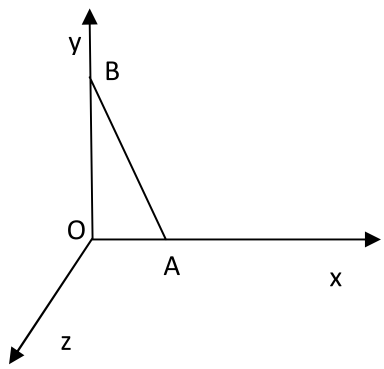
Roll No. **NAME:**

PH 101: Physics-I *Mid-Semsester Examination* *Date-19/09/15*

Duration- 2 hrs. *Total marks-30* (6×5)

Answer all six questions. Write the ***final answers*** *neatly and clearly in the space provided in the question booklet. Do not include any calculation here. You can use the answer booklet as your rough sheet. Submit both the question and answer booklets to the invigilator.*

1. The coordinates of the points O, A and B are (0,0,0), (1,0,0) and (0,2,0), respectively. A force field is given as $\vec{F} = -Ke^{-y}\,\hat{\imath} + Kxe^{-y}\,\hat{\jmath}$, where K is a constant.



i) The line integral along the path OA is

$$\int_{OA} \vec{F}.\overrightarrow{dl} = -K$$

[1]

ii)The line integral for the path AB is

$$\int_{AB} \vec{F}.\overrightarrow{dl} = K$$

[1]

iii) The line integral for the path BO is

$$\int_{BO} \vec{F}.\overrightarrow{dl} = 0$$

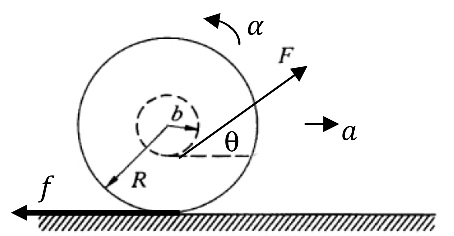
[1/2]

iv) Does a potential for the force $\vec{F}$ exist? If yes, then find the potential U, for the above force.

Yes,
$U = Kxe^{-y} + constant$ OR $U = Kxe^{-y}$

[1/2+2]

2. A yo-yo of mass M is placed vertically on a table has an axle of radius b and spool of radius R as shown. The string of the yo-yo is pulled with a constant force F making an angle θ with the horizontal. The coefficient of friction between the table and the yo-yo is μ. (Assume the moment of inertia $I = \frac{1}{2}MR^2$ about the axis of rotation and g as acceleration due to gravity)



i) Draw the friction force f on the diagram itself.

[1/2]

ii) If the centre of mass has a linear acceleration a, then write the equation for the translational motion of the centre of mass

$F\cos\theta - f = Ma$

[1]

iii) If the yo-yo has an angular acceleration α, then write the equation for the rotational motion of the yo-yo about the centre of mass.

$$bF - Rf = I\alpha$$

[1]

iv) What frictional force f, is experienced by the yo-yo during motion without slipping?

$f = \frac{F}{3R}(2b + R\cos\theta)$ **[1]**

v) What is the maximum static frictional force f_{max}, that the system can sustain before slipping?

$$f_{max} = \mu(Mg - F\sin\theta)$$

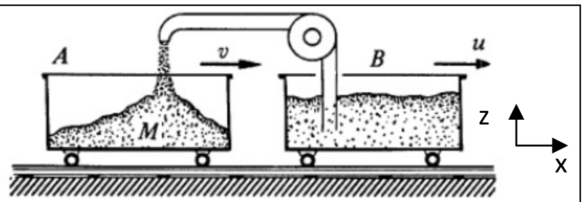
[1/2]

v) What is the maximum force F_{max}, for which the yo-yo will roll without slipping?

$$F_{max} = \frac{3\mu RMg}{2b + R\cos\theta + 3\mu R\sin\theta}$$

[1]

3. Material is blown from cart B to cart A at the rate of b kilograms per second. The material leaves the chute vertically downwards so that it has the same constant horizontal velocity u, as cart B. At a certain time t, cart A has mass M and velocity v as shown.



i) Is linear momentum of cart A conserved along the vertical direction? If yes, write the linear momentum conservation equation along the vertical direction at time t and $t + \Delta t$ (Δt is a small time interval).
NO

[1/2]

Roll No. **NAME:**

ii) Is linear momentum of cart A conserved along the horizontal direction? If yes, write the linear momentum conservation equation along the horizontal direction at time t and $t + \Delta t$. Yes, $Mv + u\,\Delta m = (M + \Delta m)(v + \Delta v)$ **[1/2+1]**
iii) Using the momentum conservation equation(s) write the instantaneous acceleration $\frac{d\vec{v}}{dt}$, of the cart A. $\frac{d\vec{v}}{dt} = \frac{u-v}{M}\, b\,\hat{x}$ **[1]**
iv) If the cart A is at rest initial $(t = 0)$ mass of the cart A is M_0, find the velocity $\vec{v}$ of the cart at a time t. $$\vec{v} = \frac{u}{1 + {M_0}/{bt}}\hat{x}$$ **[2]**

