DEPARTMENT OF MATHEMATICS

Indian Institute of Technology Guwahati

MA101: Mathematics I, July - November, 2014

Tutorial Sheet: LA - 2

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1. Examine whether the following sets are subspaces of $\mathbb{R}^n$ $(n \geq 3)$.

   $S_1 = \{[x_1, \ldots, x_n]^t \in \mathbb{R}^n : x_1 + x_2 = 4x_3\}$, $S_2 = \{[x_1, \ldots, x_n]^t \in \mathbb{R}^n : x_1 + x_2 \leq 4x_3\}$,

   $S_3 = \{[x_1, \ldots, x_n]^t \in \mathbb{R}^n : x_1 = 1 + x_2\}$ and $S_4 = \{[x_1, \ldots, x_n]^t \in \mathbb{R}^n : x_1 x_2 = 0\}$.

   Remark: If a set S is a subspace then show that the rules

   Rule 1 : $\mathbf{0} \in S$ (to claim that $S \neq \emptyset$);

   Rule 2 : $\alpha\mathbf{u} + \beta\mathbf{v} \in S$ for every $\mathbf{u}, \mathbf{v} \in S$ and $\alpha, \beta \in \mathbb{R}$.

   are satisfied. If S is **not** a subspace then show that either $\mathbf{0} \notin S$ **or** show, *by a counter-example*, that **Rule 2** is not satisfied.

2. Find all the subspaces of $\mathbb{R}^2$.

3. Show that $W = \{[x, y, z, w]^t \in \mathbb{R}^4 : w - z = y - x\}$ is a subspace of $\mathbb{R}^4$, spanned by the vectors $[1, 0, 0, -1]^t$, $[0, 1, 0, 1]^t$ and $[0, 0, 1, 1]^t$. Also, determine $\dim W$.

4. Find three vectors in $\mathbb{R}^3$ which are linearly dependent but any two of them are linearly independent.

5. Let $S = \{\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \mathbf{x}_4\}$, where $\mathbf{x}_1 = [1, 0, 0, 0]^t, \mathbf{x}_2 = [1, 1, 0, 0]^t, \mathbf{x}_3 = [1, 2, 0, 0]^t$ and $\mathbf{x}_4 = [1, 1, 1, 0]^t$. Determine all $\mathbf{x}_i \in S$ such that $\text{span}(S) = \text{span}(S \setminus \{\mathbf{x}_i\})$.

6. Let $\mathbf{u}, \mathbf{v}, \mathbf{w}$ be linearly dependent vectors in $\mathbb{R}^n$ and let $\mathbf{v}, \mathbf{w}, \mathbf{x}$ be linearly independent vectors in $\mathbb{R}^n$, where $n \geq 3$. Show that **(i)** $\mathbf{u}$ is a linear combination of $\mathbf{v}$ and $\mathbf{w}$, and **(ii)** $\mathbf{x}$ is not a linear combination of $\mathbf{u}, \mathbf{v}$, and $\mathbf{w}$.

7. Let $M = \{[x, y, z]^t \in \mathbb{R}^3 : x + y + 4z = 0\}$ and $N = \{[x, y, z]^t \in \mathbb{R}^3 : x + y + z = 0\}$. Show that M and N are subspaces of $\mathbb{R}^3$. Determine a basis for each of $M, N, M+N$ and $M \cap N$. Also, interpret your result geometrically.
