Department of
Electronics & Electrical Engineering

Lecture 8

Network Theorems for Sinusoidal Steady State Analysis

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Steps to Analyze AC Circuits

- The basic steps involved in applying network theorems to AC circuits are

  Step1: Transform the circuit to the phasor or frequency domain

  Step 2: Solve the problem using the circuit techniques such as nodal, analysis, mesh analysis, superposition theorem, etc.

  Step 3: Transform the resulting phasor to the time domain.

- The Step 1 is not necessary if the problem is specified in the frequency domain.

- In Step 2, the analysis is performed in the same manner as dc circuit analysis except that complex numbers are involved.

Nodal Analysis

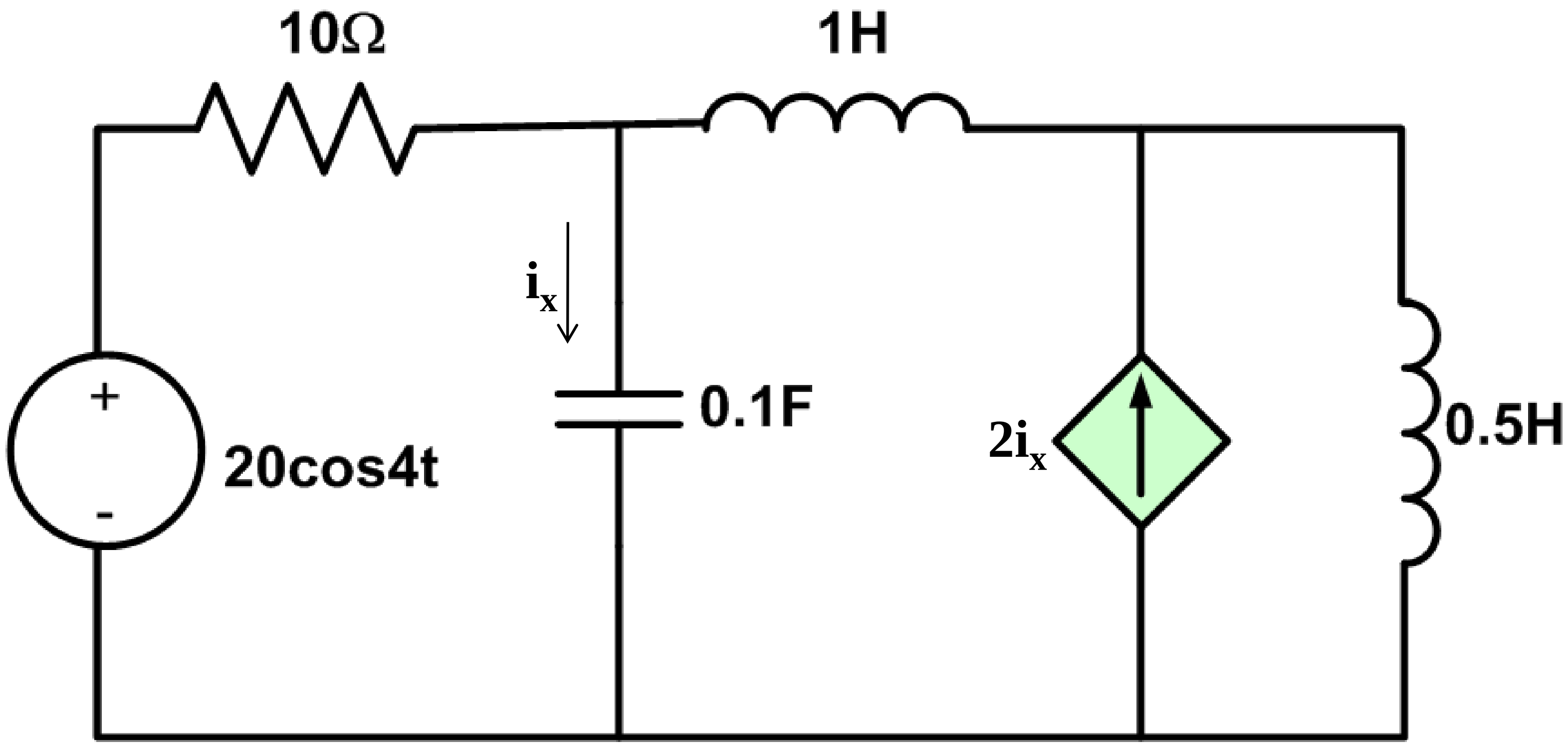

Fig.1: Network for Nodal Analysis

Nodal Analysis

- Convert the entire circuit to the frequency domain

$$20\cos 4t \Rightarrow 20\angle 0^{o},\ 1H \Rightarrow j\omega L = j4$$

$$0.5H \Rightarrow j\omega L = j2,\ \ 0.1F \Rightarrow \frac{1}{j\omega L} = -j2.5$$

$$where\ \ \omega = 4\ rad/s$$

- The frequency domain equivalent circuit is shown in Fig.2.

