EE 101
Electrical Sciences

Department of Electronics & Electrical Engineering

Lectures 12-14

Magnetic Circuits
Electromagnetism

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NOTATIONS AND CONVENTIONS

Understanding of electromagnetic fields is important to clearly understand the behavior of electric machines.

Quantity	**Symbol(s)**	**Unit(s)**	
Magneto-motive force (mmf)	F, (*NI*)	A	At
Magnetic Field Strength	H	A/m	At/m
Magnetic Flux	φ	Wb	
Magnetic Flux Density	B	T	Wb/m^2
Flux Linkage	λ	Wb	Wb t
Inductance	L, M	H	
Permeability	μ, μ_0	H/m	
Relative Permeability	μ_r		
Reluctance	$\Re$	H^{-1}	At/Wb

A: **Ampere** **t:** **turns** **m:** **meter**
Wb: **Webers** **T:** **Tesla** **H:** **Henry**

MAGNETIC INDUCTION

MAGNETIC FIELD INTENSITY (H)

A flow of electric current in a conductor creates a magnetic field around the conductor as shown in Figure 3.1.

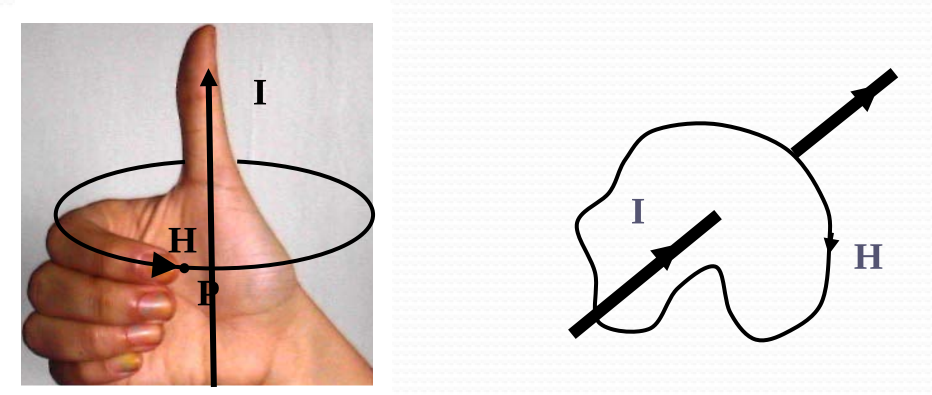
The direction of the magnetic field is defined by the Right Hand Rule illustrated in Figure 3.1 (a).

The relationship between the magnetic field intensity (H) along a closed path established by a current is specified by ***Ampere's Law***, which is expressed as:

$$\oint_C \vec{\mathrm{H}} \cdot \vec{d\ell} = I_{enc}$$

This principle can be easily adopted in many practical situations. For example, when H is constant,

$$\oint_C \vec{\mathrm{H}} \cdot \vec{d\ell} = H \oint_C d\ell = H\ell$$



(a) (b)

Figure 3.1 (a) Right Hand Rule (b) General closed loop for Ampere's Law

MAGNETIC FIELD PRODUCED BY A CONDUCTOR

Consider a long straight conductor carrying a current I as shown in Figure 3.2 (a). The field intensity (H) will be same throughout the circular path.

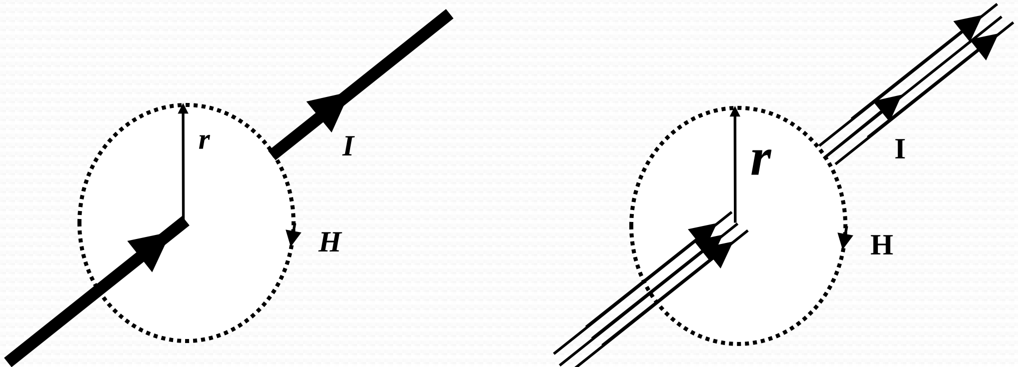
Therefore,

$$\oint_C \overrightarrow{\mathrm{H}}\, \overrightarrow{d\ell} = I_{enc}$$

$$\Rightarrow H \oint_C \overrightarrow{d\ell} = H\ell = H\,2\pi r$$

$$= I_{enc} = I$$

$$\Rightarrow H = I/\left(2\pi r\right) \quad \text{A/m or At/m}$$



(a) Single conductor (b) Multiple Copnductors

Figure 3.2 Magnetic field strength (H) around current carrying conductors

When there are N conductors as shown in Figure 3.2 (b), each carrying a current I, Then,

$$I_{enc} = NI \qquad \Rightarrow \qquad H = I/\left(2\pi r\right) \quad \text{A/m or At/m}$$

MAGNETIC FLUX (φ) AND FLUX DENSITY (B)

- In a medium of free space (or air) the field intensity produces flux density given by:

  $B = \mu_o H$ [Wb/m^2 or T (Tesla)]

  where,

  μ_o (= $4\pi\times10^{-7}$ H/m), is the permeability of free space

The flux can then be calculated as:

$$\varphi = \int_A B\,dA = BA,$$

when B is constant

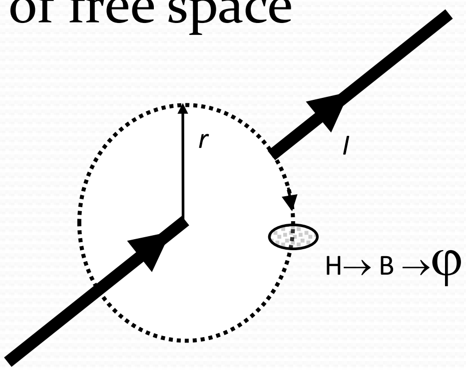


Figure 3.3 Magnetic field strength, flux, and flux density

It should be noted that both B and φ remain very small in the medium of air because of the very small value of μ_o.

MAGNETIC MATERIALS

- The magnetic flux or the flux density may be enhanced in a magnetic field by the use of magnetic materials. Consider a toroid of steel of radius ***r*** around the current carrying conductor as shown in Figure 3.4.

  Then, at a point inside the toroid,

  $$\Rightarrow H = I/(2\pi r)$$ as before.

  But,

  $B = \mu_o \mu_r H = \mu H$ [Wb/m² or T (Tesla)]

  where,

  μ_o is the permeability of free space

  μ_r is the relative permeability of steel, and

  $\mu = \mu_o \mu_r$, is the permeability of steel

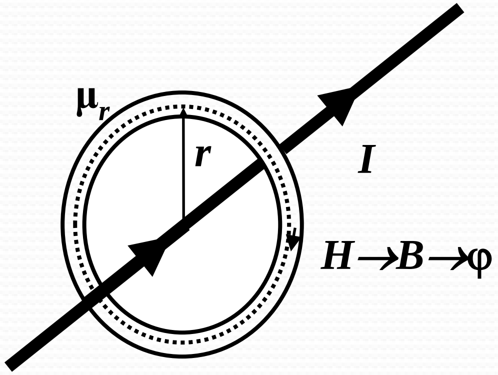
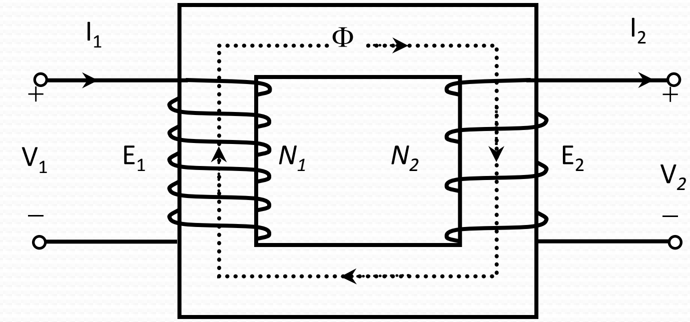


Figure 3.4 Use of magnetic material

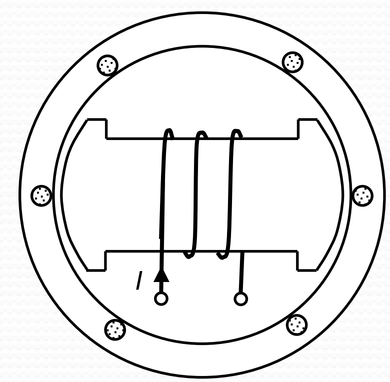
- μ_r is near unity (1) for non magnetic materials but it can be very high (2000–6000) for Ferro-magnetic materials. Thus, the use of magnetic materials can enhance the flux density (B) and therefore the flux (φ) by several magnitude of order for the same value of field intensity H.

MAGNETIC CIRCUITS IN ELECTRIC MACHINES

- In order to obtain reasonably high values of flux density and the flux, most electromagnetic machines commonly utilize:



- A structure of magnetic materials to utilize the benefit of high permeability, and coils consisting of a large number of turns to increase the total enclosed current.
- Typical electromagnetic structures of a rotating machine and a transformer are as shown.



- In such structures, the flux density inside the core structure will be several orders of magnitude higher than in the surrounding space.
- Therefore the flux outside the magnetic core can conveniently be ignored when analyzing electromagnetic machines and magnetic structures.

