1. (a) 12 μs (d) 3.5 Gbits (g) 39 pA
(b) 750 mJ (e) 6.5 nm (h) 49 kΩ
(c) 1.13 kΩ (f) 13.56 MHz (i) 11.73 pA

2.	(a) 1 MW	(e) 33 μJ	(i) 32 mm
	(b) 12.35 mm	(f) 5.33 nW	
	(c) 47. kW	(g) 1 ns	
	(d) 5.46 mA	(h) 5.555 MW	

3. (a) $(400 \text{ Hp})\left(\dfrac{745.7 \text{ W}}{1 \text{ hp}}\right) = 298.3 \quad \text{kW}$

(b) $12 \text{ ft} = (12 \text{ ft})\left(\dfrac{12 \text{ in}}{1 \text{ ft}}\right)\left(\dfrac{2.54 \text{ cm}}{1 \text{ in}}\right)\left(\dfrac{1 \text{ m}}{100 \text{ cm}}\right) = 3.658 \text{ m}$

(c) 2.54 cm = 25.4 mm

(d) $(67 \text{ Btu})\left(\dfrac{1055 \text{ J}}{1 \text{ Btu}}\right) = 70.69 \quad \text{kJ}$

(e) $285.4 \quad 10^{-15}$ s = 285.4 fs

4. (15 V)(0.1 A) = 1.5 W = 1.5 J/s.

3 hrs running at this power level equates to a transfer of energy equal to

(1.5 J/s)(3 hr)(60 min/ hr)(60 s/ min) = 16.2 kJ

5. Motor power = 175 Hp

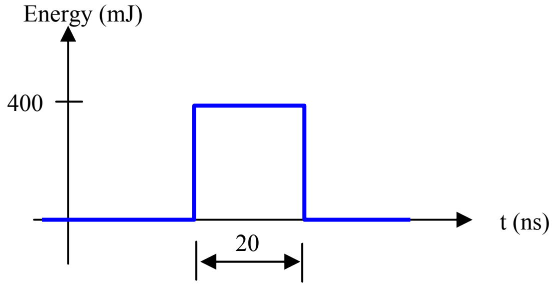
(a) With 100% efficient mechanical to electrical power conversion,
(175 Hp)[1 W/ (1/745.7 Hp)] = 130.5 kW

(b) Running for 3 hours,
Energy = (130.5×10^3 W)(3 hr)(60 min/hr)(60 s/min) = 1.409 GJ

(c) A single battery has 430 kW-hr capacity. We require
(130.5 kW)(3 hr) = 391.5 kW-hr therefore one battery is sufficient.

6. The 400-mJ pulse lasts 20 ns.

(a) To compute the peak power, we assume the pulse shape is square:



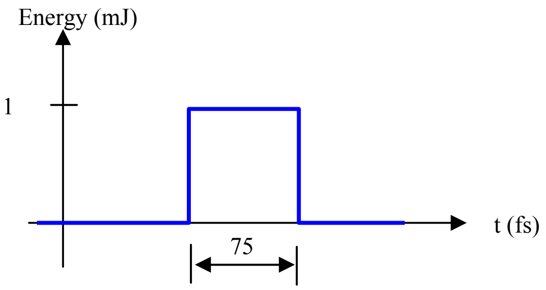
Then $P = 400\times10^{-3}/20\times10^{-9} = 20$ MW.

(b) At 20 pulses per second, the average power is

P_{avg} = (20 pulses)(400 mJ/pulse)/(1 s) = 8 W.

7. The 1-mJ pulse lasts 75 fs.

(a) To compute the peak power, we assume the pulse shape is square:

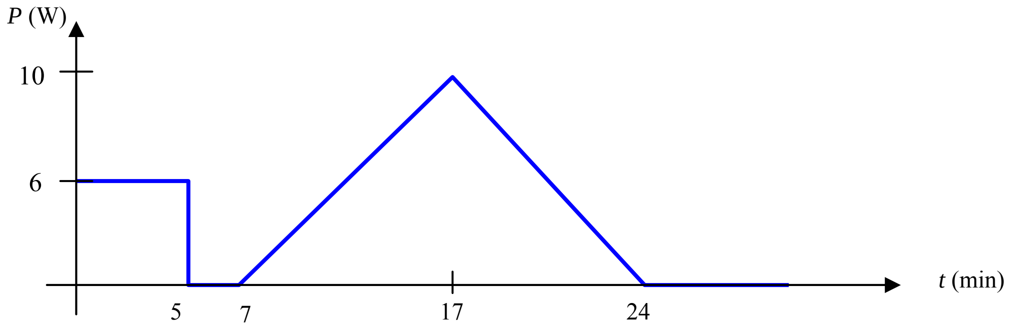


Then $P = 1\times10^{-3}/75\times10^{-15} = 13.33$ GW.

(b) At 100 pulses per second, the average power is

P_{avg} = (100 pulses)(1 mJ/pulse)/(1 s) = 100 mW.

8. The power drawn from the battery is (not quite drawn to scale):



(a) Total energy (in J) expended is
[6(5) + 0(2) + 0.5(10)(10) + 0.5(10)(7)]60 = 6.9 kJ.

(b) The average power in Btu/hr is
(6900 J/24 min)(60 min/1 hr)(1 Btu/1055 J) = 16.35 Btu/hr.

9. The total energy transferred during the first 8 hr is given by

(10 W)(8 hr)(60 min/ hr)(60 s/ min) = 288 kJ

The total energy transferred during the last five minutes is given by

$$\int_0^{300\text{ s}} \left[-\frac{10}{300}t + 10 \right] dt = -\frac{10}{600}t^2 + 10t \Big|_0^{300} = 1.5\text{ kJ}$$

(a) The total energy transferred is 288 + 1.5 = 289.5 kJ

(b) The energy transferred in the last five minutes is 1.5 kJ

10. Total charge $q = 18t^2 - 2t^4$ C.

(a) $q(2\text{ s}) = 40$ C.

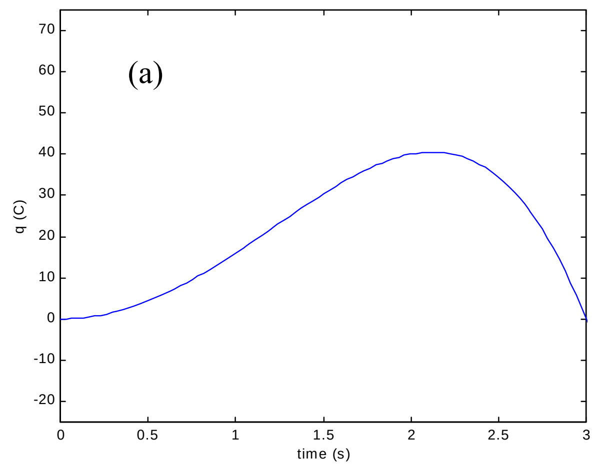
(b) To find the maximum charge within $0 \le t \le 3$ s, we need to take the first and second derivitives:

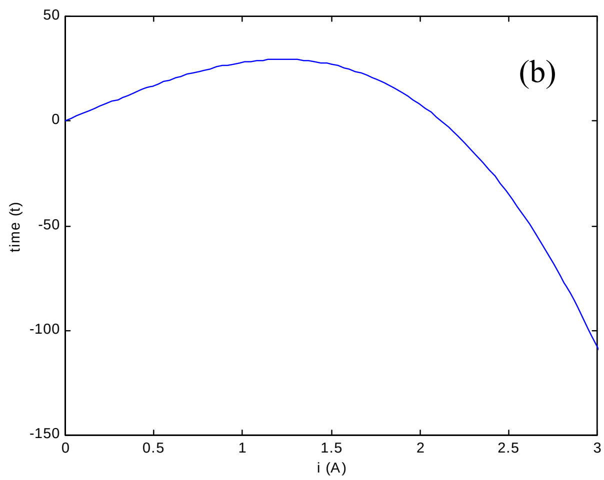
$dq/dt = 36t - 8t^3 = 0$, leading to roots at 0, ± 2.121 s
$d^2q/dt^2 = 36 - 24t^2$

substituting $t = 2.121$ s into the expression for d^2q/dt^2, we obtain a value of –14.9, so that this root represents a maximum.
Thus, we find a maximum charge $q = 40.5$ C at $t = 2.121$ s.

(c) The rate of charge accumulation at $t = 8$ s is
$dq/dt|_{t = 0.8} = 36(0.8) - 8(0.8)^3 = 24.7$ C/s.

(d) See Fig. (a) and (b).





11. Referring to Fig. 2.6*c*,

$$i_1(t) = \begin{cases} -2+3e^{-5t} \text{ A}, & t<0 \\ -2+3e^{3t} \text{ A}, & t>0 \end{cases}$$

Thus,

(a) $i_1(-0.2) =$ 6.155 A

(b) i_1 (0.2) = 3.466 A

(c) To determine the instants at which $i_1 = 0$, we must consider $t < 0$ and $t > 0$ separately:

for $t < 0$, $-2 + 3e^{-5t} = 0$ leads to $t = -0.2 \ln (2/3) = +0.0811$ s (impossible)

for $t > 0$, $-2 + 3e^{3t} = 0$ leads to $t = (1/3) \ln (2/3) = -0.135$ s (impossible)

Therefore, the current is *never* negative.

(d) The total charge passed left to right in the interval –0. 8 < t < 0.1 s is

$$q(t) = \int_{-0.8}^{0.1} i_1(t)dt$$

$$= \int_{-0.8}^{0}\left[-2+3e^{-5t}\right]dt + \int_{0}^{0.1}\left[-2+3e^{3t}\right]dt$$

$$= \left(-2t-\frac{3}{5}e^{-5t}\right)\Bigg|_{-0.8}^{0} + \left(-2t+e^{3t}\right)\Big|_{0}^{0.1}$$

$$= 33.91 \text{ C}$$

12. Referring to Fig. 2.28,

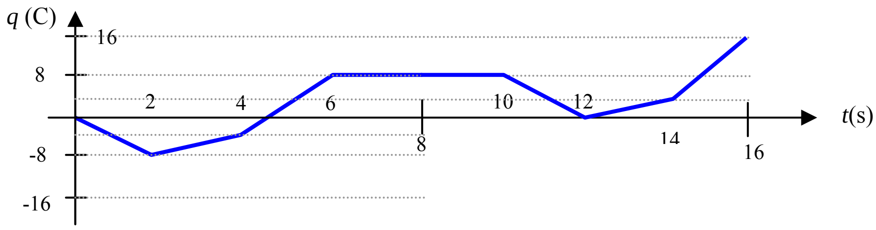
(a) The average current over one period (10 s) is

i_{avg} = [-4(2) + 2(2) + 6(2) + 0(4)]/10 = 800 mA

(b) The total charge transferred over the interval 1 < t < 12 s is

$$q_{\text{total}} = \int_1^{12} i(t)dt = -4(1) + 2(2) + 6(2) + 0(4) - 4(2) = 4 \text{ C}$$

(c) See Fig. below



13. (a) $V_{BA} = -\dfrac{2\ \text{pJ}}{-1.602\times 10^{-19}\ \text{C}} = 12.48\ \text{MV}$

(b) $V_{ED} = \dfrac{0}{-1.602\times 10^{-19}\ \text{C}} = 0$

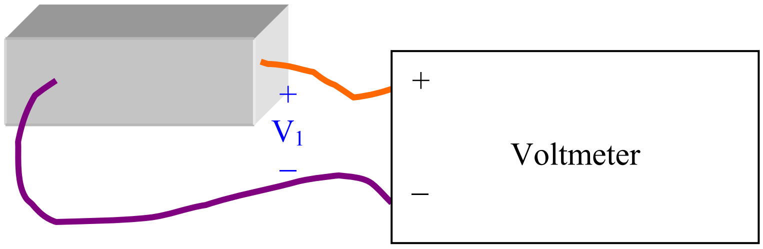
(c) $V_{DC} = -\dfrac{3\ \text{pJ}}{1.602\times 10^{-19}\ \text{C}} = -18.73\ \text{MV}$

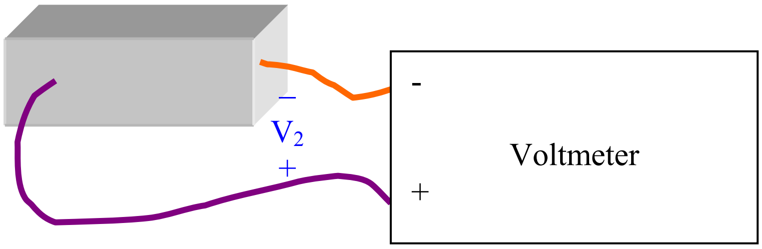
(d) It takes – 3 pJ to move $+1.602\text{x}10^{-19}$ C from D to C.
It takes 2 pJ to move $-1.602\text{x}10^{-19}$ C from B to C, or –2 pJ to move $+1.602\text{x}10^{-19}$ C from B to C, or +2 pJ to move $+1.602\text{x}10^{-19}$ C from C to B.

Thus, it requires –3 pJ + 2 pJ = –1 pJ to move $+1.602\text{x}10^{-19}$ C from D to C to B.

Hence, $V_{DB} = \dfrac{-1\ \text{pJ}}{1.602\times 10^{-19}\ \text{C}} = -6.242\ \text{MV}.$

14.





From the diagram, we see that $V_2 = -V_1 = +2.86$ V.

15. (a) P_{abs} = (+3.2 V)(-2 mA) = –6.4 mW (or +6.4 mW *supplied*)

(b) P_{abs} = (+6 V)(-20 A) = –120 W (or +120 W *supplied*)

(d) P_{abs} = (+6 V)(2 i_x) = (+6 V)[(2)(5 A)] = +60 W

(e) $P_{abs} = (4 \sin 1000t \text{ V})(-8 \cos 1000t \text{ mA})\Big|_{t = 2 \text{ ms}} = +12.11 \text{ W}$

16. $i = 3te^{-100t}$ mA and $v = [6 - 600t]\, e^{-100t}$ mV

(a) The power absorbed at t = 5 ms is

$$P_{\text{abs}} = \left[(6-600t)e^{-100t}\cdot 3te^{-100t}\right]_{t=5ms} \quad \mu\text{W}$$

$$= \quad 0.01655\ \mu\text{W} \quad = \quad 16.55\ \text{nW}$$

(b) The energy delivered over the interval $0 < t < \infty$ is

$$\int_0^\infty P_{abs}\, dt \quad = \quad \int_0^\infty 3t(6-600t)e^{-200t}\, dt \qquad \mu\text{J}$$

Making use of the relationship

$$\int_0^\infty x^n\, e^{-ax}\, dx \quad = \quad \frac{n!}{a^{n+1}}$$ where n is a positive integer and a > 0,

we find the energy delivered to be

$= 18/(200)^2 \quad - \quad 1800/(200)^3$

$= 0$

17. (a) $P_{\text{abs}} = (40i)(3e^{-100t})\Big|_{t = 8\text{ ms}} = 360\left[e^{-100t}\right]^2_{t=8ms} = 72.68\text{ W}$

(b) $P_{\text{abs}} = \left(0.2\dfrac{di}{dt}\right)i = -180\left[e^{-100t}\right]^2_{t=8ms} = -36.34\text{ W}$

(c) $P_{\text{abs}} = \left(30\int_0^t i\,dt + 20\right)\left(3e^{-100t}\right)\Big|_{t=8ms}$

$$= \left(90e^{-100t}\int_0^t 3e^{-100t'}dt' + 60e^{-100t}\right)\Bigg|_{t=8ms} = 27.63\text{ W}$$

18. (a) The short-circuit current is the value of the current at V = 0.

Reading from the graph, this corresponds to approximately 3.0 A.

(b) The open-circuit voltage is the value of the voltage at I = 0.

Reading from the graph, this corresponds to roughly 0.4875 V, estimating the curve as hitting the x-axis 1 mm behind the 0.5 V mark.

(c) We see that the maximum current corresponds to zero voltage, and likewise, the maximum voltage occurs at zero current. The maximum power point, therefore, occurs somewhere between these two points. By trial and error,

P_{max} is roughly (375 mV)(2.5 A) = 938 mW, or just under 1 W.

19. (a)

$$P\big|_{\text{first 2 hours}} = (5\text{ V})(0.001\text{ A}) = 5\text{ mW}$$
$$P\big|_{\text{next 30 minutes}} = (?\text{ V})(0\text{ A}) = 0\text{ mW}$$
$$P\big|_{\text{last 2 hours}} = (2\text{ V})(-0.001\text{ A}) = -2\text{ mW}$$

(b) Energy = (5 V)(0.001 A)(2 hr)(60 min/ hr)(60 s/ min) = 36 J

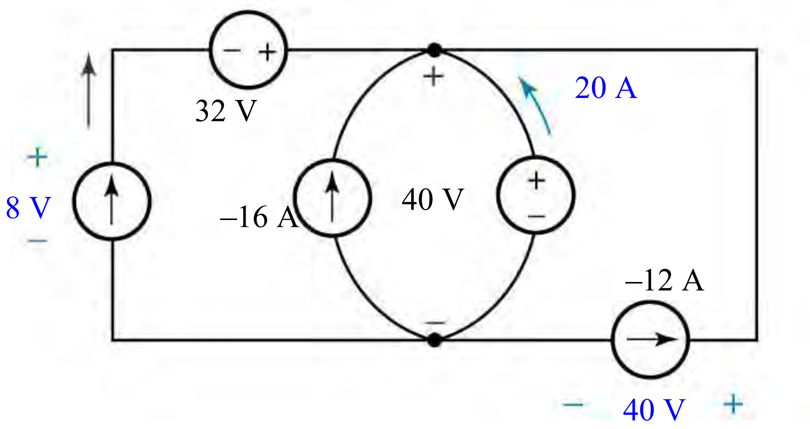
(c) 36 – (2)(0.001)(60)(60) = 21.6 J

20. Note that in the table below, only the –4-A source and the –3-A source are actually "absorbing" power; the remaining sources are supplying power to the circuit.

Source	**Absorbed Power**	**Absorbed Power**
2 V source	(2 V)(-2 A)	- 4 W
8 V source	(8 V)(-2 A)	- 16 W
-4 A source	(10 V)[-(-4 A)]	40 W
10 V source	(10 V)(-5 A)	- 50 W
-3 A source	(10 V)[-(-3 A)]	30 W

The 5 power quantities sum to –4 – 16 + 40 – 50 + 30 = 0, as demanded from conservation of energy.

21.



$P_{8V\ supplied}$	= (8)(8)	=	64 W	(source of energy)
$P_{32V\ supplied}$	= (32)(8)	=	256 W	(source of energy)
$P_{-16A\ supplied}$	= (40)(–16)	=	–640 W	
$P_{40V\ supplied}$	= (40)(20)	=	800 W	(source of energy)
$P_{-12A\ supplied}$	= (40)(–12)	=	–480 W	

Check: $\sum$ supplied power = 64 + 256 – 640 + 800 – 480 = 0 (ok)

22. We are told that $V_x = 1$ V, and from Fig. 2.33 we see that the current flowing through the dependent source (and hence through each element of the circuit) is $5V_x = 5$ A. We will compute *absorbed* power by using the current flowing *into* the positive reference terminal of the appropriate voltage (passive sign convention), and we will compute *supplied* power by using the current flowing *out of* the positive reference terminal of the appropriate voltage.

(a) The power absorbed by element "A" = (9 V)(5 A) = 45 W

(b) The power supplied by the 1-V source = (1 V)(5 A) = 5 W, and
the power supplied by the dependent source = (8 V)(5 A) = 40 W

(c) The sum of the supplied power = 5 + 40 = 45 W
The sum of the absorbed power is 45 W, so

yes, the sum of the power supplied = the sum of the power absorbed, as we expect from the principle of conservation of energy.

23. We are asked to determine the voltage v_s, which is identical to the voltage labeled v_1. The only remaining reference to v_1 is in the expression for the current flowing through the dependent source, $5v_1$.

This current is equal to $-i_2$.
Thus,

$$5\,v_1 = -i_2 = -\,5\text{ mA}$$

Therefore $v_1 = -1$ mV

and so $v_s = v_1 = -1$ mV

24. The voltage across the dependent source = $v_2 = -2i_x = -2(-0.001) = 2$ mV.

25. The battery delivers an energy of 460.8 W-hr over a period of 8 hrs.

(a) The power delivered to the headlight is therefore (460.8 W-hr)/(8 hr) = 57.6 W

(b) The current through the headlight is equal to the power it absorbs from the battery divided by the voltage at which the power is supplied, or

I = (57.6 W)/(12 V) = 4.8 A

26. The supply voltage is 110 V, and the maximum dissipated power is 500 W. The fuses are specified in terms of current, so we need to determine the maximum current that can flow through the fuse.

$$P = VI \quad \text{therefore } I_{max} = P_{max}/V = (500\text{ W})/(110\text{ V}) = 4.545\text{ A}$$

If we choose the 5-A fuse, it will allow up to (110 V)(5 A) = 550 W of power to be delivered to the application (we must assume here that the fuse absorbs zero power, a reasonable assumption in practice). This exceeds the specified maximum power.

If we choose the 4.5-A fuse instead, we will have a maximum current of 4.5 A. This leads to a maximum power of (110)(4.5) = 495 W delivered to the application.

Although 495 W is less than the maximum power allowed, this fuse will provide adequate protection for the application circuitry. If a fault occurs and the application circuitry attempts to draw too much power, 1000 W for example, the fuse will blow, no current will flow, and the application circuitry will be protected. However, if the application circuitry tries to draw its maximum rated power (500 W), the fuse will also blow. In practice, most equipment will not draw its maximum rated power continuously—although to be safe, we typically assume that it will.

27. (a) $i_{max} = 5/ 900 =$ 5.556 mA

$i_{min} = 5/ 1100 =$ 4.545 mA

(b) $p = v^2 / R$ so

$p_{min} = 25/ 1100 = 22.73$ mW

$p_{max} = 25/ 900 = 27.78$ mW

28. $p = i^2 R$, so
$p_{min} = (0.002)^2 (446.5) = 1.786$ mW and (more relevant to our discussion)
$p_{max} = (0.002)^2 (493.5) = 1.974$ mW

1.974 mW would be a correct answer, although power ratings are typically expressed as integers, so 2 mW might be more appropriate.

29. (a) $P_{abs} = i^2 R = [20e^{-12t}]^2 (1200)\ \mu W$

$= [20e^{-1.2}]^2 (1200)\ \mu W$

$=$ 43.54 mW

(b) $P_{abs} = v^2/R = [40 \cos 20t]^2 / 1200 \quad W$

$=$ $[40 \cos 2]^2 / 1200 \quad W$

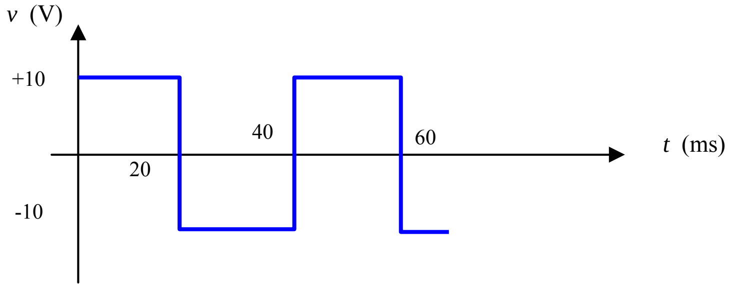
$=$ 230.9 mW

keep in mind we are using radians

(c) $P_{abs} = v\, i =$ $8t^{1.5} \quad W$

$=$ 253.0 mW

30. It's probably best to begin this problem by sketching the voltage waveform:



(a) $v_{\max} = +10$ V

(b) $v_{\text{avg}} = [(+10)(20\times10^{-3}) + (-10)(20\times10^{-3})]/(40\times10^{-3}) \;=\; 0$

(c) $i_{\text{avg}} = v_{\text{avg}}/R \;=\; 0$

(d) $$p_{abs}\Big|_{\max} = \frac{v_{\max}^2}{R} = (10)^2/50 = 2\text{ W}$$

(e) $$p_{abs}\Big|_{avg} = \frac{1}{40}\left[\frac{(+10)^2}{R}\cdot 20 + \frac{(-10)^2}{R}\cdot 20\right] = 2\text{ W}$$

31. Since we are informed that the same current must flow through each component, we begin by defining a current I flowing out of the positive reference terminal of the voltage source.

The power supplied by the voltage source is $V_s I$.
The power absorbed by resistor R_1 is I^2R_1.
The power absorbed by resistor R_2 is I^2R_2.

Since we know that the total power supplied is equal to the total power absorbed, we may write:

$$V_s I = I^2R_1 + I^2R_2$$

or

$$V_s = I R_1 + I R_2$$
$$V_s = I (R_1 + R_2)$$

By Ohm's law,

$$I = V_{R_2} / R_2$$

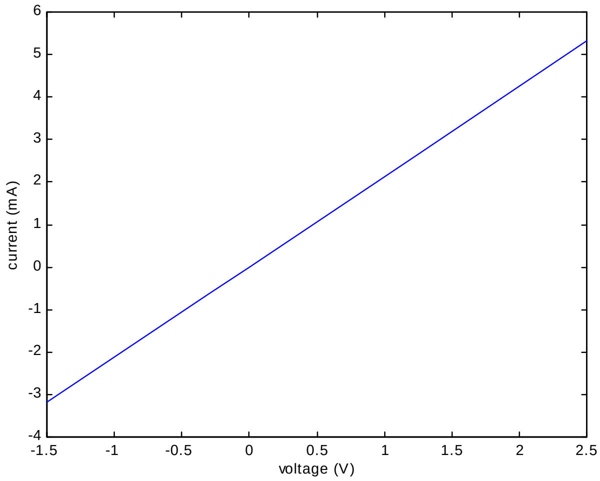
so that

$$V_s = \frac{V_{R_2}}{R_2}(R_1 + R_2)$$

Solving for V_{R_2} we find

$$V_{R_2} = V_s \frac{R_2}{(R_1 + R_2)} \qquad \textit{Q.E.D.}$$

32. (a)

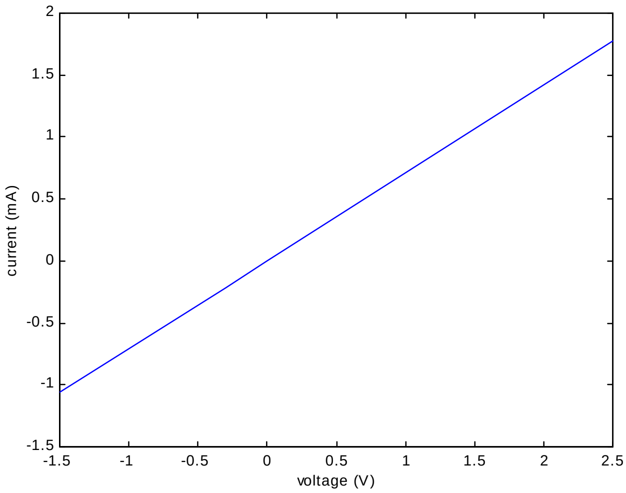


(b) W e see from our answer to part (a) that this device has a reasonably linear characteristic (a not unreasonable degree of e xperimental error is evident in the data). Thus, we choose to estimate the resistance using the two extreme points:

$$R_{\text{eff}} = [(2.5 - (-1.5)]/[5.23 - (-3.19)] \text{ k}\Omega = 475\ \Omega$$

Using the last two points instead, we find $R_{\text{eff}} = 469\ \Omega$, so that we can state with some certainty at least that a reasonable estimate of the resistance is approximately 470 Ω.

(c)



33. Top Left Circuit: $I = (5/10)$ mA = 0.5 mA, and $P_{10k} = V^2/10$ mW = 2.5 mW

Top Right Circuit: $I = -(5/10)$ mA = -0.5 mA, and $P_{10k} = V^2/10$ mW = 2.5 mW

Bottom Left Circuit: $I = (-5/10)$ mA = -0.5 mA, and $P_{10k} = V^2/10$ mW = 2.5 mW

Bottom Right Circuit: $I = -(-5/10)$ mA = 0.5 mA, and $P_{10k} = V^2/10$ mW = 2.5 mW

34. The voltage v_{out} is given by

$$v_{out} = -10^{-3}\, v_\pi\ (1000)$$

$$= -\, v_\pi$$

Since $v_\pi = v_s = 0.01 \cos 1000t$ V, we find that

$$v_{out} = -\, v_\pi = -0.01 \cos 1000t \quad \text{V}$$

35. $v_{out} = -v_\pi = -v_S = -2\sin 5t$ V

v_{out} ($t = 0$) = 0 V

v_{out} ($t = 0.324$ s) = -2sin (1.57) = -2 V

(use care to em ploy r adian m ode on your calculator or convert 1.57 radians to degrees)

36. 18 AWG wire has a resistance of 6.39 Ω / 1000 ft.

Thus, we require 1000 (53) / 6.39 = 8294 ft of wire.
(Or 1.57 miles. Or, 2.53 km).

37. We need to create a 470-Ω resistor from 28 AWG wire, knowing that the ambient temperature is 108°F, or 42.22°C.

Referring to Table 2.3, 28 AWG wire is 65.3 mΩ/ft at 20 °C, and using the equation provided we compute

$$R_2/R_1 = (234.5 + T_2)/(234.5 + T_1) = (234.5 + 42.22)/(234.5 + 20) = 1.087$$

We thus find that 28 AWG wire is (1.087)(65.3) = 71.0 mΩ/ft.

Thus, to repair the transmitter we will need

$(470\ \Omega)/(71.0 \times 10\text{-}3\ \Omega/\text{ft}) = 6620$ ft (1.25 miles, or 2.02 km).

Note: This seems like a lot of wire to be washing up on shore. We may find we don't have enough. In that case, perhaps we should take our cue from Eq. [6], and try to squash a piece of the wire flat so that it has a very small cross-sectional area…..

38. We are given that the conductivity σ of copper is 5.8×10^7 S/m.

(a) 50 ft of #18 (18 AWG) copper wire, which has a diameter of 1.024 mm, will have a resistance of $l/(\sigma A)$ ohms, where A = the cross-sectional area and l = 50 ft.

Converting the dimensional quantities to meters,

l = (50 ft)(12 in/ft)(2.54 cm/in)(1 m/100 cm) = 15.24 m

and

r = 0.5(1.024 mm)(1 m/1000 mm) = 5.12×10^{-4} m

so that

$A = \pi r^2 = \pi(5.12\times10^{-4}\text{ m})^2 = 8.236\times10^{-7}\text{ m}^2$

Thus, R = (15.24 m)/[(5.8×10^7)(8.236×10^{-7})] = 319.0 mΩ

(b) We assume that the conductivity value specified also holds true at 50°C.

The cross-sectional area of the foil is

A = (33 μm)(500 μm)(1 m/10^6 μm)(1 m/10^6 μm) = 1.65×10^{-8} m^2

So that

R = (15 cm)(1 m/100 cm)/[(5.8×10^7)(1.65×10^{-8})] = 156.7 mΩ

A 3-A current flowing through this copper in the direction specified would lead to the dissipation of

$$I^2R = (3)^2\,(156.7)\text{ mW} = 1.410\text{ W}$$

39. Since $R = \rho \ell / A$, it follows that $\rho = R A / \ell$.
From Table 2.4, we see that 28 AWG soft copper wire (cross-sectional area = 0.0804 mm^2) is 65.3 Ω per 1000 ft. Thus,

$R = 65.3\ \Omega$.

ℓ = (1000 ft)(12 in/ft)(2.54 cm/in)(10 mm/cm) = 304,800 mm.

$A = 0.0804\ mm^2$.

Thus, $\rho = (65.3)(0.0804)/304800 = 17.23\ \mu\Omega/\ mm$

or

$\rho\ =\ 1.723\ \mu\Omega\cdot cm$

which is in fact consistent with the representative data for copper in Table 2.3.

40. (a) From the text,

(1) Zener diodes,
(2) Fuses, and
(3) Incandescent (as opposed to fluorescent) light bulbs

This last one requires a few facts to be put together. We have stated that temperature can affect resistance—in other words, if the temperature changes during operation, the resistance will not remain constant and hence nonlinear behavior will be observed. Most discrete resistors are rated for up to a specific power in order to ensure that temperature variation during operation will not significantly change the resistance value. Light bulbs, however, become rather warm when operating and can experience a significant change in resistance.

(b) The energy is dissipated by the resistor, converted to heat which is transferred to the air surrounding the resistor. The resistor is unable to store the energy itself.

41. The quoted resistivity ρ of B33 copper is 1.7654 $\mu\Omega\cdot$cm.
$A = \pi r^2 = \pi(10^{-3})^2 = 10^{-6}\pi$ m^2. ℓ = 100 m.

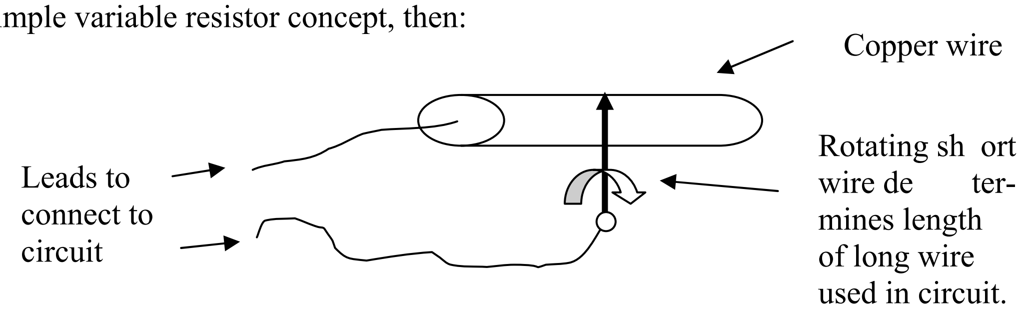
$$\text{Thus, } R = \rho\ell / A = \frac{\left(1.7654\times10^{-6}\,\Omega\text{cm}\right)\left(1\text{ m}/100\text{ cm}\right)\left(100\text{ m}\right)}{10^{-6}\pi} = 0.5619\ \Omega$$

And $P = I^2R = (1.5)^2\,(0.5619) = 1.264$ W.

42. We know that for any wire of cross-sectional area A and length ℓ, the resistance is given by R = ρ ℓ / A.

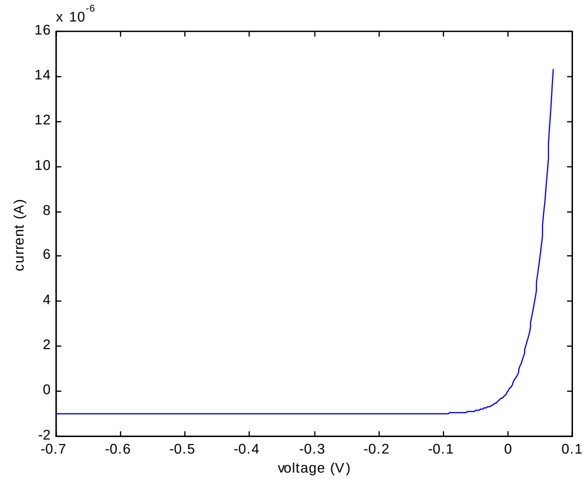
If we keep ρ fixed by choosing a material, and A fixed by choosing a wire gauge (e.g. 28 AWG), changing ℓ will change the resistance of our "device."

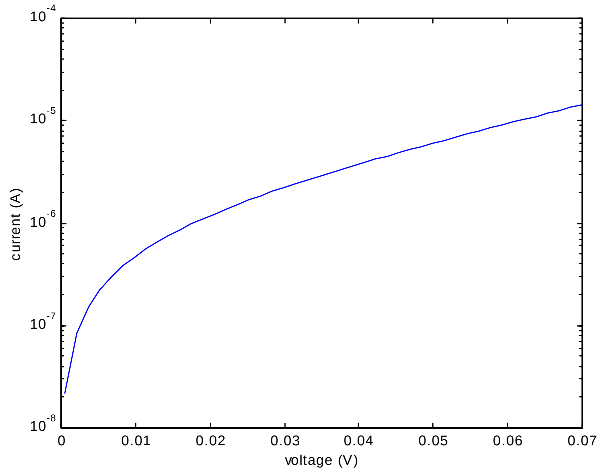
A simple variable resistor concept, then:



But this is somewhat impractical, as the leads may turn out to have almost the same resistance unless we have a very long wire, which can also be impractical. One improvement would be to replace the copper wire shown with a coil of insulated copper wire. A small amount of insulation would then need to be removed from where the moveable wire touches the coil so that electrical connection could be made.

43. (a) We need to plot the negative and positive voltage ranges separately, as the positive voltage range is, after all, exponential!





(b) To determine the resistance of the device at V = 550 mV, we compute the corresponding current:

$$I = 10^{-9}\,[e^{39(0.55)} - 1] \;=\; 2.068 \text{ A}$$

Thus, $R(0.55 \text{ V}) = 0.55/2.068 = 266 \text{ m}\Omega$

(c) $R = 1\ \Omega$ corresponds to $V = I$. Thus, we need to solve the transcendental equation

$$I = 10^{-9}\,[e^{39I} - 1]$$

Using a scientific calculator or the tried-and-true trial and error approach, we find that

$$I = 514.3 \text{ mA}$$

44. We require a 10-Ω resistor, and are told it is for a portable application, implying that size, weight or both would be important to consider when selecting a wire gauge. We have 10,000 ft of each of the gauges listed in Table 2.3 with which to work. Quick inspection of the values listed eliminates 2, 4 and 6 AWG wire as their respective resistances are too low for only 10,000 ft of wire.

Using 12-AWG wire would require (10 Ω) / (1.59 mΩ/ft) = 6290 ft.
Using 28-AWG wire, the narrowest available, would require

$$(10\ \Omega) / (65.3\ \mathrm{m}\Omega/\mathrm{ft}) = 153\ \mathrm{ft}.$$

Would the 28-AWG wire weight less? Again referring to Table 2.3, we see that the cross-sectional area of 28-AWG wire is 0.0804 mm^2, and that of 12-AWG wire is 3.31 mm^2. The volume of 12-AWG wire required is therefore 6345900 mm^3, and that of 28-AWG wire required is only 3750 mm^3.

The best (but not the only) choice for a portable application is clear: 28-AWG wire!

45. Our target is a 100-Ω resistor. We see from the plot that at $N_D = 10^{15}$ cm^{-3}, $\mu_n \sim 2\text{x}10^3$ cm^2/V-s, yielding a resistivity of 3.121 Ω-cm.

At $N_D = 10^{18}$ cm^{-3}, $\mu_n \sim 230$ cm^2/ V-s, yielding a resistivity of 0.02714 Ω-cm.

Thus, we see that the lower doping level clearly provides material with higher resistivity, requiring less of the available area on the silicon wafer.

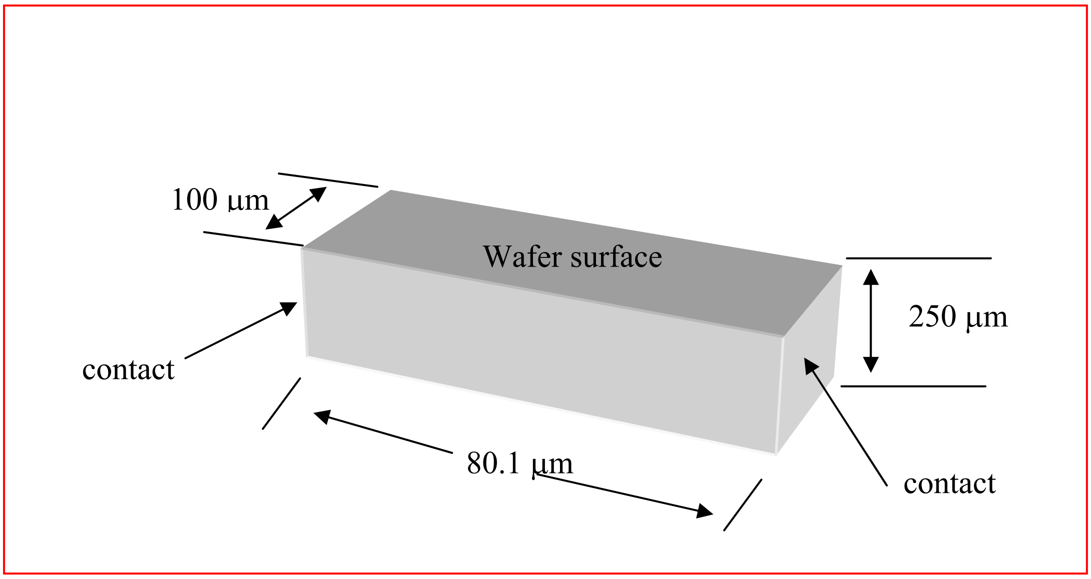
Since $R = \rho L/A$, where we know $R = 100$ Ω and $\rho = 3.121$ Ω-cm for a phosphorus concentration of 10^{15} cm^{-3}, we need only define the resistor geometry to complete the design.

We choose a geometry as shown in the figure; our contact area is arbitrarily chosen as 100 µm by 250 µm, so that only the length L remains to be specified. Solving,

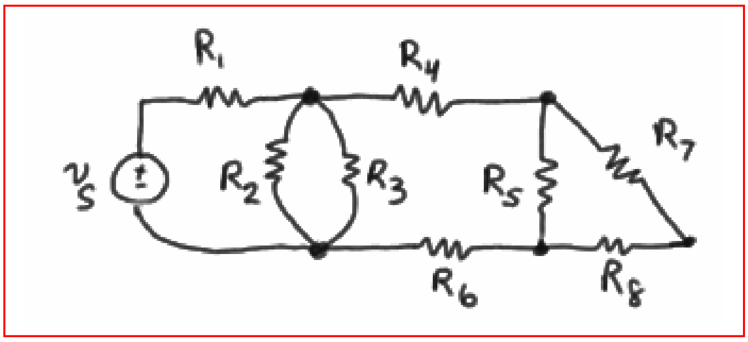
$$L = \frac{R}{\rho}A = \frac{(100\ \Omega)(100\ \mu m)(250\ \mu m)}{(3.121\ \Omega\ \text{cm})\left(10^4\ \mu\text{m/cm}\right)} = 80.1\ \mu\text{m}$$

Design summary (one possibility): $N_D = 10^{15}$ cm^{-3}
$L = 80.1$ µm
Contact width = 100 µm

(Note: this is somewhat atypical; in the semiconductor industry contacts are typically made to the top and/or bottom surface of a wafer. So, there's more than one solution based on geometry as well as doping level.)



1.



2. (*a*) $\boxed{\text{six nodes;}}$ (*b*) $\boxed{\text{nine branches.}}$

3. (*a*) Four nodes; (*b*) five branches; (*c*) path, yes – loop, no.

4. (*a*) Five nodes;

(*b*) seven branches;

(*c*) path, yes – loop, no.

5. (a) The number of nodes remains the same – $\boxed{\text{four (4).}}$

(b) The number of nodes is increased by one – to $\boxed{\text{five (5).}}$

(c)

i)	YES	
ii)	NO	– does not return to starting point
iii)	YES	
iv)	NO	– does not return to starting point
v)	NO	– point B is crossed twice

6. (a) By KCL at the bottom node: $2 - 3 + i_Z - 5 - 3 = 0$

So $\boxed{i_Z = 9\text{ A.}}$

(b) If the left-most resistor has a value of 1 Ω, then 3 V appears across the parallel network (the '+' reference terminal being the bottom node) Thus, the value of the other resistor is given by

$$R = \frac{3}{-(-5)} = \boxed{600\text{ m}\Omega\,.}$$

7. (*a*) $\boxed{3\text{ A;}}$

(*b*) $\boxed{-3\text{ A;}}$

(*c*) $\boxed{0.}$

8. By KCL, we may write:

$$5 + i_y + i_z = 3 + i_x$$

(*a*) $i_x = 2 + i_y + i_z = 2 + 2 + 0 = \boxed{4\text{ A}}$

(*b*) $i_y = 3 + i_x - 5 - i_z$

$i_y = -2 + 2 - 2\, i_y$

Thus, we find that $\boxed{i_y = 0.}$

(*c*) $5 + i_y + i_z = 3 + i_x \rightarrow 5 + i_x + i_x = 3 + i_x$ so $i_x = 3 - 5 = \boxed{\text{-2A.}}$

9. Focusing our attention on the bottom left node, we see that $\boxed{i_x = 1 \text{ A.}}$

Focusing our attention next on the top right node, we see that $\boxed{i_y = 5 \text{ A}}$.

10. We obtain the current each bulb draws by dividing its power rating by the operating voltage (115 V):

$\boxed{I_{100W} = 100/115 = 896.6 \text{ mA}}$

$\boxed{I_{60W} = 60/115 = 521.7 \text{ mA}}$

$\boxed{I_{40W} = 347.8 \text{ mA}}$

Thus, the total current draw is $\boxed{1.739 \text{ A.}}$

11. The DMM is connected in parallel with the 3 load resistors, across which develops the voltage we wish to measure. If the DMM appears as a short, then all 5 A flows through the DMM, and none through the resistors, resulting in a (false) reading of 0 V for the circuit undergoing testing. If, instead, the DMM has an infinite internal resistance, then no current is shunted away from the load resistors of the circuit, and a true voltage reading results.

12. In either case, a bulb failure will adversely affect the sign.

Still, in the parallel-connected case, at least 10 (up to 11) of the other characters will be lit, so the sign could be read and customers will know the restaurant is open for business.

13. (*a*) $v_y = 1(3v_x + i_z)$

v_x = 5 V and given that i_z = –3 A, we find that

$$v_y = 3(5) - 3 = \boxed{12\text{ V}}$$

(*b*) $v_y = 1(3v_x + i_z) = -6 = 3v_x + 0.5$

Solving, we find that $v_x = (-6 - 0.5)/3 = \boxed{-2.167\text{ V}}$.

14. (*a*) $i_x = v_1/10 + v_1/10 = 5$

$2v_1 = 50$

so $\boxed{v_1 = 25\text{ V.}}$

By Ohm's law, we see that $i_y = v_2/10$

also, using Ohm's law in combination with KCL, we may write

$$i_x = v_2/10 + v_2/10 = i_y + i_y = 5\text{ A}$$

Thus, $\boxed{i_y = 2.5\text{ A.}}$

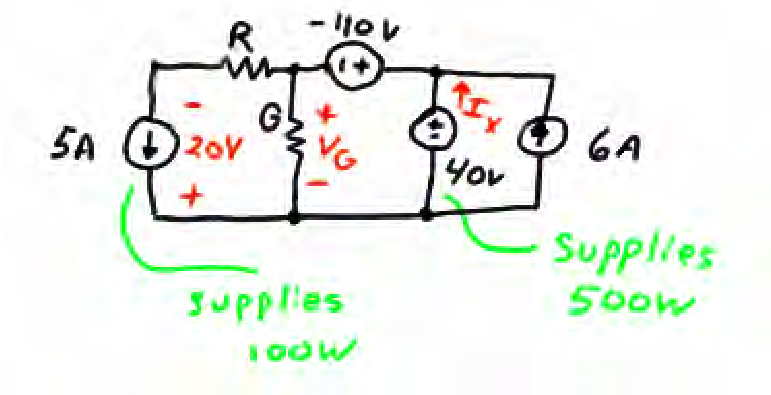
(*b*) From part (*a*), $i_x = 2\ v_1/ 10$. Substituting the new value for v_1, we find that

$\boxed{i_x = 6/10 = 600\text{ mA.}}$

Since we have found that $i_y = 0.5\ i_x$, $\boxed{i_y = 300\text{ mA.}}$

(*c*) $\boxed{\text{no value – this is impossible.}}$

15. We begin by making use of the information given regarding the power generated by the 5-A and the 40-V sources. The 5-A source supplies 100 W, so it must therefore have a terminal voltage of 20 V. The 40-V source supplies 500 W, so it must therefore provide a current I_X of 12.5 A.



(1) By KVL, $-40 + (-110) + R(5) - 20 = 0$

Thus, $\boxed{R = 34\ \Omega.}$

(2) By KVL, $-V_G - (-110) + 40 = 0$

So $V_G = 150$ V

Now that we know the voltage across the unknown conductance G, we need only to find the current flowing through it to find its value by making use of Ohm's law.

KCL provides us with the means to find this current: The current flowing into the "+" terminal of the –110-V source is 12.5 + 6 = 18.5 A.

Then, $I_x = 18.5 - 5 = 13.5$ A

By Ohm's law, $I_x = G \cdot V_G$

So $G = 13.5/150$ or $\boxed{G = 90\text{ mS}}$

16. (*a*) $-1 + 2 + 10i - 3.5 + 10i = 0$

Solving, $\boxed{i = 125 \text{ mA}}$

(*b*) $+10 + 1i - 2 + 2i + 2 - 6 + i = 0$

Solving, we find that $4i = -4$ or $\boxed{i = -1 \text{ A.}}$

17. **Circuit I.**

Starting at the bottom node and proceeding clockwise, we can write the KVL equation

$+7 - 5 - 2 - 1(i) = 0$

Which results in $\boxed{i = 0.}$

Circuit II.

Again starting with the bottom node and proceeding in a clockwise direction, we write the KVL equation

$-9 + 4i + 4i = 0$ (no current flows through either the -3 V source or the 2 Ω resistor)

Solving, we find that i = 9/8 A = $\boxed{1.125\text{ A.}}$

18. Begin by defining a clockwise current i.

$-v_S + v_1 + v_2 = 0$ so $v_S = v_1 + v_2 = i(R_1 + R_2)$

and hence $i = \dfrac{v_S}{R_1 + R_2}$.

Thus, $v_1 = R_1 i = \dfrac{R_1}{R_1 + R_2} v_S$ and $v_2 = R_2 i = \dfrac{R_2}{R_1 + R_2} v_S$.

Q.E.D.

19. Given: (1) $V_d = 0$ and (2) no current flows into either terminal of V_d.

Calculate V_{out} by writing two KVL equations.

Begin by defining current i_1 flowing right through the 100 Ω resistor, and i_2 flowing right through the 470 Ω resistor.

$-5 + 100i_1 + V_d = 0$ [1]

$-5 + 100i_1 + 470i_2 + V_{out} = 0$ [2]

Making use of the fact that in this case $V_d = 0$, we find that i1 = 5/100 A.

Making use of the fact that no current flows into the input terminals of the op amp, $i_1 = i_2$. Thus, Eq. [2] reduces to

$-5 + 570(5/100) + V_{out} = 0$ or

$\boxed{V_{out} = -23.5 \text{ V}}$ (hence, the circuit is acting as a voltage amplifier.)

20. (*a*) By KVL, $-2 + v_x + 8 = 0$

so that $\boxed{v_x = -6 \text{ V.}}$

(*b*) By KCL at the top left node,

$i_{in} = 1 + I_S + v_x/4 - 6$

or $\boxed{i_{in} = 23 \text{ A}}$

(*c*) By KCL at the top right node,

$I_S + 4\, v_x = 4 - v_x/4$

So $\boxed{I_S = 29.5 \text{ A.}}$

(*d*) The power provided by the dependent source is $8(4v_x) = \boxed{-192 \text{ W.}}$

21. (a) Working from left to right,

$v_1 = 60$ V
$v_2 = 60$ V
$i_2 = 60/20 = 3$ A
$i_4 = v_1/4 = 60/4 = 15$ A
$v_3 = 5i_2 = 15$ V

By KVL, $-60 + v_3 + v_5 = 0$
$v_5 = 60 - 15 = 45$ V
$v_4 = v_5 = 45$

$i_5 = v_5/5 = 45/5 = 9$ A
$i_3 = i_4 + i_5 = 15 + 9 = 24$ A
$i_1 = i_2 + i_3 = 3 + 24 = 27$

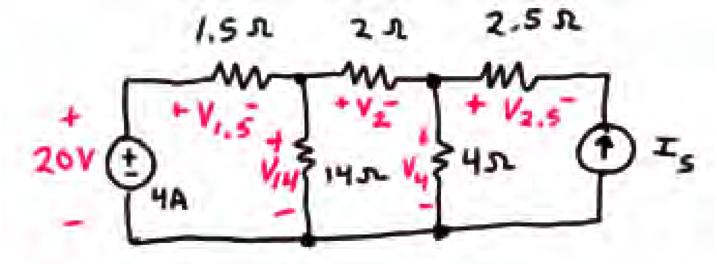
$v_1 = 60$ V	$i_1 = 27$ A
$v_2 = 60$ V	$i_2 = 3$ A
$v_3 = 15$ V	$i_3 = 24$ A
$v_4 = 45$ V	$i_4 = 15$ A
$v_5 = 45$ V	$i_5 = 9$ A

(b) It is now a simple matter to compute the power absorbed by each element:

p_1	$= -v_1 i_1$	$= -(60)(27)$	$= -1.62$ kW
p_2	$= v_2 i_2$	$= (60)(3)$	$= 180$ W
p_3	$= v_3 i_3$	$= (15)(24)$	$= 360$ W
p_4	$= v_4 i_4$	$= (45)(15)$	$= 675$ W
p_5	$= v_5 i_5$	$= (45)(9)$	$= 405$ W

and it is a simple matter to check that these values indeed sum to zero as they should.

22. Refer to the labeled diagram below.



Beginning from the left, we find

$\boxed{p_{20V} = -(20)(4) = -80 \text{ W}}$

$v_{1.5} = 4(1.5) = 6$ V therefore $\boxed{p_{1.5} = (v_{1.5})^2/ 1.5 = 24 \text{ W.}}$

$v_{14} = 20 - v_{1.5} = 20 - 6 = 14$ V $\boxed{\text{therefore } p_{14} = 14^2/ 14 = 14 \text{ W.}}$

$i_2 = v_2/2 = v_{1.5}/1.5 - v_{14}/14 = 6/1.5 - 14/14 = 3$ A

Therefore $v_2 = 2(3) = 6$ V and $\boxed{p_2 = 6^2/2 = 18 \text{ W.}}$

$v_4 = v_{14} - v_2 = 14 - 6 = 8$ V therefore $\boxed{p_4 = 8^2/4 = 16 \text{ W}}$

$i_{2.5} = v_{2.5}/ 2.5 = v_2/2 - v_4/4 = 3 - 2 = 1$ A

Therefore $v_{2.5} = (2.5)(1) = 2.5$ V and so $\boxed{p_{2.5} = (2.5)^2/2.5 = 2.5 \text{ W.}}$

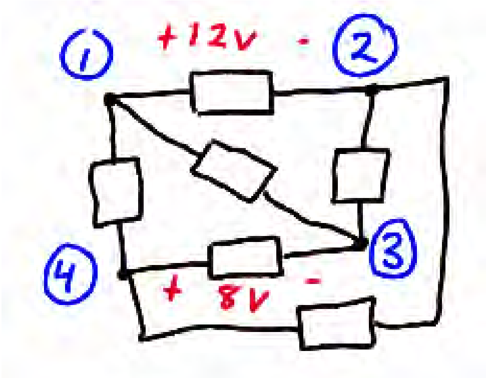
$I_{2.5} = - I_S$, thefore $I_S = -1$ A.

KVL allows us to write $-v_4 + v_{2.5} + v_{IS} = 0$

so $V_{I_S} = v_4 - v_{2.5} = 8 - 2.5 = 5.5$ V and $\boxed{p_{I_S} = -V_{I_S} I_S = 5.5 \text{ W.}}$

A quick check assures us that these power quantities sum to zero.

23. Sketching the circuit as described,



(*a*) $v_{14} = 0$.

$v_{13} = v_{43} = 8$ V
$v_{23} = -v_{12} - v_{34} = -12 + 8 = -4$ V
$v_{24} = v_{23} + v_{34} = -4 - 8 = -12$ V

(*b*) $v_{14} = 6$ V.

$v_{13} = v_{14} + v_{43} = 6 + 8 = 14$ V
$v_{23} = v_{13} - v_{12} = 14 - 12 = 2$ V
$v_{24} = v_{23} + v_{34} = 2 - 8 = -6$ V

(*c*) $v_{14} = -6$ V.

$v_{13} = v_{14} + v_{43} = -6 + 8 = 2$ V
$v_{23} = v_{13} - v_{12} = 2 - 12 = -10$ V
$v_{24} = v_{23} + v_{34} = -10 - 8 = -18$ V

24. (*a*) By KVL, $-12 + 5000I_D + V_{DS} + 2000I_D = 0$

Therefore, $\boxed{V_{DS} = 12 - 7(1.5) = 1.5 \text{ V.}}$

(*b*) By KVL, $- V_G + V_{GS} + 2000I_D = 0$

Therefore, $\boxed{V_{GS} = V_G - 2(2) = -1 \text{ V.}}$

25. Applying KVL around this series circuit,

$$-120 + 30i_x + 40i_x + 20i_x + v_x + 20 + 10i_x = 0$$

where v_x is defined across the unknown element X, with the "+" reference on top. Simplifying, we find that $100i_x + v_x = 100$

To solve further we require specific information about the element X and its properties.

(*a*) if X is a 100-Ω resistor,

$$v_x = 100i_x \text{ so we find that } 100\, i_x + 100\, i_x = 100.$$

Thus

$$i_x = 500 \text{ mA and } \boxed{p_x = v_x\, i_x = 25 \text{ W.}}$$

(*b*) If X is a 40-V independent voltage source such that $v_x = 40$ V, we find that

$$i_x = (100 - 40) / 100 = 600 \text{ mA} \quad \text{and} \quad \boxed{p_x = v_x\, i_x = 24 \text{ W}}$$

(*c*) If X is a dependent voltage source such that $v_x = 25ix$,

$$i_x = 100/125 = 800 \text{ mA and } \boxed{p_x = v_x\, i_x = 16 \text{ W.}}$$

(*d*) If X is a dependent voltage source so that $v_x = 0.8v_1$,
where $v_1 = 40i_x$, we have

$$100\, i_x + 0.8(40i_x) = 100$$

or $i_x = 100/132 = 757.6$ mA and $\boxed{p_x = v_x\, i_x = 0.8(40)(0.7576)^2 = 18.37 \text{ W.}}$

(*e*) If X is a 2-A independent current source, arrow up,

$$100(-2) + v_x = 100$$

so that $v_x = 100 + 200 = 300$ V and $\boxed{p_x = v_x\, i_x = -600 \text{ W}}$

26. (*a*) We first apply KVL:

$$-20 + 10i_1 + 90 + 40i_1 + 2v_2 = 0$$

where $v_2 = 10i_1$. Substituting,

$$70 + 70\,i_1 = 0$$

or $\boxed{i_1 = -1\text{ A.}}$

(*b*) Applying KVL,

$$-20 + 10i_1 + 90 + 40i_1 + 1.5v_3 = 0 \qquad [1]$$

where

$$v_3 = -90 - 10i_1 + 20 = -70 - 10\,i_1$$

alternatively, we could write

$$v_3 = 40i_1 + 1.5v_3 = -80i_1$$

Using either expression in Eq. [1], we find $\boxed{i_1 = 1\text{ A.}}$

(*c*) Applying KVL,

$$-20 + 10i_1 + 90 + 40i_1 - 15\,i_1 = 0$$

Solving, $\boxed{i_1 = -2\text{A.}}$

27. Applying KVL, we find that

$$-20 + 10i_1 + 90 + 40i_1 + 1.8v_3 = 0 \qquad [1]$$

Also, KVL allows us to write

$$v_3 = 40i_1 + 1.8v_3$$

$$v_3 = -50i_1$$

So that we may write Eq. [1] as

$$50i_1 - 1.8(50)i_1 = -70$$

or $i_1 = -70/-40 = 1.75$ A.

Since $v_3 = -50i_1 = -87.5$ V, no further information is required to determine its value.

The 90-V source is absorbing $(90)(i_1) = 157.5$ W of power and the dependent source is absorbing $(1.8v_3)(i_1) = -275.6$ W of power.

Therefore, *none of the conditions specified in (a) to (d) can be met by this circuit*.

28. (*a*) Define the charging current i as flowing clockwise in the circuit provided. By application of KVL,

$$-13 + 0.02i + Ri + 0.035i + 10.5 = 0$$

We know that we need a current i = 4 A, so we may calculate the necessary resistance

$$\boxed{R = [13 - 10.5 - 0.055(4)]/\ 4 = 570\ \text{m}\Omega}$$

(*b*) The total power delivered to the battery consists of the power absorbed by the 0.035-Ω resistance ($0.035i^2$), and the power absorbed by the 10.5-V ideal battery ($10.5i$). Thus, we need to solve the quadratic equation

$$0.035i^2 + 10.5i = 25$$

which has the solutions i = -302.4 A and i = 2.362 A.

In order to determine which of these two values should be used, we must recall that the idea is to charge the battery, implying that it is absorbing power, or that i as defined is positive. Thus, we choose i = 2.362 A, and, making use of the expression developed in part (*a*), we find that

$$\boxed{R = [13 - 10.5 - 0.055(2.362)]/\ 2.362 = 1.003\ \Omega}$$

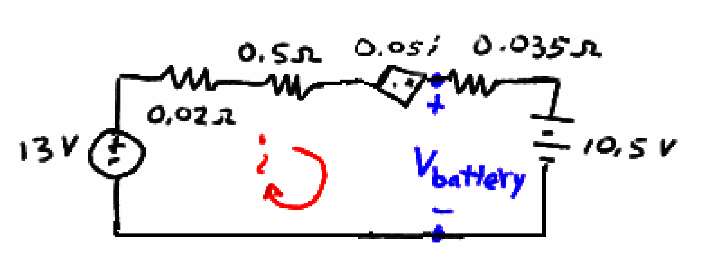
(*c*) To obtain a voltage of 11 V across the battery, we apply KVL:

$$0.035i + 10.5 = 11 \quad \text{so that } i = 14.29\text{ A}$$

From part (*a*), this means we need

$$\boxed{R = [13 - 10.5 - 0.055(14.29)]/\ 14.29 = 119.9\ \text{m}\Omega}$$

29. Drawing the circuit described, we also define a clockwise current *i*.



By KVL, we find that

$$-13 + (0.02 + 0.5 - 0.05)i + 0.035i + 10.5 = 0$$

or that $i = (13 - 10.5)/0.505 = \boxed{4.951 \text{ A}}$

and $V_{battery} = 13 - (0.02 + 0.5)i = \boxed{10.43 \text{ V.}}$

30. Applying KVL about this simple loop circuit (the dependent sources are still linear elements, by the way, as they depend only upon a sum of voltages)

$$-40 + (5 + 25 + 20)i \ - (2v_3 + v_2) + (4v_1 - v_2) \ = \ 0 \qquad [1]$$

where we have defined i to be flowing in the clockwise direction, and $v_1 = 5i$, $v_2 = 25i$, and $v_3 = 20i$.

Performing the necessary substition, Eq. [1] becomes

$$50i \ - (40i + 25i) + (20i - 25i) \ = \ 40$$

so that $i = 40/\text{-}20 \ = \ \text{-}2$ A

Computing the absorbed power is now a straightforward matter:

$p_{40\text{V}}$	$= (40)(-i)$	$=$ 80 W
$p_{5\Omega}$	$= 5i^2$	$=$ 20 W
$p_{25\Omega}$	$= 25i^2$	$=$ 100 W
$p_{20\Omega}$	$= 20i^2$	$=$ 80 W
p_{depsrc1}	$= (2v_3 + v_2)(-i) = (40i + 25i)$	$=$ -260 W
p_{depsrc2}	$= (4v_1 - v_2)(-i) = (20i - 25i)$	$=$ -20 W

and we can easily verify that these quantities indeed sum to zero as expected.

31. We begin by defining a clockwise current *i*.

(*a*) $i = 12/(40 + R)$ mA, with *R* expressed in kΩ.

We want $i^2 \cdot 25 = 2$

or $$\left(\frac{12}{40+R}\right)^2 \cdot 25 = 2$$

Rearranging, we find a quadratic expression involving *R*:

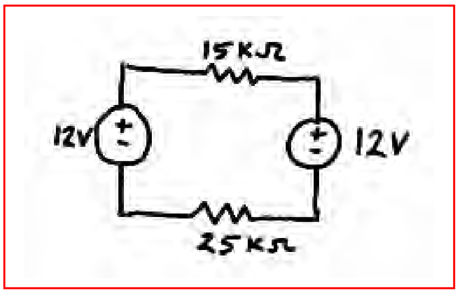
$$R^2 + 80R - 200 = 0$$

which has the solutions R = -82.43 kΩ and R = 2.426 kΩ. Only the latter is a physical solution, so

$$\boxed{R = 2.426\ \text{k}\Omega.}$$

(*b*) We require $i \cdot 12 = 3.6$ or $i = 0.3$ mA
From the circuit, we also see that $i = 12/(15 + R + 25)$ mA.
Substituting the desired value for *i*, we find that the required value of *R* is $\boxed{R = 0.}$

(*c*)



32. By KVL,

$$-12 + (1 + 2.3 + R_{\text{wire segment}})\, i = 0$$

The wire segment is a 3000–ft section of 28–AWG solid copper wire. Using Table 2.3, we compute its resistance as

$$(16.2\ \text{m}\Omega/\text{ft})(3000\ \text{ft}) = 48.6\ \Omega$$

which is certainly *not* negligible compared to the other resistances in the circuit!

Thus,

$$i = 12/(1 + 2.3 + 48.6) = \boxed{231.2\ \text{mA}}$$

33. We can apply Ohm's law to find an expression for v_o:

$$v_o = 1000(-g_m v_\pi)$$

We do not have a value for v_π, but KVL will allow us to express that in terms of v_o, which we *do* know:

$$-10\times10^{-3} \cos 5t + (300 + 50\times10^3)\, i = 0$$

where i is defined as flowing clockwise.

Thus, $v_\pi = 50\times10^3\, i = 50\times10^3\,(10\times10^{-3} \cos 5t) / (300 + 50\times10^3)$

$= 9.940\times10^{-3} \cos 5t$ V

and we by substitution we find that

$$v_o = 1000(-25\times10^{-3})(9.940\times10^{-3} \cos 5t)$$

$$= \boxed{-248.5 \cos 5t \text{ mV}}$$

34. By KVL, we find that

$$-3 + 100\, I_D + V_D = 0$$

Substituting $I_D = 3\times10^{-6}(e^{V_D / 27\times10^{-3}} - 1)$, we find that

$$-3 + 300\times10^{-6}(e^{V_D / 27\times10^{-3}} - 1) + V_D = 0$$

This is a transcendental equation. Using a scientific calculator or a numerical software package such as MATLAB®, we find

$$V_D = \boxed{246.4 \text{ mV}}$$

Let's assume such software-based assistance is unavailable. In that case, we need to "guess" a value for V_D, substitute it into the right hand side of our equation, and see how close the result is to the left hand side (in this case, zero).

GUESS	RESULT
0	–3
1	3.648×10^{12}
0.5	3.308×10^{4}
0.25	0.4001
0.245	–0.1375
0.248	0.1732
0.246	–0.0377

oops

better

At this point, the error is getting much smaller, and our confidence is increasing as to the value of V_D.

35. Define a voltage v_x, "+" reference on the right, across the dependent current source. Note that in fact v_x appears across each of the four elements. We first convert the 10 mS conductance into a 100–Ω resistor, and the 40–mS conductance into a 25–Ω resistor.

(*a*) Applying KCL, we sum the currents flowing into the right–hand node:

$$5 - v_x / 100 - v_x / 25 + 0.8\, i_x = 0 \quad [1]$$

This represents one equation in two unknowns. A second equation to introduce at this point is

$i_x = v_x /25$ so that Eq. [1] becomes

$$5 - v_x / 100 - v_x / 25 + 0.8\, (v_x / 25) = 0$$

Solving for v_x, we find v_x = 277.8 V. It is a simple matter now to compute the power absorbed by each element:

P_{5A}	$= -5\, v_x$	= –1.389 kW
$P_{100\Omega}$	$= (v_x)^2 / 100$	= 771.7 W
$P_{25\Omega}$	$= (v_x)^2 / 25$	= 3.087 kW
P_{dep}	$= -v_x(0.8\, i_x) = -0.8\, (v_x)^2 / 25$	= –2.470 kW

A quick check assures us that the calculated values sum to zero, as they should.

(*b*) Again summing the currents into the right–hand node,

$$5 - v_x / 100 - v_x / 25 + 0.8\, i_y = 0 \quad [2]$$

where $i_y = 5 - v_x/100$

Thus, Eq. [2] becomes

$$5 - v_x / 100 - v_x / 25 + 0.8(5) - 0.8\, (i_y) / 100 = 0$$

Solving, we find that v_x x = 155.2 V and i_y = 3.448 A

So that

P_{5A}	$= -5\, v_x$	= –776.0 W
$P_{100\Omega}$	$= (v_x)^2 / 100$	= 240.9 W
$P_{25\Omega}$	$= (v_x)^2 / 25$	= 963.5 W
P_{dep}	$= -v_x(0.8\, i_y)$	= –428.1 W

A quick check shows us that the calculated values sum to 0.3, which is reasonably close to zero compared to the size of the terms (small roundoff errors accumulated).

36. Define a voltage v with the "+" reference at the top node. Applying KCL and summing the currents flowing out of the top node,

$$v/5{,}000 \;+\; 4\times10^{-3} \;+\; 3i_1 \;+\; v/20{,}000 \;=\; 0 \quad [1]$$

This, unfortunately, is one equation in two unknowns, necessitating the search for a second suitable equation. Returning to the circuit diagram, we observe that

$$i_1 \;=\; 3\,i_1 \;+\; v/2{,}000$$

or

$$i_1 \;=\; -v/40{,}000 \qquad [2]$$

Upon substituting Eq. [2] into Eq. [1], Eq. [1] becomes,

$$v/5{,}000 \;+\; 4\times10^{-3} \;-\; 3v/40{,}000 \;+\; v/20{,}000 \;=\; 0$$

Solving, we find that

$$v = -22.86 \text{ V}$$

and

$$i_1 \;=\; 571.4\ \mu\text{A}$$

Since $i_x = i_1$, we find that $i_x = \boxed{571.4\ \mu\text{A}.}$

37. Define a voltage v_x with its "+" reference at the center node. Applying KCL and summing the currents into the center node,

$$8 - v_x/6 + 7 - v_x/12 - v_x/4 = 0$$

Solving, $v_x = 30$ V.

It is now a straightforward matter to compute the power absorbed by each element:

P_{8A}	$= -8\, v_x$	$= -240$ W
$P_{6\Omega}$	$= (v_x)^2 / 6$	$= 150$ W
P_{8A}	$= -7\, v_x$	$= -210$ W
$P_{12\Omega}$	$= (v_x)^2 / 12$	$= 75$ W
$P_{4\Omega}$	$= (v_x)^2 / 4$	$= 225$ W

and a quick check verifies that the computed quantities sum to zero, as expected.

38. (*a*) Define a voltage *v* across the 1–kΩ resistor with the "+" reference at the top node. Applying KCL at this top node, we find that

$$80\times10^{-3} - 30\times10^{-3} = v/1000 + v/4000$$

Solving,

$$v = (50\times10^{-3})(4\times10^{6} / 5\times10^{3}) = 40 \text{ V}$$

and $$P_{4k\Omega} = v^2/4000 = \boxed{400 \text{ mW}}$$

(*b*) Once again, we first define a voltage *v* across the 1–kΩ resistor with the "+" reference at the top node. Applying KCL at this top node, we find that

$$80\times10^{-3} - 30\times10^{-3} - 20\times10^{-3} = v/1000$$

Solving,

$$v = 30 \text{ V}$$

and $$P_{20mA} = v \cdot 20\times10^{-3} = \boxed{600 \text{ mW}}$$

(*c*) Once again, we first define a voltage *v* across the 1–kΩ resistor with the "+" reference at the top node. Applying KCL at this top node, we find that

$$80\times10^{-3} - 30\times10^{-3} - 2i_x = v/1000$$

where

$$i_x = v/1000$$

so that

$$80\times10^{-3} - 30\times10^{-3} = 2v/1000 + v/1000$$

and

$$v = 50\times10^{-3}(1000)/3 = 16.67 \text{ V}$$

Thus, $$P_{dep} = v \cdot 2i_x = \boxed{555.8 \text{ mW}}$$

(*d*) We note that $i_x = 60/1000 = 60$ mA. KCL stipulates that (viewing currents into and out of the top node)

$$80 - 30 + i_s = i_x = 60$$

Thus, $i_s = 10$ mA

and $P_{60V} = 60(-10)$ mW $= \boxed{-600 \text{ mW}}$

39. (a) To cancel out the effects of both the 80-mA and 30-mA sources, i_S must be set to

$\boxed{i_S = -50 \text{ mA.}}$

(b) Define a current is flowing out of the "+" reference terminal of the independent voltage source. Interpret "no power" to mean "zero power."

Summing the currents flowing into the top node and invoking KCL, we find that

$$80\times10^{-3} - 30\times10^{-3} - v_S/1\times10^{3} + i_S = 0$$

Simplifying slightly, this becomes

$$50 - v_S + 10^3 i_S = 0 \qquad [1]$$

We are seeking a value for v_S such that $v_S \cdot i_S = 0$. Clearly, setting $v_S = 0$ will achieve this. From Eq. [1], we also see that setting $v_S = 50$ V will work as well.

40. Define a voltage v_9 across the 9-Ω resistor, with the "+" reference at the top node.

(*a*) Summing the currents into the right-hand node and applying KCL,

$$5 + 7 = v_9 / 3 + v_9 / 9$$

Solving, we find that $v_9 = 27$ V. Since $i_x = v_9 / 9$, $\boxed{i_x = 3 \text{ A.}}$

(*b*) Again, we apply KCL, this time to the top left node:

$$2 - v_8 / 8 + 2i_x - 5 = 0$$

Since we know from part (*a*) that $i_x = 3$ A, we may calculate $\boxed{v_8 = 24 \text{ V.}}$

(*c*) $p_{5A} = (v_9 - v_8) \cdot 5 = \boxed{15 \text{ W.}}$

41. Define a voltage v_x across the 5-A source, with the "+" reference on top.

Applying KCL at the top node then yields

$$5 + 5v_1 - v_x/ (1 + 2) - v_x/ 5 = 0 \qquad [1]$$

where $v_1 = 2[v_x /(1 + 2)] = 2\, v_x / 3$.

Thus, Eq. [1] becomes

$$5 + 5(2\, v_x / 3) - v_x / 3 - v_x / 5 = 0$$

or $75 + 50\, v_x - 5\, v_x - 3\, v_x = 0$, which, upon solving, yields $v_x = -1.786$ V.

The power absorbed by the 5-Ω resistor is then simply $(v_x)^2/5 = \boxed{638.0 \text{ mW.}}$

42. Despite the way it may appear at first glance, this is actually a simple node-pair circuit. Define a voltage *v* across the elements, with the "+" reference at the top node.

Summing the currents leaving the top node and applying KCL, we find that

$$2 + 6 + 3 + v/5 + v/5 + v/5 = 0$$

or v = -55/3 = -18.33 V. The power supplied by each source is then computed as:

$$\boxed{\begin{aligned} p_{2A} &= -v(2) = 36.67 \text{ W} \\ p_{6A} &= -v(6) = 110 \text{ W} \\ p_{3A} &= -v(3) = 55 \text{ W} \end{aligned}}$$

The power absorbed by each resistor is simply $v^2/5$ = 67.22 W for a total of 201.67 W, which is the total power supplied by all sources. If instead we want the "power supplied" by the resistors, we multiply by -1 to obtain $\boxed{-201.67 \text{ W}}$. Thus, the sum of the supplied power of each circuit element is zero, as it should be.

43. Defining a voltage V_x across the 10-A source with the "+" reference at the top node, KCL tells us that $10 = 5 + I_{1\Omega}$, where $I_{1\Omega}$ is defined flowing downward through the 1-Ω resistor.

Solving, we find that $I_{1\Omega} = 5$ A, so that $V_x = (1)(5) = 5$ V.

So, we need to solve

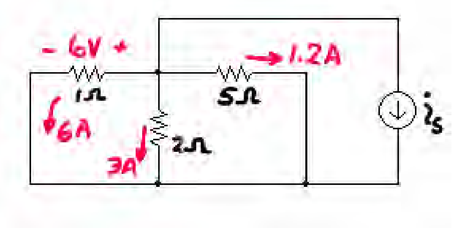
$$V_x = 5 = 5(0.5 + R_{segment})$$

with $R_{segment} = 500$ mΩ.

From Table 2.3, we see that 28-AWG solid copper wire has a resistance of 65.3 mΩ/ft. Thus, the total number of miles needed of the wire is

$$\frac{500 \text{ m}\Omega}{(65.3 \text{ m}\Omega/\text{ft})(5280 \text{ ft/mi})} = \boxed{1.450 \times 10^{-3} \text{ miles}}$$

44. Since v = 6 V, we know the current through the 1-Ω resistor is 6 A, the current through the 2-Ω resistor is 3 A, and the current through the 5-Ω resistor is 6/5 = 1.2 A, as shown below:



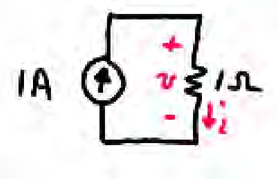
By KCL, $6 + 3 + 1.2 + i_S = 0$ or $\boxed{i_S = -10.2 \text{ A.}}$

45. (*a*) Applying KCL, $1 - i - 3 + 3 = 0$ so $\boxed{i = 1 \text{ A.}}$

(*b*) Looking at the left part of the circuit, we see 1 + 3 = 4 A flowing into the unknown current source, which, by virtue of KCL, must therefore be a 4-A current source. Thus, KCL at the node labeled with the "+" reference of the voltage *v* gives

$$4 - 2 + 7 - i = 0 \qquad \text{or} \qquad \boxed{i = 9 \text{ A}}$$

46. (*a*) We may redraw the circuit as

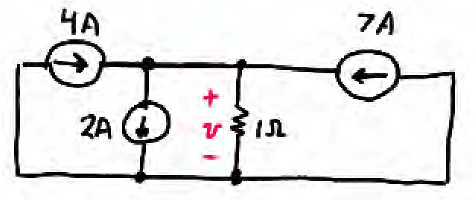


Then, we see that v = (1)(1) = $\boxed{1 \text{ V.}}$

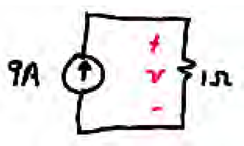
(*b*) We may combine all sources to the right of the 1-Ω resistor into a single 7-A current source. On the left, the two 1-A sources in series reduce to a single 1-A source.

The new 1-A source and the 3-A source combine to yield a 4-A source in series with the unknown current source which, by KCL, must be a 4-A current source.

At this point we have reduced the circuit to



Further simplification is possible, resulting in

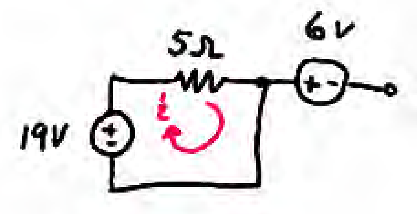


From which we see clearly that v = (9)(1) = $\boxed{9 \text{ V.}}$

47. (*a*) Combine the 12-V and 2-V series connected sources to obtain a new 12 – 2 = 10 V source, with the "+" reference terminal at the top. The result is two 10-V sources in parallel, which is permitted by KVL. Therefore,

$$\boxed{i = 10/1000 = 10 \text{ mA.}}$$

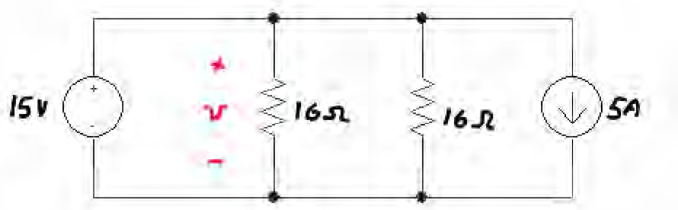
(*b*) No current flows through the 6-V source, so we may neglect it for this calculation. The 12-V, 10-V and 3-V sources are connected in series as a result, so we replace them with a 12 + 10 –3 = 19 V source as shown



Thus, $\boxed{i = 19/5 = 3.8 \text{ A.}}$

48. We first combine the 10-V and 5-V sources into a single 15-V source, with the "+" reference on top. The 2-A and 7-A current sources combine into a 7 – 2 = 5 A current source (arrow pointing down); although these two current sources may not appear to be in parallel at first glance, they actually are.

Redrawing our circuit,



we see that v = 15 V (note that we can completely the ignore the 5-A source here, since we have a voltage source directly across the resistor). Thus,

$$P_{16\Omega} = v^2/16 = \boxed{14.06 \text{ W.}}$$

Returning to the original circuit, we see that the 2 A source is in parallel with both 16 Ω resistors, so that it has a voltage of 15 V across it as well (the same goes for the 7 A source). Thus,

$$P_{2A}\Big|_{abs} = -15(2) = \boxed{-30 \text{ W}}$$
$$P_{7A}\Big|_{abs} = -15(-7) = \boxed{105 \text{ W}}$$

Each resistor draws 15/16 A, so the 5 V and 10 V sources each see a current of

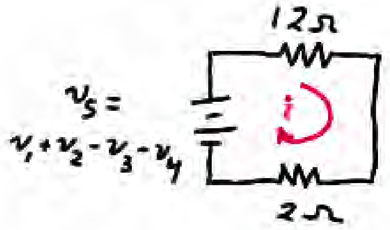
30/16 + 5 = 6.875 A flowing through them.

Thus,

$$P_{5V}\Big|_{abs} = -5(6.875) = \boxed{-34.38 \text{ W}}$$
$$P_{10V}\Big|_{abs} = -10(6.875) = \boxed{-68.75 \text{ W}}$$

which sum to -0.01 W, close enough to zero compared to the size of the terms (roundoff error accumulated).

49. We can combine the voltage sources such that



$i = v_S/ 14$

(*a*) $v_S = 10 + 10 - 6 - 6 = 20 - 12 = 8$
Therefore

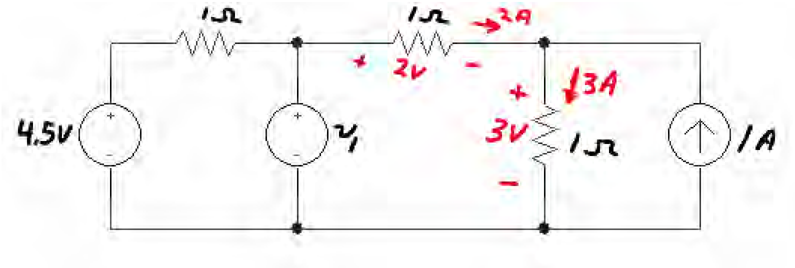
$$i = 8/14 = \boxed{571.4 \text{ mA.}}$$

(*b*) $v_S = 3 + 2.5 - 3 - 2.5 = 0$ Therefore $\boxed{i = 0.}$

(*c*) $v_S = -3 + 1.5 - (-0.5) - 0 = -1$ V
Therefore

$$i = -1/14 = \boxed{-71.43 \text{ mA.}}$$

50. We first simplify as shown, making use of the fact that we are told $i_x = 2$ A to find the voltage across the middle and right-most 1-Ω resistors as labeled.



By KVL, then, we find that $\boxed{v_1 = 2 + 3 = 5 \text{ V.}}$

51. We see that to determine the voltage v we will need v_x due to the presence of the dependent current soruce. So, let's begin with the right-hand side, where we find that

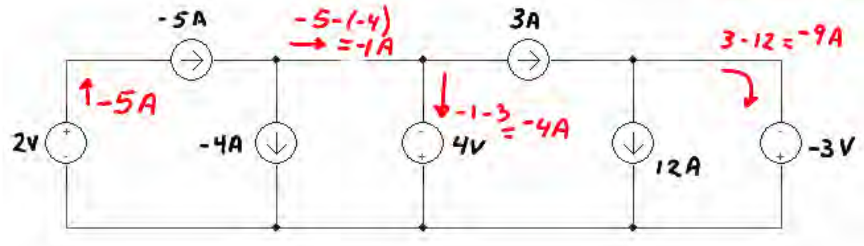
$$v_x = 1000(1 - 3) \times 10^{-3} = -2 \text{ V}.$$

Returning to the left-hand side of the circuit, and summing currents into the top node, we find that

$$(12 - 3.5) \times 10^{-3} + 0.03\, v_x = v/10\times10^3$$

or $\boxed{v = -515 \text{ V.}}$

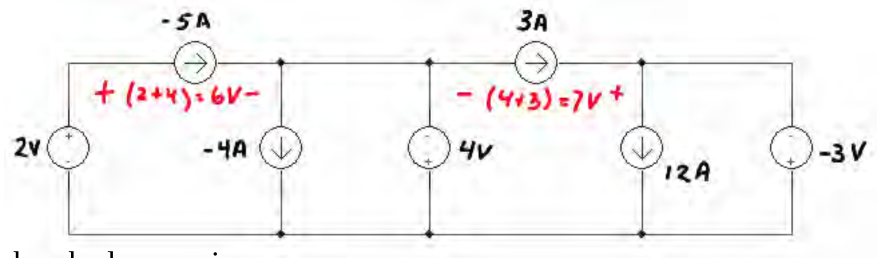
52. (*a*) We first label the circuit with a focus on determining the current flowing through each voltage source:



Then the power absorbed by each voltage source is

P_{2V}	= -2(-5)	= 10 W
P_{4V}	= -(-4)(4)	= 16 W
P_{-3V}	= -(-9)(-3)	= -27 W

For the current sources,



So that the absorbed power is

P_{-5A}	= +(-5)(6)	= -30 W
P_{-4A}	= -(-4)(4)	= 16 W
P_{3A}	= -(3)(7)	= -21 W
P_{12A}	= -(12)(-3)	= 36 W

A quick check assures us that these absorbed powers sum to zero as they should.

(*b*) We need to change the 4-V source such that the voltage across the –5-A source drops to zero. Define V_x across the –5-A source such that the "+" reference terminal is on the left. Then,

$$-2 + V_x - V_{needed} = 0$$

or $\boxed{V_{needed} = -2\text{ V.}}$

53. We begin by noting several things:
(1) The bottom resistor has been shorted out;
(2) the far-right resistor is only connected by one terminal and therefore does not affect the equivalent resistance *as seen from the indicated terminals*;
(3) All resistors to the right of the top left resistor have been shorted.

Thus, from the indicated terminals, we only see the single 1-kΩ resistor, so that $\boxed{R_{eq} = 1\text{ k}\Omega.}$

54. (*a*) We see $1\Omega \parallel (1\ \Omega + 1\ \Omega) \parallel (1\ \Omega + 1\ \Omega + 1\ \Omega)$

$= 1\Omega \parallel 2\ \Omega \parallel 3\ \Omega$

$\boxed{= 545.5\ \text{m}\Omega}$

(*b*) $1/R_{eq} = 1 + 1/2 + 1/3 + \ldots + 1/N$

Thus, $\boxed{R_{eq} = [1 + 1/2 + 1/3 + \ldots + 1/N]^{-1}}$

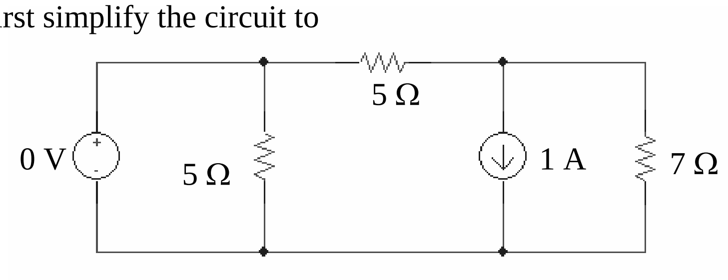
55. (*a*) 5 kΩ = $\boxed{10\ \text{k}\Omega \parallel 10\ \text{k}\Omega}$

(*b*) 57 333 Ω = $\boxed{47\ \text{k}\Omega + 10\ \text{k}\Omega + 1\ \text{k}\Omega \parallel 1\text{k}\Omega \parallel 1\text{k}\Omega}$

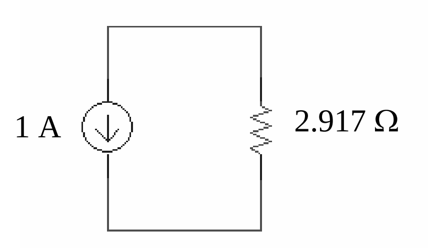
(*c*) 29.5 kΩ = $\boxed{47\ \text{k}\Omega \parallel 47\ \text{k}\Omega + 10\ \text{k}\Omega \parallel 10\ \text{k}\Omega + 1\ \text{k}\Omega}$

56. (*a*) no simplification is possible using only source and/or resistor combination techniques.

(*b*) We first simplify the circuit to



and then notice that the 0-V source is shorting out one of the 5-Ω resistors, so a further simplification is possible, noting that 5 Ω || 7 Ω = 2.917 Ω:

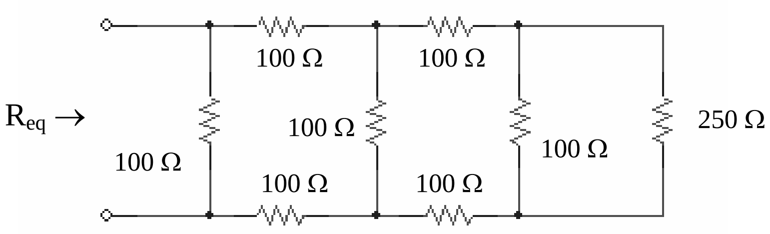


57. R_{eq} $= 1\ \text{k}\Omega + 2\ \text{k}\Omega \parallel 2\ \text{k}\Omega + 3\ \text{k}\Omega \parallel 3\ \text{k}\Omega + 4\ \text{k}\Omega \parallel 4\ \text{k}\Omega$

$= 1\ \text{k}\Omega + 1\ \text{k}\ \Omega + 1.5\ \text{k}\Omega + 2\ \text{k}\Omega$

$\boxed{= 5.5\ \text{k}\Omega.}$

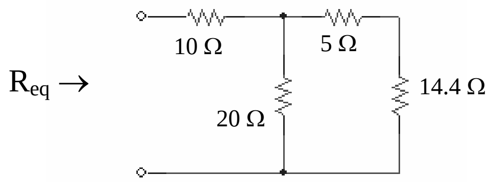
58. (a) Working from right to left, we first see that we may combine several resistors as 100 Ω + 100 Ω || 100 Ω + 100 Ω = 250 Ω, yielding the following circuit:



Next, we see 100 Ω + 100 Ω || 250 Ω + 100 Ω = 271.4 Ω,
and subsequently 100 Ω + 100 Ω || 271.4 Ω + 100 Ω = 273.1 Ω,
and, finally,

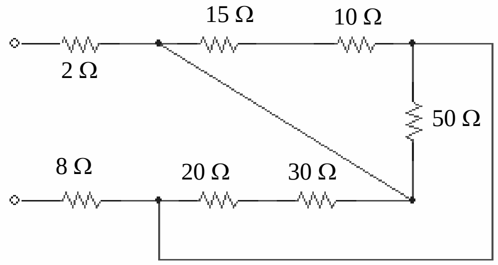
$$R_{eq} = 100\ \Omega \parallel 273.1\ \Omega = \boxed{73.20\ \Omega.}$$

(b) First, we combine 24 Ω || (50 Ω + 40 Ω) || 60 Ω = 14.4 Ω, which leaves us with

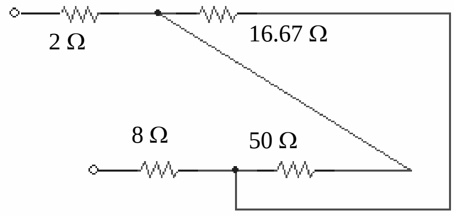


Thus, R_{eq} = 10 Ω + 20 Ω || (5 + 14.4 Ω) = $\boxed{19.85\ \Omega.}$

(c) First combine the 10-Ω and 40-Ω resistors and redraw the circuit:



We now see we have (10 Ω + 15 Ω) || 50 Ω = 16.67 Ω. Redrawing once again,



where the equivalent resistance is seen to be 2 Ω + 50 Ω || 16.67 Ω + 8 Ω = $\boxed{22.5\ \Omega.}$

59. (*a*) $R_{eq} = [(40\ \Omega + 20\ \Omega) \,||\, 30\ \Omega + 80\ \Omega] \,||\, 100\ \Omega + 10\ \Omega = \boxed{60\ \Omega.}$

(*b*) $R_{eq} = 80\ \Omega = [(40\ \Omega + 20\ \Omega) \,||\, 30\ \Omega + R] \,||\, 100\ \Omega + 10\ \Omega$

$$70\ \Omega = [(60\ \Omega \,||\, 30\ \Omega) + R] \,||\, 100\ \Omega$$

$$1/70 = 1/(20 + R) + 0.01$$

$20 + R = 233.3\ \Omega$ therefore $\boxed{R = 213.3\ \Omega.}$

(*c*) $R = [(40\ \Omega + 20\ \Omega) \,||\, 30\ \Omega + R] \,||\, 100\ \Omega + 10\ \Omega$

$$R - 10\ \Omega = [20 + R] \,||\, 100$$

$$1/(R - 10) = 1/(R + 20) + 1/100$$

$$3000 = R^2 + 10R - 200$$

Solving, we find $R = -61.79\ \Omega$ or $R = 51.79\ \Omega$.
Clearly, the first is not a physical solution, so $\boxed{R = 51.79\ \Omega.}$

60. (*a*) $25\ \Omega = \boxed{100\ \Omega \parallel 100\ \Omega \parallel 100\ \Omega \parallel 100\ \Omega}$

(*b*) $60\ \Omega = \boxed{[(100\ \Omega \parallel 100\ \Omega) + 100\ \Omega] \parallel 100\ \Omega}$

(*c*) $40\ \Omega = \boxed{(100\ \Omega + 100\ \Omega) \parallel 100\ \Omega \parallel 100\ \Omega}$

61. $R_{eq} = [(5\ \Omega \parallel 20\ \Omega) + 6\ \Omega] \parallel 30\ \Omega + 2.5\ \Omega = 10\ \Omega$

The source therefore provides a total of 1000 W and a current of 100/10 = 10 A.

$P_{2.5\Omega} = (10)^2 \cdot 2.5 = \boxed{250\text{ W}}$

$V_{30\Omega} = 100 - 2.5(10) = 75\text{ V}$
$P_{30\Omega} = 75^2/30 = \boxed{187.5\text{ W}}$

$I_{6\Omega} = 10 - V_{30\Omega}/30 = 10 - 75/30 = 7.5\text{ A}$
$P_{6\Omega} = (7.5)^2 \cdot 6 = \boxed{337.5\text{ W}}$

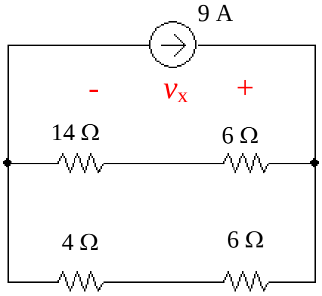
$V_{5\Omega} = 75 - 6(7.5) = 30\text{ V}$
$P_{5\Omega} = 30^2/5 = \boxed{180\text{ W}}$

$V_{20\Omega} = V_{5\Omega} = 30\text{ V}$
$P_{20\Omega} = 30^2/20 = \boxed{45\text{ W}}$

We check our results by verifying that the absorbed powers in fact add to 1000 W.

62. To begin with, the 10-Ω and 15-Ω resistors are in parallel (= 6 Ω), and so are the 20-Ω and 5-Ω resistors (= 4 Ω).

Also, the 4-A, 1-A and 6-A current sources are in parallel, so they can be combined into a single 4 + 6 – 1 = 9 A current source as shown:



Next, we note that (14 Ω + 6 Ω) || (4 Ω + 6 Ω) = 6.667 Ω
so that

$$v_x = 9(6.667) = 60 \text{ V}$$

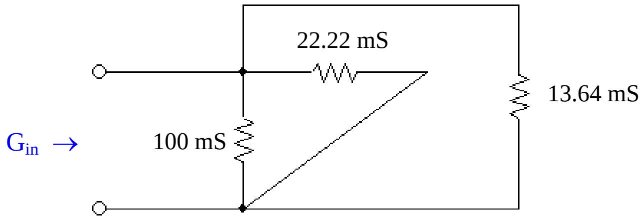
and

$$i_x = -60/10 = -6 \text{ A}.$$

63. (*a*) Working from right to left, and borrowing $x \parallel y$ notation from resistance calculations to indicate the operation $xy/(x + y)$,

$$\begin{aligned} G_{in} &= \{[(6 \parallel 2 \parallel 3) + 0.5] \parallel 1.5 \parallel 2.5 + 0.8\} \parallel 4 \parallel 5 \text{ mS} \\ &= \{[(1) + 0.5] \parallel 1.5 \parallel 2.5 + 0.8\} \parallel 4 \parallel 5 \text{ mS} \\ &= \{1.377\} \parallel 4 \parallel 5 \\ &= 0.8502 \text{ mS} = \boxed{850.2 \text{ mS}} \end{aligned}$$

(*b*) The 50-mS and 40-mS conductances are in series, equivalent to (50(40)/90 = 22.22 mS. The 30-mS and 25-mS conductances are also in series, equivalent to 13.64 mS. Redrawing for clarity,



we see that G_{in} = 10 + 22.22 + 13.64 = $\boxed{135.9 \text{ mS.}}$

64. The bottom four resistors between the 2-Ω resistor and the 30-V source are shorted out. The 10-Ω and 40-Ω resistors are in parallel (= 8 Ω), as are the 15-Ω and 60-Ω (=12 Ω) resistors. These combinations are in series.

Define a clockwise current I through the 1-Ω resistor:

$$I = (150 - 30)/(2 + 8 + 12 + 3 + 1 + 2) = 4.286 \text{ A}$$

$P_{1\Omega} = I^2 \cdot 1 = \boxed{18.37 \text{ W}}$

To compute $P_{10\Omega}$, consider that since the 10-Ω and 40-Ω resistors are in parallel, the same voltage V_x ("+" reference on the left) appears across both resistors. The current I = 4.286 A flows into this combination. Thus, $V_x = (8)(4.286) = 34.29$ V and

$P_{10\Omega} = (V_x)^2 / 10 = \boxed{117.6 \text{ W.}}$

$\boxed{P_{13\Omega} = 0}$ since no current flows through that resistor.

65. With the meter being a short circuit and no current flowing through it, we can write

$R_1 i_1 = R_2 i_2$
$R_3 i_3 = R i_R$ $\rightarrow$ $\dfrac{R_1 i_1}{R_3 i_3} = \dfrac{R_2 i_2}{R i_R}$ [1]

And since $i_1 = i_3$, and $i_2 = i_R$, Eq [1] becomes $\dfrac{R_1}{R_3} = \dfrac{R_2}{R}$, or $R = R_2 \dfrac{R_3}{R_1}$. *Q.E.D.*

66. The total resistance in the series string sums to 75 Ω.

The voltage dropped across the 2.2 Ω resistor is

$$V_{2.2\Omega} = 10\frac{2.2}{75} = \boxed{293.3 \text{ mV}}$$

and the voltage dropped across the 47 Ω resistor is

$$V_{47\Omega} = 10\frac{47}{75} = \boxed{6.267 \text{ mV}}$$

67. We first note that the 4.7 kΩ and 2.2 kΩ resistors can be combined into a single 1.5 kΩ resistor, which is then in series with the 10 kΩ resistor. Next we note that the 33 kΩ / 47 kΩ parallel combination can be replaced by a 19.39 kΩ resistance, which is in series with the remaining 33 kΩ resistor.

By voltage division, then, and noting that $V_{47k\Omega}$ is the same voltage as that across the 19.39 kΩ resistance,

$$V_{47k\Omega} = 2\frac{19.39}{10+1.5+33+19.39} = \boxed{607.0 \text{ mV}}$$

68. (a) The current downward through the 33 Ω resistor is calculated more easily if we first note that 134 Ω || 134 Ω = 67 Ω, and 67 Ω + 33 Ω = 100 Ω. Then,

$$I_{33\Omega} = 12 \frac{1/100}{1/10 + 1/10 + 1/100} = \boxed{571.4 \text{ mA}}$$

(b) The resistor flowing downward through either 134 Ω resistor is simply

$571.4/2 = \boxed{285.7 \text{ mA}}$ (by current division).

69. We first note that 20 Ω || 60 Ω = 15 Ω, and 50 Ω || 30 Ω = 18.75 Ω.
Then, 15 Ω + 22 Ω + 18.75 Ω = 55.75 Ω.

Finally, we are left with two current sources, the series combination of 10 Ω + 15 Ω, and 10 Ω || 55.75 Ω = 8.479 Ω.

Using current division on the simplified circuit,

$$I_{15\Omega} = (30-8)\frac{\frac{1}{10+15}}{\frac{1}{10+15}+\frac{1}{8.479}} = \boxed{22.12 \text{ A}}$$

70. One possible solution of many:

$v_S = 2(5.5) = 11$ V
$R_1 = R_2 = 1$ kΩ.

71. One possible solution of many:

$$\boxed{\begin{array}{l} i_S = 11 \text{ mA} \\ R_1 = R_2 = 1\text{ k}\Omega. \end{array}}$$

72. $p_{15\Omega} = (v_{15})^2 / 15\times10^3$ A

$v_{15} = 15\times10^3 (-0.3\ v_1)$

where $v_1 = [4 (5)/ (5 + 2)] \cdot 2 = 5.714$ V

Therefore $v_{15} = -25714$ V and $\boxed{p_{15} = 44.08 \text{ kW.}}$

73. Replace the top 10 kΩ, 4 kΩ and 47 kΩ resistors with 10 kΩ + 4 kΩ || 47 kΩ = 13.69 kΩ.

Define v_x across the 10 kΩ resistor with its "+" reference at the top node: then

$v_x = 5 \cdot (10\ \text{k}\Omega \parallel 13.69\ \text{k}\Omega) / (15\ \text{k}\Omega + 10 \parallel 13.69\ \text{k}\Omega) = 1.391\ \text{V}$

$i_x = v_x / 10\ \text{mA} = \boxed{139.1\ \mu\text{A}}$

$v_{15} = 5 - 1.391 = 3.609\ \text{V}$ and $p_{15} = (v_{15})^2 / 15\times10^3 = \boxed{868.3\ \mu\text{W}.}$

74. We may combine the 12-A and 5-A current sources into a single 7-A current source with its arrow oriented upwards. The left three resistors may be replaced by a 3 + 6 || 13 = 7.105 Ω resistor, and the right three resistors may be replaced by a 7 + 20 || 4 = 10.33 Ω resistor.

By current division, $i_y = 7\ (7.105)/(7.105 + 10.33) = \boxed{2.853\text{ A}}$

We must now return to the original circuit. The current into the 6 Ω, 13 Ω parallel combination is $7 - i_y = 4.147$ A. By current division,

$$i_x = 4.147\ .\ 13/\ (13 + 6) = \boxed{2.837\text{ A}}$$

$$\text{and } p_x = (4.147)^2 \cdot 3 = \boxed{51.59\text{ W}}$$

75. The controlling voltage v_1, needed to obtain the power into the 47-kΩ resistor, can be found separately as that network does not depend on the left-hand network.
The right-most 2 kΩ resistor can be neglected.

By current division, then, in combination with Ohm's law,

$$v_1 = 3000[5\times10^{-3}(2000)/(2000 + 3000 + 7000)] = 2.5\text{ V}$$

Voltage division gives the voltage across the 47-kΩ resistor:

$$0.5v_1 \frac{47}{47 + 100 \| 20} = \frac{0.5(2.5)(47)}{47 + 16.67} = 0.9228\text{ V}$$

So that $p_{47\text{k}\Omega} = (0.9928)^2 / 47\times10^3 = \boxed{18.12\ \mu\text{W}}$

76. The temptation to write an equation such as

$$v_1 = 10 \frac{20}{20 + 20}$$

must be fought!

Voltage division only applies to resistors connected in series, meaning that the *same* current must flow through *each* resistor. In this circuit, $i_1 \neq 0$, so we do not have the same current flowing through both 20 kΩ resistors.

77. (*a*) $$v_2 = V_S \frac{R_2 \| (R_3 + R_4)}{R_1 + [R_2 \| (R_3 + R_4)]}$$

$$= V_S \frac{R_2 (R_3 + R_4)/(R_2 + R_3 + R_4)}{R_1 + R_2 (R_3 + R_4)/(R_2 + R_3 + R_4)}$$

$$\boxed{= V_S \frac{R_2 (R_3 + R_4)}{R_1(R_2 + R_3 + R_4) + R_2(R_3 + R_4)}}$$

(*b*) $$v_1 = V_S \frac{R_1}{R_1 + [R_2 \| (R_3 + R_4)]}$$

$$= V_S \frac{R_1}{R_1 + R_2 (R_3 + R_4)/(R_2 + R_3 + R_4)}$$

$$\boxed{= V_S \frac{R_1 (R_2 + R_3 + R_4)}{R_1(R_2 + R_3 + R_4) + R_2(R_3 + R_4)}}$$

(*c*) $$i_4 = \left(\frac{v_1}{R_1}\right)\left(\frac{R_2}{R_2 + R_3 + R_4}\right)$$

$$= V_S \frac{R_1 (R_2 + R_3 + R_4) R_2}{R_1[R_1(R_2 + R_3 + R_4) + R_2(R_3 + R_4)(R_2 + R_3 + R_4)]}$$

$$\boxed{= V_S \frac{R_2}{R_1(R_2 + R_3 + R_4) + R_2(R_3 + R_4)}}$$

78. (*a*) With the current source open-circuited, we find that

$$v_1 = -40 \frac{500}{500 + 3000 \| 6000} = \boxed{-8\text{ V}}$$

(*b*) With the voltage source short-circuited, we find that

$$i_2 = \left(3\times10^{-3}\right) \frac{1/3000}{1/500 + 1/3000 + 1/6000} = \boxed{400\ \mu\text{A}}$$

$$i_3 = \left(3\times10^{-3}\right) \frac{500}{500 + 3000 \| 6000} = \boxed{600\ \mu\text{A}}$$

79. (*a*) The current through the 5-Ω resistor is 10/5 = 2 A. Define R as 3 || (4 + 5) = 2.25 Ω. The current through the 2-Ω resistor then is given by

$$I_S \frac{1}{1+(2+R)} = \frac{I_S}{5.25}$$

The current through the 5-Ω resistor is

$$\frac{I_S}{5.25}\left(\frac{3}{3+9}\right) = 2\text{ A}$$

so that $\boxed{I_S = 42\text{ A.}}$

(*b*) Given that I_S is now 50 A, the current through the 5-Ω resistor becomes

$$\frac{I_S}{5.25}\left(\frac{3}{3+9}\right) = 2.381\text{ A}$$

Thus, $v_x = 5(2.381) = \boxed{11.90\text{ V}}$

(*c*) $$\frac{v_x}{I_S} = \frac{\left[\frac{5I_S}{5.25}\left(\frac{3}{3+9}\right)\right]}{I_S} = \boxed{0.2381}$$

80. First combine the 1 kΩ and 3 kΩ resistors to obtain 750 Ω.
By current division, the current through resistor R_x is

$$I_{R_x} = 10\times10^{-3} \frac{2000}{2000 + R_x + 750}$$

and we know that $R_x \cdot I_{R_x} = 9$

so $9 = \dfrac{20\,R_x}{2750 + R_x}$

$9\,R_x + 24750 = 20\,R_x$ or $R_x =$ 2250 W. Thus,

$$\boxed{P_{R_x} = 9^2/ R_x = 36 \text{ mW.}}$$

81. Define $R = R_3 \parallel (R_4 + R_5)$

Then $$v_R = V_S\left(\frac{R}{R + R_2}\right)$$

$$= V_S\left(\frac{R_3\,(R_4 + R_5)/(R_3 + R_4 + R_5)}{R_3\,(R_4 + R_5)/(R_3 + R_4 + R_5) + R_2}\right)$$

$$= V_S\left(\frac{R_3\,(R_4 + R_5)}{R_2\,(R_3 + R_4 + R_5) + R_3(R_4 + R_5)}\right)$$

Thus,

$$v_5 = v_R\left(\frac{R_5}{R_4 + R_5}\right)$$

$$\boxed{= V_S\left(\frac{R_3\,R_5}{R_2\,(R_3 + R_4 + R_5) + R_3(R_4 + R_5)}\right)}$$

82. Define $R_1 = 10 + 15 \| 30 = 20\ \Omega$ and $R_2 = 5 + 25 = 30\ \Omega$.

(*a*) $I_x = I_1 . 15 / (15 + 30) = \boxed{4 \text{ mA}}$

(*b*) $I_1 = I_x . 45/15 = \boxed{36 \text{ mA}}$

(*c*) $I_2 = I_S R_1 / (R_1 + R_2)$ and $I_1 = I_S R_2 / (R_1 + R_2)$
So $I_1/I_2 = R_2/R_1$
Therefore

$$I_1 = R_2 I_2/R_1 = 30(15)/20 = 22.5 \text{ mA}$$

Thus, $I_x = I_1 \cdot 15/45 = \boxed{7.5 \text{ mA}}$

(*d*) $I_1 = I_S R_2/(R_1 + R_2) = 60(30)/50 = 36 \text{ mA}$

Thus, $I_x = I_1\ 15/45 = \boxed{12 \text{ mA.}}$

83. $v_{out} = -g_m v_\pi (100 \text{ k}\Omega \,||\, 100 \text{ k}\Omega) = -4.762\times10^3 g_m v_\pi$

where $v_\pi = (3 \sin 10t) \cdot 15/(15 + 0.3) = 2.941 \sin 10t$

Thus, $\boxed{v_{out} = -56.02 \sin 10t \text{ V}}$

84. $v_{out} = -1000 g_m v_\pi$

where $v_\pi = 3 \sin 10t \dfrac{15 \| 3}{(15 \| 3) + 0.3} = 2.679 \sin 10t$ V

therefore

$$v_{out} = -(2.679)(1000)(38\times10^{-3}) \sin 10t = \boxed{-101.8 \sin 10t \text{ V.}}$$

1. (*a*)

$$\begin{bmatrix} 0.1 & -0.3 & -0.4 \\ -0.5 & 0.1 & 0 \\ -0.2 & -0.3 & 0.4 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \\ 6 \end{bmatrix}$$

Solving this matrix equation using a scientific calculator, $\boxed{v_2 = -8.387 \text{ V}}$

(*b*) Using a scientific calculator, the determinant is equal to $\boxed{32.}$

2. (*a*)

-1

$$\begin{bmatrix} 1 & 1 & 1 \\ & 2 & 3 \\ 2 & 0 & 4 \end{bmatrix} \begin{bmatrix} v_A \\ v_B \\ v_C \end{bmatrix} = \begin{bmatrix} 27 \\ -16 \\ -6 \end{bmatrix}$$

Solving this matrix equation using a scientific calculator,

$$\boxed{\begin{aligned} v_A &= 19.57 \\ v_B &= 18.71 \\ v_C &= -11.29 \end{aligned}}$$

(*b*) Using a scientific calculator,

-1

$$\begin{vmatrix} 1 & 1\ 1 & \\ & 2 & 3 \\ 2 & 0 & 4 \end{vmatrix} = \boxed{16}$$

3.

(a) We begin by simplifying the equations prior to solution:

$$\begin{aligned} 4 &= 0.08v_1 - 0.05v_2 - 0.02v_3 \\ 8 &= -0.02v_1 - 0.025v_2 + 0.045v_3 \\ -2 &= -0.05v_1 + 0.115v_2 - 0.025v_3 \end{aligned}$$

Then, we can solve the matrix equation:

$$\begin{bmatrix} 0.08 & -0.05 & -0.02 \\ -0.02 & -0.025 & 0.045 \\ -0.05 & 0.115 & -0.025 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 8 \\ -2 \end{bmatrix}$$

to obtain $\boxed{v_1 = 264.3 \text{ V}, v_2 = 183.9 \text{ V and } v_3 = 397.4 \text{ V.}}$

(b) We may solve the matrix equation directly using MATLAB, but a better check is to invoke the symbolic processor:

```
>> e1 = '4 = v1/100 + (v1 - v2)/20 + (v1 - vx)/50';
>> e2 = '10 - 4 - (-2) = (vx - v1)/50 + (vx - v2)/40';
>> e3 = '-2 = v2/25 + (v2 - vx)/40 + (v2 - v1)/20';
>> a = solve(e1,e2,e3,'v1','v2','vx');
>> a.v1

ans =

82200/311

>> a.v2

ans =

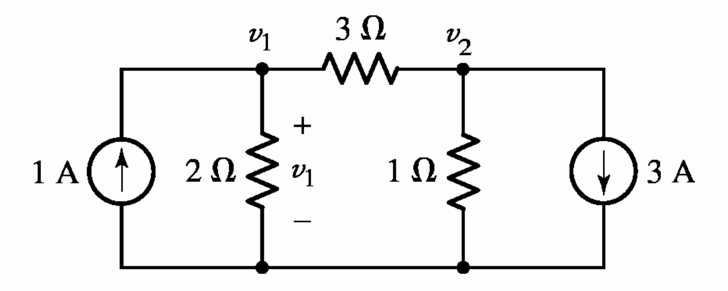
57200/311

>> a.vx

ans =

123600/311
```

4. We select the bottom node as our reference terminal and define two nodal voltages:



Ref.

Next, we write the two required nodal equations:

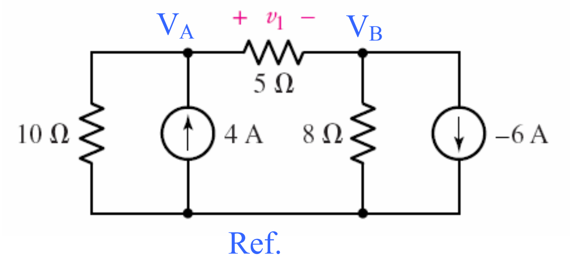
Node 1: $1 = \frac{v_1}{2} + \frac{v_1 - v_2}{3}$

Node 2: $-3 = \frac{v_2}{1} + \frac{v_2 - v_1}{3}$

Which may be simplified to: $5v_1 - 2v_2 = 6$
and $-v_1 + 4v_2 = -9$

Solving, we find that $\boxed{v_1 = 333.3 \text{ mV.}}$

5. We begin by selecting the bottom node as the reference terminal, and defining two nodal voltages V_A and V_B, as shown. (Note if we choose the upper right node, v_1 becomes a nodal voltage and falls directly out of the solution.)



We note that after completing nodal analysis, we can find v_1 as $v_1 = V_A - V_B$.

At node A: $4 = \frac{V_A}{10} + \frac{V_A - V_B}{5}$ [1]

At node B: $-(-6) = \frac{V_B}{8} + \frac{V_B - V_A}{5}$ [2]

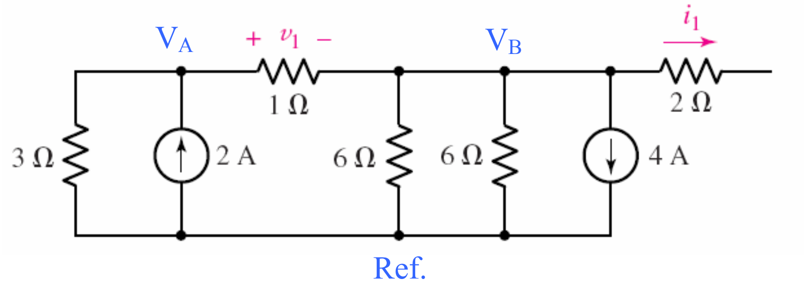
Simplifying,

$$3V_A - 2V_B = 40 \quad [1]$$
$$-8V_A + 13V_B = 240 \quad [2]$$

Solving, $V_A = 43.48$ V and $V_B = 45.22$ V, so $\boxed{v_1 = -1.740 \text{ V.}}$

6. By inspection, no current flows through the 2 Ω resistor, so $i_1 = 0$.

We next designate the bottom node as the reference terminal, and define V_A and V_B as shown:



At node A: $2 = \dfrac{V_A}{3} + \dfrac{V_A - V_B}{1}$ [1]

At node B: $-2 = \dfrac{V_B}{6} + \dfrac{V_B}{6} + \dfrac{V_B - V_A}{1}$ 2]

Note this yields V_A and V_B, not v_1, due to our choice of reference node. So, we obtain v_1 by KVL: $v_1 = V_A - V_B$.

Simplifying Eqs. [1] and [2],

$4V_A - 3V_B = 6$ [1]

$-3V_A + 4V_B = -6$ [2]

Solving, $V_A = 0.8571$ V and $V_B = -0.8571$ V, so $\boxed{v_1 = 1.714 \text{ V.}}$

7. The bottom node has the largest number of branch connections, so we choose that as our reference node. This also m akes v_P easier to find, as it will be a nodal voltag e. Working from left to right, we name our nodes 1, P, 2, and 3.

NODE 1: $10 = v_1/20 + (v_1 - v_P)/40$ [1]

NODE P: $0 = (v_P - v_1)/40 + v_P/100 + (v_P - v_2)/50$ [2]

NODE 2: $-2.5 + 2 = (v_2 - v_P)/50 + (v_2 - v_3)/10$ [3]

NODE 3: $5 - 2 = v_3/200 + (v_3 - v_2)/10$ [4]

Simplifying,

$$
\begin{array}{rrrrlr}
60v_1 & -20v_P & & & = 8000 & [1] \\
-50\,v_1 & +110\,v_P & -40v_2 & & = 0 & [2] \\
& -v_P & +6v_2 & -5v_3 & = -25 & [3] \\
& & -200v_2 & +210v_3 & = 6000 & [4]
\end{array}
$$

Solving,

$$\boxed{v_P = 171.6 \text{ V}}$$

8. The logical choice for a reference node is the bottom node, as then v_x will automatically become a nodal voltage.

NODE 1: $4 = v_1/ 100 + (v_1 - v_2)/ 20 + (v_1 - v_x)/ 50$ [1]

NODE x: $10 - 4 - (-2) = (v_x - v_1)/ 50 + (v_x - v_2)/ 40$ [2]

NODE 2: $-2 = v_2 / 25 + (v_2 - v_x)/ 40 + (v_2 - v_1)/ 20$ [3]

Simplifying,

$$4 = 0.0800v_1 - 0.0500v_2 - 0.0200v_x \quad [1]$$
$$8 = -0.0200v_1 - 0.02500v_2 + 0.04500v_x \quad [2]$$
$$-2 = -0.0500v_1 + 0.1150v_2 - 0.02500v_x \quad [3]$$

Solving,

$$\boxed{v_x = 397.4 \text{ V.}}$$

9. Designate the node between the 3-Ω and 6-Ω resistors as node X, and the right-hand node of the 6-Ω resistor as node Y. The bottom node is chosen as the reference node.

(*a*) Writing the two nodal equations, then

NODE X: $-10 = (v_X - 240)/3 + (v_X - v_Y)/6$ [1]

NODE Y: $0 = (v_Y - v_X)/6 + v_Y/30 + (v_Y - 60)/12$ [2]

Simplifying, $-180 + 1440 = 9\,v_X - 3\,v_Y$ [1]

$10800 = -360\,v_X + 612\,v_Y$ [2]

Solving, $v_X = 181.5$ V and $v_Y = 124.4$ V

Thus, $v_1 = 240 - v_X = \boxed{58.50 \text{ V}}$ and $v_2 = v_Y - 60 = \boxed{64.40 \text{ V}}$

(*b*) The power absorbed by the 6-Ω resistor is

$$(v_X - v_Y)^2 / 6 = \boxed{543.4 \text{ W}}$$

10. Only one nodal equation is required: At the node where three resistors join,

$$0.02v_1 = (v_x - 5\, i_2) / 45 + (v_x - 100) / 30 + (v_x - 0.2\, v_3) / 50 \qquad [1]$$

This, however, is one equation in four unknow ns, the other three resulting from the presence of the dependent sources. Thus, we require three additional equations:

$$i_2 = (0.2\, v_3 - v_x) / 50 \qquad [2]$$

$$v_1 = 0.2\, v_3 - 100 \qquad [3]$$

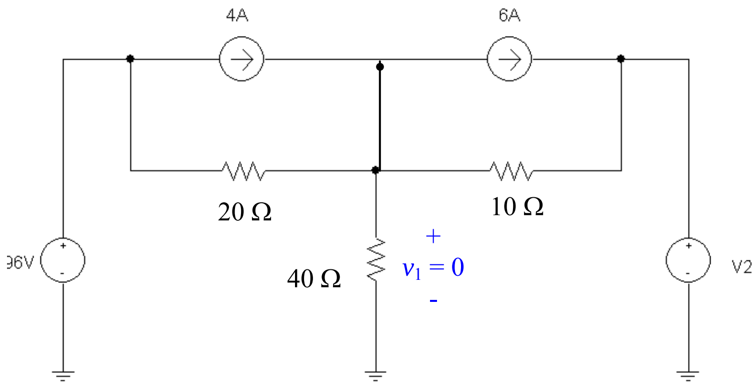
$$v_3 = 50 i_2 \qquad [4]$$

Simplifying,

$$\begin{array}{lllll} & v_1 - 0.2v_3 & & = -100 & [3] \\ & - v_3 & + 50\, i_2 & = 0 & [4] \\ -v_x & + 0.2v_3 & - 50\, i_2 & = 0 & [2] \\ 0.07556v_x - 0.02v_1 - 0.004v_3 - 0.111i_2 & & & = 33.33 & [1] \end{array}$$

Solving, we find that $\boxed{v_1 = -103..8 \text{ V}}$ and $\boxed{i_2 = -377.4 \text{ mA.}}$

11. If $v_1 = 0$, the dependent source is a short circuit and we may redraw the circuit as:



At NODE 1: $4 - 6 = v_1/40 + (v_1 - 96)/20 + (v_1 - V_2)/10$

Since $v_1 = 0$, this simplifies to

$$-2 = -96/20 - V_2/10$$

so that $\boxed{V_2 = -28 \text{ V.}}$

12. We choose the bottom node as ground to make calculation of i_5 easier. The left-most node is named "1", the top node is nam ed "2", the central node is nam ed "3" and the node between the 4-Ω and 6-Ω resistors is named "4."

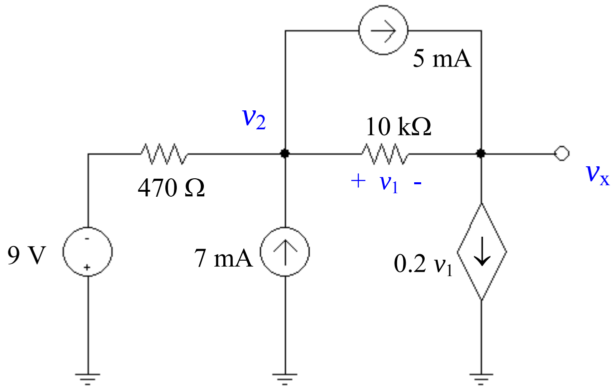
NODE 1: $-3 = v_1/2 + (v_1 - v_2)/1$ [1]
NODE 2: $2 = (v_2 - v_1)/1 + (v_2 - v_3)/3 + (v_2 - v_4)/4$ [2]
NODE 3: $3 = v_3/5 + (v_3 - v_4)/7 + (v_3 - v_2)/3$ [3]
NODE 4: $0 = v_4/6 + (v_4 - v_3)/7 + (v_4 - v_2)/4$ [4]

Rearranging and grouping terms,

$3v_1 - 2v_2 = -6$ [1]
-12 $v_1 + 19v_2 - 4v_3 - 3v_4 = 24$ [2]
$-35v_2 + 71v_3 - 15v_4 = 315$ [3]
$-42v_2 - 24v_3 + 94v_4 = 0$ [4]

Solving, we find that $v_3 = 6.760$ V and so $\boxed{i_5 = v_3/5 = 1.352 \text{ A.}}$

13. We can redraw this circuit and eliminate the 2.2-kΩ resistor as no current flows through it:



At NODE 2: $7\times10^{-3} - 5\times10^{-3} = (v_2 + 9)/ 470 + (v_2 - v_x)/ 10\times10^{-3}$ [1]

At NODE x: $5\times10^{-3} - 0.2v_1 = (v_x - v_2)/ 10\times10^{3}$ [2]

The additional equation required by the presence of the dependent source and the fact that its controlling variable is not one of the nodal voltages:

$$v_1 = v_2 - v_x \qquad [3]$$

Eliminating the variable v1 and grouping terms, we obtain:

$$10{,}470\ v_2 - 470\ v_x = -89{,}518$$

and

$$1999\ v_2 - 1999\ v_x = 50$$

Solving, we find $\boxed{v_x = -8.086 \text{ V.}}$

14. We need concern ourselves with the bottom part of this circuit only. Writing a single nodal equation,

$$-4 + 2 = v/50$$

We find that $\boxed{v = -100 \text{ V.}}$

15. We choose the bottom node as the reference terminal. Then:

Node 1: $-2 = \frac{v_1}{2} + \frac{v_1 - v_2}{1}$ [1]

Node 2: $4 = \frac{v_2 - v_1}{1} + \frac{v_2 - v_3}{2} + \frac{v_2 - v_4}{4}$ [2]

Node 3: $2 = \frac{v_3 - v_2}{2} + \frac{v_3}{5} + \frac{v_3 - v_4}{10}$ [3]

Node 4: $0 = \frac{v_4}{6} + \frac{v_4 - v_3}{10} + \frac{v_4 - v_2}{4}$ [4]

Node 5: $-1 = \frac{v_5}{2} + \frac{v_5 - v_7}{1}$ [5]

Node 6: $1 = \frac{v_6}{5} + \frac{v_6 - v_7}{2} + \frac{v_6 - v_8}{10}$ [6]

Node 7: $2 = \frac{v_7 - v_5}{1} + \frac{v_7 - v_6}{2} + \frac{v_7 - v_8}{4}$ [7]

Node 8: $0 = \frac{v_8}{6} + \frac{v_8 - v_6}{10} + \frac{v_8 - v_7}{4}$ [8]

Note that Eqs. [1-4] may be solved independently of Eqs. [5-8].

Simplifying,

$$\begin{array}{rrrrll} 3v_1 & -2v_2 & & & = -4 & [1] \\ -4v_1 & +7v_2 & -2v_3 & -v_4 & = 16 & [2] \\ & -5v_2 & +8v_3 & -v_4 & = 20 & [3] \\ & -15v_2 & -6v_3 & +31v_4 & = 0 & [4] \end{array} \quad \text{to yield} \quad \boxed{\begin{array}{l} v_1 = 3.370\text{ V} \\ v_2 = 7.055\text{ V} \\ v_3 = 7.518\text{ V} \\ v_4 = 4.869\text{ V} \end{array}}$$

and

$$\begin{array}{rrrrll} 3v_5 & & -2v_7 & & = -2 & [5] \\ & 8v_6 & -5v_7 & -v_8 & = 10 & [6] \\ -4v_5 & -2v_6 & +7v_7 & -v_8 & = 8 & [7] \\ & -6v_6 & -15v_7 & +31v_8 & = 0 & [8] \end{array} \quad \text{to yield} \quad \boxed{\begin{array}{l} v_5 = 1.685\text{ V} \\ v_6 = 3.759\text{ V} \\ v_7 = 3.527\text{ V} \\ v_8 = 2.434\text{ V} \end{array}}$$

16. We choose the center node for our common terminal, since it connects to the largest number of branches. W e na me the left node "A", the top node "B", the right node "C", and the bottom node "D". We next form a supernode between nodes A and B.

At the supernode: $5 = (V_A – V_D)/ 10 + V_A/ 20 + (V_B – V_C)/ 12.5$ [1]

At node C: $V_C = 150$ [2]

At node D: $-10 = V_D/ 25 + (V_D – V_A)/ 10$ [3]

Our supernode-related equation is $V_B – V_A = 100$ [4]

Simplifiying and grouping terms,

$$
\begin{array}{lll}
0.15\, V_A + 0.08\, V_B - 0.08\, V_C - 0.1\, V_D & = 5 & [1] \\
V_C & = 150 & [2] \\
-25\, V_A + 35\, V_D & = -2500 & [3] \\
- V_A + V_B & = 100 & [4]
\end{array}
$$

Solving, we find that $V_D = -63.06$ V. Since $v_4 = - V_D$,

$$\boxed{v_4 = 63.06 \text{ V.}}$$

17. Choosing the bottom node as the reference terminal and naming the left node "1", the center node "2" and the right node "3", we next form a supernode about nodes 1 and 2, encompassing the dependent voltage source.

At the supernode, $5 - 8 = (v_1 - v_2)/2 + v_3/2.5$ [1]
At node 2, $8 = v_2/5 + (v_2 - v_1)/2$ [2]

Our supernode equation is $v_1 - v_3 = 0.8\, v_A$ [3]
Since $v_A = v_2$, we can rewrite [3] as $v_1 - v_3 = 0.8v_2$

Simplifying and collecting terms,

$$0.5\, v_1 - 0.5\, v_2 + 0.4\, v_3 = -3 \quad [1]$$
$$-0.5\, v_1 + 0.7\, v_2 = 8 \quad [2]$$
$$v_1 - 0.8\, v_2 - v_3 = 0 \quad [3]$$

(*a*) Solving for $v_2 = v_A$, we find that $\boxed{v_A = 25.91 \text{ V}}$

(*b*) The power absorbed by the 2.5-Ω resistor is
$(v_3)^2/2.5 = (-0.4546)^2/2.5$ $\boxed{= 82.66 \text{ mW.}}$

18. Selecting the bottom node as the reference terminal, we name the left node "1", the middle node "2" and the right node "3."

NODE 1: $5 = (v_1 - v_2)/ 20 + (v_1 - v_3)/ 50$ [1]

NODE 2: $v_2 = 0.4\, v_1$ [2]

NODE 3: $0.01\, v_1 = (v_3 - v_2)/ 30 + (v_3 - v_1)/ 50$ [3]

Simplifying and collecting terms, we obtain

$$0.07\, v_1 - 0.05\, v_2 - 0.02\, v_3 = 5 \quad [1]$$
$$0.4\, v_1 - v_2 = 0 \quad [2]$$
$$-0.03\, v_1 - 0.03333\, v_2 + 0.05333\, v_3 = 0 \quad [3]$$

Since our choice of reference terminal makes the controlling variable of both dependent sources a nodal voltage, we have no need for an additional equation as we might have expected.

Solving, we find that $\boxed{v_1 = 148.2 \text{ V},}$ $v_2 = 59.26$ V, and $v_3 = 120.4$ V.

The power supplied by the dependent current source is therefore

$$(0.01\, v_1) \bullet v_3 = \boxed{177.4 \text{ W.}}$$

19. At node x: $v_x/4 + (v_x - v_y)/2 + (v_x - 6)/1 = 0$ [1]
At node y: $(v_y - kv_x)/3 + (v_y - v_x)/2 = 2$ [2]

Our additional constraint is that $v_y = 0$, so we may simplify Eqs. [1] and [2]:

14 $v_x = 48$ [1]
-2k $v_x - 3v_x = 12$ [2]

Since Eq. [1] yields $v_x = 48/14 = 3.429$ V, we find that

$$k = (12 + 3v_x)/(-2v_x) = \boxed{-3.250}$$

20. Choosing the bottom node joining the 4-Ω resistor, the 2-A current sourcee and the 4-V voltage source as our reference node, we next nam e the other node of the 4- Ω resistor node "1", and the node joining the 2- Ω r esistor and t he 2- A c urrent s ource node "2." Finally, we create a supernode with nodes "1" and "2."

At the supernode:	$-2 = v_1/4 + (v_2 - 4)/2$	[1]
Our remaining equations:	$v_1 - v_2 = -3 - 0.5i_1$	[2]
and	$i_1 = (v_2 - 4)/2$	[3]

Equation [1] simplifies to	$v_1 + 2v_2 = 0$	[1]
Combining Eqs. [2] and [3,	$4v_1 - 3v_2 = -8$	[4]

Solving these last two equations, we find that $v_2 = 727.3$ mV. Making use of Eq. [3], we therefore find that

$$\boxed{i_1 = -1.636 \text{ A.}}$$

21. We first number the nodes as 1, 2, 3, 4, and 5 moving left to right. We next select node 5 as the reference terminal. To simplify the analysis, we form a supernode from nodes 1, 2, and 3.

At the supernode,

$$-4 - 8 + 6 = v_1/40 + (v_1 - v_3)/10 + (v_3 - v_1)/10 + v_2/50 + (v_3 - v_4)/20 \qquad [1]$$

Note that since both ends of the 10-Ω resistor are connected to the supernode, the related terms cancel each other out, and so could have been ignored.

At node 4: $v_4 = 200$ [2]

Supernode KVL equation: $v_1 - v_3 = 400 + 4v_{20}$ [3]

Where the controlling voltage $v_{20} = v_3 - v_4 = v_3 - 200$ [4]

Thus, Eq. [1] becomes $-6 = v_1/40 + v_2/50 + (v_3 - 200)/20$ or, more simply,

$$4 = v_1/40 + v_2/50 + v_3/20 \qquad [1']$$

and Eq. [3] becomes $v_1 - 5v_3 = -400$ [3']

Eqs. [1'], [3'], and [5] are not sufficient, however, as we have four unknowns. At this point we need to seek an additional equation, possibly in terms of v_2. Referring to the circuit,

$$v_1 - v_2 = 400 \qquad [5]$$

Rewriting as a matrix equation,

$$\begin{bmatrix} 1/40 & 1/50 & 1/20 \\ 1 & 0 & -5 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 4 \\ -400 \\ 400 \end{bmatrix}$$

Solving, we find that
$v_1 = 145.5$ V, $v_2 = -254.5$ V, and $v_3 = 109.1$ V. Since $v_{20} = v_3 - 200$, we find that

$$\boxed{v_{20} = -90.9 \text{ V.}}$$

22. We begin by naming the top left node "1", the top right node "2", the bottom node of the 6-V source "3" and the top node of the 2- Ω resistor "4." The reference node has already been selected, and designated using a ground symbol.

By inspection, $\boxed{v_2 = 5 \text{ V.}}$

Forming a supernode with nodes 1 & 3, we find

At the supernode: $-2 = v_3/1 + (v_1 - 5)/10$ [1]

At node 4: $2 = v_4/2 + (v_4 - 5)/4$ [2]

Our supernode KVL equation: $v_1 - v_3 = 6$ [3]

Rearranging, simplifying and collecting terms,

$$v_1 + 10\, v_3 = -20 + 5 = -15 \qquad [1]$$

and

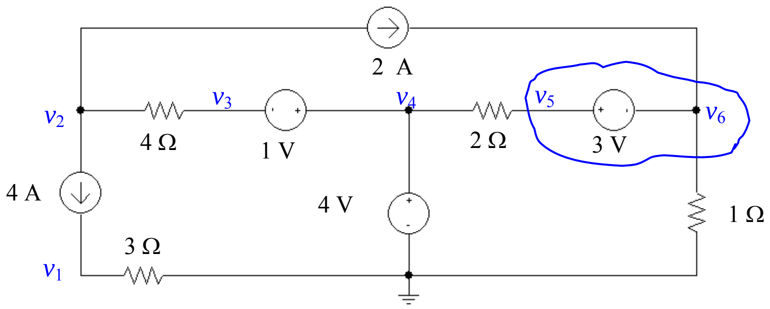
$$v_1 - v_3 = 6 \qquad [2]$$

Eq. [3] may be directly solved to obtain $\boxed{v_4 = 4.333 \text{ V.}}$

Solving Eqs. [1] and [2], we find that

$$\boxed{v_1 = 4.091 \text{ V}} \quad \text{and} \quad \boxed{v_3 = -1.909 \text{ V.}}$$

23. We begin by selecting the bottom node as the reference, naming the nodes as shown below, and forming a supernode with nodes 5 & 6.



By inspection, $v_4 = 4$ V.

By KVL, $v_3 - v_4 = 1$ so $v_3 = -1 + v_4 = -1 + 4$ or $v_3 = 3$ V.

At the supernode, $2 = v_6/1 + (v_5 - 4)/2$ [1]

At node 1, $4 = v_1/3$ therefore, $v_1 = 12$ V.

At node 2, $-4 - 2 = (v_2 - 3)/4$

Solving, we find that $v_2 = -21$ V

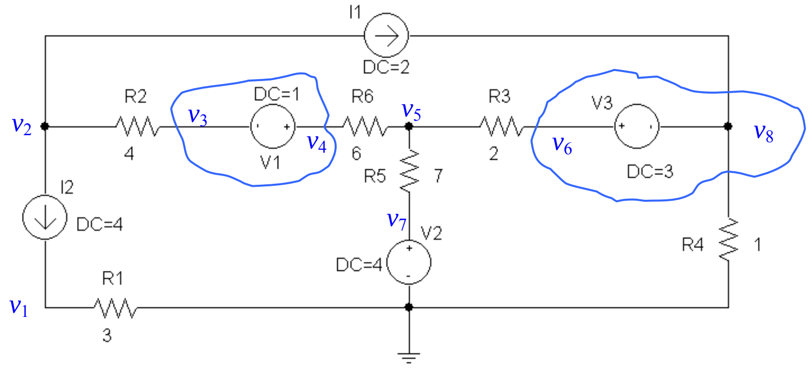
Our supernode KVL equation is $v_5 - v_6 = 3$ [2]

Solving Eqs. [1] and [2], we find that

$$v_5 = 4.667 \text{ V} \quad \text{and} \quad v_6 = 1.667 \text{ V}.$$

The power supplied by the 2-A source therefore is $(v_6 - v_2)(2) = \boxed{45.33 \text{ W.}}$

24. We begin by selecting the bottom node as the reference, naming each node as shown below, and forming two different supernodes as indicated.



Voltages in volts. Currents in amperes. Resistances in ohms.

By inspection, $v_7 = 4$ V and $v_1 = (3)(4) = 12$ V.

At node 2: $-4 - 2 = (v_2 - v_3)/4$ or $v_2 - v_3 = -24$ [1]

At the 3-4 supernode:
$0 = (v_3 - v_2)/4 + (v_4 - v_5)/6$ or $-6v_2 + 6v_3 + 4v_4 - 4v_5 = 0$ [2]

At node 5:
$0 = (v_5 - v_4)/6 + (v_5 - 4)/7 + (v_5 - v_6)/2$ or $-14v_4 + 68v_5 - 42v_6 = 48$ [3]

At the 6-8 supernode: $2 = (v_6 - v_5)/2 + v_8/1$ or $-v_5 + v_6 + 2v_8 = 4$ [4]

3-4 supernode KVL equation: $v_3 - v_4 = -1$ [5]
6-8 supernode KVL equation: $v_6 - v_8 = 3$ [6]

Rewriting Eqs. [1] to [6] in matrix form,

$$\begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ -6 & 6 & 4 & -4 & 0 & 0 \\ 0 & 0 & -14 & 68 & -42 & 0 \\ 0 & 0 & 0 & -1 & 1 & 2 \\ 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_8 \end{bmatrix} = \begin{bmatrix} -24 \\ 0 \\ 48 \\ 4 \\ -1 \\ 3 \end{bmatrix}$$

Solving, we find that
$v_2 = -68.9$ V, $v_3 = -44.9$ V, $v_4 = -43.9$ V, $v_5 = -7.9$ V, $v_6 = 700$ mV, $v_8 = -2.3$ V.

The power generated by the 2-A source is therefore $(v_8 - v_6)(2) = \boxed{133.2 \text{ W.}}$

25. With the reference terminal already specified, we name the bottom terminal of the 3-mA source node "1," the left terminal of the bottom 2.2-kΩ resistor node "2," the top terminal of the 3-mA source node "3," the "+" reference terminal of the 9-V source node "4," and the "-" terminal of the 9-V source node "5."

Since we know that 1 mA flows through the top 2.2-kΩ resistor, $v_5 = -2.2$ V. Also, we see that $v_4 - v_5 = 9$, so that $v_4 = 9 - 2.2 = 6.8$ V. Proceeding with nodal analysis,

At node 1: $-3\times10^{-3} = v_1/\ 10\times10^3 + (v_1 - v_2)/\ 2.2\times10^3$ [1]

At node 2: $0 = (v_2 - v_1)/\ 2.2\times10^3 + (v_2 - v_3)/\ 4.7\times10^3$ [2]

At node 3: $1\times10^3 + 3\times10^3 = (v_3 - v_2)/\ 4.7\times10^3 + v_3/3.3\times10^3$ [3]

Solving, $\boxed{v_1 = -8.614 \text{ V},\ v_2 = -3.909 \text{ V and } v_3 = 6.143 \text{ V.}}$

Note that we could also have made use of the supernode approach here.

26. Mesh 1: $-4 + 400i_1 + 300i_1 - 300i_2 - 1 = 0$ or $700i_1 - 300i_2 = 5$
Mesh 2: $1 + 500i_2 - 300i_1 + 2 - 2 = 0$ or $-300i_1 + 500i_2 = -3.2$

Solving, $\boxed{i_1 = 5.923 \text{ mA} \quad \text{and} \quad i_2 = -2.846 \text{ mA.}}$

27. (a) Define a clockwise mesh current i_1 in the left-most mesh; a clockwise mesh current i_2 in the central mesh, and note that i_y can be used as a mesh current for the remaining mesh.

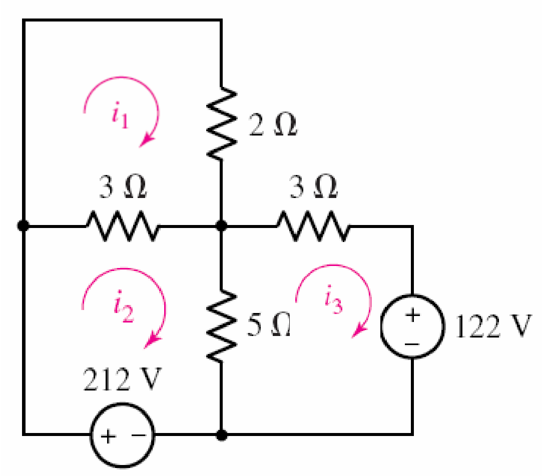
Mesh 1: $-10 + 7i_1 - 2i_2 = 0$
Mesh 2: $-2i_1 + 5i_2 = 0$
Mesh y: $-2i_2 + 9i_y = 0$

Solve the resulting matrix equation:

$$\begin{bmatrix} 7 & -2 & 0 \\ -2 & 5 & 0 \\ 0 & -2 & 9 \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \\ i_y \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \\ 0 \end{bmatrix}$$ to find that $i_1 = 1.613$ A, and $\boxed{i_y = 143.4 \text{ mA.}}$

(b) The power supplied by the 10 V source is $(10)(i_1) = 10(1.613) = \boxed{16.13 \text{ W.}}$

28. Define three mesh currents as shown:



(a) The current through the 2 Ω resistor is i_1.

Mesh 1: $5i_1 - 3i_2 = 0$	or $5i_1 - 3i_2 = 0$
Mesh 2: $-212 + 8i_2 - 3i_1 = 0$	or $-3i_1 + 8i_2 = 212$
Mesh 3: $8i_3 - 5i_2 + 122 = 0$	or $-5i_2 + 8i_3 = -122$

Solving, $\boxed{i_1 = 20.52 \text{ A},}$ $i_2 = 34.19$ A and $i_3 = 6.121$ A.

(b) The current through the 5 Ω resistor is i_3, or $\boxed{6.121 \text{ A.}}$

*** Note: since the problem statement did not specify a direction, only the current magnitude is relevant, and its sign is arbitrary.

29. We begin by defining three clockwise mesh currents i_1, i_2 and i_3 in the left-most, central, and right-most meshes, respectively. Then,

(a) Note that $i_x = i_2 - i_3$.

Mesh 1: $i_1 = 5$ A (by inspection)
Mesh 3: $i_3 = -2$ A (by inspection)

Mesh 2: $-25i_1 + 75i_2 - 20i_3 = 0$, or, making use of the above,

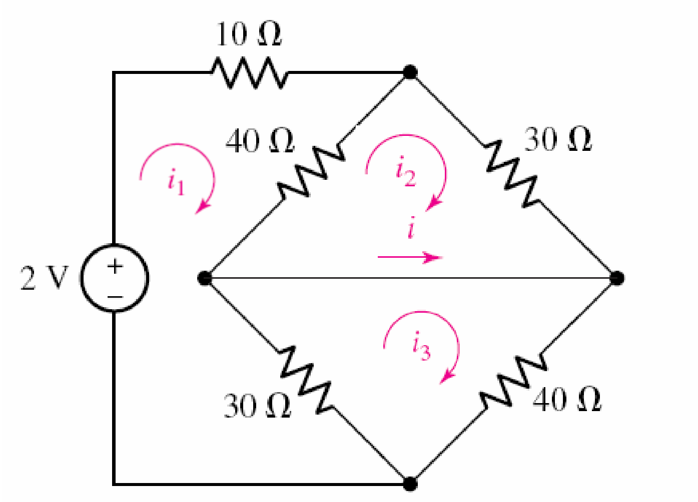
$-125 + 75i_2 + 40 = 0$ so that $i_2 = 1.133$ A.

Thus, $i_x = i_2 - i_3 = 1.133 - (-2) = \boxed{3.133 \text{ A.}}$

(b) The power absorbed by the 25 Ω resistor is

$$P_{25\Omega} = 25\,(i_1 - i_2)^2 = 25\,(5 - 1.133)^2 = \boxed{373.8 \text{ W.}}$$

30. Define three mesh currents as shown. Then,



Mesh 1: $-2 + 80i_1 - 40i_2 - 30i_3 = 0$
Mesh 2: $-40i_1 + 70i_2 = 0$
Mesh 3: $-30i_1 + 70i_3 = 0$

Solving,
$$\begin{bmatrix} 80 & -40 & -30 \\ -40 & 70 & 0 \\ -30 & 0 & 70 \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$$

we find that $i_2 = 25.81$ mA and $i_3 = 19.35$ mA. Thus, $i = i_3 - i_2 = -6.46$ mA.

31. Moving from left to right, we name the bottom three meshes, mesh "1", mesh "2," and mesh "3." In each of these three meshes we define a clockwise current. The remaining mesh current is clearly 8 A. We may then write:

MESH 1: $12\, i_1 - 4\, i_2 = 100$

MESH 2: $-4\, i_1 + 9\, i_2 - 3\, i_3 = 0$

MESH 3: $-3\, i_2 + 18\, i_3 = -80$

Solving this system of three (independent) equations in three unknowns, we find that

$$i_2 = \boxed{i_x = 2.791 \text{ A.}}$$

32. We define four clockwise mesh currents. The top mesh current is labeled i_4. The bottom left mesh current is labeled i_1, the bottom right mesh current is labeled i_3, and the remaining mesh current is labeled i_2. Define a voltage "v_{4A}" across the 4-A current source with the "+" reference terminal on the left.

By inspection, $i_3 = 5$ A and $i_a = i_4$.

MESH 1: $-60 + 2i_1 - 2i_4 + 6i_4 = 0$ or $2i_1 + 4i_4 = 60$ [1]

MESH 2: $-6i_4 + v_{4A} + 4i_2 - 4(5) = 0$ or $4i_2 - 6i_4 + v_{4A} = 20$ [2]

MESH 4: $2i_4 - 2i_1 + 5i_4 + 3i_4 - 3(5) - v_{4A} = 0$ or $-2i_1 + 10i_4 - v_{4A} = 15$ [3]

At this point, we are short an equation. Returning to the circuit diagram, we note that

$i_2 - i_4 = 4$ [4]

Collecting these equations and writing in matrix form, we have

$$\begin{bmatrix} 2 & 0 & 4 & 0 \\ 0 & 4 & -6 & 1 \\ -2 & 0 & 10 & -1 \\ 0 & 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_4 \\ v_{4A} \end{bmatrix} = \begin{bmatrix} 60 \\ 20 \\ 15 \\ 4 \end{bmatrix}$$

Solving, $i_1 = 16.83$ A, $i_2 = 10.58$ A, $i_4 = 6.583$ A and $v_{4A} = 17.17$ V.

Thus, the power dissipated by the 2-Ω resistor is

$$(i_1 - i_4)^2 \bullet (2) = \boxed{210.0 \text{ W}}$$

33. We begin our analysis by defining three clockwise mesh currents. We will call the top mesh current i_3, the bottom left mesh current i_1, and the bottom right mesh current i_2.

By inspection, $i_1 = 5$ A [1] and $i_2 = -0.01\, v_1$ [2]

MESH 3: $50\, i_3 + 30\, i_3 - 30\, i_2 + 20\, i_3 - 20\, i_1 = 0$
or $-20\, i_1 - 30\, i_2 + 100\, i_3 = 0$ [3]

These three equations are insufficient, however, to solve for the unknowns. It would be nice to be able to express the dependent source controlling variable v_1 in terms of the mesh currents. Returning to the diagram, it can be seen that KVL around mesh 1 will yield

$$- v_1 + 20\, i_1 - 20\, i_3 + 0.4\, v_1 = 0$$

or $v_1 = 20\, i_1/ 0.6 - 20\, i_3/ 0.6$ or $v_1 = (20(5)/ 0.6 - 20\, i_3/ 0.6$ [4]

Substituting Eq. [4] into Eq. [2] and then the modified Eq. [2] into Eq. [3], we find

$$-20(5) - 30(-0.01)(20)(5)/0.6 + 30(-0.01)(20)\, i_3/ 0.6 + 100\, i_3 = 0$$

Solving, we find that $i_3 = (100 - 50)/ 90 = 555.6$ mA

Thus, $v_1 = 148.1$ V, $i_2 = -1.481$ A, and the power generated by the dependent voltage source is

$$0.4\, v_1\, (i_2 - i_1) = \boxed{-383.9 \text{ W.}}$$

34. We begin by defining four clockwise mesh currents i_1, i_2, i_3 and i_4, in the meshes of our circuit, starting at the left-most mesh. We also define a voltage v_{dep} across the dependent current source, with the "+" on the top.

By inspection, $i_1 = 2\text{A}$ and $i_4 = -5$ A.

At Mesh 2: $10\, i_2 - 10(2) + 20\, i_2 + v_{dep} = 0$ [1]

At Mesh 3: $-v_{dep} + 25\, i_3 + 5\, i_3 - 5(-5) = 0$ [2]

Collecting terms, we rewrite Eqs. [1] and [2] as

$30\, i_2 + v_{dep} = 20$ [1]

$30\, i_3 - v_{dep} = -25$ [2]

This is only two equations but three unknowns, however, so we require an additional equation. Returning to the circuit diagram, we note that it is possible to express the current of the dependent source in terms of mesh currents. (We might also choose to obtain an expression for v_{dep} in terms of mesh currents using KVL around mesh 2 or 3.)

Thus, $1.5 i_x = i_3 - i_2$ where $i_x = i_1 - i_2$ so $-0.5\, i_2 - i_3 = -3$ [3]

In matrix form,

$$\begin{bmatrix} 30 & 0 & 1 \\ 0 & 30 & -1 \\ -0.5 & -1 & 0 \end{bmatrix} \begin{bmatrix} i_2 \\ i_3 \\ v_{dep} \end{bmatrix} = \begin{bmatrix} 20 \\ -25 \\ -3 \end{bmatrix}$$

Solving, we find that $i_2 = -6.333$ A so that $\boxed{i_x = i_1 - i_2 = 8.333 \text{ A.}}$

35. We define a clockwise mesh current i_1 in the bottom left mesh, a clockwise mesh current i_2 in the top left mesh, a clockwise mesh current i_3 in the top right mesh, and a clockwise mesh current i_4 in the bottom right mesh.

MESH 1: $-0.1\,v_a + 4700\,i_1 - 4700\,i_2 + 4700\,i_1 - 4700\,i_4 = 0$ [1]

MESH 2: $9400\,i_2 - 4700\,i_1 - 9 = 0$ [2]

MESH 3: $9 + 9400\,i_3 - 4700\,i_4 = 0$ [3]

MESH 4: $9400\,i_4 - 4700\,i_1 - 4700\,i_3 + 0.1\,i_x = 0$ [4]

The presence of the two dependent sources has led to the introduction of two additional unknowns (i_x and v_a) besides our four mesh currents. In a perfect world, it would simplify the solution if we could express these two quantities in terms of the mesh currents.

Referring to the circuit diagram, we see that $i_x = i_2$ (easy enough) and that $v_a = 4700\,i_3$ (also straightforward). Thus, substituting these expressions into our four mesh equations and creating a matrix equation, we arrive at:

$$\begin{bmatrix} 9400 & -4700 & -470 & -4700 \\ -4700 & 9400 & 0 & 0 \\ 0 & 0 & 9400 & -4700 \\ -4700 & 0.1 & -4700 & 9400 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 9 \\ -9 \\ 0 \end{bmatrix}$$

Solving,

$$\boxed{i_1 = 239.3\ \mu\text{A},\ i_2 = 1.077\text{ mA},\ i_3 = -1.197\text{ mA and } i_4 = -478.8\ \mu\text{A}.}$$

36. We define a clockwise mesh current i_3 in the upper right mesh, a clockwise mesh current i_1 in the lower left mesh, and a clockwise mesh current i_2 in the lower right mesh.

MESH 1: $-6 + 6\,i_1 - 2 = 0$ [1]

MESH 2: $2 + 15\,i_2 - 12\,i_3 - 1.5 = 0$ [2]

MESH 3: $i_3 = 0.1\,v_x$ [3]

Eq. [1] may be solved directly to obtain $\boxed{i_1 = 1.333 \text{ A.}}$

It would help in the solution of Eqs. [2] and [3] if we could express the dependent source controlling variable v_x in terms of mesh currents. Referring to the circuit diagram, we see that $v_x = (1)(i_1) = i_1$, so Eq. [3] reduces to

$$\boxed{i_3 = 0.1\,v_x = 0.1\,i_1 = 133.3 \text{ mA.}}$$

As a result, Eq. [1] reduces to $\boxed{i_2 = [-0.5 + 12(0.1333)]/15 = 73.31 \text{ mA.}}$

37. (a) Define a mesh current i_2 in the second mesh. Then KVL allows us to write:

MESH 1: $-9 + R\,i_1 + 47000\,i_1 - 47000\,i_2 = 0$ [1]

MESH 2: $67000\,i_2 - 47000\,i_1 - 5 = 0$ [2]

Given that $i_1 = 1.5$ mA, we may solve Eq. [2] to find that

$$i_2 = \frac{5 + 47(1.5)}{67} \text{ mA} = 1.127 \text{ mA}$$

and so

$$R = \frac{9 - 47(1.5) + 47(1.127)}{1.5 \times 10^{-3}} = \boxed{-5687\ \Omega.}$$

(b) This value of R is unique; no other value will satisfy **both** Eqs. [1] **and** [2].

38. Define three clockwise mesh currents i_1, i_2 and i_3. The bottom 1-kΩ resistor can be ignored, as no current flows through it.

MESH 1: $-4 + (2700 + 1000 + 5000)\, i_1 - 1000\, i_2 = 0$ [1]

MESH 2: $(1000 + 1000 + 4400 + 3000)\, i_2 - 1000\, i_1 - 4400\, i_3 + 2.2 - 3 = 0$ [2]

MESH 3: $(4400 + 4000 + 3000)\, i_3 - 4400\, i_2 - 1.5 = 0$ [3]

Combining terms,

$$8700\, i_1 - 1000\, i_2 = 4 \quad [1]$$
$$-1000\, i_1 + 9400\, i_2 - 4400\, i_3 = 0.8 \quad [2]$$
$$-4400\, i_2 + 11400\, i_3 = 1.5 \quad [3]$$

Solving,

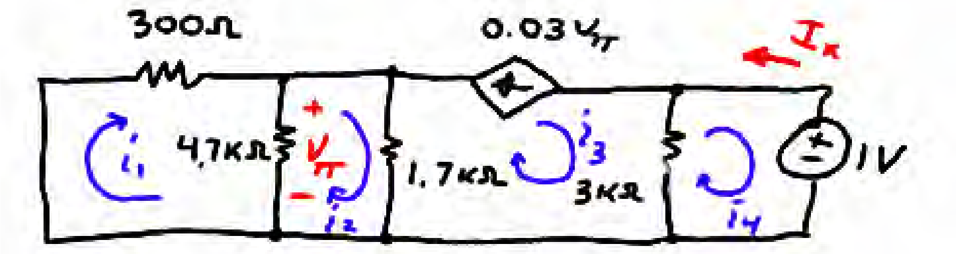
$i_1 = 487.6$ μA, $i_2 = 242.4$ μA and $i_3 = 225.1$ μA.

The power absorbed by each resistor may now be calculated:

P_{5k}	=	$5000\,(i_1)^2$ =	1.189 mW
$P_{2.7k}$	=	$2700\,(i_1)^2$ =	641.9 μW
P_{1ktop}	=	$1000\,(i_1 - i_2)^2$ =	60.12 μW
$P_{1kmiddle}$	=	$1000\,(i_2)^2$ =	58.76 μW
$P_{1kbottom}$	=	0 =	0
$P_{4.4k}$	=	$4400\,(i_2 - i_3)^2$ =	1.317 μW
P_{3ktop}	=	$3000\,(i_3)^2$ =	152.0 μW
P_{4k}	=	$4000\,(i_3)^2$ =	202.7 μW
$P_{3kbottom}$	=	$3000\,(i_2)^2$ =	176.3 μW

Check: The sources supply a total of
4(487.6) + (3 – 2.2)(242.4) + 1.5(225.1) = 2482 μW.
The absorbed powers add to 2482 μW.

39. (a) We begin by naming four mesh currents as depicted below:



Proceeding with mesh analysis, then, keeping in mind that $I_x = -i_4$,

MESH 1: $(4700 + 300)\, i_1 \;-\; 4700\, i_2 \;=\; 0$ [1]

MESH 2: $(4700 + 1700)\, i_2 - 4700\, i_1 - 1700\, i_3 \;=\; 0$ [2]

Since we have a current source on the perimeter of mesh 3, we do not require a KVL equation for that mesh. Instead, we may simply write

$i_3 = -0.03\, v_\pi$ [3*a*] where $v_\pi = 4700(i_1 - i_2)$ [3*b*]

MESH 4: $3000\, i_4 - 3000\, i_3 + 1 \;=\; 0$ [4]

Simplifying and combining Eqs. 3*a* and 3*b*,

$$5000\, i_1 - 4700\, i_2 = 0$$
$$-4700\, i_1 + 6400\, i_2 - 1700\, i_3 = 0$$
$$-141\, i_1 + 141\, i_2 - i_3 = 0$$
$$-3000\, i_3 + 3000\, i_4 = -1$$

Solving, we find that i_4 = -333.3 mA, so $\boxed{I_x = 333.3\ \mu\text{A.}}$

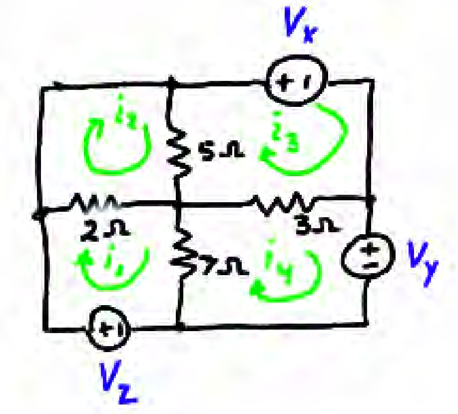
(b) At node "π" : $0.03\, v_\pi = v_\pi / 300 + v_\pi / 4700 + v_\pi / 1700$

Solving, we find that $v_\pi = 0$, therefore no current flows through the dependent source.

Hence, $\boxed{I_x = 333.3\ \mu\text{A as found in part (a).}}$

(c) $\boxed{V_s / I_x \text{ has units of resistance.}}$ It can be t hought of as the resistance "seen" by the voltage source V_s…. more on this in Chap. 5….

40. We begin by naming each mesh and the three undefined voltage sources as shown below:



MESH 1: $-V_z + 9i_1 - 2i_2 \qquad - 7i_4 = 0$

MESH 2: $-2i_1 + 7i_2 - 5i_3 \qquad = 0$

MESH 3: $V_x \qquad - 5i_2 + 8i_3 - 3i_4 = 0$

MESH 4: $V_y - 7i_1 \qquad - 3i_3 + 10i_4 = 0$

Rearranging and setting $i_1 - i_2 = 0$, $i_2 - i_3 = 0$, $i_1 - i_4 = 0$ and $i_4 - i_3 = 0$,

$$\begin{aligned} 9i_1 - 2i_2 \qquad -7i_4 &= V_z \\ -2i_1 + 7i_2 - 5i_3 \qquad &= 0 \\ -5i_2 + 8i_3 - 3i_4 &= -V_x \\ -7i_1 \quad -3i_3 + 10i_4 \qquad &= -V_y \end{aligned}$$

Since $i_1 = i_2 = i_3 = i_4$, these equations produce:

$$\begin{aligned} V_z &= 0 \\ 0 &= 0 \\ -V_x &= 0 \\ -V_y &= 0 \end{aligned}$$

This is a unique solution. Therefore, the request that nonzero values be found cannot be satisfied.

41. The "supermesh" concept is not required (or helpful) in solving this problem, as there are no current sources s hared between m eshes. Starting with the left-most m esh and moving right, we define four clockwise m esh currents i_1, i_2, i_3 and i_4. By inspection, we see that $i_1 = 2$ mA.

MESH 2: $-10 + 5000i_2 + 4 + 1000i_3 = 0$ [1]

MESH 3: $-1000i_3 + 6 + 10{,}000 - 10{,}000i_4 = 0$ [2]

MESH 4: $i_4 = -0.5i_2$ [3]

Reorganising, we find

$$
\begin{aligned}
5000\, i_2 + 1000\, i_3 \quad\quad &= 6 \quad [1] \\
9000\, i_3 - 10{,}000\, i_4 &= -6 \quad [2] \\
0.5\, i_2 \quad\quad + \quad i_4 &= 0 \quad [3]
\end{aligned}
$$

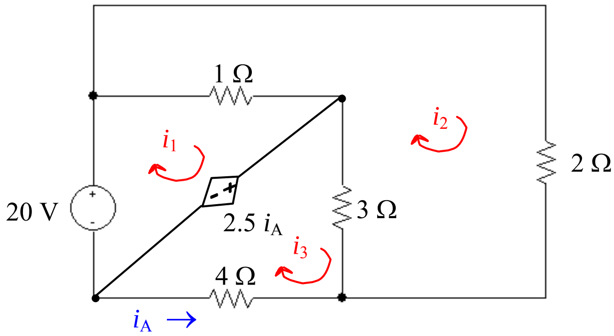
We could either subtitute Eq. [3] into Eq. [2] to reduce the number of equations, or simply go ahead and solve the system of Eqs. [1-3]. Either way, we find that

$$i_1 = 2 \text{ mA},\ i_2 = 1.5 \text{ mA},\ i_3 = -1.5 \text{ mA and } i_4 = -0.75 \text{ mA}.$$

The power generated by each source is:

P_{2mA}	$= 5000(i_1 - i_2)(i_1)$	$= 5$ mW
P_{4V}	$= 4\,(-i_2)$	$= -6$ mW
P_{6V}	$= 6\,(-i_3)$	$= 9$ mW
P_{depV}	$= 1000\, i_3\, (i_3 - i_2)$	$= 4.5$ mW
P_{depI}	$= 10{,}000(i_3 - i_4)(0.5\, i_2)$	$= -5.625$ mW

42. This circuit does not require the supermesh technique, as it does not contain any current sources. Redrawing the circuit so its planar nature and mesh structure are clear,



MESH 1: $-20 + i_1 - i_2 + 2.5\, i_A = 0$ [1]

MESH 2: $2\, i_2 + 3\, i_2 + i_2 - 3\, i_3 - i_1 = 0$ [2]

MESH 3: $-2.5\, i_A + 7\, i_3 - 3\, i_2 = 0$ [3]

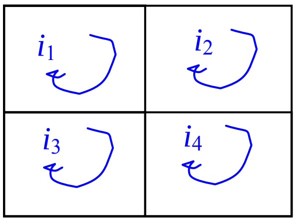
Combining terms and making use of the fact that $i_A = -i_3$,

$$i_1 - i_2 - 2.5\, i_3 = 20 \quad [1]$$
$$-i_1 + 6 i_2 - 3\, i_3 = 0 \quad [2]$$
$$-3\, i_2 + 9.5\, i_3 = 0 \quad [3]$$

Solving, $i_1 = 30.97$ A, $i_2 = 6.129$ A, and $i_3 = 1.936$ A. Since $i_A = -i_3$,

$$\boxed{i_A = -1.936 \text{ A.}}$$

43. Define four mesh currents



By inspection, $i_1 = -4.5$ A.

We form a supermesh with meshes 3 and 4 as defined above.

MESH 2: $2.2 + 3\,i_2 + 4\,i_2 + 5 - 4\,i_3 = 0$ [1]

SUPERME SH: $3\,i_4 + 9\,i_4 - 9\,i_1 + 4\,i_3 - 4\,i_2 + 6\,i_3 + i_3 - 3 = 0$ [2]

Superm esh KCL equation: $i_4 - i_3 = 2$ [3]

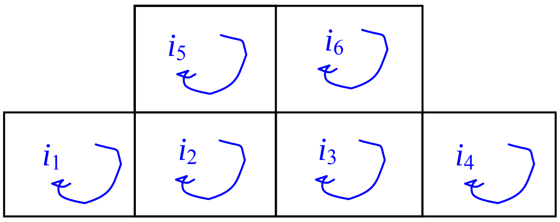
Simplifying and combining terms, we may rewrite these three equations as:

$$\begin{aligned} 7\,i_2 - 4\,i_3 \quad &= -7.2 \qquad [1] \\ -4\,i_2 + 11\,i_3 + 12\,i_4 &= -37.5 \qquad [2] \\ -\,i_3 + i_4 &= 2 \qquad [3] \end{aligned}$$

Solving, we find that $i_2 = -2.839$ A, $i_3 = -3.168$ A, and $i_4 = -1.168$ A.

The power supplied by the 2.2-V source is then $2.2\,(i_1 - i_2) = \boxed{-3.654 \text{ W.}}$

44. We begin by defining six mesh currents as depicted below:



- We form a supermesh with meshes 1 and 2 since they share a current source.
- We form a *second* supermesh with meshes 3 and 4 since they also share a current source.

1, 2 Supermesh:
$(4700 + 1000 + 10,000)\ i_1 - 2200\ i_5 + (2200 + 1000 + 4700)\ i_2 - 1000\ i_3 = 0$ [1]

3, 4 Supermesh:
$(4700 + 1000 + 2200)\ i_3 - 1000\ i_2 - 2200\ i_6 + (4700 + 10,000 + 1000)\ i_4 = 0$ [2]

MESH 5: $(2200 + 4700)\ i_5 - 2200\ i_2 + 3.2 - 1.5 = 0$ [3]

MESH 6: $1.5 + (4700 + 4700 + 2200)\ c - 2200\ i_3 = 0$ [4]

1, 2 Supermesh KCL equation: $i_1 - i_2 = 3\times10^{-3}$ [5]

3, 4 Supermesh KCL equation: $i_4 - i_3 = 2\times10^{-3}$ [6]

We can simplify these equations prior to solution in several ways. Choosing to retain six equations,

$$
\begin{array}{lllllll}
15,700\ i_1 & + 7900\ i_2 & - 1000\ i_3 & & -2200\ i5 & & = 0 \quad [1] \\
 & - 1000\ i_2 & + 7900\ i_3 & + 15,700\ i_4 & & -2200\ i_6 & = 0 \quad [2] \\
 & - 2200\ i_2 & & & + 6900\ i_5 & & = -1.7 \quad [3] \\
 & & - 2200\ i_3 & & & + 11,600\ i_6 & = -1.5 \quad [4] \\
i_1 & - i_2 & & & & & = 3\times10^{-3} \quad [5] \\
 & & - i_3 & + i_4 & & & = 2\times10^{-3} \quad [6]
\end{array}
$$

Solving, we find that $i_4 = 540.8$ mA. Thus, the voltage across the 2-mA source is

$$(4700 + 10,000 + 1000)\ (540.8\times10^{-6}) = \boxed{8.491\ \text{V}}$$

45. We define a mesh current i_a in the left-hand mesh, a mesh current i_1 in the top right mesh, and a mesh current i_2 in the bottom right mesh (all flowing clockwise).

The left-most mesh can be analysed separately to determine the controlling voltage v_a, as KCL assures us that no current flows through either the 1-Ω or 6-Ω resistor.

Thus, $-1.8 + 3\ i_a - 1.5 + 2\ i_a = 0$, which may be solved to find $i_a = 0.66$ A. Hence, $v_a = 3\ i_a = 1.98$ V.

Forming one supermesh from the remaining two meshes, we may write:

$$-3 + 2.5\ i_1 + 3\ i_2 + 4\ i_2 = 0$$

and the supermesh KCL equation: $i_2 - i_1 = 0.5\ v_a = 0.5(1.98) = 0.99$

Thus, we have two equations to solve:

$$2.5\ i_1 + 7\ i_2 = 3$$
$$-i_1 + i_2 = 0.99$$

Solving, we find that $i_1 = -413.7$ mA and the voltage across the 2.5-Ω resistor (arbitrarily assuming the left terminal is the "+" reference) is $2.5\ i_1 = \boxed{-1.034 \text{ V}}$.

46. There are only three meshes in this circuit, as the botton 22-mΩ resistor is not connected connected at its left term inal. Thus, we define three m esh currents, i_1, i_2, and i_3, beginning with the left-most mesh.

We next create a superm esh from meshes 1 and 2 (note that m esh 3 is independent, and can be analysed separately).

Thus, $-11.8 + 10\times10^{-3}\, i_1 + 22\times10^{-3}\, i_2 + 10\times10^{-3}\, i_2 + 17\times10^{-3}\, i_1 = 0$

and applying KCL to obtain an equation containing the current source,

$$i_1 - i_2 = 100$$

Combining terms and simplifying, we obtain

$$27\times10\text{-}3\, i_1 + 32\times10^{-3}\, i_2 = 11.8$$
$$i_1 - i_2 = 100$$

Solving, we find that $\boxed{i_1 = 254.2 \text{ A}}$ and $\boxed{i_2 = 154.2 \text{ A.}}$

The final mesh current is easily found: $\boxed{i_3 = 13\times10^3/\,(14 + 11.6 + 15) = 320.2 \text{ A.}}$

47. MESH 1: $-7 + i_1 - i_2 = 0$ [1]
MESH 2: $i_2 - i_1 + 2i_2 + 3i_2 - 3i_3 = 0$ [2]
MESH 3: $3i_3 - 3i_2 + xi_3 + 2i_3 - 7 = 0$ [3]

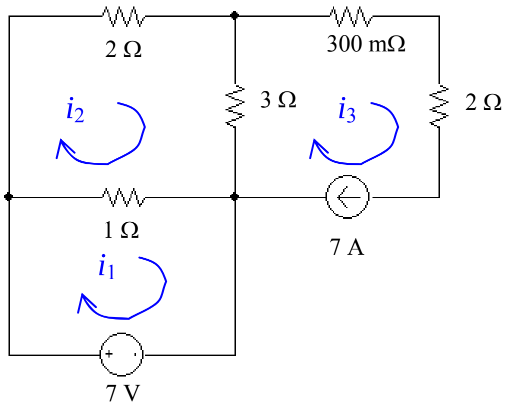
Grouping terms, we find that

$i_1 - i_2 = 7$ [1]
$-i_1 + 6i_2 - 3i_3 = 0$ [2]
$-3i_2 + (5 + x)i_3 = 7$ [3]

This, unfortunately, is four unknowns but only three equations. However, we have not yet made use of the fact that we are trying to obtain $i_2 = 2.273$ A. Solving these "four" equations, we find that

$$\boxed{x = (7 + 3\, i_2 - 5\, i_3)/\, i_3 = 4.498\ \Omega.}$$

48. We begin by redrawing the circuit as instructed, and define three mesh currents:



B y inspection, $i_3 = 7$ A.

