# MA 101 - Quiz-1-Q4 grading

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*4. Let $B$ be an $n \times n$ matrix with $\det B < 0$ such that $B^T B = I_n$. Determine $\det(B + I_n)$, where $I_n$ denotes the $n \times n$ identity matrix.*

**Answer-1:**

$$\begin{aligned}
\det(B + I_n) &= \det(B + B^T B) \\
&= \det((I_n + B^T)B) \\
&= \det(I_n + B^T)\det(B) \\
&= \det(I_n + B)\det(B) \qquad \text{(1 mark)}
\end{aligned}$$

$$\det(B + I_n) = \det(I_n + B)\det(B) \tag{1}$$
$$\Rightarrow (1 - \det(B))\det(B + I_n) = 0 \tag{2}$$

Since $\det B < 0$, $1 - \det(B) > 0$. Hence it follows from (2) that $\det(B + I_n) = 0$. (1 mark)

**Answer-2:**

$$\begin{aligned}
&B^T B = I_n \\
&\Rightarrow \det(B^T B) = 1 \\
&\Rightarrow \det(B^T)\det(B) = 1 \\
&\Rightarrow \det(B) = \pm 1
\end{aligned}$$

Since it is given that $\det B < 0$, we have $\det(B) = -1$. (1 mark).

$$\begin{aligned}
\det(B + I_n) &= \det(B + B^T B) \\
&= \det((I_n + B^T)B) \\
&= \det(I_n + B^T)\det(B) \\
&= \det(I_n + B)\det(B) \\
&= -\det(I_n + B) \\
\Rightarrow \det(B + I_n) &= 0 \qquad \text{(1 mark)}
\end{aligned}$$

As indicated, both Answer-1 and Answer-2 have 2 major parts, and each part carries 1 mark. If steps are missing or incomplete, 1 marks will be deducted for the respective part(s).

*1 mark will be deducted for each of the following:*

(i) Taking $B$ as a $2 \times 2$ or $3 \times 3$ matrix.
*Counter example:* For each odd positive integer $n$, $B = -I_n$ satisfies $\det B < 0$ and $B^T B = I_n$.

(ii) Taking $B = I_n$ or $B = -I_n$.
*Counter example:* The matrix $\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ satisfies $\det B < 0$ and $B^T B = I_n$.

(iii) Showing $\det(B) = -1$ without proper justification.

Moreover, if mistakes in (i) and (ii) is found right from the beginning of the answer, the whole answer will be awarded 0 mark even if later parts are correct.