Quantum Mechanics
Tutorial-3

1) A particle of mass m is confined to a one dimensional region $0 \leq x \leq a$. The energy eigen states and the corresponding eigen values are given by

$$\chi_n = \sqrt{\tfrac{2}{a}} \sin(\tfrac{n\pi x}{a})$$
$$E_n = \tfrac{n^2\pi^2\hbar^2}{2ma^2}.$$

The normalized wave function of the particle at $t = 0$ is given by

$$\psi(x,0) = \sqrt{\frac{8}{5\pi}}\left[1 + \cos\frac{\pi x}{a}\right]\sin\frac{\pi x}{a}.$$

(a) Find the wave function at any time t, $\psi(x,t)$.
(b) Calculate the average energy at $t = 0$ and $t = t_0$.
(c) What is the probability that the particle will be found in the left half of the box.

2) The time independent wave function of a free particle in a box is

$$\psi(x) = Ax(a - x) \quad 0 \leq x \leq a.$$

(a) Calculate A
(b) Calculate the average energy of the particle.
(c) Calculate the corresponding momentum wave function.
(d) What is the probability of finding the particle at energy $E_1 = \frac{\pi^2\hbar^2}{2ma^2}$.
Normalized energy eigenstates and the corresponding eigen values are as given in problem (1).

3) For a free particle the momentum space wave function at $t = 0$ is given by

$$\phi(p,0) = Ae^{-\alpha p^2},$$

where α is a constant.
(a) Find A .
(b) Find $\psi(x,t)$.

4) Using $[\hat{x},\hat{p}] = i\hbar$ show that
(a) $[\hat{x}^2,\hat{p}] = 2i\hbar\hat{x}$.
(b) Also, prove that $[\hat{x}^n,\hat{p}] = ni\hbar\hat{x}^{n-1}$.