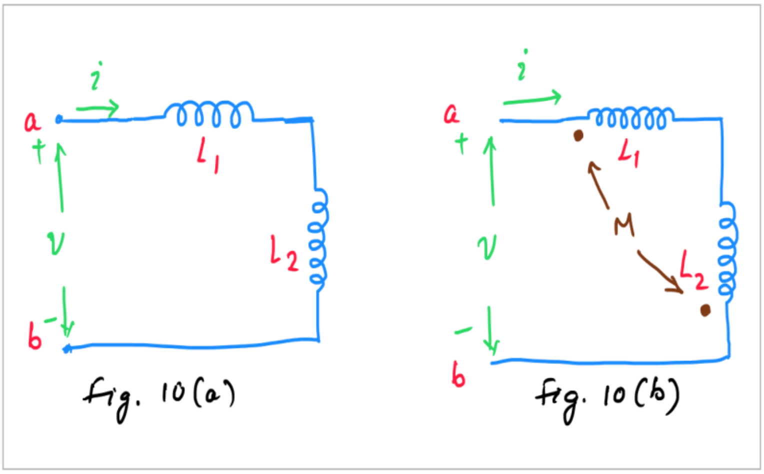
Equivalent Inductance

Equivalent inductance for more than one in inductors in series is the sum of their inductance values. Figure 10(a) shows two inductors in series. The equivalent inductance seen from the terminals ***ab*** is the sum of two inductance values ($L_{eq} = L_1 + L_2$). For finding this equivalent inductance, it is required to find the voltage current relationship. For the circuit shown in Fig. 10 (a),

$$v - L_1\frac{di}{dt} - L_2\frac{di}{dt} = 0$$
$$\Rightarrow v = (L_1 + L_2)\frac{di}{dt}$$
$$v = L_{eq}\frac{di}{dt} =\rangle L_{eq} = L_1 + L_2$$



In Fig. 10 (b), two series inductors are magnetically coupled. The mutual inductance is M. In this case, there will be two induced voltages. One will be induced in coil L_1 due to current in L_2 and the other induced voltage will be in coil L_2 due to current in L_1. As the cunnent in L_2 leaves the dotted terminal, its induced voltage in L_1 will have its negative polarity at its dotted terminal. Similarly the current enters the dotted terminal in L_1. Hence its induced voltage in L_2 will have its positive polarity at its dotted terminal. Applying KVL

$$v - L_1\frac{di}{dt} - L_2\frac{di}{dt} + M\frac{di}{dt} + M\frac{di}{dt} = 0$$
$$\Rightarrow v = (L_1 + L_2 - 2M)\frac{di}{dt}$$
$$l_{eq} = L_1 + L_2 - 2M$$

If the circuit consists of more than two inductors as mutually coupled inductors, then we can evaluate the equivalent inductance by finding the voltage and current relationship.

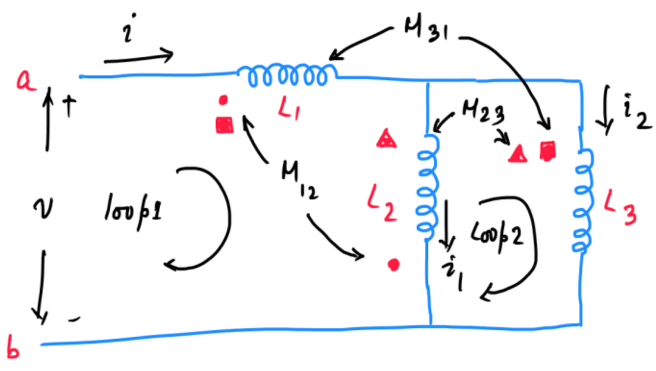


fig. 11

Fig. 11 shows a circuit with three magnetically coupled inductors. The self inductances of the three coils are L_1, L_2 and L_3. The mutual inductance between the coils L_1 and L_2 is M_{12}. Circular dots are used for this case. Triangular dots are used for mutual inductance M_{23} between the coils L_2 and L_3. Square dots are used for mutual inductance (M_{31}) between the coils L_3 and L_1. for finding the equivalent inductance at ab terminals, we can as e KVL for loops and loops.

$$i = i_1 + i_2$$

$$v = L_1\frac{di}{dt} + L_2\frac{di_1}{dt} - M_{12}\frac{di_1}{dt} - M_{12}\frac{di}{dt}$$
$$+M_{31}\frac{di_2}{dt} + M_{23}\frac{di_2}{dt}$$
$$L_2\frac{di_1}{dt} - L_3\frac{di_2}{dt} + M_{23}\frac{di_2}{dt} - M_{12}\frac{di}{dt}$$
$$-M_{31}\frac{di}{dt} - M_{23}\frac{di_1}{dt} = 0$$

Solving the three equations and finding the relation between ***v*** and ***i***, we can determine the equivalent inductance at ***ab*** terminal.