TYPES OF MAGNETIC MATERIALS : FERRO-MAGNETISM

- **Non-magnetic materials**

  These are the material whose relative permeability is near unity.

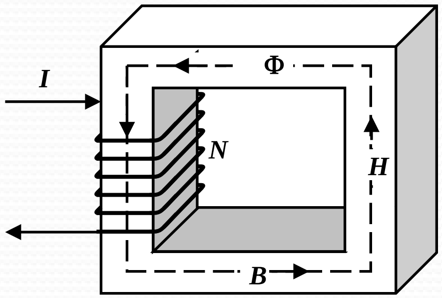
- **Ferro-magnetic materials**

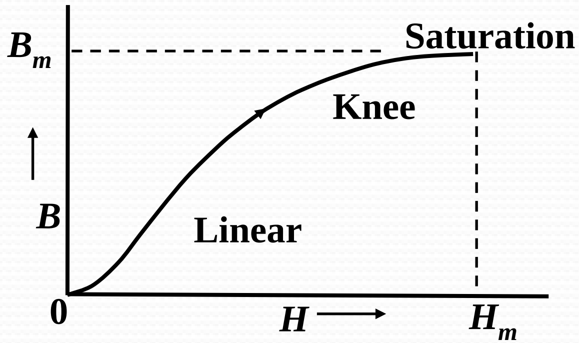
  These have very high relative permeability and experience very strong attractive force in magnetic fields.

- Consider the magnetic circuit as shown in Figure 3.9 (a).

  The current I in the coil produces field strength H, which in turn produces flux density B and the flux φ.

- The general nature of relationship between B and H for for common magnetic materials are as shown.

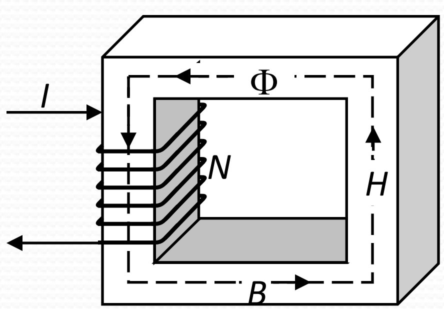




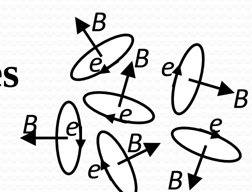
Typical magnetization curve

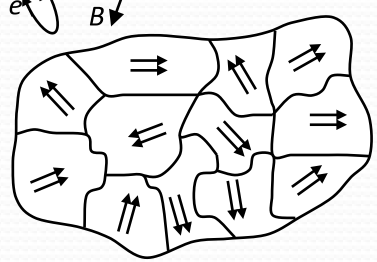
FERRO-MAGNETISM : MAGNETIZATION CURVE

- The relationship between B and H can be explained using the notions of magnetic dipoles and magnetic domains. In the absence of any current in the coil, i.e., H, the dipoles are all randomly oriented and the magnetic domains cancel each other out, resulting in no net flux and flux density.
- With the injection of current, i.e., application of H, the magnetic domains get aligned resulting in higher flux and flux density. Once all the domains are aligned, increase in I, i.e., H cannot produce further increase in B, and saturation occurs.

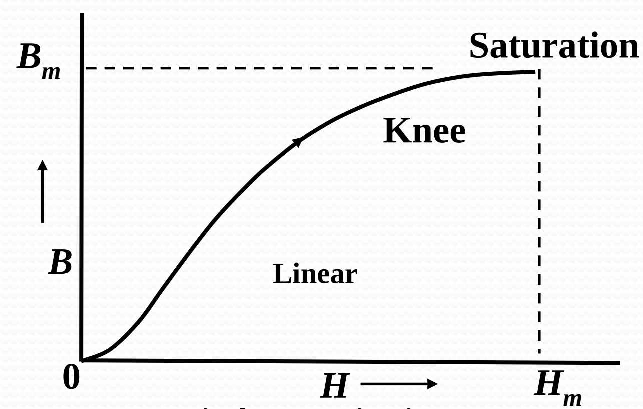


Dipoles





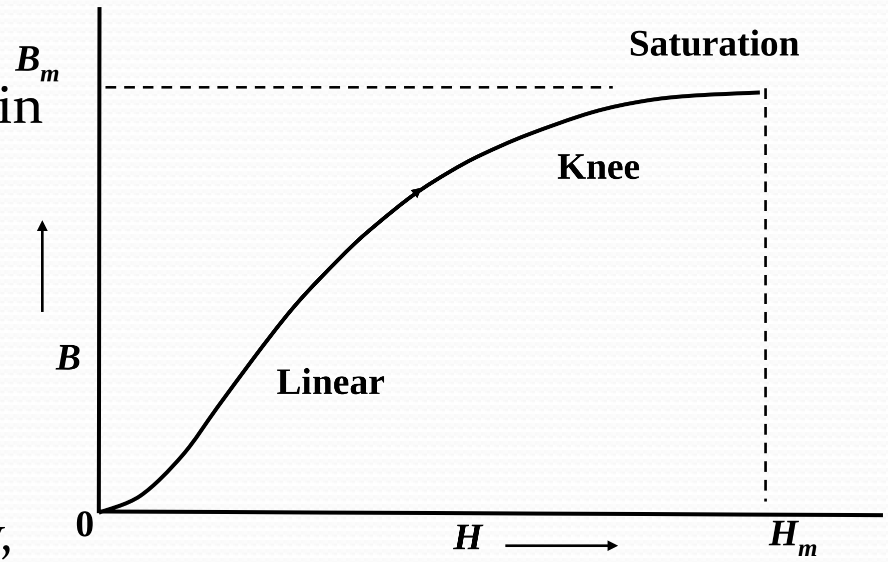
Magnetic Domains



Typical magnetization curve

MAGNETIZATION CHARACTERISTICS (B-H CURVE)

- Variation of flux density B in Ferro-magnetic material for increasing values of H as shown in the figure is called the magnetization characteristics or the B-H curve.
- Three important stages of the characteristics are indicated in the diagram.
  - ***Linear region***, where B increases almost linearly with H
  - ***Knee region***, where increase in B slows down significantly, due to reduction of domains which can be aligned.
  - ***Saturation region***, when B stops increases due to lack of domains which can be aligned and the curve practically flattens.



Typical magnetization curve

TYPICAL MAGNETIZATION (B-H) CURVES

- The B-H curves for three common magnetic materials are shown in Figure 3.11.
- Note that different materials saturate at significantly different levels of flux density.
- The initial sections of the curves (for lower values of B) are almost linear, where $B=\mu H$ with constant μ is valid for these linear regions of the curves. only.
- The permeability μ changes rapidly after the knew point (in the saturation region.

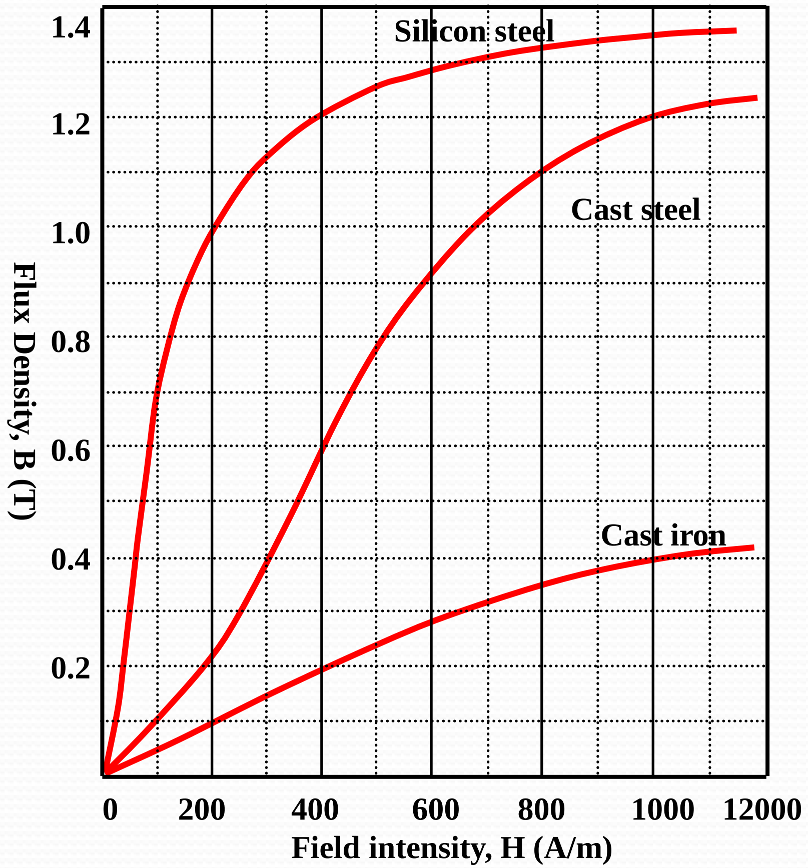


Figure 3.11 Typical magnetization curves of three materials

- Unless otherwise stated, it will be assumed in basic analyses of magnetic systems that they operate in the linear region.