MESH 1: $-7 + i_1 - i_2 = 0$ or $i_1 - i_2 = 7$ [1]

MESH 2: $(1 + 2 + 3)\, i_2 - i_1 - 3(7) = 0$ or $-i_1 + 6i_2 = 21$ [2]

There is no need for supermesh techniques for this situation, as the only current source lies on the outside perimeter of a mesh- it is not shared between meshes.

Solving, we find that $\boxed{i_1 = 12.6 \text{ A}, i_2 = 5.6 \text{ A and } i_3 = 7 \text{ A.}}$

49. (*a*) We are asked for a voltage, and have one current source and one voltage source. Nodal analysis is probably best then- the nodes can be named so that the desired voltage is a nodal voltage, or, at worst, we have one supernode equation to solve.

Name the top left node "1" and the top right node "x"; designate the bottom node as the reference terminal. Next, form a supernode with nodes "1" and "x."

At the supernode: $11 = v_1/2 + v_x/9$ [1]

and the KVL Eqn: $v_1 - v_x = 22$ [2]

Rearranging, $11(18) = 9v_1 + 2v_x$ [1]
$22 = v_1 - v_x$ [2]

Solving, $\boxed{v_x = 0}$

(*b*) We are asked for a voltage, and so may suspect that nodal analysis is preferrable; with two current sources and only one voltage source (easily dealt with using the supernode technique), nodal analysis does seem to have an edge over mesh analysis here.

Name the top left node "x," the top right node "y" and designate the bottom node as the reference node. Forming a supernode from nodes "x" and "y,"

At the supernode: $6 + 9 = v_x / 10 + v_y/20$ [1]
and the KVL Eqn: $v_y - v_x = 12$ [2]

Rearranging, $15(20) = 2v_x + v_y$ [1]
and $12 = -v_x + v_y$ [2]

Solving, we find that $\boxed{v_x = 96 \text{ V.}}$

(*c*) We are asked for a voltage, but would have to subtract two nodal voltages (not much harder than invoking Ohm's law). On the other hand, the dependent current source depends on the desired unknown, which would lead to the need for another equation if invoking mesh analysis. Trying nodal analysis,

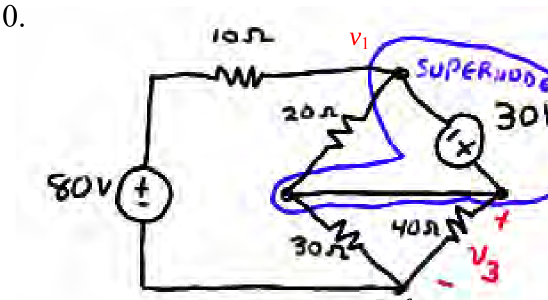
$$0.1\,v_x = (v_1 - 50)/2 + v_x/4 \qquad [1]$$

referring to the circuit we see that $v_x = v_1 - 100$. Rearranging so that we may eliminate v_1 in Eq. [1], we obtain $v_1 = v_x + 100$. Thus, Eq. [1] becomes

$$0.1\,v_x = (v_x + 100 - 50)/2 + v_x/4$$

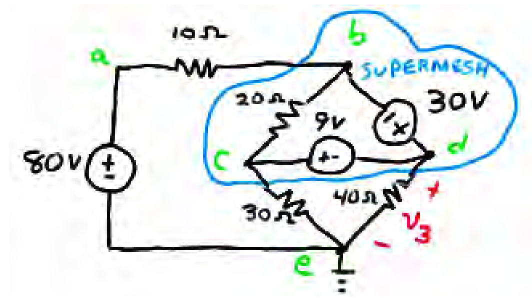
and a little algebra yields $\boxed{v_x = -38.46 \text{ V.}}$

50.



Ref.

(a) (b)



(*a*) We begin by noting that it is a voltage that is required; no current values are requested. This is a three- mesh circuit, or a four-node circuit, depending on your perspective. Either approach requires three equations…. Except that applying the supernode technique reduces the number of needed equations by one.

At the 1, 3 supernode:

$$0 = (v_1 - 80)/10 + \cancel{(v_1 - v_3)/20} + \cancel{(v_3 - v_1)/20} + v_3/40 + v_3/30$$

and $$v_3 - v_1 = 30$$

Both ends of the resistor are connected to the supernode, so we could actually just ignore it…

We simplify these two equations and collect terms, yielding

$$0.1\,v_1 + 0.05833\,v_3 = 8$$
$$-v_1 + v_3 = 30$$

Solving, we find that $\boxed{v_3 = 69.48 \text{ V}}$

(*b*) Mesh analysis would be straightforward, requiring 3 equations and a (trivial) application of Ohm's law to obtain the final answer. Nodal analysis, on the other hand, would require only two equations, and the desired voltage will be a nodal voltage.

At the b, c, d supernode: $0 = (v_b - 80)/10 + v_d/40 + v_c/30$

and: $v_d - v_b = 30$ $\qquad v_c - v_d = 9$

Simplify and collect terms:

$$0.1\,v_b + 0.03333\,v_c + 0.025\,v_d = 80$$
$$-v_b + v_d = 30$$
$$v_c - v_d = 9$$

Solving, $\boxed{v_d\ (= v_3) = 67.58 \text{ V}}$

(c) We are now faced with a dependent current source whose value depends on a mesh current. Mesh analysis in this situation requires 1 supermesh, 1 KCL equation and Ohm's law. Nodal analysis requires 1 supernode, 1 KVL equation, 1 other nodal equation, and one equation to express i_1 in terms of nodal voltages. Thus, mesh analysis has an edge here. Define the left mesh as "1," the top mesh as "2", and the bottom mesh as "3."

Mesh 1: $-80 + 10\,i_1 + 20\,i_1 - 20\,i_2 + 30\,i_1 - 30\,i_3 = 0$

2, 3 supermesh: $20\,i_2 - 20\,i_1 - 30 + 40\,i_3 + 30\,i_3 - 30\,i_1 = 0$

and: $i_2 - i_3 = 5\,i_1$

Rewriting,

$$60\,i_1 - 20\,i_2 - 30\,i_3 = 80$$
$$-50\,i_1 + 20\,i_2 + 70\,i_3 = 30$$
$$5\,i_1 - i_2 + i_3 = 0$$

Solving, $i_3 = 4.727$ A so $\boxed{v_3 = 40\,i_3 = 189 \text{ V.}}$

51. This circuit consists of 3 meshes, and no dependent sources. Therefore 3 simultaneous equations and 1 subtraction operation would be required to solve for the two desired currents. On the other hand, if we use nodal analysis, forming a supernode about the 30-V source would lead to 5 – 1 – 1 = 3 simulataneous equations as well, plus several subtraction and division operations to find the currents. Thus, mesh analysis has a slight edge here.

Define three clockwise mesh currents: i_a in the left-most mesh, i_b in the top right mesh, and i_c in the bottom right mesh. Then our mesh equations will be:

Mesh *a*: $-80 + (10 + 20 + 30)\, i_a - 20\, i_b - 30\, i_c = 0$ [1]
Mesh *b*: $-30 + (12 + 20)\, i_b - 12\, i_c - 20\, i_a = 0$ [2]
Mesh *c*: $(12 + 40 + 30)\, i_c - 12\, i_b - 30\, i_a = 0$ [3]

Simplifying and collecting terms,

$$60\, i_a - 20\, i_b - 30\, i_c = 80 \quad [1]$$
$$-20\, i_a + 32\, i_b - 12\, i_c = 30 \quad [2]$$
$$-30\, i_a - 12\, i_b + 82\, i_c = 0 \quad [3]$$

Solving, we find that i_a = 3.549 A, i_b = 3.854 A, and i_c = 1.863 A. Thus,

$$i_1 = i_a = \boxed{3.549 \text{ A}} \qquad \text{and} \quad i_2 = i_a - i_c = \boxed{1.686 \text{ A.}}$$

52. Approaching this problem using nodal analysis would require 3 separate nodal equations, plus one equation to deal with the dependent source, plus subtraction and division steps to actually find the current i_{10}. Mesh analysis, on the other hand, will require 2 mesh/supermesh equations, 1 KCL equation, and one subtraction step to find i_{10}. Thus, mesh analysis has a clear edge. Define three clockwise mesh currents: i_1 in the bottom left mesh, i_2 in the top mesh, and i_3 in the bottom right mesh.

MESH 1: $i_1 = 5$ mA *by inspection* [1]

SUPERMESH: $i_1 - i_2 = 0.4\, i_{10}$
$i_1 - i_2 = 0.4(i_3 - i_2)$
$i_1 - 0.6\, i_2 - 0.4\, i_3 = 0$ [2]

MESH 3: $-5000\, i_1 - 10000\, i_2 + 35000\, i_3 = 0$ [3]

Simplify: $0.6\, i_2 + 0.4\, i_3 = 5\times10^{-3}$ [2]
$-10000\, i_2 + 35000\, i_3 = 25$ [3]

Solving, we find $i_2 = 6.6$ mA and $i_3 = 2.6$ mA. Since $i_{10} = i_3 - i_2$, we find that

$$\boxed{i_{10} = -4 \text{ mA.}}$$

53. For this circuit problem, nodal analysis will require 3 simultaneous nodal equations, then subtraction/ division steps to obtain the desired currents. Mesh analysis requires 1 mesh equation, 1 supermesh equation, 2 simple KCL equations and one subtraction step to determine the currents. If either technique has an edge in this situation, it's probably mesh analysis. Thus, define four clockwise mesh equations: i_a in the bottom left mesh, i_b in the top left mesh, i_c in the top right mesh, and i_d in the bottom right mesh.

At the *a*, *b*, *c* supermesh: $-100 + 6\, i_a + 20\, i_b + 4\, i_c + 10\, i_c - 10\, i_d = 0$ [1]

Mesh d: $100 + 10\, id - 10\, i_c + 24\, i_d = 0$ [2]

KCL: $-\, i_a + i_b = 2$ [3]

and - $i_b + i_c = 3\, i_3 = 3\, i_a$ [4]

Collecting terms & simplifying,

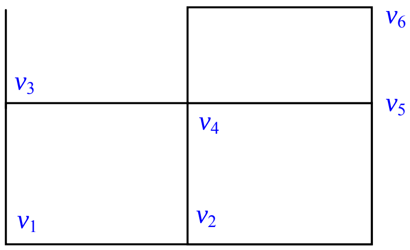
$$\begin{aligned} 6\, i_a + 20\, i_b + 14\, i_c - 10\, i_d &= 100 && [1] \\ -10\, i_c + 34\, i_d &= -100 && [2] \\ -\, i_a + i_b &= 2 && [3] \\ -3\, i_a - i_b + i_c &= 0 && [4] \end{aligned}$$

Solving,

$i_a = 0.1206$ A, $i_b = 2.121$ A, $i_c = 2.482$ A, and $i_d = -2.211$ A. Thus,

$$i_3 = i_a = \boxed{120.6 \text{ mA}} \quad \text{and} \quad i_{10} = i_c - i_d = \boxed{4.693 \text{ A.}}$$

54. With 7 nodes in this circuit, nodal analysis will require the solution of three simultaneous nodal equations (assuming we make use of the supernode technique) and one KVL equation. Mesh analysis will require the solution of three simultaneous mesh equations (one mesh current can be found by inspection), plus several subtraction and multiplication operations to finally determine the voltage at the central node. Either will probably require a comparable amount of algebraic manoeuvres, so we go with nodal analysis, as the desired unknown is a direct result of solving the simultaneous equations. Define the nodes as:



NODE 1: $-2\times10^{-3} = (v_1 - 1.3)/\ 1.8\times10^3 \quad\rightarrow\quad v_1 = -2.84$ V.

2, 4 Supernode:
$2.3\times10^{-3} = (v_2 - v_5)/\ 1\text{x}10^3 + (v_4 - 1.3)/\ 7.3\times10^3 + (v_4 - v_5)/\ 1.3\times10^3 + v_4/\ 1.5\times10^3$

KVL equation: $-v_2 + v_4 = 5.2$

Node 5: $0 = (v_5 - v_2)/\ 1\text{x}10^3 + (v_5 - v_4)/\ 1.3\text{x}10^3 + (v_5 - 2.6)/\ 6.3\text{x}10^3$

Simplifying and collecting terms,

$$
\begin{aligned}
14.235\, v_2 + 22.39\, v_4 - 25.185\, v_5 &= 35.275 \quad [1]\\
-v_2 + v_4 &= 5.2 \quad [2]\\
-8.19\, v_2 - 6.3\, v_4 + 15.79\, v_5 &= 3.38 \quad [3]
\end{aligned}
$$

Solving, we find the voltage at the central node is $\boxed{v_4 = 3.460 \text{ V.}}$

55. Mesh analysis yields current values directly, so use that approach. We therefore define four clockwise mesh currents, starting with i_1 in the left-most mesh, then i_2, i_3 and i_4 moving towards the right.

Mesh 1: $-0.8i_x + (2 + 5)\, i_1 - 5\, i_2 = 0$ [1]

Mesh 2: $i_2 = 1$ A by inspection [2]

Mesh 3: $(3 + 4)\, i_3 - 3(1) - 4(i_4) = 0$ [3]

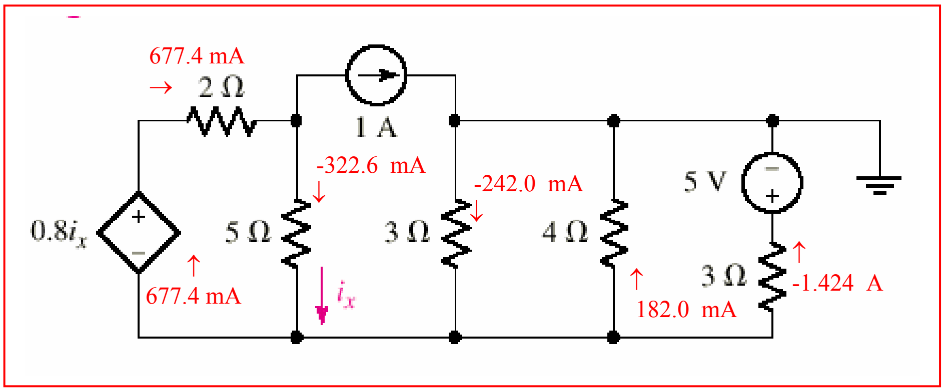
Mesh 4: $(4 + 3)\, i_4 - 4\, i_3 - 5 = 0$ [4]

Simplify and collect terms, noting that $i_x = i_1 - i_2 = i_1 - 1$

$-0.8(i_1 - 1) + 7\, i_1 - 5(1) = 0$ yields $i_1 = 677.4$ mA

Thus, [3] and [4] become: $7\, i_3 - 4\, i_4 = 3$ [3]
$-4\, i_3 + 7\, i_4 = 5$ [4]

Solving, we find that $i_3 = 1.242$ A and $i_4 = 1.424$ A. A map of individual branch currents can now be drawn:



56. If we choose to perform mesh analysis, we require 2 simultaneous equations (there are four meshes, but one mesh current is known, and we can employ the supermesh technique around the left two meshes). In order to find the voltage across the 2-mA source we will need to write a KVL equation, however. Using nodal analysis is less desirable in this case, as there will be a large number of nodal equations needed. Thus, we define four clockwise mesh currents i_1, i_2, i_3 and i_4 starting with the left-most mesh and moving towards the right of the circuit.

At the 1,2 supermesh: $2000\, i_1 + 6000\, i_2 - 3 + 5000\, i_2 = 0$ [1]
and $i_1 - i_2 = 2\times10^{-3}$ [2]

by inspection, $i_4 = -1$ mA. However, this as well as any equation for mesh four are unnecessary: we already have two equations in two unknowns and i_1 and i_2 are sufficient to enable us to find the voltage across the current source.

Simplifying, we obtain $2000\, i_1 + 11000\, i_2 = 3$ [1]
1000 $i_1 - 1000\, i_2 = 2$ [2]

Solving, $i_1 = 1.923$ mA and $i_2 = -76.92$ µA.

Thus, the voltage across the 2-mA source ("+" reference at the top of the source) is

$$v = -2000\, i_1 - 6000\,(i_1 - i_2) = \boxed{-15.85 \text{ V.}}$$

57. Nodal analysis will require 2 nodal equations (one being a "supernode" equation), 1 KVL equation, and subtraction/division operations to obtain the desired current. Mesh analysis simply requires 2 "supermesh" equations and 2 KCL equations, with the desired current being a mesh current. Thus, we define four clockwise mesh currents i_a, i_b, i_c, i_d starting with the left-most mesh and proceeding to the right of the circuit.

At the a, b supermesh: $-5 + 2\,i_a + 2\,i_b + 3\,i_b - 3\,i_c = 0$ [1]

At the c, d supermesh: $3\,i_c - 3\,i_b + 1 + 4\,i_d = 0$ [2]

and $i_a - i_b = 3$ [3]
$i_c - i_d = 2$ [4]

Simplifying and collecting terms, we obtain

$$\begin{aligned} 2\,i_a + 5\,i_b - 3\,i_c \qquad &= 5 \quad [1] \\ -3\,i_b + 3\,i_c + 4\,i_d &= -1 \quad [2] \\ i_a \quad - i_b \qquad\qquad &= 3 \quad [3] \\ i_c \quad - i_d &= 2 \quad [4] \end{aligned}$$

Solving, we find $i_a = 3.35$ A, $i_b = 350$ mA, $i_c = 1.15$ A, and $i_d = -850$ mA. As $i_1 = i_b$,

$$\boxed{i_1 = 350 \text{ mA.}}$$

58. Define a voltage v_x at the top node of the current source I_2, and a clockwise mesh current i_b in the right-most mesh.

We want 6 W dissipated in the 6-Ω resistor, which leads to the requirement $i_b = 1$ A. Applying nodal analysis to the circuit,

$I_1 + I_2 \;=\; (v_x - v_1)/\,6 \;=\; 1$

so our requirement is $I_1 + I_2 = 1$. There is no constraint on the value of v_1 other than we are told to select a nonzero value.

Thus, $\boxed{\text{we choose } I_1 = I_2 = 500 \text{ mA and } v_1 = 3.1415 \text{ V.}}$

59. Inserting the new 2-V source with "+" reference at the bottom, and the new 7-mA source with the arrow pointing down, we define four clockwise mesh currents i_1, i_2, i_3, i_4 starting with the left-most mesh and proceeding towards the right of the circuit.

Mesh 1: $(2000 + 1000 + 5000)\, i_1 - 6000\, i_2 - 2 = 0$ [1]

2, 3 Supermesh:

$2 + (5000 + 5000 + 1000 + 6000)\, i_2 - 6000\, i_1 + (3000 + 4000 + 5000)\, i_3 - 5000\, i_4 = 0$ [2]

and $i_2 - i_3 = 7\times10^{-3}$ [3]

Mesh 4: $i_4 = -1$ mA by inspection [4]

Simplifying and combining terms,

$$8000\, i_1 - 6000\, i_2 = 2 \quad [1]$$
$$1000\, i_2 - 1000\, i_3 = 7 \quad [4]$$
$$-6000\, i_1 + 17000\, i_2 + 12000\, i_3 = -7 \quad [2]$$

Solving, we find that

$$\boxed{i_1 = 2.653 \text{ A},\ i_2 = 3.204 \text{ A},\ i_3 = -3.796 \text{ A},\ i_4 = -1 \text{ mA}}$$

60. This circuit is easily analyzed by mesh analysis; it's planar, and after combining the 2A and 3 A sources into a single 1 A source, supermesh analysis is simple.

First, define clockwise mesh currents i_x, i_1, i_2 and i_3 starting from the left-most mesh and moving to the right. Next, combine the 2 A and 3 A sources temporarily into a 1 A source, arrow pointing upwards. Then, define four nodal voltages, V_1, V_2, V_3 and V_4 moving from left to right along the top of the circuit.

At the left-most mesh, $i_x = -5\, i_1$ [1]
For the supermesh, we can write $4i_1 - 2i_x + 2 + 2i_3 = 0$ [2]
and the corresponding KCL equation: $i_3 - i_1 = 1$ [3]

Substituting Eq. [1] into Eq. [2] and simplifying,

$$14\, i_1 + 2i_3 = -2$$
$$-i_1 + i_3 = 1$$

Solving, $\boxed{i_1 = -250 \text{ mA and } i_3 = 750 \text{ mA.}}$
Then, $\boxed{i_x = -5\, i_1 = 1.35 \text{ A and } i_2 = i_1 - 2 = -2.25 \text{ A}}$

Nodal voltages are straightfoward to find, then:

$$\boxed{\begin{array}{l} V_4 = 2i_3 = 1.5 \text{ V} \\ V_3 = 2 + V_4 = 3.5 \text{ V} \\ V_2 - V_3 = 2\, i_1 \text{ or } V_2 = 2\, i_1 + V_3 = 3 \text{ V} \\ V_1 - V_2 = 2\, i_x \text{ or } V_1 = 2\, i_x + V_2 = 5.5 \text{ V} \end{array}}$$

61. **Hand analysis:**

Define three clockwise mesh currents: i_1 in the bottom left mesh, i_2 in the top mesh, and i_3 in the bottom right mesh.

MESH 1: $i_1 = 5$ mA *by inspection* [1]

SUPERMESH:

$$i_1 - i_2 = 0.4\, i_{10}$$
$$i_1 - i_2 = 0.4(i_3 - i_2)$$
$$i_1 - 0.6\, i_2 - 0.4\, i_3 = 0 \quad [2]$$

MESH 3: $-5000\, i_1 - 10000\, i_2 + 35000\, i_3 = 0$ [3]

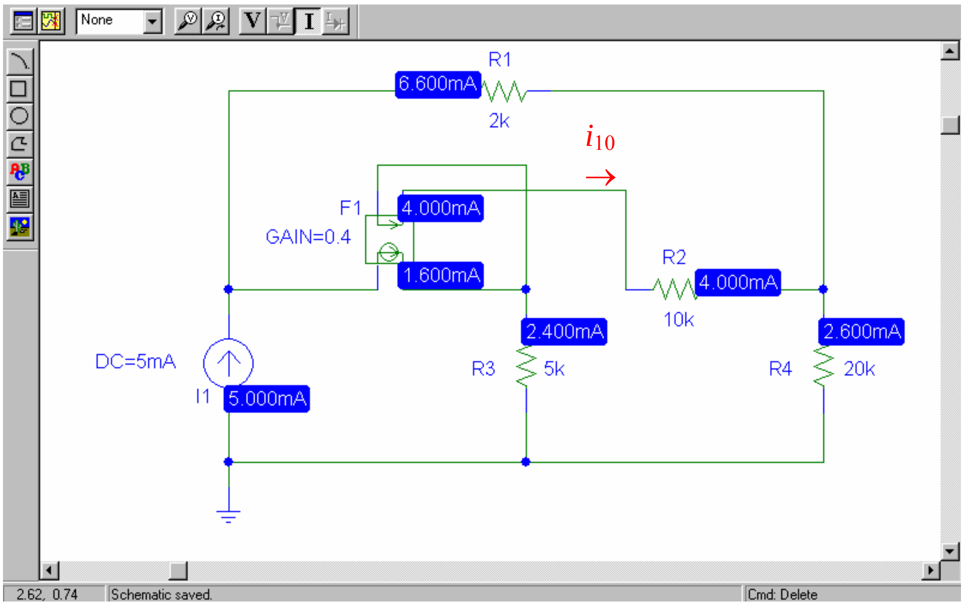
Simplify:

$$0.6\, i_2 + 0.4\, i_3 = 5\times10^{-3} \quad [2]$$
$$-10000\, i_2 + 35000\, i_3 = 25 \quad [3]$$

Solving, we find $i_2 = 6.6$ mA and $i_3 = 2.6$ mA. Since $i_{10} = i_3 - i_2$, we find that

$$\boxed{i_{10} = -4 \text{ mA.}}$$

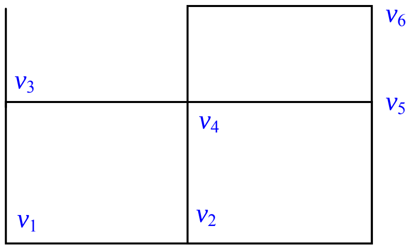
PSpice simulation results:



Summary: The current entering the right-hand node of the 10-k Ω resistor R2 is equal to 4.000 mA. Since this current is $-i_{10}$, $i_{10} = -4.000$ mA as found by hand.

62. **Hand analysis:**

Define the nodes as:



NODE 1: $-2\times10^{-3} = (v_1 - 1.3)/\ 1.8\times10^3 \quad \rightarrow \quad v_1 = -2.84$ V.

2, 4 Supernode:

$2.3\times10^{-3} = (v_2 - v_5)/\ 1\text{x}10^3 + (v_4 - 1.3)/\ 7.3\times10^3 + (v_4 - v_5)/\ 1.3\times10^3 + v_4/\ 1.5\times10^3$

KVL equation: $-v_2 + v_4 = 5.2$

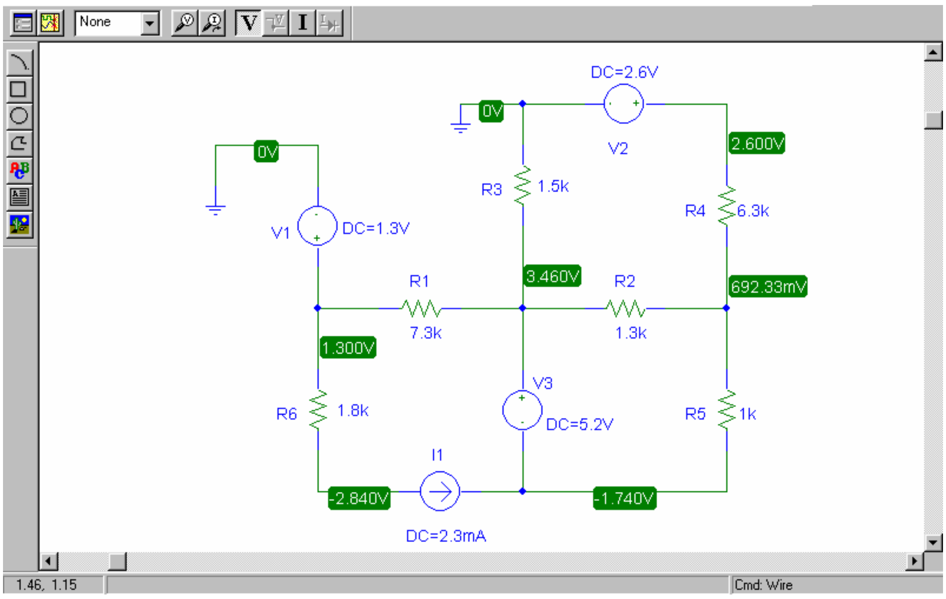
Node 5: $0 = (v_5 - v_2)/\ 1\text{x}10^3 + (v_5 - v_4)/\ 1.3\text{x}10^3 + (v_5 - 2.6)/\ 6.3\text{x}10^3$

Simplifying and collecting terms,

$$\begin{aligned} 14.235\, v_2 + 22.39\, v_4 - 25.185\, v_5 &= 35.275 \quad [1] \\ -v_2 + \quad v_4 \qquad &= 5.2 \quad [2] \\ -8.19\, v_2 \quad - 6.3\, v_4 + 15.79\, v_5 &= 3.38 \quad [3] \end{aligned}$$

Solving, we find the voltage at the central node is $\boxed{v_4 = 3.460 \text{ V.}}$

PSpice simulation results:



Summary: The voltage at the center node is found to be 3.460 V, which is in agreement with our hand calculation.

63. **Hand analysis:**

At the 1,2 supermesh: $2000\ i_1 + 6000\ i_2 - 3 + 5000\ i_2 = 0$ [1]

and $i_1 - i_2 = 2\times10^{-3}$ [2]

by inspection, $i_4 = -1$ m A. However, this as well as any equation for mesh four are unnecessary: w e already have two equations in two unknowns and i_1 and i_2 are sufficient to enable us to find the voltage across the current source.

Simplifying, we obtain $2000\ i_1 + 11000\ i_2 = 3$ [1]

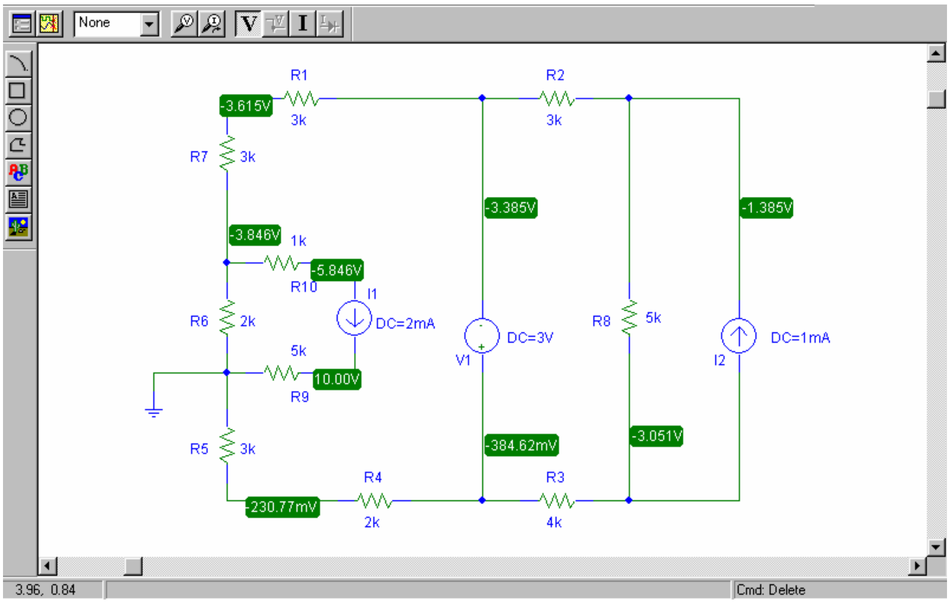
1000 i_1 - $1000\ i_2 = 2$ [2]

Solving, $i_1 = 1.923$ mA and $i_2 = -76.92$ μA.

Thus, the voltage across the 2-mA source ("+" reference at the top of the source) is

$$v = -2000\ i_1 - 6000\ (i_1 - i_2) = \boxed{-15.85 \text{ V.}}$$

PSpice simulation results:



Summary: Again arbitrarily selecting the "+" reference as the top node of the 2-mA current source, we find the voltage across it is –5.846 – 10 = -15.846 V, in agreement with our hand calculation.

64. **Hand analysis:**
Define a voltage v_x at the top node of the current source I_2, and a clockwise mesh current i_b in the right-most mesh.

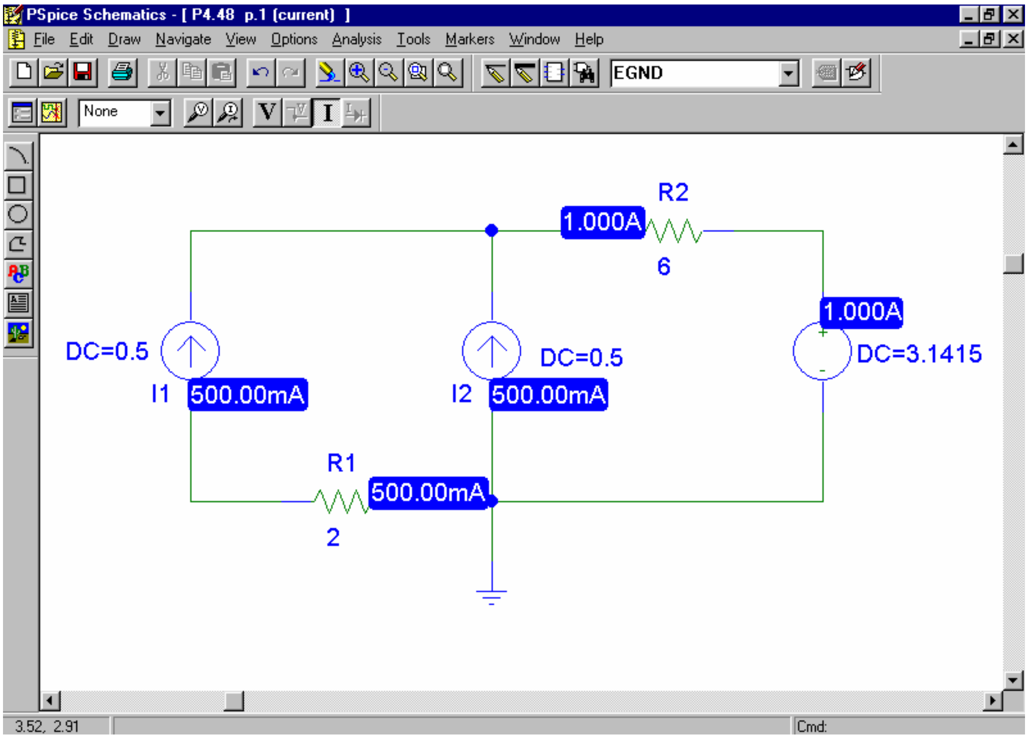
We want 6 W dissipated in the 6-Ω resistor, which leads to the requirement i_b = 1 A. Applying nodal analysis to the circuit,

$$I_1 + I_2 = (v_x - v_1)/ 6 = 1$$

so our requirement is $I_1 + I_2 = 1$. There is no constraint on the value of v_1 other than we are told to select a nonzero value.

Thus, $\boxed{\text{we choose } I_1 = I_2 = 500 \text{ mA and } v_1 = 3.1415 \text{ V.}}$

PSpice simulation results:



Summary: We see from the labeled schematic above that our choice for I_1, I_2 and V_1 lead to 1 A through the 6-Ω resistor, or 6 W dissipated in that resistor, as desired.

65. **Hand analysis:**

Define node 1 as the top left node, and node 2 as the node joining the three 2-Ω resistors. Place the "+" reference term inal of the 2-V source at the righ t. The right-most 2-Ω resistor has therefore been shorted out. Applying nodal analysis then,

Node 1: $-5\,i_1 = (v_1 - v_2)/\,2$ [1]

Node 2: $0 = (v_2 - v_1)/\,2 + v_2/\,2 + (v_2 - 2)/\,2$ [2]

and, $i_1 = (v_2 - 2)/\,2$ [3]

Simplifying and collecting terms,

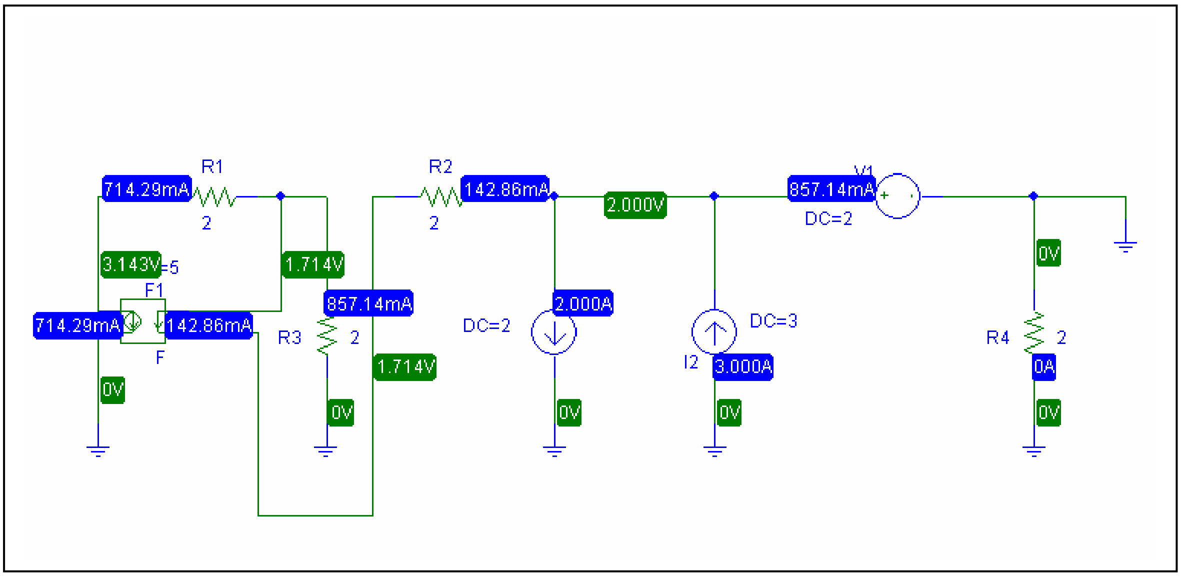
$v_1 + \quad v_2 = 10$ [1]
$-v_1 + 3\,v_2 = 2$ [2]

Solving, we find that $\boxed{v_1 = 3.143 \text{ V and } v_2 = 1.714 \text{ V.}}$

Defining clockwise m esh currents i $_a$, i $_b$, i $_c$, i $_d$ starting with the left-most m esh and proceeding right, we may easily determine that

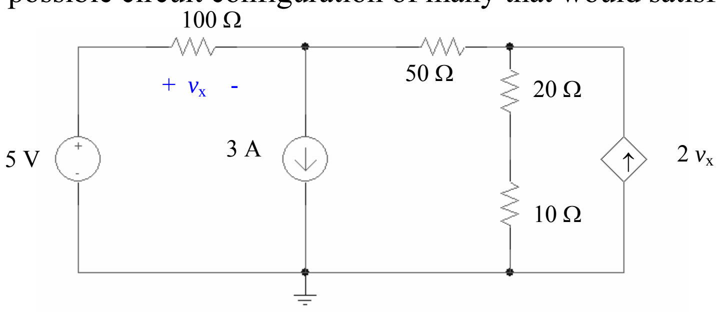
$$\boxed{\begin{array}{l} i_a = -5\,i_1 = 714.3 \text{ mA} \\ i_b = -142.9 \text{ mA} \\ i_c = i_1 - 2 = -2.143 \text{ A} \\ i_d = 3 + i_c = 857.1 \text{ mA} \end{array}}$$

PSpice simulation results:



Summary: The simulation results agree with the hand calculations.

66. (a) One possible circuit configuration of many that would satisfy the requirements:



At node 1: $-3 = (v_1 - 5)/100 + (v_1 - v2)/50$ [1]

At node 2: $2\ v_x = (v2 - v_1)/50 + v_2/30$ [2]

and, $v_x = 5 - v_1$ [3]

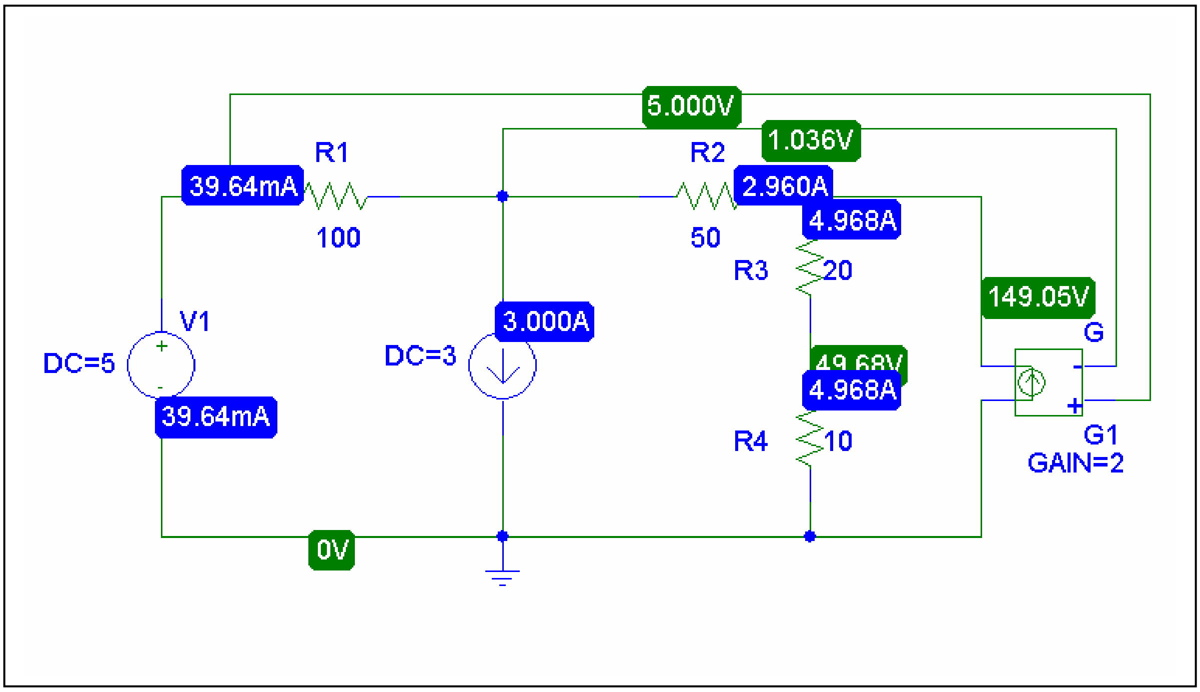
Simplifying and collecting terms,

$150 \quad v_1 - 100\ v_2 = -14750$ [1]

$2970 \quad v_1 + 80\ v_2 = 15000$ [2]

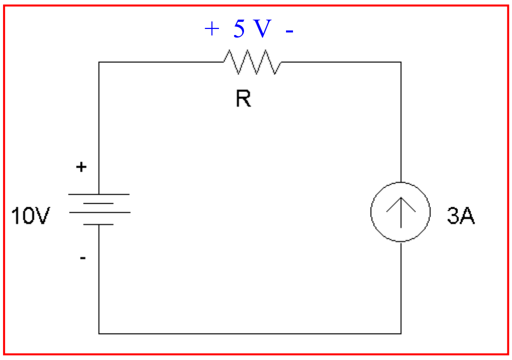
Solving, we find that $\boxed{v_1 = 1.036 \text{ V}}$ and $\boxed{v_2 = 149.1 \text{ V.}}$
The current through the 100-Ω resistor is simply $(5 - v_1)/100 = \boxed{39.64 \text{ mA}}$
The current through the 50-Ω resistor is $(v_1 - v_2)/50 = \boxed{-2.961 \text{ A,}}$
and the current through the 20- Ω and 10- Ω series combination is $v_2/30 = \boxed{4.97 \text{ A.}}$
Finally, the dependent source generates a current of $2\ v_x = \boxed{7.928 \text{ A.}}$

(b) PSpice simulation results



Summary: The simulated results agree with the hand calculations.

67. One possible solution of many:



Choose R so that 3R = 5; then the voltage across the current source will be 5 V, and so will the voltage across the resistor R.

R = 5/3 Ω. To construct this from 1-Ω resistors, note that

$$\boxed{5/3\ \Omega = 1\ \Omega + 2/3\ \Omega = 1\ \Omega + 1\ \Omega \,||\, 1\Omega \,||\, 1\Omega + 1\Omega \,||\, 1\Omega \,||\, 1\Omega}$$

```
* Solution to Problem 4.57

.OP

V1 1 0 DC 10
I1 0 4 DC 3
R1 1 2 1
R2 2 3 1
R3 2 3 1
R4 2 3 1
R5 3 4 1
R6 3 4 1
R7 3 4 1

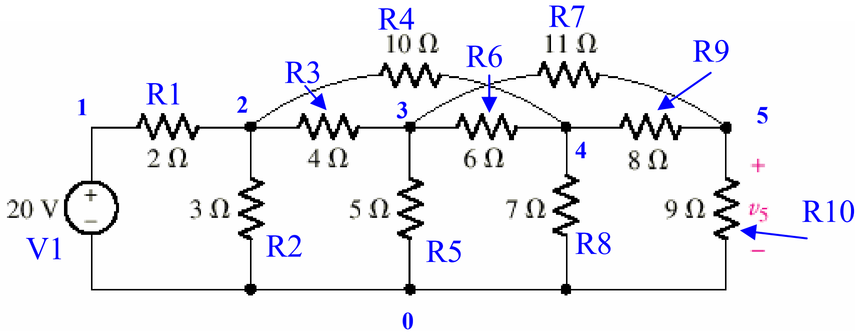
.END
```

```
****   SMALL SIGNAL BIAS SOLUTION      TEMPERATURE =   27.000 DEG C
******************************************************************************
 NODE   VOLTAGE     NODE   VOLTAGE     NODE   VOLTAGE     NODE   VOLTAGE
(   1)  10.0000  (   2)   7.0000  (   3)   6.0000  (   4)   5.0000

   VOLTAGE SOURCE CURRENTS
   NAME       CURRENT

   V1        -3.000E+00
```

68. We first name each node, resistor and voltage source:



We next write an appropriate input deck for SPICE:

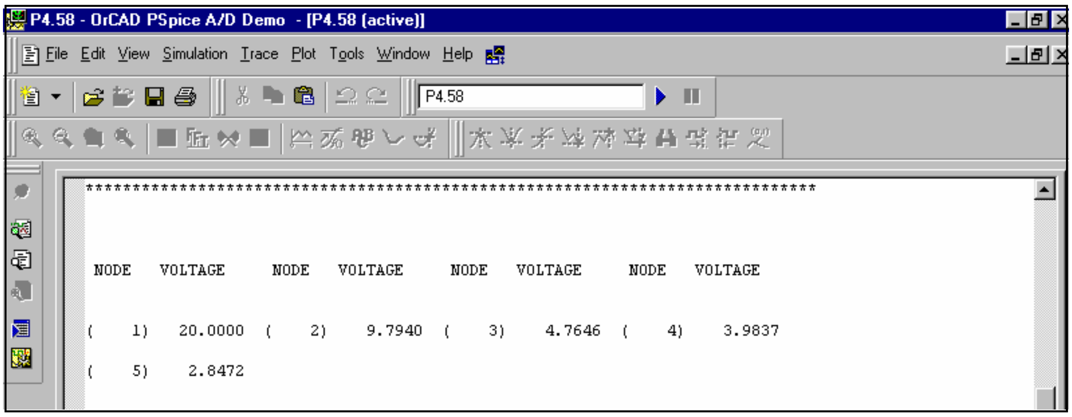
```
* Solution to Problem 4.58

.OP

V1 1 0 DC 20
R1 1 2 2
R2 2 0 3
R3 2 3 4
R4 2 4 10
R5 3 0 5
R6 3 4 6
R7 3 5 11
R8 4 0 7
R9 4 5 8
R10 5 0 9

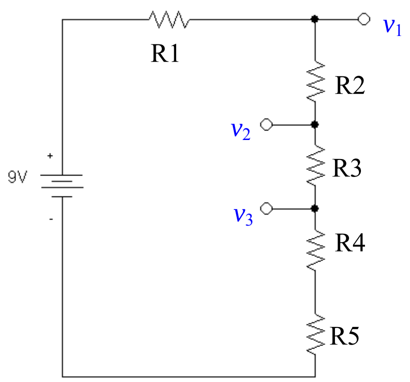
.END
```

And obtain the following output:



We see from this simulation result that the voltage v_5 = 2.847 V.

69. One possible solution of many:



All resistors are 1 Ω, except R1, which represents 5 1-Ω resistors in series.

Verify: $v_1 = 9(4/9) = 4$ V
$v_2 = 9(3/9) = 3$ V
$v_3 = 9(2/9) = 2$ V

SPICE INPUT DECK:

```
*  Solution to Problem 4.59

.OP

V1 1 0 DC 9
R1 1 2 5
R2 2 3 1
R3 3 4 1
R4 4 5 1
R5 5 0 1

.END
```

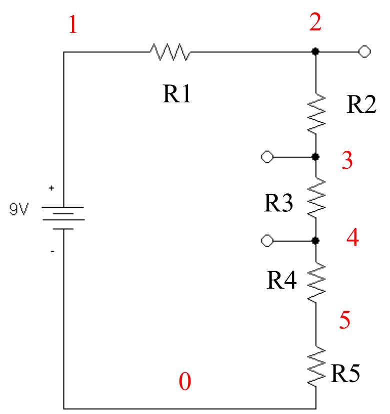
```
**** 07/29/01 21:36:26 *********** Evaluation PSpice (Nov 1999) **************

 *  Solution to Problem 4.59
 ****     SMALL SIGNAL BIAS SOLUTION       TEMPERATURE =   27.000 DEG C
**********************************************************************
*******

 NODE    VOLTAGE      NODE    VOLTAGE      NODE    VOLTAGE      NODE
VOLTAGE

(    1)    9.0000  (    2)    4.0000  (    3)    3.0000  (    4)    2.0000

(    5)    1.0000
```



70. (a) If only two bulbs are not lit (and thinking of each bulb as a resistor), the bulbs must be in parallel- otherwise, the burned out bulbs, acting as short circuits, would prevent current from flowing to the "good" bulbs.

(b) In a parallel connected circuit, each bulb "sees" 115 VAC. Therefore, the individual bulb current is 1 W / 115 V = 8.696 mA. The resistance of each "good" bulb is V/I = 13.22 kΩ. A simplified, electrically-equivalent model for this circuit would be a 115 VAC source connected in parallel to a resistor R_{eq} such that

$1/R_{eq} = 1/13.22\times10^3 + 1/13.22\times10^3 + \ldots + 1/13.22\times10^3$ (400 – 2 = 398 terms)
or $R_{eq} = 33.22\ \Omega$. We expect the source to provide 398 W.

```
* Solution to Problem 4.60

.OP

V1 1 0 AC 115 60
R1 1 0 33.22

.AC LIN 1 60 60
.PRINT AC VM(1)IM(V1)

.END
```

```
**** 07/29/01 21:09:32 *********** Evaluation PSpice (Nov 1999) **************

 * Solution to Problem 4.60

 ****     SMALL SIGNAL BIAS SOLUTION       TEMPERATURE =   27.000 DEG C
******************************************************************************

 NODE   VOLTAGE     NODE   VOLTAGE     NODE   VOLTAGE     NODE   VOLTAGE
(    1)    0.0000

   VOLTAGE SOURCE CURRENTS
   NAME         CURRENT

   V1           0.000E+00

   TOTAL POWER DISSIPATION   0.00E+00  WATTS

**** 07/29/01 21:09:32 *********** Evaluation PSpice (Nov 1999)
 * Solution to Problem 4.60


 ****     AC ANALYSIS                  TEMPERATURE =   27.000 DEG C
******************************************************************************

  FREQ       VM(1)       IM(V1)
  6.000E+01   1.150E+02   3.462E+00
```

This calculated power is not the value sought. It is an artifact of the use of an ac source, which requires that we perform an ac analysis. The supplied power is then separately computed as $(1.15\times10^2)(3.462) = 398.1$ W.

(c) The inherent series resistance of the wire connections leads to a voltage drop which increases the further one is from the voltage source. Thus, the furthest bulbs actually have less than 115 VAC across them, so they draw slightly less current and glow more dimly.

1. Define percent error as 100 $[e^x - (1 + x)]/ e^x$

x	$1 + x$	e^x	% error
0.001	1.001	1.001	5×10^{-5}
0.005	1.005	1.005	1×10^{-3}
0.01	1.01	1.010	5×10^{-3}
0.05	1.05	1.051	0.1
0.10	1.10	1.105	0.5
0.50	1.50	1.649	9
1.00	2.00	2.718	26
5.00	6.00	148.4	96

Of course, "reasonable" is a very subjective term. However, if we choose $x < 0.1$, we ensure that the error is less than 1%.

2. (a) Short-circuit the 10 V source.

Note that 6 || 4 = 2.4 Ω. By voltage division, the voltage across the 6 Ω resistor is then

$$4\frac{2.4}{3+2.4}=1.778 \text{ V}$$

So that $i_1' = \frac{1.778}{6} = \boxed{0.2963 \text{ A}}$.

(b) Short-circuit the 4 V source.

Note that 3 || 6 = 2 Ω. By voltage division, the voltage across the 6 Ω resistor is then

$$-10\frac{2}{6}=-3.333 \text{ V}$$

So that $i_1'' = \frac{-3.333}{6} = \boxed{-0.5556 \text{ A}}$.

(c) $i_1 = i_1' + i_2'' = \boxed{\text{-259.3 mA}}$

3. Open circuit the 4 A source. Then, since

(7 + 2) || (5 + 5) = 4.737 Ω, we can calculate $v_1' = (1)(4.737) = \boxed{4.737 \text{ V.}}$

To find the total current flowing through the 7 Ω resistor, we first determine the total voltage v_1 by continuing our superposition procedure. The contribution to v_1 from the 4 A source is found by first open-circuiting the 1 A source, then noting that current division yields:

$$4\frac{5}{5+(5+7+2)} = \frac{20}{19} = 1.053 \text{ A}$$

Then, $v_1'' = (1.053)(9) =$ 9.477 V. Hence, $v_1 = v_1' + v_1'' =$ 14.21 V.

We may now find the total current flowing downward through the 7 Ω resistor as

$14.21/7 = \boxed{2.03 \text{ A.}}$

4. One approach to this problem is to write a set of mesh equations, leaving the voltage source and current source as variables which can be set to zero.

We first rename the voltage source as V_x. We next define three clockwise mesh currents in the bottom three meshes: i_1, i_y and i_4. Finally, we define a clockwise mesh current i_3 in the top mesh, noting that it is equal to –4 A.

Our general mesh equations are then:

$$-V_x + 18i_1 - 10i_y = 0$$
$$-10i_1 + 15i_y - 3i_4 = 0$$
$$-3i_y + 16i_4 - 5i_3 = 0$$

** Set V_x = 10 V, i_3 = 0. Our mesh equations then become:

$$18i_1 - 10i'_y = 10$$
$$-10i_1 + 15i'_y - 3i_4 = 0$$
$$-3i'_y + 16i_4 = 0$$

Solving, i'_y = 0.6255 A.

** Set V_x = 0 V, i_3 = – 4 A. Our mesh equations then become:

$$18i_1 - 10i''_y = 0$$
$$-10i_1 + 15i''_y - 3i_4 = 0$$
$$-3i''_y + 16i_4 = -20$$

Solving, i''_y = –0.4222 A.

Thus, $i_y = i'_y + i''_y = \boxed{203.3 \text{ mA.}}$

5. We may solve this problem without writing circuit equations if we first realise that the current i_1 is composed of two terms: one that depends solely on the 4 V source, and one that depends solely on the 10 V source.

This may be written as $i_1 = 4K_1 + 10K_2$, where K_1 and K_2 are constants that depend on the circuit topology and resistor values.

We may not change K_1 or K_2, as only the source voltages may be changed. If we increase both sources by a factor of 10, then i_1 increases by the same amount.

Thus, $\boxed{4\text{ V} \rightarrow 40\text{ V and } 10\text{ V} \rightarrow 100\text{ V.}}$

6. i_A, v_B "on", $v_C = 0$: $i_x = 20$ A
i_A, v_C "on", $v_B = 0$: $i_x = -5$ A
i_A, v_B, v_C "on" : $i_x = 12$ A

so, we can write

$$\begin{aligned} i_x' + i_x'' + i_x''' &= 12 \\ i_x' + i_x'' \qquad &= 20 \\ i_x' \qquad + i_x''' &= -5 \end{aligned}$$

In matrix form,

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} i_x' \\ i_x'' \\ i_x''' \end{bmatrix} = \begin{bmatrix} 12 \\ 20 \\ -5 \end{bmatrix}$$

(a) with i_A on only, the response $i_x = i_x' = \boxed{3\text{ A.}}$
(b) with v_B on only, the response $i_x = i_x'' = \boxed{17\text{ A}}$.
(c) with v_C on only, the response $i_x = i_x''' = \boxed{-8\text{ A.}}$
(d) i_A and v_C doubled, v_B reversed: $2(3) + 2(-8) + (-1)(17) = \boxed{-27\text{ A.}}$

7. One source at a time:

The contribution from the 24-V source may be found by shorting the 45-V source and open-circuiting the 2-A source. Applying voltage division,

$$v_x' = 24\frac{20}{10+20+45\,||\,30} = 24\frac{20}{10+20+18} = 10\text{ V}$$

We find the contribution of the 2-A source by shorting both voltage sources and applying current division:

$$v_x'' = 20\left[2\frac{10}{10+20+18}\right] = 8.333\text{ V}$$

Finally, the contribution from the 45-V source is found by open-circuiting the 2-A source and shorting the 24-V source. Defining v_{30} across the 30-Ω resistor with the "+" reference on top:

$$0 = v_{30}/\,20 + v_{30}/\,(10 + 20) + (v_{30} - 45)/\,45$$

solving, $v_{30} = 11.25$ V and hence $v_x''' = -11.25(20)/(10 + 20) = -7.5$ V

Adding the individual contributions, we find that $v_x = v_x' + v_x'' + v_x''' = \boxed{10.83\text{ V}}$.

8. The contribution of the 8-A source is found by shorting out the two voltage sources and employing simple current division:

$$i_3' = -8\frac{50}{50+30} = -5\text{ A}$$

The contribution of the voltage sources may be found collectively or individually. The contribution of the 100-V source is found by open-circuiting the 8-A source and shorting the 60-V source. Then,

$$i_3'' = \frac{100}{(50+30)\ ||\ 60\ ||\ 30} = 6.25\text{ A}$$

The contribution of the 60-V source is found in a similar way as $i_3''' = -60/30 = -2$ A.

The total response is $i_3 = i_3' + i_3'' + i_3''' = \boxed{-750\text{ mA.}}$

9. (a) By current division, the contribution of the 1-A source i_2' is
$i_2' = 1\ (200)/\ 250 = 800$ mA.

The contribution of the 100-V source is $i_2'' = 100/\ 250 = 400$ mA.

The contribution of the 0.5-A source is found by current division once the 1-A source is open-circuited and the voltage source is shorted. Thus,

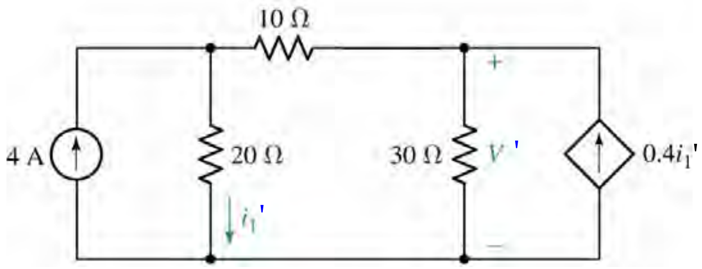
$$i_2''' = 0.5\ (50)/\ 250 = 100 \text{ mA}$$

Thus, $i_2 = i_2' + i_2'' + i_2''' = \boxed{1.3 \text{ A}}$

(b)
$$\boxed{\begin{array}{l} P_{1A} = (1)\ [(200)(1 - 1.3)] = 60 \text{ W} \\ P_{200} = (1 - 1.3)^2\ (200) = 18 \text{ W} \\ P_{100V} = -(1.3)(100) = -130 \text{ W} \\ P_{50} = (1.3 - 0.5)^2\ (50) = 32 \text{ W} \\ P_{0.5A} = (0.5)\ [(50)(1.3 - 0.5)] = 20 \text{ W} \end{array}}$$

Check: 60 + 18 + 32 + 20 = +130.

10. We find the contribution of the 4-A source by shorting out the 100-V source and analysing the resulting circuit:



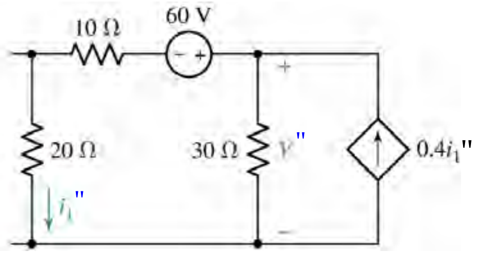
$$4 = V_1' / 20 + (V_1' - V')/ 10 \qquad [1]$$

$$0.4\, i_1' = V_1'/ 30 + (V' - V_1')/ 10 \qquad [2]$$

where $i_1' = V_1'/ 20$

Simplifying & collecting terms, we obtain $30\, V_1' - 20\, V' = 800$ [1]
$-7.2\, V_1' + 8\, V' = 0$ [2]

Solving, we find that V' = 60 V. Proceeding to the contribution of the 60-V source, we analyse the following circuit after defining a clockwise mesh current i_a flowing in the left mesh and a clockwise mesh current i_b flowing in the right mesh.



$$30\, i_a - 60 + 30\, i_a - 30\, i_b = 0 \qquad [1]$$
$$i_b = -0.4\, i_1'' = +0.4\, i_a \qquad [2]$$

Solving, we find that $i_a = 1.25$ A and so $V'' = 30(i_a - i_b) = 22.5$ V.

Thus, V = V' + V" = $\boxed{82.5 \text{ V.}}$

11. (a) Linearity allows us to consider this by viewing each source as being scaled by 25/ 10. This means that the response (v_3) will be scaled by the same factor:

$$25\, i_A'/\,10 + 25\, i_B'/\,10 = 25\, v_3'/\,10$$

$$\therefore\; v_3 = 25v_3'/\,10 = 25(80)/\,10 = \boxed{200\text{ V}}$$

(b) $i_A' = 10$ A, $i_B' = 25$ A $\rightarrow v_4' = 100$ V
$i_A'' = 10$ A, $i_B'' = 25$ A $\rightarrow v_4'' = -50$ V
$i_A = 20$ A, $i_B = -10$ A $\rightarrow v_4 = ?$

We can view this in a somewhat abstract form: the currents i_A and i_B multiply the same circuit parameters regardless of their value; the result is v_4.

Writing in matrix form, $\begin{bmatrix} 10 & 25 \\ 25 & 10 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 100 \\ -50 \end{bmatrix}$, we can solve to find

a = -4.286 and b = 5.714, so that 20a – 10b leads to $\boxed{v_4 = -142.9\text{ V}}$

12. With the current source open-circuited and the 7-V source shorted, we are left with 100k || (22k + 4.7k) = 21.07 kΩ.

Thus, $V_{3V} = 3\ (21.07)/\ (21.07 + 47) = 0.9286$ V.

In a similar fashion, we find that the contribution of the 7-V source is:

$V_{7V} = 7\ (31.97)\ /\ (31.97 + 26.7) = 3.814$ V

Finally, the contribution of the current source to the voltage V across it is:

$V_{5mA} = (5\times10^{-3})\ (\ 47\text{k} \parallel 100\text{k} \parallel 26.7\text{k}) = 72.75$ V.

Adding, we find that $V = 0.9286 + 3.814 + 72.75 = \boxed{77.49 \text{ V.}}$

13. We must find the current through the 500-kΩ resistor using superposition, and then calculate the dissipated power.

The contribution from the current source may be calculated by first noting that 1M || 2.7M || 5M = 636.8 kΩ. Then,

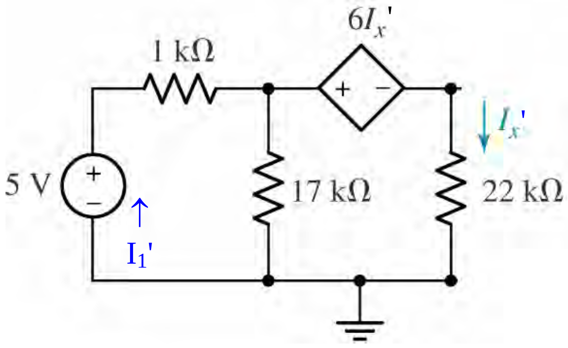
$$i_{60\mu A} = 60 \times 10^{-6} \left(\frac{3}{0.5 + 3 + 0.6368} \right) = 43.51\ \mu A$$

The contribution from the voltage source is found by first noting that 2.7M || 5M = 1.753 MΩ. The total current flowing from the voltage source (with the current source open-circuited) is –1.5/ (3.5 || 1.753 + 1) μA = -0.6919 μA. The current flowing through the 500-kΩ resistor due to the voltage source acting alone is then

$$i_{1.5V} = 0.6919\ (1.753)/\ (1.753 + 3.5)\ \text{mA} = 230.9\ \text{nA}.$$

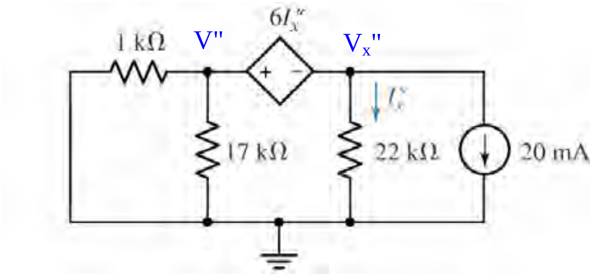
The total current through the 500-kΩ resistor is then $i_{60\mu A} + i_{1.5V}$ = 43.74 μA and the dissipated power is $(43.74\times10^{-9})^2\ (500\times10^3)$ = $\boxed{956.6\ \mu W.}$

14. We first determine the contribution of the voltage source:



Via mesh analysis, we write: $5 = 18000\ I_1' - 17000\ I_x'$
$-6\ I_x' = -17000\ I_x' + 39000\ I_x'$

Solving, we find $I_1' = 472.1$ mA and $I_x' = 205.8$ mA, so $V' = 17\times10^3\ (I_1' - I_x')$ = 4.527 V. We proceed to find the contribution of the current source:



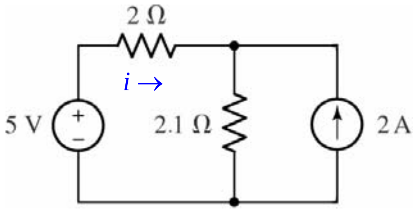
Via supernode: $-20\times10^{-3} = V_x''/\ 22\times10^3 + V''/\ 0.9444\times10^3$ [1]
and $V'' - V_x'' = 6I_x''$ or $V'' - V_x'' = 6\ V_x''/\ 22\times10^3$ [2]

Solving, we find that $V'' = -18.11$ V. Thus, $V = V' + V'' = \boxed{-13.58\text{ V}}$.

The maximum power is $V^2/\ 17\times10^3 = V^2/\ 17$ mW = 250 mW, so
$V = \sqrt{(17)(250)} = 65.19 = V' - (-18.11)$. Solving, we find $V'_{max} = 83.3$ V.
The 5-V source may then be increased by a factor of 83.3/ 4.527, so that its maximum positive value is $\boxed{92\text{ V}}$; past this value, and the resistor will overheat.

15. It is $\boxed{\text{impossible}}$ to identify the individual contribution of each source to the power dissipated in the resistor; superposition cannot be used for such a purpose.

Simplifying the circuit, we may at least determine the total power dissipated in the resistor:



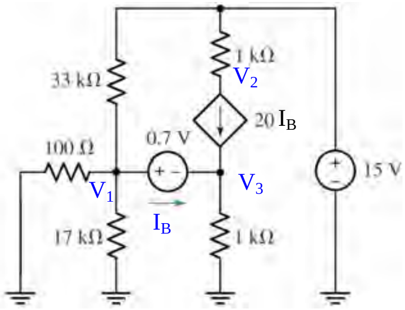
Via superposition in one step, we may write

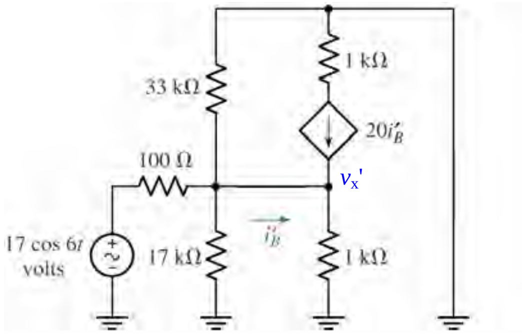
$$i = \frac{5}{2+2.1} - 2\frac{2.1}{2+2.1} = 195.1\,\text{mA}$$

Thus,

$$P_{2\Omega} = i^2 \cdot 2 = \boxed{76.15\text{ mW}}$$

16. We will analyse this circuit by first considering the combined effect of both dc sources (left), and then finding the effect of the single ac source acting alone (right).





1, 3 supernode: $V_1/ 100 + V_1/ 17\times10^3 + (V_1 – 15)/ 33\times10^3 + V_3/ 10^3 = 20 I_B$ [1]

and: $V_1 – V_3 = 0.7$ [2]

Node 2: $-20 I_B = (V_2 – 15)/ 1000$ [3]

We require one additional equation if we wish to have I_B as an unknown:

$20 I_B + I_B = V_3/ 1000$ [4]

Simplifying and collecting terms,

$10.08912 V_1 + V_3 – 20\times10^3 I_B = 0.4545$ [1]

$V_1 - V_3 = 0.7$ [2]

$V_2 + 20\times10^3 I_B = 15$ [3]

$-V_3 + 21\times103 I_B = 0$ [4]

Solving, we find that $I_B = -31.04$ μA.

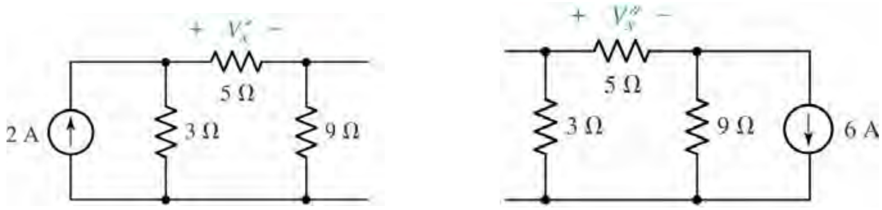
To analyse the right-hand circuit, we first find the Thévenin equivalent to the left of the wire marked i_B', noting that the 33-kΩ and 17-kΩ resistors are now in parallel. We find that $V_{TH} = 16.85 \cos 6t$ V by voltage division, and $R_{TH} = 100 \| 17k \| 33k = 99.12$ Ω. We may now proceed:

$20 i_B' = v_x' / 1000 + (v_x' – 16.85 \cos 6t)/ 99.12$ [1]
$20 i_B' + i_B'' = v_x'/ 1000$ [2]

Solving, we find that $i_B' = 798.6 \cos 6t$ mA. Thus, adding our two results, we find the complete current is

$\boxed{i_B = i_B' + I_B = -31.04 + 798.6 \cos 6t \ \mu A.}$

17.



We first consider the effect of the 2-A source separately, using the left circuit:

$$V_x' = 5\left[2\frac{3}{3+14}\right] = 1.765 \text{ V}$$

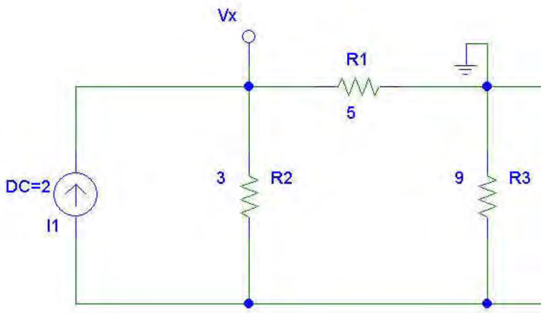
Next we consider the effect of the 6-A source on its own using the right circuit:

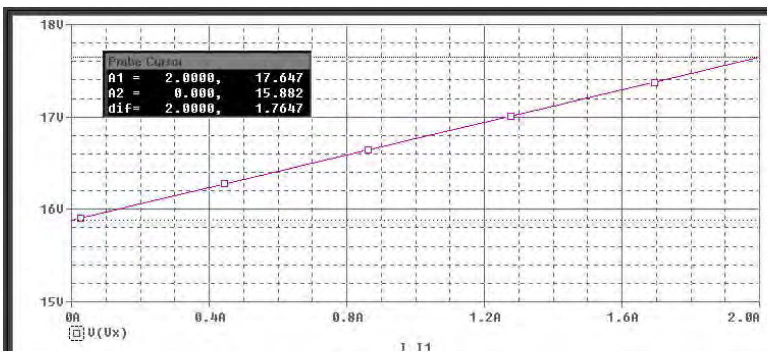
$$V_x'' = 5\left[6\frac{9}{9+8}\right] = 15.88 \text{ V}$$

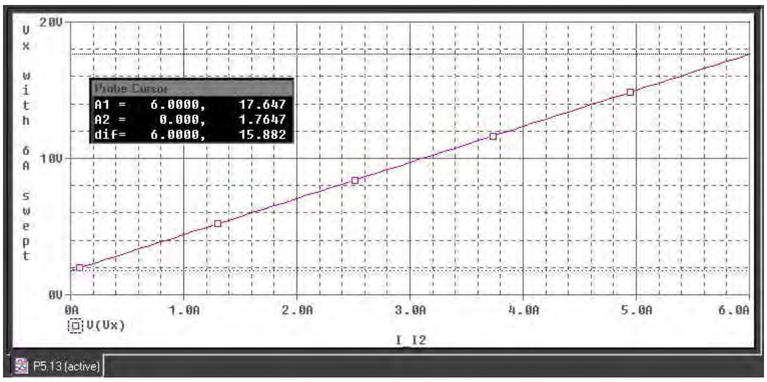
Thus, $V_x = V_x' + V_x'' = \boxed{17.65 \text{ V}}$.

(b) PSpice verification (DC Sweep)

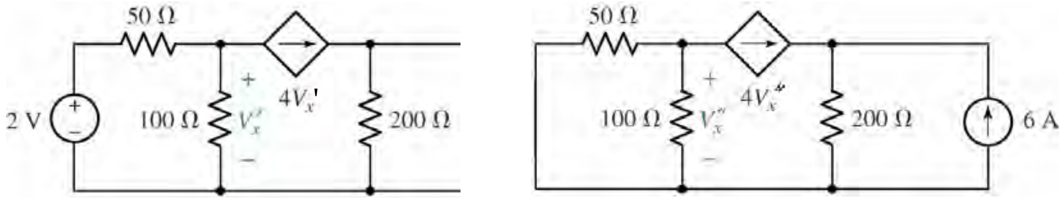
The DC sweep results below confirm that $V_x' = 1.765$ V







18.



(a) Beginning with the circuit on the left, we find the contribution of the 2-V source to V_x:

$$-4V'_x = \frac{V'_x}{100} + \frac{V'_x - 2}{50}$$

which leads to $V_x' = 9.926$ mV.

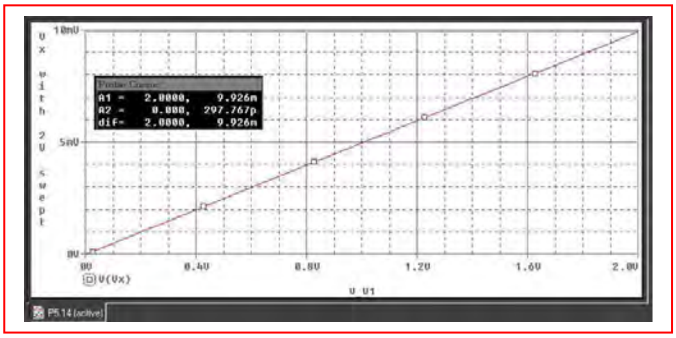
The circuit on the right yields the contribution of the 6-A source to Vx:

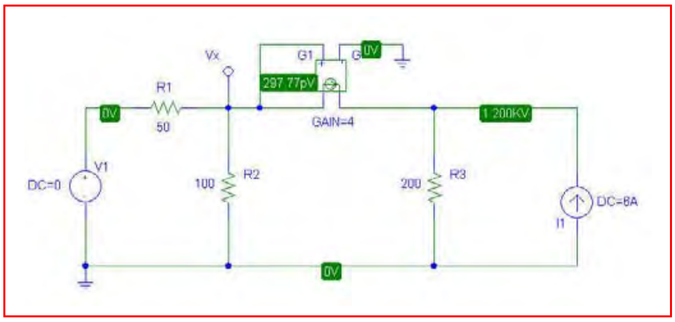
$$-4V''_x = \frac{V''_x}{100} + \frac{V''_x}{50}$$

which leads to $V_x'' = 0$.

Thus, $V_x = V_x' + V_x'' = \boxed{9.926 \text{ mV.}}$

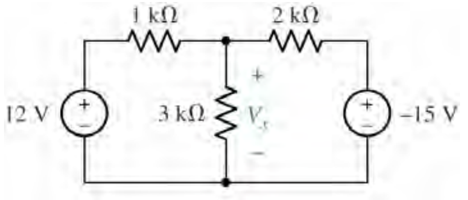
(b) PSpice verification.





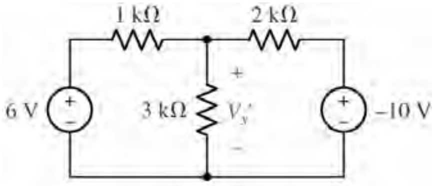
As can be seen from the two separate PSpice simulations, our hand calculations are correct; the pV-scale voltage in the second simulation is a result of numerical inaccuracy.

19.



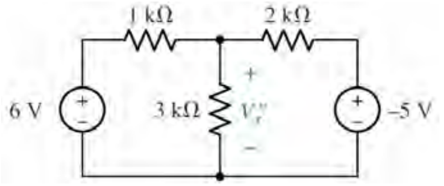
$$\frac{V_x - 12}{1} + \frac{V_x}{3} + \frac{V_x + 15}{2} = 0$$

so $\boxed{V_x = 2.455\text{ V}}$



$$\frac{V_x' - 6}{1} + \frac{V_x'}{3} + \frac{V_x' + 10}{2} = 0$$

so $\boxed{V_x' = 0.5455\text{ V}}$

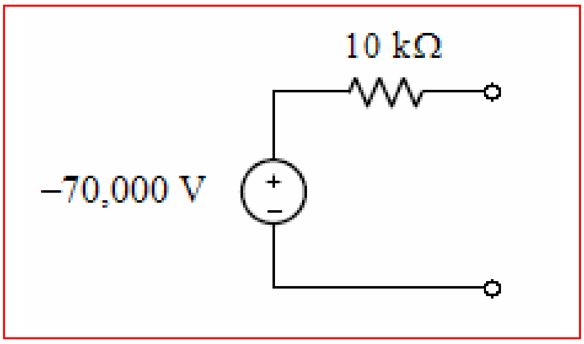


$$\frac{V_x'' - 6}{1} + \frac{V_x''}{3} + \frac{V_x'' + 5}{2} = 0$$

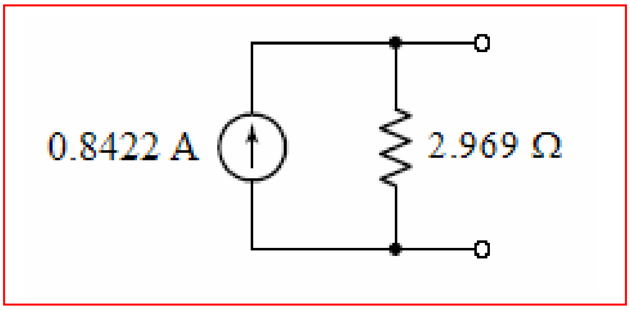
so $\boxed{V_x'' = 1.909\text{ V}}$

Adding, we find that $V_x' + V_x'' = 2.455$ V $= V_x$ as promised.

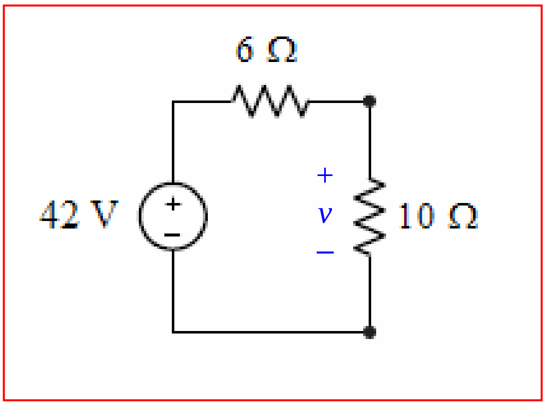
20. (a) We first recognise that the two current sources are in parallel, and hence may be replaced by a single –7 A source (arrow directed downward). This source is in parallel with a 10 kΩ resistor. A simple source transformation therefore yields a 10 kΩ resistor in series with a (–7)(10,000) = –70,000 V source (+ reference on top):



(b) This circuit requires several source transformations. First, we convert the 8 V source and 3 Ω resistor to an 8/3 A current source in parallel with 3 Ω. This yields a circuit with a 3 Ω and 10 Ω parallel combination, which may be replaced with a 2.308 Ω resistor. We may now convert the 8/3 A current source and 2.308 Ω resistor to a (8/3)(2.308) = 6.155 V voltage source in series with a 2.308 Ω resistor. This modified circuit contains a series combination of 2.308 Ω and 5 Ω; performing a source transformation yet again, we obtain a current source with value (6.155)/(2.308 + 5) = 0.8422 A in parallel with 7.308 Ω and in parallel with the remaining 5 Ω resistor. Since 7.308 Ω || 5 Ω = 2.969 Ω, our solution is:



21. (a) First we note the three current sources are in parallel, and may be replaced by a single current source having value 5 – 1 + 3 = 7 A, arrow pointing upwards. This source is in parallel with the 10 Ω resistor and the 6 Ω resistor. Performing a source transformation on the current source and 6 Ω resistor, we obtain a voltage source (7)(6) = 42 V in series with a 6 Ω resistor and in series with the 10 Ω resistor:



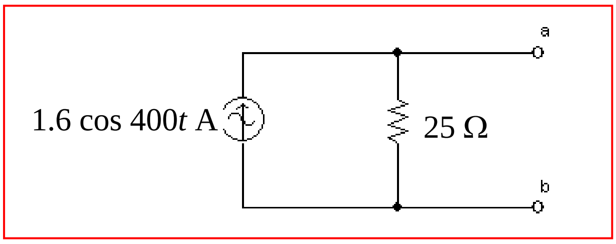
(b) By voltage division, v = 42(10)/16 = $\boxed{26.25 \text{ V.}}$

(c) Once the 10 Ω resistor is involved in a source transformation, it disappears, only to be replaced by a resistor having the same value – but whose current and voltage can be different. Since the quantity v appearing across this resistor is of interest, we cannot involve the resistor in a transformation.

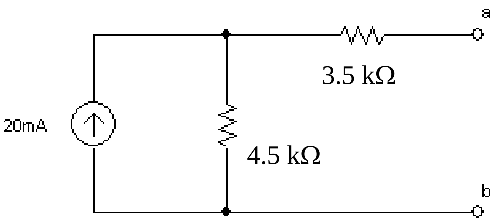
22. (a) [120 cos 400*t*] / 60 = 2 cos 400*t* A. 60 || 120 = 40 Ω.

[2 cos 400*t*] (40) = 80 cos 400*t* V. 40 + 10 = 50 Ω.

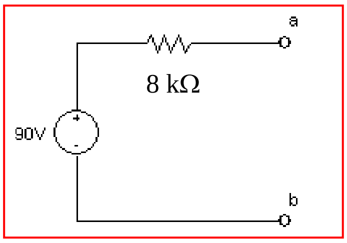
[80 cos 400*t*]/ 50 = 1.6 cos 400*t* A. 50 || 50 = 25 Ω.



(b) 2k || 3k + 6k = 7.2 kΩ. 7.2k || 12k = 4.5 kΩ



(20)(4.5) = 90 V.



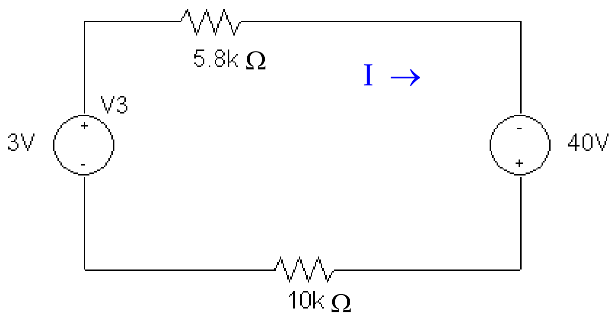
23. We can ignore the 1-kΩ resistor, at least when performing a source transformation on this circuit, as the 1-mA source will pump 1 mA through *whatever* value resistor we place there. So, we need only combine the 1 and 2 mA sources (which are in parallel once we replace the 1-kΩ resistor with a 0-Ω resistor). The current through the 5.8-kΩ resistor is then simply given by voltage division:

$$i = 3\times10^{-3} \frac{4.7}{4.7+5.8} = 1.343\,\text{mA}$$

The power dissipated by the 5.8-kΩ resistor is then $i^2 \cdot 5.8\times10^3 = \boxed{10.46 \text{ mW.}}$

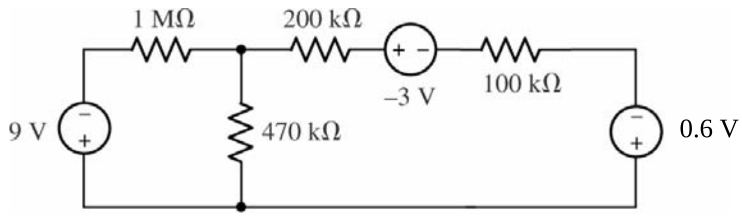
(Note that we did not "transform" either source, but rather drew on the relevant discussion to understand why the 1-kΩ resistor could be omitted.)

24. We may ignore the 10-kΩ and 9.7-kΩ resistors, as 3-V will appear across them regardless of their value. Performing a quick source transformation on the 10-kΩ resistor/ 4-mA current source combination, we replace them with a 40-V source in series with a 10-kΩ resistor:



I = 43/ 15.8 mA = 2.722 mA. Therefore, $P_{5.8\Omega} = I^{2\cdot}\, 5.8\times10^3 = \boxed{42.97 \text{ mW.}}$

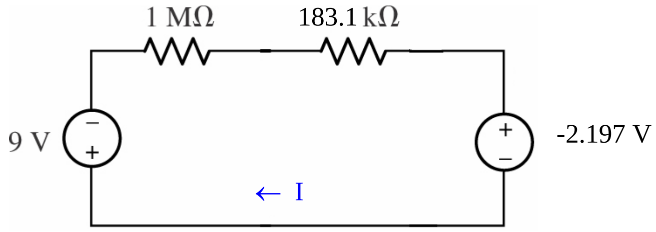
25. (100 kΩ)(6 mA) = 0.6 V



470 k || 300 k = 183.1 kΩ

$(-3 - 0.6)/\ 300\times10^3 = -12\ \mu A$

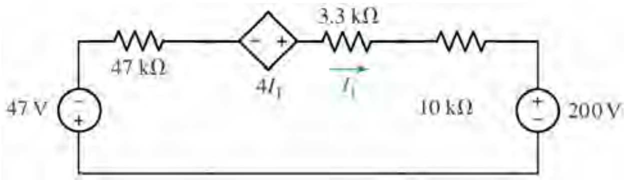
(183.1 kΩ)(-12 μA) = -2.197 V



Solving, $9 + 1183.1\times10^3\ I - 2.197 = 0$, so I = -5.750 μA. Thus,

$$P_{1M\Omega} = I^2 \cdot 10^6 = \boxed{33.06\ \mu W.}$$

26. (1)(47) = 47 V. (20)(10) = 200 V. Each voltage source "+" corresponds to its corresponding current source's arrow head.

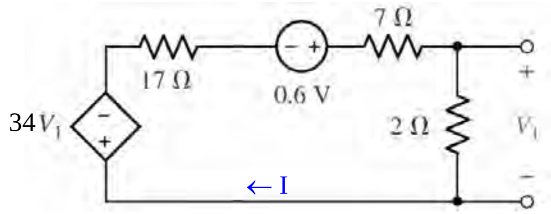


Using KVL on the simplified circuit above,

$$47 + 47\times10^3\ I_1 - 4\ I_1 + 13.3\times10^3\ I_1 + 200 = 0$$

Solving, we find that $I_1 = -247/\,(60.3\times10^3 - 4) = \boxed{-4.096\text{ mA.}}$

27. (a) $(2\ V_1)(17) = 34\ V_1$



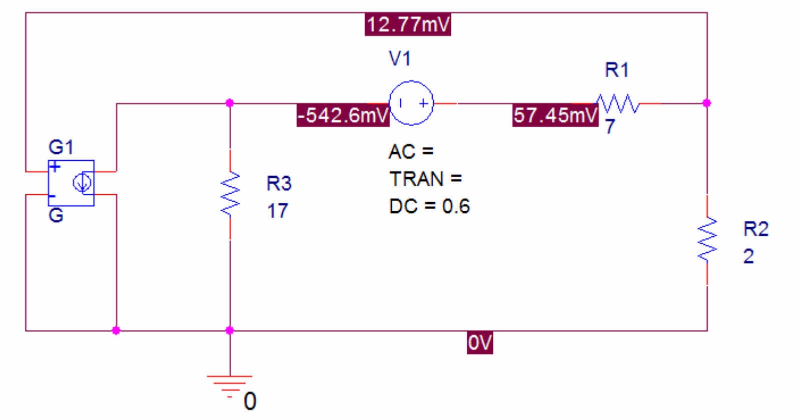
Analysing the simplified circuit above,

$$34\ V_1 - 0.6 + 7\ I + 2\ I + 17\ I\ =\ 0 \quad [1] \qquad \text{and} \quad V_1 = 2\ I \qquad [2]$$

Substituting, we find that I = 0.6/ (68 + 7 + 2 + 17) = 6.383 mA. Thus,

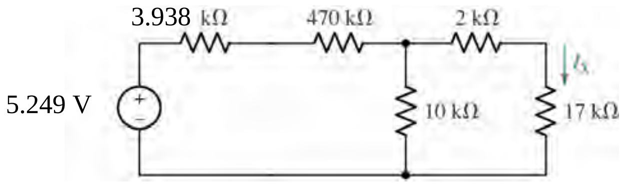
$$V_1 = 2\ I = \boxed{12.77\ \text{mV}}$$

(b)

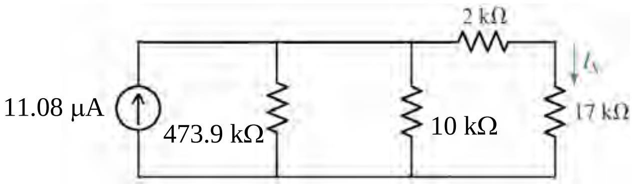


V_1 is the top-labelled voltage (12.77 mV).

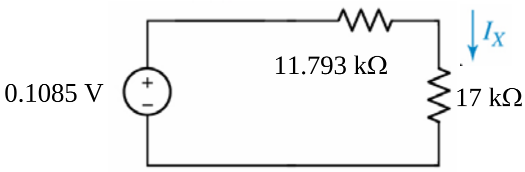
28. (a) 12/ 9000 = 1.333 mA. 9k || 7k = 3.938 kΩ. → (1.333 mA)(3.938 kΩ) = 5.249 V.



$5.249/\ 473.938\times10^3 = 11.08\ \mu A$

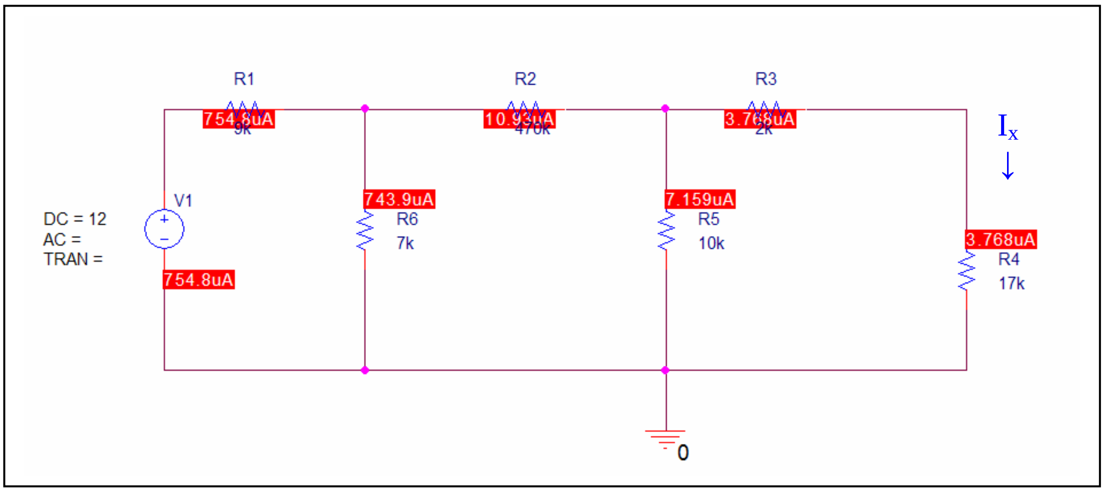


473.9 k || 10 k = 9.793 kΩ. (11.08 mA)(9.793 kΩ) = 0.1085 V

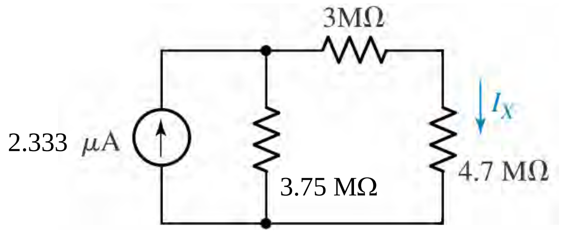


$I_x = 0.1085/\ 28.793\times10^3 = \boxed{3.768\ \mu A.}$

(b)

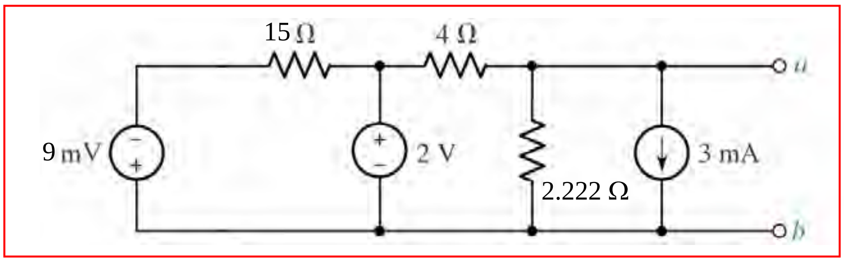


29. First, (-7 μA)(2 MΩ) = -14 V, “+” reference down. 2 MΩ + 4 MΩ = 6 MΩ.
+14 V/ 6 MΩ = 2.333 μA, arrow pointing up; 6 M || 10 M = 3.75 MΩ.



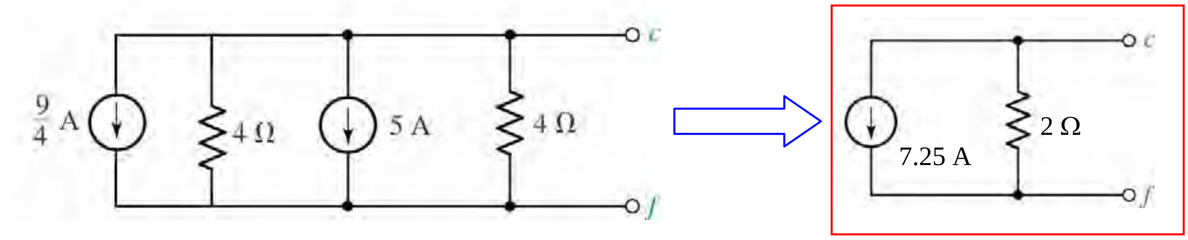
(2.333)(3.75) = 8.749 V. R_{eq} = 6.75 MΩ
∴ I_x = 8.749/ (6.75 + 4.7) μA = $\boxed{\text{764.1 nA.}}$

30. To begin, note that (1 mA)(9 Ω) = 9 mV, and 5 || 4 = 2.222 Ω.



The above circuit may not be further simplified using only source transformation techniques.

31. Label the "–" terminal of the 9-V source node **x** and the other terminal node **x'**. The 9-V source will force the voltage across these two terminals to be –9 V regardless of the value of the current source and resistor to its left. These two components may therefore be neglected from the perspective of terminals **a** & **b**. Thus, we may draw:



32. Beware of the temptation to employ superposition to compute the dissipated power- *it won't work*!

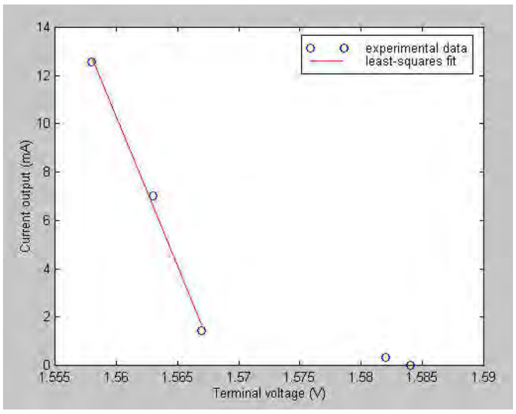
Instead, define a current I flowing into the bottom terminal of the 1-MΩ resistor. Using superposition to compute this current,

$$I = 1.8/ 1.840 + 0 + 0 \ \mu A = 978.3 \text{ nA}.$$

Thus,

$$P_{1M\Omega} = (978.3\times10^{-9})^2 (10^6) = \boxed{957.1 \text{ nW.}}$$

33. Let's begin by plotting the experimental results, along with a least-squares fit to part of the data:



Least-squares fit results:

Voltage (V)	**Current (mA)**
1.567	1.6681
1.563	6.599
1.558	12.763

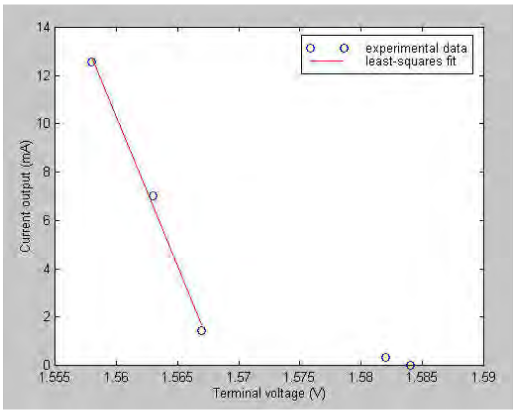
We see from the figure that we cannot draw a very good line through all data points representing currents from 1 mA to 20 mA. We have therefore chosen to perform a linear fit for the three lower voltages only, as shown. Our model will not be as accurate at 1 mA; there is no way to know if our model will be accurate at 20 mA, since that is beyond the range of the experimental data.

Modeling this system as an ideal voltage source in series with a resistance (representing the internal resistance of the battery) and a varying load resistance, we may write the following two equations based on the linear fit to the data:

$$1.567 = V_{src} - R_s\,(1.6681\times10^{-3})$$
$$1.558 = V_{src} - R_s\,(12.763\times10^{-3})$$

Solving, $\boxed{V_{src} = 1.568\text{ V}}$ and $\boxed{R_s = 811.2\text{ m}\Omega}$. It should be noted that depending on the line fit to the experimental data, these values can change somewhat, particularly the series resistance value.

34. Let's begin by plotting the experimental results, along with a least-squares fit to part of the data:



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Modeling this system as an ideal current source in parallel with a resistance R_p (representing the internal resistance of the battery) and a varying load resistance, we may write the following two equations based on the linear fit to the data:

$$1.6681\times10^{-3} \;=\; I_{src} - 1.567/\, R_p$$

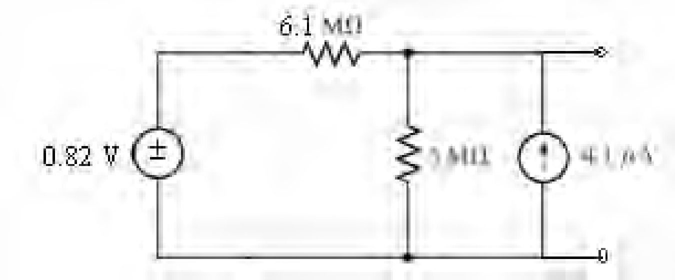
$$12.763\times10^{-3} \;=\; I_{src} - 1.558/\, R_p$$

Solving, $\boxed{I_{src} = 1.933\ \text{A}}$ and $\boxed{R_s = 811.2\ \text{m}\Omega}$. It should be noted that depending on the line fit to the experimental data, these values can change somewhat, particularly the series resistance value.

35. Working from left to right,

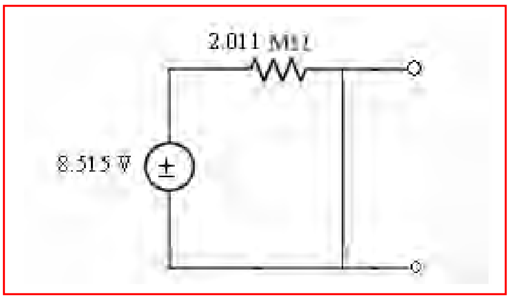
2 μA – 1.8 μA = 200 nA, arrow up.
1.4 MΩ + 2.7 MΩ = 4.1 MΩ

A transformation to a voltage source yields (200 nA)(4.1 MΩ) = 0.82 V in series with 4.1 MΩ + 2 MΩ = 6.1 MΩ, as shown below:

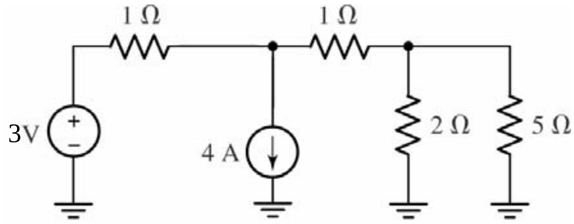


Then, 0.82 V/ 6.1 MΩ = 134.4 nA, arrow up.
6.1 MΩ || 3 MΩ = 2.011 MΩ
4.1 μA + 134.4 nA = 4.234 mA, arrow up.
(4.234 μA) (2.011 MΩ) = 8.515 V.

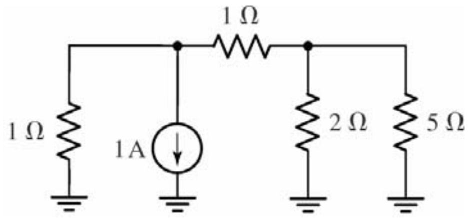
The final circuit is an 8.515 V voltage source in series with a 2.011 MΩ resistor, as shown:



36. To begin, we note that the 5-V and 2-V sources are in series:

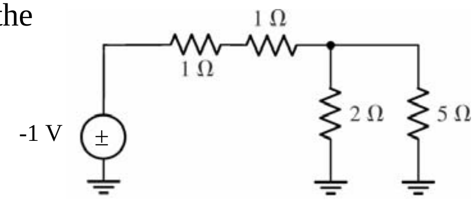


Next, noting that 3 V/ 1 Ω = 3 A, and 4 A – 3 A = +1 A (arrow down), we obtain:



The left-hand resistor and the current source are easily transformed into a 1-V source in series with a 1-Ω resistor:

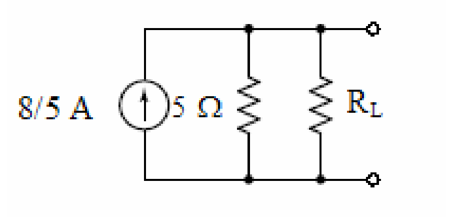
By voltage division, the voltage across the 5-Ω resistor in the circuit to the right is:



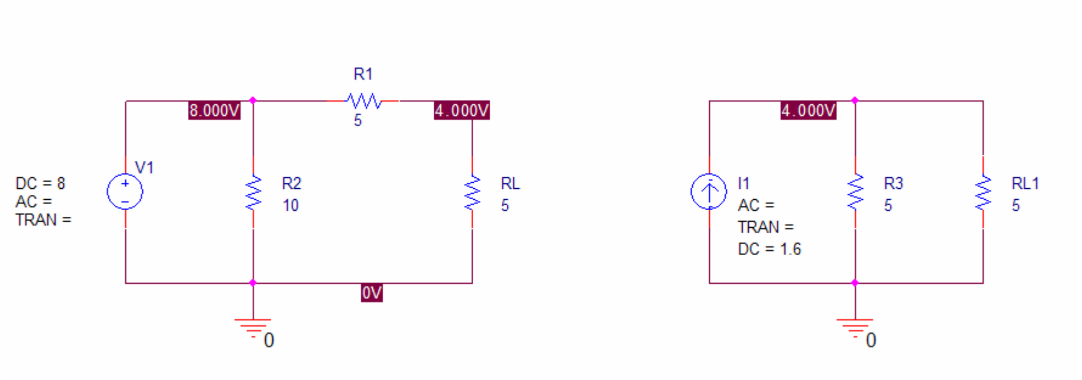
$$(-1)\ \frac{2 \,\|\, 5}{2 \,\|\, 5\ +\ 2} = -0.4167\text{ V.}$$

Thus, the power dissipated by the 5-Ω resistor is $(-0.4167)^2 / 5 = \boxed{34.73\text{ mW.}}$

37. (a) We may omit the 10 Ω resistor from the circuit, as it does not affect the voltage or current associated with R_L since it is in parallel with the voltage source. We are thus left with an 8 V source in series with a 5 Ω resistor. These may be transformed to an 8/5 A current source in parallel with 5 Ω, in parallel with R_L.



(b)



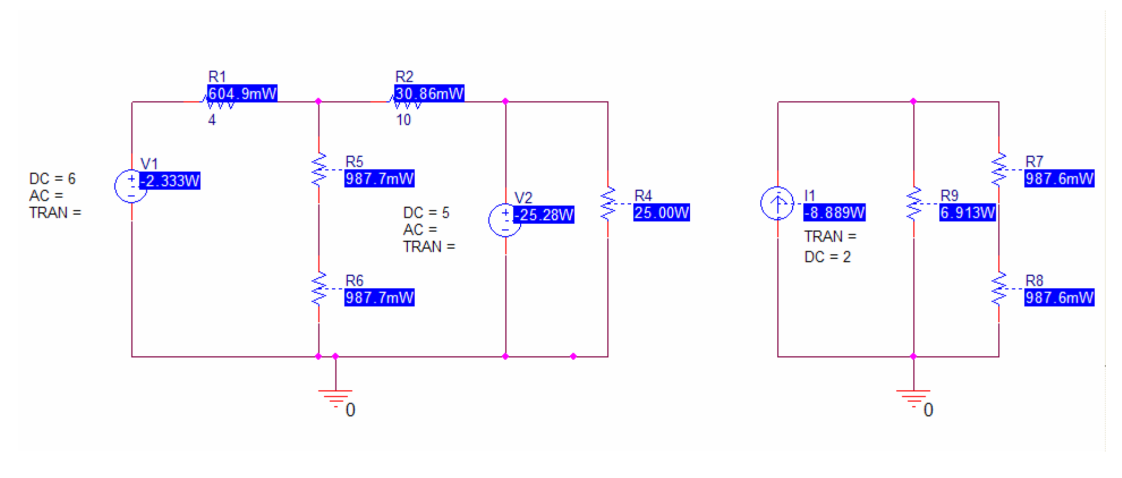
We see from simulating both circuits simultaneously that the voltage across R_L is the same (4 V).

38. (a) We may begin by omitting the 7 Ω and 1 Ω resistors. Performing the indicated source transformations, we find a 6/4 A source in parallel with 4 Ω, and a 5/10 A source in parallel with 10 Ω. These are both in parallel with the series combination of the two 5 Ω resistors. Since 4 Ω || 10 Ω = 2.857 Ω, and 6/4 + 5/10 = 2 A, we may further simplify the circuit to a single current source (2 A) in parallel with 2.857 Ω and the series combination of two 5 Ω resistors. Simple current division yields the current flowing through the 5 Ω resistors:

$$I = \frac{2(2.857)}{2.857 + 10} = 0.4444 \text{ A}$$

The power dissipated in either of the 5 Ω resistors is then I^2R = $\boxed{987.6 \text{ mW.}}$

(b) We note that PSpice will NOT allow the 7 Ω resistor to be left floating! For both circuits simulated, we observe 987.6 mW of power dissipated for the 5 Ω resistor, confirming our analytic solution.



(c) Neither does. No current flows through the 7 Ω resistor; the 1 Ω resistor is in parallel with a voltage source and hence cannot affect any other part of the circuit.

39. We obtain a $5v_3/4$ A current source in parallel with 4 Ω, and a 3 A current source in parallel with 2 Ω. We now have two dependent current sources in parallel, which may be combined to yield a single $-0.75v_3$ current source (arrow pointing upwards) in parallel with 4 Ω. Selecting the bottom node as a reference terminal, and naming the top left node V_x and the top right node V_y, we write the following equations:

$-0.75v_3 = V_x/4 + (V_x - V_y)/3$
$3 = V_y/2 + (V_y - V_x)/3$
$v_3 = V_y - V_x$

Solving, we find that $v_3 = \boxed{-2\text{ V.}}$

40. (a) $R_{TH} = 25 \parallel (10 + 15) = 25 \parallel 25 = \boxed{12.5\ \Omega.}$

$$V_{TH} = V_{ab} = 50\left(\frac{25}{10+15+25}\right) + 100\left(\frac{15+10}{15+10+25}\right) = \boxed{75\text{ V.}}$$

(b) If $R_{ab} = 50\ \Omega$,

$$P_{50\Omega} = \left[75\left(\frac{50}{50+12.5}\right)\right]^2\left(\frac{1}{50}\right) = \boxed{72\text{ W}}$$

(c) If $R_{ab} = 12.5\ \Omega$,

$$P_{12.5\Omega} = \left[75\left(\frac{12.5}{12.5+12.5}\right)\right]^2\left(\frac{1}{12.5}\right) = \boxed{112.5\text{ W}}$$

41. (a) Shorting the 14 V source, we find that R_{TH} = 10 || 20 + 10 = 16.67 Ω.

Next, we find V_{TH} by determining V_{OC} (recognising that the right-most 10 Ω resistor carries no current, hence we have a simple voltage divider):

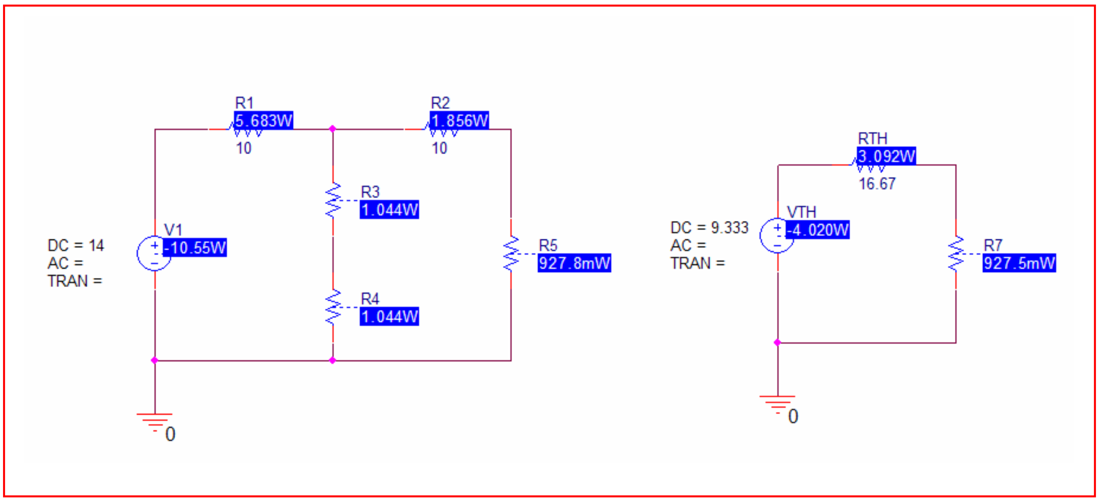
$$V_{TH} = V_{OC} = 14\frac{10+10}{10+10+10} = 9.333 \text{ V}$$

Thus, our Thevenin equivalent is a 9.333 V source in series with a 16.67 Ω resistor, which is in series with the 5 Ω resistor of interest.

(b) $I_{5\Omega}$ = 9.333/ (5 + 16.67) = 0.4307 A. Thus,

$P_{5\Omega} = (0.4307)^2 \cdot 5 = \boxed{927.5 \text{ mW}}$

(c) We see from the PSpice simulation that keeping four significant digits in calculating the Thévenin equivalent yields at least 3 digits' agreement in the results.



42. (a) Replacing the 7 Ω resistor with a short circuit, we find

I_{SC} = 15 (8)/ 10 = 12 A.

Removing the short circuit, and open-circuiting the 15 A source, we see that

R_{TH} = 2 + 8 = 10 Ω.

Thus, $V_{TH} = I_{SC}R_{TH}$ = (12)(10) = 120 V.

Our Thévenin equivalent is therefore a 120 V source in series with 10 Ω.

(b) As found above, $I_N = I_{SC}$ = 12 A, and R_{TH} = 10 Ω.

(c) Using the Thévenin equivalent circuit, we may find v_1 using voltage division:

v_1 = 120 (7)/17 = $\boxed{49.41 \text{ V.}}$

Using the Norton equivalent circuit and a combination of current division and Ohm's law, we find

$$v_1 = 7\left(12\frac{10}{17}\right) = \boxed{49.41 \text{ V}}$$

As expected, the results are equal.

(d) Employing the more convenient Thévenin equivalent model,

v_1 = 120(1)/17 = $\boxed{7.059 \text{ V.}}$

43. (a) $R_{TH} = 10 \text{ mV}/ 400 \ \mu\text{A} = \boxed{25 \ \Omega}$

(b) $R_{TH} = 110 \text{ V}/ 363.6\times10^{-3} \text{ A} = \boxed{302.5 \ \Omega}$

(c) Increased current leads to increased filament temperature, which results in a higher resistance (as measured). This means the Thévenin equivalent must apply to the specific current of a particular circuit – one model is not suitable for all operating conditions (the light bulb is nonlinear).

44. (a) We begin by shorting both voltage sources, and removing the 1 Ω resistor of interest. Looking into the terminals where the 1 Ω resistor had been connected, we see that the 9 Ω resistor is shorted out, so that

$$R_{TH} = (5 + 10) \parallel 10 + 10 = \boxed{16\ \Omega.}$$

To continue, we return to the original circuit and replace the 1 Ω resistor with a short circuit. We define three clockwise mesh currents: i_1 in the left-most mesh, i_2 in the top-right mesh, and isc in the bottom right mesh. Writing our three mesh equations,

$$\begin{array}{l} -4 + 9i_1 - 9i_2 + 3 = 0 \\ -9i_1 + 34i_2 - 10i_{sc} = 0 \\ -3 - 10i_2 + 20i_{sc} = 0 \end{array}$$

Solving using MATLAB:

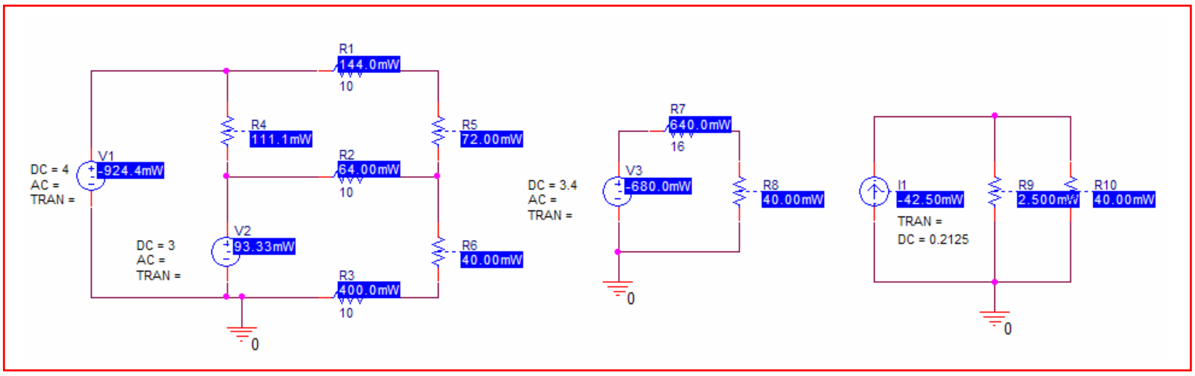
```
>> e1 = '-4 + 9*i1 - 9*i2 + 3 = 0';
>> e2 = '-9*i1 + 34*i2 - 10*isc = 0';
>> e3 = '-3 + 20*isc - 10*i2 = 0';
>> a = solve(e1,e2,e3,'i1','i2','isc');
```

we find i_{sc} = 0.2125 A, so $\boxed{I_N = 212.5 \text{ mA and } V_{TH} = I_N R_{TH} = (0.2125)(16) = 3.4 \text{ V.}}$

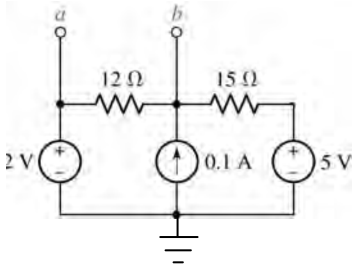
(b) Working with the Thévenin equivalent circuit, $I_{1\Omega} = V_{TH}/(R_{TH} + 1)$ = 200 mA. Thus, $P_{1\Omega} = (0.2)2.1 = \boxed{40 \text{ mW.}}$

Switching to the Norton equivalent, we find I1Ω by current division: $I_{1\Omega}$ = (0.2125)(16)/(16+1) = 200 mA. Once again, $\boxed{P_{1\Omega} = 40 \text{ mW}}$ (as expected).

(c) As we can see from simulating the original circuit simultaneously with its Thevenin and Norton equivalents, the 1 Ω resistor does in fact dissipate 40 mW, and either equivalent is equally applicable. Note all three SOURCES provide a different amount of power in total.



45. (a) Removing terminal **c**, we need write only one nodal equation:



$0.1 = \dfrac{V_b - 2}{12} + \dfrac{V_b - 5}{15}$, which may be solved to yield $V_b = 4$ V. Therefore, $V_{ab} = V_{TH} = 2 - 4 = -2$ V.

$R_{TH} = 12 \parallel 15 = \boxed{6.667\ \Omega}$. We may then calculate I_N as $I_N = V_{TH}/R_{TH}$

$= \boxed{-300 \text{ mA (arrow pointing upwards).}}$

(b) Removing terminal **a**, we again find $\boxed{R_{TH} = 6.667\ \Omega}$, and only need write a single nodal equation; in fact, it is identical to that written for the circuit above, and we once again find that $V_b = 4$ V. In this case, $V_{TH} = V_{bc} = 4 - 5 = -1$ V, so $I_N = -1/6.667$ $= \boxed{-150 \text{ mA (arrow pointing upwards).}}$

46. (a) Shorting out the 88-V source and open-circuiting the 1-A source, we see looking into the terminals x and x' a 50-Ω resistor in parallel with 10 Ω in parallel with (20 Ω + 40 Ω), so

$$R_{TH} = 50 \parallel 10 \parallel (20 + 40) = \boxed{7.317\ \Omega}$$

Using superposition to determine the voltage $V_{xx'}$ across the 50-Ω resistor, we find

$$V_{xx'} = V_{TH} = \left[88\frac{50 \parallel (20+40)}{10+[50 \parallel (20+40)]}\right] + (1)(50 \parallel 10)\left[\frac{40}{40+20+(50 \parallel 10)}\right]$$

$$= \left[88\frac{27.27}{37.27}\right] + (1)(8.333)\left[\frac{40}{40+20+8.333}\right] = \boxed{69.27\ \text{V}}$$

(b) Shorting out the 88-V source and open-circuiting the 1-A source, we see looking into the terminals y and y' a 40-Ω resistor in parallel with [20 Ω + (10 Ω || 50 Ω)]:

$$R_{TH} = 40 \parallel [20 + (10 \parallel 50)] = \boxed{16.59\ \Omega}$$

Using superposition to determine the voltage $V_{yy'}$ across the 1-A source, we find

$$V_{yy'} = V_{TH} = (1)(R_{TH}) + \left[88\frac{27.27}{10+27.27}\right]\left(\frac{40}{20+40}\right)$$

$$= \boxed{59.52\ \text{V}}$$

47. (a) Select terminal **b** as the reference terminal, and define a nodal voltage V_1 at the top of the 200-Ω resistor. Then,

$$0 = \frac{V_1 - 20}{40} + \frac{V_1 - V_{TH}}{100} + \frac{V_1}{200} \qquad [1]$$

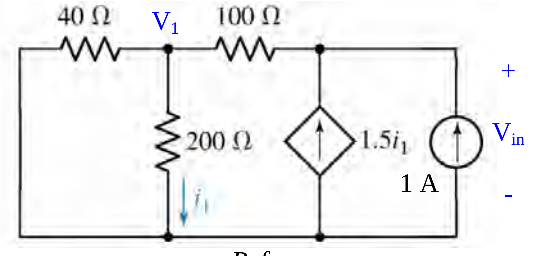
$$1.5\, i_1 = (V_{TH} - V_1)/\, 100 \qquad [2]$$

where $i_1 = V_1/\,200$, so Eq. [2] becomes $150\, V_1/\,200 + V_1 - V_{TH} = 0$ [2]

Simplifying and collecting terms, these equations may be re-written as:

$$(0.25 + 0.1 + 0.05)\, V_1 - 0.1\, V_{TH} = 5 \qquad [1]$$
$$(1 + 15/\,20)\, V_1 - V_{TH} = 0 \qquad [2]$$

Solving, we find that $\boxed{V_{TH} = 38.89\text{ V}}$. To find R_{TH}, we short the voltage source and inject 1 A into the port:



$$0 = \frac{V_1 - V_{in}}{100} + \frac{V_1}{40} + \frac{V_1}{200} \qquad [1]$$

$$1.5\, i_1 + 1 = \frac{V_{in} - V_1}{100} \qquad [2]$$

$$i_1 = V_1/\,200 \qquad [3]$$

Combining Eqs. [2] and [3] yields $1.75\, V_1 - V_{in} = -100$ [4]

Solving Eqs. [1] & [4] then results in $V_{in} = 177.8$ V, so that $R_{TH} = V_{in}/\, 1\text{ A} = \boxed{177.8\ \Omega.}$

(b) Adding a 100-Ω load to the original circuit or our Thévenin equivalent, the voltage across the load is

$$V_{100\Omega} = V_{TH}\left(\frac{100}{100 + 177.8}\right) = 14.00\text{ V}, \text{ and so } P_{100\Omega} = (V_{100\Omega})^2 /\, 100 = \boxed{1.96\text{ W.}}$$

48. We inject a current of 1 A into the port (arrow pointing up), select the bottom terminal as our reference terminal, and define the nodal voltage V_x across the 200-Ω resistor.

Then,

$$1 = V_1/100 + (V_1 - V_x)/50 \qquad [1]$$
$$-0.1\,V_1 = V_x/200 + (V_x - V_1)/50 \qquad [2]$$

which may be simplified to

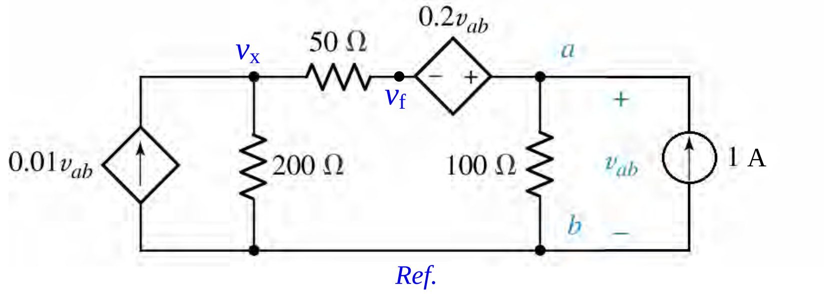
$$3\,V_1 - 2\,V_x = 100 \qquad [1]$$
$$16\,V_1 + 5\,V_x = 0 \qquad [2]$$

Solving, we find that V_1 = 10.64 V, so $R_{TH} = V_1/(1\text{ A}) = \boxed{10.64\ \Omega.}$

Since there are no independent sources present in the original network, $\boxed{I_N = 0.}$

49. With no independent sources present, $\boxed{V_{TH} = 0.}$

We decide to inject a 1-A current into the port:



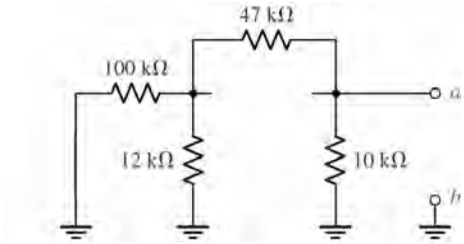
Node '**x**': $0.01\, v_{ab} = v_x/ 200 + (v_x - v_f)/ 50$ [1]
Supernode: $1 = v_{ab}/ 100 + (v_f - v_x)$ [2]
and: $v_{ab} - v_f = 0.2\, v_{ab}$ [3]

Rearranging and collecting terms,

$$-2\, v_{ab} + 5\, v_x - 4\, v_f = 0 \quad [1]$$
$$v_{ab} - 2\, v_x + 2\, v_f = 100 \quad [2]$$
$$0.8\, v_{ab} \quad - v_f = 0 \quad [3]$$

Solving, we find that v_{ab} = 192.3 V, so $R_{TH} = v_{ab}/ (1\text{ A}) = \boxed{192.3\ \Omega.}$

50. We first find R_{TH} by shorting out the voltage source and open-circuiting the current source.



Looking into the terminals **a** & **b**, we see $R_{TH} = 10 \,||\, [47 + (100 \,||\, 12)]$ $= \boxed{8.523\ \Omega.}$

Returning to the original circuit, we decide to perform nodal analysis to obtain V_{TH}:

$$-12\times10^3 = (V_1 - 12)/\,100\times10^3 + V_1/\,12\times10^3 + (V_1 - V_{TH})/\,47\times10^3 \qquad [1]$$

$$12\times10^3 = V_{TH}/\,10\times10^3 + (V_{TH} - V_1)/\,47\times10^3 \qquad [2]$$

Rearranging and collecting terms,

$$0.1146\ V_1 - 0.02128\ V_{TH} = -11.88 \quad [1]$$
$$-0.02128\ V_1 + 0.02128\ V_{TH} = 12 \quad [2]$$

Solving, we find that $V_{TH} = \boxed{83.48\text{ V.}}$

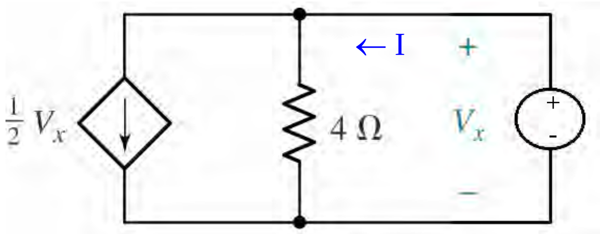
51. (a) $R_{TH} = 4 + 2 \parallel 2 + 10 = \boxed{15\ \Omega.}$
(b) same as above: $\boxed{15\ \Omega.}$

52. For Fig. 5.78a, $I_N = 12/\sim 0 \rightarrow$ $\boxed{\infty \text{ A in parallel with} \sim 0\ \Omega.}$

For Fig. 5.78b, $V_{TH} = (2)(\sim\infty) \rightarrow$ $\boxed{\infty \text{ V in series with} \sim\infty\ \Omega.}$

53. With no independent sources present, $\boxed{V_{TH} = 0.}$

Connecting a 1-V source to the port and measuring the current that flows as a result,



I = 0.5 V_x + 0.25 V_x = 0.5 + 0.25 = 0.75 A.

R_{TH} = 1/ I = 1.333 Ω.

The Norton equivalent is 0 A in parallel with 1.333 Ω.

54. Performing nodal analysis to determine V_{TH},

$$100\times10^{-3} = V_x/250 + V_{oc}/7.5\times10^3 \qquad [1]$$

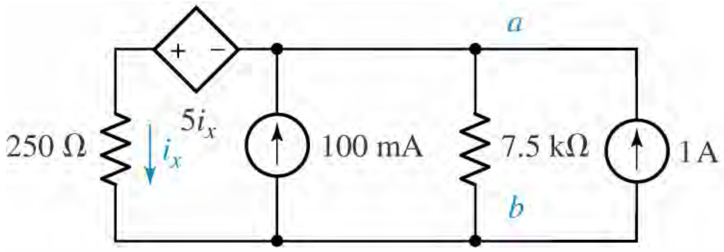
and $V_x - V_{oc} = 5\, i_x$

where $i_x = V_x/250$. Thus, we may write the second equation as

$$0.98\, V_x - V_{oc} = 0 \qquad [2]$$

Solving, we find that $V_{oc} = \boxed{V_{TH} = 23.72 \text{ V.}}$

In order to determine R_{TH}, we inject 1 A into the port:



$$V_{ab}/7.5\times10^3 + V_x/250 = 1 \qquad [1]$$

and $V_x - V_{ab} = 5\, i_x = 5V_x/250$ or

$$-V_{ab} + (1 - 5/250)\, V_x = 0 \qquad [2]$$

Solving, we find that $V_{ab} = 237.2$ V. Since $R_{TH} = V_{ab}/(1\text{ A})$, $\boxed{R_{TH} = 237.2\ \Omega.}$

Finally, $I_N = V_{TH}/R_{TH} = \boxed{100 \text{ mA.}}$

55. We first note that $V_{TH} = V_x$, so performing nodal analysis,

$$-5\,V_x = V_x/\,19 \qquad \text{which has the solution } V_x = 0\text{ V}.$$

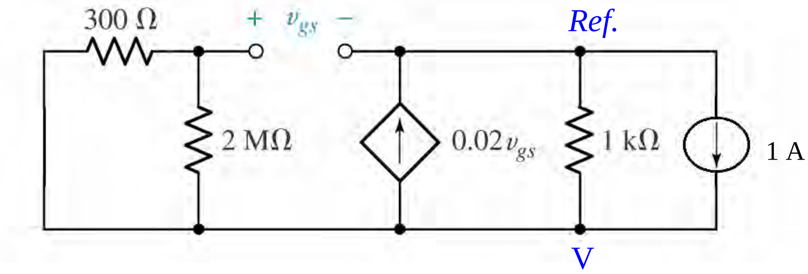
Thus, $\boxed{V_{TH} \text{ (and hence } I_N) = 0.}$ (Assuming $R_{TH} \neq 0$)

To find R_{TH}, we inject 1 A into the port, noting that $R_{TH} = V_x/\,1$ A:

$$-5\,V_x + 1 = V_x/\,19$$

Solving, we find that $V_x = 197.9$ mV, so that $\boxed{R_{TH} = R_N = 197.9\text{ m}\Omega.}$

56. Shorting out the voltage source, we redraw the circuit with a 1-A source in place of the 2-kΩ resistor:



Noting that 300 Ω || 2 MΩ ≈ 300 Ω,

$$0 = (v_{gs} - V)/300 \qquad [1]$$

$$1 - 0.02\,v_{gs} = V/1000 + (V - v_{gs})/300 \qquad [2]$$

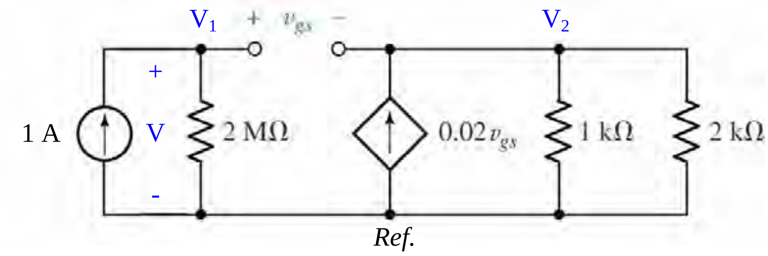
Simplifying & collecting terms,

$$v_{gs} - V = 0 \qquad [1]$$

$$0.01667\,v_{gs} + 0.00433\,V = 1 \qquad [2]$$

Solving, we find that v_{gs} = V = 47.62 V. Hence, R_{TH} = V/ 1 A = $\boxed{47.62\ \Omega.}$

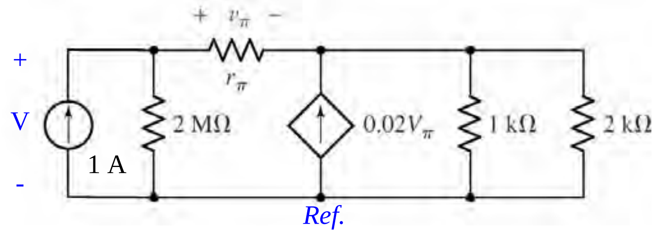
57. We replace the source v_s and the 300-Ω resistor with a 1-A source and seek its voltage:



By nodal analysis, $1 = V_1 / 2\times10^6$ so $V_1 = 2\times10^6$ V.

Since $V = V_1$, we have $R_{in} = V / 1\text{ A} = \boxed{2\text{ M}\Omega.}$

58. Removing the voltage source and the 300-Ω resistor, we replace them with a 1-A source and seek the voltage that develops across its terminals:



We select the bottom node as our reference terminal, and define nodal voltages V_1 and V_2. Then,

$$1 = V_1 / 2\times10^6 + (V_1 - V_2)/ r_\pi \qquad [1]$$

$$0.02\, v_\pi = (V_2 - V_1)/ r_\pi + V_2/ 1000 + V_2/ 2000 \qquad [2]$$

where $v_\pi = V_1 - V_2$

Simplifying & collecting terms,

$$(2\times10^6 + r_\pi)\, V_1 - 2\times10^6\, V_2 = 2\times10^6\, r_\pi \qquad [1]$$

$$-(2000 + 40\, r_\pi)\, V_1 + (2000 + 43\, r_\pi)\, V_2 = 0 \quad [2]$$

Solving, we find that $V_1 = V = 2\times10^6\left(\dfrac{666.7+14.33\,r_\pi}{2\times10^6+666.7+14.33\,r_\pi}\right)$.

Thus, $\boxed{R_{TH} = 2\times10^6 \,\|\, (666.7 + 14.33\, r_\pi)\ \Omega.}$

59. (a) We first determine v_{out} in terms of v_{in} and the resistor values only; in this case, $V_{TH} = v_{out}$. Performing nodal analysis, we write two equations:

$$0 = \frac{-v_d}{R_i} + \frac{(-v_d - v_{in})}{R_1} + \frac{(-v_d - v_o)}{R_f} \quad [1] \quad \text{and} \quad 0 = \frac{(v_o + v_d)}{R_f} + \frac{(v_o - Av_d)}{R_o} \quad [2]$$

Solving using MATLAB, we obtain:

```
>> e1 = 'vd/Ri + (vd + vin)/R1 + (vd + vo)/Rf = 0';
>> e2 = '(vo + vd)/Rf + (vo – A*vd)/Ro = 0';
>> a = solve(e1,e2,'vo','vd');
>> pretty(a.vo)

              Ri vin (–Ro + Rf A)
 – ----------------------------------------------
   R1 Ro + Ri Ro + R1 Rf + Ri Rf + R1 Ri + A R1 Ri
```

Thus, $\boxed{V_{TH} = \frac{v_{in} R_i (R_o - AR_f)}{R_1 R_o + R_i R_o + R_1 R_f + R_i R_f + R_1 R_i + AR_1 R_i}}$, which in the limit of A → ∞, approaches $-R_f/R_1$.

To find R_{TH}, we short out the independent source v_{in}, and squirt 1 A into the terminal marked v_{out}, renamed V_T. Analyzing the resulting circuit, we write two nodal equations:

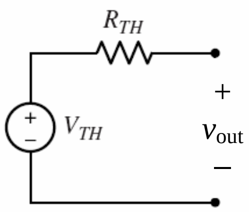
$$0 = \frac{-v_d}{R_i} - \frac{v_d}{R_1} + \frac{(-v_d - v_T)}{R_f} \quad [1] \quad \text{and} \quad 1 = \frac{(v_T + v_d)}{R_f} + \frac{(v_T - Av_d)}{R_o} \quad [2]$$

Solving using MATLAB:

```
>> e1 = 'vd/R1 + vd/Ri + (vd + VT)/Rf = 0';
>> e2 = '1 = (VT + vd)/Rf + (VT - A*vd)/Ro';
>> a = solve(e1,e2,'vd','VT');
>> pretty(a.VT)

         Ro (Ri Rf + R1 Rf + R1 Ri)
 -------------------------------------------------
 Ri Ro + R1 Ro + Ri Rf + R1 Rf + R1 Ri + A R1 Ri
```

Since $V_T/1 = V_T$, this is our Thévenin equivalent resistance (R_{TH}).

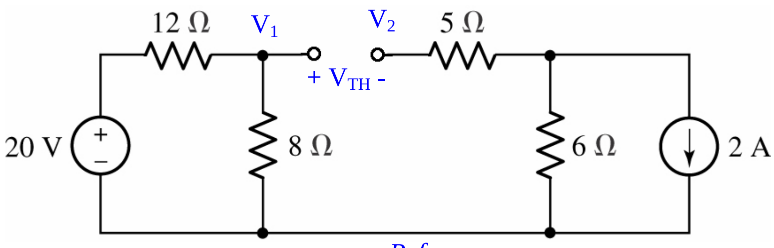


60. Such a scheme probably would lead to maximum or at least near-maximum power transfer to our home. Since we pay the utility company based on the power we use, however, this might not be such a hot idea…

61. We need to find the Thévenin equivalent resistance of the circuit connected to R_L, so we short the 20-V source and open-circuit the 2-A source; by inspection, then

$$R_{TH} = 12 \parallel 8 \quad + \quad 5 \quad + 6 = \boxed{15.8\ \Omega}$$

Analyzing the original circuit to obtain V_1 and V_2 with R_L removed:



$V_1 = 20\ 8/\ 20 = 8$ V; $V_2 = -2\ (6) = -12$ V.

We define $V_{TH} = V_1 - V_2 = 8 + 12 = 20$ V. Then,

$$P_{R_L}\Big|_{max} = \frac{V_{TH}^2}{4\,R_L} = \frac{400}{4(15.8)} = \boxed{6.329\text{ W}}$$

62. (a) $R_{TH} = 25 \parallel (10 + 15) = \boxed{12.5\ \Omega}$

Using superposition, $V_{ab} = V_{TH} = 50\dfrac{25}{15+10+25} + 100\dfrac{15+10}{50} = \boxed{75\text{ V.}}$

(b) Connecting a 50-Ω resistor,

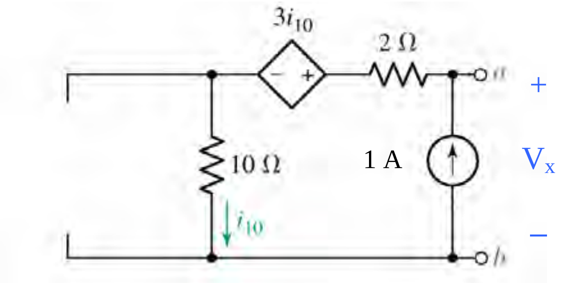
$$P_{load} = \frac{V_{TH}^2}{R_{TH} + R_{load}} = \frac{75^2}{12.5+50} = \boxed{90\text{ W}}$$

(c) Connecting a 12.5-Ω resistor,

$$P_{load} = \frac{V_{TH}^2}{4\,R_{TH}} = \frac{75^2}{4(12.5)} = \boxed{112.5\text{ W}}$$

63. (a) By inspection, we see that $i_{10} = 5$ A, so
$V_{TH} = V_{ab} = 2(0) + 3\, i_{10} + 10\, i_{10} = 13\, i_{10} = 13(5) = \boxed{65 \text{ V.}}$

To find R_{TH}, we open-circuit the 5-A source, and connect a 1-A source between terminals **a** & **b**:



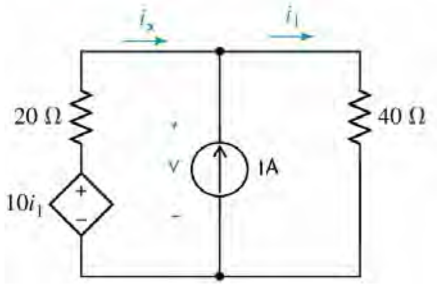
A simple KVL equation yields $V_x = 2(1) + 3i_{10} + 10\, i_{10}$.
Since $i_{10} = 1$ A in this circuit, $V_x = 15$ V.

We thus find the Thevenin equivalent resistance is $15/1 = \boxed{15\ \Omega.}$

(b) $$P_{max} = \frac{V_{TH}^2}{4\, R_{TH}} = \frac{65^2}{4(15)} = \boxed{70.42 \text{ W}}$$

64.

(a) Replacing the resistor R_L with a 1-A source, we seek the voltage that develops across its terminals with the independent voltage source shorted:



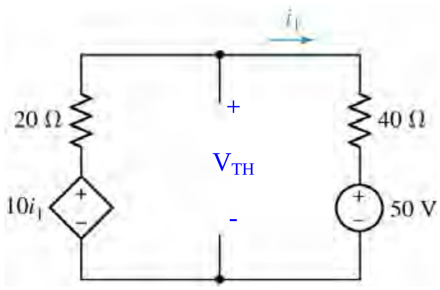
$$-10i_1 + 20i_x + 40i_1 = 0 \quad [1] \Rightarrow 30i_1 + 20i_x = 0 \ [1]$$

and $i_1 - i_x = 1$ $\quad [2] \Rightarrow \quad i_1 - i_x = 1 \quad [2]$

Solving, $i_1 = 400\,\text{mA}$

So $V = 40i_1 = 16\,\text{V}$ and $R_{TH} = \dfrac{V}{1\text{A}} = \boxed{16\,\Omega}$

(b) Removing the resistor R_L from the original circuit, we seek the resulting open-circuit voltage:



$$0 = \frac{V_{TH} - 10i_1}{20} + \frac{V_{TH} - 50}{40} \qquad [1]$$

where $i_1 = \dfrac{V_{TH} - 50}{40}$

so [1] becomes $0 = \dfrac{V_{TH}}{20} - \dfrac{1}{2}\left(\dfrac{V_{TH} - 50}{40}\right) + \left(\dfrac{V_{TH} - 50}{40}\right)$

$$0 = \frac{V_{TH}}{20} + \frac{V_{TH} - 50}{80}$$

$$0 = 4V_{TH} + V_{TH} - 50$$

$$5V_{TH} = 50$$

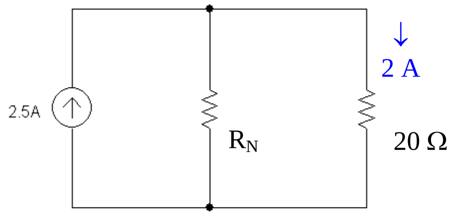
or $V_{TH} = 10\,\text{V}$

Thus, if $\quad R_L = R_{TH} = 16\,\Omega,$

$$V_{R_L} = V_{TH}\frac{R_L}{R_L + R_{TH}} = \frac{V_{TH}}{2} = \boxed{5\,\text{V}}$$

65.

(a) $I_N = 2.5\,\text{A}$



$$20i^2 = 80$$
$$i = 2\,\text{A}$$

By current division,

$$2 = 2.5\frac{R_N}{R_N + 20}$$

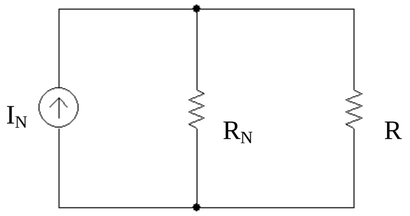
Solving, $R_N = R_{TH} = 80\,\Omega$

Thus, $V_{TH} = V_{OC} = 2.5 \times 80 = \boxed{200\,\text{V}}$

(b) $P_{\max} = \dfrac{V_{TH}{}^2}{4R_{TH}} = \dfrac{200^2}{4 \times 80} = \boxed{125\,\text{W}}$

(c) $R_L = R_{TH} = \boxed{80\,\Omega}$

66.



$10\,\mathrm{W}$ to 250Ω corresp to $200\,\mathrm{mA}$.
$20\,\mathrm{W}$ to 80Ω corresp to $500\,\mathrm{mA}$.

By Voltage ÷, $I_R = I_N \dfrac{R_N}{R+R_N}$

So $0.2 = I_N \dfrac{R_N}{250+R_N}$ [1]

$0.5 = I_N \dfrac{R_N}{80+R_N}$ [2]

Solving, $I_N = 1.7\,\mathrm{A}$ and $R_N = 33.33\Omega$

(a) If $v_L i_L$ is a maximum,

$R_L = R_N = \boxed{33.33\Omega}$

$i_L = 1.7 \times \dfrac{33.33}{33.33+33.33} \boxed{= 850\,\mathrm{mA}}$

$v_L = 33.33 i_L = \boxed{28.33\,\mathrm{V}}$

(b) If v_L is a maximum

$V_L = I_N (R_N \| R_L)$

So v_L is a maximum when $R_N \| R_L$ is a maximum, which occurs at $\boxed{R_L = \infty.}$

Then $\boxed{i_L = 0}$ and $v_L = 1.7 \times R_N = \boxed{56.66\,\mathrm{V}}$

(c) If i_L is a maximum

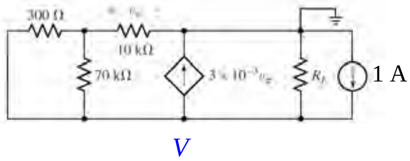
$i_L = i_N \dfrac{R_N}{R_N + R_L}$; max when $\boxed{R_L = 0\Omega}$

So $\boxed{i_L = 1.7\mathrm{A}}$

$\boxed{v_L = 0\,\mathrm{V}}$

67. There is no conflict with our derivation concerning maximum power. While a dead short across the battery terminals will indeed result in maximum current draw from the battery, and power is indeed proportional to i^2, the power delivered to the load is $i^2R_{LOAD} = i^2(0) = 0$ watts. This is the *minimum*, not the maximum, power that the battery can deliver to a load.

68. Remove R_E: $R_{TH} = R_E \| R_{in}$



bottom node: $1 - 3\times10^{-3} v_\pi = \dfrac{V - v_\pi}{300} + \dfrac{V - v_\pi}{70\times10^3}$ [1]

at other node: $0 = \dfrac{v_\pi}{10\times10^3} + \dfrac{v_\pi - V}{300} + \dfrac{v_\pi - V}{70\times10^3}$ [2]

Simplifying and collecting terms,

$210\times10^5 = 70\times10^3\, V + 300V + 63000\, v_\pi - 70\text{x}10^3\, v_\pi - 300\, v_\pi$

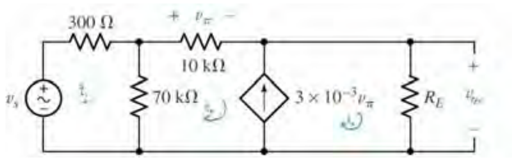
or $70.3\times10^3 V - 7300 v_\pi = 210\times10^5$ [1]

$0 = 2100 v_\pi + 70\times10^3 v_\pi - 70\times10^3 V + 300 v_\pi - 300V$

or $-69.7\times10^3 V + 72.4\times10^3 v_\pi = 0$ [2]

solving, $V = 331.9V$ So $R_{TH} = R_E \| 331.9\Omega$

Next, we determine v_{TH} using mesh analysis:



$-v_s + 70.3\times10^3 i_1 - 70\times10^3 i_2 = 0$ [1]

$80\times10^3 i_2 - 70\times10^3 i_1 + R_E i_3 = 0$ [2]

and: $i_3 - i_2 = 3\times10^{-3} v_\pi$ [3]

or $i_3 - i_2 = 3\times10^{-3}(10\times10^3) i_2$

or $i_3 - i_2 = 30 i_2$

or

$-31 i_2 + i_3 = 0$ [3]

$$\text{Solving :} \begin{bmatrix} 70.3\times10^3 & -70\times10^3 & 0 \\ -70\times10^3 & 80\times10^3 & R_E \\ 0 & -31 & 1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} v_s \\ 0 \\ 0 \end{bmatrix}$$

We seek i_3:

$$i_3 = \frac{-21.7\times10^3 v_s}{7.24\times10^6 + 21.79\times10^3 R_E}$$

$$\text{So} \quad V_{OC} = V_{TH} = R_E\, i_3 = \frac{-21.7\times10^3 R_E}{7.24\times10^6 + 21.79\times10^3 R_E} v_s$$

$$P_{8\Omega} = 8\left[\frac{V_{TH}}{R_{TH} + 8}\right]^2 = \left[\frac{-21.7\times10^3 R_E}{7.24\times10^6 + 21.79\times10^3 R_E}\right]^2 \frac{8\ vs^2}{\left[\dfrac{331.9\, R_E}{331.9 + R_E}\right]^2}$$

$$= \frac{11.35\times10^6 (331.9 + R_E)^2}{(7.24\times10^6 + 21.79\times10^3 R_E)^2} vs^2$$

$\boxed{\text{This is maximized by setting } R_E = \infty.}$

69. Thévenize the left-hand network, assigning the nodal voltage V_x at the free end of right-most 1-kΩ resistor.

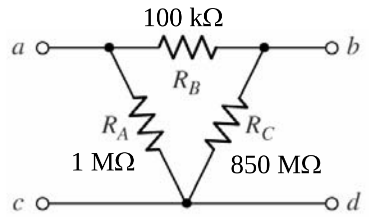
A single nodal equation: $40\times 10^{-3} = \dfrac{V_x\big|_{oc}}{7\times 10^{3}}$

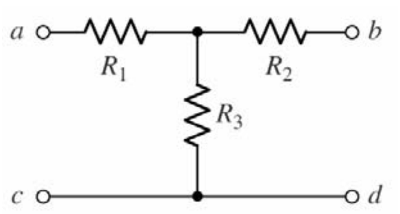
So $V_{TH} = V_x\big|_{oc} = 280\,\text{V}$

R_{TH} = 1 k + 7 k = 8 kΩ

$\boxed{\text{Select } R_1 = R_{TH} = 8\text{ k}\Omega.}$

70.





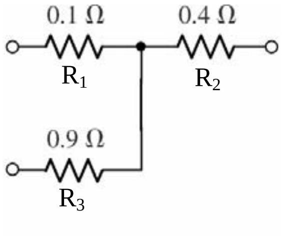
$$D = R_A + R_B + R_C = 1 + 850 + 0.1 = 851.1 \times 10^6$$

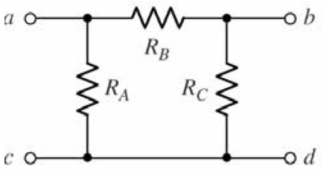
$$R_1 = \frac{R_A R_B}{D} = \frac{10^6 \times 10^5}{D} = \boxed{117.5\Omega}$$

$$R_2 = \frac{R_B R_C}{D} = \frac{10^5 \times 850 \times 10^6}{851.1 \times 10^6} = \boxed{99.87\,k\Omega}$$

$$R_3 = \frac{R_C R_A}{D} = \frac{850 \times 10^6 \times 10^6}{851.1 \times 10^6} = \boxed{998.7\,k\Omega}$$

71.





$$
\begin{aligned}
N &= R_1R_2 + R_2R_3 + R_3R_1 \\
&= 0.1\times 0.4 + 0.4\times 0.9 + 0.9\times 0.1 \\
&= 0.49\Omega
\end{aligned}
$$

$$R_A = \frac{N}{R_2} = \boxed{1.225\,\Omega}$$

$$R_B = \frac{N}{R_3} = \boxed{544.4\,\text{m}\Omega}$$

$$R_C = \frac{N}{R_1} = \boxed{4.9\,\Omega}$$

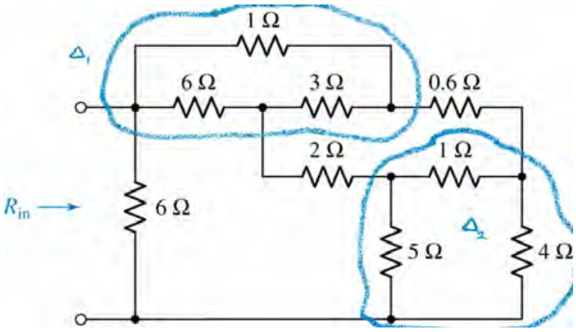
72.

$$\Delta_1 : 1+6+3=10\Omega$$

$$\frac{6\times1}{10}=0.6, \frac{6\times3}{10}=1.8 , \frac{3\times1}{10}=0.3$$

$$\Delta_2 : 5+1+4=10\Omega$$

$$\frac{5\times1}{10}=0.5 , \frac{1\times4}{10}=0.4 , \frac{5\times4}{10}=2$$



$$1.8+2+0.5=4.3\Omega$$

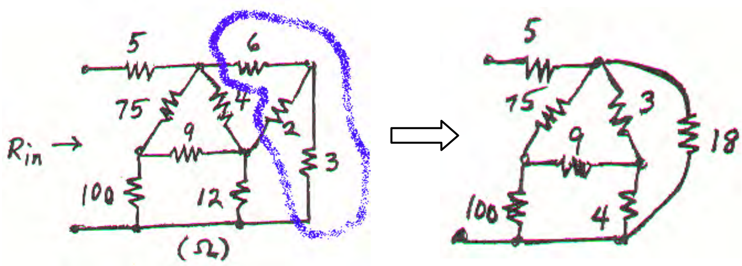
$$0.3+0.6+0.4=1.3\Omega$$

$$1.3\|4.3=0.9982\Omega$$

$$0.9982+0.6+2=3.598\Omega$$

$$3.598\|6=\boxed{2.249\Omega}$$

73.

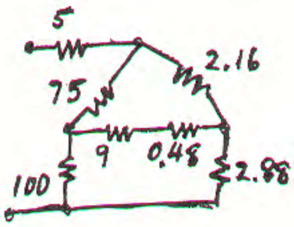


$6\times2+2\times3+3\times6=36\Omega^2$

$\dfrac{36}{6}=6\Omega,\ \dfrac{36}{2}=18\Omega,\ \dfrac{36}{3}=12\Omega$

$12\|4=3\Omega,\ 6\|12\Omega=4\Omega$

$4+3+18=25\Omega$

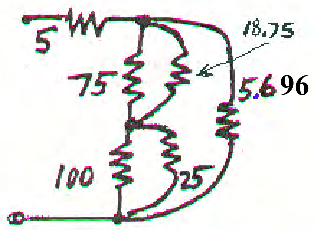


$3\times\dfrac{18}{25}=2.16\Omega$

$4\times\dfrac{18}{25}=2.88\Omega$

$4\times\dfrac{3}{25}=0.48\Omega$

$9.48\times2.16+9.48\times2.88+2.88\times2.16=54\Omega^2$



$\dfrac{54}{2.88}=18.75\Omega \qquad \dfrac{54}{9.48}=5.696\Omega$

$\dfrac{54}{2.16}=25\Omega$

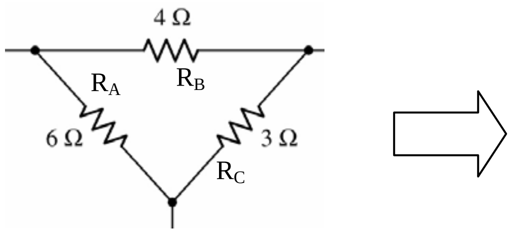
$75\|18.75=15\Omega$

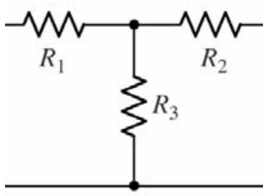
$100\|25=20\Omega$

$(15+20)\|5.696=4.899\Omega$

$\therefore R_{in}=5+4.899=\boxed{9.899\Omega}$

74. We begin by converting the Δ-connected network consisting of the 4-, 6-, and 3-Ω resistors to an equivalent Y-connected network:





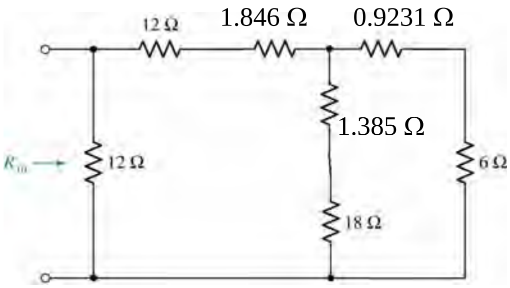
$$D = 6 + 4 + 3 = 13\Omega$$

$$R_1 = \frac{R_A R_B}{D} = \frac{6\times 4}{13} = 1.846\,\Omega$$

$$R_2 = \frac{R_B R_C}{D} = \frac{4\times 3}{13} = 923.1\,\text{m}\Omega$$

$$R_3 = \frac{R_C R_A}{D} = \frac{3\times 6}{13} = 1.385\,\Omega$$

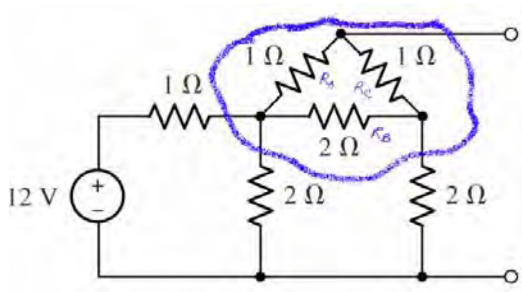
Then network becomes:



Then we may write

$$R_{in} = 12 \| [13.846 + (19.385 \| 6.9231)]$$
$$= \boxed{7.347\Omega}$$

75.

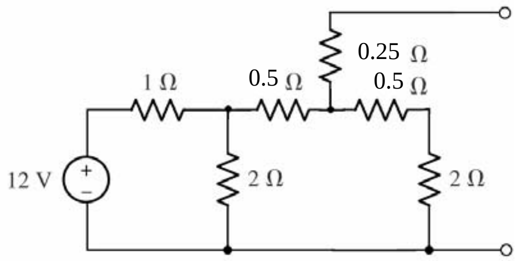


$$1+1+2=4\Omega$$

$$R_1=\frac{1\times 2}{4}=\frac{1}{2}\,\Omega$$

$$R_2=\frac{2\times 1}{4}=\frac{1}{2}\,\Omega$$

$$R_3=\frac{1\times 1}{4}=0.25\Omega$$



Next, we convert the Y-connected network on the left to a Δ-connected network:

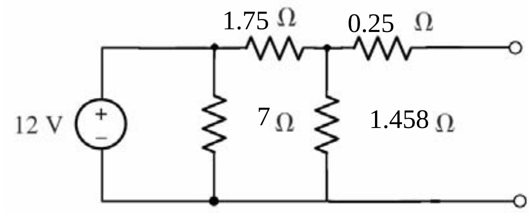
$$1\times 0.5+0.5\times 2+2\times 1=3.5\Omega^2$$

$$R_A=\frac{3.5}{0.5}=7\Omega$$

$$R_B=\frac{3.5}{2}=1.75\Omega$$

$$R_C=\frac{3.5}{1}=3.5\Omega$$

After this procedure, we have a 3.5-Ω resistor in parallel with the 2.5-Ω resistor. Replacing them with a 1.458-Ω resistor, we may redraw the circuit:



This circuit may be easily analysed to find:

$$V_{oc}=\frac{12\times 1.458}{1.75+1.458}=\boxed{5.454\,\text{V}}$$

$$R_{TH}=0.25+1.458\|1.75$$

$$\boxed{=1.045\Omega}$$

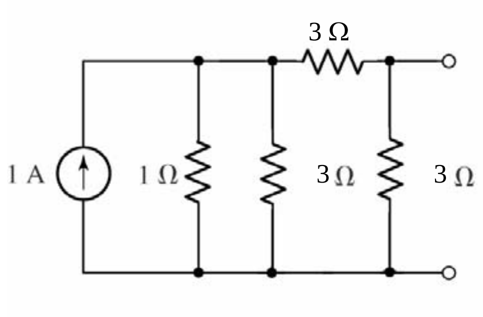
76. We begin by converting the Y-network to a Δ-connected network:

$$N = 1.1 + 1.1 + 1.1 = 3\Omega^2$$

$$R_A = \frac{3}{1} = 3\Omega$$

$$R_B = \frac{3}{1} = 3\Omega$$

$$R_C = \frac{3}{1} = 3\Omega$$



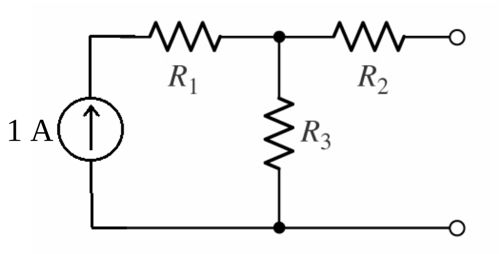
Next, we note that $1 \| 3 = 0.75\,\Omega$, and hence have a simple Δ-network. This is easily converted to a Y-connected network:

$$0.75 + 3 + 3 = 6.75\Omega$$

$$R_1 = \frac{0.75 \times 3}{6.75} = 0.3333\Omega$$

$$R_2 = \frac{3 \times 3}{6.75} = 1.333\Omega$$

$$R_3 = \frac{3 \times 0.75}{6.75} = 0.3333\Omega$$

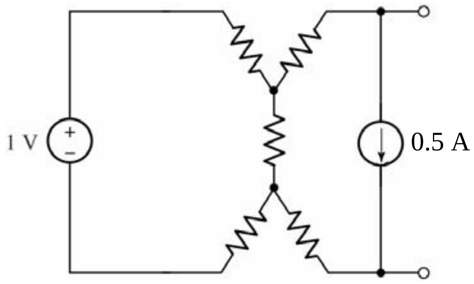


Analysing this final circuit,

$$R_N = 1.333 + 0.3333$$
$$\boxed{= 1.667\Omega}$$

$$I_N = I_{SC} = 1 \times \frac{1/3}{1/3 + 1 + 1/3}$$
$$= \frac{1}{1+3+1} = \frac{1}{5}$$
$$= 0.2\,\text{A}$$
$$\boxed{= 200\,\text{mA}}$$

77. Since 1 V appears across the resistor associated with I_1, we know that I_1 = 1 V/ 10 Ω = 100 mA. From the perspective of the open terminals, the 10-Ω resistor in parallel with the voltage source has no influence if we replace the "dependent" source with a fixed 0.5-A source:



Then, we may write:

$$-1 + (10 + 10 + 10)\, i_a - 10\,(0.5) = 0$$

so that i_a = 200 mA.

We next find that $V_{TH} = V_{ab} = 10(-0.5) + 10(i_a - 0.5) + 10(-0.5) = \boxed{-13 \text{ V.}}$

To determine R_{TH}, we first recognise that with the 1-V source shorted, $I_1 = 0$ and hence the dependent current source is dead. Thus, we may write R_{TH} from inspection:

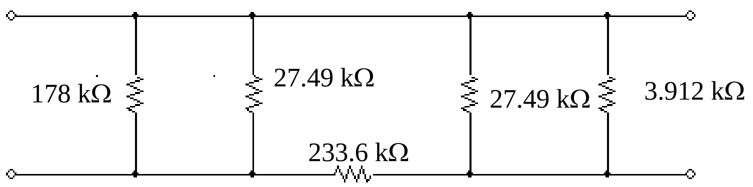
$$R_{TH} = 10 + 10 + 10 \,||\, 20 = \boxed{26.67\ \Omega.}$$

78. (a) We begin by splitting the 1-kΩ resistor into two 500-Ω resistors in series. We then have two related Y-connected networks, each with a 500-Ω resistor as a leg. Converting those networks into Δ-connected networks,

$$\Sigma = (17)(10) + (1)(4) + (4)(17) = 89\times106\ \Omega^2$$

$$89/0.5 = 178\ \text{k}\Omega; \qquad 89/\ 17 = 5.236\ \text{k}\Omega; \qquad 89/4 = 22.25\ \text{k}\Omega$$

Following this conversion, we find that we have two 5.235 kΩ resistors in parallel, and a 178-kΩ resistor in parallel with the 4-kΩ resistor. Noting that 5.235 k || 5.235 k = 2.618 kΩ and 178 k || 4 k = 3.912 kΩ, we may draw the circuit as:



We next attack the Y-connected network in the center:

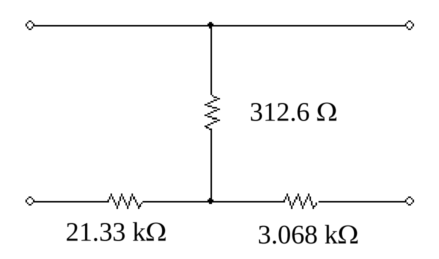
$$\Sigma = (22.25)(22.25) + (22.25)(2.618) + (2.618)(22.25) = 611.6\times106\ \Omega^2$$

$$611.6/\ 22.25 = 27.49\ \text{k}\Omega;\ 611.6/\ 2.618 = 233.6\ \text{k}\Omega$$

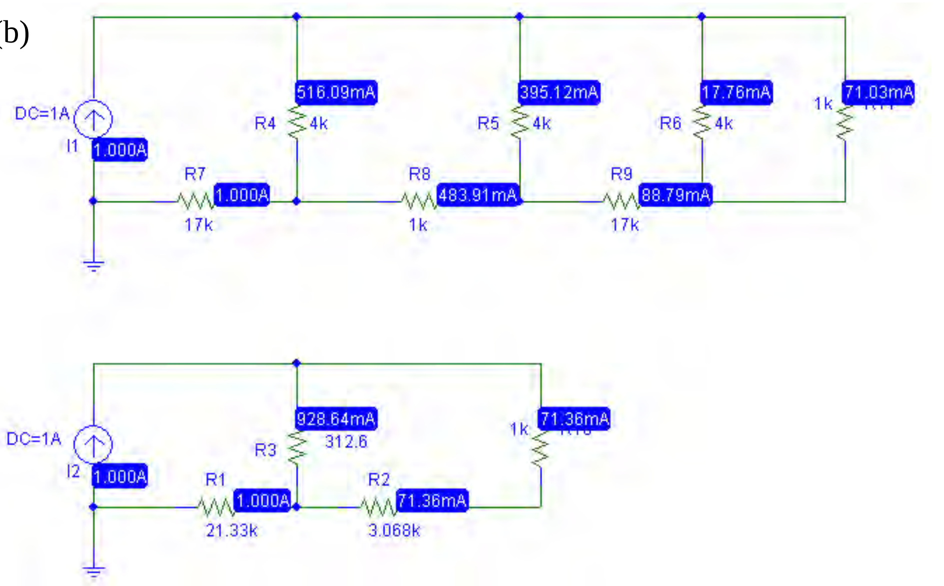
Noting that 178 k || 27.49 k = 23.81 kΩ and 27.49 || 3.912 = 3.425 kΩ, we are left with a simple Δ-connected network. To convert this to the requested Y-network,

$$\Sigma = 23.81 + 233.6 + 3.425 = 260.8\ \text{k}\Omega$$

$$(23.81)(233.6)/\ 260.8 = 21.33\ \text{k}\Omega$$
$$(233.6)(3.425)/\ 260.8 = 3.068\ \text{k}\Omega$$
$$(3.425)(23.81)/\ 260.8 = 312.6\ \Omega$$



(b)



79. (a) Although this network may be simplified, it is not possible to replace it with a three-resistor equivalent.

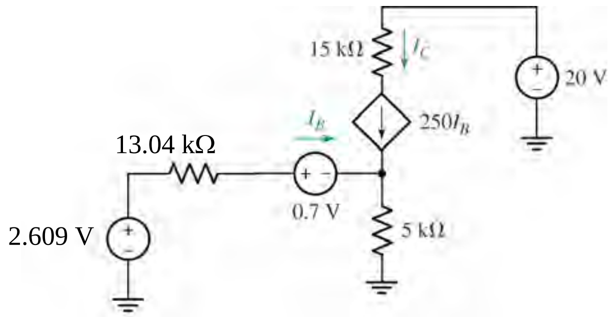
(b) See (a).

80. First, replace network to left of the 0.7-V source with its Thévenin equivalent:

$$V_{TH} = 20 \times \frac{15}{100+15} = 2.609\,\text{V}$$

$$R_{TH} = 100k \| 15k = 13.04\,k\Omega$$

Redraw:



Analysing the new circuit to find I_B, we note that $I_C = 250\, I_B$:

$$-2.609 + 13.04\times10^3 I_B + 0.7 + 5000(I_B + 250 I_B) = 0$$

$$I_B = \frac{2.609 - 0.7}{13.04\times10^3 + 251\times5000} = 1.505\,\mu\text{A}$$

$$I_C = 250 I_B = 3.764\times10^{-4}\,\text{A}$$

$$= \boxed{376.4\,\mu\text{A}}$$

81. (a) Define a nodal voltage V_1 at the top of the current source I_S, and a nodal voltage V_2 at the top of the load resistor R_L. Since the load resistor can safely dissipate 1 W, and we know that

$$P_{R_L} = \frac{V_2^2}{1000}$$

then $V_2\big|_{max} = 31.62\text{ V}$. This corresponds to a load resistor (and hence lamp) current of 32.62 mA, so we may treat the lamp as a 10.6-Ω resistor.

Proceeding with nodal analysis, we may write:

$$I_S = V_1/200 + (V_1 - 5V_x)/200 \qquad [1]$$

$$0 = V_2/1000 + (V_2 - 5V_x)/10.6 \qquad [2]$$

$$V_x = V_1 - 5V_x \quad \text{or} \quad V_x = V_1/6 \qquad [3]$$

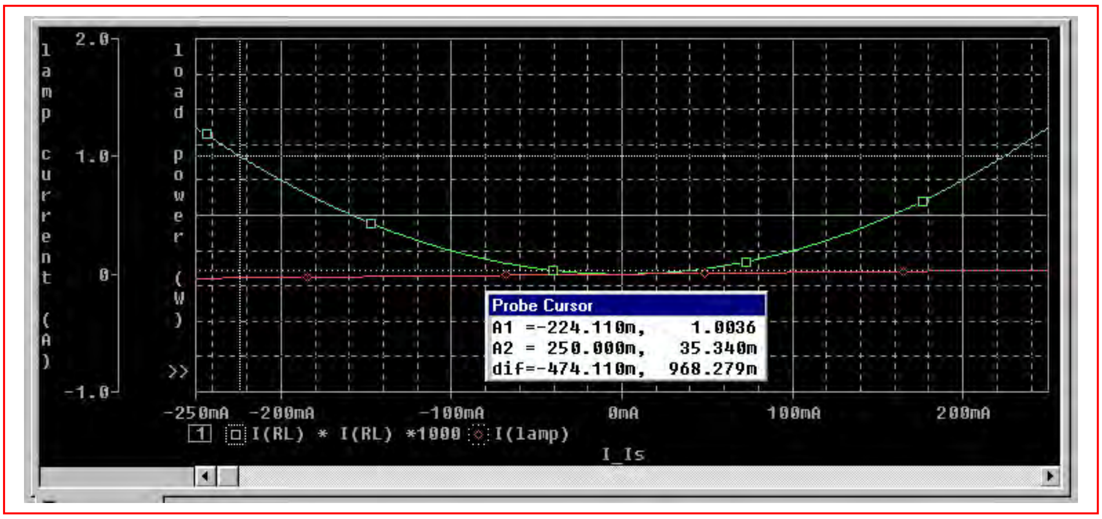
Substituting Eq. [3] into Eqs. [1] and [2], we find that

$$7V_1 = 1200 I_S \qquad [1]$$
$$-5000 V_1 + 6063.6 V_2 = 0 \qquad [2]$$

Substituting $V_2\big|_{max} = 31.62\text{ V}$ into Eq. [2] then yields $V_1 = 38.35$ V, so that

$I_S\big|_{max} = (7)(38.35)/1200 = 223.7$ mA.

(b) PSpice verification.



The lamp current does not exceed 36 mA in the range of operation allowed (*i.e.* a load power of < 1 W.) The simulation result shows that the load will dissipate slightly more than 1 W for a source current magnitude of 224 mA, as predicted by hand analysis.

