# PH 101
# Tutorial-4
# Date: 28/08/2017

1. A simple way to measure the speed of a bullet is with a ballistic pendulum. As illustrated in Fig. 1, this consists of a wooden block of mass $M$ into which the bullet is shot. The block is suspended from cables of length $l$, and the impact of the bullet causes it to swing through a maximum angle $\phi$, as shown. The initial speed of the bullet is $v$, and its mass is $m$.
(a) How fast is the block moving immediately after the bullet comes to rest? (Assume that this happens quickly.)
(b) Show how to find the velocity of the bullet by measuring $m$, $M$, $l$, and $\phi$.

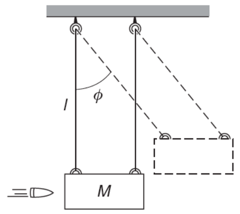

Fig. 1

2. A block of mass M on a horizontal frictionless table is connected to a spring (spring constant $k$). The block is set in motion so that it oscillates about its equilibrium point with a certain amplitude $A_0$ . The period of motion is $T_0 = 2\pi\sqrt{M/k}$
(a) A lump of sticky putty of mass $m$ is dropped onto the block. The putty sticks without bouncing. The putty hits $M$ at the instant when the velocity of $M$ is zero. Find
(1) The new period.
(2) The new amplitude.
(3) The change in the mechanical energy of the system.
(b) Repeat part (a), but this time assume that the sticky putty hits $M$ at the instant when $M$ has its maximum velocity.

3. A particle of mass $m$ and velocity $v_0$ collides elastically with a particle of mass $M$ initially at rest and is scattered through angle $\Theta$ in the center of mass C system.
(a) Find the final velocity of $m$ in the laboratory L system.
(b) Find the fractional loss of kinetic energy of $m$.

4. A "superball" of mass $m$ bounces back and forth with speed $v$ between two parallel walls, as shown in Fig. 2. The walls are initially separated by distance $l$. Gravity is neglected and the collisions are perfectly elastic.
(a) Find the time-average force $F$ on each wall.
(b) If one surface is slowly moved toward the other with speed $V \ll v$, the bounce rate will increase due to the shorter distance between collisions, and because the ball's speed increases when it bounces from the moving surface. Show that $\frac{dv}{dt} = \frac{vV}{x}, \frac{dv}{dx} = -\frac{v}{x}$ and find $v(x)$.
(c) Find the average force at distance $x$.

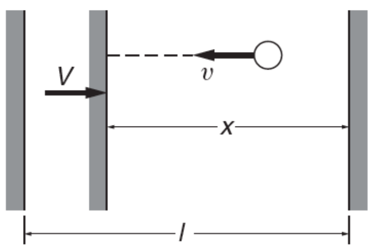

Fig. 2

5. Two balls, of mass m and mass 2m, approach from perpendicular directions with identical speeds $v$ and collide. After the collision, the more massive ball moves with the same speed $v$ but downward, perpendicular to its original direction (Refer to Fig. 3). The less massive ball moves with speed $U$ at an angle $\theta$ with respect to the horizontal. Assume that no external forces act during the collision.

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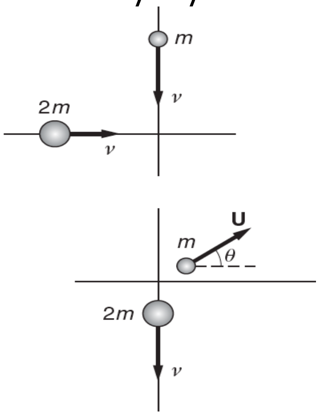



Fig. 3

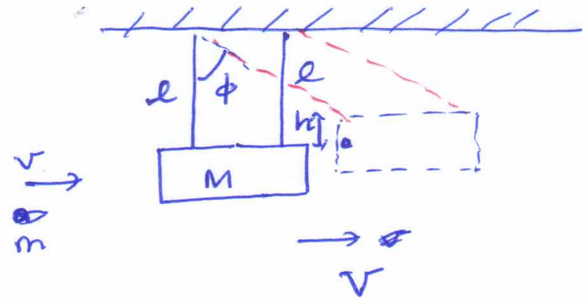
(a) Calculate the final speed U of the less massive ball and the angle θ.
(b) Determine how much kinetic energy is lost or gained by the two balls during the collision. Is this collision elastic, inelastic, or superelastic?