PERMEABILITY

- Since, $B = \mu H$, the permeability of the material at a given level of flux density can be obtained as the ratio of B/H at each point of the magnetization curve, as illustrated in Figure 3.12.
- For example, at a point where, $B = B_1$, and $H = H_1$

  $$\mu = B_1/H_1 \text{ (H/m)}$$

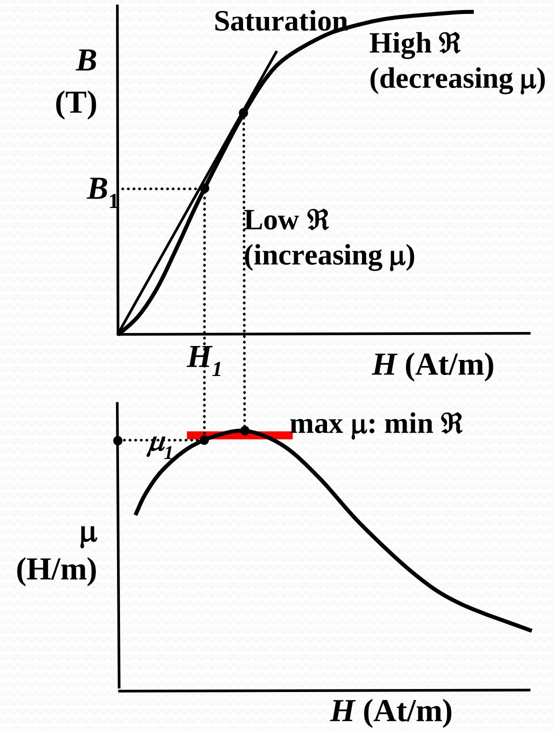


Figure 3.12 Variation of permeability

- It can be seen that:
  - The value of μ increases with H to a max value and then decreases steadily after saturation sets in.
  - The value of μ remains approximately constant as indicated in red (within narrow limits) in the operating range of the flux density B.

MAGNETIC CIRCUITS

- The flow of magnetic flux (φ) in a magnetic circuit created by the current flowing in a coil may be analyzed as the flow of current in electric circuit. Consider the magnetic circuit shown in Figure 3.13 (a)

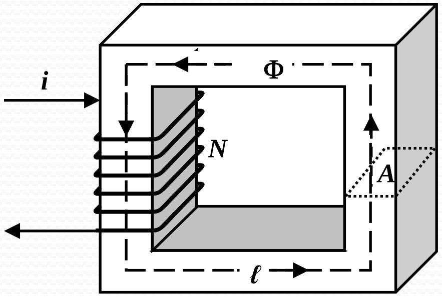
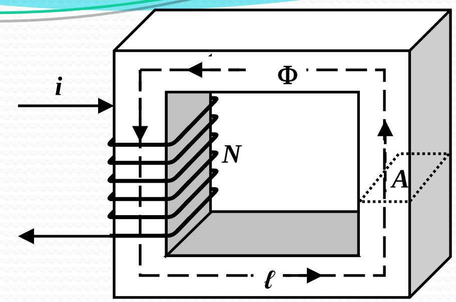


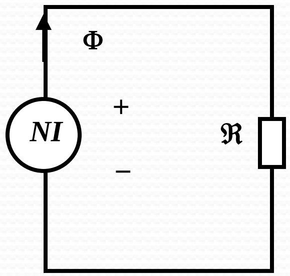
Figure 3.13 (a) An elementary magnetic circuit,

- The following common assumptions will be adopted in the basic analysis of this circuit:
  - The flux is restricted to the magnetic material (which means there are no leakage and no fringing of flux).
  - The magnetic flux density (B) is uniform within the magnetic material, which is taken as the flux density along the mean path. (B=φ /A)

ANALYSIS OF MAGNETIC CIRCUITS



(a) An elementary magnetic circuit,



(b) the magnetic equivalent circuit

- Applying Ampere's law along the mean path,
- Since, $\oint_C \vec{H} \cdot \vec{d\ell} = I_{enc}$, and

$$\oint_C \vec{H} \cdot \vec{d\ell} = Hl, \quad \& \quad I_{enc} = NI \qquad \Rightarrow \qquad Hl = NI$$

We get , $B = \mu H = \dfrac{\mu NI}{\ell}$

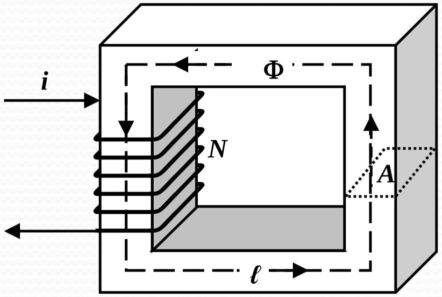
So that flux $\varphi = BA = \dfrac{\mu ANI}{\ell} = \dfrac{NI}{(\ell / \mu A)} = \dfrac{F}{\Re}$

- $F = NI$ is called the ***magnetomotive force (mmf)***, analogous to emf. Note that this quantity depends purely on the electrical properties of the winding.
- $\Re = \ell / \mu A$ is called the ***Reluctance***, analogous to Resistance.

  Note that this is purely a property of the magnetic core material and the structure.
- $\varphi = F / \Re$ or $F = \varphi\Re$ is called the ***Ohm's law*** for the magnetic circuit, which may be represented and analyzed by drawing a magnetic equivalent circuit shown in Figure (b).

DEVELOPING MAGNETIC EQUIVALENT CIRCUITS

Magnetic equivalent circuits for more elaborate magnetic circuits may be developed by adopting the following procedure.



(a) An elementary magnetic circuit,

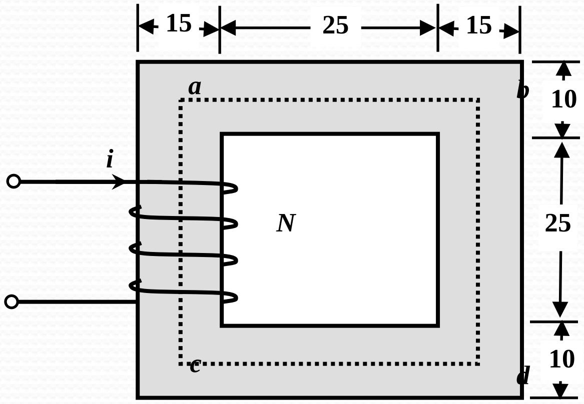
- Coils represent sources, and the right hand rule should be used to specify the polarity of this source (i.e., the direction of flux).
- Trace the mean path followed by the flux. Reluctance of various sections with different flux in them must be evaluated separately, as they will be required in the analysis of the circuit.
- When the magnetic circuit consists of two or more closed loops, then Ohm's law can be applied to each loop separately.

  $NI = \sum_i H_i \ell_i = \sum_i \varphi_i \Re_i$, for each closed loop *i*.

- Magnetic circuits can be analyzed using magnetic equivalent circuits as long as the circuit remains linear i.e., the permeability μ remains constant.

EXAMPLE

- Draw the magnetic equivalent circuit for the magnetic circuit shown in Figure where the dimensions are in cm. The depth of the core is 10 cm and μ_r of the material is known to be 1400. If the coil has 310 turns, estimate the current required in the coil to obtain a flux density of 0.8 T in the coil.



Figure

Solution - The cross-sectional areas of different sections are:

$A_{ab} = A_{cd} = 10\times10 = 100$ cm², $A_{ac} = A_{bd} = 10\times15 = 150$ cm².

The lengths of various sections are:

$$\ell_{ab} = \ell_{cd} = 25 + 2\times15/2 = 40 \text{ cm}, \qquad \ell_{ac} = \ell_{bd} = 25 + 2\times10/2 = 35 \text{ cm}$$

The relectance of various sections are:

$$\Re_{ab} = \Re_{cd} = \frac{0.4}{1400\mu_0 \times 100 \times 10^{-4}} = 22736 \text{ T}^{-1},$$

$$\Re_{ac} = \Re_{bd} = \frac{0.35}{1400\mu_0 \times 150 \times 10^{-4}} = 13263 \text{ T}^{-1}$$

EXAMPLE

Solution –

The equivalent circuit is drawn as shown in Figure (b):

Total equivalent reluctance

$$\Re_e = 2(\Re_{ab} + \Re_{cd}) = 72000\ H^{-1}$$

For a flux density of 0.8 T inside the coil,

$$\varphi_{ac} = B_{ac} \times A_{ac}$$
$$= 0.8 \times 150 \times 10^{-4} = 0.012\ Wb$$

Then,

$$Ni = \varphi_{ac} \times \Re_e = 0.012 \times 72000 = 864\ At$$

Therefore $i = 864/310 = 2.788$ A

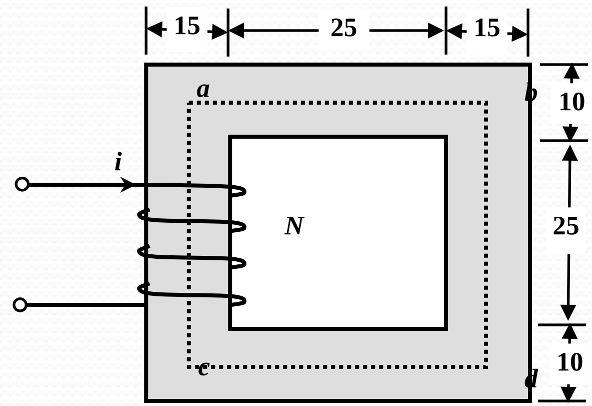


Figure a

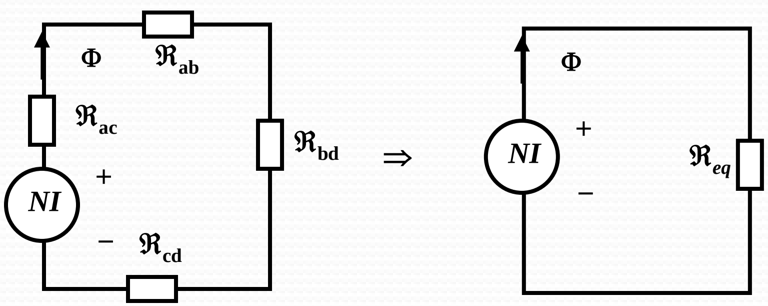


Figure (b). Magnetic equivalent circuit for Figure (a)

MAGNETIC CIRCUITS WITH AIR GAPS

- Air gaps are integral part of magnetic circuits in various electric machines, e.g., Figure 3.8 (a).