- Applying KCL at node 1

$$\frac{20 - V_1}{10} = \frac{V_1}{-j2.5} + \frac{V_1 - V_2}{j4} \qquad (1)$$

- The KCL at node 2

$$2I_x + \frac{V_1 - V_2}{j4} = \frac{V_2}{j2} \qquad (2)$$

Nodal Analysis

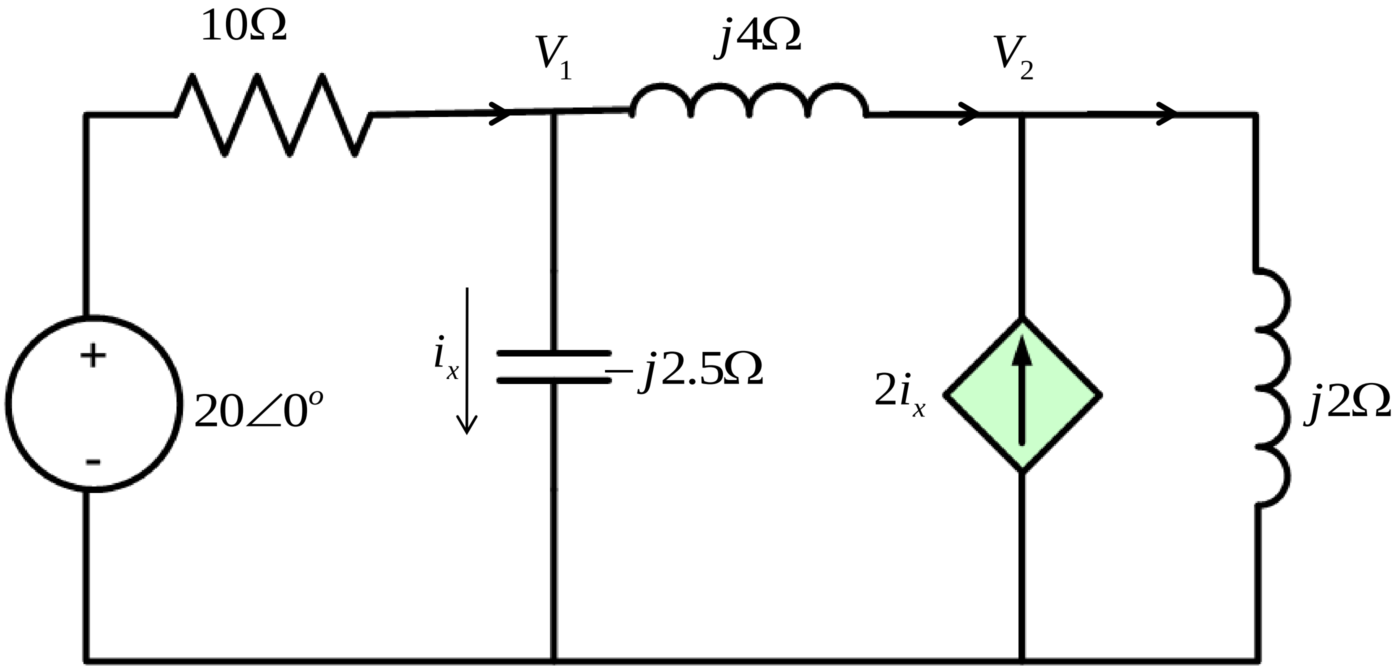

Fig.2: Frequency domain equivalent of the circuit in Fig.1

Nodal Analysis

- Solution of eq.1 and eq.2 gives

$$V_1 = 18.97\angle 18.43^o \ V$$

$$V_2 = 13.91\angle 198.3^o \ V$$

- The current

$$I_x = \frac{V_1}{-j2.5} = \frac{18.97\angle 18.43^o}{2.5\angle -90^o} = 7.59\angle 108.4^o A$$