6. (a) A particle executes Simple Harmonic motion and is located at $x = a, b$ and $c$ at time $t_0$, $2t_0$ and $3t_0$ respectively. Show that the frequency of oscillation is $\frac{1}{2\pi t_0} cos^{-1}\frac{a+c}{2b}$. [Hint: take the SHM to be of the form, $x = A\, sin(\omega t)$]
(b) A cylinder of mass $m$ is allowed to roll on a smooth horizontal table with a spring of spring constant $k$ attached to it so that it executes SHM about the equilibrium position. Find the time period of oscillations.

7. A U-tube with cross-section $A$, is filled with a liquid (having density $\rho$ and total mass $M$), the total length of the liquid column being $h$. If the liquid on one side is slightly depressed by blowing gently down, the levels of the liquid will oscillate about the equilibrium position before finally coming to rest. (a) Show that the oscillations are SHM. (b) Find the period of oscillations.

1.

During the collision, linear momentum is conserved, not the mechanical energy.




(a)

$$P_i = mv$$

$$P_f = (m+M)V$$

$$P_i = P_f \Rightarrow V = \frac{m}{m+M}v \quad , \quad V: \text{ speed of the block}$$

(b)

After the collision, energy is conserved as the block rises.

$$K_i = \frac{1}{2}(m+M)V^2$$

$$V_i = 0$$

$$\Rightarrow \boxed{E_i = \frac{1}{2}(m+M)V^2 = \frac{1}{2}\frac{m^2}{m+M}v^2}$$

$$K_f = 0 \quad , \quad V_f = (m+M)gh$$

$$\therefore \quad E_f = (m+M)gh$$

$$E_i = E_f \Rightarrow v^2 = 2\left(\frac{m+M}{m}\right)^2 gh$$

Again, $h = l(1-\cos\phi)$

Thus, $$v = \left(\frac{m+M}{m}\right)\sqrt{2gl(1-\cos\phi)}$$

clearly, knowing m, M and $l$ and measuring $\phi$ gives the speed $v$ of the bullet.

2.



(a) (1) The original period is $T_0 = 2\pi\sqrt{\frac{M}{k}}$

As the new mass is $(m+M)$, the new period is

$$T_{new} = 2\pi\sqrt{\frac{m+M}{k}}$$
$$= T_0\sqrt{\frac{m+M}{M}}$$

(2) The lump $m$ sticks at the extreme of the motion, so the amplitude is unchanged. It should be noted that the lump transfers no horizontal momentum to $M$.

(3) The mechanical energy is $E = \frac{1}{2}kA_0^2$. Because the amplitude is unchanged, the mechanical energy is also unchanged.

(b) (1) The mass is $(m+M)$, so $T = T_0\sqrt{\frac{m+M}{M}}$

(2) In this case, linear momentum is conserved, but not the mechanical energy, when the putty sticks. Say $V$ be the speed just before collision and $V'$ be the speed just after collision

Then, $(m+M)V' = MV$

$\therefore$ The new mechanical energy is, $E' = \frac{1}{2}(m+M)V'^2$

$$= \frac{1}{2}MV^2\left(\frac{M}{m+M}\right)$$
$$= \frac{1}{2}kA^2$$

where $A$ is the new amplitude. Hence:

$$\frac{1}{2}kA^2 = \frac{1}{2}MV^2\left(\frac{M}{m+M}\right) = \frac{1}{2}kA_0^2\left(\frac{M}{m+M}\right)$$

$$\Rightarrow \quad A = A_0\sqrt{\frac{M}{m+M}}$$

(3) $E' = \frac{1}{2}MV^2\left(\frac{M}{m+M}\right) = E\left(\frac{M}{m+M}\right)$

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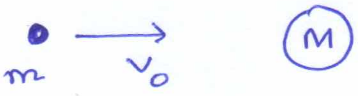
3.