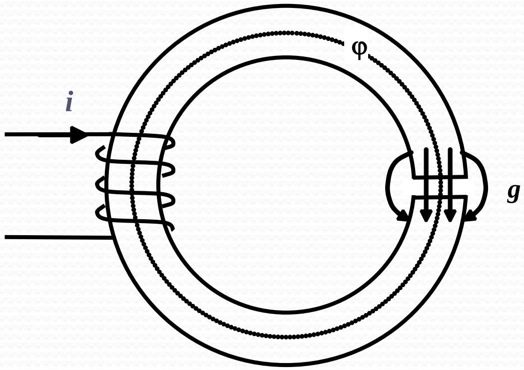


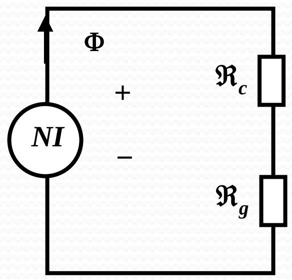
Figure 3.17 (a) Magnetic circuit with air gap

- Fringing of flux occurs in air gaps and the fringing flux often constitutes leakage flux. Although it is not easy to take account of leakage flux precisely, there are various approximate means of including their effects in the analysis. Leakage flux will be ignored for now.
- The inclusion of the effects of air gaps in magnetic circuits will be illustrated with the following example.

MAGNETIC CIRCUITS WITH AIR GAPS - EXAMPLE

An electromagnet of square cross section, shown in Figure 3.17 (a), has a coil of 1500 turns. The inner and outer radii of the core are 10 cm and 12 cm respectively and the air gap is 1 cm. If the current in the coil is 4 A and the relative permeability of the core material is 1200, determine the flux density in the circuit

Solution –



(b) Magnetic equivalent circuit

- The equivalent circuit drawn as shown in Figure (b) using the approach described earlier.

  Cross section area $A_c = A_g$ = 2 cm×2 cm = 4×10^{-4} m²

  Mean radius r (10+12)/2= 11 cm, Length of core = 2πr – 1= 68.12 cm

$$\Re_c = \frac{0.6812}{1200\times4\pi10^{-7}\times4\times10^{-4}} = 1.129\times10^{6}\ \text{T}^{-1},\quad \Re_g = \frac{0.01}{4\pi10^{-7}\times4\times10^{-4}} = 19.894\times10^{6}\ \text{T}^{-1}$$

$\Re_{eq} = \Re_c + \Re_g = 21.023\times10^{6}\ \text{H}^{-1}$

$\varphi = (1500\times4)/21.023\times10^{6} = 2.85\times10^{-4}$ Wb

$B_c = B_g = (2.85\times10^{-4})/(4\times10^{-4}) = 0.713$ T

FARADAY'S LAW (INDUCED EMF)

- Consider a simple magnetic circuit as shown Figure 3.20. Input current *i* in the coil of *N* turns establish a flux φ in the core.

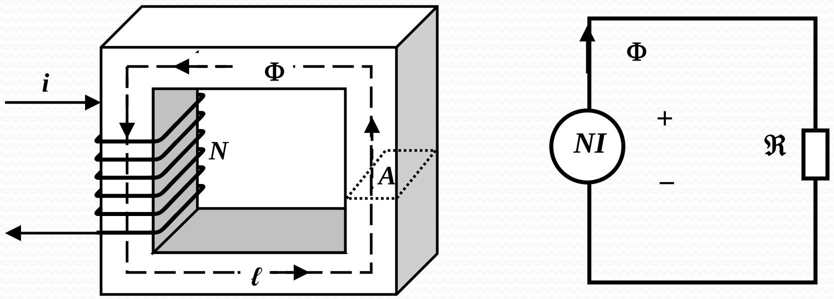


Figure 3.20 (a) An elementary magnetic circuit, (b) the magnetic equivalent circuit

- ***Flux Linkage*** (λ): When a flux φ passes through a coil of *N* turn, the flux is said to link the coil and,

  $\lambda = N\varphi$ is called the flux linkage of the coil.

- ***Faraday's law*** which is embedded in the Maxwell's Equations states that a coil with flux linkage λ will have an induced voltage ***e*** in it given by:

$$e = -\frac{d\lambda}{dt} = -N\frac{d\varphi}{dt}$$

- The –ve sign indicates that the direction of the induced voltage will be such that it will tend to oppose the current/voltage (***v***) creating the flux.

SINUSOIDAL EXCITATION OF MAGNETIC CIRCUITS

- Consider the magnetic circuit excited by a sinusoidal source shown in Figure 3.21. With a sinusoidal input current *i*, (which is due a sinusoidal voltage *v*) the mmf (***Ni***) and therefore the flux (=***Ni***/$\Re$) produced in the core are also sinusoidal.

Let $\varphi = \Phi_m \sin \omega t$

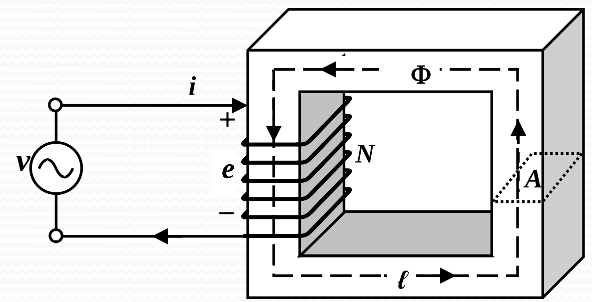


Figure 3.21 Sinusoidal excitation of magnetic circuit

- Then, $$v = e = N\frac{d\varphi}{dt} = N\Phi_m \omega \cos \omega t$$
$$= N(B_m A)(2\pi f)\cos \omega t$$
$$= 2\pi NA(B_m f)\cos \omega t$$

- Therefore, the rms value of the voltage is given by:

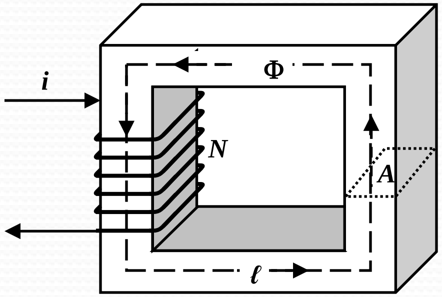
$$V = V_m/\sqrt{2} = 2\pi NA(B_m f)/\sqrt{2} = 4.44 NA(B_m f)$$

- Thus, it is seen that the product $B_m f$ is related to the input voltage and these two variables need to be treated jointly in the analysis of many magnetic circuits excited by sinusoidal sources. This observation will find useful applications in the analyses of magnetic circuits later.

INDUCTANCE (SELF INDUCTANCE)

- Consider the magnetic circuit of Figure 3.20. A current ***i*** through the coil will produce a flux φ and therefore a flux linkage **λ= *N*φ**.
- The inductance of the coil is defined as:

$$L = \frac{d\lambda}{di} \qquad (= \frac{\lambda}{i} \text{ for linear case})$$



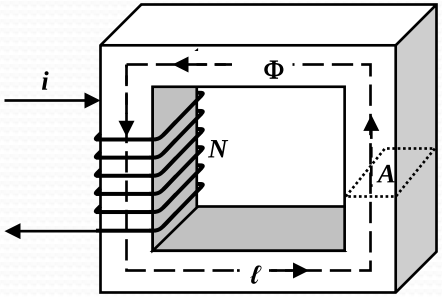
Since, $\lambda = N\varphi = N\dfrac{Ni}{\Re} = \dfrac{N^2}{\Re}i \quad \Rightarrow \quad L = \dfrac{d\lambda}{di} = \dfrac{N^2}{\Re}\ \mathrm{H}$

- Thus, the inductance ***L*** is thus determined by the coil properties and the physical properties (dimensions) of the magnetic material (ℜ=).
- For linear magnetic circuits, reluctance ℜ is constant, and therefore the inductance ***L*** is constant for a given coil.
- If the magnetic circuit consists entirely of Ferro-magnetic material, the B-H curve is hardly linear. Saturation often occurs, μ and ℜ does not remain constant so that ***L*** does not remain constant. If constant inductance ***L*** is desired, air gaps are often introduced in the magnetic circuit.

INDUCTANCE (SELF INDUCTANCE)

- For a magnetic circuit excited by *ac* source as shown in Figure 3.21, the induced emf ***e*** can be expressed as:

$$e = -\frac{d\lambda}{dt} = -\frac{d\lambda}{di}\frac{di}{dt} = -L\frac{di}{dt}$$



- This is the common form of Faraday's Law which is used in electric circuit analysis.

  Often the negative sign is dropped by considering the direction of ***e*** separately.

- If the resistance of the coil is negligibly small, then the input voltage ***v*** is completely balanced by the induced emf ***e***, so that

$$v = L\frac{di}{dt}$$

- If the resistance of the coil is incorporated in the analysis, then the input voltage is balanced by the drop in the resistance and the induced voltage in the inductor, so that: $v = ri + L\dfrac{di}{dt}$

EXAMPLE

The circular magnetic core shown in Figure 3.22 has a relative permeability of 2200. The dimensions of the core are: r_1=25 cm, r_2=20 cm, and the cross section, *A* is circular. The coil has 102 turns. Calculate the inductance of the coil.

