MA 101 QUIZ I
MATHEMATICS I 8:00–8:55AM
IIT GUWAHATI 19th August 2014

To be filled by the student

Roll No.: | | | | | | | | | | **Tutorial Group:** T | | **Sign:**

Name:

Instructions

1. Write all required information in the appropriate places above.
2. Check that you have 6 printed pages.
3. Write your answers in this booklet and only in the space marked for answers.
4. Additional space for writing/continuing your answers has been provided towards the end of the booklet. You must clearly indicate the question number(s) while using the additional space.
5. Supplementary sheets will be provided for **rough work only**. **These sheets will NOT be evaluated.**
6. No clarifications about the questions will be provided during examination.
7. This exam has 4 questions, for a total of 17 marks (points). You may attempt as many as you want.
8. The first question has 10 parts. Each part is worth **0.5** marks (points). They require only the correct answer for full credit. Remaining questions need all necessary steps for full credit.
9. The marks (points) you obtain or 15, whichever is lesser, will be your final score.

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Question Number	1	2	3	4	Total	Min(s,15)
Maximum Marks	5	4	4	4	17	———-
Marks Obtained (s)						

Jobs	Invigilator	Grader	Scrutinizer	Rechecker	Rechecker
Initials					

Space For Recheck Cribs

[5 points] 1. Write TRUE or FALSE in the spaces provided for answers. Each correct answer gets a score of $\frac{1}{2}$, each incorrect answer gets a score of $-\frac{1}{2}$ and each un-attempted question gets a score of 0.

QUESTION	ANSWER
(i) Let $A = \begin{bmatrix} 1 & 0 & 2 \\ 1 & 1 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 2 \\ 1 & 1 & 5 \\ 3 & 4 & 5 \end{bmatrix}$. Then Null$(A)$ is a proper subspace of Null(B).	
(ii) Consider the subspaces $U = \{[x, y, z, 0]^t : x, y, z \in \mathbb{R}\}$ and $V = \{[0, y, 0, w]^t : y, w \in \mathbb{R}\}$. Then $\mathbb{R}^4 = U \oplus V$.	
(iii) There exist two non-zero 2×2 matrices whose product is zero.	
(iv) If the $RREF(A)$ is R then the $RREF(-A)$ is $-R$.	
(v) The rank of the matrix $\begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix} [1 \;\; 1 \;\; 2]$ is 1.	
(vi) Let $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ be a linearly independent set of vectors in $\mathbb{R}^4$. Then for each 3×4 matrix A, the set $\{A\mathbf{v}_1, A\mathbf{v}_2, A\mathbf{v}_3, A\mathbf{v}_4\}$ is linearly dependent.	
(vii) There is a 4×5 matrix A whose columns are linearly independent.	
(viii) If A is a matrix of rank 2 then there is a matrix B such that rank$(AB) = 3$.	
(ix) The dimension of the subspace $\{[x_1, x_2, x_3, x_4]^t \in \mathbb{R}^4 : x_1 + x_2 + x_3 + x_4 = 0\}$ is 3.	
(x) Let B be the matrix obtained from $A_{4\times 4}$ by replacing each entry in the first column of A with its negative. Then dim(Null(A)) = dim(Null(B)).	

[4 points] 2. Consider the planes given by the equations $x + y + z = 1, x + 2y + 3z = 2$ and $x + 3y + cz = d$. Using Gaussian elimination, find all possible values of c and d such that these planes have (a) infinitely many common points (b) no common points and (c) a unique common point. **4**

[4 points] 3. Consider $\mathbf{u} = [1,0,0,1]^t, \mathbf{v} = [0,1,1,0]^t \in \mathbb{R}^4$.

(a) Is the set $\{\mathbf{u}, \mathbf{v}\}$ linearly independent? Justify. $\boxed{1}$

(b) Find an invertible matrix T such that $T \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$ is in reduced row echelon form. $\boxed{2}$

(c) Fill the blanks appropriately in the following paragraph. You get 1 mark if all the answers are correct and 0 mark, otherwise. No justification is required for this part. $\boxed{1}$

Let A be the sub-matrix formed by the bottom two rows of T obtained in part (b). Notice that these rows are ______________________________, as they are rows of an invertible matrix. Hence $RREF(A)$ has ____ pivotal columns and so $\dim(\mathrm{Null}(A)) =$ ____. As $\mathbf{u}, \mathbf{v} \in \mathrm{Null}(A)$ and $\{\mathbf{u}, \mathbf{v}\}$ is linearly independent, we find that $\{\mathbf{u}, \mathbf{v}\}$ is a basis of $\mathrm{Null}(A)$.

[4points] 4. (*a*) Find the inverse of $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 2 \\ 0 & -1 & -1 \end{bmatrix}$ using Gauss-Jordan method. **3**

(*b*) Consider the set S in $\mathbb{R}^3$ of all points lying on the union of x-axis and y-axis. Check whether S is a subspace or not. **1**

Extra space for answers

Extra space for answers

Extra space for answers