82. Short out all but the source operating at 10^4 rad/s, and define three clockwise mesh currents i_1, i_2, and i_3 starting with the left-most mesh. Then

$$608\, i_1 - 300\, i_2 = 3.5 \cos 10^4 t \quad [1]$$
$$-300\, i_1 + 316\, i_2 - 8\, i_3 = 0 \quad [2]$$
$$-8\, i_2 + 322\, i_3 = 0 \quad [3]$$

Solving, we find that $i_1(t) = 10.84 \cos 10^4 t$ mA
$i_2(t) = 10.29 \cos 10^4 t$ mA
$i_3(t) = 255.7 \cos 10^4 t$ μA

Next, short out all but the 7 sin 200t V source, and and define three clockwise mesh currents i_a, i_b, and i_c starting with the left-most mesh. Then

$$608\, i_a - 300\, i_b = -7 \sin 200t \quad [1]$$
$$-300\, i_a + 316\, i_b - 8\, i_c = 7 \sin 200t \quad [2]$$
$$-8\, i_b + 322\, i_c = 0 \quad [3]$$

Solving, we find that $i_a(t) = -1.084 \sin 200t$ mA
$i_b(t) = 21.14 \sin 200t$ mA
$i_c(t) = 525.1 \sin 200t$ μA

Next, short out all but the source operating at 10^3 rad/s, and define three clockwise mesh currents i_A, i_B, and i_C starting with the left-most mesh. Then

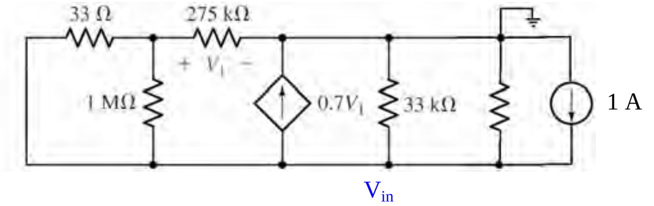
$$608\, i_A - 300\, i_B = 0 \quad [1]$$
$$-300\, i_A + 316\, i_B - 8\, i_C = 0 \quad [2]$$
$$-8\, i_B + 322\, i_C = -8 \cos 10^4 t \quad [3]$$

Solving, we find that $i_A(t) = -584.5 \cos 10^3 t$ μA
$i_B(t) = -1.185 \cos 10^3 t$ mA
$i_C(t) = -24.87 \cos 10^3 t$ mA

We may now compute the power delivered to each of the three 8-Ω speakers:

$$p_1 = 8[i_1 + i_a + i_A]^2 = 8[10.84\times10^{-3} \cos 10^4 t - 1.084\times10^{-3} \sin 200t - 584.5\times10^{-6} \cos 10^3 t]^2$$
$$p_2 = 8[i_2 + i_b + i_B]^2 = 8[10.29\times10^{-3} \cos 10^4 t + 21.14\times10^{-3} \sin 200t - 1.185\times10^{-3} \cos 10^3 t]^2$$
$$p_3 = 8[i_3 + i_c + i_C]^2 = 8[255.7\times10^{-6} \cos 10^4 t + 525.1\times10^{-6} \sin 200t - 24.87\times10^{-3} \cos 10^3 t]^2$$

83. Replacing the DMM with a possible Norton equivalent (a 1-MΩ resistor in parallel with a 1-A source):



We begin by noting that 33 Ω || 1 MΩ ≈ 33 Ω. Then,

$$0 = (V_1 - V_{in})/ 33 + V_1/ 275\times10^3 \qquad [1]$$

and

$$1 - 0.7 V_1 = V_{in}/ 10^6 + V_{in}/ 33\times10^3 + (V_{in} - V_1)/ 33 \qquad [2]$$

Simplifying and collecting terms,

$$(275\times10^3 + 33) V_1 - 275\times10^3 V_{in} = 0 \qquad [1]$$

$$22.1 V_1 + 1.001 V_{in} = 33 \qquad [2]$$

Solving, we find that V_{in} = 1.429 V; in other words, the DMM sees 1.429 V across its terminals in response to the known current of 1 A it's supplying. It therefore thinks that it is connected to a resistance of $\boxed{1.429\ \Omega.}$

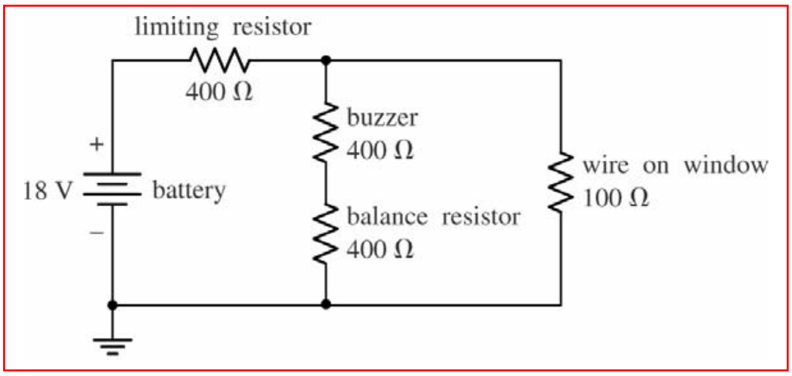
84. We know that the resistor R is absorbing maximum power. We might be tempted to say that the resistance of the cylinder is therefore 10 Ω, but this is wrong: The larger we make the cylinder resistance, the small the power delivery to R:

$$P_R = 10\, i^2 = 10\left[\frac{120}{R_{cylinder} + 10}\right]^2$$

Thus, if we are in fact delivering the maximum possible power to the resistor from the 120-V source, the resistance of the cylinder must be zero.

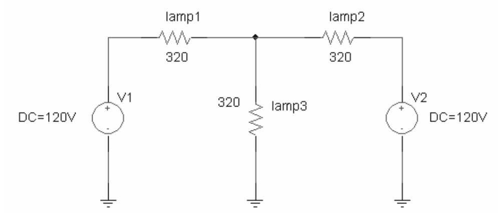
$\boxed{\text{This corresponds to a temperature of absolute zero using the equation given.}}$

85. We note that the buzzer draws 15 mA at 6 V, so that it may be modeled as a 400-Ω resistor. One possible solution of many, then, is:

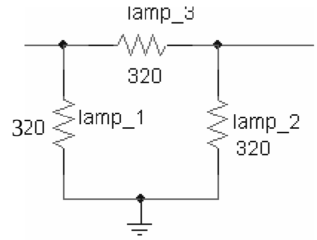


Note: construct the 18-V source from 12 1.5-V batteries in series, and the two 400-Ω resistors can be fabricated by soldering 400 1-Ω resistors in series, although there's probably a much better alternative…

86. To solve this problem, we need to assume that "45 W" is a designation that applies when 120 Vac is applied directly to a particular lamp. This corresponds to a current draw of 375 mA, or a light bulb resistance of 120/ 0.375 = 320 Ω.



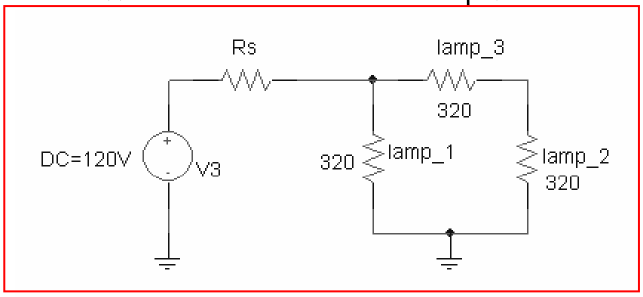
Original wiring scheme



New wiring scheme

In the original wiring scheme, Lamps 1 & 2 draw (40)2 / 320 = 5 W of power each, and Lamp 3 draws (80)2 / 320 = 20 W of power. Therefore, none of the lamps is running at its maximum rating of 45 W. We require a circuit which will deliver the same intensity after the lamps are reconnected in a Δ configuration. Thus, we need a total of 30 W from the new network of lamps.

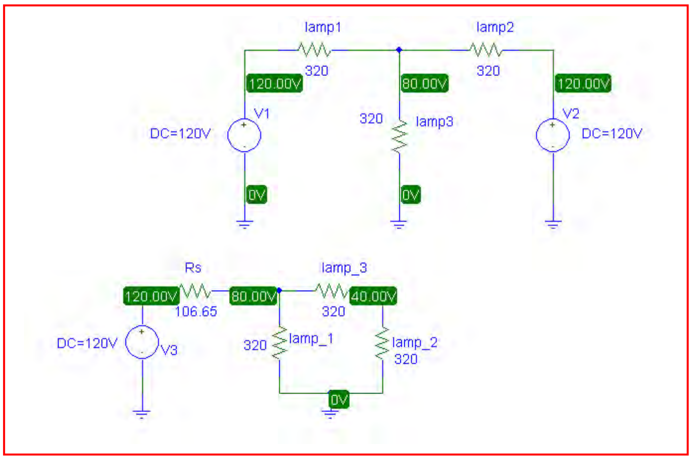
There are several ways to accomplish this, but the simplest may be to just use one 120-Vac source connected to the left port in series with a resistor whose value is chosen to obtain 30 W delivered to the three lamps.



In other words,

$$\frac{\left[120\dfrac{213.3}{\text{Rs}+213.3}\right]^2}{320} + 2\,\frac{\left[60\dfrac{213.3}{\text{Rs}+213.3}\right]^2}{320} = 30$$

Solving, we find that we require $\boxed{\text{Rs} = 106.65\ \Omega}$, as confirmed by the PSpice simulation below, which shows that both wiring configurations lead to one lamp with 80-V across it, and two lamps with 40 V across each.



87.

- Maximum current rating for the LED is 35 mA.
- Its resistance can vary between 47 and 117 Ω.
- A 9-V battery must be used as a power source.
- Only standard resistance values may be used.

One possible current-limiting scheme is to connect a 9-V battery in series with a resistor $R_{limiting}$ and in series with the LED.

From KVL,

$$I_{LED} = \frac{9}{R_{limiting} + R_{LED}}$$

The maximum value of this current will occur at the minimum LED resistance, 47 Ω. Thus, we solve

$$35\times10^{-3} = \frac{9}{R_{limiting} + 47}$$

to obtain $R_{limiting} \geq 210.1\ \Omega$ to ensure an LED current of less than 35 mA. This is not a standard resistor value, however, so we select

$$\boxed{R_{limiting} = 220\ \Omega.}$$

1. This is an inverting amplifier, therefore, $V_{out} = -\frac{R_f}{R_1}V_{in}$

So:

a) $V_{out} = -\frac{100}{10} \times 3 = \boxed{-30V}$

b) $V_{out} = -\frac{1M}{1M} \times 2.5 = \boxed{-2.5V}$

c) $V_{out} = -\frac{4.7}{3.3} \times -1 = \boxed{1.42V}$

2. This is also an inverting amplifier. The loading resistance R_s only affects the output current drawn from the op-amp. Therefore,

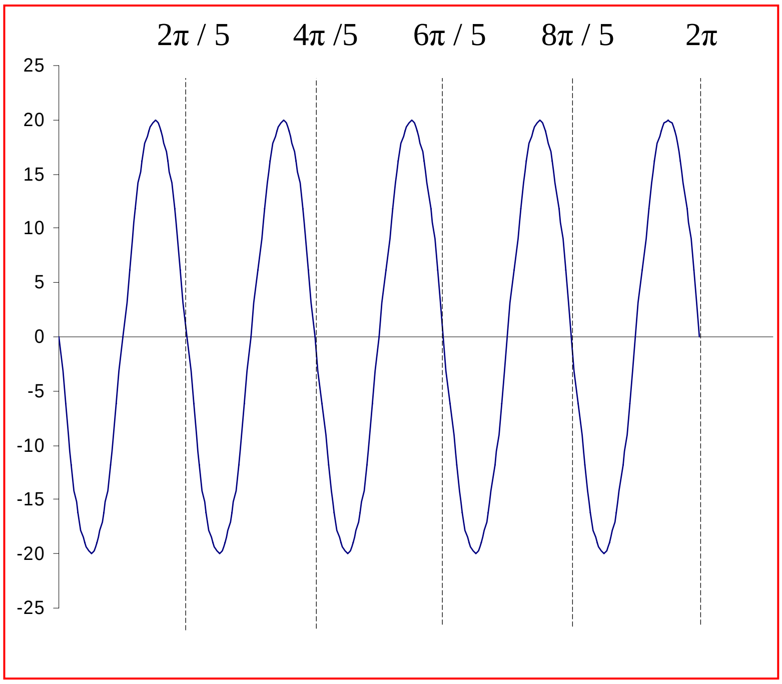
a) $V_{out} = -\frac{47}{10} \times 1.5 = \boxed{-7.05V}$

b) $V_{out} = \boxed{9V}$

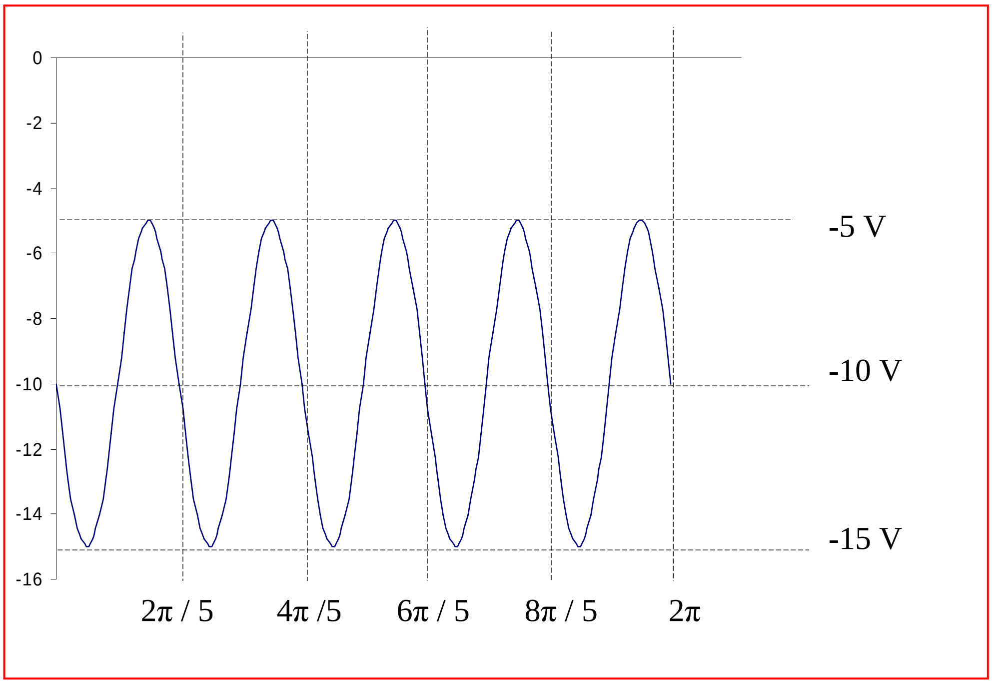
c) $V_{out} = \boxed{-680mV}$

3. For this inverting amplifier, $v_{out} = -\frac{10k}{1k} \times v_{in} = -10v_{in}$. Therefore,

a) $v_{out} = -10v_{in} = -20\sin 5t$

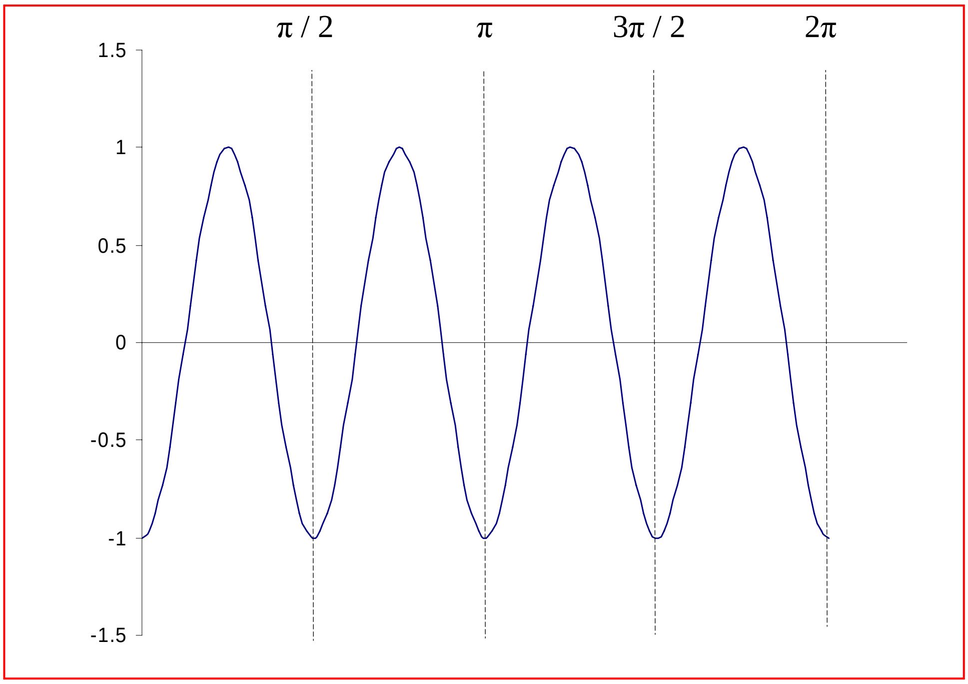


b) $v_{out} = -10v_{in} = -10 - 5\sin 5t$

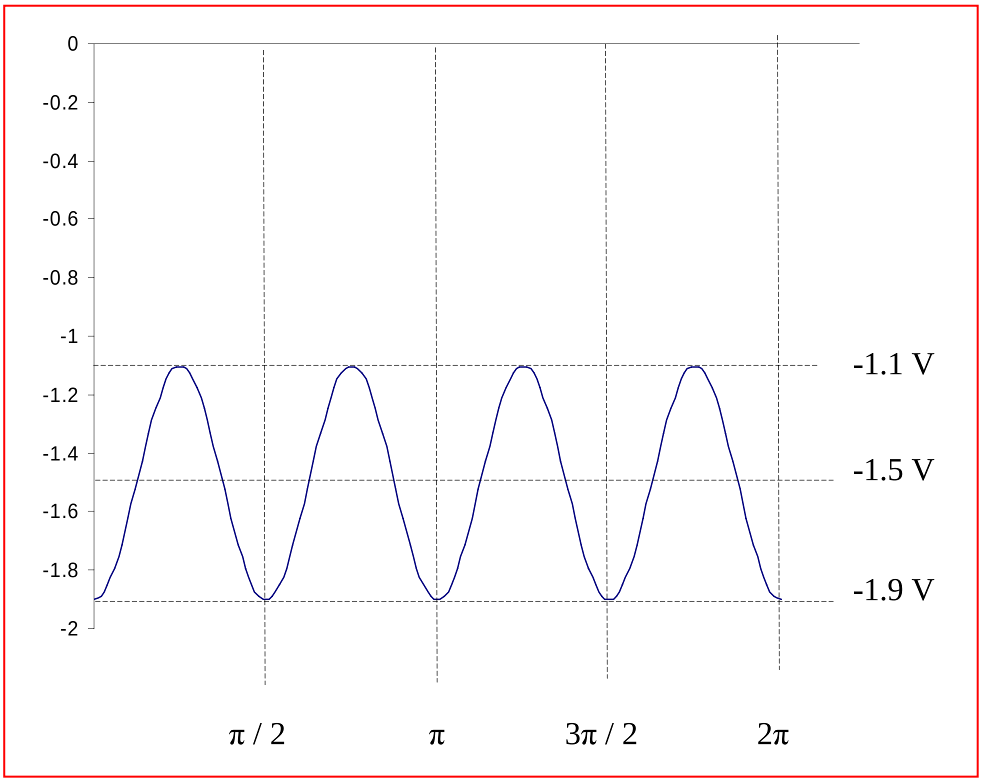


4. For this inverting amplifier, $v_{out} = -\frac{R_f}{R_1} v_{in} = -0.1 v_{in}$, hence,

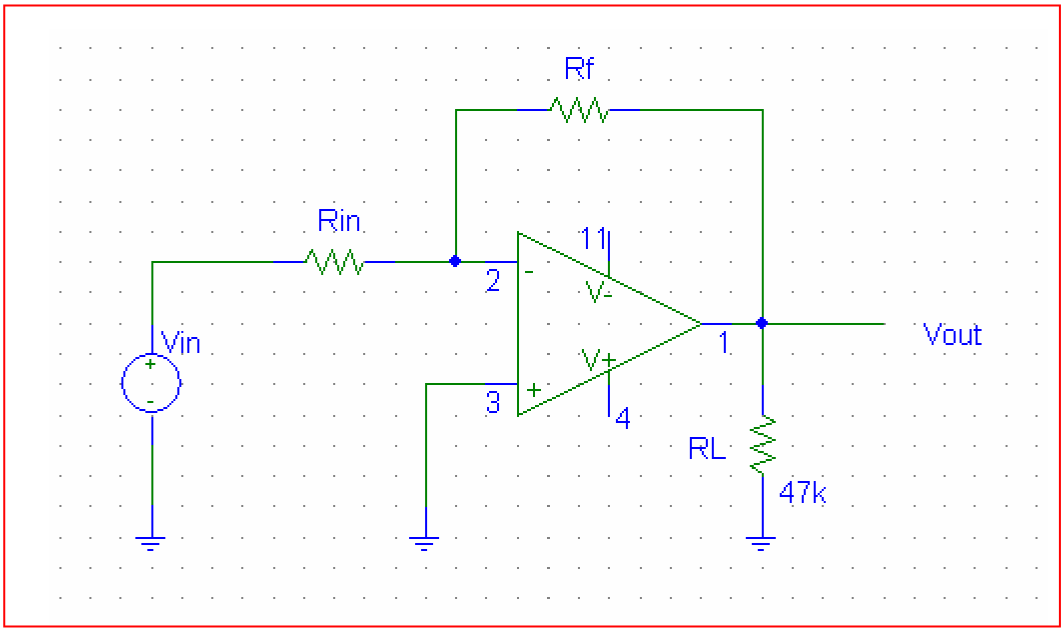
a) $v_{out} = -0.1 v_{in} = -\cos 4t$



b) $v_{out} = -0.1 v_{in} = -1.5 - 0.4 \cos 4t$



5. One possible solution is by using an inverting amplifier design, we have

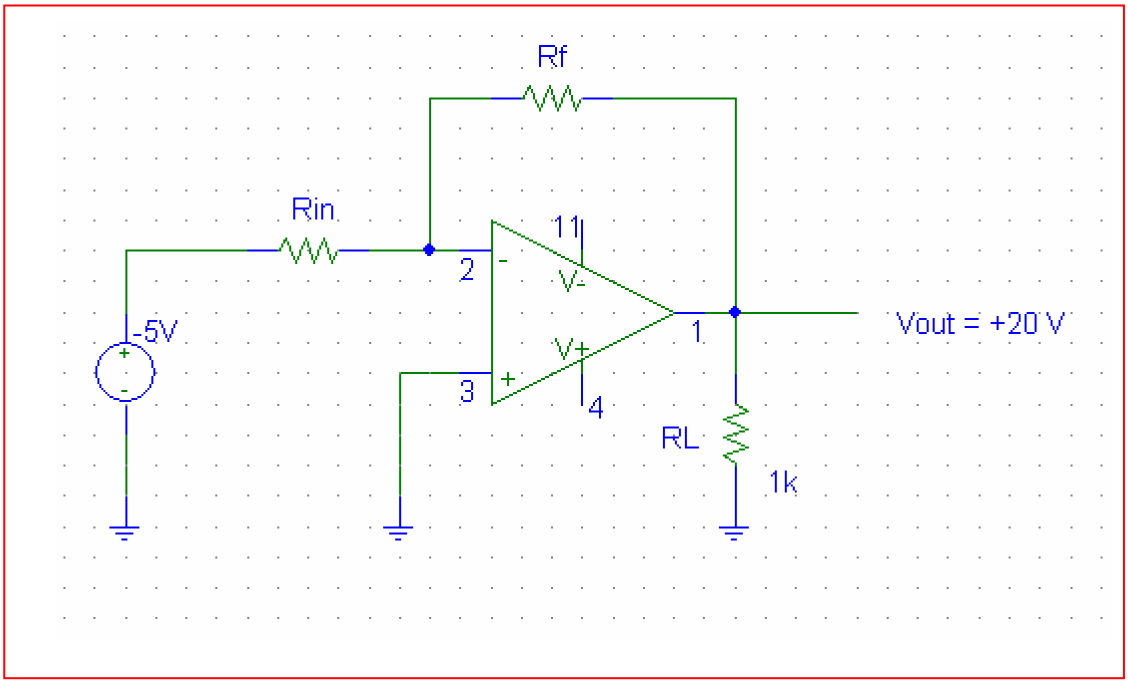


$$V_{out} = -\frac{R_f}{R_{in}} V_{in}$$

$$\Leftrightarrow \frac{R_f}{R_{in}} = -\frac{V_{out}}{V_{in}} = \frac{9}{5}$$

Using standard resistor values, we can have $\boxed{R_f = 9.1\text{k}\Omega \text{ and } R_{in} = 5.1\text{k}\Omega}$

6. One possible solution is by using an inverting amplifier design, and a -5V input to give a positive output voltage:

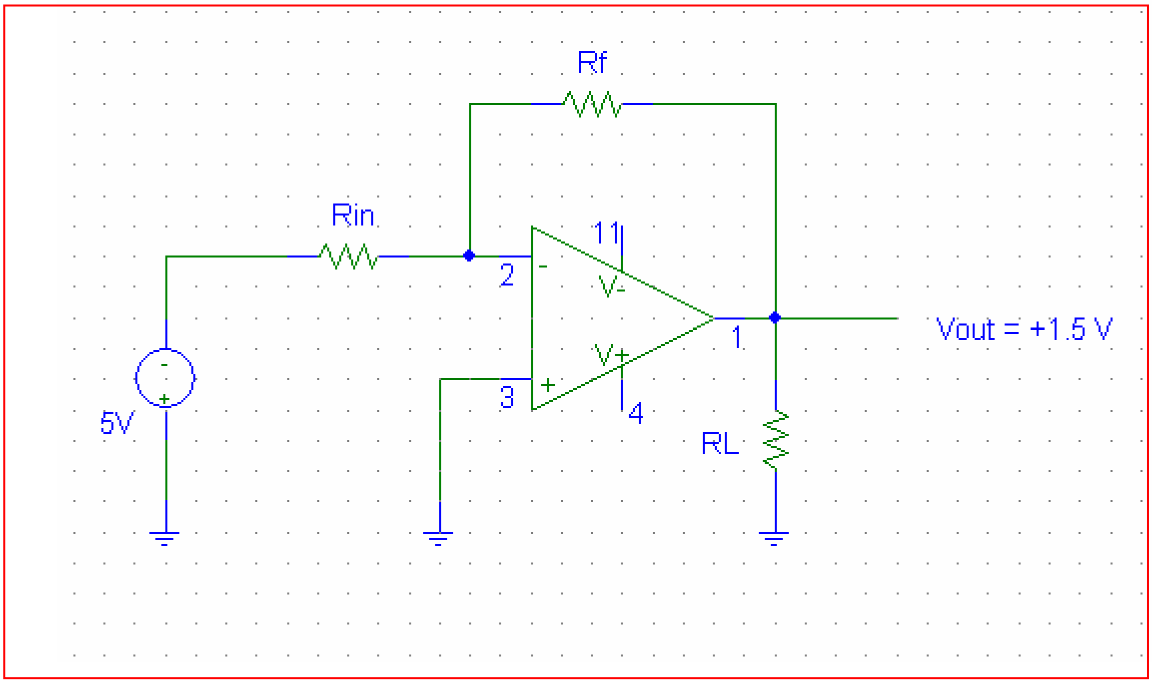


The resistance values are given by:

$$\frac{R_f}{R_{in}} = \frac{20}{5}$$

Giving possible resistor values $\boxed{R_f = 20\text{ k}\Omega \text{ and } R_{in} = 5.1\text{ k}\Omega}$

7. To get a positive output that is smaller than the input, the easiest way is to use inverting amplifier with an inverted voltage supply to give a negative voltage:

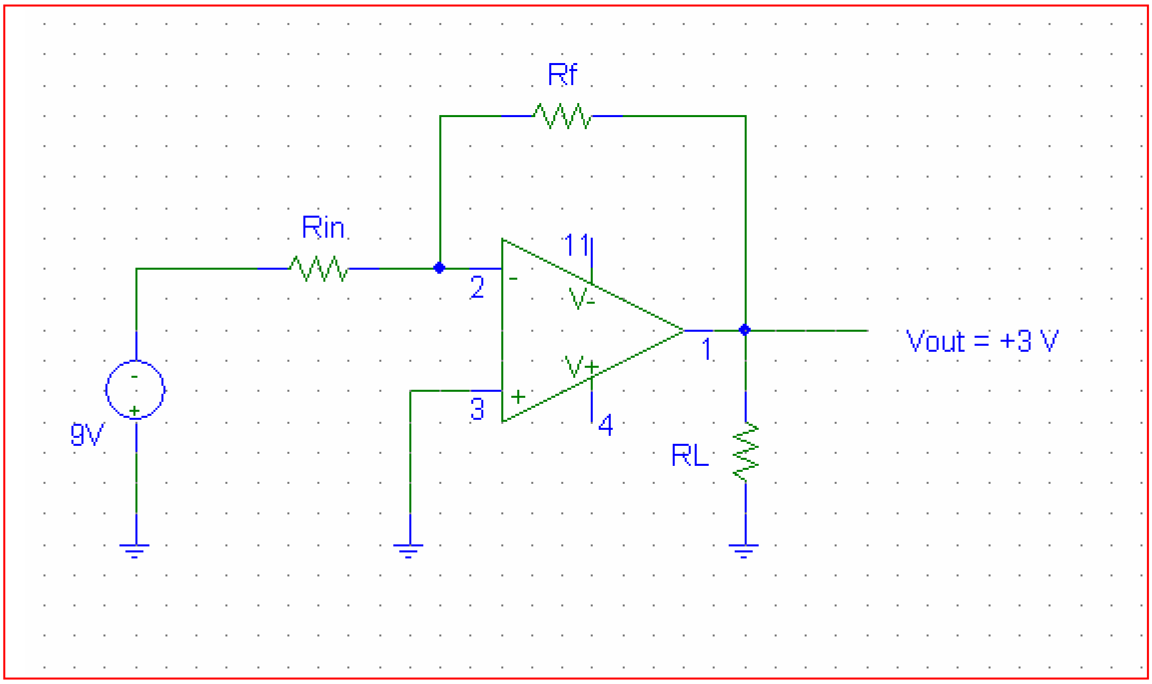


The resistances are given by:

$$\frac{R_f}{R_{in}} = \frac{1.5}{5}$$

Giving possible resistor values $\boxed{R_f = 1.5\ \text{k}\Omega \text{ and } R_{in} = 5.1\ \text{k}\Omega}$

8. Similar to question 7, the following is proposed:



The resistances are given by:

$$\frac{R_f}{R_{in}} = \frac{3}{9}$$

Giving possible resistor values $\boxed{R_f = 3.0\text{k}\Omega \text{ and } R_{in} = 9.1\ \text{k}\Omega}$

9. This circuit is a non-inverting amplifier, therefore, $V_{out} = (1+\frac{R_f}{R_1})V_{in}$

So:

a) $V_{out} = (1+\frac{47}{10})\times 300m = 1.71\,\text{V}$

b) $V_{out} = (1+\frac{1M}{1M})\times 1.5 = 3\,\text{V}$

c) $V_{out} = (1+\frac{4.7}{3.3})\times -1 = -2.42\,\text{V}$

10. This is again a non-inverting amplifier. Similar to question 9, we have:

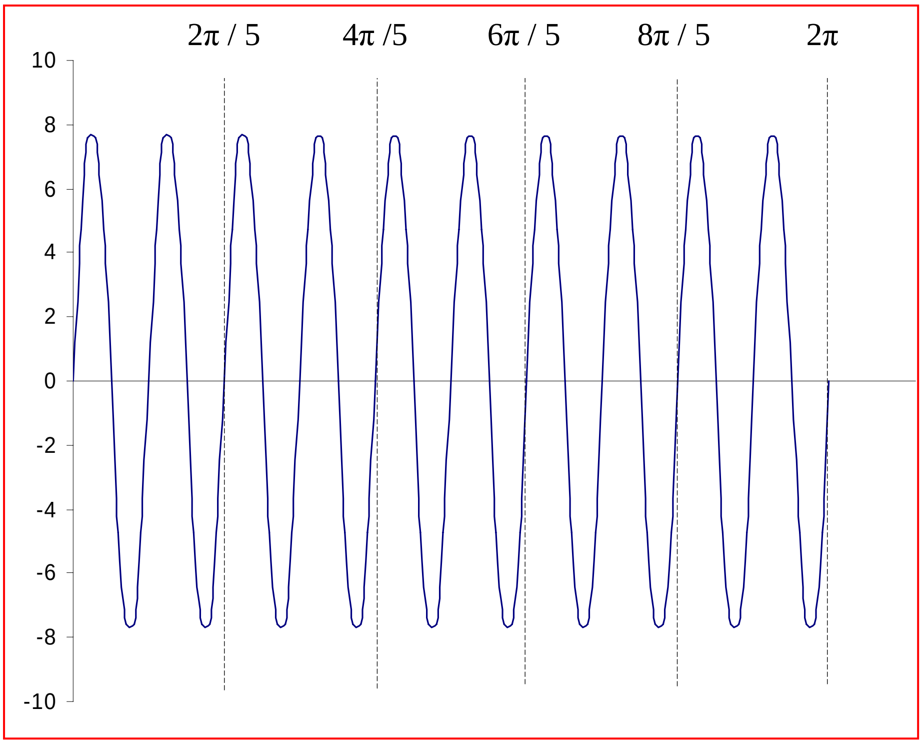
a) $V_{out} = 200m \times (1 + 4.7) = 1.14\text{ V}$

b) $V_{out} = (1 + 1) \times 9 = -18\text{ V}$

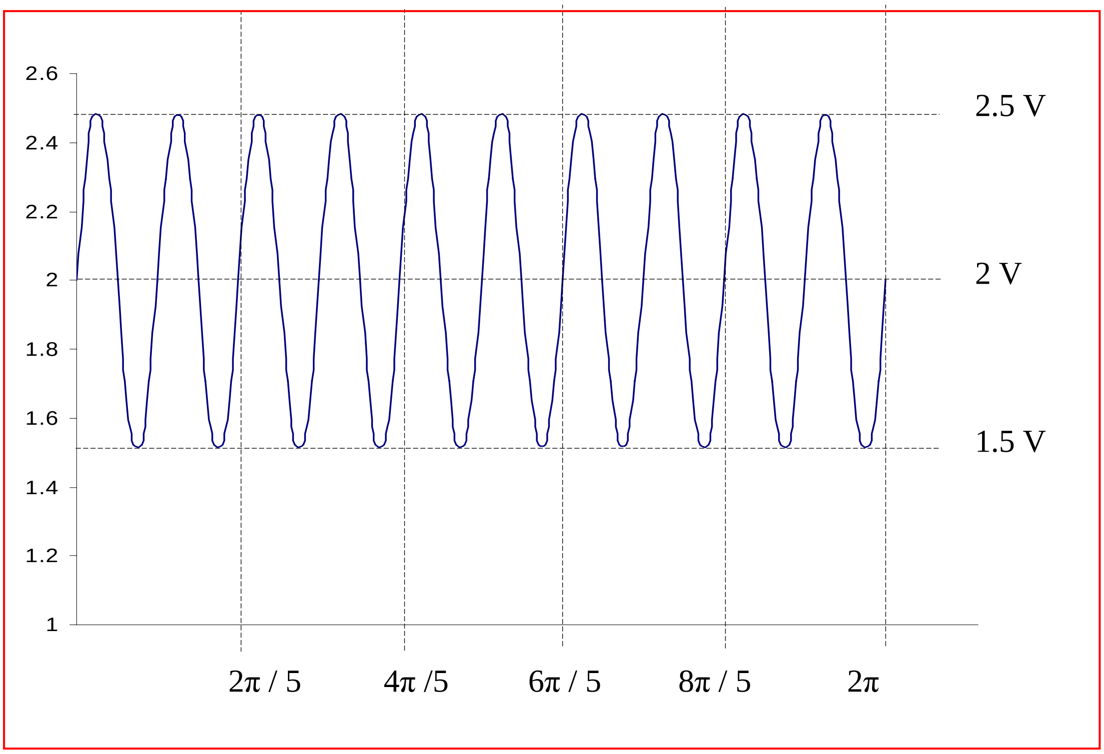
c) $V_{out} = 7.8 \times 100m = 0.78\text{ V}$

11. $v_{out} = (1+\frac{1}{1})v_{in} = 2v_{in}$ for this non inverting amplifier circuit, therefore:

a) $v_{out} = 2v_{in} = 8\sin 10t$

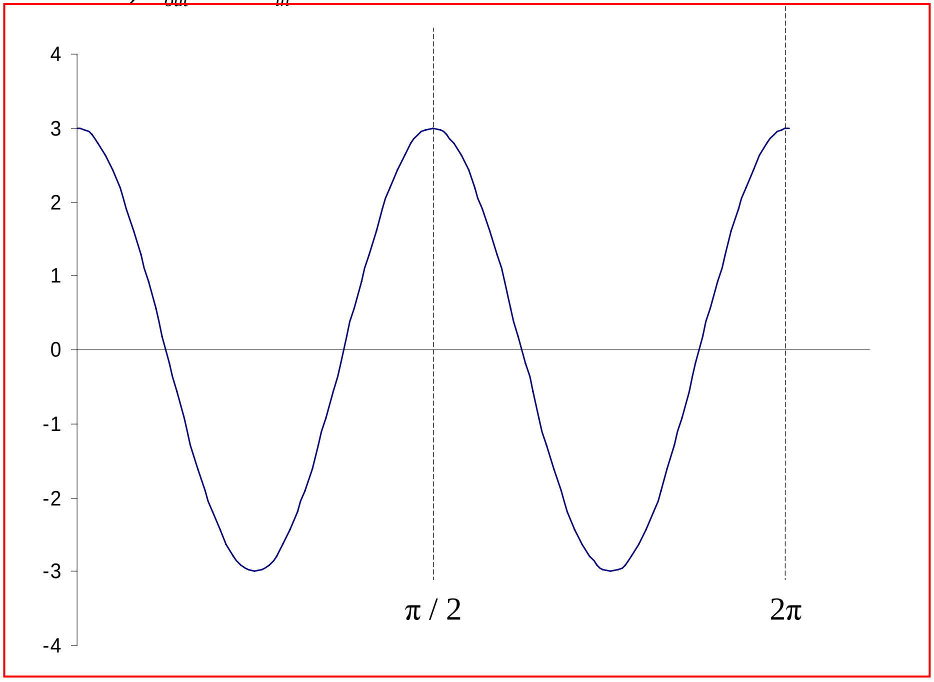


b) $v_{out} = 2v_{in} = 2 + 0.5\sin 10t$

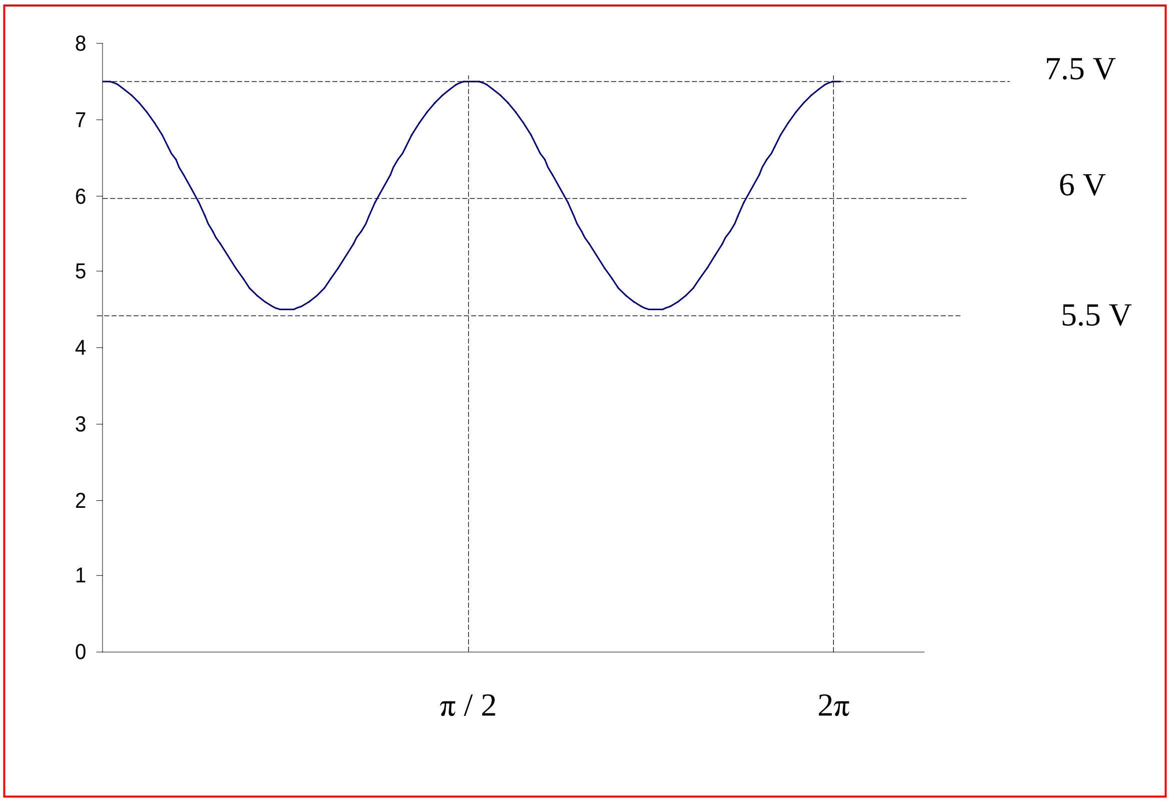


12. $v_{out} = (1+\frac{R_f}{R_{in}})v_{in} = 1.5v_{in}$ for this non inverting op-amp circuit. Hence,

a) $v_{out} = 1.5v_{in} = 3\cos 2t$



b) $v_{out} = 1.5v_{in} = 6 + 1.5\cos 2t$



13. The first step is to perform a simple source transformation, so that a 0.15-V source in series with a 150-Ω resistor is connected to the inverting pin of the ideal op amp.

Then, $v_{\text{out}} = -\frac{2200}{150}(0.15) = \boxed{-2.2 \text{ V}}$

14. In order to deliver 150 mW to the 10-kΩ resistor, we need v_{out} = $\sqrt{(0.15)(10\times10^3)} = 38.73\,\text{V}$. Writing a nodal equation at the inverting input, we find

$$0 = \frac{5}{\text{R}} + \frac{5 - v_{out}}{1000}$$

Using v_{out} = 38.73, we find that $\boxed{\text{R} = 148.2\ \Omega.}$

15. Since the 670-Ω switch requires 100 mA to activate, the voltage delivered to it by our op amp circuit must be (670)(0.1) = 67 V. The microphone acts as the input to the circuit, and provides 0.5 V. Thus, an amplifier circuit having a gain = 67/0.5 = 134 is required.

One possible solution of many: a non-inverting op amp circuit with the microphone connected to the non-inverting input terminal, the switch connected between the op amp output pin and ground, a feedback resistor R_f = 133 Ω, and a resistor R_1 = 1 Ω.

16. We begin by labeling the nodal voltages v_- and v_+ at the inverting and non-inverting input terminals, respectively. Since no current can flow into the non-inverting input, no current flows through the 40-kΩ resistor; hence, $v_+ = 0$. Therefore, we know that $v_- = 0$ as well.

Writing a single nodal equation at the non-inverting input then leads to

$$0 = \frac{(v_- - v_S)}{100} + \frac{(v_- - v_{out})}{22000}$$

or

$$0 = \frac{-v_S}{100} + \frac{-v_{out}}{22000}$$

Solving,

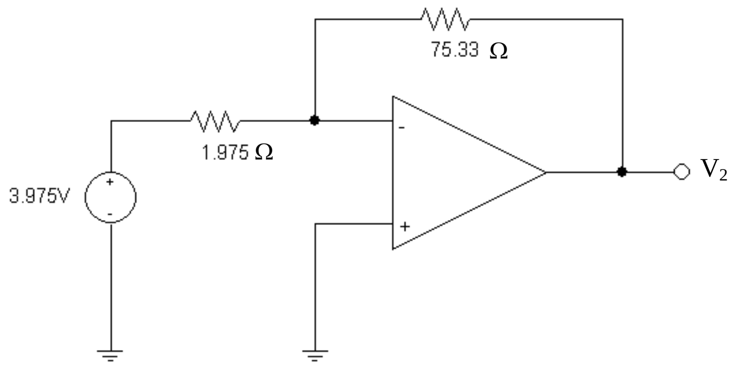
$$\boxed{v_{out} = -220\, v_S}$$

17. We first label the nodal voltage at the output pin V_o. Then, writing a single nodal equation at the inverting input terminal of the op amp,

$$0 = \frac{4 - 3}{1000} + \frac{4 - V_o}{17000}$$

Solving, we find that $V_o = 21$ V. Since no current can flow through the 300-kΩ resistor, $\boxed{V_1 = 21}$ as well.

18. A source transformation and some series combinations are well worthwhile prior to launching into the analysis. With 5 kΩ || 3 kΩ = 1.875 kΩ and (1 mA)(1.875 kΩ) = 1.875 V, we may redraw the circuit as



This is now a simple inverting amplifier with gain – R_f/ R_1 = -75.33/ 1.975 = -38.14.

Thus, V_2 = -38.14(3.975) = $\boxed{-151.6\text{ V}}$.

19. This is a simple inverting amplifier, so we may write

$$v_{\text{out}} = \frac{-2000}{1000}(2 + 2\sin 3t) = \boxed{-4(1 + \sin 3t)\text{ V}}$$

$\boxed{v_{\text{out}}(t = 3\text{ s}) = -5.648\text{ V.}}$

20. We first combine the 2 MΩ and 700 kΩ resistors into a 518.5 kΩ resistor.

We are left with a simple non-inverting amplifier having a gain of
1 + 518.5/ 250 = 3.074. Thus,

v_{out} = (3.074) v_{in} = 18 so $\boxed{v_{in} = 5.856 \text{ V.}}$

21. This is a simple non-inverting amplifier circuit, and so it has a gain of $1 + R_f / R_1$.
We want $v_{out} = 23.7 \cos 500t$ V when the input is $0.1 \cos 500t$ V, so a gain of 23.7/0.1 = 237 is required.
One possible solution of many: $\boxed{R_f = 236\ \text{k}\Omega \text{ and } R_1 = 1\ \text{k}\Omega.}$

22. Define a nodal voltage V_- at the inverting input, and a nodal voltage V_+ at the non-inverting input. Then,

At the non-inverting input: $-3\times10^{-6} = \dfrac{V_+}{1.5\times10^6}$ [1]

Thus, $V_+ = -4.5$ V, and we therefore also know that $V_- = -4.5$ V.

At the inverting input: $0 = \dfrac{V_-}{R_6} + \dfrac{V_- - V_{out}}{R_7}$ [2]

Solving and making use of the fact that $V_- = -4.5$ V,

$$v_{out} = -\frac{R_7}{R_6}(4.5) - 4.5 \boxed{= -4.5\left(\frac{R_7}{R_6}+1\right)} \text{ V}$$

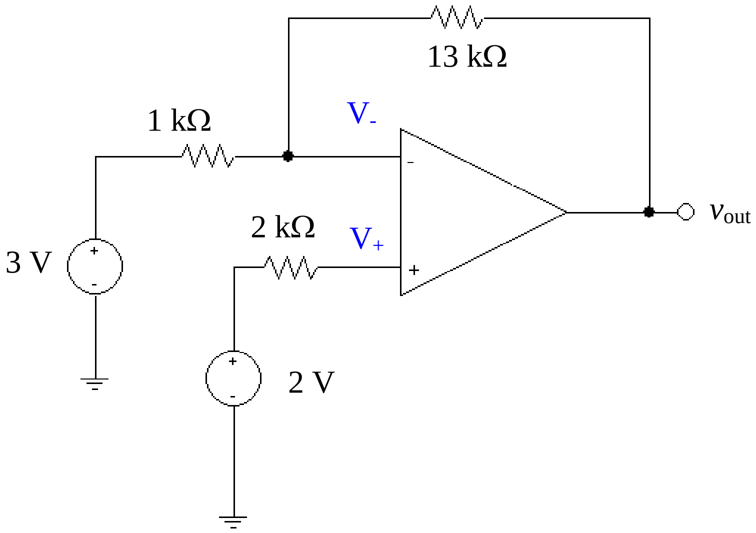
23. (a) **B** must be the non-inverting input: that yields a gain of 1 + 70/10 = 8 and an output of 8 V for a 1-V input.

(b) $R_1 = \infty$, $R_A = 0$. We need a gain of 20/10 = 2, so choose $R_2 = R_B = 1\ \Omega$.

(c) **A** is the inverting input since it has the feedback connection to the output pin.

24. It is probably best to first perform a simple source transformation:

(1 mA)(2 kΩ) = 2 V.



Since no current can flow into the non-inverting input pin, we know that $V_+ = 2$ V, and therefore also that $V_- = 2$ V. A single nodal equation at the inverting input yields:

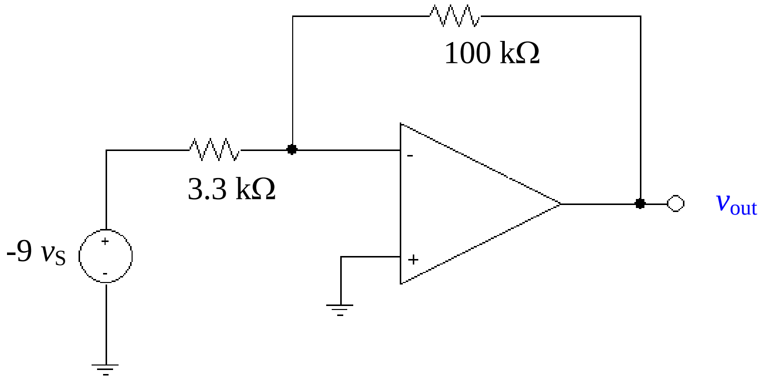
$$0 = \frac{2-3}{1000} + \frac{2-v_{out}}{13000}$$

which yields $\boxed{v_{out} = -11 \text{ V.}}$

25. We begin by find the Thévenin equivalent to the left of the op amp:

$$V_{th} = -3.3(3)\ v_\pi \ = \ -9.9\ v_\pi \ = \ -9.9\frac{1000\ v_S}{1100} \ = \ -9\ v_S$$

R_{th} = 3.3 kΩ, so we can redraw the circuit as:

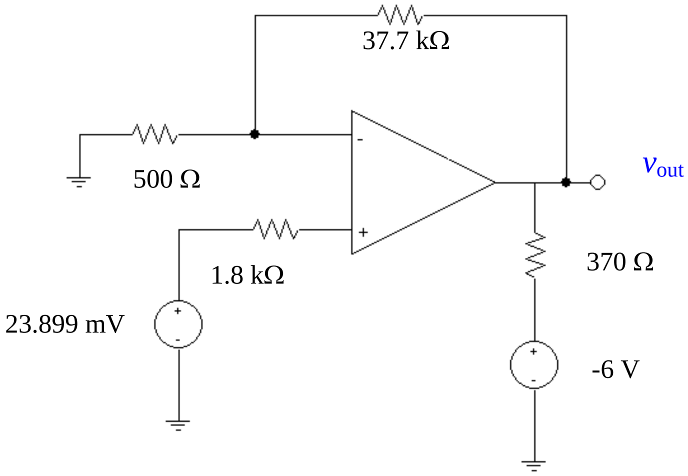


which is simply a classic inverting op amp circuit with gain of -100/3.3 = -30.3.

Thus, $v_{out} = (-30.3)(-9\ v_S) = 272.7\ v_S$

For $v_S = 5 \sin 3t$ mV, $v_{out} = 1.364 \sin 3t$ V, and $\boxed{v_{out}(0.25\text{ s}) = 0.9298\text{ V.}}$

26. We first combine the 4.7 MΩ and 1.3 kΩ resistors: 4.7 MΩ || 1.3 kΩ = 1.30 kΩ. Next, a source transformation yields $(3\times10^{-6})(1300)$ = 3.899 mV which appears in series with the 20 mV source and the 500-Ω resistor. Thus, we may redraw the circuit as



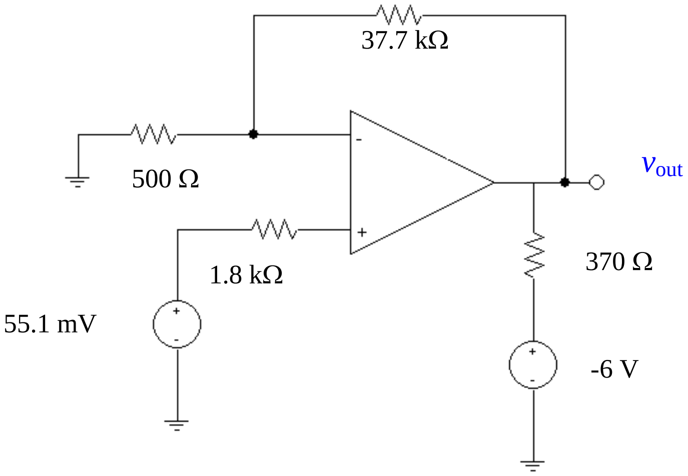
Since no current flows through the 1.8 kΩ resistor, V_+ = 23.899 mV and hence V_- = 23.899 mV as well. A single nodal equation at the inverting input terminal yields

$$0 = \frac{23.899\times10^{-3}}{500} + \frac{23.899\times10^{-3} - v_{out}}{37.7\times10^{3}}$$

Solving,

$$\boxed{v_{out} = 1.826 \text{ V}}$$

27. We first combine the 4.7 MΩ and 1.3 kΩ resistors: 4.7 MΩ || 1.3 kΩ = 1.30 kΩ. Next, a source transformation yields $(27\times10^{-6})(1300)$ = 35.1 mV which appears in series with the 20 mV source and the 500-Ω resistor. Thus, we may redraw the circuit as



Since no current flows through the 1.8 kΩ resistor, V_+ = 55.1 mV and hence V_- = 55.1 mV as well. A single nodal equation at the inverting input terminal yields

$$0 = \frac{55.1\times10^{-3}}{500} + \frac{55.1\times10^{-3} - v_{out}}{37.7\times10^{3}}$$

Solving,

$$\boxed{v_{out} = 4.21 \text{ V}}$$

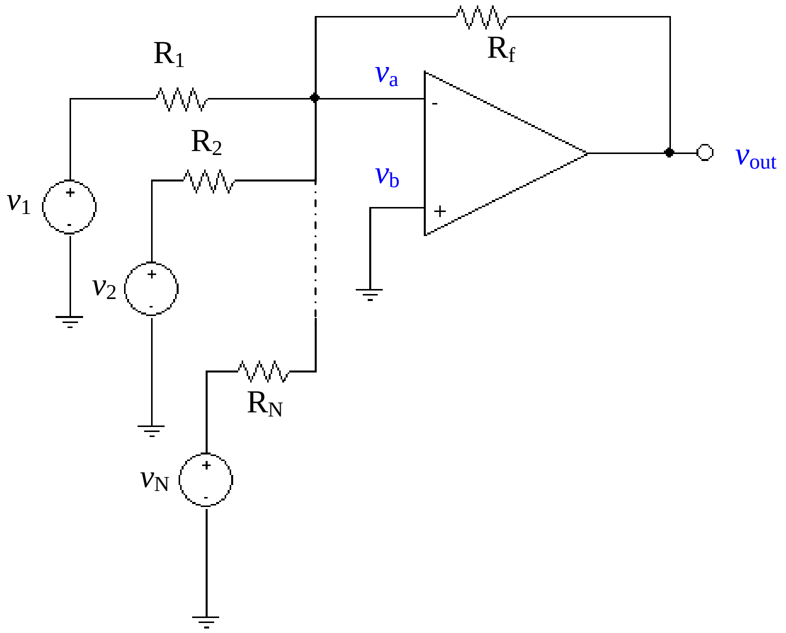
28. The 3 mA source, 1 kΩ resistor and 20 kΩ resistor may be replaced with a –3 V source ("+" reference up) in series with a 21 kΩ resistor. No current flows through either 1 MΩ resistor, so that the voltage at each of the four input terminals is identically zero. Considering each op amp circuit separately,

$$v_{\text{out}}\Big|_{\text{LEFTOPAMP}} = -(-3)\frac{100}{21} = 14.29 \text{ V}$$

$$v_{\text{out}}\Big|_{\text{RIGH OPAMP}} = -(5)\frac{100}{10} = -50 \text{ V}$$

$$v_{\text{x}} = v_{\text{out}}\Big|_{\text{LEFTOPAMP}} - v_{\text{out}}\Big|_{\text{RIGH OPAMP}} = 14.29 + 50 = \boxed{64.29 \text{ V.}}$$

29. A general summing amplifier with N input sources:



1. $v_a = v_b = 0$
2. A single nodal equation at the inverting input leads to:

$$0 = \frac{v_a - v_{out}}{R_f} + \frac{v_a - v_1}{R_1} + \frac{v_a - v_2}{R_2} + \ldots + \frac{v_a - v_N}{R_N}$$

Simplifying and making use of the fact that $v_a = 0$, we may write this as

$$\left[-\frac{1}{R_f}\prod_{i=1}^{N} R_i\right] v_{out} = \frac{v_1}{R_1}\prod_{i=1}^{N} R_i + \frac{v_2}{R_2}\prod_{i=1}^{N} R_i + \ldots + \frac{v_N}{R_N}\prod_{i=1}^{N} R_i$$

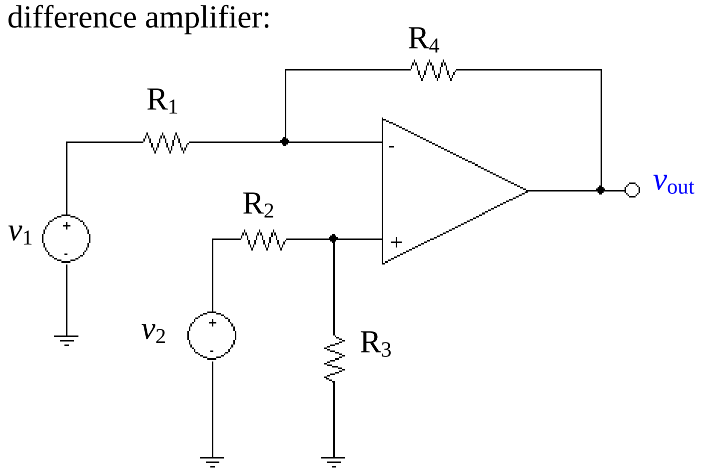
or simply

$$-\frac{v_{out}}{R_f} = \frac{v_1}{R_1} + \frac{v_2}{R_2} + \ldots + \frac{v_N}{R_N}$$

Thus,

$$\boxed{v_{out} = -R_f \sum_{i=1}^{N} \frac{v_i}{R_i}}$$

30. A general difference amplifier:



Writing a nodal equation at the inverting input,

$$0 = \frac{v_a - v_1}{R_1} + \frac{v_a - v_{out}}{R_f}$$

Writing a nodal equation at the non-inverting input,

$$0 = \frac{v_b}{R_3} + \frac{v_b - v_2}{R_2}$$

Simplifying and collecting terms, we may write

$$(R_f + R_1)\, v_a - R_1\, v_{out} = R_f\, v_1 \qquad [1]$$

$$(R_2 + R_3)\, v_b = R_3\, v_2 \qquad [2]$$

From Eqn. [2], we have $v_b = \dfrac{R_3}{R_2 + R_3} v_2$

Since va = vb, we can now rewrite Eqn. [1] as

$$-R_1\, v_{out} = R_f v_1 - \frac{(R_f + R_1)R_3}{R_2 + R_3} v_2$$

and hence

$$\boxed{v_{out} = -\frac{R_f}{R_1} v_1 + \frac{R_3}{R_1}\left(\frac{R_f + R_1}{R_2 + R_3}\right) v_2}$$

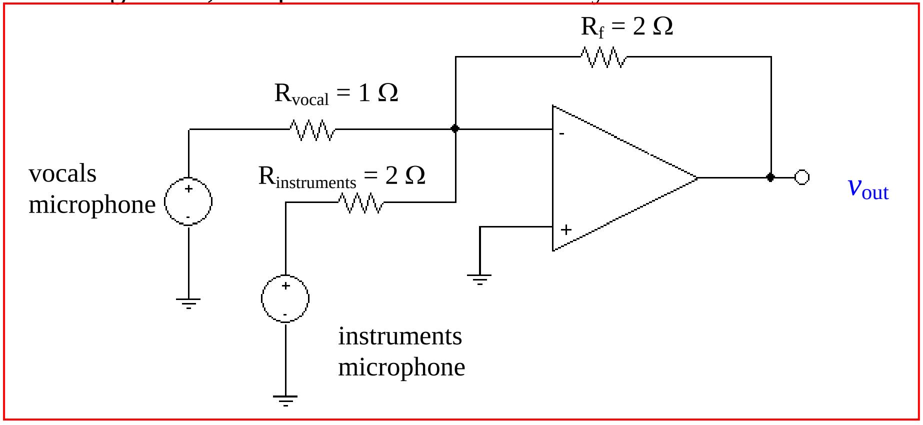
31. In total darkness, the CdS cell has a resistance of 100 kΩ, and at a light intensity L of 6 candela it has a resistance of 6 kΩ. Thus, we may compute the light-dependent resistance (assuming a linear response in the range between 0 and 6 candela) as $R_{CdS} = -15L + 100\ \Omega$.

Our design requirement (using the standard inverting op amp circuit shown) is that the voltage across the load is 1.5 V at 2 candela, and less than 1.5 V for intensities greater than 2 candela.

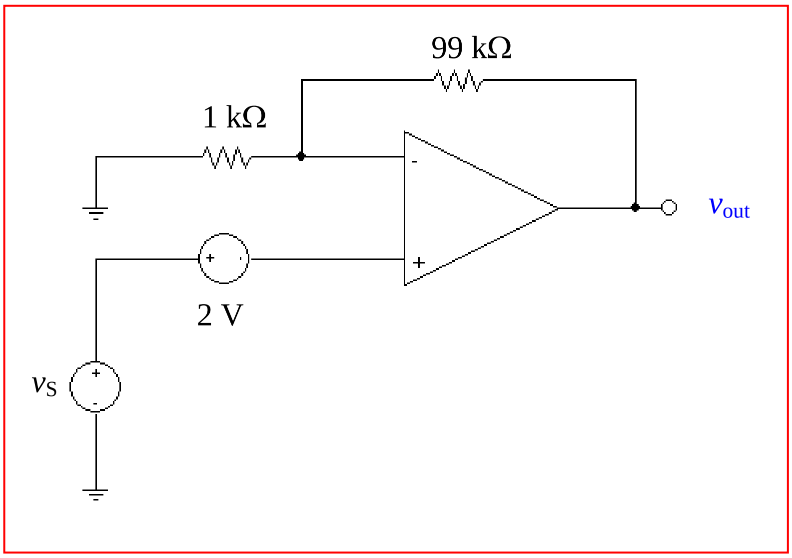
Thus, $v_{out}(2\text{ candela}) = -R_{CdS}\, v_S / R_1 = -70\ V_S / R_1 = 1.5$ (R_1 in kΩ).

$\boxed{\text{Pick } R_1 = 10\text{ k}\Omega.\ \text{Then } v_S = -0.2143\text{ V.}}$

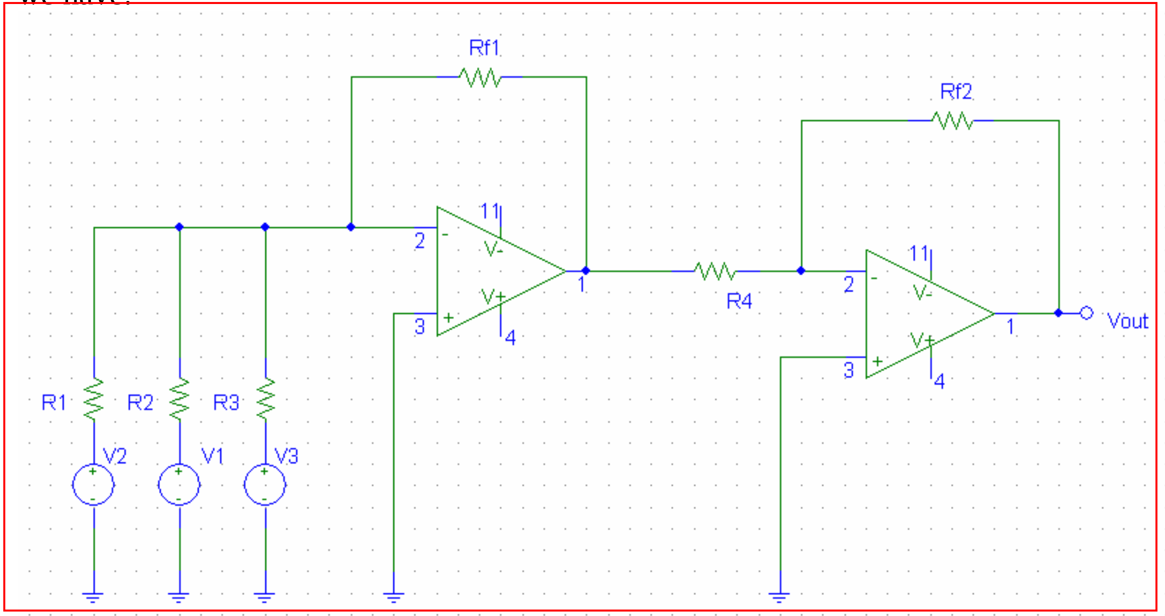
32. We want $R_f / R_{instrument} = 2K$, and $R_f / R_{vocal} = 1K$, where K is a constant not specified. Assuming K = 1, one possible solution of many is:



33. One possible solution of many:



34. To get the average voltage value, we want $v_{out} = \frac{v_1 + v_2 + v_3}{3}$. This voltage stays positive and therefore a one stage summing circuit (which inverts the voltage) is not sufficient. Using the cascade setup as shown figure 6.15 and modified for three inputs we have:



The nodal equation at the inverting input of the first op-amp gives

$$\frac{v_1}{R_1} + \frac{v_2}{R_2} + \frac{v_3}{R_3} = \frac{-v_o}{R_{f_1}}$$

If we assume $R_1=R_2=R_3=R$, then

$$v_0 = -R_{f_1} \frac{v_1 + v_2 + v_3}{R}$$

Using the nodal equation at the inverting input of the second op-amp, we have:

$$\frac{-v_{out}}{R_{f_2}} = \frac{v_o}{R_4} = \frac{-R_{f_1}}{R_4} \frac{v_1 + v_2 + v_3}{R}$$

Or,

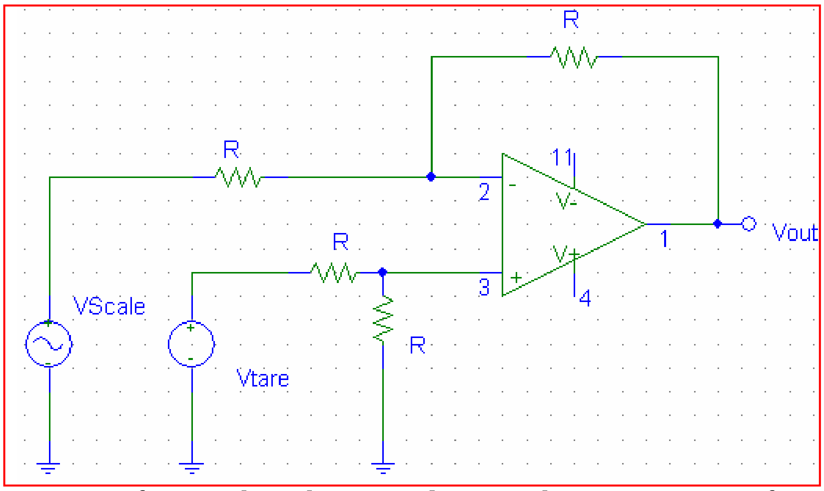
$$v_{out} = \frac{R_{f2} R_{f_1}}{R_4} \frac{v_1 + v_2 + v_3}{R}$$

For simplicity, we can take $R_{f2} = R_{f1} = R_4 = R_x$, then, to give a voltage average,

$$v_{out} = R_x \frac{v_1 + v_2 + v_3}{R} = \frac{v_1 + v_2 + v_3}{3}$$

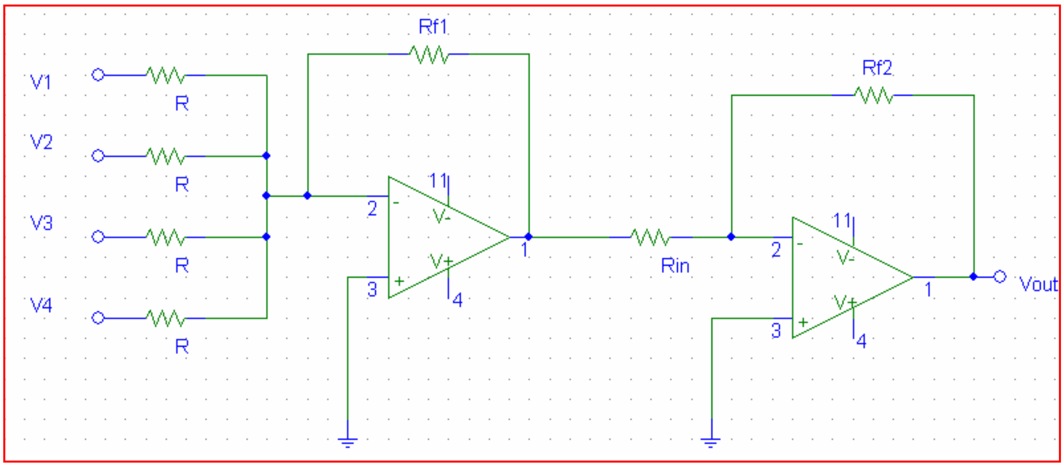
I.e. $R_x/R = 3$. Therefore, the circuit can be completed with $R_1 = R_2 = R_3 = 30$ kΩ and $R_{f2} = R_{f1} = R_4 = 10$ kΩ

35. The first stage is to subtract each voltage signal from the scale by the voltage corresponding to the weight of the pallet (V_{tare}). This can be done by using a differential amplifier:



The resistance of R can be arbitary as long as they resistances of each resistor is the same and the current rating is not exceeded. A good choice would be R = 10 kΩ.

The output voltage of the differential amps from each of the scale, $V_1 – V_4$ (now gives the weight of the items only), is then added by using a two stage summing amplifier:

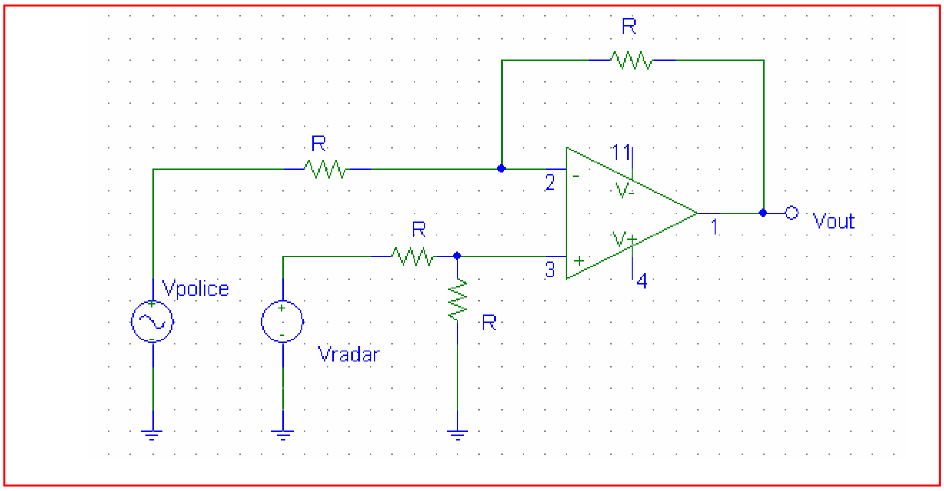


The output is given by:

$$v_{out} = \frac{R_{f2}R_{f_1}}{R_{in}} \frac{v_1 + v_2 + v_3 + v_4}{R}$$

Therefore, to get the sum of the voltages v_1 to v_4, we only need to set all resistances to be equal, so setting $R_{f2} = R_{f1} = R_{in} = R = 10$ kΩ would give an output that is proportional to the total weight of the items

36. a) Using a difference amplifier, we can provide a voltage that is the difference between the radar gun output and police speedometer output, which is proportional to the speed difference between the targeted car and the police car. Note that since a positive voltage is required which the police car is slower, the police speedometer voltage would be feed into the inverting input:

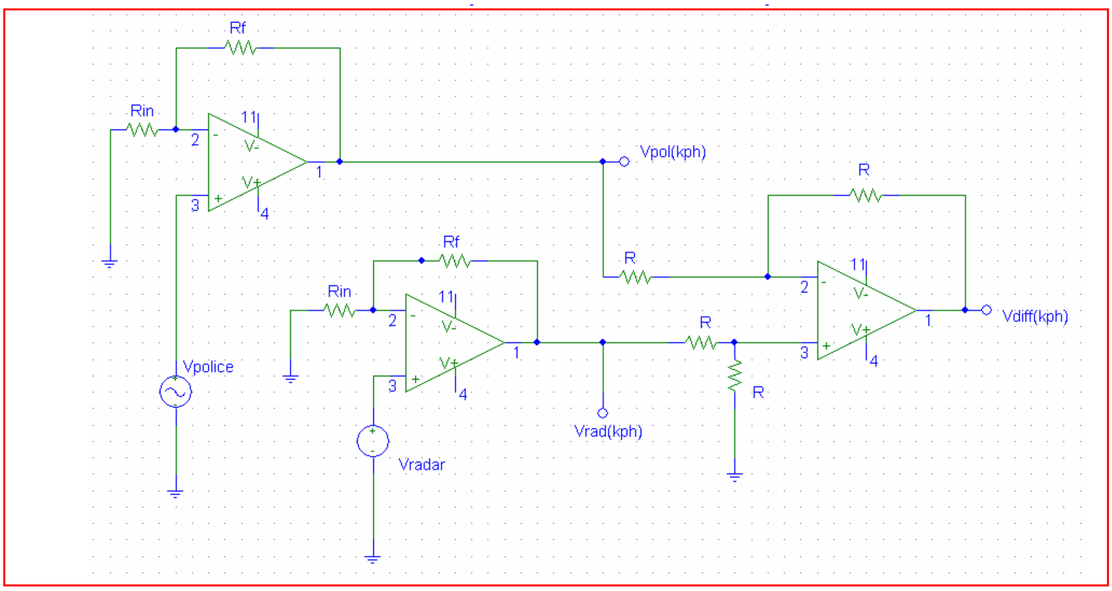


Again, R can be arbitary as long as they are equal and doesn't give an excessive current. 10 kΩ is a good choice here.

b) To convert to kph (km per hour) from mph (miles per hour), it is noted that 1 mph = 1.609 kph. Therefore, the voltage output from each device must be multiplied by 1.609. This can be done by using a non-inverting amplifier, which has an output given by:

$$v_{out} = (1 + \frac{R_f}{R_{in}})v_{in} = 1.609 v_{in}$$

This gives Rf/Rin =0.609 ≈ 61/100, i.e. $\boxed{R_f = 6.2\text{ k}\Omega \text{ and } R_{in} = 10\text{ k}\Omega}$



37. v_{out} of stage 1 is (1)(-20/ 2) = -10 V.

v_{out} of stage 2 is (-10)(-1000/ 10) = $\boxed{1000 \text{ V}}$

Note: in reality, the output voltage will be limited to a value less than that used to power the op amps.

38. We have a difference amplifier as the first amplifier stage, and a simple voltage follower as the second stage. We therefore need only to find the output voltage of the first stage: v_{out} will track this voltage. Using voltage division, then, we find that the voltage at the non-inverting input pin of the first op amp is:

$$V_2\left(\frac{R_3}{R_2 + R_3}\right)$$

and this is the voltage at the inverting input terminal also. Thus, we may write a single nodal equation at the inverting input of the first op amp:

$$0 = \frac{1}{R_1}\left[V_2\left(\frac{R_3}{R_2 + R_3}\right) - V_1\right] + \frac{1}{R_f}\left[V_2\left(\frac{R_3}{R_2 + R_3}\right) - V_{out}\Big|_{Stage\,1}\right]$$

which may be solved to obtain:

$$\boxed{V_{out} = V_{out}\Big|_{Stage\,1} = \left(\frac{R_f}{R_1} + 1\right)\frac{R_3}{R_2 + R_3}V_2 - \frac{R_f}{R_1}V_1}$$

39. The output of the first op amp stage may be found by realising that the voltage at the non-inverting input (and hence the voltage at the *inverting* input) is 0, and writing a ingle nodal equation at the inverting input:

$$0 = \frac{0 - V_{out}\big|_{stage\,1}}{47} + \frac{0-2}{1} + \frac{0-3}{7} \quad \text{which leads to} \quad V_{out}\big|_{steage\,1} = -114.1 \text{ V}$$

This voltage appears at the input of the second op amp stage, which has a gain of –3/ 0.3 = 10. Thus, the output of the second op amp stage is –10(-114.1) = 1141 V. This voltage appears at the input of the final op amp stage, which has a gain of –47/ 0.3 = -156.7.

Thus, the output of the circuit is –156.7(1141) = $\boxed{\text{-178.8 kV}}$, which is completely and utterly ridiculous.

40. The output of the top left stage is –1(10/ 2) = -5 V.
The output of the middle left stage is –2(10/ 2) = -10 V.
The output of the bottom right stage is –3(10/ 2) = -15 V.

These three voltages are the input to a summing amplifier such that

$$V_{out} = -\frac{R}{100}(-5-10-15) = 10$$

Solving, we find that $\boxed{R = 33.33\ \Omega.}$

41. Stage 1 is configured as a voltage follower: the output voltage will be equal to the input voltage. Using voltage division, the voltage at the non-inverting input (and hence at the inverting input, as well), is

$$5\frac{50}{100+50} = 1.667 \text{ V}$$

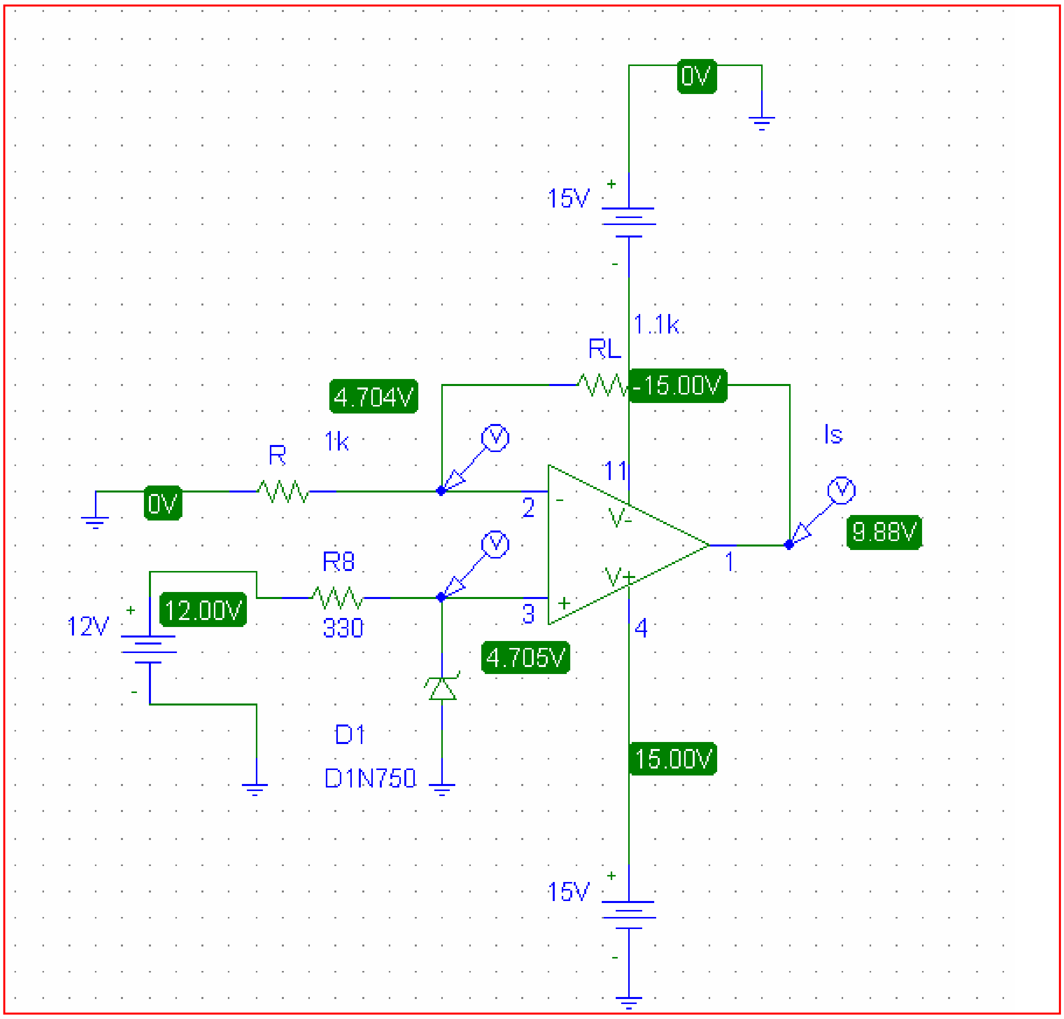
The second stage is wired as a voltage follower also, so $\boxed{v_{out} = 1.667 \text{ V.}}$

42. a) Since the voltage supply is higher than the Zener voltage of the diode, the diode is operating in the breakdown region. This means V_2 = 4.7 V, and assuming ideal op-amp, $V_1 = V_2$= 4.7 V. This gives a nodal equation at the inverting input:

$$\frac{4.7}{1k} = \frac{V_3 - 4.7}{1.1k}$$

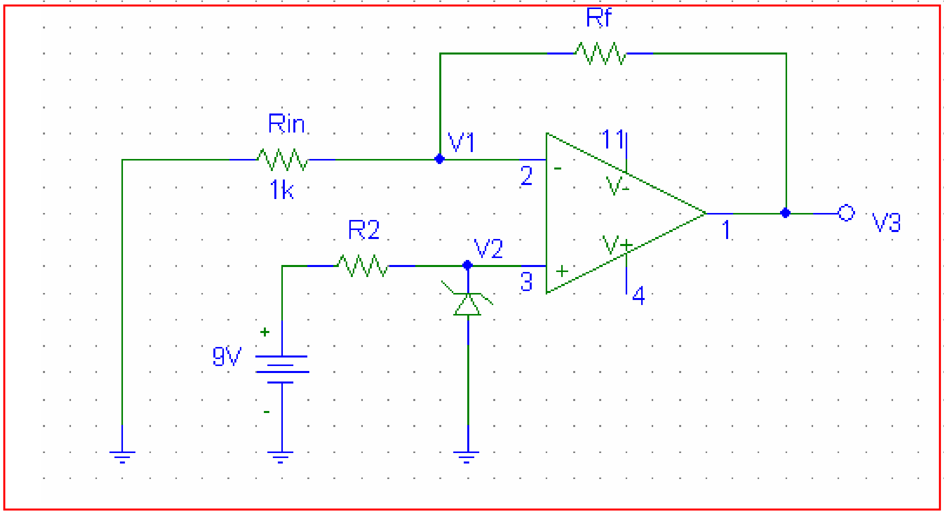
Solving this gives $\boxed{V_3 = 9.87 \text{ V}}$

b) PSpice simulation gives:



It can be seen that all voltage values are very close to what was calculated. The voltage output V_3 is 9.88Vinstead of 9.87 V. This can be explained by the fact that the operating voltage is slightly higher than the breakdown voltage, and also the non-ideal characteristics of the op-amp.

43. The following circuit can be used:



The circuit is governed by the equations:

$$V_3 = (1 + \frac{R_f}{R_{in}})V_1$$

And

$$V_1 = V_2 = V_{diode}$$

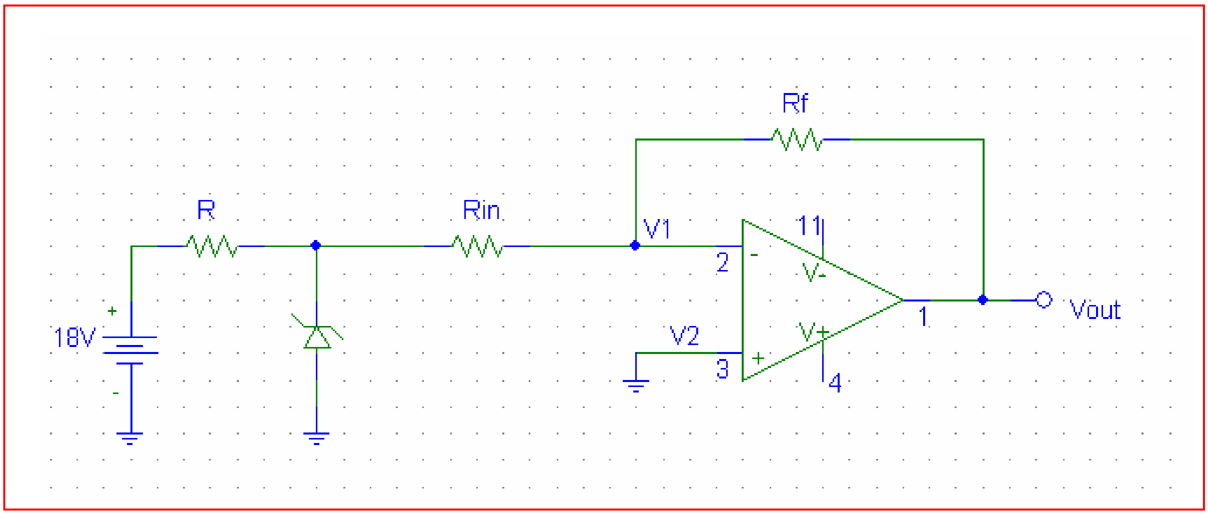
Since the diode voltage is 5.1 V, and the desired output voltage is 5.1 V, we have Rf/Rin = 0. In other words, a voltage follower is needed with $\boxed{R_f = 0\Omega}$, and R_{in} can be arbitary – $\boxed{R_{in} = 100\ k\Omega}$ would be sufficient.

The resistor value of R2 is determined by:

$$R_2 = \frac{V_s - V_{diode}}{I_{ref}}$$

At a voltage of 5.1 V, the current is 76 mA, as described in the problem. This gives $\boxed{R_2 \approx 51\ \Omega}$ using standard resistor values.

44. For the Zener diode to operate in the breakdown region, a voltage supply greater than the breakdown voltage, in this case 10 V is needed. With only 9 V batteries, the easiest way is the stack two battery to give a 18 V power supply. Also, as the input is inverted, an inverting amplifier would be needed. Hence we have the following circuit:

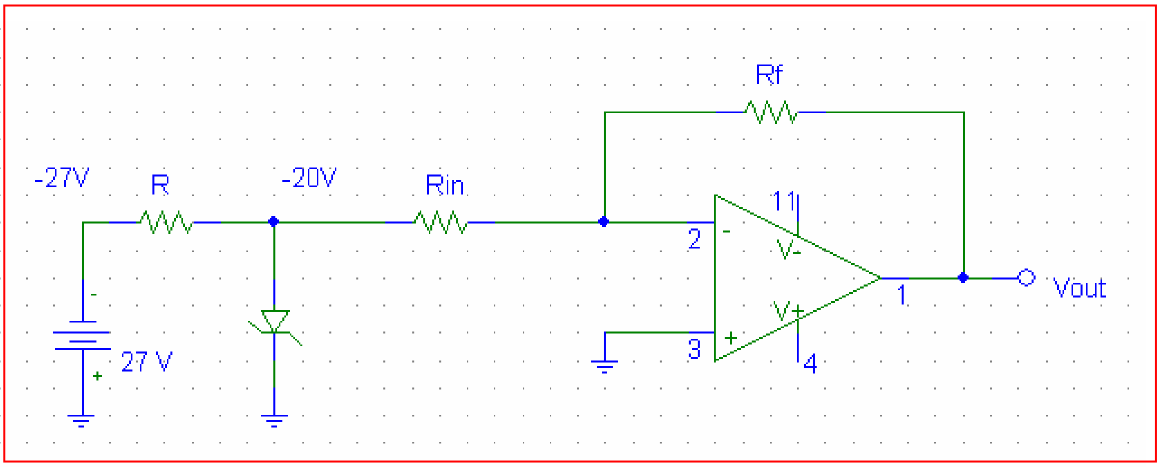


$$V_{out} = -\frac{R_f}{R_{in}} V_{in}$$

Here, the input voltage is the diode voltage = 10 V, and the desired output voltage is -2.5 V. This gives R_f/R_{in} = 25 / 100 = 50 / 200, or R_f = 51 kΩ and R_{in} = 200 kΩ using standard values. Note that large values are chosen so that most current flow through the Zener diode to provide sufficient current for breakdown condition.

The resistance R is given by R = (18-10) V /25 mA = 320 Ω = 330 Ω using standard values.

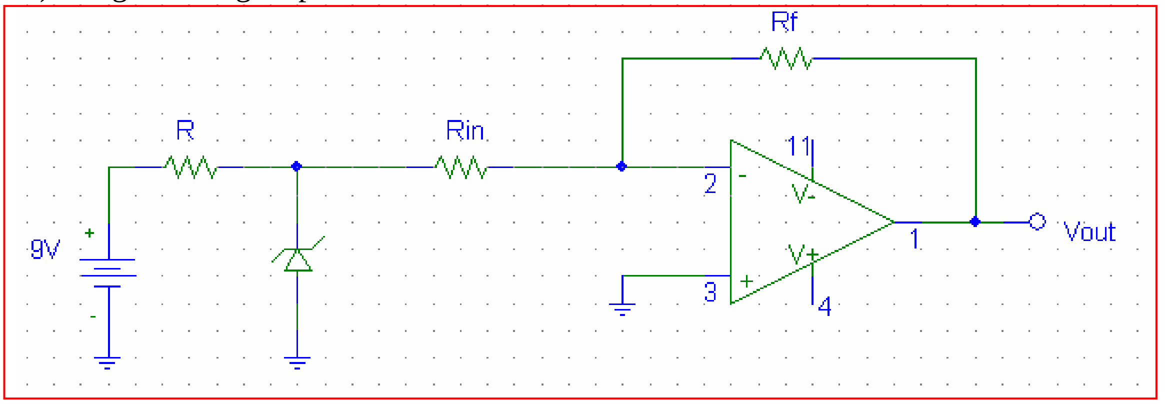
45. For a 20 V Zener diode, three 9 V batteries giving a voltage of 27 V would be needed. However, because the required voltage is smaller than the Zener voltage, a non-inverting amplifier can not be used. To use a inverting amplifier to give a positive voltage, we first need to invert the input to give a negative input:



In this circuit, the diode is flipped but so is the power supply, therefore keeping the diode in the breakdown region, giving V_{in} = -20 V. Then, using the inverting amp equation, we have R_f / R_{in} = 12/20 giving $\boxed{R_f = 120\ \text{k}\Omega \text{ and } R_{in} = 200\ \text{k}\Omega}$ using standard resistor values.

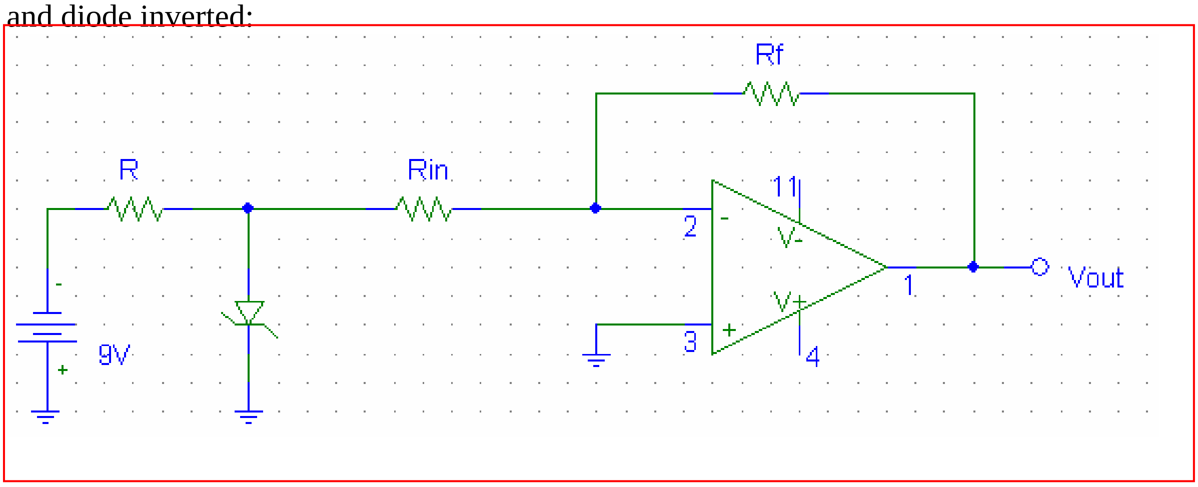
The resistance R is then given by $\boxed{R = (-20 - -27)\ \text{V} / 12.5\ \text{mA} = 560\ \Omega}$ using standard resistor values.

46. a) using inverting amplifier:



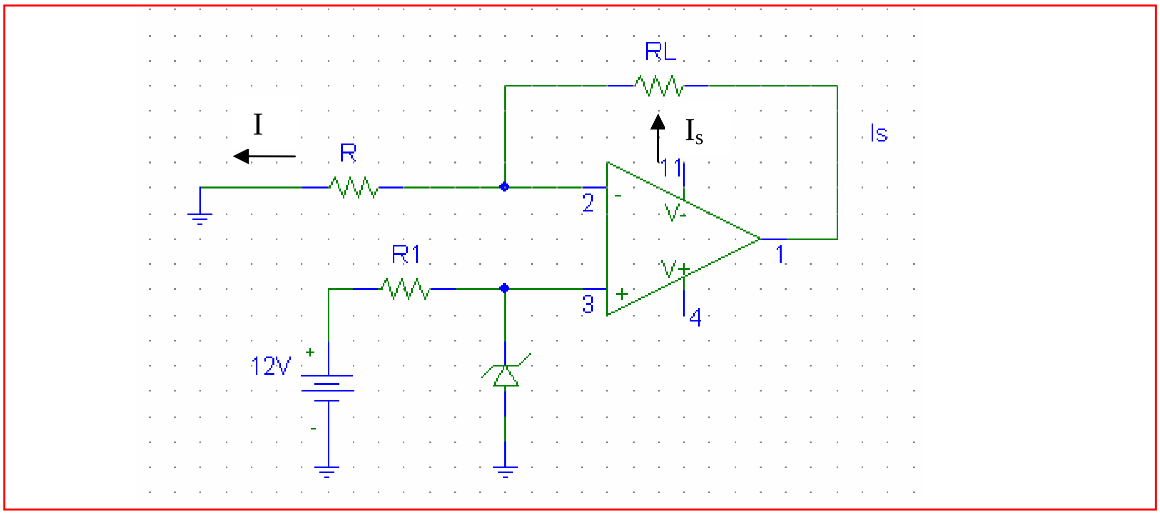
R_f/R_{in} = 5/3.3 giving R_f = 51 kΩ and R_{in} = 33 kΩ. R = (9-3.3) V / 76 mA = 75 Ω using standard resistor values.

b) To give a voltage output of +2.2 V instead, the same setup can be used, with supply and diode inverted:



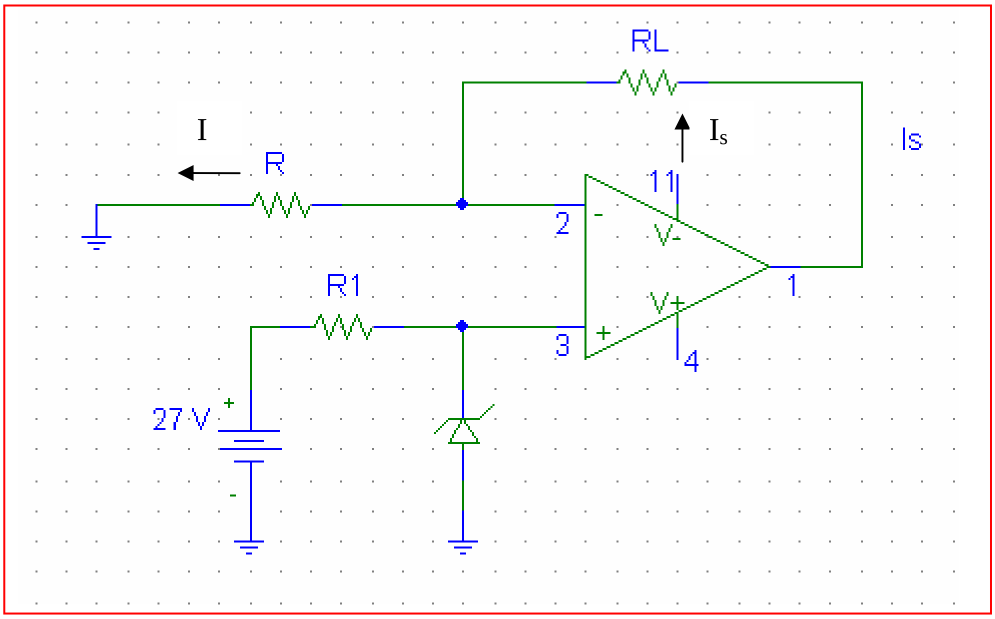
Correspondingly, the resistor values needs to be changed:
Rf / Rin = 3.3/2.2 giving R_f = 33 kΩ and R_{in} = 22 kΩ. R would be the same as before as the voltage difference between supply and diode stays the same i.e. R = 75 Ω.

47. The following setup can be used:



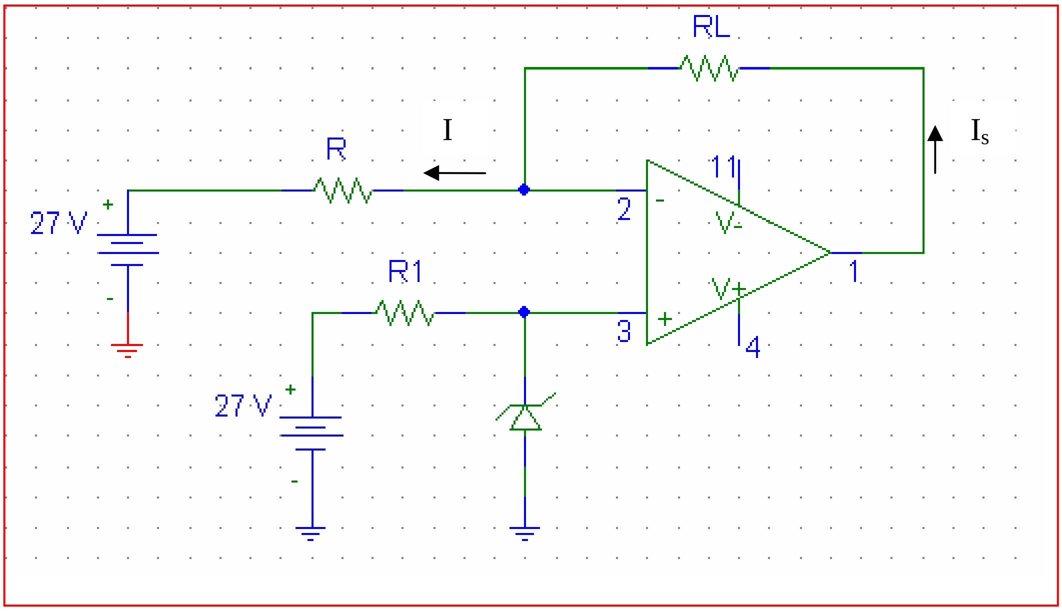
$I_s = I = 10$ V / R = 25 mA assuming ideal op-amp. This gives R = 400 Ω. Again, taking half of max current rating as the operating current, we get $R_1 = (12 – 10)$ V / 25 mA = 80 Ω = 82 Ω using standard values.

48. Using the following current source circuit, we have:



$I_s = I = 12.5$ mA = 20 V / R, assuming ideal op-amp. This gives R = 1.6 kΩ and R_1 = (27 – 20) / 12.5 mA = 560 Ω.

49. In this situation, we know that there is a supply limit at ±15 V, which is lower than the zener diode voltage. Therefore, previous designs need to be modified to suit this application. One possible solution is shown here:

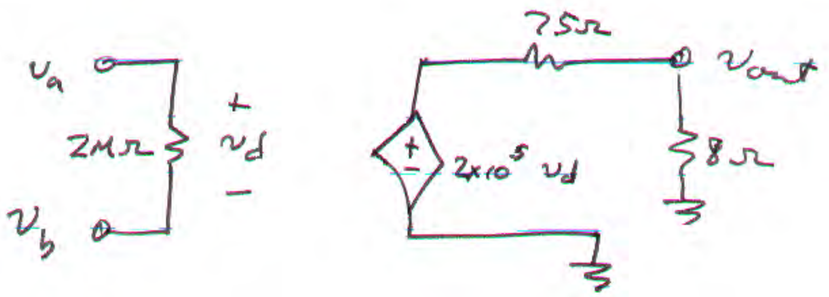


Here, we have Is = I = (27-20)/ R = 75 mA, assuming infinite op-amp input resistance. This gives R = 93.3 ≈ 91 Ω using standard values. We also have R1 = (27-20)/ 12.5 mA = 560 Ω.

Now look at the range of possible loads. The maximum output voltage is approximately equal to the supply voltage, i.e. 15 V. Therefore, the minimum load is given by R_L = (20 – 15) V / 75 mA = 66.67 Ω. Similarly, the maximum load is given by RL = (20 - -15) V/ 75 mA = 466.67 Ω. i.e. this design is suitable for

$$466.67\ \Omega > R_L > 66.67\ \Omega.$$

50.



(a) $v_a = v_b = 1$ nV $\therefore$ $v_d = 0$ and $v_{out} = 0$. Thus, $\boxed{P_{8\Omega} = 0 \text{ W.}}$

(b) $v_a = 0$, $v_b = 1$ nV $\therefore$ $v_d = -1$ nV

$v_{out} = (2\times10^5)(-1\times10^{-9})\dfrac{8}{75+8} = -19.28\ \mu\text{V}$. Thus, $P_{8\Omega} = \dfrac{v_{out}^2}{8} = \boxed{46.46 \text{ pW.}}$

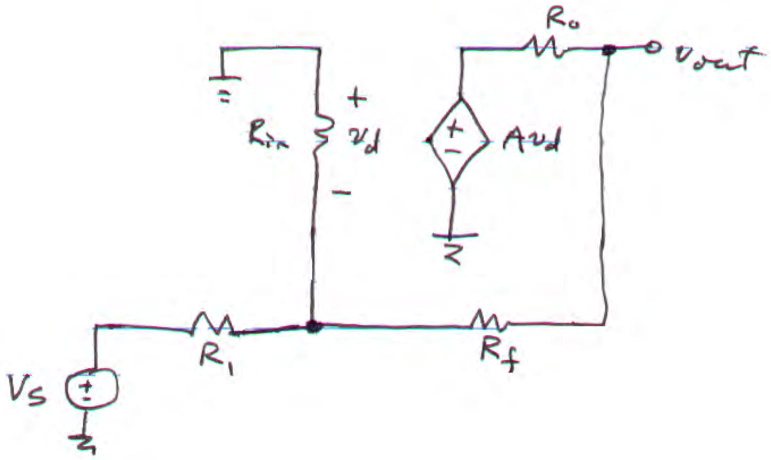
(c) $v_a = 2$ pV, $v_b = 1$ fV $\therefore$ $v_d = 1.999$ pV

$v_{out} = (2\times10^5)(1.999\times10^{-12})\dfrac{8}{75+8} = 38.53$ nV. Thus, $P_{8\Omega} = \dfrac{v_{out}^2}{8} = \boxed{185.6 \text{ aW}}.$

(c) $v_a = 50\ \mu$V, $v_b = -4\ \mu$V $\therefore$ $v_d = 54\ \mu$V

$v_{out} = (2\times10^5)(54\times10^{-6})\dfrac{8}{75+8} = 1.041$ V. Thus, $P_{8\Omega} = \dfrac{v_{out}^2}{8} = \boxed{135.5 \text{ mW.}}$

51.



Writing a nodal equation at the "-v_d" node,

$$0 = \frac{-v_d}{R_{in}} + \frac{-v_d - V_S}{R_1} + \frac{-v_d - v_{out}}{R_f} \qquad [1]$$

or $\quad (R_1R_f + R_{in}R_f + R_{in}R_1)\, v_d + R_{in}R_1 v_{out} = -R_{in}R_f V_S \quad [1]$

Writing a nodal equation at the "v_{out}" node,

$$0 = \frac{-v_{out} - Av_d}{R_o} + \frac{v_{out} - (-v_d)}{R_f} \qquad [2]$$

Eqn. [2] can be rewritten as:

$$v_d = \frac{-(R_f + R_o)}{R_o - AR_f}\, v_{out} \qquad [2]$$

so that Eqn. [1] becomes:

$$v_{out} = -\frac{R_{in}(AR_f - R_o)V_S}{AR_{in}R_1 + R_fR_1 + R_{in}R_f + R_{in}R_1 + R_oR_1 + R_oR_{in}}$$

where for this circuit, $A = 10^6$, $R_{in} = 10\ \text{T}\Omega$, $R_o = 15\ \Omega$, $R_f = 1000\ \text{k}\Omega$, $R_1 = 270\ \text{k}\Omega$.

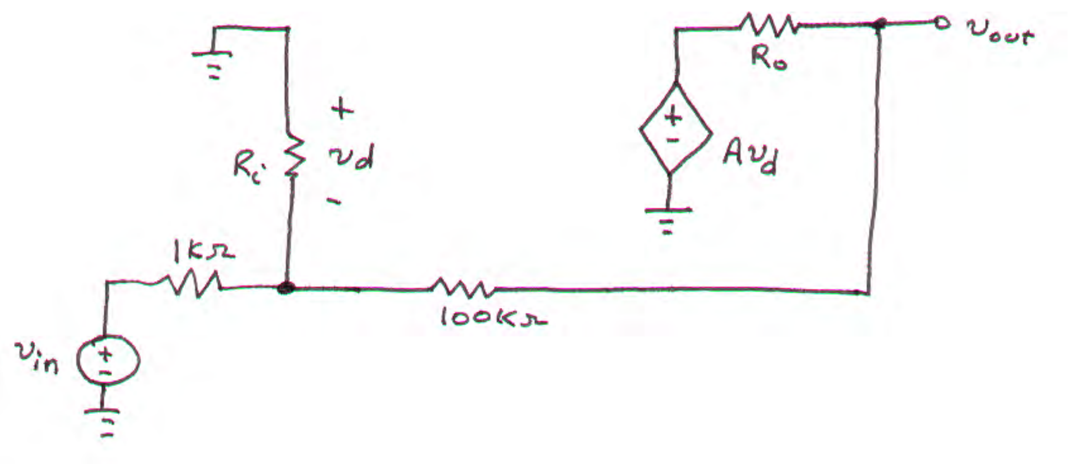
(a) –3.704 mV; (b) 27.78 mV; (c) –3.704 V.

52. $v_{out} = Av_d = A\dfrac{R_i}{16+R_i}\left(80\times10^{15}\right)\sin 2t\ \ \text{V}$

(a) A = 10^5, R_i = 100 MΩ, R_o value irrelevant. $\boxed{v_{out} = 8 \sin 2t \text{ nV}}$

(b) A = 10^6, R_i = 1 TΩ, R_o value irrelevant. $\boxed{v_{out} = 80 \sin 2t \text{ nV}}$

53.



(a) Find v_{out}/ v_{in} if $R_i = \infty$, $R_o = 0$, and A is finite.

The nodal equation at the inverting input is

$$0 = \frac{-v_d - v_{in}}{1} + \frac{-v_d - v_{out}}{100} \qquad [1]$$

At the output, with $R_o = 0$ we may write $v_{out} = Av_d$ so $v_d = v_{out}/ A$. Thus, Eqn. [1] becomes

$$0 = \frac{v_{out}}{A} + v_{in} + \frac{v_{out}}{100A} + \frac{v_{out}}{100}$$

from which we find

$$\boxed{\frac{v_{out}}{v_{in}} = \frac{-100A}{101 + A}} \qquad [2]$$

(b) We want the value of A such that $v_{out}/ v_{in} = -99$ (the “ideal” value would be –100 if A were infinite). Substituting into Eqn. [2], we find

$$\boxed{A = 9999}$$

54. (a) $\delta = 0$ V $\therefore$ $v_d = 0$, and $\boxed{P_{8\Omega} = 0 \text{ W.}}$

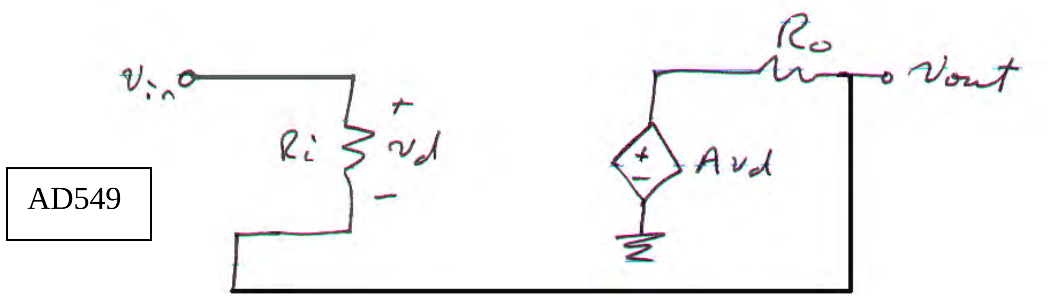
(b) $\delta = 1$ nV, so $v_d = 5 - (5 + 10^{-9}) = -10^{-9}$ V
Thus,

$$v_{out} = (2\times10^5)v_d \frac{8}{8+75} = -19.28\ \mu\text{V and } P_{8\Omega} = (v_{out})^2/\ 8 = \boxed{46.46\text{ pW.}}$$

(c) $\delta = 2.5$ μV, so $v_d = 5 - (5 + 2.5\times10^{-6}) = -2.5\times10^{-6}$V
Thus,

$$v_{out} = (2\times10^5)v_d \frac{8}{8+75} = -48.19\text{ mV and } P_{8\Omega} = (v_{out})^2/\ 8 = \boxed{290.3\ \mu\text{W.}}$$

55.



Writing a single nodal equation at the output, we find that

$$0 = \frac{v_{out} - v_{in}}{R_i} + \frac{v_{out} - Av_d}{R_o} \qquad [1]$$

Also, $v_{in} - v_{out} = v_d$, so Eqn. [1] becomes

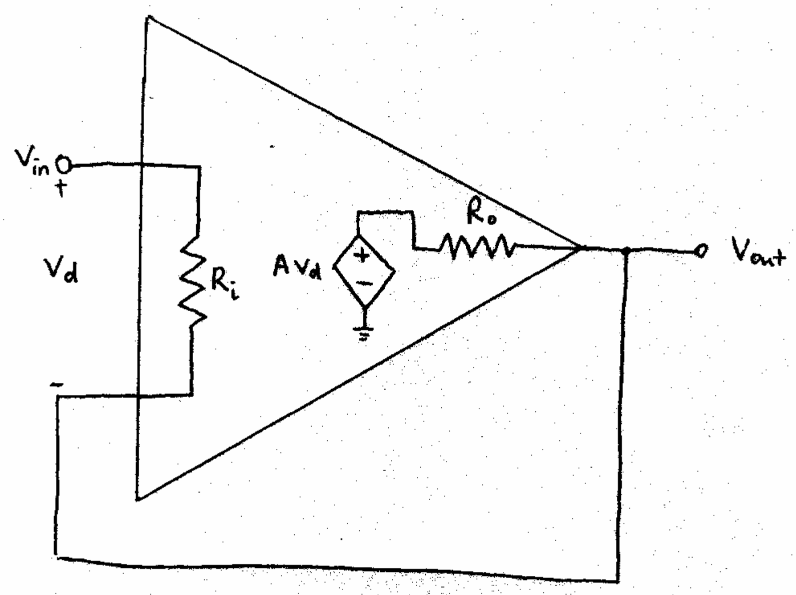
$$0 = (v_{out} - v_{in})\, R_o + (v_{out} - Av_{in} + Av_{out})\, R_i$$

and

$$v_{out} = \frac{(R_o + AR_i)}{R_o + (A+1)\, R_i}\, v_{in}$$

To within 4 significant figures (and more, actually), when v_{in} = -16 mV, $\boxed{v_{out} = -16 \text{ mV}}$ (this is, after all, a voltage follower circuit).

56. The Voltage follower with a finite op-amp model is shown below:



Nodal equation at the op-amp output gives:

$$\frac{V_{out} - V_{in}}{R_i} = \frac{AV_d - V_{out}}{R_o}$$

But in this circuit, $V_d = V_{in} - V_{out}$. Substitution gives:

$$\frac{V_{out} - V_{in}}{R_i} = \frac{A(V_{in} - V_{out}) - V_{out}}{R_o} = \frac{AV_{in} - (A+1)V_{out}}{R_o}$$

Further rearranging gives:

$$R_o V_{out} + R_i (A+1) V_{out} = R_o V_{in} + R_i A V_{in}$$

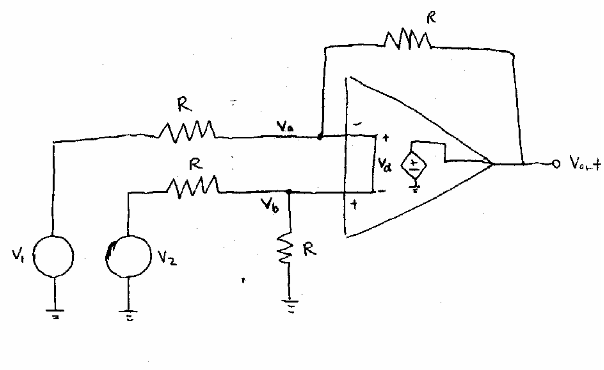
$$\Leftrightarrow \quad \boxed{V_{out} = \frac{R_o + R_i A}{R_o + R_i (A+1)} V_{in}}$$

This is the expression for the voltage follower in non-ideal situation. In the case of ideal op-amp, $A \to \infty$, and so $A+1 \to A$. This means the denominator and the numerator would cancel out to give $V_{out} = V_{in}$, which is exactly what we expected.

57. a) By definition, when the op-amp is at common mode, $v_{out} = A_{CM}v_{in}$. Therefore, a model that can represent this is:

This model relies on that fact that A_{CM} is much smaller than the differential gain A, and therefore when the inputs are different, the contribution of A_{CM} is negligible. When the inputs are the same, however, the differential term Av_d vanishes, and so $v_{out} = A_{CM}v_2$, which is correct.

b) The voltage source in the circuit now becomes $10^5v_d+10v_2$. Assuming $R_o = 0$, the circuit in figure 6.25 becomes:



The circuit is governed by the following equations:

$$v_d = v_b - v_a \qquad \text{so} \qquad \frac{10^5 v_d + 10v_b - v_a}{R} = \frac{v_a - v_1}{R}$$

$$v_b = v_2 / 2 \ \text{(from voltage divider)}$$

Rearranging gives:

$$10^5(v_2/2 - v_a) + 10(v_2/2) - v_a = v_a - v_1$$

$$50000v_2 - 10^5 v_a + 5v_2 - v_a = v_a - v_1$$

$$v_a = \frac{50005v_2 + v_1}{(10^5+2)}$$

Then, the output is given by:

$$v_{out} = 10^5 v_d + 10v_b = 10^5 \times (v_2/2 - v_a) + 5v_2$$

in this case, $v_1 = 5 + 2\sin t$, $v_2 = 5$. This gives $v_a=0.50004v_2+9.99980\times10^{-6}v_1$. Thus

$v_{out} = 10^5(-0.00004v_2 - 9.99980\times10^{-6}v_1)+5v_2 = 1.00008v_2 - 0.99998v_1 = 0.0005 - 1.99996 \sin t$

c) If the common mode gain is 0, than the equation for v_a becomes

$$v_a = \frac{50000v_2 + v_1}{(10^5+2)}$$

Giving $v_a=0.49999v_2+9.99980\times10^{-6}v_1$. The output equation becomes

$v_{out} = 10^5 v_d = 10^5 \times (v_2/2 - v_a) = 0.99998v_2 - 0.99998v_1 = 1.99996 \sin t$

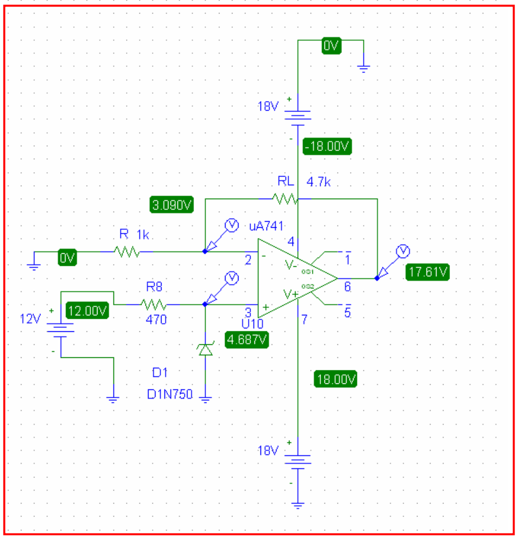
58. Slew rate is the rate at which output voltage can respond to changes in the input. The higher the slew rate, the faster the op-amp responds to changes. Limitation in slew rate – i.e. when the change in input is faster than the slew rate, causes degradation in performance of the op-amp as the change is delayed and output distorted.

59. a) V_2 = 4.7 V from the Zener diode, $V_1 = V_2$ = 4.7 V assuming ideal op-amp, and V_3 is given by the nodal equation at the inverting input:

$$\frac{V_3 - V_1}{4.7k} = \frac{V_1}{1k}$$

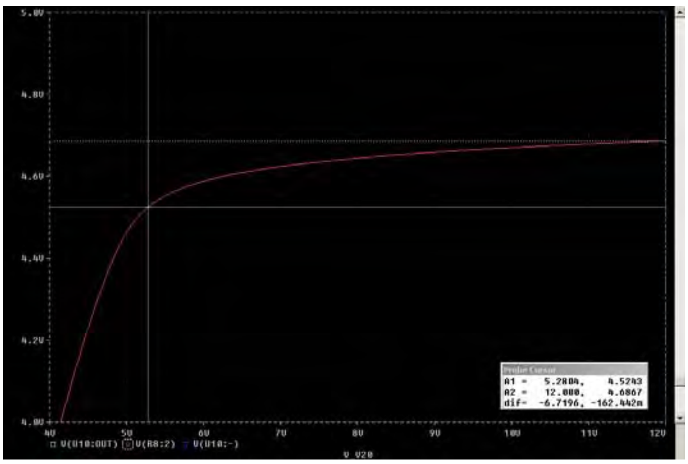
Solving gives V_3 = 26.79 V

b) The simulation result is shown below



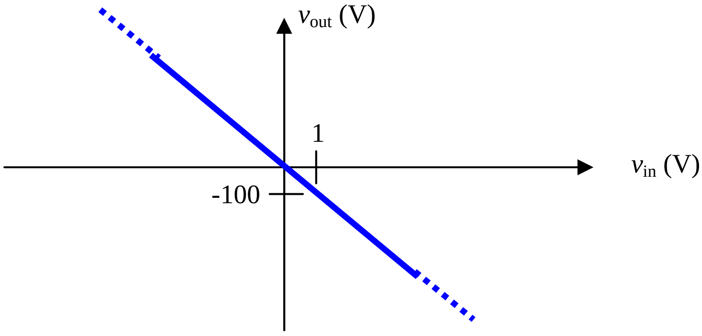
There are considerable discrepancies between calculated and simulated voltages. In particular, V_1 = 3.090 V is considerably lower than the expected 4.7 V. This is due to the non-ideal characteristics of uA741 which has a finite input resistance, inducing a voltage drop between the two input pins. A more severe limitation, however, is the supply voltage. Since the supply voltage is 18V, the output cannot exceed 18 V. This is consistent with the simulation result which gives V_3 = 17.61 V but is quite different to the calculated value as the mathematical model does not account for supply limitations.

c) By using a DC sweep, the voltage from the diode (i.e. V_2) was monitored as the battery voltage changes from 12 V to 4V.

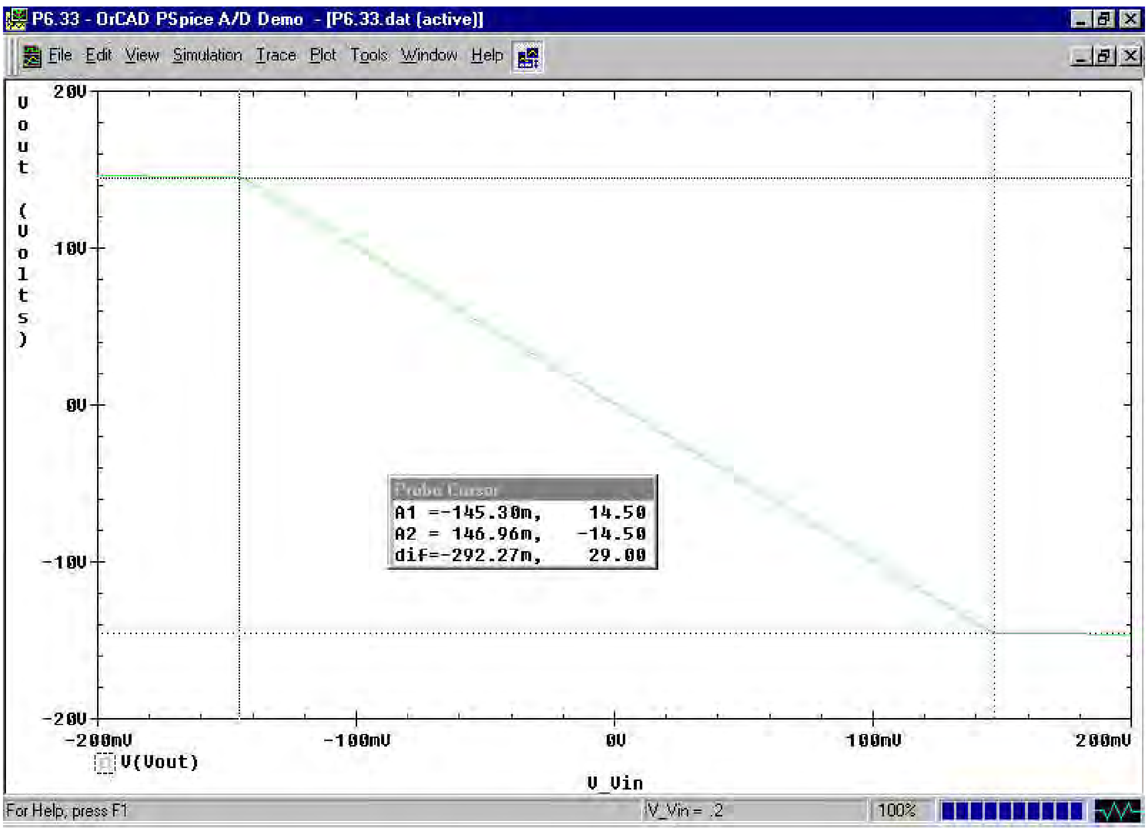


It can be seen that the diode voltage started dropping when batteries drop below 10 V. However, the diode can still be considered as operating in the breakdown region, until it hit the knee of the curve. This occurs at V_{supply} = 5.28 V

60. The ideal op amp model predicts a gain v_{out}/ v_{in} = -1000/ 10 = -100, regardless of the value of v_{in}. In other words, it predicts an input-output characteristic such as:



From the PSpice simulation result shown below, we see that the ideal op amp model is reasonably accurate for $|v_{in}| \times 100 < 15$ V (the supply voltage, assuming both have the same magnitude), but the onset of saturation is at ±14.5 V, or $|v_{in}| \sim 145$ mV. Increasing $|v_{in}|$ past this value does not lead to an increase in $|v_{out}|$.

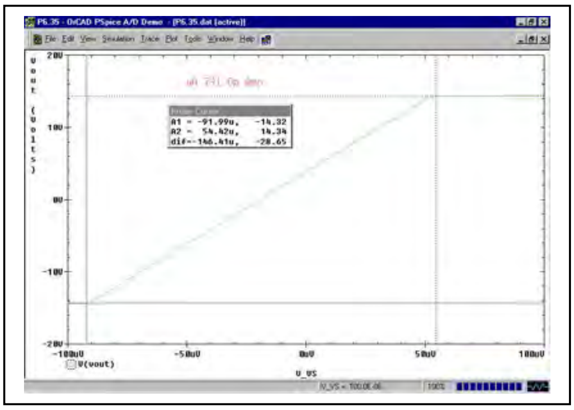


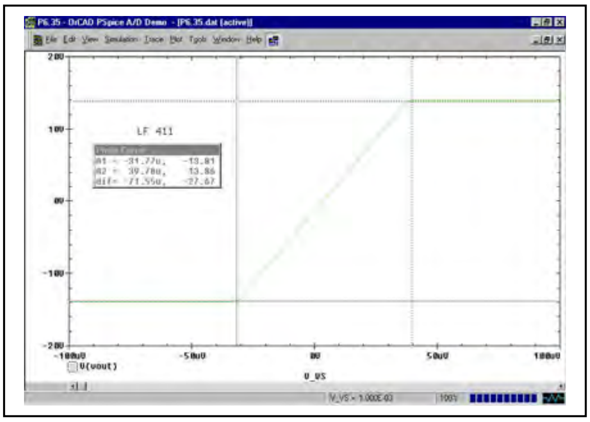
61. $\boxed{\text{Positive voltage supply, negative voltage supply, inverting input, ground, output pin.}}$

62. This op amp circuit is an open-loop circuit; there is no external feedback path from the output terminal to either input. Thus, the output should be the open-loop gain times the differential input voltage, minus any resistive losses.

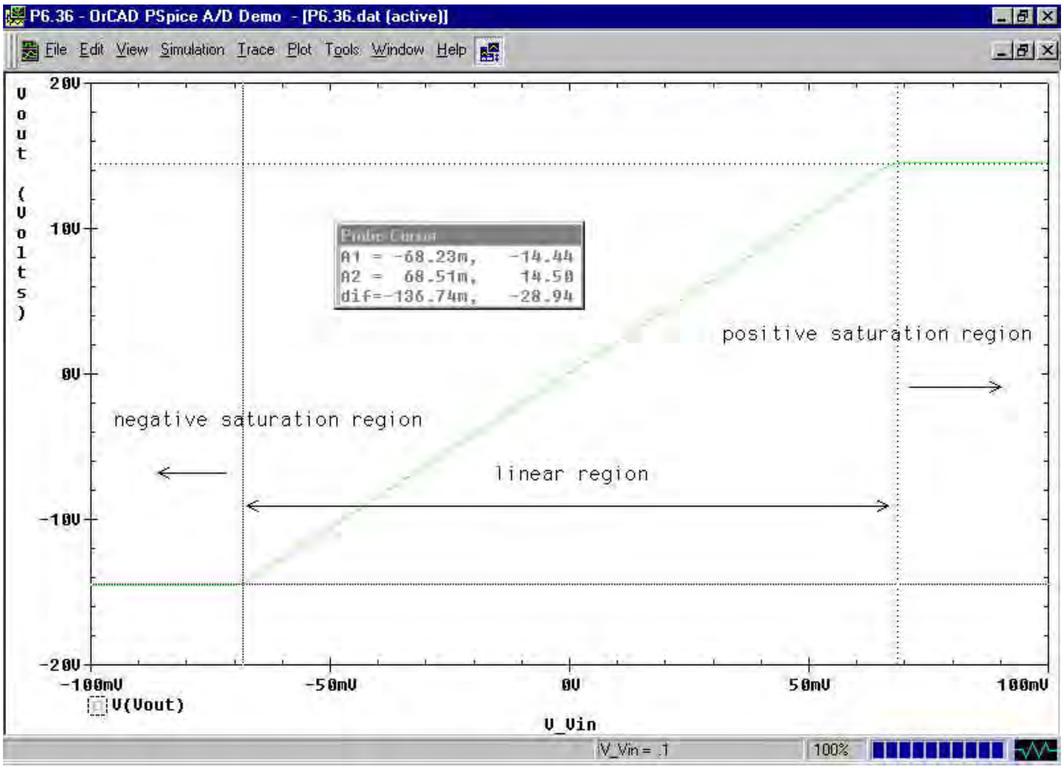
From the simulation results below, we see that all three op amps saturate at a voltage magnitude of approximately 14 V, corresponding to a differential input voltage of 50 to 100 μV, except in the interest case of the LM 324, which may be showing some unexpected input offset behavior.

op amp	**onset of negative saturation**	**negative saturation voltage**	**onset of positive saturation**	**positive saturation voltage**
μA 741	-92 μV	-14.32 V	54.4 mV	14.34 V
LM 324	41.3 μV	-14.71 V	337.2 mV	13.87 V
LF 411	-31.77 μV	-13.81 V	39.78 mV	13.86 V



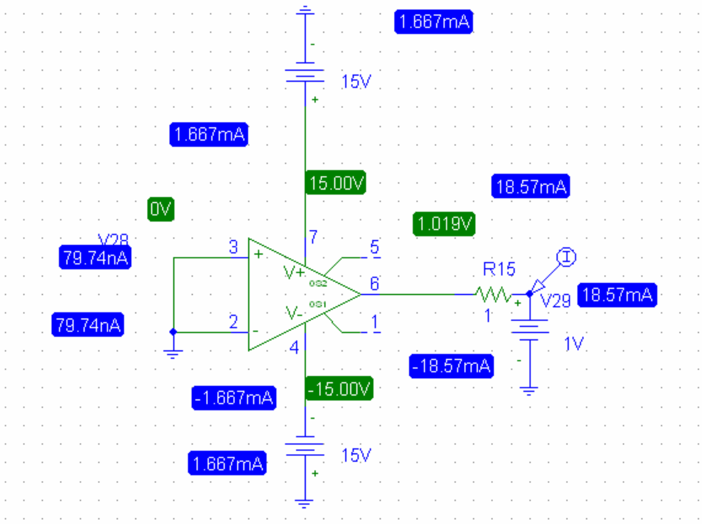


63. This is a non-inverting op amp circuit, so we expect a gain of 1 + 1000/4.7 = 213.8. With ±15 V DC supplies, we need to sweep the input just above and just below this value divided by the gain to ensure that we see the saturation regions. Performing the indicated simulation and a DC sweep from –0.1 V to +0.1 V with 0.001 V steps, we obtain the following input-output characteristic:



Using the cursor tool, we see that the linear region is in the range of
$-68.2 \text{ mV} < V_{in} < 68.5 \text{ mV}$.
The simulation predicts a gain of 7.103 V/ 32.87 mV = 216.1, which is reasonably close to the value predicted using the ideal op amp model.

64. To give a proper simulation, the inputs are grounded to give an input of 0. This gives:



As can be seen, a current of 18.57 mA is drawn from the uA741. Assuming the output voltage from the op-amp before R_o is 0, we have Ro = (1-18.57m)/18.57m = 52.9 Ω. This is close to the value given in table 6.3. There is difference between the two as here we are still using the assumption that the voltage output is independent to the loading circuit. This is illustrated by the fact that as the supplied voltage to the 1 ohm resistor changes, the voltage at the output pin actually increases, and is always higher than the voltage provided by the battery, as long as the supplied to the op-amp is greater than the battery voltage. When the supply voltage drops to 1V, the output current increased greatly and gave an output resistance of only 8 Ω. This suggests that the inner workings of the op-amp depend on both the supply and the loading.

For LF411, a current of 25.34 mA is drawn from the op-amp. This gives a output resistance of 38.4 Ω. This value is quite different to the 1 Ω figure given in the table.

65. Based on the detailed model of **the LF 411 op amp**, we can write the following nodal equation at the inverting input:

$$0 = \frac{-v_d}{R_{in}} + \frac{v_x - v_d}{10^4} + \frac{Av_d - v_d}{10^6 + R_o}$$

Substituting values for the LF 411 and simplifying, we make appropriate approximations and then solve for v_d in terms of v_x, finding that

$$v_d = \frac{-10^6}{199.9 \times 10^6} v_x = -\frac{v_x}{199.9}$$

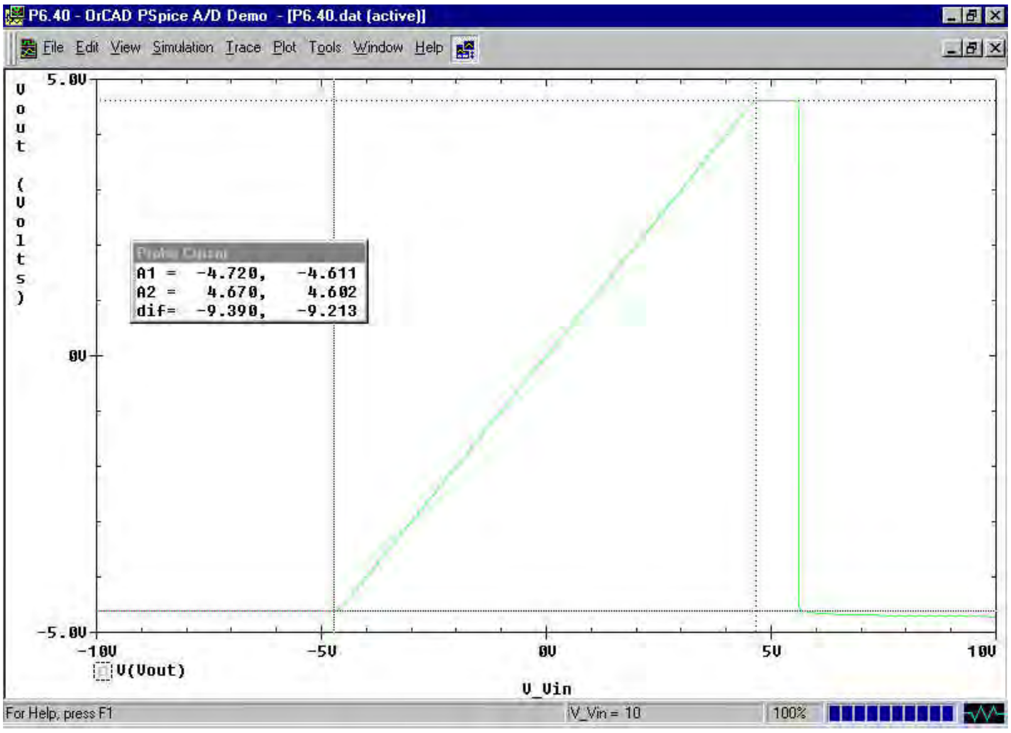
With a gain of –1000/10 = -100 and supply voltage magnitudes of 15 V, we are effectively limited to values of $|v_x| < 150$ mV.

For v_x = -10 mV, PSpice predicts v_d = 6 μV, where the hand calculations based on the detailed model predict 50 μV, which is about one order of magnitude larger. For the same input voltage, PSpice predicts an input current of -1 μA, whereas the hand calculations predict 99.5v_x mA = -995 nA (which is reasonably close).

66. (a) The gain of the inverting amplifier is –1000. At a sensor voltage of –30 mV, the predicted output voltage (assuming an ideal op amp) is +30 V. At a sensor voltage of +75 mV, the predicted output voltage (again assuming an ideal op amp) is –75 V. Since the op amp is being powered by dc sources with voltage magnitude equal to 15 V, the output voltage range will realistically be limited to the range $\boxed{-15 < V_{out} < 15 \text{ V.}}$

(b) The peak input voltage is 75 mV. Therefore, $15/\ 75\times10^{-3} = 200$, and we should set the resistance ratio $\boxed{R_f/\ R_1 < 199}$ to ensure the op amp does not saturate.

67. (a)

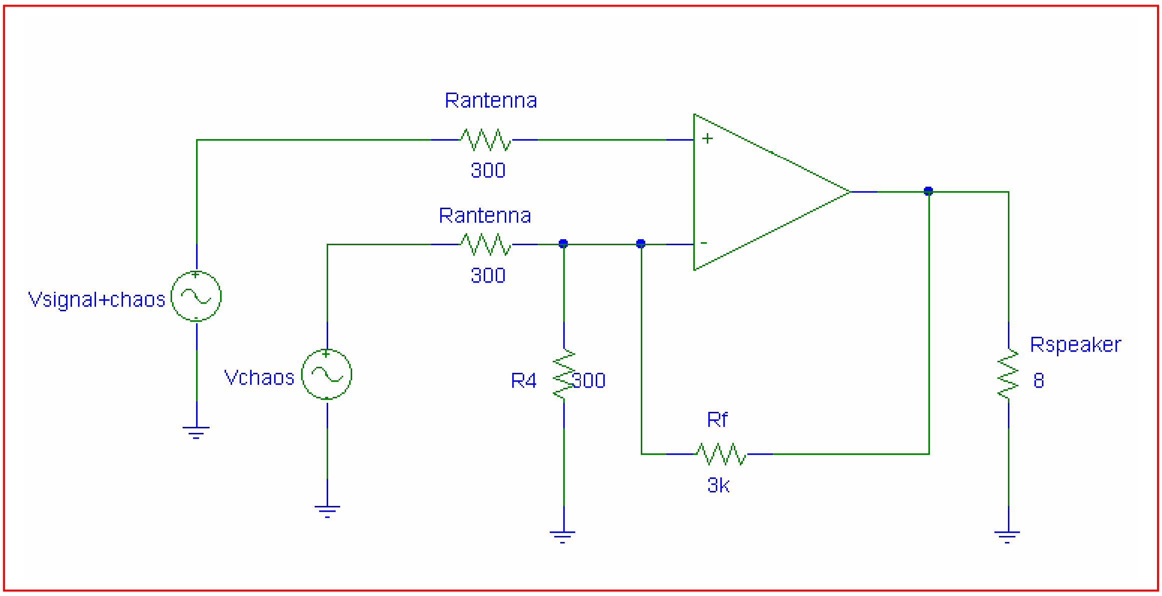


We see from the simulation result that negative saturation begins at V_{in} = –4.72 V, and positive saturation begins at V_{in} = +4.67 V.

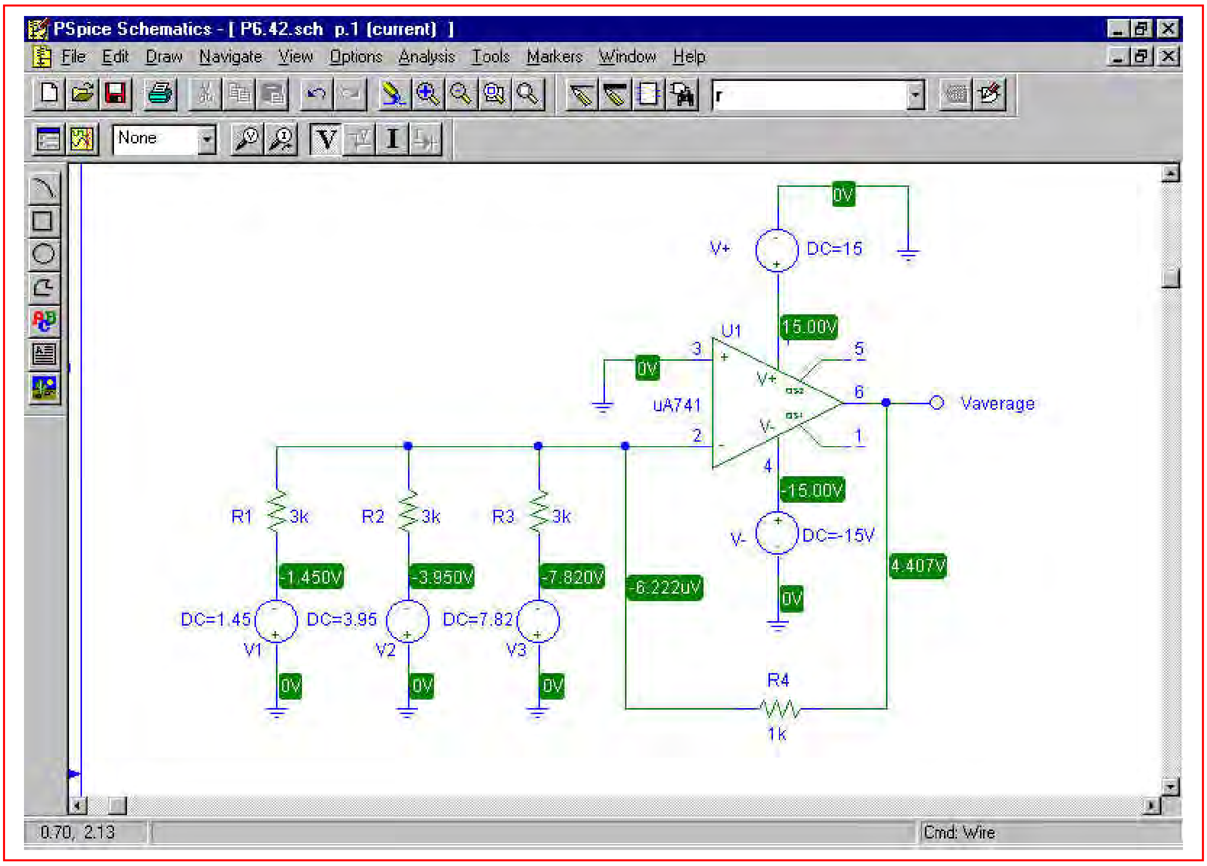
(b) Using a 1 pΩ resistor between the output pin and ground, we obtain an output current of 40.61 mA, slightly larger than the expected 35 mA, but not too far off.

68. We assume that the strength of the separately-broadcast chaotic "noise" signal is received at the appropriate intensity such that it may precisely cancel out the chaotic component of the total received signal; otherwise, a variable-gain stage would need to be added so that this could be adjusted by the user. We also assume that the signal frequency is separate from the "carrier" or broadcast frequency, and has already been separated out by an appropriate circuit (in a similar fashion, a radio station transmitting at 92 MHz is sending an audio signal of between 20 and 20 kHz, which must be separated from the 92 MHz frequency.)

One possible solution of many (all resistances in ohms):



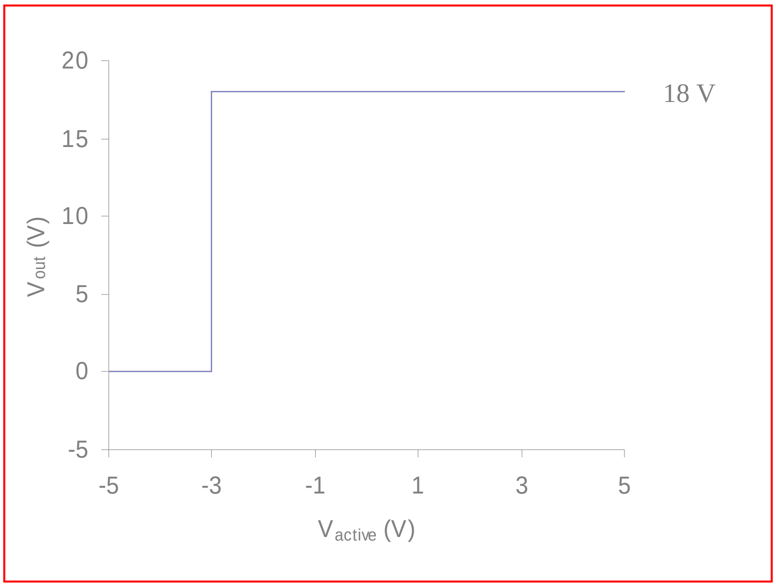
69. One possible solution of many:



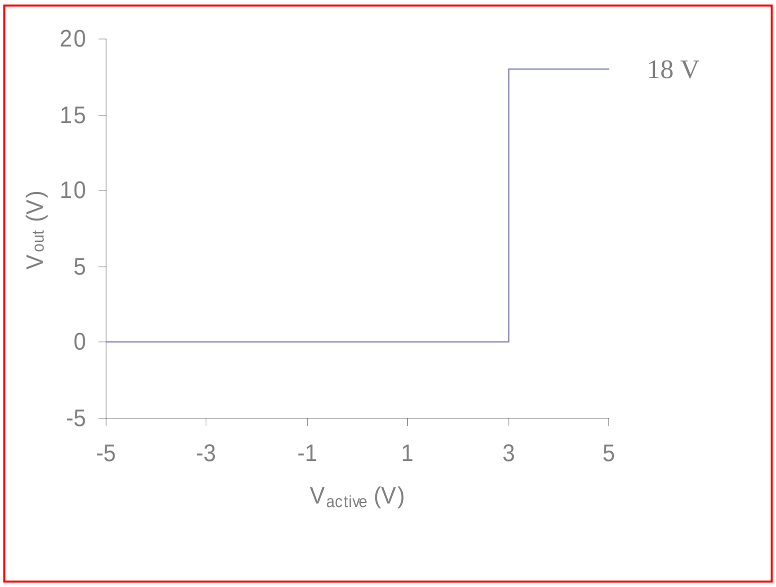
This circuit produces an output equal to the average of V_1, V_2, and V_3, as shown in the simulation result: $V_{average} = (1.45 + 3.95 + 7.82)/ 3 = 4.407$ V.

70. Assuming ideal situations (ie slew rate = infinite)

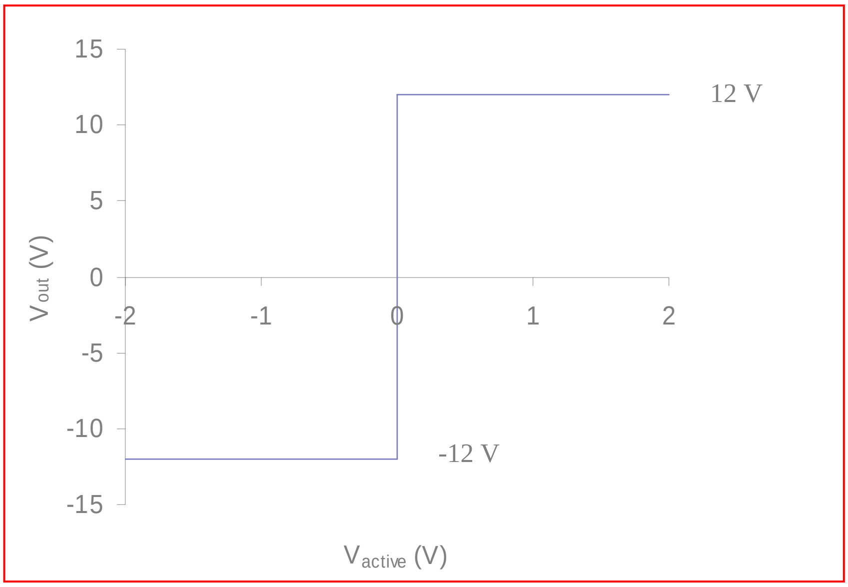
a)



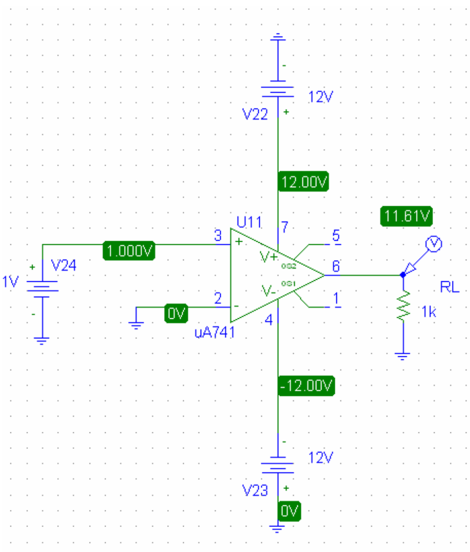
b)

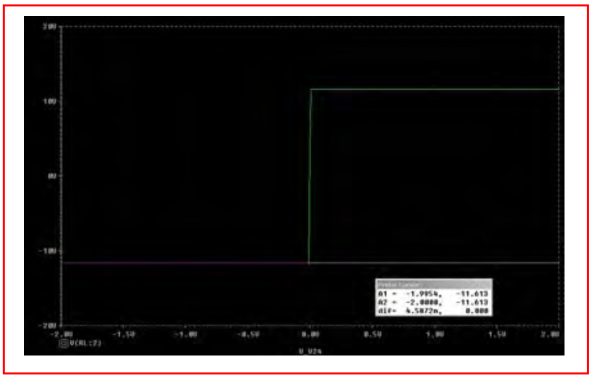


71. a)



b) The simulation is performed using the following circuit:

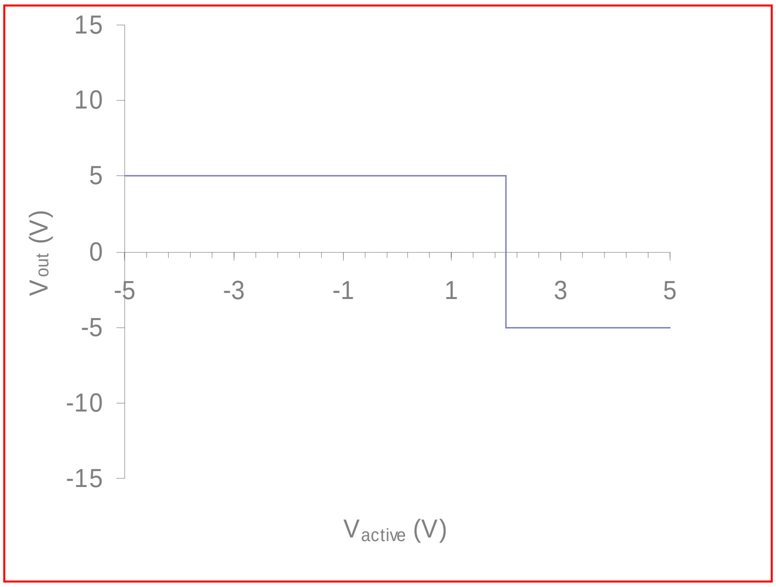




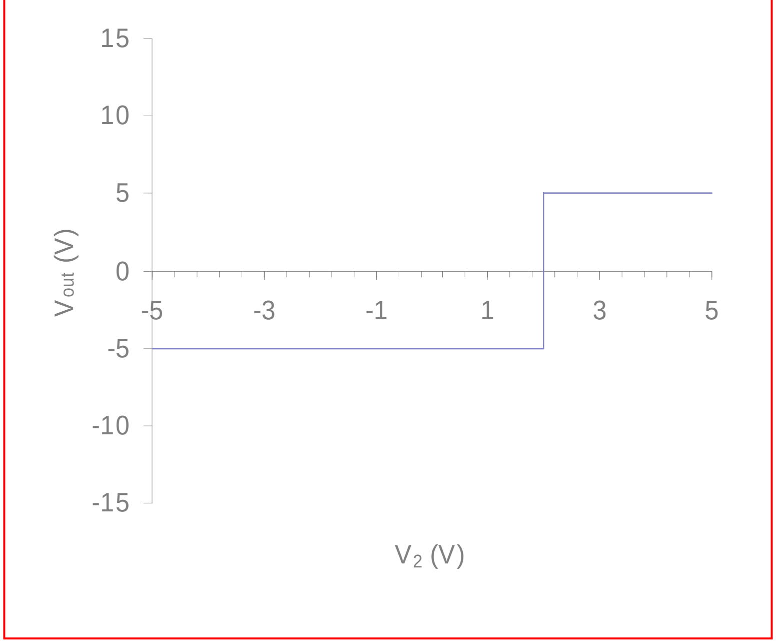
Where R_L = load resistor which is needed for the voltage probe to perform properly. The battery is swept from -2V to +2 V and the voltage sweep is displayed on the next page.

It can be seen that the sweep is very much identical to what was expected, with a discontinuity at 0V. The only difference is the voltage levels which are +11.61V and –11.61 V instead of ±12 V. This is because the output of an op-amp or comparator can never quite reach the supplied voltage.

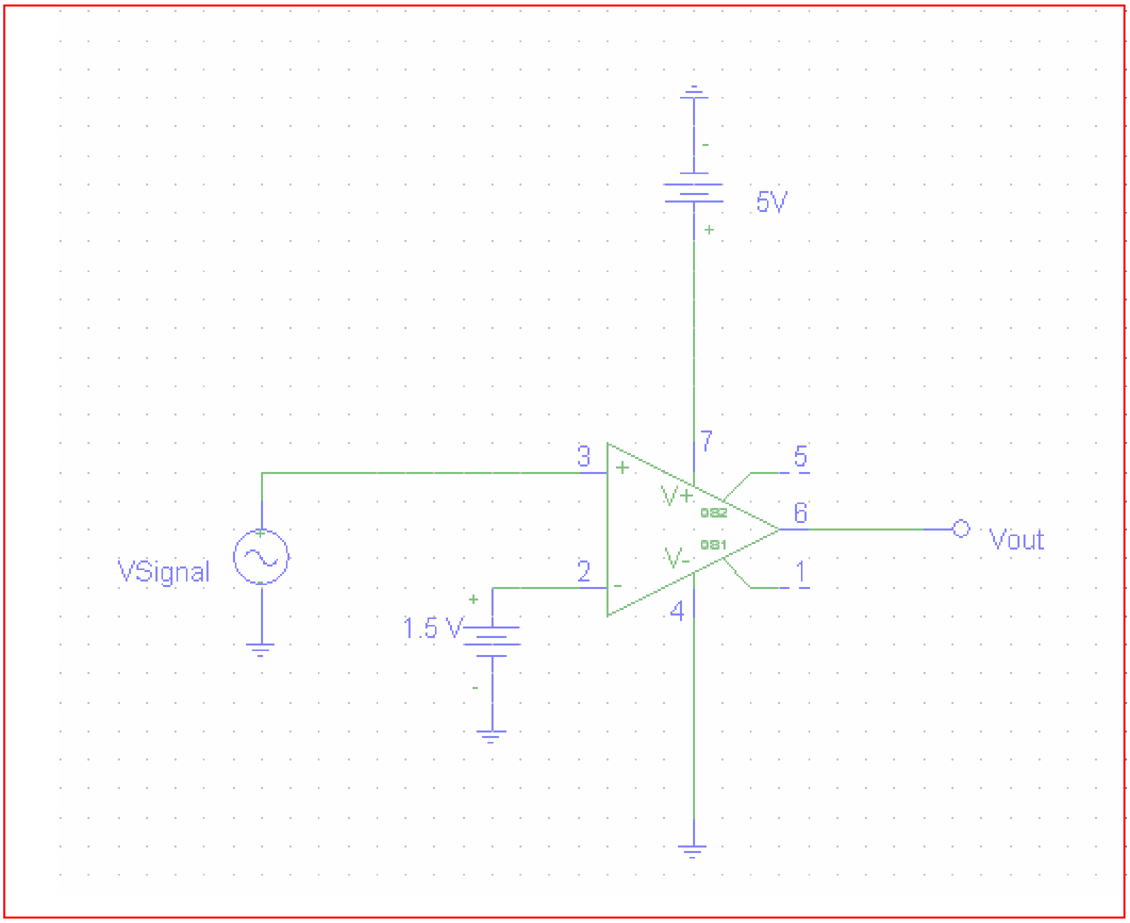
72. a)


15
10
5
0
-5
-10
-15
-5
-3
-1
1
3
5
V out (V)
V active (V)

b)


15
10
5
0
-5
-10
-15
-5
-3
-1
1
3
5
V out (V)
V 2 (V)

73. The following comparator setup would give a logic 0 for voltages below 1.5 V and logic 1 for voltages above 1.5 V



74. The voltage output of the circuit is given by

$$v_{out} = \frac{R_4}{R_3}\left(\frac{1+R_2/R_1}{1+R_4/R_3}\right)v_+ - \frac{R_2}{R_1}v_-$$

a) When $R_1 = R_3$ and $R_2 = R_4$, the equation reduces to

$$v_{out} = \frac{R_4}{R_3}(v_+ - v_-)$$

When $v_+ = v_-$, $v_{out} = 0$, thus $A_{CM} = 0$. Hence CMRR = ∞

b) If R_1, R_2, R_3 and R_4 are all different, then when $v_+ = v_- = v$,

$$v_{out} = \left(\frac{R_4}{R_3}\left(\frac{1+R_2/R_1}{1+R_4/R_3}\right) - \frac{R_2}{R_1}\right)v$$

Simplifying the algebra gives

$$v_{out} = \frac{R_1R_4 - R_3R_4}{R_1R_3 + R_1R_4}v$$

If v_+ and v_- are different, it turns out that it is impossible to separate v_{out} and v_d completely. Therefore, it is not possible to obtain A or CMRR in symbolic form.

75. a) The voltage at node between R_1 and R_2 is

$$V_1 = V_{ref}\left(\frac{R_2}{R_1 + R_2}\right)$$

by treating it as a voltage divider. Similarly, the voltage at node between R_{Gauge} and R_3 is:

$$V_2 = V_{ref}\left(\frac{R_3}{R_3 + R_{Gauge}}\right)$$

Therefore, the output voltage is

$$V_{out} = V_1 - V_2 = V_{ref}\left(\frac{R_2}{R_1 + R_2} - \frac{R_3}{R_3 + R_{Gauge}}\right)$$

b) If $R_1 = R_2 = R_3 = R_{gauge}$ then the two terms in the bracket cancels out, giving V_{out} = 0.

c) The amplifier has a maximum gain of 1000 and minimum gain of 2. Therefore to get a voltage of 1V at maximum loading, the voltage input into the amplifier must fall between 0.001 and 0.5, i.e. 0.5 > V_{out} > 0.001.

To simplify the situation, let $R_1 = R_2 = R_3 = R$, then at maximum loading,

$$V_{out} = V_{ref}\left(\frac{R}{R+R} - \frac{R}{R + R_{Gauge} + \Delta R}\right) = 12\left(\frac{1}{2} - \frac{R}{R + 5k + 50m}\right)$$

Using this we can set up two inequalities according to the two limits. The first one is:

$$\left(6 - \frac{12R}{R + 5k + 50m}\right) \geq 0.001$$

Solving gives

$$5.999 \geq \frac{12R}{R + 5000.05}$$

$$4998.38 \geq R$$

Similarly, the lower gain limit gives: $5.5 \leq \dfrac{12R}{R + 5000.05}$

$$\Rightarrow 4230 \leq R$$

This gives 4998.38 > R > 4230. Using standard resistor values, the only possible resistor values are R = 4.3 kΩ and R = 4.7 kΩ.

If we take R = 4.7 kΩ, then

$$V_{out} = 12\left(\frac{1}{2} - \frac{4.7k}{4.7k + 5k + 50m}\right) = 0.1855$$

Giving a gain of 5.388. This means a resistor value of R = 50.5/(5.388 – 1) = 11.5 kΩ or 11kΩ using standard value is needed between pin 1 and pin 8 of the amplifier.

1. $i = C\dfrac{dv}{dt}$

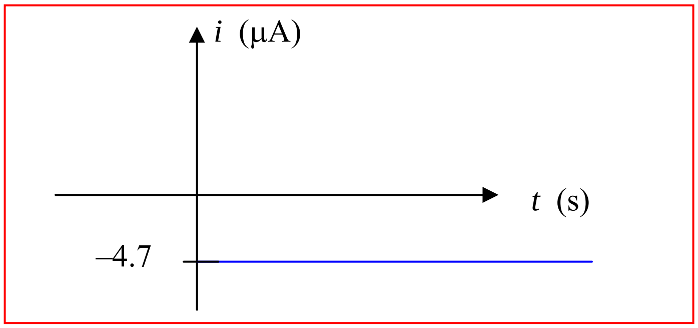
(a) $\boxed{i = 0}$ (DC)

(b) $i = C\dfrac{dv}{dt} = -\left(10\times10^{-6}\right)\left(115\sqrt{2}\right)\left(120\pi\right)\sin 120\pi t = \boxed{-613\sin 120\pi t\ \text{mA}}$

(c) $i = C\dfrac{dv}{dt} = -\left(10\times10^{-6}\right)\left(4\times10^{-3}\right)e^{-t} = \boxed{-40e^{-t}\ \text{nA}}$

2. $i = C\dfrac{dv}{dt}$

$v = \dfrac{6-0}{0-6}t + 6 = 6 - t$, therefore $i = C\dfrac{dv}{dt} = -4.7 \times 10^{-6}$ μA



3. $$i = C\frac{dv}{dt}$$

(a) $$\frac{dv}{dt} = 30\left[e^{-t} - te^{-t}\right] \qquad \text{therefore} \quad i = 10^{-3}\frac{dv}{dt} = \boxed{30(1-t)e^{-t} \quad \text{mA}}$$

(b)

$$\frac{dv}{dt} = 4\left[-5e^{-5t}\sin 100t + 100e^{-5t}\cos 100t\right]$$

$$\text{therefore} \quad i = 10^{-3}\frac{dv}{dt} = \boxed{4e^{-5t}\left(100\cos 100t - 5\sin 100t\right) \quad \text{mA}}$$

4. $W = \frac{1}{2}CV^2$

(a) $\left(\frac{1}{2}\right)\left(2000\times10^{-6}\right)1600 = \boxed{1.6\text{ J}}$

(b) $\left(\frac{1}{2}\right)\left(25\times10^{-3}\right)\left(35\right)^2 = \boxed{15.3\text{ J}}$

(c) $\left(\frac{1}{2}\right)\left(10^{-4}\right)\left(63\right)^2 = \boxed{198\text{ mJ}}$

(d) $\left(\frac{1}{2}\right)\left(2.2\times10^{-3}\right)\left(2500\right) = \boxed{2.75\text{ J}}$

(e) $\left(\frac{1}{2}\right)\left(55\right)\left(2.5\right)^2 = \boxed{171.9\text{ J}}$

(f) $\left(\frac{1}{2}\right)\left(4.8\times10^{-3}\right)\left(50\right)^2 = \boxed{6\text{ J}}$

5. (a) $$C = \frac{\varepsilon A}{d} = \frac{8.854 \times 10^{-12}\left(78.54 \times 10^{-6}\right)}{100 \times 10^{-6}} = \boxed{6.954\, pF}$$

(b) $$Energy, E = \frac{1}{2}CV^2 \;\therefore V = \sqrt{\frac{2E}{C}} = \sqrt{\frac{2\left(1 \times 10^{-3}\right)}{6.954 \times 10^{-12}}} = \boxed{16.96 \text{ kV}}$$

(c) $$E = \frac{1}{2}CV^2 \;\therefore C = \frac{2E}{V^2} = \frac{2\left(2.5 \times 10^{-6}\right)}{\left(100^2\right)} = 500\, pF$$

$$C = \frac{\varepsilon A}{d} \;\therefore \varepsilon = \frac{Cd}{A} = \frac{\left(500 \times 10^{-12}\right)\left(100 \times 10^{-6}\right)}{\left(78.54 \times 10^{-6}\right)} = 636.62\, pF.m^{-1}$$

$$\backslash\text{Relative permittivity}: \quad \frac{\varepsilon}{\varepsilon_0} = \frac{636.62 \times 10^{-12}}{8.854 \times 10^{-12}} = \boxed{71.9}$$

6. (a) For $V_A = -1V$, $W = \sqrt{\frac{2K_s\varepsilon_0}{qN}(V_{bi} - V_A)} = \sqrt{\frac{2(11.8)(8.854\times10^{-12})}{(1.6\times10^{-19})(1\times10^{24})}(0.57+1)}$

$$= 45.281\times10^{-9}\,m$$

$$C_j = \frac{11.8(8.854\times10^{-12})(1\times10^{-12})}{45.281\times10^{-9}} = \boxed{2.307\ fF}$$

(b) For $V_A = -5V$, $W = \sqrt{\frac{2K_s\varepsilon_0}{qN}(V_{bi} - V_A)} = \sqrt{\frac{2(11.8)(8.854\times10^{-12})}{(1.6\times10^{-19})(1\times10^{24})}(0.57+5)}$

$$= 85.289\times10^{-9}\,m$$

$$C_j = \frac{11.8(8.854\times10^{-12})(1\times10^{-12})}{85.289\times10^{-9}} = \boxed{1.225\ fF}$$

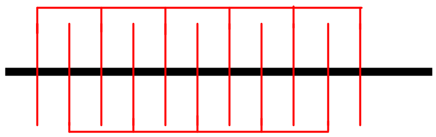
(c) For $V_A = -10V$,

$$W = \sqrt{\frac{2K_s\varepsilon_0}{qN}(V_{bi} - V_A)} = \sqrt{\frac{2(11.8)(8.854\times10^{-12})}{(1.6\times10^{-19})(1\times10^{24})}(0.57+10)}$$

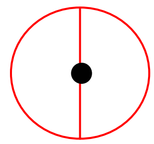
$$= 117.491\times10^{-9}\,m$$

$$C_j = \frac{11.8(8.854\times10^{-12})(1\times10^{-12})}{117.491\times10^{-9}} = \boxed{889.239aF}$$

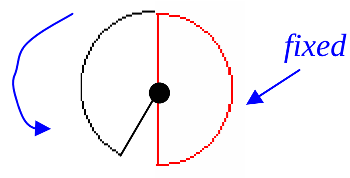
7. We require a capacitor that may be manually varied between 100 and 1000 pF by rotation of a knob. Let's choose an air di electric for simplicity of cons truction, and a series of 11 half-plates:



Top view



Side view with no overlap between plates



Side view with a small overlap between plates.

Constructed as shown, the half-plates are in parallel, so that each of the 10 pairs must have a capacitance of 1000/ 10 = 100 pF when rotated such that they overlap completely. If we arbitrarily select an area of 1 cm^2 for each half-plate, then the g ap spacing between each plate is $d = \varepsilon A/C = (8.854\times10^{-14}$ F/cm)(1 cm^2)/ $(100\times10^{-12}$ F) = 0.8854 mm. This is tight, but not impossible to achieve. The final step is to determine the amount of overlap which corresponds to 100 pF for the total capacitor structure. A capacitance of 100 pF is equal to 10% of the capacitance when all of the plate areas are aligned, so we need a pie-shaped wedge having an area of 0.1 cm^2. If the m iddle figure above corresponds to an angle of 0^o and the case of perfect alignm ent (maximum capacitance) corresponds to an angle of 180^o, we need to set out minimum angle to be 18^o.

8. (a) Energy stored $= \int_{t_0}^{t} v \cdot C\dfrac{dv}{dt} = C\int_{0}^{2\times 10^{-3}} 3e^{-\frac{t}{5}} \cdot \left(-\dfrac{3}{5}e^{-\frac{t}{5}}\right)dt = \boxed{-1.080\mu J}$

(b) $V_{max} = 3$ V

Max. energy at t=0, $= \dfrac{1}{2}CV^2 = 1.35mJ \therefore 37\% E_{max} = 499.5\mu J$

V at 37% E_{max} = 1.825 V

$v(t) = 1.825 = 3e^{-\frac{t}{5}} \therefore t = 2.486s \Rightarrow \approx \boxed{2s}$

(c) $i = C\dfrac{dv}{dt} = 300\times 10^{-6}\left(-\dfrac{3}{5}e^{-\frac{1.2}{5}}\right) = \boxed{-141.593\mu A}$

(d) $P = vi = 2.011\left(-120.658\times 10^{-6}\right) = \boxed{-242.6\mu W}$

9. (a) $$v = \frac{1}{C}.\frac{\pi}{2}\left(1\times10^{-3}\right)^2 = \frac{1}{47\times10^{-6}}.\frac{(3.14159)}{2}\left(1\times10^{-3}\right)^2 = \boxed{33.421mV}$$

(b) $$v = \frac{1}{C}.\left(\frac{\pi}{2}\left(1\times10^{-3}\right)^2 + 0\right) = \frac{1}{47\times10^{-6}}.\frac{(3.14159)}{2}\left(1\times10^{-3}\right)^2 = \boxed{33.421mV}$$

(c) $$v = \frac{1}{C}.\left(\frac{\pi}{2}\left(1\times10^{-3}\right)^2 + \frac{\pi}{4}\left(1\times10^{-3}\right)^2\right) = \frac{1}{47\times10^{-6}}.\left(\frac{3\pi}{4}\left(1\times10^{-3}\right)^2\right) = \boxed{50.132mV}$$

10. $$V = \frac{1}{C}\int_0^{200ms} i\,dt = \frac{1}{C}\left[\left(-\frac{7\times10^{-3}}{\pi}\cos\pi t\right)\right]_0^{200ms} = \frac{0.426}{C}$$

$$E = \frac{1}{2}CV^2 = 3\times10^{-6} = \frac{181.086\times10^{-9}}{2C} \therefore C = \frac{181.086\times10^{-9}}{2\left(3\times10^{-6}\right)} = \boxed{30181\mu F}$$

11.

(a) $c = 0.2\mu\text{F},\ v_c = 5 + 3\cos^2 200t\text{V};\ \therefore i_c = 0.2 \times 10^{-6}(3)(-2)200\sin 200t\cos 200t$

$\therefore i_c = \boxed{-0.12\sin 400t\text{mA}}$

(b) $w_c = \frac{1}{2}cv_c^2 = \frac{1}{2} \times 2 \times 10^{-7}(5 + 3\cos^2 200t)^2 \therefore w_{c\max} = 10^{-7} \times 64 = \boxed{6.4\mu\text{J}}$

(c) $v_c = \frac{1}{0.2} \times 10^6 \int_0^t 8e^{-100t} \times 10^{-3}\,dt = 10^3 \times 40(-0.01)(e^{-100t} - 1) = \boxed{400(1 - e^{100t})\text{V}}$

(d) $\boxed{v_c = 500 - 400e^{-100t}\ \text{V}}$

12. $v_c(0) = 250\text{V},\ c = 2\text{mF}\ (a)\ v_c(0.1) = 250 + 500\int_0^{0.1} 5dt$

$\therefore v_c(0.1) = 500\text{V}; v_c(0.2) = 500\int_{0.1}^{0.2} 10dt = 1000\text{V}$

$\therefore v_c(0.6) = 1750\text{V}, v_c(0.9) = 2000\text{V}$

$\therefore 0.9 < t < 1: v_c = 2000 + 500\int_{0.9}^{t} 10dt = 2000 + 5000(t - 0.9)$

$\therefore v_c = 2100 = 2000 + 5000(t_2 - 0.9) \therefore t_2 = 0.92 \therefore \boxed{0.9 < t < 0.92s}$

13.

