PH 101
Tutorial-3
Date: 21/08/2017

1. (a) Prove that the centre of mass, CM of any two particles always lies on the line joining them, as illustrated in Fig. 1.
   (b) Prove that the distances from the CM to m_1 and m_2 are in the ratio m_2/m_1. Explain why if m_1 is much greater than m_2, the CM lies very close to the position of m_1.

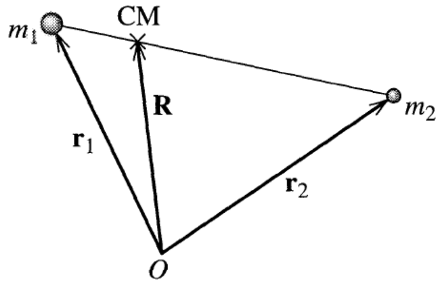


Fig. 1

2. (a) A water molecule H_2O consists of a central oxygen atom bound to two hydrogen atoms. The two hydrogen–oxygen bonds subtend an angle of 104.5°, and each bond has a length of 0.097 nm. Find the center of mass of the water molecule.
   (b) A uniform thin sheet of metal is cut in the shape of a semicircle of radius R and lies in the xy-plane with its center at the origin and diameter lying along the x-axis. Find the position of the CM using polar coordinates.
3. A boat of mass M is at rest in still water and a man of mass m is sitting at the bow. The man stands up, walks to the stern of the boat and then sits down again. If the water offers a resistance to the motion of the boat proportional to the velocity of the boat, show that the boat will eventually come to rest at its original position. [Refer to Fig. 2: x is the displacement of the boat at time t and v is the velocity. ξ is the displacement of the man relative to the boat at time t]

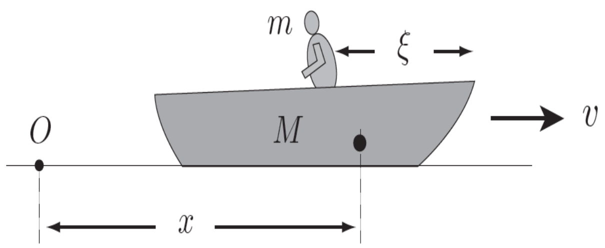


Fig. 2

4. A fire engine directs a water jet onto a wall at an angle θ with the wall. Calculate the pressure exerted by the jet on the wall assuming that the collision with the wall is elastic, in terms of ρ, the density of water, A the area of the nozzle, and v the jet velocity
5. On the Earth, a mirror of area 1 m^2 is held perpendicular to the Sun's rays.
   (a) What is the force on the mirror due to photons from the Sun, assuming that the mirror is a perfect reflector? The momentum flux density from the Sun's photons is $J_{sun} = 4.6\times\frac{10^{-6}kg}{m.s^2}$

(b) Find how the force varies with angle if the mirror is tilted at angle α from the perpendicular.

6. A one-dimensional stream of particles of mass m with density λ particles per unit length, moving with speed v, reflects back from a surface, leaving with a different speed v', as shown in Fig. 3. Find the force on the surface.

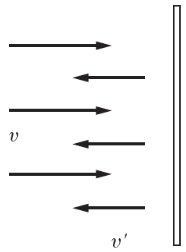


Fig. 3

7. (a) A heavy uniform chain of length L and mass M hangs vertically above a horizontal table, its lower end just touching the table. When it falls freely, show that the pressure on the table at any instant during the fall is three times the weight of the portion on the table.
(b) A body of mass $4m$ is at rest when it explodes into three fragments of masses $2m$, m and m. After the explosion the two fragments of mass m are observed to be moving with the same speed in directions making 120^0 with each other. Find the proportion of the total kinetic energy carried by each fragment. (Refer to Fig. 4)

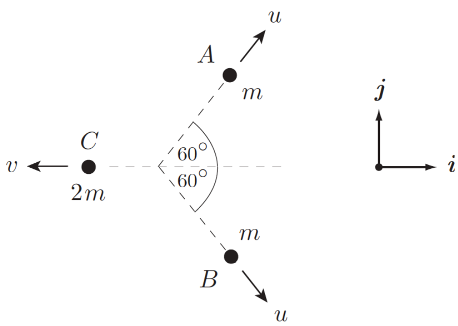
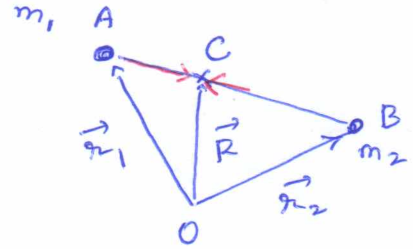


Fig.4

1.

