Question 1
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Question: Write the matrix $\begin{bmatrix}1 & 1\\1 & 2\end{bmatrix}$ as a product of elementary matrices.

Solution 1:

$$A = \begin{bmatrix}1 & 1\\1 & 2\end{bmatrix} \xrightarrow{R_2-R_1} \begin{bmatrix}1 & 1\\0 & 1\end{bmatrix} \xrightarrow{R_1-R_2} \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$$

$\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} \xrightarrow{R_2-R_1} \begin{bmatrix}1 & 0\\-1 & 1\end{bmatrix} = E_1$ and $\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} \xrightarrow{R_1-R_2} \begin{bmatrix}1 & -1\\0 & 1\end{bmatrix} = E_2$ **1 mark**

So $E_2E_1A = I \Rightarrow A = E_1^{-1}E_2^{-1} = \begin{bmatrix}1 & 0\\1 & 1\end{bmatrix}\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}$ **1 mark**

Solution 2:

$$\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} \xrightarrow{R_1+R_2} \begin{bmatrix}1 & 1\\0 & 1\end{bmatrix} \xrightarrow{R_2+R_1} \begin{bmatrix}1 & 1\\1 & 2\end{bmatrix} = A$$

$\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} \xrightarrow{R_2+R_1} \begin{bmatrix}1 & 0\\1 & 1\end{bmatrix} = E_1$ and $\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} \xrightarrow{R_1+R_2} \begin{bmatrix}1 & 1\\0 & 1\end{bmatrix} = E_2$ **1 mark**

So $A = E_1E_2 = \begin{bmatrix}1 & 0\\1 & 1\end{bmatrix}\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}$ **1 mark**

1 mark has been deducted for writing $A = IE_1E_2$.

Solution 3:

$$\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} \xrightarrow{R_1+R_2} \begin{bmatrix}1 & 1\\0 & 1\end{bmatrix} \xrightarrow{R_2+R_1} \begin{bmatrix}1 & 1\\1 & 2\end{bmatrix} = A$$

So $A = E_{R_2+R_1}E_{R_1+R_2}$ **1 mark**

Solution 4:

$\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} \xrightarrow{R_2+R_1} \begin{bmatrix}1 & 0\\1 & 1\end{bmatrix}$ and $\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} \xrightarrow{R_1+R_2} \begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}$

Now $\begin{bmatrix}1 & 1\\1 & 2\end{bmatrix} = \begin{bmatrix}1 & 0\\1 & 1\end{bmatrix}\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}$ **2 marks**

1 mark has been deducted for showing exactly one factor matrix is elementary.

No marks given for not showing the factor matrices are elementary.

Question: Write the matrix $\begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$ as a product of elementary matrices.

Solution 1:

$$A = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix} \xrightarrow{R_2 - R_1} \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \xrightarrow{R_1 - 2R_2} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \xrightarrow{R_2 - R_1} \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} = E_1$ and $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \xrightarrow{R_1 - 2R_2} \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix} = E_2$ **1 mark**

So $E_2 E_1 A = I \Rightarrow A = E_1^{-1} E_2^{-1} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ **1 mark**

Solution 2:

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \xrightarrow{R_1 + 2R_2} \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \xrightarrow{R_2 + R_1} \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix} = A$$

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \xrightarrow{R_2 + R_1} \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = E_1$ and $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \xrightarrow{R_1 + 2R_2} \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = E_2$ **1 mark**

So $A = E_1 E_2 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ **1 mark**

1 mark has been deducted for writing $A = I E_1 E_2$.

Solution 3:

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \xrightarrow{R_1 + 2R_2} \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \xrightarrow{R_2 + R_1} \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix} = A$$

So $A = E_{R_2 + R_1} E_{R_1 + 2R_2}$ **1 mark**

Solution 4:

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \xrightarrow{R_2 + R_1} \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ and $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \xrightarrow{R_1 + 2R_2} \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

Now $\begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ **2 marks**

1 mark has been deducted for showing exactly one factor matrix is elementary.

No marks given for not showing the factor matrices are elementary.