Three-phase System with Delta Load

For a Y-connected load the line voltage is √3 times the phase voltage and line current is same as the phase current. In a delta connected system, the loads are connected back to back as shown in fig. 7.

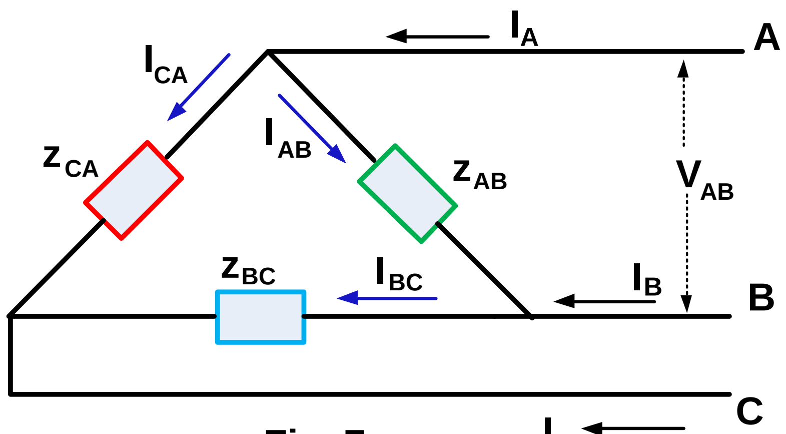

Fig. 7

The line voltages and the phase voltages are same. They are not distinguishable. The relation between the line and the phase quantities are

$$V_{AB} = V_L = V_p$$

$$\begin{aligned} I_A &= I_{AB} - I_{CA} \\ &= I_p L0^o - I_p L - 240^\circ \\ &= I_p \sin\omega t - I_p \sin(\omega t - 240^0) \\ &= I_p[\sin\omega t - (\sin\omega t.\cos 240^0 - \cos\omega t \cdot \sin 240^0) \\ &= I_p\left[\sin\omega t + \frac{1}{2}\sin\omega t - \frac{\sqrt{3}}{2}\cos\omega t\right] \\ &= \sqrt{3} I_P\left[\frac{\sqrt{3}}{2}\sin\omega t - \frac{1}{2}\cos\omega t\right] \\ &= \sqrt{3} I_p \sin(\omega t - 30^0) \end{aligned}$$

$$I_L = \sqrt{3} I_p$$

In a delta connected load the phase voltage and the line voltage are same. The line current is √3 times the phase current. One can convert a delta connected load to a star connected load. by using star-delta transformation.

Star-Delta Transformation

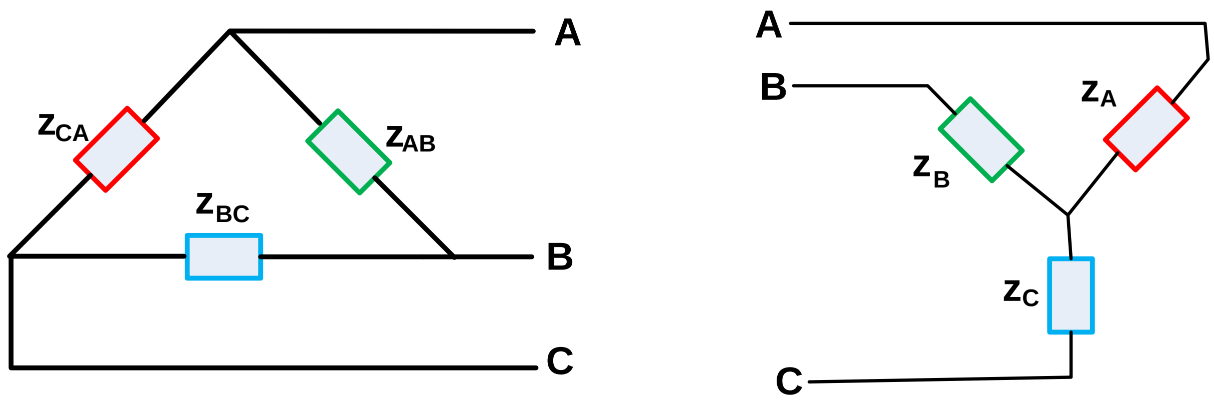

Fig. 8

The impedance seen between the lines A and B in the star connected load is $Z_A + Z_B$ (series the combination of Z_A and Z_B). In the delta connected load, the impedance seen between A and B is Z_{AB} in parallel with the series combination of Z_{CA} and Z_{BC}.