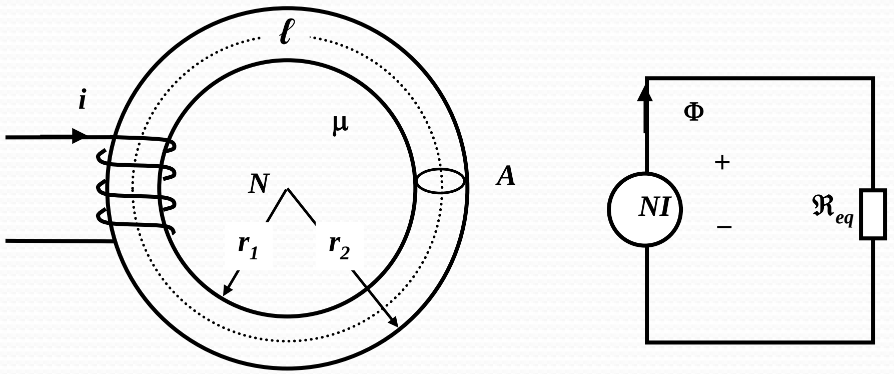


Figure 3.22 (a) Magnetic circuit (b) Magnetic Equivalent circuit

Solution

Mean radius *r*

= [(25+20)/2]×10⁻² = 0.225 m

Diameter of core section

= r_1-r_2 = 5 cm = 0.05 m

⇒ Cross section area *A*

= πr^2 = $\pi\, 0.025^2$ m²

Reluctance of core: $\Re = \dfrac{\ell}{\mu A} = \dfrac{2\pi \times 0.225}{2200 \times 4\pi \times 10^{-7} \times \pi \times 0.025^2} = 260435\ \mathrm{H^{-1}}$

The magnetic equivalent circuit is as shown in Figure 3.22(b).

Inductance, $L = \dfrac{N^2}{\Re} = \dfrac{102^2}{260435} = 0.04\ \mathrm{H}$

MUTUAL INDUCTANCE

- Consider the magnetic circuit with two coils of self inductances L_1 and L_2 shown in Figure 3.24(a). Let a current i_1 in Coil 1 produces flux Φ_1 and Coil 2 is left open. A portion Φ_{21} of Φ_1 links both coil 1 as well as coil 2, while the portion Φ_{11} links only coil 1.

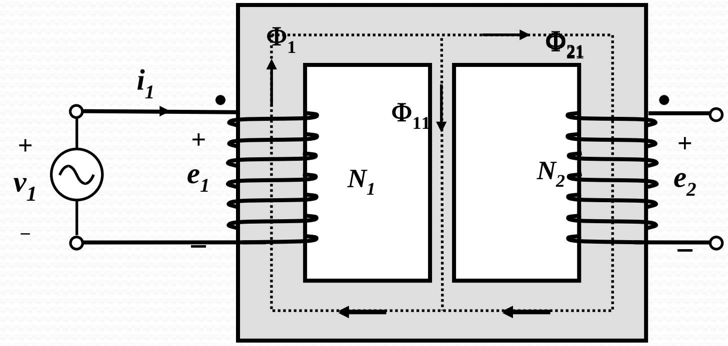


Figure 3.24(a) Magnetic circuit with two coils

- Then, $$e_1 = L_1 \frac{di_1}{dt}$$

$$e_{21} = N_2 \frac{d\Phi_{21}}{dt} = N_2 \frac{d\Phi_{21}}{di_1}\frac{di_1}{dt} = M_{21}\frac{di_1}{dt}$$

where, $$M_{21} = N_2 \frac{d\Phi_{21}}{di_1} = \frac{d\lambda_{21}}{di_1}$$

is called the mutual inductance of Coil 2 with respect to Coil 1.

- Φ_{12} may be just the leakage flux in some circuits instead of a distinct and significant flux as shown in Figure 3.24 (a).

- Similarly, $$M_{12} = N_1 \frac{d\Phi_{12}}{di_2} = \frac{d\lambda_{12}}{di_2}$$

is called the mutual inductance of Coil 1 with respect to Coil 2.

MUTUAL INDUCTANCE - 2

- These two equations can be combined to get,

$$M_{12}M_{21} = N_1 \frac{d\Phi_{12}}{di_2} N_2 \frac{d\Phi_{21}}{di_1}$$

- If a fraction k_{21} of Φ_1 links Coil 2, and a fraction k_{12} of Φ_2 links Coil 1,

  i.e., $\Phi_{21} = k_{21}\Phi_1$, and, $\Phi_{12} = k_{12}\Phi_2$,

  We can write: $M_{12}M_{21} = N_1(k_{21}\frac{d\Phi_2}{di_2})N_2(k_{12}\frac{d\Phi_1}{di_1}) = k_{12}k_{21}(N_1\frac{d\Phi_1}{di_1})(N_2\frac{d\Phi_2}{di_2}) = k_{12}k_{21}L_1L_2$

- For linear systems, it can be shown that M_{12}=M_{21}.

  Let (i) M_{12}=M_{21}=M, (say), and (ii) $k_{12}\times k_{21}$=k^2,

  Then, the above equation reduces to,

$$M^2 = k^2 L_1 L_2 \text{ or } M = k\sqrt{L_1 L_2}$$

  where, ***k*** is called the coefficient of coupling (or the coupling factor).

- Then, the voltage induced in the second coil when one coil is excited by a current can be simply written as:

$$e_{12} = M_{12}\frac{di_2}{dt} = M\frac{di_2}{dt}, \qquad e_{21} = M_{21}\frac{di_1}{dt} = M\frac{di_1}{dt}$$

ENERGY IN MAGNETIC CIRCUITS

- Consider a lossless magnetic circuit shown in Figure 3.25.
- If the current input is *i* at a voltage of *v*, electrical energy input in time interval *dt* is:

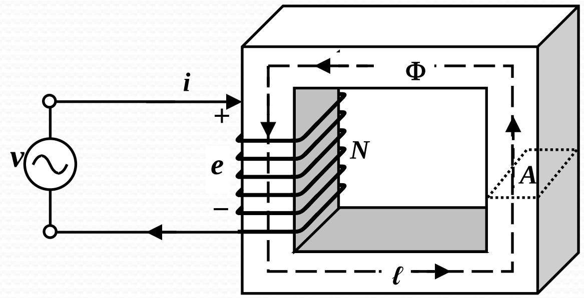


Figure 3.25 Lossless magnetic circuit

$$dW_i = vi\ dt = -ei\ dt$$

- But, $e = -\dfrac{d\lambda}{dt}$,

therefore, $dW_i = -ei\ dt = i\ d\lambda$

Also, since $\lambda = N\varphi$, $d\ W_i = i\ d\lambda = Ni\ d\varphi$

- Therefore the energy input to a magnetic circuit to establish flux φ in N turn coil can be written as:

$$W_i = \int i\ d\lambda = \int Ni\ d\varphi$$

- This expression can be used to calculate the total electrical energy input to a magnetic system.

MAGNETIC FIELD ENERGY -MAGNETIC STORED ENERGY

- In a lossless system, since there is no output, the input energy will be stored as magnetic field energy (W_f or W_m). Therefore,

$$W_f = \int i \, d\lambda = \int Ni \, d\varphi$$

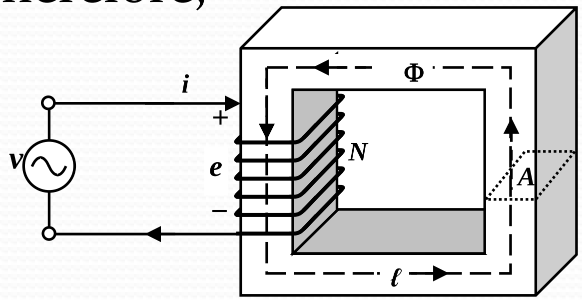


Figure 3.25 Lossless magnetic circuit

- Noting that $Ni = \varphi\Re$ for linear circuits, the field energy may be evaluated as:

$$W_f = \int Ni \, d\varphi = \int \varphi\Re \, d\varphi = \tfrac{1}{2}\varphi^2\Re \quad (= \tfrac{1}{2} Ni \; \varphi)$$

- Further, if we note that , $L = \dfrac{\lambda}{i} = \dfrac{N\varphi}{i} \quad \Rightarrow \quad N\varphi = iL$

  the stored energy can also be written as,

$$W_f = \tfrac{1}{2} N\varphi \, i = \tfrac{1}{2} \; i^2 L$$

- It should be noted that these expressions for stored magnetic field energy are for linear systems only, since we assume constant reluctance $\Re$.
- For non-linear magnetic circuits proper integration must be carried out to evaluate the stored energy.

ENERGY DENSITY

- The stored magnetic field energy (W_f or W_m) has been expressed as:

$$W_f = \int i \, d\lambda = \int Ni \, d\varphi$$

- Noting that Ni=Hl and φ=BA, so that dφ= AdB, the field energy can also be written as:

$$W_f = A\ell \int H \, dB$$

- Note that (Al) is the volume of the magnetic material and (W_f/Al) is the energy per unit volume. Therefore,

$$\frac{W_f}{A\ell} = \int H \, dB$$ is called the energy density, usually denoted as "***w***".

- Noting further that $B = \mu H \Rightarrow H = B/\mu$, for linear magnetic circuits, the energy density can be evaluated as:

$$w_i = \int H \, dB = \int (B/\mu) dB = \frac{B^2}{2\mu} \quad \text{J/m}^3.$$

for linear magnetic circuits, where μ remains constant.

EXAMPLE

The circular magnetic core shown in Figure 3.22 has a relative permeability of 2200. The dimensions of the core are: r_1=25 cm, r_2=20 cm, and the cross section, A is circular. The coil has 102 turns and a resistance of 4 Ω.

