LOGIC GATES AND BOOLEAN ALGEBRA

Digital (logic) circuits operate in the binary mode when each input and output voltage is either a 0 or 1.

0 and 1 represent predefined voltage ranges.

For example, for the TTL (Transistor Transistor Logic), have

Logic	Input Range	Output Range
0	0-0.8 V	0-0.4 V
1	2-5 V	2.4-5V

*Note that the output range is **tighter** than the input range to ensure reliability*

Combinatorial Circuits designed/analyzed using **Boolean Algebra**,

In Boolean Algebra, variables can only take on the logical values 0 (FALSE) or 1 (TRUE)

The **Basic Functions** in Boolean Algebra are NOT, OR and AND defined as follows

NOT $B=\bar{A}$ (NOT) A B=1 if A=0 or B=0 if A=1

OR $C=A+B$ C=0 only if A=B=0; else C=1

AND $C=AB$ C=1 if A=B=1; else C=0

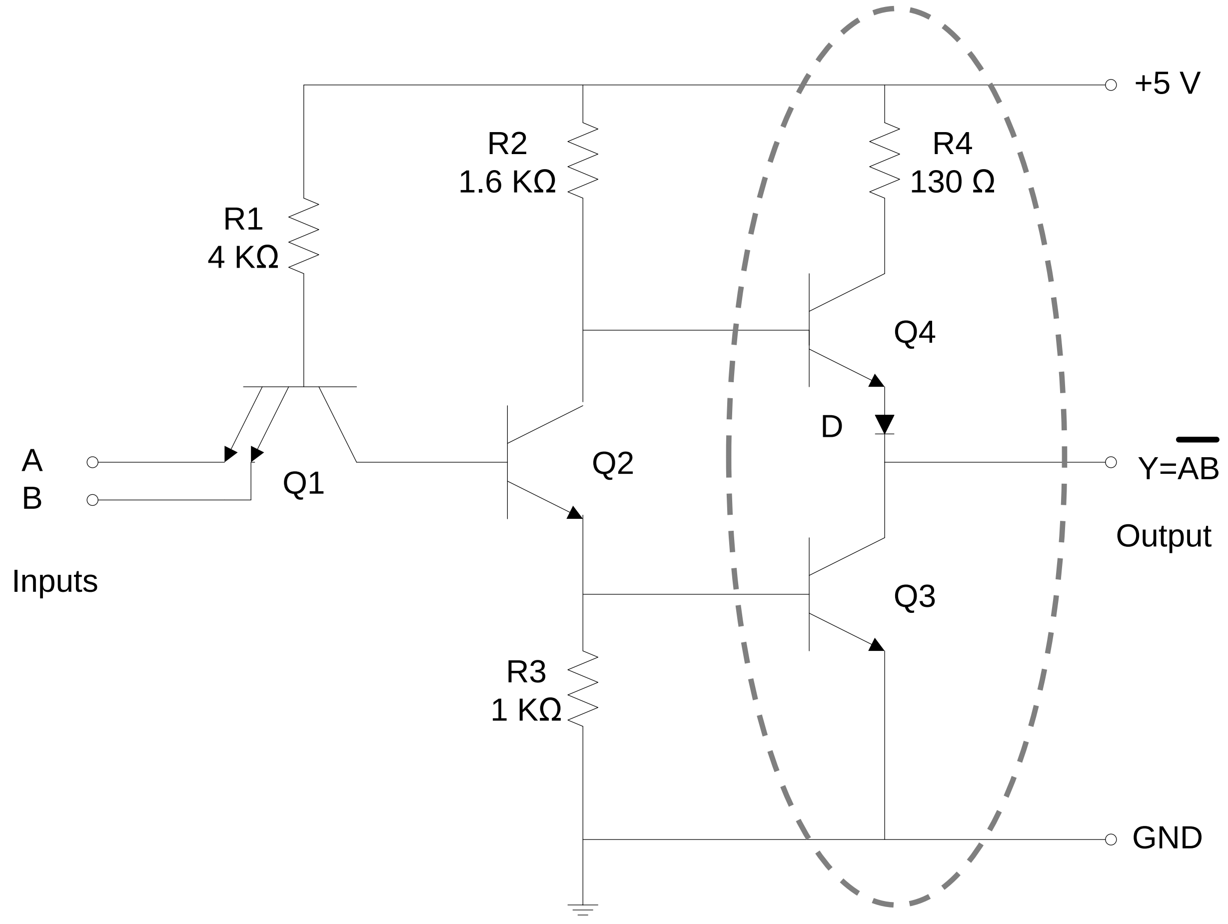
In Boolean Algebra, 1 and 0 are NOT NUMBERS. They merely represent the logical variables TRUE and FALSE

Logic Gates

Digital circuits called logic gates can be constructed from diodes, transistors, and resistors in such a way that the circuit output is the result of a basic logic operation (OR, AND, NOT) performed on the circuits.

Logic Gates which do more complicated logical operations are also widely available, e.g. NOR, NAND, Ex-OR etc..