- Transforming $\boldsymbol{i_x}$ to the time domain gives

$$i_x = 7.59\cos(4t + 108.4^o)A \qquad (3)$$

Mesh Analysis

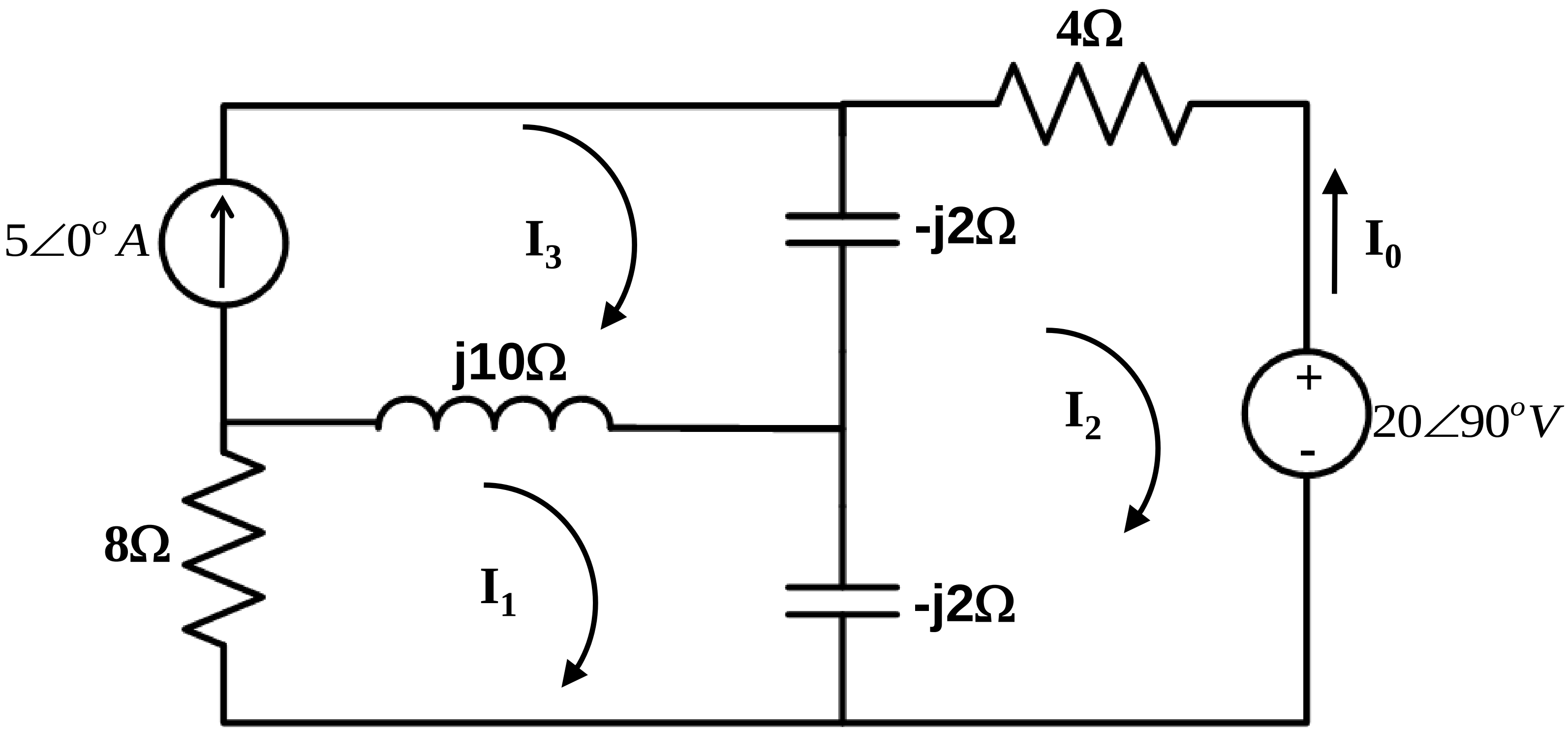

Fig.3: The network for nodal analysis

Mesh Analysis

- Applying KVL to mesh 1

$$\left(8+j10-j2\right)I_1-\left(-j2\right)I_2-j10I_3=0 \tag{3}$$

- The KVL for mesh 2 is

$$\left(4-j2-j2\right)I_2-\left(-j2\right)I_1-\left(-j2\right)I_3+20\angle 90^o=0 \tag{4}$$

- For mesh 3

$$I_3=5 \tag{5}$$

- Substituting eq.5 in eq.3 and eq.4 gives

$$\left(8+j8\right)I_1+j2I_2=j50 \tag{6}$$

$$j2I_1+\left(4-j4\right)I_2=-j20-j10 \tag{7}$$

- Solving eq6. and eq.7 gives

$$I_2=6.12\angle -35.22^o\,A$$

$$\Rightarrow I_0=-I_2=6.12\angle 144.78^o\,A$$

Superposition Theorem

- Since AC circuits are linear, the superposition theorem applies to AC circuits the same way it applies to dc circuits.
- ***The theorem becomes important if the circuit has sources operating at different frequencies. In this case, since the impedances depend in frequency, it is required to have a different frequency domain circuit for each frequency.***
- In case of sources with different frequencies, the total response must be obtained by adding the individual responses in time domain. It is incorrect to add the responses in the phasor or frequency domain

