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(3) Two rockets of proper length Lo, are approaching the earth from opposit directions at velocities $\frac{c}{2}$. Find out the length of on rocket with respect to the observer in the other rocket.

Solution :

According to the velocity addition therem:

$$u' = \frac{u - v}{1 - \frac{uv}{c^2}}.$$

where v : is the velocity of the 'prime' frame.

u : is the velocity of the rocket in rest frame.

In present case : let us calculate the velocity u' with respect to the frame in rocket 'A'.

Hence $v = \frac{c}{2}$, and $u = -\frac{c}{2}$ [velocity of 'B' with respect to earth].

Therefore : velocity of the rocket 'B' with respect to the rocket 'A' is

$$u' = \frac{-\frac{c}{2} - \frac{c}{2}}{1 + \frac{c^2}{4c^2}} = -\frac{4c}{5}$$

$$\boxed{u' = -\frac{4c}{5}}$$

Now proper length of the rocket 'B' is Lo.

Hence length of the rocket 'B' with respect to the observe in 'A' will be

$$L = \frac{L_0}{\gamma} = \frac{L_0}{\frac{1}{\sqrt{1 - \frac{u'^2}{c^2}}}} = \sqrt{1 - \left(\frac{16}{25}\right)}\, L_0$$

$$\approx \cancel{0.60\,L_0}\ 0.60\,L_0$$

Q.2

~~Initial~~

$E_\gamma = E_0$

$E'_\gamma = E'_0$

$p_\gamma = \frac{E_0}{c}$

$p'_\gamma = \frac{E'_0}{c}$

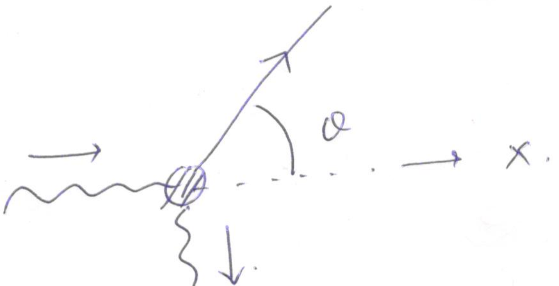
~~Final~~

$E_e = m_e c^2$

$E'_e = m'_e c^2$

$p_e = 0$

$p'_e = m'_e v'$; $m'_e = \frac{m_e}{\sqrt{1-\frac{v'^2}{c^2}}}$



Energy conservation: $E_0 + m_e c^2 = E'_0 + m'_e c^2 \quad \ldots (1)$

Momentum conservation: $\frac{E_0}{c} = \cancel{\frac{E'_0}{c}\cos\phi +} m'_e v' \cos\theta$

$(2) \longrightarrow \boxed{\frac{E_0}{c} = m'_e v' \cos\theta}$ along x-direction

$(3) \longrightarrow m'_e v' \sin\theta = \frac{E'_0}{c}$ along y-direction

$\boxed{p'_e \sin\theta = \frac{E'_0}{c}}$

For electron we know: $m'^2_e c^4 = c^2 p'^2_e + m_e^2 c^4 \Rightarrow E'_e = m'_e c^2 = T^e_{KE} + m_e c^2$

Therefore, kinetic engy of electron: $T^e_{KE} = E'_e - m_e c^2 = (m'_e c^2 - m_e c^2)$

using (1) $\Rightarrow \boxed{T^e_{KE} = (E_0 - E'_0)}$

Now from (2) & (3) $\Rightarrow p'^2_e c^2 = E'^2_0 + E_0^2$

From (1) $\Rightarrow E_0 + m_e c^2 - E'_0 = \sqrt{c^2 p'^2_e + m_e^2 c^4}$

$= \sqrt{E'^2_0 + E_0^2 + m_e^2 c^4}$

Squaring both side : $\boxed{E'_0 = \frac{E_0 m_e c^2}{E_0 + m_e c^2}}$

Therefore : $$\boxed{T_{KE}^{e} = E_0 - E_0' = \frac{E_0^2}{E_0 + m_e c^2}}$$

$$\frac{③}{②} \Rightarrow \tan\theta = \frac{E_0'}{E_0} = \frac{m_e c^2}{E_0 + m_e c^2}$$

$$\theta = \tan^{-1}\left[\frac{m_e c^2}{E_0 + m_e c^2}\right]$$

Numerical part

$$m_e c^2 = 9.11 \times 10^{-31} \times 9 \times 10^{16}\ \mathrm{kg\,m^2\,s^{-2}}$$

$$= 81.99 \times 10^{-15}\ \mathrm{J} = \frac{81.99}{1.6} \times \frac{10^{-15}}{10^{-19}}$$

$$= \frac{81.99}{1.6} \times 10^{4}\ \mathrm{eV} = 5.1 \times 10^{5}\ \mathrm{eV}$$

$$= 5.1 \times 10^{2}\ \mathrm{KeV}$$

$$T_{Ke}^{E} = \frac{(100)^2}{(100) + (500.1)} = \frac{10^4}{600.1} = 0.002 \times 10^4\ \mathrm{keV}$$

$$= 20\ \mathrm{eV}.$$

$$\theta = \tan^{-1}\left[\frac{5.1 \times 10^2}{100 + 500.1}\right] = \tan^{-1}\left[\frac{500.1}{600.1}\right] = \tan^{-1}[0.83]$$

$$= 0.69\ \mathrm{rad}$$

$$\approx 40^{\circ}.$$

Q.3 Uncertainity in position of an electron

$$\Delta x = 10^{-12}\ cm = 10^{-14}\ m.$$

Now from uncertainity relation:

$$\Delta p \geq \frac{\hbar}{2}\frac{1}{\Delta x}$$

Hence uncertainity in energy:

$$\boxed{\Delta E_{non\text{-}rel} = \frac{\Delta p^2}{2m_e}}$$ For non-relativistic case.

$$\boxed{\Delta E_{rel} = \sqrt{c^2 \Delta p^2 + m_e^2 c^4}}$$ for relativistic case.

using the numerical inputs

$$\left.\begin{aligned} \Delta E_{non\text{-}rel} &\simeq 9.6\ MeV. \\ \Delta E_{rel} &\approx 15.8\ MeV. \end{aligned}\right\}$$

Therefore, one 'MeV' electron can not exist inside the nucleus.

Any of the expression for energy, (relativistic or non-relati... award full marks.