(a) $$w_c = \frac{1}{2}Cv^2 = \frac{1}{2}\times 10^{-6} v^2 = 2\times 10^{-2} e^{-1000t} \therefore v = \pm 200 e^{-500t}\,\text{V}$$

$$i = Cv' = 10^{-6}(\pm 200)(-500)e^{-500t} = \mp 0.1 e^{-500t}$$

$$\therefore \text{R} = \frac{-v}{i} = \frac{200}{0.1} = \boxed{2k\Omega}$$

(b) $$\text{P}_R = i^2\text{R} = 0.01\times 2000 e^{-1000t} = 20e^{-1000t}\,\text{W}$$

$$\therefore \text{W}_\text{R} = \int_0^\infty 20e^{-1000t}dt = -0.02e^{-1000t}\Big|_0^\infty = \boxed{0.02\text{J}}$$

14. (a) Left circuit:

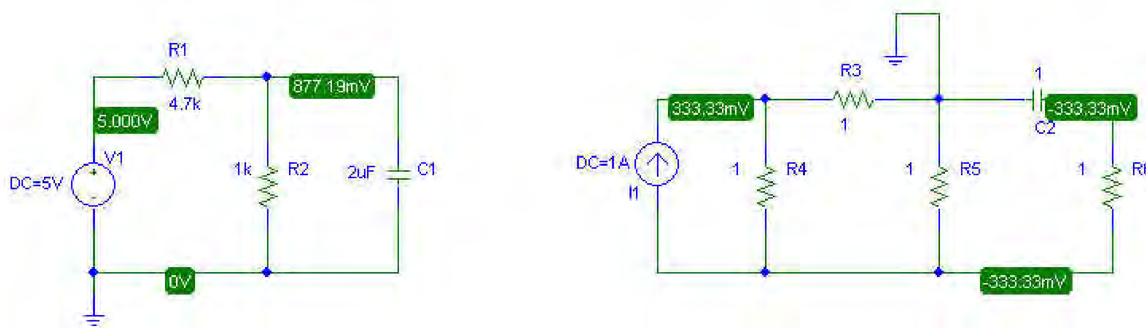
By Voltage division, $V_C = \frac{1k}{4.7k + 1k}(5) = \boxed{0.877V}$

Right circuit:

$$V_1 = 1(1 // 2) = \frac{2}{3}V$$

By Voltage Division, $V_2 = \frac{1}{3}V \therefore \boxed{V_C = -\frac{1}{3}V}$

(b)



15. $v = L\dfrac{di}{dt}$

(a) $\boxed{v = 0}$ since i = constant (DC)

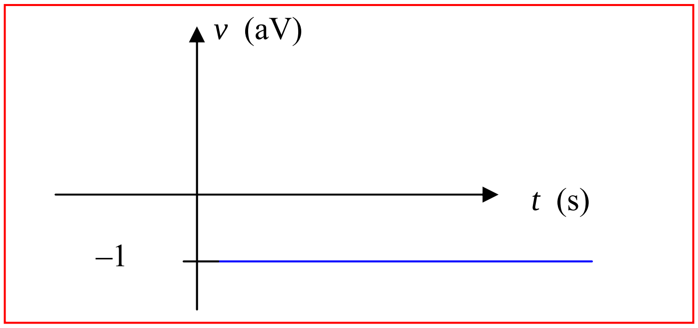
(b) $v = -10^{-8}\left(115\sqrt{2}\right)(120\pi)\sin 120\pi t = \boxed{-613\sin 120\pi t \ \ \mu\text{V}}$

(c) $v = -10^{-8}\left(115\sqrt{2}\right)\left(24\times 10^{-3}\right)e^{-6t} = \boxed{-240e^{-6t}\ \ \text{pV}}$

16. $v = L\dfrac{di}{dt}$

$$i = \left[\frac{(6-0)\times10^{-9}}{(0-6)\times10^{-3}}\right]t + 6\times10^{-9} = 6\times10^{-9} - 10^{-6}t \text{, therefore}$$

$$v = L\frac{di}{dt} = -\left(10^{-12}\right)\left(10^{-6}\right) = -10^{-18}\text{ V} = -1\text{ aV}$$



17. $v = L\dfrac{di}{dt}$

(a) $L\dfrac{di}{dt} = \left(5\times10^{-6}\right)30\times10^{-9}\left[e^{-t} - te^{-t}\right] = \boxed{150\left(1-t\right)e^{-t} \quad \text{fV}}$

(b)

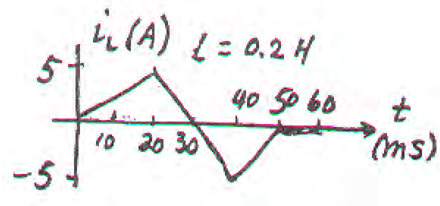
$$L\frac{di}{dt} = \left(5\times10^{-6}\right)\left(4\times10^{-3}\right)\left[-5e^{-5t}\sin 100t + 100e^{-5t}\cos 100t\right]$$

therefore $\boxed{v = \;\; 100e^{-5t}\left(20\cos 100t - \sin 100t\right) \quad \text{pV}}$

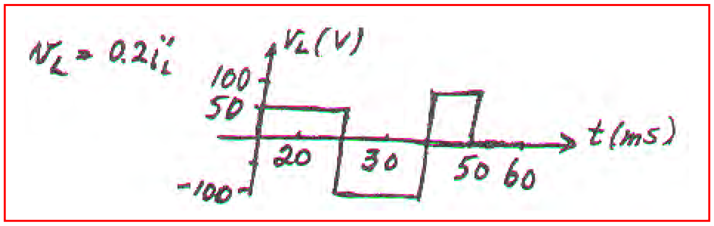
18. $W = \frac{1}{2}LI^2$. Maximum energy corresponds to maximum current flow, so

$$W_{\max} = \frac{1}{2}\left(5\times10^{-3}\right)\left(1.5\right)^2 = \boxed{5.625 \text{ mJ}}$$

19.



(a)



(b) $\quad P_L = v_L i_L \;\therefore\; P_{L\max} = (-100)(-5) = 500\,\text{W at } t = \boxed{40^- \text{ ms}}$

(c) $\quad P_{L\min} = 100(-5) = -500\,\text{W at } \boxed{t = 20^+ \text{ and } 40^+ \text{ ms}}$

(d) $\quad W_L = \frac{1}{2} L i_L^2 \;\therefore\; W_L(40\text{ms}) = \frac{1}{2} \times 0.2(-5)^2 = \boxed{2.5\text{J}}$

20.

$$\mathrm{L} = 50\times10^{-3}, t<0: \ i=0;\ t>0\ \ i = 80te^{-100t}\mathrm{mA} = 0.08te^{-100t}\mathrm{A}$$

$$\therefore \mathrm{i}' = 0.08\mathrm{e}^{-100\mathrm{t}} - 8te^{-100t} \ \therefore 0.08 = 8t, \boxed{t_m, = 0.01\mathrm{s}}, |i|_{\max} = 0.08\times0.01e^{-1}$$

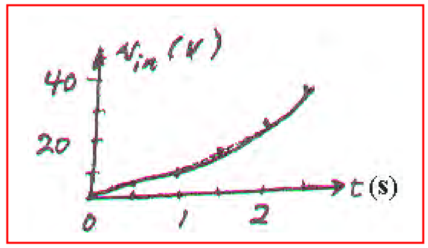
$$\therefore \boxed{|i|_{\max} = 0.2943\mathrm{mA}};\ v = 0.05i' = e^{-100t}(0.004-0.4t)$$

$$\therefore v' = e^{-100t}(-0.4) - 100e^{-100t}(0.004-0.4t) \therefore -0.4 = 0.4 - 40t,\ t = \frac{0.8}{40} = 0.02s$$

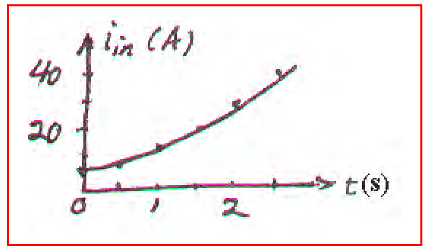
$$v = e^{-2}(0.004-0.008) = -0.5413\mathrm{mV}\ \text{this is minimum} \therefore |v|_{\max} = \boxed{0.004\mathrm{V}\ \text{at}\ \ \mathrm{t{=}0}}$$

21.

(a) $t > 0:\ i_s = 0.4t^2\text{A} \therefore v_{in} = 10i_s + 5i_s' = \boxed{4t^2 + 4t\text{V}}$



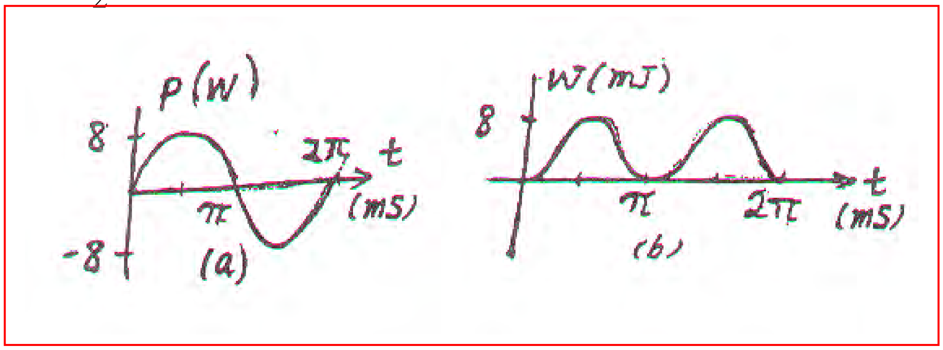
(b) $i_{in'} = 0.1v_s + \frac{1}{5}\int_0^t 40tdt + 5 = \boxed{4t + 4t^2 + 5\text{A}}$



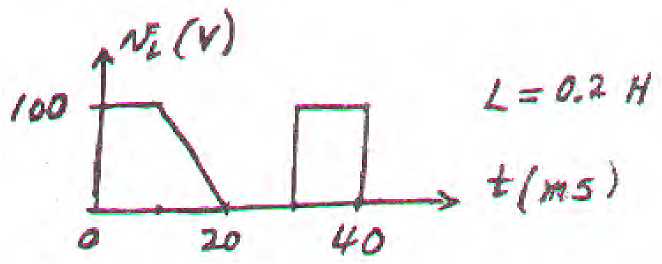
22. $v_L = 20\cos 1000t\text{V},\ \text{L} = 25\text{mH},\ i_L(0) = 0$

(a) $i_L = 40\int_0^t 20\ \cos\ 1000t dt = 0.8\sin\ 1000t\text{A} \therefore \boxed{p = 8\sin 2000t\ \text{ W}}$

(b) $w = \frac{1}{2} \times 25 \times 10^{-3} \times 0.64\sin^2 1000t = \boxed{8\sin^2 1000t\ \text{ mJ}}$



23.



(a) $0 < t < 10\text{ ms}:\ i_L = -2 + 5\int_0^t 100dt = -2 + 500t \therefore i_L(10\text{ms}) = 3\text{A}, i_L(8\text{ms}) = \boxed{2\text{A}}$

(b) $i_L(0) = 0 \therefore i_L(10\text{ms}) = 500 \times 0.01 = 5\text{A} \therefore i_L(20\text{ms}) = 5 + 5\int_{0.01}^{0.02} 10^4(0.02 - t)dt$

$\therefore i_L(20\text{ms}) = 5 + 5\times10^4(0.02t - 0.5t)_{0.01}^{0.02} = 5 + 5\times10^4(0.0002 - 0.00015) = 7.5\text{A}$

$\therefore w_L = \frac{1}{2}\times0.2\times7.5^2 = \boxed{5.625\text{J}}$

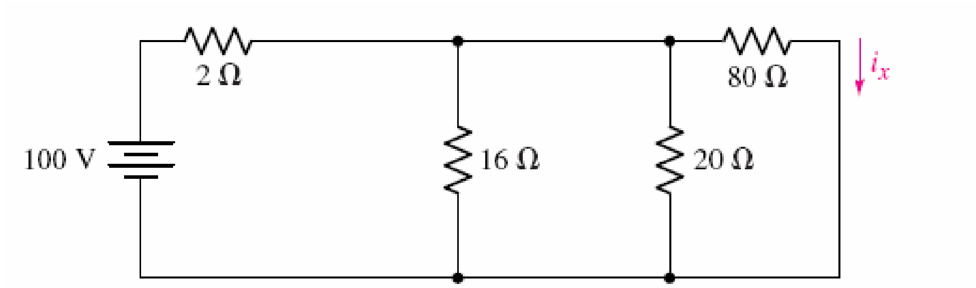
(c) If the circuit has been connected for a long time, L appears like short circuit.

$$V_{8\Omega} = \frac{8}{2+8}(100V) = 80V$$

$$I_{2\Omega} = \frac{20V}{2\Omega} = 10A$$

$$\therefore \boxed{i_x = \frac{80V}{80\Omega} = 1A}$$

24. After a very long time connected only to DC sources, the inductors act as short circuits. The circuit may thus be redrawn as



And we find that $i_x = \left(\dfrac{\frac{80}{9}}{80 + \frac{80}{9}} \right) \left(\dfrac{100}{2+8} \right) = \boxed{1\ \text{A}}$

25. $\text{L} = 5\text{H}, \text{V}_L = 10(e^{-t} - e^{-2t})\text{V}, i_L(0) = 0.08\text{A}$

(a) $v_L(1) = 10(e^{-1} - e^{-2}) = \boxed{2.325^+\,\text{V}}$

(b) $i_L = 0.08 + 0.2\int_0^t 10(e^{-t} - e^{-2t})dt = 0.08 + 2(-e^{-t} + 0.5e^{-2t})_0^t$

$i_L = 0.08 + 2(-e^{-t} + 0.5e^{-2t} + 1 - 0.5) = 1.08 + e^{-2t} - 2e^{-t} \therefore i_L(1) = \boxed{0.4796\text{A}}$

(c) $i_L(\infty) = \boxed{1.08\text{A}}$

26.

(a)

$$v_x = 120 \times \frac{40}{12+20+40} + 40 \times 5 \times \frac{12}{12+20+40}$$

$$= \frac{200}{3} + \frac{100}{3} = \boxed{100\text{V}}$$

(b)

$$v_x = \frac{120}{12+15\|60} \times \frac{15}{15+60} \times 40 + 40 \times 5 \frac{15\|12}{15\|12+60}$$

$$= \frac{120}{12+12} \times \frac{1}{5} \times 40 + 200 \frac{6.667}{66.667}$$

$$= 40 + 20 = \boxed{60\text{V}}$$

27.

(a) $w_L = \frac{1}{2} \times 5 \times 1.6^2 = \boxed{6.4\text{J}}$

(b) $w_c = \frac{1}{2} \times 20 \times 10^{-6} \times 100^2 = \boxed{0.1\text{J}}$

(c) Left to right (magnitudes): 100, 0, 100, 116, 16, 16, 0 (V)

(d) Left to right (magnitudes): 0, 0, 2, 2, 0.4, 1.6, 0 (A)

28.

(a)

$$v_s = 400t^2\text{V}, t > 0;\ i_L(0) = 0.5\text{A};\ t = 0.4s$$

$$v_c = 400 \times 0.16 = 64\text{V}, w_c \frac{1}{2} \times 10^{-5} \times 64^2 = \boxed{20.48\text{mJ}}$$

(b) $$i_L = 0.5 + 0.1\int_0^{0.4} 400t^2 dt = 0.5 + 40 \times \frac{1}{3} \times 0.4^3 = 1.3533\text{A}$$

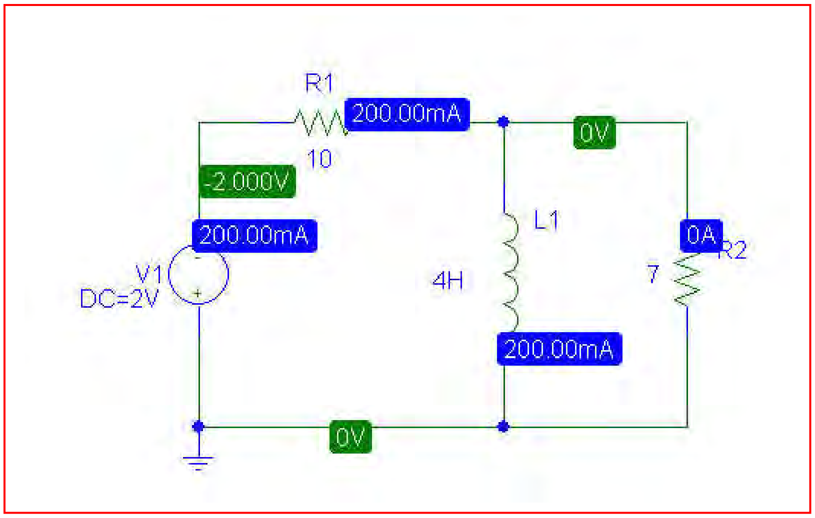
$$\therefore w_L = \frac{1}{2} \times 10 \times 1.3533^2 = \boxed{9.1581\text{J}}$$

(c) $$i_R = 4t^2,\ \text{P}_R = 100 \times 16t^4\ \therefore w_R = \int_0^{0.4} 1600t^4 dt = 320 \times 0.4^5 = \boxed{3.277\text{J}}$$

29. (a) $\boxed{P_{7\Omega} = 0W}; P_{10\Omega} = \dfrac{V^2}{R} = \dfrac{(2)^2}{10} = \boxed{0.4W}$

(b) PSpice verification

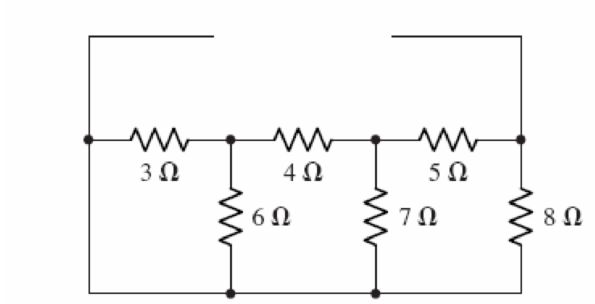
We see from the PSpice simulation that the voltage across the 10-Ω resistor is –2 V, so that it is dissipating 4/10 = 400 mW.



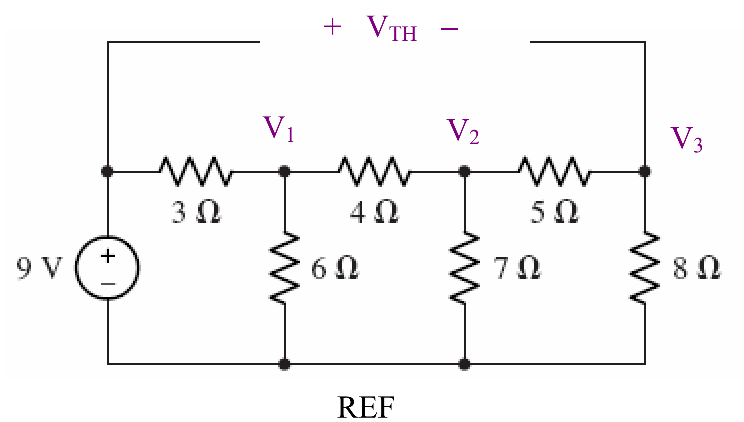
The 7-Ω resistor has zero volts across its terminals, and hence dissipates zero power.

Both results agree with the hand calculations.

30. (a) We find R_{TH} by first short-circuiting the voltage source, removing the inductor, and looking into the open terminals.



Simplifying the network from the right, 3 || 6 + 4 = 6 Ω, which is in parallel with 7 Ω. 6 || 7 + 5 = 8.23 Ω. Thus, R_{TH} = 8.23 || 8 = $\boxed{4.06\ \Omega.}$ To find V_{TH}, we remove the inductor:



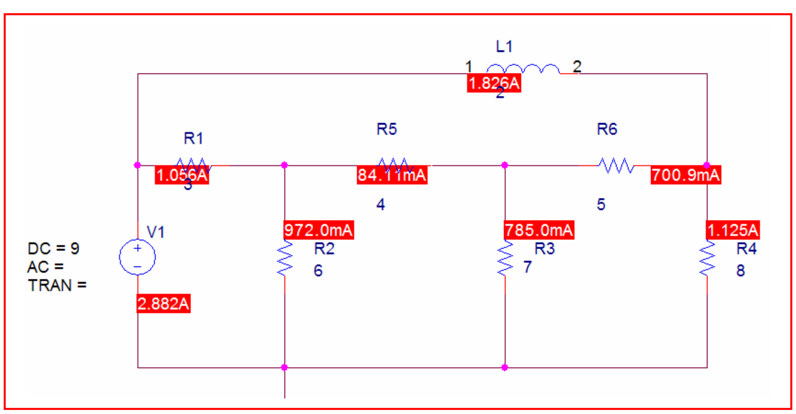
Writing the nodal equations required:

$(V_1 - 9)/3 + V_1/6 + (V_1 - V_2)/4 = 0$
$(V_2 - V_1)/4 + V_2/7 + (V_2 - V_3)/5 = 0$
$V_3/8 + (V_3 - V_2)/5 = 0$

Solving, $V_3 = 1.592$ V, therefore $V_{TH} = 9 - V_3 = \boxed{7.408\text{ V.}}$

(b) $i_L = 7.408/4.06 = \boxed{1.825\text{ A}}$ (inductor acts like a short circuit to DC).

(c)



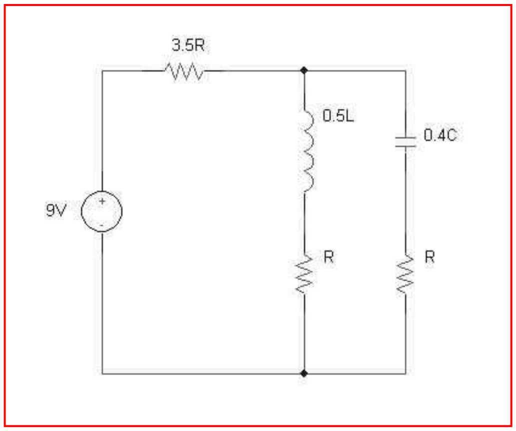
31.

$$C_{equiv} \equiv 10\mu + \left(\frac{1}{\frac{1}{10\mu} + \frac{1}{10\mu}} \right) \text{in series with } 10\mu \text{ in series with } 10\mu + \left(\frac{1}{\frac{1}{10\mu} + \frac{1}{10\mu}} \right)$$

$$\equiv \boxed{4.286\mu F}$$

32. $L_{equiv} \equiv (77p // (77p + 77p)) + 77p + (77p // (77p + 77p)) = \boxed{179.\dot{6}\, pH}$

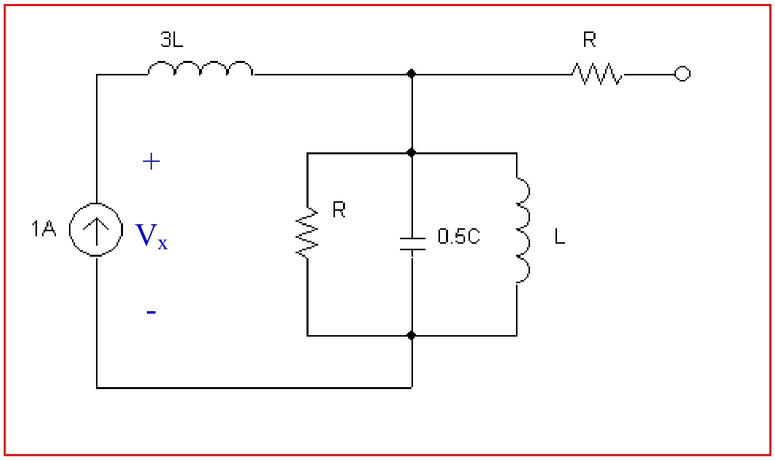
33. (a) Assuming all resistors have value R, all inductors have value L, and all capacitors have value C,



(b) At dc, 20µF is open circuit; 500µH is short circuit.

Using voltage division, $V_x = \dfrac{10k}{10k + 15k}(9) = \boxed{3.6V}$

34. (a) As all resistors have value R, all inductors value L, and all capacitors value C,



(b) $\boxed{V_x = 0\text{ V}}$ as L is short circuit at dc.

35. $C_{equiv} = \{ [(100\text{ n} + 40\text{ n}) \parallel 12\text{ n}] + 75\text{ n}\} \parallel \{7\ \mu + (2\ \mu \parallel 12\ \mu)\}$

$$\boxed{C_{equiv} \equiv 85.211nF}$$

36. $L_{equiv} = \{[(17\text{ p} \parallel 4\text{ n}) + 77\text{ p}] \parallel 12\text{ n}\} + \{1\text{ n} \parallel (72\text{ p} + 14\text{ p})\}$

$$\boxed{L_{equiv} \equiv 172.388\, pH}$$

37. $C_T - C_x = (7 + 47 + 1 + 16 + 100) = 171\mu F$

$$E_{C_T - C_x} = \frac{1}{2}(C_T - C_x)V^2 = \frac{1}{2}(171\mu)(2.5)^2 = 534.375\mu J$$

$$E_{C_x} = E_{C_T} - E_{C_T - C_x} = (534.8 - 534.375)\mu J = 425nJ$$

$$\therefore E_{C_x} = 425n = \frac{1}{2}C_x V^2 \Rightarrow C_x = \frac{425n(2)}{(2.5)^2} = \boxed{136nF}$$

38.

(a) For all L = 1.5H, $L_{equiv} = 1.5 + \left(\dfrac{1}{\dfrac{1}{1.5} + \dfrac{1}{1.5}} \right) + \left(\dfrac{1}{\dfrac{1}{1.5} + \dfrac{1}{1.5} + \dfrac{1}{1.5}} \right) = \boxed{2.75H}$

(b) For a general network of this type, having N stages (and all L values equiv),

$$\boxed{L_{equiv} = \sum_{N=1}^{n} \frac{L^N}{NL^{N-1}}}$$

39.

(a) $$L_{equiv} = 1 + \left(\frac{1}{\frac{1}{2} + \frac{1}{2}} \right) + \left(\frac{1}{\frac{1}{3} + \frac{1}{3} + \frac{1}{3}} \right) = \boxed{3H}$$

(b) For a network of this type having 3 stages,

$$L_{equiv} = 1 + \frac{1}{\frac{2+2}{(2)^2}} + \frac{1}{\frac{3+3}{(3)^2} + \frac{1}{3}} = 1 + \frac{(2)^2}{2(2)} + \frac{(3)^3}{3(3)^2}$$

Extending for the general case of N stages,

$$L_{equiv} = 1 + \frac{1}{\frac{1}{2} + \frac{1}{2}} + \frac{1}{\frac{1}{3} + \frac{1}{3} + \frac{1}{3}} + \ldots + \frac{1}{\frac{1}{\text{N}} + \ldots \frac{1}{\text{N}}}$$

$$= 1 + \frac{1}{2(1/2)} + \frac{1}{3(1/3)} + \ldots + \frac{1}{\text{N}(1/\text{N})} = \boxed{\text{N}}$$

40. $$C_{equiv} = \frac{(3p)(0.25p)}{3p + 0.25p} = \boxed{0.231\,pF}$$

41. $$L_{equiv} = \frac{(2.\dot{3}n)(0.\dot{3}n)}{2.\dot{6}n} = \boxed{0.291\dot{6}nH}$$

42. (a) $\boxed{\text{Use 2 x 1μH in series with 4 x 1μH in parallel.}}$

(b) $\boxed{\text{Use 2 x 1μH in parallel, in series with 4 x 1μH in parallel.}}$

(c) $\boxed{\text{Use 5 x 1μH in parallel, in series with 4 x 1μH in parallel.}}$

43.

(a) $R = 10\Omega : 10 \| 10 \| 10 = \frac{10}{3}, \frac{10}{3} + 10 + 10 \| 10 = \frac{55}{3}$

$\therefore R_{eq} = \frac{55}{3} \| 30 = \boxed{11.379\Omega}$

(b) $L = 10\text{H} \therefore L_{eq} = \boxed{11.379\text{H}}$

(c) $C = 10\text{F} : \frac{1}{1/30 + 1/10 + 1/20} = 5.4545$

$\therefore C_{eq} = 5.4545 + \frac{10}{3} = \boxed{8.788\text{F}}$

44.

(a) $oc: \mathrm{L}_{eq} = 6\|1+3 = \boxed{3.857\mathrm{H}}$

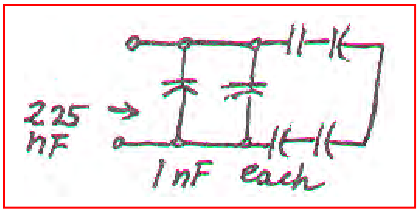
$sc: \mathrm{L}_{eq} = (3\|2+1)\|4 = 2.2| \ |4 = \boxed{1.4194\mathrm{H}}$

(b) $oc: 1 + \dfrac{1}{1/4+1/2} = \dfrac{7}{3}, c_{eq} = \dfrac{1}{3/7+1/2} = \boxed{1.3125\mathrm{F}}$

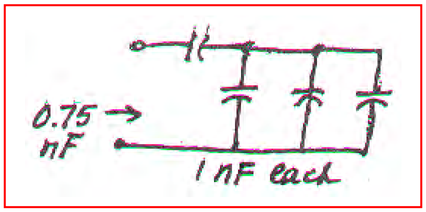
$sc: \dfrac{1}{1/5+1} = \dfrac{5}{6}, \mathrm{C}_{eq} = 4 + \dfrac{5}{6} = \boxed{4.833\mathrm{F}}$

45.

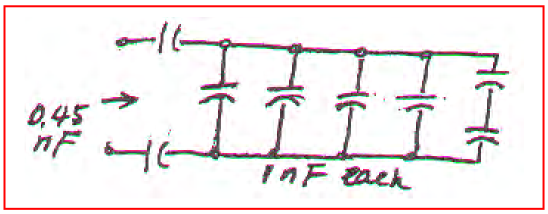
(a)



(b)



(c)



46. $i_s = 60e^{-200t}\text{mA},\ i_1(0) = 20\text{mA}$

(a) $6\|4 = 2.4\text{H} \therefore v = \text{L}_{eq} i_s' = 2.4 \times 0.06(-200)e^{-200t}$

or $\boxed{v = -28.8e^{-200t}\text{V}}$

(b) $i_1 = \dfrac{1}{6}\displaystyle\int_o^t -28.8e^{-200t}dt + 0.02 = \dfrac{4.8}{200}(e^{-200t} - 1) + 0.02$

$= \boxed{24e^{-200t} - 4\text{mA}(t > 0)}$

(c) $i_2 = i_s - i_1 = 60e^{-200t} - 24e^{-200t} + 4 = \boxed{36e^{-200t} + 4\text{mA}(t > 0)}$

47. $v_s = 100e^{-80t}V, v_1(0) = 20\text{V}$

(a) $i = C_{eq} v_s' = 0.8\times10^{-6}(-80)100e^{-80t} = \boxed{-6.4\times10^{-3}e^{-80t}\text{A}}$

(b) $v_1 = 10^6(-6.4\times10^{-3})\int_o^t e^{-80t}dt + 20 = \dfrac{6400}{80}(e^{-80t}-1)+20$

$\therefore v_1 = \boxed{80e^{-80t}-60\text{V}}$

(c) $v_2 \dfrac{10^6}{4}(-6.4\times10^{-3})\int_o^t e^{-80t}dt + 80 = \dfrac{1600}{80}(e^{-80t}-1)+80$

$= \boxed{20e^{-80t}+60\text{V}}$

48.

(a)

$$\boxed{\begin{aligned}&\frac{v_c - v_s}{20} + 5\times 10^{-6} v_c' + \frac{v_c - v_L}{10} = 0\\&\frac{v_L - v_c}{10} + \frac{1}{8\times 10^{-3}}\int_o^t v_L\, dt + 2 = 0\end{aligned}}$$

(b)

$$\boxed{\begin{aligned}&20 i_{20} + \frac{1}{5\times 10^{-6}}\int_o^t (i_{20} - i_L)\, dt + 12 = v_s\\&\frac{1}{5\times 10^{-6}}\int_o^t (i_L - i_{20})\, dt - 12 + 10 i_L + 8\times 10^{-3} i_L' = 0\end{aligned}}$$

49.

$v_c(t)$: 30mA: $0.03 \times 20 = 0.6\text{V}, v_c = 0.6\text{V}$

9V: $v_c = 9\text{V}$, 20mA: $v_c = -0.02 \times 20 = 0.4\text{V}$

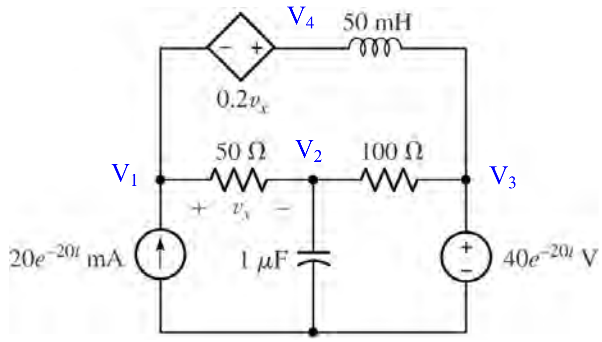
$0.04\cos 10^3 t$: $v_c = 0$

$\therefore \boxed{v_c(t) = 9.2\text{V}}$

$v_L(t)$: 30mA, 20mA,

9V: $v_L = 0$; $0.04\cos 10^3 t$: $v_L = -0.06 \times 0.04(-1000)\sin 10^3 t = \boxed{2.4\sin 10^3 t\text{V}}$

50. We begin by selecting the bottom node as the reference and assigning four nodal voltages:



Ref.

1, 4 Supernode: $$\boxed{20\times10^{-3}\, e^{-20t} = \frac{V_1 - V_2}{50} + 0.02\times10^{3}\int_0^t \left(V_4 - 40e^{-20t}\right)dt'} \quad [1]$$

and: $V_1 - V_4 = 0.2\, V_x$ or $\boxed{0.8V_1 + 0.2\, V_2 - V_4 = 0}$ [2]

Node 2: $$\boxed{0 = \frac{V_2 - V_1}{50} + \frac{V_2 - 40e^{-20t}}{100} + 10^{-6}\frac{dV_2}{dt}} \quad [3]$$

51. (a) $R_i = \infty,\ R_o = 0,\ A = \infty \therefore v_i = 0 \therefore i = Cv_s'$

also $0 + Ri + v_o = 0 \therefore v_o = -RCv_s'$

$-v_i + Ri - Av_i = 0,\ \boxed{v_s = \frac{1}{c}\int idt + v_i}$

(b) $v_o = -Av_i \therefore v_i = \frac{-1}{A}v_o \therefore i = \frac{1+A}{R}v_i$

$\therefore v_s = \frac{1}{c}\int idt - \frac{1}{A}v_o = -\frac{1}{A}v_o + \frac{1+A}{RC}\int -\frac{v_o}{A}dt$

$\therefore Av_s' = -v_o' - \frac{1+A}{RC}v_o$ or $\boxed{v_o' + \frac{1+A}{RC}v_o + Av_s' = 0}$

52. Place a current source in parallel with a 1-MΩ resistor on the positive input of a buffer with output voltage, *v*. This feeds into an integrator stage with input resistor, R_2, of 1-MΩ and feedback capacitor, C_f, of 1 μF.

$$i = C_f \frac{dv_{c_f}}{dt} = 1.602 \times 10^{-19} \times \frac{ions}{\sec}$$

$$0 = \frac{V_a - V}{1 \times 10^6} + C_f \frac{dv_{c_f}}{dt} = \frac{V_a - V}{1 \times 10^6} + 1.602 \times 10^{-19} \frac{ions}{\sec}$$

$$0 = \frac{-V}{R_2} + C_f \frac{dv_{c_f}}{dt} = \frac{-V}{1 \times 10^6} + 1.602 \times 10^{-19} \frac{ions}{\sec}$$

Integrating current with respect to t, $\frac{1}{R_2}\int_0^t v dt' = C_f \left(V_{c_f} - V_{c_f}(0)\right)$

$$\frac{1.602 \times 10^{-19} \times ions}{R_2} = C_f V_{c_f}$$

$$V_{c_f} = V_a - V_{out} \Rightarrow V_{out} = \frac{-R_1}{R_2 C_f} \times 1.602 \times 10^{-19} \times ions \Rightarrow V_{out} = \frac{-1}{C_f} \times 1.602 \times 10^{-19} \times ions$$

$R_1 = 1$ MΩ, $C_f = 1$μF

53. $\mathrm{R} = 0.5\mathrm{M}\Omega,\ \mathrm{C} = 2\mu\mathrm{F},\ \mathrm{R}_i = \infty,\ \mathrm{R}_o = 0,\ v_o = \cos 10t - 1\mathrm{V}$

(a) Eq. (16) is: $\left(1+\frac{1}{\mathrm{A}}\right)v_o = -\frac{1}{\mathrm{RC}}\int_o^t \left(v_s + \frac{v_o}{\mathrm{A}}\right)dt - v_c(0)$

$\therefore \left(1+\frac{1}{\mathrm{A}}\right)v_o' = -\frac{1}{\mathrm{RC}}\left(v_s + \frac{v_o}{\mathrm{A}}\right) \therefore \left(1+\frac{1}{\mathrm{A}}\right)(-10\sin 10t) = -1\left(v_s + \frac{1}{\mathrm{A}}\cos 10t - \frac{1}{\mathrm{A}}\right)$

$\therefore v_s = \left(1+\frac{1}{\mathrm{A}}\right)10\sin 10t + \frac{1}{\mathrm{A}} - \frac{1}{\mathrm{A}}\cos 10t$ Let $\mathrm{A} = 2000$

$\therefore \boxed{v_s = 10.005\sin 10t + 0.0005 - 0.0005\cos 10t}$

(b) Let $\mathrm{A} = \infty \therefore v_s = \boxed{10\sin 10t\mathrm{V}}$

54. Create a op-amp based differentiator using an ideal op amp with input capacitor C_1 and feedback resistor R_f followed by inverter stage with unity gain.

$$V_{out} = +\frac{R}{R} R_f C_1 \frac{dvs}{dt} = 60 \times \frac{1mV}{rpm} / \min$$

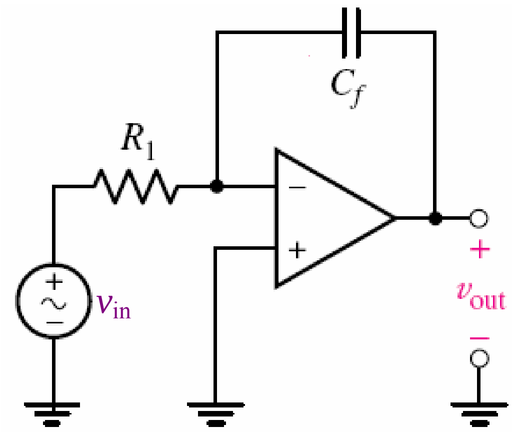
$R_f C_1$=60 so choose R_f= 6 MΩ and C_1 = 10 μF.

55. (a) $$0 = \frac{1}{L}\int v dt + \frac{V_a - V_{out}}{R_f}$$

$$V_a = V = 0, \therefore \frac{1}{L}\int v_L dt = \frac{V_{out}}{R_f} \Rightarrow \boxed{V_{out} = \frac{-R_f}{L}\int_0^t v_s dt'}$$

(b) In practice, capacitors are usually used as capacitor values are more readily available than inductor values.

56. One possible solution:



$$v_{out} = -\frac{1}{R_1 C_f}\int v_{in}\,dt$$

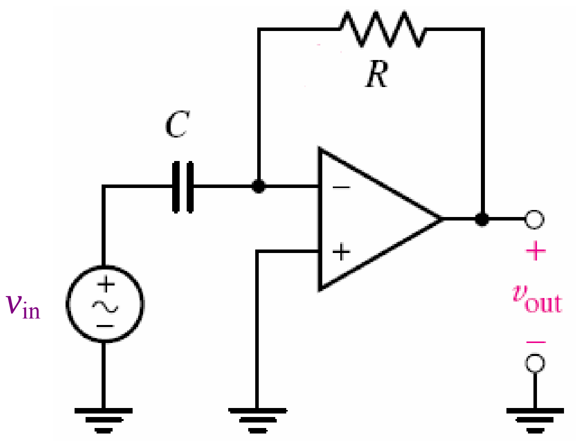
we want $v_{out} = 1$ V for $v_{in} = 1$ mV over 1 s.

In other words, $1 = -\frac{1}{R_1 C_f}\int_0^1 10^{-3}\,dt = -\frac{10^{-3}}{R_1 C_f}$

Neglecting the sign (we can reverse terminals of output connection if needed), we therefore need $R_1C_f = 10^{-3}$.

Arbitrarily selecting $C_f = 1\ \mu F$, we find $R_1 = 1\ k\Omega$.

57. One possible solution of many:



$$v_{out} = -RC\frac{dv_{in}}{dt}$$

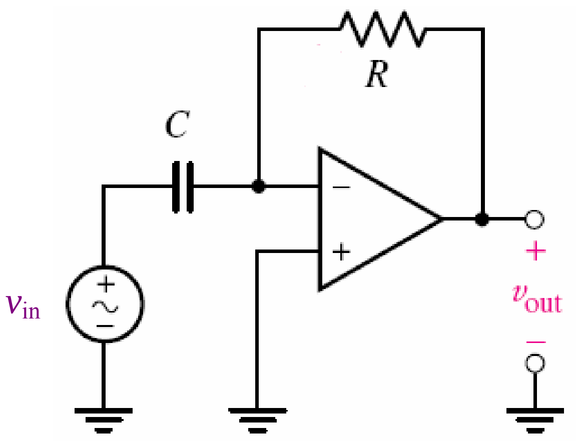
$$\text{maximum}\left(\frac{dv_{in}}{dt}\right) .= \frac{100\text{ mV}}{60\text{s}}$$

In other words, $|v_{out}| = 1\text{ V} = RC\left(\dfrac{100\text{ mV}}{60\text{s}}\right)$

$$or\ RC = 600$$

Arbitrarily selecting $\boxed{\text{C = 1000 μF, we find that R = 600 kΩ.}}$

58. One possible solution of many:



$$v_{out} = -RC\frac{dv_{in}}{dt}$$

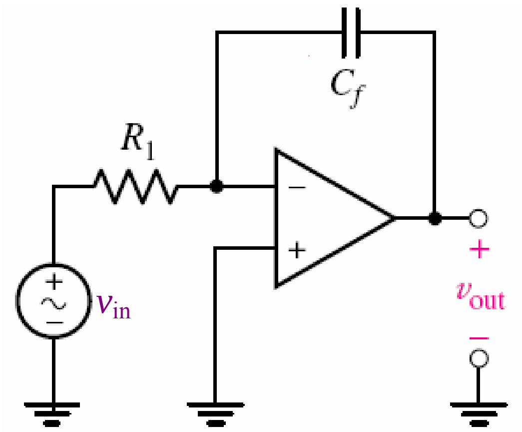
At 1 litre/s, $\left(\frac{dv_{in}}{dt}\right) = \frac{100 \text{ mV}}{\text{s}}$.

In other words, $|v_{out}| = 1 \text{ V} = RC\left(\frac{100 \text{ mV}}{1\text{s}}\right)$

$or\ RC = 10$

Arbitrarily selecting $\boxed{C = 10\ \mu F, \text{ we find that } R = 1\ M\Omega.}$

59. One possible solution:



The power into a 1 Ω load is I^2, therefore energy = W = $I^2\Delta t$.

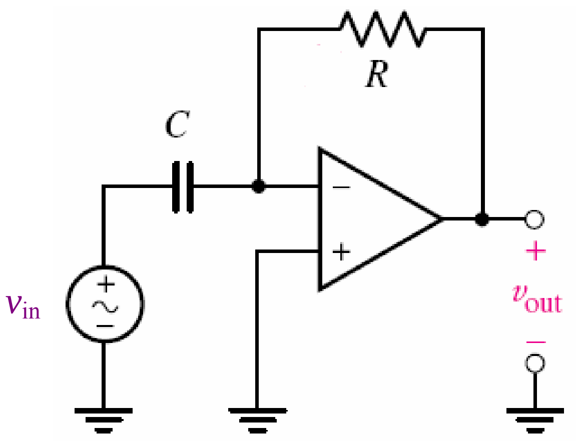
$$|v_{out}| = \frac{1}{R_1 C_f}\int I^2 dt$$

we want $v_{out} = 1$ mV for $v_{in} = 1$ mV (corresponding to 1 A^2).

Thus, $10^{-3} = RC\left(10^{-3}\right)$, so $RC = 1$

$\boxed{\text{Arbitrarily selecting C = 1 μF, we find that we need R = 1 MΩ.}}$

60. One possible solution of many:



$$v_{out} = -RC\frac{dv_{in}}{dt}$$

Input: 1 mV = 1 mph, 1 mile = 1609 metres.
Thus, on the input side, we see 1 mV corresponding to 1609/3600 m/s.

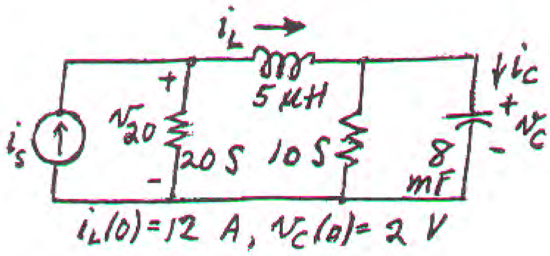
Output: 1 mV per m/s^2. Therefore,

$$|v_{out}| = 2.237RC = 1$$
$$so \;\; RC = 0.447$$

Arbitrarily selecting C = 1 μF, we find that R = 447 kΩ.

61.

(a)



(b)

$$20v_{20} + \frac{1}{5\times10^{-6}}\int_o^t (v_{20} - v_c)dt + 12 = i_s$$

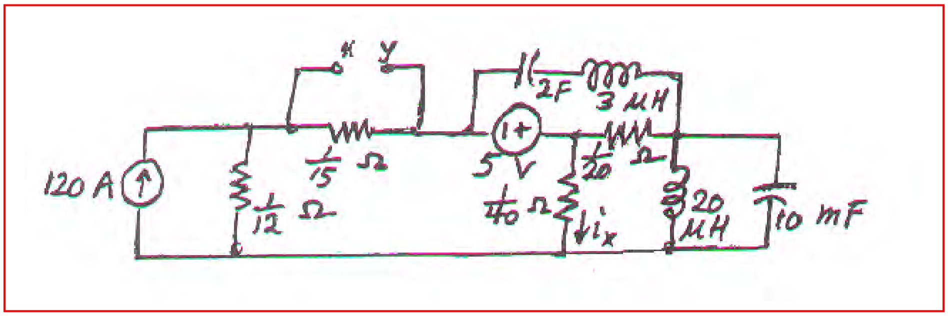
$$\frac{1}{5\times10^{-6}}\int_o^t (v_c - v_{20})dt - 12 + 10v_c + 8\times10^{-3}v_c' = 0$$

(c)

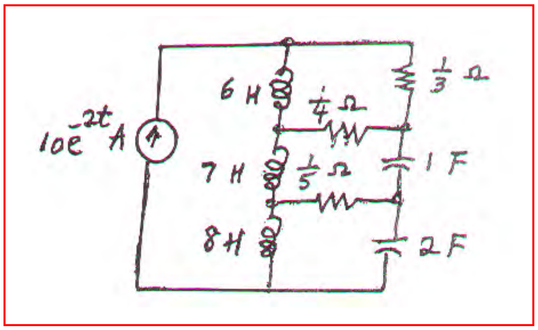
$$\frac{i_L - i_s}{20} + 5\times10^{-6}\, i_L' + \frac{i_L - i_c}{10} = 0$$

$$\frac{i_c - i_L}{10} + \frac{1}{8\times10^{-3}}\int_o^t i_c dt + 2 = 0$$

62.

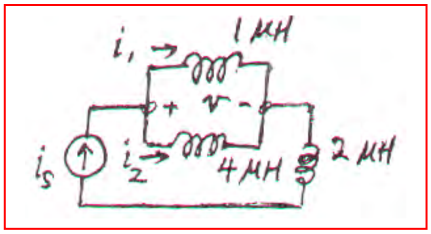


63.



64.

(a)



(b) "Let $i_s = 100e^{-80t}$ A and i_1 (0) = 20 A in the circuit of (new) Fig. 7.62.

(a) Determine *v(t)* for all *t*.
(b) Find $i_1(t)$ for $t \geq 0$.
(c) Find $v_2(t)$ for $t \geq 0$."

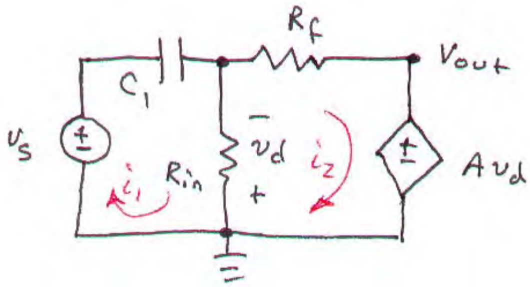
(c) (a) $$L_{eq} = 1\|4 = 0.8\mu\text{H} \therefore v(t) = L_{eq}\, i_s' = 0.8\times10^{-6}\times100(-80)r^{-80t}\,\text{V}$$

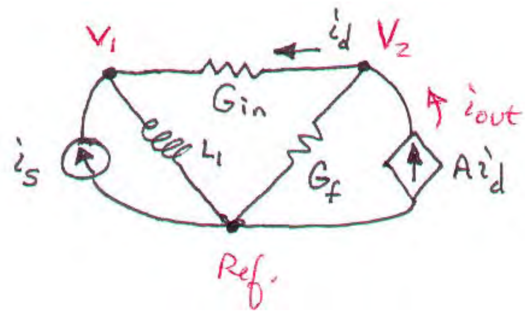
$$\therefore \text{v(t)} = \boxed{-6.43^{\text{-80t}}\text{mV}}$$

(b) $$i_1(t) = 10^6\int_o^t -6.4\times10^{-3}e^{-80t}dt + 20 \therefore i_1(t) = \frac{6400}{80}(e^{-80t}-1) = \boxed{80e^{-80t} - 60\text{A}}$$

(c) $$i_2(t) = i_s - i_1(t) \therefore i_2(t) = \boxed{20e^{-80t} + 60\text{A}}$$

65.





In creating the dual of the original circuit, we have lost both v_s and v_{out}. However, we may write the dual of the original transfer function: i_{out}/ i_S. Performing nodal analysis,

$$i_S = \frac{1}{L_1}\int_0^t V_1 dt' + G_{in}(V_1 - V_2) \qquad [1]$$

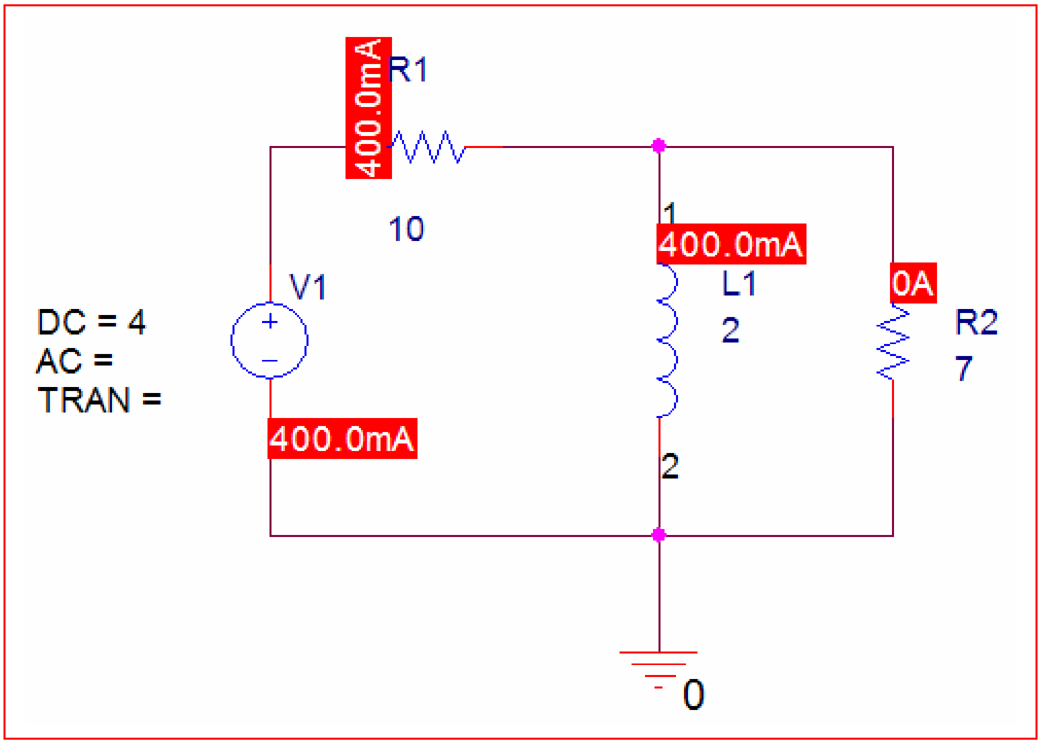
$$i_{out} = A i_d = G_f V_2 + G_{in}(V_2 - V_1) \qquad [2]$$

Dividing, we find that

$$\boxed{\frac{i_{out}}{i_S} = \frac{G_{in}(V_2 - V_1) + G_f V_2}{\frac{1}{L_1}\int_0^t V_1 dt' + G_{in}(V_1 - V_2)}}$$

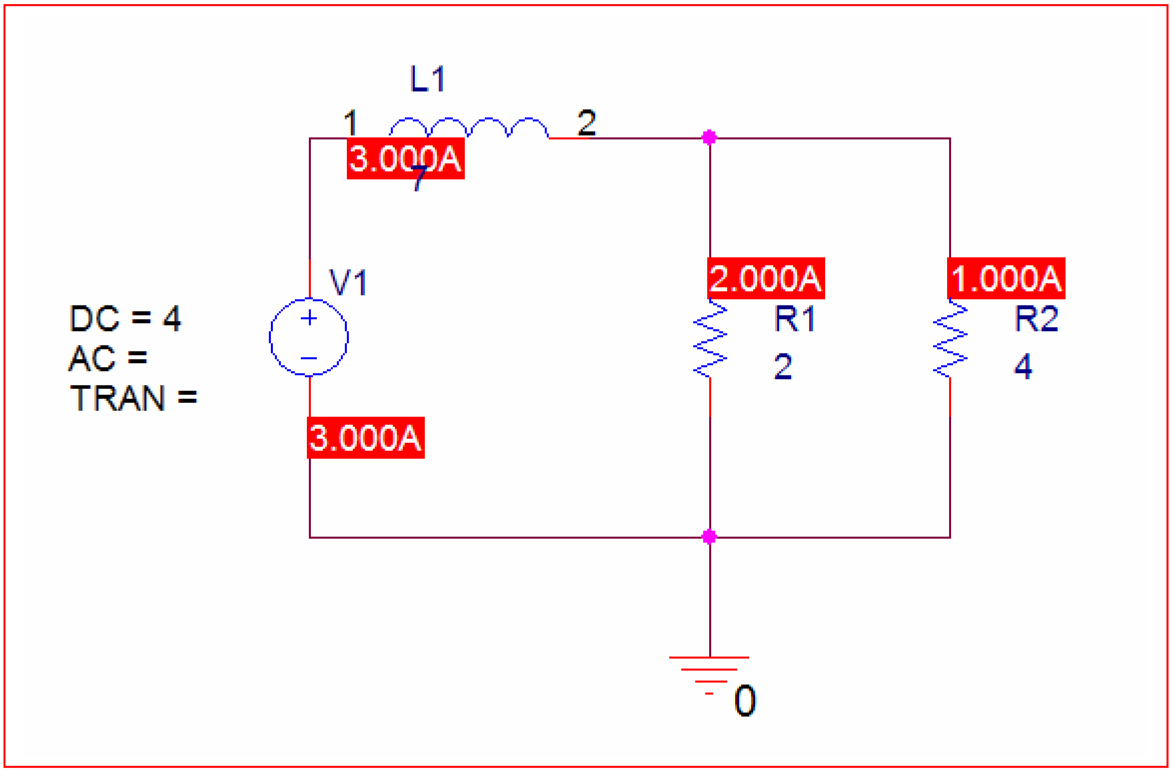
66. $I_L = 4/10 = 400$ mA. $W = \frac{1}{2} L I_L^2 = \boxed{160 \text{ mJ}}$

PSpice verification:



67. $I_L = 4/(4/3) = 3$ A. $W = \frac{1}{2} L I_L^2 = \boxed{31.5 \text{ J}}$

PSpice verification:



68. We choose the bottom node as the reference node, and label the nodal voltage at the top of the dependent source V_A.

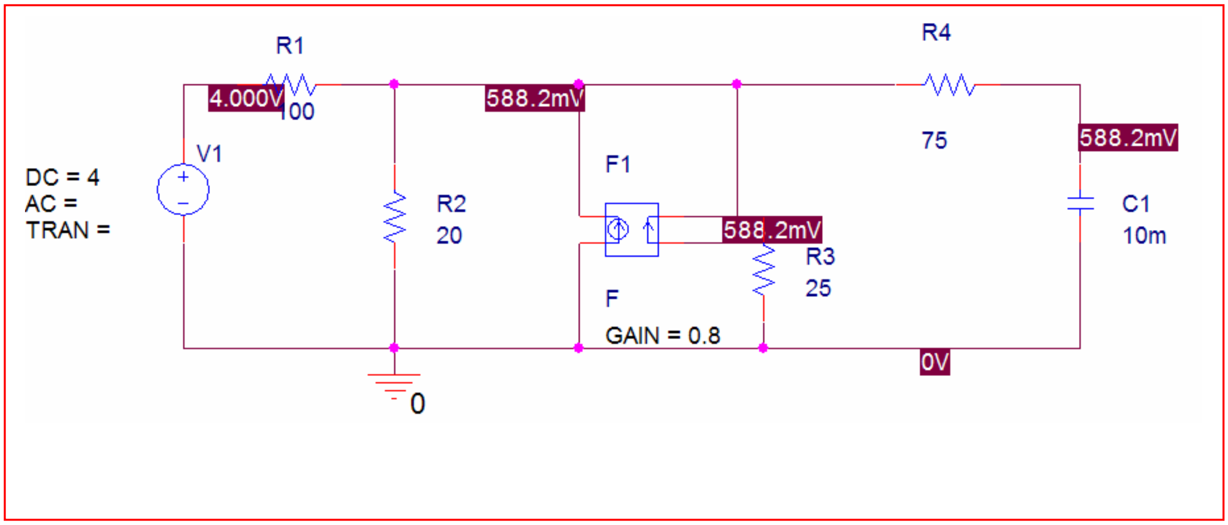
Then, by KCL,

$$\frac{V_A - 4}{100} + \frac{V_A}{20} + \frac{V_A}{25} = 0.8\frac{V_A}{25}$$

Solving, we find that V_A = 588 mV.

Therefore, V_C, the voltage on the capacitor, is 588 mV (no DC current can flow through the 75 Ω resistor due to the presence of the capacitor.)

Hence, the energy stored in the capacitor is $\frac{1}{2}CV^2 = \frac{1}{2}\left(10^{-3}\right)\left(0.588\right)^2 = \boxed{173\ \mu\text{J}}$

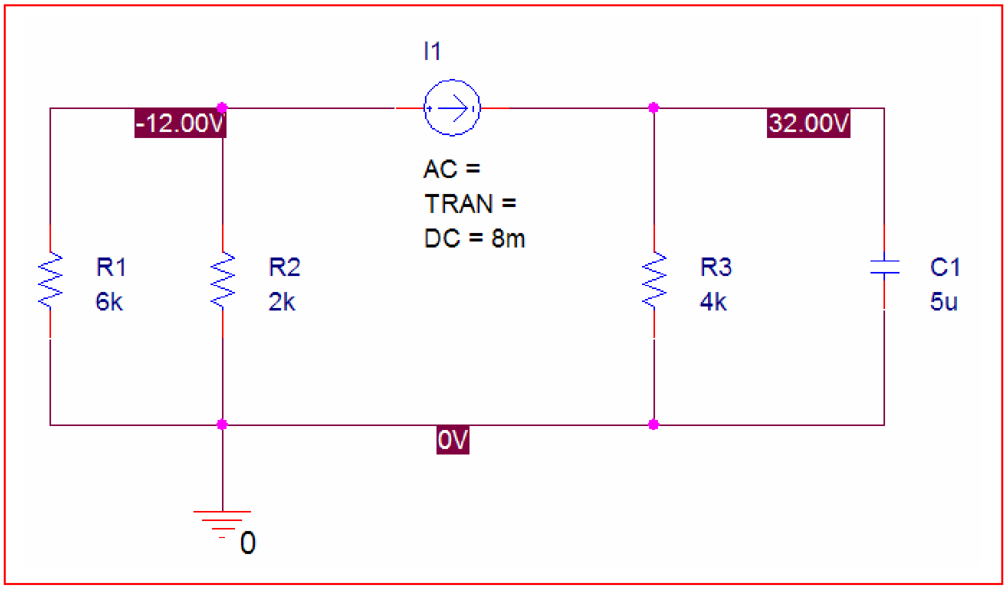


69. By inspection, noting that the capacitor is acting as an open circuit,

the current through the 4 kΩ resistor is 8 mA. Thus, Vc = (8)(4) = 32 V.

Hence, the energy stored in the capacitor = $\frac{1}{2}CV^2 = \frac{1}{2}\left(5\times10^{-6}\right)\left(32\right)^2 = \boxed{2.56\text{ mJ}}$

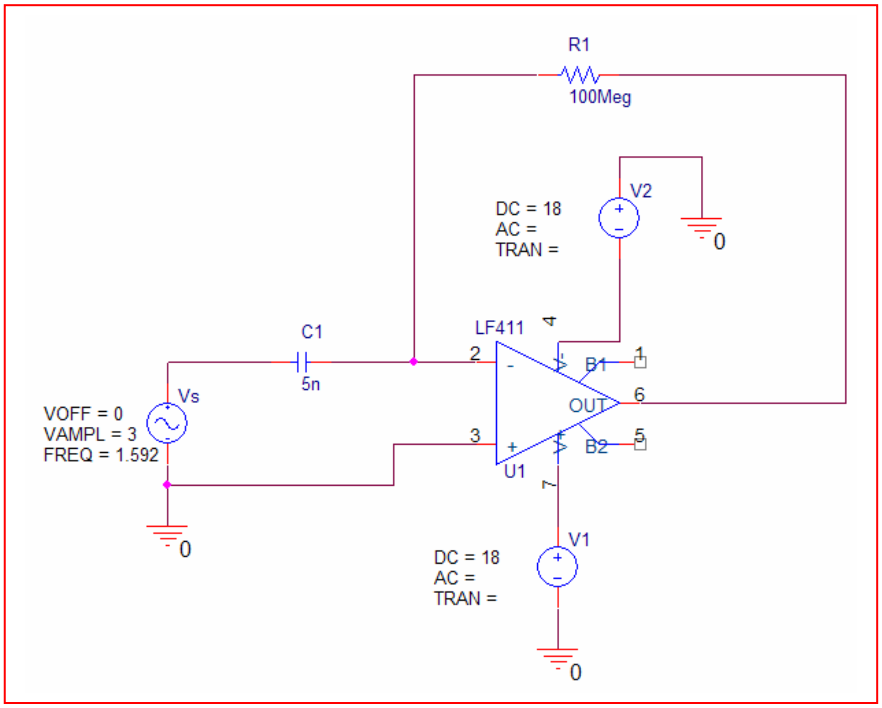
PSpice verification:

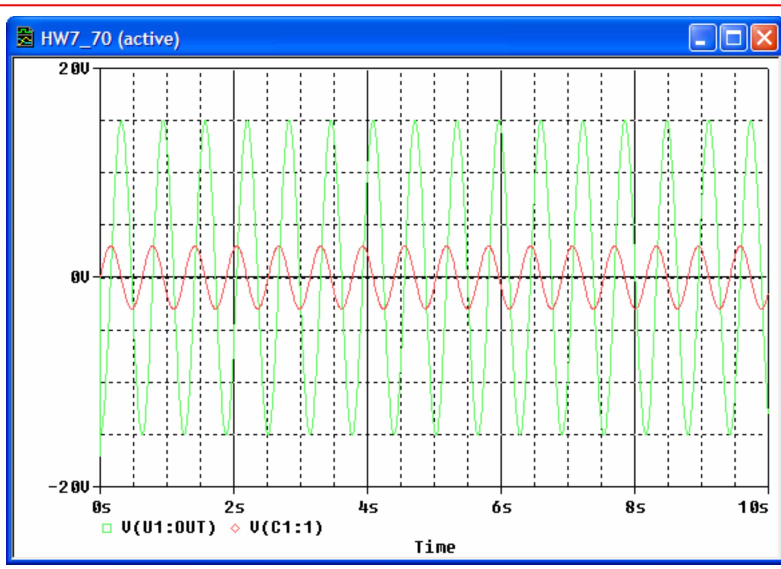


70. $C_1 = 5$ nF, $R_f = 100$ MΩ.

$$v_{out} = -R_f C_1 \frac{dv_s}{dt} = -\left(5\times10^{-9}\right)\left(10^{8}\right)\left(30\cos 100t\right) = \boxed{-15\cos 10t \text{ V}}$$

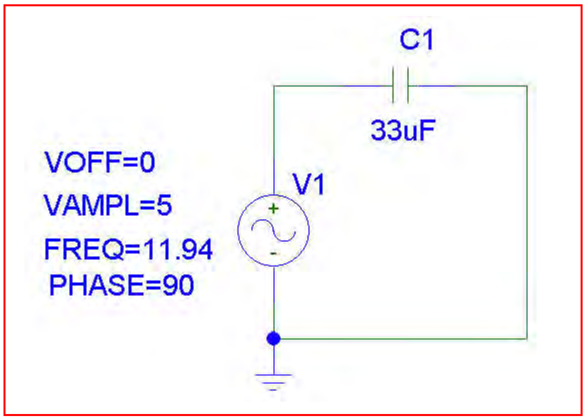
Verifying with PSpice, choosing the LF411 and ±18 V supplies:

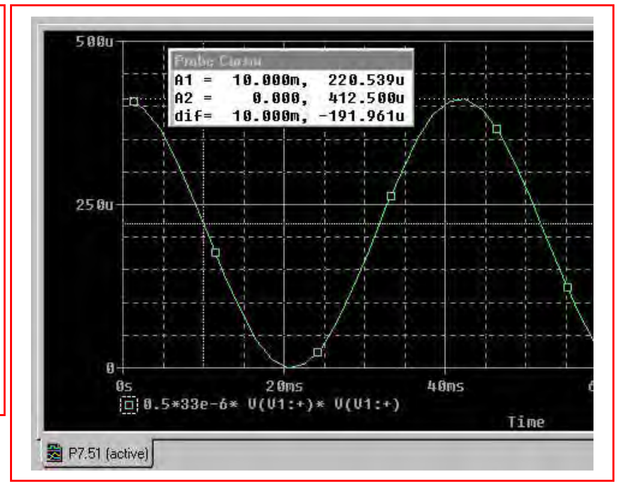




71. PSpice verification

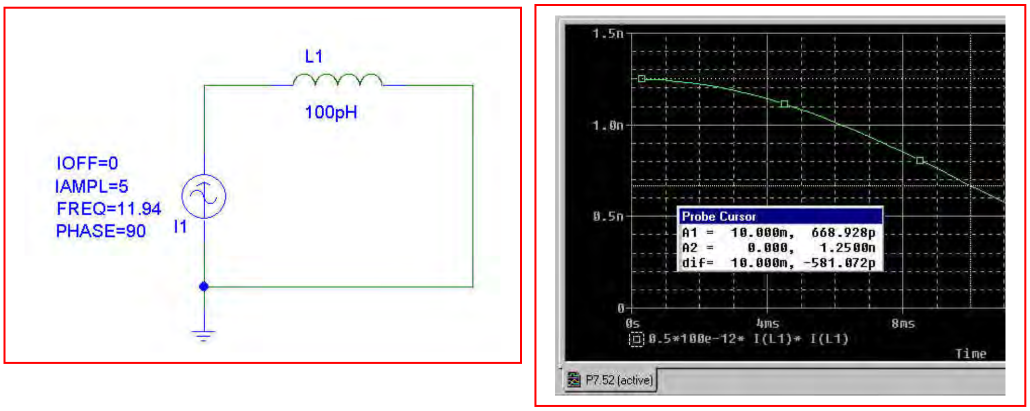
$w = \frac{1}{2} Cv^2 = 0.5\,(33\times10^{-6})[5 \cos (75\times10^{-2})]^2 = 220.8\ \mu\text{J}$. This is in agreement with the PSpice simulation results shown below.





72. PSpice verification

$w = \frac{1}{2} Li^2 = 0.5\,(100\times10^{-12})[5 \cos (75\times10^{-2})]^2 = 669.2$ pJ. This is in agreement with the PSpice simulation results shown below.



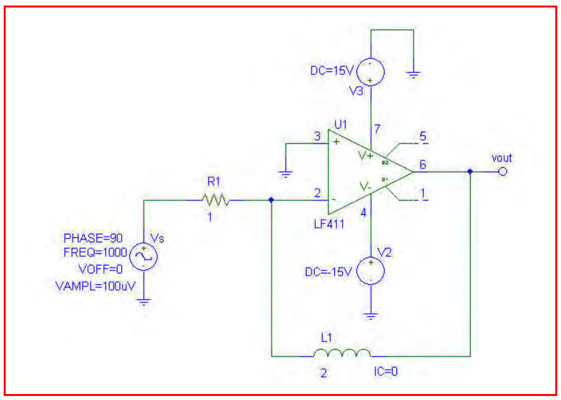
73. $$0 = \frac{V_a - V_s}{R_1} + \frac{1}{L}\int v_{L_f}\,dt$$

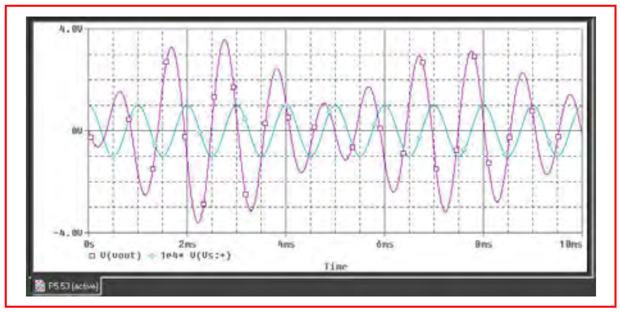
$$V_a = V_b = 0, \qquad 0 = \frac{-V_s}{R_1} + \frac{1}{L}\int v_{L_f}\,dt$$

$$V_{L_f} = V_a - V_{out} = 0 - V_{out} = \frac{L}{R_1}\frac{dVs}{dt}$$

$$V_{out} = -\frac{L_f}{R_1}\frac{dVs}{dt} = -\frac{L_f}{R_1}\frac{d}{dt}\left(A\cos 2\pi 10^3 t\right) \Rightarrow L_f = 2R_1; \text{Let\_R} = 1\ \Omega \text{ and } L = 1\text{ H.}$$

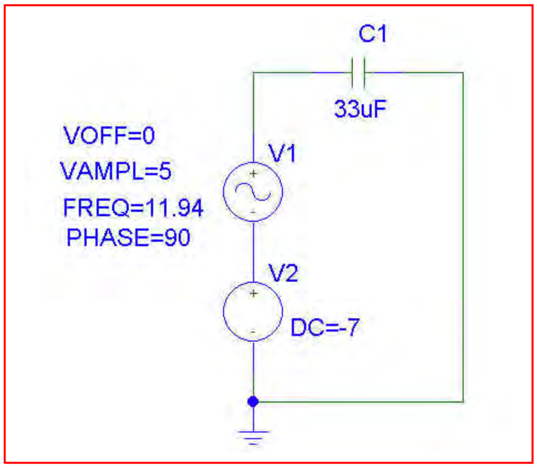
PSpice Verification: clearly, something rather odd is occuring in the simulation of this particular circuit, since the output is not a pure sinusoid, but a combination of several sinusoids.

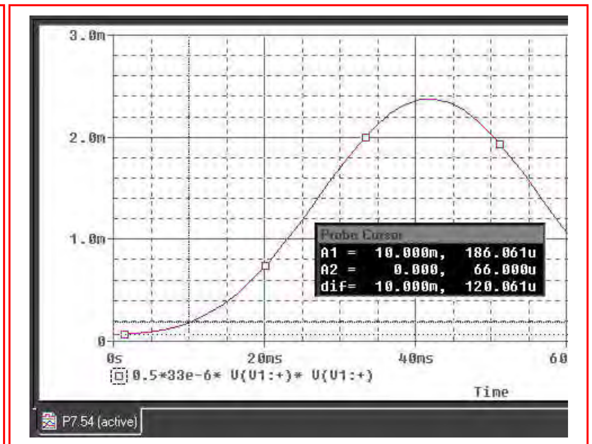




74. PSpice verification

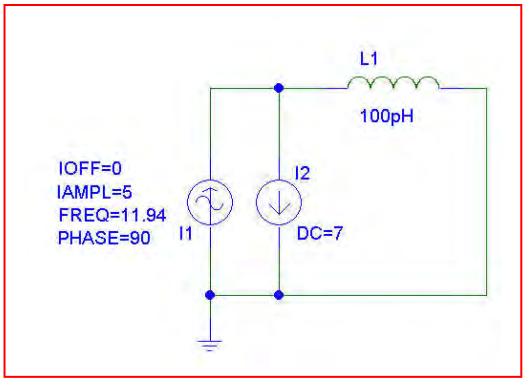
$w = \frac{1}{2} Cv^2 = 0.5\,(33\times10^{-6})[5\cos(75\times10^{-2}) - 7]^2 = 184.2\ \mu\text{J}$. This is in reasonable agreement with the PSpice simulation results shown below.

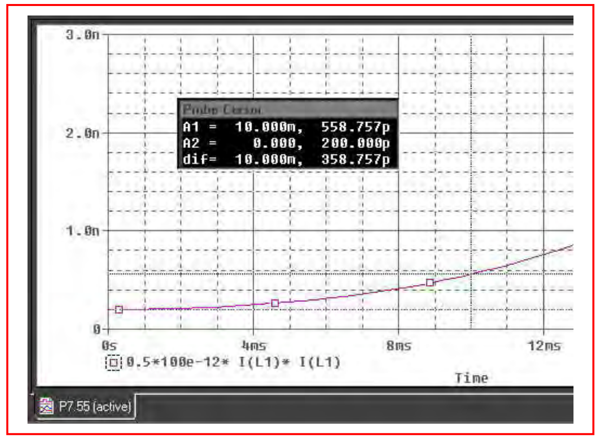




75. PSpice verification

$w = \frac{1}{2} Li^2 = 0.5\,(100\times10^{-12})[5\cos(75\times10^{-2}) - 7]^2 = 558.3$ pJ. This is in agreement with the PSpice simulation results shown below.





1. $i(t) = i(0)e^{-R/L\, t} = 2e^{-4.7\times10^{9}\, t}$ mA

(a) $i(100\text{ ps}) = 2e^{-4.7\times10^{9}\left(100\times10^{-12}\right)} = \boxed{1.25\text{ mA}}$

(b) $i(212.8\text{ ps}) = 2e^{-4.7\times10^{9}\left(212.8\times10^{-12}\right)} = \boxed{736\ \mu\text{A}}$

(c) $v_R = -iR$

$v_R(75\text{ ps}) = -2\left(4700\right)e^{-4.7\times10^{9}\left(75\times10^{-12}\right)} = \boxed{-6.608\text{ V}}$

(d) $v_L(75\text{ ps}) = v_R(75\text{ ps}) = \boxed{-6.608\text{ V}}$

2.

$W = \frac{1}{2}Li^2 = 100 \text{ mJ at } t = 0.$

$Thus, i(0) = \sqrt{0.1} = 316 \text{ mA}$

and $i(t) = i(0)e^{-\frac{R}{L}t} = 316e^{-t/2} \text{ mA}$

(a) At $t = 1$ s, $i(t) = 316e^{-1/2}$ mA $= \boxed{192 \text{ mA}}$

(b) At $t = 5$ s, $i(t) = 316e^{-5/2}$ mA $= \boxed{25.96 \text{ mA}}$

(c) At $t = 10$ s, $i(t) = 316e^{-10/2}$ mA $= \boxed{2.13 \text{ mA}}$

(d) At $t = 2$ s, $i(t) = 316e^{-1}$ mA $=$ 116.3 mA . Thus, the energy remaining is

$W(2) = \frac{1}{2}Li(2)^2 = \boxed{13.53 \text{ mJ}}$

3. We know that $i(t) = i(0)e^{-\frac{R}{L}t} = 2\times10^{-3}e^{-\frac{100}{L}t}$, and that i(500 μs) = 735.8 μA.

Thus, $$L = \frac{-100\left(500\times10^{-6}\right)}{\ln\left(\frac{i(500\times10^{-6})}{2\times10^{-3}}\right)} = \frac{-100\left(500\times10^{-6}\right)}{\ln\left(\frac{735.8\times10^{-6}}{2\times10^{-3}}\right)} = \boxed{50\text{ mH}}$$

4. We know that $i(t) = i(0)e^{-\frac{R}{L}t} = 1.5e^{-\frac{R}{3\times10^{-3}}t}$, and that $i(2) = 551.8$ mA.

Thus, $$R = -\frac{3\times10^{-3}}{2}\ln\left(\frac{i(2)}{1.5}\right) = -\frac{3\times10^{-3}}{2}\ln\left(\frac{0.5518}{1.5}\right) = \boxed{1.50\ \text{m}\Omega}$$

5. We know that $i(t) = i(0)e^{-\frac{R}{L}t} = 1.5e^{-\frac{R}{3\times10^{-3}}t}$, and that $W(0) = 1$ J; $W(10^{-3}) = 100$ mJ.

At $t = 0$, $\frac{1}{2}\left(3\times10^{-3}\right)\left[i\left(0\right)\right]^2 = 1$ therefore $i(0) = 25.82$ A.

At $t = 1$ ms, $\frac{1}{2}\left(3\times10^{-3}\right)\left[i\left(10^{-3}\right)\right]^2 = 0.1$ therefore $i(10^{-3}) = 8.165$ A.

Thus, $$R = -\frac{3\times10^{-3}}{t}\ln\left(\frac{i(t)}{i(0)}\right) = -\frac{3\times10^{-3}}{0.001}\ln\left(\frac{8.165}{25.82}\right) = \boxed{3.454\ \Omega}$$

6.

(a) Since the inductor current can't change instantaneously, we simply need to find i_L while the switch is closed. The inductor is shorting out both of the resistors, so $\boxed{i_L(0^+) = 2 \text{ A.}}$

(b) The instant after the switch is thrown, we know that 2 A flows through the inductor. By KCL, the simple circuit must have 2 A flowing through the 20-Ω resistor as well. Thus,

$$\boxed{v = 4(20) = 80 \text{ V.}}$$

7. (a) Prior to the switch being thrown, the 12-Ω resistor is isolated and we have a simple two-resistor current divider (the inductor is acting like a short circuit in the DC circuit, since it has been connected in this fashion long enough for any transients to have decayed). Thus, the current i_L through the inductor is simply 5(8)/ (8 + 2) = $\boxed{\text{4 A.}}$ The voltage v must be $\boxed{\text{0 V.}}$

(b) The instant just after the switch is thrown, the inductor current must remain the same, so $\boxed{i_L = 4\text{ A.}}$ KCL requires that the same current must now be flowing through the 12-Ω resistor, so v = 12(-4) = $\boxed{\text{-48 V.}}$

8.

(a) $i_L(0) = 4.5\text{mA},\ \text{R/L} = \dfrac{10^3}{4\times10^{-3}} = \dfrac{10^6}{4}$

$\therefore i_L = 4.5e^{-10^6 t/4}\text{mA} \therefore i_L(5\mu\, s) = 4.5e^{-1.25}$

$\boxed{= 1.289 \text{ mA.}}$

(b) $i_{SW}(5\ \mu s) = 9 - 1.289 = \boxed{7.711 \text{ mA.}}$

9.

(a) $i_L(0) = \frac{100}{50} = 2\text{A} \therefore i_L(t) = 2e^{-80t/0.2}$
$= \boxed{2e^{-400t}\text{A}, t > 0}$

(b) $i_L(0.01) = 2e^{-4} = \boxed{36.63\text{mA}}$

(c) $2e^{-400t_1} = 1,\ e^{400t_1} = 2,\ t_1 = \boxed{1.7329\text{ms}}$

10. (a)

$$L\frac{di}{dt}+5i=0 \qquad [1]$$

$v_R = -2i$ so Eq. [1] can be written as

$$L\frac{d\left(\frac{-v_R}{2}\right)}{dt}-5\left(\frac{-v_R}{2}\right)=0 \quad \text{or}$$

$$\boxed{2.5\frac{dv_R}{dt}+2.5v_R=0}$$

(b) Characteristic equation is 2.5**s** + 2.5 = 0, or **s** + 1 = 0

Solving, $\boxed{\mathbf{s}=-1,\ v_R(t)=Ae^{-t}}$

(c) At $t = 0^-$, $i(0^-) = 5$ A $= i(0^+)$. Thus, $v_R(t)=-10e^{-t}, t>0$

$$\boxed{\begin{aligned} &v_R(0^-)=\frac{2}{3}(10)= \quad 6.667\text{ V} \\ &v_R(0^+)=-10\text{ V} \\ &v_R(1)=-10e^{-1}= \quad -3.679\text{ V}\end{aligned}}$$

11.

(a) $\frac{i}{\mathrm{I}_o} = e^{-t/\tau}, \frac{t}{\tau} = \ell n \frac{\mathrm{I}_o}{i}, \frac{\mathrm{I}_o}{i} = 10 \therefore \frac{t}{\tau} = \ell n 10 = 2.303;$

$\frac{\mathrm{I}_o}{i} = 100, \frac{t}{\tau} = 4.605; \ \frac{\mathrm{I}_o}{i} = 1000, \frac{t}{\tau} = \boxed{6.908}$

(b) $\frac{i}{\mathrm{I}_o} = e^{-t/\tau}, \frac{d(i/\mathrm{I}_o)}{d(t/\tau)} = -e^{t/\tau}; \quad \mathrm{t}\ t/\tau = 1, \frac{d()}{d()} = a - e^{-1}$

Now, $y = m(x-1) + b = -e^{-1}(x-1) + e^{-1}\left(\frac{t}{\tau} = x, \frac{i}{\mathrm{I}_o} = y\right)$

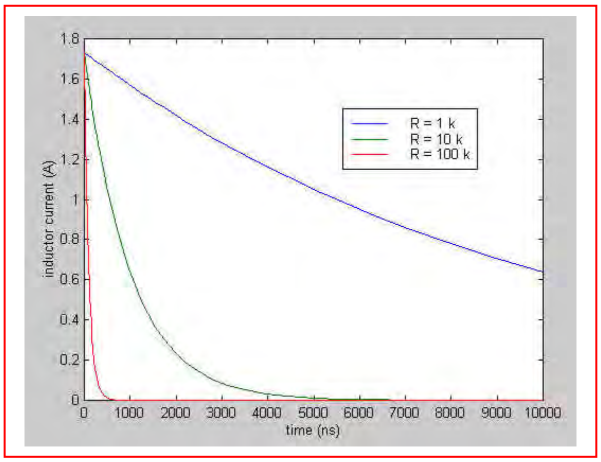
At $y = 0, e^{-1}(x-1) = e^{-1} \therefore x = 2 \therefore t/\tau = \boxed{2}$

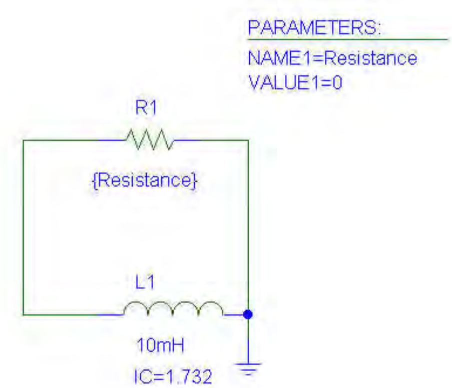
12. Reading from the graph current is at 0.37 at 2 ms

$$\boxed{\begin{array}{l} \therefore \quad \tau = 2\text{ ms} \\ \\ \mathrm{I}_0 = 10\text{ A} \end{array}}$$

13. $w = ½ Li^2$, so an initial energy of 15 mJ in a 10-mH inductor corresponds to an initial inductor current of 1.732 A. For R = 1 kΩ, τ = L/R = 10 μs, so $i_L(t) = 1.732\ e^{-0.1t}$ A. For R = 10 kΩ, τ = 1 μs so $i_L(t) = 1.732\ e^{-t}$. For R = 100 kΩ, τ = 100 ns or 0.1 μs so $i_L(t) = 1.732\ e^{-10t}$ A. For each current expression above, it is assumed that time is expressed in microseconds.

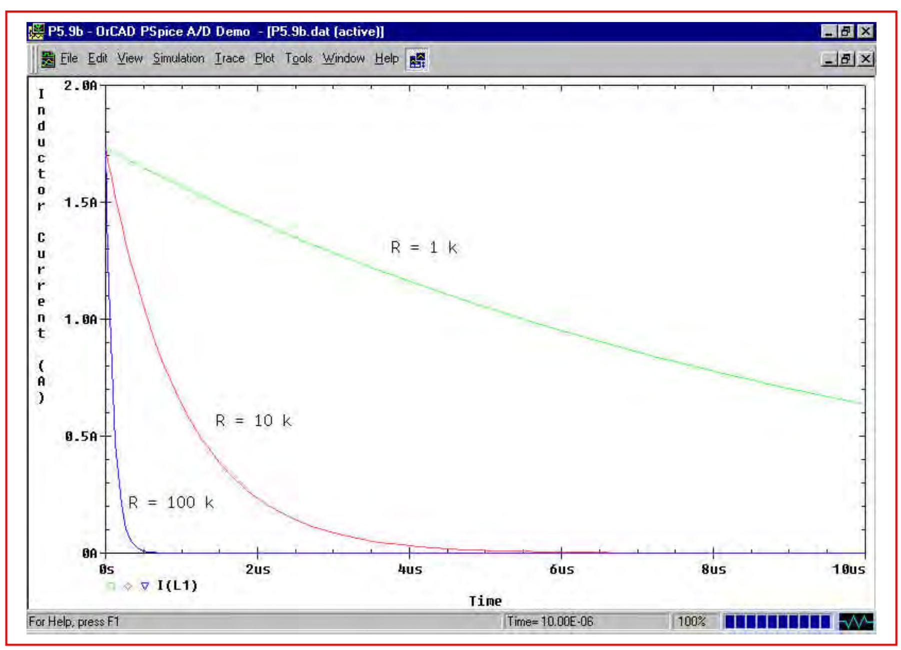
To create a sketch, we first realise that the maximum current for any of the three cases will be 1.732 A, and after one time constant (10, 1, or 0.1 μs), the current will drop to 36.79% of this value (637.2 mA); after approximately 5 time constants, the current will be close to zero.





Sketch based on hand analysis **Circuit used for PSpice verification**

As can be seen by comparing the two plots, which probably should have the same x-axis scale labels for easier comparison, the PSpice simulation results obtained using a parametric sweep do in fact agree with our hand calculations.



14.

(a) $$\tau = \frac{3.3\times10^{-6}}{1\times10^{6}} = \boxed{3.3\times10^{-12}}$$

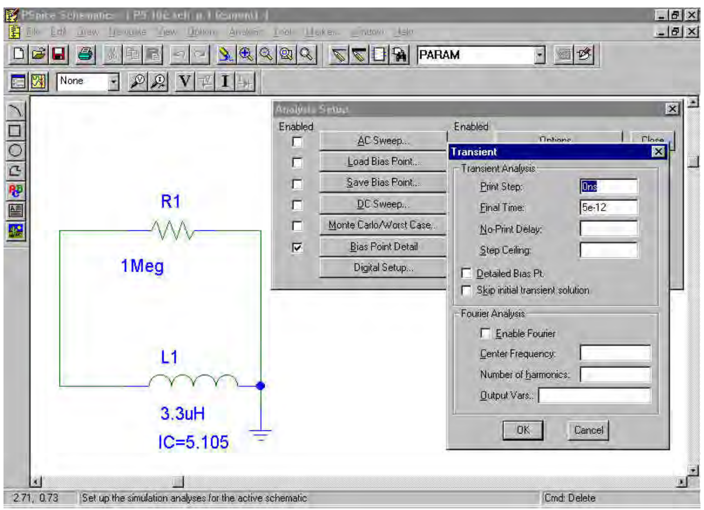
(b)

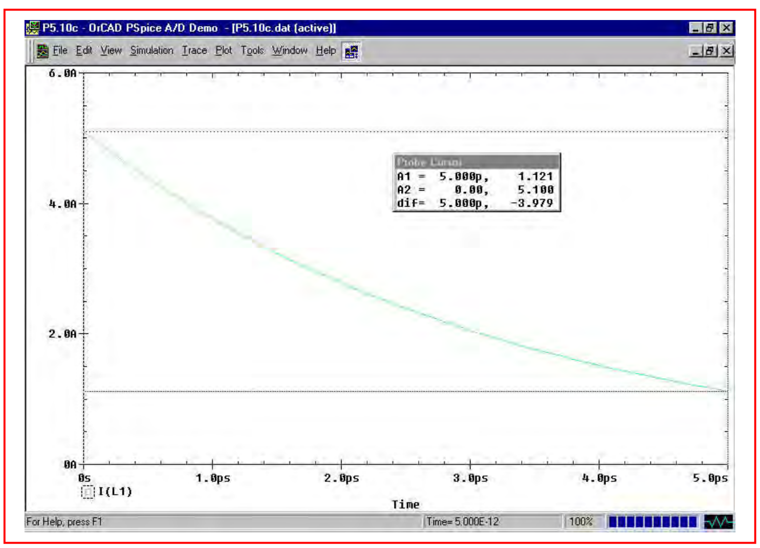
$$\omega = \frac{1}{2}.L.I_0^{\ 2}$$

$$I_0 = \sqrt{\frac{2\times43\times10^{-6}}{3.3\times10^{-6}}} = 5.1\text{ A}$$

$$i(5\,ps) = 5.1e^{-1\times10^{6}\times5\times10^{-12}\;/\;3.3\times10^{-6}} = \boxed{1.12\text{ A}}$$

(c)





From the PSpice simulation, we see that the inductor current is 1.121 A at t = 5 ps, in agreement with the hand calculation.

15. Assume the source Thévenin resistance is zero, and assume the transient is measured to 5τ. Then,

$$\tau = \frac{L}{R} \quad \therefore 5\tau = \frac{5L}{R} = 100\times10^{-9} \text{ secs}$$

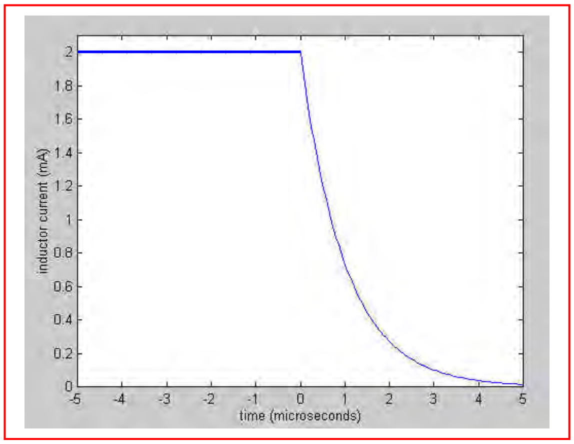
$$\therefore R > \frac{(5)(125.7)10^{-6}}{10^{-7}}$$

so R must be greater than $\boxed{6.285\ \text{k}\Omega.}$

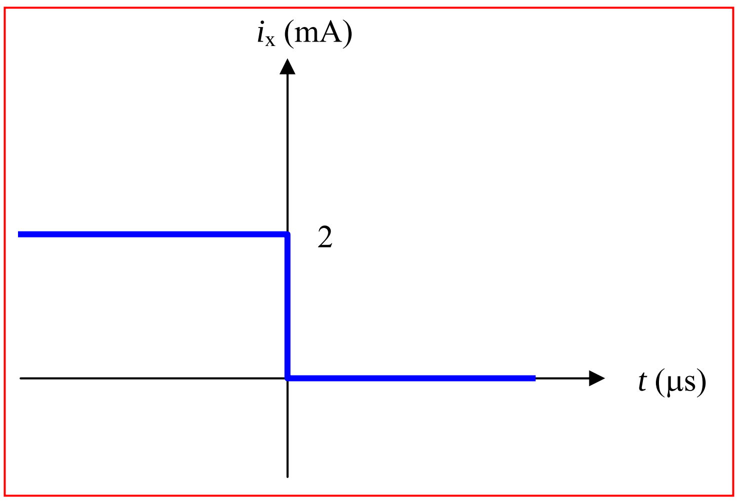
(If 1τ assumed then $R > \frac{6.285}{5} = 125.7\Omega$)

16. For $t < 0$, we have a current divider with $i_L(0^-) = i_x(0^-) = 0.5$ [10 (1/ (1 + 1.5)] mA = 2 mA. For $t > 0$, the resistor through which i_x flows is shorted, so that $i_x(t > 0) = 0$. The remaining 1-kΩ resistor and 1-mH inductor network exhibits a decaying current such that $i_L(t) = 2e^{-t/\tau}$ mA where $\tau = L/R = 1$ μS.

(a)



(b)



17. $\frac{1}{2}C\left[v(0)\right]^2 = 10^{-3}$ so $v(0) = \sqrt{\frac{2\times10^{-3}}{10^{-6}}} = 44.72$ V

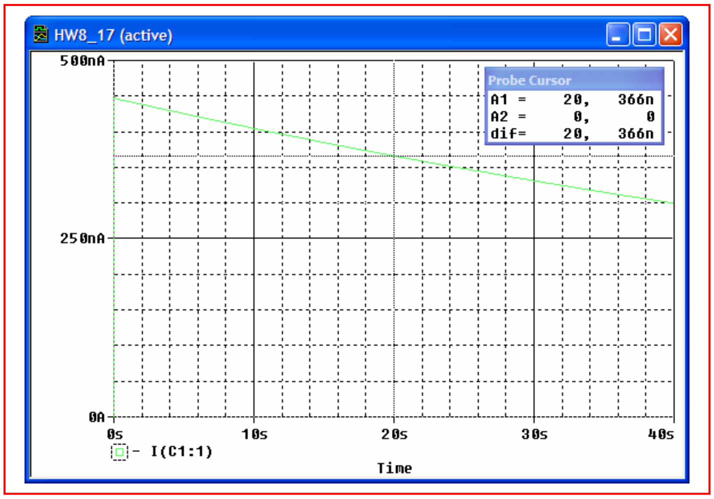
(a) $\tau = \text{RC} = \boxed{100 \text{ s}}$

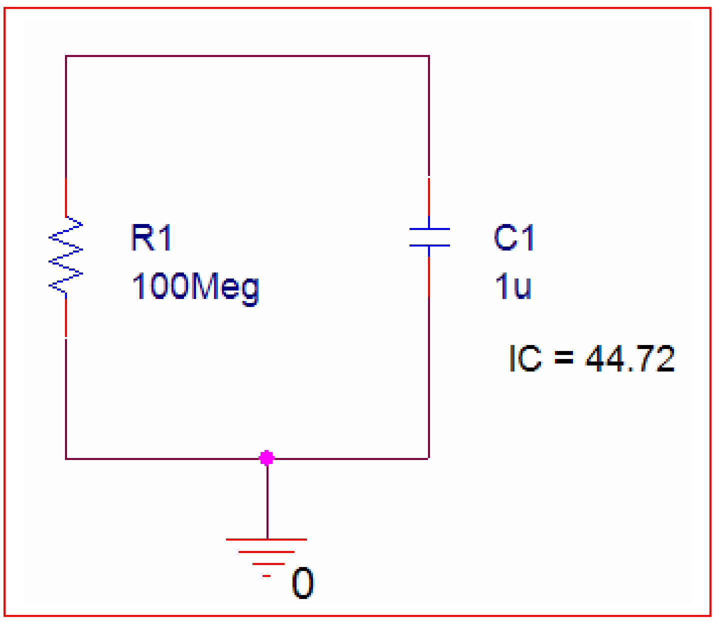
(b) $v(t) = v(0)e^{-t/RC} = 44.72e^{-0.01t}$ V

Thus, $v(20) = = 44.72e^{-0.01(20)} =$ 36.62 V .

Since $i = \frac{v}{R} = \frac{36.62}{100\times10^6} = \boxed{366 \text{ nA}}$

(c)





18. If $i(0) = 10$ A, $v(0) = 10$ V. $v(t) = v(0)e^{-t/RC} = 10e^{-t/2}$ V

(a) At $t = 1$ s, $v(1) = 10e^{-1/2} = \boxed{6.065 \text{ V}}$

(b) At $t = 2$ s, $v(2) = 10e^{-1} = \boxed{3.679 \text{ V}}$

(c) At $t = 5$ s, $v(5) = 10e^{-2.5} = \boxed{821 \text{ mV}}$

(d) At $t = 10$ s, $v(10) = 10e^{-5} = \boxed{67.4 \text{ mV}}$

19. Referring to Fig. 8.62, we note that τ = RC = 4 s. Thus,

$$v(t) = v(0)e^{-t/RC} = 5e^{-0.25t} \quad \text{V.}$$

(a) $v(1\text{ ms}) = 5e^{-0.25(0.001)} = \boxed{4.999\text{ V}}$

(b) $v(2\text{ ms}) = 5e^{-0.25(0.002)} = 4.998\text{ V}$

Therefore i(2 ms) = 4.998 / 1000 = $\boxed{4.998\text{ mA}}$

(c) $v(4\text{ ms}) = 5e^{-0.25(0.004)} = 4.995\text{ V}$

$$W = \frac{1}{2}Cv^2 = \boxed{49.9\text{ mJ}}$$

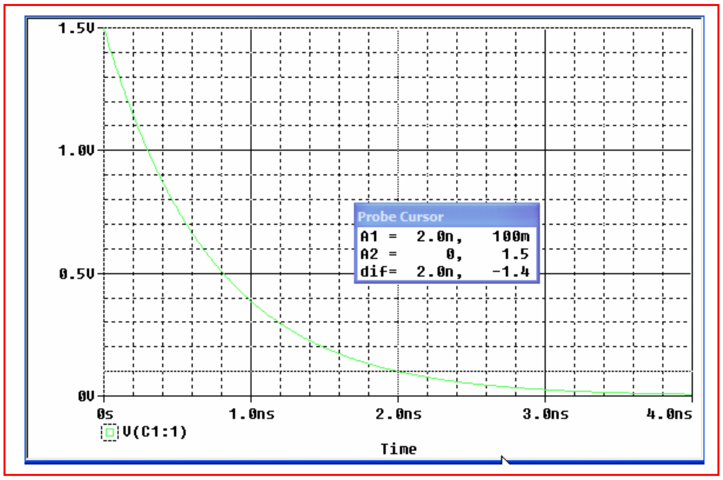
20. (a) $v(t) = v(0)e^{-t/RC} = 1.5e^{-t/RC}$ V

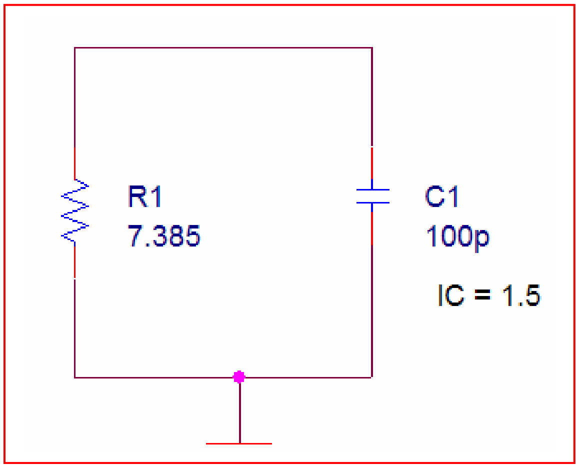
$$\frac{-t}{RC} = \ln\left[\frac{v(t)}{1.5}\right]$$

Thus,

$$R = \frac{-t}{C\ln\left[\frac{v(t)}{1.5}\right]} = \frac{-2\times10^{-9}}{10^{-10}\ln\left[\frac{0.1}{1.5}\right]} = \boxed{7.385\ \Omega}$$

(b)





We see from the PSpice simulation that our predicted voltage at 2 ns agrees with the information used to calculate R = 7.385 Ω.

21. The film acts as an intensity integrator. Assuming that we may model the intensity as a simple decaying exponential,

$$\phi(t) = \phi_o e^{-t/\tau}$$

where the time constant $\tau = R_{TH}C$ represents the effect of the Thévenin equivalent resistance of the equipment as it drains the energy stored in the two capacitors, then the intensity of the image on the film Φ is actually proportional to the integrated exposure:

$$\Phi = K \int_0^{\text{exposure time}} \phi_o e^{-t/\tau} dt$$

where K is some constant. Solving the integral, we find that

$$\Phi = -K \phi_o \tau \left[e^{-(\text{exposure time})/\tau} - 1 \right]$$

The maximum value of this intensity function is $-K\phi_o \tau$.

With 150 ms yielding an image intensity of approximately 14% of the maximum observed and the knowledge that at 2 s no further increase is seen leads us to estimate that $1 - e^{-150\times10^{-3}/\tau} = 0.14$, assuming that we are observing single-exponential decay behavior and that the response speed of the film is not affecting the measurement. Thus, we may extract an estimate of the circuit time constant as $\tau = 994.5$ ms.

This estimate is consistent with the additional observation that at $t = 2$ s, the image appears to be saturated.

With two 50-mF capacitors connected in parallel for a total capacitance of 100 mF, we may estimate the Thévenin equivalent resistance from $\tau = RC$ as $R_{th} = \tau / C$ = $\boxed{9.945\ \Omega.}$

22.

(a) $v_c(0) = 8(50\|200) \times \dfrac{30}{50} = 192\text{V}$

$v_c(t) = 192e^{-3000t/24} = \boxed{192e^{-125t}\text{V}}$

(b) $0.1 = e^{-125t} \;\therefore \boxed{t = 18.421\text{ms}}$

23.

(a) $v_c = 80e^{-10^6 t/100} = 80e^{-10^4 t}\text{V},\ t > 0;\ 0.5 = e^{-10^4 t}\ \therefore \boxed{t = 69.31\mu\,s}$

(b) $w_c = \dfrac{1}{2}C80^2 e^{-20,000t} = \dfrac{1}{4}C80^2\ \therefore \boxed{t = 34.66\mu\,s}$

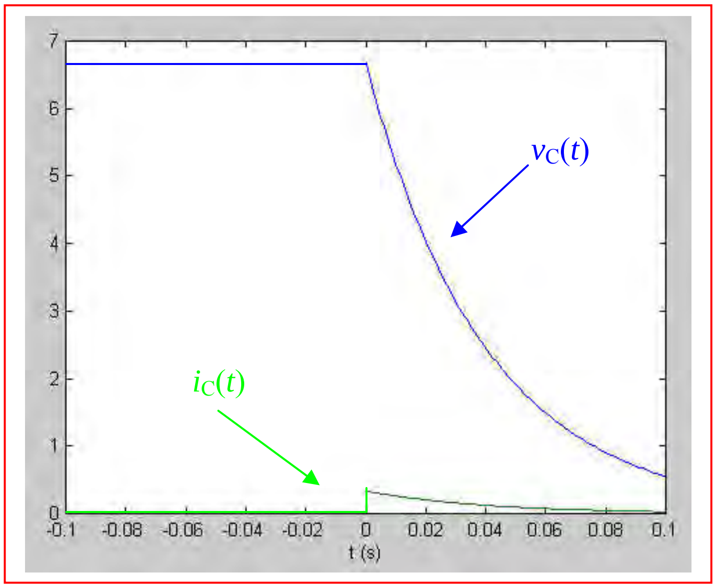
24.

$$t<0: \boxed{i_c(t)=0,}\ 10=5000i_s+10^4\ i_s \therefore i_s=\frac{2}{3}\text{mA}$$

$$\therefore v_c(t)=\frac{20}{3}=\boxed{6.667\text{V}}$$

$$t>0:\ i_s=0 \therefore v_c(t)=6.667e^{-t/2\times10^4\times2\times10^{-6}}$$

$$\therefore \boxed{v_c(t)=6.667e^{-25t}\text{V}} \therefore i_c(t)=\frac{-6.667}{20\times10^3}e^{-25t}=\boxed{0.3333e^{-25t}\text{mA}}$$



25.

$$v(0^+) = 20V$$
$$i(0^+) = 0.1A$$

$$v(1.5ms) = 20e^{-1.5\times10^{-3}/50\times20\times10^{-6}} = 4.5V$$
$$i(1.5ms) = 0A$$

$$v(3ms) = 20e^{-3\times10^{-3}/50\times20\times10^{-6}} = 1V$$
$$i(3ms) = 0A$$

26.

(a) $i_L(0^-) = \frac{1}{2} \times 60 = \boxed{30 \text{ mA},}\ i_x(0^-) = \frac{2}{3} \times 30 = \boxed{20 \text{ mA}}$

(b) $i_L(0^+) = \boxed{30 \text{ mA},}\ i_x(0^+) = \boxed{-30 \text{ mA}}$

(c) $i_L(t) = 30e^{-250t/0.05} = 30e^{-5000t}\text{mA},\ i_L(0.3\text{ms})$
$= 30e^{-1.5} = \boxed{6.694\text{mA}} = -i_x$

Thus, $\boxed{i_x = -6.694 \text{ mA.}}$

27.

(a) $i_L(0) = 4\text{A} \therefore i_L(t) = 4e^{-500t}\text{A} \ (0 \le t \le 1\text{ms})$

$i_L(0.8\text{ms}) = 4e^{-0.4} = \boxed{2.681\text{A}}$

(b) $i_L(1\text{ms}) = 4e^{-0.5} = 2.426\text{A}$

$\therefore i_L(t) = 2.426e^{-250(t-0.001)}$

$\therefore i_L(2\text{ms}) = 2.426e^{-0.25} = \boxed{1.8895^{-}\text{A}}$

28.

(a) $i_L = 40e^{-50,000t}\text{mA} \therefore 10 = 40e^{-50,000t}, \ \therefore t_1 = \boxed{27.73\mu s}$

(b) $i_L(10\mu s) = 40e^{-0.5} = 24.26\text{mA} \therefore i_L$
$= 24.26e^{-(1000+R)50t}(t>10\mu s)$
$\therefore 10 = 24.26e^{-(1000+R)5\times10^{-6}} \ \therefore \ell n2.426 = 0.8863$
$= 0.25(1000+\text{R})10^{-3}, 1000+\text{R} = 0.8863\times4\times10^3 \therefore \text{R} = \boxed{2545^+\Omega}$

29.

(a) $i_1(0) = 20\text{mA},\ i_2(0) = 15\text{mA}$

$\therefore v(t) = 40e^{-50000t} + 45e^{-100000t}\text{V} \therefore \boxed{v(0) = 85\text{V}}$

(b) $v(15\mu s) = 40e^{-0.75} + 45e^{-1.5} = \boxed{28.94\text{V}}$

(c) $\dfrac{85}{10} = 40e^{-50000t} + 45e^{-100000t}.$ Let $e^{-50000t} = x$

$\therefore 45x^2 + 40x - 8.5 = 0$

$\therefore x = \dfrac{-40 \pm \sqrt{1600 + 1530}}{90} = 0.17718, < 0$

$\therefore e^{-50000t} = 0.17718, \boxed{t = 34.61\mu s}$

30. $t < 0 : v_R = \frac{2R_1R_2}{R_1 + R_2}, \downarrow i_L(0) = \frac{2R_1}{R_1 + R_2}$

$t > 0 : i_L(t) = \frac{2R_1}{R_1 + R_2}\ e^{-50R_2t}\ \therefore v_R = \frac{2R_1R_2}{R_1 + R_2}\ e^{-50R_2t}$

$\therefore v_R(0^+) = 10 = \frac{2R_1R_2}{R_1 + R_2} \therefore R_1 \| R_2 = 5\Omega$. Also, $v_R(1\text{ms})$

$= 5 = 10e^{-50R_2/1000} \therefore 0.05R_2 = 0.6931 \therefore \boxed{R_2 = 13.863\Omega}$

$\therefore \frac{1}{13.863} + \frac{1}{R_1} = \frac{1}{5} \therefore \boxed{R_1 = 7.821\Omega}$

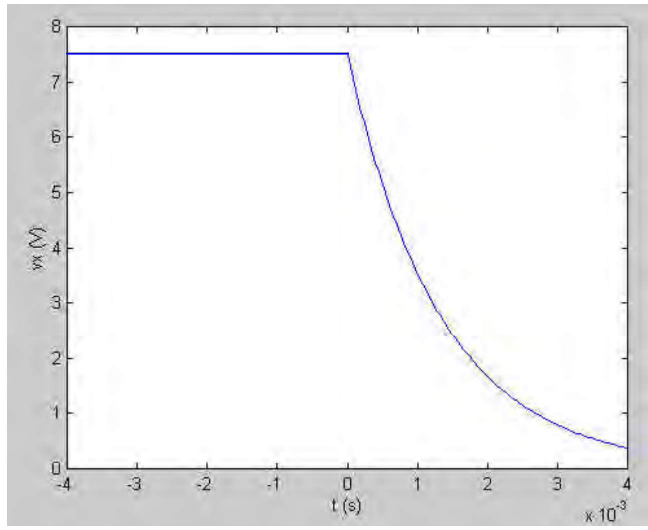
31.

(a) $i_L(0) = \dfrac{24}{60} = 0.4\text{A} \therefore \boxed{i_L(t) = 0.4e^{-750t}\text{A},\ t > 0}$

(b) $v_x = \dfrac{5}{6} \times 24 = 20\text{V},\ t < 0$

$v_x(0^+) = 50 \times 0.4 \times \dfrac{3}{8} = 7.5\text{V}$

$\therefore v_x(t) = 7.5e^{-750t}\text{V},\ t > 0$



32.

$$v_{in} = \frac{3i_L}{4} \times 20 + 10i_L = 25i_L$$

$$v_{in} \therefore \frac{v_{in}}{i_L} = 25\Omega \therefore i_L = 10e^{-25t/0.5} = \boxed{10e^{-50t}\text{A}, \, t > 0}$$

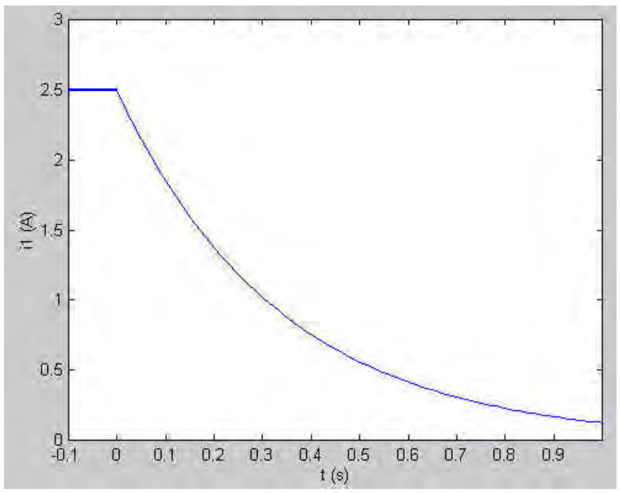
33.

$$i_L(0) = \frac{64}{4 + 40\|8} \times \frac{40}{48} = 5\text{A}$$

$$\therefore i_L = 5e^{-24t/8} = 5e^{-3t}\text{A}$$

$$\therefore i_1(t) = 2.5e^{-3t}\text{A},\ t{>}0;\ \boxed{i_1(-0.1) = 2.5\ \text{A}}$$

$$\boxed{i_1(0.03) = 2.285^-\ \text{A},\ i_1(0.1) = 1.852\ \text{A}}$$



34.

(a) $i_L(0) = 4\text{A} \therefore i_L = 4e^{-100t}\text{A},\ 0 < t < 15\,\text{ms}$

$\therefore i_L(15\text{ms}) = 4e^{-1.5} = \boxed{0.8925^{+}\,\text{A}}$

(b) $t > 15\,\text{ms}:\ i_L = 0.8925^{+}e^{-20(t-0.015)}\text{A}$

$\therefore i_L(30\text{ms}) = 0.8925^{+}e^{-0.3} = \boxed{0.6612\text{A}}$

35.

(a) $i_1(0^+) = i_1(0^-) = 10\text{A},\ i_2(0^+) = i_2(0^-) = 20\text{A} \therefore i(0^+) = \boxed{30\text{A}}$

(b) $\tau = \mathrm{L}_{eq} / \mathrm{R}_{eq} = \dfrac{0.08}{48} = \dfrac{5}{3}\text{ms} = \boxed{1.6667\text{ms}}$

(c) $i_1(0^-) = 10\text{A},\ i_2(0^-) = 20\text{A}\ ;\ \boxed{i(t) = 30e^{-600t}\text{A}}$

(d) $v = -48i = \boxed{-1440e^{-600t}\text{V}}$

(e)
$$i_1 = 10(-440)\int_0^t e^{-600t}dt + 10 = 24e^{-600t}\Big|_0^t + 10 = 24e^{-600t} - 14\text{A}$$
$$i_2 = 2.5(-1440)\int_0^t e^{-600t}dt + 20$$
$$= 6e^{-600t}\Big|_0^t + 20 = \boxed{6e^{-600t} + 14\text{A}}$$

(f)
$$\mathrm{W}_L(0) = \frac{1}{2}\times 0.1\times 10^2 + \frac{1}{2}\times 0.4\times 20^2 = 5 + 80 = 85\text{J}$$
$$\mathrm{W}_\mathrm{L}(\infty) = \frac{1}{2}\times 0.1\times 14^2 + \frac{1}{2}\times 0.4\times 14^2 = 9.8 + 39.2 = 49\text{J}$$
$$\mathrm{W}_\mathrm{R} = \int_0^\infty i^2 48dt = \int_0^\infty 900\times 48e^{-1200t}dt = \frac{900\times 48}{-1200}(-1) = 36\text{J}$$
$$\boxed{\therefore 49 + 36 = 85 \text{ checks}}$$

36.

(a) $v_c(0) = 100 \times \frac{2}{2+2} \times \frac{2}{3} = 33.33\text{V};\ i_1(0^-) = \frac{100}{2+2} \times \frac{2}{3} = 16.667\text{mA}$

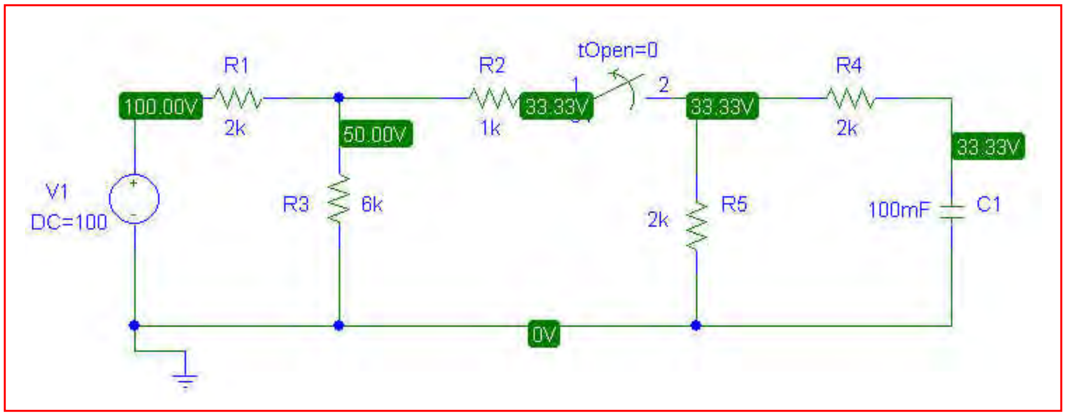
$\therefore \boxed{\text{v}_c(9:59) = 33.33\text{V},\ i_1(9:59) = 16.667\text{mA}}$

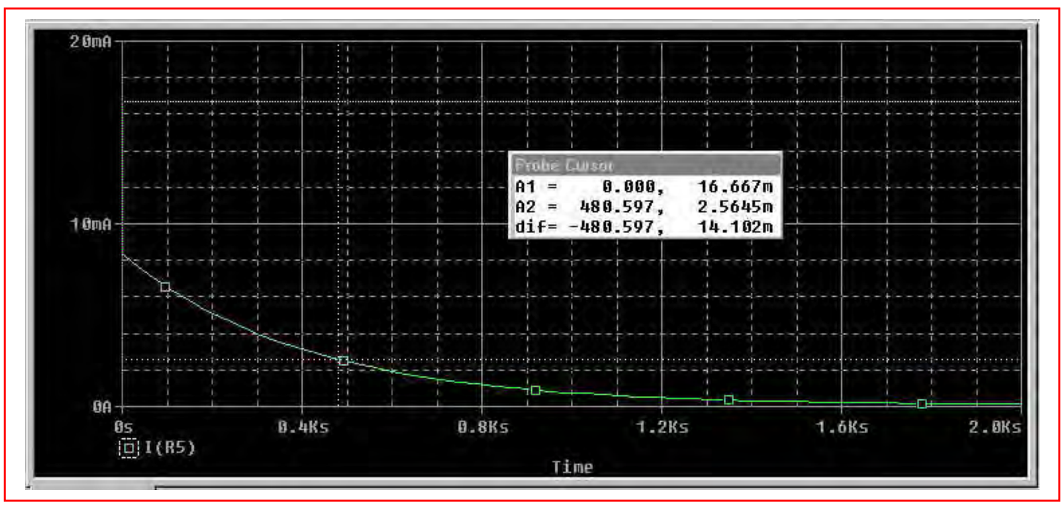
(b) $v_c(t) = 33.33e^{-t/400}, t > 10:00 \therefore v_c(10:05) = 33.33e^{-300/400}$

$\boxed{= 15.745^+\,\text{V},}\ i_1(10:05) = \frac{15.745}{4000} = \boxed{3.936\text{mA}}$

(c) $\tau = 400$ s, so $1.2\tau = 480$ s. $v_C(1.2\tau) = 33.33\ e^{-1.2} = 10.04$ V.
Using Ohm's law, we find that $i_1(1.2\tau) = v_C(1.2\tau)/\ 4000 = \boxed{2.51\text{ mA.}}$

(d) PSpice Verification:





We see from the DC analysis of the circuit that our initial value is correct; the Probe output confirms our hand calculations, especially for part (c).

37.

$t > 0: \dfrac{25i_x}{20} = 1.25i_x \therefore 34 = 100(1.25i_x - 0.8i_x + i_x) + 25i_x \therefore i_x = 0.2\text{A}$

(a) $i_s(0^-) = (1.25 - 0.8 + 1)0.2 = \boxed{0.290\,\text{A}}$

(b) $i_x(0^-) = \boxed{0.2\text{A}}$

(c) $v_c(t) = 25 \times 0.2e^{-t} = 5e^{-t}\text{V} \therefore i_x(0^+) = \dfrac{5}{100} = \boxed{0.05\text{A}}$

(d) $0.8i_x(0^+) = 0.04\text{A} \therefore i_s(0^+) = \dfrac{34}{120} - 0.04 \times \dfrac{20}{120} = \dfrac{33.2}{120} = \boxed{0.2767\text{A}}$

(e) $i_x(0.4) = \dfrac{1}{100} \times 5e^{-0.4} = \boxed{0.03352\text{A}}$

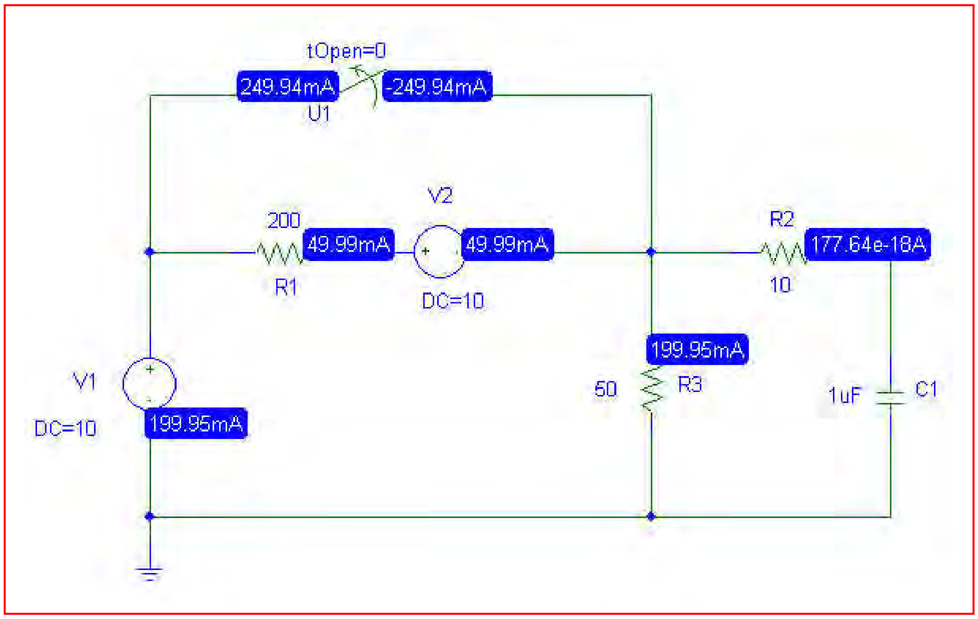
38.

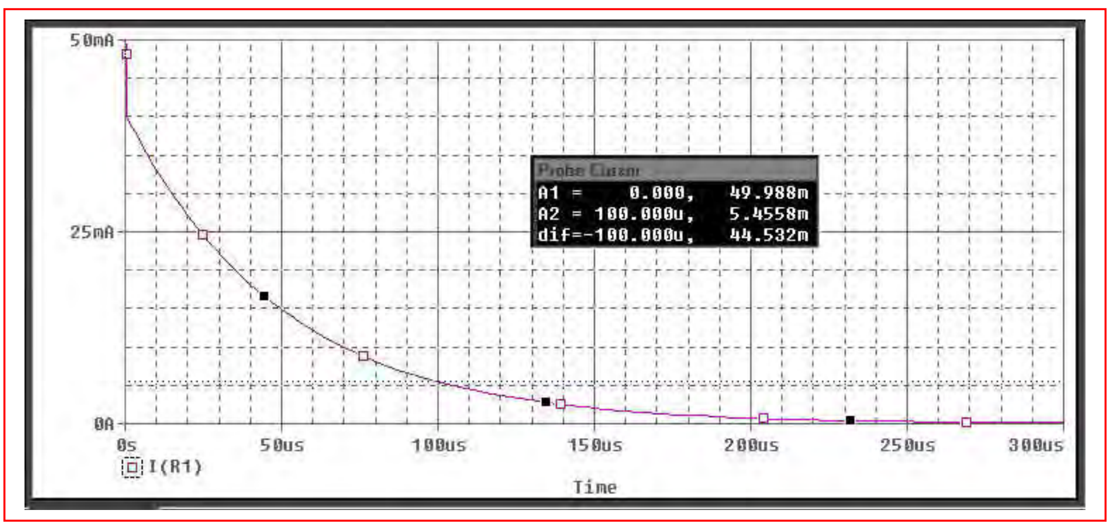
(a) $v_c(0) = 10\text{V} \therefore v_c(t) = 10e^{-10^6 t/(10+50\|200)} = \boxed{10e^{-20000t}\text{V}}$

(b) $i_A(-100\mu s) = i_A(0^-) = \dfrac{10}{200} = \boxed{50\text{mA}}$

$$i_A(100\mu s) = 10e^{-2}\left(\frac{1}{10+40}\right)\frac{50}{250} = \boxed{5.413\text{mA}}$$

(c) PSpice Verification.





From the DC simulation, we see that PSpice verifies our hand calculation of i_A = 50 mA. The transient response plotted using Probe indicates that at 100 μs, the current is approximately 5.46 mA, which is within acceptable round-off error compared to the hand calculated value.

39.

(a) $i_1(t) = 8(-1)\dfrac{12}{12+4} = \boxed{-6\text{mA } (t<0)}$

(b) $4\|12\|6 = 2k\Omega,\ v_c(0) = 48\text{V}$

$\therefore v_c(t) = 48e^{-10^6 t/5\times2\times10^3} = 48e^{-100t}\text{V},\ t>0$

$\therefore i(t) = \boxed{12e^{-100t}\text{mA},\ t>0}$

40.

(a) $v_{CLeft}(0) = 20\text{V},\ v_{CRIGHT}(0) = 80\text{V}$

$\therefore v_{CL} = 20e^{-10^6 t/8}, v_{CR} = 80e^{-10^6 t/0.8}$

$\therefore v_{out} = v_{CR} - v_{CL} = \boxed{80e^{-1,250,000t} - 20e^{-125,000t}\text{V},\ t>0}$

(b) $v_{out}(0^+) = \boxed{60\text{V}};\ v_{out}(1\mu s) = 80e^{-1.25} - 20e^{-0.125} = \boxed{5.270\text{V}}$

$v_{out}(5\mu s) = 80e^{-6.25} - 20e^{-0.625} = \boxed{-10.551\text{V}}$

41. (a) $t<0$: $\dfrac{v_c - 0.25v_c}{5} + \dfrac{v_c}{10} + \dfrac{v_c - 40}{4} = 0 \therefore v_c = 20\text{V}\,(t<0)$

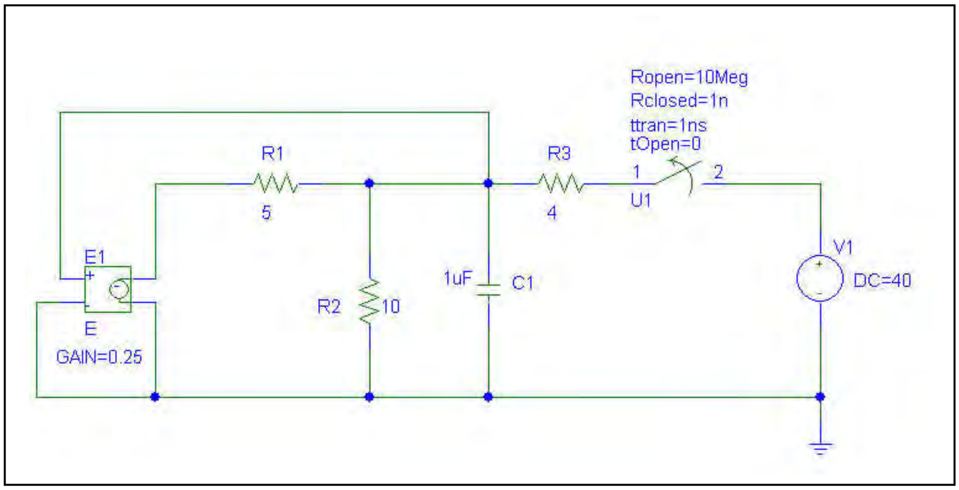
$t>0$: Apply $v_c = 1\text{V} \therefore \dfrac{1-0.25}{5} + 0.1 - i_{in} = 0.25\text{A}$

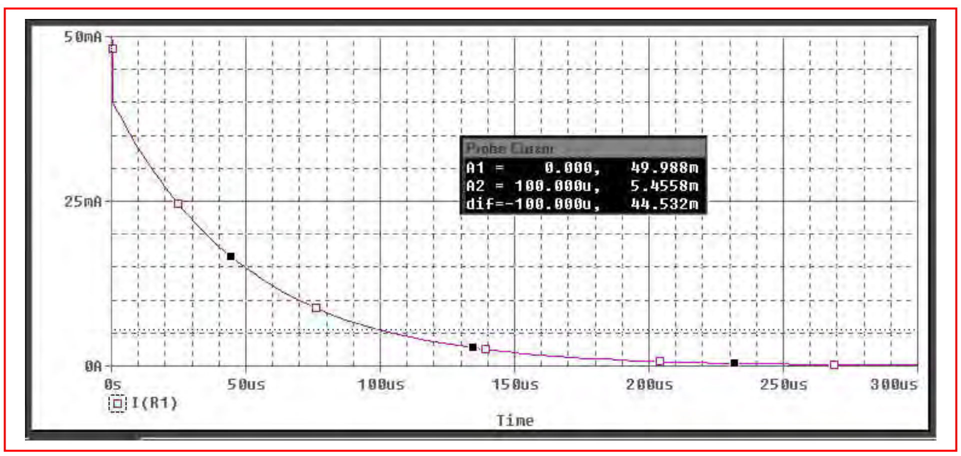
$$\therefore \text{R}_{eq} = \frac{1}{0.25} = 4\Omega$$

$$\therefore v_c(t) = 20e^{-10^6 t/4} = \boxed{20e^{-250,000t}\text{V}\,(t>0)}$$

(b) $v_C(3\ \mu\text{s}) = \boxed{9.447\ \text{V}}$

(c) PSpice verification. Note that the switch parameters had to be changed in order to perform this simulation.





As can be seen from the simulation results, our hand calculations are accurate.

42. $t < 0: \; v_c(0) = 60\text{V}$

$$0 < t < 1\text{ms}: \; v_c = 60e^{-10^6 t/(R_o+1000)} \therefore \frac{50}{60} e^{-500/(R_o+1000)}$$

$$\therefore \frac{500}{R_o + 1000} = \ell n\; 1.2 = 0.18232 \therefore \frac{R_o}{500} + 2 = 5.4848, R_o = \boxed{1742.4\Omega}$$

$$\therefore v_c(1\text{ms}) = 60e^{-1000/2742.4} = 41.67\text{V}$$

$$t > 1\text{ms}: v_c = 41.67e^{-10^6(t-10^{-3})} /(1742.4 + R_1 \| 1000)$$

$$\therefore 25 = 41.67e^{-1000()} \therefore 0.5108 = \frac{.1000}{1742.4 + R_1 \| 1000}, 1742.4 + R_1 \| 1000$$

$$= 1957.6, \; R_1 \| 1000 = 215.2 \frac{1}{R_1} + 10^{-3} = \frac{1}{215.2} \therefore R_1 = \boxed{274.2\Omega}$$

43.

(a) With the switch closed, define a nodal voltage V_1 at the top of the 5-kΩ resistor. Then,

$$0 = (V_1 - 100)/ 2 + (V_1 - V_C)/ 3 + V_1/ 5 \qquad [1]$$
$$0 = V_C/ 10 + (V_C - V_1)/ 3 + (V_C - 100) \qquad [2]$$

Solving, we find that $V_C = v_C(0^-) = \boxed{99.76 \text{ V.}}$

(b) $t>0: \; R_{eq} = 10 \| 6.5 = 3.939 k\Omega \;\therefore\; v_c = 87.59 e^{-10^7 t/3939} = \boxed{87.59 e^{-2539t} \text{V} \; (t>0)}$

44. $t < 0$:

$$12 = 4i_1 + 20i_1 \therefore i_1 = 0.5\text{mA} \therefore v_c(0) = 6i_1 + 20i_1 = 26i_1$$

$$v_c(0) = 13\text{V}$$

$$t > 0: \text{ Apply} \leftarrow 1\text{mA} \therefore 1 + 0.6i_1 = i_1 \therefore i_1 = 2.5\text{mA}; \pm v_{in} = 30i_1 = 75\text{V} \therefore \text{R}_{eq} = 75k\Omega$$

$$\therefore v_c(t) = 13e^{-t/75\times10^3\times2\times10^{-9}} = 13e^{-10^6 t/150} = 13e^{-6667t}$$

$$\therefore i_1(t) = \frac{v_o}{3\times10^4} = \boxed{0.4333e^{-6667t}\text{mA} \ (t > 0)}$$

45.

(a) $v_1(0^-) = \boxed{100\text{V.}}\ v_2(0^-) = 0,\ v_R(0^-) = \boxed{0}$

(b) $v_1(0^-) = \boxed{100\text{V.}}\ v_2(0^+) = 0,\ v_R(0^+) = \boxed{100\text{V}}$

(c) $\tau = \dfrac{20\times 5}{20+5}\times 10^{-6}\times 2\ 10^4 = \boxed{8\times 10^{-2}\, s}$

(d) $v_R(t) = \boxed{100e^{-12.5t}\text{V},\ t>0}$

(e) $i(t) = \dfrac{v_R(t)}{2\times 10^4} = \boxed{5e^{-12.5t}\text{mA}}$

(f) $v_1(t) = \dfrac{10^6}{20}\displaystyle\int_o^t -5\times 10^{-3} e^{-12.5t} dt + 100 = \dfrac{10^3}{50} e^{-12.5t}\Big|_o^t + 100 = \boxed{-20e^{-12.5t} + 80\text{V}}$

$v_2(t) = \dfrac{1000}{5}\displaystyle\int_o^t 5\, e^{-12.5t} dt + 0 = -80e^{-12.5t}\Big|_o^t + 0 = \boxed{-80e^{-12.5t} + 80\text{V}}$

(g) $w_{c1}(\infty) = \dfrac{1}{2}\times 20\times 10^{-6}\times 80^2 = 64\text{mJ},\ w_{c2}(\infty)\dfrac{1}{2}\times 5\times 10^{-6}\times 80^2 = \boxed{16\text{mJ}}$

$w_{c1}(0) = \dfrac{1}{2}\times 20\times 10^{-6}\times 100^2 = \boxed{100\text{mJ}},\ w_{c2}(0) = 0$

$w_R = \displaystyle\int_o^\infty 25\times 10^{-6} e^{-25t}\times 2\times 10^4 dt = \dfrac{25}{-25}\times 2\times 10^4(-1)10^{-6} = \boxed{20\text{mJ}}$

$\boxed{64+16+20 = 100 \text{ checks}}$

46.

(a) $t < 0$: $i_s = 1\text{mA} \therefore \pm\ v_c(0) = 10\text{V}, \downarrow i_L(0) = -1\text{mA} \therefore v_x(0) = \boxed{10\text{V}, t < 0}$

(b) $t > 0$: $v_c(t) = 10e^{-t/10^4 \times 20 \times 10^{-9}} = 10e^{-5000t}\text{V}$

$\text{i}_\text{L}(t) = -10^{-3}e - 10^{3t/0.1} = -10^{-3}e^{-10000t}\text{A} \therefore \pm v_L(t) = e^{-10000t}\text{V},\ t > 0$

$\therefore v_x = v_c - v_L(t) = \boxed{10e^{-5000t} - e^{-10000t}\text{V},\ t > 0}$

47.

(a) $t<0:\ v_s = 20\text{V} \therefore v_c = 20\text{V},\ i_L = 20\text{mA} \therefore \boxed{i_x(t) = 20\text{mA},\ t<0}$

(b) $t>0:\ v_s = 0 \therefore i_L(t) = 0.02e^{-10000t}\text{A};\ v_c(t) = 20e^{-t/2\times10^{-8}10^4} = 20e^{-5000t}\text{V}$

$\downarrow i_c(t) = 2\times10^{-8}\times 20(-5000)\,e^{-5000t} = -2e^{-5000t}\text{mA}$

$i_x(t) = i_L(t) + i_c(t) = 0.02e^{-10000t} - 0.002e^{-5000t}\text{A} = \boxed{20e^{-10000t} - 2e^{-5000t}\text{mA}}$

48.

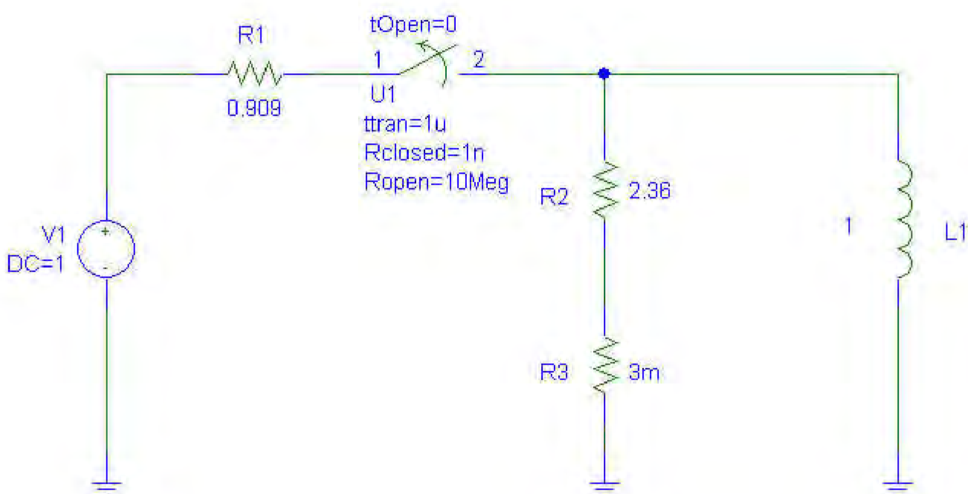
$$i_L(0^-) = \frac{V}{R} = \frac{1}{0.909} = 1.1 \text{ A}$$

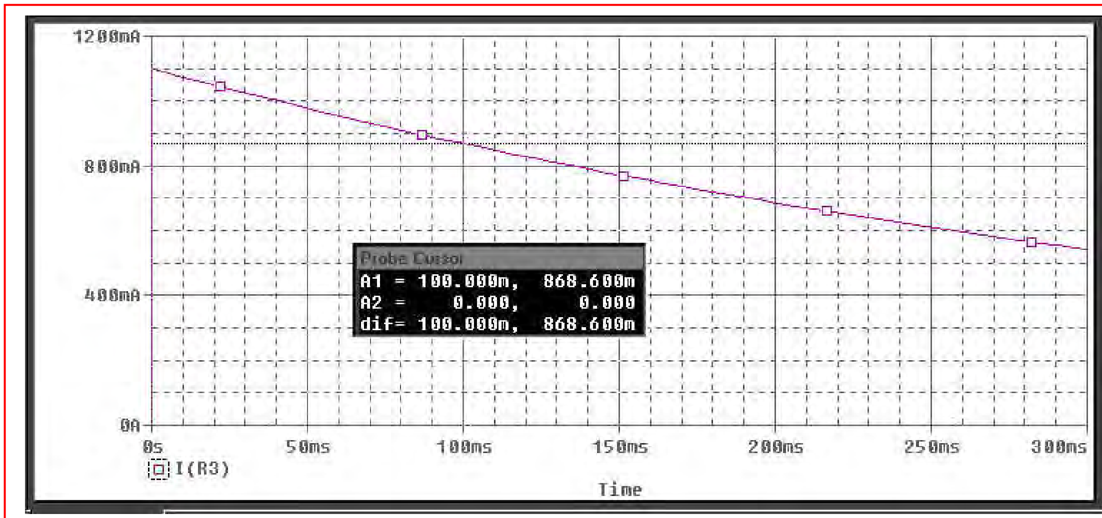
$$t > 0: \; i_L(t) = e^{-2.363t} \text{ A}$$

$$i_L(0.1s) = 1.1e^{-2.363\times 0.1} = 0.8685 \text{ A}$$

∴ since the current has dropped to less than 1 A prior to $t = 100$ ms, the fuse does not blow.

PSpice verification: Note that the switch properties were changed.





We see from the simulation result that the current through the fuse (R3) is 869 mA, in agreement with our hand calculation.

49. $\boxed{v(t) = 6u(t) - 6u(t-2) + 3u(t-4)}$ V

50. $\boxed{i(t) = 2u(t) + 2u(t-2) - 8u(t-3) + 6u(t-4) \text{ A}}$

51. (a) $f(-1) = 6 + 6 - 3 = \boxed{9}$

(b) $f(0^-) = 6 + 6 - 3 = \boxed{9}$

(c) $f(0^+) = 6 + 6 - 3 = \boxed{9}$

(d) $f(1.5) = 0 + 6 - 3 = \boxed{3}$

(e) $f(3) = 0 + 6 - 3 = \boxed{3}$

52. (a) $g(-1) = 0 - 6 + 3 = \boxed{-3}$

(b) $g(0^+) = 9 - 6 + 3 = \boxed{6}$

(c) $g(5) = 9 - 6 + 3 = \boxed{6}$

(d) $g(11) = 9 - 6 + 3 = \boxed{6}$

(e) $g(30) = 9 - 6 + 3 = \boxed{6}$

53. (a) $v_A = 300u(t-1)\,\text{V},\; v_B = -120u(t+1)\,\text{V};\; i_c = 3u(-t)\text{A}$

$t = -1.5:\; i_1(-1.5) = 3\times\frac{100}{300} = \boxed{1\text{A}}$

$t = 0.5:\; i_1(-0.5) = \frac{-120}{300} + 1 = \boxed{0.6\text{A};}$

$t = 0.5:\; i_1 = -\frac{120}{300} = -0.4\text{A};\; t = 1.5:\; i_1 = \frac{300}{300} - \frac{120}{300} = \boxed{0.6\text{A}}$

54.

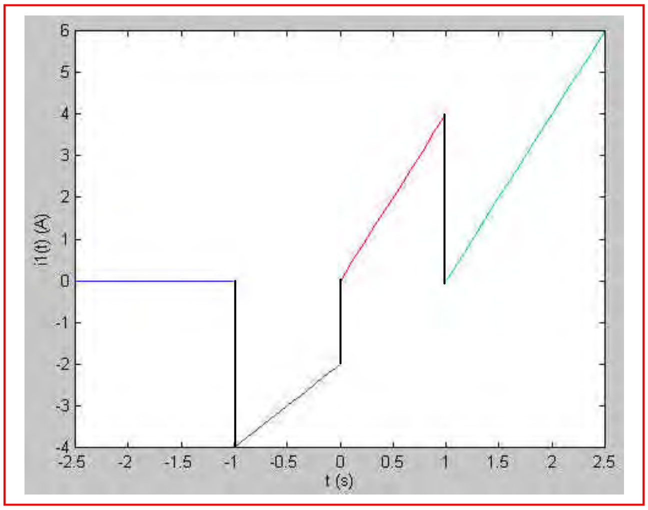
` $v_A = 600tu(t+1)\,v,\ v_B = 600(t+1)u(t)\,\text{V},\ i_c = 6(t-1)u(t-1)\text{A}$

(a) $t = -1.5: i_1 = 0;\ t = -0.5: i_1 = 600(-0.5)/300 = \boxed{-1\text{A}}$

$$t = 0.5: i_1 = \frac{600(0.5)}{300} + \frac{600(1.5)}{300} = \boxed{4\text{A}}$$

$$t = 1.5: i_1 = \frac{600(1.5)}{300} + \frac{600(2.5)}{300} + \frac{1}{3} \times 6 \times 0.5 = 3 + 5 + 1 = \boxed{9\text{A}}$$

(b)



55.

(a) $2u(-1) - 3u(1) + 4u(3) = -3 + 4 = \boxed{1}$

(b) $[5 - u(2)]\,[2 + u(1)]\,[1 - u(-1)]$
$= 4 \times 3 \times 1 = \boxed{12}$

(c) $4e^{-u(1)}u(1) = 4e^{-1} = \boxed{1.4715^{+}}$

56.

(a) $$t<0:\ i_x = \frac{100}{50} + 0 + 10 \times \frac{20}{50} = \boxed{6\text{A}}$$

$$t>0:\ i_x = 0 + \frac{60}{30} + 0 = \boxed{2\text{A}}$$

(b) $t < 0$: The voltage source is shorting out the 30-Ω resistor, so $\boxed{i_x = 0.}$

$t > 0$: $i_x = 60/\,30 = \boxed{2\text{ A}}$.

57. $t = -0.5:$ $50 \| 25 = 16.667,\ i_x = \frac{200}{66.67} - 2\frac{1/50}{1/50 + 1/25 + 1/50} = 3 - \frac{1}{2} = \boxed{2.5\text{A}}$

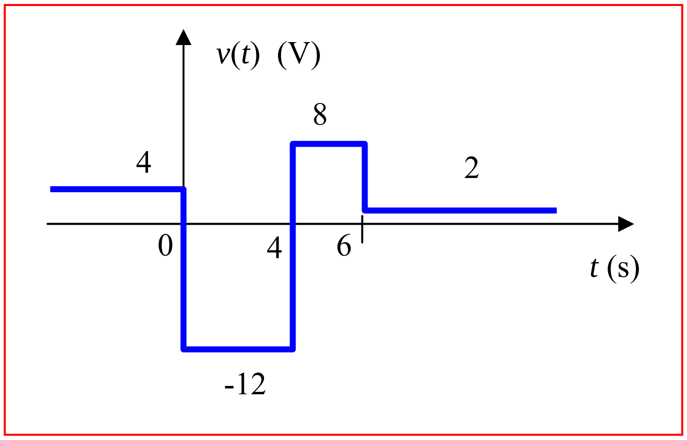
$t = 0.5:$ $i_x = \frac{200}{66.67} = \boxed{3\text{A}}$

$t = 1.5:$ $i_x = 3 - \frac{100}{66.67} \times \frac{1}{3} = \boxed{2.5\text{A}}$

$t = 2.5:$ $i_x = \frac{200 - 100}{50} = \boxed{2\text{A}}$

$t = 3.5:$ $i_x = -\frac{100}{50} = \boxed{-2\text{A}}$

58. $\boxed{v(t) = 4 - 16u(t) + 20u(t-4) - 6u(t-6)\,\text{V}}$



59. (a) $7\ u(t) - 0.2\ u(t) + 8(t-2) + 3$

$\boxed{v(1) = 9.8 \text{ volts}}$

(b) $\boxed{\text{Resistor of value } 2\Omega}$

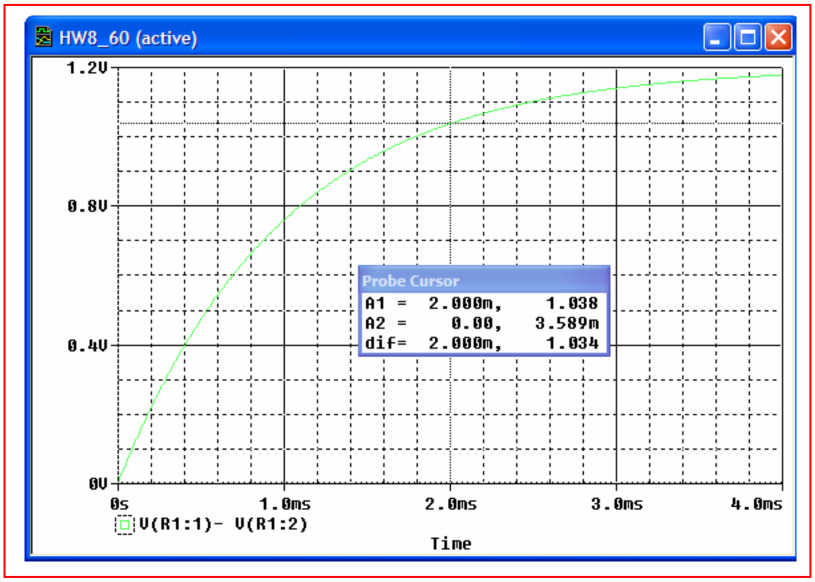
60.

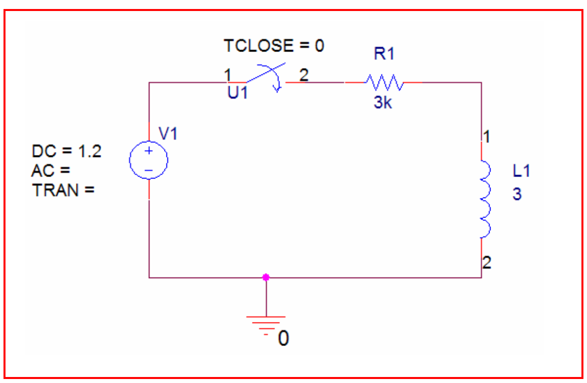
$$i(t) = \frac{V_o}{R}\left(1 - e^{-\frac{R}{L}t}\right)u(t) \text{ A} \quad \text{and} \quad v_R(t) = i(t)R$$

(a) $$v_R(t) = V_o\left(1 - e^{-\frac{R}{L}t}\right)u(t) \text{ V} = \boxed{1.2\left(1 - e^{-1000t}\right)u(t) \text{ V}}$$

(b) $$v_R(2\times10^{-3}) = 1.2\left(1 - e^{-2}\right) \text{ V} = \boxed{1.038 \text{ V}}$$

(c)





61.

(a) $\boxed{i_L(t) = (2 - 2e^{-200000t})u(t) \text{ mA}}$

(b) $v_L(t) = \text{L}i_L' = 15\times10^{-3}\times10^{-3}(-2)$

$(-200000e^{-200000t})\ u(t) = \boxed{6e^{-200000t}u(t)\text{V}}$

62.

(a) $i_L(t) = 2 + 2(1 - e^{-2.5t})u(t)\text{ A} \therefore i_1(-0.5) = \boxed{2\text{A}}$

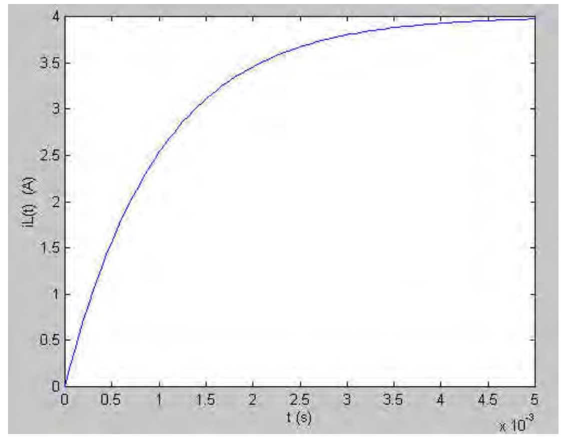
(b) $i_L(0.5) = 2 + 2(1 - e^{-1.25}) = \boxed{3.427\text{A}}$

(c) $i_L(1.5) = 2 + 2(1 - e^{-3.75}) = \boxed{3.953\text{A}}$

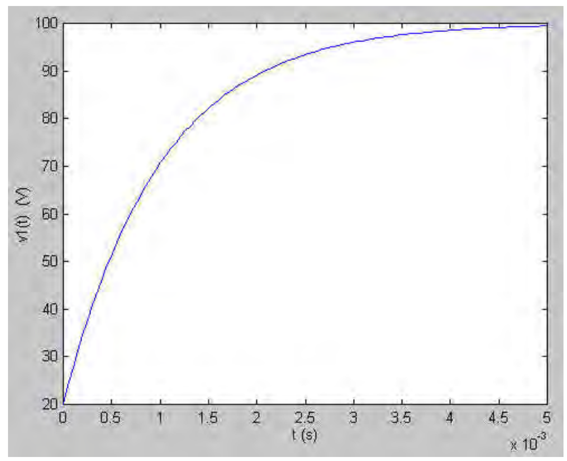
63.

(a) $i_L(t) = (4 - 4e^{-20t/0.02})u(t)$

$\therefore \boxed{i_L(t) = 4(1 - e^{-1000t})u(t)\text{A}}$



(b) $\boxed{v_1(t) = (100 - 80e^{-1000t})u(t)\text{V}}$

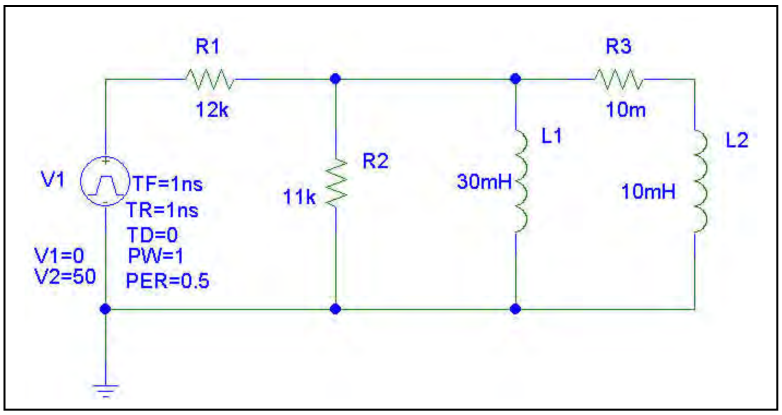


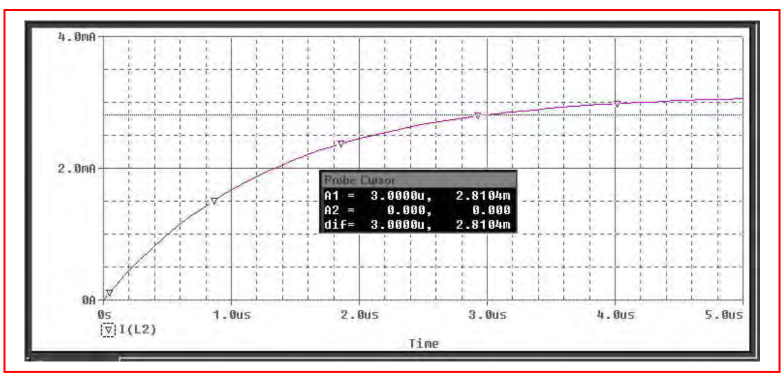
64. (a) $\boxed{\text{0 W}}$

(b) The total inductance is 30 || 10 = 7.5 mH. The Thévenin equivalent resistance is 12 || 11 = 5.739 kΩ. Thus, the circuit time constant is L/R = 1.307 μs. The final value of the total current flowing into the parallel inductor combination is 50/12 mA = 4.167 mA. This will be divided between the two inductors, so that $i(\infty)$ = (4.167)(30)/ (30 + 10) = 3.125 mA.

We may therefore write $i(t) = 3.125[1 - e^{-10^6 t/1.307}]$ A. Solving at t = 3 μs, we find $\boxed{\text{2.810 A.}}$

(c) PSpice verification





We see from the Probe output that our hand calculations are correct by verifying using the cursor tool at t = 3 μs.

65. $\tau = L/R_{TH} = \dfrac{30\times10^{-6}}{5\,||\,10} = \dfrac{30\times10^{-6}}{3.333} = 9\times10^{-6}$ s

(a) $i(t) = i_f(t) + i_n(t)$

$i_n = Ae^{-10^6 t/9}$ and $i_f = \dfrac{9}{5}$

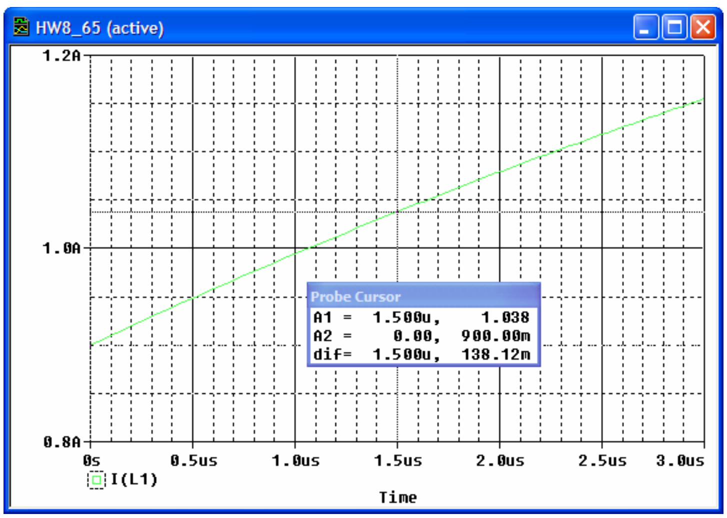
Thus, $i(t) = \dfrac{9}{5} + Ae^{-\frac{10^6}{9}t}$

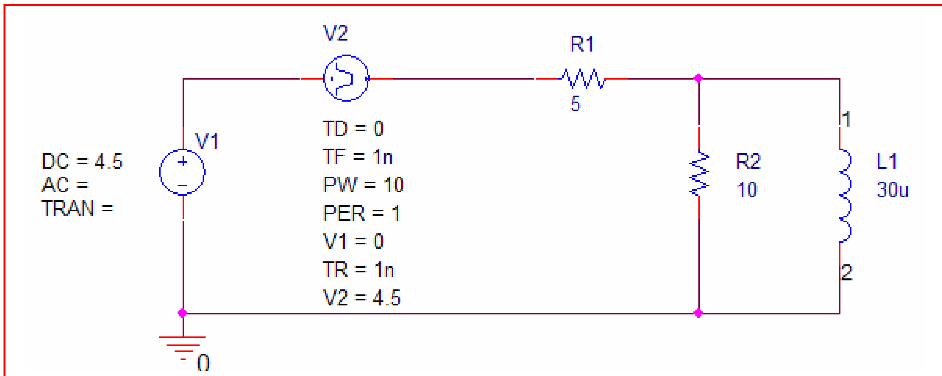
At $t = 0$, $i(0^-) = i(0^+) = 4.5/5$. Thus, $A = –4.5/5 = 0.9$
so

$$\boxed{i(t) = \frac{9}{5} - 0.9e^{-\frac{10^6}{9}t} \text{ A}}$$

(b) At $t = 1.5$ μs, $i = i(t) = \dfrac{9}{5} - 0.9e^{-\frac{1.5}{9}t} = \boxed{1.038 \text{ A}}$

(c)





66. $\tau = L/R_{eq} = \dfrac{45\times10^{-3}}{10\,||\,5} = \dfrac{45\times10^{-3}}{3.333} = 0.0135$ s

(a) $v_R(t) = v_f + v_n$

$v_f(t) = 0$ since inductor acts as a short circuit.

Thus, $v_R(t) = v_n = Ae^{-74.07t}$.

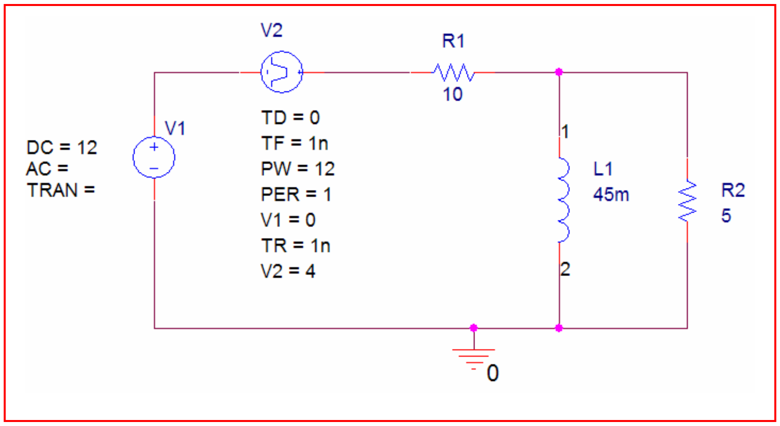
At $t = 0$, $i_L = 12/10 = 1.2$ A $= i_L(0^-) = i_L(0^+)$.

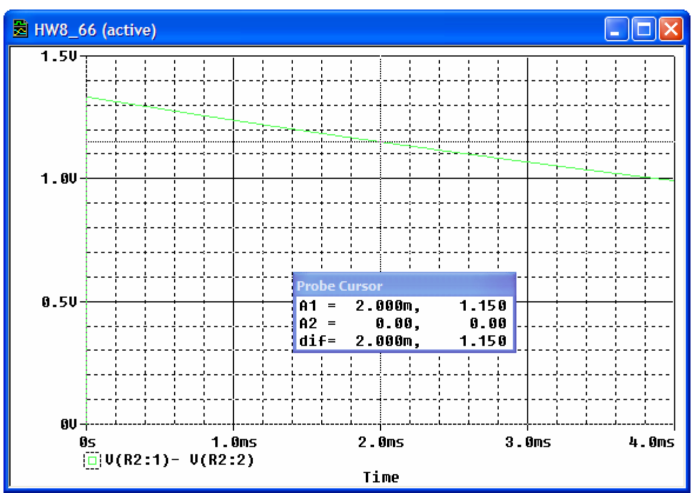
Writing KVL for this instant in time, $16 – 10(1.2 + v_R/5) = v_R$

Therefore $v_R(0^+) = \dfrac{4}{3}$ V and hence $\boxed{v_R(t) = \dfrac{4}{3}e^{-74.07t}\ \text{V}}$

(b) At $t = 2$ ms, $v_R(2\text{ ms}) = \dfrac{4}{3}e^{-74.07\left(2\times10^{-3}\right)}$ V $= \boxed{1.15\text{ V}}$

(c)





67. $\tau = \dfrac{L}{R_{eq}} = \dfrac{5\times 10^{-3}}{100} = 50\ \mu s$

$v_1(t) = v_{1f} + v_{1n}$ where $v_{1f} = 6$ V since the inductor acts as a short circuit

Therefore $v_1(t) = 6 + Ae^{-\frac{10^6 t}{50}}$.

At $t = 0^-$, $i_L = 0 = i_L(0^+)$. Thus, $v_1(0^+) = 0$ since no current flows through the resistor.

Hence $v_1(t) = 6\left(1 - e^{-\frac{10^6 t}{50}}\right)$ V.

At $t = 27\ \mu s$, $v_1(27\times 10^{-6}) = 6\left(1 - e^{-\frac{27}{50}}\right) = \boxed{2.5\text{ V}}$

68.

(a) $\boxed{i_L(t) = 10A,\ t < 0}$

(b) $i_L(t) = 8 + 2e^{-5t/0.5}$

$\therefore \boxed{i_L(t) = 8 + 2e^{-10t}\text{A},\ t > 0}$

69.

(a) $\boxed{i_L(t) = 2\text{A},\ t>0}$

(b) $i_L(t) = 5 - e^{-4t/0.1}$

$\therefore \boxed{i_L(t) = 5 - 3e^{-40t}\ \text{A},\ t>0}$

70.

(a) 0, $\boxed{0}$

(b) $\boxed{0,\ 200\text{V}}$

(c) 1A, $\boxed{100\text{V}}$

(d) $\tau = \dfrac{50\times10^{-3}}{200} = \dfrac{1}{4}\text{ms} \therefore i_L = 1(1-e^{-4000t})u(t)\text{A},\ i_L(0.2\text{ms}) = \boxed{0.5507\text{A}}$

$v_1(t) = (100+100e^{-4000t})u(t)\text{V},\ v_1(0.2\text{ms}) = \boxed{144.93\text{V}}$

71. $\dfrac{di}{dt}+\mathrm{P}i=\mathrm{Q},\ i=e^{-Pt}\int \mathrm{Q}e^{Pt}dt+Ae^{-Pt},\ \mathrm{R}=125\Omega,\ \mathrm{L}=5\mathrm{H}$

$\therefore \mathrm{L}\dfrac{di}{dt}\mathrm{LP}i=\mathrm{LQ}\therefore \mathrm{LP}=5\mathrm{P}=\mathrm{R}=125\therefore \mathrm{P}=25$

(a) $\mathrm{Q}(t)=\dfrac{10}{\mathrm{L}}=2\therefore i=e^{-25t}\int_o^t 2e^{25t}dt+\mathrm{A}e^{-25t}=e^{-25t}\times\dfrac{2}{25}e^{25t}\Big|_o^t+\mathrm{A}e^{-25t}$

$\therefore i=\dfrac{2}{25}+\mathrm{A}e^{-25t},\ i(0)=\dfrac{10}{125}=\dfrac{2}{25}\therefore \mathrm{A}=0\therefore i=\dfrac{2}{25}=\boxed{0.08\mathrm{A}}$

(b) $\mathrm{Q}(t)=\dfrac{10u(t)}{5}=2u(t)\therefore i=e^{-25t}\int_o^t 2e^{25t}dt+\mathrm{A}e^{-25t}=\dfrac{2}{25}+\mathrm{A}e^{-25t}$

$i(0)=0\therefore \mathrm{A}=-\dfrac{2}{25}\therefore \boxed{i(t)=0.08(1-e^{-25t})\mathrm{A},\ t>0}$

(c) $\mathrm{Q}(t)=\dfrac{10+10u(t)}{5}=2+2u(t)\therefore \boxed{i=0.16-0.08e^{-25t}\mathrm{A},\ t>0}$

(d) $\mathrm{Q}(t)=\dfrac{10u(t)\cos 50t}{5}=2u(t)\cos 50t\therefore i=e^{-25t}\int_o^t 2\cos 50t\times e^{25t}dt+Ae^{-25t}$

$$\therefore i=2e^{-25t}\left[\frac{e^{25t}}{50^2+25^2}(25\cos 50t+50\sin 50t)\right]_o^t+\mathrm{A}e^{-25t}$$

$$=2e^{-25t}\left[\frac{e^{25t}}{3125}(25\cos 50t+50\sin 50t)-\frac{1}{3125}\times 25\right]+\mathrm{A}e^{-25t}$$

$$=\frac{2}{125}\cos 50t+\frac{4}{125}\sin 50t-\frac{2}{125}e^{-25t}+\mathrm{A}e^{-25t}$$

$$i(0)=0\therefore 0=\frac{2}{125}-\frac{2}{125}+\mathrm{A}\therefore \mathrm{A}=0$$

$$\therefore \boxed{i(t)=0.016\cos 50t+0.032\sin 50t-0.016e^{-25t}\mathrm{A},\ t>0}$$

72.

(a) $i_L(t) = \dfrac{100}{20} - \dfrac{100}{5} = \boxed{-15\text{A},\ t<0}$

(b) $i_L(0^+) = i_L(0^-) = \boxed{-15\text{A}}$

(c) $i_L(\infty) = \dfrac{100}{20} = \boxed{5\text{A}}$

(d) $i_L(t) = \boxed{5 - 20e^{-40t}\text{A},\ t>0}$

73. $i_L(0^-) = \frac{18}{60+30} \times \frac{1}{2} = 0.1\text{A} \therefore i_L(0^+) = 0.1\text{A}$

$\text{i}_\text{L}(\infty) = 0.1 + 0.1 = 0.2\text{A}$

$\therefore i_L(t) = 0.2 - 0.1e^{-9000t}\text{A},\ t > 0$

$\therefore i_L(t) = 0.1u(-t) + (0.2 - 0.1e^{-9000t})u(t)\text{A}$

or, $\boxed{i_L(t) = 0.1 + (0.1 - 0.1e^{-9000t})u(t)\text{A}}$

74.

(a) $i_x(0^-) = \frac{30}{7.5} \times \frac{3}{4} = 3\text{A},\ i_L(0^-) = \boxed{4\text{A}}$

(b) $i_x(0^+) = i_L(0^+) = \boxed{4\text{A}}$

(c) $i_x(\infty) = i_L(\infty) = 3\text{A}$

$\therefore i_x(t) = 3 + 1e^{-10t/0.5} = 3 + e^{-20t}\text{A} \therefore i_x(0.04)$

$= 3 + e^{-0.8} = \boxed{3.449\text{A}}$

75.

(a) $i_x(0^-) = i_L(0^-) = \dfrac{30}{10} = \boxed{3\text{A}}$

(b) $i_x(0^+) = \dfrac{30}{30+7.5} \times \dfrac{30}{40} + 3 \times \dfrac{15}{10+15} = \boxed{2.4\text{A}}$

(c) $i_x(\infty) = \dfrac{30}{7.5} \times \dfrac{30}{40} = 3\text{A} \therefore i_x(t) = 3 - 0.6e^{-6t/0.5}$

$= 3 - 0.6e^{-12t} \therefore i_x(0.04) = 3 - 0.6e^{-0.48} = \boxed{2.629\text{A}}$

76. $\text{OC}: v_x = 0,\ v_{oc} = 4u(t)\text{V}$

$$\text{SC}: 0.1u(t) = \frac{v_x - 0.2v_x}{40} + \frac{v_x}{60}, 12u(t) = 0.6v_x + 2v_x$$

$$\therefore v_x = \frac{12u(t)}{2.6} \therefore i_{ab} = \frac{v_x}{60} = \frac{12u(t)}{2.6\times 60} = \frac{u(t)}{13}$$

$$\therefore R_{th} = 4\times 13 = 52\Omega \therefore i_L = \frac{4u(t)}{52}(1-e^{-52t/0.2})u(t) = \frac{u(t)}{13}(1-e^{-260t})u(t)$$

$$\therefore v_x = 60i_L = \boxed{4.615^{+}(1-e^{-260t})u(t)\text{V}}$$

77.

(a) $\text{OC}: -100 + 30i_1 + 20i_1 = 0,\ i_1 = 2\text{A}$

$\therefore v_{oc} = 80u(t)\text{V}$

$\text{SC}: i_1 = 10\text{A}, \downarrow i_{sc} = 10 + \dfrac{20 \times 10}{20} = 20\text{A}$

$\therefore \text{R}_{th} = 4\Omega \therefore i_L(t) = \boxed{20(1 - e^{-40t})u(t)\text{A}}$

(b) $v_L = 0.1 \times 20 \times 40e^{-40t}u(t) = 80e^{-40t}u(t)$

$\therefore i_1(t) = \dfrac{100u(t) - 80e^{-40t}u(t)}{10} = \boxed{10 - 8e^{-40t}u(t)\text{A}}$

78. $\tau = R_{eq}C = (5)(2) = 10$ s

Thus, $v_n(t) = Ae^{-0.2t}$.

$v = v_n + v_f = Ae^{-0.1t} + Be^{-5t}$.

At $t = 0$, $v(0) = 0$ since no source exists prior to $t = 0$. Thus, A + B = 0 [1].

As $t \to \infty$, $v(\infty) \to 0$. We need another equation.

$$i(0) = C\frac{dv}{dt} = \frac{4.7}{5}(2) \quad \text{therefore} \quad \left.\frac{dv}{dt}\right|_{t=0^+} = \frac{4.7}{5} \quad \text{or} \quad -0.1A - 5B = \frac{4.7}{5} \quad [2]$$

Solving our two equations, we find that A = –B = 0.192.

Thus, $\boxed{v(t) = 0.192\left(e^{-0.1t} - e^{-5t}\right)}$

79. Begin by transforming the circuit such that it contains a 9.4 cos 4*t* $u(t)$ V voltage source in series with a 5 Ω resistor, in series with the 2 F capacitor.

Then we find that

$$9.4\cos 4t = 5i + v$$
$$= (5)(2)\frac{dv}{dt} + v$$

or $\frac{dv}{dt} + 0.1v = 0.94\cos 4t$, so that $v(t) = e^{-0.1t}\int\left(0.94\cos 4t\right)e^{0.1t}dt + Ae^{-0.1t}$

Performing the integration, we find that

$$v(t) = 0.94\left[\frac{10\cos 4t + 400\sin 4t}{1+1600}\right] + Ae^{-0.1t}.$$

At $t = 0$, $v = 0$, so that $A = -\frac{0.94}{1601}(10)$

and $\boxed{v(t) = \frac{0.94}{1601}\left[-10e^{-0.1t} + 10\cos 4t + 400\sin 4t\right]}$

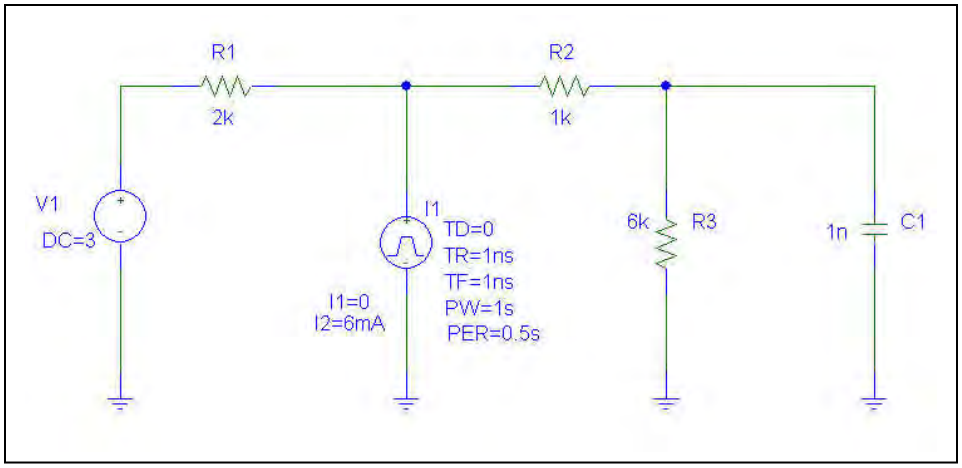
80. (a) $v_c(0^-) = \frac{6}{9}\times 3 = 2\text{V} = v_c(0^+)$

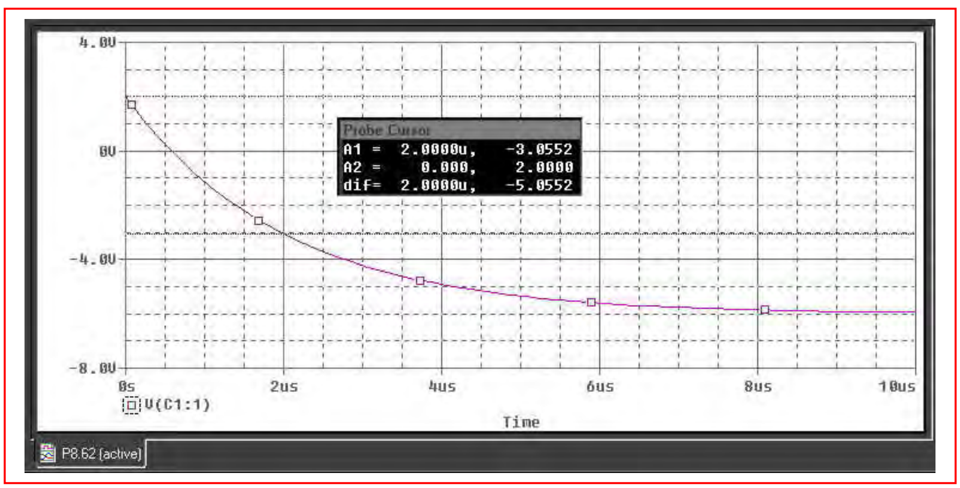
$$v_c(\infty) = 2 - 6(2\|7)\frac{6}{7} = -6\text{V}$$

$$\therefore v_c(t) = -6 + 8e^{-10^9 t/2\times 10^3} = -6 + 8e^{-500000t}\text{V},\ t>0$$

$$v_c(-2\mu s) = v_c(0^-) = 2\text{V},\ v_c(2\mu s) = -6 + 8e^{-1} = \boxed{-3.057\text{V}}$$

(b) PSpice verification.