Superposition Theorem

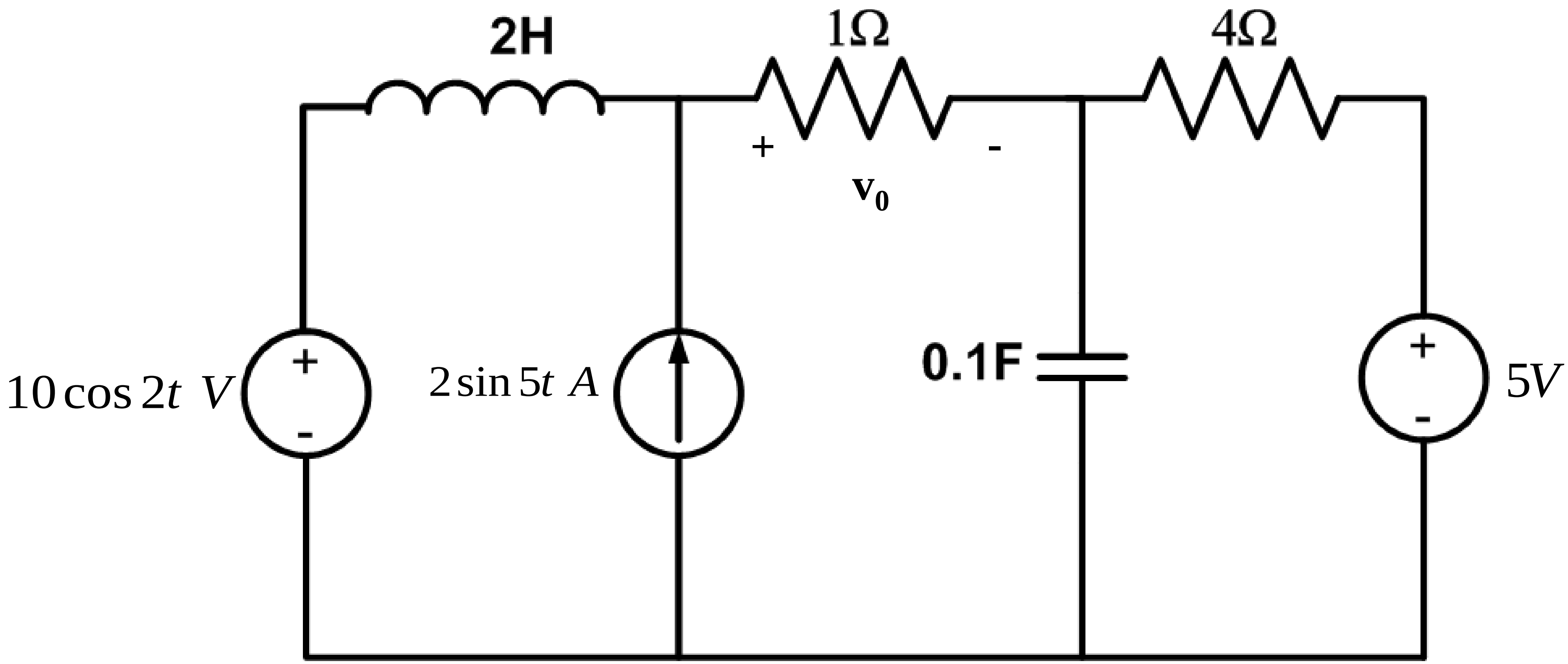

Fig.4: The Network for Superposition Theorem

Superposition Theorem

- Since the circuit operates at three different frequencies, the problem is divided into single frequency problems. Let

$$v_0 = v_1 + v_2 + v_3 \qquad (8)$$

$$where$$

$$v_1 \ is\ due\ to\ 5\ V\ \ dc\ voltage\ source$$

$$v_2\ \ is\ due\ to\ the\ 10\cos 2t\ \ V\ \ voltage\ source$$

$$v_3\ \ is\ due\ to\ the\ \ 2\sin 5t\ \ A\ current\ \ source$$

- To find v1, set all sources except the 5V dc source. In steady state, a capacitor is an open circuit to dc while an inductor is a short circuit to dc. The equivalent circuit is shown Fig.5a. By voltage division

$$-v_1 = \frac{1}{1+4} \times 5 = 1V \qquad (9)$$

Superposition Theorem

- To find v_2, the 5v voltage source is open circuited and the current source is short circuited. The equivalent circuit is shown in Fig.5b.

$$10\cos 2t \Rightarrow 10\angle 0^o,\ \ \omega\text{=2rad/s}$$

$$2H \Rightarrow j\omega L = j4\Omega$$

$$0.1F \Rightarrow \frac{1}{j\omega C} = -j5\Omega$$

The parallel combination of -j5Ω and 4Ω is

$$Z = \frac{-j5\times 4}{4-j5} = 2.439 - j1.951$$

- By voltage division

$$V_2 = \frac{1}{1+j4+Z}\times\left(10\angle 0^o\right) = \frac{10}{3.439+j2.049} = 2.498\angle -30.79^o$$

In time domain

$$v_2 = 2.498\cos(2t-30.79^o) \qquad (10)$$

Superposition Theorem

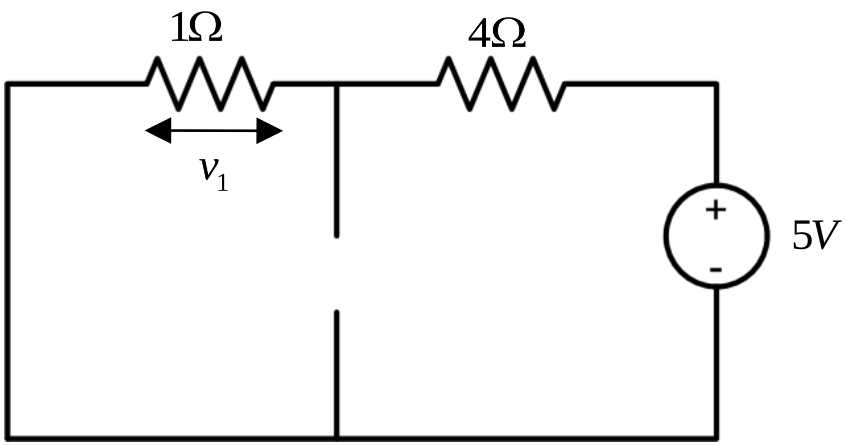

Fig.5a:

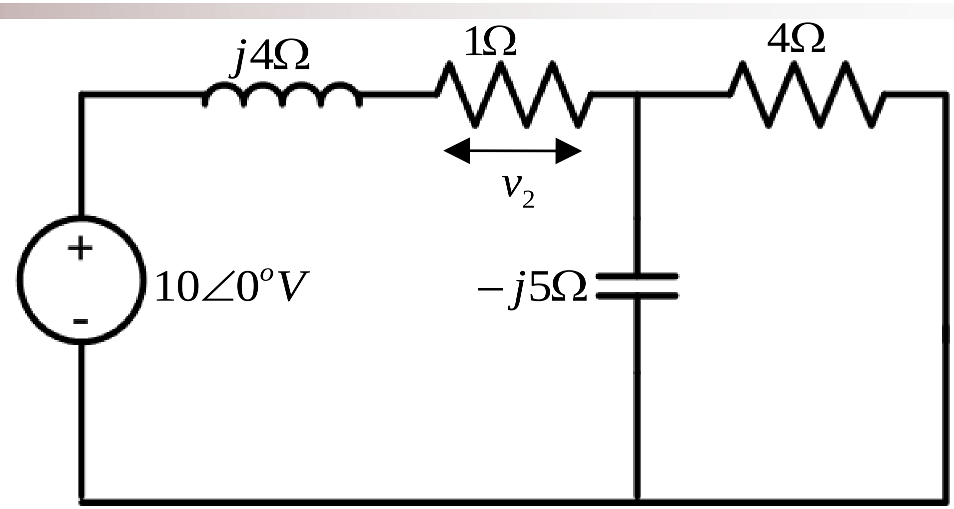

Fig.5b:

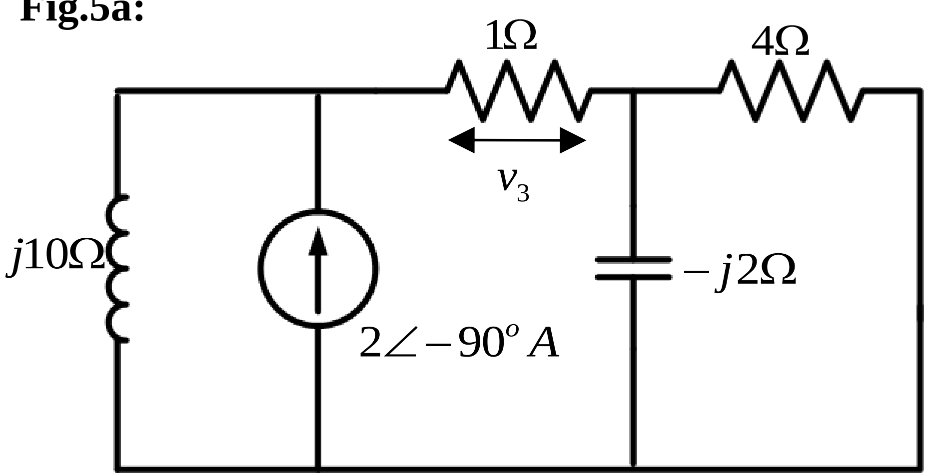

Fig.5c:

Superposition Theorem

- To obtain v3, set the voltage sources to zero (Fig.5c) and transform what is left to the frequency domain

$$2\cos 5t \Rightarrow 2\angle -90^o,\ \omega\text{=5rad/s}$$

$$2H \Rightarrow j\omega L = j10\Omega$$

$$0.1F \Rightarrow \frac{1}{j\omega C} = -j2\Omega$$

The parallel combination of -j2Ω and 4Ω is

$$Z = \frac{-j2\times 4}{4-j2} = 0.8 - j1.6$$

- By current division

$$I_3 = \frac{j10}{j10+1+Z}\left(2\angle -90^o\right)A,\ \ v_3 = I_1\times 1 = \frac{j10}{1.8+j8.4}\left(-j2\right) = 2.328\angle -80^o V \qquad (11)$$