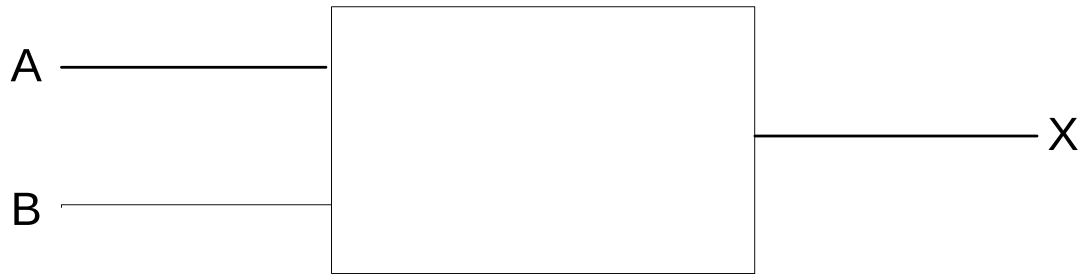
TTL NAND Gate



Totem Pole or *Active Pull-Up* Output

Truth Table

A truth table is a tabular representation for describing how a logic circuit’s output depends on the logic levels present at the circuit’s inputs



Truth Table

Binary Counting Sequence

A	B	X
0	0	0
0	1	1
1	0	1
1	1	1

No. of input combinations= 2^N for N- Inputs

NOT Operation

$$X = \overline{A}$$

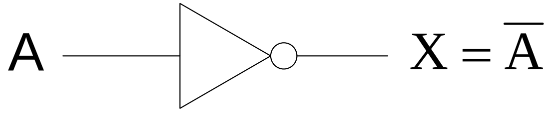
Function of only one variable

Called – NOT A
or Complement of A
or Inverse of A

$$A = 0 \Rightarrow \quad X = \overline{A} = 1$$

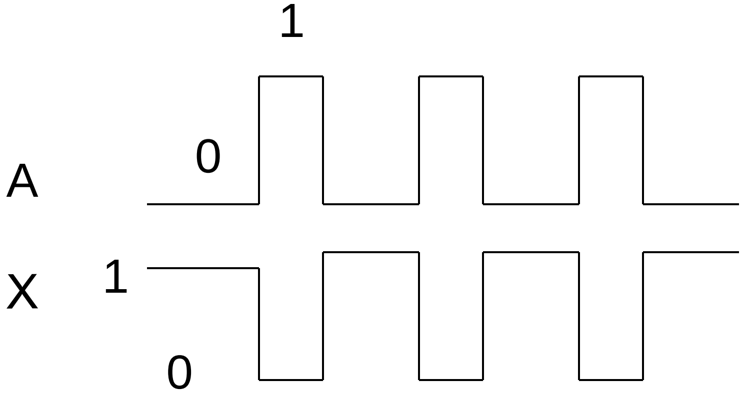
$$A = 1 \Rightarrow \quad X = \overline{A} = 0$$

NOT gate or Inverter



A	$X = \overline{A}$
0	1
1	0

Truth Table



Function of Two Binary Variables

X = f(A,B)

Examples: OR, AND

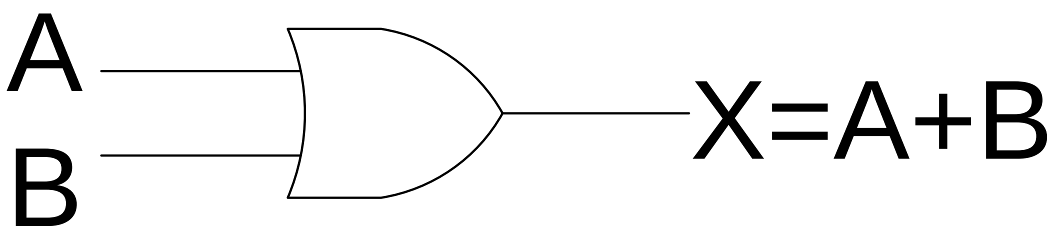
NOR (Not-OR)

NAND (Not-AND)