4. Consider earth, of mass M_E, is revolving around the sun, of mass M_s, under the gravitational force F of the sun. Assume sun is fixed at the origin. At any instant it was found that the earth is at a distance $\vec{r}$ from the sun and moving with a velocity $\vec{v} = v_r\hat{r} + v_\theta\hat{\theta}$. (Assume G as the universal gravitational constant.)
i) Is linear momentum of earth conserved? If yes, what is the instantaneous linear momentum $\vec{p}$ that remains constant? NO **[1/2]**
ii) What is the value of $\nabla \times \vec{F}$? ZERO **[1/2]**
iii) Is angular momentum of earth conserved? If yes, write the magnitude of instantaneous angular momentum L that remains constant. Yes, $L = M_E v_\theta r$ **[1/2+1/2]**
iv) Is total mechanical energy of earth conserved? If yes, write the value of instantaneous total mechanical energy E that remains constant. Yes, $E = \frac{1}{2}M_E\left(v_r^2 + v_\theta^2\right) - \frac{GM_SM_E}{r}$ **[1/2+1/2]**

v) Find the value of $\dot{r}^2$ as function of r and other previously known conserved quantities, if any, such as p, L and E.

$$\dot{r}^2 = \frac{2E}{M_E} + \frac{2GM_S}{r} - \frac{L^2}{r^2 M_E^2}$$

[1]

vi) What is the farthest distance (r_{max}) of the earth from the sun?

$$r_{max} = \frac{-2GM_S + \sqrt{\left(4G^2M_S^2 + {8EL^2}/{M_E^3}\right)}}{4E/M_E} = \frac{-2GM_EM_S + \sqrt{4G^2M_E^2M_S^2 + {8EL^2}/{M_E}}}{4E}$$

[1]

5. A cloud of interstellar gas of total mass M is moving freely in an empty space. Initially the cloud has the form of a uniform sphere of radius a rotating with angular velocity $\vec{\omega}$ in some arbitrary direction passing through its centre. Later the cloud is observed to change its shape to that of a uniform circular disc of radius b which is also rotating about an axis through the centre and perpendicular to its plane.

i) How much is the external torque $\vec{\tau}$ acting on the system?
ZERO

[1/2]

ii) What is the final angular velocity $\vec{\Omega}$ of the cloud?

$$\vec{\Omega} = \frac{4a^2}{5b^2}\vec{\omega}$$

[2 $^1/_2$]

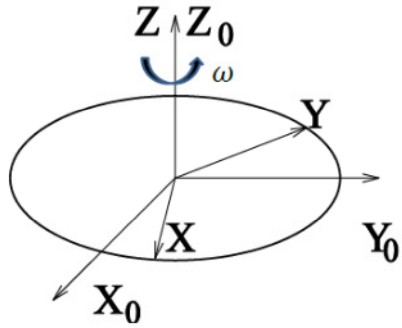
iii) If T_i and T_f are the initial and final kinetic energies of the cloud, find $\Delta T = T_f - T_i$, in terms of M, a, b and ω.

$\Delta T = T_f - T_i = \frac{4Ma^4}{25b^2}\omega^2 - \frac{1}{5}Ma^2\omega^2$ OR

$$\Delta T = T_f - T_i = \frac{Ma^2\omega^2}{25b^2}(4a^2 - 5b^2)$$

[2]

Roll No. **NAME:**

6. A merry-go-round (MGR) having two orthogonal axes (X, Y) painted on it and is rotating with constant angular velocity ω about the vertical $Z-$ axis. . MGR is fixed on earth which is assumed to be an inertial frame X_0, Y_0 and Z_0. A bug of mass m is crawling outward without slipping along the X −axis with a constant velocity v_0. Assume that at $t = 0, (X, Y, Z)$ coincides with (X_0, Y_0, Z_0) and the bug is at the origin.	

i) Write the pseudo-force $\vec{F}_{pseudo}$ experienced by the bug as seen from the MGR frame?

$$\vec{F}_{pseudo} = m\omega^2 x\,\hat{x} - 2m\omega v_0\hat{y} \;\; OR \; \vec{F}_{pseudo} = m\omega^2 v_0 t\hat{x} - 2m\omega v_0\hat{y}$$

[1+1]

ii) What is the force, $\vec{F}_{B,MGR}$, exerted by the MGR on the bug as seen from the MGR frame?

$$\vec{F}_{B,MGR} = -m\omega^2 x\hat{x} + 2m\omega v_0\hat{y} + mg\hat{z} \; OR \; \vec{F}_{B,MGR} = -m\omega^2 v_0 t\hat{x} + 2m\omega v_0\hat{y} + mg\hat{z}$$

[1]

iii) Write the force $\vec{F}_{B,MGR}$ in the inertial frame(X_0, Y_0, Z_0).

$$\vec{F}_{B,MGR} = -(2m\omega v_0 \sin\omega t + m\omega^2 v_0 t\cos\omega t)\hat{x}_0 + (2m\omega v_0\cos\omega t - m\omega^2 v_0 t\sin\omega t)\hat{y}_0 + mg\hat{z}_0$$

[2]