

The Given equations are,

$$A + B = C + D \dots\dots\dots(1)$$

$$ikA - ikB = \kappa C - \kappa D \dots\dots\dots(2)$$

$$Ce^{\kappa L} + De^{-\kappa L} = A'e^{ikL} \dots\dots\dots(3)$$

$$\kappa Ce^{\kappa L} - \kappa De^{-\kappa L} = ikA'e^{ikL} \dots\dots\dots(4)$$

Now, eliminating B from (1) and (2) and writing A in terms of C and D, we get,

$$2ikA = (ik + \kappa)C + (ik - \kappa)D$$

$$\Rightarrow A = \frac{1}{2}[(1 + \frac{\kappa}{ik})C + (1 - \frac{\kappa}{ik})D] \dots\dots\dots(5)$$

Now, from (3) & (4) writing the value of C & D in terms of  $A'$  respectively, we get,

$$2\kappa Ce^{\kappa L} = (ik + \kappa)A'e^{ikL}$$

$$\Rightarrow C = \frac{1}{2}[(\frac{ik}{\kappa} + 1)A'e^{ikL}e^{-\kappa L}] \dots\dots\dots(6)$$

And

$$2D\kappa e^{-\kappa L} = (\kappa - ik)A'e^{ikL}$$

$$\Rightarrow D = \frac{1}{2}[(1 - \frac{ik}{\kappa})A'e^{ikL}e^{\kappa L}] \dots\dots\dots(7)$$

Putting the value of C & D from (6) & (7) in (5) we can get the value of A in terms of  $A'$ .

$$\Rightarrow A = \frac{1}{4}[(1 + \frac{\kappa}{ik})(1 + \frac{ik}{\kappa})e^{-\kappa L} + (1 - \frac{\kappa}{ik})(1 - \frac{ik}{\kappa})e^{\kappa L}]A'e^{ikL} \dots\dots\dots(8)$$

$$\Rightarrow 4ik\kappa A = [(2ik\kappa + \kappa^2 - k^2)e^{-\kappa L} + (2ik\kappa - \kappa^2 + k^2)e^{\kappa L}]A'e^{ikL}$$

$$\Rightarrow 4ik\kappa A = [(k^2 - \kappa^2)(e^{\kappa L} - e^{-\kappa L}) + 2ik\kappa(e^{\kappa L} + e^{-\kappa L})]A'e^{ikL}$$

Using,

$$\frac{e^{\kappa L} - e^{-\kappa L}}{2} = \sinh(\kappa L), \quad \& \quad \frac{e^{\kappa L} + e^{-\kappa L}}{2} = \cosh(\kappa L),$$

We have,

$$\Rightarrow 2ik\kappa A = [(k^2 - \kappa^2)\sinh(\kappa L) + 2ik\kappa \cosh(\kappa L)]A'e^{ikL}$$

Now the ratio between the transmission & incident amplitudes is given by,

$$\Rightarrow \frac{A'}{A} = \frac{2ik\kappa e^{-ikL}}{(k^2 - \kappa^2) \sinh(\kappa L) + 2ik\kappa \cosh(\kappa L)} \dots\dots\dots(9)$$

The transmission co-efficient is given by,

$$\begin{aligned} T = \frac{|A'|^2}{|A|^2} &= \frac{\{2ik\kappa e^{-ikL}\} \{-2ik\kappa e^{ikL}\}}{\{(k^2 - \kappa^2) \sinh(\kappa L)\}^2 - \{2ik\kappa \cosh(\kappa L)\}^2} \\ &= \frac{4k^2\kappa^2}{(k^2 - \kappa^2)^2 \sinh^2(\kappa L) + 4k^2\kappa^2 \cosh^2(\kappa L)} \dots\dots\dots(10) \end{aligned}$$

Now, using

$$\cosh^2(\kappa L) = 1 + \sinh^2(\kappa L) \dots\dots\dots(11)$$

We get,

$$\begin{aligned} T &= \frac{4k^2\kappa^2}{4k^2\kappa^2 + (k^2 + \kappa^2)^2 \sinh^2(\kappa L)} \\ &= \frac{1}{1 + \frac{1}{4} \left( \frac{k^2 + \kappa^2}{k\kappa} \right)^2 \sinh^2(\kappa L)} \dots\dots\dots(12) \end{aligned}$$

Again,

$$\left( \frac{k^2 + \kappa^2}{k\kappa} \right)^2 = \left( \frac{V}{\sqrt{E(V-E)}} \right)^2 = \frac{V}{E} \times \frac{V}{(V-E)} = \frac{1}{\varepsilon} \times \frac{1}{(1-\varepsilon)} \dots\dots\dots(13)$$

Therefore,

$$\begin{aligned} T &= \frac{1}{1 + \frac{1}{4\varepsilon(1-\varepsilon)} \left( \frac{e^{\kappa L} - e^{-\kappa L}}{2} \right)^2} \\ \Rightarrow T &= \left\{ 1 + \frac{(e^{\kappa L} - e^{-\kappa L})^2}{16\varepsilon(1-\varepsilon)} \right\}^{-1} \dots\dots\dots(14) \end{aligned}$$

This is the required transmission co-efficient.