Ex-OR

OR operation $X = A+B$

OR gate

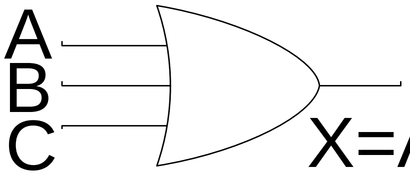


A	B	X
0	0	0
0	1	1
1	0	1
1	1	1

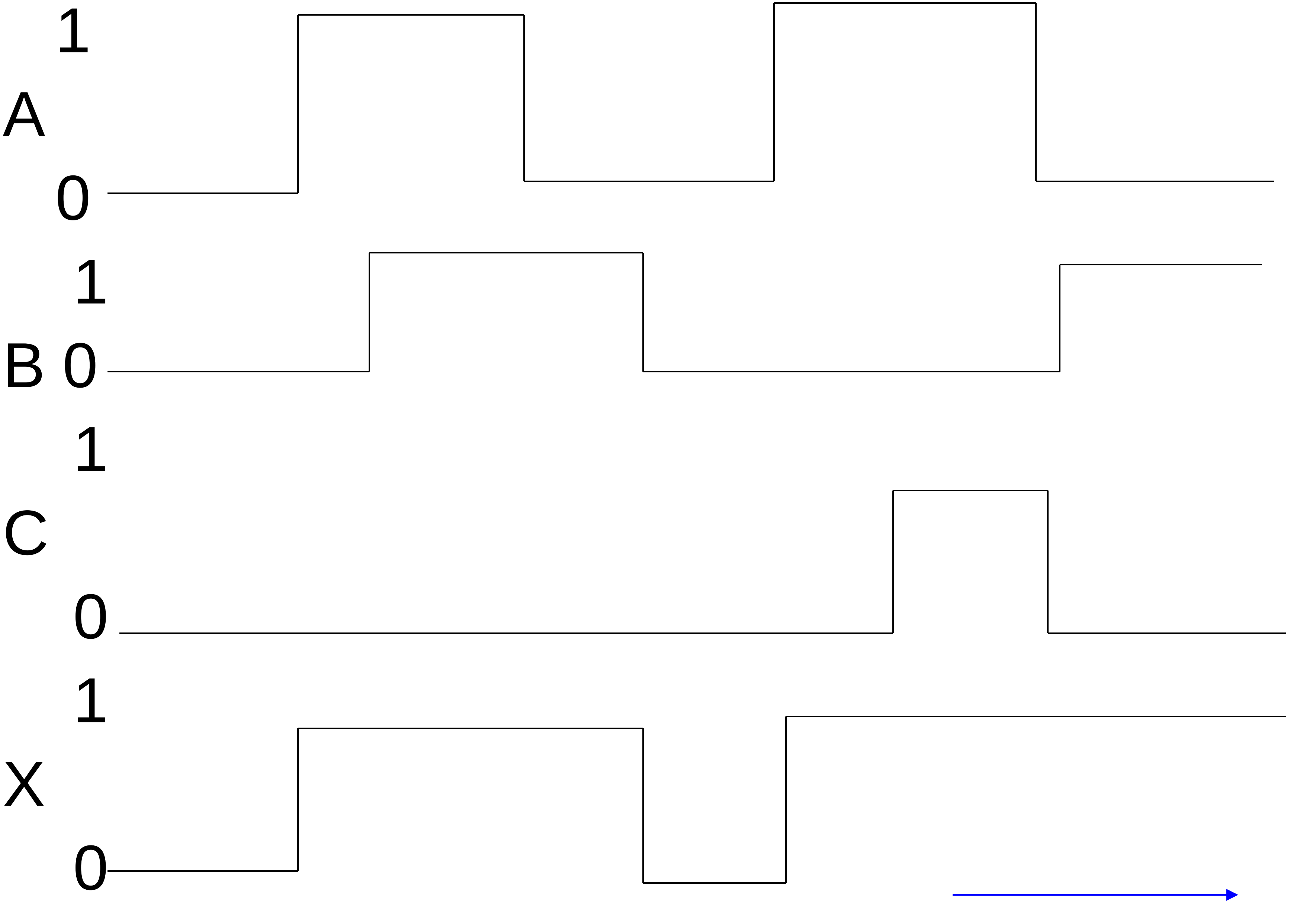
Truth Table

Truth Table

A	B	C	X
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1



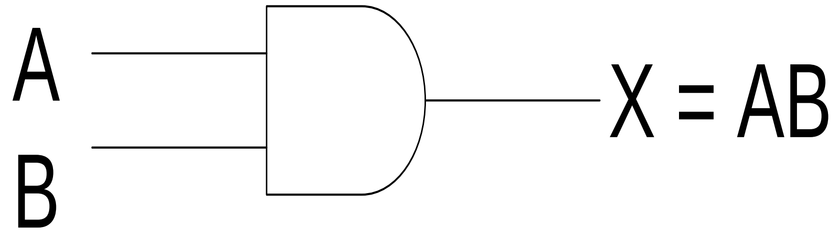
X=A+B+C



OR Operation

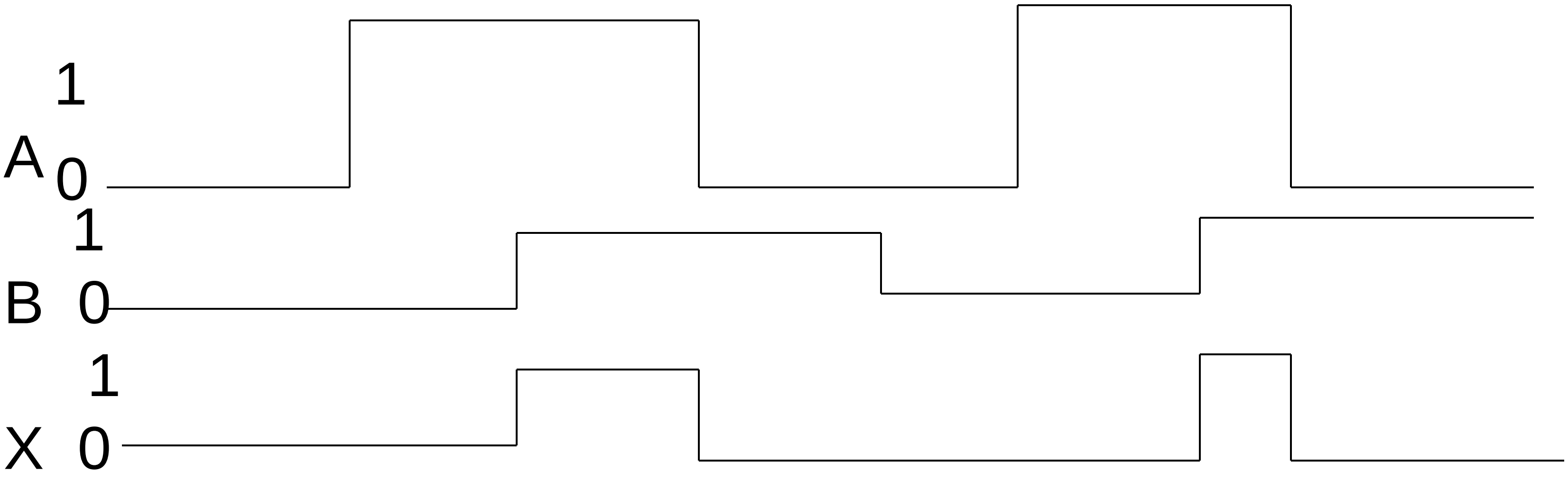
AND Operation $X = AB$

AND gate



A	B	X
0	0	0
0	1	0
1	0	0
1	1	1

Truth Table



AND Operation

NOR and NAND gates

NOR - NOT-OR

Combination of NOT and OR

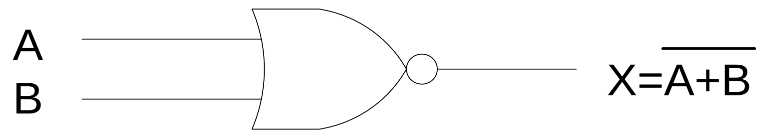
OR followed by NOT

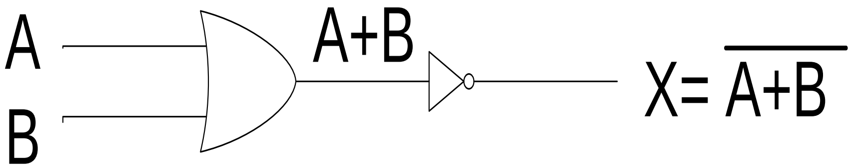
NAND – NOT-AND

Combination of NOT and AND