As can be seen from the plot above, the PSpice simulation results confirm our hand calcu lations of $v_C(t < 0) = 2$ V and $v_C(t = 2\ \mu s) = -3.06$ V

81. $\tau = RC = 2\times10^{-3}\left(50\right) = 0.1$

$v_n = Ae^{-10t}$

$v_C(t) = v_{Cn} + v_{Cf} = Ae^{-10t} + 4.5 \quad \text{since } v_C\left(\infty\right) = 4.5 \text{ V}$

Since $\quad v_C(0^-) = v_C(0^+) = 0$

$v_C(t) = -4.5e^{-10t} + 4.5 = \boxed{4.5\left(1-e^{-10t}\right)}$

82. $i_A(0^-) = \frac{10}{1} = 10\text{mA},\ i_A(\infty) = 2.5\text{mA},\ v_c(0) = 0$

$$i_A(0^+) = \frac{10}{1.75} \times \frac{1}{4}\ 1.4286\text{mA} \therefore \boxed{i_A = 10\text{mA},\ t<0}$$

$$i_A = 2.5 + (1.4286 - 2.5)\,e^{-10^8 t/1.75\times10^3} = \boxed{2.5 - 1.0714e^{-57140t}\text{mA},\ t>0}$$

83. $i_A(0^-) = \dfrac{10}{4} = 2.5\text{mA},\ i_A(\infty) = 10\text{mA}$

$v_c(0) = 7.5\text{V} \therefore i_A(0^+) = \dfrac{10}{1} + \dfrac{7.5}{1} = 17.5\text{mA}$

$\boxed{i_A = 10 + 7.5e^{-10^8 t/10^3} = 10 + 7.5e^{-10^5 t}\text{mA},\ t>0,\ i_A = 2.5\text{mA}\ t<0}$

84.

(a) $i_{in}(-1.5) = \boxed{0}$

(b) $i_{in}(1.5) = \boxed{0}$

85.

(a)

$$v_s = -12u(-t) + 24u(t)\text{V}$$

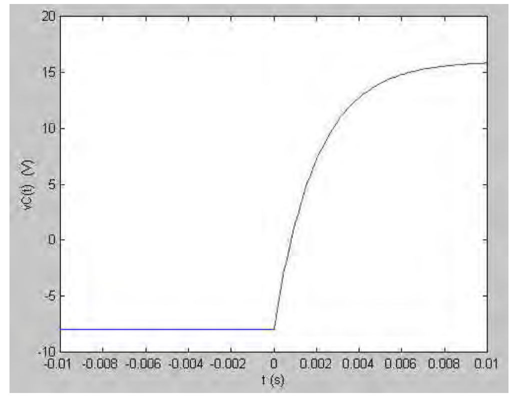
$$t<0:\ v_c(0^-) = -8\text{V} \therefore v_c(0^+) = -8\text{V}$$

$$t>0:\ v_c(\infty) = \frac{2}{3}\times 24 = 16\text{V}$$

$$\text{RC} = \frac{200}{30}\times 10^3 \times 3\times 10^{-7} = 2\times 10^{-3}$$

$$\therefore v_c(t) = 16 - 24e^{-500t}\text{V},\ t>0$$

$$\therefore \boxed{v_c(t) = -8u(-t) + (16 - 24e^{-500t})u(t)}$$

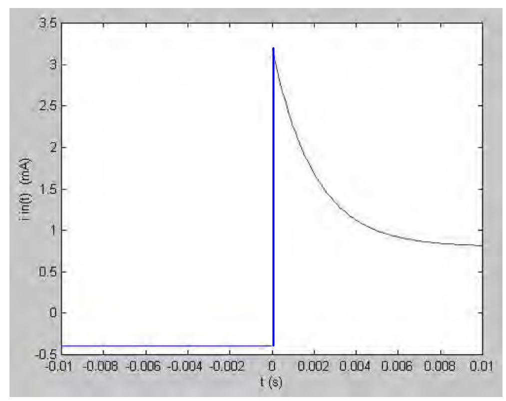


(b)

$$i_{in}(0^-) = \frac{-12}{30} = -0.4\text{mA},\ i_{in}(0^+) = \frac{24+8}{10} = 3.2\text{mA}$$

$$i_{in}(\infty) = \frac{24}{30} = 0.8\text{mA}$$

$$\boxed{i_{in}(t) = -0.4u(t) + (0.8 + 2.4e^{-500t})u(t)\text{mA}}$$



86. $\text{OC}: \dfrac{-v_x}{100} - \dfrac{v_x}{100} + \dfrac{3-v_x}{100} = 0 \therefore v_x = 1,\ v_{oc} = 3-1 = 2\text{V}$

$\text{SC}: v_x = 3\text{V} \therefore i_{sc} = \dfrac{v_x}{100} + \dfrac{v_x}{100} = 0.06\text{A}$

$\therefore \text{R}_{th} = v_{oc} / i_{sc} = 2/0.06 = 33.33\Omega$

$\therefore v_c = v_{oc}(1 - e^{-t/R_{th}C}) = 2(1 - e^{-10^6 t/33.33})$

$\boxed{= 2(1 - e^{-30{,}000t})\,\text{V},\ t > 0}$

87.

$v_c(0^-) = 10\text{V} = v_c(0^+),\ i_{in}(0^-) = 0$

$i_{in}(0^+) = 0 \therefore i_{in}(t) = 0$ for all t

$0 < t < 0.5s:\ v_c = 10(1 - e^{-2.5t})\text{V}$

$\boxed{v_c(0.4) = 6.321\text{V}},\ v_c(0.5) = 7.135\text{V}$

$$t > 0.5:\ \frac{20-10}{12} = \frac{5}{6}\text{A} \therefore v_c(\infty) = 10 + 8 + \frac{5}{6} = \frac{50}{3}\text{V},\ 4\|8 = \frac{8}{3}\Omega$$

$$v_c(t) = \frac{50}{3} + \left(7.135 - \frac{50}{3}\right)e^{-0.375\times 20(t-0.5)} = 16.667 - 9.532e^{-7.5(t-0.5)}\text{V}$$

$$\therefore v_c(0.8) = 16.667 - 9.532e^{-7.5(0.3)} = \boxed{15.662\text{V}}$$

88.

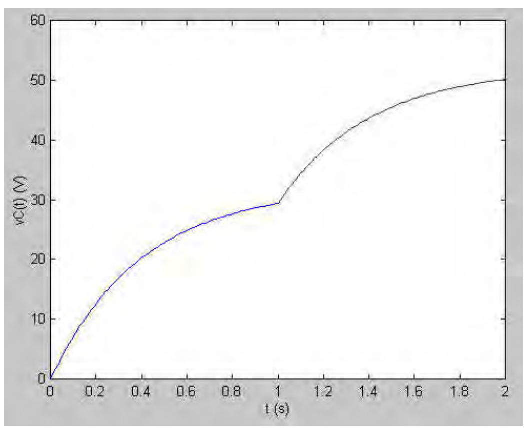
(a) For $t < 0$, there are no active sources, and so $\boxed{v_C = 0.}$

For $0 < t < 1$, only the 40-V source is active. $R_{th} = 5k \,\|\, 20\text{ k} = 4\text{ k}\Omega$ and hence $\tau = R_{th}\, C = 0.4$ s. The "final" value (assuming no other source is ever added) is found by voltage division to be $v_C(\infty) = 40(20)/(20 + 5) = 32$ V. Thus, we may write $v_C(t) = 32 + [0 - 32]\, e^{-t/\,0.4}$ V $= \boxed{32(1 - e^{-2.5t})\text{ V.}}$

For $t > 1$, we now have two sources operating, although the circuit time constant remains unchanged. We define a new time axis temporarily: $t' = t - 1$. Then $v_C(t' = 0^+) = v_C(t = 1) = 29.37$ V. This is the voltage across the capacitor when the second source kicks on. The new final voltage is found to be $v_C(\infty) = 40(20)/\,(20 + 5) + 100(5)/\,(20 + 5) = 52$ V.

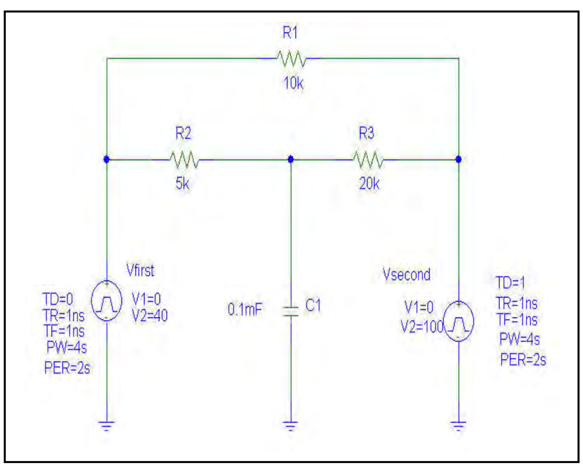
Thus, $v_C(t') = 52 + [29.37 - 52]\, e^{-2.5t'} = \boxed{52 - 22.63\, e^{-2.5(t-1)}\text{ V.}}$

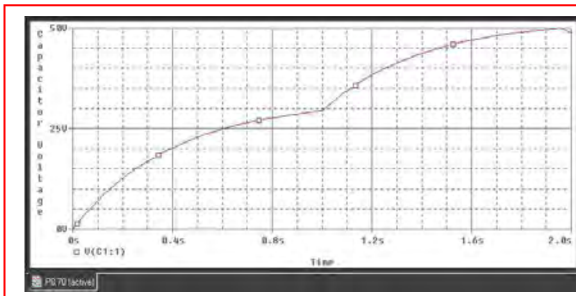
(b)



For $t < 0$, $v_C = 0$.

(c)





We see from the simulation results that our hand calculations and sketch are indeed correct.

89.

(a) $t<0:\ 8(10+20)=240\text{V} = \boxed{v_R(t)=80\text{V},\ t<0}$

(b) $t<0:\ v_c(t)=8\times 30=240\text{V} \therefore v_c(0^+)=240\text{V}$

$$t=(\infty):\ v_c(\infty)=\frac{1}{2}\times 8(10+10)=80\text{V}$$

$$\therefore v_c(t)=80+160e^{-t/10\times10^{-6}}=80+160e^{-100000t}\text{V}$$

$$\therefore v_R(t)=\boxed{80+160e^{-100000t}\text{V},\ t>0}$$

(c) $t<0:\ v_R(t)=\boxed{80\text{V}}$

(d)

$$v_c(0^-)=80\text{V},\ v_c(\infty)=240\text{V} \therefore v_c(t)=240-160e^{-t/50\times10^{-6}}=240-160e^{-20000t}\text{V}$$

$$v_R(0^-)=80\text{V},\ v_R(0^+)=8\frac{20}{30+20}\times 10+\frac{80}{50}\times 10=32+16=48\text{V}$$

$$v_R(\infty)=80\text{V} \therefore v_R(t)=\boxed{80-32e^{-20000t}\text{V},\ t>0}$$

90. $t < 0: \; v_c = 0$

$0 < t < 1\text{ms}: \; v_c = 9(1 - e^{-10^6 t/(R_1+100)})$

$\therefore 8 = 9(1 - e^{-1000/(R_1+100)}), \; \frac{1}{9} = e^{-1000/(R_1+100)}$

$\therefore \frac{1000}{R_1 + 100} = 2.197, \; R_1 = \boxed{355.1\Omega}$

$t > 1\text{ms}: \; v_c = 8e^{-10^6 t'/(R_2+100)}, \; t' = t - 10^{-3} \; \therefore 1 - 8e^{-1000}(R_2 + 100)$

$\therefore \frac{1000}{R_2 + 100} = 2.079, \; R_2 = 480.9 - 100 = \boxed{380.9\Omega}$

91. $v_{x,L} = 200e^{-2000t}\text{V}$

$$v_{x,c} = 100(1 - e^{-1000t})\text{V}$$

$$v_x = v_{x,L} - v_{x,c} = 0$$

$$\therefore 200e^{-2000t} = 100 - 100e^{-1000t}$$

$$\therefore 100e^{-1000t} + 200(e^{-1000t})^2 - 100 = 0,$$

$$e^{-1000t} = \frac{-100 \pm \sqrt{10,000 + 80,000}}{400} = -0.25 \pm 0.75$$

$$\therefore e^{-1000t} = 0.5,\ t = \boxed{0.6931\text{ms}}$$

92.

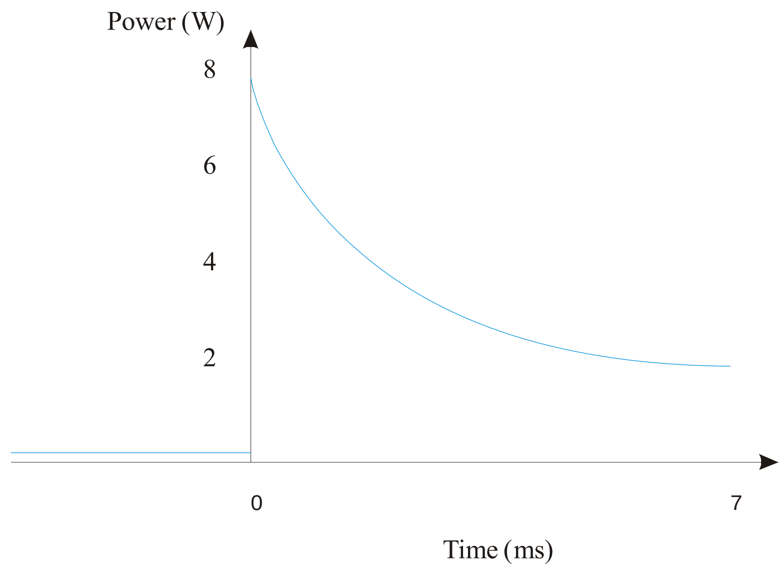
$$P(t<0) = I^2R = 0.001^2 \times 10^3 = 0.001 \text{ W}$$

$$V_{init} = I.R = 7 \times 10^{-3} \times 900 = 6.3 \text{ V}$$

$$P_{init} = \frac{V^2}{R} = 0.08 \text{ W}$$

$$V_{final} = 7 \times 10^{-3} \times 900\Omega // 1000\Omega = 3.3 \text{ V}$$

$$P_{final} = \frac{V^2}{R} = 0.02 \text{ W}$$



93. For $t < 0$, the voltage across all three capacitors is simply 9 (4.7)/ 5.7 = 7.421 V. The circuit time constant is $\tau = RC = 4700\,(0.5455\times10^{-6}) = 2.564$ ms.

When the circuit was first constructed, we assume no energy was stored in any of the capacitors, and hence the voltage across each was zero. When the switch was closed, the capacitors began to charge according to ½ Cv^2. The capacitors charge with the same current flowing through each, so that by KCL we may write

$$C_1 \frac{dv_1}{dt} = C_2 \frac{dv_2}{dt} = C_3 \frac{dv_3}{dt}$$

With no initial energy stored, integration yields the relationship $C_1v_1 = C_2v_2 = C_3v_3$ throughout the charging (*i.e.* until the switch is eventually opened). Thus, just prior to the switch being thrown at what we now call $t = 0$, the total voltage across the capacitor string is 7.421 V, and the individual voltages may be found by solving:

$$\begin{aligned} v_1 + v_2 + v_3 &= 7.421 \\ 10^{-6}\,v_1 - 2\times10^{-6}\,v_2 &= 0 \\ 2\times10^{-6}\,v_2 - 3\times10^{-6}\,v_3 &= 0 \end{aligned}$$

so that $v_2 = 2.024$ V.

With the initial voltage across the 2-uF capacitor now known, we may write

$$v(t) = 2.024\, e^{-t/\,2.564\times10^{-3}}\ \text{V}$$

(a) $v(t = 5.45\text{ ms}) = \boxed{241.6\text{ mV.}}$

(b) The voltage across the entire capacitor string can be written as $7.421\, e^{-t/\,2.564\times10^{-3}}$ V. Thus, the voltage across the 4.7-kΩ resistor at $t = 1.7$ ms = 3.824 V and the dissipated power is therefore $\boxed{3.111\text{ mW.}}$

(c) Energy stored at $t = 0$ is ½ $Cv^2 = 0.5(0.5455\times10^{-6})(7.421)^2 = \boxed{15.02\ \mu\text{J.}}$

94. voltage follower $\therefore v_o(t) = v_2(t)$

$$v_2(t) = 1.25u(t)\text{V} = v_o(t)$$

$$v_x(t) = 1.25e^{-10^6/0.5\times 200}u(t)$$

$$= \boxed{1.25e^{-10,000t}\,u(t)\text{V}}$$

95. This is a voltage follower $\therefore v_o(t) = v_2(t)$, where $v_2(t)$ is defined at the non-inverting input. The time constant of the RC input circuit is 0.008(1000+250) = 10 s.

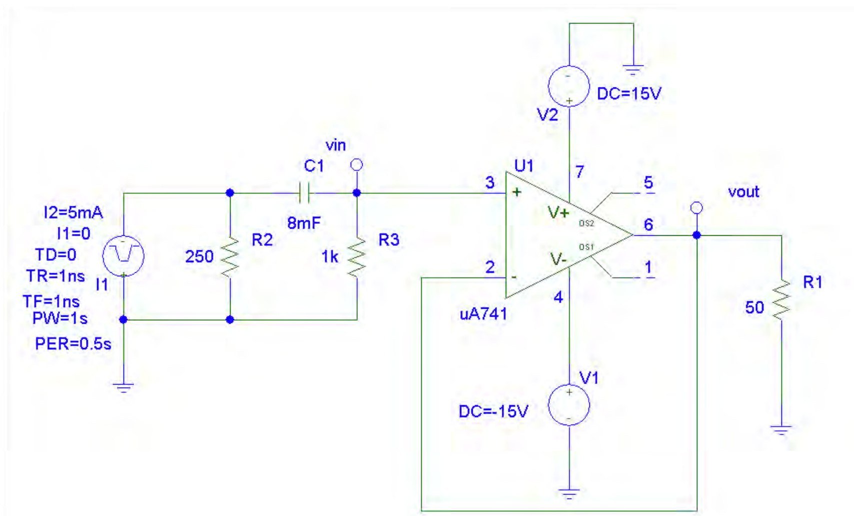
Considering initial conditions: $v_C(0^-) = 0 \ \therefore\ v_C(0^+) = 0$.
Applying KVL at $t = 0^+$,

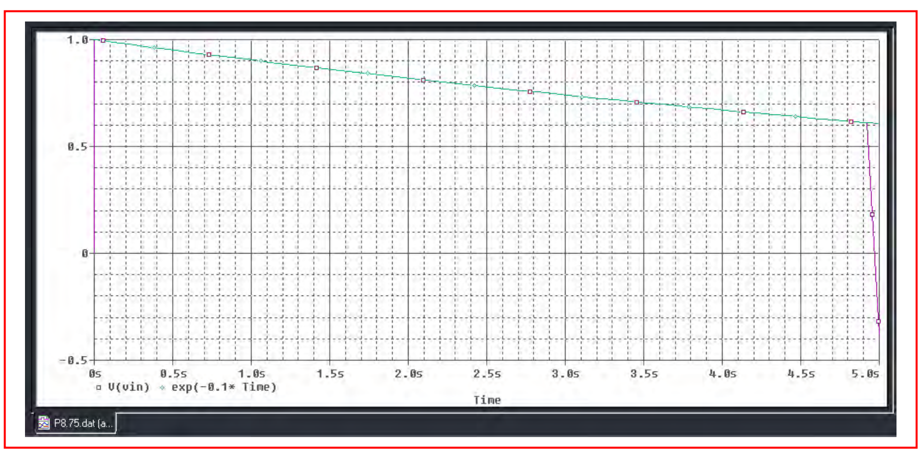
$$5 \quad v_{250}/250 + v_2/1000.$$

Since at $t = 0^+$ $v_{250} = v_2$, we find that $v_2(0^+) = 1$ V.
As $t \to \infty$, $v_2 \to 0$, so we may write,

$$v_o(t) = v_2(t) = \boxed{1.0e^{-t/10}\ u(t)\text{ V.}}$$

PSpice verification:

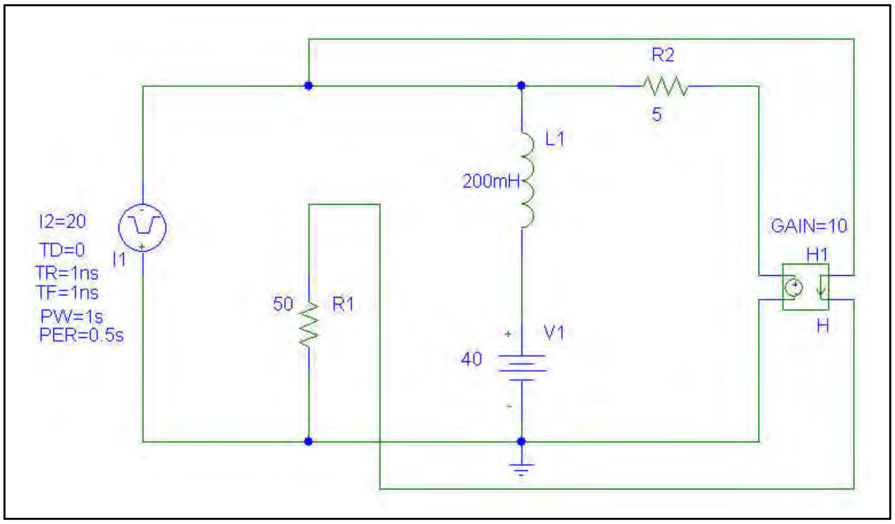


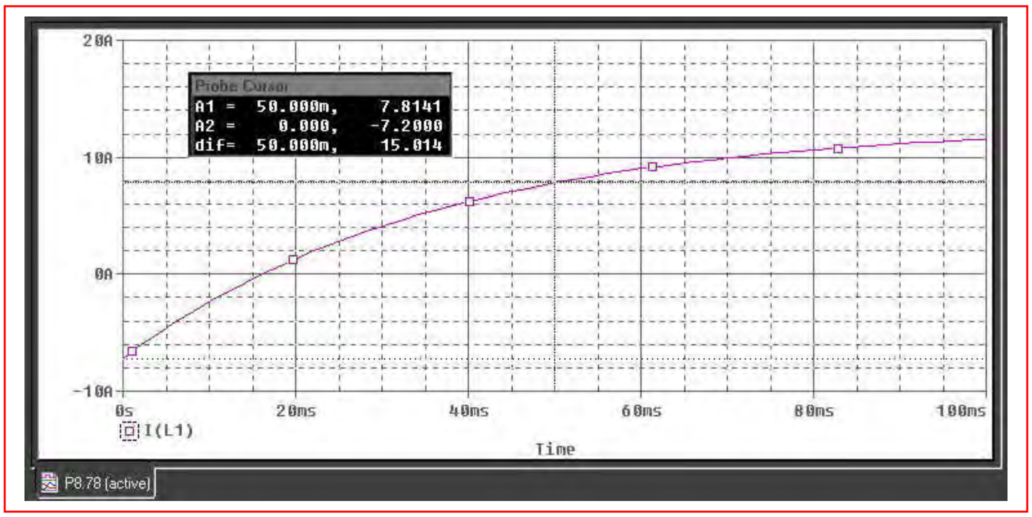


In plotting both the hand-derived result and the PSpice simulation result, we see that the ideal op amp approximation holds very well for this particular circuit. Although the 741 contains internal capacitors, it does not introduce any shorter time constants than that of the input circuit.

96. For $t < 0$, the current source is an open circuit and so $i_1 = 40/\ 50 = 0.8$ A.
The current through the 5-Ω resistor is $[40 – 10(0.8)]/\ 5 = 7.2$ A, so the inductor current is equal to $\boxed{-7.2\text{ A}}$

PSpice Simulation



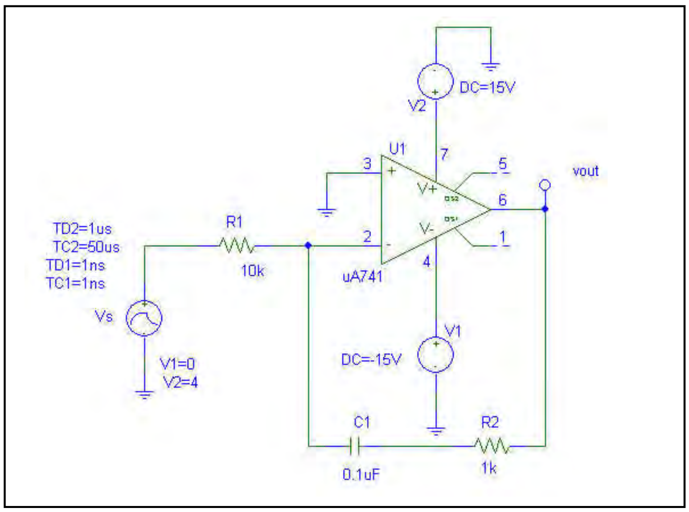


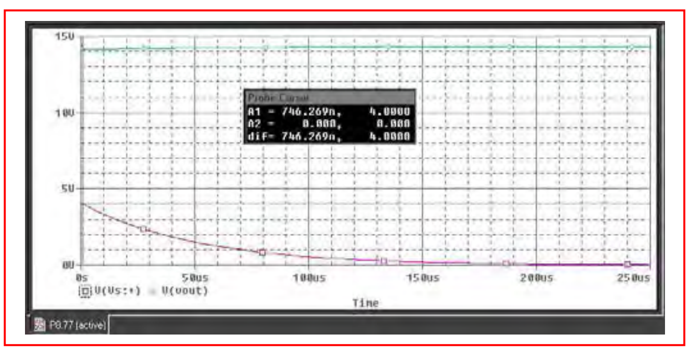
From the PSpice simulation, we see that our $t < 0$ calculation is indeed correct, and find that the inductor current at $t = 50$ ms is $\boxed{7.82\text{ A.}}$

97. (a)
$$v_1 = 0 \text{ (virtual gnd)} \;\therefore i = \frac{4}{10^4} e^{-20{,}000t} u(t)\text{A}$$
$$\therefore v_c = 10^7 \int_o^t \frac{4}{10^4} e^{-20{,}000t} dt = -0.2e^{-20{,}000t} \Big|_o^t$$
$$\therefore v_c(t) = 0.2(1 - e^{-20{,}000t})u(t)$$
$$\therefore v_R(t) = 10^3\, i(t) = 0.4e^{-20{,}000t} u(t)\text{V}$$
$$\therefore v_o(t) = -v_c(t) - v_R(t) = (-0.2 + 0.2e^{-20{,}000t} - 0.4e^{-20{,}000t})u(t)$$

And we may write $\boxed{v_o(t) = -0.2[1 + e^{-20\times 10^3 t}]u(t) \text{ V.}}$

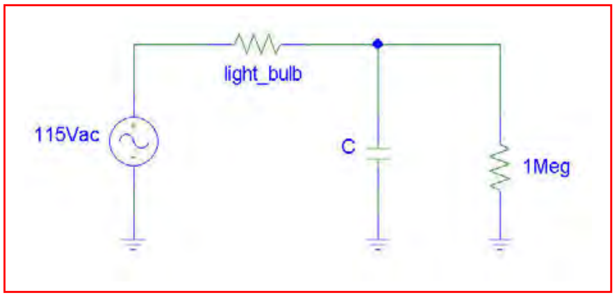
(b) PSpice verification:





We can see from the simulation result that our ideal op amp approximation is not providing a great deal of accuracy in modeling the transient response of an op amp in this particular circuit; the output was predicted to be negative for $t > 0$.

98. One possible solution of many: implement a capacitor to retain charge; assuming the light is left on long enough to fully charge the capacitor, the stored charge will run the lightbulb after the wall switch is turned off. Taking a 40-W light bulb connected to 115 V, we estimate the resistance of the light bulb (which changes with its temperature) as 330.6 Ω. We define "on" for the light bulb somewhat arbitrarily as 50% intensity, taking intensity as proportional to the dissipated power. Thus, we need at least 20 W (246 mA or 81.33 V) to the light bulb for 5 seconds after the light switch is turned off.



The circuit above contains a 1-MΩ resistor in parallel with the capacitor to allow current to flow through the light bulb when the light switch is on. In order to determine the required capacitor size, we first recognise that it will see a Thevenin equivalent resistance of 1 MΩ || 330.6 Ω = 330.5 Ω. We want $v_C(t = 5\text{s}) = 81.33 = 115\ e^{-5/\tau}$, so we need a circuit time constant of t = 14.43 s and a capacitor value of τ / R_{th} = $\boxed{43.67 \text{ mF.}}$

99. Assume at least 1 μA required otherwise alarm triggers.

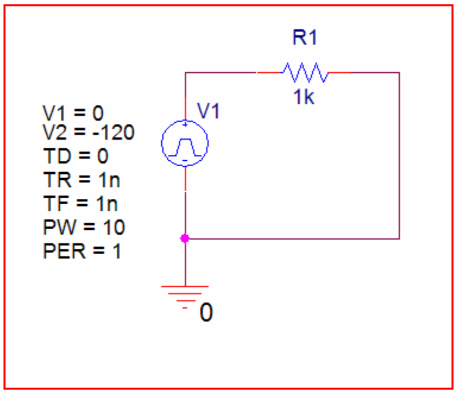
Add capacitor C.

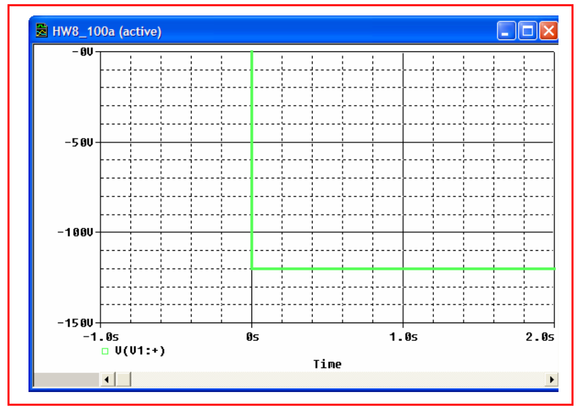
$v_c(1) = 1\text{ volt}$

$v_c(0) = \dfrac{1000}{1002.37}.1.5 = 1.496\text{ volts}$

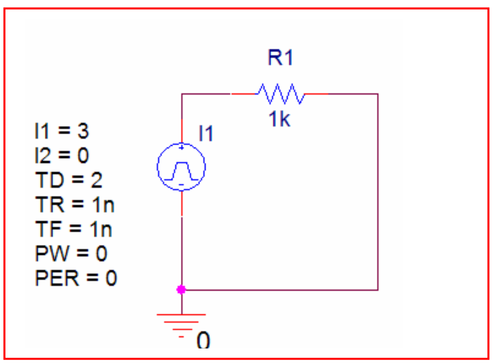
$\therefore$ We have $1 = 1.496e^{-\frac{1}{10^6 C}}$ or $C = \dfrac{1}{10^6\,\ell n(1.496)} = \boxed{2.48\mu\text{F}}$

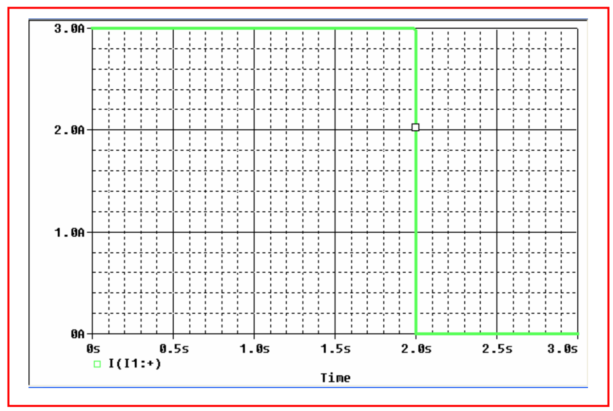
100. (a) Note that negative times are not permitted in PSpice. The only way to model this situation is to shift the time axis by a fixed amount, e.g., $t' = t + 1$.



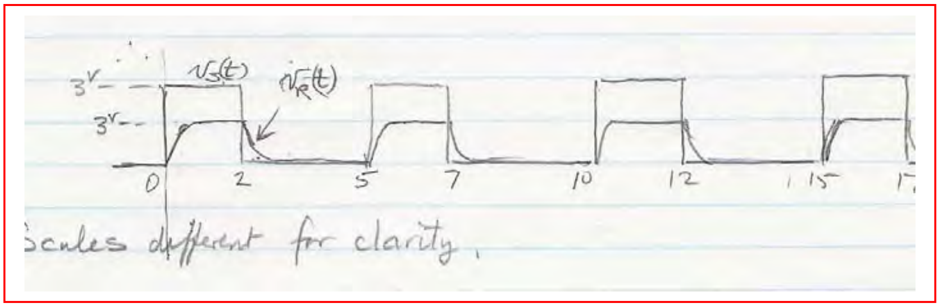


(b) Negative times are not permitted in PSpice. The only way to model this situation is to shift the time axis by a fixed amount, e.g., $t' = t + 2$.

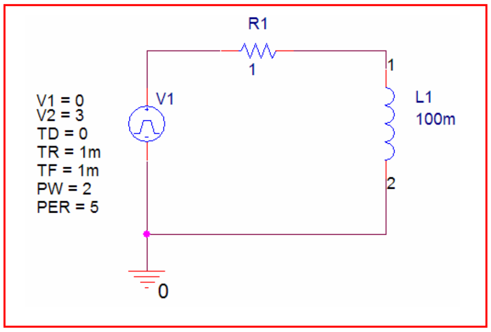


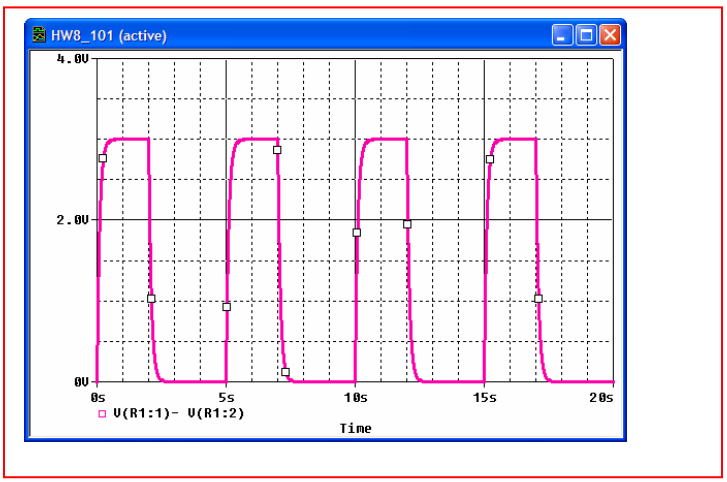


101. (a) $\tau = L/R = 0.1$ s. This is much less than either the period or pulsewidth.

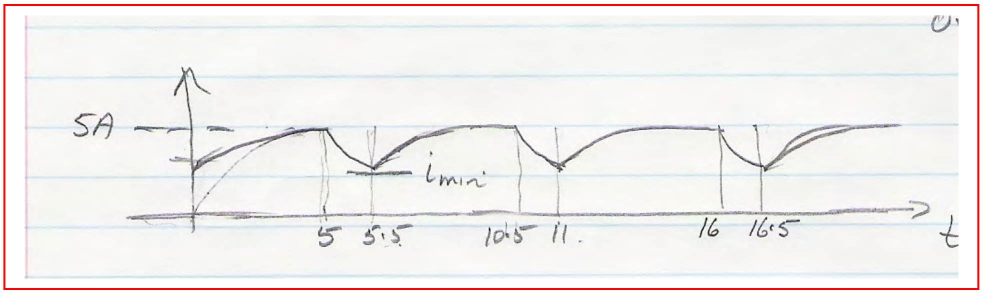


(b)

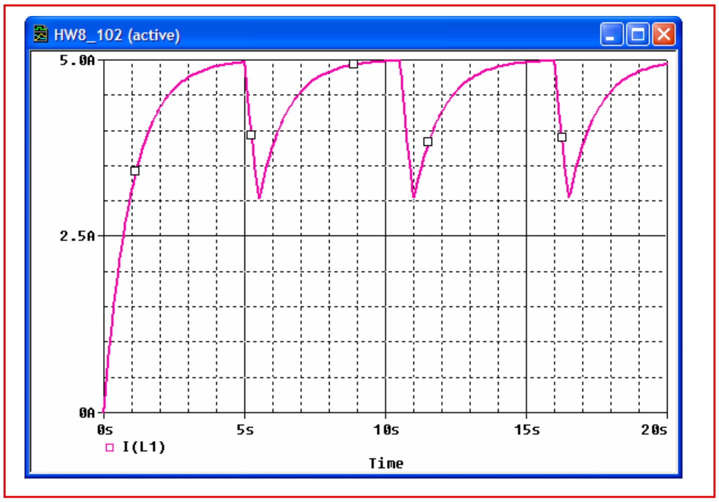


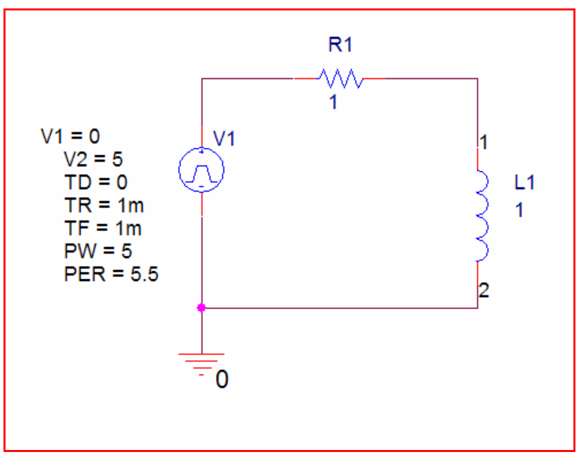


102. (a) $\tau = L / R = 1$ s. This is much less than either the period or pulsewidth.



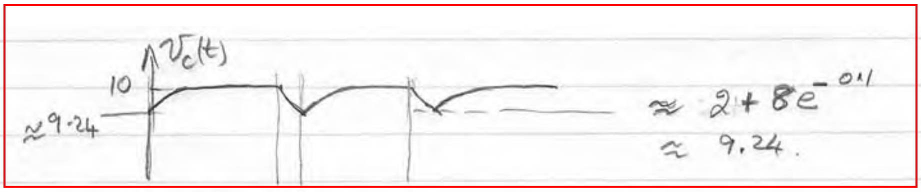
(b)



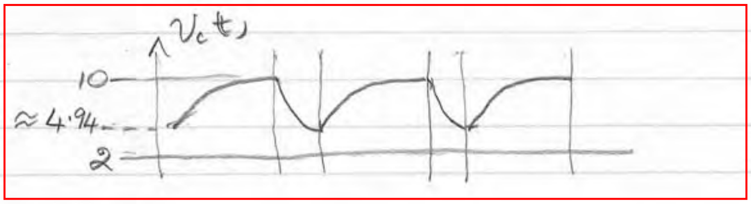


103.

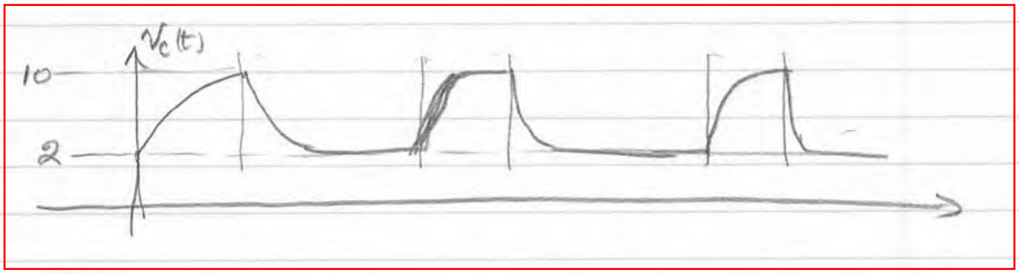
(a) 0.1 RC



(b) RC



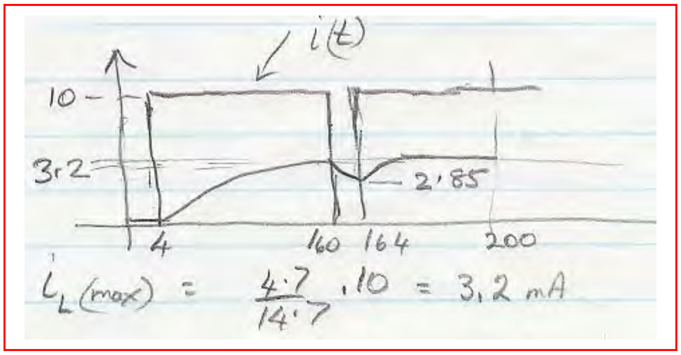
(c) 10RC



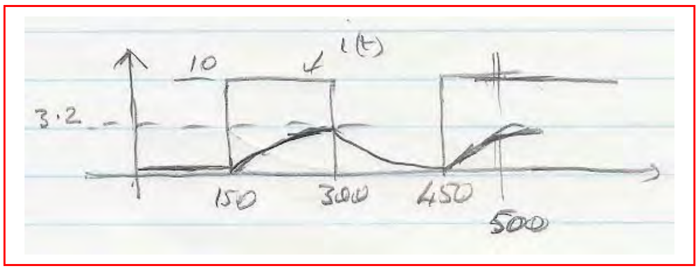
104. $\tau = \frac{L}{R_{eq}} = \frac{500\times10^{-6}}{14.7\times10^{3}} = 34$ ns

The transient response will therefore have the form $Ae^{-29.4\times10^{6}t}$.

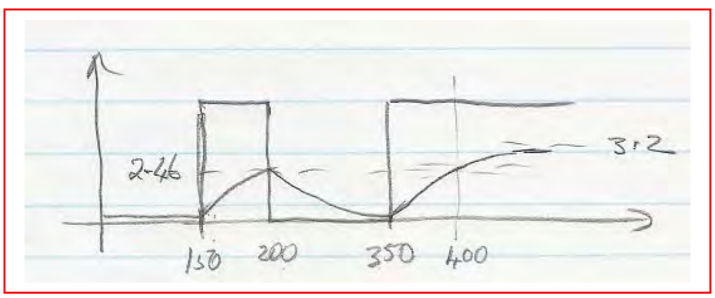
(a)



(b)



(c)



1. Parallel RLC circuit:

(a) $$\alpha = \frac{1}{2RC} = \frac{1}{(2)(4 \| 10)(10^{-6})} = \frac{1}{(2)(2.857)(10^{-6})} = \boxed{175\times10^{3}\ \text{s}^{-1}}$$

(b) $$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(2\times10^{-3})(10^{-6})}} = \boxed{22.4\ \text{krad/s}}$$

(c) The circuit is overdamped since $\alpha > \omega_0$.

2. Parallel RLC circuit:

(a) For an underdamped response, we require $\alpha < \omega_0$, so that

$$\frac{1}{2RC} < \frac{1}{\sqrt{LC}} \quad \text{or} \quad R > \frac{1}{2}\sqrt{\frac{L}{C}}; \quad R > \frac{1}{2}\sqrt{\frac{2}{10^{-12}}}\ .$$

Thus, $\boxed{R > 707\ \text{k}\Omega.}$

(b) For critical damping,

$$\boxed{R = \frac{1}{2}\sqrt{\frac{L}{C}} = 707\ \text{k}\Omega}$$

3. Parallel RLC circuit:

(a)

$$\alpha = \frac{1}{2RC} = \frac{1}{(2)(4\,||\,10)(10^{-6})} = \frac{1}{(2)(1)(10^{-9})} = \boxed{5\times10^{8}\ \text{s}^{-1}}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(10^{-12})(10^{-9})}} = 3.16\times10^{13}\ \text{rad/s} = \boxed{31.6\ \text{Trad/s}}$$

(b)

$$\mathbf{s}_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} = -0.5\times10^{9} \pm j\sqrt{10^{21} - (0.25)(10^{18})} = \boxed{-0.5 \pm j31.62\ \text{Grad/s}}$$

(c) $\boxed{\text{The circuit is underdamped since } \alpha < \omega_0.}$

4. Parallel RLC circuit:

(a) For an underdamped response, we require $\alpha < \omega_0$, so that

$$\frac{1}{2RC} < \frac{1}{\sqrt{LC}} \quad \text{or} \quad R > \frac{1}{2}\sqrt{\frac{L}{C}}; \quad R > \frac{1}{2}\sqrt{\frac{10^{-15}}{2\times 10^{-18}}}.$$

Thus, $\boxed{R > 11.18\ \Omega.}$

(b) For critical damping,

$$\boxed{R = \frac{1}{2}\sqrt{\frac{L}{C}} = 11.18\ \Omega}$$

(c) For overdamped, $\boxed{R < 11.18\ \Omega.}$

5. $\omega_o \text{L} = 10\Omega, s_1 = -6s^{-1},\ s_2 = -8s^{-1}$

$$\therefore -6 = \alpha + \sqrt{\alpha^2 - \omega_o^2}, -8 = -\alpha - \sqrt{\alpha^2 - \omega_o^2} \text{ adding,}$$

$-14 = -2\alpha \therefore \alpha = 7s^{-1}$

$$\therefore -6 = -7 + \sqrt{49 - \omega_o^2} \therefore \omega_o^2 = 48\frac{1}{\text{LC}}, \omega_o = 6.928$$

rad/s $\therefore 6.928\,\text{L} = 10, \boxed{\text{L} = 1.4434\text{H},}$

$$\text{C} = \frac{1}{48\text{L}} = \boxed{14.434\text{mF}}, \frac{1}{2\text{RC}} = 7 \therefore \text{R} = \boxed{4.949\Omega}$$

6. $i_c = 40e^{-100t} - 30e^{-200t}\text{mA, C} = 1\text{mF, } v(0) = -0.25\text{V}$

(a) $v(t) = \frac{1}{\text{C}}\int_o^t i_c dt - 0.25 = \int_o^t (40e^{-100t} - 30e^{-200t})\, dt - 0.25$

$\therefore v(t) = -0.4(e^{-100t} - 1) + 0.15\,(e^{-200t} - 1) - 0.25$

$\therefore v(t) = \boxed{-0.4e^{-100t} + 0.15e^{-200t}\text{V}}$

(b) $s_1 = -100 = -\alpha + \sqrt{\alpha^2 - \omega_o^2},\ s_2 = -200 = -\alpha - \sqrt{\alpha^2 - \omega_o^2}$

$\therefore -300 = -2\alpha,\ \alpha = 150s - 1$

$\therefore 150 + \frac{1}{2\text{R}10^{-3}}, \text{R} = \frac{500}{150} = 3.333\Omega$ Also,

$-200 = -150 - \sqrt{22500 - \omega_o^2}\ \therefore \omega_o^2 = 20000$

$\therefore 20000 = \frac{1}{\text{LC}} = \frac{100}{\text{L}},\ \text{L} = 0.5\text{H}$

$\therefore \text{i}_\text{R}(t) = \frac{v}{\text{R}} = \boxed{0.12e^{-100t} + 0.045e^{-200t}\text{A}}$

(c) $(i)t = -i_R(t) - i_c(t) = (0.12 - 0.04)e^{-100t} + (-0.045 + 0.03)e^{-200t}$

$\therefore i(t) = \boxed{80e^{-100t} - 15e^{-200t}\text{mA, } t > 0}$

7. Parallel RLC with $\omega_o = 70.71 \times 10^{12}$ rad/s. L = 2 pH.

(a) $$\omega_o^2 = \frac{1}{LC} = (70.71 \times 10^{12})^2$$

So $C = \dfrac{1}{(70.71 \times 10^{12})^2 (2 \times 10^{-12})} = \boxed{\quad \text{aF}}$

(b) $$\alpha = \frac{1}{2RC} = 5 \times 10^9 \ s^{-1}$$

So $R = \dfrac{1}{(10^{10})(100 \times 10^{-18})} = \boxed{\quad \text{M}\Omega}$

(c) α is the neper frequency: $\boxed{5 \text{ Gs}^{-1}}$

(d) $$S_1 = -\alpha + \sqrt{\alpha^2 - \omega_o^2} = \boxed{-5 \times 10^9 + j70.71 \times 10^{12} \ s^{-1}}$$

$$S_2 = -\alpha - \sqrt{\alpha^2 - \omega_o^2} = \boxed{-5 \times 10^9 - j70.71 \times 10^{12} \ s^{-1}}$$

(e) $$\zeta = \frac{\alpha}{\omega_o} = \frac{5 \times 10^9}{70.71 \times 10^{12}} = \boxed{7.071 \times 10^{-5}}$$

8. Given: $L = 4R^2C$, $\alpha = \dfrac{1}{2RC}$

Show that $v(t) = e^{-\alpha t}(A_1 t + A_2)$ is a solution to

$$C\frac{d^2v}{dt^2} + \frac{1}{R}\frac{dv}{dt} + \frac{1}{L}v = 0 \qquad [1]$$

$$\frac{dv}{dt} = e^{-\alpha t}(A_1) - \alpha e^{-\alpha t}(A_1 t + A_2)$$
$$= (A_1 - \alpha A_1 t - \alpha A_2)e^{-\alpha t} \qquad [2]$$
$$\frac{d^2v}{dt^2} = (A_1 - \alpha A_1 t - \alpha A_2)(-\alpha e^{-\alpha t}) - \alpha A_1 e^{-\alpha t}$$
$$= -\alpha(A_1 - \alpha A_2 + A_1 - \alpha A_1 t)e^{-\alpha t}$$
$$= -\alpha(2A_1 - \alpha A_2 - \alpha A_1 t)e^{-\alpha t} \qquad [3]$$

Substituting Eqs. [2] and [3] into Eq. [1], and using the information initially provided,

$$-\frac{1}{2RC}(2A_1)e^{-\alpha t} + \left(\frac{1}{2RC}\right)^2 (A_1 t + A_2)e^{-\alpha t} + \frac{1}{RC}(A_1)e^{-\alpha t}$$
$$-\frac{1}{2RC}(A_1 t + A_2)e^{-\alpha t} + \frac{1}{4R^2C^2}(A_1 t + A_2)e^{-\alpha t}$$
$$= 0$$

Thus, $v(t) = e^{-\alpha t}(A_1 t + A_2)$ is in fact a solution to the differential equation.

Next, with $v(0) = \boxed{A_2 = 16}$

and $\left.\dfrac{dv}{dt}\right|_{t=0} = (A_1 - \alpha A_2) = (A_1 - 16\alpha) = 4$

we find that $\boxed{A_1 = 4 + 16\alpha}$

9. Parallel RLC with ω_o = 800 rad/s, and α = 1000 s^{-1} when R = 100 Ω.

$$\alpha = \frac{1}{2RC} \quad \text{so} \quad C = 5\mu\text{F}$$

$$\omega_o^2 = \frac{1}{LC} \quad \text{so} \quad L = 312.5 \text{ mH}$$

Replace the resistor with 5 meters of 18 AWG copper wire. From Table 2.3, 18 AWG soft solid copper wire has a resistance of 6.39 Ω/1000ft. Thus, the wire has a resistance of

$$(5\,\text{m})\left(\frac{100\,\text{cm}}{1\,\text{m}}\right)\left(\frac{1\,\text{in}}{2.54\,\text{cm}}\right)\left(\frac{1\,\text{ft}}{12\,\text{in}}\right)\left(\frac{6.39\,\Omega}{1000\,\text{ft}}\right)$$
$$= 0.1048\,\Omega \;\text{ or }\; 104.8\,\text{m}\Omega$$

(a) The resonant frequency is unchanged, so $\boxed{\omega_o = 800\,\text{rad/s}}$

(b) $\alpha = \dfrac{1}{2RC} = 954.0\times10^3\ s^{-1}$

(c)
$$\zeta_{old} = \frac{\alpha_{old}}{\omega_o}$$
$$\zeta_{new} = \frac{\alpha_{new}}{\omega_o}$$

Define the percent change as $\dfrac{\zeta_{new} - \zeta_{old}}{\zeta_{old}}\times 100$

$$= \frac{\alpha_{new} - \alpha_{old}}{\alpha_{old}}\times 100$$
$$= \boxed{95300\%}$$

10. $L = 5\text{H},\ \text{R} = 8\Omega,\ \text{C} = 12.5\text{mF},\ v(0^+) = 40\text{V}$

(a) $i(0^+) = 8\text{A}: \alpha = \dfrac{1}{2\text{RC}} = \dfrac{1000}{2\times 8\times 12.5} = 5,\ \omega_o^2 = \dfrac{1}{\text{LC}} = 16,$

$\omega_o = 4\ s_{1,2} = -5 \pm \sqrt{25-16} = -2,\ -8 \therefore v(t) = \text{A}_1 e^{-2t} + \text{A}_2 e^{-8t}$

$\therefore 40 = \text{A}_1 + \text{A}_2\ v'(0^+) = \dfrac{1000}{12.5}\left(-i_L(0^+) - \dfrac{40}{8}\right) = 80(-8-5) = -1040$

$v/s = -2\text{A}_1 - 8\text{A}_2 \therefore -520 = -\text{A}_1 - 4\text{A}_2 \therefore -3\text{A}_2 = -480,\ \text{A}_2 = 160, \text{A}_1 = -120$

$\therefore v(t) = \boxed{-120e^{-2t} + 160e^{-8t}\text{V},\ t > 0}$

(b) $i_c(0^+) = 8\text{A}$ Let $(\ t) = \text{A}_3 e^{-2t} + \text{A}_4 e^{-8t};\ i_R(0^+) = \dfrac{v(0^+)}{\text{R}} = \dfrac{40}{8} = 5\text{A}$

$\therefore i(0^+) = \text{A}_3 + \text{A}_4 = -i_R(0^+) - i_c(0^+) = -8 - 5 = -13\text{A};$

$i(0^+) = -2\text{A}_3 - 8\text{A}_4 = \dfrac{40}{5} = 8\ \text{A}/s \therefore 4 = -\text{A}_3 - 4\text{A}_4$

$\therefore -3\text{A}_4 = -13 + 4,\ \text{A}_4 = 3,\ \text{A}_3 = -16 \therefore i(t) = \boxed{-16e^{-2t} + 3e^{-8t}\text{A},\ t > 0}$

11. (a) $$R_C = \frac{1}{2}\sqrt{\frac{L}{C}} = \frac{1}{2}\sqrt{\frac{10^{-3}}{10^{-4}}} = \frac{1}{2}\sqrt{10} = \quad 1.581\ \Omega$$

Therefore

$$R \quad = 0.1R_C = \boxed{158.1\ \text{m}\Omega}$$

(b) $$\alpha = \frac{1}{2RC} = 3.162\times10^4\ \text{s}^{-1} \quad \text{and} \quad \omega_0 = \frac{1}{\sqrt{LC}} = 3.162\times10^3\ \text{rad/s}$$

Thus, $$\mathbf{s}_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} = \quad -158.5\ \text{s}^{-1} \ \text{ and } \ -6.31\times10^4\ \text{s}^{-1}$$

So we may write $i(t) = A_1 e^{-158.5t} + A_2 e^{-6.31\times10^4 t}$

With $i(0^-) = i(0^+) = 4\ \text{A}$ and $v(0^-) = v(0^+) = 10\ \text{V}$

$$A_1 + A_2 = 4 \qquad [1]$$

Noting

$$v(0^+) = L\left.\frac{di}{dt}\right|_{t=0} = 10$$

$$10^{-3}\left(-158.5A_1 - 6.31\times10^4 A_2\right) = 10 \quad [2]$$

Solving Eqs. [1] and [2] yields $A_1 = 4.169$ A and $A_2 = -0.169$ A

So that $$\boxed{i(t) = 4.169e^{-158.5t} - 0.169e^{-6.31\times10^4 t} \quad \text{A}}$$

12. (a) $\alpha = \dfrac{1}{2RC} = 500\ \text{s}^{-1}$ and $\omega_0 = \dfrac{1}{\sqrt{LC}} = 100$ rad/s

Thus, $\mathbf{s}_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} =\ -10.10\ \text{s}^{-1}$ and $-989.9\ \text{s}^{-1}$

So we may write $i_R(t) = A_1 e^{-10.1t} + A_2 e^{-989.9t}$ [1]

With $i(0^-) = i(0^+) = 2$ mA and $v(0^-) = v(0^+) = 0$

$A_1 + A_2 = 0$ [2]

We need to find $\left.\dfrac{di_R}{dt}\right|_{t=0}$. Note that $\dfrac{di_R(t)}{dt} = \dfrac{1}{R}\dfrac{dv}{dt}$ [3] and $i_C = C\dfrac{dv}{dt} = -i - i_R$.

Thus, $i_C(0^+) = C\left.\dfrac{dv}{dt}\right|_{t=0^+} = -i(0^+) - i_R(0^+) = -2\times10^{-3} - \dfrac{v(0^+)}{R} = -2\times10^{-3}$ [4]

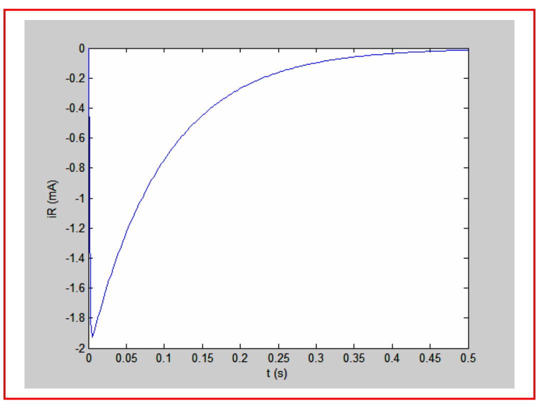
Therefore, we may write based on Eqs. [3] and [4]:

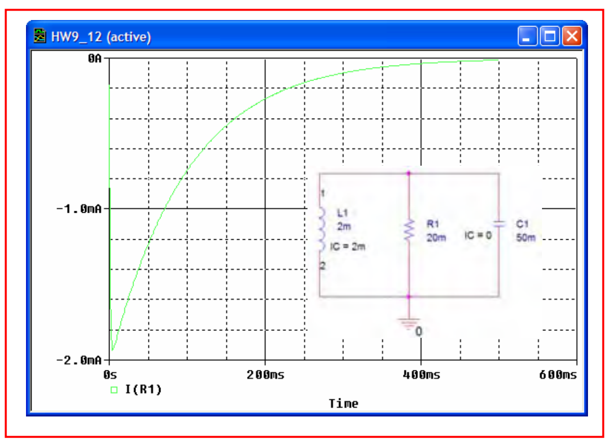
$\left.\dfrac{di_R}{dt}\right|_{t=0} = (50)(-0.04) = -2$ [5]. Taking the derivative of Eq. [1] and combining with Eq. [5] then yields: $\mathbf{s}_1 A_1 + \mathbf{s}_2 A_2 = -2$ [6].

Solving Eqs. [2] and [6] yields $A_1 = -2.04$ mA and $A_2 = 2.04$ mA

So that $\boxed{i_R(t) = -2.04\left(e^{-10.1t} - e^{-989.9t}\right)\ \text{mA}}$

(b) (c) We see that the simulation agrees.





13. $i(0) = 40\text{A},\ v(0) = 40\text{V},\ \text{L} = \dfrac{1}{80}\text{H},\ \text{R} = 0.1\Omega,\ \text{C} = 0.2\text{F}$

(a) $\alpha = \dfrac{1}{2\times 0.1\times 0.2} = 25,\ \omega_o^2 = \dfrac{80}{0.2} = 400,$

$\omega_o = 20,\ s_{1,2} = -25 \pm \sqrt{625-400} = 10,\ -40$

$\therefore v(t) = \text{A}_1\, e^{-10t} + \text{A}_2\, e^{-40t}\ \therefore 40 = \text{A}_1 + \text{A}_2;$

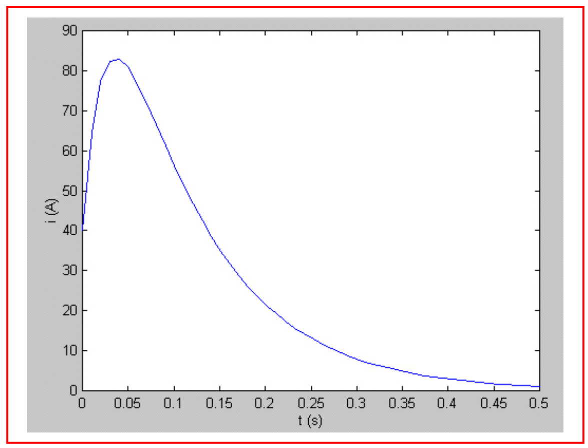
$v'(0^+) = -10\text{A}_1 - 40\text{A}_2\ v'(0^+) = \dfrac{1}{\text{C}}\left(i(0) - \dfrac{v(0)}{\text{R}}\right) = -2200$

$\therefore -\text{A}_1 - 4\text{A}_2 = -220 \therefore -3\text{A}_2 = -180 \therefore \text{A}_2 = 60,\ \text{A}_1 = -20$

$\therefore v(t) = \boxed{-20e^{-10t} + 60e^{-40t}\text{V},\ t>0}$

(b) $i(t) = -v/\text{R} - \text{C}\dfrac{dv}{dt} = 200e^{-10t} - 600e^{-40t} - 0.2(-20)(-10)e^{-10t} - (0.2)(60)(-40)e^{-40t}$

$= 160e^{-10t} - 120e^{-40t}\ \text{A}$



14. (a) $\alpha = \dfrac{1}{2RC} = 6.667\times10^{8}\ \text{s}^{-1}$ and $\omega_0 = \dfrac{1}{\sqrt{LC}} = 10^{5}$ rad/s

Thus, $\mathbf{s}_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} = \ -7.5\ \text{s}^{-1}$ and $-1.333\times10^{9}\ \text{s}^{-1}$. So we may write

$i_C(t) = A_1 e^{-7.5t} + A_2 e^{-1.333\times10^9 t}$ [1] With $i(0^-) = i(0^+) = 0$ A and $v(0^-) = v(0^+) = 2$ V,

$i_C(0^+) = -i_R(0^+) = -\dfrac{2}{15\times10^{-6}} = -0.133\times10^{6}$ so that

$$A_1 + A_2 = -0.133\times10^{6} \quad [2]$$

We need to find $\left.\dfrac{di_R}{dt}\right|_{t=0}$. We know that $L\left.\dfrac{di}{dt}\right|_{t=0} = 2$ so $\left.\dfrac{di}{dt}\right|_{t=0} = \dfrac{2}{2\times10^{-6}} = 10^{6}$. Also,

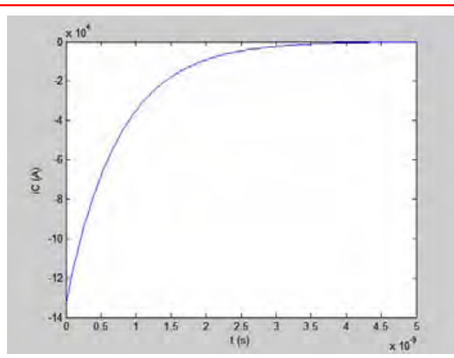
$C\dfrac{dv}{dt} = i_C$ and $\dfrac{di_R}{dt} = \dfrac{1}{R}\dfrac{dv}{dt}$ so $\dfrac{di_R}{dt} = \dfrac{i_C}{CR} = \dfrac{1}{CR}\left(A_1 e^{-7.5t} + A_2 e^{-1.333\times10^9 t}\right)$.

Using $\dfrac{di}{dt} + \dfrac{di_R}{dt} + \dfrac{di_C}{dt} = 0$ so $\left.\dfrac{di_C}{dt}\right|_{t=0} = -7.5A_1 - 1.33\times10^{9} A_2 = -10^{6} - \dfrac{1}{CR}(A_1 + A_2)$ [3]

Solving Eqs. [2] and [3] yields $A_1 = -0.75$ mA and $A_2 = -0.133$ MA (very different!)

So that $\boxed{i_C(t) = -\left(0.75\times10^{-3} e^{-7.5t} + 0.133\times10^{6} e^{-1.333\times10^9 t}\right) \ \text{A}}$

(b)



15. (a) $\alpha = \dfrac{1}{2RC} = 0.125\ \text{s}^{-1}$ and $\omega_0 = \dfrac{1}{\sqrt{LC}} = 0.112$ rad/s

Thus, $\mathbf{s}_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} = \ -0.069\ \text{s}^{-1}$ and $-0.181\ \text{s}^{-1}$. So we may write

$$v(t) = A_1 e^{-0.069t} + A_2 e^{-0.181t} \quad [1]$$

With $i_C(0^-) = i_C(0^+) = -8$ A and $v(0^-) = v(0^+) = 0$,

$$A_1 + A_2 = 0 \qquad [2]$$

We need to find $\left.\dfrac{di_R}{dt}\right|_{t=0}$. We know that

$i_C(t) = C\dfrac{dv}{dt} = 4\left[-0.069A_1e^{-0.069t} - 0.181A_2e^{-0.181t}\right]$. So,

$$i_C(0) = 4\left[-0.069A_1 - 0.181A_2\right] = -8 \qquad [3]$$

Solving Eqs. [2] and [3] yields $A_1 = -17.89$ V and $A_2 = 17.89$ V

So that $\boxed{v(t) = -17.89\left[e^{-0.069t} - e^{-0.181t}\right]\ \text{V}}$

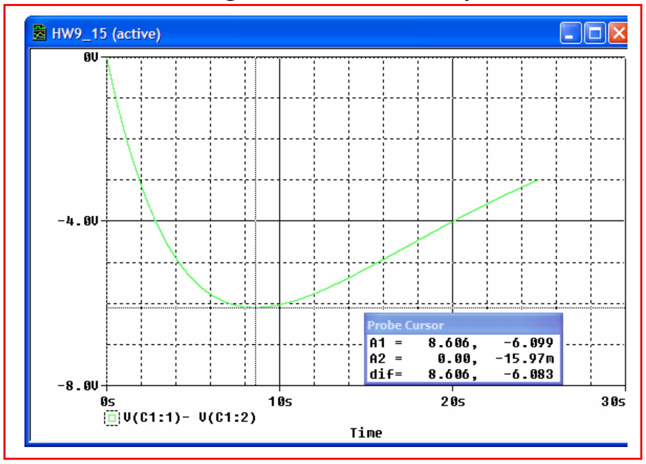
(b) $\dfrac{dv}{dt} = 1.236e^{-0.069t} - 8.236e^{-0.181t}$. We set this equal to 0 and solve for t_m:

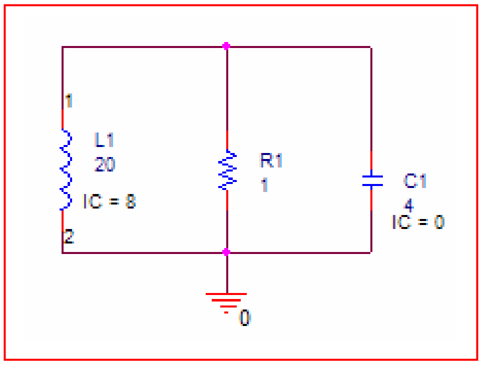
$$\frac{3.236}{1.236} = \frac{e^{-0.069t_m}}{e^{-0.181t_m}} = e^{0.112t_m}\text{, so that } \boxed{t_m = 8.61\ \text{s}.}$$

Substituting into our expression for the voltage, the peak value is

$$\boxed{v(8.61)\ =\ -6.1\ \text{V}}$$

(c) The simulation agrees with the analytic results.





16. $i_L(0) = \frac{100}{50} = 2\text{A},\ v_c(0) = 100\text{V}$

$$\alpha = \frac{10^6}{2\times 50\times 2.5} = 4000,\ w_o^2 = \frac{3\times 10^{6+3}}{100\times 2.5} = 12\times 10^6$$

$$\sqrt{16-12}\times 10^3 = 200,\ s_{1,2} = -4000 \pm 2000$$

$$\therefore i_L(t) = \text{A}_1 e^{-2000t} + \text{A}_2 e^{-6000t},\ t>0\ \therefore \text{A}_1 + \text{A}_s = 2$$

$$i_L'(0^+) = \frac{-10^3\times 3}{100}\times 100 = -3000 = -2000\text{A}_1 - 6000\text{A}_2 \therefore -1.5 = -\text{A}_1 - 3\text{A}_2 \therefore 0.5 = -2\text{A}_2$$

$$\therefore \text{A}_2 = -0.25,\ \text{A}_1 = 2.25 \therefore i_L(t) = 2.25e^{-2000t} - 0.25e^{-6000t}\text{A},\ t>0$$

$$t>0:\ i_L(t) = 2\text{A} \therefore i_L(t) = \boxed{2u(-t) + (2.25e^{-2000t} - 0.25e^{-6000t})u(t)\text{A}}$$

17.

$$i_L(0)=\frac{12}{5+1}=2\text{A},\ v_c(0)=2\text{V}$$

$$\alpha=\frac{1000}{2\times1\times2}=250,\ \omega_o^2=\frac{1000\times45}{2}=22500$$

$$s_{1,2}=-250\pm\sqrt{250^2-22500}=-50,\,-450\ s^{-1}$$

$$\therefore i_L=\text{A}_1e^{-50t}+\text{A}_2e^{-450t}\ \therefore \text{A}_1+\text{A}_2=2;\ i_L'(0^+)=45(-2)=-50\text{A}_1-450\text{A}_2$$

$$\therefore \text{A}_1+9\text{A}_2=1.8\ \therefore -8\text{A}_2=0.2\ \therefore \text{A}_2=-0.025,\ \text{A}_1=2.025(\text{A})$$

$$\therefore i_L(t)=\boxed{2.025e^{-50t}-0.025e^{-450t}\text{A},\ t>0}$$

18.

(a) $\alpha = \frac{1}{2RC} = \frac{1440}{72} = 20,\ \omega_o^2 = \frac{1440}{10} = 144$

$s_{1,2} = -20 \pm \sqrt{400 - 144} = -4, -36:\ v = A_1 e^{-4t} + A_2 e^{-36t}$

$v(0) = 18 = A_1 + A_2,\ v'(0) = 1440\left(\frac{1}{2} - \frac{18}{36}\right) = 0$

$\therefore 0 = -4A_1 - 36A_2 = -A_1 - 9A_2 = \therefore 18 = -8A_2,\ A_2 = -2.25,\ A_1 = 20.25$

$\therefore v(t) = \boxed{20.25e^{-4^+} - 2.25e^{-36t}\text{V},\ t>0}$

(b) $i(t) = \frac{v}{36} + \frac{1}{1440}v' = 0.5625e^{-4t} - 0.0625e^{-36t} - 0.05625e^{-4t} + 0.05625e^{-36t}$

$\therefore i(t) = \boxed{0.50625e^{-4t} - 0.00625e^{-36t}\text{A},\ t>0}$

(c) $v_{\max}$ at $t = 0 \therefore v_{\max} = 18\text{V} \therefore 0.18 = 20.25e^{-4t_s} - 2.25e^{-36t_s}$

Solving using a scientific calculator, we find that $\boxed{t_s = 1.181\text{ s.}}$

19. Referring to Fig. 9.43,

$$L = 1250 \text{ mH}$$

so $\omega_o = 1\big/\sqrt{LC} = 4 \text{ rad/s}$ Since α > ω_o, this circuit is over damped.

$$\alpha = \frac{1}{2RC} = 5 \ s^{-1}$$

The capacitor stores 390 J at $t = 0^-$:

$$W_c = \frac{1}{2} C v_c^2$$

$$\text{So } v_c(0^-) = \sqrt{\frac{2W_c}{C}} = 125 \text{ V} = v_c(0^+)$$

The inductor initially stores zero energy,

so $i_L(0^-) = i_L(0^+) = 0$

$$S_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -5 \pm 3 = -8, -2$$

Thus, $v(t) = Ae^{-8t} + Be^{-2t}$

Using the initial conditions, $v(0) = 125 = A + B$ [1]

$$i_L(0^+) + i_R(0^+) + i_c(0^+) = 0 + \frac{v(0^+)}{2} + i_c(0^+) = 0$$

$$\text{So } i_c(0^+) = -\frac{v(0^+)}{2} = -\frac{125}{2} = -62.5 \text{ V}$$

$$i_c = C\frac{dv}{dt} = 50 \times 10^{-3}[-8Ae^{-8t} - 2Be^{-2t}]$$

$$i_c(0^+) = -62.5 = -50 \times 10^{-3}(8A + 2B) \qquad [2]$$

Solving Eqs. [1] and [2], $A = 150$ V
$B = -25$ V

Thus, $\boxed{v(t) = 166.7e^{-8t} - 41.67e^{-2t},\ t > 0}$

20. (a) We want a response $v = Ae^{-4t} + Be^{-6t}$

$$\alpha = \frac{1}{2RC} = 5\,s^{-1}$$

$$S_1 = -\alpha + \sqrt{\alpha^2 - \omega_o^2} = -4 = -5 + \sqrt{25 - \omega_o^2}$$

$$S_2 = -\alpha - \sqrt{\alpha^2 - \omega_o^2} = -6 = -5 - \sqrt{25 - \omega_o^2}$$

Solving either equation, we obtain $\omega_o = 4.899$ rad/s

Since $\omega_o^2 = \frac{1}{LC},\ L = \frac{1}{\omega_o^2 C} = \boxed{833.3 \text{ mH}}$

(b) If $i_R(0^+) = 10$ A and $i_c(0^+) = 15$ A, find A and B.

with $i_R(0^+) = 10 \text{ A},\ v_R(0^+) = v(0^+) = v_c(0^+) = 20 \text{ V}$

$$v(0) = A + B = 20 \qquad [1]$$

$$i_c = C\frac{dv}{dt} = 50\times10^{-3}(-4Ae^{-4t} - 6Be^{-6t})$$

$$i_c(0^+) = 50\times10^{-3}(-4A - 6B) = 15 \qquad [2]$$

Solving, $\boxed{A = 210 \text{ V},\ B = -190 \text{ V}}$

Thus, $v = 210e^{-4t} - 190e^{-6t},\ t > 0$

21. Initial conditions: $i_L(0^-) = i_L(0^+) = 0$ $\qquad i_R(0^+) = \dfrac{50}{25} = 2 \text{ A}$

(a) $v_c(0^+) = v_c(0^-) = 2(25) = \boxed{50 \text{ V}}$

(b) $i_c(0^+) = -i_L(0^+) - i_R(0^+) = 0 - 2 = \boxed{-2 \text{ A}}$

(c) $t > 0$: parallel (source-free) RLC circuit

$$\alpha = \frac{1}{2RC} = 4000\, s^{-1} \qquad s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2}$$

$$\omega_o = \frac{1}{\sqrt{LC}} = 3464 \text{ rad/s} \qquad = -2000,\ -6000$$

Since $\alpha > \omega_0$, this system is overdamped. Thus,

$$v_c(t) = Ae^{-2000t} + Be^{-6000t}$$

$$i_c = C\frac{dv}{dt} = (5\times10^{-6})(-2000\,Ae^{-2000t} - 6000\,Be^{-6000t})$$

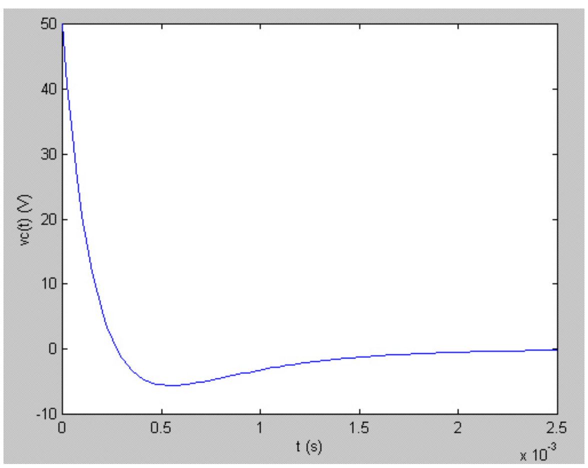
$$i_c(0^+) = -0.01A - 0.03B = -2 \qquad [1]$$

and $v_c(0^+) = A + B = 50$ [2]

Solving, we find $A = -25$ and $B = 75$

so that $\boxed{v_c(t) = -25e^{-2000t} + 75e^{-6000t},\ t > 0}$

(d)



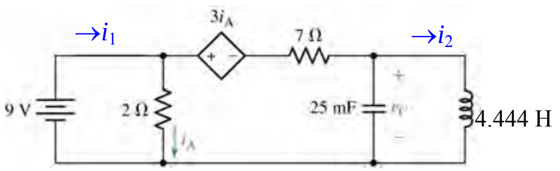
(e) $-25e^{-2000t} + 75e^{-6000t} = 0 \Rightarrow \boxed{t = 274.7\ \mu s}$

using a scientific calculator

(f) $|v_c|_{max} = -25 + 75 = 50 \text{ V}$

So, solving $|-25e^{-2000t_s} + 75e^{-6000t_s}| = 0.5$ in view of the graph in part (d),
we find $\boxed{t_s = 1.955 \text{ ms}}$ using a scientific calculator's equation solver routine.

22. Due to the presence of the inductor, $v_c(0^-) = 0$. Performing mesh analysis,

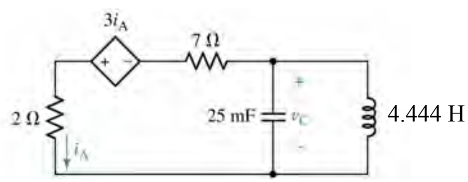


$$-9 + 2i_1 - 2i_2 = 0 \qquad [1]$$
$$2i_2 - 2i_1 + 3i_A + 7i_2 = 0 \qquad [2]$$
and $i_1 - i_2 = i_A$

Rearranging, we obtain $2i_1 - 2i_2 = 0$ and $-4i_1 + 6\, i_2 = 0$. Solving, $i_1 = 13.5$ A and $i_2 = 9$ A.

(a) $i_A(0^-) = i_1 - i_2 = \boxed{4.5 \text{ A}}$ and $i_L(0^-) = i_2 = 9$ A

(b) $t > 0$:



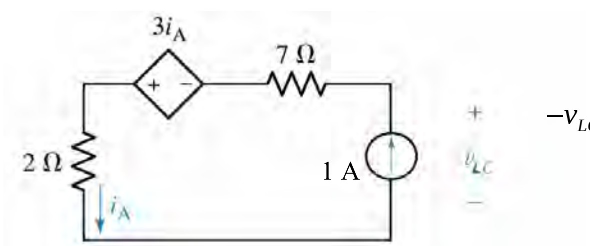
around left mesh:

$$-v_c(0^+) + 7i_A(0^+) - 3i_A(0^+) + 2i_A(0^+) = 0$$

so, $\boxed{i_A(0^+) = 0}$

(c) $\boxed{v_c(0^-) = 0}$ due to the presence of the inductor.

(d)



$$-v_{LC} + 7 - 3(1) + 2 = 0$$

$$v_{LC} = 6 \text{ V} \;\therefore\; \boxed{R_{TH} = \frac{6}{1} = 6\Omega}$$

(e)
$$\alpha = \frac{1}{2RC} = 3.333\ s^{-1}$$
$$\omega_o = \frac{1}{\sqrt{LC}} = 3 \text{ rad/s}$$
$$S_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -1.881,\ -4.785$$

Thus,
$$i_A(t) = Ae^{-1.881t} + Be^{-4.785t}$$
$$i_A(0^+) = 0 = A + B \qquad [1]$$

To find the second equation required to determine the coefficients, we write:

$$i_L = -i_c - i_R$$
$$= -C\frac{dv_c}{dt} - i_A \;=\; -25\times10^{-3}\left[-1.881(6A)e^{-1.881t} - 4.785(6B)e^{-4.785t}\right]$$
$$- Ae^{-1.881t} - Be^{-4.785t}$$

$$i_L(0^+) = 9 = -25\times10^{-3}[-1.881(6A) - 4.785(6B)] - A - B$$
or $\quad 9 = -0.7178A - 0.2822B \qquad [2]$

Solving Eqs. [1] and [2], $A = -20.66$ and $B = +20.66$

So that $\boxed{i_A(t) = 20.66[e^{-4.785t} - e^{-1.881t}]}$

23. Diameter of a dime: approximately 8 mm. Area = $\pi r^2 = 0.5027\text{cm}^2$

Capacitance $= \dfrac{\varepsilon_r \varepsilon_o A}{d} = \dfrac{(88)(8.854\times10^{-14}\,\text{F/cm})(0.5027\text{cm}^2)}{0.1\text{cm}}$

$= 39.17\text{pF}$

$L = 4\mu\text{H}$

$$\omega_o = \frac{1}{\sqrt{LC}} = 79.89\,\text{Mrad/s}$$

For an over damped response, we require $\alpha > \omega_o$.

Thus, $\dfrac{1}{2RC} > 79.89\times10^6$

$$R < \frac{1}{2(39.17\times10^{-12})(79.89\times10^6)}$$

or $\boxed{R < 159.8\Omega}$

*Note: The final answer depends quite strongly on the choice of ε_r.

24. (a) For critical damping, $R = \frac{1}{2}\sqrt{\frac{L}{C}} = \frac{1}{2}\sqrt{\frac{10^{-3}}{12\times10^{-6}}} = \boxed{4.564\ \Omega}$.

(b) $$\alpha = \frac{1}{2RC} = \frac{1}{2(4.564)(12\times10^{-6})} = 9.129\times10^{3}\ \text{s}^{-1}$$

Thus, $$v_C(t) = e^{-9.129\times10^{3}t}(A_1 t + A_2) \qquad [1]$$

At $t = 0$, $v_C(0) = A_1(0) + A_2 = 12 \quad \therefore\ A_2 = 12$ V.

Taking the derivative of Eq. [1],

$$\frac{dv_C(t)}{dt} = e^{-9.129\times10^{3}t}\left[-9.129\times10^{3}A_1 t + A_1 - 9.129\times10^{3}(12)\right]$$

and also $i_C = -(i_R + i_L)$, so

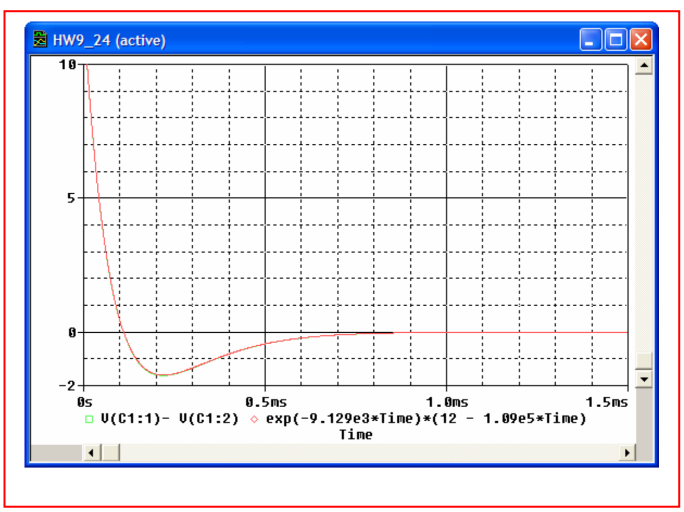
$$\left.\frac{dv_C}{dt}\right|_{t=0} = -\frac{1}{C}\left(\frac{v_C(0)}{R} + 0\right) = -\frac{1}{12\times10^{-6}}\left(\frac{12}{4.565} + 0\right) A_1 - 9.129\times10^{3}(12)$$

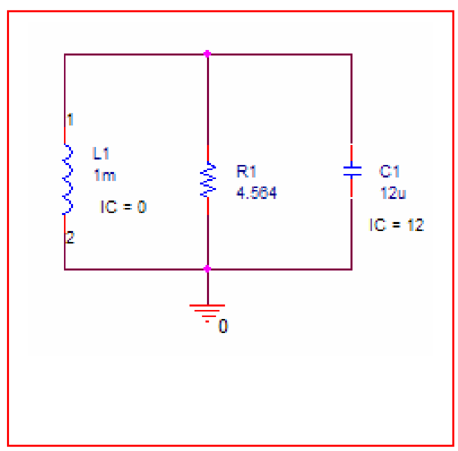
Solving,

$A_1 = -109.6\times10^{3}$ V, so we may write

$$\boxed{v_C(t) = e^{-9.129\times10^{3}t}\left(-109.6\times10^{3}t + 12\right).}$$

(c) We see from plotting both the analytic result in Probe and the simulated voltage, the two are in excellent agreement (the curves lie on top of one another).





25. (a) For critical damping, $R = \frac{1}{2}\sqrt{\frac{L}{C}} = \frac{1}{2}\sqrt{\frac{10^{-8}}{10^{-3}}} = \boxed{1.581\ \text{m}\Omega}$.

(b) $$\alpha = \frac{1}{2RC} = \frac{1}{2\left(1.581\times10^{-3}\right)\left(10^{-3}\right)} = 3.162\times10^{5}\ \text{s}^{-1}$$

Thus, $$i_L(t) = e^{-3.162\times10^{5}t}\left(A_1 t + A_2\right) \qquad [1]$$

At $t = 0,\ i_L(0) = A_1(0) + A_2 = 10 \qquad \therefore\ A_2 = 10\ \text{A}$.

Taking the derivative of Eq. [1],

$$\frac{di_L(t)}{dt} = e^{-3.162\times10^{5}t}\left[-3.162\times10^{5}A_1 t + A_1 - 3.162\times10^{5}(10)\right] \qquad [2]$$

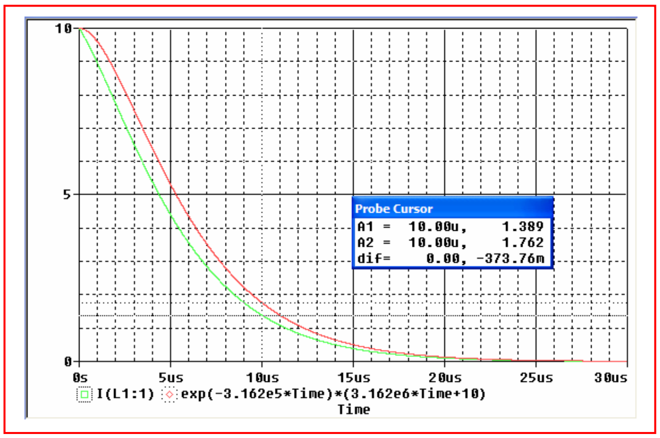
and also $L\left.\frac{di_L}{dt}\right|_{t=0} = v_C(0) = 0$ [3], so

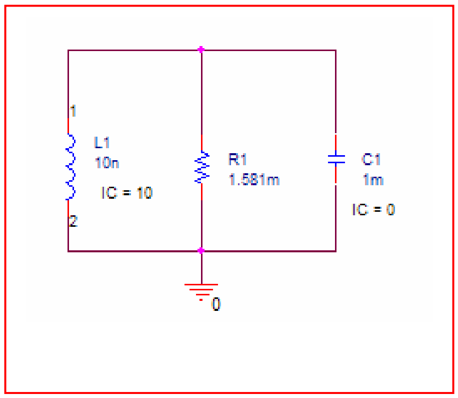
Solving Eqs. [2] and [3],

$A_1 = \left(3.162\times10^{5}\right)(10) = 3.162\times10^{6}\ \text{V}$, so we may write

$$\boxed{i_L(t) = e^{-3.162\times10^{5}t}\left(3.162\times10^{6}t + 10\right).}$$

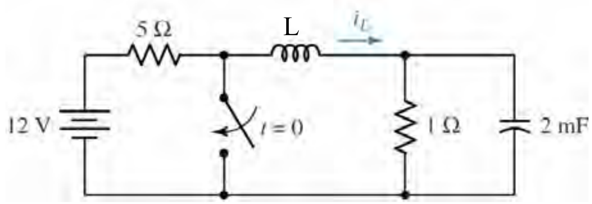
(c) We see from plotting both the analytic result in Probe and the simulated voltage, the two are in reasonable agreement (some numerical error is evident).





26. It is unlikely to observe a critically damped response in real-life circuits, as it would be virtually impossible to obtain the exact values required for R, L and C. However, using carefully chosen components, it is possible to obtain a response which is for all intents and purposes very close to a critically damped response.

27.

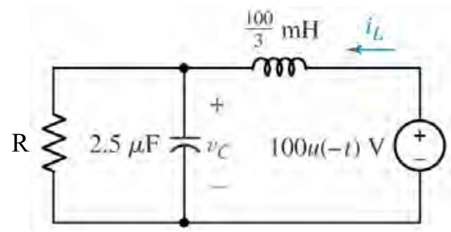


crit. damp. (a) $\mathrm{L} = 4\mathrm{R}^2\mathrm{C} = 4\times1\times2\times10^{-3} = \boxed{8\mathrm{mH}}$

(b)
$$\alpha = \omega_o \frac{1}{2\mathrm{RC}} = \frac{1000}{2\times1\times2} = 250 \therefore i_L = e^{-250t}(\mathrm{A}_1 t + \mathrm{A}_2)$$
$$i_L(0) = 2\mathrm{A},\ v_c(0) = 2\mathrm{V} \therefore i_L = e^{-250t}(\mathrm{A}_1 t + 2)$$
$$\text{Then } 8\times10^{-3}\, i_L'(0^+) = -2 = 8\times10^{-3}(\mathrm{A}_1 - 500),\ = e^{-1.25}\ (1.25+2) = \boxed{0.9311\mathrm{A}}$$

(c)
$$i_{L\max} : (250t_m + 2) = 0,\ 1 = 250t_m + 2,\ t_m < 0\ \text{No!}$$
$$\therefore t_m = 0,\ i_{L\max} = 2\mathrm{A} \therefore 0.02 = e^{-250t_s}(250t_s + 2);\ \text{SOLVE: } t_s = \boxed{23.96\mathrm{ms}}$$

28.



crit. damp. (a) $L = 4R^2C = \frac{100}{3} \times 10^{-3} = 4R^2 \times 10^{-6} \therefore \boxed{R = 57.74\Omega}$

(b)

$$\omega_o = \alpha = 10^3 / \sqrt{\frac{1}{30} \times 2.5} = 3464 s^{-1}$$

$$\therefore v_c(t) = e^{-3464t}(A_1 t + A_2)\ v_c(0) = 100\text{V}$$

$$i_L(0) = \frac{100}{57.74} = 1.7321\text{A} \therefore 100 = A_2$$

$$v_c'(0^+) = \frac{10^6}{2.5}\left(1.7321 - \frac{100}{57.74}\right) = 0 = A_1 - 3464A_2 \therefore A_1 = 3.464 \times 10^5$$

$$\therefore v_c(t) = \boxed{e^{-3464t}(3.464 \times 10^5 t + 100)\,\text{V},\ t > 0}$$

29. Diameter of a dime is approximately 8 mm. The area, therefore, is $\pi r^2 = 0.5027\ \text{cm}^2$.

The capacitance is $\dfrac{\varepsilon_r \varepsilon_o A}{d} = \dfrac{(88)(8.854\times 10^{-14})(0.5027)}{0.1}$

$$= 39.17\ \text{pF}$$

w ith $L = 4\mu\text{H},\ \omega_o = \dfrac{1}{\sqrt{LC}} = 79.89\ \text{Mrad/s}$

For critical damping, we require $\dfrac{1}{2RC} = \omega_o$

or $R = \dfrac{1}{2\omega_o C} = \boxed{159.8\,\Omega}$

30. $L = 5\text{mH},\ C = 10^{-8}\ \text{F, crit. damp. } v(0) = -400\text{V},\ i(0) = 0.1\text{A}$

(a) $L = 4R^2C = 5\times10^{-3} = 4R^2\,10^{-8}\ \therefore R = \boxed{353.6\Omega}$

(b) $\alpha = \dfrac{10^8}{2\times353.6} = 141,420\ \therefore i = e^{-141,420t}(A_1 t + A_2)$

$\therefore A_2 = 0.1 \therefore = e^{-141,421t}(A_1 t + 0.1),\ 5\times10^{-3}$

$(A_1 - 141,420\times0.1) = -400\ \therefore A_1 = -65,860$

$\therefore i = e^{-141,421t}(-65,860t + 0.1).\ i' = 0$

$\therefore e^{-\alpha t}(+65860) + 141,420e^{-\alpha t}(-65,860t_m + 0.1) = 0$

$\therefore t_m = 8.590\,\mu s \therefore i(t_m) = e^{-141,420\times8.590\times10^{-6}}$

$(-65,860\times8.590\times10^{-6} + 0.1) = -0.13821\text{A}$

$\therefore |\text{i}|_{\max} = |i(t_m)| = \boxed{0.13821\text{A}}$

(c) $\therefore i_{\max} = i(0) = \boxed{0.1\text{A}}$

31. Critically damped parallel RLC with $\alpha = 10^{-3}\ s^{-1},\ R = 1\,\text{M}\Omega$.

We know $\frac{1}{2RC} = 10^{-3}$, so $C = \frac{10^3}{2\times 10^6} = 500\ \mu\text{F}$

Since α = ωo, $\omega_o = \frac{1}{\sqrt{LC}} = 10^{-3}$

or $\frac{1}{LC} = 10^{-6}$

so $L = 2$ GH (!)

$$L = \frac{\mu N^2 A}{S} = 2\times 10^9$$

If So $$\frac{(4\pi\times 10^{-7}\,\text{H/m})\left[\left(\frac{50\text{ turns}}{\text{cm}}\right).s\right]^2 (0.5\,\text{cm})^2.\pi.\left(\frac{1\text{ m}}{100\text{ cm}}\right)}{s} = 2\times 10^9$$

$$(4\pi^2\times 10^{-9})(50)^2(0.5)^2\, s = 2\times 10^9$$

So $\boxed{s = 8.106\times 10^{13}\,\text{cm}}$

32.

$$\alpha = \frac{1}{2\text{RC}} = \frac{4}{2\times 2} = 1,\ \omega_o^2 = \frac{1}{\text{LC}} = \frac{4\times 13}{2} = 26,\ \omega_d = \sqrt{26-1} = 5$$

$$\therefore v_c(t) = e^{-t}(B_1 \cos 5t + B_2 \sin 5t)$$

(a) $i_L(0^+) = i_L(0) = \boxed{4\text{A}}$

(b) $v_c(0^+) = v_c(0) = \boxed{0}$

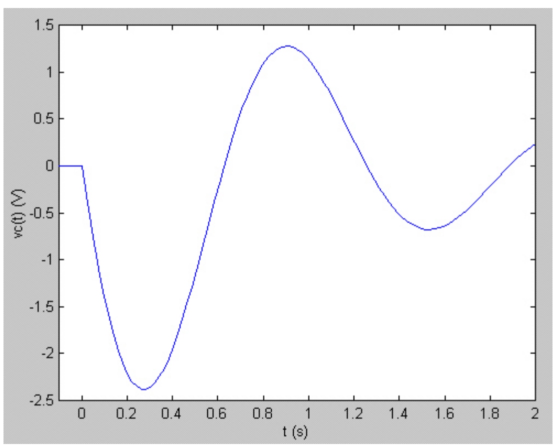
(c) $i_L'(0^+) = \dfrac{1}{\text{L}} v_c(0^+) = \boxed{0}$

(d) $v_c'(0^+) = \dfrac{1}{c}[-i_L(0^+) - i_R(0^+)] = 4\left[-4 - \dfrac{v_c(0^+)}{2}\right] = 4(-4+0) = \boxed{-16\ \text{V/s}}$

(e) $\therefore (e)\, 0 = 1(\text{B}_1) \therefore \text{B}_1 = 0,\ v_c(t) = \text{B}_2 e^{-t}\sin 5t,\ v_c'(0^+) = \text{B}_2(5) = -16$

$\therefore \text{B}_2 = -3.2,\ v_c(t) = \boxed{-3.2e^{-t}\sin 5t\,\text{V},\ t>0}$

(f)



33.

$$\alpha = \frac{1}{2RC} = \frac{10^6}{100\times 2.5} = 4000,\ \omega_o^2 = \frac{1}{LC} = \frac{10^{6+3}}{50} = 2\times 10^7$$

$$\omega_d = \sqrt{20\times 10^6 - 16\times 10^6} = 2000$$

$$\therefore i_c = e^{-4000t}(B_1 \cos 2000t + B_2 \sin 2000t)$$

$$i_L(0) = 2A,\ v_c(0) = 0 \therefore i_c(0^+) = -2A;\ i_c'(0^+) = -i_L'(0^+) - i_R'(0^+)$$

$$\therefore i_c'(0^+) = -\frac{1}{L}v_c(0) - \frac{1}{R}v_c'(0^+) = 0 - \frac{1}{RC}i_c(0^+) = \frac{2\times 10^6}{125}$$

$$\therefore B_1 = -2A,\ \frac{2\times 10^6}{125} = 16,000 = 2000B_2 + (-2)(-4000) \therefore B_2 = 4$$

$$\therefore \boxed{i_c(t) = e^{-4000t}(-2\cos 2000t + 4\sin 2000t)A,\ t>0}$$

34.

(a) $\alpha = \dfrac{1}{2RC} = \dfrac{100}{12.5} = 8,\ \omega_o^2 = \dfrac{1}{LC} = \dfrac{100}{L},\ \omega_d^2 = 36 = \omega_o^2 - 64$

$\therefore \omega_o^2 = 100 = \dfrac{100}{L} \therefore \boxed{L = 1\text{H}}$

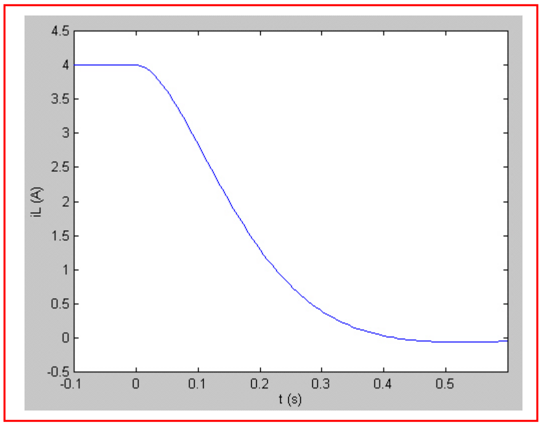
(b) $t<0:\ i_L(t) = 4\text{A};\ t>0:\ i_L(t) = e^{-8t}(\text{B}_1\cos 6t + \text{B}_2 \sin 6t)$

$i_L(0) = 4\text{A} \therefore \text{B}_1 = 4\text{A},\ i_L = e^{-8t}(4\cos 6t + \text{B}_2\sin 6t)\ v_c(0) = 0$

$i_L'(0^+) = t\ v_c(0^+) = 0 \therefore 6\text{B}_2 - 8(4) = 0,\ \text{B}_2 = 16/3$

$\therefore i_L(t) = \boxed{4u(-t) + e^{-8t}(4\cos 6t + 5.333\sin 6t)\ u(t)\ \text{A}}$

(c)



35.

(a) $$\alpha \frac{1}{2RC} = \frac{10^{9-3}}{2\times 20\times 5} = 5000,\ \omega_o^2 = \frac{1}{LC} = \frac{10^9}{1.6\times 5} = 1.25\times 10^8$$

$$\omega_d = \sqrt{\omega_o^2 - \alpha^2} = \sqrt{125\times 10^6 - 25\times 10^6} = 10,000$$

$$\therefore v_c(t) = e^{-5000t}(\mathrm{B}_1 \cos 10^4 t + \mathrm{B}_2 \sin 10^4 t)$$

$$v_c(0) = 200\text{V},\ i_L(0) = 10\text{mA} \therefore v_c(t) = e^{-5000t}(200\cos 10^4 t + \mathrm{B}_2 \sin 10^4 t)$$

$$v_c'(0^+) = \frac{1}{c} i_c(0^+) = \frac{10^9}{5}\left[i_L(0) - \frac{v_c(0)}{20,000}\right]$$

$$= \frac{10^9}{5}\left(10^{-2} - \frac{200}{20,000}\right) = 0 = 10^4\,\mathrm{B}_2 - 200(5000)$$

$$\therefore \mathrm{B}_2 = 100\text{V} \therefore v_c(t) = \boxed{e^{-5000t}(200\cos 10^4 t + 100\sin 10^4 t)\ \text{V},\ t>0}$$

(b) $$i_{sw} = 10^{-2} - i_L,\ i_L = \frac{1}{\mathrm{R}} v_c + \mathrm{C} v_c'$$

$$v_c' = e^{-5000t}[10^4(-200\sin + 100\cos] - 5000(200\cos + 100\sin)]$$

$$= e^{-500t}[10^6(-2\sin - 0.5\cos)] = -2.5\times 10^6 e^{-5000t} \sin 10^4 t\ v/s$$

$$\therefore i_L = e^{-5000t}\left[\frac{1}{20,000}(200\cos + 100\sin) - 5\times 10^{-9} \times 2.5\times 10^6 e^{-5000t} \sin 10^4 t\right]$$

$$= e^{-5000t}(0.01\cos 10^4 t - 0.0075\sin 10^4 t)\,\text{A}$$

$$\therefore i_{sw} = \boxed{10 - e^{-5000t}(10\cos 10^4 t - 7.5\sin 10^4 t)\,\text{mA},\ t>0}$$

36.

(a)

$$\alpha = \frac{1}{2RC} = \frac{10^6}{2000\times 25} = 20,\ \omega_o^2 = \frac{1}{LC} = \frac{1.01\times 10^6}{25} = 40,400$$

$$\omega_d = \sqrt{\omega_o^2 - \alpha^2} = \sqrt{40,400 - 400} = 200$$

$$\therefore v = e^{-20t}\,(A_1 \cos 200t + A_2 \sin 200t)$$

$$v(0) = 10\text{V},\ i_L(0) = 9\text{mA} \therefore A_1 = 10\text{V}$$

$$\therefore v = e^{-20t}(10\cos 200t + A_2 \sin 200t)\ \text{V},\ t > 0$$

$$v'(0^+) = 200A_2 - 20\times 10 = 200(A_2 - 1) = \frac{1}{C} i_o(0^+)$$

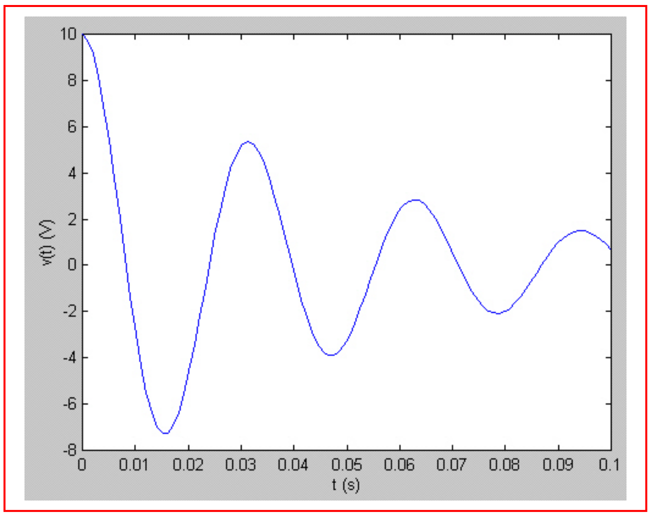
$$= \frac{10^6}{25}(-10^{-3}) = -40 \therefore A_2 = 1 - 0.2 = 0.8$$

$$\therefore \boxed{v(t) = e^{-20t}(10\cos 200t + 0.8\sin 200t)\ \text{V},\ t > 0}$$

(b)

$$v = 10.032e^{-20t}\cos\,(200t - 4.574°)\text{V}$$

$$T = \frac{2\pi}{200} = 3.42\text{ms}$$



37.

$$\alpha = \frac{1}{2RC} = \frac{10^{6-3}}{2\times 5} = 100s^{-1},\ \omega_o^2 = \frac{1}{LC} = 1.01\times 10^6$$

$$\therefore \omega_d = \sqrt{101\times 10^4 - 10^4} = 100;\ i_L(0) = \frac{60}{10} = 6\text{mA}$$

$$v_c(0) = 0 \therefore v_c(t) = e^{-100t}\,(\text{A}_1 \cos 1000t + \text{A}_2 \sin 1000t),\ t > 0$$

$$\therefore \text{A}_1 = 0,\ v_c(t) = \text{A}_2 e^{-100t} \sin 1000t$$

$$v_c'(0^+) = \frac{1}{C} i_c(0^+) = 10^6[-i_1(0^+) - \frac{1}{5000} v_c(0^+)] = 10^6$$

$$(-6\times 10^{-3}) = -6000 = 1000\,\text{A}_2 \therefore \text{A}_2 = -6$$

$$\therefore v_c(t) = -6e^{-100t} \sin 1000t\text{V},\ t > 0 \therefore i_1(t) = -\frac{1}{10^4}$$

$$v_c(t) = -10^{-4}(-6)\,e^{-100t} \sin 1000t\text{A}$$

$$\therefore \boxed{i_1(t) = 0.6e^{-100t} \sin 1000t\ \text{mA},\ t > 0}$$

38. We replace the 25-Ω resistor to obtain an underdamped response:

$$\alpha = \frac{1}{2RC} \quad \text{and} \quad \omega_0 = \frac{1}{\sqrt{LC}}; \text{ we require } \alpha < \omega_0.$$

Thus, $$\frac{1}{10 \times 10^{-6} R} < 3464 \quad \text{or} \quad \boxed{R > 34.64 \text{ m}\Omega.}$$

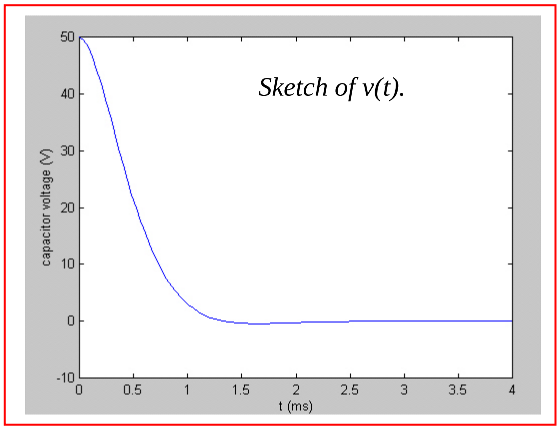
For R = 34.64 Ω (1000× the minimum required value), the response is:

$v(t) = e^{-\alpha t}(A \cos \omega_d t + B \sin \omega_d t)$ where $\alpha = 2887 \text{ s}^{-1}$ and $\omega_d = 1914$ rad/s.
$i_L(0^+) = i_L(0^-) = 0$ and $v_C(0^+) = v_C(0^-) = (2)(25) = 50$ V = A.

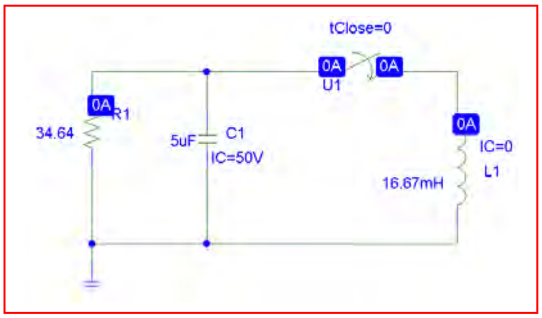
$$i_L(t) = L\frac{dv_L}{dt} = L\frac{dv_C}{dt}$$
$$= L\left[e^{-\alpha t}\left(-A\omega_d t \sin \omega_d t + B\omega_d t \cos \omega_d t\right) - \alpha e^{-\alpha t}\left(A\cos\omega_d t + B\sin\omega_d t\right)\right]$$

$$i_L(0^+) = 0 = \frac{50 \times 10^{-3}}{3}\left[B\,\omega_d - \alpha A\right], \text{ so that } B = 75.42 \text{ V}.$$

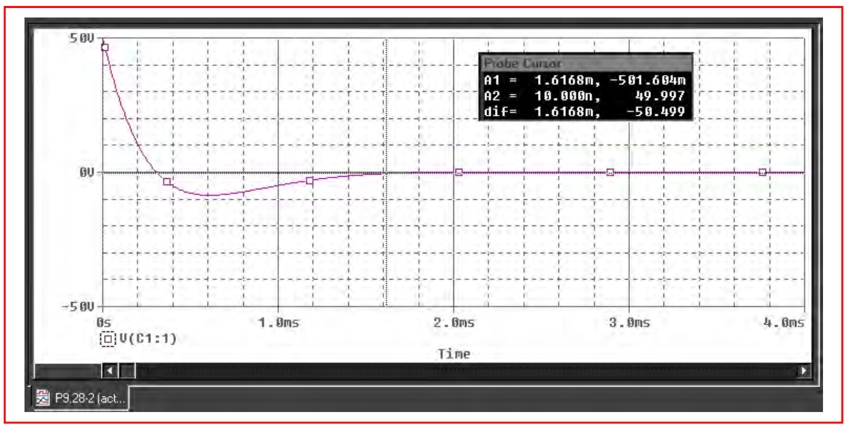
Thus, $$v(t) = e^{-2887t}(50 \cos 1914t + 75.42 \sin 1914t) \text{ V}.$$



PSpice schematic for t > 0 circuit.



From PSpice the settling time using R = 34.64 Ω is approximately 1.6 ms.



39.

$v(0) = 0;\ i(0) = 10\text{A}$

$v = e^{\alpha t}(\text{A}\cos\omega_d t + \text{B}\sin\omega_d t) \therefore \text{A} = 0,$

$v = \text{B}e^{-\alpha t}\sin\omega_d t$

$v' = e^{-\alpha t}\ [-\alpha\,\text{B}\sin\omega_d t + \omega_d \text{B}\cos\omega_d t] = 0$

$$\therefore \tan\omega_d t = \frac{\omega_d}{\alpha},\ t_{m1} = \frac{1}{\omega_d}\tan^{-1}\frac{\omega_d}{\alpha}$$

$$t_{m2} = t_{m1} + \frac{1}{2}\text{T}_d = t_{m1} + \frac{\pi}{\omega_d};$$

$v_{m1} = \text{B}e^{-\alpha t_{m1}}\sin\omega_d t_{m1}\ v_{m2} = -\text{B}e^{-\alpha t_{m1} - \alpha\pi/\omega_d}$

$$\sin\omega_d\ t_{m1} \therefore \frac{v_{m2}}{\text{V}_{m1}} = -e^{-\alpha\pi/\omega_d};\ \text{let}\ \left|\frac{v_{m2}}{v_{m1}}\right| = \frac{1}{100}$$

$$\therefore e^{\alpha\pi/\omega_d} = 100,\ \alpha = \frac{\omega_d}{\pi}\ell n\ 100;\ \alpha = \frac{1}{2\text{RC}} = \frac{21}{\text{R}},$$

$$\omega_0^2 = \frac{1}{\text{LC}} = 6 \therefore \omega_d = \sqrt{6 - 441/\text{R}^2} \therefore \frac{21}{\text{R}}\ \frac{\ell n 100}{\pi\text{R}}\ \sqrt{6\text{R}^2 - 441}$$

$$\therefore \text{R} = \sqrt{1/6\left[441 + \left(\frac{21\pi}{100}\right)^2\right]} = 10.3781\Omega \text{ To keep}$$

$$\left|\frac{v_{m2}}{v_{m1}}\right| < 0.01,\ \text{chose}\ \boxed{\text{R} = 10.3780\Omega}\ v'(0^+) = \omega_d$$

$$\text{B} = \text{B}\sqrt{6 - \left(\frac{21}{10.378}\right)^2} = 4\text{R}\left(10 + \frac{0}{10.3780}\right) \therefore \text{B} = 1.380363$$

$$\alpha = \frac{21}{10.378} = 2.02351;\ \omega_d = \sqrt{6 - \left(\frac{21}{10.378}\right)^2} = 1.380363$$

$\therefore v = 304.268e^{-2.02351t}\sin 1.380363t\ v\ t_{m1} = 0.434s,$

$v_{m1} = 71.2926v$ Computed values show

$\boxed{t_s = 2.145\,\text{sec}};\ v_{m2} = 0.7126 < 0.01 v_{m1}$

40. (a) For $t < 0$ s, we see from the circuit that the capacitor and the resistor are shorted by the presence of the inductor. Hence, $i_L(0^-) = 4$ A and $v_C(0^-) = 0$ V.

When the 4-A source turns off at $t = 0$ s, we are left with a parallel RLC circuit such that $\alpha = 1/2RC = 0.4\ s^{-1}$ and $\omega_0 = 5.099$ rad/s. Since $\alpha < \omega_0$, the response will be underdamped with $\omega_d = 5.083$ rad/s. Assume the form $i_L(t) = e^{-\alpha t}$ (C cos $\omega_d t$ + D sin $\omega_d t$) for the response.

With $i_L(0^+) = i_L(0^-) = 4$ A, we find C = 4 A. To find D, we first note that

$$v_C(t) = v_L(t) = L\,\frac{di_L}{dt}$$

and so $v_C(t)$ = (2/13) [$e^{-\alpha t}$ (-Cω_d sin $\omega_d t$ + Dω_d cos $\omega_d t$) - $\alpha e^{-\alpha t}$ (C cos $\omega_d t$ + D sin $\omega_d t$)]

With $v_C(0^+)$ = 0 = (2/13) (5.083D – 0.4C), we obtain D = 0.3148 A.

Thus, $i_L(t) = e^{-0.4t}$ (4 cos 5.083t + 0.3148 sin 5.083t) A and $\boxed{i_L(2.5) = 1.473 \text{ A.}}$

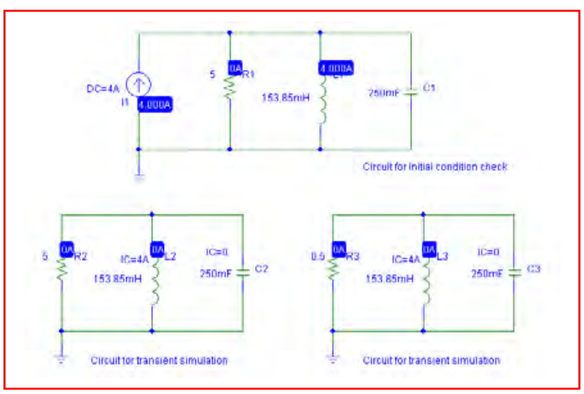
(b) $\alpha = 1/2RC = 4\ s^{-1}$ and $\omega_0 = 5.099$ rad/s. Since $\alpha < \omega_0$, the new response will still be underdamped, but with $\omega_d = 3.162$ rad/s. We still may write

$v_C(t)$ = (2/13) [$e^{-\alpha t}$ (-Cω_d sin $\omega_d t$ + Dω_d cos $\omega_d t$) - $\alpha e^{-\alpha t}$ (C cos $\omega_d t$ + D sin $\omega_d t$)]

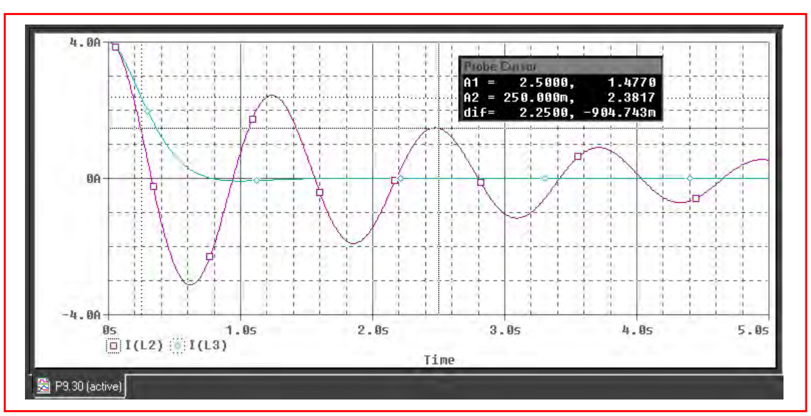
and so with $v_C(0^+)$ = 0 = (2/13) (3.162D – 4C), we obtain D = 5.06 A.

Thus, $i_L(t) = e^{-4t}$ (4 cos 3.162t + 5.06 sin 3.162t) A and $\boxed{i_L(.25) = 2.358 \text{ A.}}$

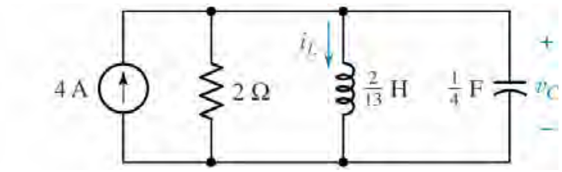
(c)



We see from the simulation result below that our hand calculations are correct; the slight disagreement is due to numerical inaccuracy. Changing the step ceiling from the 10-ms value employed to a smaller value will improve the accuracy.



41. (a,b) For $t < 0$ s, we see from the circuit below that the capacitor and the resistor are shorted by the presence of the inductor. Hence, $i_L(0^-) = 4$ A and $v_C(0^-) = 0$ V.



When the 4-A source turns off at $t = 0$ s, we are left with a parallel RLC circuit such that $\alpha = 1/2RC = 1\ s^{-1}$ and $\omega_0 = 5.099$ rad/s. Since $\alpha < \omega_0$, the response will be underdamped with $\omega_d = 5$ rad/s. Assume the form $i_L(t) = e^{-\alpha t}(C \cos \omega_d t + D \sin \omega_d t)$ for the response.

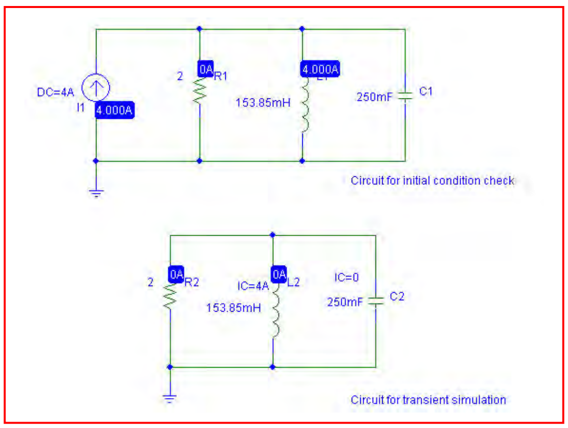
With $i_L(0^+) = i_L(0^-) = 4$ A, we find C = 4 A. To find D, we first note that

$$v_C(t) = v_L(t) = L \frac{di_L}{dt}$$

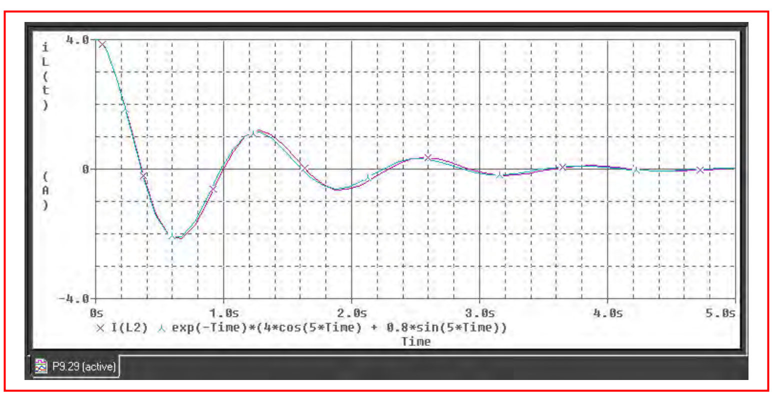
and so $v_C(t) = (2/13)\left[e^{-\alpha t}(-C\omega_d \sin \omega_d t + D\omega_d \cos \omega_d t) - \alpha e^{-\alpha t}(C \cos \omega_d t + D \sin \omega_d t)\right]$

With $v_C(0^+) = 0 = (2/13)(5D - 4)$, we obtain D = 0.8 A.

Thus, $i_L(t) = e^{-t}(4 \cos 5t + 0.8 \sin 5t)$ A



We see that the simulation result confirms our hand analysis; there is only a slight difference due to numerical error between the simulation result and our exact expression.



(c) Using the cursor tool, the settling time is approximately $\boxed{4.65\text{ s.}}$

Probe Cursor		
A1 =	4.6493,	-40.658m
A2 =	1.0000m,	3.9999
dif=	4.6483,	-4.0406

42.

$v_c(0) = 50 + 80 \times 2 = 210\,\text{V},\ i_L(0) = 0,\ \alpha = \dfrac{\text{R}}{2\text{L}} = \dfrac{80}{4} = 20$

$\omega_o^2 = \dfrac{100}{2} = 500 : \omega_d = \sqrt{500 - 20^2} = 10$

$\therefore v_c(t) = e^{-20t}(\text{A}_1 \cos 10t + \text{A}_2 \sin 10t) \therefore \text{A}_1 = 210\,\text{V}$

$\therefore v_c(t) = e^{-20t}(210 \cos 10t + \text{A}_2 \sin 10t);\ v_c'(0^+) = \dfrac{1}{\text{C}} i_c(0^+) = 0$

$\therefore 0 = 10\text{A}_2 - 20(210),\ \text{A}_2 = 420 \therefore v_c(t) = e^{-20t}(210 \cos 10t + 420 \sin 10t)$

$\therefore v_c(40\text{ms}) = e^{-0.8}(210 \cos 0.4 + 420 \sin 0.4) = \boxed{160.40\,\text{V}}$

Also, $(i_L = e^{-20t}\ \text{B}_1 \cos 10t + \text{B}_2 \sin 10t),$

$i_L(0^+) = \dfrac{1}{\text{L}} v_L(0^+) = \dfrac{1}{2}[0 - v_c(0^+)] = \dfrac{1}{2} \times 210$

$\therefore i_L'(0^+) = -105 = 10\text{B}_2 \therefore \text{B}_2 = 10.5$

$\therefore i_L(t) = -10.5 e^{-20t} \sin 10t\,\text{A},\ t > 0$

$\therefore v_R(t) = 80 i_L = 840 e^{-20t} \sin 10t\text{V}$

$\therefore v_R(40\text{ms}) = -840 e^{-0.8} \sin 0.4 = \boxed{-146.98\,\text{V}}$

$v_L(t) = -v_c(t) - v_c(t) - v_R(t) \therefore v_L$

$(40\text{ms}) = -160.40 + 146.98 = \boxed{-13.420\,\text{V}}$

[check: $v_L = e^{-20t}(-210 \cos - 420 \sin + 840 \sin)$

$= e^{-20t}(-210 \cos 10t + 420 \sin 10t)\,\text{V},\ t > 0$

$\therefore v_L(40\text{ms}) = e^{-0.8}(-210 \cos - 420 \sin + 840 \sin) = e^{-20t}$

$(-210 \cos 10t + 420 \sin 10t)\text{V},\ t > 0$

$\therefore \text{V}_L(40\text{ms}) = e^{-0.8}$

$(420 \sin 0.4 - 210 \cos 0.4) = -13.420\,\text{V}$ Checks]

43. Series:

$$\alpha = \frac{R}{2\text{L}} = \frac{2}{1/2} = 4,\ \omega_o^2 = \frac{1}{\text{LC}} = \frac{4}{0.2} = 20,\ \omega_d = \sqrt{20-16} = 2$$

$$\therefore i_L = e^{-4t}(\text{A}_1 \cos 2t + \text{A}_2 \sin 2t);\ i_L(0) = 10\text{A},\ v_c(0) = 20\text{V}$$

$$\therefore \text{A}_1 = 10;\ i_L'(0^+) = \frac{1}{\text{L}} v_L(0^+) = 4(20-20) = 0$$

$$\therefore i_L'(0^+) = 2\text{A}_2 - 4\times 10 \therefore \text{A}_2 = 20$$

$$\therefore \boxed{i_L(t) = e^{-4t}(10\cos 2t + 20\sin 2t)\text{A},\ t>0}$$

44. (a) crit. damp; $\alpha^2 = \dfrac{R^2}{4L^2} = \omega_o^2 = \dfrac{1}{LC} \therefore L = \dfrac{1}{4}R^2C$

$\therefore L = \dfrac{1}{4} \times 4 \times 10^{4-6} = 0.01\text{H},\ \alpha = \dfrac{200}{0.02} = 10^4 = \omega_o$

$\therefore v_c(t) = e^{-10000t}(A_1 t + A_2);\ v_c(0) = -10\text{V},\ i_L(0) = -0.15\text{A}$

$\therefore A_2 = -10,\ v_c(t) = e^{-10000t}(A_1 t - 0);\ v_c'(0^+) = -\dfrac{1}{C}$

$i_L(0) = -10^6(-0.15) = 150{,}000$

Now, $(v_c'\ 0^+) = A_1 + 10^5 = 150{,}000 \therefore A_1 = 50{,}000$

$\therefore \boxed{v_c(t) = e^{-10{,}000t}(50{,}000t - 10)\ \text{V}, t > 0}$

(b) $v_c'(t) = e^{-10{,}000t}[50{,}000 - 10{,}000(50{,}000t - 10)] = \therefore$

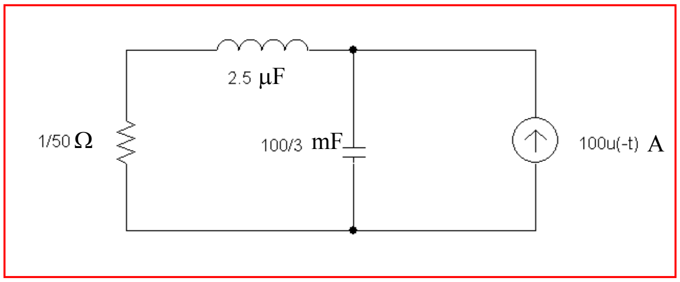
$5 = 50{,}000t_m - 10 \therefore t_m = \dfrac{15}{50{,}000} = 0.3\text{ms}$

$\therefore v_c(t_m) = e^{-3}(15 - 10) = 5e^{-3} = 0.2489\text{V}$

$v_c(0) = -10\text{V} \therefore |v_c|_{\max} = \boxed{10\text{V}}$

(c) $\boxed{v_{c,\max} = 0.2489\text{V}}$

45. "Obtain an expression for $v_c(t)$ in the circuit of Fig. 9.8 (dual) that is valid for all t".



$$\alpha = \frac{R}{2L} = \frac{0.02\times10^6}{2\times2.5} = 4000,\ \omega_o^2 = \frac{10^6\times3}{2.5\times10} = 1.2\times10^7$$

$$\therefore s_{1,2} = -4000 \pm \sqrt{16\times10^6 - 12\times10^6} = -2000, -6000$$

$$\therefore v_c(t) = A_1e^{-2000t} + A_2e^{-6000t};\ v_c(0) = \frac{1}{50}\times100 = 2\text{V}$$

$$i_L(0) = 100\text{A} \therefore 2 = A_1 + A_2, v_c'(0^+) = \frac{1}{C}$$

$$(-i_L(0)) = -\frac{3}{100}\times10^3\times100 = -3000v/s$$

$$\therefore -3000 = -200A_1 - 600A_2, -1.5 = -A_1 - 3A_2$$

$$\therefore 0.5 = -2A_2, = -0.25,\ A_1 = 2.25$$

$$\therefore \boxed{v_c(t) = (2.25e^{-200t} - 0.25e^{-6000t})\ u(t) + 2u(-t)\ \text{V}}\ \text{(checks)}$$

46. (a)

$$\alpha = \frac{R}{2L} = \frac{2}{2} = 1,\ \omega_o^2 = \frac{1}{LC} = 5,\ \omega_d = \sqrt{\omega_o^2 - \alpha^2} = 2$$

$$\therefore i_L = e^{-t}(B_1 \cos 2t + B_2 \sin 2t),\ i_L(0) = 0,\ v_c(0) = 10V$$

$$\therefore B_1 = 0,\ i_L = B_2\, e^{-t} \sin 2t$$

$$i_1(0) = \frac{1}{1} v_L(0^+) = v_R(0^+) - V_c(0^+) = 0 - 10 = 2B_2$$

$$\therefore \boxed{B_2 = 5 \therefore i_L = -5e^{-t} \sin 2t\text{A},\ t>0}$$

(b)

$$i_L' = -5[e^{-t}(2\cos 2t - \sin 2t)] = 0$$

$$\therefore 2\cos 2t = \sin 2t, \tan 2t = 2$$

$$\therefore t_1 = 0.5536s,\ i_L(t_1) = -2.571\text{A}$$

$$2t_2 = 2 \times 0.5536 + \pi,\ t_2 = 2.124,$$

$$i_L(t_2) = 0.5345 \therefore |i_L|_{\max} = 2.571\text{A}$$

and $0.5345\,\boxed{\text{A}}$

47. (a)

$$\alpha = \frac{\mathrm{R}}{2\mathrm{L}} = \frac{250}{10} = 25,\ \omega_o^2 = \frac{1}{\mathrm{LC}} = \frac{10^6}{2500} = 400$$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -25 \pm 15 = -10, -40$$

$$\therefore i_L = \mathrm{A}_1 e^{-10t} + \mathrm{A}_2 e^{-40t},\ i_L(0) = 0.5\mathrm{A},\ v_c(0) = 100\mathrm{V}$$

$$\therefore 0.5 = \mathrm{A}_1 + \mathrm{A}_2,\ i_L'(0^+) = \frac{1}{5} v_L(0^+) = \frac{1}{5}$$

$$(100 - 25 - 100) = -5\ \mathrm{A}/s = -10\mathrm{A}_1 - 40\mathrm{A}_2$$

$$\therefore 5 = 10\,\mathrm{A}_1 + 40\ (0.5 - \mathrm{A}_1) = 10\mathrm{A}_1 - 40$$

$$\mathrm{A}_1 + 20 \therefore -30\mathrm{A}_1 = -15,\ \mathrm{A}_1 = 0.5,\ \mathrm{A}_2 = 0$$

$$\therefore i_L(t) = \boxed{0.5e^{-10t}\mathrm{A},\ t > 0}$$

(b)

$$v_c = \mathrm{A}_3 e^{-10t} + \mathrm{A}_4 e^{-40t} \therefore 100 = \mathrm{A}_3 + \mathrm{A}_4;$$

$$v_c' = \frac{1}{c} i_c'(0^+) \frac{10^6}{500}(-0.5) = -1000$$

$$\therefore -10\mathrm{A}_3 - 40\mathrm{A}_4 = -1000 \therefore -3\mathrm{A}_4 = 0,\ \mathrm{A}_4 = 0,\ \mathrm{A}_3 = 100$$

$$\therefore v_c(t) = \boxed{100e^{-10t}\mathrm{V}\ t > 0}$$

48. Considering the circuit as it exists for $t < 0$, we conclude that $v_C(0^-) = 0$ and $i_L(0^-) = 9/4 = 2.25$ A. For $t > 0$, we are left with a parallel RLC circuit having $\alpha = 1/2RC = 0.25\ s^{-1}$ and $\omega_o = 1/\sqrt{LC} = 0.3333$ rad/s. Thus, we expect an underdamped response with $\omega_d = 0.2205$ rad/s:

$$i_L(t) = e^{-\alpha t}(A \cos \omega_d t + B \sin \omega_d t)$$

$$i_L(0^+) = i_L(0^-) = 2.25 = A$$

so $i_L(t) = e^{-0.25t}(2.25 \cos 0.2205t + B \sin 0.2205t)$

In order to determine B, we must invoke the remaining boundary condition. Noting that

$v_C(t) = v_L(t) = L\dfrac{di_L}{dt}$

$= (9)(-0.25)e^{-0.25t}(2.25 \cos 0.2205t + B \sin 0.2205t)$
$\quad + (9)\, e^{-0.25t}[-2.25(0.2205) \sin 0.2205t + 0.2205B \cos 0.2205t]$

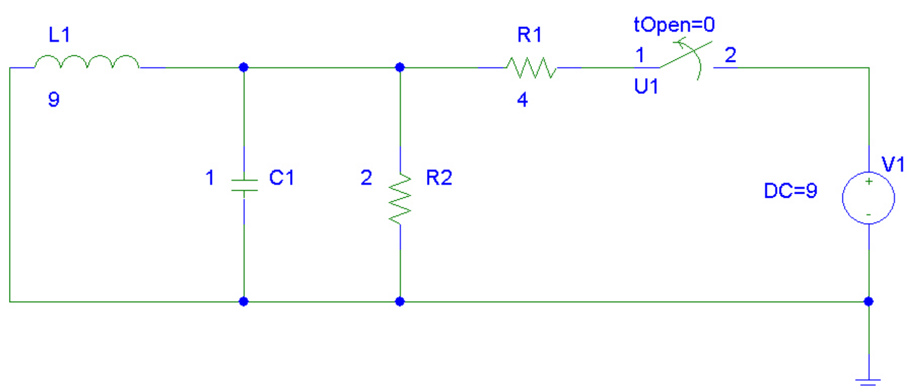
$v_C(0^+) = v_C(0^-) = 0 = (9)(-0.25)(2.25) + (9)(0.2205B)$

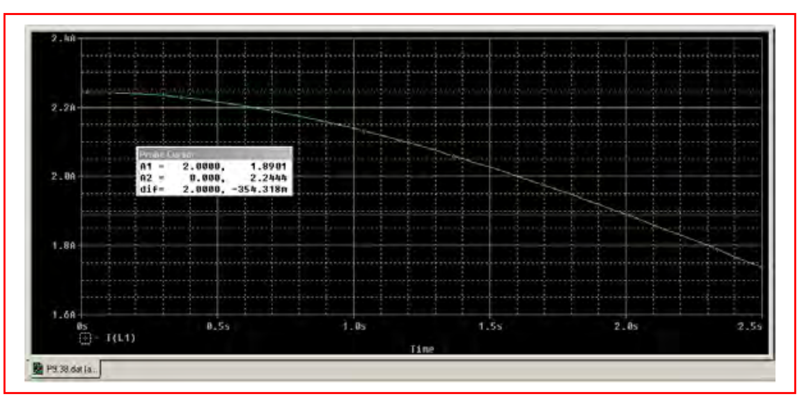
so B = 2.551 and

$$i_L(t) = e^{-0.25t}[2.25 \cos 0.2205t + 2.551 \sin 0.2205t] \text{ A}$$

Thus, $i_L(2) = 1.895$ A

This answer is borne out by PSpice simulation:





49. We are presented with a series RLC circuit having $\alpha = R/2L = 4700\ s^{-1}$ and $\omega_0 = 1/\sqrt{LC} = 447.2$ rad/s; therefore we expect an overdamped response with $s_1 = -21.32\ s^{-1}$ and $s_2 = -9379\ s^{-1}$.

From the circuit as it exists for $t < 0$, it is evident that $i_L(0^-) = 0$ and $v_C(0^-) = 4.7$ kV

Thus, $$v_L(t) = A\,e^{-21.32t} + B\,e^{-9379t} \qquad [1]$$

With $i_L(0^+) = i_L(0^-) = 0$ and $i_R(0^+) = 0$ we conclude that $v_R(0^+) = 0$; this leads to $v_L(0^+) = -v_C(0^-) = -4.7$ kV and hence A + B = -4700 [2]

Since $v_L = L\dfrac{di}{dt}$, we may integrate Eq. [1] to find an expression for the inductor current:

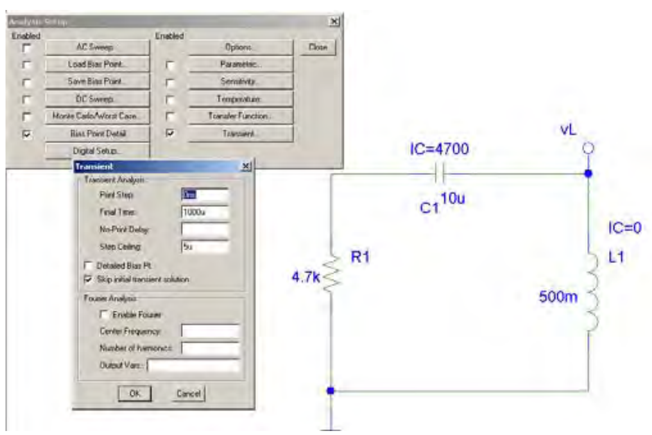
$$i_L(t) = \frac{1}{L}\left[-\frac{A}{21.32}e^{-21.32t} - \frac{B}{9379}e^{-9379t}\right]$$

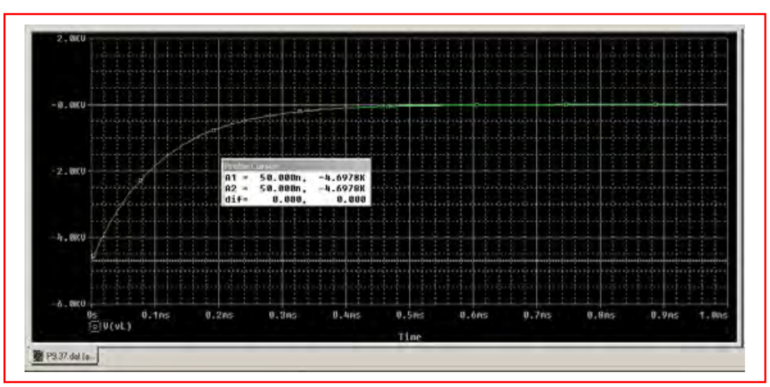
At $t = 0^+$, $i_L = 0$ so we have $\dfrac{1}{500\times10^{-3}}\left[-\dfrac{A}{21.32} - \dfrac{B}{9379}\right] = 0$ [3]

Simultaneous solution of Eqs. [2] and [3] yields A = 10.71 and B = -4711. Thus,

$$v_L(t) = 10.71e^{-21.32t} - 4711\,e^{-9379t}\ \text{V}, \quad t > 0$$

and the peak inductor voltage magnitude is 4700 V.





50. With the 144 mJ originally stored via a 12-V battery, we know that the capacitor has a value of 2 mF. The initial inductor current is zero, and the initial capacitor voltage is 12 V. We begin by seeking a (painful) current response of the form

$$i_{bear} = Ae^{s_1 t} + Be^{s_2 t}$$

Using our first initial condition, $i_{bear}(0^+) = i_L(0^+) = i_L(0^-) = 0 = A + B$

$$di/dt = As_1\, e^{s_1 t} + Bs_2\, e^{s_2 t}$$

$v_L = L di/dt = ALs_1\, e^{s_1 t} + BLs_2\, e^{s_2 t}$

$v_L(0^+) = ALs_1 + BLs_2 = v_C(0^+) = v_C(0^-) = 12$

What else is known? We know that the bear stops reacting at $t = 18$ μs, meaning that the current flowing through its fur coat has dropped just below 100 mA by then (not a long shock).

Thus, A $\exp[(18\times10^{-6})s_1]$ + B $\exp[(18\times10^{-6})s_2]$ $= 100\times10^{-3}$

Iterating, we find that $R_{bear} = \boxed{119.9775\ \Omega.}$

This corresponds to A = 100 mA, B = -100 mA, $s_1 = -4.167\ s^{-1}$ and $s_2 = -24\times10^6\ s^{-1}$

51. Considering the circuit at $t < 0$, we note that $i_L(0^-) = 9/4 = 2.25$ A and $v_C(0^-) = 0$.

For a critically damped circuit, we require $\alpha = \omega_o$, or $\frac{1}{2RC} = \frac{1}{\sqrt{LC}}$, which, with L = 9 H and C = 1 F, leads to the requirement that $\boxed{R = 1.5\ \Omega}$ (so $\alpha = 0.3333\ s^{-1}$).

The inductor energy is given by $w_L = ½\ L\ [i_L(t)]^2$, so we seek an expression for $i_L(t)$:

$$i_L(t) = e^{-\alpha t}\,(At + B)$$

Noting that $i_L(0^+) = i_L(0^-) = 2.25$, we see that B = 2.25 and hence

$$i_L(t) = e^{-0.3333t}\,(At + 2.25)$$

Invoking the remaining initial condition requires consideration of the voltage across the capacitor, which is equal in this case to the inductor voltage, given by:

$$v_C(t) = v_L(t) = L\frac{di_L}{dt} = 9(-0.3333)\,e^{-0.3333t}\,(At + 2.25) + 9A\,e^{-0.3333t}$$

$$v_C(0^+) = v_C(0^-) = 0 = 9(-0.333)(2.25) + 9A \text{ so } A = 0.7499 \text{ amperes and}$$

$$i_L(t) = e^{-0.3333t}\,(0.7499t + 2.25)\text{ A}$$

Thus, $i_L(100\text{ ms}) = 2.249$ A and so $w_L(100\text{ ms}) = \boxed{22.76\text{ J}}$

52. Prior to $t = 0$, we find that $v = (10 + i_1)\left(\frac{50}{15}\right)$ and $i_1 = \frac{v}{5}$

Thus, $v\left(1 - \frac{10}{15}\right) = \frac{500}{15}$ so $v = 100$ V.

Therefore, $v_C(0^+) = v_C(0^-) = 100$ V, and $i_L(0^+) = i_L(0^-) = 0$.

The circuit for $t > 0$ may be reduced to a simple series circuit consisting of a 2 mH inductor, 20 nF capacitor, and a 10 Ω resistor; the dependent source delivers exactly the current to the 5 Ω that is required.

Thus, $$\alpha = \frac{R}{2L} = \frac{10}{2\left(2\times10^{-3}\right)} = 2.5\times10^{3}\ \text{s}^{-1}$$

and $$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\left(2\times10^{-3}\right)\left(20\times10^{-9}\right)}} = 1.581\times10^{5}\ \text{rad/s}$$

With $\alpha < \omega_0$ we find the circuit is underdamped, with

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2} = 1.581\times10^{5}\ \text{rad/s}$$

We may therefore write the response as

$$i_L(t) = e^{-\alpha t}\left(B_1 \cos\omega_d t + B_2 \sin\omega_d t\right)$$

At $t = 0,\ i_L = 0 \quad \therefore B_1 = 0$.

Noting that $\frac{di_L}{dt} = \frac{d}{dt}\left(e^{-\alpha t} B_2 \sin\omega_d t\right) = B_2 e^{-\alpha t}\left(-\alpha \sin\omega_d t + \omega_d \cos\omega_d t\right)$ and

$L\left.\frac{di_L}{dt}\right|_{t=0} = -100$ we find that B_2 = -0.316 A.

Finally, $\boxed{i_L(t) = -316 e^{-2500t} \sin 1.581\times10^{5} t\ \text{mA}}$

53. Prior to $t = 0$, we find that $v_C = 100$ V, since 10 A flows through the 10 Ω resistor.

Therefore, $v_C(0^+) = v_C(0^-) = 100$ V, and $i_L(0^+) = i_L(0^-) = 0$.

The circuit for $t > 0$ may be reduced to a simple series circuit consisting of a 2 mH inductor, 20 nF capacitor, and a 10 Ω resistor; the dependent source delivers exactly the current to the 5 Ω that is required to maintain its current.

Thus, $$\alpha = \frac{R}{2L} = \frac{10}{2\left(2\times10^{-3}\right)} = 2.5\times10^{3}\ \text{s}^{-1}$$

and $$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\left(2\times10^{-3}\right)\left(20\times10^{-9}\right)}} = 1.581\times10^{5}\ \text{rad/s}$$

With $\alpha < \omega_0$ we find the circuit is underdamped, with

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2} = 1.581\times10^{5}\ \text{rad/s}$$

We may therefore write the response as

$$v_C(t) = e^{-\alpha t}\left(B_1 \cos\omega_d t + B_2 \sin\omega_d t\right)$$

At $t = 0,\ v_C = 100 \quad \therefore B_1 = 100$ V.

Noting that $C\dfrac{dv_C}{dt} = i_L$ and

$$\frac{d}{dt}\left[e^{-\alpha t}\left(100\cos\omega_d t + B_2 \sin\omega_d t\right)\right]$$

$$= e^{-\alpha t}\left[-\alpha\left(100\cos\omega_d t + B_2\sin\omega_d t\right) - 100\omega_d \sin\omega_d t + B_2\omega_d \cos\omega_d t\right]$$

which is equal to zero at $t = 0$ (since $i_L = 0$)

we find that $B_2 = 1.581$ V .

Finally, $$\boxed{v_C(t) = e^{-2500t}\left[100\cos\left(1.581\times10^{5}t\right) + 1.581\sin\left(1.581\times10^{5}t\right)\right]\ \text{V}}$$

54. Prior to $t = 0$, $i_1 = 10/4 = 2.5$ A, $v = 7.5$ V, and $v_g = -5$ V.
Thus, $v_C(0^+) = v_C(0^-) = 7.5 + 5 = 12.5$ V and $i_L = 0$

After $t = 0$ we are left with a series RLC circuit where $i_1 = -\frac{i_L}{4}$. We may replace the dependent current source with a 0.5 Ω resistor. Thus, we have a series RLC circuit with R = 1.25 Ω, C = 1 F, and L = 3 H.

Thus, $$\alpha = \frac{R}{2L} = \frac{1.25}{6} = 0.208 \text{ s}^{-1}$$

and $$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{3}} = 577 \text{ mrad/s}$$

With $\alpha < \omega_0$ we find the circuit is underdamped, so that
$$\omega_d = \sqrt{\omega_0^2 - \alpha^2} = 538 \text{ mrad/s}$$

We may therefore write the response as
$$i_L(t) = e^{-\alpha t}\left(B_1 \cos \omega_d t + B_2 \sin \omega_d t\right)$$
At $t = 0,\ i_L = 0 \quad \therefore B_1 = 0$ A.

Noting that $L\left.\frac{di_L}{dt}\right|_{t=0} = -v_C(0)$ and

$$\frac{di_L}{dt} = \frac{d}{dt}\left[e^{-\alpha t}\left(B_2 \sin \omega_d t\right)\right] = B_2 e^{-\alpha t}\left[-\alpha \sin \omega_d t + \omega_d \cos \omega_d t\right] = \frac{v_C(t)}{L} = \frac{-12.5}{3} \quad (t = 0)$$
,
we find that $B_2 = -7.738$ V.

Finally, $$\boxed{i_L(t) = 1.935 e^{-0.208t} \sin 0.538t \text{ A for } t > 0 \text{ and } 2.5 \text{ A}, t < 0}$$

55. Prior to $t = 0$, $i_1 = 10/4 = 2.5$ A, $v = 7.5$ V, and $v_g = -5$ V.
Thus, $v_C(0^+) = v_C(0^-) = 12.5$ V and $i_L = 0$

After $t = 0$ we are left with a series RLC circuit where $i_1 = -\frac{i_L}{4}$. We may replace the dependent current source with a 0.5 Ω resistor. Thus, we have a series RLC circuit with R = 1.25 Ω, C = 1 mF, and L = 3 H.

Thus,
$$\alpha = \frac{R}{2L} = \frac{1.25}{6} = 0.208 \text{ s}^{-1}$$
and
$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\left(3\times10^{-3}\right)}} = 18.26 \text{ rad/s}$$

With $\alpha < \omega_0$ we find the circuit is underdamped, so that
$$\omega_d = \sqrt{\omega_0^2 - \alpha^2} = 18.26 \text{ rad/s}$$

We may therefore write the response as
$$v_C(t) = e^{-\alpha t}\left(B_1 \cos\omega_d t + B_2 \sin\omega_d t\right)$$
At $t = 0,\ v_C = 12.5 \quad \therefore B_1 = 12.5 \text{ V}$.

Noting that
$$\frac{dv_C}{dt} = \frac{d}{dt}\left[e^{-\alpha t}\left(B_1 \cos\omega_d t + B_2 \sin\omega_d t\right)\right]$$
$$= -\alpha e^{-\alpha t}\left[12.5\cos\omega_d t + B_2 \sin\omega_d t\right] + e^{-\alpha t}\left[-12.5\omega_d \sin\omega_d t + \omega_d B_2 \cos\omega_d t\right]$$
and this expression is equal to 0 at $t = 0$,
we find that $B_2 = 0.143$ V.

Finally,
$$\boxed{v_C(t) = e^{-0.208t}\left[12.5\cos 18.26t + 0.143\sin 18.26t\right] \text{ V for } t > 0 \text{ and } 12.5 \text{ V}, t < 0}$$

56. (a)

$$\text{Series, driven: } \alpha = \frac{\text{R}}{2\text{L}} = \frac{100}{0.2} = 500,$$

$$\omega_o^2 = \frac{1}{\text{LC}} = \frac{10\times10^6}{40} = 250,000$$

$$\therefore \text{Crit. damp } i_L(f) = 3(1-2) = -3,$$

$$i_L(0) = 3,\ v_c(0) = 300\text{V}$$

$$\therefore i_L = -3 + e^{-500t}\ (\text{A}_1 t + \text{A}_2) \therefore 3 = -3 + \text{A}_2,\ \text{A}_2 = 6\text{A}$$

$$i_L(0^+) = \text{A}_1 - 300 = \frac{1}{\text{L}}[v_c(0) - v_R(0^+)] = 0$$

$$\therefore \text{A}_1 = 3000\,e^{-5000t}\ \therefore i_L(t) = -3 + e^{-500t}$$

$$(3000t + 6),\ t > 0$$

$$\therefore i_L(t) = \boxed{3u(-t) + [-3 + e^{-500t}(3000t+6)]u(t)\text{A}}$$

(b) $e^{-500t_o}(3000t_o + 6) = 3$; by SOLVE, $t_o = \boxed{3.357\text{ms}}$

57.

$$v_c(0) = 0,\ i_L(0) = 0,\ \alpha = \frac{R}{2L} = \frac{2}{0.5} = 4,\ \omega_o^2 = \frac{1}{LC} = 4 \times 5 = 20$$

$$\therefore \omega_d = \sqrt{20 - 16} = 2 \therefore i_L(t) = e^{-4t}\,(A_1 \cos 2t + A_2 \sin 2t) + i_{L,f}$$

$$i_{L,f} = 10\text{A} \therefore i_L(t) = 10 + e^{-4t}(A_1 \cos 2t + A_2 \sin 2t)$$

$$\therefore 0 = 10 + A_1,\ A_1 = -10,\ i_L(t) = 10 + e^{-4t}(A_2 \sin 2t - 10 \cos 2t)$$

$$i_L(0^+) = \frac{1}{L} v_L(0^+) = 4 \times 0 = 0 \therefore i_L(0^+) = 0 = 2A_2 + 40,\ A_2 = -20$$

$$\boxed{i_L(t) = 10 - e^{-4t}\,(20 \sin 2t + 10 \cos 2t)\ \text{A},\ t > 0}$$

58.

$$\alpha = \frac{R}{2L} = \frac{250}{10} = 25,\ \omega_o^2 = \frac{1}{LC} = \frac{10^6}{2500} = 400$$

$$s_{1,2} = -25 \pm \sqrt{625-400} = -10, -40$$

$$i_L(0) = 0.5\text{A},\ v_c(0) = 100\text{V},\ i_{L,f} = -0.5\text{A}$$

$$\therefore i_L(t) = -0.5 + A_1 e^{-10t} + A_2 e^{-40t}\ \text{A}$$

$$t = 0^+:\ v_L(0^+) = 100 - 50\times 1 - 200\times 0.5 = -50\text{V} \therefore -50 = 5i_L'(0^+)$$

$$\therefore i_L'(0^+) = -10 \therefore -10 = -10A_1 - 40A_2,\ 0.5 = -0.5 + A_1 + A_2$$

$$\therefore A_1 + A_2 = 1 \therefore -10 = -10A_2 - 40(-1+A_1) = -50A_1 + 40,\ \ A_1 = 1, A_2 = 0$$

$$\therefore i_L(t) = \boxed{-0.5 + 1e^{-10t}\text{A},}\ t>0;\ i_L(t) = \boxed{0.5\text{A},\ t>0}$$

59.

$$\alpha = \frac{1}{2RC} = \frac{10^6}{100\times 2.5} = 4000,\ \omega_o^2 = \frac{1}{LC} = \frac{10^{6+3}}{50} = 20\times 10^6$$

$$\therefore \omega_d = \sqrt{\omega_o^2 - \alpha^2} = 2000,\ i_L(0) = 2\text{A},\ v_c(0) = 0$$

$$i_{c,f} = 0,\ (v_{c,f} = 0) \therefore i_c = e^{-400t}\ (\text{A}_1 \cos 2000t + \text{A}_2 \sin 2000t)$$

$$\text{work with } v_c:\ v_c(t) = e^{-4000t}(\text{B}_1 \cos 2000t + \text{B}_2 \sin 2000t) \therefore \text{B}_1 = 0$$

$$\therefore v_c = \text{B}_2 e^{-4000t} \sin 2000t,\ v_c'(0^+) = \frac{1}{\text{C}} i_c(0^+) = \frac{10^6}{2.5}(2\times 1) = 8\times 10^5$$

$$\therefore 8\times 10^5 = 2000\text{B}_2,\ \text{B}_2 = 400,\ v_c = 400e^{-4000t} \sin 2000t$$

$$\therefore i_c(t) = \text{C}v_c' = 2.5\times 10^{-6} \times 400e^{-4000t}\ (-4000 \sin 200^t + 2000 \cos 200t)$$

$$= 10^{-6+3+3} e^{-4000t}\ (-4 \sin 2000t + 2 \cos 2000t)$$

$$= \boxed{e^{-4000t}\ (2 \cos 2000t - 4 \sin 2000t)\,\text{A},\ t > 0}$$

60. (a)

$$\alpha = \frac{1}{2RC} = \frac{8\times10^6}{2\times4\times10^3} = 1000,\ \omega_o^2 = \frac{8\times10^6\times13}{4} = 26\times10^6$$

$$\therefore \omega_d = \sqrt{26-1}\times10^3 = 5000,\ v_c(0) = 8\text{V}$$

$$i_L(0) = 8\text{mA},\ v_{c,f} = 0$$

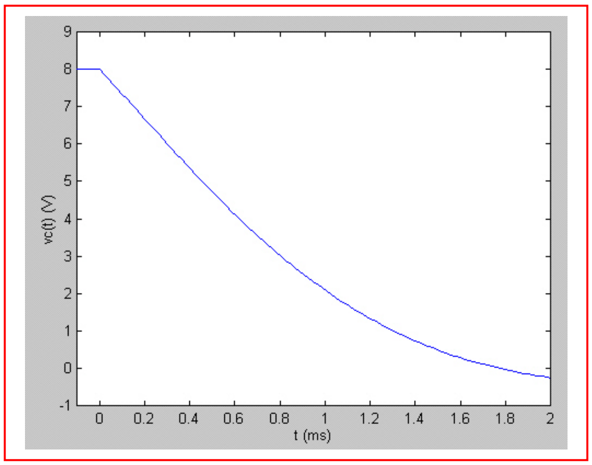
$$\therefore v_c = e^{-1000t}(\text{A}_1\cos1000t + \text{A}_2\sin5000t)$$

$$\therefore \text{A}_1 = 8;\ v_c'(0^+) = \frac{1}{\text{C}}i_c(0^+) = 8\times10^6(0.01 - \frac{8}{4000} - 0.008) = 0$$

$$\therefore 5000\text{A}_2 - 1000\times8 = 0,\ \text{A}_2 = 1.6$$

So $\boxed{v_c(t) = e^{-1000t}(8\cos 1000t + 1.6\sin 1000t)\text{ V},\ t > 0}$

(b)



61.

$$\alpha = \frac{R}{2L} = \frac{1}{1} = 1,\ \omega_o^2 = \frac{1}{LC} = 1 \therefore \text{crit. damp}$$

$$v_c(0) = \frac{5}{6} \times 12 = 10\text{V},\ i_L(0) = 2\text{A},\ v_{c,f} = 12\text{V}$$

$$\therefore v_c(t) = 12 + e^{-t}(\text{A}_1 t - 2);\ v_c'(0^+) = \frac{1}{\text{C}}\, i_c(0^+) = \frac{1}{2} \times i_L(0^+) = 1$$

$$\therefore 1 = \text{A}_1 + 2;\ \text{A}_1 = -1 \therefore \boxed{v_c(t) = 12 - e^{-t}(t+2)\,\text{V},\ t > 0}$$

62. (a)

$$v_s = 10u(-t)\ \text{V}: \alpha = \frac{1}{2\text{RC}} = \frac{10^6}{2000 \times 0.5} = 1000$$

$$\omega_o^2 = \frac{1}{\text{LC}} = \frac{2 \times 10^6 \times 3}{8} = 0.75 \times 10^6 \therefore s_{1,2} = -500,\ -1500$$

$$\therefore v_c = \text{A}_1 e^{-500t} + \text{A}_2 e^{-1500t},\ v_o(0) = 10\text{V},\ i_L(0) = 10\text{mA}$$

$$\therefore A_1 + \text{A}_2 = 10,\ v_c'(0^+) = 2 \times 10^6 [i_L(0) - i_R(0^+)] = 2 \times 10^6$$

$$\left(0.01 - \frac{10}{1000}\right) = 0 \therefore -500\text{A}_1 - 1500\,\text{A}_2 = 0,$$

$$-\text{A}_1 - 3\text{A}_2 = 0;\ \text{add:} -2\,\text{A}_2 = 10,\ \text{A}_2 = -5,\ \text{A}_1 = 15$$

$$\therefore v_c(t) = 15e^{-500t} - 5e^{-1500t}\ \text{V}\ t > 0$$

$$\therefore i_R(t) = \boxed{15e^{-500t} - 5e^{-1500t}\text{mA},\ t > 0}$$

(b)

$$v_s = 10u(t)\,\text{V},\ v_{c,f} = 10,\ v_c = 10 + \text{A}_3 e^{-500t} + \text{A}_4 e^{-1500t},$$

$$v_c(0) = 0,\ i_L(0) = 0 \therefore \text{A}_3 + \text{A}_4 = -10\text{V},\ v_c'(0^+) = 2 \times 10^6$$

$$[i_L(0) - i_R(0^+)] = 2 \times 10^6 (0 - 0) = 0 = -500\text{A}_3 - 1500\text{A}_4$$

$$\therefore -\text{A}_3 - 3\text{A}_4 = 0,\ \text{add:}\ -2\text{A}_4 = -10,\ \text{A}_4 = 5 \therefore \text{A}_3 = -15$$

$$\therefore v_c(t) = 10 - 15e^{-500t} + 5e^{-1500t}\text{V},\ t > 0$$

$$\therefore i_R(t) = \boxed{10 - 15e^{-500t} + 5e^{1500t}\text{mA},\ t > 0}$$

63. (a)

$$v_s(t) = 10u(-t)\,\text{V}: \alpha = \frac{1}{2\text{RC}} = \frac{10^6}{1000} = 1000$$

$$\omega_o^2 = \frac{1}{\text{LC}} = \frac{10^6 \times 3}{4} \therefore s_{1,2} = -1000 \pm \sqrt{10^6 - \frac{3}{4}\times 10^6} = -500,\ -1500$$

$$v_{c,f} = 0 \therefore v_c = \text{A}_1 e^{-500t} + \text{A}_2 e^{-1500t},\ v_c(0) = 10\text{V},\ i_L(0) = 0$$

$$\therefore 10 = \text{A}_1 + \text{A}_2,\ v_c' = 10^6\ i_c(0^+) = 10^6\left[0 - \frac{10}{500}\right] = -2\times 10^4$$

$$\therefore -2\times 10^4 = -500\text{A}_1 - 1500\text{A} \therefore 40 = \text{A}_1 + 3\text{A}_2 \therefore 30 = 2\text{A}_2,\ \text{A}_2 = 15,\ \text{A}_1 = -5$$

$$\therefore v_c = -5e^{-500t} + 15e^{-1500t}\text{V},\ t>0\ \therefore i_s = i_c = \text{C}v_c'$$

$$\therefore i_s = 10^{-6}(2500e^{-500t} - 22{,}500e^{-1500t})$$

$$= \boxed{2.5e^{-500t} - 22.5e^{-1500t}\text{mA},\ t>0}$$

(b)

$$v_s(t) = 10u(t)\,\text{V} \therefore v_{c,f} = 10\text{V},\ v_c(0) = 0,\ i_L(0) = 0$$

$$\therefore v_c = 10 + A_3 e^{-500t} + \text{A}_4 e^{-1500t} \therefore \text{A}_3 + \text{A}_4 = -10$$

$$v_c'(0^+) = 10^6 i_c(0^+) = 10^6\left(0 + \frac{10}{500}\right) = 2\times 10^4 = -500\,\text{A}_3 - 1500\,\text{A}_4$$

$$\therefore -\text{A}_3 - 3\text{A}_4 = 40,\ \text{add: } \cancel{2}\ \ \text{A}_4 = 30,\ \text{A}_4 = -15,\ \text{A}_3 = 5,$$

$$v_c = 10 + 5e^{-500t} - 15e^{-1500t}\text{V},\ i_s = i_c =$$

$$10^{-6}(-2500e^{-500t} + 22{,}500e^{-1500t}) = \boxed{25e^{-500t} + 22.5e^{-1500t}\ \text{mA},\ t>0}$$

64. Considering the circuit at $t < 0$, we see that $i_L(0^-) = 15$ A and $v_C(0^-) = 0$.
The circuit is a series RLC with $\alpha = R/2L = 0.375$ s^{-1} and $\omega_0 = 1.768$ rad/s. We therefore expect an underdamped response with $\omega_d = 1.728$ rad/s. The general form of the response will be

$$v_C(t) \;=\; e^{-\alpha t}\,(A \cos \omega_d t + B \sin \omega_d t) + 0 \qquad\qquad (v_C(\infty) = 0)$$

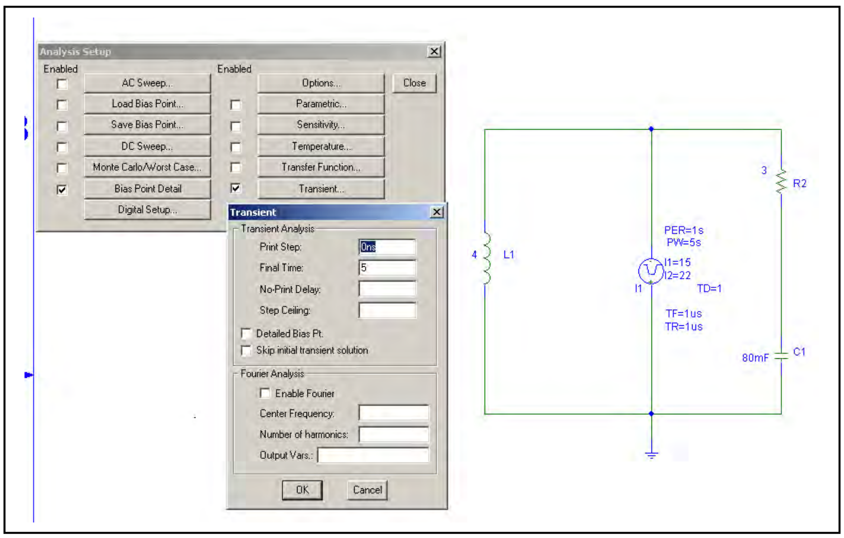
$v_C(0^+) = v_C(0^-) = 0 = A$ and we may therefore write $v_C(t) = Be^{-0.375t} \sin (1.728t)$ V

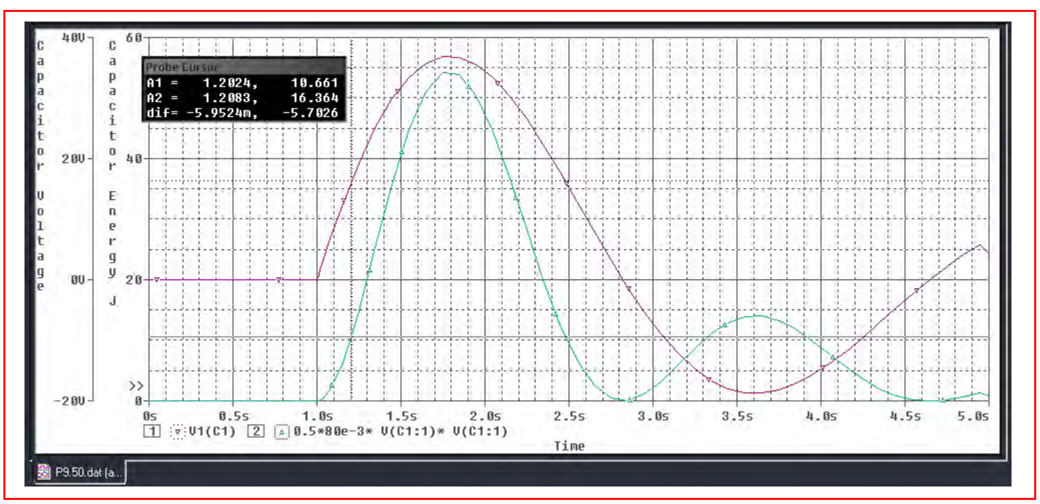
$$i_C(t) = -i_L(t) = C\frac{dv_C}{dt} = (80\times10^{-3})(-0.375B\, e^{-0.375t} \sin 1.728t$$

At $\quad t = 0^+$, $i_C = 15 + 7 - i_L(0^+) = 7 = (80\times10^{-3})(1.728B)$ so that B = 50.64 V.

Thus, $\quad v_C(t) = 50.64\, e^{-0.375t} \sin 1.807t$ V and $v_C(t = 200 \text{ ms}) = 16.61$ V.

The energy stored in the capacitor at that instant is ½ Cv_C^2 = $\boxed{11.04 \text{ J}}$





65. (a) $v_S(0^-) = v_C(0^-) = 2(15) = \boxed{30\text{ V}}$

(b) $i_L(0^+) = i_L(0^-) = 15$ A

Thus, $i_C(0^+) = 22 - 15 = 7$ A and $v_S(0^+) = 3(7) + v_C(0^+) = \boxed{51\text{ V}}$

(c) As $t \to \infty$, the current through the inductor approaches 22 A, so $v_S(t\to \infty,) = \boxed{44\text{ A.}}$

(d) We are presented with a series RLC circuit having $\alpha = 5/2 = 2.5\text{ s}^{-1}$ and $\omega_o = 3.536$ rad/s. The natural response will therefore be underdamped with $\omega_d = 2.501$ rad/s.

$i_L(t) = 22 + e^{-\alpha t}(\text{A}\cos \omega_d t + \text{B}\sin \omega_d t)$
$i_L(0^+) = i_L(0^-) = 15 = 22 + \text{A}$ so A = -7 amperes

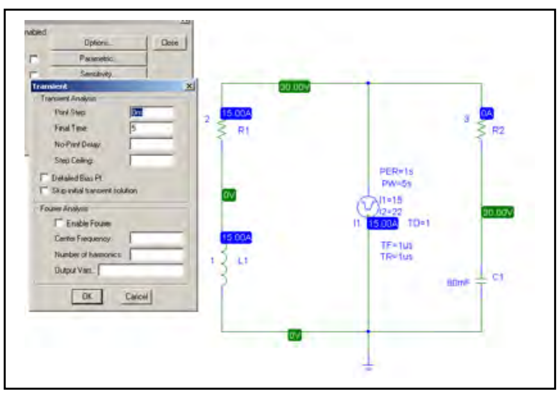
Thus, $i_L(t) = 22 + e^{-2.5t}(-7\cos 2.501\text{t} + \text{B}\sin 2.501\text{t})$

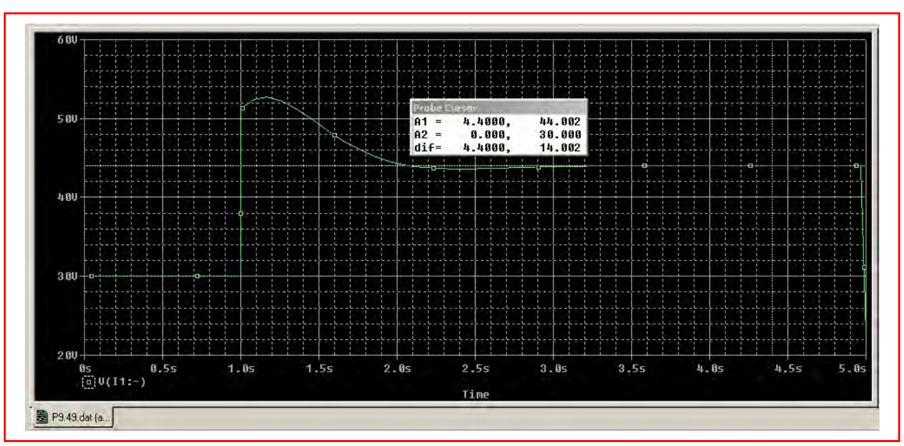
$$v_S(t) = 2\,i_L(t) + L\frac{di_L}{dt} = 2i_L + \frac{di_L}{dt} = 44 + 2e^{-2.5t}(-7\cos 2.501t + \text{B}\sin 2.501t)$$
$$-\,2.5e^{-2.5t}(-7\cos 2.501t + \text{B}\sin 2.501t) + e^{-2.5t}[7(2.501)\sin 2.501t + 2.501\text{B}\cos 2.501t)]$$

$v_S(t) = 51 = 44 + 2(-7) - 2.5(-7) + 2.501\text{B}$ so B = 1.399 amperes and hence

$$v_S(t) = 44 + 2e^{-2.5t}(-7\cos 2.501t + 1.399\sin 2.501t)$$
$$-2.5e^{-2.5t}(-7\cos 2.501t + 1.399\sin 2.501t) + e^{-2.5t}[17.51\sin 2.501t + 3.499\cos 2.501t)]$$

and $v_S(t)$ at $t = 3.4$ s = $\boxed{44.002\text{ V.}}$ This is borne out by PSpice simulation:





66. For $t < 0$, we have 15 A dc flowing, so that $i_L = 15$ A, $v_C = 30$ V, $v_{3\Omega} = 0$ and $v_S = 30$ V. This is a series RLC circuit with $\alpha = R/2L = 2.5\ s^{-1}$ and $\omega_0 = 3.536$ rad/s. We therefore expect an underdamped response with $\omega_d = 2.501$ rad/s.

<u>$0 < t < 1$</u> $\qquad v_C(t) = e^{-\alpha t}(A \cos \omega_d t + B \sin \omega_d t)$

$v_C(0^+) = v_C(0^-) = 30 = A$ so we may write $v_C(t) = e^{-2.5t}(30 \cos 2.501t + B \sin 2.501t)$

$$\frac{dv_C}{dt} = -2.5e^{-2.5t}(30 \cos 2.501t + B \sin 2.501t) + e^{-2.5t}[-30(2.501)\sin 2.501t + 2.501B \cos 2.501t]$$

$$i_C(0^+) = C\left.\frac{dv_C}{dt}\right|_{t=0^+} = 80\times10^{-3}[-2.5(30) + 2.501B] = -i_L(0^+) = -i_L(0^-) = -15 \text{ so } B = -44.98 \text{ V}$$

Thus, $v_C(t) = e^{-2.5t}(30 \cos 2.501t - 44.98 \sin 2.501t)$ and

$i_C(t) = e^{-2.5t}(-15 \cos 2.501t + 2.994 \sin 2.501t)$.

Hence, $v_S(t) = 3\, i_C(t) + v_C(t) = e^{-2.5t}(-15 \cos 2.501t - 36 \sin 2.501t)$

Prior to switching, $v_C(t = 1) = -4.181$ V and $i_L(t = 1) = -i_C(t = 1) = -1.134$ A.

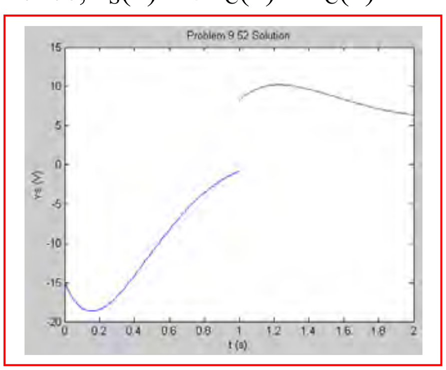
<u>$t > 2$:</u> Define $t' = t - 1$ for notational simplicity. Then, with the fact that $v_C(\infty) = 6$ V, our response will now be $v_C(t') = e^{-\alpha t'}(A' \cos \omega_d t' + B' \sin \omega_d t') + 6$.
With $v_C(0^+) = A' + 6 = -4.181$, we find that $A' = -10.18$ V.

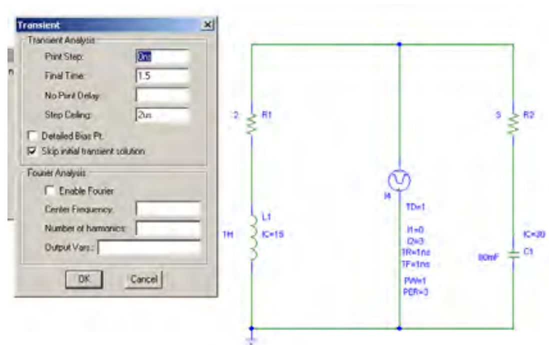
$$i_C(0^+) = C\left.\frac{dv_C}{dt'}\right|_{t'=0^+} = (80\times10^{-3})[(-2.5)(-10.18) + 2.501B')] = 3 - i_L(0^+) \text{ so } B' = 10.48 \text{ V}$$

Thus, $v_C(t') = e^{-2.5t}(-10.18 \cos 2.501t' + 10.48 \sin 2.501t')$ and

$i_C(t') = e^{-2.5t}(4.133 \cos 2.501t' - 0.05919 \sin 2.501t')$.

