Full solution to Quiz-I (PH 101)

1.

(a) $\vec{v} = \dot{r}\,\hat{r} + (r\dot{\theta})\,\hat{\theta}$

$$\Rightarrow \boxed{\vec{v} = (a\omega e^{\omega t})\,\hat{r} + (a\omega e^{\omega t})\,\hat{\theta}}$$

$$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\,\hat{r} + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\,\hat{\theta}$$

$$= (a\omega^2 e^{\omega t} - a\omega^2 e^{\omega t})\,\hat{r} + (0 + 2\omega^2 a e^{\omega t})\,\hat{\theta}$$

$$\Rightarrow \boxed{\vec{a} = 2\omega^2 a e^{\omega t}\,\hat{\theta}}$$

(b)

$$v_{cm} = \frac{\sum m_i v_i}{\sum m_i} = \frac{4mv_0 + (m)(0)}{5m} = \frac{4}{5}v_0$$

2.

$$x = A\sin(\omega t + \alpha)$$

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{16} = \frac{\pi}{8}$$

when $t = 2s$, $x = 0$

$$\therefore\ 0 = A\sin\left(\frac{\pi}{8}\times 2 + \alpha\right)$$

Since $A \neq 0$ $\quad \sin\left(\frac{\pi}{4} + \alpha\right) = 0 \Rightarrow \frac{\pi}{4} + \alpha = 0 \Rightarrow \alpha = -\frac{\pi}{4}$

Now,

$$v = \frac{dx}{dt} = A\omega\cos(\omega t + \alpha)$$

when $t = 4$, $v = 4$

$$\therefore\ 4 = A\,\frac{\pi}{8}\cos\left(\frac{\pi}{8}4 - \frac{\pi}{4}\right)$$

$$\Rightarrow \boxed{A = \frac{32\sqrt{2}}{\pi}}$$

Other forms

~~$A = 45.2\pi$~~

~~$A = 45.2\pi$~~ $\quad \boxed{A = \frac{45.25}{\pi}}$ or $\boxed{A = 14.40}$

3.

The centre of mass of the hanging part of the chain is located at a distance $L/6$ below the edge of the table. The mass of the hanging part of the chain is $M/3$. The work done to pull the hanging part on the table is

$$W = \frac{Mg}{3}\,\frac{L}{6} = \frac{MgL}{18}$$

Other method

Consider an element of length dx of the hanging part at a distance x below the edge. The mass of the length dx is $\frac{M}{L}\,dx$.

Thus,
$$dW = \frac{M\,dx}{L}\,g\,x$$

$$W = \int_0^{L/3} \frac{Mg}{L}\,x\,dx = \frac{MgL}{18} \quad //$$