AND followed by NOT

NOR gate

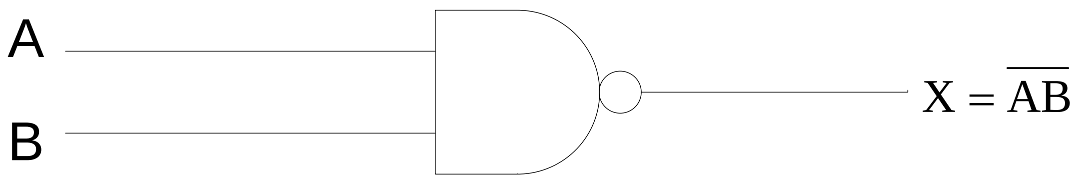


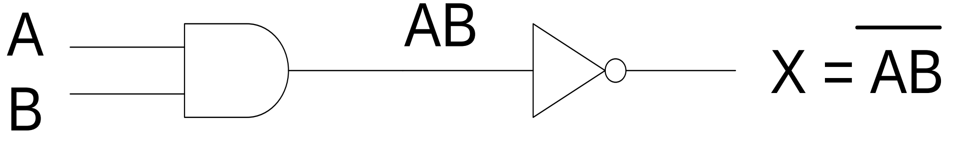


A	B	A+B	$\overline{A+B}$
0	0	0	1
0	1	1	0
1	0	1	0
1	1	1	0

Truth Table for NOR Gate

NAND gate



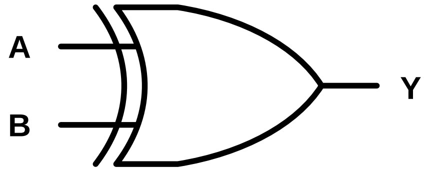


A	B	AB	$\overline{AB}$
0	0	0	1
0	1	0	1
1	0	0	1
1	1	1	0

Truth Table for NAND Gate

Ex-OR Gate

Symbol $\oplus$



$$Y = A \oplus B$$
$$= A\overline{B} + \overline{A}B$$

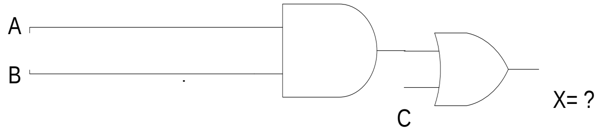
Comparator: Output is LOW if both the inputs are identical; else output is HIGH

Half-Adder: Y is the sum of A and B (in binary) *ignoring the carry out that will be generated by* 1+1