Calculate the magnetic field energy stored when connected to 10 V dc source using different approaches

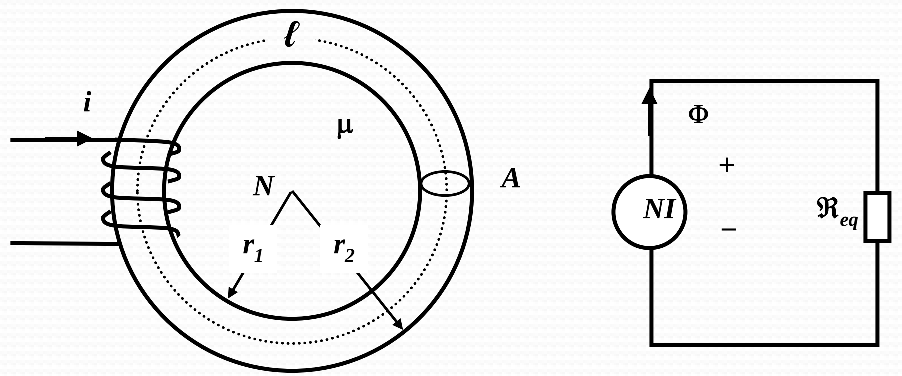


Figure 3.22 (a) Magnetic circuit (b) Magnetic Equivalent circuit

Solution

From earlier example,

$\Re$ = 260435 H^{-1}, and L = 0.4 H.

With 10 V dc source,

i =2.5 A, $\varphi = Ni/\Re = 9.79\times10^{-4}$ Wb,

$B = \varphi/A = 0.50$ T, $\lambda = N\varphi = 0.10$ Wb t

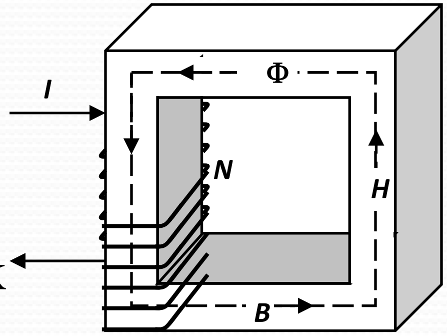
Therefore, $$W_f = 1/2\varphi^2\Re = 1/2\times(9.79\times10^{-4})^2\times200435 = 0.125\text{ J}$$

or, $$W_f = 1/2 i^2 L = 1/2\times(2.5)^2\times0.4 = 0.125\text{ J}$$

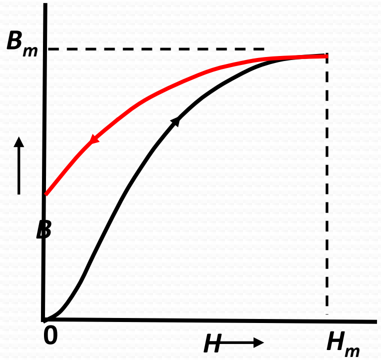
etc.

MAGNETIZATION CURVE: AN EXTENSION

- Consider a simple magnetic circuit shown in Figure (a) excited by a variable input current.
- When the field strength in a magnetic circuit is increased by increasing the current through the coil, the flux density increases as shown by the dark line in Figure (b) as discussed earlier.
- If the field strength is now reduced by decreasing the current through the coil, the decrease in flux density does not retrace the path taken during the increasing filed strength but it traces a different path during decreasing field strength as shown by the red line in Figure (b). In fact some flux density is maintained even when the current and therefore the field strength is reduced to zero. This is called “Remnant” or “Retention”.



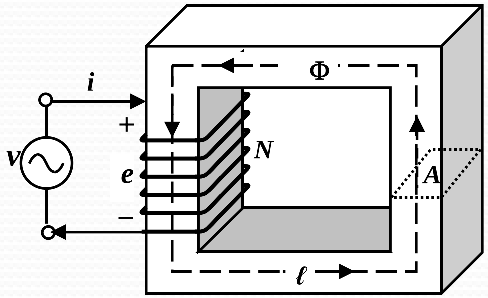
(a) Magnetic circuit



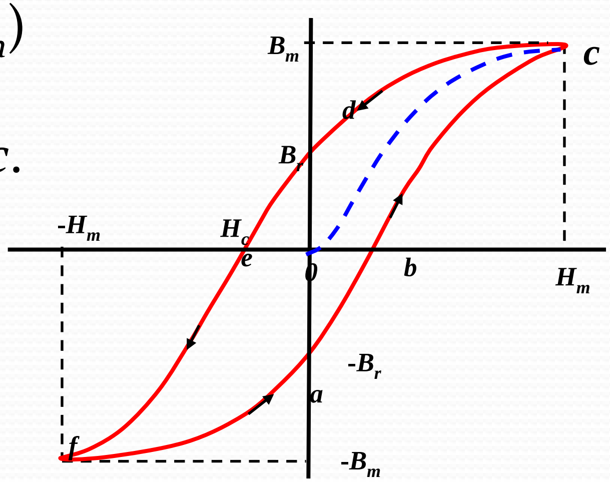
(b) Magnetization for increasing and decreasing current

MAGNETIZATION CURVE: AN EXTENSION

- If a magnetic circuit is excited by AC source as shown, the input current and therefore the field strength follows a cycle, (i) increasing slowly to a maximum value (H_m) at *c*, (ii) decreasing slowly to zero at *d*, (iii) reversing the direction (*i.e.* getting negative) and increasing to a maximum value in the reverse direction ($-H_m$) at *f*, and (iv) finally reversing the direction to trace back to maximum positive value (H_m) at *c*.



- A current flow in the opposite direction, i.e., a negative field-strength is required to reduce the flux density to zero. Such field strength is called the Coercive Force.



- The path followed by the magnetization curve, and the retention and the coercive force are characteristics of specific magnetic materials and can vary significantly from material to material

HYSTERESIS LOOP

- The path traced by the B-H curve for such a cycle of variation in the field strength is shown in the Figure and is called the Hysteresis Loop. The shape of this loop determines a part of magnetic losses.
- The actual shape of the hysteresis loop depends on the magnetic material.

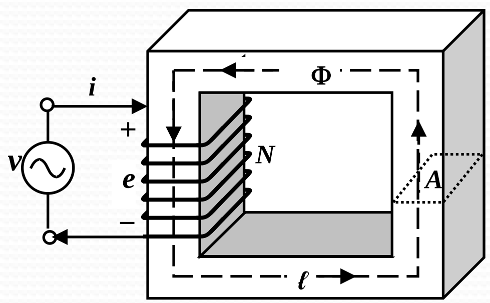
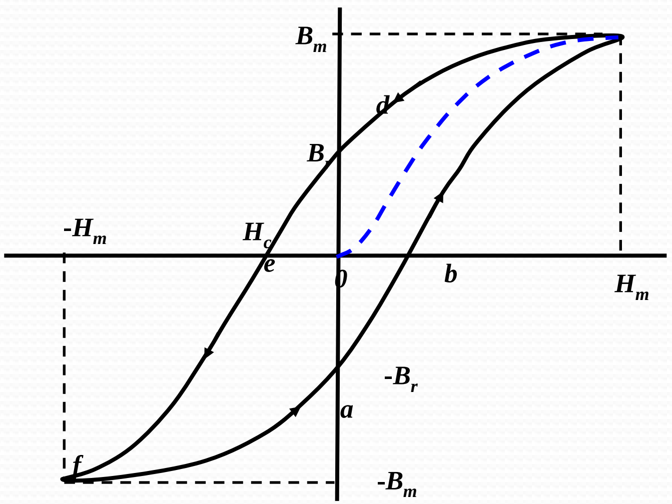


Figure 3.27 (a) Magnetic circuit



(b) Hysteresis loop

MAGNETIC LOSSES - HYSTERESIS LOSS

- Magnetic losses, also called “core losses” or “iron losses”, consist of “Hysteresis loss” and “Eddy current loss”
- For magnetic circuits excited by AC sources, the flux density in the core traverses a complete Hysteresis loop for each cycle of the input current as indicated in Figure 3.28. This process incurs a core loss known as “Hysteresis loss”.
- Hysteresis loss per unit volume of the core can be shown to be equal to the area of the Hysteresis loop by tracing one full cycle of the loop and tallying the energy input during the process.
- Energy input when the flux density changes from $-B_r$ to B_m is:

$$\int_{-B_r}^{B_m} H dB = \text{area } abcdea$$

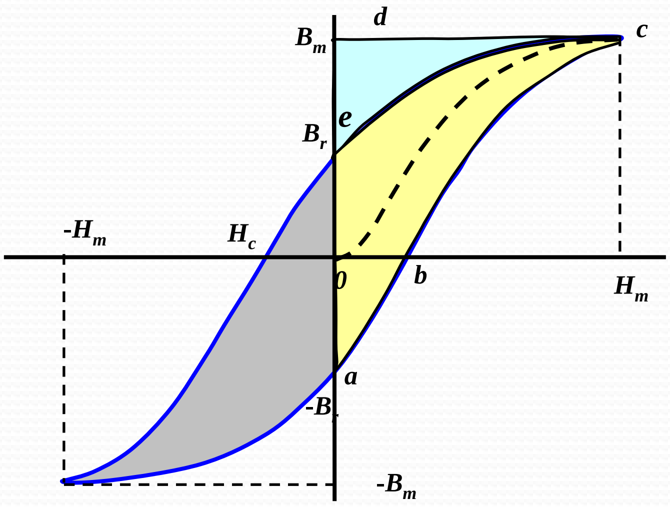


Figure 3.28 Hysteresis loop and Hysteresis loss

HYSTERESIS LOSS

- Energy released when the flux density changes from B_m to B_r is:

  $\int_{B_m}^{B_r} HdB \quad = \text{area } cde$

- Net energy input in each cycle, *i.e.*, energy loss per cycle is,

  $w \quad = 2 \times \text{area } abcea$

  $\quad = \text{area of the hysteresis loop}$

- The shape/area of the hysteresis loop and therefore the hysteresis loss depends on the core material.

