EE 101: Electrical Sciences

Tutorial-11, Nov. 17,2017

Question 1(Pre-tutorial question):
For the network shown below, find

(a) The resonant frequency ω_0

(b) $Z_{in}(j\omega_0)$

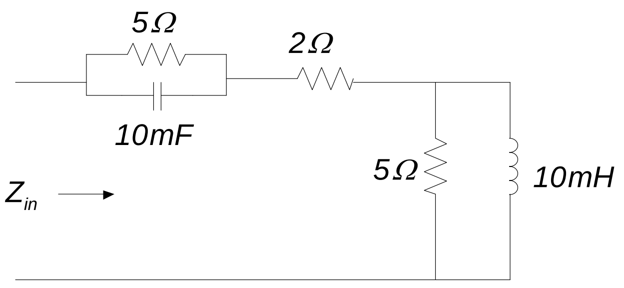

Figure 1

Solution: $Z_{in} = \dfrac{-j500}{5\omega - j100} + 2 + \dfrac{j5\omega}{500 + j\omega}$

(a) At resonance $I_m(Z_{in}) = 0$

$$\Rightarrow \frac{-2500\omega_0}{25\omega_0^2 + 10^4} + \frac{2500\omega_0}{25\times 10^4 + \omega_0^2} = 0$$

$$\Rightarrow -25\times 10^4 - \omega_0^2 + 25\omega_0^2 + 10^4 = 0$$

$$\Rightarrow \omega_0^2 = 10^4 \Rightarrow \omega_0 = 100rad/s$$

$$(b) Z_{in}(j\omega_0) = Re\{Z_{in}(j\omega_0)\} = 2 + \frac{5\times 10^4}{25\omega_0^2 + 10^4} + \frac{5\omega_0^2}{\omega_0^2 + 500}$$

$$= 2 + \frac{5}{26} + \frac{5}{26} = 2.385\Omega$$

Question 2: A series resonant network consists of a 50Ω resistor, a $4mH$ and a $0.1\mu F$ capacitor. Calculate the values of:

(a) ω_0

(b) f_0

(c) Q_0

(d) Bandwidth

(e) ω_1

(f) ω_2

(g) Z_{in} at 45 krad/s and

(h) The ratio of magnitudes of capacitor impedance to resistor at $45krad/s$.

Solution: $\omega_0 = \dfrac{1}{\sqrt{LC}} = 50krad/s$

$$f_0 = \frac{\omega_0}{2\pi} = 7.96kHz$$

$Q_0 = \omega_0 L / R = 4$

Bandwidth $B = \frac{\omega_0}{Q_0} = 12.5 krad / s$

$$\omega_2, \omega_1 = w_0 \left[1 + \left(\frac{1}{2Q_0} \right)^2 \mp \frac{1}{2Q_0} \right]$$

$\omega_2 = 56.65 krad / s,$

$\omega_1 = 44.15 krad / s$

$$Z_{in} = R_s + j \left(\omega L_s - \frac{1}{\omega C_s} \right)$$

At $\omega = 45 krad / s, \quad Z_{in} = 65.4 \angle -40.2^\circ$

$$\frac{Z_c}{R} = \frac{1}{\omega CR} = \frac{1}{45 \times 10^{-4} \times 50} = 4.44$$

Question 3: An AC-coupled amplifier is shown below.

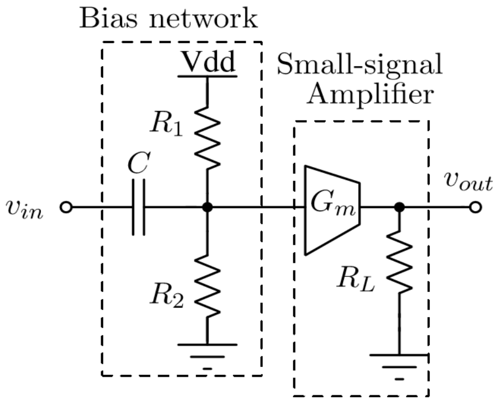

Figure 2: An AC-coupled amplifier

This circuit can be seen as a cascade of two networks. Assuming an ideal transconductor, derive the (frequency-dependent) transfer function of the above amplifier.

Solution:

Small signal voltage gain of the amplifier, $= \mathbf{G_m\ R_L}$

The resistors R_1 and R_2 come in parallel in the small-signal equivalent circuit. The parallel combination of the R_1 and R_2 forms a high-pass filter with the capacitor C.

Transfer function of this high-pass filter network is: $\frac{\mathbf{j\omega RC}}{\mathbf{1+j\omega RC}}$

Where, $\mathbf{R} = \frac{\mathbf{R_1 R_2}}{\mathbf{R_1 + R_2}}$

Overall transfer function = $\frac{\mathbf{j\omega RC}}{\mathbf{1+j\omega RC}}$ $\mathbf{x\ G_m\ R_L}$