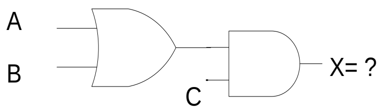
Truth Table

A	B	**Y**
0	0	0
0	1	1
1	0	1
1	1	0

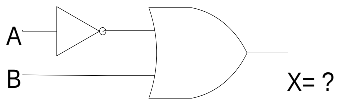
Describing Logic Circuits Algebraically



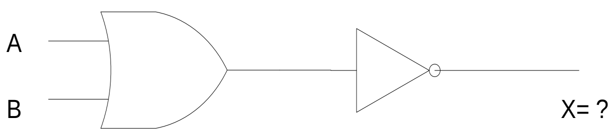
X=AB+C



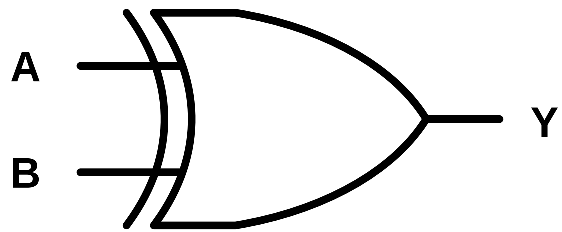
X=(A+B)C

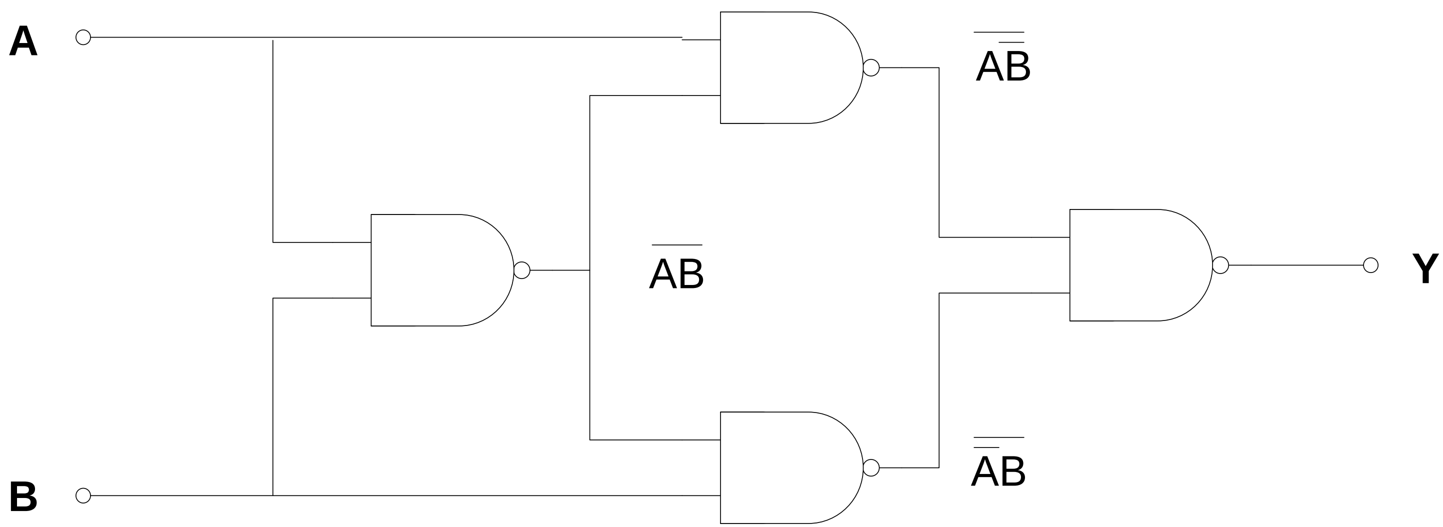


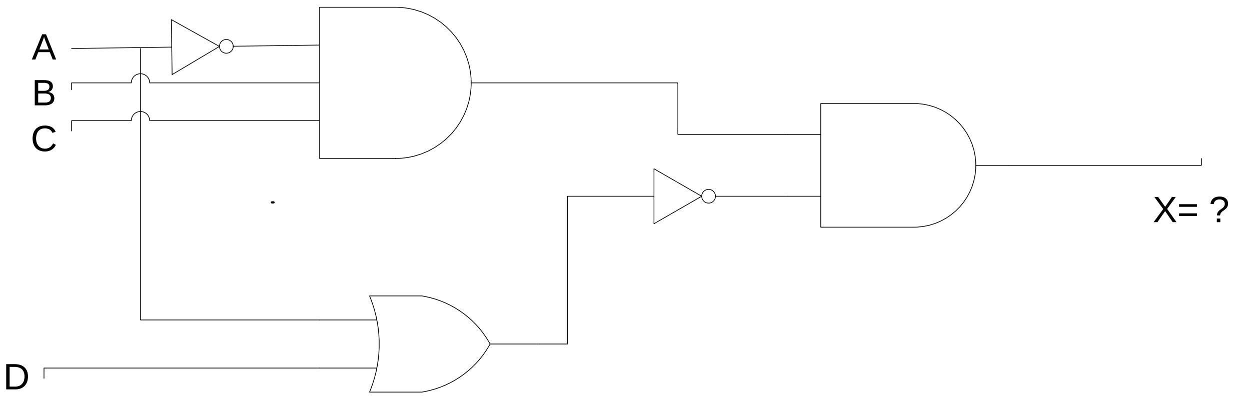
$$X = \overline{A} + B$$



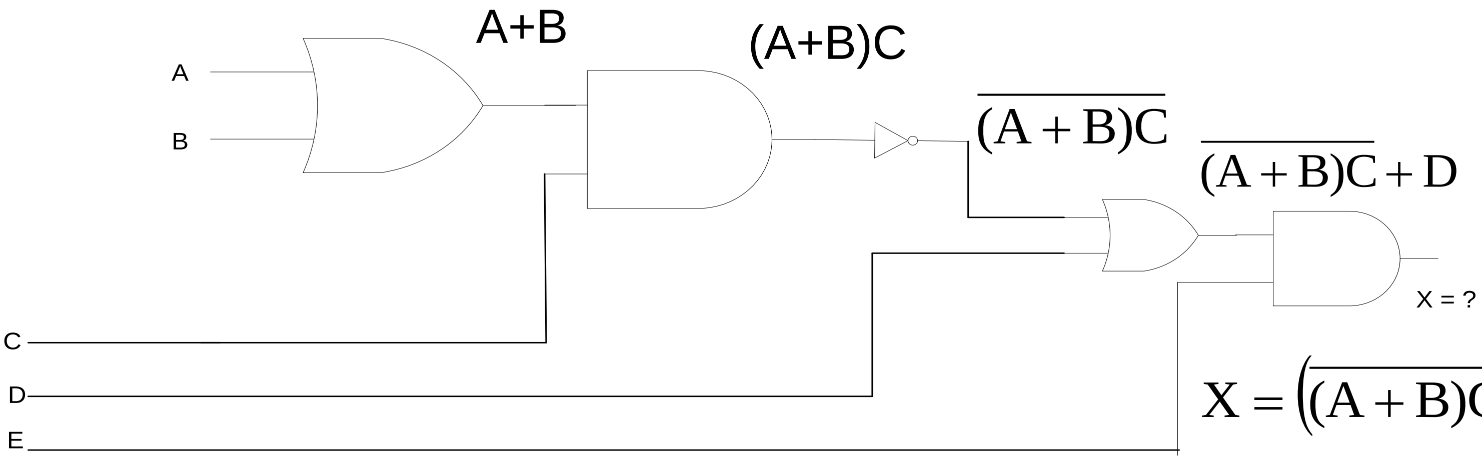
$$X = (\overline{A + B})$$


A
B
Y


A
B
$\overline{A\overline{B}}$
$\overline{AB}$
$\overline{\overline{A}B}$
Y

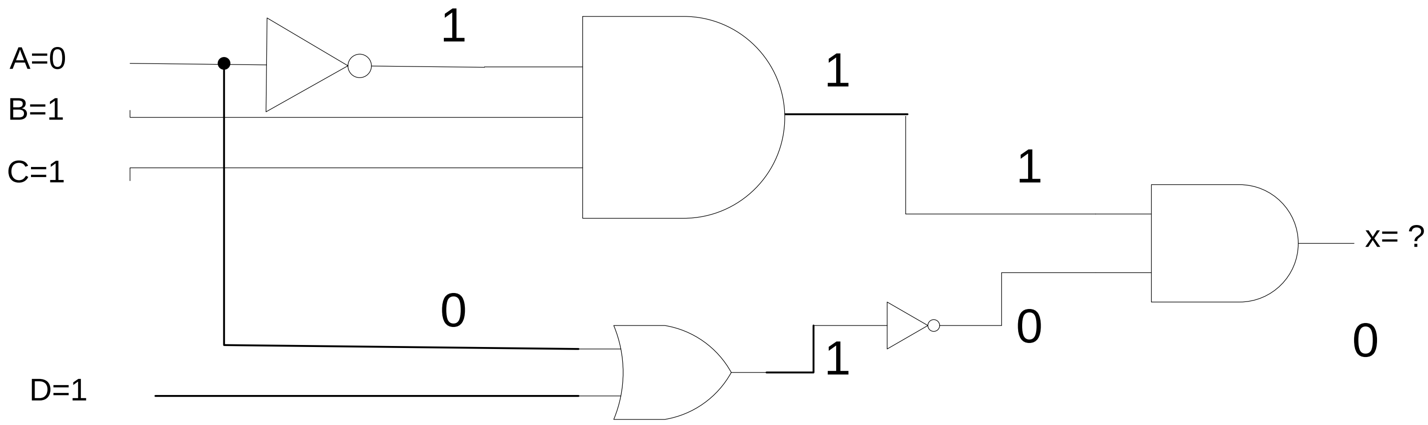


$$X = \overline{A}BC(\overline{A + D})$$



$$X=\left(\overline{(A+B)C}+D\right)E$$

Determining output level from a diagram



Boolean Algebraic Theorems : Useful in simplifying expression of logic variables

A variable can have only two values, A = 1 or A= 0

$$\overline{\overline{A}} = A \qquad (1)$$

Also,

Duals

Interchange + and .

Interchange 0 and 1

$$A + 0 = A \qquad (2)$$

$$A + 1 = 1 \qquad (3)$$

$$A + A = A \qquad (4)$$

$$A + \overline{A} = 1 \qquad (5)$$

$$A.1 = A \qquad (6)$$

$$A.0 = 0 \qquad (7)$$

$$A.A = A \qquad (8)$$

$$A.\overline{A} = 0 \qquad (9)$$

$$A(B+C)=AB+AC \qquad (10)$$ **Distributive Law**

Dual $$A+BC=(A+B)(A+C) \qquad (11)$$

Proof : RHS= $(A+B)(A+C)$

$$= AA+AB+CA+BC$$

$$= A+AB+CA+BC$$

$$= A(1+B)+CA+BC$$

$$= A+CA+BC$$

$$= A(1+C)+BC$$

$$= A+BC$$

$$= LHS$$

$$A + AB = A \qquad (12)$$

$$A + \overline{A}B = A + B \qquad (13)$$

$$AB + A\overline{B} = A \qquad (14)$$

$$AB + \overline{A}C = (A + C)(\overline{A} + B) \qquad (15)$$

$$AB + \overline{A}C + BC = AB + \overline{A}C \qquad (16)$$

Duals

$$A.(A + B) = A \qquad (17)$$

$$A.(\overline{A} + B) = A.B \qquad (18)$$

$$(A + B).(A + \overline{B}) = A \qquad (19)$$

$$(A + B).(\overline{A} + C) = AC + \overline{A}B \qquad (20)$$

$$(A + B).(\overline{A} + C).(B + C) = (A + B).(\overline{A} + C) \qquad (21)$$

Some sample proofs

