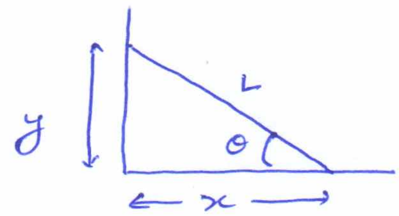
Solution to Tutorial 2

1.

(a)



$$x^2 + y^2 = L^2$$

$$\Rightarrow \quad 2x\dot{x} + 2y\dot{y} = 0$$

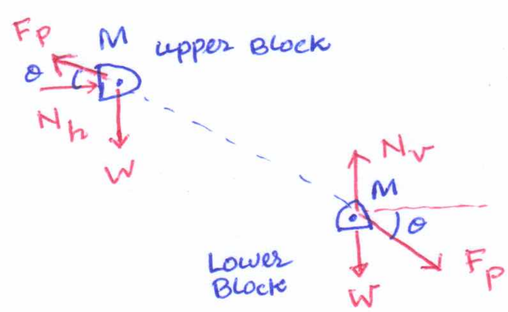
$$\Rightarrow \quad \dot{x}^2 + x\ddot{x} + \dot{y}^2 + y\ddot{y} = 0$$

When the pole is at rest $\dot{x} = 0 = \dot{y}$

At rest the condition becomes: $x\ddot{x} + y\ddot{y} = 0$

$$\Rightarrow \quad \boxed{\ddot{x} = -\frac{y}{x}\ddot{y} = -(\tan\theta)\ddot{y}} \rightarrow (i)$$

(b) Take the pole to be massless $\Rightarrow$ net force on the pole must be 0.



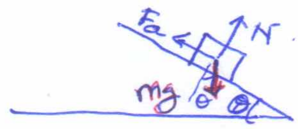
Upper Block : $M\ddot{y} = F_p \sin\theta - W \rightarrow (ii)$

Lower Block : $M\ddot{x} = F_p \cos\theta \rightarrow (iii)$

Solving ~~(i)~~, (ii) and (iii) and using (i) one can obtain:

$$\boxed{\ddot{x} = g\sin\theta\cos\theta \qquad \ddot{y} = -g\cos^2\theta}$$

2. (a) The block to remain motionless $\Rightarrow$ it has 0 acceleration



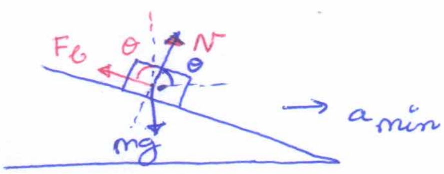
$$N - mg\cos\theta = 0$$

$$mg\sin\theta - F_a = 0$$

$$F_a = \mu N$$

$$\therefore \quad mg\sin\theta = \mu N = \mu mg\cos\theta \Rightarrow \boxed{\mu = \tan\theta}$$

(b) Minimum acceleration



Block's horizontal acceleration is

$$ma_{min} = N\cos\theta - F_b\sin\theta \rightarrow (i)$$

In the limit $F_b = \mu N \rightarrow (ii)$

The block has zero vertical acceleration:

$$N\sin\theta + F_b\cos\theta - mg = 0 \rightarrow (iii)$$

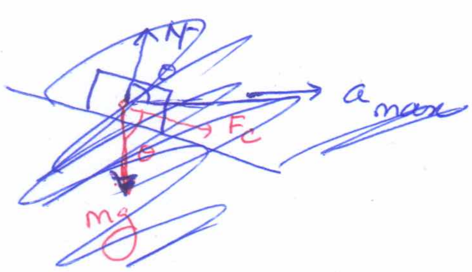
Using ~~combining~~ (i), (ii) and (iii), it is straightforward to show:

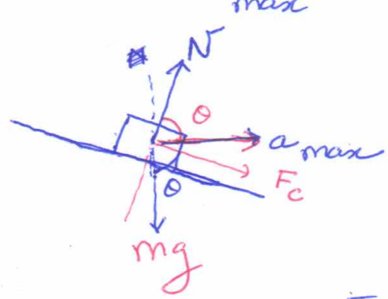
$$\boxed{a_{min} = \left(\frac{\cos\theta - \mu\sin\theta}{\sin\theta + \mu\cos\theta}\right) g}$$