- The final output is summation of eq.9, eq.10 and eq.11

$$v_0(t) = -1 + +2.498\cos(2t - 30.79^o) + 2.33\sin(5t + 10^o)V \qquad (12)$$

Thevenin and Norton Equivalent Circuits

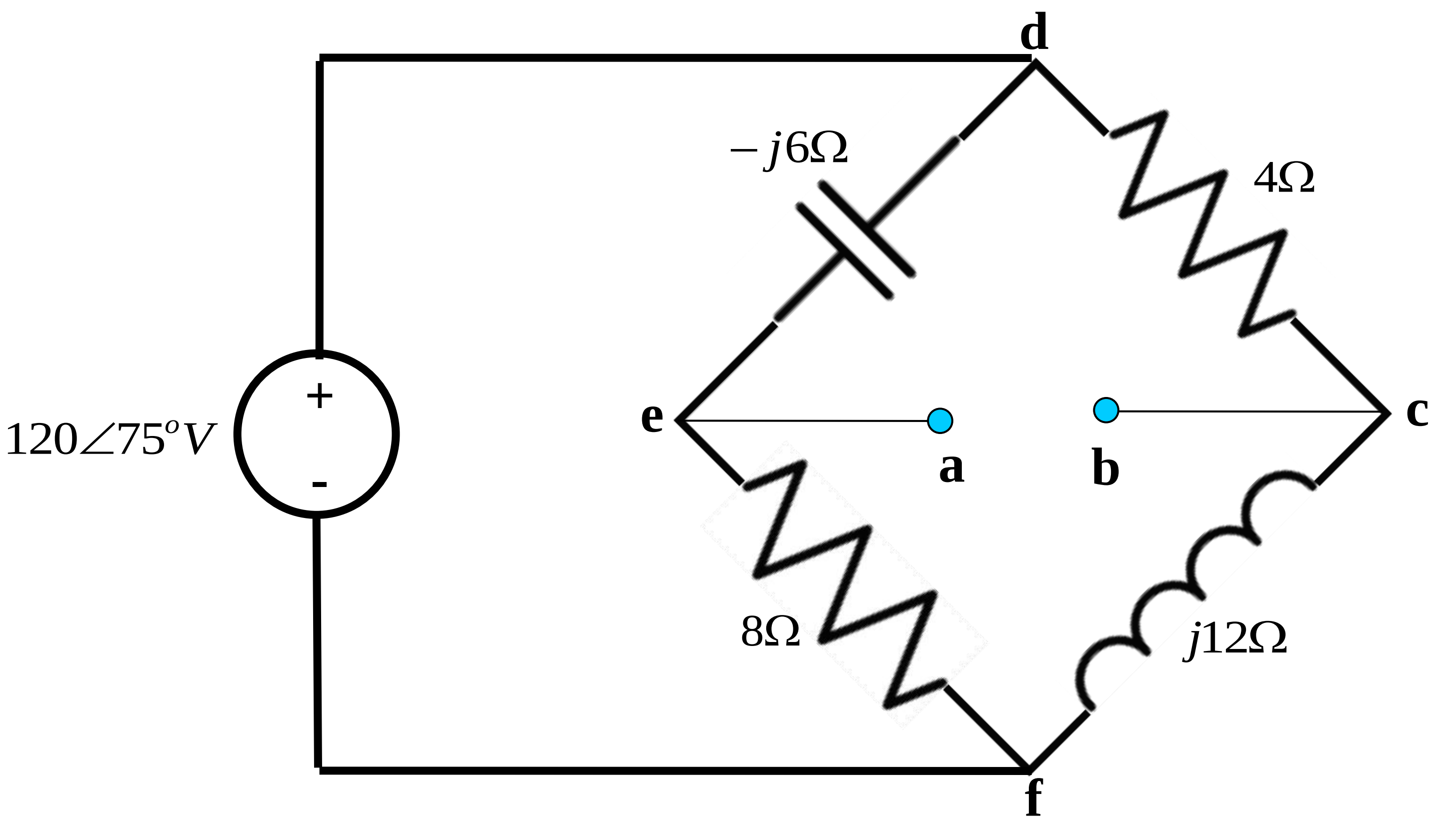

Fig.6: The Network for Thevenin and Norton's Theorem

Thevenin and Norton's Theorem

- The value of Thevenin's impedance Zth is obtained by setting the voltage source to zero, Fig.7a. From Fig.7a it is seen that 8Ohm and –j6Ohm are in parallel:

$$Z_1 = \frac{-j6\times 8}{8-j6} = 2.88 - j3.84\Omega \qquad (13)$$

- The 4Ohm resistance is in parallel with the j12 reactance:

$$Z_2 = \frac{j12\times 4}{4+j12} = 3.6 + j1.2\ \Omega \qquad (14)$$

- The Thevenin impedance is the series combination of Z1 and Z2, i.e.

$$Z_{th} = Z_1 + Z_2 = 6.48 - j2.64 \qquad (15)$$

- To find Vth, consider the circuit in Fig.7b, currents I1 and I2 are obtained as

$$I_1 = \frac{120\angle 75^o}{8-j6} A,\ I_2 = \frac{120\angle 75^o}{4+j12} \qquad (16)$$

