Solution to Mid-Sem Exam. paper (PH 101)

PART - I

1. (c)

In addition to the frictional force, an inertial force $-m\vec{a}$ would also act on the block.

2. (a)

Newton's 2nd law gives:

$$mg - mbv = m\dot{v} \Rightarrow \dot{v} + bv - g = 0$$

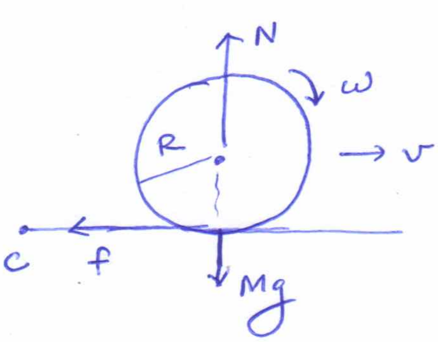
3. (c)

Conservation of momentum gives a speed 0·5 m/s to the plank in the direction opposite to that of the boy. Relative to an observer on the ground, the boy's speed will be:

$$1{\cdot}0 - 0{\cdot}5 = 0{\cdot}5 \text{ m/s}$$

4. (c)

Take angular momentum about any point on the path, say c in the sketch.



$$L_i = M v_0 R$$

$$L_f = MvR + I\omega$$
$$= MvR + \frac{2}{5}MR^2\omega$$

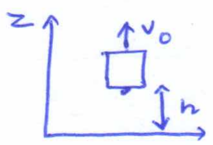
Rolling starts at $t = t_r$ when $\omega(t_r) = v(t_r)/R$

$$L_f = \left(MR + \frac{2}{5}MR\right)v(t_r) = \frac{7}{5}MR\,v(t_r) = L_i = Mv_0R$$

$$\Rightarrow v(t_r) = \frac{5}{7}v_0 \qquad \text{Here } v_0 = 14 \text{ m/s}$$

$$\Rightarrow v = 10 \text{ m/s}$$

5. (b)



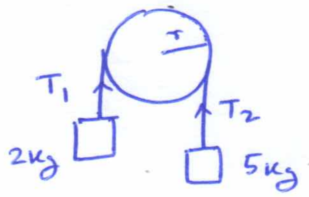
At $t=0$, the elevator starts moving upward with uniform speed v_0, so the height above the ground at time t is $z = v_0 t$. At $t = t_1 (= 2s)$, $h = v_0 t_1$.
At $t = t_1$, the marble is released; it is at height h. Its height z at t is then

$$z = h + v_0 (t - t_1) - \frac{1}{2} g (t - t_1)^2. \quad \text{At } t = t_2 \ (= 4s),\ h = 0$$

$$\Rightarrow \quad h = \frac{1}{2} g \frac{t_1}{t_2} (t_2 - t_1)^2.$$ Here $t_1 = 2s$, $t_2 = 4s$, $g = 10 m/s^2$

$\therefore h = 10 m$

6. (b)



$$5a = 5g - T_2$$
$$2a = T_1 - 2g$$
$$a = \alpha r \quad \text{with} \quad \alpha = \frac{(T_2 - T_1) r}{\frac{1}{2} m r^2}$$
$$\Rightarrow a = \frac{2(T_2 - T_1)}{m} \qquad m = 0.4 kg$$

This leads to (b)

7. (d) $\quad g = k/r^2 \Rightarrow \dfrac{dg}{g} = -\dfrac{2dr}{r} \Rightarrow$ ~~g~~ decreases by 2%.

8. (a) Total energy $E = KE + PE = \frac{1}{2} m v^2 - k/r$

$$\frac{mv^2}{r} = \frac{k}{r^2} \Rightarrow \frac{1}{2} m v^2 = \frac{1}{2} k/r$$

Thus, $E = -k/2r$ //

9. (b)

P.E. $\quad V = -\int F dx = K \int x^2 dx = \frac{1}{3} K x^3$

Energy conservation eq^n, while the particle is in the region $x \geq 0$ is

$$\frac{1}{2} m v^2 + \frac{1}{3} K x^3 = E \quad \text{where } v = \dot{x}, \ E \text{ is } \cancel{\text{to}} \text{ the total energy.}$$

Consider ~~the~~ the motion arising from the initial condition $v = u$ when $x = 0$. In this case $E = \frac{1}{2} m u^2$. Therefore we have

$$\frac{1}{2} m v^2 + \frac{1}{3} K x^3 = \frac{1}{2} m u^2.$$

The maximum value of x is attained when $v = 0$, i.e. when x satisfies the equation $0 + \frac{1}{3} K x^3 = \frac{1}{2} m u^2$. Thus, the farthest point along the x-axis reached by the particle is $x = \left(\frac{3mu^2}{2K}\right)^{1/3}$

10.