(a)



The vector pointing from m_1 to the CM is

$$\vec{AC} = \vec{R} - \vec{r}_1 = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1+m_2} - \vec{r}_1$$

$$= \frac{m_2}{m_1+m_2}\left(\vec{r}_2 - \vec{r}_1\right)$$

$$= \frac{m_2}{M}\,\vec{AB}$$

Since $\vec{AC}$ and $\vec{AB}$ are in the same ~~dx~~ direction, the three points A, B and C are collinear. Since $AC < AB$, the CM lies between A and B on the line joining them.

(b) By similar argument,

$$\vec{BC} = \frac{m_1}{M}\,\vec{BA}$$

$\Rightarrow$ The ratio of the two lengths is

$$\frac{AC}{BC} = \frac{m_2}{m_1}$$

If $m_1 >> m_2$ then $AC << BC$ and C is very close to A.

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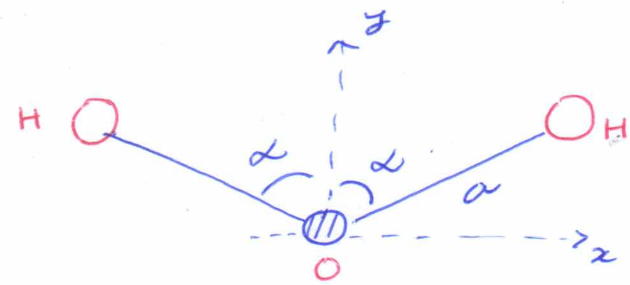
2.

(a)



The CM lies along the y-axis, by symmetry. Take the origin at the Oxygen atom, so that the y-co-ordinate of the oxygen atom is $y_0 = 0$. The y-co-ordinate of each hydrogen atom is

$$y_H = a \cos\alpha = 0.097 \text{ nm} \times \cos(52.25°) = 0.059 \text{ nm}$$

Thus, $$\bar{Y} = \frac{1}{M}\left(2M_H y_H + M_O y_O\right)$$

$$M = 2M_H + M_O$$

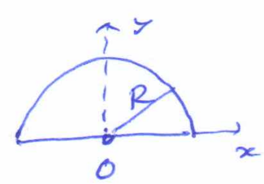
$$M_H = 1 \text{ amu}, \quad M_O = 16 \text{ amu}$$

Hence, $$\bar{Y} = \frac{2}{(2+16)}\left[2 \times 0.059\right] = \underline{0.0066 \text{ nm}}$$ //

⇒ The CM is very near to the massive oxygen atom.

(b) Say, M is the mass of the metal sheet.

Mass density, $\sigma = \frac{M}{A}$ where $A = \pi R^2/2$



CM position is $$\vec{R} = \int \frac{\sigma \vec{r}}{M}\, dA$$
$$= \int \vec{r}\, \frac{dA}{A}$$

where the integral runs over the area of the sheet in the plane $z=0$. Clearly $Z = 0$. By symmetry $X = 0$. Hence, $$Y = \frac{1}{A}\int y\, dA = \frac{2}{\pi R^2}\int_0^R r\,dr \int_0^\pi (r\sin\theta)\, d\theta$$
$$= (4/3\pi) R \quad //$$

3.

The true velocity of the man in the +ve x direction is $v - \frac{d\xi}{dt} = v - \dot{\xi}$

Linear momentum of the system in +ve x-direction is:

$$P = Mv + m(v - \dot{\xi}) = (M+m)v - m\dot{\xi}$$

The only horizontal force acting on the system is the resistance force R exerted by the water.

$$R = -kv \quad , \quad k \text{ is a constant}$$

Thus,

$$\frac{dP}{dt} = R$$

$$\Rightarrow (M+m)\dot{v} - m\ddot{\xi} = -kv$$

$$\Rightarrow \dot{v} + \frac{k}{M+m} v = \frac{m}{M+m}\ddot{\xi}$$

$$\Rightarrow \dot{v} + cv = \frac{m}{M+m}\ddot{\xi} \quad , \quad c = \frac{k}{M+m} \text{, a const.}$$

Integrating above eq^n with respect to t, we obtain

$$\dot{x} + cx = \left(\frac{m}{M+m}\right)\dot{\xi} + \alpha \quad , \quad \alpha \text{ is a constant of integration}$$