(a) To convert from Laboratory (L) to centre of mass (C) co-ordinate system, subtract the centre of mass velocity $(\vec{\dot{R}} =) \vec{V}_c$ from every L system velocity vector



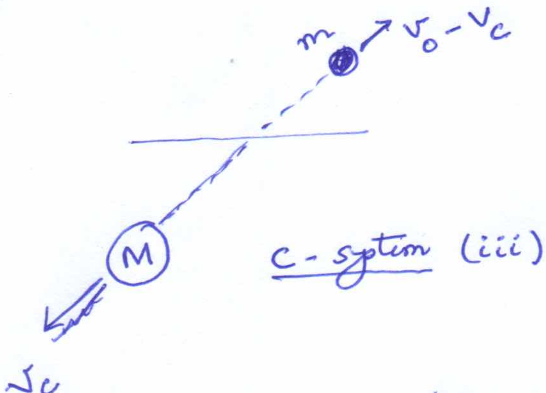

L- system (i)


C - system (ii)

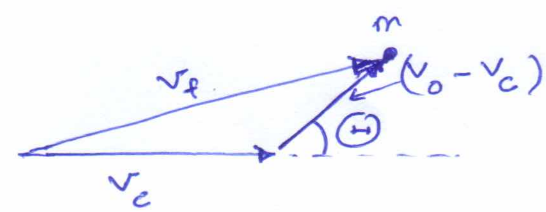
$$V_c = \frac{m v_0}{m+M}$$

(sketch (iii))

In an elastic collision, the speeds in C are unchanged. To convert from C to L, add $\vec{V}_c$ to every velocity vector, as shown for mass m (sketch iv)




C - system (iii)




L - system (iv)

$v_f$ is the velocity of m in the L system after the collision.

$$v_0 - V_c = \frac{M v_0}{m+M}$$

$$v_f^2 = V_c^2 + (v_0 - V_c)^2 - 2(v_0 - V_c) V_c \cos(\pi - \Theta)$$

$$\Rightarrow \quad v_f = \left(\frac{v_0}{m+M}\right) \sqrt{m^2 + M^2 + 2mM \cos\Theta}$$

(b) $K_0 = \frac{1}{2} m v_0^2$, $K_f = \frac{1}{2} m v_f^2$

$$\therefore \quad \frac{K_0 - K_f}{K_0} = \frac{2mM(1 - \cos\Theta)}{(m+M)^2}$$

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(Please see Lecture Notes (by A.K. Sarma) 5 and 6, page no. 22 - 24)

4. (4)

(a) The time-average force is the average rate of momentum transfer to a wall. Consider the situation when the walls are stationary.

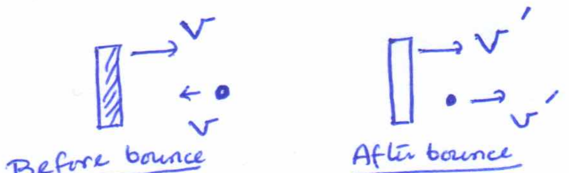
In a single collision (elastic)

$$\Delta p = 2mv$$

The time between the collisions is $\Delta T = \dfrac{2l}{v}$

$\therefore$ The average force, $\bar{F} = \dfrac{\Delta p}{\Delta T} = \dfrac{mv^2}{l}$

(b) Consider now the case when one wall is moving. To an observer moving with the wall, the superball approaches with speed $v+V$ and leaves with the same speed (elastic collision)




Lab frame


moving frame

Converting back to the Lab frame by adding $V$ we obtain:

$$v' = (v+V)+V$$
$$= v+2V$$

$\therefore \Delta v = v' - v = 2V$

Time between collision is

$$\Delta T = \frac{2x}{v}.$$

In the limit $\Delta T \to 0$ we get

$$\frac{dv}{dt} = \frac{2V}{2x/v} = \frac{vV}{x} \quad //$$

$$\frac{dv}{dx} = \frac{dv/dt}{dx/dt} = -\frac{1}{V}\frac{dv}{dt}$$ [(−) sign because $x$ decreases with time]

$$\Rightarrow \quad \frac{dv}{dx} = -\frac{v}{x}$$

Integrating we obtain $v(x) = v_0\, l/x$

(c) $$\bar{F} = \frac{2mv}{2x/v} = \frac{mv^2}{x} = \frac{m v_0^2 l^2}{x^3}$$

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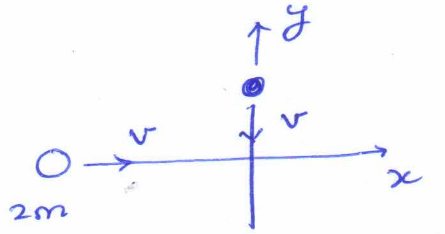
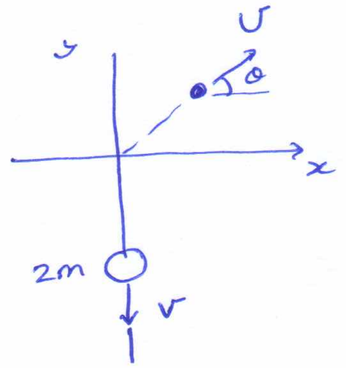
5.