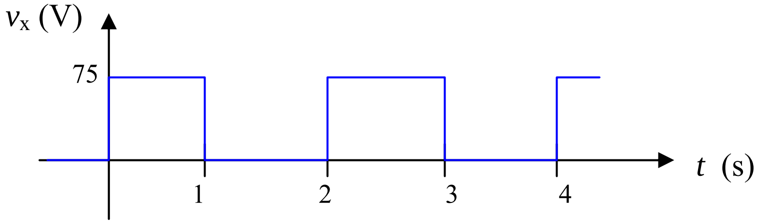
Hence, $v_S(t') = 3\, i_C(t') + v_C(t') = e^{-2.5t}(2.219 \cos 2.501t' + 10.36 \sin 2.501t')$





We see that our hand calculations are supported by the PSpice simulation.

67. It's probably easiest to begin by sketching the waveform v_x:



(a) The source current (= $i_L(t)$) = $\boxed{0}$ at $t = 0^-$.

(b) $i_L(t)$ = $\boxed{0}$ at $t = 0^+$

(c) We are faced with a series RLC circuit having α = R/2L = 2000 rad/s and ω_0 = 2828 rad/s. Thus, an underdamped response is expected with ω_d = 1999 rad/s.

The general form of the expected response is $i_L(t) = e^{-\alpha t}$ (A cos $\omega_d t$ + B sin $\omega_d t$)

$i_L(0^+) = i_L(0^-) = 0 = $ A so A = 0. This leaves $i_L(t) = \mathrm{B}\, e^{-2000t} \sin 1999t$

$$v_L(t) = \mathrm{L}\frac{di_L}{dt} = \mathrm{B}[(5\times10^{-3})(-2000\, e^{-2000t} \sin 1999t + 1999\, e^{-2000t} \cos 1999t)]$$

$v_L(0^+) = v_x(0^+) - v_C(0^+) - 20\, i_L(0^+) = \mathrm{B}\,(5\times10^{-3})(1999)$ so B = 7.504 A.

Thus, $i_L(t) = 7.504\, e^{-2000t} \sin 1999t$ and $i_L(1\text{ ms}) = \boxed{0.9239\text{ A.}}$

(d) Define $t' = t - 1$ ms for notational convenience. With no source present, we expect a new response but with the same general form:

$$i_L(t') = \mathrm{e}^{-2000t'}\,(\mathrm{A}' \cos 1999t' + \mathrm{B}' \sin 1999t')$$

$v_L(t) = \mathrm{L}\dfrac{di_L}{dt}$, and this enables us to calculate that $v_L(t = 1\text{ ms}) = -13.54$ V. Prior to the pulse returning to zero volts, $-75 + v_L + v_C + 20\, i_L = 0$ so $v_C(t' = 0) = 69.97$ V.

$i_L(t' = 0) = \mathrm{A}' = 0.9239$ and $-v_x + v_L + v_C + 20\, i_L = 0$ so that B' = -7.925.
Thus, $i_L(t')$ = e-2000 t' (0.9239 cos $1999t'$ – 7.925 sin $1999t'$) and
hence $i_L(t = 2\text{ ms}) = i_L(t' = 1\text{ ms}) = \boxed{-1.028\text{ A.}}$

68. The key will be to coordinate the decay dictated by α, and the oscillation period determined by ω_d (and hence partially by α). **One possible solution of many:**

Arbitrarily set $\omega_d = 2\pi$ rad/s.

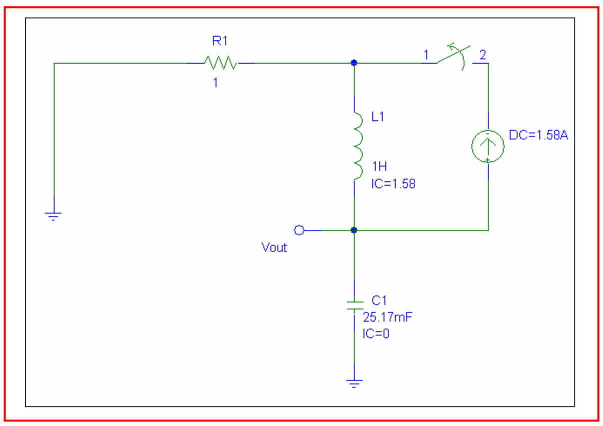
We want a capacitor voltage $v_C(t) = e^{-\alpha t}$ (A cos $2\pi t$ + B sin $2\pi t$). If we go ahead and decide to set $v_C(0^-) = 0$, then we can force A = 0 and simplify some of our algebra.

Thus, $v_C(t) = \text{B } e^{-\alpha t} \sin 2\pi t$. This function has max/min at t = 0.25 s, 0.75 s, 1.25 s, *etc*. Designing so that there is no strong damping for several seconds, we pick $\alpha = 0.5 \text{ s}^{-1}$. Choosing a series RLC circuit, this now establishes the following:

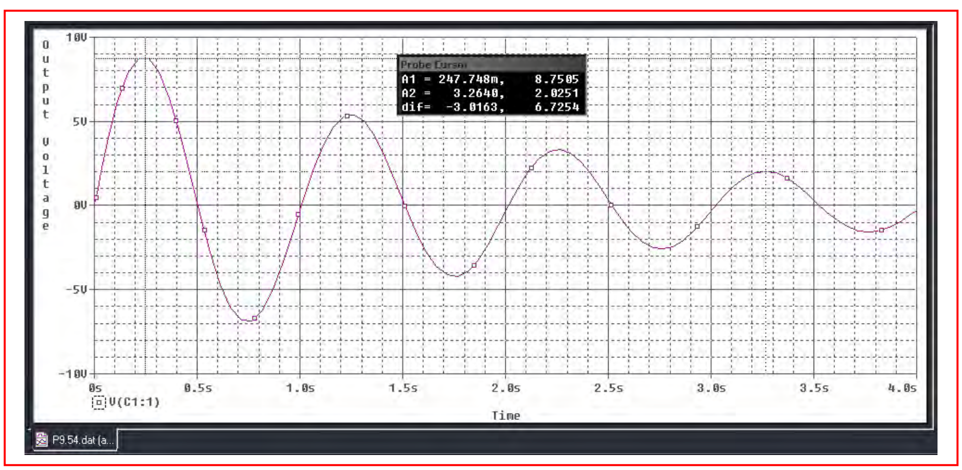
$$R/2L = 0.5 \text{ so } \quad R = L \text{ and}$$

$$\omega_d = \sqrt{\omega_0^2 - \left(\frac{1}{2}\right)^2} = 39.73 \text{ rad/s} \quad = \quad \frac{1}{LC}$$

Arbitrarily selecting R = 1 Ω, we find that L = 1 H and C = 25.17 mF. We need the first peak to be at least 5 V. Designing for B = 10 V, we ∴ need $i_L(0^+) = 2\pi(25.17\times10^{-3})(10)$ = 1.58 A. Our final circuit, then is:



And the operation is verified by a simple PSpice simulation:



69. The circuit described is a series RLC circuit, and the fact that oscillations are detected tells us that it is an underdamped response that we are modeling. Thus,

$i_L(t) = e^{-\alpha t}(\text{A} \cos \omega_d t + \text{B} \sin \omega_d t)$ where we were given that $\omega_d = 1.825\times10^6$ rad/s.

$\omega_0 = \dfrac{1}{\sqrt{LC}} = 1.914\times10^6$ rad/s, and so $\omega_d^{\,2} = \omega_0^{\,2} - \alpha^2$ leads to $\alpha^2 = 332.8\times10^9$

Thus, $\alpha = \text{R}/2\text{L} = 576863\ \text{s}^{-1}$, and hence R = $\boxed{1003\ \Omega.}$

Theoretically, this value must include the "radiation resistance" that accounts for the power lost from the circuit and received by the radio; there is no way to separate this effect from the resistance of the rag with the information provided.

70. For $t < 0$, $i_L(0^-) = 3$ A and $v_C(0^-) = 25(3) = 75$ V. This is a series RLC circuit with α = R/2L = 5000 s^{-1} and ω_0 = 4000 rad/s. We therefore expect an overdamped response with s_1 = -2000 s^{-1} and s_2 = -8000 s^{-1}. The final value of v_C = -50 V.

For $t > 0$, $v_C(t) = A\, e^{-2000t} + B\, e^{-8000t} - 50$

$v_C(0^+) = v_C(0^-) = 75 = A + B - 50$

so $A + B = 125$ [1]

$$\frac{dv_C}{dt} = -2000\, Ae^{-2000t} - 8000\, Be^{-8000t}$$

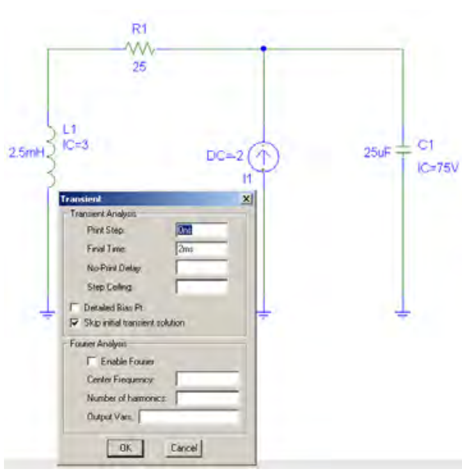
$$i_C(0^+) = C\left.\frac{dv_C}{dt}\right|_{t=0^+} = 3 - 5 - i_L(0^-) = -5 = -25\times10^{-6}\,(2000A + 8000B)$$

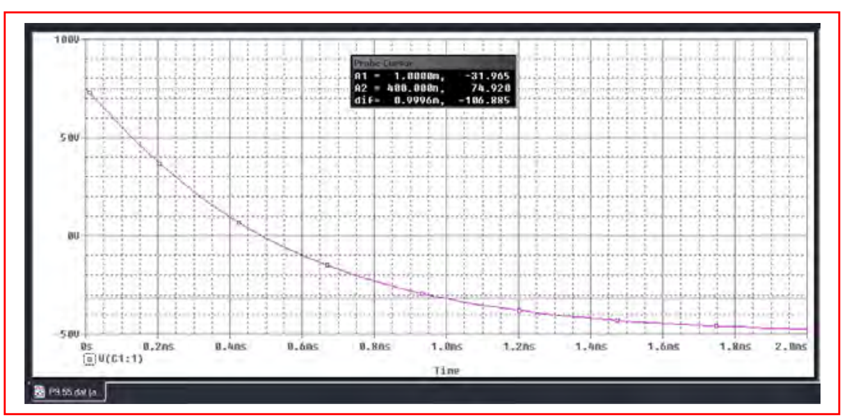
Thus, $2000A + 8000B = 5/25\times10^{-6}$ [2]

Solving Eqs. [1] and [2], we find that A = 133.3 V and B = -8.333 V. Thus,

$$v_C(t) = 133.3\, e^{-2000t} - 8.333\, e^{-8000t} - 50$$

and $v_C(1\text{ ms})$ = $\boxed{-31.96\text{ V.}}$ This is confirmed by the PSpice simulation shown below.





71. $\alpha = 0$ (this is a series RLC with R = 0, or a parallel RLC with R = ∞)
$\omega_o^2 = 0.05$ therefore ω_d = 0.223 rad/s. We anticipate a response of the form:
$v(t)$ = A cos $0.2236t$ + B sin $0.2236t$

$v(0^+) = v(0^-) = 0 = A$ therefore $v(t)$ = B sin $0.2236t$

dv/dt = 0.2236B cos $0.2236t$; $i_C(t) = Cdv/dt$ = 0.4472B cos $0.2236t$

$i_C(0^+) = 0.4472B = -i_L(0^+) = -i_L(0^-) = -1\times10^{-3}$ so B = -2.236×10^{-3} and thus

$v(t)$ = -2.236 sin $0.2236t$ mV

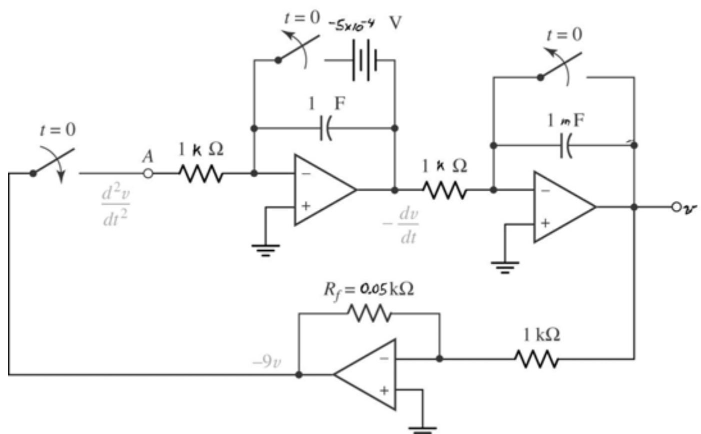
In designing the op amp stage, we first write the differential equation:

$$\frac{1}{10}\int_0^t v\,dt' + 10^{-3} + 2\frac{dv}{dt} = 0 \qquad (i_C + i_L = 0)$$

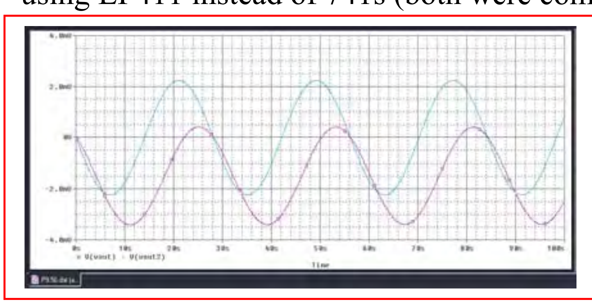
and then take the derivative of both sides:

$$\frac{d^2v}{dt^2} = -\frac{1}{20}v$$

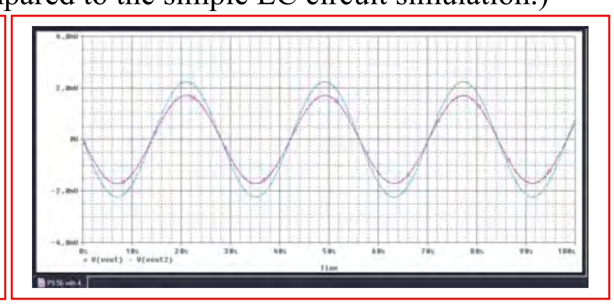
With $\left.\frac{dv}{dt}\right|_{t=0^+} = (0.2236)(-2.236\times10^{-3}) = -5\times10^{-4}$, one possible solution is:



PSpice simulations are very sensitive to parameter values; better results were obtained using LF411 instead of 741s (both were compared to the simple LC circuit simulation.)



Simulation using 741 op amps



Simulation using LF411 op amps

72. $\alpha = 0$ (this is a series RLC with R = 0, or a parallel RLC with R = ∞)
$\omega_o^2 = 50$ therefore ω_d = 7.071 rad/s. We anticipate a response of the form:
$v(t)$ = A cos 7.071t + B sin 7.071t, knowing that $i_L(0^-) = 2$ A and $v(0^-) = 0$.

$v(0^+) = v(0^-) = 0 = A$ therefore $v(t)$ = B sin 7.071t

dv/dt = 7.071B cos 7.071t; $i_C(t) = C dv/dt$ = 0.007071B cos 7.071t

$i_C(0^+)$ = 0.007071B = $-i_L(0^+) = -i_L(0^-) = -2$ so B = -282.8 and thus

$v(t)$ = -282.8 sin 7.071t V

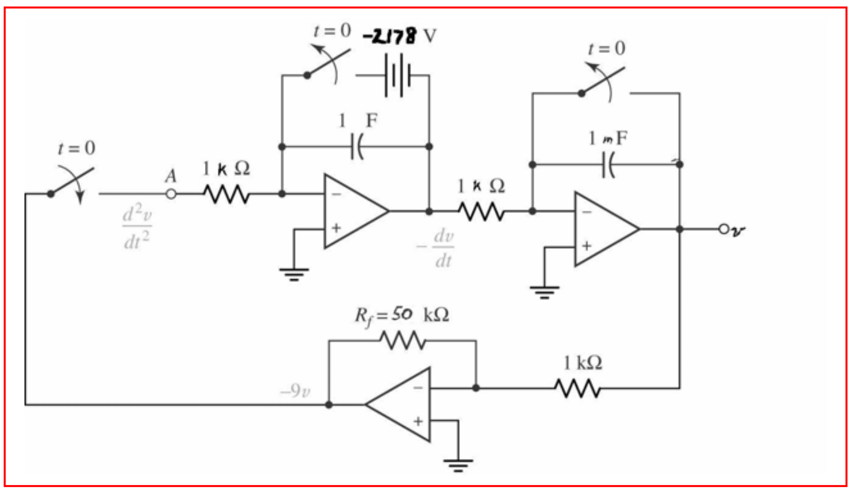
In designing the op amp stage, we first write the differential equation:

$$\frac{1}{20}\int_0^t v\,dt' + 2 + 10^{-3}\frac{dv}{dt} = 0 \qquad (i_C + i_L = 0)$$

and then take the derivative of both sides:

$$\frac{d^2v}{dt^2} = -50v$$

With $\left.\frac{dv}{dt}\right|_{t=0^+} = (7.071)(-282.8) = -2178$, one possible solution is:



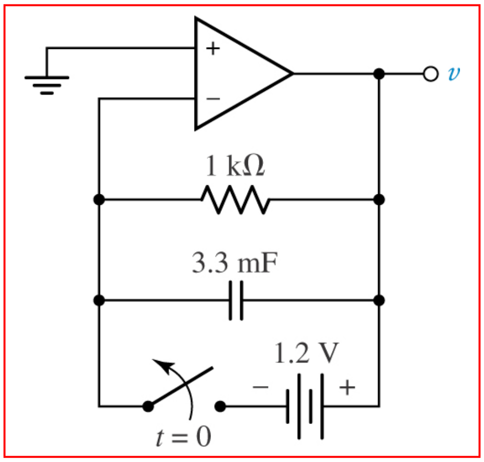
73.

$$\frac{v}{1000} + 3.3\times 10^{-3}\frac{dv}{dt} = 0$$

(a) or

$$\boxed{\frac{dv}{dt} = -\frac{1}{3.3}v}$$

(b) One possible solution:



74. We see either a series RLC with R = 0 or a parallel RLC with R = ∞; either way, $\alpha = 0$. $\omega_0^2 = 0.3$ so $\omega_d = 0.5477$ rad/s (combining the two inductors in parallel for the calculation). We expect a response of the form $i(t) = \text{A} \cos \omega_d t + \text{B} \sin \omega_d t$.

$i(0^+) = i(0^-) = \text{A} = 1\times10^{-3}$
$di/dt = -\text{A}\omega_d \sin \omega_d t + \text{B}\omega_d \cos \omega_d t$
$v_L = 10di/dt = -10\text{A}\omega_d \sin \omega_d t + 10\text{B}\omega_d \cos \omega_d t$
$v_L(0^+) = v_C(0^+) = v_C(0^-) = 0 = 10\text{B}(0.5477)$ so that B = 0
and hence $i(t) = 10^{-3} \cos 0.5477t$ A

The differential equation for this circuit is

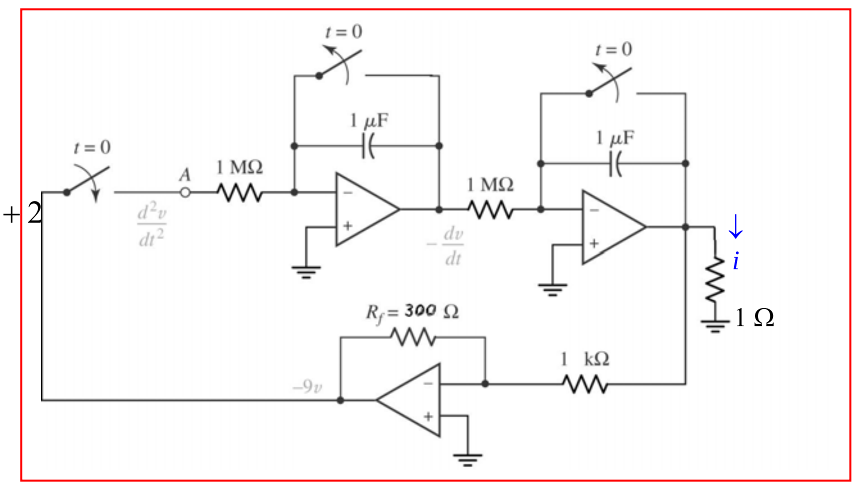
$$\text{and } \left.\frac{di}{dt}\right|_{t=0^+} = 0$$

$$\frac{1}{10}\int_0^t v\,dt' + 10^{-3} + \frac{1}{2}\int_0^t v\,dt' + 2$$

or

$$\frac{d^2v}{dt^2} = -0.3v$$

One possible solution is:



75. (a) $v_R = v_L$

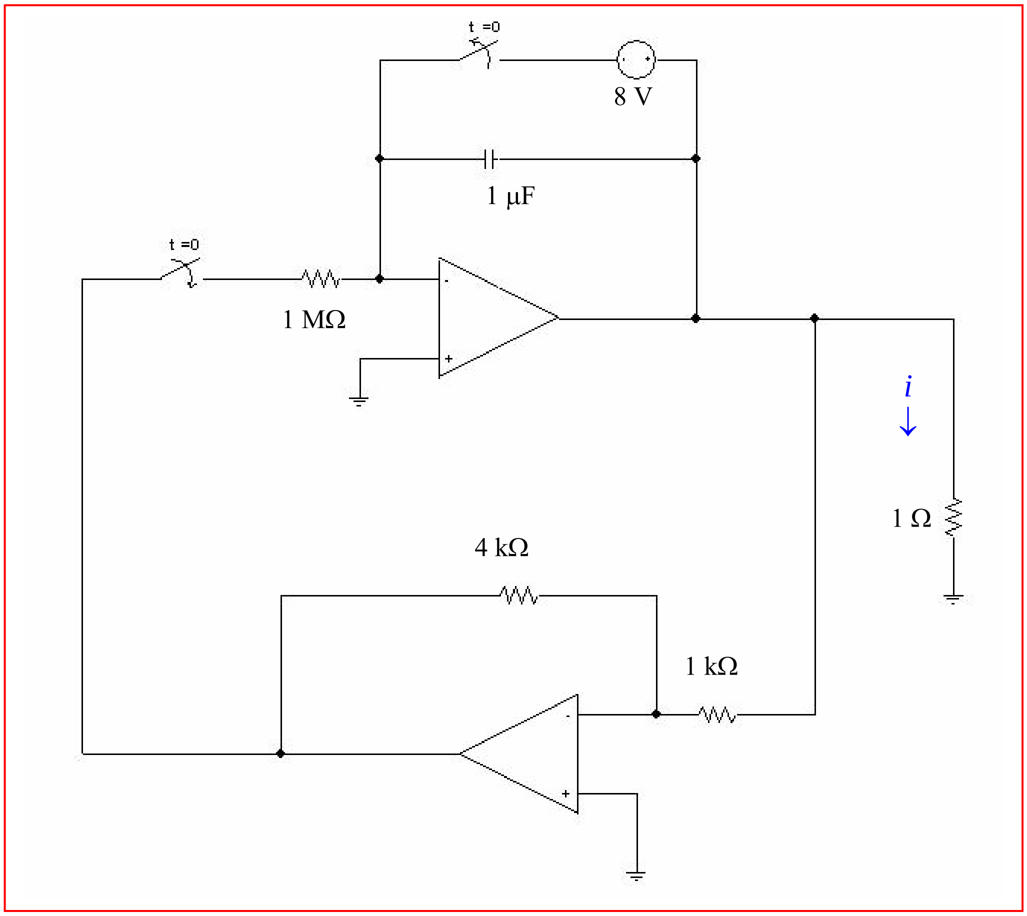
$$20(-i_L) = 5\frac{di_L}{dt} \quad \text{or} \quad \boxed{\frac{di_L}{dt} = -4i_L}$$

(b) We expect a response of the form $i_L(t) = A\,e^{-t/\tau}$ where $\tau = L/R = 0.25$.

We know that $i_L(0^-) = 2$ amperes, so A = 2 and $i_L(t) = 2\,e^{-4t}$

$$\left.\frac{di_L}{dt}\right|_{t=0^+} = -4(2) = -8 \text{ A/s.}$$

One possible solution, then, is



1.

(a) $\mathrm{T} = 4(7.5 - 2.1)10^{-3} = 21.6 \times 10^{-3},\ \omega = \dfrac{2\pi 10^3}{21.6} = 290.9t \text{ rad/s}$

$\therefore\ f(t) = 8.5\sin(290.9t + \Phi) \therefore 0 = 8.5\sin(290.9 \times 2.1 \times 10^{-3} + \Phi)$

$\therefore \Phi = -0.6109^{\text{rad}} + 2\pi = 5.672^{\text{rad}} \text{ or } 325.0°$

$\therefore\ f(t) = \boxed{8.5\sin(290.9t + 325.0°)}$

(b) $8.5\sin(290.9t + 325.0°) = 8.5$

$\cos(290.9t + 235°) = \boxed{8.5\cos(290.9t - 125°)}$

(c) $8.5\cos(-125°)\cos\omega t + 8.5\sin 125°$

$\sin\omega t = \boxed{-4.875^{+}\cos 290.9t + 6.963\sin 290.9t}$

2.

(a) $-10\cos\omega t + 4\sin\omega t + A\mathrm{Cos}(wt + \Phi),\ A > 0,\ -180° < \Phi \leq 180°$

$A = \sqrt{116} = 10.770,\ A\cos\Phi = -10,\ A\sin\Phi = -4 \therefore \tan\Phi = 0.4,\ 3^d$ quad

$\therefore \Phi = 21.80° = 201.8°, \text{too large} \therefore \Phi = 201.8° - 360° = \boxed{-158.20°}$

(b) $200\cos(5t + 130°) = F\cos 5t + G\sin 5t \therefore F = 200\cos 130° = \boxed{-128.6}$

$G = -200\sin 130° = \boxed{-153.2}$

(c) $i(t) = 5\cos 10t - 3\sin 10t = 0,\ 0 \leq t \leq 1\text{ s}$

$\therefore \frac{\sin 10t}{\cos 10t} = \frac{5}{3},\ 10t = 1.0304,$

$\boxed{t = 0.10304\text{ s}}$; also, $10t = 1.0304 + \pi$, $\boxed{t = 0.4172\text{ s}}$; $10t = 1.0304 + 2\pi$, $\boxed{t = 0.7314\text{ s}}$

(d) $0 < t < 10\text{ms},\ 10\cos 100\pi t \geq 12\sin 100\pi t$; let $10\cos 100\pi t = 12\sin 100\pi t$

$\therefore \tan 100\pi t = \frac{10}{12},\ 100\pi t = 0.6947 \therefore t = 2.211\text{ ms} \therefore \boxed{0 < t < 2.211\text{ ms}}$

3.

(a) Note that $A\cos x + B\sin x = \sqrt{A^2 + B^2}\cos\left(x + \tan^{-1}\left(\frac{-B}{A}\right)\right)$. For *f*(*t*), the angle is in the second quadrant; most calculators will return –30.96°, which is off by 180°.

$$f(t) = -50\cos\omega t - 30\sin\omega t = 58.31\cos(\omega t + 149.04°)$$

$$g(t) = 55\cos\omega t - 15\sin\omega t = 57.01\cos(\omega t + 15.255°)$$

$$\therefore \text{ampl. of } f(t) = \boxed{58.31,}\ \text{ampl. of } g(t) = \boxed{57.01}$$

(b) $f(t)$ leads $g(t)$ by $149.04° - 15.255° = \boxed{133.8°}$

4. $i(t) = \mathrm{A}\cos(\omega t - \theta)$, and

$\mathrm{L}(di/dt) + \mathrm{R}i = \mathrm{V}_m \cos\omega t$

$\therefore \quad [-\omega \mathrm{A}\sin(\mathrm{L}\omega t - \theta)] + \mathrm{RA}\cos(\omega t - \theta) = \mathrm{V}_m \cos\omega t$

$-\omega \mathrm{LA}\sin\omega t\cos\theta + \omega\mathrm{LA}\cos\omega t\sin\theta + \mathrm{RA}\cos\omega t\cos\theta + \mathrm{RA}\sin\omega t\sin\theta$

$= \mathrm{V}_m \cos\omega t$

$\therefore\ \omega\mathrm{LA}\cos\theta = \mathrm{RA}\sin\theta$

and $\omega LA\sin\theta + \mathrm{RA}\cos\theta = \mathrm{V}_m$

Thus, $\tan\theta = \dfrac{\omega \mathrm{L}}{R}$ *

and $\omega\mathrm{LA}\dfrac{\omega\mathrm{L}}{\sqrt{\mathrm{R}^2+\omega^2\mathrm{L}^2}} + \mathrm{RA}\dfrac{\mathrm{R}}{\sqrt{\mathrm{R}^2+\omega^2\mathrm{L}^2}} = \mathrm{V}_m$

so that $\left(\dfrac{\omega^2\mathrm{L}^2}{\sqrt{\mathrm{R}^2+\omega^2\mathrm{L}^2}} + \dfrac{\mathrm{R}^2}{\sqrt{\mathrm{R}^2+\omega^2\mathrm{L}^2}}\right)\mathrm{A} = \mathrm{V}_m$

Thus, $\left(\sqrt{\mathrm{R}^2+\omega^2\mathrm{L}^2}\right)\mathrm{A} = \left(\mathrm{V}_m\right)$ and therefore we may write $\mathrm{A} = \dfrac{\mathrm{V}_m}{\sqrt{\mathrm{R}^2+\omega^2\mathrm{L}^2}}$ *

5. f = 13.56 MHz so $\omega = 2\pi f = \boxed{85.20 \text{ Mrad/s.}}$

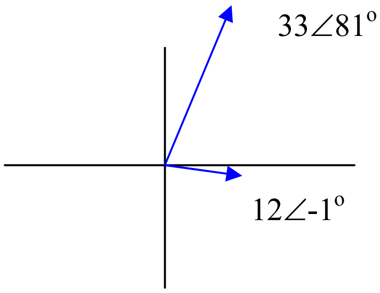
Delivering 300 W (peak) to a 5-Ω load implies that $\frac{V_m^2}{5} = 300$ so $\boxed{V_m = 38.73 \text{ V.}}$

Finally, $(85.2\times10^6)(21.15\times10^{-3}) + \phi = n\pi$, n = 1, 3, 5, …

Since $(85.2\times10^6)(21.15\times10^{-3}) = 1801980$, which is 573588π, we find that

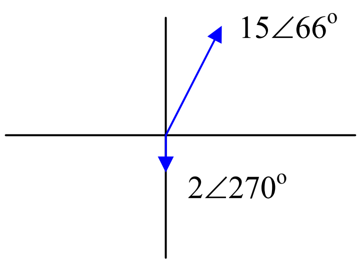
$$\phi = 573589\pi - (85.2\times10^6)(21.15\times10^{-3}) = 573589\pi - 573588\pi = \boxed{\pi}$$

6. (a) $-33 \sin(8t - 9^o) \rightarrow -33\angle(-9-90)^o = 33\angle 81^o$
$12 \cos(8t - 1^o) \rightarrow 12\angle -1^o$



$-33 \sin(8t - 9^o)$ leads $12 \cos(8t - 1^o)$ by $81 - (-1) = 82^o$.

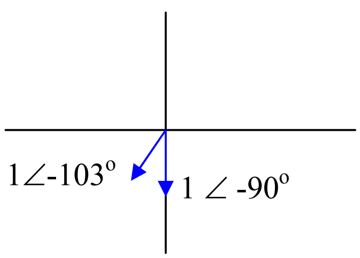
(b) $15 \cos(1000t + 66^o) \rightarrow 15 \angle 66^o$
$-2 \cos(1000t + 450^o) \rightarrow -2 \angle 450^o = -2 \angle 90^o = 2 \angle 270^o$



$15 \cos(1000t + 66^o)$ leads $-2 \cos(1000t + 450^o)$ by $66 - -90 = 156^o$.

(c) $\sin(t - 13^o) \rightarrow 1\angle -103^o$
$\cos(t - 90^o) \rightarrow 1 \angle -90^o$

$\cos(t - 90^o)$ leads $\sin(t - 13^o)$ by $66 - -90 = 156^o$.

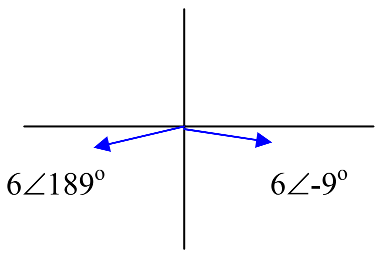


(d) $\sin t \rightarrow 1 \angle -90^o$
$\cos(t - 90^o) \rightarrow 1 \angle -90^o$

These two waveforms are *in phase*. Neither leads the other.

7. (a) $6\cos(2\pi 60t - 9^\circ) \rightarrow 6\angle\text{-}9^\circ$

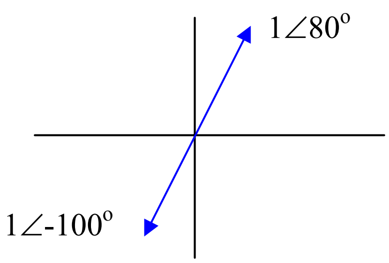
$-6\cos(2\pi 60t + 9^\circ) \rightarrow 6\angle 189^\circ$



$-6\cos(2\pi 60t + 9^\circ)$ lags $6\cos(2\pi 60t - 9^\circ)$ by $360 - 9 - 189 = 162^\circ$.

(b) $\cos(t - 100^\circ) \rightarrow 1\angle -100^\circ$

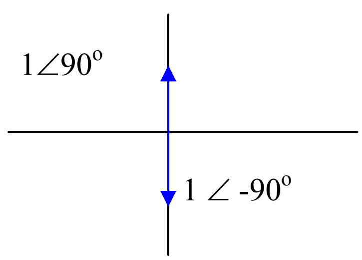
$-\cos(t - 100^\circ) \rightarrow -1 \angle -100^\circ = 1\angle 80^\circ$



$-\cos(t - 100^\circ)$ lags $\cos(t - 100^\circ)$ by 180°.

(c) $-\sin t \rightarrow -1 \angle -90^\circ = 1\angle 90^\circ$

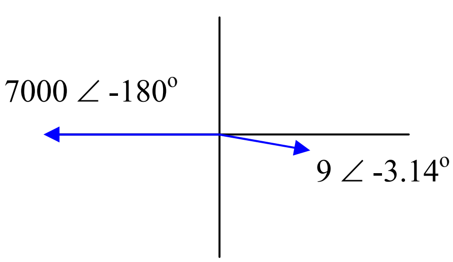
$\sin t \rightarrow 1\angle -90^\circ$



$-\sin t$ lags $\sin t$ by 180°.

(d) $7000\cos(t - \pi) \rightarrow 7000 \angle -\pi = 7000\angle -180^\circ$

$9\cos(t - 3.14^\circ) \rightarrow 9 \angle -3.14^\circ$



$7000\cos(t - \pi)$ lags $9\cos(t - 3.14^\circ)$ by $180 - 3.14 = 176.9^\circ$.

8. $v(t) = V_1 \cos \omega t \; - \; V_2 \sin \omega t$ [1]

We assume this can be written as a single cosine such that

$$v(t) = V_m \cos(\omega t + \phi) = V_m \cos \omega t \cos \phi - V_m \sin \omega t \sin \phi \quad [2]$$

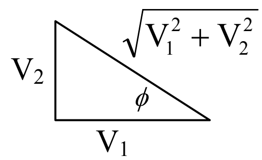
Equating terms on the right hand sides of Eqs. [1] and [2],

$$V_1 \cos \omega t - V_2 \sin \omega t = (Vm \cos \phi) \cos \omega t - (Vm \sin \phi) \sin \omega t$$

yields

$$V_1 = V_m \cos \phi \text{ and } V_2 = V_m \sin \phi$$

Dividing, we find that $\dfrac{V_2}{V_1} = \dfrac{V_m \sin \phi}{V_m \cos \phi} = \tan \phi$ and $\boxed{\phi = \tan^{-1}(V_2 / V_1)}$



Next, we see from the above sketch that we may write $V_m = V_1 / \cos \phi$ or

$$V_m = \frac{V_1}{V_1 \Big/ \sqrt{V_1^2 + V_2^2}} = \boxed{\sqrt{V_1^2 + V_2^2}}$$

Thus, we can write $v(t) = V_m \cos(\omega t + \phi) = \sqrt{V_1^2 + V_2^2} \cos[\omega t + \tan^{-1}(V_2 / V_1)]$.

9. (a) In the range $0 \le t \le 0.5$, $v(t) = t/0.5$ V.
Thus, $v(0.4) = 0.4/0.5 = \boxed{0.8 \text{ V.}}$

(b) Remembering to set the calculator to radians, $\boxed{0.7709 \text{ V.}}$

(c) $\boxed{0.8141 \text{ V.}}$

(d) $\boxed{0.8046 \text{ V.}}$

10. (a)

$$V_{rms} = \left[\frac{V_m^2}{T} \int_0^T \cos^2 \omega t \, dt \right]^{1/2}$$

$$= \left[\frac{V_m^2}{T} \int_0^T \cos^2 \frac{2\pi t}{T} \, dt \right]^{1/2}$$

$$= \left[\frac{V_m^2}{2T} \int_0^T \left(1 + \cos \frac{4\pi t}{T} \right) dt \right]^{1/2}$$

$$= \left[\frac{V_m^2}{2T} \int_0^T dt \; + \; \frac{V_m^2}{2T} \int_0^T \cos \frac{4\pi t}{T} \, dt \right]^{1/2}$$

$$= \left[\frac{V_m^2}{2T} T \; + \; \frac{V_m^2}{8\pi} \cos u \Big|_0^{4\pi} \right]^{1/2}$$

$$= \frac{V_m}{\sqrt{2}} \quad *$$

(b)

$$\boxed{V_m = 110\sqrt{2} = 155.6 \text{ V}, \; 115\sqrt{2} = 162.6 \text{ V}, \; 120\sqrt{2} = 169.7 \text{ V}}$$

11. We begin by defining a clockwise current *i*. Then, KVL yields

$$-2\times10^{-3}\cos 5t + 10i + v_C = 0.$$

Since $i = i_C = C\dfrac{dv_C}{dt}$, we may rewrite our KVL equation as

$$30\frac{dv_C}{dt} + v_C = 2\times10^{-3}\cos 5t \qquad [1]$$

We anticipate a response of the form $v_C(t) = A\cos(5t + \theta)$. Since $\dfrac{dv_C}{dt} = -5A\sin(5t+\theta)$,

we now may write Eq. [1] as $-150A\sin(5t + \theta) + A\cos(5t + \theta) = 2\times10^{-3}\cos 5t$. Using a common trigonometric identity, we may combine the two terms on the left hand side into a single cosine function:

$$\sqrt{(150A)^2 + A^2}\cos\left(5t + \theta + \tan^{-1}\frac{150A}{A}\right) = 2\times10^{-3}\cos 5t$$

Equating terms, we find that A = 13.33 μV and $\theta = -\tan^{-1} 150 = -89.62^\circ$. Thus,

$$\boxed{v_C(t) = 13.33\cos(5t - 89.62^\circ)\ \mu\text{V}.}$$

12. KVL yields

$$-6\cos 400t + 100i + v_L = 0.$$

Since $v_L = L\frac{di}{dt} = 2\frac{di}{dt}$, we may rewrite our KVL equation as

$$2\frac{di}{dt} + 100i = 6\cos 400t \qquad [1]$$

We anticipate a response of the form $i(t) = A\cos(400t + \theta)$. Since

$$\frac{di}{dt} = -400A\sin(400t+\theta),$$

we now may write Eq. [1] as

$$-800A\sin(400t + \theta) + 100A\cos(400t + \theta) = 6\cos 400t.$$

Using a common trigonometric identity, we may combine the two terms on the left hand side into a single cosine function:

$$\sqrt{(800A)^2 + (100A)^2}\cos\left(400t + \theta + \text{ an}^{-1}\frac{800A}{100A}\right) = 6\cos 400t$$

Equating terms, we find that A = 7.442 mA and $\theta = -\tan^{-1} 8 = -82.88^{\circ}$. Thus,

$i(t) = 7.442\cos(400t - 82.88^{\circ})$ mV, so $v_L = L\frac{di}{dt} = 2\frac{di}{dt} = \boxed{5.954\cos(400t + 7.12^{\circ})}$

13. $20\cos 500t$ V $\rightarrow 20\angle 0^{\circ}$ V. 20 mH $\rightarrow j10\ \Omega$.

Performing a quick source transformation, we replace the voltage source/20-Ω resistor series combination with a $1\angle 0^{\circ}$ A current source in parallel with a 20-Ω resistor.

20 || 60k = 19.99 Ω. By current division, then,

$$\mathbf{I}_L = \frac{19.99}{19.99 + 5 + j10} = 0.7427\angle -21.81^{\circ} \text{ A. Thus, } i_L(t) = \boxed{742.7 \cos(500t - 21.81^{\circ}) \text{ mA.}}$$

14.

At $x - x$: $R_{th} = 80 \| 20 = 16\Omega$

$$v_{oc} = -0.4\,(15 \| 85)\,\frac{80}{85}\cos 500t$$

$\therefore v_{oc} = 4.8\cos 500t$ V

(a) $$i_L = \frac{4.8}{\sqrt{16^2 + 10^2}}\cos\left(500t - \tan^{-1}\frac{10}{15}\right)$$

$$= \boxed{0.2544\cos(500t - 32.01°)\text{ A}}$$

(b) $v_L = \text{L}i_L' = 0.02 \times 0.02544\,(-500)$

$\sin(500t - 32.01°) = -2.544\sin(500t - 32.01°)$ V

$\therefore v_L = 2.544\cos(500t + 57.99°)$ V, i_x

$$= \boxed{31.80\cos(500t + 57.99°)\text{ mA}}$$

15.

(a) $$i = \frac{100}{\sqrt{500^2 + 800^2}} \cos\left(10^5 t - \frac{800}{500}\right) = 0.10600 \cos(10^5 t - 57.99°)\,\text{A}$$

$$p_R = 0 \text{ when } i = 0 \therefore 10^5 t - \frac{57.99°}{180}\pi = \frac{\pi}{2},\ t = \boxed{25.83\mu s}$$

(b) $$\pm v_L = \text{L}i' = 8 \times 10^{-3} \times 0.10600(-10^5)\ \sin(10^5 t - 57.99°)$$

$$\therefore v_L = -84.80 \sin(10^5 t - 57.99°)$$

$$\therefore p_L = v_L i = -8.989 \sin(10^5 t - 57.99°)$$

$$\cos(10^5 t - 57.99°) = -4.494\ \sin(2 \times 16^5 t - 115.989°)$$

$$\therefore p_L = 0 \text{ when } 2 \times 10^5 t - 115.989° = 0°, 180°,$$

$$\therefore t = \boxed{10.121 \text{ or } 25.83\mu s}$$

(c) $$p_s = v_s\, i_L = 10.600 \cos 10^5 t \cos(10^5 t - 57.99°)$$

$$\therefore p_s = 0 \text{ when } 10^5 t = \frac{\pi}{2},\ t = \boxed{15.708\mu s} \text{ and also } t = \boxed{25.83\mu s}$$

16. $v_s = 3\cos 10^5 t$ V, $i_s = 0.1\cos 10^5 t$ A

v_s in series with $30\Omega \to 0.1\cos 10^5 t \text{ A} \| 30\Omega$

Add, getting $0.2\cos 10^5 t \text{ A} \| 30\,\Omega$

change to 6 cos $10^5 t$ V in series with 30Ω; $30\Omega + 20\Omega = 50\Omega$

$$\therefore i_L = \frac{6}{\sqrt{50^2 + 10^2}} \cos\left(10^5 t - \tan^{-1}\frac{10}{50}\right) = 0.11767\cos(10^5 t - 11.310°)\text{ A}$$

At $t = 10\ \mu$s, $10^5 t = 1 \therefore i_L = 0.1167\cos(1^{\text{rad}} - 11.310°) = \boxed{81.76\text{mA}}$

$\therefore v_L = 0.11767 \times 10\cos(1^{\text{rad}} - 11.30° + 90°) = \boxed{-0.8462\text{V}}$

17. cos500 t V → $1\angle 0^{\circ}$ V. 0.3 mH → $j0.15\ \Omega$.

Performing a quick source transformation, we replace the voltage source-resistor series combination with at $0.01\angle 0^{\circ}$ A current source in parallel with a 100-Ω resistor. Current division then leads to

$$\left(0.01+0.2\mathbf{I}_{\mathrm{L}}\right)\frac{100}{100+j0.15} = \mathbf{I}_{\mathrm{L}}$$

$$1 + 20\mathbf{I}_{\mathrm{L}} = (100 + j0.15)\,\mathbf{I}_{\mathrm{L}}$$

Solving, we find that $\mathbf{I}_{\mathrm{L}} = 0.0125\angle -0.1074^{\circ}$ A,

so that $i_{\mathrm{L}}(t) = \boxed{12.5\cos(500t - 0.1074^{\circ})\text{ mA.}}$

18. $v_{s1} = V_{s2} = 120\cos 120\pi t \text{ V}$

$$\frac{120}{60} = 2\text{A},\ \frac{120}{12} = 1\text{A},\ 2+1 = 3\text{A},\ 60 \| 120 = 40\Omega$$

$$3\times 40 = 120\text{ V}, \omega L = 12\pi = 37.70\Omega$$

$$\therefore i_L = \frac{120}{\sqrt{40^2 + 37.70^2}} \cos\left(120\pi t - \tan^{-1}\frac{37.70}{40}\right)$$

$$= 2.183\cos(120\pi t - 43.30°)\text{ A}$$

(a) $$\therefore \omega_L = \frac{1}{2}\times 0.1\times 2.183^2 \cos^2(120\pi t - 43.30°)$$

$$= \boxed{0.2383\cos^2(120\pi t - 43.30°)\text{ J}}$$

(b) $$\omega_{L,av} = \frac{1}{2}\times 0.2383 = \boxed{0.11916\text{ J}}$$

19. $v_{s1} = 120\cos 400t$ V, $v_{s2} = 180\cos 200t$ V

Performing two quick source transformations,

$\frac{120}{60} = 2$ A, $\frac{180}{120} = 1.5$ A, and noting that $60 \| 120 = 40\ \Omega$,

results in two current sources (with different frequencies) in parallel, and also in parallel with a 40 Ω resistor and the 100 mH inductor.

Next we employ superposition. Open-circuiting the 200 rad/s source first, we perform a source transformation to obtain a voltage source having magnitude $2 \times 40 = 80$ V. Applying Eqn. 10.4,

$$i_L' = \frac{80}{\sqrt{40^2 + 400^2(0.1)^2}}\cos\left(400t - \tan^{-1}\frac{400(0.1)}{40}\right)$$

Next, we open-circuit the 400 rad/s current source, and perform a source transformation to obtain a voltage source with magnitude $1.5 \times 40 = 60$ V. Its contribution to the inductor current is

$$i_L'' = \frac{60}{\sqrt{40^2 + 200^2(0.1)^2}}\cos\left(200t - \tan^{-1}\frac{200(0.1)}{40}\right)\text{ A}$$

so that $\boxed{i_L = 1.414\cos(400t - 45°) + 1.342\cos(200t - 26.57°)\text{ A}}$

20.

$$R_i = \infty,\ R_o = 0,\ A = \infty,\ \text{ideal},\quad {}_1C_1\,R = \frac{L}{R}$$

$$i_{upper} = -\frac{V_m \cos \omega t}{R},\ i_{lower} = \frac{v_{out}}{R_1}$$

$$\therefore i_{c1} = i_{upper} + i_{lower} = \frac{i}{R_1}(v_{out} - V_m \cos \omega t) = -C_1 v'_{out}$$

$$\therefore V_m \cos \omega t = v_{out} + R_1 C_1 v'_{out} = v_{out} + \frac{L}{R} v'_{out}$$

For RL circuit, $V_m \cos \omega t = v_r + L\frac{d}{dt}\left(\frac{v_R}{R}\right)$

$$\therefore V_m \cos \omega t = v_R + \frac{L}{R} v'_R$$

By comparison, $v_R = v_{out}$ *

21.

(a) $$V_m \cos \omega t = \mathrm{R}i + \frac{1}{\mathrm{C}}\int idt \text{ (ignore I.C)}$$

$$\therefore \boxed{-\omega V_m \sin \omega t = \mathrm{R}i' + \frac{1}{C}i}$$

(b) Assume $i = \mathrm{A}\cos(\omega t + \Phi)$

$$\therefore -\omega V_m \sin \omega t = -\mathrm{R}\omega \mathrm{A}\sin(\omega t + \Phi) + \frac{\mathrm{A}}{\mathrm{C}}\cos(\omega t + \Phi)$$

$$\therefore -\omega V_m \sin \omega t = -\mathrm{R}\omega \mathrm{A}\cos\Phi \sin\omega t - \mathrm{R}\omega \mathrm{A}\sin\Phi\cos\omega t + \frac{\mathrm{A}}{\mathrm{C}}\cos\omega t\cos\Phi - \frac{\mathrm{A}}{\mathrm{C}}\sin\omega t\sin\Phi$$

Equating terms on the left and right side,

[1] $\mathrm{R}\omega \mathrm{A}\sin\Phi = \frac{\mathrm{A}}{\mathrm{C}}\cos\Phi \therefore \tan\Phi = \frac{1}{\omega \mathrm{CR}}$ so $\Phi = \tan^{-1}(1/\omega \mathrm{CR})$, and

[2] $$-\omega V_m = -\mathrm{R}\omega \mathrm{A}\frac{\omega \mathrm{CR}}{\sqrt{1+\omega^2\mathrm{C}^2\mathrm{R}^2}} - \frac{\mathrm{A}}{\mathrm{C}}\frac{1}{\sqrt{1+\omega^2\mathrm{C}^2\mathrm{R}^2}}$$

$$\therefore \omega V_m = \frac{\mathrm{A}}{\mathrm{C}}\left[\frac{\mathrm{R}^2\omega^2\mathrm{C}^2+1}{\sqrt{1+\omega^2\mathrm{C}^2\mathrm{R}^2}}\right] = \frac{\mathrm{A}}{\mathrm{C}}\sqrt{1+\omega^2\mathrm{C}^2\mathrm{R}^2} \therefore \mathrm{A} = \frac{\omega \mathrm{C}V_m}{\sqrt{1+\omega^2\mathrm{C}^2\mathrm{R}^2}}$$

$$\therefore \boxed{i = \frac{\omega \mathrm{C}V_m}{\sqrt{1+\omega^2\mathrm{C}^2\mathrm{R}^2}}\cos\left(\omega t + \tan^{-1}\frac{1}{\omega \mathrm{CR}}\right)}$$

22. (a) $7 \angle -90^{o} = \boxed{-j\,7}$

(b) $3 + j + 7 \angle -17^{o} = 3 + j + 6.694 - j\,2.047 = \boxed{9.694 - j\,1.047}$

(c) $14e^{j15^{o}} = 14 \angle 15^{o} = 14 \cos 15^{o} + j\,14 \sin 15^{o} = \boxed{13.52 + j\,3.623}$

(d) $1 \angle 0^{o} = \boxed{1}$

(e) $-2\,(1 + j\,9) = -2 - j\,18 = \boxed{18.11 \angle -96.34^{o}}$

(f) $3 = \boxed{3 \angle 0^{o}}$

23. (a) $3 + 15 \angle -23^o = 3 + 13.81 - j\,5.861 = \boxed{16.81 - j\,5.861}$

(b) $(j\,12)(17 \angle 180^o) = (12 \angle 90^o)(17 \angle 180^o) = 204 \angle 270^o = \boxed{-j\,204}$

(c) $5 - 16(9 - j\,5)/\,(33 \angle -9^o) = 5 - (164 \angle -29.05^o)/\,(33 \angle -9^o)$

$= 5 - 4.992 \angle -20.05^o = 5 - 4.689 - j\,1.712 = \boxed{0.3109 + j\,1.712}$

24. (a) $5\angle 9^\circ - 9\angle -17^\circ = 4.938 + j\,0.7822 - 8.607 + j\,2.631 = -3.668 + j\,3.414$

$= \boxed{5.011\angle 137.1^\circ}$

(b) $(8 - j\,15)(4 + j\,16) - j = 272 + j\,68 - j = 272 + j\,67 = \boxed{280.1\angle 13.84^\circ}$

(c) $(14 - j\,9)/(2 - j\,8) + 5\angle -30^\circ = (16.64\angle -32.74^\circ)/(8.246\angle -75.96^\circ) + 4.330 - j\,2.5$

$= 1.471 + j\,1.382 + 4.330 - j\,2.5 = 5.801 - j\,1.118 = \boxed{5.908\angle -10.91^\circ}$

(d) $17\angle -33^\circ + 6\angle -21^\circ + j\,3 = 14.26 - j\,9.259 + 5.601 - j\,2.150 + j\,3$

$= 19.86 - j\,8.409 = \boxed{21.57\angle -22.95^\circ}$

25. (a) $e^{j14^\circ} + 9 \angle 3^\circ - (8 - j\,6)/\,j^2 = 1 \angle 14^\circ + 9 \angle 3^\circ - (8 - j\,6)/\,(-1)$

$= 0.9703 + j\,0.2419 + 8.988 + j\,0.4710 + 8 - j\,6 = 17.96 - j\,5.287 = \boxed{18.72 \angle -16.40^\circ}$

(b) $(5 \angle 30^\circ)/\,(2 \angle -15^\circ) + 2\,e^{j5^\circ}/\,(2 - j\,2)$

$= 2.5 \angle 45^\circ + (2 \angle 5^\circ)/\,(2.828 \angle -45^\circ) = 1.768 + j\,1.768 + 0.7072 \angle 50^\circ$

$= 1.768 + j\,1.768 + 0.4546 + j\,0.5418$

$= 2.224 + j\,2.310 = \boxed{3.207 \angle 46.09^\circ}$

26.

(a) $5\angle -110° = \boxed{-1.7101 - j4.698}$

(b) $6e^{j160°} = \boxed{-5.638 + j2.052}$

(c) $(3 + j6)(2\angle 50°) = \boxed{-5.336 + j12.310}$

(d) $-100 - j40 = \boxed{107.70\angle -158.20°}$

(e) $2\angle 50° + 3\angle -120° = \boxed{1.0873\angle -101.37°}$

27.

(a) $40\angle -50° - 18\angle 25° = \boxed{39.39\angle -76.20°}$

(b) $3 + \frac{2}{j} + \frac{2 - j5}{1 + j2} = \boxed{4.050^{-}\angle -69.78°}$

(c) $(2.1\angle 25°)^3 = 9.261\angle 75° = \boxed{2.397 + j8.945^{+}}$

(d) $0.7e^{j0.3} = 0.7\angle 0.3^{\text{rad}} = \boxed{0.6687 + j0.2069}$

28.

$$i_c = 20e^{(40t+30°)} \text{ A} \therefore v_c = 100\int 20e^{j(40t+30°)}dt$$

$$v_c = -j50e^{j(40t+30°)},\ i_R = -j10e^{j(40t+30°)} \text{ A}$$

$$\therefore i_L = (20 - j10)e^{j(40t+30°)}, v_L = j40 \times 0.08(20 - j10)e^{j(40t+30°)}$$

$$\therefore v_L = (32 + j64)e^{j(40t+30°)} \text{ V} \therefore v_s = (32 + j64 - j50)e^{j(40t+30°)}$$

$$\therefore v_s = \boxed{34.93e^{j(40t-53.63°)} \text{ V}}$$

29.

$$i_L = 20e^{j(10t+25°)} \text{ A}$$

$$v_L = 0.2\frac{d}{dt}[20e^{j(10t+25°)}] = j40e^{(10t=25°)}$$

$$v_R = 80e^{j(10t+25°)}$$

$$v_s = (80 + j40)e^{j(10t+25°)}, i_c = 0.08(80 + j40)\, j10e^{j(10t+25°)}$$

$$\therefore i_c = (-32 + j64)e^{j(10t+25°)} \;\therefore i_s = (-12 + j64)e^{j(10t+25°)}$$

$$\therefore i_s = \boxed{65.12e^{j(10t+125.62°)} \text{ A}}$$

30. $80\cos(500t - 20°)\text{ V} \rightarrow 5\cos(500t + 12°)\text{ A}$

(a) $v_s = 40\cos(500t + 10°) \therefore i_{out} = \boxed{2.5\cos(500t + 42°)\text{ A}}$

(b) $v_s = 40\sin(500t + 10°) = 40\cos(500t - 80°)$

$\therefore i_{out} = \boxed{2.5\cos(500t - 48°)\text{ A}}$

(c) $v_s = 40e^{j(500t+10°)} = 40\cos(500t + 10°)$

$+ j40\sin(500t + 10°) \therefore i_{out} = \boxed{2.5e^{j(500t+42°)}\text{ A}}$

(d) $v_s = (50 + j20)e^{j500t} = 53.85^{+} e^{j21.80° + j500t}$

$\therefore i_{out} = \boxed{3.366e^{j(500t+53.80°)}\text{ A}}$

31.

(a) $12\sin(400t+110°)\text{ A} \rightarrow \boxed{12\angle 20°\text{A}}$

(b) $-7\sin 800t - 3\cos 800t \rightarrow j7-3$
$= -3 + j7 = \boxed{7.616\angle 113.20°\text{ A}}$

(c) $4\cos(200t-30°) - 5\cos(200t+20°)$
$\rightarrow 4\angle -30° - 5\angle 20° = \boxed{3.910\angle -108.40°\text{ A}}$

(d) $\omega = 600,\ t = 5\text{ms}: 70\angle 30°\text{ V}$
$\rightarrow 70\cos(600\times 5\times 10^{-3\text{rad}} + 30°) = \boxed{-64.95\ \text{ V}}$

(e) $\omega = 600,\ t = 5\text{ms}: 60 + j40\text{ V} = 72.11\angle 146.3°$
$\rightarrow 72.11\cos(3^{\text{rad}} + 146.31°) = \boxed{53.75\text{ V}}$

32. $\omega = 4000,\ t = 1\text{ms}$

(a) $\mathbf{I}_x = 5\angle -80° \text{ A}$

$\therefore i_x = 5\cos(4^{\text{rad}} - 80°) = \boxed{-4.294 \text{ A}}$

(b) $\mathbf{I}_x = -4 + j1.5 = 4.272\angle 159.44° \text{ A}$

$\therefore i_x = 4.272\cos(4^{\text{rad}} + 159.44°) = \boxed{3.750^{-} \text{ A}}$

(c) $v_x(t) = 50\sin(250t - 40°)$

$= 50\cos(250t - 130°) \rightarrow \mathbf{V}_x = \boxed{50\angle -130° \text{ V}}$

(d) $v_x = 20\cos 108t - 30\sin 108t$

$\rightarrow 20 + j30 = \boxed{36.06\angle 56.31° \text{ V}}$

(e) $v_x = 33\cos(80t - 50°) + 41\cos(80t - 75°)$

$\rightarrow 33\angle -50° + 41\angle -75° = \boxed{72.27\angle -63.87° \text{ V}}$

33. $\mathrm{V}_1 = 10\angle 90°\,\mathrm{mV},\ \omega = 500;\ \mathrm{V}_2 = 8\angle 90°\,\mathrm{mV},$

$\omega = 1200,\ \mathrm{M\ by}\ -5,\ t = 0.5\mathrm{ms}$

$$v_{out} = (-5)\ [10\cos(500 \times 0.5 \times 10^{-3\mathrm{rad}} + 90°)$$

$$+ 8\cos(1.2 \times 0.5 + 90°)]$$

$$= 50\sin 0.25^{\mathrm{rad}} + 40\sin 0.6^{\mathrm{rad}} = \boxed{34.96\mathrm{mV}}$$

34. Begin with the inductor:

$(2.5\angle 40^o)\,(j500)\,(20\times 10^{-3}) = 25\angle 130^o$ V across the inductor and the 25-Ω resistor.
The current through the 25-Ω resistor is then $(25\angle 130^o) / 25 = 1\angle 130^o$ A.

The current through the unknown element is therefore $2.5\angle 40^o + 1\angle 130^o =$
$2.693\angle 61.80^o$ A; this is the same current through the 10-Ω resistor as well.
Armed with this information, KVL provides that

$$\mathbf{V}_s = 10(26.93\angle 61.8^o) + (25\angle -30^o) + (25\angle 130^o) = 35.47\angle 58.93^o$$

and so $\boxed{v_s(t) = 35.47 \cos(500t + 58.93^o) \text{ V.}}$

35. $\omega = 5000$ rad/s.

(a) The inductor voltage = $48\angle 30^\circ = j\omega L\,\mathbf{I}_L = j(5000)(1.2\times10^{-3})\,\mathbf{I}_L$
So $\mathbf{I}_L = 8\angle{-60}^\circ$ and the total current flowing through the capacitor is
$10\angle 0^\circ - \mathbf{I}_L = 9.165\angle 49.11^\circ$ A and the voltage $\mathbf{V}_1$ across the capacitor is

$\mathbf{V}_1 = (1/j\omega C)(9.165\angle 49.11^\circ) = -j2\,(9.165\angle 49.11^\circ) = 18.33\angle{-40.89}^\circ$ V.

Thus, $\boxed{v_1(t) = 18.33\cos(5000t - 40.89^\circ)\text{ V.}}$

(b) $\mathbf{V}_2 = \mathbf{V}_1 + 5(9.165\angle 49.11^\circ) + 60\angle 120^\circ = 75.88\angle 79.48^\circ$ V
$\therefore \boxed{v_2(t) = 75.88\cos(5000t + 79.48^\circ)\text{ V}}$

(c) $\mathbf{V}_3 = \mathbf{V}_2 - 48\angle 30^\circ = 75.88\angle 79.48^\circ - 48\angle 30^\circ = 57.70\angle 118.7^\circ$ V
$\therefore \boxed{v_3(t) = 57.70\cos(5000t + 118.70^\circ)\text{ V}}$

36. $\mathbf{V}_R = 1\angle 0^o$ V, $\mathbf{V}_{series} = (1 + j\omega - j/\omega)(1\angle 0^o)$

$V_R = 1$ and $V_{series} = \sqrt{1+(\omega - 1/\omega)^2}$

We desire the frequency w at which $V_{series} = 2V_R$ or $V_{series} = 2$

Thus, we need to solve the equation $1+(\omega - 1/\omega)^2 = 4$

or $\omega^2 - \sqrt{3}\,\omega - 1 = 0$

Solving, we find that $\boxed{\omega = 2.189 \text{ rad/s.}}$

37. With an operating frequency of $\omega = 400$ rad/s, the impedance of the 10-mH inductor is $j\omega L = j4\ \Omega$, and the impedance of the 1-mF capacitor is $-j/\omega C = -j2.5\ \Omega$.

$\therefore \mathrm{V}_c = 2\angle 40°(-j2.5) = 5\angle -50°\ \mathrm{A}$

$\therefore \mathrm{I}_L = 3 - 2\angle 40° = 1.9513\angle -41.211°\ \mathrm{A}$

$\therefore \mathrm{V}_L = 4 \times 1.9513\angle 90° - 4.211° = 7.805^{+}\angle 48.79°\ \mathrm{V}$

$\therefore \mathrm{V}_x = \mathrm{V}_L - \mathrm{V}_c = 7.805^{+}\angle 48.79° - 5\angle -50°$

$\therefore \mathrm{V}_x = 9.892\angle 78.76°\ \mathrm{V}$, $\boxed{v_x = 9.892\cos(400t + 78.76°)\ \mathrm{V}}$

38.

If $I_{si} = 2\angle 20°\,\text{A},\ I_{s2} = 3\angle -30°\ \text{A} \rightarrow V_{out} = 80\angle 10°\,\text{V}$

$I_{s1} = I_{s2} = 4\angle 40°\,\text{A} \rightarrow V_{out} = 90 - j30\,\text{V}$

Now let $I_{s1} = 2.5\angle -60°\,\text{A}$ and $I_{s2} = 2.5\angle 60°\,\text{A}$

Let $V_{out} = AI_{s1} + BI_{s2}\ \therefore 80\angle 10° = A(2\angle 20°) + B(3\angle -30°)$

and $90 - j30 = (A+B)(4\angle 40°)\ \therefore A + B = \dfrac{90 - j30}{4\angle 40°} = 12.415^{+} - j20.21$

$\therefore \dfrac{80\angle 10°}{2\angle 20°} = A + B\dfrac{3\angle -30°}{2\angle 20}\ \therefore A = 40\angle -10° - B(1.5\angle -50°)$

$\therefore 12.415^{+} - j20.21 - B = 40\angle -10° - B(1.5\angle -50°)$

$\therefore 12.415^{+} - j20.21 - 40\angle -10° = B(1 - 1.5\angle -50°)$

$= B(1.1496\angle +88.21°)$

$\therefore B = \dfrac{30.06\angle -153.82°}{1.1496\angle +88.21°} = 26.148\angle 117.97°$

$\therefore A = 12.415^{+} - j20.21 - 10.800 + j23.81$

$= 49.842\angle -60.32°$

$V_{out} = (49.842\angle -60.32°)(2.5\angle -60°)$

$+(26.15\angle 117.97°)(2.5\angle 60°)$

$= \boxed{165.90\angle -140.63°\,\text{V}}$

39. We begin by noting that the series connection of capacitors can be replaced by a single equivalent capacitance of value $C = \dfrac{1}{1 + \frac{1}{2} + \frac{1}{3}} = 545.5\ \mu\text{F}$. Noting $\omega = 2\pi f$,

(a) $\omega = 2\pi$ rad/s, therefore $\mathbf{Z}_C = -j/\omega C = \dfrac{-j10^6}{2\pi(545.5)} = \boxed{-j291.8\ \Omega}$.

(b) $\omega = 200\pi$ rad/s, therefore $\mathbf{Z}_C = -j/\omega C = \dfrac{-j10^6}{200\pi(545.5)} = \boxed{-j2.918\ \Omega}$.

(c) $\omega = 2000\pi$ rad/s, therefore $\mathbf{Z}_C = -j/\omega C = \dfrac{-j10^6}{2000\pi(545.5)} = \boxed{-j291.8\ \text{m}\Omega}$.

(d) $\omega = 2\times10^9\pi$ rad/s, therefore $\mathbf{Z}_C = -j/\omega C = \dfrac{-j10^6}{2\times10^9\pi(545.5)} = \boxed{-j291.8\ \text{n}\Omega}$.

40. We begin by noting that the parallel connection of inductors can be replaced by a single equivalent inductance of value $L = \frac{1}{1+5} = \frac{5}{6}$ nH. In terms of impedance, then, we have

$$\mathbf{Z} = \frac{5\left(j\omega\frac{5}{6}\times10^{-9}\right)}{5 + j\omega\frac{5}{6}\times10^{-9}}$$

Noting $\omega = 2\pi f$,

(a) $\omega = 2\pi$ rad/s, therefore $\boxed{\mathbf{Z} = j5.236\times10^{-9}\ \Omega}$ (the real part is essentially zero).

(b) $\omega = 2\times10^{3}\pi$ rad/s, therefore $\boxed{\mathbf{Z} = 5.483\times10^{-12} + j5.236\times10^{-6}\ \Omega.}$

(c) $\omega = 2\times10^{6}\pi$ rad/s, therefore $\boxed{\mathbf{Z} = 5.483\times10^{-6} + j5.236\times10^{-6}\ \Omega.}$

(d) $\omega = 2\times10^{9}\pi$ rad/s, therefore $\boxed{\mathbf{Z} = 2.615 + j2.497\ \Omega.}$

(e) $\omega = 2\times10^{12}\pi$ rad/s, therefore $\boxed{\mathbf{Z} = 5 + j4.775\times10^{-3}\ \Omega\ .}$

41.

(a) $\omega = 800: 2\mu\text{F} \to -j625,\ 0.6\text{H} \to j480$

$$\therefore Z_{in} = \frac{300(-j625)}{300 - j625} + \frac{600(j480)}{600 + j480}$$

$$= \boxed{478.0 + j175.65\Omega}$$

(b) $$\omega = 1600: Z_{in} = \frac{300(-j312.5)}{300 - j312.5}$$

$$+ \frac{600(j960)}{600 + j960} = \boxed{587.6 + j119.79\Omega}$$

42.

At $\omega = 100$ rad/s, 2 mF $\rightarrow$ $-j5\ \Omega$; 0.1 H $\rightarrow$ $j10\ \Omega$.

(a) $(10 + j10) \| (-j5) = \dfrac{50 - j50}{10 + j5} = \dfrac{10 - j10}{2 + j1}\ \dfrac{2 - j1}{2 - j1}$

$= 2 - j6\,\Omega \therefore \mathrm{Z}_{in} = 20 + 2 - j6 = \boxed{22 - j6\ \Omega}$

(b) $\mathrm{SC}a,b: 20 \| 10 = 6.667,\ (6.667 - j5) \| j10$

$= \dfrac{50 + j66.67}{6.667 + j5} = \dfrac{150 + j200}{20 + j15} = \dfrac{30 + j40}{4 + j3} \times \dfrac{4 - j3}{4 - j3}$

$= Z_{in} \therefore Z_{in} (1.2 + j1.6)(4 - j3) = \boxed{9.6 + j2.8\,\Omega}$

43.

$$\omega = 800: 2\mu\text{F} \rightarrow -j625,\ 0.6\text{H} \rightarrow j480$$

$$\therefore Z_{in} = \frac{300(-j625)}{300 - j625} + \frac{600(j480)}{600 + j480}$$

$$= 478.0 + j175.65\Omega$$

$$\therefore \mathbf{I} = \frac{120}{478.0 + j175.65} \times \frac{-j625}{300 - j625}$$

$$\text{or } \mathbf{I} = 0.2124\angle -45.82^\circ \text{ A}$$

Thus, $\boxed{i(t) = 212.4 \cos(800t - 45.82^\circ) \text{ mA.}}$

44.

(a) $3\Omega + 2\text{mH}: \mathbf{V} = (3\angle -20°)\ (3 + j4) = \boxed{15\angle 33.13°\ \text{V}}$

(b) $3\Omega + 125\mu\text{F}: \mathbf{V} = (3\angle -20°)\ (3 - j4) = \boxed{15\angle -73.3°\ \text{V}}$

(c) $3\Omega\ 2\text{mH}\ 125\mu\text{F}: \mathbf{V} = (3\angle -20°)\ 3 = \boxed{9\angle -20°\ \text{V}}$

(d) same: $\omega = 4000 \therefore \mathbf{V} = (3\angle -20°)\ (3 + j8 - j2)$

$\therefore \mathbf{V} = (3\angle -20°)\ (3 + j6) = \boxed{20.12\angle 43.43°\ \text{V}}$

45.

(a) $\mathrm{C} = 20\mu\mathrm{F},\ \omega = 100$

$$\mathbf{Z}_{in} = \frac{1}{\frac{1}{200} + \frac{1}{j1000} + j1000 \times 20 \times 10^{-6}} = \frac{1}{0.005 - j0.01 + j0.002}$$

$$\therefore \mathbf{Z}_{in} = \frac{1}{0.005 + j0.001} = \boxed{196.12\angle -11.310°\Omega}$$

(b) $$\omega = 100 \text{ rad/s} \therefore \mathbf{Z}_{in} = \frac{1}{0.005 - j0.001 + j100C}$$

$$\left|\mathbf{Z}_{in}\right| = 125 = \frac{1}{\sqrt{0.005^2 + (100C - 0.001)^2}}$$

or $64 \times 10^{-6} = 0.005^2 + (100C - 0.001)^2$

so $6.245 \times 10^{-3} = \sqrt{39 \times 10^{-6}} = 100C - 0.001$

or $\boxed{C = 72.45\ \mu\mathrm{F}}$

(c) $$\mathrm{C} = 20\mu\mathrm{F} \therefore \mathbf{Z}_{in} = \frac{1}{0.0005 - j0.1/\omega + j2 \times 10^{-5}\omega} = 100\angle = \frac{1}{0.01\angle}$$

$$\therefore 0.005^2 + \left(2 \times 10^{-5}\omega - \frac{0.1}{\omega}\right)^2 = 0.0001, \left(2 \times 10^{-5} - \frac{0.1}{\omega}\right)^2 = 7.5 \times 10^{-5}$$

$$\therefore 2 \times 10^{-5} - \frac{0.01}{\omega} \mp 866.0 \times 10^{-5} = 0 \therefore 2 \times 10^{-5}\omega^2 \mp 866.0 \times 10^{-5}\omega - 0.1 = 0$$

$$\text{use } - \text{ sign: } \omega = \frac{866.0 \times 10^{-5} \pm \sqrt{7.5 \times 10^{-5} + 8 \times 10^{-6}}}{4 \times 10^{-5}} = 444.3 \text{ and } < 0$$

$$\text{use } + \text{ sign: } \omega = \frac{-866.0 \times 10^{-5} \pm \sqrt{7.5 \times 10^{-5} + 8 \times 10^{-6}}}{4 \times 10^{-5}} = 11.254 \text{ and } <0$$

$$\therefore \omega = \boxed{11.254} \text{ and } \boxed{444.3 \text{ rad/s}}$$

46.

(a)
$$\left|\frac{1}{\frac{1}{jx}+\frac{1}{30}}\right| = 25 = \frac{1}{0.04} \therefore \frac{1}{900}+\frac{1}{x^2} = 0.0016$$
$$\therefore \text{X} = 45.23\,\Omega = 0.002\omega, \boxed{\omega = 2261\,\text{rad/s}}$$

(b)
$$\angle \text{Y}_{in} = -25° = \angle \quad \text{f}\left(\frac{1}{30} - j\frac{1}{x}\right) = \tan^{-1}\frac{-30}{x}$$
$$\therefore x = 64.34 = 0.02\omega,\ \omega = \boxed{3217\text{rad/s}}$$

(c)
$$Z_{in} = \frac{30(j0.02\omega)}{30+j0.02\omega} \times \frac{30-j0.092\omega}{30-j0.02\omega} = \frac{0.012\omega^2 + j18\omega}{900+0.0004\omega^2}$$
$$\therefore 0.012\omega^2 = 25\ (900+0.0004\omega^2)$$
$$\therefore 0.012\omega^2 = 0.01\omega^2 + 22,500,\ \omega = \boxed{3354\text{rad/s}}$$

(d)
$$18\omega = 10(900+0.0004\omega^2), 0.004\omega^2 - 18\omega + 9000 = 0,$$
$$\omega^2 - 4500\omega + 2.25\times10^6 = 0$$
$$\omega = \frac{4500 \pm \sqrt{20.25\times10^6 - 9\times10^6}}{2} = \frac{4500\pm3354}{2} = \boxed{572.9,\ 3927\text{rad/s}}$$

47. With an operating frequency of $\omega = 400$ rad/s, the impedance of the 10-mH inductor is $j\omega L = j4\ \Omega$, and the impedance of the 1-mF capacitor is $-j/\omega C = -j2.5\ \Omega$.

$$\therefore \mathrm{V}_c = 2\angle 40°(-j2.5) = 5\angle -50°\ \mathrm{A}$$

$$\therefore \mathrm{I}_L = 3 - 2\angle 40° = 1.9513\angle -41.211°\ \mathrm{A}$$

$$\mathrm{I}_L = \frac{2\angle 40°(\mathrm{R}_2 - j2.5)}{\mathrm{R}_1 + j4}$$

$$\therefore \mathrm{R}_1 + j4 = \frac{2\angle 40°(\mathrm{R}_2 - j2.5)}{1.9513\angle -41.21°}$$

$$= 1.0250\angle 81.21°(R_2 - j2.5)$$

$$= \mathrm{R}_2(1.0250\angle 81.21°) + 2.562\angle -8.789°$$

$$= 0.15662\mathrm{R}_2 + j1.0130\mathrm{R}_2 + 2.532 - j0.3915$$

$$\therefore \mathrm{R}_1 = 2.532 + 0.15662\mathrm{R}_2,\ 4 = 1.0130\mathrm{R}_2 - 0.395^-$$

$$\therefore \boxed{\mathrm{R}_2 = 4.335^+\Omega,\ \mathrm{R}_1 = 3.211\Omega}$$

48. $\omega = 1200$ rad/s.

(a) $\omega = 1200$

$$\mathbf{Z}_{in} = \frac{-jx\times(200+j80)}{200+j(80-x)} = \frac{(80x - j200x)[200+j(x-80)]}{40,000+6400-160x+x^2}$$

$$X_{in} = 0 \therefore -40,000x + 80x^2 - 6400x = 0$$

$$\therefore 46,400 = 80x,\ x = 580\,\Omega = \frac{1}{1200C} \therefore \boxed{C = 1.437\ \mu F}$$

(b) $$\mathbf{Z}_{in} = \frac{80X - j200X}{200+j(80-X)} \left|\mathbf{Z}_{in}\right| = 100$$

$$\therefore \frac{6400X^2 + 40,000X^2}{40,000+6400-160X+X^2} = 10,000$$

$$\therefore 0.64X^2 + 4X^2 = X^2 - 160X + 46,400$$

$$\therefore 3.64X^2 + 160X - 46,400 = 0,$$

$$X = \frac{-160 \pm \sqrt{25,600+675,600}}{7.28} = \frac{-160\pm 837.4}{7.28}$$

$$\therefore X = 93.05^{-}(>0) = \frac{1}{1200C} \therefore \boxed{C = 8.956\mu F}$$

49. At $\omega = 4$ rad/s, the 1/8-F capacitor has an impedance of $-j/\omega C = -j2\ \Omega$, and the 4-H inductor has an impedance of $j\omega L = j16\ \Omega$.

(a) Terminals *ab* open circuited: $\mathbf{Z}_{in} = 8 + j16 \parallel (2 - j2) = \boxed{10.56 - j1.92\ \Omega}$

(b) Terminals *ab* short-circuited: $\mathbf{Z}_{in} = 8 + j16 \parallel 2 = \boxed{9.969 + j0.2462\ \Omega}$

50. $f = 1$ MHz, $\omega = 2\pi f = 6.283$ Mrad/s

2	μF	$\rightarrow -j0.07958\ \Omega$	$= \mathbf{Z}_1$
3.2	μH	$\rightarrow j20.11\ \Omega$	$= \mathbf{Z}_2$
1	μF	$\rightarrow -j0.1592\ \Omega$	$= \mathbf{Z}_3$
1	μH	$\rightarrow j6.283\ \Omega$	$= \mathbf{Z}_4$
20	μH	$\rightarrow j125.7\ \Omega$	$= \mathbf{Z}_5$
	200 pF	$\rightarrow -j795.8\ \Omega$	$= \mathbf{Z}_6$

The three impedances at the upper right, $\mathbf{Z}_3$, 700 kΩ, and $\mathbf{Z}_3$ reduce to $-j0.01592\ \Omega$

Then we form $\mathbf{Z}_2$ in series with $\mathbf{Z}_{eq}$: $\mathbf{Z}_2 + \mathbf{Z}_{eq} = j20.09\ \Omega$.

Next we see $10^6 \parallel (\mathbf{Z}_2 + \mathbf{Z}_{eq}) = j20.09\ \Omega$.

Finally, $\mathbf{Z}_{in} = \mathbf{Z}_1 + \mathbf{Z}_4 + j20.09 = \boxed{j26.29\ \Omega.}$

51. As in any true design problem, there is more than one possible solution. Model answers
f ollow:

(a) Using at least 1 inductor, $\omega = 1$ rad/s. $\mathbf{Z} = 1 + j4\ \Omega$.

Construct this using a single 1 Ω resistor in series with a 4 H inductor.

(b) Force $jL = j/C$, so that $C = 1/L$. Then we construct the network using a single 5 Ω resistor, a 2 H inductor, and a 0.5 F capacitor, all in series (any values for these last two will suffice, provided they satisfy the C = 1/L requirement).

(c) $\mathbf{Z} = 7\angle 80^\circ\ \Omega$. $R = \text{Re}\{\mathbf{Z}\} = 7\cos 80^\circ = 1.216\ \Omega$, and $X = \text{Im}\{\mathbf{Z}\} = 7\sin 80^\circ = 6.894\ \Omega$.

We can obtain this impedance at 100 rad/s by placing a resistor of value 1.216 Ω in series with an inductor having a value of $L = 6.894/\omega = 68.94$ mH.

(d) A single resistor having value R = 5 Ω is the simplest solution.

52. As in any true design problem, there is more than one possible solution. Model answers
f ollow:

(a) $1 + j4$ kΩ at $\omega = 230$ rad/s may be constructed using a 1 kΩ resistor in series with an inductor L and a capacitor C such that $j230L - j/(230C) = 4000$. Selecting arbitrarily C = 1 F yields a required inductance value of L = 17.39 H.

Thus, one design is a 1 kΩ resistor in series with 17.39 H in series with 1 F.

(b) To obtain a purely real impedance, the reactance of the inductor must cancel the reactance of the capacitor, In a series string, this is obtained by meeting the criterion ωL
= $1/\omega C$, or $L = 1/\omega^2 C = 1/100C$.

Select a 5 MΩ resistor in series with 1 F in series with 100 mH.

(c) If $\mathbf{Z} = 80\angle{-22^\circ}$ Ω is constructed using a series combination of a single resistor R and single capacitor C, $R = \text{Re}\{\mathbf{Z}\} = 80\cos(-22^\circ) = 74.17$ Ω. $X = -1/\omega C = \text{Im}\{\mathbf{Z}\} =$ $80\sin(-22^\circ) = -29.97$ Ω. Thus, C = 667.3 μF.

(d) The simplest solution, independent of frequency, is a single 300 Ω resistor.

53. Note that we may replace the three capacitors in parallel with a single capacitor having value $10^{-3} + 2\times10^{-3} + 4\times10^{-3} = 7$ mF.

(a) $\omega = 4\pi$ rad/s. $\mathbf{Y} = j4\pi C = \boxed{j87.96 \text{ mS}}$

(b) $\omega = 400\pi$ rad/s. $\mathbf{Y} = j400\pi C = \boxed{j8.796 \text{ S}}$

(c) $\omega = 4\pi\times10^3$ rad/s. $\mathbf{Y} = j4\pi\times10^3 C = \boxed{j879.6 \text{ S}}$

(d) $\omega = 4\pi\times10^{11}$ rad/s. $\mathbf{Y} = j4\pi\times10^{11} C = \boxed{j8.796\times10^9 \text{ S}}$

54. (a) $\boxed{\text{Susceptance is 0}}$

(b) $B = \omega C = \boxed{100\text{ S}}$

(c) $\mathbf{Z} = 1 + j100\ \Omega$, so $\mathbf{Y} = \dfrac{1}{1+j100} = \dfrac{1-j100}{1+100^2} = G + jB$, where $\boxed{B = -9.999\text{ mS.}}$

55.

$2\,\text{H} \rightarrow j2,\ 1\text{F} \rightarrow -j1$ Let $\mathbf{I}_{\in} = \angle 0°\,\text{A}$

$\therefore \mathbf{V}_L = j2\text{V} \therefore \mathbf{I}_c = \mathbf{I}_{in} + 0.5\,\mathbf{V}_L = 1 + j1$

$\therefore \mathbf{V}_{in} = j2 + (1 + j1)(-j1) = 1 + j1$

$$\therefore \mathbf{V}_{in} = \frac{1\angle 0°}{\mathbf{V}_{in}} = \frac{1}{1 + j1}\,\frac{1 - j1}{1 - j1} = 0.5 - j0.5$$

$$\text{Now}\ 0.5\ \text{S} \rightarrow \boxed{2\,\Omega},\ -j0.5\ \text{S} = \frac{1}{j2} \rightarrow \boxed{2\ \text{H}}$$

56.

(a) $\omega = 500,\ Z_{inRLC} = 5 + j10 - j1 = 5 + j9$

$$\therefore \mathrm{Y}_{inRLC} = \frac{1}{5+j9} = \frac{5-j9}{106} \therefore \mathrm{Y}_c = \frac{9}{106} = 500\mathrm{C}$$

$$\therefore \mathrm{C} = \frac{9}{53{,}000} = \boxed{169.8\ \mu\mathrm{F}}$$

(b) $$\mathrm{R}_{in,ab} = \frac{106}{5} = \boxed{21.2\,\Omega}$$

(c) $\omega = 1000$ rad/s $\therefore$

$\mathbf{Z}_S = 5 + j2 - j5 = 5 - j3 = 5.831\angle -30.96^\circ\ \Omega$

and $\mathbf{Z}_C = -j58.89\ \Omega$.

Thus,

$$\mathbf{Y}_{in,ab} = \frac{1}{\mathbf{Z}_S} + \frac{1}{\mathbf{Z}_C} = \boxed{0.1808\angle 35.58^\circ\ \mathrm{S}}$$

$$= \boxed{147.1 + j105.2\ \mathrm{mS}}$$

57.

(a) $R_{in} = 550\,\Omega : Z_{in} = 500 + \dfrac{j0.1\omega}{100 + j0.001\omega}$

$\therefore Z_{in} = \dfrac{50,000 + j0.6\omega}{100 + j0.001\omega} \times \dfrac{100 - j0.001\omega}{100 - j0.001\omega}$

$\therefore Z_{in} = \dfrac{5\times10^6 + 0.0006\omega^2 + j(60\omega - 50\omega)}{10^4 + 10^{-6}\omega^2}$

$\therefore R_{in} = \dfrac{5\times10^6 + 0.006\omega^2}{10^4 + 10^{-6}\omega^2} = 550 \therefore 5.5\times10^6$

$+5.5\times10^{-4}\omega^2 = 5\times10^6 \times 10^{-4}\omega^2$

$\therefore 0.5\times10^{-4}\omega^2 = 0.5\times10^6,\ \omega^2 = 10^{10},\ \omega = \boxed{10^5\text{ rad/s}}$

(b) $X_{in} = 50\,\Omega = \dfrac{10\omega}{10^4 + 10^{-6}\omega^2} = 0.5\times10^6 + 0.5\times10^{-4}\omega^2 - 10\omega$

$= 0,\ \omega^2 - 2\times10^5\omega + 10^{10} = 0$

$\therefore \omega = \dfrac{2\times10^5 \pm \sqrt{4\times10^{10} - 4\times10^{10}}}{2} = 10^5 \therefore \omega = \boxed{10^5\text{ rad/s}}$

(c) $G_{in} = 1.8\times10^{-3} : Y_{in} = \dfrac{100 + j0.001\omega}{50,000 + j0.6\omega} \times \dfrac{50,000 - j0.6\omega}{50,000 - j0.6\omega}$

$= \dfrac{5\times10^6 + 6\times10^{-4}\omega^2 + j(50\omega - 6\omega)}{25\times10^8 + 0.36\omega^2}$

$\therefore 1.8\times10^3 = \dfrac{5\times10^6 + 6\times10^{-4}\omega^2}{25\times10^8 + 0.36\omega^2}$

$\therefore 5\times10^6 + 6\times10^{-4}\omega^2 = 4.5\times10^6 + 648\times10^{-6}\omega^2$

$\therefore 0.5\times10^6 = 48\times10^{-6}\omega^2 \therefore \omega = \boxed{102.06\text{ krad/s}}$

(d) $B_{in} = 1.5\times10^{-4} = \dfrac{-10\omega}{25\times10^8 + 0.36\omega^2}$

$\therefore 10\omega = 37.5\times10^4 + 54\times10^{-6}\omega^2$

$\therefore 54\times10^{-6}\omega^2 - 10\omega + 37.5\times10^4 = 0,$

$\omega = 10 \pm \dfrac{\sqrt{100-81}}{108\times10^{-6}} = \boxed{52.23\text{ and }133.95\text{ krad/s}}$

58.

(a) $$V_1 = \frac{I_1}{Y_1} = \frac{0.1\angle 30°}{(3+j4)10^{-3}} = 20\angle -23.13° \therefore |V_1| = \boxed{20\,\text{V}}$$

(b) $$V_2 = V_1 \therefore |V_2| = \boxed{20\text{V}}$$

(c) $$I_2 = Y_2\ V_2 = (5+j2)10^{-3} \times 20\angle -23.13° = 0.10770\angle -1.3286°\,\text{A}$$
$$\therefore I_3 = I_1 + I_2 = 0.1\angle 30° + 0.10770\angle -1.3286° = 0.2\angle 13.740°\,\text{A}$$
$$\therefore V_3 = \frac{I_3}{Y_3} = \frac{0.2\angle 13.740°}{(2-j4)10^{-3}} = 44.72\angle 77.18°\ \text{V} \therefore |V_3| = \boxed{44.72\text{V}}$$

(d) $$V_{in} = V_1 + V_3 + 20\angle -23.13° + 44.72\angle 77.18° = 45.60\angle 51.62°$$
$$\therefore |V_{in}| = \boxed{45.60\text{V}}$$

59.

(a) $50\mu\text{F} \rightarrow -j20\,\Omega \therefore \text{Y}_{in} = 0.1 + j0.05$

$$\text{Y}_{in} = \frac{1}{\text{R}_1 - j\dfrac{1000}{\text{C}}} \therefore \text{R}_1 - j\frac{1000}{\text{C}} = \frac{1}{0.1 + j0.05} = 8 - j4$$

$$\therefore \text{R}_1 = 8\,\Omega \text{ and } \text{C}_1 = \frac{1}{4\omega} = \boxed{250\,\mu\text{F}}$$

(b) $$\omega = 2000 : 50\mu\text{F} \rightarrow -j10\,\Omega \;\therefore \text{Y}_{in} = 0.1 + j0.1 = \frac{1}{\text{R}_1 - j\dfrac{500}{\text{C}_1}}$$

$$\therefore \text{R}_1 - j\frac{500}{\text{C}_1} = 5 - j5 \therefore \text{R}_1 = 5\,\Omega,\ \text{C}_1 = \boxed{100\,\mu\text{F}}$$

60.

(a) $Z_{in} = 1 + \frac{10}{j\omega} = \frac{10 + j\omega}{j\omega}$

$\therefore Y_{in} \frac{j\omega}{10 + j\omega} \times \frac{10 - j\omega}{10 - j\omega}$

$$\boxed{\begin{aligned} &\therefore Y_{in} = \frac{\omega^2 + j10\omega}{\omega^2 + 100} \\ &G_{in} = \frac{\omega^2}{\omega^2 + 100},\ B_{in} = \frac{10\omega}{\omega^2 + 100} \end{aligned}}$$

ω	G_{in}	B_{in}
0	0	0
1	0.0099	0.0099
2	0.0385	0.1923
5	0.2	0.4
10	0.5	0.5
20	0.8	0.4
∞	1	0

61. As in any true design problem , there is m ore than one possible solution. Model answers f ollow:

(a) $\mathbf{Y} = 1 - j4$ S at $\omega = 1$ rad/s.
Construct this using a 1 S conductance in parallel with an induc tance L such that $1/\omega L = 4$, or L = 250 mH.

(b) $\mathbf{Y} = 200$ m S (purely real at $\omega = 1$ rad/s). This can be constructed using a 200 mS conductance (R = 5 Ω), in pa rallel with an ind uctor L and capacitor C such th at $\omega C - 1/\omega L = 0$. Arbitrarily selecting L = 1 H, we find that C = 1 F.

One solution therefore is a 5 Ω resistor in parallel with a 1 F capacitor in parallel with a 1 H inductor.

(c) $\mathbf{Y} = 7\angle 80^{o}$ µS = G + jB at $\omega = 100$ rad/s. G = Re{ $\mathbf{Y}$} = 7cos80^{o} = 1.216 S (an 822.7 mΩ resistor). B = Im {$\mathbf{Y}$} = 7sin8 0^{o} = 6.894 S. W e m ay rea lize th is suscep tance by placing a capacito r C in parallel with the resis tor such th at $j\omega C = j6.894$, or C = 68.94 mF.

One solution therefore is an 822.7 mΩ resistor in parallel with a 68.94 mF.

(d) The simplest solution is a single conductance G = 200 mS (a 5 Ω resistor).

62. As in any true design problem, there is more than one possible solution. Model answers follow:

(a) $\mathbf{Y} = 1 - j4$ pS at $\omega = 30$ rad/s.

Construct this using a 1 pS conductance (a 1 TΩ resistor) in parallel with an inductor L such that $-j4\times10^{-12} = -j/\omega L$, or L = 8.333 GH.

(b) We may realise a purely real admittance of 5 μS by placing a 5 μS conductance (a 200 kΩ resistor) in parallel with a capacitor C and inductance L such that $\omega C - 1/\omega L = 0$. Arbitrarily selecting a value of L = 2 H, we find a value of C = 1.594 μF.

One possible solution, then, is a 200 kΩ resistor in parallel with a 2 H inductor and a 1.594 μF capacitor.

(c) $\mathbf{Y} = 4\angle{-10^\circ}$ nS = G + jB at $\omega = 50$ rad/s. G = Re{**Y**} = $4\times10^{-9}\cos(-10^\circ) = 3.939$ nS (an 253.9 MΩ resistor). B = Im{**Y**} = $4\times10^{-9}\sin(-10^\circ) = -6.946\times10^{-10}$ S. We may realize this susceptance by placing an inductor L in parallel with the resistor such that $-j/\omega L = -j6.946\times10^{-10}$, or L = 28.78 μH.

One possible solution, then, is a 253.9 MΩ resistor in parallel with a 28.78 μH inductor.

(d) The simplest possible solution is a 60 nS resistor (a 16.67 MΩ resistor).

63.

$$-j5 = \frac{v_1}{3} + \frac{V_1 - V_2}{-j5} + \frac{v_1 - V_2}{j3},\ -j75 = 5V_1 + j3V_1 - j3V_2 - j5V_1 + j5V_2$$

$$\therefore (5 - j2)\,V_1 + j2V_2 = -j75 \qquad (1)$$

$$\frac{v_2 - V_1}{j3} + \frac{V_2 - V_1}{-j5} + \frac{V_2}{6} = 10$$

$$-j10V_2 + j10V_1 + j6V_2 - j6V_1 + 5V_2 = 300 \therefore\ j4V_1 + (5 - j4)\,V_2 = 300 \qquad (2)$$

$$\therefore V_2 = \frac{\begin{vmatrix} 5 - j2 & -j75 \\ j4 & 300 \end{vmatrix}}{\begin{vmatrix} 5 - j2 & j2 \\ j4 & 5 - j4 \end{vmatrix}} = \frac{1500 - j600 - 300}{17 - j30 + 8} = \frac{1200 - j600}{25 - j30} = \boxed{34.36\angle 23.63^\circ\ \text{V}}$$

64.

$$j3\mathrm{I}_B - j5(\mathrm{I}_B - \mathrm{I}_D) = 0 \therefore -2\mathrm{I}_B + j5\mathrm{I}_D = 0$$

$$3(\mathrm{I}_D + j5) - j5(\mathrm{I}_D - \mathrm{I}_B) + 6(\mathrm{I}_D + 10) = 0$$

$$\therefore j5\mathrm{I}_B + (9 - j5)\mathrm{I}_D = -60 - j15$$

$$\mathrm{I}_B = \frac{\begin{vmatrix} 0 & j5 \\ -60 - j15 & 9 - j5 \end{vmatrix}}{\begin{vmatrix} -j2 & j5 \\ j5 & 9 - j5 \end{vmatrix}} = \frac{-75 + j300}{15 - j18}$$

$$= \boxed{13.198\angle 154.23^\circ \text{ A}}$$

65.

$$v_{s1} = 20\cos 1000t \text{ V}, v_{s2} = 20\sin 1000t \text{ V}$$

$$\therefore \mathbf{V}_{s1} = 20\angle 0° \text{ V}, \ \mathbf{V}_{s2} = -j20\text{V}$$

$$0.01\text{H} \rightarrow j10\,\Omega, \ 0.1\text{mF} \rightarrow -j10\,\Omega$$

$$\therefore \frac{v_x - 20}{j10} + \frac{v_x}{25} + \frac{v_x + j20}{-j10} = 0, \ 0.04v_x + j2 - 2 = 0,$$

$$\mathbf{V}_x = 25(2 - j2) = 70.71\angle -45° \text{ V}$$

$$\therefore \boxed{v_x(t) = 70.71\cos(1000t - 45°)\text{ V}}$$

66.

(a) Assume $V_3 = 1\text{V} \therefore V_2 = 1 - j0.5\text{V},\ I_2 = 1 - j0.5\,\text{mA}$

$\therefore V_1 = 1 - j0.5 + (2 - j0.5)(-j0.5) = 0.75 - j1.5\text{V}$

$\therefore I_1 = 0.75 - j1.5\,\text{mA},\ \therefore I_{in} = 0.75 - j1.5 + 2 - j0.5 = 2.75 - j2\,\text{mA}$

$\therefore V_{in} = 0.75 - j1.5 - j1.5 + (2.75 - j2)(-j0.5)$

$= -0.25 - j2.875\,\text{V} \therefore V_3 = \dfrac{100}{-j0.25 - j2.875} = \boxed{34.65^{+}\angle 94.97°\text{V}}$

(b) $-j0.5 \rightarrow -jx$ Assume $\mathbf{V}_3 = \ \text{V} \therefore \mathbf{I}_3 = 1\text{A},$

$\mathbf{V}_2 = 1 - j\text{X},\ \mathbf{I}_2 = 1 - j\text{X}, \rightarrow \mathbf{I}_{12} = 2 - j\text{X}$

$\therefore \mathbf{V}_1 = 1 - j\text{X} + (2 - j\text{X})(-j\text{X}) = 1 - \text{X}^2 - j3\text{X},\ I_1 = 1 - \text{X}^2 - j3\text{X},\ \mathbf{I}_{in} = 3 - \text{X}^2 - j4X$

$\therefore \mathbf{V}_{in} = 1 - \text{X}^2 - j3\text{X} - 4\text{X}^2 + j\text{X}^3 - j3\text{X} = 1 - 5\text{X}^2 + j(\text{X}^3 - 6\text{X}) \therefore \text{X}^3 - 6\text{X} = 0$

$\therefore \text{X}^2 = 6,\ \text{X} = \sqrt{6},\ \mathbf{Z}_c = \boxed{-j2.449\,\text{k}\Omega}$

67. Define three clockwise mesh currents i_1, i_2, i_3 with i_1 in the left mesh, i_2 in the top right m esh, and i_3 in the bottom right mesh.

Mesh 1: $-10\angle 0^\circ + (1 + 1 - j0.25)\mathbf{I}_1 - \mathbf{I}_2 - (-j0.25)\mathbf{I}_3 = 0$

Mesh 2: $-\mathbf{I}_1 + (1 + 1 + j4)\mathbf{I}_2 - \mathbf{I}_3 = 0$

Mesh 3: $(-j0.25 + 1 + 1)\mathbf{I}_3 - \mathbf{I}_2 - (-j0.25\mathbf{I}_1) = 0$

$$\mathbf{I}_x = \frac{\begin{vmatrix} 2-j0.25 & -1 & 10 \\ -1 & 2+j4 & 0 \\ j0.25 & -1 & 0 \end{vmatrix}}{\begin{vmatrix} 2-j0.25 & -1 & j0.25 \\ -1 & 2+j4 & -1 \\ j0.25 & -1 & 2-j0.25 \end{vmatrix}}$$

$$\therefore \mathbf{I}_x = \frac{10(1+1-j0.5)}{j0.25(2-j0.5)+(-2+j0.25+j0.25)+(2-j0.25)(4+1-j0.5+j8-1)}$$

$$= \frac{20-j5}{8+j15} \therefore \mathbf{I}_x = 1.217\angle -75.96^\circ \text{ A}, \boxed{i_x(t) = 1.2127\cos(100t - 75.96^\circ)\text{ A}}$$

68.

$$\mathrm{V}_1 - 10 - j0.25\mathrm{V}_1 + j0.25\mathrm{V}_x + \mathrm{V}_1 - \mathrm{V}_2 = 0$$

$$\therefore (2 - j0.25)\,\mathrm{V}_1 - \mathrm{V}_2 + j0.25\,\mathrm{V}_x = 10$$

$$\mathrm{V}_2 - \mathrm{V}_1 + \mathrm{V}_2 - \mathrm{V}_x + j4\mathrm{V}_2 = 0$$

$$-\mathrm{V}_1 + (2 + j4)\,\mathrm{V}_2 - \mathrm{V}_x = 0$$

$$-j0.25\mathrm{V}_x + j0.25\mathrm{V}_1 + \mathrm{V}_x + \mathrm{V}_x - \mathrm{V}_2$$

$$\therefore\ j0.25\mathrm{V}_1 - \mathrm{V}_2 + (2 - j0.25)\,\mathrm{V}_x = 0$$

$$\mathrm{V}_x = \frac{\begin{vmatrix} 2 - j0.25 & -1 & 10 \\ -1 & 2 + j4 & 0 \\ j0.25 & -1 & 0 \end{vmatrix}}{\begin{vmatrix} -j0.25 & -1 & j0.25 \\ -1 & 2 + j4 & -1 \\ j0.25 & -1 & 2 - j0.25 \end{vmatrix}}$$

$$= \frac{10\,(1 + 1 - j0.5)}{j0.25\,(2 - j0.5) + (-2 + j0.25 + j0.25) + (2 - j0.25)\,(4 + 1 - j0.5 + j8 - 1)}$$

$$= \frac{20 - j5}{8 + j15} = 1.2127\angle -75.96°\ \mathrm{V}$$

$$\therefore \boxed{v_x = 1.2127\cos(100t - 75.96°)\ \mathrm{V}}$$

69.

(a) $R_1 = \infty,\ R_o = 0,\ A = -V_o / V_i >> 0$

$$I = \frac{V_1 + AV_i}{R_f} = j\omega C_1 (V_s - V_i)$$

$$\therefore V_i(1 + A + j\omega C_1 R_f) = j\omega C_1 R_f V_s$$

$$V_o = -AV_i \therefore -\frac{V_o}{A}(1 + A + j\omega C_1 R_f) = j\omega C_1 R_f V_s$$

$$\therefore \boxed{\frac{V_o}{V_s} = -\frac{j\omega C_1 R_f A}{1 + A + j\omega C_1 R_f}} \text{As } A \to \infty, \frac{V_o}{V_s} \to -j\omega C_1 R_f$$

(b) $$R_f \| C_f = \frac{1}{j\omega C_f + \frac{1}{R_f}} = \frac{R_f}{1 + j\omega C_f R_f}$$

$$I = \frac{(V_1 + AV_i)}{R_f}(1 + j\omega C_f R_f) = (V_s - V_i)\, j\omega C_1,\ V_o = -AV_i$$

$$\therefore V_i (1 + A)(1 + j\omega C_f R_f) = V_s j\omega C_1 R_f - j\omega C_1 R_f V_i,$$

$$V_i [(1 + A)(1 + j\omega C_f R_f) + j\omega C_1 R_f] = j\omega C_1 R_f V_s$$

$$\therefore -\frac{V_o}{A}[(1 + A)(1 + j\omega C_f R_f) + j\omega C_1 R_f] = j\omega C_1 R_f V_s$$

$$\therefore \boxed{\frac{V_o}{V_s} = \frac{-j\omega C_1 R_f A}{(1 + A)(1 + j\omega C_f R_f) + j\omega C_1 R_f}} \text{As } A \to \infty, \frac{V_o}{V_s} \to \frac{-j\omega C_1 R_f}{1 + j\omega C_f R_f}$$

70. Define the nodal voltage $v_1(t)$ at the junction between the two dependent sources. The voltage source may be replaced by a $3\angle\text{-}3^{\circ}$ V source, the 600-μF capacitor by a $-j/\,0.6\ \Omega$ impedance, the 500-μF capacitor by a $-j2\ \Omega$ impedance, and the inductor by a $j2\ \Omega$ impedance.

$$5\mathbf{V}_2 + 3\mathbf{V}_2 = \frac{\mathbf{V}_1 - 3\angle -3^{\circ}}{100 - j/0.6} + \frac{(\mathbf{V}_1 - \mathbf{V}_2)}{-j2} \quad [1]$$

$$-5\mathbf{V}_2 = \frac{(\mathbf{V}_2 - \mathbf{V}_1)}{-j2} + \frac{\mathbf{V}_2}{j2} \quad [2]$$

Solving, we find that $\mathbf{V}_2 = 9.81\ \angle -13.36^{\circ}$ mV.
Converting back to the time domain,

$$\boxed{v_2(t) = 9.81\cos(10^3 t - 13.36^{\circ})\ \text{mV}}$$

71. Define three clockwise mesh currents: $i_1(t)$ in the left-most mesh, $i_2(t)$ in the bottom right
m esh, and $i_3(t)$ in the top right mesh. The 15-μF capacitor is replaced with a $-j/\ 0.15\ \Omega$ impedance, the inductor is replaced by a $j20\ \Omega$ impedance, the 74 μF capacitor is replaced by a $-j1.351\ \Omega$ impedance, the current source is replaced by a $2\angle 0^\circ$ mA source, and the voltage source is replaced with a $5\angle 0^\circ$ V source.

Around the 1, 2 supermesh: $(1 + j20)\ \mathbf{I}_1 + (13 - j1.351)\ \mathbf{I}_2 - 5\ \mathbf{I}_3 = 0$

and

$$-\mathbf{I}_1 + \mathbf{I}_2 = 2\times 10^{-3}$$

Mesh 3: $5\angle 0^\circ - 5\ \mathbf{I}_2 + (5 - j6.667)\ \mathbf{I}_3 = 0$

Solving, we find that $\mathbf{I}_1 = 148.0\angle 179.6^\circ$ mA. Converting to the time domain,

$$i_1(t) = 148.0\cos(10^4 t + 179.6^\circ)\ \mu\text{A}$$

Thus, $P_{1\Omega} = [i_1(1\text{ ms})]^2 \cdot 1$

$= (16.15\times 10^{-3})(1)\text{ W} = \boxed{16.15\text{ mW}}$.

72. We define an additional clockwise mesh current $i_4(t)$ flowing in the upper right-hand mesh. The inductor is replaced by a $j0.004\ \Omega$ impedance, the 750 μF capacitor is replaced by a $-j/\ 0.0015\ \Omega$ impedance, and the 1000 μF capacitor is replaced by a $-j/\ 2\ \Omega$ impedance. We replace the left voltage source with a a $6\ \angle\ \text{-}13^{\circ}$ V source, and the right voltage source with a $6\ \angle\ 0^{\circ}$ V source.

$$(1 - j/\ 0.0015)\ \mathbf{I}_1 + j/0.0015\mathbf{I}_2\ \ - \mathbf{I}_3 \qquad = \ 6\ \angle -13^{\circ}\ [1]$$

$$(0.005 + j/\ 0.0015)\ \mathbf{I}_1 + \ (j0.004 - j/0.0015)\ \mathbf{I}_2 \qquad - \ j0.004\ \mathbf{I}_4 \ = \ 0 \qquad [2]$$

$$-\mathbf{I}_1 \qquad + (1 - j500)\ \mathbf{I}_3 \qquad + \qquad j500\ \mathbf{I}_4 \ = \ -6\ \angle\ 0^{\circ}\ [3]$$

$$-j0.004\ \mathbf{I}_2 + j500\mathbf{I}_3 + (j0.004 - j500)\ \mathbf{I}_4 \ = \ 0 \qquad [4]$$

Solving, we find that

$\mathbf{I}_1 = 2.002\angle -6.613^{\circ}$ mA, $\mathbf{I}_2 = 2.038\ \angle -6.500^{\circ}$ mA, and $\mathbf{I}_3 = 5.998\ \angle\ 179.8^{\circ}$ A.

Converting to the time domain,

$$\boxed{\begin{aligned} i_1(t) &= 1.44 \cos(2t - 6.613^{\circ})\ \text{mA} \\ i_2(t) &= 2.038 \cos(2t - 6.500^{\circ})\ \text{mA} \\ i_3(t) &= 5.998 \cos(2t + 179.8^{\circ})\ \text{A} \end{aligned}}$$

73. We replace the voltage source with a $115\sqrt{2}\ \angle 0^{\circ}$ V source, the capacitor with a $-j/\ 2\pi C_1\ \Omega$ impedance, and the inductor with a $j0.03142\ \Omega$ impedance.

Defi ne $\mathbf{Z}$ such that $\mathbf{Z}^{-1} = 2\pi C_1 - j/0.03142 + 1/20$

By voltage division, we can write that $6.014\ \angle 85.76^{\circ} = 115\sqrt{2}\ \dfrac{\mathbf{Z}}{\mathbf{Z}+20}$

Thus, $\mathbf{Z} = 0.7411\ \angle\ 87.88^{\circ}\ \Omega$. This allows us to solve for C_1:

$2\pi C_1 - 1/0.03142 = -1.348$ so that $\boxed{C_1 = 4.85\ \text{F.}}$

74. Defining a clockwise mesh current $i_1(t)$, we replace the voltage source with a $115\sqrt{2}\,\angle 0^{\circ}$ V source, the inductor with a $j2\pi L$ Ω impedance, and the capacitor with a $-j1.592$ Ω impedance.

Ohm's law then yields $\mathbf{I}_1 = \dfrac{115\sqrt{2}}{20 + j(2\pi L - 1.592)} = 8.132\angle 0^{\circ}$

Thus, $20 = \sqrt{20^2 + (2\pi L - 1.592)^2}$ and we find that $\boxed{L = 253.4 \text{ mH.}}$

75. (a) By nodal analysis:

$$0 = (\mathbf{V}_\pi - 1)/\,R_s + \mathbf{V}_\pi / R_B + \mathbf{V}_\pi / r_\pi + j\omega C_\pi \mathbf{V}_\pi + (\mathbf{V}_\pi - \mathbf{V}_{out})\, j\omega C_\mu \quad [1]$$

$$-g_m\mathbf{V}_\pi = (\mathbf{V}_{out} - \mathbf{V}_\pi)\, j\omega C_\mu + \mathbf{V}_{out} / R_C + \mathbf{V}_{out} / R_L \quad [2]$$

Simplify and collect terms:

$$\left[\left(\frac{1}{R_S} + \frac{1}{R_B} + \frac{1}{r_\pi}\right) + j\omega\left(C_\pi + C_\mu\right)\right]\mathbf{V}_\pi \;-\; j\omega C_\mu \mathbf{V}_{out} = \frac{1}{R_S} \quad [1]$$

$$(-g_m + j\omega C_\mu)\,\mathbf{V}_\pi \;-\; (j\omega C_\mu + 1/R_C + 1/R_L)\,\mathbf{V}_{out} = 0 \quad [2]$$

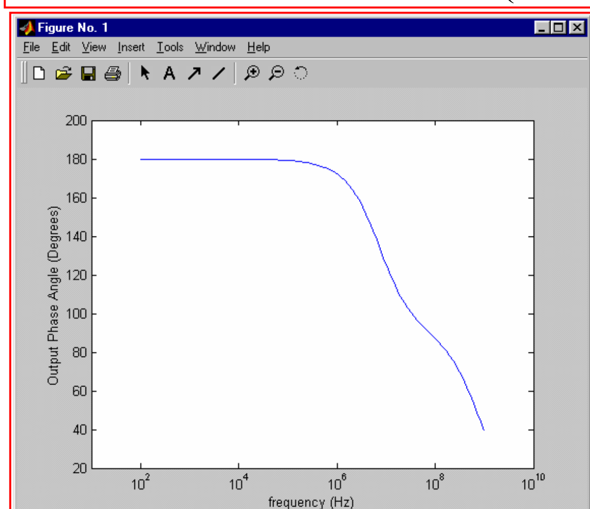
Define $\dfrac{1}{R_S'} = \dfrac{1}{R_S} + \dfrac{1}{R_B} + \dfrac{1}{r_\pi}$ and $R_L' = R_C \,||\, R_L$

Then $$\Delta = \frac{-1}{R_S' R_L'} + \omega^2\left(2C_\mu^2 + C_\mu C_\pi\right) - j\omega\left(g_m C_\mu + \frac{C_\mu + C_\pi}{R_L'} + \frac{C_\mu}{R_S'}\right)$$

And $$\mathbf{V}_{out} = \frac{g_m R_S - j\omega C_\mu / R_S}{\dfrac{-1}{R_S' R_L'} + \omega^2\left(2C_\mu^2 + C_\mu C_\pi\right) - j\omega\left(g_m C_\mu + \dfrac{C_\mu + C_\pi}{R_L'} + \dfrac{C_\mu}{R_S'}\right)}$$

Therefore, $$\boxed{\mathrm{ang}(\mathbf{V}_{out}) = \tan^{-1}\left(\frac{-j\omega C_\mu}{g_m R_S^2}\right) - \tan^{-1}\left(\frac{-\omega\left(g_m C_\mu + \dfrac{C_\mu + C_\pi}{R_L'} + \dfrac{C_\mu}{R_S'}\right)}{\dfrac{-1}{R_S' R_L'} + \omega^2\left(2C_\mu^2 + C_\mu C_\pi\right)}\right)}$$

(b)



(c) The output is ~180° out of phase with the input for $f < 10^5$ Hz; only for $f = 0$ is it exactly 180° out of phase with the input.

76.

$$\text{OC}: -\frac{V_x}{20} + \frac{100 - V_x}{-j10} - 0.02V_x = 0$$

$$j10 = (0.05 + j0.1 + 0.02)\ V_x, V_x = \frac{j10}{0.07 + j0.1}$$

$$\therefore V_x = 67.11 + j46.98$$

$$\therefore V_{ab,oc} = 100 - V_x = 32.89 - j46.98 = \boxed{57.35\angle -55.01^\circ\ \text{V}}$$

$$\text{SC}: V_x = 100 \therefore \downarrow I_{SC} = 0.02 \times 100 + \frac{100}{20} = 7\text{A}$$

$$\therefore Z_{th} = \frac{57.35\angle -55.01^\circ}{7} = \boxed{4.698 - j6.711\Omega}$$

77.

Let $\mathbf{I}_{in} = 1\angle 0$. Then $\mathbf{V}_L = j2\omega \mathbf{I}_{in} = j2\omega \therefore 0.5\mathbf{V}_L = j\omega$

$$\therefore \mathbf{V}_{in} = (1 + j\omega)\frac{1}{j\omega} + j2\omega$$

$$= 1 + \frac{1}{j\omega} + j2\omega$$

$$\therefore \mathbf{Z}_{in} = \frac{\mathbf{V}_{in}}{1} = 1 + \frac{1}{j\omega} + j2\omega \quad \text{so} \quad \boxed{\mathbf{Y}_{\text{in}} = \frac{\omega}{\omega + j\left(2\omega^2 - 1\right)}}$$

At $\omega = 1$, $\mathbf{Z}_{in} = 1 - j1 + j2 = 1 + j$

$$\therefore \mathbf{Y}_{in} = \frac{1}{1 + j1} = 0.5 - j0.5$$

$\boxed{\text{R = 1/0.5 = 2 }\Omega \quad \text{and} \quad \text{L = 1/0.5 = 2 H.}}$

78.

(a) $\mathrm{V}_s : \dfrac{(1-j1)1}{2-j1}\times\dfrac{2+j1}{2+j1}=\dfrac{3-j1}{5}\therefore \mathrm{V}_1=\dfrac{-15}{j2+0.6-j0.2}\times 0.6-j0.2$

$\therefore \mathrm{V}_1 = 5\angle 90° \therefore \boxed{v_1(t)=5\cos(1000t+90°)\ \mathrm{V}}$

(b) I_s:

$j2\|1=\dfrac{j2}{1+j2}\dfrac{1-j2}{1-j2}=0.8+j0.4\therefore \mathrm{V}_1$

$= j25\dfrac{0.8+j0.4}{1-j1+0.8+j0.4}=\dfrac{-10+j20}{1.8-j0.6}=11.785^{+}\angle 135°\ \mathrm{V}$

so $\boxed{v_1(t) = 11.79\cos(1000t+135^{\circ})\ \mathrm{V}.}$

79.

$$\text{OC}: \text{V}_L = 0 \therefore \text{V}_{ab,oc} = 1\angle 0^\circ\ \text{V}$$

$$\text{SC}: \downarrow \text{I}_N \therefore \text{V}_L = j2\text{I}_N \therefore 1\angle 0^\circ = -j1[0.25(j2\text{I}_N) + \text{I}_N] + j2\text{I}_N$$

$$\therefore 1 = (0.5 - j + j2)\,\text{I}_N = (0.5 + j1)\,\text{I}_N$$

$$\therefore \text{I}_N = \frac{1}{0.5 + j1} = 0.4 - j0.8 \therefore Y_N = \frac{\text{I}_N}{1\angle 0^\circ} = 0.4 - j0.8$$

$$\therefore \text{R}_N = \frac{1}{0.4} = \boxed{2.5\,\Omega,}\ \frac{1}{j\omega \text{L}_N} = \frac{1}{j\text{L}_N} = -j0.8,\ \text{L}_N = \frac{1}{0.8} = \boxed{1.25\text{H}}$$

$$\text{I}_N = 0.4 - j0.8 = \boxed{0.8944\angle -63.43^\circ\ \text{A}}$$

80. To solve this problem, we employ superposition in order to separate sources having different frequencies. First considering the sources operating at w = 200 rad/s, we open-circuit the 100 rad/s current source. This leads to $\mathbf{V}_L' = (j)(2\angle 0) = j2$ V. Therefore, $v_L'(t)$ = 2cos(200t + 90°) V. For the 100 rad/s source, we find

$$\mathbf{V}_L'' = \frac{j}{2}(1\angle 0),\ v_L'' = 0.5\cos(100t + 90°)\ \text{V}$$

$$\therefore \boxed{v_L(t) = 2\cos(200t + 90°) + 0.5\cos(100t + 90°)\ \text{V}}$$

81.

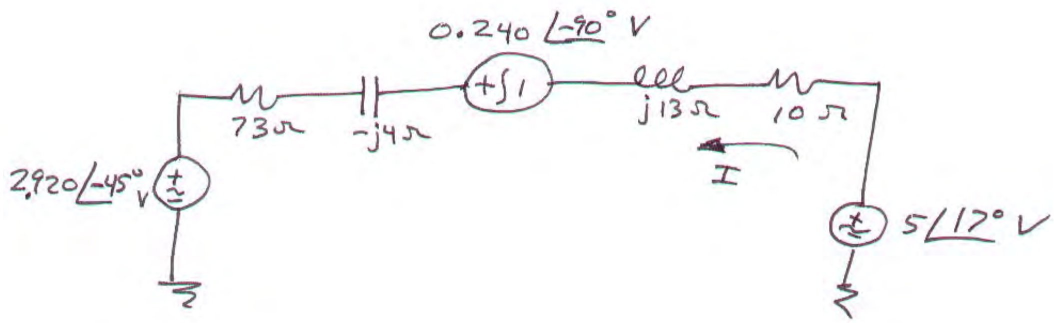
Use superposition. Left: $V_{ab} = 100\dfrac{j100}{j100 - j300}$

$= -50\angle 0°$ V Right: $V_{ab} = j100\dfrac{-j300}{-j300 + j100} = j150\,\text{V}$

$\therefore \boxed{V_{th} = -50 + j150 = 158.11\angle 108.43°\,\text{V}}$

$Z_{th} = j100 \| -j300 = \dfrac{30,000}{-j200} = \boxed{j150\,\Omega}$

82. This problem is easily solved if we first perform two source transformations to yield a circuit containing only voltage sources and impedances:



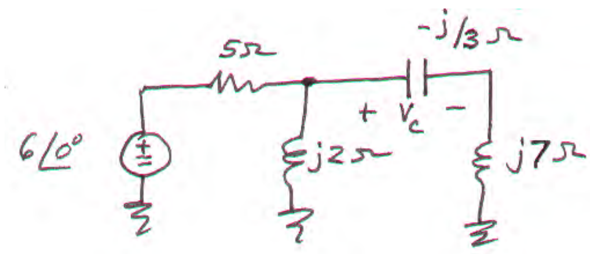
Then $$\mathbf{I} = \frac{5\angle 17^\circ + 0.240\angle\text{-}90^\circ - 2.920\angle\text{-}45^\circ}{73 + 10 + j13 - j4}$$

$= (4.264\angle\ 50.42^\circ)/\ (83.49\ \angle\ 6.189^\circ) = 51.07\ \angle\ 44.23$ mA

Converting back to the time domain, we find that

$\boxed{i(t) = 51.07 \cos(10^3 t + 43.23^\circ) \text{ mA}}$

83.



(a) There are a number of possible approaches: Thévenizing everything to the left of the capacitor is one of them.

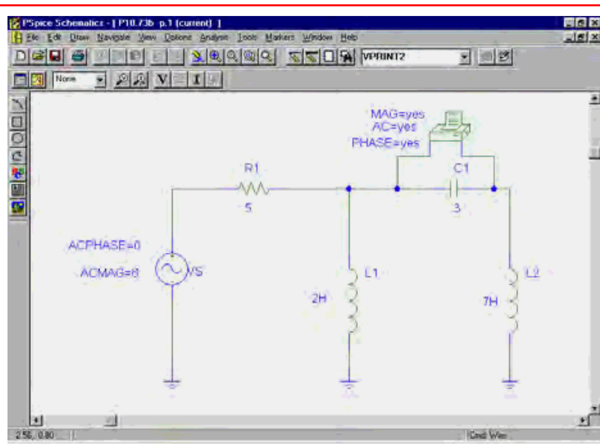
$$\mathbf{V}_{TH} = 6(j2)/\,(5 + j2) = 2.228 \angle 68.2^\circ \text{ V}$$
$$\mathbf{Z}_{TH} = 5 \,||\, j2 = j10/\,(5 + j2) = 1.857 \angle 68.2^\circ \,\Omega$$

Then, by simple voltage division, we find that

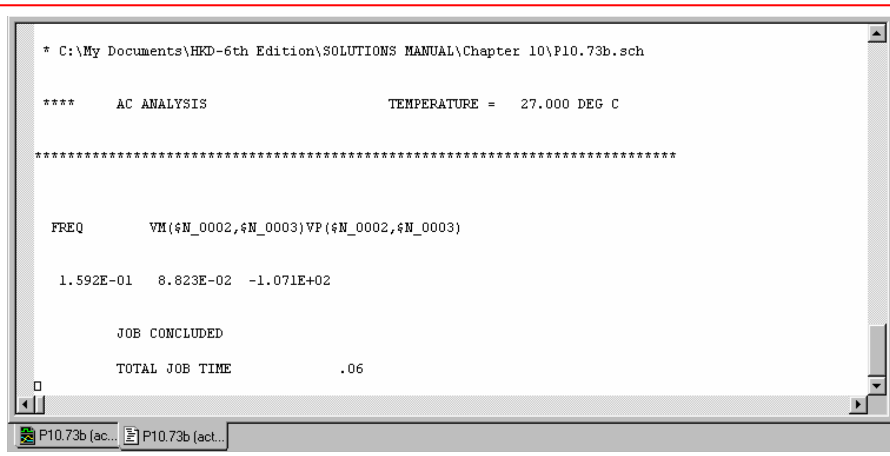
$$\mathbf{V}_C = (2.228 \angle 68.2^\circ)\,\frac{-j/3}{1.857\angle 68.2^\circ - j/3 + j7}$$
$$= 88.21 \angle{-107.1^\circ} \text{ mV}$$

Converting back to the time domain, $\boxed{v_C(t) = 88.21 \cos(t - 107.1^\circ) \text{ mV.}}$

(b) **PSpice verification**.



Running an ac sweep at the frequency $f = 1/2\pi = 0.1592$ Hz, we obtain a phasor magnitude of 88.23 mV, and a phasor angle of -107.1°, in agreement with our calculated result (the slight disagreement is a combination of round-off error in the hand calculations and the rounding due to expressing 1 rad/s in Hz.



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* C:\My Documents\HKD-6th Edition\SOLUTIONS MANUAL\Chapter 10\P10.73b.sch

****     AC ANALYSIS                    TEMPERATURE =   27.000 DEG C

******************************************************************************

 FREQ        VM($N_0002,$N_0003)VP($N_0002,$N_0003)

  1.592E-01   8.823E-02  -1.071E+02

          JOB CONCLUDED

          TOTAL JOB TIME            .06
```

84. (a) Performing nodal analysis on the circuit,

Node 1: $1 = \mathbf{V}_1/\,5 + \mathbf{V}_1/\,(-j10) + (\mathbf{V}_1 - \mathbf{V}_2)/\,(-j5) + (\mathbf{V}_1 - \mathbf{V}_2)/\,j10$ [1]

Node 2: $j0.5 = \mathbf{V}_2/\,10 + (\mathbf{V}_2 - \mathbf{V}_1)/\,(-j5) + (\mathbf{V}_2 - \mathbf{V}_1)/\,j10$ [2]

Simplifying and collecting terms,

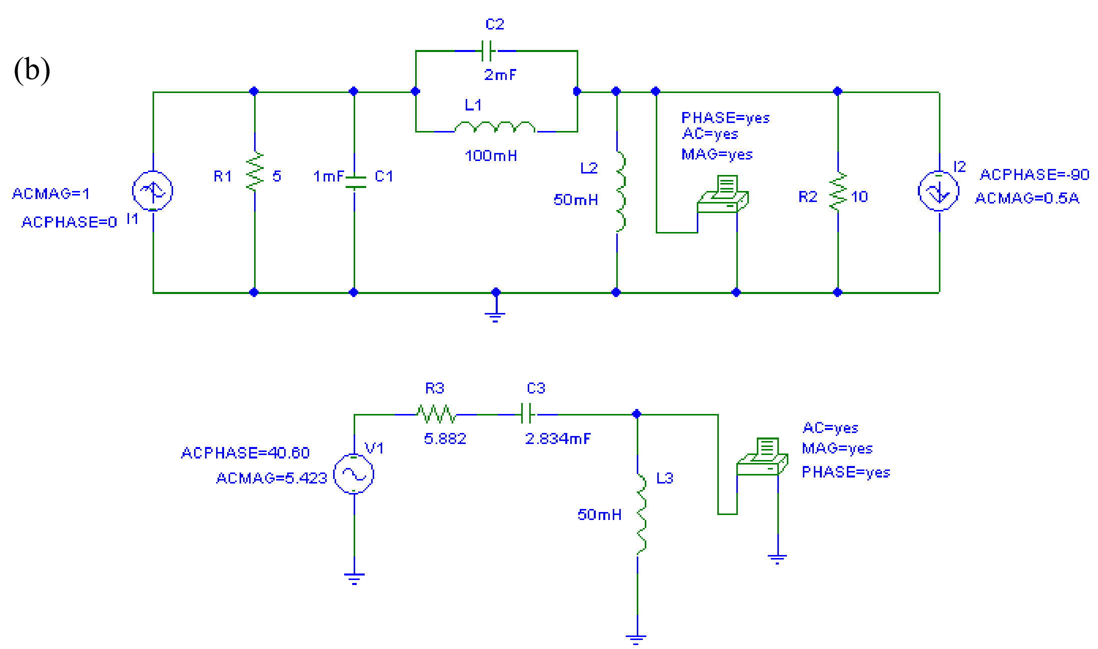
$(0.2 + j0.2)\,\mathbf{V}_1 - j0.1\,\mathbf{V}_2 = 1$ [1]

$-j\,\mathbf{V}_1 + (1 + j)\,\mathbf{V}_2 = j5$ [2]

Solving, we find that $\mathbf{V}_2 = \mathbf{V}_{TH} = \boxed{5.423 \angle 40.60^\circ \text{ V}}$

$\mathbf{Z}_{TH} = 10 \,||\, [(j10 \,||\, -j5) + (5 \,||\, -j10)] = 10 \,||\, (-j10 + 4 - j2) = \boxed{5.882 - j3.529\ \Omega.}$

(b)



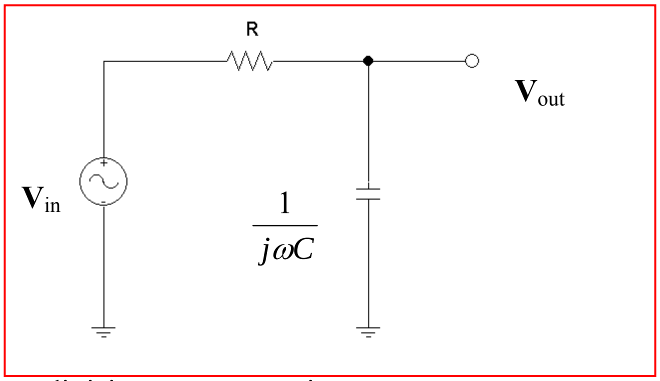

```
FREQ        VM($N 0002,0)     VP($N 0002,0)

1.592E+01   4.474E+00         1.165E+02

FREQ        VM($N_0005,0)     VP($N_0005,0)

1.592E+01   4.473E+00         1.165E+02
```

85. Consider the circuit below:



Using voltage division, we may write:

$$\mathbf{V}_{\text{out}} = \mathbf{V}_{\text{in}} \frac{1/j\omega C}{R + 1/j\omega C}, \quad \text{or} \quad \frac{\mathbf{V}_{\text{out}}}{\mathbf{V}_{\text{in}}} = \frac{1}{1 + j\omega RC}$$

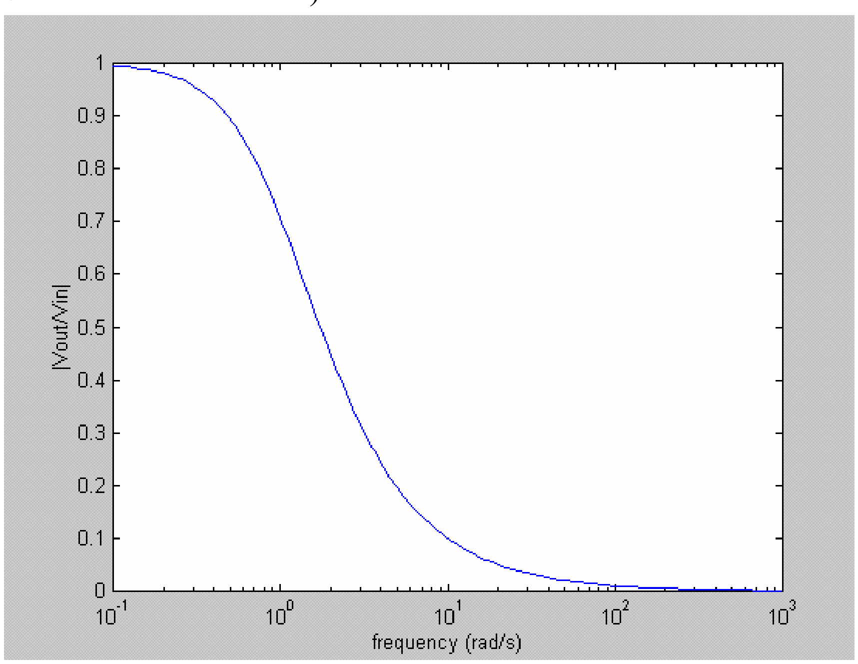
The magnitude of this ratio (consider, for example, an input with unity magnitude and zero phase) is

$$\left|\frac{\mathbf{V}_{\text{out}}}{\mathbf{V}_{\text{in}}}\right| = \frac{1}{\sqrt{1 + (\omega RC)^2}}$$

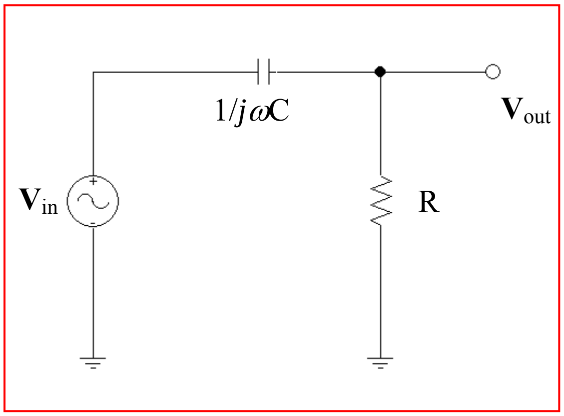
As $\omega \to 0$, this magnitude $\to 1$, its maximum value.

As $\omega \to \infty$, this magnitude $\to 0$; the capacitor is acting as a short circuit to the ac signal.

Thus, low frequency signals are transferred from the input to the output relatively unaffected by this circuit, but high frequency signals are attenuated, or "filtered out." This is readily apparent if we plot the magnitude as a function of frequency (assuming R = 1 Ω and C = 1 F for convenience):



86. Consider the circuit below:



Using voltage division, we may write:

$$\mathbf{V}_{\text{out}} = \mathbf{V}_{\text{in}} \frac{R}{R + 1/\,j\omega C}, \quad \text{or} \quad \frac{\mathbf{V}_{\text{out}}}{\mathbf{V}_{\text{in}}} = \frac{j\omega RC}{1 + j\omega RC}$$

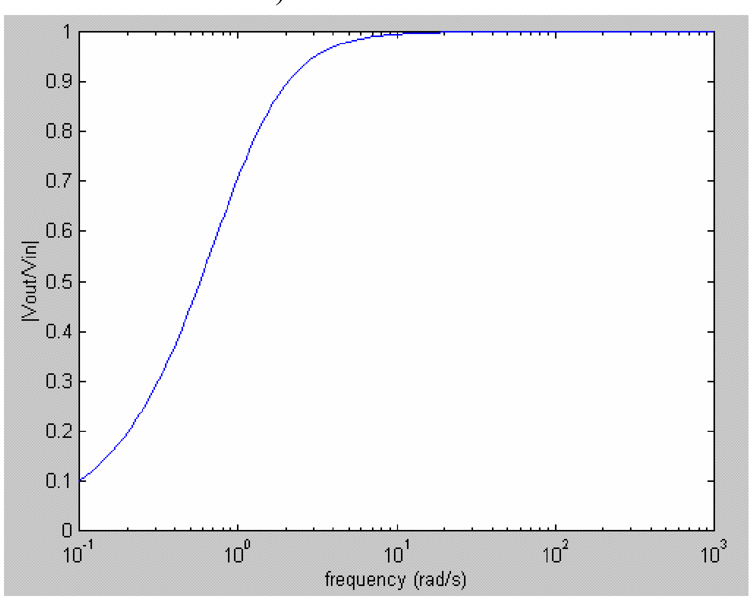
The magnitude of this ratio (consider, for example, an input with unity magnitude and zero phase) is

$$\left|\frac{\mathbf{V}_{\text{out}}}{\mathbf{V}_{\text{in}}}\right| = \frac{\omega RC}{\sqrt{1 + (\omega RC)^2}}$$

As $\omega \to \infty$, this magnitude $\to$ 1, its maximum value.

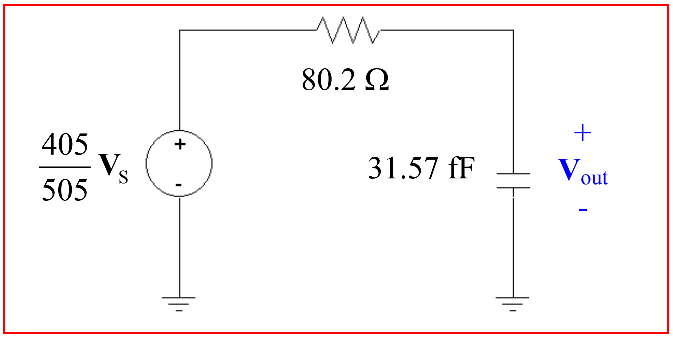
As $\omega \to 0$, this magnitude $\to$ 0; the capacitor is acting as an open circuit to the ac signal.

Thus, high frequency signals are transferred from the input to the output relatively unaffected by this circuit, but low frequency signals are attenuated, or "filtered out." This is readily apparent if we plot the magnitude as a function of frequency (assuming R = 1 Ω and C = 1 F for convenience):



87. (a) Removing the capacitor temporarily, we easily find the Thévenin equivalent:

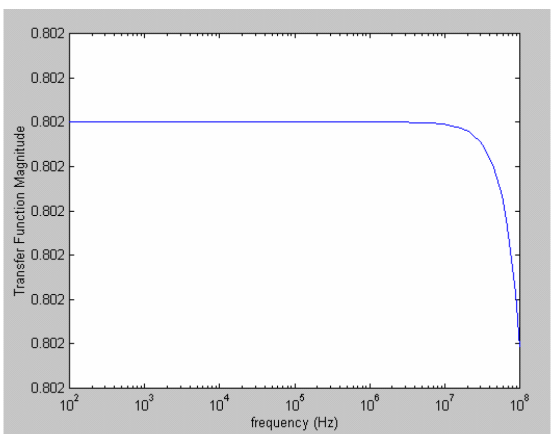
$$\mathbf{V}_{th} = (405/505)\,\mathbf{V}_S \quad \text{and} \quad R_{th} = 100 \,||\, (330 + 75) = 80.2\ \Omega$$



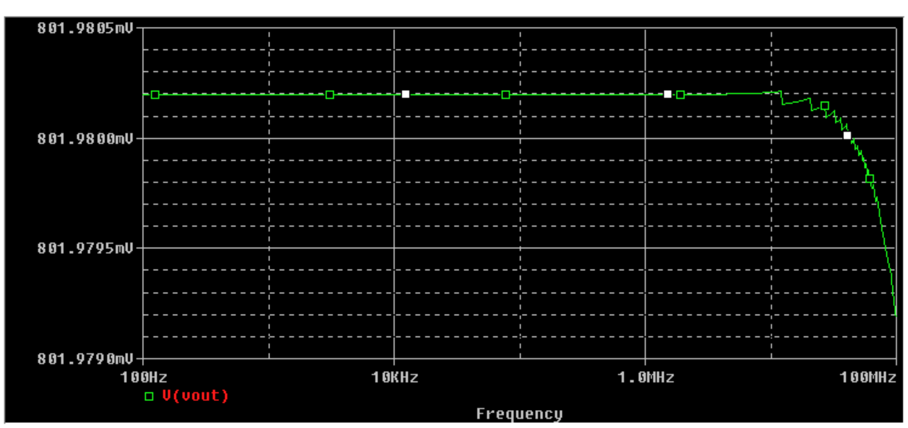
(b) $$\mathbf{V}_{out} = \frac{405}{505}\mathbf{V}_S \frac{1/j\omega C}{80.2 + 1/j\omega C} \quad \text{so} \quad \frac{\mathbf{V}_{out}}{\mathbf{V}_S} = \left(\frac{405}{505}\right)\frac{1}{1 + j2.532\times 10^{-12}\,\omega}$$

and hence $$\boxed{\left|\frac{\mathbf{V}_{out}}{\mathbf{V}_S}\right| = \frac{0.802}{\sqrt{1 + 6.411\times 10^{-24}\,\omega^2}}}$$

(c)



Both the MATLAB plot of the frequency response and the PSpice simulation show essentially the same behavior; at a frequency of approximately 20 MHz, there is a sharp roll-off in the transfer function magnitude.



88. From the derivation, we see that

$$\frac{\mathbf{V}_{out}}{\mathbf{V}_{in}} = \frac{-g_m(R_C \| R_L) + j\omega(R_C \| R_L)C_\mu}{1 + j\omega(R_C \| R_L)C_\mu}$$

so that

$$\left|\frac{\mathbf{V}_{out}}{\mathbf{V}_{in}}\right| = \left[\frac{g_m^{\ 2}\left(\frac{R_C R_L}{R_C + R_L}\right)^2 + \omega^2\left(\frac{R_C R_L}{R_C + R_L}\right)^2 C_\mu^{\ 2}}{1 + \omega^2\left(\frac{R_C R_L}{R_C + R_L}\right)^2 C_\mu^{\ 2}}\right]^{1/2}$$

This function has a maximum value of $g_m (R_C \| R_L)$ at $\omega = 0$. Thus, the capacitors reduce the gain at high frequencies; this is the frequency regime at which they begin to act as short circuits. Therefore, the maximum gain is obtained at frequencies at which the capacitors may be treated as open circuits. If we do this, we may analyze the circuit of Fig. 10.25*b* without the capacitors, which leads to

$$\left.\frac{\mathbf{V}_{out}}{\mathbf{V}_S}\right|_{\text{low frequency}} = -g_m\left(\frac{R_C R_L}{R_C + R_L}\right)\frac{(r_\pi \| R_B)}{R_S + r_\pi \| R_B} = -g_m\left(\frac{R_C R_L}{R_C + R_L}\right)\frac{r_\pi R_B}{R_S(r_\pi + R_B) + r_\pi R_B}$$

The resistor network comprised of r_π, R_S, and R_B acts as a voltage divider, leading to a reduction in the gain of the amplifier. In the situation where $r_\pi \| R_B >> R_S$, then it has minimal effect and the gain will equal its "maximum" value of $-g_m (R_C \| R_L)$.

(b) If we set $R_S = 100\ \Omega$, $R_L = 8\ \Omega$, $R_C |_{max} = 10\ k\Omega$ and $r_\pi g_m = 300$, then we find that

$$\frac{\mathbf{V}_{out}}{\mathbf{V}_S} = -g_m\,(7.994)\frac{r_\pi \| R_B}{100 + r_\pi \| R_B}$$

We seek to maximize this term within the stated constraints. This requires a large value of g_m, but also a large value of $r_\pi \| R_B$. This parallel combination will be less than the smaller of the two terms, so even if we allow $R_B \to \infty$, we are left with

$$\frac{\mathbf{V}_{out}}{\mathbf{V}_S} \approx -(7.994)\frac{g_m r_\pi}{100 + r_\pi} = \frac{-2398}{100 + r_\pi}$$

Considering this simpler expression, it is clear that if we select r_π to be small, (*i.e.* $r_\pi << 100$), then g_m will be large and the gain will have a maximum value of approximately –23.98.

(c) Referring to our original expression in which the gain $\mathbf{V}_{out}/\mathbf{V}_{in}$ was computed, we see that the critical frequency $\omega_C = [(R_C \| R_L)\, C_\mu]^{-1}$. Our selection of maximum R_C, $R_B \to \infty$, and $r_\pi << 100$ has not affected this frequency.

89. Considering the $\omega = 2\times10^4$ rad/ s source first, we make the following replacements:

$100 \cos(2\times10^4 t + 3^\circ)$ V $\rightarrow 100 \angle 3^\circ$ V

33 μF $\rightarrow$ $-j1.515\ \Omega$ 112 μH $\rightarrow j2.24\ \Omega$ 92 μF $\rightarrow -j0.5435\ \Omega$

Then

$$(\mathbf{V}_1' - 100\angle 3^\circ)/\ 47\times10^3 + \mathbf{V}_1'/(-j1.515) + (\mathbf{V}_1' - \mathbf{V}_2')/(56\times10^3 + j4.48) = 0 \quad [1]$$

$$(\mathbf{V}_2' - \mathbf{V}_1')/(56\times10^3 + j4.48) + \mathbf{V}_2'/(-j0.5435) = 0 \quad [2]$$

Solving, we find that

$$\mathbf{V}_1' = 3.223\angle -87^\circ \text{ mV} \quad \text{and } \mathbf{V}_2' = 31.28\angle -177^\circ \text{ nV}$$

Thus, $v_1'(t) = 3.223\cos(2\times10^4 t - 87^\circ)$ mV and $v_2'(t) = 31.28\cos(2\times10^4 t - 177^\circ)$ nV

Considering the effects of the $\omega = 2\times10^5$ rad/ s source next,

$100\cos(2\times10^5 t - 3^\circ)$ V $\rightarrow 100\angle -3^\circ$ V

33 μF $\rightarrow$ $-j0.1515\ \Omega$ 112 μH $\rightarrow j22.4\ \Omega$ 92 μF $\rightarrow -j0.05435\ \Omega$

Then

$$\mathbf{V}_1''/ -j0.1515 + (\mathbf{V}_1'' - \mathbf{V}_2'')/(56\times10^3 + j44.8) = 0 \quad [3]$$

$$(\mathbf{V}_2'' - \mathbf{V}_1'')/(56\times10^3 + j44.8) + (\mathbf{V}_2'' - 100\angle 3^\circ)/\ 47\times10^3 + \mathbf{V}_2''/(-j0.05435) = 0 \quad [4]$$

Solving, we find that

$$\mathbf{V}_1'' = 312.8\angle 177^\circ \text{ pV and } \mathbf{V}_2'' = 115.7\angle -93^\circ \text{ μV}$$

Thus,

$$v_1''(t) = 312.8\cos(2\times10^5 t + 177^\circ) \text{ pV and } v_2''(t) = 115.7\cos(2\times10^5 t - 93^\circ) \text{ μV}$$

Adding, we find

$$\boxed{\begin{aligned} v_1(t) &= 3.223\times10^{-3}\cos(2\times10^4 t - 87^\circ) + 312.8\times10^{-12}\cos(2\times10^5 t + 177^\circ) \text{ V and} \\ v_2(t) &= 31.28\times10^{-9}\cos(2\times10^4 t - 177^\circ) + 115.7\times10^{-12}\cos(2\times10^5 t - 93^\circ) \text{ V} \end{aligned}}$$

90. For the source operating at $\omega = 4$ rad/s,
7 cos 4$t \to 7\angle 0^\circ$ V, 1 H $\to j4\ \Omega$, 500 mF $\to -j0.5\ \Omega$, 3 H $\to j12\ \Omega$, and 2 F $\to -j/8\ \Omega$.

Then by mesh analysis, (define 4 clockwise mesh currents $\mathbf{I}_1$, $\mathbf{I}_2$, $\mathbf{I}_3$, $\mathbf{I}_4$ in the top left, top right, bottom left and bottom right meshes, respectively):

$$\begin{array}{lcl} (9.5 + j4)\,\mathbf{I}_1 - j4\,\mathbf{I}_2 - 7\,\mathbf{I}_3 - 4\,\mathbf{I}_4 & = 0 & [1] \\ -j4\,\mathbf{I}_1 + (3 + j3.5)\,\mathbf{I}_2 - 3\,\mathbf{I}_4 & = -7 & [2] \\ -7\,\mathbf{I}_1 + (12 - j/8)\,\mathbf{I}_3 + j/8\,\mathbf{I}_4 & = 0 & [3] \\ -3\,\mathbf{I}_2 + j/8\,\mathbf{I}_3 + (4 + j11.875)\,\mathbf{I}_4 & = 0 & [4] \end{array}$$

Solving, we find that $\mathbf{I}_3 = 365.3\ \angle\ -166.1^\circ$ mA and $\mathbf{I}_4 = 330.97\ \angle\ 72.66^\circ$ mA.

For the source operating at $\omega = 2$ rad/s,
5.5 cos 2$t \to 5.5\angle 0^\circ$ V, 1 H $\to j2\ \Omega$, 500 mF $\to -j\ \Omega$, 3 H $\to j6\ \Omega$, and 2 F $\to -j/4\ \Omega$.

Then by mesh analysis, (define 4 clockwise mesh currents $\mathbf{I}_A$, $\mathbf{I}_B$, $\mathbf{I}_C$, $\mathbf{I}_D$ in the top left, top right, bottom left and bottom right meshes, respectively):

$$\begin{array}{lcl} (9.5 + j2)\,\mathbf{I}_A - j2\,\mathbf{I}_B - 7\,\mathbf{I}_C - 4\,\mathbf{I}_D & = 0 & [1] \\ -j2\,\mathbf{I}_A + (3 + j)\,\mathbf{I}_B - 3\,\mathbf{I}_D & = -7 & [2] \\ -7\,\mathbf{I}_A + (12 - j/4)\,\mathbf{I}_C + j/4\,\mathbf{I}_D & = 0 & [3] \\ -3\,\mathbf{I}_2 + j/4\,\mathbf{I}_C + (4 + j5.75)\,\mathbf{I}_D & = 0 & [4] \end{array}$$

Solving, we find that $\mathbf{I}_C = 783.8\ \angle\ -4.427^\circ$ mA and $\mathbf{I}_D = 134\ \angle\ -25.93^\circ$ mA.

$\mathbf{V}_1' = -j0.25\,(\mathbf{I}_3 - \mathbf{I}_4) = 0.1517\angle 131.7^\circ$ V and $\mathbf{V}_1'' = -j0.25(\mathbf{I}_C - \mathbf{I}_D) = 0.1652\angle -90.17^\circ$ V

$\mathbf{V}_2' = (1 + j6)\,\mathbf{I}_4 = 2.013\angle 155.2^\circ$ V and $\mathbf{V}_2'' = (1 + j6)\,\mathbf{I}_D = 0.8151\angle 54.61^\circ$ V

Converting back to the time domain,

$$\boxed{\begin{array}{lcl} v_1(t) & = & 0.1517\cos(4t + 131.7^\circ) + 0.1652\cos(2t - 90.17^\circ)\text{ V} \\ v_2(t) & = & 2.013\cos(4t + 155.2^\circ) + 0.8151\cos(2t + 54.61^\circ)\text{ V} \end{array}}$$

91.

(a) $$\mathrm{I}_L = \frac{100}{j2.5 + \dfrac{-2}{2-j1}} = \frac{100(2-j1)}{2.5+j3} = \boxed{57.26\angle -76.76°\,(2.29in)}$$

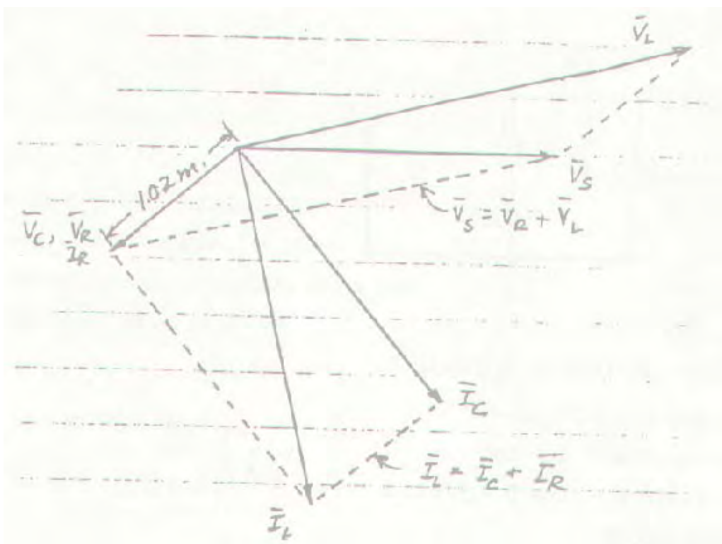
$$\mathrm{I}_R = (57.26\angle -76.76°)\frac{-j1}{2-j1} = \boxed{25.61\angle -140.19°\,(1.02in)}$$

$$\mathrm{I}_c = (57.26\angle -76.76°)\frac{2}{2-j1} = \boxed{51.21\angle -50.19°\,(2.05in)}$$

$$\mathrm{V}_L = 2.5\times 57.26\angle 90° - 76.76° = \boxed{143.15\angle 13.24°\,(2.86in)}$$

$$\mathrm{V}_R = 2\times 25.61\angle -140.19° = \boxed{51.22\angle -140.19°\,(1.02in)}$$

$$\mathrm{V}_c = \boxed{51.21\angle -140.19°\,(1.02in)}$$



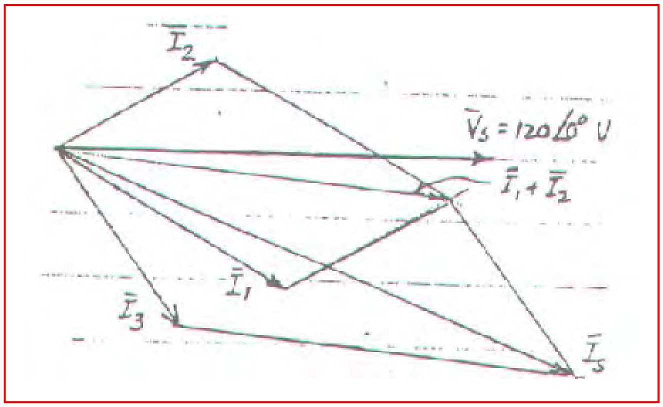
92.

(a)
$$\mathbf{I}_1 = \frac{120}{40\angle 30°} = \boxed{3\angle -30°\text{ A}}$$

$$\mathbf{I}_2 = \frac{120}{50 - j30} = \boxed{2.058\angle 30.96°\text{ A}}$$

$$\mathbf{I}_3 = \frac{120}{30 + j40} = \boxed{2.4\angle -53.13°\text{ A}}$$

(b)



(c)
$$\mathbf{I}_s = \mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3$$
$$= \boxed{6.265\angle -22.14°\text{ A}}$$

93.

$|I_1| = 5A, |I_2| = 7A$

$I_1 + I_2 = 10\angle 0°$, I_1 lags V, I_2 leads V

I_1 lags I_2. Use 2.5A / *in*

[Analytically: $5\angle\alpha + 7\angle\beta = 10$

$= 5\cos\alpha + j5\sin\alpha + 7\cos\beta + j7\sin\beta$

$\therefore \sin\alpha = -1.4\sin\beta$

$\therefore 5\sqrt{1-1.4^2\sin^2\beta} + 7\sqrt{1-1\sin^2\beta} = 10$

By SOLVE, $\boxed{\alpha = -40.54° \; \beta = 27.66°]}$

94. $\mathbf{V}_1 = 100\angle 0^{\circ}$ V, $|\mathbf{V}_2| = 140$ V, $|\mathbf{V}_1 + \mathbf{V}_2| = 120$ V.
Let 50 V = 1 inch. From the sketch, for $\angle\mathbf{V}_2$ positive,
$\boxed{\mathbf{V}_2 = 140\angle 122.5^{\circ}.}$ We may also have $\boxed{\mathbf{V}_2 = 140\angle -122.5^{\circ} \text{ V}}$

[Analytically: $|100 + 140\angle\alpha| = 120$
so $|\ 100 + 140 \cos\alpha + j140 \sin\alpha\ | = 120$
Using the "Solve" routine of a scientific calculator,
$\alpha = \pm 122.88^{\circ}$.]

1.

$$\mathbf{Z}_c = \frac{10^6}{j500 \times 25} = -j80\,\Omega,\ \frac{50(-j80)}{50-j80} = 42.40\angle -32.01°\,\Omega$$

$$\therefore \mathbf{V} = 84.80\angle -32.01°\,\text{V},\ \mathbf{I}_R = 1.696\angle -32.01°\,\text{A}$$

$$\mathbf{I}_c = 1.0600\angle 57.99°\,\text{A}$$

$$p_s(\pi/2\text{ms}) = 84.80\cos(45° - 32.01°)\,2\cos 45° = \boxed{116.85\,\text{W}}$$

$$p_R = 50 \times 1.696^2 \cos^2(45° - 32.01°) = \boxed{136.55\,\text{W}}$$

$$p_c = 84.80\cos(45° - 32.01°) = 1.060\cos(45° + 57.99°) = \boxed{-19.69\,\text{W}}$$

2.

(a) $$4\text{H}: i = 2t^2 - 1 \therefore v = Li' = 4(4t) = 16t, w_L = \frac{1}{2}Li^2 = \frac{1}{2}\times 4(4t^4 - 4t^2 + 1)$$

$$\therefore w_L = 8t^4 - 8t^2 + 2 \therefore w_L(3) - w_L(1) = 8\times 3^4 - 8\times 3^2 + 2 - 8\times 1 + 8\times 1 - 2 = \boxed{576\text{ J}}$$

(b) $$0.2\,\text{F}: v_c = \frac{1}{0.2}\int_1^t (2t^2 - 1)\,dt + 2 = 5\left(\frac{2}{3}t^3 - t\right)_1^t + 2 = 5\left(\frac{2}{3}t^3 - t\right) - 5\left(\frac{2}{3} - 1\right) + 2$$

$$\therefore v_c(2) = \frac{10}{3}\times 8 - 10 - \frac{10}{3} + 5 + 2 = \frac{61}{3}\text{V} \therefore \text{P}_c(2) = \frac{61}{3}\times 7 = \boxed{142.33\,\text{W}}$$

3. $v_c(0) = -2\text{V},\ i(0) = 4\text{A},\ \alpha = \dfrac{\text{R}}{2\text{L}} = 2,\ \omega_o^2 = \dfrac{1}{\text{LC}} = 3,\ s_{1,2} = -2 \pm 1 = -1, -3$

(a) $i = \text{A}e^{-t} + \text{B}e^{-3t} \therefore \text{A} + \text{B} = 4;\ i(0^+) = \dfrac{1}{1} v_L(0^+) = (-4 \times 4 \times +2) = -14$

$\therefore -\text{A} - 38 = -14 \therefore \text{B} = 5,\ \text{A} = -1,\ i = -e^{-t} + 5e^{-3t}\ \text{A}$

$\therefore +v_c = 3\int_o^t (-e^{-t} + 5e^{-3t})\,dt - 2 = 3(e^{-t} - 5e^{-3t})\Big|_o^t - 2 = e^{-t} - 3 - 5e^{-3t} + 5 - 2$

$\therefore v_c = 3e^{-t} - 5e^{-3t} \therefore \text{P}_c(0^+) = (3-5)(-1+5) = \boxed{-8\ \text{W}}$

(b) $\text{P}_c(0.2) = (3e^{-0.2} - 5e^{-0.6})(-e^{0.2} + 5e^{-0.6}) = \boxed{-0.5542\ \text{W}}$

(c) $\text{P}_c(0.4) = (3e^{-0.4} - 5e^{-1.2})(5e^{-1.2} - e^{-0.4}) = \boxed{0.4220\ \text{W}}$

4. We assume the circuit has already reached sinusoidal steady state by t = 0.

2.5 kΩ → 2.5 kΩ, 1 H → j1000 Ω, 4 μF → -j250 Ω, 10 kΩ → 10 kΩ

$$\mathbf{Z}_{eq} = j1000 \parallel -j250 \parallel 10000 = 11.10 - j333.0\ \Omega$$

$$\mathbf{V}_{eq} = \frac{(20\angle 30)(11.10 - j333.0)}{2500 + 11.10 - j333.0} = 2.631\angle -50.54^{\circ}\ \text{V}$$

$$\mathbf{I}_{10k} = \frac{\mathbf{V}_{eq}}{10000} = 0.2631\angle -50.54^{\circ}\ \text{mA} \qquad \mathbf{I}_{1\,H} = \frac{\mathbf{V}_{eq}}{j1000} = 2.631\angle -140.5^{\circ}\ \text{mA}$$

$$\mathbf{I}_{4\,\mu F} = \frac{\mathbf{V}_{eq}}{-j250} = 10.52\angle 39.46^{\circ}\ \text{mA} \quad \mathbf{V}_{2.5k} = \frac{(20\angle 30)(2500)}{2500 + 11.10 - j333.0} = 19.74\angle 37.55^{\circ}\ \text{V}$$

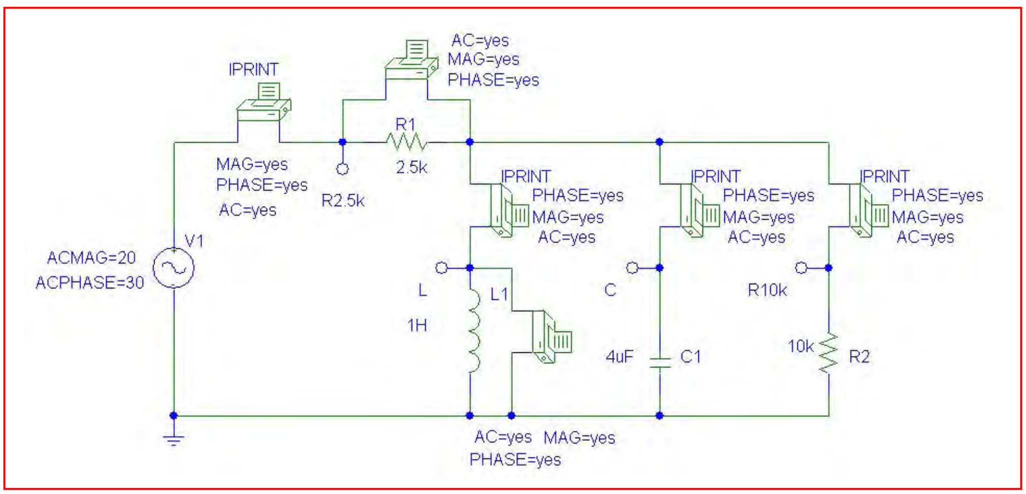
Thus,

$$P_{2.5k} = \frac{\left[19.74\cos 37.55^{\circ}\right]^2}{2500} = \boxed{97.97\ \text{mW}}$$

$$P_{1\,H} = \left[2.631\cos(-50.54)^{\circ}\right]\left[2.631\times 10^{-3}\cos(-140.5^{\circ})\right] = \boxed{-3.395\ \text{mW}}$$

$$P_{4\,\mu F} = \left[2.631\cos(-50.54^{\circ})\right]\left[10.52\times 10^{-3}\cos(39.46^{\circ})\right] = \boxed{13.58\ \text{mW}}$$

$$P_{2.5k} = \frac{\left[2.631\cos(-50.54^{\circ})\right]^2}{10000} = \boxed{279.6\ \mu\text{W}}$$



FREQ	IM(V_PRINT1)	IP(V_PRINT1)
1.592E+02	7.896E-03	3.755E+01

FREQ	VM(R2_5k,\$N_0002)	VP(R2_5k,\$N_0002)
1.592E+02	1.974E+01	3.755E+01

FREQ	IM(V_PRINT2)	IP(V_PRINT2)
1.592E+02	2.628E-03	-1.405E+02

FREQ	VM(L,0)	VP(L,0)
1.592E+02	2.629E+00	-5.054E+01

FREQ	IM(V_PRINT11)	IP(V_PRINT11)
1.592E+02	1.052E-02	3.946E+01

FREQ	IM(V_PRINT12)	IP(V_PRINT12)
1.592E+02	2.629E-04	-5.054E+01

5.

$$i_s \to 5\angle 0°\,\text{A},\ \text{C} \to -j4\,\Omega,\ \mathbf{Z}_{in} = 8\|(3-j4) = \frac{40\angle -53.13°}{11-j4}$$

$$= 3.417\angle -33.15° \therefore \mathbf{V}_s = 17.087\angle -33.15°,$$

$$v_s = 17.087\cos(25t - 33.15°)\,\text{V} \therefore$$

$$\text{P}_{s,abs}(0.1) = -17.087\cos(2.5^{\text{rad}} - 33.147°)\times 5\cos 2.5^{\text{rad}} = \boxed{-23.51\ \text{W}}$$

$$i_8 = \frac{17.087}{8}\cos(25t - 33.15°) \therefore$$

$$i_8(0.1) = 2.136\cos(2.5^{\text{rad}} - 33.15°) = -0.7338\ \text{A}$$

$$\therefore \text{P}_{8,abs} = 0.7338^2 \times 8 = \boxed{4.307\ \text{W}};$$

$$\mathbf{I}_3 = \frac{17.087\angle -33.15°}{3-j4} = 3.417\angle 19.98°\,\text{A}$$

$$\therefore i_3(0.1) = 3.417\cos(2.5^{\text{rad}} + 19.98°) = -3.272\,\text{A} \therefore$$

$$\text{P}_{3,abc} = 3.272^2 \times 3 = \boxed{32.12\,\text{W}}$$

$$\mathbf{V}_c = -j4(3.417\angle 19.983°) = 13.67\angle -70.02°,$$

$$v_c(0.1) = 13.670\cos(2.5^{\text{rad}} - 70.02°) = 3.946\,\text{V}$$

$$\therefore \text{P}_{c,abc} = 3.946(-3.272) = \boxed{-12.911\ \text{W}} \qquad (\Sigma = 0)$$

6. For $t > 0$, $i(t) = 8e^{-R/L\,t} = 8e^{-2t}$.

(a) $p(0^+) = (8)^2(1) = \boxed{64\text{ W}}$

(b) at $t = 1$ s, $i = 8e^{-2} = 1.083$ A; $p(1) = i^2R = \boxed{1.723\text{ W.}}$

(c) at $t = 2$ s, $i = 8e^{-4} = 146.5$ mA; $p(2) = i^2R = \boxed{21.47\text{ mW}}$

7. $v(t) = (3)(6000)e^{-t/30\times10^{-3}}$

(a) $p(0^+) = v^2(0^+)/R = (18\times10^3)^2 / 6000 = \boxed{54 \text{ kW}}$

(b) $p(0.03) = v^2(0.03)/R = (18\times10^3 e^{-1})^2 / 6000 = \boxed{7.308 \text{ kW}}$

(c) $p(0.09) = v^2(0.09)/R = (18\times10^3 e^{-3})^2 / 6000 = \boxed{134 \text{ W}}$

8. (a) $p = (30\times10^{3})^{2}\,(1.2\times10^{-3}) = \boxed{1.080\text{ MW}}$

(b) $W = (1.080\times10^{6})(150\times10^{-6}) = \boxed{162\text{ J}}$

9. W $= \frac{1}{2}CV^2$. The initial voltage, $v(0^+)$, is therefore

$$v(0^+) = \sqrt{\frac{2W}{C}} = \sqrt{\frac{2(100\times10^{-3})}{100\times10^{-3}}} = \sqrt{2}\text{ V} \text{ and so } v(t) = \sqrt{2}e^{-t/RC} = \sqrt{2}e^{-t/0.12}\text{ V.}$$

The instantaneous power dissipated at t = 120 mS is therefore

$$p(120\text{ ms}) = \frac{v^2(120\text{ ms})}{R} = \frac{2e^{-2}}{1.2} = \boxed{226\text{ mW}}$$

The energy dissipated over the first second is given by

$$\int_0^1 \frac{v^2(t)}{R}dt = \int_0^1 \frac{2e^{-2t/RC}}{R}dt = -\frac{RC}{2}\left(\frac{2}{R}\right)\left[e^{-2/RC} - 1\right] \approx 100\text{ mJ}$$

$\Delta T = Q/mc$, where Q = 100 mJ, c = 0.9 kJ/kg•K, and m = 10^{-3} kg.

Thus, the final temperature

$$= 271.15 + 23 + \frac{100\times10^{-6}\text{ kJ}}{\left(10^{-3}\text{ kG}\right)\left(0.9\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)} = 271.15 + 23 + 0.1111$$

$= \boxed{\text{294.3 K, representing a temperature increase of 0.1111 K.}}$

10. (a) $p = (276)(130) = \boxed{358.8 \text{ mW}}$

(b) $v(t) = 2.76\cos 1000t$ V (given); we need to know $\boxed{\text{the I-V relationship}}$ for this (nonlinear) device.

11.

$$\mathbf{Z}_{in} = 4 + \frac{j5(10 - j5)}{10} = 4 + 2.5 + j5 = 6.5 + j5 \ \Omega$$

$$\therefore \mathbf{I}_s = \frac{100}{6.5 + j5} = 12.194\angle -37.57^\circ \text{ A}$$

$$\therefore \mathrm{P}_{s,abs} = -\frac{1}{2} \times 100 \times 12.194 \cos 37.57^\circ = -483.3 \text{ W}$$

$$\mathrm{P}_{4,abs} = \frac{1}{2}(12.194)^2\, 4 = \boxed{297.4 \text{ W}},$$

$$\mathrm{P}_{cabs} = \boxed{0}$$

$$\mathbf{I}_{10} = \frac{100}{6.5 + j5}\,\frac{j5}{10} = 6.097\angle 52.43^\circ \ \text{ so}$$

$$\mathrm{P}_{10,abs} = \frac{1}{2}(6.097)^2 \times 10 = \boxed{185.87 \text{ W}}$$

$$\mathrm{P}_L = \boxed{0} \qquad (\Sigma = 0)$$

12.

$$\mathbf{V} = (10 + j10)\frac{40\angle 30°}{5\angle 50° + 8\angle -20°} = 52.44\angle 69.18°\ \text{V}$$

$$\text{P}_{10,gen} = \frac{1}{2} \times 10 \times 52.44 \cos 69.18° = \boxed{93.19\ \text{W}}$$

$$\text{P}_{j10,gen} = \frac{1}{2} \times 10 \times 52.44 \cos(90° - 69.18°) = \boxed{245.1\ \text{W}}$$

$$\text{P}_{5\angle 50 abs} = \frac{1}{2}\left(\frac{52.44}{5}\right)^2 \cos(50°) = \boxed{176.8\ \text{W}}$$

$$\text{P}_{8\angle -20 abs} = \frac{1}{2}\left(\frac{52.44}{8}\right)^2 \cos(-20°) = \boxed{161.5\ \text{W}} \qquad (\Sigma_{gen} = \Sigma_{abs})$$

13.

$$\mathbf{Z}_R = 3 + \frac{1}{0.1 - j0.3} = 3 + 1 + j3 = 4 + j3\,\Omega$$

Ignore 30° on $\mathbf{V}_s$, $\mathbf{I}_R = 5\,\dfrac{2 + j5}{6 + j8}$, $|\mathbf{I}_R| = \dfrac{5\sqrt{29}}{10}$

(a) $$\text{P}_{3\Omega} = \frac{1}{2}\left(\frac{5\sqrt{29}}{10}\right)^2 \times 3 = \boxed{10.875\ \text{W}}$$

(b) $$\text{V}_s = 5\angle 0^\circ \frac{(2 + j5)(4 + j3)}{6 + j8} = 13.463\angle 51.94^\circ\ \text{V}$$

$$\therefore \text{P}_{s,gen} = \frac{1}{2} \times 13.463 \times 5 \cos 51.94^\circ = \boxed{20.75\ \text{W}}$$

14.

$$\boxed{P_{j10} = P_{-j5} = 0,}$$

$$\frac{\mathbf{V}_{10} - 50}{j10} + \frac{\mathbf{V}_{10}}{10} + \frac{\mathbf{V}_{10} - j50}{-j5} = 0$$

$$\therefore \mathbf{V}_{10}(-j0.1 + 0.1 + j0.2) + j5 + 10 = 0$$

$$\therefore \mathbf{V}_{10} = 79.06\angle 16.57^\circ \text{ V}$$

$$P_{10\Omega} = \frac{1}{2}\frac{79.06^2}{10} = \boxed{312.5 \text{ W};}$$

$$\mathbf{I}_{50} = \frac{79.06\angle 161.57^\circ - 50}{j10} = 12.75\angle 78.69^\circ \text{ A}$$

$$\therefore P_{50V} = \frac{1}{2} \times 50 \times 12.748\cos 78.69^\circ = \boxed{62.50 \text{ W}}$$

$$\mathbf{I}_{j50} = \frac{79.06\angle 161.57^\circ - j50}{-j5} = 15.811\angle -7.57^\circ:$$

$$P_{j50} = \frac{1}{2} \times 50 \times 15.811\cos(90^\circ + 71.57^\circ) = \boxed{-375.0 \text{ W}}$$

15.

$$\frac{\mathbf{V}_x - 20}{2} + \frac{\mathbf{V}_x - \mathbf{V}_c}{3} = 2\mathbf{V}_c \qquad [1]$$

and

$$0 = \frac{\mathbf{V}_c}{-j2} + \frac{\mathbf{V}_c - \mathbf{V}_x}{3} \qquad [2]$$

which simplify to

$5\mathbf{V}_x - 14\mathbf{V}_c = 60$ [1] and

$j2\mathbf{V}_x + (3 - j2)\mathbf{V}_c = 0$ [2]

Solving,

$\mathbf{V}_x = 9.233\angle -83.88°\text{ V}$ and $\mathbf{V}_c = 5.122\angle -140.2°\text{ V}$

$$\text{P}_{gen} = \frac{1}{2} \times 9.233 \times (2 \times 5.122)\cos(-83.88° + 140.2°) = \boxed{26.22\text{ W}}$$

16.

(a) $X_{in} = 0 \therefore \mathbf{Z}_L = \boxed{R_{th} + j0}$

(b) R_L, X_L independent $\therefore \boxed{\mathbf{Z}_L = \mathbf{Z}_{th}^* = R_{th} - jX_{th}}$

(c) R_L fixed $\therefore P_L = \dfrac{1}{2}\dfrac{|V_{th}|^2}{(R_{th} + R_L)^2 + (X_{th} + X_L)^2} \times R_L \therefore \boxed{\mathbf{Z}_L = R_L - jX_{th}}$

(d) X_L fixed, Let $X_L + X_{th} = a \therefore f = \dfrac{2P_L}{|\mathbf{V}_{th}|^2} = \dfrac{RL}{(R_{th} + R_L)^2 + a^2}$

$$\frac{df}{dR_L} = \frac{R_{th} + R_L^2 + a^2 - 2R_L(R_{th} + R_L)}{\left[(R_{th} + R_L)^2 + a^2\right]^2} = 0$$

$$R_{th}^2 + 2R_{th}R_L + R_L^2 + a^2 - 2R_{th}R_L = 2R_L^2 = 0$$

$$\therefore R_L = \sqrt{R_{th}^2 + a^2} = \boxed{\sqrt{R_{th}^2 + (X_{th} + X_L)^2}}$$

(e) $X_L = 0 \therefore R_L = \boxed{\sqrt{R_{th}^2 + X_{th}^2}} = |\mathbf{Z}_{th}|$

17.

$$\mathbf{V}_{th} = 120\frac{-j10}{10+j5} = 107.3\angle -116.6°\,\text{V}$$

$$\mathbf{Z}_{th} = \frac{-j10(10+j15)}{10+j5} = 8-j14\,\Omega$$

(a) $\mathbf{Z}_{TH} = \left(\mathbf{Z}_L\right)^* = \boxed{8+j14\ \Omega}$

(b) $$\mathbf{I}_L = \frac{\mathbf{V}_{TH}}{\mathbf{Z}_{TH}+\left(\mathbf{Z}_{TH}\right)^*} = \frac{107.3\angle -116.6°}{16}.$$

$$\mathbf{V}_L = \mathbf{V}_{TH}\frac{\left(\mathbf{Z}_{TH}\right)^*}{\mathbf{Z}_{TH}+\left(\mathbf{Z}_{TH}\right)^*} = \frac{\left(107.3\angle -116.6°\right)\left(16.12\angle -60.26°\right)}{16}$$

$$\text{P}_{L,\max} = \frac{1}{2}\left[\frac{(107.3)(16.12)}{16}\right]\left[\frac{107.3}{10}\right]\cos\left(-116.6° - 60.26° + 116.6°\right) = \boxed{179.8\ \text{W}}$$

18.

$$R_L = |\mathbf{Z}_{th}| \therefore R_L = \sqrt{8^2 + 14^2} = \boxed{16.125\,\Omega}$$

$$P_L = \frac{1}{2}\frac{107.33^2}{(8+16.125)^2 + 14^2} \times 16.125 = \boxed{119.38\,\text{W}}$$

19.

$$-j9.6 = -4.8\,\mathrm{I}_x - j1.92\ \mathrm{I}_x - +4.8\mathrm{I}_x$$

$$\therefore \mathrm{I}_x = \frac{9.6}{1.92} = 5$$

$$\therefore \mathrm{V} = (0.6 \times 5)8 = 24\,\mathrm{V}$$

$$\therefore \mathrm{P}_o = \frac{1}{2} \times 24 \times 1.6 \times 5 = \boxed{96 \quad (gen)\mathrm{W}}$$

20.

(a) $$\mathbf{Z}_{th} = 80 \| j60 = \frac{j480}{80 + j60} \frac{80 - j60}{80 - j60}$$

$$= 28.8 + j38.4\,\Omega \therefore \mathbf{Z}_{L\max} = \boxed{28.8 - j38.4\,\Omega}$$

(b) $$\mathbf{V}_{th} = 5(28.8 + j38.4) = 144 + j192\,\text{V},$$

$$\therefore \mathbf{I}_L = \frac{144 + j192}{2 \times 28.8}$$

and $$\text{P}_{L,\max} = \frac{1}{2} \frac{144^2 + 192^2}{4 \times 28.8^2} \times 28.8 = \boxed{250\text{ W}}$$

21. $\mathbf{Z}_{eq} = (6 - j8) \parallel (12 + j9) = 8.321 \angle -19.44^o$ W

$\mathbf{V}_{eq} = (5 \angle -30^o)(8.321 \angle -19.44^o) = 41.61 \angle -49.44^o$ V

$P_{total} = ½ (41.61)(5) \cos(-19.44^o) = 98.09$ W

$\mathbf{I}_{6-j8} = \mathbf{V}_{eq} / (6 - j8) = 4.161 \angle 3.69^o$ A
$\mathbf{I}_{4+j2} = \mathbf{I}_{8+j7} = \mathbf{V}_{eq} / 12+j9 = 2.774 \angle -86.31^o$ A

$P_{6-j8} = ½ (41.61)(4.161) \cos(-49.44^o - 3.69^o) = \boxed{51.94 \text{ W}}$

$P_{4+j2} = ½ (2.774)2 (4) = \boxed{15.39 \text{ W}}$
$P_{8+j7} = ½ (2.774)2 (8) = \boxed{30.78 \text{ W}}$

Check: $\Sigma = 98.11$ W (okay)

22.

$$\mathbf{V}_{th} = 100\frac{j10}{20+j10} = 20+j40,\ \mathbf{Z}_{th} = \frac{j10(20)}{20+j10} = 4+j8\,\Omega$$

$$\therefore \mathrm{R}_L = |\mathbf{Z}_{th}| \therefore \mathrm{R}_L = \boxed{8.944\,\Omega}$$

$$\therefore \mathrm{P}_{L,\max} = \frac{1}{2}\frac{20^2+40^2}{(4+8.944)^2+64}\times 8.944 \boxed{= 38.63\,\mathrm{W}}$$

23. We may write a single mesh equation: $170\angle 0^\circ = (30 + j10)\,\mathbf{I}_1 - (10 - j50)(-\lambda \mathbf{I}_1)$
Solving,

$$\mathbf{I}_1 = \frac{170\angle 0^\circ}{30 + j10 + 10\lambda - j50\lambda}$$

(a) $\lambda = 0$, so $\mathbf{I}_1 = \dfrac{170\angle 0^\circ}{30 + j10} = 5.376\angle -18.43^\circ$ A and, with the same current flowing through both resistors in this case,

$P_{20} = ½\,(5.376)^2\,(20) = \boxed{289.0 \text{ W}}$
$P_{10} = ½\,(5.376)^2\,(10) = \boxed{144.5 \text{ W}}$

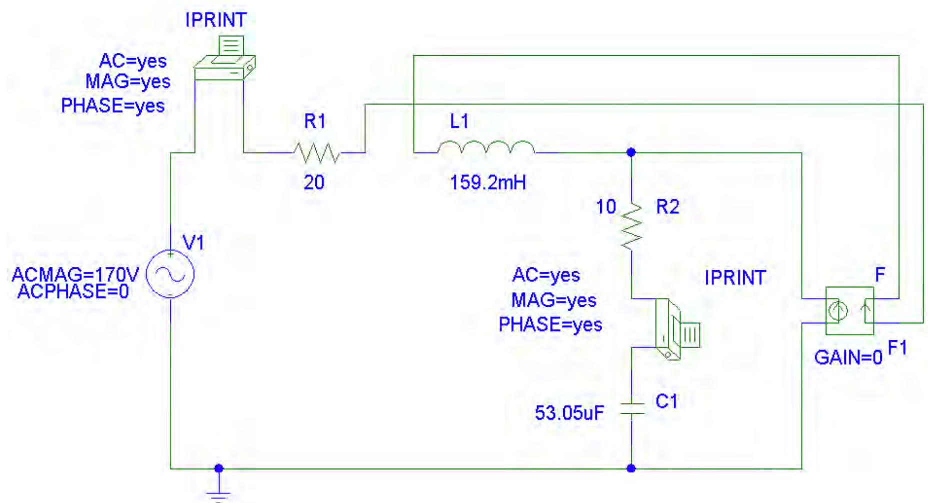
(b) $\lambda = 1$, so $\mathbf{I}_1 = \dfrac{170\angle 0^\circ}{40 - j40} = 3.005\angle 45^\circ$ A

$P_{20} = ½\,(3.005)^2\,(20) = \boxed{90.30 \text{ W}}$

The current through the 10-Ω resistor is $\mathbf{I}_1 + \lambda\mathbf{I}_1 = 2\,\mathbf{I}_1 = 6.01 \angle 45^\circ$ so

$P_{10} = ½\,(6.01)^2\,(10) = \boxed{180.6 \text{ W}}$

(c)



(a)

FREQ	IM(V_PRINT3)	IP(V_PRINT3)
6.000E+01	5.375E+00	-1.846E+01

FREQ	IM(V_PRINT4)	IP(V_PRINT4)
6.000E+01	5.375E+00	-1.846E+01

(b)

FREQ	IM(V_PRINT3)	IP(V_PRINT3)
6.000E+01	6.011E+00	4.499E+01

FREQ	IM(V_PRINT4)	IP(V_PRINT4)
6.000E+01	3.006E+00	4.499E+01

24. (a) Waveform (a): $I_{avg} = \dfrac{(10)(1) + (-5)(1) + 0(1)}{3} = \boxed{1.667\text{ A}}$

Waveform (b): $I_{avg} = \dfrac{\frac{1}{2}(20)(1) + 0(1)}{2} = \boxed{5\text{ A}}$

Waveform (c):

$$I_{avg} = \frac{1}{1\times10^{-3}}\int_0^{10^{-3}} 8\sin\frac{2\pi t}{4\times10^{-3}}\,dt = -\left(8\times10^{3}\right)\left(\frac{4\times10^{-3}}{2\pi}\right)\cos\left(\frac{\pi t}{2\times10^{-3}}\right)\Bigg|_0^{10^{-3}}$$

$$= -\frac{16}{\pi}(0-1) = \boxed{\frac{16}{\pi}\text{ A}}$$

(b) Waveform (a): $I^2_{avg} = \dfrac{(100)(1) + (25)(1) + (0)(1)}{3} = \boxed{41.67\text{ A}^2}$

Waveform (b): $i(t) = -20\times10^3\, t + 20$

$i^2(t) = 4\times10^8\, t^2 - 8\times10^5\, t + 400$

$$I^2_{avg} = \frac{1}{2\times10^{-3}}\int_0^{10^{-3}} \left(4\times10^8 t^2 - 8\times10^5 t + 400\right)dt$$

$$= \frac{1}{2\times10^{-3}}\left[\frac{4\times10^8}{3}\left(10^{-3}\right)^3 - \frac{8\times10^5}{2}\left(10^{-3}\right)^2 + 400\left(10^{-3}\right)\right] = \frac{0.1333}{2\times10^{-3}} = \boxed{66.67\text{ A}^2}$$

Waveform (c):

$$I^2_{avg} = \frac{1}{1\times10^{-3}}\int_0^{10^{-3}} 64\sin^2\frac{2\pi t}{4\times10^{-3}}\,dt = \left(64\times10^3\right)\left[\frac{t}{2} - \frac{\sin\pi\times10^3 t}{2\pi\times10^3}\right]\Bigg|_0^{10^{-3}}$$

$$= \left(64\times10^3\right)\left[\frac{10^{-3}}{2} - \frac{\sin\pi}{2\pi\times10^3}\right] = \boxed{32\text{ A}^2}$$

25. At $\omega = 120\pi$, 1 H $\rightarrow$ $j377\ \Omega$, and 4 μF $\rightarrow$ $-j663.1\ \Omega$

Define $\mathbf{Z}_{eff} = j377 \parallel -j663.1 \parallel 10\,000 = 870.5 \angle 85.01^\circ\ \Omega$

$$\mathbf{V}_{2.5k} = \frac{\left(400\sqrt{2}\angle -9^\circ\right)2500}{2500 + 870.5\angle 85.01^\circ} = 520.4 \angle -27.61^\circ \text{ V}$$

$$\mathbf{V}_{10k} = \frac{\left(400\sqrt{2}\angle -9^\circ\right)\left(870.5\angle 85.01^\circ\right)}{2500 + 870.5\angle 85.01^\circ} = 181.2 \angle 57.40^\circ \text{ V}$$

Thus,

$P_{2.5k} = ½ (520.4)2 / 2\,500 = \boxed{54.16 \text{ W}}$

$P_{10k} = ½ (181.2)2 / 10\,000 = \boxed{1.642 \text{ W}}$

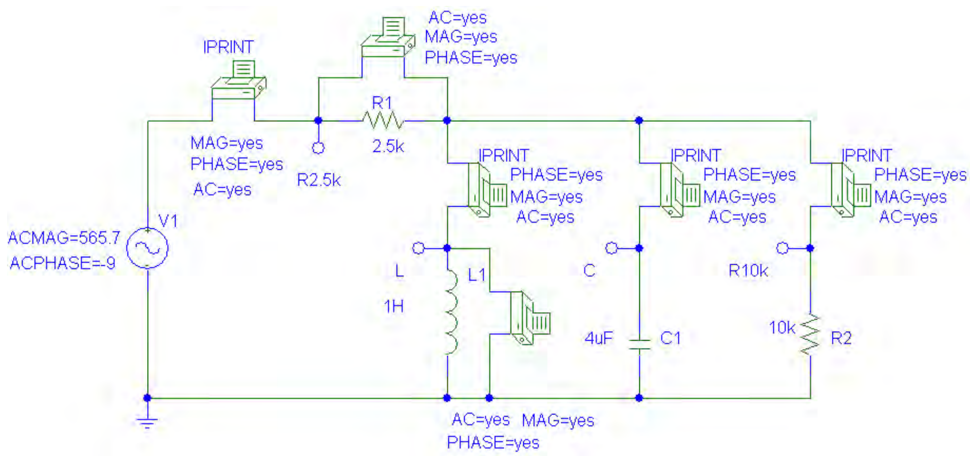
$P_{1H} = \boxed{0}$

$P_{4\mu F} = \boxed{0}$ (A total absorbed power of 55.80 W.)

To check, the average power delivered by the source:

$$\mathbf{I}_{source} = \frac{400\sqrt{2}\angle -9^\circ}{2500 + 870.5\angle 85.01^\circ} = 0.2081 \angle -27.61^\circ \text{ A}$$

and $P_{source} = ½\,(400\sqrt{2})(0.2081)\cos(-9^\circ + 27.61^\circ) = 55.78$ W (checks out).



FREQ	IM(V_PRINT1)	IP(V_PRINT1)	FREQ	VM(L,0)	VP(L,0)
6.000E+01	2.081E-01	-2.760E+01	6.000E+01	1.812E+02	5.740E+01
FREQ	VM(R2_5k,$N_0002)	VP(R2_5k,$N_0002)	FREQ	IM(V_PRINT11)	IP(V_PRINT11)
6.000E+01	5.204E+02	-2.760E+01	6.000E+01	2.732E-01	1.474E+02
FREQ	IM(V_PRINT2)	IP(V_PRINT2)	FREQ	IM(V_PRINT12)	IP(V_PRINT12)
6.000E+01	4.805E-01	-3.260E+01	6.000E+01	1.812E-02	5.740E+01

26. (a) $$\sqrt{\frac{1}{T}\int_0^T \frac{144}{2}\left(1+\cos 2000t\right)dt} = \sqrt{\frac{144}{2}} = \boxed{8.485}$$

(b) $$\sqrt{\frac{1}{T}\int_0^T \frac{144}{2}\left(1-\cos 2000t\right)dt} = \sqrt{\frac{144}{2}} = \boxed{8.485}$$

(c) $$\sqrt{\frac{1}{T}\int_0^T \frac{144}{2}\left(1+\cos 1000t\right)dt} = \sqrt{\frac{144}{2}} = \boxed{8.485}$$

(d) $$\sqrt{\frac{1}{T}\int_0^T \frac{144}{2}\left[1+\cos\left(1000t-176^\circ\right)\right]dt} = \sqrt{\frac{144}{2}} = \boxed{8.485}$$

27. (a) $$\sqrt{\frac{1}{T}\int_0^T \frac{4}{2}\left(1+\cos 20t\right)dt} = \frac{2}{\sqrt{2}} = \boxed{1.414}$$

(b) $$\sqrt{\frac{1}{T}\int_0^T \frac{4}{2}\left(1-\cos 20t\right)dt} = \frac{2}{\sqrt{2}} = \boxed{1.414}$$

(c) $$\sqrt{\frac{1}{T}\int_0^T \frac{4}{2}\left(1+\cos 10t\right)dt} = \frac{\sqrt{2}}{2} = \boxed{1.414}$$

(d) $$\sqrt{\frac{1}{T}\int_0^T \frac{4}{2}\left[1+\cos\left(10t-64^\circ\right)\right]dt} = \frac{\sqrt{2}}{2} = \boxed{1.414}$$

28. T = 3 s; integrate from 1 to 4 s; need only really integrate from 1 to 3 s as function is zero between $t = 3$ and $t = 4$ s.

$$V_{rms} = \sqrt{\frac{1}{3}\int_1^3 (10)^2\,dt} = \sqrt{\frac{100}{3}t\Big|_1^3} = \sqrt{\frac{100(2)}{3}} = \boxed{8.165 \text{ V}}$$

29. T = 3 s; integrate from 2 to 5 s; need only really integrate from 2 to 3 s as function is zero between $t = 3$ and $t = 4$ s.

$$I_{rms} = \sqrt{\frac{1}{3}\int_2^3 (7)^2\,dt} = \sqrt{\frac{49}{3}t\Big|_2^3} = \sqrt{\frac{49(1)}{3}} = \boxed{4.041\text{ A}}$$

30. (a) $\boxed{1\text{ V}}$

(b) $$V_{rms} = \sqrt{V_{1_{eff}}^2 + V_{2_{eff}}^2} = \sqrt{1^2 + \left(\frac{1}{\sqrt{2}}\right)^2} = \boxed{1.225\text{ V}}$$

(c) $$V_{rms} = \sqrt{V_{1_{eff}}^2 + V_{2_{eff}}^2} = \sqrt{1^2 + \left(\frac{1}{\sqrt{2}}\right)^2} = \boxed{1.225\text{ V}}$$

31.

(a) $v = 10 + 9\cos 100t + 6\sin 100t$

$$\therefore \text{V}_{eff} = \sqrt{100 + \frac{1}{2}\times 81 + \frac{1}{2}\times 36} = \sqrt{158.5} = \boxed{12.590\text{ V}}$$

(b) $$\text{F}_{eff} = \sqrt{\frac{1}{4}(10^2 + 20^2 + 10^2)} = \sqrt{150} = \boxed{12.247}$$

(c) $$\text{F}_{avg} = \frac{(10)(1)+(20)(1)+(10)(1)}{4} = \frac{40}{4} = \boxed{10}$$

32.

(a) $g(t) = 2 + 3\cos 100t + 4\cos(100t - 120^{\circ})$

3 $\angle 0 + 4\angle\text{-}120^{\circ} = 3.606\ \angle\text{-}73.90^{\circ}$ so $G_{eff} = \sqrt{4 + \dfrac{3.606^2}{2}} = \boxed{3.240}$

(b) $h(t) = 2 + 3\cos 100t + 4\cos(101t - 120^{\circ})$

$\therefore H_{eff} = \sqrt{2^2 + \frac{1}{2}3^2 + \frac{1}{2}4^2} = \sqrt{16.5} = \boxed{4.062}$

(c) $f(t) = 100t,\ 0 < t < 0.1 \therefore F_{eff} = \sqrt{\dfrac{1}{0.3}\displaystyle\int_0^{0.1} 10^6 t^2 dt}$

$= \sqrt{\dfrac{10}{3} \times 10^6 \times \dfrac{1}{3} \times 10^{-3}} = \boxed{33.33}$

33. $f(t) = (2 - 3\cos 100t)^2$

(a) $f(t) = 4 - 12\cos 100t + 9\cos^2 100t$

$\therefore\ f(t) = 4 - 12\cos 100t + 4.5 + 4.5\cos 200t \ \therefore \mathrm{F}_{av} = 4 + 4.5 = \boxed{8.5}$

(b) $\mathrm{F}_{eff} = \sqrt{8.5^2 + \frac{1}{2}\times 12^2 + \frac{1}{2}\times 4.5^2} = \boxed{12.43}$

34. (a) $$i_{\text{eff}} = \left[\frac{1}{3}\left(10^2 + (-5)^2\right) + 0\right]^{1/2} = \boxed{6.455 \text{ A}}$$

(b) $$i_{\text{eff}} = \left[\frac{1}{2}\left(\int_0^1 [-20t + 20]\, dt\right) + 0\right]^{1/2} = \sqrt{5} = \boxed{2.236 \text{ A}}$$

(c) $$i_{\text{eff}} = \left[\frac{1}{1}\left(\int_0^1 8\sin\left(\frac{2\pi}{4}t\right) dt\right)\right]^{1/2} = \sqrt{\left[-8\left(\frac{2}{\pi}\right)\cos\left(\frac{\pi t}{2}\right)\right]_0^1} = \boxed{2.257 \text{ A}}$$

35.

(a) $A = B = 10\text{V},\ C = D = 0 \therefore 10\angle 0° + 10\angle -45° = 18.48\angle -22.50°$

$$\therefore P = \frac{1}{2}\times\frac{1}{4}\times 18.48^2 = \boxed{42.68\ \text{W}}$$

(b) $A = C = 10\text{V},\ B = D = 0,\ v_s = 10\cos 10t + 10\cos 40t,$

$$P = \frac{1}{2}\frac{10^2}{4} + \frac{1}{2}\frac{10^2}{4} = \boxed{25}\quad \text{W}$$

(c) $v_s = 10\cos 10t - 10\sin\ (10t + 45°) \rightarrow 10 - 10\angle -45° = 7.654\angle 67.50°$

$$\therefore P = \frac{1}{2}\frac{7.654^2}{4} = \boxed{7.322}\quad \text{W}$$

(d) $v = 10\cos 10t + 10\sin(10t + 45°) + 10\cos 40t;$

$10\angle 0° + 10\angle -45° = 18.48\angle -22.50°$

$$\therefore P = \frac{1}{2}\times 18.48^2\times\frac{1}{4} + \frac{1}{2}\times 10^2\times\frac{1}{4} = \boxed{55.18\ \text{W}}$$

(e) $// + 10dc \therefore P_{av} = 55.18 + \frac{10^2}{4} = \boxed{80.18\ \text{W}}$

36. $\mathbf{Z}_{eq} = R \parallel j0.3\omega = \dfrac{j0.3R\omega}{R + j0.3R\omega}$. By voltage division, then, we write:

$$\mathbf{V}_{100mH} = 120\angle 0 \frac{j0.1\omega}{j0.1\omega + \dfrac{j0.3R\omega}{R + j0.3\omega}} = 120\angle 0 \frac{-0.03\omega^2 + j0.1\omega R}{-0.03\omega^2 + j0.4R\omega}$$

$$\mathbf{V}_{300mH} = 120\angle 0 \frac{\dfrac{j0.3R\omega}{R + j0.3\omega}}{j0.1\omega + \dfrac{j0.3R\omega}{R + j0.3\omega}} = 120\angle 0 \frac{j36R\omega}{-0.03\omega^2 + j0.4R\omega}$$

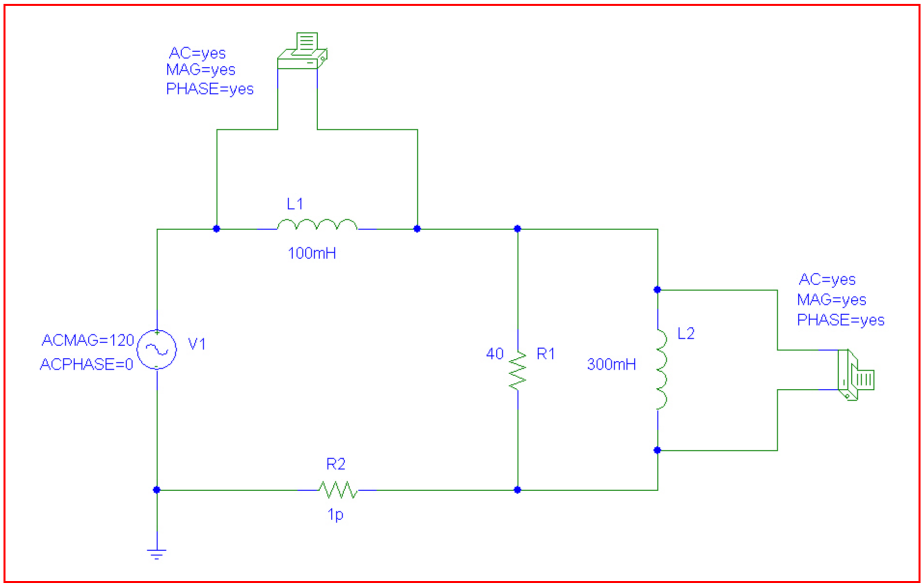
(a) We're interested in the value of R that would lead to equal voltage magnitudes, or

$$\left| j36R\omega \right| = \left| (120)\left(-0.03\omega^2 + j0.1\omega R\right) \right|$$

Thus, $36R\omega = \sqrt{12.96\omega^4 + 144\omega^2 R^2}$ or $\boxed{R = 0.1061\ \omega}$

(b) Substituting into the expression for $\mathbf{V}_{100mH}$, we find that $V_{100mH} = \boxed{73.47\text{ V},}$ independent of frequency.

To verify with PSpice, simulate the circuit at 60 Hz, or $\omega = 120\pi$ rad/s, so R = 40 Ω. We also include a miniscule (1 pΩ) resistor to avoid inductor loop warnings. We see from the simulation results that the two voltage magnitudes are indeed the same.




