Quiz-2 (PH101)

Time - 45 Minutes, Marks: 10, Date: 11th November, 2014

1. [Marks: 3] A rigid body consisting of four particles of mass m are placed at $(a,0,0)$, $(-a,0,0)$, $(0,a,0)$, $(0,-a,0)$ and two particles of mass $2m$ is placed at $(0,0,c)$ and $(0,0,-c)$. Find the ratio c/a for which every axis is a principal axis.

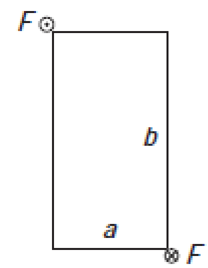
Solution:
Clearly, by symmetry, the three coordinate axes are also principal axes of this body. Then,

$$\begin{aligned} I_{xx} &= I_{yy} = 2ma^2 + 4mc^2 \\ I_{zz} &= 4ma^2. \end{aligned}$$

If every axis is a principal axis then all three principal moments must be identical. Thus

$$\begin{aligned} I_{zz} &= I_{xx} \\ 4ma^2 &= 2ma^2 + 4mc^2 \\ c &= \frac{a}{\sqrt{2}} \end{aligned}$$

2. [Marks: 3] A flat uniform rectangle with sides of length a and b sits in space, not rotating. You strike the corners at the ends of one diagonal, with equal and opposite forces. Show that the resulting initial $\vec{\omega}$ points along the other diagonal.



Solution:
Set up coordinate system with rectangle in XY plane with origin at COM of the rectangle and axis parallel to the edges. The principal moments are

$$I_{xx} = \frac{1}{12}Mb^2 \qquad I_{yy} = \frac{1}{12}Ma^2 \qquad I_{zz} = I_{xx} + I_{yy}$$

Let $\tan\theta = a/b$. Clearly the angular impulse is delivered in the direction that makes an angle θ with the X axis. Thus $\vec{L} = L\left(\hat{x}\cos\theta + \hat{y}\sin\theta\right)$. Let $\vec{\omega} = \hat{x}\omega_x + \hat{y}\omega_y$. Then,

$$\vec{L} = \hat{x} I_{xx}\omega_x + \hat{y} I_{yy}\omega_y$$

and

$$\begin{aligned} \frac{I_{yy}\omega_y}{I_{xx}\omega_x} &= \tan\theta \\ \therefore \quad \frac{\omega_y}{\omega_x} &= \frac{b^2}{a^2}\tan\theta = \cot\theta \\ \therefore \quad \frac{\omega_y}{\omega_x} &= \tan\left(90-\theta\right) \end{aligned}$$

Thus, $\vec{\omega}$ makes the angle of $90-\theta$ with X axis and thus passes through the other diagonal.

3. [Marks: 4] A train of proper length L and speed $3c/5$ approaches a tunnel of length L. At the moment the front of the train enters the tunnel, a person starts walking from the front end of the train towards the back. She arrives at the back end of the train right when the back end leaves the tunnel.

   (a) How much time does this take in the ground frame?

(b) What is the person's speed with respect to the ground?

(c) How much time elapses on the person's watch?

Solution

Let v_t be the velocity of the train. Let $\gamma_t = \left(1 - v_t^2/c^2\right)^{-1/2} = 5/4$. Then the events are:

- Event A: Train enters tunnel, person starts walking.
- Event B: Train (back end) leaves tunnel, person reaches back end.

(a) The train appears to have length of L/γ_t. Then the back end of the train travels a distance of $L\left(1 + 1/\gamma_t\right) = \frac{9}{5}L$. The time between two events in ground frame is

$$T = \frac{9L}{5v_t} = 3\frac{L}{c}$$

(b) The person has traveled a distance of L in time T. Thus the speed of the person is

$$v_p = \frac{1}{3}v_t = \frac{1}{5}c.$$

Now let $\gamma_p = \left(1 - v_t^2/c^2\right)^{-1/2} = \frac{5}{\sqrt{24}}$

(c) The person's watch is dilated and hence time between two events will be

$$T_p = \frac{\sqrt{24}}{5}T = \frac{6\sqrt{6}}{5}\frac{L}{c}$$