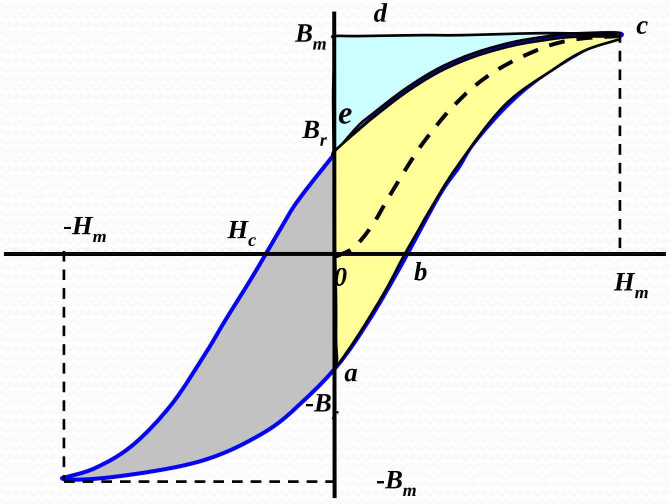


Figure 3.28 Hysteresis loop and Hysteresis loss

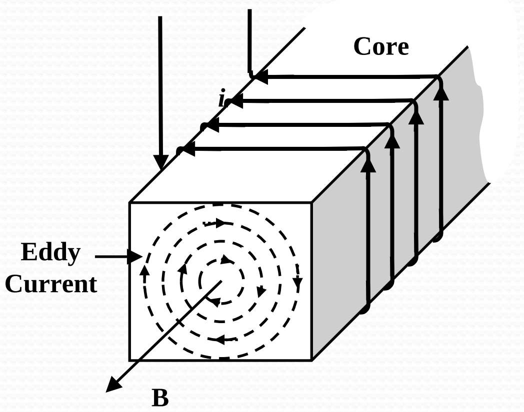
- Hysteresis loss is commonly estimated using the empirical formulae given by Steinmetz formulae :

  $P_h = K_h B_m^{\ n} f$ where,

  B_m is the maximum flux density in the core,
  f is the frequency of the source,
  K_h is a constant, and
  n is the Steinmetz index (1.5~2.5) for common core materials)

EDDY CURRENT LOSS

- When a time varying flux is created in a magnetic core, it will produce induced voltage in any perceivable closed path as shown in Figure (a). The direction of these “Eddy” currents will be such that they tend to oppose the original flux responsible for such currents.
- The eddy currents flowing in these closed paths incur power losses, which are called “***Eddy current losses***”. The magnitude of the “Eddy current” and therefore the “Eddy loss” will depend on (i) the magnitude of the induced voltage in each closed path, and (ii) the resistance of such closed paths. Therefore, it is very difficult to quantify Eddy losses.



(a) In solid core material

Eddy current and eddy current paths

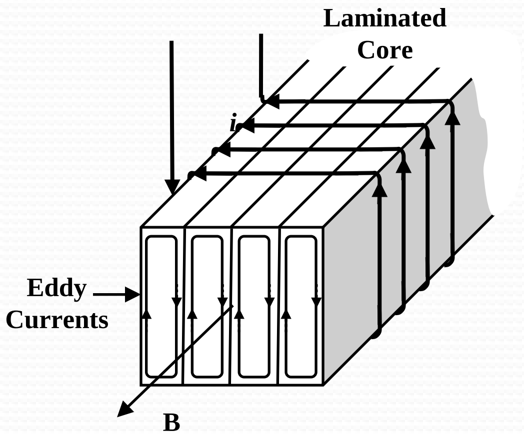
EDDY CURRENT LOSS

- Eddy current loss (P_e) in a magnetic circuits excited by a sinusoidal source is commonly estimated using the empirical formulae:

$$P_e = K_e B_m^{\,2} f^{\,2}$$

where, B_m is the maximum flux density in the magnetic core,
f is the frequency of the source, and
K_e is a constant.

- Eddy currents and therefore eddy losses are reduced by using laminations of high resistivity core material, instead of solid core material. This increases (i) the effective length of eddy current paths, and (ii) the resistance of these eddy current paths. Reduced "Eddy currents" lead to reduced "Eddy current losses".



(b) In laminated core material
Eddy current and eddy current paths

SINUSOIDAL EXCITATION OF MAGNETIC CIRCUITS (REVISIT)

- Both ***Eddy current loss*** as well as ***Hysteresis loss*** depend on frequency f and the maximum flux density B_m. It should be noted that these two variables are not fully independent.
- For a magnetic circuit excited by a sinusoidal input it was shown earlier that

$$V = V_m / \sqrt{2} = 2\pi NA(B_m f)/\sqrt{2} = 4.44 NA(B_m f)$$

- Thus, it is seen that the product $B_m f$ is jointly related to the input voltage, and these two variables cannot vary independently for a given input voltage.

EXAMPLE

An electromagnet is known to have hysteresis loss of 180 W when excited by 60 Hz, 120 V source. If the Steinmetz index of the core material is 1.6, estimate the hysteresis loss when the electromagnet is connected to 120 V, 50 Hz source.

Solution

Let B_{m1} and B_{m2} be the maximum flux densities under the two conditions.

Then, $P_{h1}=K_h B_{m1}^{1.6} f_1$, and $P_{h2}=K_h B_{m2}^{1.6} f_2 = ?$

Therefore, $$\frac{P_{h2}}{P_{h1}} = \frac{B_{m2}^{1.6}}{B_{m1}^{1.6}} \frac{f_2}{f_1},$$

and $P_{h1}= 180$ W, $f_1 = 60$ Hz, and $f_2 = 50$ Hz

The voltage equation under the two conditions yield,

$$V_1 = 120 \text{ V} = K\, B_{m1} f_1, \quad \text{and} \quad V_2 = 120 \text{ V} = K\, B_{m2} f_2$$

$$\Rightarrow B_{m1} f_1 = B_{m2} f_2 \quad \Rightarrow B_{m2}/B_{m1} = f_1/f_2 = 60/50 = 1.2$$

Therefore, $$P_{h2} = \frac{B_{m2}^{1.6}}{B_{m1}^{1.6}} \frac{f_2}{f_1} P_{h1} = 1.2^{1.6} \times \frac{50}{60} \times 180 = 200.8\ W$$

PRINCIPLES OF ELECTROMECHANICAL ENERGY CONVERSION

- Electromechanical devices (machines) convert electrical energy into mechanical energy and vice versa. Most of these devices utilize magnetic field as a medium.
- Conservation of energy has to be satisfied by all these processes.

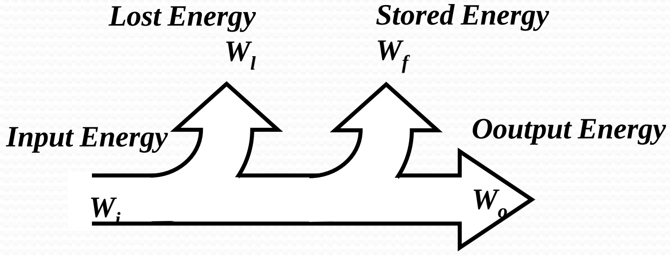
$W_i = W_o + W_f + W_\ell$ where,

W_i is the input (electrical) energy,

W_o is the output (mechanical) energy,

W_f is the field or stored energy,

W_l is the energy lost in the system.



Energy balance in electro-mechanical devices

- The flow of energy in the process is as shown in the Figure. This process is reversible except for the losses.
- Ignoring losses, the energy balance equation reduces to:

$$W_i = W_o + W_f$$

- Incremental analysis between two states gives

$$dW_i = dW_o + dW_f$$

PRODUCTION OF FORCE

- Consider an electro-magnetic system with one fixed and one movable part separated by a gap ***x*** as shown in Figure 3.31 (a). If the total reluctance is $\Re_1$, the flux in the structure for an input current *i*, is given by,

  $$\varphi = (Ni)/\Re$$

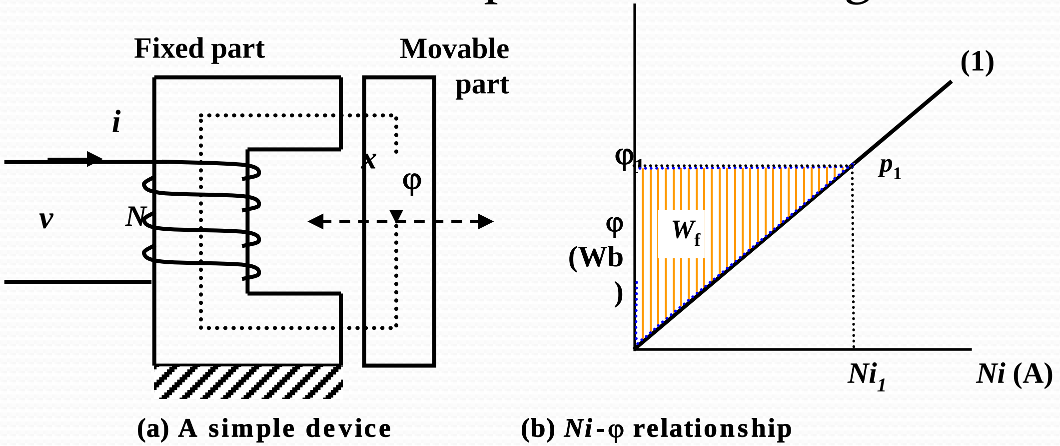


Figure 3.31 Analysis of electromechanical device

- The relationship between Ni and φ is a straight line (1) with slope $1/\Re_1$ as shown in Figure 3.31 (b). If the system is operating at point $\boldsymbol{p_1}$ with input current i_1 and flux φ_1, the stored energy is , $W_f = \frac{1}{2} N i_1 \ \varphi_1$ which is the area indicated in the diagram.