Since the whole system starts from rest with $x = 0$, it follows that $\alpha = 0$. Thus we obtain:

$$\dot{x} + cx = \left(\frac{m}{M+m}\right)\dot{\xi} \quad \rightarrow (1)$$

This is the Eq^n of motion satisfied by x. The function $\xi(t)$, the motion of the man is supposed to be known. Solving (1) [Eq^n (1) is a 1st order ODE with integrating factor ~~e^{t}~~ e^{ct}] we find that

$$x = \left(\frac{m}{M+m}\right) e^{-ct} \int_0^t \dot{\xi}(t')\, e^{ct'} dt + D e^{-ct}$$

where D is a second constant of integration. The initial condition $x = 0$ when $t = 0 \Rightarrow D = 0$

~~So Soln to 3~~

Solution to problem 3 contd.

Thus, the displacement x of the boat at time t is

$$\boxed{x = \left(\frac{m}{M+m}\right) e^{-ct} \int_0^t \dot{\xi}(t')\, e^{ct'} dt'}$$

Now we wish to show that the boat ~~ev~~ eventually regains its original position. Let us suppose that the man has taken his seat at the back of the boat by time τ. Then for $t \geq \tau$, $\dot{\xi} = 0$ and the integration can be restricted to the range $0 \leq t' \leq \tau$. The solution for x for $t > \tau$ can be written as:

$$x = \left(\frac{m}{M+m}\right) e^{-ct} \int_0^\tau \dot{\xi}(t')\, e^{+ct'} dt'$$

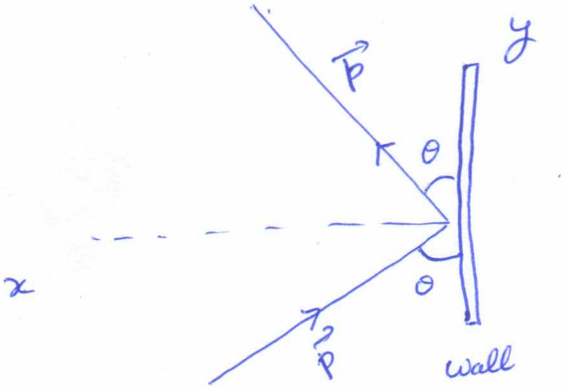
$$= \left[\frac{m}{M+m} \int_0^\tau \dot{\xi}(t')\, e^{ct'} dt' \right] e^{-ct}$$

The expression is inside $[\cdots]$ looks complicated but since the limits of integration are now constants, it is simply a constant, say x_0. Hence, for $t \geq \tau$, the solution for x has the form:

$$\boxed{x = x_0\, e^{-ct}}$$

$x \to 0$ as $t \to \infty$, which is the required result!

4.



Say A is the area of the ~~noz~~ nozzle through which jet comes. The mass of water in the jet per second is $\rho A v$.

The momentum associated with this volume of water is

$$p = (\rho A v) v = \rho A v^2.$$

The momentum after hitting the wall will also be equal to $\rho A v^2$.

The change of x component of momentum is

$$\Delta p_x = p \sin\theta - (-p\sin\theta) = 2p\sin\theta$$

The change in y. component of momentum is

$$\Delta p_y = p\cos\theta - p\cos\theta = 0$$

Then, $\Delta p = \Delta p_x = 2p\sin\theta = 2\rho A v^2 \sin\theta$

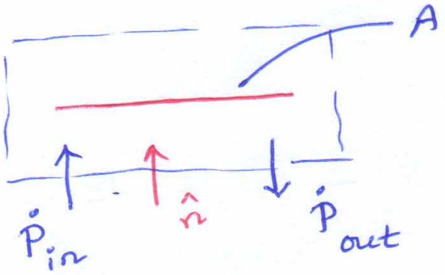
The pressure exerted on the wall will be

$$P = \frac{\Delta p}{A} = 2\rho v^2 \sin\theta \quad //$$

5.

(a)

Say momentum flow $\vec{\dot{P}}$ through a surface area $\vec{A}$ is $\vec{\dot{P}} = \vec{J} \cdot \vec{A}$ where $\vec{J}$ is the momentum flux density.

