

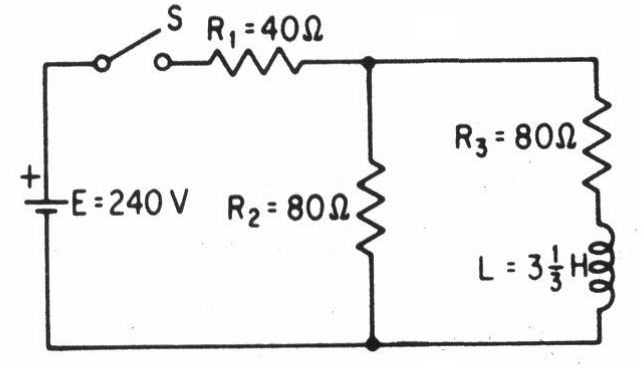
$$=\frac{1}{(1/8)+(1/j10)+(1/(-j0.5))}=\frac{1}{0.125-j0.1+j2}=\frac{1}{1.90\angle86.2^\circ}=0.52\angle-86.2^\circ$$

$V_2 = 2\angle0^\circ\times0.52\angle-86.2^\circ = 1.04\angle-86.2^\circ$

$v_2 = 1.04\cos(10t-86.2^\circ)$

Therefore, $v = v_1 + v_2 = 4.64\sin(5t-80.1^\circ)+1.04\cos(10t-86.2^\circ)$

Solution to Q2



Using operational impedances:

The current supplied by the source:

$$i = \frac{E}{R_1 + \dfrac{R_2 \times (R_3 + pL)}{R_2 + R_3 + pL}}$$

The current through L,

$$\begin{aligned} i_L &= i \times \frac{R2}{R_2 + R_3 + pL} = \frac{E}{R_1 + \dfrac{R_2(R_3 + pL)}{R_2 + R_3 + pL}} \times \frac{R_2}{R_2 + R_3 + pL} \\ &= \frac{ER_2}{R_1(R_2 + R_3 + pL) + R_2(R_3 + pL)} = \frac{ER_2}{(R_1R_2 + R_2R_3 + R_3R_1) + (R_1 + R_2)pL} \\ &= \frac{ER_2 / (R_1R_2 + R_2R_3 + R_3R_1)}{1 + p(L / [(R_1R_2 + R_2R_3 + R_3R_1) / (R_1 + R_2)]} \end{aligned}$$

Therefore,

$$\begin{aligned} i_L(t) &= ER_2 / (R_1R_2 + R_2R_3 + R_3R_1)(1 - e^{-[(R_1R_2 + R_2R_3 + R_3R_1)/L(R_1 + R_2)]t}) \\ &= 1.5(1 - e^{-32t}) \end{aligned}$$

Then,

$$\begin{aligned} i_L(0) &= 0 \quad \text{A} \\ i_L(\infty) &= 1.5 \quad \text{A} \\ \tau &= 1/32 = 0.03125 \quad \text{s} \end{aligned}$$

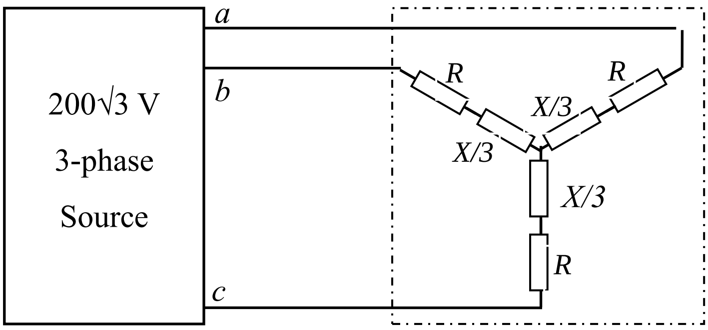
And time taken to establish 1 A current in the inductor is given by

$$\begin{aligned} 1 &= 1.5(1 - e^{-32t}) \\ t &= 0.0343 \quad \text{s} \end{aligned}$$

(Note, since 1 A is 2/3 of steady state, the time taken is about one time constant.)

Solution to Q3

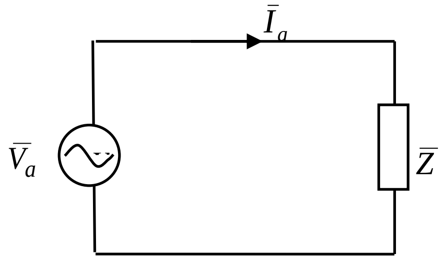
Converting the X delta to equivalent star, the system becomes:



Therefore the one phase equivalent circuits can be drawn as shown, where:

$V_a = 200\angle 0°$ V, and

$Z = R+X/3 = 6 + j8 = 10\angle 53.13°$ Ω



Therefore,

$I_a = 200\angle 0°/10\angle 53.13° = 20\angle -53.13°$ A

Hence<

Current supplied = 20 A

Power factor = cos 53.13° = 0.6 (lag)

Power consumed by the resistors = $3\times 20^2\times 6$ = 7.2 kW

Reactive power consumed by the reactors: = $3\times 20^2\times 8$ = 9.6 kVAr

Total power drawn by the load S = (7.2+j9.6) kVA

Voltage across each R = 6×20 = 120 V

Voltage across each X/3 in star configuration = 20×8 = 160 V

Voltage across each X in delta configuration = √3×160 = 277.1 V

(You can try many other ways of computing.)