Indian Institute of Technology Guwahati
Department of Mathematics

MA 101 – Mathematics-I **Date:** 03-Aug-2017
Tutorial Sheet-1 **Time:** 08:00 – 09:00

Linear Algebra

Topics Covered:
Row-Echlon form, Gaussian elimination, Elementary row operations, Equivalence of systems of linear equations, Row equivalent matrices

1. Let A be an $m \times n$ matrix and B be the matrix obtained by interchanging two rows of A. Is it possible to obtain the matrix B by a finite sequence of elementary row operations of the other two types (applied to A)? Justify your answer. *Understanding concepts*

2. For what values of $\lambda \in \mathbb{R}$, the following system of equations has (i) no solution, (ii) a unique solution, and (iii) infinitely many solutions? *Calculations*

$$\begin{aligned} (i) \quad & x + y + \lambda z = 1,\ x + \lambda y + z = 1, \lambda x + y + z = -2. \\ (ii) \quad & x - 2y + 3z = 2,\ x + y + z = \lambda, 2x - y + 4z = \lambda^2. \end{aligned}$$

3. Let A and B be two 2×3 real matrices that are in reduced row echelon form. If the systems $Ax = 0$ and $Bx = 0$ are equivalent then is it always true that $A = B$? Justify your answer.

Understanding concepts

4. Check whether the following systems of linear equations are equivalent, by reducing the corresponding augmented matrices to the row-echlon form. *Calculations*

$$\begin{array}{rl} System - I & \\ 7x + 2y - 3z & = 33 \\ 3x - y + 5z & = -14 \\ 2x + 4y - z & = 27 \end{array} \qquad \begin{array}{rl} System - II & \\ 3x + 4y + 2z & = 20 \\ x + 3y - z & = 20 \\ x + 2y + 3z & = 3 \\ 4y + z & = 17 \end{array}$$

5. Let A, B be a 2×2 real matrices, such that $AB = BA = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. Then find out the image of the following under A: *Understanding concepts / geometrical interpretation*

   (a) a straight line in a plane
   (b) a straight line in a plane passing through origin
   (c) a pair of two straight lines in a plane parallel to each other
   (d) a pair of two straight lines in a plane perpendicular to each other

6. Let $A = \begin{bmatrix} 1 & \beta \\ \gamma & \delta \end{bmatrix}$ be a real matrix. Then show that *Understanding concepts*

   (a) if $\delta \neq \beta\gamma$ then the system $Ax = 0$ has only the trivial solution $[0, 0]^t \in \mathbb{R}^2$.
   (b) if $\delta = \beta\gamma$ then the system $Ax = 0$ has infinitely many solution in $\mathbb{R}^2$.

7. Using Gaussian elimination process, check whether the following system of linear equations have a solution! If yes, describe the solution set. *Calculations*

   (a)

$$\begin{aligned} 2y + z &= -8 \\ x - 2y - 3z &= 0 \\ -x + y + 2z &= 3 \end{aligned}$$

   (b)

$$\begin{aligned} x - 2y - 6z &= 12 \\ 2x + 4y + 12z &= -17 \\ x - 4y - 12z &= 22. \end{aligned}$$

   (c)

$$\begin{aligned} x - y + 2x &= -3 \\ 4x + 4y - 2z &= 1 \\ -2x + 2y - 4z &= 6 \end{aligned}$$

Additional problems

1. In the context of Question 3, does the same statement hold for arbitrary $m \times n$ matrices? Does the same statement hold for systems $Ax = v$ and $Bx = v$, where v is a non-zero $m \times 1$ column-matrix?

2. Let A be an $n \times n$ real matrix such that $A^m = 0$, for some $m \in N$ (such a matrix is called nilpotent matrix). Show that there exists a matrix B such that $B(I_n + A) = I_n$.