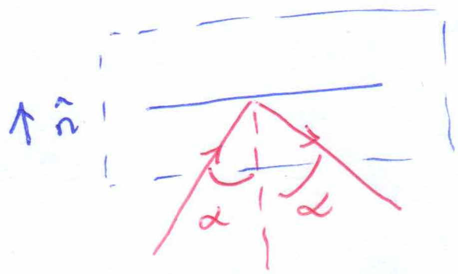


$$\vec{\dot{P}}_{net} = \vec{\dot{P}}_{in} - \vec{\dot{P}}_{out} = 2JA\,\hat{n}$$

$$\Rightarrow \quad \vec{F} = 2JA\,\hat{n}$$

$$F = 2JA = 9.2 \times 10^{-6} \text{ kg m/s}^2 \,//$$

(b)



$$\vec{J}_{in} \cdot \vec{A} = JA \cos\alpha$$

$$\vec{J}_{out} \cdot \vec{A} = -JA \cos\alpha$$

$$\therefore \vec{F} = \vec{\dot{P}}_{in} - \vec{\dot{P}}_{out} = 2JA\cos\alpha\,\hat{n}$$

$$= 9.2 \times 10^{-6} \cos\alpha\,\hat{n}$$

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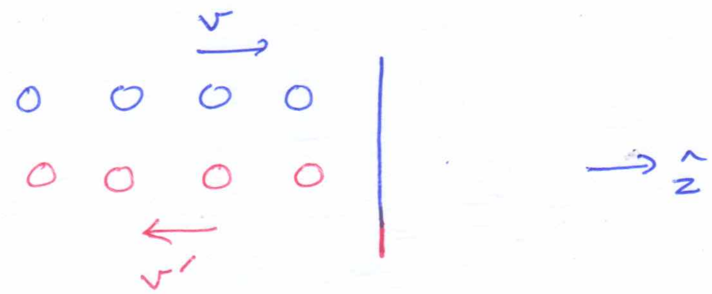
6.



The rate at which incoming particles strike the surface is = (number of particles per unit length) × (speed)

$= \lambda v$

$\therefore \quad \dot{\vec{P}}_{in} = (mv) \times \lambda v \, \hat{z}$, the rate at which momentum arrives at the surface

In steady conditions, the rate at which particles leave must be equal at which particles arrive.

If they leave with speed v' with λ' particles per unit length, then $\lambda v = \lambda' v'$

The reflected particles carry momentum in the opposite direction at rate $m v' \lambda' v' = m v' \lambda v$

Thus the total force $= \lambda m (v^2 + v v')$ //

7.

(a) The pressure on the table consists of two parts:

(i) the weight of the coil on the table producing the pressure and

(ii) the destruction of momentum producing the pressure.

Let's first consider part (ii):

Say, a length x be coiled up on the table. Since the ~~coil is~~ chain is falling freely under gravity, the velocity of the chain will be $\sqrt{2gx}$.

In a small time interval δt, the length which reaches the table is $\delta t\sqrt{2gx}$

Thus, the momentum destroyed in time δt is

$$\delta p = \delta t \frac{M}{L}\sqrt{2gx}\sqrt{2gx} = \delta t \frac{M}{L} 2gx$$

$\Rightarrow$ The rate of destruction of momentum is

$$\frac{\delta p}{\delta t} = \frac{M}{L} 2gx$$

Pressure due to part (i) will be $\frac{Mg}{L}x$

Total pressure on the table $= \frac{M}{L}2gx + \frac{M}{L}gx = \frac{3Mgx}{L}$

$\equiv$ three times the weight of the coil on the table

Note:
(Area is taken be unity)

~~Q~~. 7(c)

Total Linear momentum of the fragments must be zero as the body is initially at rest.

Again, since the two fragments of mass m have equal speeds, the third velocity must lie along the bisector of the angle between their paths, as already illustrated in Fig. 5.

Say, speed of fragments A and B be u while that of C be v.

Then,

$$mu \cos 60^\circ + mu \cos 60^\circ - 2mv = 0$$

$$\Rightarrow v = \frac{1}{2}u$$

$\therefore$ Kinetic energies are

$$T^A = T^B = \frac{1}{2}mu^2$$

$$T^C = \frac{1}{2}(2m)\left(\frac{1}{2}u\right)^2 = \frac{1}{4}mu^2$$

Total K.E., $T = \frac{5}{4}mu^2$

Thus,

$$\frac{T^A}{T} = \frac{2}{5}$$

$$\frac{T^B}{T} = \frac{2}{5}$$

$$\frac{T^C}{T} = \frac{1}{5} \quad //$$