Thevenin and Norton's Theorem

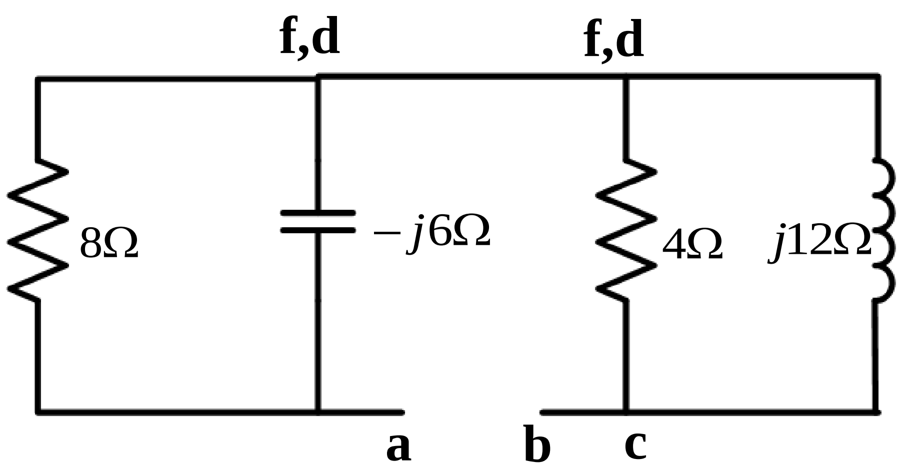

Fig.7a: The Network for Thevenin Theorem for finding $\mathbf{Z_{th}}$

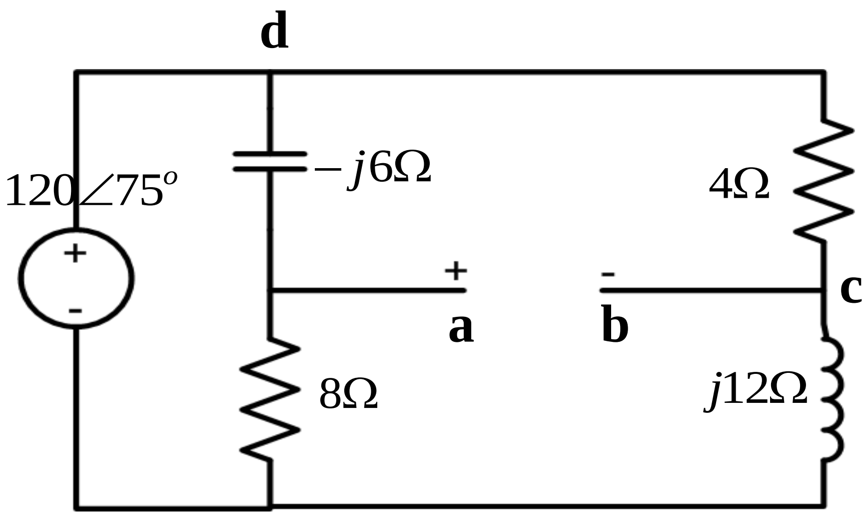

Fig.7b: The Network for Thevenin Theorem for finding $\mathbf{V_{th}}$

Thevenin and Norton's Theorem

- Applying KVL around loop **bcdeab** in Fig.7b gives

$$V_{th} - 4I_2 + (-j6)I_1 = 0 \qquad (17)$$

$$V_{th} = 4I_2 + j6I_1 = \frac{480\angle 75^o}{4+j12} + \frac{720\angle 75^o + 90^o}{8-j6}$$