$$Z_A + Z_B = Z_{AB} \,||\, (Z_{BC} + Z_{CA})$$
$$= \frac{Z_{AB}(Z_{BC} + Z_{CA})}{Z_{AB} + Z_{BC} + Z_{CA}}$$

Similarly,

$$Z_B + Z_c = \frac{Z_{BC}(Z_{AB} + Z_{CA})}{Z_{AB} + Z_{BC} + Z_{CA}}$$
$$Z_c + Z_A = \frac{Z_{CA}(z_{AB} + Z_{BC})}{Z_{AB} + z_{13c} + Z_{CA}}$$

Solving these three equations, the star connected impedances can be represented with equivalent delta connected impedances as

$$Z_A = \frac{Z_{AB}Z_{CA}}{Z_{AB} + Z_{BC} + Z_{CA}}$$
$$Z_B = \frac{Z_{AB}Z_{BC}}{Z_{AB} + Z_{BC} + Z_{CA}}$$
$$Z_c = \frac{Z_{BC}Z_{CA}}{Z_{AB} + Z_{BC} + Z_{CA}}$$

Example: A balanced three-phase three-wire system has a line voltage of 500 V. Two balanced Y-connected loads are present. One is a capacitive load with 7-j2 per phase and the other is an inductive load of 4+j2 a per phase. Find (a) the phase voltage, (b) the line current, (c) the total power drawn by the load. (d) the power factor at which the source is operating.

Solution:

(a) As this is a Y-connected system, the phase voltage will be

$$v_p = \frac{V_L}{\sqrt{3}} = \frac{500}{\sqrt{3}} V = 288.67V$$

(b) Two loads are connected in parallel. The per phase load $\mathbf{Z_P}$ can be estimated as

$$Z_1 = 7 - j2 \qquad Z_2 = 4 + j2$$
$$Z_P = \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{32 + j6}{11}$$
$$I_P = I_L = \frac{V_P}{Z_P} = 97.53L - 10 \cdot 6^0$$

(c) Total power is

$$P = 3V_pI_p\cos\theta = 3 \times 288 \cdot 67 \times 97 \cdot 53 \cos(-10.6^0)$$
$$= 83 \text{ KW}$$

(d) The source power factor is

$$P \cdot F. = cos\theta = cos(-10 \cdot 6^0)$$
$$= 0 \cdot 983 \text{ lagging}$$

In part (a) and part (b) of the above question, the magnitudes of voltage and current were asked. This question can be modified by asking the phasor voltage, V_{BN} and the phasor current I_B. Given line voltage is V_{AB} is equal to 500 v.

Solution: The magnitude of the voltages and currents will be same as found in the previous case. The difference will be the extra information which is the phase angle of the voltage and the current. In this question V_{AB} is the reference phasor as its angle is zero. The phase voltage V_{BN} will lag the line voltage by an angle of 150 degree. Similarly, the line current I_B will lag the line voltage V_{AB} by an angle of 160.6 degree. Hence

$$V_{BN} = 288 \cdot 67L - 150^0 V$$
$$I_B = 97 \cdot 53L - 160.6^0 A$$

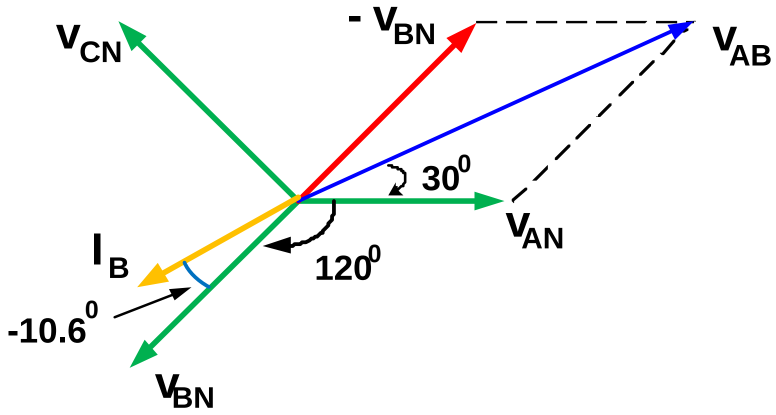

Fig. 9

Three-Phase Power Measurement

Wattmeter is the instrument used for power measurement. Fig. 10 shows a wattmeter (W) and its connection with a single-phase load. It has two coils, one is called as current coil (cc) and the other is the voltage coil (vc). The current coil is connected in series and the voltage coil is connected across the load or supply whose power is intended to be measured.