(a)

conservation of angular momentum gives:

$I_1 \omega_1 = I_2 \omega_2$; $I_2 = \frac{1}{3} I_1 \Rightarrow \omega_2 = 3\omega_1$

$\therefore$ Ratio of KE $= \dfrac{\frac{1}{2} I_2 \omega_2^2}{\frac{1}{2} I_1 \omega_1^2} = \dfrac{3}{1}$

11. (c)

$E = \dfrac{p^2}{2m} + \frac{1}{2} m\omega^2 x^2$, $E =$ const. of motion

12. (d)

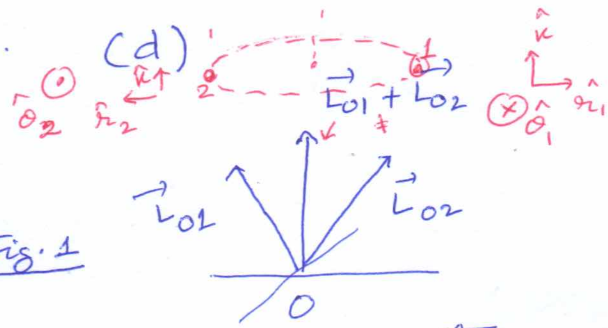


Fig. 1

The angular momentum about the origin is the sum of contributions from each object. Since they have the same mass, the angular momentum vectors are shown in Fig. 1. The components that lie in the x-y plane ~~cancl~~ cancel leaving only a non-zero z-component.

$\vec{L}_0 = \vec{L}_{0,1} + \vec{L}_{0,2} = 2m\omega r^2 \hat{k}$ //

{ Explicit calculations can also be done to get the same answer. }

$\vec{L}_0 = \vec{L}_{0,1} + \vec{L}_{0,2} = (r\hat{r}_1 + h\hat{k}) \times mr\omega\hat{\theta}_1 + (r\hat{r}_2 + h\hat{k}) \times (mr\omega\hat{\theta}_2)$

$= 2mr^2\omega\hat{k} + hmr\omega(-\hat{r}_1 - \hat{r}_2)$

$= 2mr^2\omega\hat{k} + hmr\omega(-\hat{r}_1 + \hat{r}_1) = 2mr^2\omega\hat{k}$ //

13. (c)

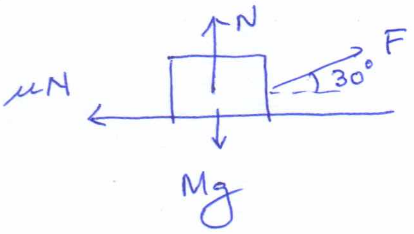
Moving up, the tension in the string or the effective weight of the bob becomes

$mg' = m\left(g + \frac{1}{16} g\right) = \frac{17}{16} mg$. $T = 2\pi\sqrt{l/g}$

$T' = 2\pi\sqrt{l/g'}$

$\Rightarrow T' = \sqrt{g/g'}\, T = \frac{4}{\sqrt{17}} T$

14. (b)



$N + F \sin 30^\circ = Mg$

$F \cos 30^\circ - \mu N = 0$

$\therefore N = \left(100 - \frac{F}{2}\right)$

$F\frac{\sqrt{3}}{2} - (0.4)\left(100 - \frac{F}{2}\right) = 0$

$\Rightarrow F = 37.5$ Newton //

15. (c)

$\frac{1}{2} a (0\cdot 6)^2 = 2 \Rightarrow a \approx 11 \, m/s^2$, acc^n of the ~~ele~~ elevator upwards

The passenger is in an accelerated frame, i.e. non-inertial frame. Hence a pseudo force $-3 \times 11 = -33$ N acts downwards.

Thus, $T = (3 \times 10) + 33 = 63$ N

16. (b)

Say weight of ~~wa~~ wagon $W_1 = \omega_1$
and weight of wagon $W_2 = \omega_2$

Then, $\omega_2 v = \omega_1 v_1 + \omega_2 v_2$ $\quad$ v_1 = final velocity of W_1, v_2 = final velocity of W_2

$\Rightarrow \boxed{v = v_1 + v_2} \to$ (i) since $\omega_1 = \omega_2 = \omega$, equal weight

Again, $\frac{1}{2} \omega_2 v^2 = \frac{1}{2} \omega_1 v_1^2 + \frac{1}{2} \omega_2 v_2^2$

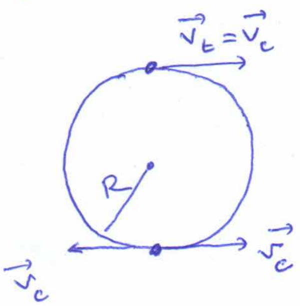
$\Rightarrow \boxed{v^2 = v_1^2 + v_2^2} \to$ (ii)

(i) and (ii) can both be correct if $v_1 = v$ and $v_2 = 0$

hence ~~as~~ ans is (b) //

17. (a)

All the points on the rim perform simultaneously two motions : (i) translational motion with the entire wheel (ii) the rotational motion around the axis of the wheel.