$$= 37.95\angle 3.43^o + 72\angle 201.87^o = 37.95\angle 220.31^o \ V$$

Example 1

- Compute V1 and V1 in the network shown in Fig.8

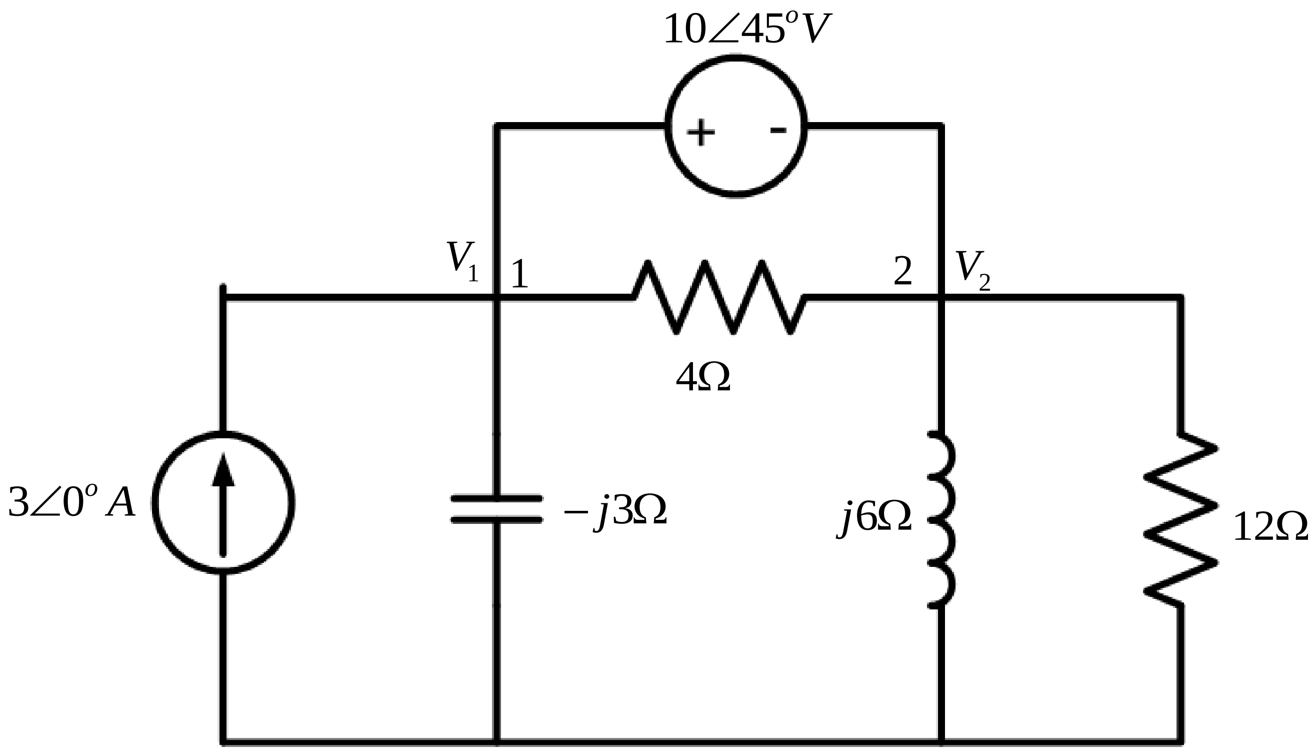

Fig.8: The Network for Example 1

Example 1

- Node 1 and Node 2 form a supernode, Fig.9.

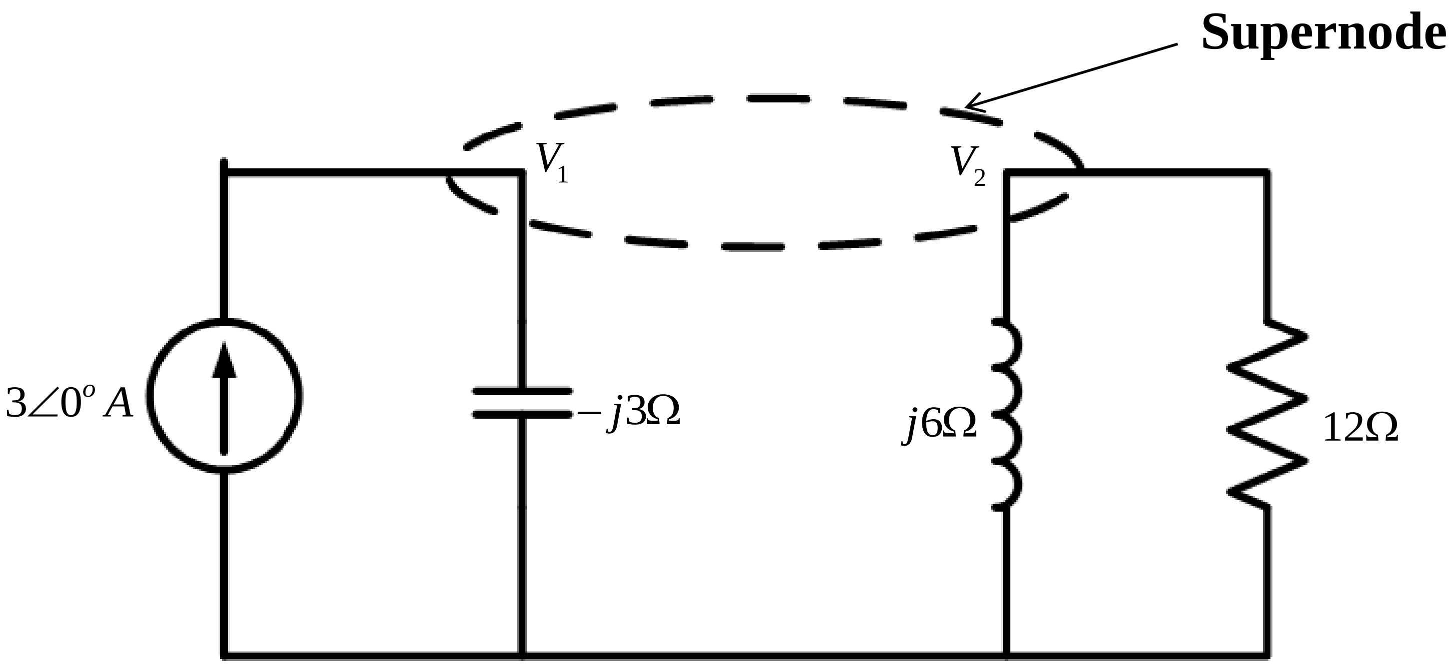

Fig.9: Supernode

Example 1

- Applying KCL to supernode gives

$$3 = \frac{V_1}{-j3} + \frac{V_2}{j6} + \frac{V_2}{12}$$

$$36 = j4V_1 + (1 - j2)V_2 \qquad (18)$$

- A voltage source is connected between nodes 1 and 2, hence

$$V_1 = V_2 + 10\angle 45^o \qquad (19)$$

- Substituting eq.19 into eq.18 gives

$$36 - 40\angle 135^o = \left(1 + j2\right)V_2$$

$$V_2 = 31.41\angle -87.18^o V$$

$$V_1 = V_2 + 10\angle 45^o = 25.78\angle -70.48^o$$