(12) $A + AB = A(1+B) = A.1 = A$

(13) $A + \overline{A}B = A(B+1) + \overline{A}B = AB + A + \overline{A}B = A + B(A + \overline{A}) = A + B$

(14) $AB + A\overline{B} = A(B + \overline{B}) = A.1 = A$

De Morgan's Theorem:

$$\overline{A.B.C..........} = \overline{A} + \overline{B} + \overline{C} + ... \quad (22)$$

$$\overline{A + B + C..........} = \overline{A}.\ \overline{B}.\ \overline{C}\ .... \quad (23)$$

Ex. Simplify

$$Z = \overline{\left(\overline{A} + C\right).\left(B + \overline{D}\right)}$$

$$= \overline{\left(\overline{A} + C\right)} + \overline{\left(B + \overline{D}\right)}$$

Using De Morgan’s Theorem

$$= \left(\overline{\overline{A}}.\overline{C}\right) + \overline{B}.\overline{\overline{D}}$$

$$= A\overline{C} + \overline{B}D$$

Ex. Simplify

$$Z = \overline{A}C(\overline{\overline{A}BD}) + \overline{A}B\overline{C}\overline{D} + A\overline{B}C$$

$$= \overline{A}C(\overline{\overline{A}} + \overline{B} + \overline{D}) + \overline{A}B\overline{C}\overline{D} + A\overline{B}C$$

$$= \overline{A}C\left(A + \overline{B} + \overline{D}\right) + \overline{A}B\overline{C}\overline{D} + A\overline{B}C$$

$$= \overline{A}CA + \overline{A}\overline{B}C + \overline{A}C\overline{D} + \overline{A}B\overline{C}\overline{D} + A\overline{B}C$$

$$= 0 + \overline{B}C(\overline{A} + A) + \overline{A}C\overline{D} + \overline{A}B\overline{C}\overline{D}$$

$$= \overline{B}C + \overline{A}\,\overline{D}(C + B\overline{C})$$

$$= \overline{B}C + \overline{A}\,\overline{D}(C + B)$$

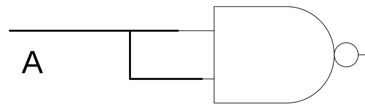
Universality of NAND and NOR gates

- All the Boolean expressions can be implemented using various combinations of OR, AND and NOT gates.
- Again NAND or NOR gates in proper combination can be used to perform each of the Boolean operations OR, AND and NOT.
- Any logic expression can be implemented using **ONLY** NAND or NOR gates.
- Therefore, NAND and NOR Gates are known as Universal Gates