Caution
This 'θ' is different from ~~inclined~~ wedge angle θ

This $\theta = 90^\circ - \theta_{wedge}$

(c) Maximum acc^n





$$m a_{max} = N\cos\theta + F_c\sin\theta \rightarrow (i)$$

no vertical acc^n

$\Rightarrow$

$$N\sin\theta - F_c\cos\theta - mg = 0 \rightarrow (ii)$$

In the limiting case

$$F_c = \mu N \rightarrow (iii)$$

Using (i), (ii) and (iii) one can get:

$$\boxed{a_{max} = \left(\frac{\cos\theta + \mu\sin\theta}{\sin\theta - \mu\cos\theta}\right) g}$$

3.

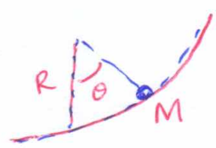
Here, it is assumed, $r = R =$ constant

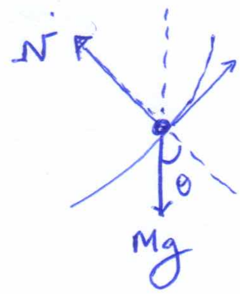
$\therefore \quad \dot{r} = 0$

$\Rightarrow \ddot{r} = 0$

$\dot{\theta} = v/R =$ constant

$\Rightarrow \ddot{\theta} = 0$





Radial Eq^n of motion: $\quad M\frac{v^2}{R} = N - Mg\cos\theta \quad \rightarrow (i)$

Tangential Eq^n of motion: $\quad F - Mg\sin\theta = 0 \quad \rightarrow (ii)$

The automobile begins to skid when, tangential force $\quad F - Mg\sin\theta \leq 0$

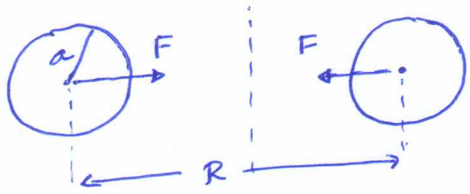
Now, the max^m value of F is μN. In the limiting case, $\quad \mu N = Mg\sin\theta$

From (i)

$$M\frac{v^2}{R} = Mg\left(\frac{\sin\theta}{\mu} - \cos\theta\right)$$

$$\Rightarrow \boxed{\frac{\sin\theta}{\mu} - \cos\theta = \frac{v^2}{Rg}}$$

4.



Each sphere orbits in a circle of radius $R/2$ and experiences a radial gravitational attraction F:

$$F = \frac{GMM}{R^2}$$

Radial eq^n of motion:

$$M\frac{R}{2}\omega^2 = \frac{GM^2}{R^2} \Rightarrow \omega = \sqrt{\frac{2GM}{R^3}}$$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{R^3}{2GM}}$$

Say, ρ is the density, then $M = \frac{4}{3}\pi a^3 \rho$

~~$a_{max} = R/2$ (spheres touching)~~ T can be made smaller by making M larger; we can make a large. However, $a_{max} = R/2$ (sphere touching)

$$\therefore T_{min} = 2\pi\sqrt{\frac{R^3}{2G\rho\, \frac{4}{3}\pi (R/2)^3}}$$

$$= 2\pi\sqrt{\frac{3}{G\rho\pi}}$$

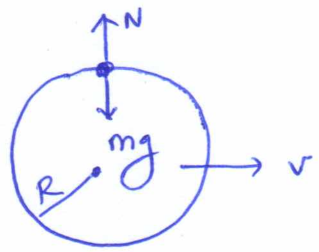
$$= \sqrt{\frac{12\pi}{G\rho}}$$

$$G = 6.67 \times 10^{-11}\ \mathrm{kg}^{-1}\,\mathrm{m}^3\,\mathrm{s}^{-2}$$

$$\rho = 21.5\ \mathrm{g\,cm^{-3}} = 21.5 \times 10^3\ \mathrm{kg\,m^{-3}}$$

One should get $T_{min} = 85.5$ mins

The speed of the pebble is $v_p = v_w$, speed of the ~~where~~ wheel as long as the pebble is in contanct with wheel.