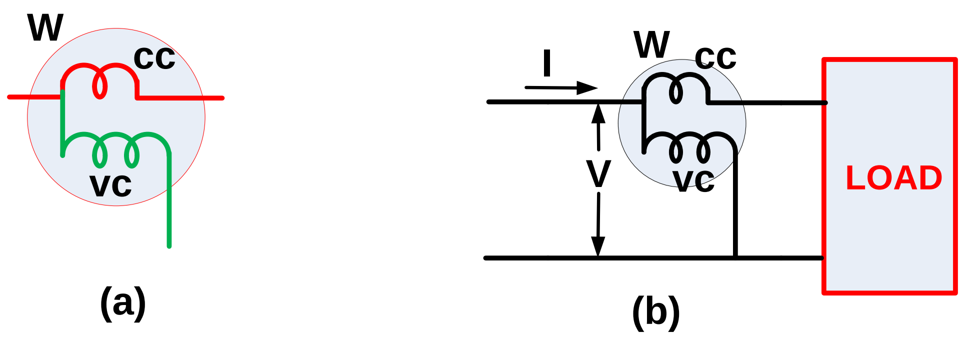

Fig. 10

For a three-phase system three wattmeters are required. Three-phase power can also be measured using two wattmeters. In addition to real power, the power factor and the reactive power can be estimated from the readings of the two wattmeters. Two wattmeter method for measurement of three-phase power is depicted in Fig. 11.

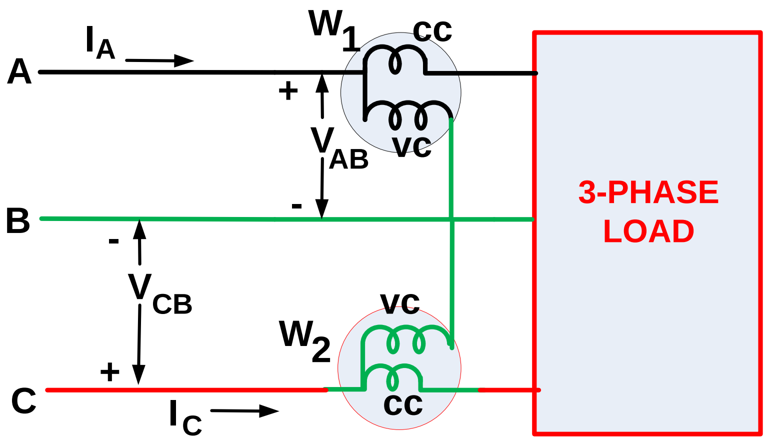

Fig. 11

The current I_A flows through the current coil of W_1 and V_{AB} is the voltage sensed by the voltage coil of W_1. Similarly, I_C flows through the current coil of W_2 and V_{CB} is the voltage across its voltage coil. From Fig.12, one can see

$$W_1 = V_{AB} I_A \cos(30^0 + \theta) = V_L I_L \cos(30^0 + \theta)$$
$$W_2 = V_{CB} I_c \cos(30^0 - \theta) = V_L I_L \cos(30^0 - \theta)$$
$$W_1 + W_2 = V_L I_L \{\cos(30^0 + \theta) + \cos(30^0 - \theta)\}$$
$$= V_L I_L \{2\cos 30^0 . \cos\theta\}$$
$$= V_L I_L \left\{2 \times \frac{\sqrt{3}}{2} \cdot \cos\theta\right\}$$
$$= \sqrt{3} V_L I_L \cos\theta = P = Total\ Power$$

$$W_1 - W_2 = V_L I_L sin\theta \qquad \tan\theta = \frac{1}{\sqrt{3}}\left(\frac{W_1 - W_2}{W_1 + W_2}\right)$$

$$\text{Power Factor} = \tan^{-1}\left\{ \frac{1}{\sqrt{3}}\left(\frac{W_1 - W_2}{W_1 + W_2}\right)\right\}$$

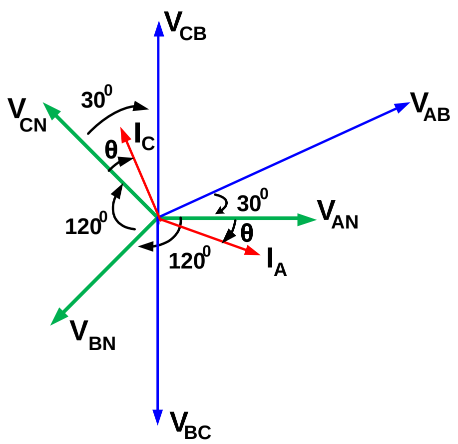

Fig. 12