Using NAND Gates

Using NAND Gates

NOT

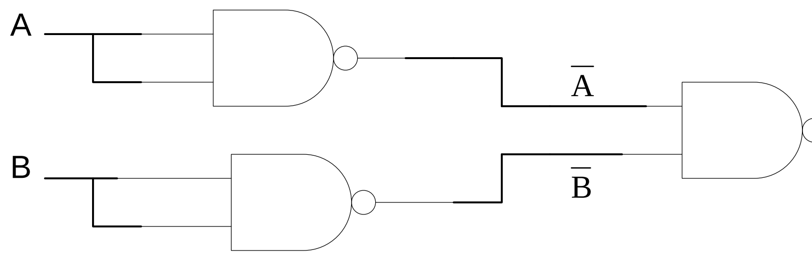


$$X = \overline{A.A} = \overline{A}$$

AND

$$X = \overline{\overline{AB}.\,\overline{AB}} = \overline{\overline{AB}} = AB$$

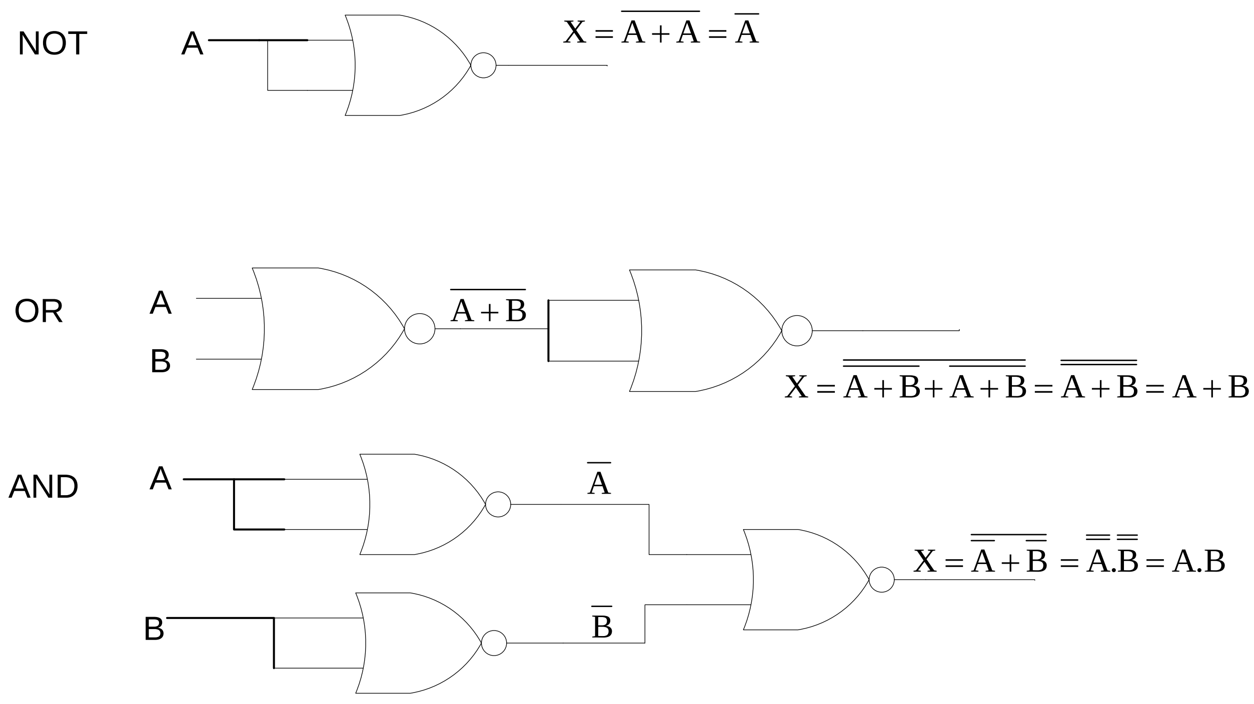
OR



$$X = \overline{\overline{A}.\,\overline{B}} = \overline{\overline{A}} + \overline{\overline{B}} = A + B$$

Using NOR Gates

Using NOR



COMBINATIONAL LOGIC CIRCUITS
or **COMBINATORIAL CIRCUITS**

The circuits made up of combination of logic gates discussed earlier can be classified as combinational logic circuits because at any time, the logic level at the output depends on the combination of logic levels present at the inputs.

A combinational circuit has no memory, so its output depends only on the current value of its inputs.

Standard Forms For Logical Expression

For Logic circuit simplification and design, we try to express the logical expression in one of two standard forms -

Sum-of-Products Form

or Product-of-Sums Form

Sum-of-Products (SOP)

Examples: (i) $ABC+A\bar{B}\bar{C}$ (ii) $AB+A\bar{B}\bar{C}+C\bar{D}+\bar{D}$

Note that each expression consists of two or more **AND** terms that are **ORed** together and that -

(i) The same variable used never appears twice in a product. (Because A.A=A and $A\bar{A}=0$)

(ii) One complement sign can not cover more than one variable in a term (e.g. we can not have $\overline{ABC}$ as we can then use De Morgan’s Law to simplify)

Example 1

$Z=(\bar{A}+BC)(B+CD)$

Express the function as sum of products.

Solution

$Z=(\bar{A}+BC)(B+CD)$

$=\bar{A}B+\bar{A}CD+BBC+BC.C.D$

$=\bar{A}B+\bar{A}CD+BC+BCD$

Example 2

Express the following function as sum of products

$$Z=(A+\overline{BC})(\overline{D+BE})$$

$$=(A+\overline{B}+\overline{C})(\overline{D}.\overline{BE})$$

$$=(A+\overline{B}+\overline{C})[\overline{D}(\overline{B}+\overline{E})]$$

$$=(A+\overline{B}+\overline{C})(\overline{B}\overline{D}+\overline{D}\overline{E})$$

$$=A\overline{B}\overline{D}+A\overline{D}\overline{E}+\overline{B}\ \overline{B}\overline{D}+\overline{B}\overline{D}\overline{E}+\overline{B}\ \overline{C}\ \overline{D}+\overline{C}\overline{D}\overline{E}$$

$$=A\overline{B}\overline{D}+A\overline{D}\overline{E}+\overline{B}\ \overline{D}+\overline{B}\overline{D}\overline{E}+\overline{B}\ \overline{C}\ \overline{D}+\overline{C}\overline{D}\overline{E}$$

Standard SOP Form

A SOP Form in which each product term involves all the variables (complemented or uncomplemented).