$$v_p = R\omega = v_w$$

(a) From the force diagram above,

$$m\frac{v^2}{R} = mg - N, \qquad v = v_p = v_w$$

where, $N \geq 0$

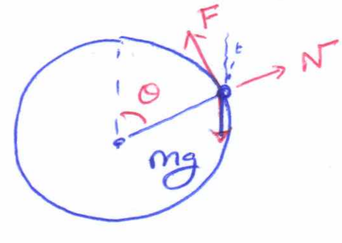
The pebble flies off when $N = 0$.

$$\therefore \quad \frac{mv^2}{R} > mg \Rightarrow \boxed{v > \sqrt{Rg}}$$

is the required condition for the pebble to fly off the wheel immediately.

(b) When in contact, the radial eq^n of motion for the pebble is

$$\frac{mv^2}{R} = mg\cos\theta - N \to (i)$$



Now, $N \geq 0$

$$\Rightarrow \cos\theta_{max} = \frac{v^2}{Rg}.$$

However, there is a more stringent criterion based on the frictional force F, as follows.

There is no tangential acceleration:

$$0 = mg\sin\theta - F \Rightarrow F = mg\sin\theta \to (ii)$$

$$F \leq \mu N \to (iii)$$

From (i), (ii) and (iii) one can obtain:

$$g \sin\theta \le \mu g \cos\theta - \mu v^2/R$$

For $\mu = 1$

$$g \sin\theta \le g \cos\theta - v^2/R$$

$$\Rightarrow \sin\theta \le \cos\theta - v^2/Rg$$

$$v^2/Rg = \cos\theta_{max} - \sin\theta_{max}$$

$$= \sqrt{2}\cos(\theta_{max} + \pi/4)$$

$$\Rightarrow \cos(\theta_{max} + \pi/4) = \frac{v^2}{\sqrt{2}\,Rg}$$

$$\Rightarrow \theta_{max} = \cos^{-1}\left(v^2/\sqrt{2}Rg\right) - \pi/4$$

6.

$$m\frac{dv}{dt} = -kv^2$$

$$\Rightarrow \int_{v_0}^{v} \frac{dv}{v^2} = -\frac{k}{m}\int_0^t dt$$

$$\Rightarrow \frac{1}{v} = \frac{1}{v_0} + \frac{kt}{m}$$

$$= \frac{1}{v_0}\left(1 + \frac{kv_0 t}{m}\right)$$

$$= \frac{1}{v_0}\left(1 + t/\tau\right),$$

$\tau = \dfrac{m}{kv_0}$, is a characteristic time

$$d = \int v\, dt$$

$$= \int_0^t \left(\frac{v_0}{1+t/\tau}\right) dt = v_0\tau \ln(1 + t/\tau)$$

$$\therefore \boxed{d = v_0\tau \ln(1 + t/\tau)}.$$

For short time, $t \ll \tau$, using $\ln(1+x) \approx x$ we obtain

$$d \approx v_0\tau(t/\tau) = v_0 t$$

7.

$$[h] = ML^2T^{-1}$$
$$[G] = M^{-1}L^3T^{-2}$$
$$[c] = LT^{-1}$$

(a) Planck length $L_p = h^\alpha G^\beta c^\gamma$

~~$[L_p] = [h^\alpha$~~

$$L = (ML^2T^{-1})^\alpha (M^{-1}L^3T^{-2})^\beta (LT^{-1})^\gamma$$
$$= M^{\alpha-\beta} L^{2\alpha+3\beta+\gamma} T^{-(\alpha+2\beta+\gamma)}$$

$\Rightarrow$
$$\alpha - \beta = 0$$
$$2\alpha + 3\beta + \gamma = 1$$
$$+\alpha + 2\beta + \gamma = 0$$

$\Downarrow$

$$\alpha = 1/2$$
$$\beta = 1/2$$
$$\gamma = -3/2$$

$$L_p = \sqrt{\frac{hG}{c^3}} \approx 4.1 \times 10^{-35}\ m$$

~~to~~ Similar way one can find

(b) $M_p = \sqrt{\frac{hc}{G}} \approx 5.4 \times 10^{-8}\ kg$

(c) $T_p = \sqrt{\frac{hG}{c^5}} \approx 1.3 \times 10^{-43}\ s$

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