The velocity at any point on the rim is the vector sum of $\vec{v}_c$, the velocity of c.m. and $\vec{v}_t$ the tangential velocity of the point. $|\vec{v}_c| = |\vec{v}_t| = \omega R$. At the top $\vec{v}_c \parallel \vec{v}_t$. The velocity of the point at the top is $v = v_c + v_t = 2\omega R$

At the point in contact with the floor $\vec{v}_t$ is antiparallel to $\vec{v}_c$. Therefore $v = \omega R - \omega R = 0$ //

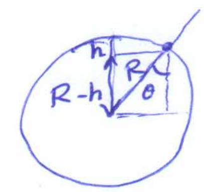
18. (c)

Eq^n of motion of the marble during sliding:

$$mg\cos\theta - N = \frac{mv^2}{R}$$

The point where it leaves the surface, $N=0$

Hence, $\cos\theta = \frac{v^2}{Rg} = \frac{R-h}{R} \rightarrow (i)$



Also, $mgh = \frac{1}{2}mv^2 \Rightarrow v^2 = 2gh \rightarrow (ii)$

From (i) and (ii) we obtain: $h = \frac{R}{3}$ //

19. (b)

$$P = \vec{F}.\vec{v} = m\frac{d\vec{v}}{dt}.\vec{v} = \frac{1}{2}m\frac{dv^2}{dt} = \text{constant}$$

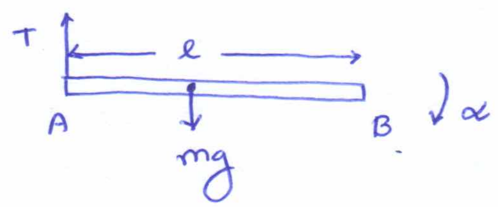
$$\Rightarrow v^2 = \frac{2Pt}{m} \Rightarrow v = \frac{ds}{dt} = \left(\frac{2Pt}{m}\right)^{1/2} \Rightarrow s \propto t^{3/2} \text{ //}$$

20. (b)

PART - II

1.

(a) When one of the string is cut, rod experiences an angular acceleration α. If T is the tension in the string, then



$$mg - T = m\frac{l}{2}\alpha \rightarrow (1)$$

$$v = \omega\frac{l}{2}, \quad \frac{dv}{dt} = \alpha\frac{l}{2}$$

Taking moments about A

$$mg\frac{l}{2} = I\alpha = m\frac{l^2}{3}\alpha$$

$$\Rightarrow \quad \alpha = \frac{3}{2}\,g/l \rightarrow (2)$$

Putting this in (1) we obtain:

$$T = mg - m\frac{l}{2}\alpha$$
$$= mg - \frac{3}{4}mg$$
$$= \frac{1}{4}mg$$

$$\Rightarrow \quad T = \frac{1}{4}mg \quad //$$

(b) The three masses are equal, $m_1 = m_2 = m_3 = m$

Their positions are

$$\vec{r}_1 = a\,(1,0,0), \quad \vec{r}_2 = a\,(0,1,2), \quad \vec{r}_3 = a\,(0,2,1)$$

Therefore,

$$I_{xx} = \sum m_\alpha(y_\alpha^2 + z_\alpha^2) = ma^2(0+5+5) = 10\,ma^2$$
$$I_{yy} = \sum m_\alpha(x_\alpha^2 + z_\alpha^2) = ma^2(1+4+1) = 6\,ma^2$$
$$I_{zz} = \sum m_\alpha(x_\alpha^2 + y_\alpha^2) = ma^2(1+1+4) = 6\,ma^2$$
$$I_{xy} = -\sum m_\alpha x_\alpha y_\alpha = -ma^2(0+0+0) = 0$$
$$I_{xz} = -\sum m_\alpha x_\alpha z_\alpha = -ma^2(0+0+0) = 0$$
$$I_{yz} = -\sum m_\alpha y_\alpha z_\alpha = -ma^2(0+2+2) = -4ma^2$$

Thus,

$$I = 2ma^2\begin{pmatrix} 5 & 0 & 0 \\ 0 & 3 & -2 \\ 0 & -2 & 3 \end{pmatrix} \equiv \begin{pmatrix} 10 & 0 & 0 \\ 0 & 6 & -4 \\ 0 & -4 & 6 \end{pmatrix} \quad //$$

2.

Initial energy :

$$K_i = \frac{1}{2} M v_0^2 \quad , \quad V_i = 0$$

$$E_i = K_i + V_i = \frac{1}{2} M v_0^2$$

Final energy :

$$K_f = 0 \quad , \quad V_f = \frac{1}{2} k \ell^2$$

$$E_f = \frac{1}{2} k \ell^2$$

The friction force $F_{friction} = \mu N$ where $N = Mg$

$$E_f - E_i = \text{work on the system}$$

$$= \int_0^{\ell} F_{friction}\, dx'$$

$$= - \int_0^{\ell} \mu Mg\, dx'$$

$$= - Mg \int_0^{\ell} \mu\, dx' = - Mg \int_0^{\ell} b x'\, dx'$$

$$= - \frac{1}{2} Mg b \ell^2$$

Thus, $\frac{1}{2} k \ell^2 - \frac{1}{2} M v_0^2 = - \frac{1}{2} Mg b \ell^2$

$$\Rightarrow \quad \ell = v_0 \sqrt{\frac{M}{k + Mgb}}$$

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