Each individual term is referred to as a **minterm**.

Example $$Z=\overline{A}BC+A\overline{B}\,\overline{C}+A\overline{B}C$$

Express in the standard SOP form

$$Z = A + \overline{B}C$$

$$= A(B+\overline{B})(C+\overline{C}) + (A+\overline{A})\overline{B}\,C$$

$$= (AB + A\overline{B})(C+\overline{C}) + A\overline{B}C + \overline{A}\overline{B}C$$

$$= ABC + A\overline{B}C + AB\overline{C} + A\overline{B}\,\overline{C} + A\overline{B}C + \overline{A}\overline{B}C$$

$$= ABC + A\overline{B}C + AB\overline{C} + A\overline{B}\,\overline{C} + \overline{A}\overline{B}C$$

Product of Sum (POS)

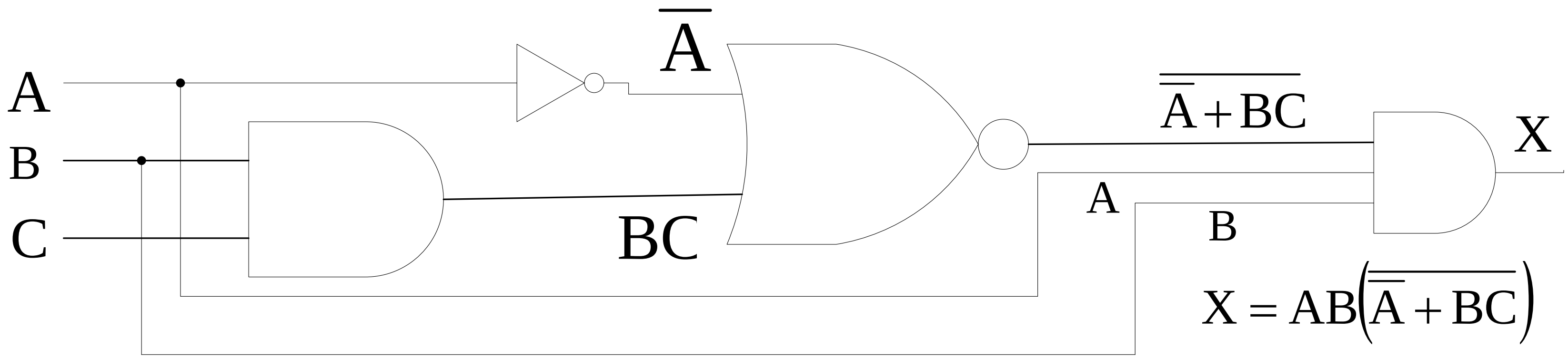
It consists of two or more OR terms (Sums) that are ANDed together. Each OR term contains one or more variables in complemented or uncomplemented form.

e.g. $(A+\overline{B}+C)(A+C)$

Standard POS : When a function is expressed as a POS, where each term consists of a sum, the sum involving all of the variables in either complemented or uncomplemented form.

e.g. $\mathbf{Z=(\overline{A}+B+\overline{C})(A+B+C)}$

Simplifying Logic Circuits



$$X = AB(\overline{\overline{A} + BC})$$

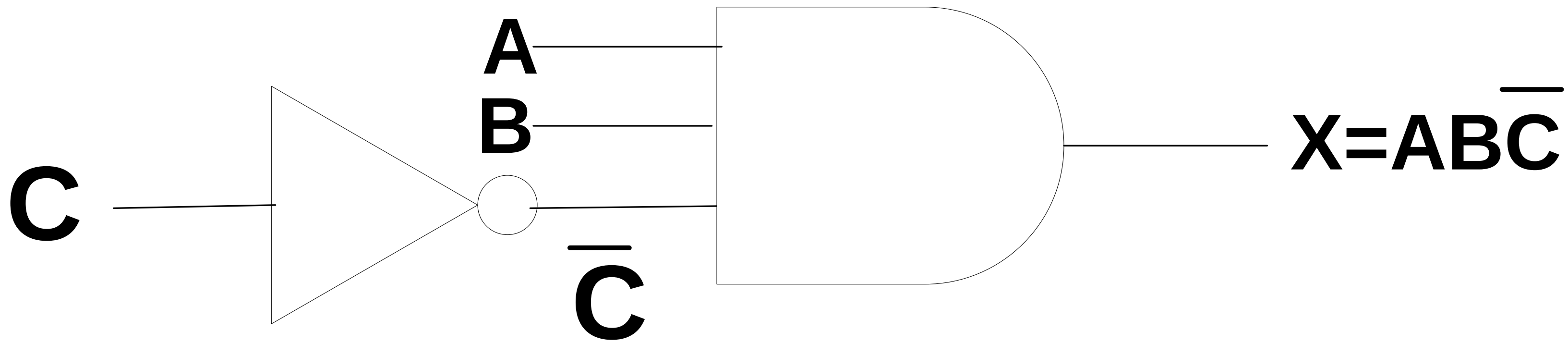
$$= AB(\overline{\overline{A}} \,.\, \overline{BC})$$

$$= AB[\, A \,.\, (\overline{B} + \overline{C})]$$

$$= AB\,A(\overline{B} + \overline{C})$$

$$= AB(\overline{B} + \overline{C})$$

$$= AB\overline{B} + AB\overline{C} = AB\overline{C}$$



A
B
C
$\overline{C}$
$X = AB\overline{C}$