- Let the force experienced by the movable part at a distance of ***x*** be F_m, and it moves towards the fixed part by an incremental distance of ***dx***.

  The incremental energy output, $dW_o = F_m dx$

PRODUCTION OF FORCE

- In this process, since the air gap has decreased, the reluctance should be reduced to, say $\Re_2$, ($\Re_2 < \Re_1$) and the relationship between the flux and the mmf is given by:

  $$\varphi = \frac{Ni}{\Re_2},$$ which is straight line with slope $1/\Re_2$ and may represented by the line (2) in Figure 3.32.

- And the operating point has to move from p_1 to somewhere in line (2). Consequently, there will be changes in various energy components – input energy and stored energy.

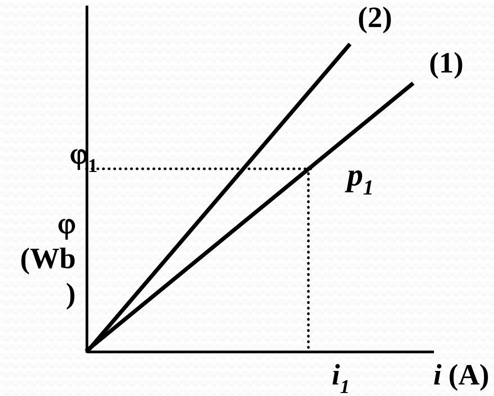


Figure 3.32 Change in Ni-φ relationship

- Using incremental analysis of the various energy components:

$$dW_i = dW_o + dW_f \quad \Rightarrow \quad dW_0 = dW_i - dW_f \quad \Rightarrow \quad F_m dx = dW_i - dW_f$$

- This relationship can be used to derive the expression for the force F_m, by evaluating the energy components dW_i and dW_f corresponding to the new operating point in line (2). This will be done for two different processes.

PRODUCTION OF FORCE

The production of force will be analyzed from two different processes.

I.Constant Flux

- If the flux φ is held constant while the movable part moves a distance *dx* under the force F_m, then the operating point changes from p_1 to p_2 is as indicated in Figure 3.33.

Then, $dW_i = i\ d\lambda = Ni\ d\varphi = Ni(\varphi_2 - \varphi_1) = 0$

($\varphi_2 = \varphi_1$)   since the flux is held constant.)

- Therefore, $F_m\ dx = dW_0 - dW_f = -dW_f$

$$\Rightarrow \quad F_m = -\frac{dW_f}{dx}$$

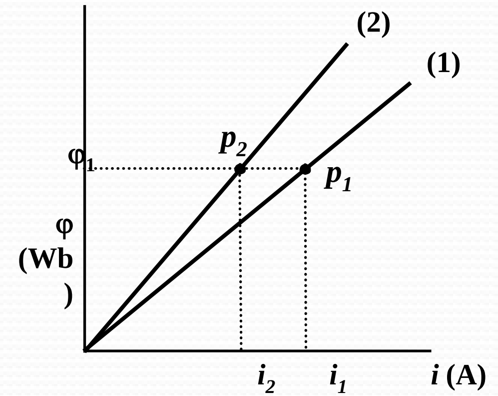


Figure 3.33 Change of operating point for constant flux process

- The force developed is proportional to the rate of decrement of the stored energy.

PRODUCTION OF FORCE

II. Constant Current

- If the input current is held constant during the process, the operating point moves from p_1 to p_3 as indicated in Figure 3.34, and the flux changes from φ_1 to φ_2, so that,

$$d\varphi = \varphi_2 - \varphi_1$$

- Under this condition,

(i) $dW_i = i\, d\lambda = Ni_1\, d\varphi = Ni_1(\varphi_2 - \varphi_1)$., and

(ii) $dW_f = W_{f2} - W_{f1} = \frac{1}{2} Ni_1\, \varphi_2 - \frac{1}{2} Ni_1\, \varphi_1$

$$= \tfrac{1}{2} Ni_1\,(\varphi_2 - \varphi_1) = \tfrac{1}{2} dW_i$$

Thus, $dW_i = 2\, dW_f$

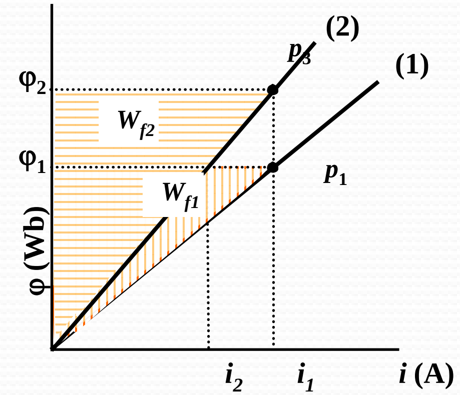


Figure 3.34 Change of operating point in constant current process

- Therefore, $F_m dx = dW_i - dW_f = 2dW_f - dW_f = dW_f$

And, $F_m = \dfrac{dW_f}{dx}$

- Thus the force developed is the rate of increment of the field energy.

PRODUCTION OF FORCE

- In both the cases, the evaluation of force F_m requires the expression for the magnetic field energy W_f, which may be a function of either
  - flux **φ**, and the gap distance ***x***, or
  - current ***i*** and gap distance ***x***.

- When the flux is held constant, $F_m = -\frac{dW_f}{dx} = -\frac{\partial W_f(\varphi, x)}{\partial x}$

  Since, $W_f = \frac{1}{2}\varphi^2\Re$, this approach usually takes the form,

$$F_m = -\frac{dW_f}{dx} = -\frac{\partial W_f(\varphi, x)}{\partial x} = -\frac{\partial[\frac{1}{2}\varphi^2\Re(x)]}{\partial x} = -\frac{1}{2}\varphi^2\frac{d\Re(x)}{dx}$$

- When the current is held constant, $F_m = \frac{dW_f}{dx} = \frac{\partial W_f(i, x)}{\partial x}$

  Since, $W_f = \frac{1}{2}i^2L$, this approach usually takes the form,

$$F_m = \frac{dW_f}{dx} = \frac{\partial W_f(\varphi, x)}{\partial x} = \frac{\partial[\frac{1}{2}i^2L(x)]}{\partial x} = \frac{1}{2}i^2\frac{dL(x)}{dx}$$

EXAMPLE

Determine the minimum amount of current required to develop a force of 80 N on the moving part of the electromagnet shown in Figure 3.35 (a) at a distance of 2 mm from the pole faces of the fixed electromagnet having 500 turns. Each pole face cross sectional area is 10 cm². Ignore the reluctance of the core materials.

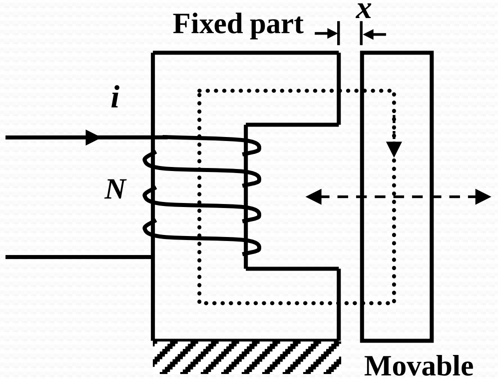


Figure 3.35 (a) A simple electromagnet

Solution

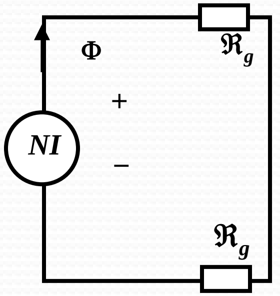
- The magnetic equivalent circuit for the system can be drawn as shown in Figure 3.35 (b).
- For a gap ***x***, the total reluctance is:

$$\Re_{eq} = \frac{2x}{4\pi\times10^{-7}\times10\times10^{-4}} = 1.59\times10^{9}\,x \quad \mathrm{H}^{-1}$$,

- If the flux is held constant, the field energy can be expressed as: $W_f(\varphi, x) = \frac{1}{2}\varphi^2\Re(x)$

  Then,

$$F_m = -\frac{dW_f}{dx} = -\frac{\partial[\frac{1}{2}\varphi^2\Re(x)]}{\partial x} = -\frac{1}{2}\varphi^2\frac{d\Re(x)}{dx} = -\frac{1}{2}\varphi^2\times1.59\times10^{9}$$



(b) Magnetic equivalent circuit

EXAMPLE

$$\Rightarrow \quad \frac{1}{2}\varphi^2 \times 1.59\times10^9 = 80 \text{ N, at } x = 2 \text{ mm}$$

$$\Rightarrow \quad \varphi = 0.317\times10^{-3} \text{ Wb, at } x = 2 \text{ mm}$$

Also, at x = 2 mm, $\varphi = \dfrac{NI}{\Re} = \dfrac{500\ I}{1.59\times10^9\times2\times10^{-3}}$

Therefore, $\dfrac{500\ I}{1.59\times10^9\times2\times10^{-3}} = 0.317\times10^{-3}$

$$\Rightarrow \quad I = 2.0 \text{ A}$$

- This problem can similarly be solved using the approach that utilizes the other expression for energy in terms of the inductance *L*.

Thank you!