(a) By conservation of momentum




$$2mv\,\hat{i} - mv\,\hat{j} = mU\cos\theta\,\hat{i} + mU\sin\theta\,\hat{j} - 2mv\,\hat{j}$$

$$\Rightarrow \quad 2v = U\cos\theta$$

$$v = U\sin\theta$$

$$\Rightarrow \quad \tan\theta = \frac{1}{2}$$

$$\sin\theta = \frac{1}{\sqrt{2^2+1}} = \frac{1}{\sqrt{5}}$$

$$U = \frac{v}{\sin\theta} = \sqrt{5}\,v$$

$$\tan\theta = \frac{1}{2} \Rightarrow \theta \approx 27^\circ$$

(b)

$$E_i = \frac{1}{2}(2m)v^2 + \frac{1}{2}mv^2 = \frac{3}{2}mv^2$$

$$E_f = \frac{1}{2}(2m)v^2 + \frac{1}{2}mU^2$$

$$= \cancel{1}\,mv^2 + \frac{5}{2}mv^2$$

$$= \frac{7}{2}mv^2$$

The collision is super elastic !

6.

(a)
$$a = A \sin \omega t_0$$
$$b = A \sin 2\omega t_0$$
$$c = A \sin 3\omega t_0$$

$$a + c = 2A \sin 2\omega t_0 \cos \omega t_0$$

$$\therefore \quad \frac{a+c}{2b} = \cos \omega t_0$$

$$\Rightarrow \quad \omega = \frac{1}{t_0} \cos^{-1}\left(\frac{a+c}{2b}\right)$$

$$\Rightarrow f = \frac{1}{2\pi t_0} \cos^{-1}\left(\frac{a+c}{2b}\right) \quad //$$

(b) Total energy is constant

$$K_{trans} + K_{rot} + V = \text{const}$$

$$\Rightarrow \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2 + \frac{1}{2} k x^2 = \text{cons}$$

Here, $I = \frac{1}{2} m R^2$, $\omega = v/R$

Thus, $\frac{3}{4} m \left(\frac{dx}{dt}\right)^2 + \frac{1}{2} k x^2 = \text{const.}$

Differentiating we obtain

$$\frac{3}{2} m \frac{d^2x}{dt^2} \frac{dx}{dt} + kx \frac{dx}{dt} = 0$$

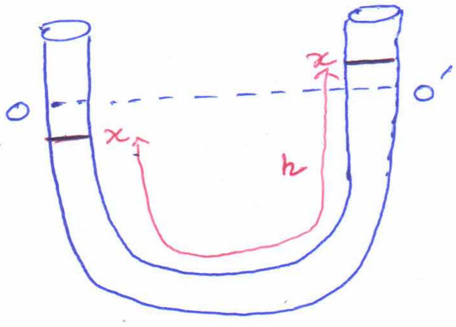
$$\Rightarrow \quad \frac{d^2x}{dt^2} + \left(\frac{2k}{3m}\right) x = 0$$

This is the eq$^n$ for SHM

with $\omega^2 = \frac{2k}{3m}$

$$\therefore \quad T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{3m}{2k}} \quad //$$

7. (a) Let the liquid level in the left limb be depressed by $x$, so that it is elevated by the same height in the right limb. ~~Th~~ $\rho$ is the density of the liquid, $A$ the cross-section of the tube, $M$ the total mass.




Say $m$ is the mass of the liquid corresponding to the length $2x$.

The eq$^n$ of motion is:

$$M\frac{d^2x}{dt^2} = -mg = -(2xA\rho)g$$

$$\Rightarrow \frac{d^2x}{dt^2} = -\frac{2A\rho g}{M}x = -\frac{2A\rho g x}{hA\rho} = -\frac{2g}{h}x = -\omega^2 x$$

This is the eq$^n$ of SHM

(b) Time period

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{h}{2g}}$$

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