Energy in Mutually Coupled Circuits

For the circuit shown in Fig. 12, the right hand side is open circuited. The current i_1 is increased from zero to some constant value I_1 during the time interval $t = 0$ *to* t_1 Power delivered to L_1 from left is

$$v_1 i_1 = L_1\frac{di_1}{dt} i_1$$

Power entering from right side circuit is

$$v_2 i_2 = 0$$

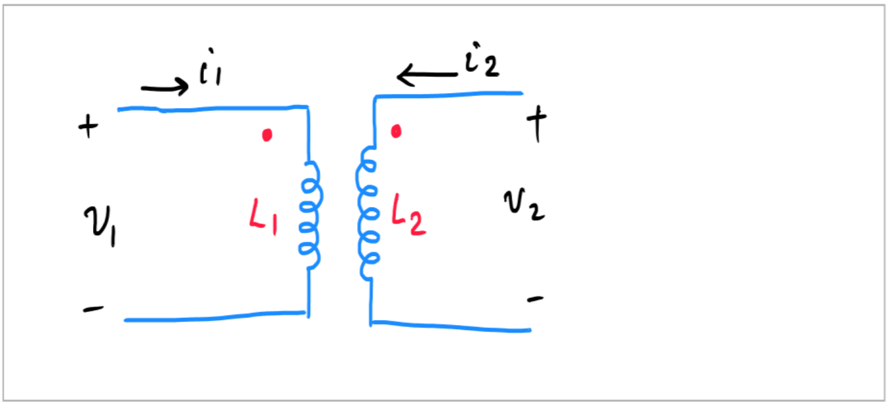


Fig. 12

Energy stoned in L_1 at $t = t_1$ is

$$\int_0^{t_1} v_1 i_1 dt = \int_0^{I_1} L_1 \frac{di_1}{dt} i_1 dt = \int_0^{I_1} L_1 i_1 di_1 = \frac{1}{2} L_1 I_1^2$$

Then i_1 is kept constant at a value I_1 during the time interval $t = t_1 \rightarrow t_2$ and allowing i_2 to change from 0 at $t = t_1$ to a constant value I_2 at $t = t_2$. The energy delivered to L_2 from the night side source will be

$$\int_{t_1}^{t_2} v_2 i_2 dt = \int_0^{I_2} L_2 i_2 di_2 = \frac{1}{2} L_2 I_2^2$$

During the time interval $t_1 \rightarrow t_2$, i_1 remains constant at I_1. It will not produce any induced voltage in L_2 as this is not a time varying current. But the current i_2 varies from 0 to I_2 in the internal t_1 to t_2. Hence, i_2 will produce an induced voltage in L_1 which can be given as V_1'

$$v_1^{/} = M \frac{di_2}{dt}$$

I_1 is the current through v_1'. As the current is entering into v_1', energy will be delivered to L_1 from the night hand sounee. This energy will be given as

$$\int_{t_1}^{t_2} v_1^{/} I_1 dt = \int_{t_1}^{t_2} M \frac{di_2}{dt} I_1 dt = M I_1 \int_0^{I_2} di_2 = M I_1 I_2$$

Total energy delivered by the right hand side source in the interval t_1 to t_2 will be

$$\frac{1}{2}L_2I_2^2 + MI_1I_2$$

Total energy delivered when ***i*,** and ***i*₂** have reached constant values during the interval ***t* = 0** to ***t*₂** will be

$$W_{Total} = \frac{1}{2}L_1I_1^2 + \frac{1}{2}L_2I_2^2 + MI_1I_2$$

If one current enters a dotted terminal while the other leaves a dotted terminal, the sign of the mutual energy term will be negative.

$$W_{Total} = \frac{1}{2}L_1I_1^2 + \frac{1}{2}L_2I_2^2 - MI_1I_2$$

Energy stored expression can be valid for instantaneous values of ***i*,** and ***i***$_2$.

$$W(t) = \frac{1}{2}L_1[(i_1(t)]^2 + \frac{1}{2}L_2[(i_2(t)]^2 \pm M[i_1(t)][i_2(t)]$$

Since ***W(t)*** represents the energy stored within a passive network, it cannot be negative for any value of ***i*,**, ***i***$_2$, ***L***$_1$, ***L***$_2$ or ***M***. An upper bound for the value of ***M*** can be found from the value of ***W(t)***.

$$W = \frac{1}{2}L_1i_1^2 + \frac{1}{2}L_2i_2^2 - Mi_1i_2$$
$$= \frac{1}{2}\left(\sqrt{L_1}i_1 - \sqrt{L_2}i_2\right)^2 + \sqrt{L_1L_2}i_1i_2 - Mi_1i_2$$

The first term is positive as it is a square term. Its minimum value is zero. With this condition the energy can be nonnegative if

$$\sqrt{L_1L_2} \geq M$$
$$\Rightarrow M \leq \sqrt{L_1L_2}$$

This is the upper limit for the magnitude of the mutual inductance. The degree to which ***M*** approaches its maximum value is described by the coefficient of coupling given as

$$K = Coeff.\ of\ Coupling = \frac{M}{\sqrt{L_1L_2}}$$
$$0 \leq k \leq 1$$