Quantum Mechanics
Tutorial-2

1) At time $t = 0$, a particle is represented by the wave function

$$\psi(x,0) = \begin{cases} \frac{Ax}{a}, & \text{if } 0 \leq x \leq a \\ \frac{A(b-x)}{(b-a)}, & \text{if } a \leq x \leq b \\ 0 & \text{otherwise} \end{cases}$$

where $A$, $a$, and $b$ are constants.
(a) Normalize $\psi(x,0)$.
(b) Sketch $\psi(x,0)$, as a function of $x$.
(c) Where is the particle most likely to be found, at $t = 0$. ?
(d) What is the probaility of finding the particle to the left of $a$.
(e) What is the expectation value of $x$.

2) A particle of mass $m$ is in the state $\psi = Ae^{-a(\frac{mx^2}{\hbar}+it)}$.
(a) Find A.
(b) For what potential energy function $V(x)$ does $\psi$ satisfy the Schrödinger equation.
(c) Compute the uncertainties $\Delta x$ and $\Delta p$. Show that the product is consistent with the uncertainty relation.
(d) Calculate the corresponding probaility current.

3) A free particle has the initial wave function $\psi(x,0) = Ae^{-\alpha|x|}$ where $A$ and $\alpha$ are positive constant.
(a) Normalize $\psi(x,0)$.
(b) Find the momentum space wavefunction $\phi(p)$ using the inverse Fourier transformation $\phi(p) = \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{\infty} \psi(x,0)e^{-ipx/\hbar}$ .
(c) Show that in the momentum space $\phi(p)$ is normalized to unity.