```
FREQ        VM($N_0002,$N_0003)VP($N_0002,$N_0003)
6.000E+01   7.349E+01  -3.525E+01

FREQ        VM($N_0001,$N_0002)VP($N_0001,$N_0002)
6.000E+01   7.347E+01   3.527E+01
```

37.

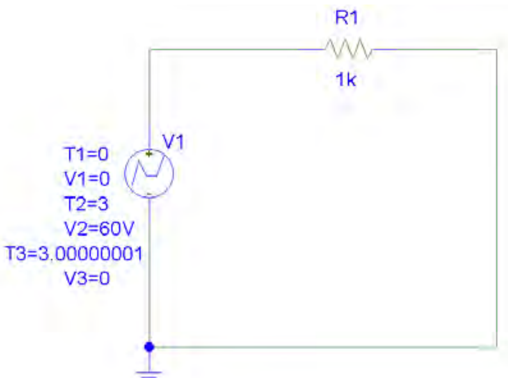
(a) $V_{av,1} = \boxed{30\text{V}}$

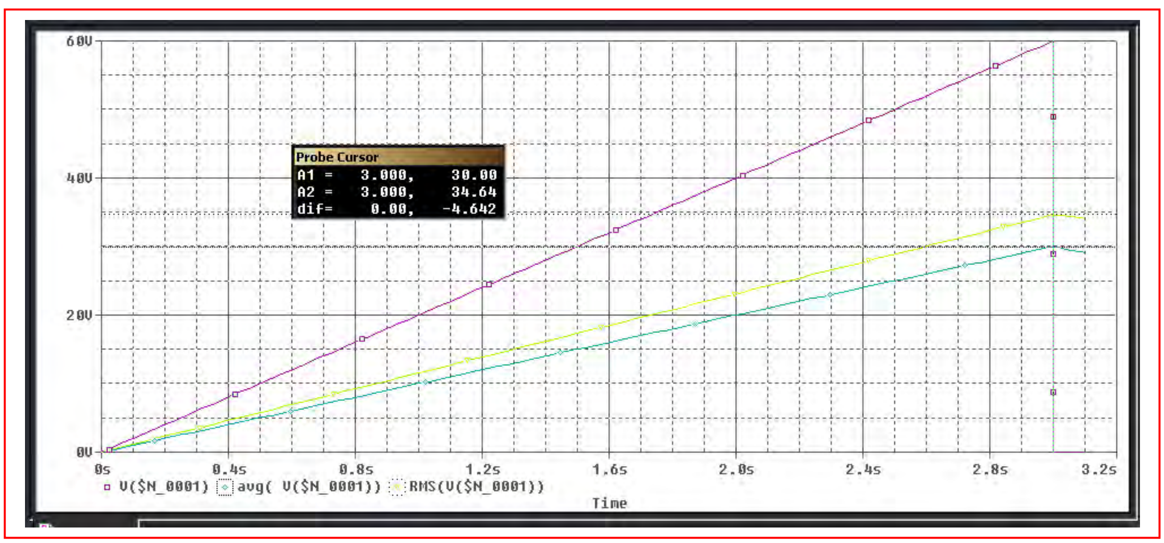
$$V_{av,2} = \frac{1}{3}(10+30+50) = \boxed{30\text{V}}$$

(b) $$V_{eff,1} = \sqrt{\frac{1}{3}\int_0^3 (20t)^2\,dt} = \sqrt{\frac{1}{3}\times 400 \times \frac{1}{3} \times 27} = \sqrt{1200} = \boxed{34.64\text{V}}$$

$$V_{eff,2} = \sqrt{\frac{1}{3}(10^2+30^2+50^2)} = \sqrt{\frac{1}{3}\times 3500} = \boxed{34.16\text{ V}}$$

(c) PSpice verification for Sawtooth waveform of Fig. 11.40*a*:





38. $\mathbf{Z}_{\text{eff}} = \text{R} \,||\, \left(\dfrac{-j10^6}{3\omega}\right) = \dfrac{-jR10^6}{3\omega R - j10^6}$

$$\mathbf{I}_{\text{SRC}} = \frac{120\angle 0}{-j\dfrac{10^6}{\omega} - j\dfrac{R10^6}{3\omega R - j10^6}} = \frac{120\omega\left(3\omega \text{R} - j10^6\right)}{-j10^6\left(3\omega R - j10^6\right) - j\omega R10^6}$$

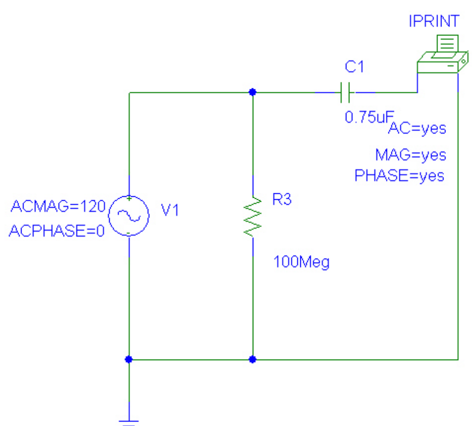
$$\mathbf{I}_{3\mu\text{F}} = \mathbf{I}_{\text{SRC}}\frac{R}{R - j\dfrac{10^6}{3\omega}}$$

(a) For the two current magnitudes to be equal, we must have $\left|\dfrac{R}{R - j\dfrac{10^6}{3\omega}}\right| = 1$. This is only true when $\boxed{\text{R} = \infty;}$ otherwise, current is shunted through the resistor and the two capacitor currents will be unequal.

(b) In this case, the capacitor current is

$$120\angle 0 \frac{1}{-j\dfrac{10^6}{\omega} - j\dfrac{10^6}{3\omega}} = j90\omega\ \mu\text{A, or} \qquad \boxed{90\omega\cos(\omega t + 90^\circ)\ \mu\text{A}}$$

(c) PSpice verification: set f = 60 Hz, simulate a single 0.75- µF capacitor, and include a 100-MΩ resistor in parallel with the capacitor to prevent a floating node. This should resit in a rms current amplitude of 33.93 mA, which it does.



FREQ	IM(V_PRINT3)	IP(V_PRINT3)
6.000E+01	3.393E-02	9.000E+01

39.

$$v(t) = 10t[u(t) - u(t-2)] + 16e^{-0.5(t-3)}\ [u(t-3) - u(t-5)]\ \text{V}$$

Find eff. value separately

$$\text{V}_{1,eff} = \sqrt{\frac{1}{5}\int_0^2 100t^2 dt} = \sqrt{\frac{20}{3}\times 8} = 7.303$$

$$\text{V}_{2,eff} = \sqrt{\frac{1}{5}\int_3^5 256e^{-(t-3)} dt} = \sqrt{\frac{256}{5} e^3 (-e^{-t})_3^5} = 6.654$$

$$\therefore \text{V}_{eff} = \sqrt{7.303^2 + 6.654^2} = \boxed{9.879}$$

$$\text{V}_{eff} = \sqrt{\frac{1}{5}\left[\int_0^2 100t^2 dt + \int_3^5 256e^3 e^{-t} dt\right]}$$

$$= \sqrt{\frac{1}{5}\left[\frac{100}{3}\times 8 + 256e^3\,(e^{-3} - e^{-5})\right]}$$

$$= \sqrt{\frac{1}{5}\left[\frac{800}{3} + 256\,(1 - e^{-2})\right]} = 9.879\ \text{V}\ \text{OK}$$

40. The peak instantaneous power is 250 mW. The combination of elements yields

$\mathbf{Z} = 1000 + j1000\ \Omega = 1414\ \angle 45^\circ\ \Omega.$

Arbitrarily designate $\mathbf{V} = V_m \angle 0$, so that $\mathbf{I} = \dfrac{V_m \angle 0}{\mathbf{Z}} = \dfrac{V_m \angle -45^\circ}{1414}$ A.

We may write $p(t) = \frac{1}{2} V_m I_m \cos\phi + \frac{1}{2} V_m I_m \cos(2\omega t + \phi)$ where ϕ = the angle of the current (-45°). This function has a maximum value of $\frac{1}{2} V_m I_m \cos\phi + \frac{1}{2} V_m I_m$.

Thus, $0.250 = \frac{1}{2} V_m I_m (1 + \cos\phi) = \frac{1}{2}(1414)\, I_m^2\, (1.707)$
and $I_m = 14.39$ mA.

In terms of rms current, the largest rms current permitted is $14.39 / \sqrt{2} = \boxed{10.18 \text{ mA rms.}}$

41. $\mathbf{I} = 4\angle 35° \text{ A rms}$

(a) $\mathbf{V} = 20\mathbf{I} + 80\angle 35° \quad \text{Vrms}, \ P_{s,gen} = 80 \times 10 \cos 35° = \boxed{655.3 \ \text{W}}$

(b) $P_R = |\mathbf{I}|^2 R = 16 \times 20 = \boxed{320 \ \text{W}}$

(c) $P_{Load} = 655.3 - 320 = \boxed{335.3 \text{ W}}$

(d) $AP_{s,gen} = 80 \times 10 = \boxed{800 \ \text{VA}}$

(e) $AP_R = P_R = \boxed{320 \text{ VA}}$

(f) $\mathbf{I}_L = 10\angle 0° - 4\angle 35° = 7.104\angle -18.84° \text{ A rms}$

$\therefore AP_L = 80 \times 7.104 = \boxed{568.3 \text{ V A}}$

(g) $PF_L = \cos\theta_L = \dfrac{P_L}{AP_L} = \dfrac{335.3}{568.3} = \boxed{0.599}$

since I_L lags V, PF_L is $\boxed{\text{lagging}}$

42.

(a) $$I_s = \frac{120}{4 + \dfrac{j192}{12 + j16}} = 9.214\angle -26.25° \text{ A rms}$$

$$\therefore \text{PF}_s = \cos 26.25 = \boxed{0.8969 \text{ lag}}$$

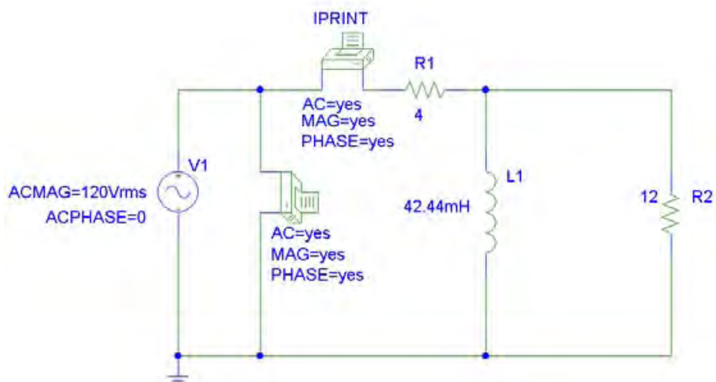
(b) $$P_s = 120 \times 9.214 \times 0.8969 = \boxed{991.7 \text{W}}$$

(c) $$Z_L = 4 + \frac{j48}{3 + j4} = 4 + \frac{1}{25}(192 + j144)$$

$$\therefore Z_L = 11.68 + j5.76 \Omega, \; Y_L = \frac{11.68 - j5.76}{11.68^2 + 5.76^2}$$

$$\therefore j120\pi \text{C} = \frac{j5.76}{11.68^2 + 5.76^2}, \; \text{C} = \boxed{90.09 \mu\text{F}}$$

(d) PSpi ce verification



FREQ VM($N_0003,0) VP($N_0003,0)
6.000E+01 1.200E+02 0.000E+00

FREQ IM(V_PRINT1) IP(V_PRINT1)
6.000E+01 9.215E+00 -2.625E+01 ; (a) and (b) are correct

Next, add a 90.09-μF capacitor in parallel with the source:

FREQ IM(V_PRINT1) IP(V_PRINT1)
6.000E+01 8.264E+00 -9.774E-05 ;(c) is correct (-9.8×10^{-5} degrees is essentially zero, for unity PF).

43.

$$\mathbf{Z}_A = 5 + j2\,\Omega,\ \mathbf{Z}_B = 20 - j10\,\Omega,\ \mathbf{Z}_c = 10\angle 30° = 8.660 + j5\ \Omega$$

$$\mathbf{Z}_D = 10\angle -60° = 5 - j8.660\ \Omega$$

$$\mathbf{I}_1 = \frac{\begin{vmatrix} 200 & -20 + j10 \\ 0 & 33.66 - j13.660 \end{vmatrix}}{\begin{vmatrix} 25 - j8 & -20 + j10 \\ -20 + j10 & 33.66 - j13.660 \end{vmatrix}} = \frac{7265\angle 22.09°}{480.9\angle -26.00°} = 15.11\angle 3.908°\ \text{A rms}$$

$$\mathbf{I}_2 = \frac{\begin{vmatrix} 25 - j8 & 200 \\ -20 + j10 & 0 \end{vmatrix}}{480.9\angle -26.00°} = \frac{200(20 - j10)}{480.9\angle 20.00°} = 9.300\angle -0.5681°\ \text{A rms}$$

$$\text{AP}_A = \left|\mathbf{I}_1\right|^2 \left|\mathbf{Z}_A\right| = 15.108^2\sqrt{29} = \boxed{1229\ \text{VA}}$$

$$\text{AP}_B = \left|\mathbf{I}_1 - \mathbf{I}_2\right|^2 \left|\mathbf{Z}_B\right| = 5.881^2 \times 10\sqrt{5} = \boxed{773.5\,\text{VA}}$$

$$\text{AP}_C = \left|\mathbf{I}_2\right| 2 \left|\mathbf{Z}_C\right| = 9.3^2 \times 10 = \boxed{86.49\,\text{VA}}$$

$$\text{AP}_D = \left|\mathbf{I}_2\right|^2 \left|\mathbf{Z}_1\right| = 9.3^2 \times 10 = \boxed{864.9\,\text{VA}}$$

$$\text{AP}_S = 200\left|\mathbf{I}_1\right| = 200 \times 15.108 = \boxed{3022\,\text{VA}}$$

44. $\mathbf{Z}_1 = 30\angle 15°\,\Omega,\ \mathbf{Z}_2 = 40\angle 40°\,\Omega$

(a) $\mathbf{Z}_{tot} = 30\angle 15° + 40\angle 40° = 68.37\angle 29.31°\,\Omega$

$\therefore \text{PF} = \cos 29.3° = \boxed{0.8719 \text{ lag}}$

(b) $\mathbf{V} = \mathbf{IZ}_{tot} = 683.8\angle 29.31^{\circ}\,\Omega$ so

$\mathbf{S} = \mathbf{VI}^* = \left(683.8\angle 29.31^{\circ}\right)\left(10\angle 0\right) = 6838\angle 29.31^{\circ}$ VA .

Thus, the apparent power = $\boxed{\text{S} = 6.838 \text{ kVA.}}$

(c) The impedance has a positive angle; it therefore has a net $\boxed{\text{inductive}}$ character.

45. $\theta_1 = \cos^{-1}(0.92) = 23.07^\circ$, $\theta_2 = \cos^{-1}(0.8) = 36.87^\circ$, $\theta_3 = 0$

$$\mathbf{S}_1 = \frac{100\angle 23.07^\circ}{0.92} = 100 + j42.59 \text{ VA}$$

$$\mathbf{S}_2 = \frac{250\angle 36.87^\circ}{0.8} = 250 + j187.5 \text{ VA}$$

$$\mathbf{S}_3 = \frac{500\angle 0^\circ}{1} = 500 \text{ VA}$$

$$\mathbf{S}_{total} = \mathbf{S}_1 + \mathbf{S}_2 + \mathbf{S}_3 = 500 + j230.1 \text{ VA} = 550.4\angle 24.71^\circ \text{ VA}$$

(a) $$\mathbf{I}_{eff} = \frac{S_{total}}{V_{eff}} = \frac{550.4}{115} = \boxed{4.786 \text{ A rms}}$$

(b) PF of composite load = cos (24.71°) = $\boxed{0.9084 \textit{ lagging}}$

46.

$$AP_L = 10{,}000 \text{ VA, } PF_L = 0.8 \text{ lag, } |\mathbf{I}_L| = 40 \text{ A rms}$$

$$\text{Let } \mathbf{I}_L = 40\angle 0° \text{ A rms; } P_L = 10{,}000 \times 0.8 = 8000 \text{ W}$$

$$\text{Let } \mathbf{Z}_L = R_L + jX_L \therefore R_L = \frac{8000}{40^2} = 5\ \Omega$$

$$\cos\theta_L = 0.8 \text{ lag} \therefore \theta_L = \cos^{-1} 0.8 = 36.87°$$

$$\therefore X_L = 5\tan 36.87° = 3.75\Omega,\ \mathbf{Z}_L = 5 + j3.75,\ \mathbf{Z}_{tot} = 5.2 + j3.75\Omega$$

$$\therefore \mathbf{V}_s = 40(5.2 + j3.75) = 256.4\angle 35.80° \text{ V; } \mathbf{Y}_{tot} = \frac{1}{5.2 + j3.75}$$

$$= 0.12651 - j0.09124\text{S},\ \mathbf{Y}_{new} = 0.12651 + j(120\pi C - 0.09124),$$

$$PF_{new} = 0.9 \text{ lag}, \theta_{new} = 25.84° \therefore \tan 25.84° = 0.4843$$

$$= \frac{0.09124 - 120\pi C}{0.12651} \therefore$$

$$C = \boxed{79.48\mu F}$$

47. $\mathbf{Z}_{eff} = j100 + j300 \| 200 = 237 \angle 54.25°$. PF = cos 54.25° = 0.5843 *lagging*.

(a) Raise PF to 0.92 *lagging* with series capacitance

$$\mathbf{Z}_{new} = j100 + jX_C + j300 \| 200 = 138.5 + j(192.3 + X_C)\ \Omega$$

$$\tan^{-1}\left(\frac{192.3 + X_C}{138.5}\right) = \cos^{-1} 0.92 = 23.07°$$

Solving, we find that $X_C = -133.3\ \Omega = -1/\omega C$, so that C = $\boxed{7.501\ \mu F}$

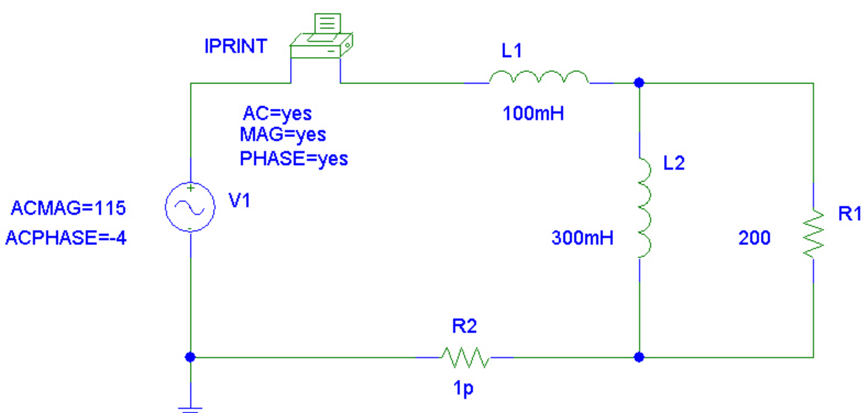
(b) Raise PF to 0.92 *lagging* with parallel capacitance

$$\mathbf{Z}_{new} = j100 \| jX_C + j300 \| 200 = \frac{-100\,X_C}{j(100 + X_C)} + 138.5 + j92.31\ \Omega$$

$$= 138.5 + j\left(92.31 + \frac{100X_C}{100 + X_C}\right)\ \Omega$$

$$\tan^{-1}\left(\frac{92.31 + \frac{100X_C}{100 + X_C}}{138.5}\right) = \cos^{-1} 0.92 = 23.07°$$

Solving, we find that $X_C = -25\ \Omega = -1/\omega C$, so that C = $\boxed{40\ \mu F}$



General circuit for simulations. Results agree with hand calculations

	FREQ	IM(V_PRINT1)	IP(V_PRINT1)	θ PF
With no compensation:	1.592E+02	4.853E-01	-5.825E+01	54.25° 0. 5843 lag
With series compensation:	1.592E+02	7.641E-01	-2.707E+01	23.07° 0. 9200 lag
With parallel compensation:	1.592E+02	7.641E-01	-2.707E+01	23.07° 0. 9200 lag

48.

$$\mathbf{Z}_{in} = -j10 + \frac{20(1+j2)}{3+j2} = 10.769 - j3.846 = 11.435^{+}\angle -19.65°\ \Omega$$

$$\therefore \mathbf{I}_s = \frac{100}{11.435\angle -19.654°} = 8.745\angle 19.65°$$

$$\therefore \mathbf{S}_s = -\mathbf{V}_s\mathbf{I}_s^* = -100 \times 8.745\angle -19.65° = \boxed{-823.5 + j294.1\text{VA}}$$

$$\mathbf{I}_{20} = 8.745\angle 19.65° \times \frac{10+j20}{30+j20} = 5.423\angle 49.40°$$

$$\therefore \mathbf{S}_{20} = 20 \times 5.432^2 = \boxed{588.2 + j0\ \text{VA}}$$

$$\mathbf{I}_{10} = \frac{20 \times 5.423\angle 49.40}{10+j20} = 4.851\angle -14.04°$$

$$\mathbf{S}_{10} = 10 \times 4.851^2 = \boxed{235.3 + j0\ \text{VA}}$$

$$\mathbf{S}_{j20} = j20 \times 4.851^2 = \boxed{j470.6\ \text{VA}},$$

$$\mathbf{S}_{-j10} = -j10 \times 8.745^2 = \boxed{-j764.7\ \text{VA}}, \quad \Sigma = 0$$

49.

$$\frac{\mathbf{V}_x - 100}{6+j4} + \frac{\mathbf{V}_x}{-j10} + \frac{\mathbf{V}_x - j100}{5} = 0$$

$$\therefore \mathbf{V}_x \left(\frac{1}{6+j4} + j0.1 + 0.2 \right) = \frac{100}{6+j4} + j20$$

$$\therefore \mathbf{V}_x = 53.35^{-} \angle 42.66° \text{ V}$$

$$\therefore \mathbf{I}_1 = \frac{100 - 53.35^{-} \angle 42.66°}{6+j4} = 9.806 \angle -64.44° \text{ A}$$

$$\therefore \mathbf{S}_{1.gen} = \frac{1}{2} \times 100 \times 9.806 \angle 64.44° = \boxed{211.5 + j442.3 \text{ VA}}$$

$$\mathbf{S}_{6,abs} = \frac{1}{2} \times 6 \times 9.806^2 = \boxed{288.5 + j0 \text{ VA}}$$

$$\mathbf{S}_{j4,abs} = \frac{1}{2}(j4)9.806^2 = \boxed{0 + j192.3 \text{ VA}}$$

$$\mathbf{I}_2 = \frac{j100 - 53.35^{-} \angle 42.66°}{5} = 14.99 \angle 121.6°,$$

$$\mathbf{S}_{5abs} = \frac{1}{2} \times 5 \times 14.99^2 = \boxed{561.5 + j0 \text{ VA}}$$

$$\mathbf{S}_{2,gen} = \frac{1}{2}(j100)14.99 \angle -121.57° = \boxed{638.4 - j392.3 \text{ VA}}$$

$$\mathbf{S}_{-j10,abs} = \frac{1}{2}\left(\frac{53.35}{10} \right)(-j10) = 0 - j142.3 \text{ VA} = \boxed{142.3 \angle -90° \text{ VA}} \quad \Sigma = 0$$

50.

(a) $500\,\text{VA, PF} = 0.75 \text{ lead} \therefore$

$$\mathbf{S} = 500\angle -\cos^{-1} 0.75 = \boxed{375 - j330.7 \text{ VA}}$$

(b) $500\text{W, PF} = 0.75 \text{ lead} \therefore$

$$\mathbf{S} = 500 - \frac{500}{j.075}\sin(\cos^{-1} 0.75) = \boxed{500 - j441.0 \text{ VA}}$$

(c) $-500 \text{ VAR, PF} = 0.75 (\text{lead}) \therefore \theta = -\cos^{-1} 0.75 = -41.41°$

$\therefore \text{P}\, 500 / \tan 41.41° = 566.9\text{W},$

$$\boxed{\mathbf{S} = 566.9 - j500 \text{ VA}}$$

51. $\mathbf{S}_s = 1600 + j500\,\text{VA (gen)}$

(a) $\mathbf{I}_s^* = \dfrac{1600 + j500}{400} = 4 + j1.25 \therefore \mathbf{I}_s = 4 - j1.25$

$\mathbf{I}_c = \dfrac{400}{-j120} = j3.333\,\text{A rms} \therefore \mathbf{I}_L = \mathbf{I}_s - \mathbf{I}_c = 4 - j1.25 - j3.333$

$\therefore \mathbf{I}_L = 4 - j4.583\,\text{A rms} \therefore$

$\mathbf{S}_L = 400(4 + j4.583) = \boxed{1600 + j1833 \quad \text{VA}}$

(b) $\text{PF}_L = \cos\left(\tan^{-1}\dfrac{1833.3}{1600}\right) = \boxed{0.6575^+ \ \text{lag}}$

(c) $\mathbf{S}_s = 1600 + j500 = 1676\angle 17.35^\circ \ \text{VA} \therefore \text{PF}_s = \cos 17.35^\circ = \boxed{0.9545 \ \text{lag}}$

52. $(\cos^{-1} 0.8 = 36.87°,\ \cos^{-1} 0.9 = 25.84°)$

(a)
$$\begin{aligned}\mathbf{S}_{tot} &= 1200\angle 36.87° + 1600\angle 25.84° + 900\\ &= 960 + j720 + 1440 + j697.4 + 900\\ &= 3300 + j1417.4 = 3592\angle 23.25°\text{ VA}\\ \therefore \mathbf{I}_s &= \frac{3591.5}{230} = \boxed{15.62\text{ A rms}}\end{aligned}$$

(b) $\text{PF}_s = \cos 23.245° = \boxed{0.9188}$

(c) $\mathbf{S} = \boxed{3300 + j1417\text{ VA}}$

53.

(a) $P_{s,tot} = 20 + 25 \times 0.8 + 30 \times 0.75 = \boxed{70 \text{ kW}}$

(b)
$$\mathbf{I}_1 = \frac{20,000}{250} = 80\angle 0° \text{ A rms}$$
$$|\mathbf{I}_2| = 25,000/250 = 100 \text{ A rms}$$
$$\angle\mathbf{I}_2 = -\cos^{-1} 0.8 = -36.87 \therefore \mathbf{I}_2 = 100\angle -36.87° \text{ A rms}$$
$$AP_3 = \frac{30,000}{0.75} = 40,000 \text{ VA}, \quad |\mathbf{I}_3| = \frac{40,000}{250} = 160 \text{ A rms}$$
$$\angle\mathbf{I}_3 = -\cos^{-1} 0.75 = -41.41° \therefore \mathbf{I}_3 = 160\angle -41.41° \text{ A rms}$$
$$\therefore \mathbf{I}_s = 80\angle 0° + 100\angle -36.87° + 160\angle -41.41° = 325.4\angle -30.64° \text{ A rms}$$
$$\therefore AP_s = 250 \times 325.4 = \boxed{81,360 \text{ VA}}$$

(c) $$PF_3 = \frac{70,000}{81,360} = \boxed{0.8604 \text{ lag}}$$

54. 200 kW average power and 280 kVAR reactive result in a power factor of PF = cos $(\tan^{-1}(280/200)) = 0.5813$ *lagging*, which is pretty low.

(a) 0.65 peak = 0.65(200) = 130 kVAR
Excess = 280 – 130 = 150 kVAR, for a cost of (12)(0.22)(150) = $\boxed{\$396 \text{ / year.}}$

(b) Target = **S** = P + *j*0.65 P
$\theta = \tan^{-1}(0.65P/P) = 33.02^{\circ}$, so target PF = $\cos\theta = \boxed{0.8385}$

(c) A single 100-kVAR increment costs \$200 to install. The excess kVAR would then be 280 – 100 – 130 = 50 kVAR, for an annual penalty of \$332. This would result in a first-year savings of \$64.

A single 200-kVAR increment costs \$395 to install, and would remove the entire excess kVAR. The savings would be \$1 (wow) in the first year, but \$396 each year thereafter.

The single 200-kVAR increment is the most economical choice.

55. Perhaps the easiest approach is to consider the load and the compensation capacitor separately. The load draws a complex power $\mathbf{S}_{load} = P + jQ$. The capacitor draws a purely reactive complex power $\mathbf{S}_C = -jQ_C$.

$\theta_{load} = \tan^{-1}(Q/P)$, or $Q = P \tan \theta_{load}$

$$Q_C = S_C = V_{rms} \left| \frac{\mathbf{V}_{rms}}{(-j/\omega C)} \right| = \left| \omega C \mathbf{V}_{rms}^2 \right| = \omega C V_{rms}^2$$

$$\mathbf{S}_{total} = \mathbf{S}_{load} + \mathbf{S}_C = P + j(Q - Q_C)$$

$$\theta_{new} = \text{ang}(\mathbf{S}_{total}) = \tan^{-1}\left(\frac{Q\text{-}Q_C}{P}\right), \text{ so that } Q - Q_C = P \tan \theta_{new}$$

Substituting, we find that $Q_C = P \tan \theta_{load} - P \tan \theta_{new}$
or

$$\omega C V_{rms}^2 = P\,(\tan \theta_{load} - \tan \theta_{new})$$

Thus, noting that $\theta_{old} = \theta_{load}$,

$$\boxed{C = \frac{P\left(\tan \theta_{old} - \tan \theta_{new}\right)}{\omega V_{rms}^2}}$$

56. $\mathbf{V} = 339\ \angle{-66^\circ}$ V, $\omega = 100\pi$ rad/ s, connected to $\mathbf{Z} = 1000\ \Omega$.

(a) $V_{eff} = \dfrac{339}{\sqrt{2}} = \boxed{239.7 \text{ V rms}}$

(b) $p_{max} = 339^2 / 1000 = \boxed{114.9 \text{ W}}$

(c) $p_{min} = \boxed{0 \text{ W}}$

(d) Apparent power $= V_{eff}\, I_{eff} = \left(\dfrac{339}{\sqrt{2}}\right)\left(\dfrac{339/\sqrt{2}}{1000}\right) = \dfrac{V_{eff}^2}{1000} = \boxed{57.46 \text{ VA}}$

(e) Since the load is purely resistive, it draws $\boxed{\text{zero reactive power.}}$

(f) $S = \boxed{57.46 \text{ VA}}$

57. $\mathbf{V} = 339\ \angle\text{-}66^\circ$ V, $\omega = 100\pi$ rad/s to a purely inductive load of 150 mH ($j47.12\ \Omega$)

(a) $$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}} = \frac{339\angle -66^\circ}{j47.12} = 7.194\angle -156^\circ \text{ A}$$

so $$\mathbf{I}_{eff} = \frac{7.194}{\sqrt{2}} = \boxed{5.087 \text{ A rms}}$$

(b) $p(t) = ½\, V_m I_m \cos\phi + ½\, V_m I_m \cos(2\omega t + \phi)$

where ϕ = angle of current – angle of voltage

$p_{max} = ½\, V_m I_m \cos\phi + ½\, V_m I_m = (1 + \cos(-90^\circ))\,(339)(7.194)/\,2 = \boxed{1219 \text{ W}}$

(c) $p_{min} = ½\, V_m I_m \cos\phi - ½\, V_m I_m = \boxed{-1219 \text{ W}}$

(d) apparent power $= V_{eff} I_{eff} = \dfrac{339}{\sqrt{2}}(5.087) = \boxed{1219 \text{ VA}}$

(e) reactive power $= Q = V_{eff} I_{eff} \sin(\theta - \phi) = \boxed{1219 \text{ VA}}$

(f) complex power $= \boxed{j1219\ \text{VA}}$

58. 1 $\quad$ H $\rightarrow j\ \Omega$, 4 μF $\rightarrow -j250\ \Omega$

$$\mathbf{Z}_{\text{eff}} = j \,||\, -j250 \,||\, 10^4\ \Omega = 1.004\ \angle 89.99^\circ\ \Omega$$

$$\mathbf{V}_{10\text{k}} = \frac{(5\angle 0)\ (1.004\angle\ 89.99^\circ\ )}{2500 + (1.004\angle 89.99^\circ\ )} = 2.008\ \angle 89.97^\circ\ \ \text{mV}$$

(a) $\quad p_{\max} = (0.002)^2 / 10\times 10^3 = \boxed{400\ \text{pW}}$

(b) $\boxed{0\ \text{W}}$ (purely resistive elements draw no reactive power)

(c) apparent power $= V_{\text{eff}}I_{\text{eff}} = ½\ V_m I_m = ½\ (0.002)^2 / 10000 = \boxed{200\ \text{pVA}}$

(d) $$\mathbf{S}_{\text{source}} = \frac{1}{2}(5\angle 0)\left(\frac{5\angle 0}{2500\angle 0.02292}\right) = \boxed{0.005\ \angle\text{-}0.02292^\circ\ \ \text{VA}}$$

59. (a) At $\omega = 400$ rad/s, 1 μF → $-j2500$ Ω, 100 mH → $j40$ Ω
Defi ne $\mathbf{Z}_{eff} = -j2500 \parallel (250 + j40) = 256 \angle 3.287^\circ$ Ω

$$\mathbf{I}_S = \frac{12000\angle 0}{20 + 256\angle 3.287^\circ} = 43.48 \angle -3.049^\circ \text{ A rms}$$

$$\mathbf{S}_{source} = (12000)(43.48) \angle 3.049^\circ = \boxed{521.8 \angle 3.049^\circ \text{ kVA}}$$

$$\mathbf{S}_{20\Omega} = (43.48)^2 (20) \angle 0 = \boxed{37.81 \angle 0 \quad \text{kVA}}$$

$$\mathbf{V}_{eff} = \frac{(12000\angle 0)(256\angle 3.287^\circ)}{20 + 256\angle 3.287^\circ} = 11130 \angle 0.2381^\circ \text{ V rms}$$

$$\mathbf{I}_{1\mu F} = \frac{\mathbf{V}_{eff}}{-j2500} = 4.452 \angle 90.24^\circ \text{ A rms}$$

so $\mathbf{S}_{1\mu F} = (11130)(4.452) \angle -90^\circ = \boxed{49.55 \angle -90^\circ \text{ kVA}}$

$$\mathbf{V}_{100mH} = \frac{(11130\angle 0.2381^\circ)(j40)}{250 + j40} = 1758 \angle 81.15^\circ \text{ V rms}$$

$$\mathbf{I}_{100mH} = \frac{\mathbf{V}_{100mH}}{j40} = 43.96 \angle -8.852^\circ \text{ A rms}$$

so $\mathbf{S}_{100\mu H} = (1758)(4.43.96) \angle 90^\circ = \boxed{77.28 \angle 90^\circ \text{ kVA}}$

$$\mathbf{V}_{250\Omega} = \frac{(11130\angle 0.2381^\circ)(250)}{250 + j40} = 10990 \angle -8.852^\circ \text{ V rms}$$

so $\mathbf{S}_{250\Omega} = (10990)^2 / 250 = \boxed{483.1 \angle 0^\circ \text{ kVA}}$

(b) $37.81 \angle 0 + 49.55 \angle -90^\circ + 77.28 \angle 90^\circ + 483.1 \angle 0^\circ = \boxed{521.6 \angle 3.014^\circ \text{ kVA,}}$ which is within rounding error of the complex power delivered by the source.

(c) The apparent power of the source is 521.8 kVA. The apparent powers of the passive elements sum to 37.81 + 49.55 + 77.28 + 483.1 = 647.7 kVA, so $\boxed{\text{NO!}}$ Phase angle is important!

(d) $P = \mathbf{V}_{eff} \mathbf{I}_{eff} \cos(\text{ang } \mathbf{V}_S - \text{ang } \mathbf{I}_S) = (12000)(43.48) \cos(3.049^\circ) = \boxed{521 \text{ kW}}$

(e) $Q = \mathbf{V}_{eff} \mathbf{I}_{eff} \sin(\text{ang } \mathbf{V}_S - \text{ang } \mathbf{I}_S) = (12000)(43.48) \sin(3.049^\circ) = \boxed{27.75 \text{ kVAR}}$

60. (a) Peak current $= 28\sqrt{2} = \boxed{39.6\text{ A}}$

(b) $\theta_{load} = \cos^{-1}(0.812) = +35.71^o$ (since lagging PF). Assume ang (**V**) = 0°.

$$p(t) = \left(2300\sqrt{2}\right)\left(39.6\right)\cos\left(120\pi t\right)\cos\left(120\pi t - 35.71^o\right)$$

at $t = 2.5$ ms, then, $p(t) = \boxed{71.89\text{ kW}}$

(c) $P = V_{eff}\,I_{eff}\cos\theta = (2300)(28)\cos(35.71^o) = \boxed{52.29\text{ kW}}$

(d) $\mathbf{S} = V_{eff}\,I_{eff}\angle\theta = \boxed{64.4\angle 35.71^o\text{ kVA}}$

(e) apparent power $= |\mathbf{S}| = \boxed{64.4\text{ kVA}}$

(f) $|\mathbf{Z}_{load}| = |\mathbf{V}/\mathbf{I}| = 2300/28 = 82.14\ \Omega$. Thus, $\mathbf{Z}_{load} = \boxed{82.14\angle 35.71^o\ \Omega}$

(g) $Q = V_{eff}\,I_{eff}\sin\theta = \boxed{37.59\text{ kVAR}}$

1.

$$\mathbf{Z}_c = \frac{10^6}{j500 \times 25} = -j80\,\Omega,\ \frac{50(-j80)}{50-j80} = 42.40\angle -32.01°\,\Omega$$

$$\therefore \mathbf{V} = 84.80\angle -32.01°\,\text{V},\ \mathbf{I}_R = 1.696\angle -32.01°\,\text{A}$$

$$\mathbf{I}_c = 1.0600\angle 57.99°\,\text{A}$$

$$p_s(\pi/2\text{ms}) = 84.80\cos(45° - 32.01°)\,2\cos 45° = \boxed{116.85\,\text{W}}$$

$$p_R = 50 \times 1.696^2 \cos^2(45° - 32.01°) = \boxed{136.55\,\text{W}}$$

$$p_c = 84.80\cos(45° - 32.01°) = 1.060\cos(45° + 57.99°) = \boxed{-19.69\,\text{W}}$$

2.

(a) $4\text{H}: i = 2t^2 - 1 \therefore v = Li' = 4(4t) = 16t, w_L = \frac{1}{2} Li^2 = \frac{1}{2} \times 4(4t^4 - 4t^2 + 1)$

$\therefore w_L = 8t^4 - 8t^2 + 2 \therefore w_L(3) - w_L(1) = 8 \times 3^4 - 8 \times 3^2 + 2 - 8 \times 1 + 8 \times 1 - 2 = \boxed{576 \text{ J}}$

(b) $0.2\,\text{F}: v_c = \frac{1}{0.2}\int_1^t (2t^2 - 1)\, dt + 2 = 5\left(\frac{2}{3}t^3 - t\right)_1^t + 2 = 5\left(\frac{2}{3}t^3 - t\right) - 5\left(\frac{2}{3} - 1\right) + 2$

$\therefore v_c(2) = \frac{10}{3} \times 8 - 10 - \frac{10}{3} + 5 + 2 = \frac{61}{3}\text{V} \therefore \text{P}_c(2) = \frac{61}{3} \times 7 = \boxed{142.33\,\text{W}}$

3. $v_c(0) = -2\text{V},\ i(0) = 4\text{A},\ \alpha = \dfrac{\text{R}}{2\text{L}} = 2,\ \omega_o^2 = \dfrac{1}{\text{LC}} = 3,\ s_{1,2} = -2 \pm 1 = -1, -3$

(a) $i = \text{A}e^{-t} + \text{B}e^{-3t} \therefore \text{A} + \text{B} = 4;\ i(0^+) = \dfrac{1}{1} v_L(0^+) = (-4 \times 4 \times +2) = -14$

$\therefore -\text{A} - 38 = -14 \therefore \text{B} = 5,\ \text{A} = -1,\ i = -e^{-t} + 5e^{-3t}\ \text{A}$

$\therefore +v_c = 3\int_o^t (-e^{-t} + 5e^{-3t})\, dt - 2 = 3(e^{-t} - 5e^{-3t})\,\big|_o^t - 2 = e^{-t} - 3 - 5e^{-3t} + 5 - 2$

$\therefore v_c = 3e^{-t} - 5e^{-3t} \therefore \text{P}_c(0^+) = (3-5)(-1+5) = \boxed{-8\,\text{W}}$

(b) $\text{P}_c(0.2) = (3e^{-0.2} - 5e^{-0.6})(-e^{0.2} + 5e^{-0.6}) = \boxed{-0.5542\,\text{W}}$

(c) $\text{P}_c(0.4) = (3e^{-0.4} - 5e^{-1.2})(5e^{-1.2} - e^{-0.4}) = \boxed{0.4220\,\text{W}}$

4. We assume the circuit has already reached sinusoidal steady state by t = 0.

2.5 kΩ → 2.5 kΩ, 1 H → j1000 Ω, 4 μF → -j250 Ω, 10 kΩ → 10 kΩ

$$\mathbf{Z}_{eq} = j1000 \parallel -j250 \parallel 10000 = 11.10 - j333.0\ \Omega$$

$$\mathbf{V}_{eq} = \frac{(20\angle 30)(11.10 - j333.0)}{2500 + 11.10 - j333.0} = 2.631\angle -50.54^\circ\ \text{V}$$

$$\mathbf{I}_{10k} = \frac{\mathbf{V}_{eq}}{10000} = 0.2631\angle -50.54^\circ\ \text{mA} \qquad \mathbf{I}_{1\,H} = \frac{\mathbf{V}_{eq}}{j1000} = 2.631\angle -140.5^\circ\ \text{mA}$$

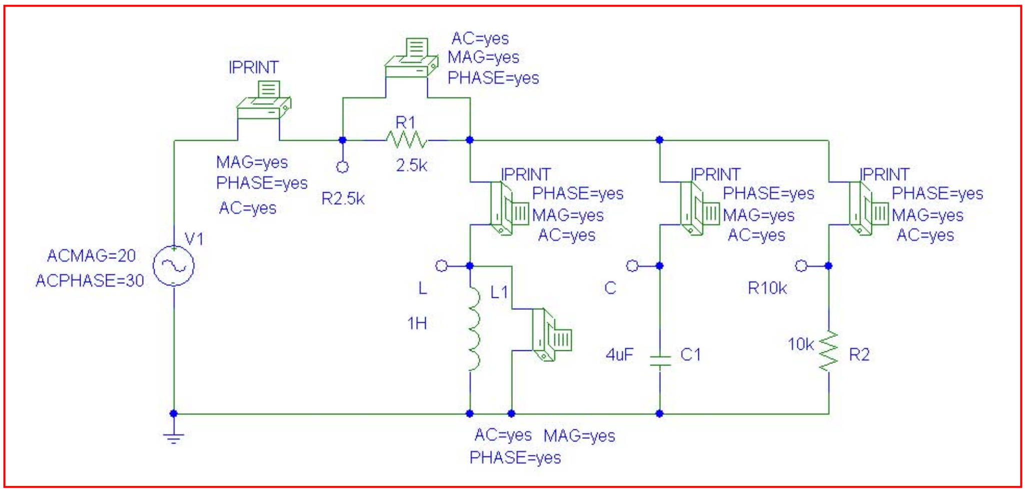
$$\mathbf{I}_{4\,\mu F} = \frac{\mathbf{V}_{eq}}{-j250} = 10.52\angle 39.46^\circ\ \text{mA} \quad \mathbf{V}_{2.5k} = \frac{(20\angle 30)(2500)}{2500 + 11.10 - j333.0} = 19.74\angle 37.55^\circ\ \text{V}$$

Thus, $$P_{2.5k} = \frac{[19.74\cos 37.55^\circ]^2}{2500} = \boxed{97.97\ \text{mW}}$$

$$P_{1\,H} = [2.631\cos(-50.54)^\circ][2.631\times 10^{-3}\cos(-140.5^\circ)] = \boxed{-3.395\ \text{mW}}$$

$$P_{4\,\mu F} = [2.631\cos(-50.54^\circ)][10.52\times 10^{-3}\cos(39.46^\circ)] = \boxed{13.58\ \text{mW}}$$

$$P_{2.5k} = \frac{[2.631\cos(-50.54^\circ)]^2}{10000} = \boxed{279.6\ \mu\text{W}}$$




```
FREQ       IM(V_PRINT1)   IP(V_PRINT1)
  1.592E+02  7.896E-03     3.755E+01

FREQ       VM(R2_5k,$N_0002)VP(R2_5k,$N_0002)
  1.592E+02  1.974E+01     3.755E+01

FREQ       IM(V_PRINT2)   IP(V_PRINT2)
  1.592E+02  2.628E-03     -1.405E+02

FREQ       VM(L,0)         VP(L,0)
  1.592E+02  2.629E+00     -5.054E+01

FREQ       IM(V_PRINT11)  IP(V_PRINT11)
  1.592E+02  1.052E-02     3.946E+01

FREQ       IM(V_PRINT12)  IP(V_PRINT12)
  1.592E+02  2.629E-04     -5.054E+01
```

5.

$$i_s \to 5\angle 0°\text{ A, C} \to -j4\,\Omega,\ \mathbf{Z}_{in} = 8\|(3-j4) = \frac{40\angle -53.13°}{11-j4}$$

$$= 3.417\angle -33.15° \therefore \mathbf{V}_s = 17.087\angle -33.15°,$$

$$v_s = 17.087\cos(25t - 33.15°)\text{ V} \therefore$$

$$\text{P}_{s,abs}(0.1) = -17.087\cos(2.5^{\text{rad}} - 33.147°)\times 5\cos 2.5^{\text{rad}} = \boxed{-23.51\ \text{W}}$$

$$i_8 = \frac{17.087}{8}\cos(25t - 33.15°) \therefore$$

$$i_8(0.1) = 2.136\cos(2.5^{\text{rad}} - 33.15°) = -0.7338\ \text{A}$$

$$\therefore \text{P}_{8,abs} = 0.7338^2 \times 8 = \boxed{4.307\ \text{W}};$$

$$\mathbf{I}_3 = \frac{17.087\angle -33.15°}{3-j4} = 3.417\angle 19.98°\text{ A}$$

$$\therefore i_3(0.1) = 3.417\cos(2.5^{\text{rad}} + 19.98°) = -3.272\text{ A} \therefore$$

$$\text{P}_{3,abc} = 3.272^2 \times 3 = \boxed{32.12\,\text{W}}$$

$$\mathbf{V}_c = -j4(3.417\angle 19.983°) = 13.67\angle -70.02°,$$

$$v_c(0.1) = 13.670\cos(2.5^{\text{rad}} - 70.02°) = 3.946\text{ V}$$

$$\therefore \text{P}_{c,abc} = 3.946(-3.272) = \boxed{-12.911\ \text{W}} \qquad (\Sigma = 0)$$

6. For $t > 0$, $i(t) = 8e^{-\frac{R}{L}t} = 8e^{-2t}$.

(a) $p(0^+) = (8)^2(1) = \boxed{64 \text{ W}}$

(b) at $t = 1$ s, $i = 8e^{-2} = 1.083$ A; $p(1) = i^2R = \boxed{1.723 \text{ W.}}$

(c) at $t = 2$ s, $i = 8e^{-4} = 146.5$ mA; $p(2) = i^2R = \boxed{21.47 \text{ mW}}$

7. $v(t) = (3)(6000)e^{-t/30\times10^{-3}}$

(a) $p(0^+) = v^2(0^+)/R = (18\times10^3)^2 / 6000 = \boxed{54 \text{ kW}}$

(b) $p(0.03) = v^2(0.03)/R = (18\times10^3 e^{-1})^2 / 6000 = \boxed{7.308 \text{ kW}}$

(c) $p(0.09) = v^2(0.09)/R = (18\times10^3 e^{-3})^2 / 6000 = \boxed{134 \text{ W}}$

8. (a) $p = (30\times10^{3})^{2}\,(1.2\times10^{-3}) = \boxed{1.080\text{ MW}}$

(b) $W = (1.080\times10^{6})(150\times10^{-6}) = \boxed{162\text{ J}}$

9. W = $\frac{1}{2}CV^2$. The initial voltage, $v(0^+)$, is therefore

$$v(0^+) = \sqrt{\frac{2W}{C}} = \sqrt{\frac{2(100\times10^{-3})}{100\times10^{-3}}} = \sqrt{2}\ \text{V} \ \text{ and so } \ v(t) = \sqrt{2}e^{-t/RC} = \sqrt{2}e^{-t/0.12}\ \text{V}.$$

The instantaneous power dissipated at t = 120 mS is therefore

$$\text{p(120 ms)} = \frac{v^2(120\text{ ms})}{R} = \frac{2e^{-2}}{1.2} = \boxed{226\text{ mW}}$$

The energy dissipated over the first second is given by

$$\int_0^1 \frac{v^2(t)}{R}dt = \int_0^1 \frac{2e^{-2t/RC}}{R}dt = -\frac{RC}{2}\left(\frac{2}{R}\right)\left[e^{-2/RC} - 1\right] \approx \ 100\text{ mJ}$$

ΔT = Q/mc, where Q = 100 mJ, c = 0.9 kJ/kg•K, and m = 10^{-3} kg.

Thus, the final temperature

$$= 271.15 + 23 + \frac{100\times10^{-6}\ \text{kJ}}{\left(10^{-3}\ \text{kG}\right)\left(0.9\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)} = 271.15 + 23 + 0.1111$$

$$= \boxed{294.3\text{ K, representing a temperature increase of 0.1111 K.}}$$

10. (a) $p = (276)(130) = \boxed{358.8 \text{ mW}}$

(b) $v(t) = 2.76\cos 1000t$ V (given); we need to know $\boxed{\text{the I-V relationship}}$ for this (nonlinear) device.

11.

$$\mathbf{Z}_{in} = 4 + \frac{j5(10 - j5)}{10} = 4 + 2.5 + j5 = 6.5 + j5\ \ \Omega$$

$$\therefore \mathbf{I}_s = \frac{100}{6.5 + j5} = 12.194\angle -37.57°\ \text{A}$$

$$\therefore \text{P}_{s,abs} = -\frac{1}{2} \times 100 \times 12.194 \cos 37.57° = -483.3\ \text{W}$$

$$\text{P}_{4,abs} = \frac{1}{2}(12.194)^2\, 4 = \boxed{297.4\ \text{W}},$$

$$\text{P}_{cabs} = \boxed{0}$$

$$\mathbf{I}_{10} = \frac{100}{6.5 + j5}\ \frac{j5}{10} = 6.097\angle 52.43°\ \ \text{so}$$

$$\text{P}_{10,abs} = \frac{1}{2}(6.097)^2 \times 10 = \boxed{185.87\ \text{W}}$$

$$\text{P}_L = \boxed{0} \qquad (\Sigma = 0)$$

12.

$$\mathbf{V} = (10 + j10)\frac{40\angle 30^\circ}{5\angle 50^\circ + 8\angle -20^\circ} = 52.44\angle 69.18^\circ \text{ V}$$

$$\text{P}_{10,gen} = \frac{1}{2} \times 10 \times 52.44 \cos 69.18^\circ = \boxed{93.19 \text{ W}}$$

$$\text{P}_{j10,gen} = \frac{1}{2} \times 10 \times 52.44 \cos(90^\circ - 69.18^\circ) = \boxed{245.1 \text{ W}}$$

$$\text{P}_{5\angle 50 abs} = \frac{1}{2}\left(\frac{52.44}{5}\right)^2 \cos(50^\circ) = \boxed{176.8 \text{ W}}$$

$$\text{P}_{8\angle -20 abs} = \frac{1}{2}\left(\frac{52.44}{8}\right)^2 \cos(-20^\circ) = \boxed{161.5 \text{ W}} \qquad (\Sigma_{gen} = \Sigma_{abs})$$

13.

$$\mathbf{Z}_R = 3 + \frac{1}{0.1 - j0.3} = 3 + 1 + j3 = 4 + j3\,\Omega$$

Ignore $30°$ on $\mathbf{V}_s$, $\mathbf{I}_R = 5\,\frac{2 + j5}{6 + j8}$, $|\mathbf{I}_R| = \frac{5\sqrt{29}}{10}$

(a) $$\text{P}_{3\Omega} = \frac{1}{2}\left(\frac{5\sqrt{29}}{10}\right)^2 \times 3 = \boxed{10.875\ \text{W}}$$

(b) $$\text{V}_s = 5\angle 0° \frac{(2 + j5)(4 + j3)}{6 + j8} = 13.463\angle 51.94°\,\text{V}$$

$$\therefore \text{P}_{s,gen} = \frac{1}{2} \times 13.463 \times 5 \cos 51.94° = \boxed{20.75\ \text{W}}$$

14.

$$\boxed{P_{j10} = P_{-j5} = 0,}$$

$$\frac{\mathbf{V}_{10} - 50}{j10} + \frac{\mathbf{V}_{10}}{10} + \frac{\mathbf{V}_{10} - j50}{-j5} = 0$$

$$\therefore \mathbf{V}_{10}(-j0.1 + 0.1 + j0.2) + j5 + 10 = 0$$

$$\therefore \mathbf{V}_{10} = 79.06\angle 16.57° \text{ V}$$

$$P_{10\Omega} = \frac{1}{2}\frac{79.06^2}{10} = \boxed{312.5 \text{ W};}$$

$$\mathbf{I}_{50} = \frac{79.06\angle 161.57° - 50}{j10} = 12.75\angle 78.69° \text{ A}$$

$$\therefore P_{50V} = \frac{1}{2} \times 50 \times 12.748 \cos 78.69° = \boxed{62.50 \text{ W}}$$

$$\mathbf{I}_{j50} = \frac{79.06\angle 161.57° - j50}{-j5} = 15.811\angle -7.57° :$$

$$P_{j50} = \frac{1}{2} \times 50 \times 15.811 \cos(90° + 71.57°) = \boxed{-375.0 \text{ W}}$$

15.

$$\frac{\mathbf{V}_x - 20}{2} + \frac{\mathbf{V}_x - \mathbf{V}_c}{3} = 2\mathbf{V}_c \qquad [1]$$

and

$$0 = \frac{\mathbf{V}_c}{-j2} + \frac{\mathbf{V}_c - \mathbf{V}_x}{3} \qquad [2]$$

which simplify to

$5\mathbf{V}_x - 14\mathbf{V}_c = 60$ [1] and

$j2\mathbf{V}_x + (3 - j2)\mathbf{V}_c = 0$ [2]

Solving,

$\mathbf{V}_x = 9.233\angle -83.88°\text{ V}$ and $\mathbf{V}_c = 5.122\angle -140.2°\text{ V}$

$$\text{P}_{gen} = \frac{1}{2} \times 9.233 \times (2 \times 5.122)\cos(-83.88° + 140.2°) = \boxed{26.22\text{ W}}$$

16.

(a) $X_{in} = 0 \therefore \mathbf{Z}_L = \boxed{R_{th} + j0}$

(b) R_L, X_L independent $\therefore \boxed{\mathbf{Z}_L = \mathbf{Z}_{th}^* = R_{th} - jX_{th}}$

(c) R_L fixed $\therefore P_L = \dfrac{1}{2}\dfrac{|V_{th}|^2}{(R_{th}+R_L)^2 + (X_{th}+X_L)^2} \times R_L \therefore \boxed{\mathbf{Z}_L = R_L - jX_{th}}$

(d) X_L fixed, Let $X_L + X_{th} = a \therefore f = \dfrac{2P_L}{|\mathbf{V}_{th}|^2} = \dfrac{RL}{(R_{th}+R_L)^2 + a^2}$

$$\frac{df}{dR_L} = \frac{R_{th} + R_L^2 + a^2 - 2R_L(R_{th}+R_L)}{\left[(R_{th}+R_L)^2 + a^2\right]^2} = 0$$

$$R_{th}^2 + 2R_{th}R_L + R_L^2 + a^2 - 2R_{th}R_L = 2R_L^2 = 0$$

$$\therefore R_L = \sqrt{R_{th}^2 + a^2} = \boxed{\sqrt{R_{th}^2 + (X_{th}+X_L)^2}}$$

(e) $X_L = 0 \therefore R_L = \boxed{\sqrt{R_{th}^2 + X_{th}^2}} = |\mathbf{Z}_{th}|$

17.

$$\mathbf{V}_{th} = 120\frac{-j10}{10+j5} = 107.3\angle -116.6^\circ \text{ V}$$

$$\mathbf{Z}_{th} = \frac{-j10\,(10+j15)}{10+j5} = 8-j14\,\Omega$$

(a) $$\mathbf{Z}_{TH} = \left(\mathbf{Z}_L\right)^* = \boxed{8+j14\ \Omega}$$

(b) $$\mathbf{I}_L = \frac{\mathbf{V}_{TH}}{\mathbf{Z}_{TH}+\left(\mathbf{Z}_{TH}\right)^*} = \frac{107.3\angle -116.6^\circ}{16}.$$

$$\mathbf{V}_L = \mathbf{V}_{TH}\frac{\left(\mathbf{Z}_{TH}\right)^*}{\mathbf{Z}_{TH}+\left(\mathbf{Z}_{TH}\right)^*} = \frac{\left(107.3\angle -116.6^\circ\right)\left(16.12\angle -60.26^\circ\right)}{16}$$

$$\text{P}_{L,\max} = \frac{1}{2}\left[\frac{(107.3)(16.12)}{16}\right]\left[\frac{107.3}{10}\right]\cos\left(-116.6^\circ-60.26^\circ+116.6^\circ\right) = \boxed{179.8\text{ W}}$$

18.

$$R_L = |\mathbf{Z}_{th}| \therefore R_L = \sqrt{8^2 + 14^2} = \boxed{16.125\,\Omega}$$

$$P_L = \frac{1}{2}\frac{107.33^2}{(8+16.125)^2 + 14^2} \times 16.125 = \boxed{119.38\,\text{W}}$$

19.

$$-j9.6 = -4.8\,\mathrm{I}_x - j1.92\ \mathrm{I}_x - +4.8\mathrm{I}_x$$

$$\therefore\ \mathrm{I}_x = \frac{9.6}{1.92} = 5$$

$$\therefore\ \mathrm{V} = (0.6\times 5)8 = 24\,\mathrm{V}$$

$$\therefore\ \mathrm{P}_o = \frac{1}{2}\times 24\times 1.6\times 5 = \boxed{96\ \mathrm{W}\ \ (gen)}$$

20.

(a) $$\mathbf{Z}_{th} = 80 \| j60 = \frac{j480}{80 + j60}\,\frac{80 - j60}{80 - j60}$$

$$= 28.8 + j38.4\,\Omega \;\therefore\; \mathbf{Z}_{L\,\max} = \boxed{28.8 - j38.4\,\Omega}$$

(b) $$\mathbf{V}_{th} = 5(28.8 + j38.4) = 144 + j192\,\text{V},$$

$$\therefore \mathbf{I}_L = \frac{144 + j192}{2 \times 28.8}$$

$$\text{and } \mathrm{P}_{L,\max} = \frac{1}{2}\,\frac{144^2 + 192^2}{4 \times 28.8^2} \times 28.8 = \boxed{250\text{ W}}$$

21. $\mathbf{Z}_{eq} = (6 - j8) \parallel (12 + j9) = 8.321 \angle -19.44^o$ W

$\mathbf{V}_{eq} = (5 \angle -30^o)(8.321 \angle -19.44^o) = 41.61 \angle -49.44^o$ V

$P_{total} = ½ (41.61)(5) \cos(-19.44^o) = 98.09$ W

$\mathbf{I}_{6-j8} = \mathbf{V}_{eq} / (6 - j8) = 4.161 \angle 3.69^o$ A
$\mathbf{I}_{4+j2} = \mathbf{I}_{8+j7} = \mathbf{V}_{eq} / 12+j9 = 2.774 \angle -86.31^o$ A

$P_{6-j8} = ½ (41.61)(4.161) \cos(-49.44^o - 3.69^o) = \boxed{51.94 \text{ W}}$

$P_{4+j2} = ½ (2.774)2 (4) = \boxed{15.39 \text{ W}}$
$P_{8+j7} = ½ (2.774)2 (8) = \boxed{30.78 \text{ W}}$

Check: $\Sigma = 98.11$ W (okay)

22.

$$\mathbf{V}_{th} = 100\frac{j10}{20+j10} = 20+j40,\ \mathbf{Z}_{th} = \frac{j10(20)}{20+j10} = 4+j8\,\Omega$$

$$\therefore \text{R}_L = |\mathbf{Z}_{th}| \ \therefore \text{R}_L = \boxed{8.944\,\Omega}$$

$$\therefore \text{P}_{L,\max} = \frac{1}{2}\,\frac{20^2+40^2}{(4+8.944)^2+64}\times 8.944 \boxed{= 38.63\,\text{W}}$$

23. We may write a single mesh equation: $170\angle 0^\circ = (30 + j10)\,\mathbf{I}_1 - (10 - j50)(-\lambda\mathbf{I}_1)$
Solving,

$$\mathbf{I}_1 = \frac{170\angle 0^\circ}{30 + j10 + 10\lambda - j50\lambda}$$

(a) $\lambda = 0$, so $\mathbf{I}_1 = \dfrac{170\angle 0^\circ}{30 + j10} = 5.376\angle{-18.43^\circ}$ A and, with the same current flowing through both resistors in this case,

$P_{20} = ½\,(5.376)^2\,(20) = \boxed{289.0 \text{ W}}$
$P_{10} = ½\,(5.376)^2\,(10) = \boxed{144.5 \text{ W}}$

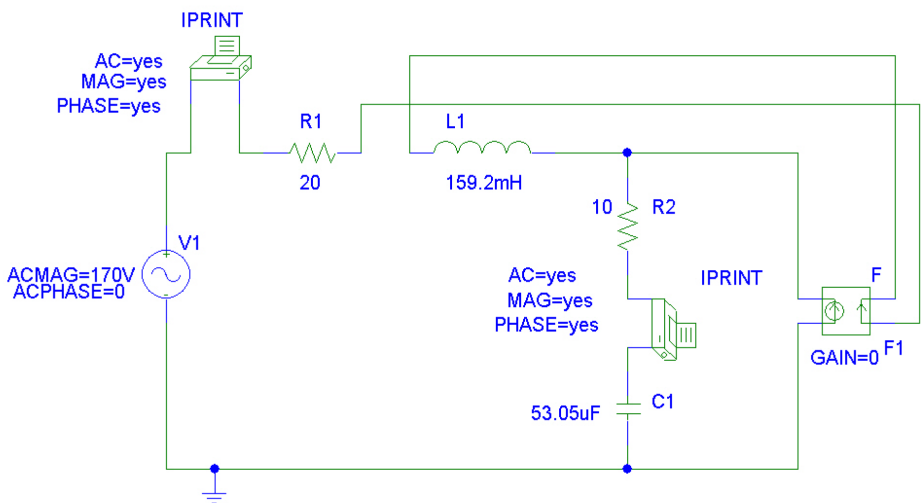
(b) $\lambda = 1$, so $\mathbf{I}_1 = \dfrac{170\angle 0^\circ}{40 - j40} = 3.005\angle 45^\circ$ A

$P_{20} = ½\,(3.005)^2\,(20) = \boxed{90.30 \text{ W}}$

The current through the 10-Ω resistor is $\mathbf{I}_1 + \lambda\mathbf{I}_1 = 2\,\mathbf{I}_1 = 6.01\angle 45^\circ$ so

$P_{10} = ½\,(6.01)^2\,(10) = \boxed{180.6 \text{ W}}$

(c)



(a)

FREQ	IM(V_PRINT3)	IP(V_PRINT3)
6.000E+01	5.375E+00	-1.846E+01

FREQ	IM(V_PRINT4)	IP(V_PRINT4)
6.000E+01	5.375E+00	-1.846E+01

(b)

FREQ	IM(V_PRINT3)	IP(V_PRINT3)
6.000E+01	6.011E+00	4.499E+01

FREQ	IM(V_PRINT4)	IP(V_PRINT4)
6.000E+01	3.006E+00	4.499E+01

24. (a) Waveform (a): $I_{avg} = \dfrac{(10)(1)+(-5)(1)+0(1)}{3} = \boxed{1.667 \text{ A}}$

Waveform (b): $I_{avg} = \dfrac{\frac{1}{2}(20)(1)+0(1)}{2} = \boxed{5 \text{ A}}$

Waveform (c):

$$I_{avg} = \frac{1}{1\times10^{-3}}\int_0^{10^{-3}} 8\sin\frac{2\pi t}{4\times10^{-3}}\,dt = -\left(8\times10^{3}\right)\left(\frac{4\times10^{-3}}{2\pi}\right)\cos\left(\frac{\pi t}{2\times10^{-3}}\right)\Bigg|_0^{10^{-3}}$$

$$= -\frac{16}{\pi}(0-1) = \boxed{\frac{16}{\pi}\text{ A}}$$

(b) Waveform (a): $I^2_{avg} = \dfrac{(100)(1)+(25)(1)+(0)(1)}{3} = \boxed{41.67 \text{ A}^2}$

Waveform (b): $i(t) = -20\times10^3\, t + 20$

$i^2(t) = 4\times10^8\, t^2 - 8\times10^5\, t + 400$

$$I^2_{avg} = \frac{1}{2\times10^{-3}}\int_0^{10^{-3}} \left(4\times10^8 t^2 - 8\times10^5 t + 400\right)dt$$

$$= \frac{1}{2\times10^{-3}}\left[\frac{4\times10^8}{3}\left(10^{-3}\right)^3 - \frac{8\times10^5}{2}\left(10^{-3}\right)^2 + 400\left(10^{-3}\right)\right] = \frac{0.1333}{2\times10^{-3}} = \boxed{66.67 \text{ A}^2}$$

Waveform (c):

$$I^2_{avg} = \frac{1}{1\times10^{-3}}\int_0^{10^{-3}} 64\sin^2\frac{2\pi t}{4\times10^{-3}}\,dt = \left(64\times10^3\right)\left[\frac{t}{2} - \frac{\sin\pi\times10^3 t}{2\pi\times10^3}\right]\Bigg|_0^{10^{-3}}$$

$$= \left(64\times10^3\right)\left[\frac{10^{-3}}{2} - \frac{\sin\pi}{2\pi\times10^3}\right] = \boxed{32 \text{ A}^2}$$

25. At $\omega = 120\pi$, 1 H $\rightarrow j377\ \Omega$, and 4 µF $\rightarrow -j663.1\ \Omega$

Define $\mathbf{Z}_{eff} = j377 \parallel -j663.1 \parallel 10\,000 = 870.5 \angle 85.01^\circ\ \Omega$

$$\mathbf{V}_{2.5k} = \frac{\left(400\sqrt{2}\angle -9^\circ\right)2500}{2500 + 870.5\angle 85.01^\circ} = 520.4\angle -27.61^\circ \text{ V}$$

$$\mathbf{V}_{10k} = \frac{\left(400\sqrt{2}\angle -9^\circ\right)\left(870.5\angle 85.01^\circ\right)}{2500 + 870.5\angle 85.01^\circ} = 181.2\angle 57.40^\circ \text{ V}$$

Thus, $P_{2.5k} = ½ (520.4)2 / 2\,500 = \boxed{54.16 \text{ W}}$

$P_{10k} = ½ (181.2)2 / 10\,000 = \boxed{1.642 \text{ W}}$

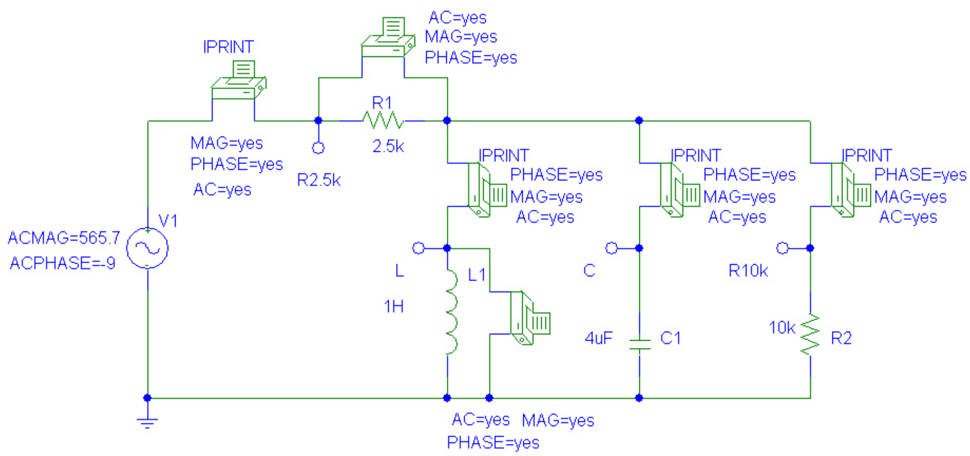
$P_{1H} = \boxed{0}$

$P_{4\mu F} = \boxed{0}$ (A total absorbed power of 55.80 W.)

To check, the average power delivered by the source:

$$\mathbf{I}_{source} = \frac{400\sqrt{2}\angle -9^\circ}{2500 + 870.5\angle 85.01^\circ} = 0.2081\angle -27.61^\circ \text{ A}$$

and $P_{source} = ½\,(400\sqrt{2})(0.2081)\cos(-9^\circ + 27.61^\circ) =$ 55.78 W (checks out).



FREQ	IM(V_PRINT1)	IP(V_PRINT1)	FREQ	VM(L,0)	VP(L,0)
6.000E+01	2.081E-01	-2.760E+01	6.000E+01	1.812E+02	5.740E+01
FREQ	VM(R2_5k,$N_0002)	VP(R2_5k,$N_0002)	FREQ	IM(V_PRINT11)	IP(V_PRINT11)
6.000E+01	5.204E+02	-2.760E+01	6.000E+01	2.732E-01	1.474E+02
FREQ	IM(V_PRINT2)	IP(V_PRINT2)	FREQ	IM(V_PRINT12)	IP(V_PRINT12)
6.000E+01	4.805E-01	-3.260E+01	6.000E+01	1.812E-02	5.740E+01

26. (a) $\sqrt{\frac{1}{T}\int_0^T \frac{144}{2}\left(1+\cos 2000t\right)dt} = \sqrt{\frac{144}{2}} = \boxed{8.485}$

(b) $\sqrt{\frac{1}{T}\int_0^T \frac{144}{2}\left(1-\cos 2000t\right)dt} = \sqrt{\frac{144}{2}} = \boxed{8.485}$

(c) $\sqrt{\frac{1}{T}\int_0^T \frac{144}{2}\left(1+\cos 1000t\right)dt} = \sqrt{\frac{144}{2}} = \boxed{8.485}$

(d) $\sqrt{\frac{1}{T}\int_0^T \frac{144}{2}\left[1+\cos\left(1000t-176^{\circ}\right)\right]dt} = \sqrt{\frac{144}{2}} = \boxed{8.485}$

27. (a) $\sqrt{\frac{1}{T}\int_0^T \frac{4}{2}\left(1+\cos 20t\right)dt} = \frac{2}{\sqrt{2}} = \boxed{1.414}$

(b) $\sqrt{\frac{1}{T}\int_0^T \frac{4}{2}\left(1-\cos 20t\right)dt} = \frac{2}{\sqrt{2}} = \boxed{1.414}$

(c) $\sqrt{\frac{1}{T}\int_0^T \frac{4}{2}\left(1+\cos 10t\right)dt} = \frac{\sqrt{2}}{2} = \boxed{1.414}$

(d) $\sqrt{\frac{1}{T}\int_0^T \frac{4}{2}\left[1+\cos\left(10t-64^\circ\right)\right]dt} = \frac{\sqrt{2}}{2} = \boxed{1.414}$

28. T = 3 s; integrate from 1 to 4 s; need only really integrate from 1 to 3 s as function is zero between t = 3 and t = 4 s.

$$V_{rms} = \sqrt{\frac{1}{3}\int_1^3 (10)^2 \, dt} = \sqrt{\frac{100}{3} t \Big|_1^3} = \sqrt{\frac{100(2)}{3}} = \boxed{8.165 \text{ V}}$$

29. T = 3 s; integrate from 2 to 5 s; need only really integrate from 2 to 3 s as function is zero between t = 3 and t = 4 s.

$$I_{rms} = \sqrt{\frac{1}{3}\int_2^3 (7)^2\,dt} = \sqrt{\frac{49}{3}t\Big|_2^3} = \sqrt{\frac{49(1)}{3}} = \boxed{4.041\text{ A}}$$

30. (a) $\boxed{1\text{ V}}$

(b) $V_{rms} = \sqrt{V_{1_{eff}}^2 + V_{2_{eff}}^2} = \sqrt{1^2 + \left(\frac{1}{\sqrt{2}}\right)^2} = \boxed{1.225\text{ V}}$

(c) $V_{rms} = \sqrt{V_{1_{eff}}^2 + V_{2_{eff}}^2} = \sqrt{1^2 + \left(\frac{1}{\sqrt{2}}\right)^2} = \boxed{1.225\text{ V}}$

31.

(a) $v = 10 + 9\cos 100t + 6\sin 100t$

$$\therefore \mathrm{V}_{eff} = \sqrt{100 + \frac{1}{2}\times 81 + \frac{1}{2}\times 36} = \sqrt{158.5} = \boxed{12.590\ \mathrm{V}}$$

(b) $$\mathrm{F}_{eff} = \sqrt{\frac{1}{4}(10^2 + 20^2 + 10^2)} = \sqrt{150} = \boxed{12.247}$$

(c) $$\mathrm{F}_{\mathrm{avg}} = \frac{(10)(1) + (20)(1) + (10)(1)}{4} = \frac{40}{4} = \boxed{10}$$

32.

(a) $g(t) = 2 + 3\cos 100t + 4\cos(100t - 120^{\circ})$

$3\angle 0 + 4\angle{-120^{\circ}} = 3.606\angle{-73.90^{\circ}}$ so $G_{eff} = \sqrt{4+\dfrac{3.606^2}{2}} = \boxed{3.240}$

(b) $h(t) = 2 + 3\cos 100t + 4\cos(101t - 120^{\circ})$

$\therefore H_{eff} = \sqrt{2^2 + \frac{1}{2}3^2 + \frac{1}{2}4^2} = \sqrt{16.5} = \boxed{4.062}$

(c) $f(t) = 100t,\ 0 < t < 0.1 \therefore F_{eff} = \sqrt{\dfrac{1}{0.3}\int_0^{0.1} 10^6 t^2 dt}$

$= \sqrt{\dfrac{10}{3}\times 10^6 \times \dfrac{1}{3} \times 10^{-3}} = \boxed{33.33}$

33. $f(t) = (2 - 3\cos 100t)^2$

(a) $f(t) = 4 - 12\cos 100t + 9\cos^2 100t$

$\therefore f(t) = 4 - 12\cos 100t + 4.5 + 4.5\cos 200t \;\therefore \mathrm{F}_{av} = 4 + 4.5 = \boxed{8.5}$

(b) $\mathrm{F}_{eff} = \sqrt{8.5^2 + \frac{1}{2}\times 12^2 + \frac{1}{2}\times 4.5^2} = \boxed{12.43}$

34. (a) $$i_{\text{eff}} = \left[\frac{1}{3}\left(10^2 + (-5)^2\right) + 0\right]^{1/2} = \boxed{6.455\text{ A}}$$

(b) $$i_{\text{eff}} = \left[\frac{1}{2}\left(\int_0^1 [-20t + 20]\,dt\right) + 0\right]^{1/2} = \sqrt{5} = \boxed{2.236\text{ A}}$$

(c) $$i_{\text{eff}} = \left[\frac{1}{1}\left(\int_0^1 8\sin\left(\frac{2\pi}{4}t\right)dt\right)\right]^{1/2} = \sqrt{\left[-8\left(\frac{2}{\pi}\right)\cos\left(\frac{\pi t}{2}\right)\right]_0^1} = \boxed{2.257\text{ A}}$$

35.

(a) $A = B = 10\text{V},\ C = D = 0 \therefore 10\angle 0° + 10\angle -45° = 18.48\angle -22.50^{\circ}$

$\therefore P = \frac{1}{2}\times\frac{1}{4}\times 18.48^2 = \boxed{42.68\ \text{W}}$

(b) $A = C = 10\text{V},\ B = D = 0,\ v_s = 10\cos 10t + 10\cos 40t,$

$P = \frac{1}{2}\frac{10^2}{4} + \frac{1}{2}\frac{10^2}{4} = \boxed{25\ \text{W}}$

(c) $v_s = 10\cos 10t - 10\sin\ (10t + 45°) \rightarrow 10 - 10\angle -45° = 7.654\angle 67.50^{\circ}$

$\therefore P = \frac{1}{2}\frac{7.654^2}{4} = \boxed{7.322\ \text{W}}$

(d) $v = 10\cos 10t + 10\sin(10t + 45°) + 10\cos 40t;$

$10\angle 0° + 10\angle -45° = 18.48\angle -22.50^{\circ}$

$\therefore P = \frac{1}{2}\times 18.48^2 \times\frac{1}{4} + \frac{1}{2}\times 10^2 \times\frac{1}{4} = \boxed{55.18\ \text{W}}$

(e) $// + 10dc \therefore P_{av} = 55.18 + \frac{10^2}{4} = \boxed{80.18\ \text{W}}$

36. $\mathbf{Z}_{eq} = \text{R} \parallel j0.3\omega = \dfrac{j0.3R\omega}{R + j0.3R\omega}$. By voltage division, then, we write:

$$\mathbf{V}_{100mH} = 120\angle 0 \; \frac{j0.1\omega}{j0.1\omega + \dfrac{j0.3R\omega}{R + j0.3\omega}} = 120\angle 0 \frac{-0.03\omega^2 + j0.1\omega R}{-0.03\omega^2 + j0.4R\omega}$$

$$\mathbf{V}_{300mH} = 120\angle 0 \; \frac{\dfrac{j0.3R\omega}{R + j0.3\omega}}{j0.1\omega + \dfrac{j0.3R\omega}{R + j0.3\omega}} = 120\angle 0 \frac{j36R\omega}{-0.03\omega^2 + j0.4R\omega}$$

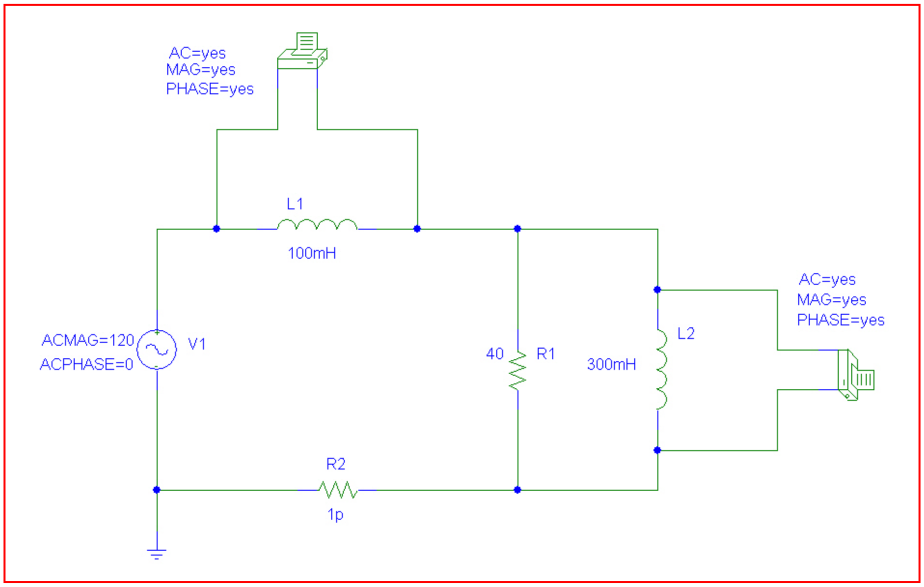
(a) We're interested in the value of R that would lead to equal voltage magnitudes, or

$$\left| j36R\omega \right| = \left| (120)\left(-0.03\omega^2 + j0.1\omega \text{R}\right) \right|$$

Thus, $36\text{R}\omega = \sqrt{12.96\omega^4 + 144\omega^2 R^2}$ or $\boxed{\text{R} = 0.1061\ \omega}$

(b) Substituting into the expression for $\mathbf{V}_{100mH}$, we find that $\text{V}_{100mH} = \boxed{73.47 \text{ V},}$ independent of frequency.

To verify with PSpice, simulate the circuit at 60 Hz, or $\omega = 120\pi$ rad/s, so R = 40 Ω. We also include a miniscule (1 pΩ) resistor to avoid inductor loop warnings. We see from the simulation results that the two voltage magnitudes are indeed the same.




```
FREQ        VM($N_0002,$N_0003)VP($N_0002,$N_0003)
6.000E+01   7.349E+01  -3.525E+01

FREQ        VM($N_0001,$N_0002)VP($N_0001,$N_0002)
6.000E+01   7.347E+01   3.527E+01
```

37.

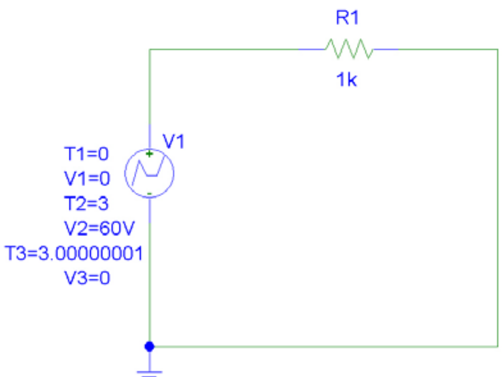
(a) $V_{av,1} = \boxed{30\text{V}}$

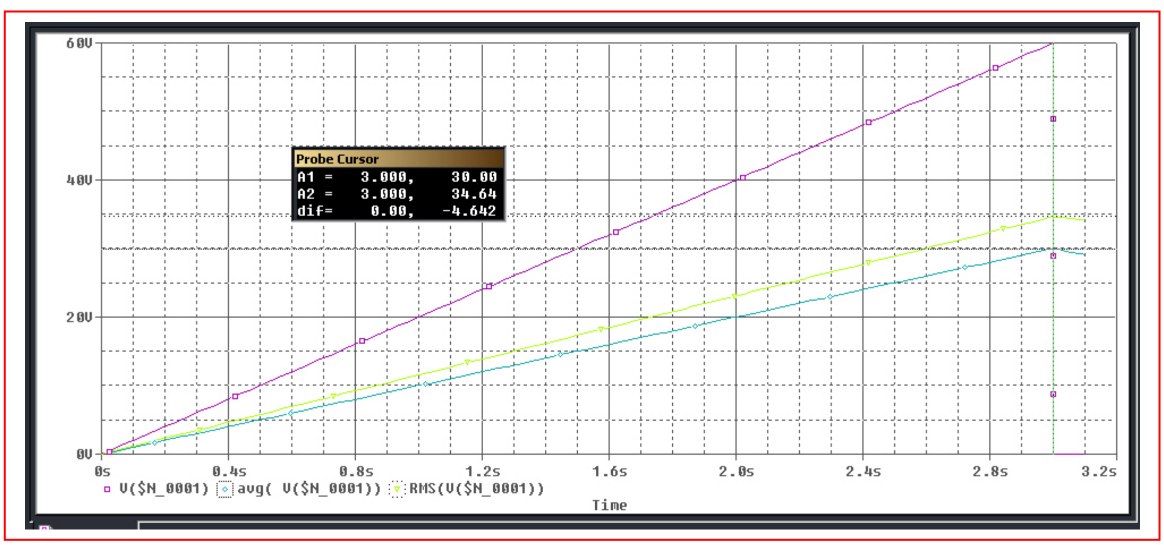
$$V_{av,2} = \frac{1}{3}(10+30+50) = \boxed{30\text{V}}$$

(b) $$V_{eff,1} = \sqrt{\frac{1}{3}\int_0^3 (20t)^2\,dt} = \sqrt{\frac{1}{3}\times 400\times\frac{1}{3}\times 27} = \sqrt{1200} = \boxed{34.64\text{V}}$$

$$V_{eff,2} = \sqrt{\frac{1}{3}(10^2+30^2+50^2)} = \sqrt{\frac{1}{3}\times 3500} = \boxed{34.16\text{ V}}$$

(c) PSpice verification for Sawtooth waveform of Fig. 11.40*a*:





38. $\mathbf{Z}_{\text{eff}} = \mathrm{R} \parallel \left(\dfrac{-j10^6}{3\omega} \right) = \dfrac{-jR10^6}{3\omega R - j10^6}$

$$\mathbf{I}_{\text{SRC}} = \frac{120\angle 0}{-j\dfrac{10^6}{\omega} - j\dfrac{R10^6}{3\omega R - j10^6}} = \frac{120\omega\left(3\omega \mathrm{R} - j10^6\right)}{-j10^6\left(3\omega R - j10^6\right) - j\omega R10^6}$$

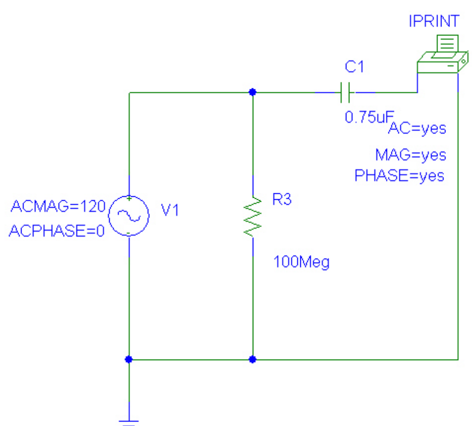
$$\mathbf{I}_{3\mu\text{F}} = \mathbf{I}_{\text{SRC}} \frac{R}{R - j\dfrac{10^6}{3\omega}}$$

(a) For the two current magnitudes to be equal, we must have $\left| \dfrac{R}{R - j\dfrac{10^6}{3\omega}} \right| = 1$. This is only true when $\boxed{\mathrm{R} = \infty;}$ otherwise, current is shunted through the resistor and the two capacitor currents will be unequal.

(b) In this case, the capacitor current is

$$120\angle 0 \frac{1}{-j\dfrac{10^6}{\omega} - j\dfrac{10^6}{3\omega}} = j90\omega\ \mu\text{A, or} \qquad \boxed{90\omega \cos(\omega t + 90^\circ)\ \mu\text{A}}$$

(c) PSpice verification: set f = 60 Hz, simulate a single 0.75-μF capacitor, and include a 100-MΩ resistor in parallel with the capacitor to prevent a floating node. This should resit in a rms current amplitude of 33.93 mA, which it does.



FREQ	IM(V_PRINT3)	IP(V_PRINT3)
6.000E+01	3.393E-02	9.000E+01

39.

$$v(t) = 10t[u(t) - u(t-2)] + 16e^{-0.5(t-3)}\ [u(t-3) - u(t-5)]\ \text{V}$$

Find eff. value separately

$$\text{V}_{1,eff} = \sqrt{\frac{1}{5}\int_0^2 100t^2 dt} = \sqrt{\frac{20}{3}\times 8} = 7.303$$

$$\text{V}_{2,eff} = \sqrt{\frac{1}{5}\int_3^5 256e^{-(t-3)} dt} = \sqrt{\frac{256}{5} e^3 (-e^{-t})_3^5} = 6.654$$

$$\therefore \text{V}_{eff} = \sqrt{7.303^2 + 6.654^2} = \boxed{9.879}$$

$$\text{V}_{eff} = \sqrt{\frac{1}{5}\left[\int_0^2 100t^2 dt + \int_3^5 256e^3 e^{-t} dt\right]}$$

$$= \sqrt{\frac{1}{5}\left[\frac{100}{3}\times 8 + 256e^3\,(e^{-3} - e^{-5})\right]}$$

$$= \sqrt{\frac{1}{5}\left[\frac{800}{3} + 256\,(1 - e^{-2})\right]} = 9.879\ \text{V OK}$$

40. The peak instantaneous power is 250 mW. The combination of elements yields

$\mathbf{Z} = 1000 + j1000\ \Omega = 1414\ \angle 45^{\circ}\ \Omega$.

Arbitrarily designate $\mathbf{V} = V_m \angle 0$, so that $\mathbf{I} = \dfrac{V_m\angle 0}{\mathbf{Z}} = \dfrac{V_m\angle -45^{\circ}}{1414}$ A.

We may write $p(t) = ½\ V_m\ I_m \cos\phi + ½\ V_m\ I_m \cos(2\omega t + \phi)$ where ϕ = the angle of the current (-45°). This function has a maximum value of $½\ V_mI_m \cos\phi + ½\ V_mI_m$.

Thus, $0.250 = ½\ V_mI_m (1 + \cos\phi) = ½\ (1414)\ I_m^2\ (1.707)$
and $I_m = 14.39$ mA.

In terms of rms current, the largest rms current permitted is $14.39 / \sqrt{2} = \boxed{10.18\text{ mA rms.}}$

41. $\mathbf{I} = 4\angle 35°$ A rms

(a) $\mathbf{V} = 20\mathbf{I} + 80\angle 35°$ Vrms, $P_{s,gen} = 80 \times 10 \cos 35° = \boxed{655.3 \text{ W}}$

(b) $P_R = |\mathbf{I}|^2 R = 16 \times 20 = \boxed{320 \text{ W}}$

(c) $P_{Load} = 655.3 - 320 = \boxed{335.3 \text{ W}}$

(d) $AP_{s,gen} = 80 \times 10 = \boxed{800 \text{ VA}}$

(e) $AP_R = P_R = \boxed{320 \text{ VA}}$

(f) $\mathbf{I}_L = 10\angle 0° - 4\angle 35° = 7.104\angle -18.84°$ A rms

$\therefore AP_L = 80 \times 7.104 = \boxed{568.3 \text{ VA}}$

(g) $PF_L = \cos\theta_L = \dfrac{P_L}{AP_L} = \dfrac{335.3}{568.3} = \boxed{0.599}$

since I_L lags V, PF_L is $\boxed{\text{lagging}}$

42.

(a) $$\mathrm{I}_s = \frac{120}{4 + \dfrac{j192}{12 + j16}} = 9.214\angle -26.25° \text{ A rms}$$

$$\therefore \mathrm{PF}_s = \cos 26.25 = \boxed{0.8969 \text{ lag}}$$

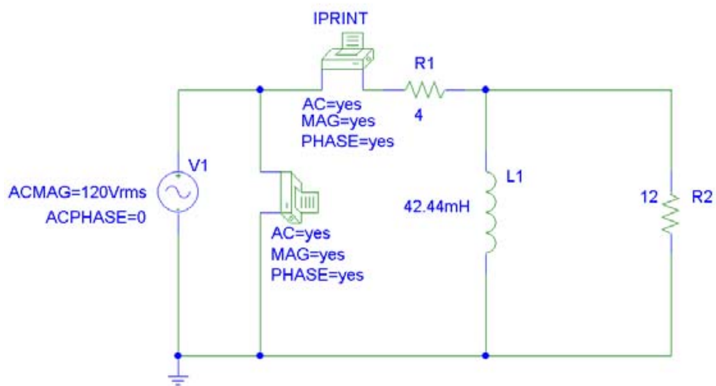
(b) $$\mathrm{P}_s = 120 \times 9.214 \times 0.8969 = \boxed{991.7\text{W}}$$

(c) $$\mathrm{Z}_L = 4 + \frac{j48}{3 + j4} = 4 + \frac{1}{25}(192 + j144)$$

$$\therefore \mathrm{Z}_L = 11.68 + j5.76\,\Omega,\ \mathrm{Y}_L = \frac{11.68 - j5.76}{11.68^2 + 5.76^2}$$

$$\therefore j120\pi\,\mathrm{C} = \frac{j5.76}{11.68^2 + 5.76^2},\ \mathrm{C} = \boxed{90.09\,\mu\text{F}}$$

(d) PSpice verification



FREQ VM($N_0003,0) VP($N_0003,0)
6.000E+01 1.200E+02 0.000E+00

FREQ IM(V_PRINT1) IP(V_PRINT1)
6.000E+01 9.215E+00 -2.625E+01 ; (a) and (b) are correct

Next, add a 90.09-μF capacitor in parallel with the source:

FREQ IM(V_PRINT1) IP(V_PRINT1)
6.000E+01 8.264E+00 -9.774E-05 ;(c) is correct (-9.8×10^{-5} degrees is essentially zero, for unity PF).

43.

$$\mathbf{Z}_A = 5 + j2\,\Omega,\ \mathbf{Z}_B = 20 - j10\,\Omega,\ \mathbf{Z}_c = 10\angle 30°\ \Omega = 8.660 + j5\ \Omega$$

$$\mathbf{Z}_D = 10\angle -60° = 5 - j8.660\ \Omega$$

$$\mathbf{I}_1 = \frac{\begin{vmatrix} 200 & -20 + j10 \\ 0 & 33.66 - j13.660 \end{vmatrix}}{\begin{vmatrix} 25 - j8 & -20 + j10 \\ -20 + j10 & 33.66 - j13.660 \end{vmatrix}} = \frac{7265\angle 22.09°}{480.9\angle -26.00°} = 15.11\angle 3.908°\ \text{A rms}$$

$$\mathbf{I}_2 = \frac{\begin{vmatrix} 25 - j8 & 200 \\ -20 + j10 & 0 \end{vmatrix}}{480.9\angle -26.00°} = \frac{200(20 - j10)}{480.9\angle 20.00°} = 9.300\angle -0.5681°\ \text{A rms}$$

$$\text{AP}_A = |\mathbf{I}_1|^2\,|\mathbf{Z}_A| = 15.108^2\sqrt{29} = \boxed{1229\ \text{VA}}$$

$$\text{AP}_B = |\mathbf{I}_1 - \mathbf{I}_2|^2\,|\mathbf{Z}_B| = 5.881^2 \times 10\sqrt{5} = \boxed{773.5\,\text{VA}}$$

$$\text{AP}_C = |\mathbf{I}_2|\,2\,|\mathbf{Z}_C| = 9.3^2 \times 10 = \boxed{86.49\,\text{VA}}$$

$$\text{AP}_D = |\mathbf{I}_2|^2\,|\mathbf{Z}_1| = 9.3^2 \times 10 = \boxed{864.9\,\text{VA}}$$

$$\text{AP}_S = 200|\mathbf{I}_1| = 200 \times 15.108 = \boxed{3022\,\text{VA}}$$

44. $\mathbf{Z}_1 = 30\angle 15°\,\Omega,\ \mathbf{Z}_2 = 40\angle 40°\,\Omega$

(a) $\mathbf{Z}_{tot} = 30\angle 15° + 40\angle 40° = 68.37\angle 29.31°\,\Omega$

$\therefore$ PF = cos 29.3° = $\boxed{0.8719 \text{ lag}}$

(b) $\mathbf{V} = \mathbf{IZ}_{tot} = 683.8\angle 29.31^\circ\,\Omega$ so

$$\mathbf{S} = \mathbf{VI}^* = \left(683.8\angle 29.31^\circ\right)\left(10\angle 0\right) = 6838\angle 29.31^\circ \text{ VA}.$$

Thus, the apparent power = $\boxed{\text{S} = 6.838 \text{ kVA.}}$

(c) The impedance has a positive angle; it therefore has a net $\boxed{\text{inductive}}$ character.

45. $\theta_1 = \cos^{-1}(0.92) = 23.07^o$, $\theta_2 = \cos^{-1}(0.8) = 36.87^o$, $\theta_3 = 0$

$$\mathbf{S}_1 = \frac{100\angle 23.07^o}{0.92} = 100 + j42.59 \text{ VA}$$

$$\mathbf{S}_2 = \frac{250\angle 36.87^o}{0.8} = 250 + j187.5 \text{ VA}$$

$$\mathbf{S}_3 = \frac{500\angle 0^o}{1} = 500 \text{ VA}$$

$$\mathbf{S}_{\text{total}} = \mathbf{S}_1 + \mathbf{S}_2 + \mathbf{S}_3 = 500 + j230.1 \text{ VA} = 550.4\angle 24.71^o \text{ VA}$$

(a) $\mathbf{I}_{\text{eff}} = \dfrac{S_{\text{total}}}{V_{\text{eff}}} = \dfrac{550.4}{115} = \boxed{4.786 \text{ A rms}}$

(b) PF of composite load = cos (24.71^o) = $\boxed{0.9084 \textit{ lagging}}$

46.

$$\text{AP}_L = 10{,}000 \text{ VA, PF}_L = 0.8 \text{ lag, } |\mathbf{I}_L| = 40 \text{A rms}$$

$$\text{Let } \mathbf{I}_L = 40\angle 0° \text{ A rms; P}_L = 10{,}000 \times 0.8 = 8000 \text{ W}$$

$$\text{Let } \mathbf{Z}_L = \text{R}_L + j\text{X}_L \therefore \text{R}_L = \frac{8000}{40^2} = 5 \ \Omega$$

$$\cos\theta_L = 0.8 \text{ lag} \therefore \theta_L = \cos^{-1} 0.8 = 36.87°$$

$$\therefore \text{X}_L = 5\tan 36.87° = 3.75\Omega, \ \mathbf{Z}_L = 5 + j3.75, \ \mathbf{Z}_{tot} = 5.2 + j3.75\Omega$$

$$\therefore \mathbf{V}_s = 40(5.2 + j3.75) = 256.4\angle 35.80° \text{ V; } \mathbf{Y}_{tot} = \frac{1}{5.2 + j3.75}$$

$$= 0.12651 - j0.09124\text{S}, \ \mathbf{Y}_{new} = 0.12651 + j(120\pi\text{C} - 0.09124),$$

$$\text{PF}_{new} = 0.9 \text{ lag}, \theta_{new} = 25.84° \therefore \tan 25.84° = 0.4843$$

$$= \frac{0.09124 - 120\pi\text{C}}{0.12651} \therefore$$

$$\text{C} = \boxed{79.48\mu\text{F}}$$

47. $\mathbf{Z}_{eff} = j100 + j300 \,||\, 200 = 237 \angle 54.25^o$. PF = cos 54.25° = 0.5843 *lagging*.

(a) Raise PF to 0.92 *lagging* with series capacitance

$$\mathbf{Z}_{new} = j100 + jX_C + j300 \,||\, 200 = 138.5 + j(192.3 + X_C)\ \Omega$$

$$\tan^{-1}\left(\frac{192.3 + X_C}{138.5}\right) = \cos^{-1} 0.92 = 23.07^o$$

Solving, we find that $X_C = -133.3\ \Omega = -1/\omega C$, so that C = $\boxed{7.501\ \mu F}$

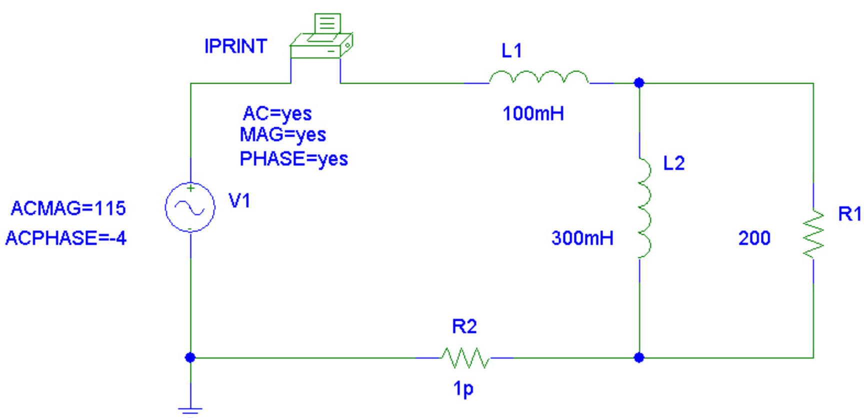
(b) Raise PF to 0.92 *lagging* with parallel capacitance

$$\mathbf{Z}_{new} = j100 \,||\, jX_C + j300 \,||\, 200 = \frac{-100\, X_C}{j(100 + X_C)} + 138.5 + j92.31\ \Omega$$

$$= 138.5 + j\left(92.31 + \frac{100 X_C}{100 + X_C}\right)\ \Omega$$

$$\tan^{-1}\left(\frac{92.31 + \frac{100X_C}{100 + X_C}}{138.5}\right) = \cos^{-1} 0.92 = 23.07^o$$

Solving, we find that $X_C = -25\ \Omega = -1/\omega C$, so that C = $\boxed{40\ \mu F}$



General circuit for simulations. Results agree with hand calculations

	FREQ	IM(V_PRINT1)	IP(V_PRINT1)	θ	PF
With no compensation:	1.592E+02	4.853E-01	-5.825E+01	54.25°	0.5843 lag
With series compensation:	1.592E+02	7.641E-01	-2.707E+01	23.07°	0.9200 lag
With parallel compensation:	1.592E+02	7.641E-01	-2.707E+01	23.07°	0.9200 lag

48.

$$\mathbf{Z}_{in} = -j10 + \frac{20(1+j2)}{3+j2} = 10.769 - j3.846 = 11.435^{+}\angle -19.65° \ \Omega$$

$$\therefore \mathbf{I}_s = \frac{100}{11.435\angle -19.654°} = 8.745\angle 19.65°$$

$$\therefore \mathbf{S}_s = -\mathbf{V}_s\mathbf{I}_s^* = -100 \times 8.745\angle -19.65° = \boxed{-823.5 + j294.1\text{VA}}$$

$$\mathbf{I}_{20} = 8.745\angle 19.65° \times \frac{10+j20}{30+j20} = 5.423\angle 49.40°$$

$$\therefore \mathbf{S}_{20} = 20 \times 5.432^2 = \boxed{588.2 + j0 \text{ VA}}$$

$$\mathbf{I}_{10} = \frac{20 \times 5.423\angle 49.40}{10+j20} = 4.851\angle -14.04°$$

$$\mathbf{S}_{10} = 10 \times 4.851^2 = \boxed{235.3 + j0 \text{ VA}}$$

$$\mathbf{S}_{j20} = j20 \times 4.851^2 = \boxed{j470.6 \text{ VA}},$$

$$\mathbf{S}_{-j10} = -j10 \times 8.745^2 = \boxed{-j764.7 \text{ VA}}, \quad \Sigma = 0$$

49.

$$\frac{\mathbf{V}_x - 100}{6 + j4} + \frac{\mathbf{V}_x}{-j10} + \frac{\mathbf{V}_x - j100}{5} = 0$$

$$\therefore \mathbf{V}_x \left(\frac{1}{6 + j4} + j0.1 + 0.2 \right) = \frac{100}{6 + j4} + j20$$

$$\therefore \mathbf{V}_x = 53.35^{-} \angle 42.66° \text{ V}$$

$$\therefore \mathbf{I}_1 = \frac{100 - 53.35^{-} \angle 42.66°}{6 + j4} = 9.806 \angle -64.44° \text{ A}$$

$$\therefore \mathbf{S}_{1.gen} = \frac{1}{2} \times 100 \times 9.806 \angle 64.44° = \boxed{211.5 + j442.3 \text{ VA}}$$

$$\mathbf{S}_{6,abs} = \frac{1}{2} \times 6 \times 9.806^2 = \boxed{288.5 + j0 \text{ VA}}$$

$$\mathbf{S}_{j4,abs} = \frac{1}{2}(j4) 9.806^2 = \boxed{0 + j192.3 \text{ VA}}$$

$$\mathbf{I}_2 = \frac{j100 - 53.35^{-} \angle 42.66°}{5} = 14.99 \angle 121.6°,$$

$$\mathbf{S}_{5abs} = \frac{1}{2} \times 5 \times 14.99^2 = \boxed{561.5 + j0 \text{ VA}}$$

$$\mathbf{S}_{2,gen} = \frac{1}{2}(j100) 14.99 \angle -121.57° = \boxed{638.4 - j392.3 \text{ VA}}$$

$$\mathbf{S}_{-j10,abs} = \frac{1}{2} \left(\frac{53.35}{10} \right) (-j10) = 0 - j142.3 \text{ VA} = \boxed{142.3 \angle -90° \text{ VA}} \quad \Sigma = 0$$

50.

(a) $500\,\text{VA},\ \text{PF} = 0.75 \text{ lead} \therefore$

$$\mathbf{S} = 500\angle -\cos^{-1} 0.75 = \boxed{375 - j330.7\ \text{VA}}$$

(b) $500\text{W},\ \text{PF} = 0.75 \text{ lead} \therefore$

$$\mathbf{S} = 500 - \frac{500}{j.075}\sin(\cos^{-1} 0.75) = \boxed{500 - j441.0\ \text{VA}}$$

(c) $-500\ \text{VAR},\ \text{PF} = 0.75(\text{lead}) \therefore \theta = -\cos^{-1} 0.75 = -41.41°$

$\therefore \text{P}\,500/\tan 41.41° = 566.9\text{W},$

$$\boxed{\mathbf{S} = 566.9 - j500\ \text{VA}}$$

51. $\mathbf{S}_s = 1600 + j500\,\text{VA (gen)}$

(a) $\mathbf{I}_s^* = \dfrac{1600 + j500}{400} = 4 + j1.25 \therefore \mathbf{I}_s = 4 - j1.25$

$\mathbf{I}_c = \dfrac{400}{-j120} = j3.333\,\text{A rms} \therefore \mathbf{I}_L = \mathbf{I}_s - \mathbf{I}_c = 4 - j1.25 - j3.333$

$\therefore \mathbf{I}_L = 4 - j4.583\,\text{A rms} \therefore$

$\mathbf{S}_L = 400\,(4 + j4.583) = \boxed{1600 + j1833\ \ \text{VA}}$

(b) $\text{PF}_L = \cos\left(\tan^{-1}\dfrac{1833.3}{1600}\right) = \boxed{0.6575^+\ \text{lag}}$

(c) $\mathbf{S}_s = 1600 + j500 = 1676\angle 17.35^\circ\ \text{VA} \therefore \text{PF}_s = \cos 17.35^\circ = \boxed{0.9545\ \text{lag}}$

52. $(\cos^{-1} 0.8 = 36.87°,\ \cos^{-1} 0.9 = 25.84°)$

(a) $$\mathbf{S}_{tot} = 1200\angle 36.87° + 1600\angle 25.84° + 900$$
$$= 960 + j720 + 1440 + j697.4 + 900$$
$$= 3300 + j1417.4 = 3592\angle 23.25°\ \text{VA}$$
$$\therefore \mathbf{I}_s = \frac{3591.5}{230} = \boxed{15.62\ \text{A rms}}$$

(b) $$\text{PF}_s = \cos 23.245° = \boxed{0.9188}$$

(c) $$\mathbf{S} = \boxed{3300 + j1417\ \text{VA}}$$

53.

(a) $P_{s,tot} = 20 + 25 \times 0.8 + 30 \times 0.75 = \boxed{70 \text{ kW}}$

(b)
$$\mathbf{I}_1 = \frac{20,000}{250} = 80\angle 0° \text{ A rms}$$
$$|\mathbf{I}_2| = 25,000/250 = 100 \text{ A rms}$$
$$\angle \mathbf{I}_2 = -\cos^{-1} 0.8 = -36.87 \therefore \mathbf{I}_2 = 100\angle -36.87° \text{ A rms}$$
$$AP_3 = \frac{30,000}{0.75} = 40,000 \text{ VA}, \quad |\mathbf{I}_3| = \frac{40,000}{250} = 160 \text{ A rms}$$
$$\angle \mathbf{I}_3 = -\cos^{-1} 0.75 = -41.41° \;\therefore\; \mathbf{I}_3 = 160\angle -41.41° \text{ A rms}$$
$$\therefore \mathbf{I}_s = 80\angle 0° + 100\angle -36.87° + 160\angle -41.41° = 325.4\angle -30.64° \text{ A rms}$$
$$\therefore AP_s = 250 \times 325.4 = \boxed{81,360 \text{ VA}}$$

(c) $PF_3 = \dfrac{70,000}{81,360} = \boxed{0.8604 \text{ lag}}$

54. 200 kW average power and 280 kVAR reactive result in a power factor of
PF = cos (tan^{-1} (280/200) = 0.5813 *lagging*, which is pretty low.

(a) 0.65 peak = 0.65(200) = 130 kVAR
Excess = 280 – 130 = 150 kVAR, for a cost of (12)(0.22)(150) = $\boxed{\$396 \text{ / year.}}$

(b) Target = **S** = P + *j*0.65 P
$\theta = \tan^{-1}(0.65P/P) = 33.02^{o}$, so target PF = $\cos\theta = \boxed{0.8385}$

(c) A single 100-kVAR increment costs \$200 to install. The excess kVAR would then be 280 – 100 – 130 = 50 kVAR, for an annual penalty of \$332. This would result in a first-year savings of \$64.

A single 200-kVAR increment costs \$395 to install, and would remove the entire excess kVAR. The savings would be \$1 (wow) in the first year, but \$396 each year thereafter.

$\boxed{\text{The single 200-kVAR increment is the most economical choice.}}$

55. Perhaps the easiest approach is to consider the load and the compensation capacitor separately. The load draws a complex power $\mathbf{S}_{load} = P + jQ$. The capacitor draws a purely reactive complex power $\mathbf{S}_C = -jQ_C$.

$\theta_{load} = \tan^{-1}(Q/P)$, or $Q = P \tan \theta_{load}$

$$Q_C = S_C = V_{rms}\left|\frac{\mathbf{V}_{rms}}{(-j/\omega C)}\right| = \left|\omega C \mathbf{V}_{rms}^2\right| = \omega C V_{rms}^2$$

$$\mathbf{S}_{total} = \mathbf{S}_{load} + \mathbf{S}_C = P + j(Q - Q_C)$$

$$\theta_{new} = \text{ang}(\mathbf{S}_{total}) = \tan^{-1}\left(\frac{Q\text{-}Q_C}{P}\right)$$, so that $Q - Q_C = P \tan \theta_{new}$

Substituting, we find that $Q_C = P \tan \theta_{load} - P \tan \theta_{new}$
or

$$\omega C V_{rms}^2 = P(\tan \theta_{load} - \tan \theta_{new})$$

Thus, noting that $\theta_{old} = \theta_{load}$,

$$\boxed{C = \frac{P\left(\tan \theta_{old} - \tan \theta_{new}\right)}{\omega V_{rms}^2}}$$

56. **V** = 339 $\angle$-66° V, ω = 100π rad/ s, connected to **Z** = 1000 Ω.

(a) $V_{eff} = \dfrac{339}{\sqrt{2}} = \boxed{239.7 \text{ V rms}}$

(b) $p_{max} = 339^2 / 1000 = \boxed{114.9 \text{ W}}$

(c) $p_{min} = \boxed{0 \text{ W}}$

(d) Apparent power $= V_{eff} I_{eff} = \left(\dfrac{339}{\sqrt{2}}\right)\left(\dfrac{339/\sqrt{2}}{1000}\right) = \dfrac{V_{eff}^2}{1000} = \boxed{57.46 \text{ VA}}$

(e) Since the load is purely resistive, it draws $\boxed{\text{zero reactive power.}}$

(f) $S = \boxed{57.46 \text{ VA}}$

57. $\mathbf{V} = 339\ \angle\text{-}66^\circ$ V, $\omega = 100\pi$ rad/s to a purely inductive load of 150 mH ($j47.12\ \Omega$)

(a) $\mathbf{I} = \dfrac{\mathbf{V}}{\mathbf{Z}} = \dfrac{339\angle -66^\circ}{j47.12} = 7.194\angle -156^\circ$ A

so $\mathbf{I}_{eff} = \dfrac{7.194}{\sqrt{2}} = \boxed{5.087 \text{ A rms}}$

(b) $p(t) = ½\, V_m I_m \cos\phi + ½\, V_m I_m \cos(2\omega t + \phi)$

where ϕ = angle of current – angle of voltage

$p_{max} = ½\, V_m I_m \cos\phi + ½\, V_m I_m = (1 + \cos(-90^\circ))\,(339)(7.194)/\,2 = \boxed{1219 \text{ W}}$

(c) $p_{min} = ½\, V_m I_m \cos\phi - ½\, V_m I_m = \boxed{-1219 \text{ W}}$

(d) apparent power $= V_{eff}\, I_{eff} = \dfrac{339}{\sqrt{2}}(5.087) = \boxed{1219 \text{ VA}}$

(e) reactive power = Q $= V_{eff}\, I_{eff} \sin(\theta - \phi) = \boxed{1219 \text{ VA}}$

(f) complex power $= \boxed{j1219 \text{ VA}}$

58. $1\ \text{H} \rightarrow j\ \Omega,\ 4\ \mu\text{F} \rightarrow -j250\ \Omega$

$$\mathbf{Z}_{\text{eff}} = j \parallel -j250 \parallel 10^4\ \Omega = 1.004\ \angle 89.99^\circ\ \Omega$$

$$\mathbf{V}_{10k} = \frac{(5\angle 0)\ (1.004\angle\ 89.99^\circ\ )}{2500 + (1.004\angle 89.99^\circ\ )} = 2.008\ \angle 89.97^\circ\ \ \text{mV}$$

(a) $p_{\max} = (0.002)^2 / 10\times 10^3 = \boxed{400\ \text{pW}}$

(b) $\boxed{0\ \text{W}}$ (purely resistive elements draw no reactive power)

(c) apparent power $= V_{\text{eff}}I_{\text{eff}} = ½\ V_mI_m = ½\ (0.002)^2 / 10000 = \boxed{200\ \text{pVA}}$

(d) $$\mathbf{S}_{\text{source}} = \frac{1}{2}(5\angle 0)\left(\frac{5\angle 0}{2500\angle 0.02292}\right) = \boxed{0.005\ \angle\text{-}0.02292^\circ\ \ \text{VA}}$$

59. (a) At $\omega = 400$ rad/s, 1 μF → $-j2500\ \Omega$, 100 mH → $j40\ \Omega$
Define $\mathbf{Z}_{\text{eff}} = -j2500 \parallel (250 + j40) = 256 \angle 3.287^{\circ}\ \Omega$

$$\mathbf{I}_S = \frac{12000\angle 0}{20 + 256\angle 3.287^{\circ}} = 43.48 \angle -3.049^{\circ} \text{ A rms}$$

$$\mathbf{S}_{\text{source}} = (12000)(43.48) \angle 3.049^{\circ} = \boxed{521.8 \angle 3.049^{\circ} \text{ kVA}}$$

$$\mathbf{S}_{20\Omega} = (43.48)^2 (20) \angle 0 = \boxed{37.81 \angle 0 \quad \text{kVA}}$$

$$\mathbf{V}_{\text{eff}} = \frac{(12000\angle 0)(256\angle 3.287^{o})}{20 + 256\angle 3.287^{o}} = 11130 \angle 0.2381^{\circ} \text{ V rms}$$

$$\mathbf{I}_{1\mu F} = \frac{\mathbf{V}_{\text{eff}}}{-j2500} = 4.452 \angle 90.24^{\circ} \text{ A rms}$$

so $\mathbf{S}_{1\mu F} = (11130)(4.452) \angle -90^{\circ} = \boxed{49.55 \angle -90^{\circ} \text{ kVA}}$

$$\mathbf{V}_{100\text{mH}} = \frac{(11130\angle 0.2381^{o})(j40)}{250 + j40} = 1758 \angle 81.15^{\circ} \text{ V rms}$$

$$\mathbf{I}_{100\text{mH}} = \frac{\mathbf{V}_{100\text{mH}}}{j40} = 43.96 \angle -8.852^{\circ} \text{ A rms}$$

so $\mathbf{S}_{100\mu H} = (1758)(4.43.96) \angle 90^{\circ} = \boxed{77.28 \angle 90^{\circ} \text{ kVA}}$

$$\mathbf{V}_{250\Omega} = \frac{(11130\angle 0.2381^{o})(250)}{250 + j40} = 10990 \angle -8.852^{\circ} \text{ V rms}$$

so $\mathbf{S}_{250\Omega} = (10990)^2 / 250 = \boxed{483.1 \angle 0^{\circ} \text{ kVA}}$

(b) $37.81 \angle 0 + 49.55 \angle -90^{\circ} + 77.28 \angle 90^{\circ} + 483.1 \angle 0^{\circ} = \boxed{521.6 \angle 3.014^{\circ} \text{ kVA,}}$ which is within rounding error of the complex power delivered by the source.

(c) The apparent power of the source is 521.8 kVA. The apparent powers of the passive elements sum to 37.81 + 49.55 + 77.28 + 483.1 = 647.7 kVA, so $\boxed{\text{NO!}}$ Phase angle is important!

(d) $P = \mathbf{V}_{\text{eff}}\, \mathbf{I}_{\text{eff}} \cos(\text{ang } \mathbf{V}_S - \text{ang } \mathbf{I}_S) = (12000)(43.48) \cos(3.049^{\circ}) = \boxed{521 \text{ kW}}$

(e) $Q = \mathbf{V}_{\text{eff}}\, \mathbf{I}_{\text{eff}} \sin(\text{ang } \mathbf{V}_S - \text{ang } \mathbf{I}_S) = (12000)(43.48) \sin(3.049^{\circ}) = \boxed{27.75 \text{ kVAR}}$

60. (a) Peak current $= 28\sqrt{2} = \boxed{39.6\text{ A}}$

(b) $\theta_{load} = \cos^{-1}(0.812) = +35.71^o$ (since lagging PF). Assume ang $(\mathbf{V}) = 0^o$.

$$p(t) = \left(2300\sqrt{2}\right)(39.6)\cos(120\pi t)\cos(120\pi t - 35.71^o)$$

at t = 2.5 ms, then, $p(t) = \boxed{71.89\text{ kW}}$

(c) $P = V_{eff} I_{eff} \cos\theta = (2300)(28)\cos(35.71^o) = \boxed{52.29\text{ kW}}$

(d) $\mathbf{S} = V_{eff} I_{eff} \angle\theta = \boxed{64.4 \angle 35.71^o\text{ kVA}}$

(e) apparent power = $|\mathbf{S}| = \boxed{64.4\text{ kVA}}$

(f) $|\mathbf{Z}_{load}| = |\mathbf{V}/\mathbf{I}| = 2300/28 = 82.14\ \Omega$. Thus, $\mathbf{Z}_{load} = \boxed{82.14 \angle 35.71^o\ \Omega}$

(g) $Q = V_{eff} I_{eff} \sin\theta = \boxed{37.59\text{ kVAR}}$

1. $V_{bc} = V_{be} + V_{ec} = 0.7 – 10 = \boxed{-9.3 \text{ V}}$

$V_{eb} = - V_{be} = \boxed{-0.7 \text{ V}}$

$V_{cb} = V_{ce} + V_{eb} = 10 – 0.7 = \boxed{9.3 \text{ V}}$

2. (a) $V_{gd} = V_{gs} + V_{sd} = -1 - 5 = \boxed{-6\text{ V}}$

(b) $V_{sg} = V_{sd} + V_{dg} = -4 - 2.5 = \boxed{-6.5\text{ V}}$

3. (a) positive phase sequence

$\mathbf{V}_{an} = |\mathbf{V}_p| \angle 0^o$ $\mathbf{V}_{dn} = |\mathbf{V}_p| \angle -180^o$
$\mathbf{V}_{bn} = |\mathbf{V}_p| \angle -60^o$ $\mathbf{V}_{en} = |\mathbf{V}_p| \angle -240^o$
$\mathbf{V}_{cn} = |\mathbf{V}_p| \angle -120^o$ $\mathbf{V}_{fn} = |\mathbf{V}_p| \angle -300^o$

(b) negative phase sequence

$\mathbf{V}_{an} = |\mathbf{V}_p| \angle 0^o$ $\mathbf{V}_{dn} = |\mathbf{V}_p| \angle 180^o$
$\mathbf{V}_{bn} = |\mathbf{V}_p| \angle 60^o$ $\mathbf{V}_{en} = |\mathbf{V}_p| \angle 240^o$
$\mathbf{V}_{cn} = |\mathbf{V}_p| \angle 120^o$ $\mathbf{V}_{fn} = |\mathbf{V}_p| \angle 300^o$

4. (a) $\mathbf{V}_{yz} = \mathbf{V}_{yx} + \mathbf{V}_{xz}$

$$= -110 \angle 20^o + 160 \angle -50^o$$
$$= -103.4 - j37.62 + 102.8 - j122.6 = -0.6 - j160.2$$
$$= \boxed{160.2 \angle -90.21^o \text{ V}}$$

(b) $\mathbf{V}_{az} = \mathbf{V}_{ay} + \mathbf{V}_{yz}$

$$= 80 \angle 130^o + 160.2 \angle -90.21^o$$
$$= -51.42 + j61.28 - 0.6 - j160.2 = -52.02 - j98.92$$
$$= \boxed{111.8 \angle -117.7^o \text{ V}}$$

(c) $$\frac{\mathbf{V}_{zx}}{\mathbf{V}_{xy}} = \frac{-160\angle -50^o}{110\angle 20^o} = \frac{160\angle 130^o}{110\angle 20^o} = \boxed{1.455\angle 110^o}$$

5. (a) $\mathbf{V}_{25} = \mathbf{V}_{24} + \mathbf{V}_{45} = -80 \angle 120^o + 60 \angle 75^o$

$= 40 - j69.28 + 15.53 + j57.96 = 55.53 - j11.32$

$= \boxed{56.67 \angle -11.52^o \text{ V}}$

(b) $\mathbf{V}_{13} = \mathbf{V}_{12} + \mathbf{V}_{25} + \mathbf{V}_{53} = 100 + 55.53 - j11.32 + j120$

$= 155.53 + j108.7$

$= \boxed{189.8 \angle 34.95^o \text{ V}}$

6. $\mathbf{V}_{12} = 9\angle 87^\circ$ V = 0.4710 + j8.988 V, $\mathbf{V}_{23} = 8\angle 45^\circ$ V = 5.657 + j5.657 V

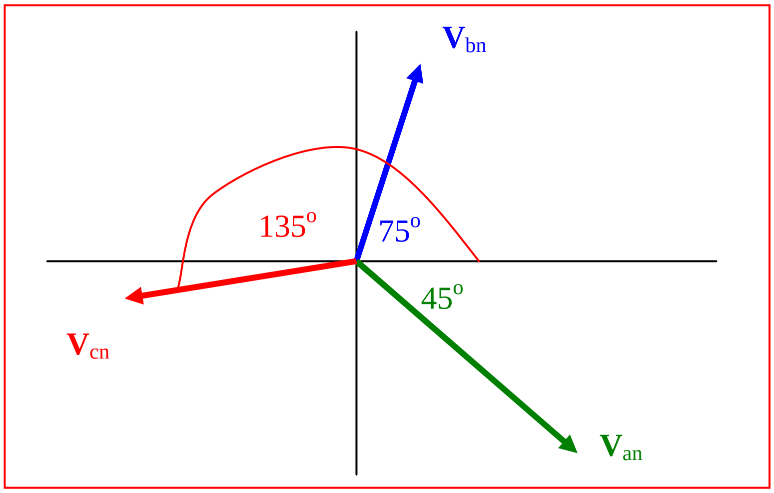
(a) $\mathbf{V}_{21} = -\mathbf{V}_{12} = 9\angle\left(180^\circ + 87^\circ\right)$ V $= 9\angle\left(267^\circ\right)$ V $= \boxed{9\angle\left(-93^\circ\right) \text{ V}}$

(b) $\mathbf{V}_{32} = -\mathbf{V}_{23} = 8\angle\left(180^\circ + 45^\circ\right)$ V $= 8\angle\left(225^\circ\right)$ V $= \boxed{8\angle\left(-135^\circ\right) \text{ V}}$

(c) $\mathbf{V}_{12} - \mathbf{V}_{32} = \mathbf{V}_{12} + \mathbf{V}_{23} = 0.4710 + j8.988 + 5.657 + j5.657 = 6.128 + j14.65$ V

$= \boxed{15.88\angle 67.29^\circ \text{ V}}$

7. (a)



(b) The phase sequence is negative, since sequence is acbacb….
A positive sequence would be abcabc…

8. The temptation is to extend the procedure for voltages, but without the specific circuit topology, we do not have sufficient information to determine $\mathbf{I}_{31}$.

9. The temptation is to extend the procedure for voltages, but without the specific circuit topology, we do not have sufficient information to determine $\mathbf{I}_{31}$.

10.

$230/460\,\text{V rms}\ \bar{Z}_{AN}:\ \bar{S} = 10\angle 40°\,\text{kVA};\ \bar{Z}_{NB}:\ 8\angle 10°\,\text{kVA};$

$\bar{Z}_{AB}:\ 4\angle -80°\,\text{kVA}\qquad \text{Let } \bar{V}_{AN} = 230\angle 0°\,\text{V}\ \therefore \bar{S}_{AN} = \bar{V}_{AN}\bar{I}^*_{AN}, \bar{I}^*_{AN} = \dfrac{10,000\angle 40°}{230} = 43.48\angle 40°\,\text{A}$

$\therefore \bar{I}_{AN} = 43.48\angle -40°\,\text{A},\ \bar{S}_{AB} = \bar{V}_{AB}\bar{I}^*_{AB}\ \therefore\ \bar{I}^*_{AB} = \dfrac{4000\angle -80°}{460} = 8.696\angle -80°, \bar{I}_{AB} = 8.696\angle 80° \therefore\ \bar{I}_{aA} = \bar{I}_{AN} + \bar{I}_{AB}$

$\therefore \bar{I}_{aA} = 43.48\angle 40° + 8.696\angle 80° = 39.85^-\angle -29.107°\ \therefore \text{I}_{aA} = \boxed{39.85^-\ \text{A}}$

$\bar{I}^*_{NB} = \dfrac{8000\angle 10°}{230} = 34.78\angle 10°,\ \bar{I}_{NB} = 34.78\angle -10°\,\text{A}$

$\therefore \bar{I}_{bB} = -34.78\angle -10° - 8.696\angle 80° = 35.85^+\angle -175.96°,\ \therefore \text{I}_{bB} = \boxed{35.85^+\ \text{A}}$

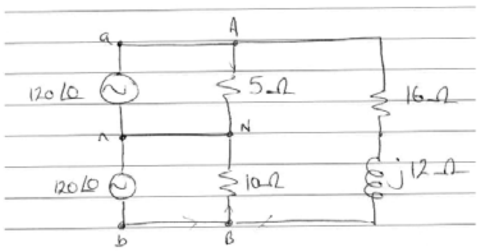
$\bar{I}_{nN} = -43.48\angle -40° + 34.78\angle -10° = 21.93\angle 87.52°,\ \text{I}_{nN} = \boxed{21.93\,\text{A}}$

11. (a) $\mathbf{I}_{nN} = 0$ since the circuit is balanced.

$$\mathbf{I}_{AN} = 12\angle 0 \qquad \mathbf{I}_{AB} = \frac{240\angle 0}{16 + j12} = 12\angle -36.9^{\circ}$$

$$\mathbf{I}_{aA} = \mathbf{I}_{AN} + \mathbf{I}_{AB} = 12 + 9.596 - j7.205 = \boxed{22.77\angle -18.45^{\circ}\text{ A}}$$

(b)



$\mathbf{I}_{AN} = 24\angle 0^{\circ}$ A
$\mathbf{I}_{BN} = -12\angle 0^{\circ}$ A
$\mathbf{I}_{nN} = -12\angle 0^{\circ}$ A

The voltage across the 16-Ω resistor and j12-Ω impedance has not changed, so $\mathbf{I}_{AB}$ has not changed from above.

$$\mathbf{I}_{aA} = \mathbf{I}_{AN} + \mathbf{I}_{AB} = 24\angle 0^{\circ} + 12\angle -36.9^{\circ} = \boxed{34.36\angle -12.10^{\circ}\text{ A}}$$

$$\mathbf{I}_{bB} = \mathbf{I}_{BN} - \mathbf{I}_{AB} = -12\angle 0^{\circ} - 12\angle -36.9^{\circ} = \boxed{7.595\angle -108.5^{\circ}\text{ A}}$$

$$\mathbf{I}_{nN} = \mathbf{I}_{BN} - \mathbf{I}_{AN} = -12 - 24 = \boxed{36\angle 180^{\circ}\text{ A}}$$

12.

(a)
$$\Delta = \begin{vmatrix} 21+j3 & -10 & -10-j3 \\ -10 & 19+j2 & -8-j2 \\ -10-j3 & -8-j2 & 36+j5 \end{vmatrix} = (21+j3)(674+j167-60-j32)$$

$$+10(-360-j50-74-j44)-(10+j3)(80+j20+184+j77)$$

$$\therefore \Delta = 5800+j1995 = 6127\angle 18.805°$$

$$\begin{vmatrix} 720 & -10 & -10-j3 \\ 720 & 19+j2 & -8-j2 \\ 0 & -8-j2 & 36+j5 \end{vmatrix} = 720(614+j135+434+j94) = 720\times 1072.7\angle 12.326°$$

$$\therefore \bar{\mathrm{I}}_{aA} = \frac{720\times 1072.7\angle 12.326°}{6127\angle 18.805°} = \boxed{126.06\angle -6.479°\ \mathrm{A}}$$

(b)
$$\begin{vmatrix} 21+j3 & 720 & -10-j3 \\ -10 & 720 & -8-j2 \\ -10-j3 & 0 & 36+j5 \end{vmatrix} = 720(1084+j247) \quad \therefore \bar{\mathrm{I}}_{Bb} = \frac{720(1084+j247)}{6127\angle 18.805°} = 130.65^{-}\angle -5.968°\ \mathrm{A}$$

$$\therefore \mathrm{I}_{nN} = 130.65^{-}\angle -5.968° - 126.06\angle -6.479° = \boxed{4.730\angle 7.760°\ \mathrm{A}}$$

(c) $$\mathrm{P}_{\omega,tot} = 126.06^2\times 1 + 130.65^2\times 1 + 4.730^2\times 10 = 15.891+17.069+0.224 = \boxed{33.18\ \mathrm{kW}}$$

(d) $$\mathrm{P}_{gen,tot} = 720\times 126.06\cos 6.479° + 720\times 130.65^{-}\cos 5.968° = 90.18+93.56 = \boxed{183.74\ \mathrm{kW}}$$

13. $\bar{V}_{AN} = 220\,\text{Vrms},\ 60\,\text{Hz}$

(a) $\text{PF} = 1\ \therefore\ \bar{I}_{AN} = \dfrac{220\angle 0°}{5 + j2} = 40.85^{+}\angle -21.80°\,\text{A};\ \bar{I}_{AB} = j377\text{C} \times 440$

$\therefore\ \bar{I}_{aA} = 40.85\cos 21.80° + j(377\text{C}440 - 40.85\sin 21.80°)$

$\therefore\ \text{C} = \dfrac{40.85\sin 21.80°}{377 \times 440} = \boxed{91.47\,\mu\text{F}}$

(b) $\bar{I}_{AB} = 377 \times 91.47 \times 10^{-6} \times 440 = 15.172\,\text{A}\ \therefore\ \text{VA} = 440 \times 15.172 = \boxed{6.676\,\text{kVA}}$

14. (a) $\mathbf{I}_{aA} = \mathbf{I}_{AN} + \mathbf{I}_{AB} = \dfrac{200\angle 0}{12 + j3} + \dfrac{400\angle 0}{R_{AB}} = 15.69 - j3.922 + \dfrac{400}{R_{AB}}$

Since we know that $|\mathbf{I}_{aA}|$ = 30 A rms = 42.43 A,

$$42.43 = \sqrt{\left(15.69 + \frac{400}{R_{AB}}\right)^2 + 3.922^2}$$

or $R_{AB} = \boxed{15.06\ \Omega}$

(b) $\mathbf{I}_{aA} = \mathbf{I}_{AN} + \mathbf{I}_{AB} = \dfrac{200\angle 0}{12 + j3} + \dfrac{400\angle 0}{-jX_{AB}} = 15.69 - j3.922 + \dfrac{j400}{X_{AB}}$

In order for the angle of $\mathbf{I}_{aA}$ to be zero, $\dfrac{400}{X_{AB}}$ = 3.922, so that X_{AB} = $\boxed{102\ \Omega \text{ capacitive.}}$

15. $+$ seq. $\bar{V}_{BC} = 120\angle 60°$ V rms, $R_w = 0.6\,\Omega$ $\quad P_{load} = 5\,\text{kVA},\ 0.6$ lag

(a) $\bar{V}_{AN} = \frac{120}{\sqrt{3}}\angle 150°\,\text{V} \therefore \bar{S}_{AN} = \frac{5000}{3}\times 0.8 + j\frac{5000}{3}\,0.6$

$\therefore \bar{S}_{AN} = \frac{120}{\sqrt{3}}\angle 150°\ \bar{I}^*_{aA} \therefore \bar{I}^*_{aA} = 24.06\ \angle -113.13°\ \text{A}$

$\therefore \bar{I}_{aA} = 24.06\angle 113.13° \therefore P_{wire} = 3\times 24.06^2 \times 0.6 = \boxed{1041.7\,\text{W}}$

(b) $\bar{V}_{aA} = 0.6\times 24.06\angle 113.13° = 14.434\angle 113.13°\,\text{V}$

$\therefore \bar{V}_{an} = \bar{V}_{aA} + \bar{V}_{AN} = 14.434\angle 113.13° + \frac{120}{\sqrt{3}}\angle 158° = \boxed{81.29\angle 143.88°\,\text{V}}$

16. $\uparrow \bar{V}_{an} = 2300\angle 0°\ V_{\text{rms}},\ R_w = 2\ \Omega, +seq., \bar{S}_{tot} = 100 + j30\,\text{kVA}$

(a) $\frac{1}{3}(100{,}000 + j30{,}000) = 2300\,I^*_{aA}\ \therefore\ \bar{I}_{aA} = \boxed{15.131\angle -16.699°\ \text{A}}$

(b) $\bar{V}_{AN} = 2300 - 2 \times 15.131\angle -16.699° = \boxed{2271\angle 0.2194°\ \text{V}}$

(c) $\bar{Z}_p = \bar{V}_{AN} / \bar{I}_{aA} = \frac{2271\angle 0.2194°}{15.131\angle -16.699°} = \boxed{143.60 + j43.67\,\Omega}$

(d) $\text{trans. eff.} = \frac{143.60}{145.60} = 0.9863,\ \text{or}\ \boxed{98.63\%}$

17. $\uparrow \bar{Z}_p = 12 + j5\Omega,\ \bar{I}_{bB} = 20\angle 0°$ A rms, +seq., PF $= 0.935$

(a) $\theta = \cos^{-1} 0.935 = 20.77° \therefore \dfrac{5}{12 + R_w} = \tan\ 20.77°,\ \ R_w = \boxed{1.1821\Omega}$

(b) $\bar{V}_{BN} = I_{bB}Z_p = 20(12 + j5) = 240 + j100\,\text{V} \ \ \therefore \bar{V}_{bn} = 20(13.1821 + j5) = \boxed{281.97\angle 20.77°\,\text{V}}$

(c) $\bar{V}_{AB} = \sqrt{3}\left|\bar{V}_{BN}\right| / \angle V_{BN} + 150° = \boxed{450.3\angle 172.62°\,\text{V}}$

(d) $\bar{S}_{source} = 3\ \bar{V}_{Bn}\bar{I}^*_{bB} = 3 \times 281.97\angle -20.77°(20)$

$= \boxed{15.819 - j6.000\,\text{kVA}}$

18. 125 mH → $j(2\pi)(60)(0.125) = j47.12\ \Omega$ 75 Ω → 75 Ω

55 μF → $-j/(2\pi)(60)(55\times10^{-6}) = -j48.23\ \Omega$

The per-phase current magnitude |**I**| is then $I = \dfrac{125}{\sqrt{75^2 + (47.12 - 48.23)^2}} = \boxed{1.667\text{ A.}}$

The power in each phase = $(1.667)^2$ (75) = 208.4 W, so that the total power taken by the load is 3(208.4) = $\boxed{625.2\text{ W.}}$

The power factor of the load is $\cos\left(\dfrac{47.12 - 48.23}{75}\right) = \boxed{1.000}$

This isn't surprising, as the impedance of the inductor and the impedance of the capacitor essentially cancel each other out as they have approximately the same magnitude but opposite sign and are connected in series.

19.

$\uparrow$ Bal.,+ seq. $\text{Z}_{AN} = 8 + j6\,\Omega,\ \overline{\text{Z}}_{BN} = 12 - j16\ \Omega,\ \overline{\text{Z}}_{CN} = 5 + j0,\ \overline{\text{V}}_{AN} = 120\angle 0°\ \text{V rms}$

$$\text{R}_w = 0.5\,\Omega\ (a) - \overline{\text{I}}_{nN} = \frac{120\angle 0°}{8.5 + j6} + \frac{120\angle -120°}{12.5 - j16} + \frac{120\angle 120°}{5.5} = 6.803\angle 83.86°\ \text{A}$$

$$\therefore\ \overline{\text{I}}_{nN} = \boxed{6.803\angle -96.14°\ \text{A rms}}$$

20. Working on a per-phase basis, the line current magnitude is simply

$$|\mathbf{I}| = \frac{40}{\sqrt{(R_w + 5)^2 + 10^2}}$$

(a) $R_W = 0$

Then $|\mathbf{I}| = \dfrac{40}{\sqrt{25 + 10^2}} = \boxed{3.578\ \text{A}}$, and the power delivered to each phase of the load is $(3.578)^2(5) = 64.01$ W. The total power of the load is therefore $3(64.01) = \boxed{192.0\ \text{W.}}$

(b) $R_W = 3\ \Omega$

Then $|\mathbf{I}| = \dfrac{40}{\sqrt{64 + 10^2}} = \boxed{3.123\ \text{A}}$, and the power delivered to each phase of the load is $(3.123)^2(5) = 48.77$ W. The total power of the load is therefore $3(48.77) = \boxed{146.3\ \text{W.}}$

21. $\uparrow \bar{Z}_p = 75\angle 25°\ \Omega \| 25\,\mu\text{F},\ \bar{V}_{an} = 240\angle 0°\ \text{V rms},\ 60\,\text{Hz},\ R_w = 2\,\Omega$

(a) $\bar{Z}_{cap} = -j\dfrac{10^6}{377\times 25} = -j\ 106.10\,\Omega \ \therefore \bar{Z}_p = \dfrac{75\angle 25°(-j106.10)}{75\angle 25° - j106.10} = 75.34 - j23.63\,\Omega$

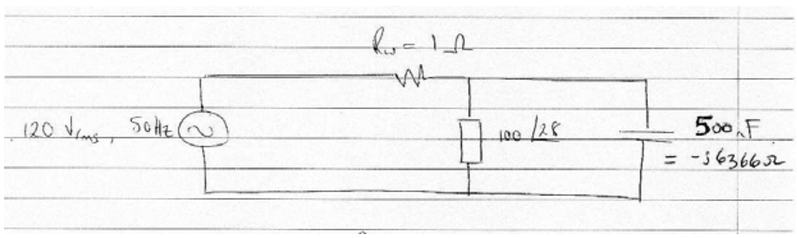
$\therefore \bar{Z}_{p+w} = 77.34 - j23.63 \ \therefore \bar{I}_{aA} = \dfrac{240}{77.34 - j23.63} = \boxed{2.968\angle 16.989°\ \text{A}}$

(b) $P_w = 3(2.968)^2 \times 2 = \boxed{52.84\,\text{W}}$

(c) $P_{load} = 3(2.968)^2 75.34 = \boxed{1990.6\,\text{W}}$

(d) $PF_{source} = \cos 16.989° = \boxed{0.9564\ \text{lead}}$

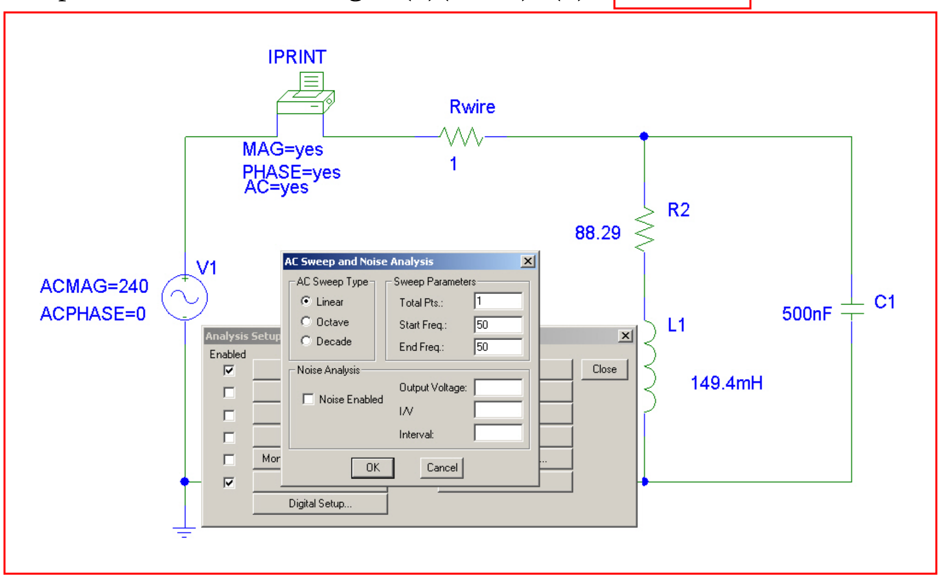
22. Working on a per-phase basis and noting that the capacitor corresponds to a –*j*6366-Ω impedance,



-*j*6366 || 100 ∠ 28° = 89.59 + *j*46.04 Ω so that the current flowing through the combined load is

$$|\mathbf{I}| = \frac{240}{\sqrt{90.59^2 + 46.04^2}} = \boxed{2.362 \text{ A rms}}$$

The power in each phase is $(2.362)^2$ (90.59) = 505.4 W, so that the power deliverd to the total load is 3(505.4) = $\boxed{1.516 \text{ kW}}$.
The power lost in the wiring is $(3)(2.362)^2$ (1) = $\boxed{16.74 \text{ W.}}$



Simulation Result:

FREQ	IM(V_PRINT1)	IP(V_PRINT1)
5.000E+01	1.181E+00	-2.694E+01

23. $\uparrow$ Bal., $R_w = 0,\ \bar{Z}_p = 10 + j5\,\Omega,\ f = 60\ \text{Hz}$

(a) $10 + j5 = 11.180\angle 26.57° \ \therefore \text{PF} = \cos 26.57° = \boxed{0.8944}$

(b) $$\text{PF} = 0.93 \text{ lag},\ \theta = 21.57°,\ \bar{Y}_p = \frac{1}{11.180\angle 26.57°} = 0.08 - j0.04\,\text{S}$$

$$\bar{Y}p' = 0.08 + j(377\text{C} - 0.04) \ \therefore \frac{377\text{C} - 0.04}{0.08} = -\tan 21.57° = -0.3952$$

$$\therefore 377\text{C} = 0.04 - 0.08 \times 0.3952 = 0.00838 \therefore \text{C} = \boxed{22.23\,\mu\text{F}}$$

(c) $$V_{L,load} = 440\,\text{V rms},\ \bar{Z}_c = \frac{-j10^6}{120\pi 22.23} = -j119.30\,\Omega,\ I_c = \frac{440/\sqrt{3}}{119.30} = 2.129\,\text{A}$$

$$\therefore \text{VAR} = 2.129 \times \frac{440}{\sqrt{3}} = \boxed{540.9\ \text{VAR}\,(cap.)}$$

24. Working from the single-phase equivalent,

$$\mathbf{V}_{an}\ \text{rms} = \frac{1}{\sqrt{3}}\left(\frac{115\angle 0^o}{\sqrt{2}}\right) = 46.9\ \angle 0^o\ \text{V rms}$$

1.5 H $\rightarrow j565\ \Omega$, 100 μF $\rightarrow -j26.5\ \Omega$ and 1 kΩ $\rightarrow$ 1 kΩ.
These three impedances appear in parallel, with a combined value of $27.8 \angle -88.4^o\ \Omega$.

Thus, $|\mathbf{I}_{rms}| = 46.9/\ 27.8 = \boxed{1.69\ \text{A rms}}$

$\mathbf{Z}_{load} = 27.8 \angle 88.4^o = 0.776 - j\ 27.8\ \Omega$, so $P_{load} = (3)(1.69)^2\ (0.776) = \boxed{2.22\ \text{W.}}$

25.

$\mathrm{R}_w = 0,\ \overline{\mathrm{V}}_{an} = 200\angle 60°\ \mathrm{V\,rms.}\ \mathrm{S}_p = 2 - j1\ \mathrm{kVA} + \mathrm{seq.}$

(a) $\overline{\mathrm{V}}_{bc} = 220\sqrt{3}\angle -30° = \boxed{346.4\angle -30°\ \mathrm{V}}$

(b) $\overline{\mathrm{S}}_{BC} = 2000 - j1000 = \overline{\mathrm{V}}_{BC}\overline{\mathrm{I}}_{BC}^{*} = 346.4\angle -30°\,\overline{\mathrm{I}}_{BC}^{*}$

$\therefore\ \overline{\mathrm{I}}_{BC}^{*} = 6.455^{-}\angle 3.435°,\ \overline{\mathrm{I}}_{BC} = 6.455^{-}\angle -3.435°$

$$\therefore\ \overline{\mathrm{Z}}_p = \frac{200\sqrt{3}\angle -30°}{6.455^{-}\angle -3.435°} = 53.67\angle -26.57° = \boxed{48 - j24\ \Omega}$$

(c) $\overline{\mathrm{I}}_{aA} = \overline{\mathrm{I}}_{AB} - \overline{\mathrm{I}}_{CA} = 6.455^{-}\angle 120° - 3.43° - 6.455^{-}\angle -120° - 3.43° = \boxed{11.180\angle 86.57°\ \mathrm{A\,rms}}$

26. $\uparrow 15\text{kVA},\ 0.8\,\text{lag},\ +\text{seq.},\ \overline{\text{V}}_{BC} = 180\angle 30°\ \text{V rms},\ \text{R}_w = 0.75\Omega$

(a) $\overline{\text{V}}_{BC} = 180\angle 30° \ \therefore \overline{\text{V}}_{AB} = 180\angle 150°\,\text{V},\ \overline{\text{S}}_p = 5000\angle \cos^{-1} 0.8 = 5000\angle 36.87° = 180\angle 30°\,\overline{\text{I}}^*_{BC}$

$\therefore \overline{\text{I}}_{BC} = 27.78\angle -6.87°$ and $\overline{\text{I}}_{AB} = 27.78\angle 113.13°\,\text{A} \ \therefore \overline{\text{I}}_{bB} = \overline{\text{I}}_{BC} - \overline{\text{I}}_{AB}$

$\therefore \overline{\text{I}}_{bB} = 27.78(1\angle -6.87° - 1\angle 113.13°) = 48.11\angle -36.87°\,\text{A} \ \therefore \overline{\text{V}}_{bC} = 0.75(\overline{\text{I}}_{bB} - \overline{\text{I}}_{cC})$

$\therefore \overline{\text{V}}_{bC} = 0.75 \times 48.11(1\angle -36.87° - 1\angle -156.87°) + 180\angle 30° = \boxed{233.0\angle 20.74°\,\text{V}}$

(b) $\text{P}_{wire} = 3 \times 48.11^2 \times 0.75 = 5208\,\text{W}$

$\overline{\text{S}}_{gen} = 5208 + 15{,}000\angle 36.87° = \boxed{17.208 + j9.000\,\text{kVA}}$

27. $\uparrow$ Bal., $\overline{S}_L = 3 + j1.8$ kVA, $\overline{S}_{gen} = 3.45 + j1.8$ kVA, $R_w = 5\Omega$

(a) $P_w = 450\,\text{W} \therefore \frac{1}{3} \times 450 = I_{aA}^2 \times 5 \;\therefore I_{aA} = \boxed{5.477\,\text{A rms}}$

(b) $I_{AB} = \frac{1}{\sqrt{3}} \times 5.477 = \boxed{3.162\,\text{A rms}}$

(c) Assume $\overline{I}_{AB} = 3.162\angle 0°$ and +seq. $\therefore \frac{1}{3}(3000 + j1800) = \overline{V}_{AB}\overline{I}_{AB}^{*} = \overline{V}_{AB}(3.162\angle 0°)$

$\therefore \overline{V}_{AB} = 368.8\angle 30.96°\,\text{V} \;\therefore \overline{V}_{an} = \overline{V}_{aA} + \overline{V}_{AB} - \overline{V}_{bB} + \overline{V}_{bn}$

$\overline{V}_{aA} = 5\overline{I}_{aA} = 5 \times 5.477\angle -30° = 27.39\angle -30°, \overline{V}_{bB} = 27.39\angle -150°$

$\therefore \overline{V}_{an} = 27.39\angle -30° - 27.39\angle -150° + 368.8\angle 30.96° + V_{an}(1\angle -120°)$

$$\therefore \overline{V}_{an} = \frac{27.39\angle -30° - 27.39\angle -150° + 368.8\angle 30.96°}{1 - 1\angle -120°} = 236.8\angle -2.447° \therefore V_{an} = \boxed{236.8\,\text{V rms}}$$

28. If a total of 240 W is lost in the three wires marked R_w, then 80 W is lost in each 2.3-Ω segment. Thus, the line current is $\sqrt{\frac{80}{2.3}} = 5.898\text{ A rms}$. Since this is a D-connected load, the phase current is $1/\sqrt{3}$ times the line current, or $\boxed{3.405 \text{ A rms.}}$

In order to determine the phase voltage of the source, we note that

$$P_{total} = \sqrt{3}\left|\mathbf{V}_{line}\right| \cdot \left|\mathbf{I}_{line}\right| \cdot \text{PF} \quad = \sqrt{3}\ \left|\mathbf{V}_{line}\right| (5.898)\left(\frac{\sqrt{2}}{2}\right) = 1800$$

where $|\mathbf{V}_{line}| = \dfrac{(1800)(2)}{\sqrt{2}\,\sqrt{3}\,(5.898)} = 249.2\text{ V}$

This is the voltage at the load, so we need to add the voltage lost across the wire, which (taking the load voltage as the reference phase) is $\left[5.898\angle -\cos^{-1}\left(\frac{1}{\sqrt{2}}\right)\right](R_W)$

= 13.57 $\angle$-45° V. Thus, the line voltage magnitude of the source is
$|249.2 \angle 0^o + 13.57 \angle -45^o|$ = $\boxed{259.0 \text{ V rms.}}$

29. Bal., +seq.

(a) $\overline{V}_{an} = 120\angle 0° \therefore \overline{V}_{ab} = 120\sqrt{3}\angle 30°, etc., \overline{I}_{AB} = \dfrac{120\sqrt{3}\angle 30°}{10} = 20.78\angle 30°\ \text{A}$

$$\overline{I}_{BC} = \frac{120\sqrt{3}\angle -90°}{j5} = -41.57\ A;\ \overline{I}_{CA} = \frac{120\sqrt{3}\angle 150°}{-j10} = 20.78\angle -120°\ \text{A}$$

$$\overline{I}_{aA} = \overline{I}_{AB} - \overline{I}_{CA} = 20.78(1\angle 30° - 1\angle -120°) = \boxed{40.15\angle 45°\ \text{A rms}}$$

(b) $\overline{I}_{bB} = -41.57 - 20.78\angle 30° = \boxed{60.47\angle -170.10°\ \text{A rms}}$

(c) $I_{cC} = 20.78\angle -120° + 41.57 = \boxed{36.00\angle -30°\ \text{A rms}}$

(d) $\overline{S}_{tot} = \overline{V}_{AB}\overline{I}_{AB}^* + \overline{V}_{BC}\overline{I}_{BC}^* + \overline{V}_{CA}\overline{I}_{CA}^* = 120\sqrt{3}\angle 30° \times 20.78\angle -30° + 120\sqrt{3}\angle -90°(-41.57) +$
$120\sqrt{3}\angle 150° \times 20.78\angle 120° = 4320 + j0 + 0 + j8640 + 0 - j4320 = \boxed{4320 + j4320\ \text{VA}}$

30. $\mathbf{I}_{AB} = \dfrac{200\angle 0}{10 \parallel j30} = \dfrac{200\angle 0}{9.49\angle 18.4^o} = 21.1\angle -18.4^o$ A

$|\mathbf{I}_A| = \sqrt{3}\, I_{AB} = 36.5$ A

The power supplied by the source = (3) $|I_A|^2$ (0.2) + (3) $(200)^2$ / 10 = $\boxed{12.8 \text{ kW}}$

Define transmission efficiency as $\eta = 100 \times P_{load} / P_{source}$. Then η = $\boxed{93.8\%}$.

$\mathbf{I}_A$ leads $\mathbf{I}_{AB}$ by 30^o, so that $\mathbf{I}_A = 36.5 \angle 11.6^o$.

$\mathbf{V}_{R_w} = (0.2)(36.5\angle 11.6^o) = 7.3\angle 11.6^o$ V

With $\mathbf{V}_{AN} = \dfrac{200}{\sqrt{3}}\angle 30^o$, and noting that $\mathbf{V}_{an} = \mathbf{V}_{AN} + \mathbf{V}_{R_w} = 122 \angle 28.9^o$, we may now compute the power factor of the source as

$$PF = \cos(\text{ang}(\mathbf{V}_{an}) - \text{ang}(\mathbf{I}_A)) = \cos(28.9^o - 11.6^o) = \boxed{0.955.}$$

31. $\uparrow$ Bal., $\overline{V}_{an} = 140\angle 0°\ V_{rms}, +\text{seq.}, R_w = 0, \overline{S}_L = 15 + j9$ kVA

(a) $\overline{V}_{ab} = \overline{V}_{AB} = \sqrt{3}\ 140\angle 30° = \boxed{242.5^{-}\angle 30°\ V}$

(b) $\overline{V}_{AB}\overline{I}_{AB}^{*} = 5000 + j3000 = 242.5^{-}\angle 30°\overline{I}_{AB}^{*}\ \therefore\ \overline{I}_{AB} = \boxed{24.05^{-}\angle -0.9638°\ \text{A rms}}$

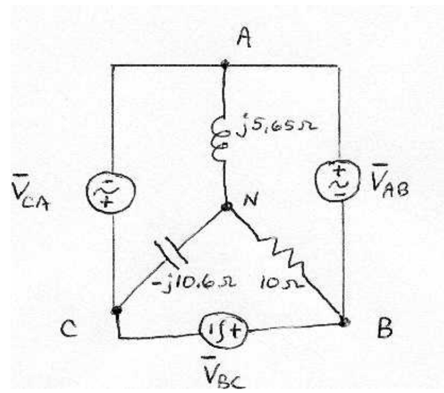
(c) $\overline{I}_{aA} = \overline{I}_{AB} - \overline{I}_{CA} = 24.05^{-}\angle -0.9638° - 24.05^{-}\angle 119.03° = \boxed{41.65^{-}\angle -30.96°\ \text{A rms}}$

32. 15 mH $\rightarrow$ j5.65 Ω, 0.25 mF $\rightarrow$ -j10.6 Ω

$\mathbf{V}_{AB} = 120\sqrt{3}\,\angle 30^\circ$ V

$\mathbf{V}_{BC} = 120\sqrt{3}\,\angle -90^\circ$ V

$\mathbf{V}_{CA} = 120\sqrt{3}\,\angle -210^\circ$ V



Defining three clockwise mesh currents $\mathbf{I}_1$, $\mathbf{I}_2$ and $\mathbf{I}_3$ corresponding to sources $\mathbf{V}_{AB}$, $\mathbf{V}_{BC}$ and $\mathbf{V}_{CA}$, respectively, we may write:

$\mathbf{V}_{AB} = (10 + j5.65)\,\mathbf{I}_1 - 10\,\mathbf{I}_2 + j5.65\,\mathbf{I}_3$ [1]

$\mathbf{V}_{BC} = -10\,\mathbf{I}_1 + (10 - j10.6)\,\mathbf{I}_2 + j10.6\,\mathbf{I}_3$ [2]

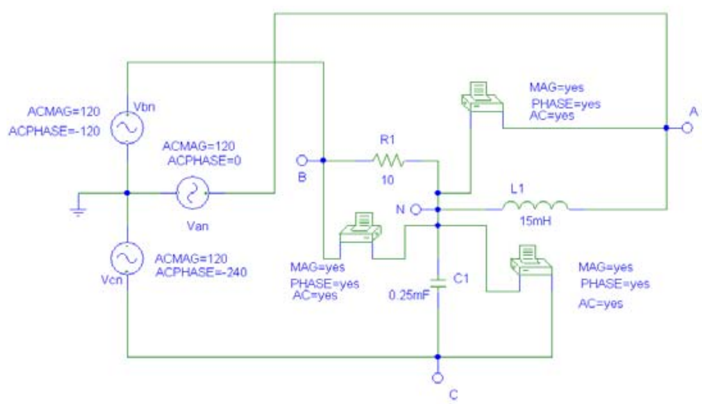
$\mathbf{V}_{CA} = -\,j5.65\,\mathbf{I}_1 + j10.6\,\mathbf{I}_2 + (j5.65 - j10.6)\,\mathbf{I}_3$ [3]

Solving using MATLAB or a scientific calculator, we find that
$\mathbf{I}_1 = 53.23\,\angle\,-5.873^\circ$ A, $\mathbf{I}_2 = 40.55\,\angle\,20.31^\circ$ A, and $\mathbf{I}_3 = 0$

(a) $\mathbf{V}_{AN} = j5.65(\mathbf{I}_1 - \mathbf{I}_3) = 300.7\,\angle\,84.13^\circ$ V, so $V_{AN} = 300.7$ V

(b) $\mathbf{V}_{BN} = 10(\mathbf{I}_2 - \mathbf{I}_1) = 245.7\,\angle\,127.4^\circ$ V, so $V_{BN} = 245.7$ V

(c) $\mathbf{V}_{CN} = -j10.6\,(-\mathbf{I}_2) = 429.8\,\angle\,110.3^\circ$ V, so $V_{CN} = 429.8$ V



PSpice Simulation Results
(agree with hand calculations)

FREQ	VM(A,N)	VP(A,N)
6.000E+01	3.007E+02	8.410E+01

FREQ	VM(B,N)	VP(B,N)
6.000E+01	2.456E+02	1.274E+02

FREQ	VM(C,N)	VP(C,N)
6.000E+01	4.297E+02	1.103E+02

33. $\uparrow R_{line} = 1\Omega$

(a)

$$120\sqrt{3} = 207.8\ \bar{I}_1 = \frac{\begin{vmatrix} 207.8\angle 30° & -1 & -j10 \\ 207.8\angle -90° & 2+j5 & -j5 \\ 0 & -j5 & 10-j5 \end{vmatrix}}{\begin{vmatrix} 12 & -1 & -j10 \\ -1 & 2+j5 & -j5 \\ -10 & -j5 & 10-j5 \end{vmatrix}} = \frac{207.8\begin{vmatrix} 1\angle 30° & -1 & -10 \\ -j1 & 2+j5 & -j5 \\ 0 & -j5 & 10-j5 \end{vmatrix}}{12(70+j40)+(-10-j45)-10(20+j55)}$$

$$\therefore \bar{I}_1 = \frac{207.8[1\angle 30°(70+j40)+j1(-10-j45)]}{630-j115} = \frac{21.690\angle 34.86°}{630-j115} = \boxed{33.87\angle 45.20° = \bar{I}_{aA}}$$

$$\therefore \bar{I}_2 = \frac{\begin{vmatrix} 12 & 1\angle 30° & -10 \\ -1 & -j1 & -j5 \\ -10 & 0 & 10-j5 \end{vmatrix} 207.8}{630-j115} = \frac{207.8[-1\angle 30°(-10-j45)-j1(20-j60)]}{630-j115}$$

$$= \frac{16{,}136\angle 162.01°}{630-j115} = 25.20\angle 172.36°\text{ A}$$

(b) $\therefore \bar{I}_{cC} = \boxed{25.20\angle -7.641°\text{ A}}$

(c) $\therefore \bar{I}_{bB} = -\bar{I}_{aA} - \bar{I}_{CC} = -33.87\angle 45.20° - 25.20\angle -7.641° = \boxed{53.03\angle -157.05°\text{ A rms}}$

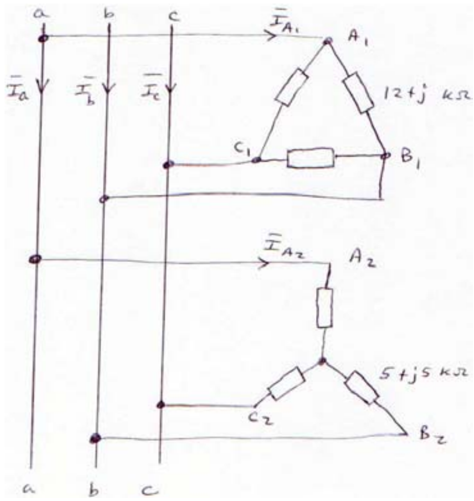
(d)

$$\bar{S} = 120\sqrt{3}\angle 30°(33.87\angle -45.20°) + 120\sqrt{3}\angle 90°(25.20\angle 7.641°)$$
$$= 6793 - j1846.1 - 696.3 + j5190.4 = \boxed{6096 + j3344\text{ VA}}$$

34. $|\mathbf{V}_{line}|$ = 240 V. Set $\mathbf{V}_{ab} = 240\angle 0^\circ$ V. Then $\mathbf{V}_{an} = \dfrac{240}{\sqrt{3}}\angle -30^\circ$.

$$\mathbf{I}_{A2} = \frac{\frac{240}{\sqrt{3}}\angle -30^\circ}{5 + j3} = 23.8\angle -61.0^\circ \text{ A}$$

$$\mathbf{I}_{A1B1} = \frac{\frac{240}{\sqrt{3}}\angle 0^\circ}{(12 + j)\times 10^3} = 20.0\angle -4.76^\circ \text{ mA}$$



$\mathbf{I}_{phase}$ leads $\mathbf{I}_{line}$ by 30°, so

$\mathbf{I}_{A1} = 20\sqrt{3}\angle -34.8^\circ$ mA $= 34.6\angle -34.8^\circ$ mA

$\mathbf{I}_a = \mathbf{I}_{A1} + \mathbf{I}_{A2}$ = 11.5 – j20.8 + 28.4 – j19.7 mA = $\boxed{56.9\angle -45.4^\circ \text{ mA}}$

The power factor at the source = cos (45.4° – 30°) = $\boxed{\text{0.964 lagging.}}$

The power taken by the load = $(3)(20\times 10^{-3})^2\,(12\times 10^3) + (3)(23.8\times 10^{-3})^2\,(5000)$ = $\boxed{\text{22.9 W.}}$

35. Define **I** flowing from the '+' terminal of the source. Then,

$$\mathbf{I} \;=\; \frac{200\angle 0}{10+(j10 \,||\, 20)} = \frac{200\angle 0}{16.12\angle 29.74^\circ} = 12.41\angle -29.74^\circ$$

(a) $\mathbf{V}_{xy}$ = 10 **I** = 124.1 ∠-29.74° V. Thus, P_{xy} = (12.41)(124.1) = $\boxed{1.54 \text{ kW}}$

(b) P_{xz} = (200)(12.41) cos (29.74°) = $\boxed{2.155 \text{ kW}}$

(c) $\mathbf{V}_{yz}$ = 200 ∠0 – 124.1 ∠-29.74° = 110.9 ∠ 33.72° V
Thus, P_{yz} = (110.9)(12.41) cos (33.72° + 29.74°) = $\boxed{614.9 \text{ W}}$

$\boxed{\text{No reversal of meter leads is required for any of the above measurements.}}$

36. 1 H $\rightarrow$ j377 Ω, 25 μF $\rightarrow$ -j106 Ω

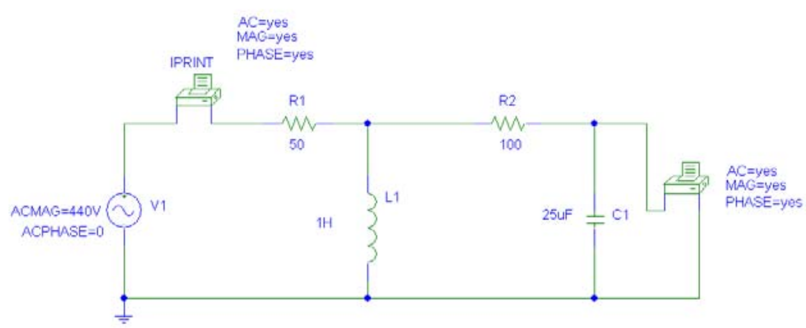
$$\mathbf{I}_1 = \frac{440\angle 0}{50 + \left[j377 \| (100\text{-}j106)\right]} = 1.86\angle 21^\circ \text{ A}$$

$$\mathbf{I}_C = \mathbf{I}\,\frac{j377}{j377 + 100 - j106} = 2.43\angle 41.3^\circ \text{ A}$$

$$\mathbf{V}_2 = (106\angle\text{-}90^\circ)(2.43\angle\text{-}41.3^\circ) = 257\angle\text{-}48.7^\circ \text{ V}$$

$P_{measured}$ = (257)(1.86) cos (21° + 48.7°) = $\boxed{\text{166 W.}}$

No reversal of meter leads is needed. PSpice verification:



FREQ	VM($N_0002,0)	VP($N_0002,0)
6.000E+01	2.581E+02	-4.871E+01
FREQ	IM(V_PRINT1)	IP(V_PRINT1)
6.000E+01	1.863E+00	2.103E+01

37. 2.5 A peak = 1.77 A rms. 200 V peak = 141 V rms. 100 μF → -*j*20 Ω.

Define the clockwise mesh current $\mathbf{I}_1$ in the bottom mesh, and the clockwise mesh current $\mathbf{I}_2$ in the top mesh. $\mathbf{I}_C = \mathbf{I}_1 - \mathbf{I}_2$.

Since $\mathbf{I}_2 = -177\angle{-90^o}$, we need write only one mesh equation:

$$141\angle 0^o = (20 - j40^o)\,\mathbf{I}_1 + (-20 + j20)\,\mathbf{I}_2$$

so that $\mathbf{I}_1 = \dfrac{141\angle 0 + (-20 + j20)(1.77\angle -90^o)}{20 - j40} = 4.023\angle 74.78^o$ A

and $\mathbf{I}_C = \mathbf{I}_1 - \mathbf{I}_2 = 2.361\angle 63.43^o$ A. $\mathbf{I}_{meter} = -\mathbf{I}_1 = 4.023\angle{-105.2^o}$
$\mathbf{V}_{meter} = 20\,\mathbf{I}_C = 47.23\angle 63.43^o$ V

Thus, Pmeter = $(47.23)(4.023)\cos(63.43^o + 105.2^o) = \boxed{-186.3 \text{ W.}}$
Since this would result in pegging the meter, we would need to swap the potential leads.

38. (a) Define three clockwise mesh currents $\mathbf{I}_1$, $\mathbf{I}_2$ and $\mathbf{I}_3$ in the top left, bottom left and right-hand meshes, respectively. Then we may write:

$$\begin{aligned}
100\angle 0 &= (10 - j10)\,\mathbf{I}_1 & & - (10 - j10)\,\mathbf{I}_3 \\
50\angle 90^\circ &= & (8 + j6)\,\mathbf{I}_2 & - (8 + j6)\,\mathbf{I}_3 \\
0 &= -(10 - j10)\,\mathbf{I}_1 & - (8 + j6)\,\mathbf{I}_2 & + (48 + j6)\,\mathbf{I}_3
\end{aligned}$$

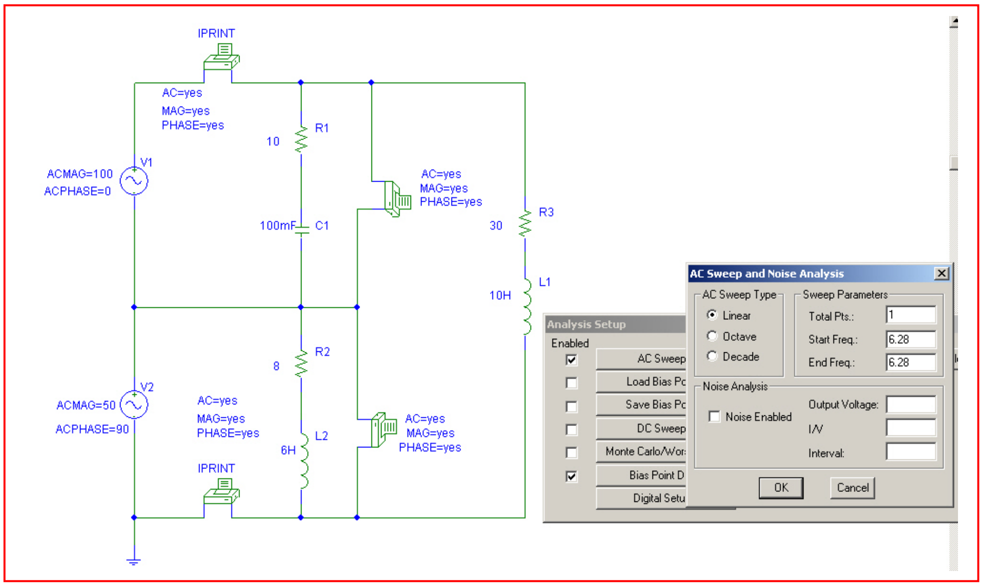
Solving, we find that $\mathbf{I}_1 = 10.12\angle 32.91^\circ$ A, $\mathbf{I}_2 = 7.906\angle 34.7^\circ$ and $\mathbf{I}_3 = 3.536\angle 8.13^\circ$ A.

Thus, $P_A = (100)(10.12)\cos(-32.91^\circ) = \boxed{849.6\text{ W}}$

and $P_B = (5)(7.906)\cos(90^\circ - 34.7^\circ) = \boxed{225.0\text{ W}}$

(b) Yes, the total power absorbed by the combined load (1.075 kW) is the sum of the wattmeter readings.

PSpice verification:




```
FREQ        IM(V_PRINT1)          IP(V_PRINT1)
  6.280E+00   1.014E+01            6.144E-02

FREQ        IM(V_PRINT2)          IP(V_PRINT2)
  6.280E+00   4.268E-01            1.465E+02

FREQ        VM($N_0002,$N_0006)          VP($N_0002,$N_0006)
  6.280E+00   1.000E+02                  0.000E+00

FREQ        VM($N_0004,$N_0006)          VP($N_0004,$N_0006)
  6.280E+00   5.000E+01  -               9.000E+01
```

39. This circuit is equivalent to a Y-connected load in parallel with a Δ-connected load.

For the Y-connected load, $\mathbf{I}_{\text{line}} = \dfrac{\frac{200}{\sqrt{3}}\angle -30^o}{25\angle 30^o} = 4.62\angle -60^o$ A

$P_Y = (3)\left(\dfrac{200}{\sqrt{3}}\right)(4.62)\cos 30^o = 1.386\text{ kW}$

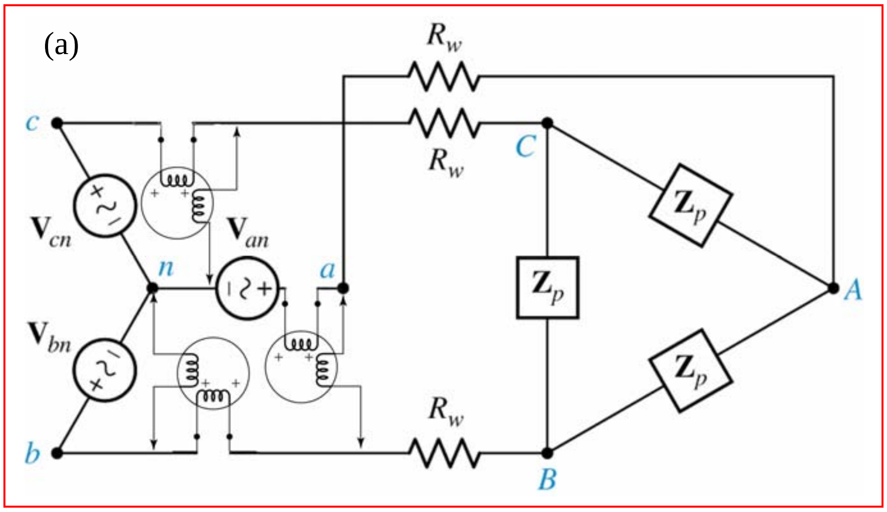
For the Δ-connected load, $\mathbf{I}_{\text{line}} = \dfrac{200\angle 0}{50\angle -60^o} = 4\angle 60^o$ A

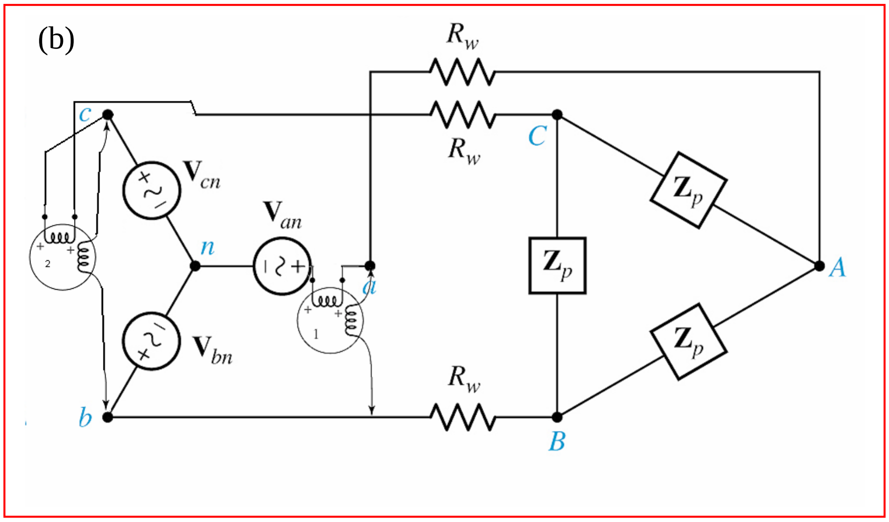
$P_\Delta = (3)(200)(4\cos 60^o) = 1.2$ kW

$P_{\text{total}} = P_Y + P_\Delta = 2.586$ kW

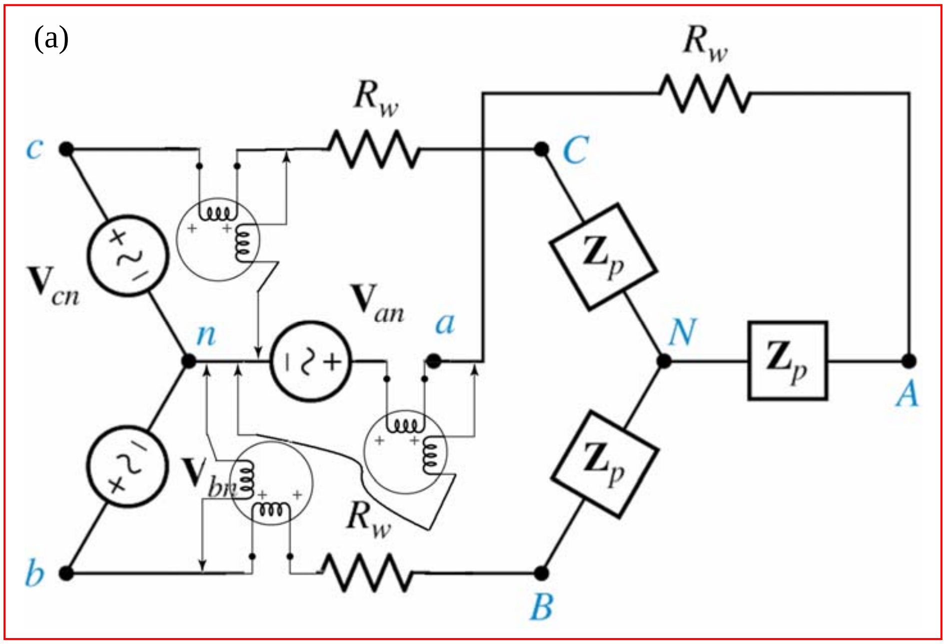
$P_{\text{wattmeter}} = P_{\text{total}} / 3 = \boxed{862\text{ W}}$

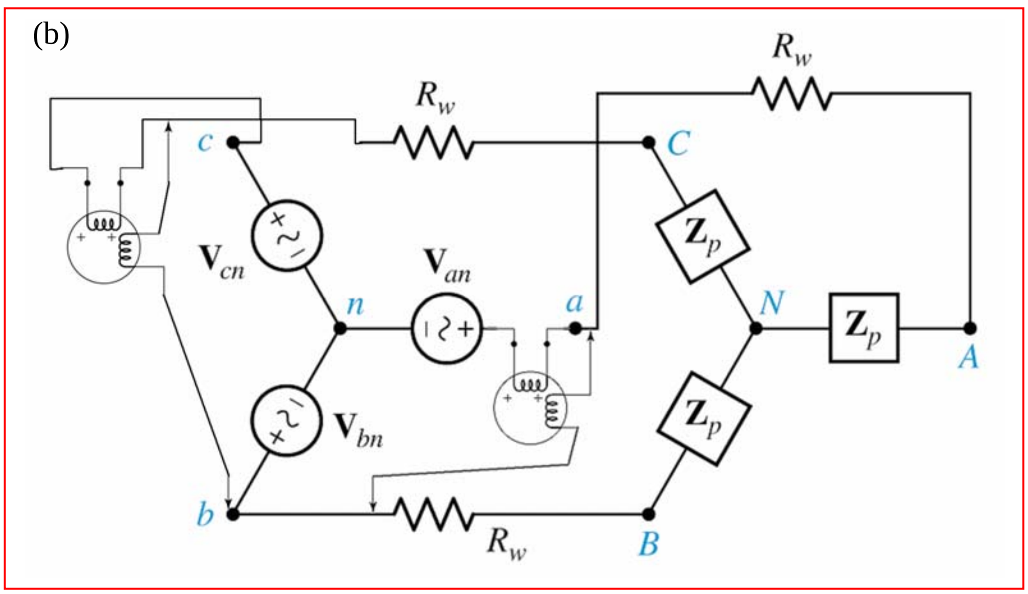
40. We assume that the wire resistance cannot be separated from the load, so we measure from the source connection:





41. We assume that the wire resistance cannot be separated from the load, so we measure from the source connection:





1. $v_2(t) = M_{21}\dfrac{di_1(t)}{dt} = -M_{21}(400)(120\pi)\sin(120\pi t)$

Taking peak values and noting sign is irrelevant, $100 = M_{21}(400)(120\pi)$.

Thus, $\boxed{M_{21} = 663.1\ \mu\text{H}}$

2.

$$v_1 = M_{12}\frac{di_2}{dt} \quad \text{therefore}$$

$$i_2 = \frac{1}{M_{12}}\int v_1 dt = \frac{1}{M_{12}}\left(\frac{115\sqrt{2}}{120\pi}\right)\sin\left(120\pi t - 16^{\circ}\right)$$

Equating peak values, $M_{12} = \frac{1}{45}\left(\frac{115\sqrt{2}}{120\pi}\right) = \boxed{9.59 \text{ mH}}$

3.

$$\boxed{\begin{array}{l}\text{1 and 3, 2 and 4}\\ \text{1 and 4, 2 and 3}\\ \text{3 and 1, 2 and 4}\end{array}}$$

4. (a) $v_1 = -L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$

Substituting in i_1 = 30 sin 80t and i_2 = 30 cos 80t, we find that

v_1 = –2400 cos 80t – 1200 sin 80t

$$= -\sqrt{2400^2 + 1200^2} \cos\left(80t - \tan^{-1}\frac{1200}{2400}\right)$$

$$= \boxed{-2683 \cos (80t - 26.57^{\circ}) \text{ V}}$$

(b) $v_2 = -L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}$

Substituting in i_1 = 30 sin 80t and i_2 = 30 cos 80t, we find that

v_2 = 7200 sin 80t + 1200 cos 80t

$$= \sqrt{7200^2 + 1200^2} \cos\left(80t - \tan^{-1}\frac{7200}{2400}\right)$$

$$= \boxed{7299 \cos (80t - 80.54^{\circ}) \text{ V}}$$

5. (a) $v_1 = -\left(L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} \right)$

Substituting in i_1 = 3 cos 800*t* nA and i_2 = 2 cos 800*t* nA,we find that

$$v_1 = -\left[-(22\times10^{-6})(3)(800)\times10^{-9} \sin 800t - (5\times10^{-6})(2)(800)\times10^{-9} \sin 800t \right]$$

$$= \boxed{60.8 \sin 800t \;\text{ pV}}$$

(b) $v_2 = +\left(L_2 \frac{di_2}{dt} + M \frac{di_1}{dt} \right)$

Substituting in i_1 = 3 cos 800*t* nA and i_2 = 2 cos 800*t* nA,we find that

$$v_1 = -(15\times10^{-6})(2)(800)\times10^{-9} \sin 800t - (5\times10^{-6})(3)(800)\times10^{-9} \sin 800t$$

$$= \boxed{36 \sin 800t \;\text{ pV}}$$

6. $$8\frac{di_1}{dt} + 0.4\frac{di_2}{dt} = 5e^{-t} \qquad [1]$$

$$0.4\frac{di_1}{dt} + 8\frac{di_2}{dt} = 3e^{-2t} \qquad [2]$$

Let $i_1 = ae^{-t} + be^{-2t}$ and $i_2 = ce^{-t} + de^{-2t}$

Then from Eq. [1] we have

–8a – 0.4c = 5 [3] and –16b – 0.8d = 0 [4]

And from Eq. [2] we have

–0.4a – 8c = 0 [5] and –0.8b – 16d = 3 [6]

Solving, we find that a = –0.6266, b = 0.0094, c = 0.03133, and d = –0.1880

(a) $\frac{di_1}{dt} = \frac{d}{dt}\left[-0.6266e^{-t} + 0.0094e^{-2t}\right] = \boxed{0.6266e^{-t} - 0.0188e^{-2t} \text{ A/s}}$

(b) $\frac{di_2}{dt} = \frac{d}{dt}\left[0.0313e^{-t} - 0.1880e^{-2t}\right] = \boxed{-0.0313e^{-t} + 0.376e^{-2t} \text{ A/s}}$

(c) $\boxed{i_1 = -0.6266e^{-t} + 0.0094e^{-2t} \text{ A}}$

7. $$\left(-2\frac{di_1}{dt}+1.5\frac{di_2}{dt}\right)\times 10^{-3} = 2e^{-t} \qquad [1]$$

$$\left(-1.5\frac{di_1}{dt}+2\frac{di_2}{dt}\right) = 4e^{-3t} \qquad [2]$$

Let $i_1 = ae^{-t}+be^{-3t}$ and $i_2 = ce^{-t}+de^{-3t}$

Then from Eq. [1] we have

2a – 1.5c = 2×10^3 [3] and 6b – 4.5d = 0 [4]

And from Eq. [2] we have

1.5a – 2c = 0 [5] and 4.5b – 6d = 4×10^3 [6]

Solving, we find that a = 2286, b = -1143, c = 1714, and d = –1524

(a) $\frac{di_1}{dt} = \frac{d}{dt}\left[2286e^{-t} - 1143e^{-3t}\right] = \boxed{-2286e^{-t} + 3429e^{-3t}\ \text{A/s}}$

(b) $\frac{di_2}{dt} = \frac{d}{dt}\left[1714e^{-t} - 1524e^{-3t}\right] = \boxed{-1714e^{-t} + 4572e^{-3t}\ \text{A/s}}$

(c) $\boxed{i_2 = 1714e^{-t} + 4572e^{-3t}\ \text{A}}$

8.

(a) $-\boldsymbol{V}_2 = j\omega 0.4 \; 1\angle 0$

$\boldsymbol{V}_2 = -j100\pi \times 0.4 \times 1\angle 0 = 126 \angle 90^\circ$ V

Thus, $\boxed{v(t) = 126 \cos(100\pi t + 90^\circ) \text{ V}}$

(b) Define $\mathbf{V}_2$ across the 2-H inductor with + reference at the dot, and a clockwise currents $\mathbf{I}_1$ and $\mathbf{I}_2$, respectively, in each mesh. Then,

$\mathbf{V} = -\mathbf{V}_2$ and we may also write

$$\mathbf{V}_2 = j\omega L_2 \mathbf{I}_2 + j\omega M \mathbf{I}_1 \quad \text{or} \quad -\mathbf{V} = j\omega L_2 \frac{\mathbf{V}}{10} + j\omega M$$

Solving for $\mathbf{V}$,

$$\mathbf{V} = \frac{-(j100\pi)(0.4)}{1+(j100\pi)(2)} = \frac{125.7\angle -90^o}{1+j62.83} = \frac{125.7\angle -90^o}{62.84\angle 89.09^o} = 2.000\angle -179.1^o$$

Thus,

$$\boxed{v(t) = 2\cos(100\pi t - 179.1^o) \text{ V.}}$$

(c) Define $\mathbf{V}_1$ across the left inductor, and $\mathbf{V}_2$ across the right inductor, with the "+" reference at the respective dot; also define two clockwise mesh currents $\mathbf{I}_1$ and $\mathbf{I}_2$. Then,

$$\boldsymbol{V}_1 = j\omega L_1 \boldsymbol{I}_1 + j\omega M \boldsymbol{I}_2$$
$$\boldsymbol{V}_2 = j\omega L_2 \boldsymbol{I}_2 + j\omega M \boldsymbol{I}_1$$

Now $\boldsymbol{I}_1 = \dfrac{1\angle 0 - \boldsymbol{V}_1}{4}$ and $\boldsymbol{V}_{out} = -\boldsymbol{V}_2$

and $\boldsymbol{I}_2 = \dfrac{\boldsymbol{V}_{out}}{10}$

$$\Rightarrow \boldsymbol{V}_1 = j\omega L_1 \left[\frac{1\angle 0 - \boldsymbol{V}_1}{4}\right] + j\omega M \frac{\boldsymbol{V}_{out}}{10} \quad \text{EQN 1}$$

$$-\boldsymbol{V}_{out} = j\omega L_2 \frac{\boldsymbol{V}_{out}}{10} + j\omega M \left[\frac{1\angle 0 - \boldsymbol{V}_1}{4}\right] \quad \text{EQN 2}$$

$$\begin{bmatrix} 1-\dfrac{j\omega L_1}{4} & \dfrac{-j\omega M}{10} \\ \dfrac{j\omega M}{4} & -1+\dfrac{j\omega L_2}{10} \end{bmatrix} \begin{bmatrix} \boldsymbol{V}_1 \\ \boldsymbol{V}_{out} \end{bmatrix} = \begin{bmatrix} \dfrac{j\omega L_1 \, 1\angle 0}{4} \\ \dfrac{j\omega M \, 1\angle 0}{4} \end{bmatrix}$$

$$\begin{bmatrix} 1-j39 & -j12.6 \\ j31.4 & -1+j62.8 \end{bmatrix} \begin{bmatrix} \boldsymbol{V}_1 \\ \boldsymbol{V}_{out} \end{bmatrix} = \begin{bmatrix} 39.3j \\ 31.4j \end{bmatrix}$$

Solving, we find that $\mathbf{V}_{out}$ (= $\mathbf{V}$) = 1.20 $\angle$ -2.108° V and hence

$$\boxed{v(t) = 1.2\cos(100\pi t - 2.108^o) \text{ V.}}$$

9.

(a) $100 = (50 + j200)\bar{I}_1 + j300\bar{I}_2, (2000 + j500)\bar{I}_2 + j300\bar{I}_1 = 0$

$\therefore \bar{I}_2 = \frac{-j3}{20 + j5}, 100 = \left(50 + j200 + \frac{900}{20 + j5}\right)\bar{I}_1$

$\therefore 100 = \frac{900 + j4250}{20 + j5}\bar{I}_1 \therefore \bar{I}_1 = 0.4745^1 \angle -64.01° \text{ A}$

$\therefore P_{S,abS} = -\frac{1}{2} \times 100 \times 0.4745 \cos 64.01° = \boxed{-10.399 \text{ W}}$

(b) $P_{50} = \frac{1}{2} \times 50 \times 0.4745^2 = 5.630 \text{ W}, P_{2000} = \frac{1}{2} \times 2000 \times 0.4745^2 \times \left|\frac{-j3}{20 + j5}\right|^2 = \boxed{4.769 \text{ W}}$

(c) $\boxed{\text{0 each}}$

(d) $\boxed{0}$

10. $i_{S1} = 4t \text{ A}, i_{S2} = 10t \text{ A}$

(a) $v_{AG} = 20 \times 4 + 4 \times 10 = \boxed{120 \text{ V}}$

(b) $v_{CG} = -4 \times 6 = \boxed{-24 \text{ V}}$

(c) $v_{BG} = 3 \times 10 + 4 \times 4 - 6 \times 4 = 30 + 16 - 24 = \boxed{22 \text{ V}}$

11.

(a) $\bar{V}_{ab,oc} = \dfrac{100}{50 + j200}(-j300) = 145.52\angle -165.96°\ \text{V}$

$100 = (50 + j200)\,\bar{I}_1 + j300\,\bar{I}_{2SC},\ j500\,\bar{I}_{2SC} + j300\,\bar{I}_1 = 0$

$\therefore \bar{I}_1 = -\dfrac{5}{3}\bar{I}_{2SC},\ 100 = \left[(50 + j200)\left(-\dfrac{5}{3}\right) + j300\right]\bar{I}_{2SC}\ \therefore \bar{I}_{2SC} = 1.1142\angle 158.199°\ \text{A}$

$\therefore \bar{Z}_{th} = \bar{V}_{ab,bc} / \bar{I}_{2SC} = \dfrac{145.52\angle -165.96°}{1.1142\angle 158.199°} = 130.60\angle 35.84° = \boxed{105.88 + j76.47\,\Omega}$

(b) $\bar{Z}_L = 105.88 - j76.47\,\Omega\ \therefore \left|\bar{I}_L\right| = \dfrac{145.52}{2 \times 105.88} = 0.6872\ \text{A}$

$\therefore P_{L\max} = \dfrac{1}{2} \times 0.6872^2 \times 105.88 = \boxed{25.00\ \text{W}}$

12.

KVL Loop 1 $100\angle 0 = 2(\mathbf{I}_1 - \mathbf{I}_2) + j\omega 3\,(\mathbf{I}_1 - \mathbf{I}_3) + j\omega 2\,(\mathbf{I}_2 - \mathbf{I}_3)$

KVL Loop 2 $2(\mathbf{I}_2 - \mathbf{I}_1) + 10\mathbf{I}_2 + j\omega 4\,(\mathbf{I}_2 - \mathbf{I}_3) + j\omega 2\,(\mathbf{I}_1 - \mathbf{I}_3) = 0$

KVL Loop 3 $5\mathbf{I}_3 + j\omega 3\,(\mathbf{I}_3 - \mathbf{I}_1) + j\omega 2\,(\mathbf{I}_3 - \mathbf{I}_2) + j\omega 4\,(\mathbf{I}_3 - \mathbf{I}_2) + j\omega 2\,(\mathbf{I}_3 - \mathbf{I}_1) = 0$

$\therefore$ LINEAR EQUATIONS

$$\begin{bmatrix} 2+j\omega 3 & -2+j\omega 2 & -j\omega 5 \\ -2+j\omega 2 & 12+j\omega 4 & -j\omega 6 \\ -j\omega 5 & j\omega 2 & 5+j11 \end{bmatrix}\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \\ \mathbf{I}_3 \end{bmatrix} = \begin{bmatrix} 100\angle 0 \\ 0 \\ 0 \end{bmatrix}$$

Since $\omega = 2\pi f = 2\pi(50) = 314.2$ rad/s, the matrix becomes

$$\begin{bmatrix} 2+j942.6 & -2+j628.4 & -j1571 \\ -2+j628.4 & 12+j1257 & -j1885 \\ -j1571 & j628.4 & 5+j3456 \end{bmatrix}\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \\ \mathbf{I}_3 \end{bmatrix} = \begin{bmatrix} 100\angle 0 \\ 0 \\ 0 \end{bmatrix}$$

Solving using a scientific calculator or MATLAB, we find that

$\mathbf{I}_1 = 278.5 \angle -89.65^\circ$ mA, $\mathbf{I}_2 = 39.78 \angle -89.43^\circ$ mA, $\mathbf{I}_3 = 119.4 \angle -89.58^\circ$ mA.

Returning to the time domain, we thus find that

$i_1(t) = 278.5 \cos(100\pi t - 89.65^\circ)$ mA, $i_2(t) = 39.78 \cos(100\pi t - 89.43^\circ)$ mA, and $i_3(t) = 119.4 \cos(100\pi t - 89.58^\circ)$ mA.

13.

$$v_s = \frac{10t^2 u(t)}{t^2 + 0.01} = 0.01 i'_S \;\; \therefore i'_S = \frac{1000t^2}{t^2 + 0.01} u(t)$$

$$v_x = 0.015 i'_S = \frac{15t^2}{t^2 + 0.01} u(t), \;\; 100 v_x = \frac{1500t^2}{t^2 + 0.01} u(t)$$

$$\therefore i_C = 100 \times 10^{-6} v'_x = 10^{-4} \frac{d}{dt}\left(\frac{15t^2}{t^2 + 0.01} u(t)\right) = 15 \times 10^{-4} \frac{(t^2 + 0.01)2t - t^2 \times 2t}{(t^2 + 0.01)^2} u(t)$$

$$\therefore i_C = 15 \times 10^{-4} \frac{0.02t}{(t^2 + 0.01)^2} \quad \therefore \boxed{i_C(t) = \frac{30t}{(t^2 + 0.01)^2}\ \mu\text{A}, \qquad t > 0}$$

14.

(a) $\boxed{v_A(t) = \mathrm{L}_1 i_1' - \mathrm{M} i_2',\ v_B(t) = \mathrm{L}_1 i_1' - \mathrm{M} i_2' + \mathrm{L}_2 i_2' - \mathrm{M} i_1'}$

(b)

$$\mathbf{V}_1(j\omega) = j\omega \mathrm{L}_1 \mathbf{I}_\mathrm{A} + j\omega \mathrm{M}(\mathbf{I}_\mathrm{B} + \mathbf{I}_\mathrm{A})$$

$$\mathbf{V}_2(j\omega) = j\omega \mathrm{L}_2 (\mathbf{I}_\mathrm{B} + \mathbf{I}_\mathrm{A}) + j\omega \mathrm{M}\mathbf{I}_\mathrm{A}$$

15.

(a) $100 = j5\omega\,(\mathbf{I}_1 - \mathbf{I}_2) + j3\omega\mathbf{I}_2 + 6(\mathbf{I}_1 - \mathbf{I}_3)$ [1]

$(4 + j4\omega)\mathbf{I}_2 + j3\omega\,(\mathbf{I}_1 - \mathbf{I}_2) + j2\omega\,(\mathbf{I}_3 - \mathbf{I}_2) + j6\omega\,(\mathbf{I}_2 - \mathbf{I}_3) - j2\omega\,\mathbf{I}_2 + j5\omega\,(\mathbf{I}_2 - \mathbf{I}_1)$
$- j3\omega\,\mathbf{I}_2 = 0$ [2]

$6\,(\mathbf{I}_3 - \mathbf{I}_1) + j6\omega\,(\mathbf{I}_3 - \mathbf{I}_2) + j2\omega\,\mathbf{I}_2 + 5\,\mathbf{I}_3 = 0$ [3]

Collecting terms,

$(6 + j5\omega)\,\mathbf{I}_1 - j2\omega\,\mathbf{I}_2 - 6\,\mathbf{I}_3 = 100$ [1]

$-j2\omega\,\mathbf{I}_1 + (4 + j5\omega)\,\mathbf{I}_2 - j4\omega\,\mathbf{I}_3 = 0$ [2]

$-6\,\mathbf{I}_1 - j4\omega\,\mathbf{I}_2 + (11 + j6\omega)\,\mathbf{I}_3 = 0$ [3]

(b) For $\omega = 2$ rad/s, we find

$(6 + j10)\,\mathbf{I}_1 - j4\,\mathbf{I}_2 - 6\,\mathbf{I}_3 = 100$

$-j4\,\mathbf{I}_1 + (4 + j10)\,\mathbf{I}_2 - j8\,\mathbf{I}_3 = 0$

$-6\,\mathbf{I}_1 - j8\,\mathbf{I}_2 + (11 + j12)\,\mathbf{I}_3 = 0$

Solving, $\boxed{\mathbf{I}_3 = 4.32 \angle -54.30^{\circ}\text{ A}}$

16.

(a)

$$\mathbf{V}_a = j\omega L_1 \mathbf{I}_a + j\omega M \mathbf{I}_b \qquad \mathbf{I}_a = \mathbf{I}_1$$
$$\mathbf{V}_b = j\omega L_2 \mathbf{I}_b + j\omega M \mathbf{I}_a \qquad \mathbf{I}_b = -\mathbf{I}_2$$

$$\begin{aligned}\mathbf{V}_1 &= \mathbf{I}_1 R_1 + \mathbf{V}_a \\ &= \mathbf{I}_1 R_1 + j\omega L_1 \mathbf{I}_a + j\omega M \mathbf{I}_b \\ &= \boxed{\mathbf{I}_1 R_1 + j\omega L_1 \mathbf{I}_1 - j\omega M \mathbf{I}_2}\end{aligned}$$

$$\begin{aligned}\mathbf{V}_2 &= \mathbf{I}_2 R_2 - \mathbf{V}_b \\ &= \mathbf{I}_2 R_2 - j\omega L_2 \mathbf{I}_b - j\omega M \mathbf{I}_a \\ &= \boxed{\mathbf{I}_2 R_2 + j\omega L_2 \mathbf{I}_2 - j\omega M \mathbf{I}_1}\end{aligned}$$

(b) Assuming that the systems connecting the transformer are fully isolated.

$$\mathbf{V}_a = j\omega L_1 \mathbf{I}_a + j\omega M\mathbf{I}_b \qquad \mathbf{I}_a = -\mathbf{I}_1$$
$$\mathbf{V}_b = j\omega L_2\mathbf{I}_b + j\omega M\mathbf{I}_a \qquad \mathbf{I}_b = -\mathbf{I}_2$$

$$\begin{aligned}\mathbf{V}_1 &= \mathbf{I}_1 R - \mathbf{V}_a \\ &= \mathbf{I}_1 R - j\omega L_1 \mathbf{I}_a - j\omega M \mathbf{I}_b \\ &= \boxed{\mathbf{I}_1 R + j\omega L_1 \mathbf{I}_1 + j\omega M \mathbf{I}_2}\end{aligned}$$

$$\begin{aligned}\mathbf{V}_2 &= \mathbf{V}_b + \mathbf{I}_b R_2 \\ &= -\mathbf{I}_2 R_2 + j\omega L_2\mathbf{I}_b + j\omega M \mathbf{I}_a \\ &= \boxed{-\mathbf{I}_2 R_2 - j\omega L_2 \mathbf{I}_2 - j\omega M \mathbf{I}_1}\end{aligned}$$

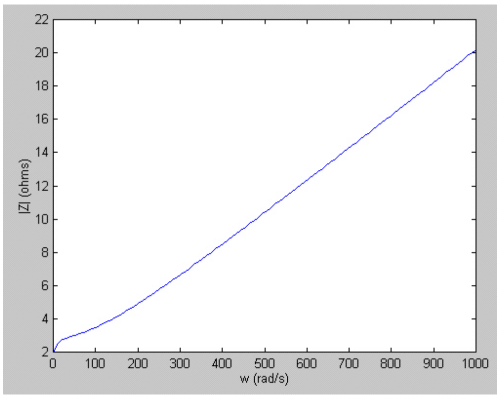
17.
(a)

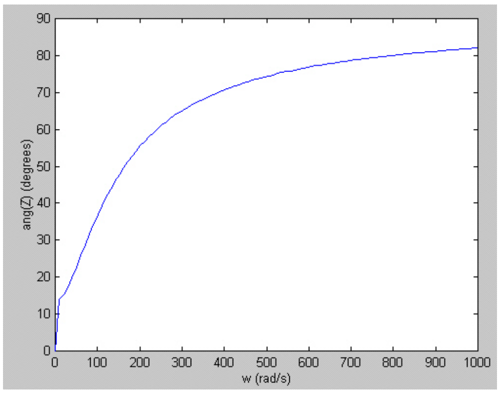
$$\mathbf{Z} = 2 + j\omega 0.1 + \frac{\omega^2 (0.2)^2}{5 + j\omega 0.5}$$

$$= 2 + j\omega 0.1 + \frac{5\omega^2 (0.2)^2}{5^2 + (\omega 0.5)^2} - \frac{j\omega 0.5\ \omega^2 (0.2)^2}{5^2 + (\omega 0.5)^2}$$

$$\boxed{= 2 + \frac{0.2\omega^2}{25 + 0.25\omega^2} + j\omega\left[0.1 - \frac{0.02\omega^2}{25 + 0.25\omega^2}\right]}$$

(b)





(c) $\mathbf{Z}_{in}(j\omega)$ at $\omega = 50$ is equal to $2 + 0.769 + j(50)(0.023) = \boxed{2.77 + j1.15\ \Omega.}$

18.

$$\mathbf{Z}_{in} = \mathbf{Z}_{11} + \frac{\omega^2 M^2}{\mathbf{Z}_{22}}$$

$$= j\omega 50\times10^{-3} + \frac{\omega^2 M^2}{8 + j\omega 10\times10^{-3}}$$

$$\Rightarrow \mathbf{Z}_{in} = j\omega 50\times10^{-3} + \frac{\omega^2 M^2 8}{8^2 + (\omega 10\times10^{-3})^2} - \frac{j\omega 10\times10^{-3}\omega^2 M}{8^2 + (\omega 10\times10^{-3})^2}$$

$$= \frac{\omega^2 M^2 8}{8^2 + (\omega 10\times10^{-3})^2} + j\omega\left[50\times10^{-3} - \frac{10\times10^{-3}\omega^2 M^2}{8^2 + (\omega 10\times10^{-3})^2}\right]$$

In this circuit the real power delivered by the source is all consumed at the speaker, so

$$P = \frac{V_{rms}{}^2}{R} \Rightarrow 3.2 = \left(\frac{20}{\sqrt{2}}\right)^2 \times \frac{1}{\frac{\omega^2 M^2 8}{8^2(\omega 10\times10^{-3})^2}}$$

$$\Rightarrow \frac{\omega^2 M^2 8}{8^2 + (\omega 10\times10^{-3})^2} = \frac{20^2}{2\times3.2} \quad \boxed{= 62.5\text{ W}}$$

19. $i_{S1} = 2\cos 10t$ A, $i_{S2} = 1.2\cos 10t$ A

(a) $v_1 = 0.6(-20\sin 10t) - 0.2(-12\sin 10t) + 0.5(-32\sin 10t) + 9.6\cos 10t$

$\therefore v_1 = 9.6\cos 10t - 25.6\sin 10t = \boxed{27.34\cos(10t + 69.44°)\ \text{V}}$

(b) $v_2 = 0.8(-12\sin 10t) - 0.2(-20\sin 10t) - 16\sin 10t + 9.6\cos 10t$

$\therefore v_2 = 9.6\cos 10t - 21.6\sin 10t = \boxed{23.64\cos(10t + 66.04°)\ \text{V}}$

(c) $\text{P}_{S1} = \frac{1}{2} \times 27.34 \times 2\cos 69.44° = \boxed{9.601\ \text{W}}$, $\text{P}_{S2} = \frac{1}{2} \times 23.64 \times 1.2\cos 66.04° = \boxed{5.760\ \text{W}}$

20.

$$\mathbf{V}_a = j\omega 8\mathbf{I}_a + j\omega 4\mathbf{I}_b$$

$$* \quad \mathbf{V}_b = j\omega 10\mathbf{I}_b + j\omega 4\mathbf{I}_a = j\omega 10\mathbf{I}_b + j\omega 5\mathbf{I}_c$$

$$\mathbf{V}_c = j\omega 6\mathbf{I}_c + j\omega 5\mathbf{I}_b$$

Also $\mathbf{I} = -\mathbf{I}_a = -\mathbf{I}_b = \mathbf{I}_c$

Now examine equation *.

$$-j\omega 10\mathbf{I} - j\omega 4\mathbf{I} = -j\omega 10\mathbf{I} + j\omega 5\mathbf{I}_c$$

$\therefore$ the only solution to this circuit is $\mathbf{I}$ = and hence

$\boxed{v(t) = 120\cos\omega t \text{ V.}}$

21.

$$100 = j10\bar{\mathrm{I}}_1 - j15\bar{\mathrm{I}}_2$$

$$0 = j200\bar{\mathrm{I}}_2 - j15\bar{\mathrm{I}}_1 - j15\bar{\mathrm{I}}_L$$

$$0 = (5 + j10)\,\bar{\mathrm{I}}_L - j15\bar{\mathrm{I}}_2$$

$$\therefore \bar{\mathrm{I}}_2 = \frac{5 + j10}{j15}\,\bar{\mathrm{I}}_L = \frac{1 + j2}{j3}\,\bar{\mathrm{I}}_L \quad \therefore 0 = j200\left(\frac{1 + j2}{j3} - j15\right)\bar{\mathrm{I}}_L - j15\bar{\mathrm{I}}_1$$

$$\therefore 0 = \left(j\frac{400}{3} - j15 + \frac{200}{3}\right)\bar{\mathrm{I}}_L - j15\bar{\mathrm{I}}_1 \quad \therefore \bar{\mathrm{I}}_1 = \frac{j118.33 + 66.67}{j15}\,\bar{\mathrm{I}}_L$$

$$\therefore 100 = \left[\frac{2}{3}(66.67 + j118.33) - 5 - j10\right]\bar{\mathrm{I}}_L = (39.44 + j68.89)\,\bar{\mathrm{I}}_L$$

$$\therefore \bar{\mathrm{I}}_L = \boxed{1.2597\angle -60.21^\circ\ \mathrm{A}}$$

22. $i_s = 2\cos 10t$ A, $t = 0$

(a) $a - b$ O.C. $\therefore w(0) = \frac{1}{2} \times 5 \times 2^2 + \frac{1}{2} \times 4 \times 2^2 = 10 + 8 = \boxed{18 \text{ J}}$

(b) $a - b$ S.C. $\omega = 10$, $\bar{\text{I}}_S = 2\angle 0°$ A, $\text{M} = \frac{1}{2}\sqrt{12} = \sqrt{3}$ H

$(j30 + 5)\,\bar{\text{I}}_2 - j10\sqrt{3} \times 2$, $\therefore \bar{\text{I}}_2 = \dfrac{j20\sqrt{3}}{5 + j30} = 1.1390\angle 9.462°\text{A}$ $\therefore i_2 = 1.1390\cos(10t + 9.462°)$ A

$\therefore i_2(0) = 1.1235^-$ $\therefore w(0) = 10 + 8 - \sqrt{3} \times 2 \times 1.1235 + \frac{1}{2} \times 3 \times 1.1235^2 = \boxed{16.001 \text{ J}}$

23.

$\bar{V}_s = 12\angle 0°$ V rms, $\omega = 100$ rad/s

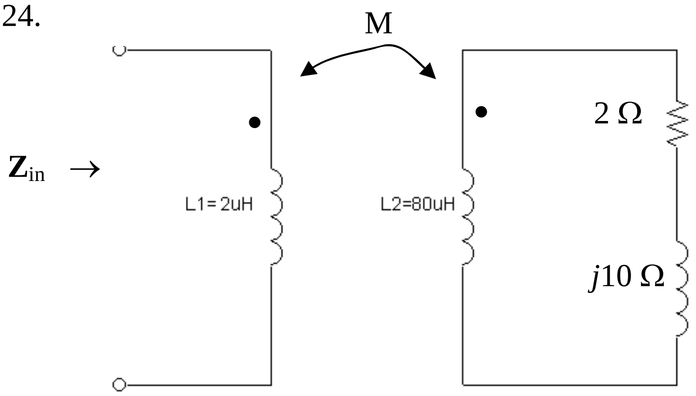
$12 = (6 + j20)\ \bar{I}_1 + j100(0.4\text{K})\ \bar{I}_2,\ (24 + j80)\ \bar{I}_2 + j40\text{K}\bar{I}_1 = 0$

$$\therefore\ \bar{I}_1 = \frac{3 + j10}{-j5\text{K}}\ \bar{I}_2\quad \therefore 12 = \left[(6 + j20)\frac{3 + j10}{-j5\text{K}} + j40\text{K}\right]\bar{I}_2$$

$$\therefore 12 = \frac{18 - 200 + j60 + j60 + 200\text{K}^2}{-j5\text{K}}\ \bar{I}_2\quad \therefore\ \bar{I}_2 = \frac{-j60\text{K}}{-182 + 200\text{K}^2 + j120}$$

$$\therefore\ P_{24} = \frac{60^2\text{K}^2 24}{(200\text{K}^2 - 182)^2 + 120^2} = \frac{86,400\ \text{K}^2}{40,000\text{K}^4 - 72,800\text{K}^2 + 47,524} = \boxed{\frac{2.16\text{K}^2}{\text{K}^4 - 1.82\text{K}^2 + 1.1881}\ \text{W}}$$

24.



$$k = \frac{M}{\sqrt{L_1 L_2}}$$

$$\omega = 250k\ rad/s$$

$$M = \sqrt{L_1 L_2} = \sqrt{2 \times 80} \times 10^{-6}$$
$$= 12.6 \mu\text{H}$$

$$\mathbf{Z}_{in}, = \boldsymbol{Z}_{11} + \frac{\omega^2 M^2 R_{22}}{R_{22}^2 + X_{22}^2} + \frac{-jM^2\omega^2 X_{22}}{R_{22}^2 + X_{22}^2}$$

$$\boldsymbol{Z}_{11} = j \times 250 \times 10^3 \times 2 \times 10^{-6}$$
$$= j0.5$$

$$R_{22} = 2\Omega$$
$$X_{22} = (250 \times 10^3)(80 \times 10^{-6})$$
$$= 20$$

Thus, $\mathbf{Z}_{in} = j0.5 + 19.8/404 - j198/404$

$$= \boxed{0.049 + j0.010\ \Omega.}$$

25. $\omega = 100$ rad/s

(a)

$$\mathrm{K}_1 \to j50\Omega,\ \mathrm{K}_2 \to j20\Omega,\ 1\mathrm{H} \to \mathrm{j}100\,\Omega$$

$$100 = \mathrm{j}200\ \bar{\mathrm{I}}_1 - j50\bar{\mathrm{I}}_2 - j20\bar{\mathrm{I}}_3$$

$$0 = (10 + j100)\bar{\mathrm{I}}_2 - j50\bar{\mathrm{I}}_1$$

$$0 = (20 + j100)\ \bar{\mathrm{I}}_3 - j20\bar{\mathrm{I}}_1$$

$$\therefore \bar{\mathrm{I}}_3 = \frac{j2}{2+j10}\bar{\mathrm{I}}_1, \bar{\mathrm{I}}_2 = \frac{j5}{1+j10}\bar{\mathrm{I}}_1\ \ \therefore 10 = \left[j20 - j5\frac{j5}{1+j10} - j2\frac{j2}{2+j10} \right]\bar{\mathrm{I}}_1$$

$$\therefore 10 = \left(j20 + \frac{25}{1+j10} + \frac{4}{2+j10} \right)\bar{\mathrm{I}}_1\ \ \therefore \bar{\mathrm{I}}_1 = 0.5833\angle -88.92^\circ \text{ A},\ \bar{\mathrm{I}}_2 = 0.2902\angle -83.20^\circ \text{ A},$$

$$\bar{\mathrm{I}}_3 = 0.11440\angle -77.61^\circ \text{ A} \quad \therefore \mathrm{P}_{10\Omega} = 0.2902^2 \times 10 = \boxed{0.8422 \text{ W}}$$

(b) $\mathrm{P}_{20} = 0.1144^2 \times 20 = \boxed{0.2617 \text{ W}}$

(c) $\mathrm{P}_{gen} = 100 \times 0.5833 \cos 88.92^\circ = \boxed{1.1039 \text{ W}}$

26.

(a)

$$k = \frac{M}{\sqrt{L_1 L_2}}$$

$$\Rightarrow M = 0.4\sqrt{5\times 1.8}$$

$$= \boxed{1.2\text{H}}$$

(b)

$$\boldsymbol{I}_1 + \boldsymbol{I}_2 = \boldsymbol{I}_3$$

$$\Rightarrow \boldsymbol{I}_2 = \boldsymbol{I}_3 - \boldsymbol{I}_1$$

$$= \boxed{5\times 10^{\frac{-t}{5}} - 4\times 10^{\frac{-t}{10}}}$$

(c) The total energy stored at $t = 0$.

$$\boldsymbol{I}_1 = 4A \qquad \boldsymbol{I}_2 = 1A$$

$$W\ total = \frac{1}{2}L_1\boldsymbol{I}_1^2 + \frac{1}{2}L_2\boldsymbol{I}_2^2 + M_{12}\boldsymbol{I}_1\boldsymbol{I}_2$$

$$= \frac{1}{2}\times 5\times 16 + \frac{1}{2}\times 1.8\times 1 - 1.2\times 4\times 1$$

$$= 40 + 0.9 - 4.8$$

$$= \boxed{36.1\text{J}}$$

27.

$$\mathrm{K} \to j1000\mathrm{K}\sqrt{\mathrm{L_1L_2}},\ \mathrm{L_1} \to j1000\mathrm{L_1},\ \mathrm{L_2} \to j1000\mathrm{L_2}$$

$$\therefore \bar{\mathrm{V}}_s = (2 + j1000\mathrm{L_1})\,\bar{\mathrm{I}}_1 - j1000\mathrm{K}\,\sqrt{\mathrm{L_1L_2}}\ \bar{\mathrm{I}}_2$$

$$0 = -j1000\mathrm{K}\sqrt{\mathrm{L_1L_2}}\ \bar{\mathrm{I}}_1 + (40 + j1000\mathrm{L_2})\,\bar{\mathrm{I}}_2$$

$$\omega = 1000 \text{ rad/s} \quad \therefore \bar{\mathrm{I}}_1 = \frac{40 + j1000\mathrm{L_2}}{j1000\mathrm{K}\sqrt{\mathrm{L_1L_2}}}\,\bar{\mathrm{I}}_2$$

$$\therefore \bar{\mathrm{V}}_s = \frac{(2 + j1000\mathrm{L_1})(40 + j1000\mathrm{L_2}) + 10^6\mathrm{K^2L_1L_2}}{j1000\mathrm{K}\sqrt{\mathrm{L_1L_2}}}\,\bar{\mathrm{I}}_2$$

$$\therefore \bar{\mathrm{I}}_2 = \frac{j1000\mathrm{K}\sqrt{\mathrm{L_1L_2}}}{80 + j40{,}000\mathrm{L_1} + j2000\mathrm{L_2} - 10^6\mathrm{L_1L_2}(1-\mathrm{K}^2)}$$

$$\therefore \frac{\bar{\mathrm{V}}_2}{\bar{\mathrm{V}}_s} = \frac{j40{,}000\mathrm{K}\sqrt{\mathrm{L_1L_2}}}{80 - 10^6\mathrm{L_1L_2}(1-\mathrm{K}^2) + j(40{,}000\mathrm{L_1} + 2000\mathrm{L_2})}$$

(a) $\mathrm{L_1} = 10^{-3},\ \mathrm{L_2} = 25\times 10^{-3},\ \mathrm{K} = 1 \quad \therefore \dfrac{\bar{\mathrm{V}}_2}{\bar{\mathrm{V}}_s} = \dfrac{j40\times 5}{80 - 0 + j(40+50)} = \dfrac{j200}{80 + j90} = \boxed{1.6609\angle 41.63^\circ}$

(b) $\mathrm{L_1} = 1,\ \mathrm{L_2} = 25,\ \mathrm{K} = 0.99 \quad \therefore \dfrac{\mathrm{V_2}}{\mathrm{V_3}} = \dfrac{j40{,}000\times 0.99\times 5}{80 - 25\times 10^6(1-0.99^2) + j(40{,}000 + 50{,}000)}$

$$\therefore \frac{\bar{\mathrm{V}}_2}{\bar{\mathrm{V}}_S} = \frac{j198{,}000}{80 - 497{,}500 + j90{,}000} = \boxed{0.3917\angle -79.74^\circ}$$

(c) $\mathrm{L_1} = 1,\ \mathrm{L_2} = 25,\ \mathrm{K} = 1 \quad \therefore \dfrac{\mathrm{V_2}}{\mathrm{V}_s} = \dfrac{j40{,}000\times 5}{80 - 0 + j90{,}000} = \dfrac{j200{,}000}{80 + j90{,}000} = \boxed{2.222\angle 0.05093^\circ}$

28.

(a) $L_{AB,CDOC} = 10 \text{ mH}, \ L_{CD,ABOC} = 5 \text{ mH}$

$L_{AB,CDSC} = 8 \text{ mH}$

$\therefore L_1 = 10 \text{ mH}, \ L_2 = 5 \text{ mH}, \ 8 = 10 - M + M \| (5 - M) \ (\text{mH})$

$\therefore 8 = 10 - M + \dfrac{M(5-M)}{5}, \ \therefore 5M = (10-8)5 + 5M - M^2 \ \therefore M = 3.162 \text{ mH } (= \sqrt{10})$

$\therefore K = \dfrac{3.162}{\sqrt{50}} \ \therefore \boxed{K = 0.4472}$

(b) Dots at A and D, $i_1 = 5$ A, $w_{tot} = 100$ mJ

$\therefore 100 \times 10^{-3} = \dfrac{1}{2} \times 10 \times 10^{-3} \times 25 + \dfrac{1}{2} \times 5 \times 10^{-3} i_2^2 - \sqrt{10} \times 5 i_2 \times 10^{-3}$

$100 = 125 + 2.5 i_2^2 - 5\sqrt{10}\, i_2 \ \therefore i_2^2 - 2\sqrt{10}\, i_2 + 10 = 0, \ i_2 = \dfrac{2\sqrt{10} \pm \sqrt{40-40}}{2} = \sqrt{10}$

$\therefore i_2 = \boxed{3.162 \text{ A}}$

29. Define coil voltages v_1 and v_2 with the "+" reference at the respective dot. Also define two clockwise mesh currents i_1 and i_2. We may then write:

$$v_1 = L_1 \frac{d\boldsymbol{I}_1}{dt} + M\frac{d\boldsymbol{I}_2}{dt} \qquad M = k\sqrt{L_1 L_2}$$

$$v_2 = L_2 \frac{d\boldsymbol{I}_2}{dt} + M\frac{d\boldsymbol{I}_1}{dt} \qquad \omega = 2\pi 60\, rad/s$$

or, using phasor notation,

$$\boldsymbol{V}_1 = j\omega L_1\, \boldsymbol{I}_1 + j\omega M\, \boldsymbol{I}_2$$
$$\boldsymbol{V}_2 = j\omega L_2\, \boldsymbol{I}_2 + j\omega M\, \boldsymbol{I}_1$$

$$100\angle 0 = 50\boldsymbol{I}_1 + j\omega L_1\boldsymbol{I}_1 + j\omega M\, \boldsymbol{I}_2$$
$$-25\boldsymbol{I}_2 = j\omega L_2\boldsymbol{I}_2 + j\omega M\, \boldsymbol{I}_1$$

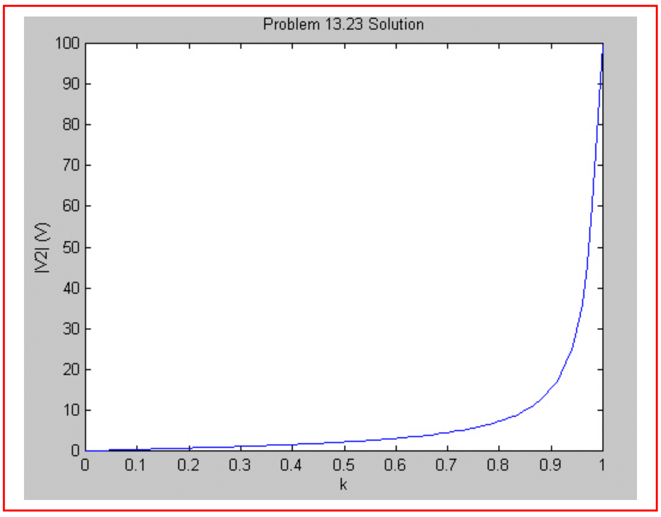
Rearrange: $[50 + j\omega L_1]\boldsymbol{I}_1 + j\omega M\boldsymbol{I}_2 = 100\angle 0$

$j\omega M\boldsymbol{I}_1 [-25 + j\omega L_2]\boldsymbol{I}_2 = 0$

or
$$\begin{bmatrix} 50 + j\omega L_1 & j\omega M \\ j\omega M & -25 + j\omega L_2 \end{bmatrix} \begin{bmatrix} \boldsymbol{I}_1 \\ \boldsymbol{I}_2 \end{bmatrix} = \begin{bmatrix} 100\angle 0 \\ 0 \end{bmatrix}$$

We can solve for $\mathbf{I}_2$ and $\mathbf{V}_2 = -25\mathbf{I}_2$:

$$\boxed{\mathbf{V}_2 = -\frac{j1.658}{k\sqrt{\mathrm{L}_1\mathrm{L}_2} + 1}}$$



30.

$$i_1 = 2\cos 500t \text{ A} \quad \text{W}_{\text{max}} \text{ at } t = 0$$

$$\therefore w_{\text{max}} = \frac{1}{2}\times 4\times 2^2 + \frac{1}{2}\times 6\times 2^2 + \frac{1}{2}\times 5\times 2^2 + 3\times 2^2$$

$$= 8 + 12 + 10 + 12 = \boxed{42 \text{ J}}$$

31. (a) Reflected impedance = $\dfrac{\omega^2 M^2}{Z_{22}}$.

$Z_{22} = 2 + 7\angle 32^0 + j\omega 10^{-2}$ where $\omega = 100\pi$

Thus, the reflected impedance is $\boxed{4.56 - j3.94 \text{ n}\Omega \text{ (essentially zero).}}$

(b) $Z_{in} = Z_{11}$ + reflected impedance = $10 + j\omega(20\times10^{-2}) + (4.56 - j3.94)\times10^{-9}$

$= \boxed{10 + j62.84\ \Omega}$ (essentially Z11 due to small reflected impedance)

32. Reflected impedance = $\dfrac{\omega^2 M^2}{Z_{22}} = \dfrac{\omega^2 M^2}{3.5 + j(\omega L_2 + X_L)}$.

We therefore require $1 + j\omega\left(3\times10^{-3}\right) = \dfrac{\omega^2 10^{-6}}{3.5 + j(10^{-3}\omega + X_L)}$.

Thus,

$$X_L = -j\left[\frac{\omega^2 10^{-6}}{1 + j\omega\left(3\times10^{-3}\right)} - 3.5 - j10^{-3}\omega\right] = -0.448 + j3.438.$$

$\boxed{\text{This is physically impossible; to be built, } X_L \text{ must be a real number.}}$

33. $\boxed{M = 5\text{ H.}}$

$L_1 - M = 4$ H, therefore $L_1 = 9$ H
$L_2 - M = 6$ H, therefore $L_2 = 11$ H.

34. $L_z = L_1 - M = 300 - 200 = \boxed{100 \text{ mH}}$

$L_y = L_2 - M = 500 - 200 = \boxed{300 \text{ mH}}$

$L_x = M = \boxed{200 \text{ mH}}$

35.

(a) All DC: $L_{1-2} = 2 - 1 = 1$ H

(b) AB SC: $L_{1-2} = -1 + 2 \| 8 = \boxed{0.6\text{ H}}$

(c) BC SC: $L_{1-2} = 2 + (-1) \| 9 = 2 - 9/8 = \boxed{0.875\text{ H}}$

(d) AC SC: $L_{1-2} = (2-1) \| (1+2) = 1 \| 3 = \boxed{0.750\text{ H}}$

36.

(a) $$\frac{\mathbf{I}_L}{\mathbf{V}_S} = \frac{1}{15 + j3\omega + \dfrac{j2\omega(20 + j\omega)}{20 + j3\omega}}\left(\frac{j2\omega}{20 + j3\omega}\right)$$

$$\boxed{= \frac{j2\omega}{300 - 11\omega^2 + j145\omega}}$$

(b) $v_s(t) = 100u(t),\ i_s(0) = 0,\ i_L(0) = 0,\ s_{1,2} = \dfrac{-145 \pm \sqrt{145^2 - 13,200}}{22} = -2.570,\ -10.612$

$i_L = i_{Lf} + i_{Ln},\ i_{Lf} = 0,\ \ \therefore i_L = \text{A}e^{-2.57t} + \text{B}e^{-10.61t},\ \ \therefore 0 = \text{A} + \text{B}$

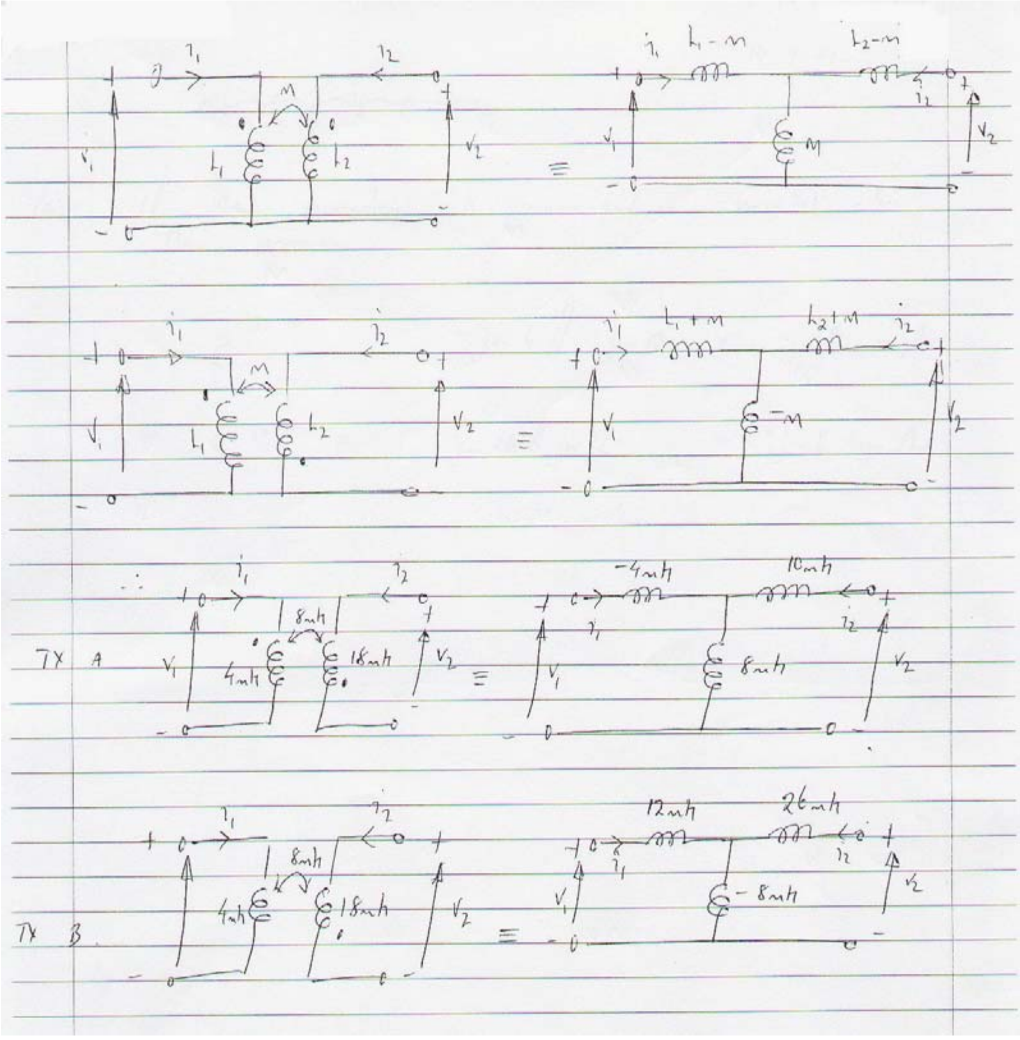
$100 = 15i_s + 5i'_s - 2i'_L,\ 0 = 20i_L + 3i'_L - 2i'_s$ At $t = 0^+$: $100 = 0 + 5i'_s(0^+) - 2i'_L(0^+)$ and

$0 = 0 + 3i'_L(0^+) - 2i'_s(0^+)\ \ \therefore i'_s(0^+) = 1.5i'_L(0^+)\ \ \therefore 100 = 7.5i'_L(0^+) - 2i'_L(0^+) = 5.5i'_L(0^+)$

$\therefore i'_L(0^+) = 18.182$ A/s $\quad \therefore 18.182 = -2.57\text{A} - 10.61\text{B} = -2.57\text{A} + 10.61\text{A} = 8.042\text{A}$

$\therefore \text{A} = 2.261,\ \text{B} = -2.261,\ \boxed{i_L(t) = 2.261(e^{-2.57t} - e^{-10.612t})\ \text{A},\ t > 0}$

37.



(a) Open-Circuit

$$\mathbf{Z}_{oc}^{T\times A} = j\omega 4 \ M\Omega$$
$$\mathbf{Z}_{oc}^{T\times B} = j\omega 4 \ M\Omega$$

(b) Short-Circuit

$$\mathbf{Z}_{SS}^{T\times A} = \mathbf{Z}_{SS}^{T\times B} = -j\omega 4\, M\Omega + j\omega 8 \| j\omega 10\, M\Omega$$

(c) If the secondary is connected in parallel with the primary

$$\mathbf{Z}_{in}^{T\times A} = -j\omega 4 \| -j\omega 10 + j\omega 8\, M\Omega$$
$$\mathbf{Z}_{in}^{T\times B} = j\omega 26 \| j\omega 12 - j\omega 8\, M\Omega$$

38. Define three clockwise mesh currents $\mathbf{I}_1$, $\mathbf{I}_2$, and $\mathbf{I}_3$ beginning with the left-most mesh.

$\mathbf{V}_s \;=\; j8\omega\,\mathbf{I}_1 - j4\omega\,\mathbf{I}_2$
$0 \;=\; -4j\omega\,\mathbf{I}_1 + (5 + j6\omega)\,\mathbf{I}_2 - j2\omega\,\mathbf{I}_3$
$0 \;=\; -j2\omega\,\mathbf{I}_2 + (3 + j\omega)\,\mathbf{I}_3$

Solving, $\mathbf{I}_3 \;=\; j\omega\,/\,(15 + j17\omega)$. Since $\mathbf{V}_o = 3\,\mathbf{I}_3$,

$$\boxed{\frac{\mathbf{V}_o}{\mathbf{V}_S} \;=\; \frac{j3\omega}{15 + j17\omega}}$$

39. $L_{eq} = 2/3 + 1 + 2 + 6/5 = 4.867$ H

$Z(j\omega) = 10\,j\omega(4.867)/(10 + j\omega 4.867)$

$\boxed{= j4.867\omega/(1 + j0.4867\omega)\ \Omega.}$

40.

$\omega = 100$ rad/s

$\bar{V}_s = 100\angle 0°$ V rms

(a) $$Z_{ina-b} = 20 + j600 + \frac{j400(10 - j200)}{10 + j200} = 20 + j600 + \frac{80,000 + j4,000}{10 + j200}$$

$$= \boxed{210.7\angle 73.48^{o}\text{ V and } \mathbf{V}_{oc} = 0.}$$

(b) $$\bar{V}_{OC,cd} = \frac{100(j400)}{20 + j1000} = \boxed{39.99\angle 1.146°\text{ V rms}}$$

$$\bar{Z}_{incd},\ \bar{V}_S = 0 = -j200 + \frac{j400(20 + j600)}{20 + j1000} = -j200 + \frac{-240,000 + j8,000}{20 + j1,000} = \boxed{40.19\angle 85.44°\,\Omega}$$

41. $L_1 = 1\text{ H},\ L_2 = 4\text{ H},\ K = 1,\ \omega = 1000\text{ rad/s}$

(a) $\bar{Z}_L = 1000\,\Omega \quad \therefore \bar{Z}_{in} = j1000 + \dfrac{10^6 \times 1 \times 4}{j4000 + 100} = \boxed{24.98 + j0.6246\,\Omega}$

(b) $\bar{Z}_L = j1000 \times 0.1 \quad \therefore \bar{Z}_{in} = j1000 + \dfrac{4 \times 10^6}{j4000 + j100} = \boxed{j24.39\,\Omega}$

(c) $\bar{Z}_L = -j100 \quad \therefore \bar{Z}_{in} = j1000 + \dfrac{4 \times 10^6}{j4000 - j100} = \boxed{-j25.46\,\Omega}$

42.

$\mathrm{L}_1 = 6\ \mathrm{H},\ \mathrm{L}_2 = 12\ \mathrm{H},\ \mathrm{M} = 5\ \mathrm{H}$

#1, $\mathrm{L}_{inAB,CDOC} = \boxed{6\ \mathrm{H}}$

#2, $\mathrm{L}_{inCD,ABOC} = \boxed{12\ \mathrm{H}}$

#3, $\mathrm{L}_{inAB,CDSC} = 1 + 7 \| 5 = \boxed{3.917\ \mathrm{H}}$

#4, $\mathrm{L}_{inCD,ABSC} = 7 + 5 \| 1 = \boxed{7.833\ \mathrm{H}}$

#5, $\mathrm{L}_{inAC,BDSC} = 7 + 1 = \boxed{8\ \mathrm{H}}$

#6, $\mathrm{L}_{inAB,ACSC,BDSC} = 7 \| 1 + 5 = \boxed{5.875\ \mathrm{H}}$

#7, $\mathrm{L}_{inAD,BCSC} = 11 + 17 = \boxed{28\ \mathrm{H}}$

#8, $\mathrm{L}_{inAB,ADSC} = -5 + 11/17 = \boxed{1.6786\ \mathrm{H}}$

43.

$$\mathbf{Z}_{in} = \mathbf{Z}_{11} + \frac{\omega^2 M^2}{R_{22} + jX_{22}}$$

$$\frac{1}{\omega C} = 31.83 \Rightarrow \omega = \frac{1}{31.83 \times C} = 314\, rad/s$$

ie. a 50Hz system

$$\mathbf{Z}_{in} = 20 + j\omega 100 \times 10^{-3} + \frac{\omega^2 k^2 L_1 L_2}{2 - j31.83}$$

$$\mathbf{Z}_{in} = 20 + j\omega 100 \times 10^{-3} + \frac{\omega^2 k^2 L_1 L_2 2}{2^2 + 31.83^2} - \frac{j\omega^2 k^2 L_1 L_2 31.83}{2^2 + 31.83^2}$$

$$= 20 + j31.4 + \left[\frac{493}{1020} - j\frac{7840}{1020}\right] k^2$$

$$= 20 + j31.4 + [0.483 - j7.69] k^2$$

(a) $\mathbf{Z}_{in}(k = 0) = \boxed{20 + j31.4 \quad \Omega}$

(b) $\mathbf{Z}_{in}(k = 0.5) = \boxed{20.2 + j27.6 \ \Omega}$

(c) $\mathbf{Z}_{in}(k = 0.9) = \boxed{20.4 + j24.5 \ \Omega}$

(d) $\mathbf{Z}_{in}(k = 1.0) = \boxed{20.5 + j23.7 \ \Omega}$

44. $\uparrow L_1 \rightarrow 125\text{ H}, L_2 \rightarrow 20\text{ H}, K = 1, \therefore M = \sqrt{2500} = 50\text{ H}, j\omega M = j5000\,\Omega$

(a) $\bar{Z}_{ina-b} = 20 + j7500 + \dfrac{j5000(10 - j3000)}{10 + j2000}$

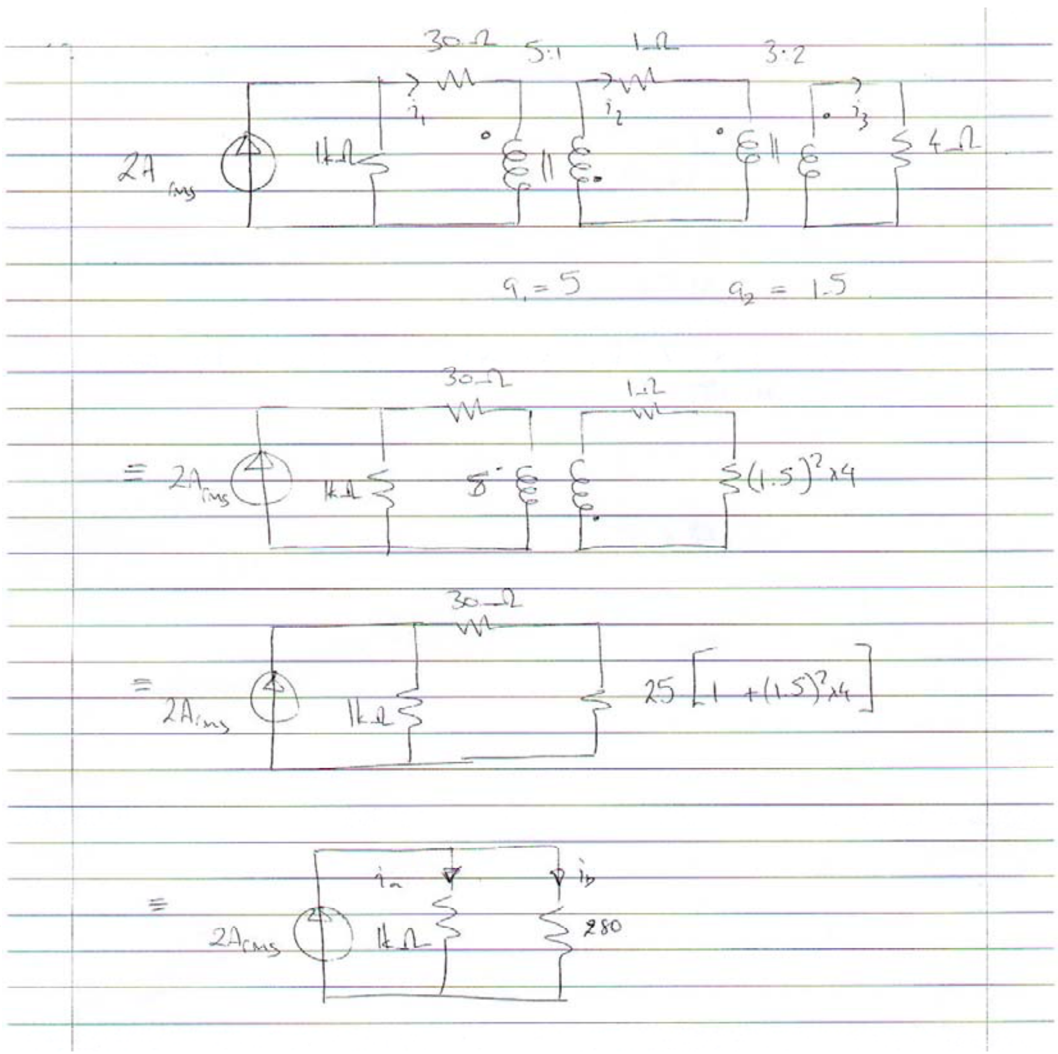
$= 20 + j7500 + \dfrac{15\times 10^6 + j50,000}{10 + j2000} = 82.499\angle 0.2170°\,\Omega$

$= 82.498 + j0.3125^{-}\ \Omega\ V_{OC} = \boxed{0}$

(b) $\bar{V}_{OC,cd} = \dfrac{100(j5000)}{20 + j12,500} = \boxed{39.99995\angle 0.09167°\text{ V rms}}$

$\bar{Z}_{incd},\ V_S = 0 = -j3000 + \dfrac{j5000(20 + j7500)}{20 + j12,500} = \boxed{3.19999 + j0.00512\,\Omega}$

45.



$\therefore \boldsymbol{I}_1 = 1.56\text{A}$

$\Rightarrow \boldsymbol{I}_2 = 5 \times 1.56 = 7.8\text{A}$

$\Rightarrow \boldsymbol{I}_3 = 1.5 \times 7.8A = 11.7\text{A}$

$$\Rightarrow P(1k) = I_a^2 R$$
$$= 0.438^2 \times 1 \times 10^3$$
$$= \boxed{192\text{W}}$$

$$\Rightarrow P(30\Omega) = I_1^2 R = (1.56)^2 \times 30$$
$$= \boxed{73\text{W}}$$

$$\Rightarrow P(1\Omega) = I_2^2 R \quad = 7.8^2 \times 1$$
$$= \boxed{60.8\text{W}}$$

$$\Rightarrow P(4\Omega) = I_3^2 R \quad = 11.7^2 \times 4$$
$$= \boxed{548\text{W}}$$

46.

(a) R_L sees $10 \times 4^2 = 160\,\Omega \quad \therefore$ use $R_L = 160\,\Omega$

$$P_{L\max} = \left(\frac{100}{20}\right)^2 \times 10 = \boxed{250\text{ W}}$$

(b) $R_L = 100\,\Omega$

$$I_2 = I_1/4,\ V_2 = 4\ V_1 \ \therefore I_X = \frac{V_2 - V_1}{40} = \frac{3V_1}{40}$$

$$\therefore 100 = 10\left(I_1 \frac{3V_1}{40}\right) + V_1,\ \frac{I_1}{4} = \frac{3V_1}{40} + \frac{4V_1}{100}$$

$$\therefore I_1 = 0.46V_1 \ \therefore 100 = 10(0.46V_1 - 0.075V_1) + V_1 = 4.85\ V_1 \ \therefore V_1 = \frac{100}{4.85}$$

$$\therefore V_2 = 4V_1 = \frac{400}{4.85} = 82.47\text{ V} \quad \therefore P_L = \frac{82.47^2}{100} = \boxed{68.02\text{ W}}$$

47.

$$\bar{I}_2 = \frac{\bar{V}_2}{8} \quad \therefore \bar{I}_1 = \frac{\bar{V}_2}{40}, \ \bar{V}_1 = 5V_2$$

$$\therefore 100 = 300(C + 0.025)\bar{V}_2 + 5\bar{V}_2$$

$$\therefore \bar{V}_2 = \frac{100}{12.5 + 300C}$$

(a) $C = 0 \ \therefore \bar{V}_2 = 8\,\text{V} \quad \therefore P_L = \frac{8^2}{8} = \boxed{8\text{ W}}$

(b) $C = 0.04 \ \therefore \bar{V}_2 = \frac{100}{24.5} \quad \therefore P_L = \left(\frac{100}{24.5}\right)^2 \frac{1}{8} = \boxed{2.082\text{ W (neg. } \mathit{fdbk}\text{)}}$

(c) $C = -0.04 \ \therefore \bar{V}_2 = \frac{100}{0.5} = 200\text{ V} \quad \therefore P_L = \frac{200^2}{8} = \boxed{5000\text{ W (pos. } \mathit{fdbk}\text{)}}$

48.

Apply $\overline{\mathrm{V}}_{ab} = 1\ \mathrm{V} \quad \therefore \overline{\mathrm{I}}_x = 0.05\ \mathrm{A},\ \overline{\mathrm{V}}_2 = 4\ \mathrm{V}$

$\therefore 4 = 60\ \overline{\mathrm{I}}_2 + 20 \times 0.05 \ \therefore \overline{\mathrm{I}}_2 = \dfrac{4-1}{60} = 0.05\ \mathrm{A}$

$\therefore \overline{\mathrm{I}}_1 = 0.2\ \mathrm{A} \quad \therefore \overline{\mathrm{I}}_{in} = 0.25\ \mathrm{A} \quad \therefore \boxed{\mathrm{R}_{th} = 4\ \Omega,\ \ \overline{\mathrm{V}}_{th} = 0}$

49.

$$P_{gen} = 1000 \text{ W}, \ P_{100} = 500 \text{ W}$$

$$\therefore I_L = \sqrt{\frac{500}{100}} = \sqrt{5} \text{ A}, \ \ V_L = 100\sqrt{5} \text{ V}$$

$$I_S = \frac{1000}{100} = 10 \text{ A} \quad \therefore V_1 = 100 - 40 = 60 \text{ V}$$

Now, $P_{25} = 1000 - 500 - 10^2 \times 4 = 100\text{W} \quad \therefore I_X = \sqrt{\frac{100}{25}} = 2 \text{ A}$; also

$$\text{I}x = b\sqrt{5} = 2, b = \frac{2}{\sqrt{5}} = \boxed{0.8944}$$

Around center mesh: $60a = 2 \times 25 + 100\sqrt{5}\,\frac{1}{0.8944} \ \therefore a = \frac{300}{60} = \boxed{5}$

50.

(a) $3\times\left(\frac{4}{3}\right)^2=\frac{16}{3}\ \Omega,\ \frac{16}{3}+2=\frac{22}{3}\ \Omega,\ \frac{22}{3}(3)^2=66\,\Omega$

$66+25=91\Omega\ \frac{100}{91}=\boxed{1.0989\angle 0°\ \text{A}=\bar{I}_1}$

(b) $\bar{I}_2=3\bar{I}_1=\boxed{3.297\angle 0°\ \text{A}}$

(c) $\bar{I}_3=-\frac{4}{3}\times 3.297=\boxed{4.396\angle 180°\ \text{A}}$

(d) $P_{25}=25\times 1.0989^2=\boxed{30.19\ \text{W}}$

(e) $P_2=3.297^2\times 2=\boxed{21.74\ \text{W}}$

(f) $P_3=4.396^2\times 3=\boxed{57.96\ \text{W}}$

51.

$$\bar{V}_1 = 2.5\ \bar{V}_2,\ \bar{I}_1 = 0.4\bar{I}_2,\ \bar{I}_{50} = \bar{I}_2 + 0.1\ \bar{V}_2$$

$$\therefore 60 = 40(0.4\ \bar{I}_2) - 2.5\bar{V}_2 \quad \therefore\ \bar{I}_2 = \frac{60 + 2.5\ \bar{V}_2}{16}$$

$$\text{Also, } 60 = 50(\bar{I}_2 + 0.1\ \bar{V}_2) + \bar{V}_2 = 50\bar{I}_2 + 6\bar{V}_2$$

$$\therefore 60 = 50\left(\frac{60 + 2.5\ V_2}{16}\right) + 6\ \bar{V}_2 = 187.5 + (7.8125 + 6)\ \bar{V}_2$$

$$\therefore \bar{V}_2 = \frac{60 - 187.5}{13.8125} = \boxed{-9.231\ \text{V}}$$

52.

$$\frac{400}{5^2} = 16\ \Omega,\ 16 \| 48 = 12\Omega,\ 12 + 4 = 16\Omega$$

$$\frac{16}{2^2} = 4\Omega \quad \therefore \mathrm{I}_s = \frac{10}{4+1} = 2\ \mathrm{A} \quad \therefore \mathrm{P}_1 = 4\ \mathrm{W}$$

$$\frac{2}{2} = 1\ \mathrm{A} \quad \therefore \mathrm{P}_4 = 4\ \mathrm{W},\ 10 - 2 \times 1 = 8\ \mathrm{V}$$

$$8 \times 2 = 16\ \mathrm{V},\ 16 - 4 \times 1 = 12\ \mathrm{V},\ 12^2 / 48 = 3\ \mathrm{W} = \mathrm{P}_{48},\ 12 \times 5 = 60\ \mathrm{V}$$

$$\mathrm{P}_{400} = \frac{60^2}{400} = \boxed{9\ \mathrm{W}}$$

53.

$$I_1 = 2I_2,\ 2I_2 = I_s + I_x \quad \therefore I_x + I_s - 2I_2 = 0$$

$$100 = 3I_s + \frac{1}{2}(4I_2 + 20I_2 - 20I_x)$$

$$\therefore 10I_x - 3I_s - 12I_2 = -100$$

$$100 = 3\ I_s - 5I_x + 20I_2 - 20I_x$$

$$\therefore 25I_x - 3I_s - 20I_2 = -100$$

$$\therefore I_X = \frac{\begin{vmatrix} 0 & 1 & -2 \\ -100 & -3 & -12 \\ -100 & -3 & -20 \end{vmatrix}}{\begin{vmatrix} 1 & 1 & -2 \\ 10 & -3 & -12 \\ 25 & -3 & -20 \end{vmatrix}} = \frac{0 + 100(-26) - 100(-18)}{1(60-36) - 10(-20-6) + 25(-12-6)} = \frac{-800}{-166} = \boxed{4.819\text{ A}}$$

54.

(a) $$50 \| 10 = \frac{25}{3}\ \Omega \quad \therefore \mathrm{V}_{AB} = 1 \times 4 \times \frac{25}{3} = \frac{100}{3}\ \mathrm{V}$$

$$\therefore \mathrm{P}_{10AB} = \left(\frac{100}{3}\right)^2 \frac{1}{10} = \frac{1000}{9} = \boxed{111.11\ \mathrm{W}}$$

$$\mathrm{V}_{CD} = 1 \times 3 \times \frac{25}{3} = 25\ \mathrm{V},\ \mathrm{P}_{10CD} = \frac{25^2}{10} = \boxed{62.5\ \mathrm{W}}$$

(b) Specify 3 A and 4 A in secondaries

$$\mathrm{I}_{AB} = \mathrm{I}_f + 4$$

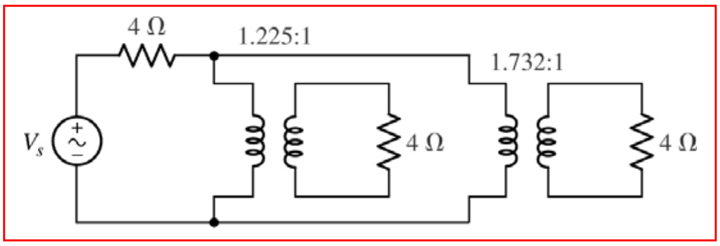
$$\mathrm{I}_{CD} = -\mathrm{I}_b - 3 \quad \therefore \frac{25}{3}(\mathrm{I}_f + 4) = \frac{25}{3}(-\mathrm{I}_f - 3)$$

$$\therefore 2\mathrm{I}_f = -7,\ \mathrm{I}_f = -3.5\ \mathrm{A}$$

$$\therefore \mathrm{V}_{AB} = \mathrm{V}_{CD} = \frac{25}{3}(-3.5 + 4) = \frac{25}{6}\mathrm{V}$$

$$\therefore \mathrm{P}_{10AB} = \mathrm{P}_{10CD} = \left(\frac{25}{6}\right)^2 \frac{1}{10} = \boxed{1.7361\ \mathrm{W}}$$

55. **Corrections required to the problem text**: both speakers that comprise the load are 4-Ω devices. We desire a circuit that will connect the signal generator (whose Thévenin resistance is 4 Ω) to the individual speakers such that one speaker receives twice the power delivered to the other. One possible solution of many:



We can see from analysing the above circuit that the voltage across the right-most speaker will be $\dfrac{1.732}{1.225}$ *or* $\sqrt{2}$ times that across the left speaker. Since power is proportional to voltage *squared*, twice as much power is delivered to the right speaker.

56. (a) We assume $\mathbf{V}_{secondary} = 230\angle 0^o$ V as a phasor reference. Then,

$$\mathbf{I}_{\text{unity PF load}} = \frac{8000}{230}\angle 0^o = 34.8\angle 0^o \text{ A} \qquad \text{and}$$

$$\mathbf{I}_{\text{0.8 PF load}} = \frac{15000}{230}\angle\left(-\cos^{-1} 0.8\right) = 65.2\angle -36.9^o \text{ A}$$

$$\text{Thus, } \mathbf{I}_{\text{primary}} = \frac{230}{2300}\left(34.8\angle 0^o + 65.2\angle -36.9^o\right)$$

$$= 0.1\,(86.9 - j39.1) = \boxed{9.5\angle -24.3^o \text{ A}}$$

(b) The magnitude of the secondary current is limited to $25\times10^3/230 = 109$ A. If we include a new load operating at 0.95 PF lagging, whose current is

$$\mathbf{I}_{\text{0.95 PF load}} = |\,\mathbf{I}_{\text{0.95 PF load}}\,| \angle\,(-\cos^{-1} 0.95) = |\,\mathbf{I}_{\text{0.95 PF load}}\,| \angle -18.2^o \text{ A},$$

then the new total secondary current is

$$86.9 - j39.1 + |\,\mathbf{I}_{\text{0.95 PF load}}\,| \cos 18.2^o - j\,|\,\mathbf{I}_{\text{0.95 PF load}}\,| \sin 18.2^o \text{ A}.$$

Thus, we may equate this to the maximum rated current of the secondary:

$$109 = \sqrt{\left(86.9 + |\,\mathbf{I}_{\text{0.95 PF load}}\,| \cos 18.2^o\right)^2 + \left(39.1 + |\,\mathbf{I}_{\text{0.95 PF load}}\,| \sin 18.2^o\right)^2}$$

Solving, we find

$$|\,\mathbf{I}_{\text{0.95 PF load}}\,|^2 = \frac{-189 \pm \sqrt{189^2 + (4)(2800)}}{2}$$

So, $|\mathbf{I}_{\text{0.95 PF load}}| =$ 13.8 A (or –203 A, which is nonsense).

This transformer, then, can deliver to the additional load a power of

$$13.8\times0.95\times230 = \boxed{3 \text{ kW.}}$$

57. After careful examination of the circuit diagram, we (fortunately or unfortunately) determine that the meter determines individual IQ based on age alone. A simplified version of the circuit then, is simply a 120 V ac source, a 28.8-kΩ resistor and a $(24^2)R_A$ resistor all connected in series. The IQ result is equal to the power (W) dissipated in resistor R_A divided by 1000.

$$P = \left(\frac{120}{28.8 \times 10^3 + 576 R_A} \right)^2 \times 576 R_A$$

Thus, $$\boxed{IQ = \frac{1}{1000} \left(\frac{120}{28.8 \times 10^3 + 576 \times \text{Age}} \right)^2 \times 576 \times \text{Age}}$$

(a) Implementation of the above equation with a given age will yield the "measured" IQ.

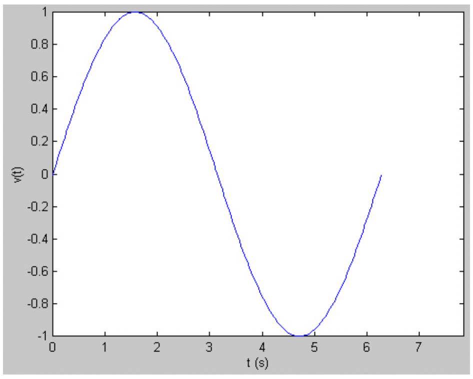
(b) The maximum IQ is achieved when maximum power is delivered to resistor R_A, which will occur when $576R_A = 28.8\times10^3$, or the person's age is $\boxed{\text{50 years.}}$

(c) Well, now, this arguably depends on your answer to part (a), and your own sense of ethics. Hopefully you'll do the right thing, and simply write to the Better Business Bureau. And watch less television.

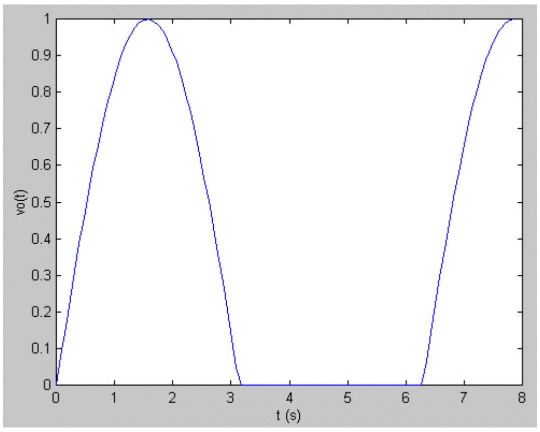
58. We require a transformer that converts 240 V ac to 120 V ac, so that a turns ratio of 2:1 is needed. We attach a male european plug to the primary coil, and a female US plug to the secondary coil. Unfortunately, we are not given the current requirements of the CD writer, so that we will have to over-rate the transformer to ensure that it doesn't overheat. Checking specifications on the web for an example CD writer, we find that the power supply provides a dual DC output: 1.2 A at 5 V, and 0.8 A at 12 V. This corresponds to a total DC power delivery of 15.6 W. Assuming a moderately efficient ac to DC converter is being used (*e.g.* 80% efficient), the unit will draw approximately 15.6/0.8 or 20 W from the wall socket. Thus, the secondary coil should be rated for at least that (let's go for 40 W, corresponding to a peak current draw of about 333 mA). Thus, we include a 300-mA fuse in series with the secondary coil and the US plug for safety.

59. $\boxed{\text{You need to purchase (and wire in) a three-phase transformer rated at } \left(\sqrt{3}\right)(208)(10) \ = \ 3.6\,\text{kVA. The turns ratio for each phase needs to be 400:208 or 1.923.}}$

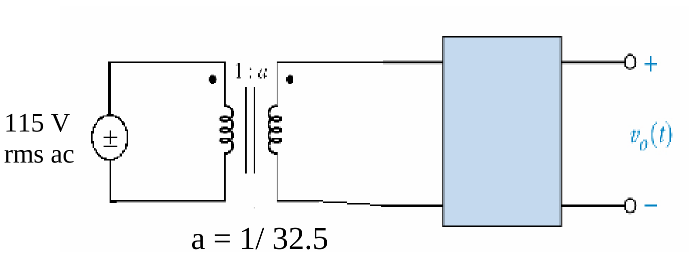
60. (a) The input to the left of the unit will have the shape:



and the output voltage will be:



We need to reduce the magnitude from 115-V (rms) to a peak voltage of 5 V. The corresponding peak voltage at the input will be $115\sqrt{2}$ = 162.6 V, so we require a transformer with a turns ratio of 162.6:5 or about 32.5:1, connected as shown:



(b) If we wish to reduce the "ripple" in the output voltage, we can connect a capacitor in parallel with the output terminals. The necessary size will depend on the maximum allowable ripple voltage and the minimum anticipated load resistance. When the input voltage swings negative and the output voltage tries to reduce to follow, current will flow out of the capacitor to reduce the amount of voltage drop that would otherwise occur.

1. (a) $\boxed{\mathbf{s} = 0;}$

(b) $\boxed{\mathbf{s} = \pm j9 \text{ s}^{-1};}$

(c) $\boxed{\mathbf{s} = -8 \text{ s}^{-1};}$

(d) $\boxed{\mathbf{s} = -1000 \pm j1000 \text{ s}^{-1};}$

(e) $v(t) = 8 + 2 \cos t$ mV cannot be attributed a single complex frequency. In a circuit analysis problem, superposition will need to be invoked, where the original function $v(t)$ is expressed as $v(t) = v_1(t) + v_2(t)$, with $v_1(t) = 8$ mV and $v_2(t) = 2 \cos t$ mV. The complex frequency of $v_1(t)$ is $\boxed{\mathbf{s} = 0,}$ and the complex frequency of $v_2(t)$ is $\boxed{\mathbf{s} = \pm 2 \text{ s}^{-1}.}$

2. (a) $\boxed{\mathbf{s} = 0}$

(b) $\boxed{\mathbf{s} = \pm j77 \text{ s}^{-1}}$

(c) $\boxed{\mathbf{s} = -5 \text{ s}^{-1}}$

(d) $\boxed{\mathbf{s} = 0.5 \text{ s}^{-1}, -5 \pm j8 \text{ s}^{-1}}$

3. (a) $\boxed{8e^{-t}}$

(b) $\boxed{19}$

(c) $9 + j7 = \boxed{11.4\angle 37.87^{\circ}}$

(d) $e^{-j\omega t} \rightarrow \boxed{1\angle 0^{\circ}}$

(e) $\cos 4t \rightarrow \boxed{1\angle 0^{\circ}}$

(f) $\sin 4t \rightarrow \boxed{1\angle 0^{\circ}}$

(g) $\boxed{88\angle 9^{\circ}}$

4. (a) $(6 - j)^* = \boxed{6 + j}$

(b) $(9)^* = \boxed{9}$

(c) $(-j30)^* = \boxed{+j30}$

(d) $(5\,e^{-j6})^* = \boxed{5\,e^{+j6}}$

(e) $(24 \angle -45^\circ)^* = \boxed{24 \angle 45^\circ}$

(f) $$\left(\frac{4 - j18}{3.33 + j}\right)^* = \boxed{\left(\frac{4 + j18}{3.33 - j}\right)} = \frac{18.44 \angle 77.47^\circ}{3.477 \angle -16.72^\circ} = 5.303 \angle 94.19^\circ$$

(g) $$\left(\frac{5 \angle 0.1^\circ}{4 - j7}\right)^* = \left(\frac{5 \angle 0.1^\circ}{8.062 \angle -60.26^\circ}\right)^* = \left(0.6202 \angle 60.36^\circ\right)^* = \boxed{0.6202 \angle -60.36^\circ}$$

(h) $(4 - 22 \angle 92.5^\circ)^* = (4 + 0.9596 - j21.98)^* = (4.9596 - j21.98)^* = \boxed{4.9596 + j21.98}$

5. $\mathbf{Q} = 9\angle 43^{\circ}\ \mu\text{C}$, $\mathbf{s} = j20\pi$. Thus, $q = 9\cos(20\pi t + 43^{\circ})\ \mu\text{C}$.

(a) At $t = 1$, $q(1) = q(1) = 9\cos(20\pi + 43^{\circ})\ \mu\text{C} = \boxed{6.582\ \mu\text{C}.}$

(b) Maximum = $\boxed{9\ \mu\text{C}}$

(c) NO. The indication would be a **negative real part** in the complex frequency.

6. (a) The missing term is $\mathbf{V}_x^* e^{(-2-j60)t} = (8 + j100)e^{(-2-j60)t}$. We can tell it is missing since $v_x(t)$ is not purely real as written; the complex conjugate term above was omitted.

(b) $\mathbf{s} = -2 \pm j60\ \text{s}^{-1}$

(c) This means simply that the sine term amplitude is larger than the cosine term amplitude.

(d) This indicates that the source is oscillating more strongly than it is decaying.

7. $\text{Re}\{\,\mathbf{i}(t)\} = i(t)$. No units provided.

(a) $\bar{i}_x(t) = (4 - j7)e^{(-3+j15)t} = (8.062\angle -60.26°)\,e^{-3t}e^{j15t} = 8.062e^{-3t}e^{j(15t-60.26°)}$

$\therefore i_x(t) = \text{Re}\,\bar{i}_x(t) = \boxed{8.062e^{-3t}\cos(15t - 60.26°)}$

(b) $\bar{i}_y(t) = (4 + j7)e^{-3t}(\cos 15t - j\sin 15t) = 8.062e^{-3t}e^{-j15t+j60.26°}$

$\therefore i_y(t) = \boxed{8.062e^{-3t}\cos(15t - 60.26°)}$

(c) $\bar{i}_A(t) = (5 - j8)e^{(-1.5t+j12)t} = 9.434e^{-j57.99°}e^{-1.5t}e^{j12t} = 9.434e^{-1.5t}e^{j(125-57.99°)}$

$\therefore \text{Re}\;\bar{i}_A(0.4) = 9.434e^{-0.6}\cos(4.8^{rad} - 57.99°) = \boxed{-4.134}$

(d) $\bar{i}_B(t) = (5 + j8)e^{(-1.5+j12)t} = 9.434e^{j57.99°}e^{-1.5t}e^{-j12t} = 9.434e^{-1.5t}e^{-j(12t-57.99°)}$

$\therefore \text{Re}\;\bar{i}_B(0.4) = \boxed{-4.134}$

8. (a) $\omega = 279$ Mrad/s, and $\omega = 2\,\pi f$. Thus, $f = \omega/2\pi = \boxed{44.4 \text{ MHz}}$

(b) If the current $i(t) = 2.33 \cos(279\times10^6\, t)$ fA flows through a precision 1-TΩ resistor, the voltage across the resistor will be $10^{12}\, i(t) = 2.33 \cos(279\times10^6\, t)$ mV. We may write this as $0.5(2.33) \cos(279\times10^6\, t) + j\,(0.5)2.33 \sin(279\times10^6\, t) + 0.5(2.33) \cos(279\times10^6\, t) - j\,(0.5)2.33 \sin(279\times10^6\, t)$ mV

$= \boxed{1.165\, e^{\,j279\times106\, t} + 1.165\, e^{\,-j279\times106\, t} \text{ mV}}$

9. (a) $\mathbf{v}_s(0.1) = (20 - j30)\, e^{(-2 + j50)(0.1)} = (36.06 \angle -56.31^\circ)\, e^{(-0.2 + j5)}$

$= 36.06e^{-0.2} \angle [-56.31^\circ + j5(180)/\pi] = \boxed{29.52 \angle 230.2^\circ \text{ V}}$ (or $29.52 \angle -129.8^\circ$ V).

(b) $\text{Re}\{\, \mathbf{v}_s \,\} = \boxed{36.06\, e^{-2t} \cos(50t - 56.31^\circ) \text{ V.}}$

(c) $\text{Re}\{\, \mathbf{v}_s(0.1) \,\} = 29.52 \cos(230.2^\circ) = \boxed{-18.89 \text{ V}}.$

(d) The complex frequency of this waveform is $\boxed{\mathbf{s} = -2 + j50 \text{ s}^{-1}}$

(e) $\mathbf{s}^* = (-2 + j50)^* = \boxed{-2 - j50 \text{ s}^{-1}}$

10. *Let* $v_{S_{forced}} = \left(10\angle 3^\circ\right)e^{st}$. Let $i_{forced} = \mathbf{I}_m e^{st}$.

(a) $v_S(t) = Ri + L\dfrac{di}{dt}$, so $v_{S_{forced}}(t) = Ri_{forced} + L\dfrac{di_{forced}}{dt}$, a superposition of our actual voltages and currents with corresponding imaginary components.

Substituting, $10\angle 3^\circ e^{st} = R\mathbf{I}e^{st} + L\mathbf{s}e^{st}\mathbf{I}$ [1]

$$\text{or } \mathbf{I} = \frac{10\angle 3^o}{R + \mathbf{s}L} = \frac{10\angle 3^o}{100 + \left(-2 + j10\right)2\times 10^{-3}} = 0.1\angle 2.99^o$$

Thus, $i(t) = \text{Re}\{\mathbf{I}e^{st}\} = \boxed{0.1e^{-2t}\cos\,(10t + 2.99^o)\text{ A.}}$

(b) By Ohm's law, $\boxed{v_1(t) = 100i(t) = 10\,e^{-2t}\cos\,(10t + 2.99^o)\text{ V.}}$

We obtain $v_2(t)$ by recognising from Eq. [1] that $\mathbf{V}_2 e^{st} = L\mathbf{s}e^{st}\mathbf{I}$,
or

$$\mathbf{V}_2 = (2\times 10^{-3})(-2 + j10)\left(0.1\angle 2.99^o\right) = 2.04\angle 104.3^o \text{ mV}$$

Thus, $\boxed{v_2(t) = 2.04e^{-2t}\cos\,(10t + 104.3^o)\text{ mV}}$

11. (a) Let the complex frequency be $\sigma + j\omega$. $\mathbf{V} = V_m \angle \theta$. $\mathbf{I} = I_m \angle \theta$

RESISTOR $v = Ri$

$V_m e^{\sigma t} e^{j(\omega t + \theta)} = RI_m e^{\sigma t} e^{j(\omega t + \theta)}$

Thus, $V_m \angle \theta = RI_m \angle \theta$ or $\mathbf{V} = R\mathbf{I}$

which defines an $\boxed{\text{impedance R.}}$

INDUCTOR $v(t) = L\dfrac{di}{dt}$. Let $i = I_m e^{st} = I_m e^{\sigma t} e^{j(\omega t + \theta)}$.

$v(t) = (\sigma + j\omega) L I_m e^{\sigma t} e^{j(\omega t + \theta)} = V_m e^{\sigma t} e^{j(\omega t + \theta)}$

Thus, $V_m \angle \theta = (\sigma + j\omega) L I_m \angle \theta$ or $\mathbf{V} = \mathbf{Z}_L \mathbf{I}$

which defines $\boxed{\text{an impedance } \mathbf{Z}_\text{L} = \mathbf{s}L = (\sigma + j\omega)L.}$

CAPACITOR $i(t) = C\dfrac{dv}{dt}$. Let $v = V_m e^{st} = V_m e^{\sigma t} e^{j(\omega t + \phi)}$.

$i(t) = (\sigma + j\omega) C V_m e^{\sigma t} e^{j(\omega t + \theta)} = I_m e^{\sigma t} e^{j(\omega t + \theta)}$

Thus, $I_m \angle \theta = \left[(\sigma + j\omega)C\right](V_m \angle \theta)$ or $\mathbf{V} = \mathbf{Z}_C \mathbf{I}$

which defines $\boxed{\text{an impedance } \mathbf{Z}_\text{C} = \dfrac{1}{(\sigma + j\omega)C} = \dfrac{1}{\mathbf{s}C}}$

(b) $\boxed{\mathbf{Z}_\text{R} = 100\ \Omega. \quad \mathbf{Z}_\text{L} = (-2 + j10)(0.002) = 20.4\angle 101.3^\circ\ \Omega}$.

(c) $\boxed{\text{Yes.}}$ $\mathbf{Z}_R \to R;\ \mathbf{Z}_L \to j\omega L;\ \mathbf{Z}_C \to \dfrac{1}{j\omega C}$

12. (a) $\mathbf{s} = 0 + j120\pi = \boxed{+j120\pi}$

(b) We first construct an s-domain voltage $\mathbf{V}(\mathbf{s}) = 179 \angle 0^\circ$ with $\mathbf{s}$ given above. The equation for the circuit is

$$v(t) \;=\; 100\, i(t) + \mathrm{L}\frac{di}{dt} = \;100\, i(t) + 500\times 10^{-6}\frac{di}{dt}$$

and we assume a response of the form $\mathbf{I}e^{\mathbf{s}t}$.

Substituting, we write $(179 \angle 0^\circ)\, e^{\mathbf{s}t} = \; 100\, \mathbf{I}e^{\mathbf{s}t} + \mathbf{s}\mathrm{L}\, \mathbf{I}e^{\mathbf{s}t}$

Supressing the exponential factor, we may write

$$\mathbf{I} \;=\; \frac{179\angle 0^\circ}{100 + \mathbf{s}500\times 10^{-6}} \;=\; \frac{179\angle 0^\circ}{100 + j120\pi\left(500\times 10^{-6}\right)} \;=\; \frac{179\angle 0^\circ}{100\angle 0.108^\circ} = 1.79 \angle -0.108^\circ \text{ A}$$

Converting back to the time domain, we find that

$$\boxed{i(t) \;=\; 1.79 \cos\,(120\pi t - 0.108^\circ) \text{ A.}}$$

13.

(a) $v_s = 10e^{-2t}\cos(10t + 30°)\text{ V} \;\therefore s = -2 + j10,\; \overline{\text{V}}_s = 10\angle 30°\text{ V}$

$$\overline{\text{Z}}_c = \frac{10}{-2+j10} = \frac{5}{-1+j5}\,\frac{-1-j5}{26} = \frac{-5-j25}{26},\; \overline{\text{Z}}_c \parallel 5 = \frac{(-25-j125)/26}{(-5-j25+130)/26}$$

$$\therefore \overline{\text{Z}}_c \parallel 5 = \frac{-25-j125}{125-j25} = \frac{-1-j5}{5-j1} = -j1 \;\therefore \overline{\text{Z}}_{in} = 5 + 0.5(-2+j10) - j1 = 4 + j4\,\Omega$$

$$\therefore \overline{\text{I}}_x = \frac{10\angle 30°}{4+j4} \times \frac{(-5-j25)/26}{5+(-5-j25)/26} = \frac{10\angle 30°}{4+j4}\,\frac{-5-j25}{130-5-j25} = \frac{5\angle 30°}{2+j2}\,\frac{-5-j25}{125-j25} = \frac{1\angle 30°}{2+j2}\,\frac{-1-j5}{5-j1}$$

$$\therefore \overline{\text{I}}_x = \frac{1\angle 30°}{2\sqrt{2}\angle 45°}(-j1) = \boxed{0.3536\angle -105°\text{ A}}$$

(b) $i_x(t) = \boxed{0.3536e^{-2t}\cos(10t - 105°)\text{ A}}$

14. (a) $\mathbf{s} = 0 + j100\pi = \boxed{+j100\pi}$

(b) We first construct an s-domain voltage $\mathbf{V}(\mathbf{s}) = 339 \angle 0^\circ$ with $\mathbf{s}$ given above. The equation for the circuit is

$$v(t) = 2000\, i(t) + v_C(t) = 2000\,C\frac{dv_C}{dt} + v_C(t) = 0.2\frac{dv_C}{dt} + v_C(t)$$

and we assume a response of the form $\mathbf{V}_C e^{st}$.

Substituting, we write $(339 \angle 0^\circ)\, e^{st} = 0.2\mathbf{s}\, \mathbf{V}_C e^{st} + \mathbf{V}_C e^{st}$

Supressing the exponential factor, we may write

$$\mathbf{V}_C = \frac{339\angle 0^\circ}{1+0.2\mathbf{s}} = \frac{339\angle 0^\circ}{1+j100\pi(0.2)} = \frac{339\angle 0^\circ}{62.84\angle 89.09^\circ} = 5.395 \angle -89.09^\circ \text{ A}$$

Converting back to the time domain, we find that

$$v_C(t) = 5.395 \cos(100\pi t - 89.09^\circ) \text{ V.}$$

and so the current is $i(t) = C\frac{dv_C}{dt} = -0.1695 \sin(100\pi t - 89.09^\circ)$ A

$$= \boxed{169.5 \cos(100\pi t + 0.91^\circ) \text{ mA.}}$$

15. $i_{S1} = 20e^{-3t} \cos 4t \text{ A}, \; i_{S2} = 30e^{-3t} \sin 4t \text{ A}$

(a) $\bar{\text{I}}_{S1} = 20\angle 0°, \; \bar{\text{I}}_{S2} = -j30, \bar{s} = -3 + j4$

$$\therefore \bar{\text{Z}}_c = \frac{10}{-3+j4} \, \frac{-3-j4}{-3-j4} = 0.4(-3-j4) = -1.2 - j1.6, \; \bar{\text{Z}}_L = -6 + j8$$

$$\therefore \bar{\text{V}}_x = 20\frac{5(7.2+j6.4)}{-2.2+j6.4} \times \frac{-6+j8}{-7.2+j6.4} - j30\frac{(-6+j8)(3.8-j1.6)}{-2.2+j6.4}$$

$$= \frac{-600+j800-j30(-22.8+12.8+j30.4+j9.6)}{-2.2+j6.4} = \frac{-600+j800-j30(-10+j40)}{-2.2+j6.4}$$

$$= \frac{-600+1200+j1000}{-2.2+j6.4} = \frac{600+j1000}{-2.2+j6.4} = \boxed{185.15\angle -47.58° \text{ V}}$$

(b) $v_x(t) = \boxed{185.15 e^{-3t} \cos(4t - 47.58°) \text{ V}}$

16. (a) If $v(t) = 240\sqrt{2}\,e^{-2t}\cos 120\pi t$ V, then $\mathbf{V} = 240\sqrt{2}\,\angle 0^\circ$ V where $\mathbf{s} = -2 + j120\pi$.

Since R = 3 mΩ, the current is simply $\mathbf{I} = \dfrac{240\sqrt{2}\,\angle 0^\circ}{3\times 10^{-3}} = 113.1\,\angle 0^\circ$ kA. Thus,

$i(t) = \boxed{113.1e^{-2t}\cos 120\pi t \text{ kA}}$

(b) Working in the time domain, we may directly compute

$i(t) = v(t)\,/\,3\times 10^{-3} = (240\sqrt{2}\,e^{-2t}\cos 120\pi t\,)\,/\,3\times 10^{-3} = \boxed{113.1e^{-2t}\cos 120\pi t \text{ kA}}$

(c) A 1000-mF capacitor added to this circuit corresponds to an impedance

$$\frac{1}{\mathbf{s}C} = \frac{1}{(-2 + j120\pi)(1000\times 10^{-3})} = \frac{1}{-2 + j120\pi}\ \Omega$$ in parallel with the 3-mΩ resistor. However, since the capacitor has been added in parallel (it would have been more interesting if the connection were in series), the same voltage still appears across its terminals, and so

$\boxed{i(t) = 113.1e^{-2t}\cos 120\pi t \text{ kA}}$ as before.

17. $\mathsf{L}\{K\,u(t)\} = \int_{0^-}^{\infty} Ke^{-\mathbf{s}t}u(t)dt = K\int_{0^-}^{\infty} e^{-\mathbf{s}t}u(t)dt = K\int_{0}^{\infty} e^{-\mathbf{s}t}dt = \left.\frac{-K}{\mathbf{s}}e^{-\mathbf{s}t}\right|_0^{\infty}$

$$= \lim_{t\to\infty}\left(\frac{-K}{\mathbf{s}}e^{-\mathbf{s}t}\right) + \lim_{t\to 0}\left(\frac{K}{\mathbf{s}}e^{-\mathbf{s}t}\right)$$

If the integral is going to converge, then $\lim_{t\to\infty}\left(e^{-\mathbf{s}t}\right) = 0$ (i.e. $\mathbf{s}$ must be finite). This leads to the first term dropping out (l'Hospital's rule assures us of this), and so

$$\mathsf{L}\{K\,u(t)\} = \boxed{\frac{K}{\mathbf{s}}}$$

18. (a) $\mathcal{L}\{3\,u(t)\} = \int_{0^-}^{\infty} 3e^{-st}u(t)dt = 3\int_{0^-}^{\infty} e^{-st}u(t)dt = 3\int_{0}^{\infty} e^{-st}dt = \left.\frac{-3}{\mathbf{s}}e^{-st}\right|_{0}^{\infty}$

$$= \lim_{t\to\infty}\left(\frac{-3}{\mathbf{s}}e^{-st}\right) + \lim_{t\to 0}\left(\frac{3}{\mathbf{s}}e^{-st}\right)$$

If the integral is going to converge, then $\lim_{t\to\infty}\left(e^{-st}\right) = 0$ (i.e. $\mathbf{s}$ must be finite). This leads to the first term dropping out (l'Hospital's rule assures us of this), and so

$$\mathcal{L}\{3\,u(t)\} = \boxed{\frac{3}{\mathbf{s}}}$$

(b) $\mathcal{L}\{3\,u(t-3)\} = \int_{0^-}^{\infty} 3e^{-st}u(t-3)dt = 3\int_{3}^{\infty} e^{-st}dt = \left.\frac{-3}{\mathbf{s}}e^{-st}\right|_{3}^{\infty}$

$$= \lim_{t\to\infty}\left(\frac{-3}{\mathbf{s}}e^{-st}\right) + \left(\frac{3}{\mathbf{s}}e^{-3\mathbf{s}}\right)$$

If the integral is going to converge, then $\lim_{t\to\infty}\left(e^{-st}\right) = 0$ (i.e. $\mathbf{s}$ must be finite). This leads to the first term dropping out (l'Hospital's rule assures us of this), and so

$$\mathcal{L}\{3\,u(t-3)\} = \boxed{\frac{3}{\mathbf{s}}e^{-3\mathbf{s}}}$$

(c)

$$\mathcal{L}\{3\,u(t-3)-3\} = \int_{0^-}^{\infty}\left[3u(t-3)-3\right]e^{-st}dt = 3\int_{3}^{\infty} e^{-st}dt - 3\int_{0^-}^{\infty} e^{-st}dt$$

$$= \left.\frac{-3}{\mathbf{s}}e^{-st}\right|_{3}^{\infty} - \left.\frac{-3}{\mathbf{s}}e^{-st}\right|_{0^-}^{\infty}$$

Based on our answers to parts (a) and (b), we may write

$$\mathcal{L}\{3\,u(t-3)-3\} = \frac{3}{\mathbf{s}}e^{-3\mathbf{s}} - \frac{3}{\mathbf{s}} = \boxed{\frac{3}{\mathbf{s}}\left(e^{-3\mathbf{s}} - 1\right)}$$

(d)

$$\mathcal{L}\{3\,u(3-t)\} = 3\int_{0^-}^{\infty} e^{-st}u(3-t)dt = 3\int_{0^-}^{3} e^{-st}dt = \left.\frac{-3}{\mathbf{s}}e^{-st}\right|_{0^-}^{3}$$

$$= \frac{-3}{\mathbf{s}}\left(e^{-3\mathbf{s}} - 1\right) = \boxed{\frac{3}{\mathbf{s}}\left(1 - e^{-3\mathbf{s}}\right)}$$

19. (a) $\mathsf{L}\{2+3\,u(t)\} \;=\; \int_{0^-}^{\infty} e^{-\mathbf{s}t}\left[2+3u(t)\right]dt \;=\; \int_{0}^{\infty} 5e^{-\mathbf{s}t}dt \;=\; \left.\frac{-5}{\mathbf{s}}e^{-\mathbf{s}t}\right|_{0}^{\infty}$

$$= \lim_{t\to\infty}\left(\frac{-5}{\mathbf{s}}e^{-\mathbf{s}t}\right) + \lim_{t\to 0}\left(\frac{5}{\mathbf{s}}e^{-\mathbf{s}t}\right)$$

If the integral is going to converge, then $\lim_{t\to\infty}\left(e^{-\mathbf{s}t}\right) = 0$ (i.e. $\mathbf{s}$ must be finite). This leads to the first term dropping out (l'Hospital's rule assures us of this), and so

$$\mathsf{L}\{2+3\,u(t)\} = \boxed{\frac{5}{\mathbf{s}}}$$

(b) $\mathsf{L}\{3\,e^{-8t}\} \;=\; \int_{0^-}^{\infty} 3\,e^{-8t}e^{-\mathbf{s}t}dt \;=\; \int_{0^-}^{\infty} 3e^{-(8+\mathbf{s})t}dt \;=\; \left.\frac{-3}{\mathbf{s}+8}e^{-(8+\mathbf{s})t}\right|_{0^-}^{\infty}$

$$= \lim_{t\to\infty}\left(\frac{-3}{\mathbf{s}+8}e^{-(\mathbf{s}+8)t}\right) + \lim_{t\to 0}\left(\frac{3}{\mathbf{s}+8}e^{-(\mathbf{s}+8)t}\right) = 0 + \frac{3}{\mathbf{s}+8} = \boxed{\frac{3}{\mathbf{s}+8}}$$

(c) $\mathsf{L}\{u(-t)\} \;=\; \int_{0^-}^{\infty} e^{-\mathbf{s}t}u(-t)dt \;=\; \int_{0^-}^{0} e^{-\mathbf{s}t}u(-t)dt \;=\; \int_{0^-}^{0^-}(0)\,e^{-\mathbf{s}t}dt \;=\; \boxed{0}$

(d) $\mathsf{L}\{K\} \;=\; \int_{0^-}^{\infty} Ke^{-\mathbf{s}t}dt \;=\; K\int_{0^-}^{\infty} e^{-\mathbf{s}t}dt \;=\; K\int_{0}^{\infty} e^{-\mathbf{s}t}dt \;=\; \left.\frac{-K}{\mathbf{s}}e^{-\mathbf{s}t}\right|_{0}^{\infty}$

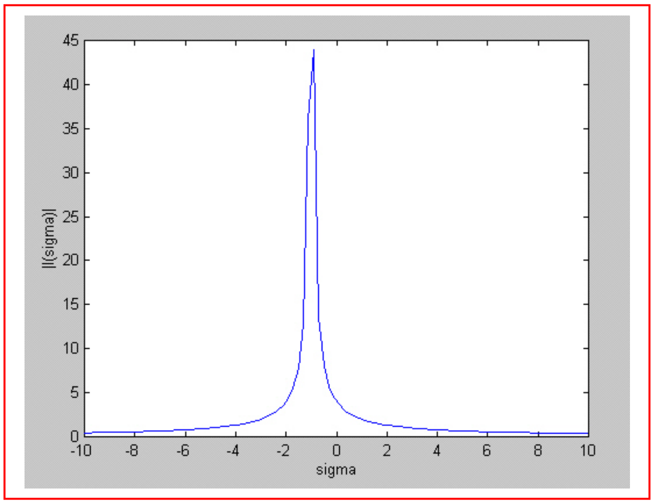
$$= \lim_{t\to\infty}\left(\frac{-K}{\mathbf{s}}e^{-\mathbf{s}t}\right) + \lim_{t\to 0}\left(\frac{K}{\mathbf{s}}e^{-\mathbf{s}t}\right)$$

If the integral is going to converge, then $\lim_{t\to\infty}\left(e^{-\mathbf{s}t}\right) = 0$ (i.e. $\mathbf{s}$ must be finite). This leads to the first term dropping out (l'Hospital's rule assures us of this), and so

$$\mathsf{L}\{K\} = \boxed{\frac{K}{\mathbf{s}}}$$

20. (a) The frequency-domain representation of the voltage across the resistor is (1)**I**(**s**) where $\mathbf{I}(\mathbf{s}) = \mathcal{L}\{4e^{-t}\,u(t)\} = \dfrac{4}{\mathbf{s}+1}$ A. Thus, the voltage is $\boxed{\dfrac{4}{\mathbf{s}+1}\ \text{V}}$.

(b)



21. (a)

$$\mathcal{L}\{5\,u(t)-5\,u(t-2)\} \;=\; \int_{0^-}^{\infty}[5\,u(t)-5\,u(t-2)]e^{-\mathbf{s}t}dt$$

$$= \;\; 5\int_0^{\infty} e^{-\mathbf{s}t}dt \;-\; 5\int_2^{\infty} e^{-\mathbf{s}t}dt \;=\; \left.\frac{-5}{\mathbf{s}}e^{-\mathbf{s}t}\right|_0^{\infty} \;+\; \left.\frac{5}{\mathbf{s}}e^{-\mathbf{s}t}\right|_2^{\infty}$$

$$= \lim_{t\to\infty}\left(\frac{-5}{\mathbf{s}}e^{-\mathbf{s}t}\right) + \lim_{t\to 0}\left(\frac{5}{\mathbf{s}}e^{-\mathbf{s}t}\right) + \lim_{t\to\infty}\left(\frac{-5}{\mathbf{s}}e^{-\mathbf{s}t}\right) - \left(\frac{5}{\mathbf{s}}e^{-2\mathbf{s}}\right)$$

If the integral is going to converge, then $\lim_{t\to\infty}\left(e^{-\mathbf{s}t}\right) = 0$ (i.e. $\mathbf{s}$ must be finite). This leads to the first and third terms dropping out (l'Hospital's rule assures us of this), and so

$$\mathcal{L}\{5\,u(t)-5\,u(t-2)\} \;=\; \boxed{\frac{5}{\mathbf{s}}\left(1-e^{-2\mathbf{s}}\right)}$$

(b) The frequency domain current is simply one ohm times the frequency domain voltage, or

$$\boxed{\frac{5}{\mathbf{s}}\left(1-e^{-2\mathbf{s}}\right)}$$

22.

(a) $f(t) = t+1 \therefore \mathrm{F}(s) = \int_{0^-}^{\infty} (t+1)\, e^{-(\sigma+j\omega)t} dt \therefore \boxed{\sigma > 0}$

(b) $f(t) = (t+1)\ u(t) \therefore \mathrm{F}(s) = \int_{0^-}^{\infty} (t+1)\, e^{-(\sigma+j\omega)t} dt \therefore \boxed{\sigma > 0}$

(c) $f(t) = e^{50t} u(t) \therefore \mathrm{F}(s) = \int_{0^-}^{\infty} e^{50t}\, e^{-(\sigma+j\omega)t} dt \therefore \boxed{\sigma > 50}$

(d) $f(t) = e^{50t} u(t-5) \therefore \mathrm{F}(s) = \int_{0^-}^{\infty} e^{50t} u(t-5)\, e^{-(\sigma+j\omega)t} dt \therefore \boxed{\sigma > 50}$

(e) $f(t) = e^{-50t} u(t-5) \therefore \mathrm{F}(s) = \int_{0^-}^{\infty} e^{-50t} u(t-5)\, e^{-(\sigma+j\omega)t} dt \therefore \boxed{\sigma < 50}$

23.

(a)

$$f(t) = 8e^{-2t}[u(t+3) - u(t-3)]$$

$$\mathrm{F}(s) = \int_0^{\infty} f(t)e^{-\mathbf{s}t}dt = \int_0^3 8e^{(-2+\mathbf{s})t}dt = \boxed{\frac{8}{2+\mathbf{s}}[1 - e^{-6-3\mathbf{s}}]}$$

(b)

$$f(t) = 8e^{2t}[u(t+3) - u(t-3)]$$

$$\mathrm{F}(\mathbf{s}) = \int_{0^-}^{\infty} f(t)e^{-\mathbf{s}t}dt = \int_0^3 8e^{(2-\mathbf{s})t}dt$$

$$= \frac{8}{2-\mathbf{s}}[e^{6-3\mathbf{s}} - 1] = \boxed{\frac{8}{\mathbf{s}-2}\left[1 - e^6 e^{-3\mathbf{s}}\right]}$$

(c)

$$f(t) = 8e^{-2|t|}[u(t+3) - u(t-3)]$$

$$\mathrm{F}(\mathbf{s}) = \int_{0^-}^{\infty} f(t)e^{-\mathbf{s}t}dt = \int_{0^-}^3 8e^{(-2-\mathbf{s})t}dt = \boxed{\frac{8}{\mathbf{s}+2}\left[1 - e^{-6-3\mathbf{s}}\right]}$$

24. (a) $\mathsf{L}\left\{\mathsf{L}^{-1}\left(\frac{1}{\mathbf{s}}\right)\right\} = \boxed{\frac{1}{\mathbf{s}}}$

(b) $\mathsf{L}\left\{1+u(t)+[u(t)]^2\right\} = \frac{1}{\mathbf{s}}+\frac{1}{\mathbf{s}}+\frac{1}{\mathbf{s}} = \boxed{\frac{3}{\mathbf{s}}}$

(c) $\mathsf{L}\left\{t\,u(t)-3\right\} = \boxed{\frac{1}{\mathbf{s}^2}-\frac{3}{\mathbf{s}}}$

(d) $\mathsf{L}\left\{1 - \delta(t)+\delta(t-1)-\delta(t-2)\right\} = \boxed{\frac{1}{\mathbf{s}}-1+e^{-\mathbf{s}}-e^{-2\mathbf{s}}}$

25. (a) $f(t) = \boxed{e^{-3t}\, u(t)}$

(b) $f(t) = \boxed{\delta(t)}$

(c) $f(t) = \boxed{t\, u(t)}$

(d) $f(t) = \boxed{275\ \delta(t)}$

(e) $f(t) = \boxed{u(t)}$

26.

$$\boxed{\begin{aligned} \mathcal{L}\{f_1(t) + f_2(t)\} &= \int_{0^-}^{\infty}[f_1(t) + f_2(t)]e^{-st}dt = \int_{0^-}^{\infty} f_1(t)e^{-st}dt + \int_{0^-}^{\infty} f_2(t)e^{-st}dt \\ &= \mathcal{L}\{f_1(t)\} + \mathcal{L}\{f_2(t)\} \end{aligned}}$$

27.

(a) $$f(t) = 2u(t-2) \;\therefore\; \mathrm{F}(s) = 2\int_2^{\infty} e^{-st}dt + \frac{-2}{s}e^{st}\Big|_2^{\infty} = \frac{2}{s}e^{-2s};\; s = 1+j2$$

$$\therefore \mathrm{F}(1+j2) = \frac{2}{1+j2}e^{-2}\; e^{-j4} = \boxed{0.04655^{+} + j0.11174}$$

(b) $$f(t) = 2\delta(t-2) \;\therefore\; \mathrm{F}(s) = 2e^{-2s},\; \mathrm{F}(1+j2) = 2e^{-2}\,e^{-j4} = \boxed{-0.17692 + j0.2048}$$

(c) $$f(t) = e^{-t}u(t-2) \;\therefore\; \mathrm{F}(s) = \int_2^{-\infty} e^{-(s+1)t}\,dt = \frac{1}{-s+1}e^{-(s+1)t}\Big|_2^{\infty} = \frac{1}{s+1}e^{-2s-2}$$

$$\therefore \mathrm{F}(1+j2) = \frac{1}{2+j2}e^{-2}\,e^{-2}\,e^{-j^4} = \boxed{(0.4724 + j6.458)10^{-3}}$$

28. (a) $\int_{-\infty}^{\infty} 8\sin 5t\,\delta(t-1)\,dt \quad = \quad 8\sin 5\times 1 = \boxed{-7.671}$

(b) $\int_{-\infty}^{\infty} (t-5)^2\,\delta(t-2)dt \quad = \quad (2-5)^2 = \boxed{9}$

(c) $\int_{-\infty}^{\infty} 5e^{-3000t}\,\delta(t-3.333\times 10^{-4})dt \quad = \quad 5e^{-3000(3.333\times 10^{-4})} = \boxed{1.840}$

(d) $\int_{-\infty}^{\infty} K\delta(t-2)dt \quad = \boxed{K}$

29.

(a) $f(t) = [u(5-t)]\,[u(t-2)]\,u(t), \;\therefore\; \mathrm{F}(s)\int_{0^-}^{\infty}[u(5-t)]\,[u(t-2)]\,u(t)e^{-st}dt$

$$\therefore \mathrm{F}(s) = \int_2^5 e^{-st}dt = -\frac{1}{s}e^{-st}\Big|_2^5 = \boxed{\frac{1}{s}(e^{-2s}-e^{-5s})}$$

(b) $f(t) = 4u(t-2) \;\therefore\; \mathrm{F}(s) = 4\int_2^{\infty}e^{-st}dt = \boxed{\frac{4}{s}e^{-2s}}$

(c) $f(t) = 4e^{-3t}u(t-2) \;\therefore\; \mathrm{F}(s) = 4\int_2^{\infty}e^{-(s+3)t}dt = \frac{-4}{s+3}e^{-(s+3)t}\Big|_2^{\infty}$

$$\therefore \boxed{\mathrm{F}(s) = \frac{4}{s+3}e^{-2s-6}}$$

(d) $f(t) = 4\delta(t-2) \;\therefore\; \mathrm{F}(s) = 4\int_{0^-}^{\infty}\delta(t-2)\,e^{-st}dt = 4\int_2^{2^+}e^{-2s}\,\delta(t-2)\,dt = \boxed{4e^{-2s}}$

(e) $f(t) = 5\delta(t)\sin(10t+0.2\pi) \;\therefore\; \mathrm{F}(s) = 5\int_{0^-}^{0^+}\delta(t)\;[\sin 0.2\pi]\text{ X }1dt = 5\sin 36°$

$$\therefore \mathrm{F}(s) = \boxed{2.939}$$

30. (a) $\int_{-\infty}^{\infty} \cos 500t\, \delta(t)\, dt \quad = \quad \cos 500 \times 0 = \boxed{1}$

(b) $\int_{-\infty}^{\infty} (t)^5 \delta(t-2) dt \quad = \quad (2)^5 = \boxed{32}$

(c) $\int_{-\infty}^{\infty} 2.5e^{-0.001t} \delta(t-1000) dt \quad = \quad 2.5e^{-0.001(1000)} = \boxed{0.9197}$

(d) $\int_{-\infty}^{\infty} -K^2 \delta(t-c) dt \quad = \quad \boxed{-K^2}$

31.

(a) $f(t) = 2\, u(t-1)\, u(3-t)\, u(t^3)$

$$\mathbf{F(s)} = \int_1^3 e^{-st}\, dt = -\frac{2}{\mathbf{s}} e^{-st}\Big|_1^3 = \boxed{\frac{2}{\mathbf{s}}\left(e^{-\mathbf{s}} - e^{-3\mathbf{s}}\right)}$$

(b) $$f(t) = 2u(t-4) \therefore \mathrm{F}(s) = 2\int_4^\infty e^{-st}\, dt = \frac{-2}{s}(0 - e^{-4s}) = \boxed{\frac{2}{s} e^{-4s}}$$

(c) $$f(t) = 3e^{-2t}\, u(t-4) \therefore \mathrm{F}(s) = 3\int_4^\infty e^{-(s+2)t} dt = \boxed{\frac{3}{s+2} e^{-4s-8}}$$

(d) $$f(t) = 3\delta(t-5) \therefore \mathrm{F}(s) = 3\int_{0^-}^\infty \delta(t-5)\, e^{-st} dt = \boxed{3e^{-5s}}$$

(e) $$f(t) = 4\delta(t-1)\, [\cos \pi t - \sin \pi t]$$

$$\therefore \mathrm{F}(s) = 4\int_{0^-}^\infty \delta(t-1)\, [\cos \pi t - \sin \pi t]\, e^{-st} dt \therefore \mathrm{F}(s) = \boxed{-4e^{-s}}$$

32. (a) $\mathbf{F}(\mathbf{s}) = 3 + 1/\mathbf{s}$; $\boxed{f(t) = 3\delta(t) + u(t)}$

(b) $\mathbf{F}(\mathbf{s}) = 3 + 1/\mathbf{s}^2$; $\boxed{f(t) = 3\delta(t) + tu(t)}$

(c) $\mathbf{F}(\mathbf{s}) = \dfrac{1}{(\mathbf{s}+3)(\mathbf{s}+4)} = \dfrac{1}{(\mathbf{s}+3)} - \dfrac{1}{(\mathbf{s}+4)}$; $\boxed{f(t) = \left[e^{-3t} - e^{-4t}\right]u(t)}$

(d) $\mathbf{F}(\mathbf{s}) = \dfrac{1}{(\mathbf{s}+3)(\mathbf{s}+4)(\mathbf{s}+5)} = \dfrac{1/2}{(\mathbf{s}+3)} - \dfrac{1}{(\mathbf{s}+4)} + \dfrac{1/2}{(\mathbf{s}+5)}$;

$$\boxed{f(t) = \left[\frac{1}{2}e^{-3t} - e^{-4t} + \frac{1}{2}e^{-5t}\right]u(t)}$$

33. (a) $\mathbf{G}(\mathbf{s}) = 90 - 4.5/\mathbf{s}$; $\boxed{g(t) = 90\delta(t) - 4.5u(t)}$

(b) $\mathbf{G}(\mathbf{s}) = 11 + 2/\mathbf{s}$; $\boxed{g(t) = 11\delta(t) + 2u(t)}$

(c) $\mathbf{G}(\mathbf{s}) = \dfrac{1}{(\mathbf{s}+1)^2}$; $\boxed{g(t) = te^{-t}u(t)}$

(d) $\mathbf{G}(\mathbf{s}) = \dfrac{1}{(\mathbf{s}+1)(\mathbf{s}+2)(\mathbf{s}+3)} = \dfrac{1/2}{(\mathbf{s}+1)} - \dfrac{1}{(\mathbf{s}+2)} + \dfrac{1/2}{(\mathbf{s}+3)}$; $\boxed{g(t) = \left[\dfrac{1}{2}e^{-t} - e^{-2t} + \dfrac{1}{2}e^{-3t}\right]u(t)}$

34. (a) $f(t) = \boxed{5\,u(t) - 16\,\delta(t) + e^{-4.4t}\,u(t)}$

(b) $f(t) = \boxed{\delta(t) - u(t) + t\,u(t)}$

(c) $\mathbf{F(s)} = \dfrac{5}{\mathbf{s}+7} + \dfrac{88}{\mathbf{s}} + \dfrac{a}{\mathbf{s}+6} + \dfrac{b}{\mathbf{s}+1}$

where $a = \left.\dfrac{17}{\mathbf{s}+1}\right|_{\mathbf{s}=-6} = -3.4$ and $b = \left.\dfrac{17}{\mathbf{s}+6}\right|_{\mathbf{s}=-1} = 3.4$.

Thus,

$$f(t) = \boxed{5\,e^{-7t}\,u(t) + 88\,u(t) - 3.4\,e^{-6t}\,u(t) + 3.4\,e^{-t}\,u(t)}$$

Check with MATLAB:

```
EDU» T1 = '5/(s+7)';
EDU» T2 = '88/s';
EDU» T3 = '17/(s^2 + 7*s + 6)';
EDU» T = symadd(T1,T2);
EDU» P = symadd(T,T3);
EDU» p = ilaplace(P)

p =

5*exp(-7*t)+88-17/5*exp(-6*t)+17/5*exp(-t)

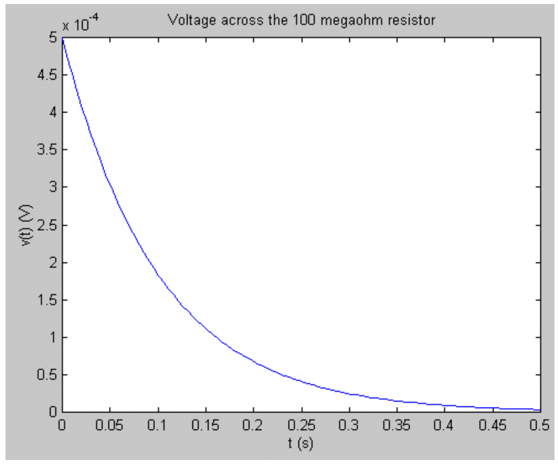
EDU» pretty(p)

        5 exp(-7 t) + 88 - 17/5 exp(-6 t) + 17/5 exp(-t)
```

35. If $\mathbf{V}(\mathbf{s}) = \dfrac{5}{\mathbf{s}}$, then $v(t) = 5\,u(t)$ V. The voltage at $t = 1$ ms is then simply 5 V, and the current through the 2-kΩ resistor at that instant in time is $\boxed{\text{2.5 mA.}}$

36. $\mathbf{I}(\mathbf{s}) = \dfrac{5}{\mathbf{s}+10}$ pA, so $i(t) = 5\,e^{-10t}\,u(t)$ pA. The voltage across the 100-MΩ resistor is therefore $500\,e^{-10t}\,u(t)$ μV.

(a) The voltage as specified has zero value for $t < 0$, and a peak value of 500 μV.



(b) $i(0.1\text{ s})$ = 1.839 pA, so the power absorbed by the resistor at that instant = i^2R = $\boxed{338.2\text{ aW.}}$ (A pretty small number).

(c) $500\,e^{-10t_{1\%}} = 5$

Taking the natural log of both sides, we find $t_{1\%} = \boxed{460.5\text{ ms}}$

37.

(a) $\mathbf{F}(\mathbf{s}) = \frac{\mathbf{s}+1}{\mathbf{s}} + \frac{2}{\mathbf{s}+1} = 1 + \frac{1}{\mathbf{s}} + \frac{2}{\mathbf{s}+1} \Leftrightarrow \boxed{\delta(t) + u(t) + 2e^{-t}u(t)}$

(b) $\mathbf{F}(\mathbf{s}) = (e^{-\mathbf{s}}+1)^2 = e^{-2\mathbf{s}} + 2e^{-\mathbf{s}} + 1 \Leftrightarrow \boxed{\delta(t-2) + 2\delta(t-1) + \delta(t)}$

(c) $\mathbf{F}(\mathbf{s}) = 2e^{-(\mathbf{s}+1)} = 2e^{-1}\, e^{-2\mathbf{s}} \Leftrightarrow \boxed{2e^{-1}\,\delta(t-1)}$

(d) $\mathbf{F}(\mathbf{s}) = 2e^{-3\mathbf{s}} \cosh 2\mathbf{s} = e^{-3\mathbf{s}}\,(e^{2\mathbf{s}} + e^{-2\mathbf{s}}) = e^{-\mathbf{s}} + e^{-5\mathbf{s}} \Leftrightarrow \boxed{\delta(t-1) + \delta(t-5)}$

38. $\mathbf{N(s)} = 5\mathbf{s}$.

(a) $\mathbf{D(s)} = \mathbf{s}^2 - 9$ so $\dfrac{\mathbf{N(s)}}{\mathbf{D(s)}} = \dfrac{5\mathbf{s}}{\mathbf{s}^2 - 9} = \dfrac{5\mathbf{s}}{(\mathbf{s}+3)(\mathbf{s}-3)} = \dfrac{a}{(\mathbf{s}+3)} + \dfrac{b}{(\mathbf{s}-3)}$

where $a = \left.\dfrac{5\mathbf{s}}{(\mathbf{s}-3)}\right|_{\mathbf{s}=-3} = \dfrac{-15}{-6} = 2.5$ and $b = \left.\dfrac{5\mathbf{s}}{(\mathbf{s}+3)}\right|_{\mathbf{s}=3} = \dfrac{15}{6} = 2.5$. Thus,

$f(t) = \boxed{[2.5\, e^{-3t} + 2.5\, e^{3t}]\, u(t)}$

(b) $\mathbf{D(s)} = (\mathbf{s}+3)(\mathbf{s}^2 + 19\mathbf{s} + 90) = (\mathbf{s}+3)(\mathbf{s}+10)(\mathbf{s}+9)$ so

$$\frac{\mathbf{N(s)}}{\mathbf{D(s)}} = \frac{5\mathbf{s}}{(\mathbf{s}+3)(\mathbf{s}+10)(\mathbf{s}+9)} = \frac{a}{(\mathbf{s}+3)} + \frac{b}{(\mathbf{s}+10)} + \frac{c}{(\mathbf{s}+9)}$$

$a = \left.\dfrac{5\mathbf{s}}{(\mathbf{s}+10)(\mathbf{s}+9)}\right|_{\mathbf{s}=-3} = \dfrac{-15}{(7)(6)} = -0.3571$, $b = \left.\dfrac{5\mathbf{s}}{(\mathbf{s}+3)(\mathbf{s}+9)}\right|_{\mathbf{s}=-10} = \dfrac{-50}{(-7)(-1)} = -7.143$

$c = \left.\dfrac{5\mathbf{s}}{(\mathbf{s}+3)(\mathbf{s}+10)}\right|_{\mathbf{s}=-9} = \dfrac{-45}{(-6)(1)} = 7.5.$ $\therefore f(t) = \boxed{[-0.3571\, e^{-3t} - 7.143\, e^{-10t} + 7.5\, e^{-9t}]\, u(t)}$

(c) $\mathbf{D(s)} = (4\mathbf{s} + 12)(8\mathbf{s}^2 + 6\mathbf{s} + 1) = 32(\mathbf{s}+3)(\mathbf{s}+0.5)(\mathbf{s}+0.25)$ so

$$\frac{\mathbf{N(s)}}{\mathbf{D(s)}} = \left(\frac{5}{32}\right)\frac{\mathbf{s}}{(\mathbf{s}+3)(\mathbf{s}+0.5)(\mathbf{s}+0.25)} = \frac{a}{(\mathbf{s}+3)} + \frac{b}{(\mathbf{s}+0.5)} + \frac{c}{(\mathbf{s}+0.25)}$$

$a = \left(\dfrac{5}{32}\right)\left.\dfrac{\mathbf{s}}{(\mathbf{s}+0.5)(\mathbf{s}+0.25)}\right|_{\mathbf{s}=-3} = -0.06818$, $b = \left(\dfrac{5}{32}\right)\left.\dfrac{\mathbf{s}}{(\mathbf{s}+3)(\mathbf{s}+0.25)}\right|_{\mathbf{s}=-0.5} = 0.125$

$c = \left(\dfrac{5}{32}\right)\left.\dfrac{\mathbf{s}}{(\mathbf{s}+3)(\mathbf{s}+0.5)}\right|_{\mathbf{s}=--0.25} = -0.05682$

$$\therefore f(t) = \boxed{[-0.06818\, e^{-3t} + 0.125\, e^{-0.5t} - 0.05682 e^{-0.25t}]\, u(t)}$$

(d) Part (a):

```
EDU» N = [5 0];
EDU» D = [1 0 -9];
EDU» [r p y] = residue(N,D)

r =
    2.5000
    2.5000

p =
     3
    -3

y =
     []
```

Part (b):

```
EDU» N = [5 0];
EDU» D = [1 22 147 270];
EDU» [r p y] = residue(N,D)
r =
  -7.1429
   7.5000
  -0.3571

p =
 -10.0000
  -9.0000
  -3.0000

y =
     []
```

Part (c):

```
EDU» N = [5 0];
EDU» D = [32 120 76 12];
EDU» [r p y] = residue(N,D)
r =
  -0.0682
   0.1250
  -0.0568

p =
  -3.0000
  -0.5000
  -0.2500

y =
     []
```

39.

(a) $\mathrm{F}(s) = \dfrac{5}{s+1} \leftrightarrow \boxed{5e^{-t}u(t)}$

(b) $\mathrm{F}(s) = \dfrac{5}{s+1} - \dfrac{2}{s+4} \leftrightarrow \boxed{(5e^{-t} - 2e^{-4t})u(t)}$

(c) $\mathrm{F}(s) = \dfrac{18}{(s+1)(s+4)} = \dfrac{6}{s+1} - \dfrac{6}{s+4} \leftrightarrow \boxed{6(e^{-t} - e^{-4t})u(t)}$

(d) $\mathrm{F}(s) = \dfrac{18s}{(s+1)(s+4)} = \dfrac{-6}{s+1} + \dfrac{24}{s+4} \leftrightarrow \boxed{6(4e^{-4t} - e^{-t})u(t)}$

(e) $\mathrm{F}(s) = \dfrac{18s^2}{(s+1)(s+4)} = 18 + \dfrac{6}{s+1} - \dfrac{96}{s+4} \leftrightarrow \boxed{18\delta(t) + 6(e^{-t} - 16e^{-4t})u(t)}$

40. $\mathbf{N(s)} = 2\mathbf{s}^2$.

(a) $\mathbf{D(s)} = \mathbf{s}^2 - 1$ so $\dfrac{\mathbf{N(s)}}{\mathbf{D(s)}} = \dfrac{2\mathbf{s}^2}{\mathbf{s}^2 - 1} = \dfrac{2\mathbf{s}^2}{(\mathbf{s}+1)(\mathbf{s}-1)} = \dfrac{a}{(\mathbf{s}+1)} + \dfrac{b}{(\mathbf{s}-1)} + 2$

where $a = \left.\dfrac{2\mathbf{s}^2}{(\mathbf{s}-1)}\right|_{\mathbf{s}\,=\,-1} = \dfrac{2}{-2} = -1$ and $b = \left.\dfrac{2\mathbf{s}^2}{(\mathbf{s}+1)}\right|_{\mathbf{s}\,=\,1} = \dfrac{2}{2} = 1$. Thus,

$$\boxed{f(t) = [2\delta(t) - e^{-t} + e^{t}]\, u(t)}$$

(b) $\mathbf{D(s)} = (\mathbf{s}+3)(\mathbf{s}^2 + 19\mathbf{s} + 90) = (\mathbf{s}+3)(\mathbf{s}+10)(\mathbf{s}+9)$ so

$$\frac{\mathbf{N(s)}}{\mathbf{D(s)}} = \frac{2\mathbf{s}^2}{(\mathbf{s}+3)(\mathbf{s}+10)(\mathbf{s}+9)} = \frac{a}{(\mathbf{s}+3)} + \frac{b}{(\mathbf{s}+10)} + \frac{c}{(\mathbf{s}+9)}$$

$$a = \left.\frac{2\mathbf{s}^2}{(\mathbf{s}+10)(\mathbf{s}+9)}\right|_{\mathbf{s}\,=\,-3} = \frac{18}{(7)(6)} = 0.4286,\quad b = \left.\frac{2\mathbf{s}^2}{(\mathbf{s}+3)(\mathbf{s}+9)}\right|_{\mathbf{s}\,=\,-10} = \frac{200}{(-7)(-1)} = 28.57$$

$$c = \left.\frac{2\mathbf{s}^2}{(\mathbf{s}+3)(\mathbf{s}+10)}\right|_{\mathbf{s}\,=\,-9} = \frac{162}{(-6)(1)} = -27. \quad \therefore \boxed{f(t) = [0.4286\, e^{-3t} + 28.57\, e^{-10t} - 27\, e^{-9t}]\, u(t)}$$

(c) $\mathbf{D(s)} = (8\mathbf{s} + 12)(16\mathbf{s}^2 + 12\mathbf{s} + 2) = 128(\mathbf{s}+1.5)(\mathbf{s}+0.5)(\mathbf{s}+0.25)$ so

$$\frac{\mathbf{N(s)}}{\mathbf{D(s)}} = \left(\frac{2}{128}\right)\frac{\mathbf{s}^2}{(\mathbf{s}+1.5)(\mathbf{s}+0.5)(\mathbf{s}+0.25)} = \frac{a}{(\mathbf{s}+1.5)} + \frac{b}{(\mathbf{s}+0.5)} + \frac{c}{(\mathbf{s}+0.25)}$$

$$a = \left(\frac{2}{128}\right)\left.\frac{\mathbf{s}^2}{(\mathbf{s}+0.5)(\mathbf{s}+0.25)}\right|_{\mathbf{s}\,=\,-1.5} = 0.02813,\quad b = \left(\frac{2}{128}\right)\left.\frac{\mathbf{s}^2}{(\mathbf{s}+1.5)(\mathbf{s}+0.25)}\right|_{\mathbf{s}\,=\,-0.5} = -0.01563$$

$$c = \left(\frac{2}{128}\right)\left.\frac{\mathbf{s}^2}{(\mathbf{s}+1.5)(\mathbf{s}+0.5)}\right|_{\mathbf{s}\,=\,-0.25} = 0.003125$$

$$\therefore \boxed{f(t) = 0.02813\, e^{-1.5t} - 0.01563\, e^{-0.5t} + 0.003125 e^{-0.25t}]\, u(t)}$$

(d) Part (a):

```
EDU» N = [2 0 0];
EDU» D = [1 0 -1];
EDU» [r p y] = residue(N,D)

r =
   -1.0000
    1.0000

p =
   -1.0000
    1.0000

y =
    2
```

Part (b):

```
EDU» N = [2 0 0];
EDU» D = [1 22 147 270];
EDU» [r p y] = residue(N,D)
r =
   28.5714
  -27.0000
    0.4286

p =
  -10.0000
   -9.0000
   -3.0000

y =
    []
```

Part (c):

```
EDU» N = [2 0 0];
EDU» D = [128 288 160 24];
EDU» [r p y] = residue(N,D)
r =
    0.0281
   -0.0156
    0.0031

p =
   -1.5000
   -0.5000
   -0.2500

y =
    []
```

41.

(a) $\mathrm{F}(s) = \frac{2}{s} - \frac{3}{s+1}$ so $f(t) = \boxed{2\,u(t) - 3\,e^{-t}\,u(t)}$

(b) $\mathrm{F}(s) = \frac{2s+10}{s+3} = 2 + \frac{4}{s+3} \leftrightarrow \boxed{2\delta(t) + 4e^{-3t}u(t)}$

(c) $\mathrm{F}(s) = 3e^{-0.8s} \leftrightarrow \boxed{3\delta(t-0.8)}$

(d) $\mathrm{F}(s) = \frac{12}{(s+2)(s+6)} = \frac{3}{s+2} - \frac{3}{s+6} \leftrightarrow \boxed{3(e^{-2t} - e^{-6t})u(t)}$

(e) $\mathrm{F}(s) = \frac{12}{(s+2)^2(s+6)} = \frac{3}{(s+2)^2} + \frac{\mathrm{A}}{s+2} + \frac{0.75}{s+6}$

Let $s = 0 \;\therefore\; \frac{12}{4\times 6} = \frac{3}{4} + \frac{\mathrm{A}}{2} + \frac{0.75}{6} \;\therefore\; \mathrm{A} = -0.75$

$\therefore \mathrm{F}(s) = \frac{3}{(s+2)^2} - \frac{0.75}{s+2} + \frac{0.75}{s+6} \leftrightarrow \boxed{(3te^{-2t} - 0.75e^{-2t} + 0.75e^{-6t})u(t)}$

42. $$\mathbf{F}(\mathbf{s}) = 2 - \frac{1}{\mathbf{s}} + \frac{\pi}{\mathbf{s}^3 + 4\mathbf{s}^2 + 5\mathbf{s} + 2}$$

$$= 2 - \frac{1}{\mathbf{s}} + \frac{\pi}{(\mathbf{s}+2)(\mathbf{s}+1)^2}$$

$$= 2 - \frac{1}{\mathbf{s}} + \frac{a}{(\mathbf{s}+2)} + \frac{b}{(\mathbf{s}+1)^2} + \frac{c}{(\mathbf{s}+1)}$$

where $a = \left.\frac{\pi}{(\mathbf{s}+1)^2}\right|_{\mathbf{s}=-2} = \pi$

$$b = \left.\frac{\pi}{(\mathbf{s}+2)}\right|_{\mathbf{s}=-1} = \pi$$

and $c = \left.\frac{d}{ds}\left[(\mathbf{s}+1)^2 \frac{\pi}{(\mathbf{s}+2)(\mathbf{s}+1)^2}\right]\right|_{\mathbf{s}=-1} = \frac{d}{ds}\left[\frac{\pi}{(\mathbf{s}+2)}\right]_{\mathbf{s}=-1} = \left[-\frac{\pi}{(\mathbf{s}+2)^2}\right]_{\mathbf{s}=-1} = -\pi$

Thus, we may write

$$\boxed{f(t) = 2\,\delta(t) - u(t) + \pi e^{-2t}\,u(t) + \pi t e^{-t}\,u(t) - \pi e^{-t}\,u(t)}$$

43. (a) $\mathbf{F(s)} = \dfrac{(\mathbf{s}+1)(\mathbf{s}+2)}{\mathbf{s}(\mathbf{s}+3)} = 1 + \dfrac{a}{\mathbf{s}} + \dfrac{b}{(\mathbf{s}+3)}$

$$a = \left.\frac{(\mathbf{s}+1)(\mathbf{s}+2)}{(\mathbf{s}+3)}\right|_{\mathbf{s}=0} = \frac{2}{3} \quad \text{and} \quad b = \left.\frac{(\mathbf{s}+1)(\mathbf{s}+2)}{\mathbf{s}}\right|_{\mathbf{s}=-3} = \frac{(-2)(-1)}{-3} = -\frac{2}{3}$$

so

$$\boxed{f(t) = \delta(t) + \frac{2}{3}u(t) - \frac{2}{3}e^{-3t}\,u(t)}$$

(b) $\mathbf{F(s)} = \dfrac{(\mathbf{s}+2)}{\mathbf{s}^2(\mathbf{s}^2+4)} = \dfrac{a}{\mathbf{s}^2} + \dfrac{b}{\mathbf{s}} + \dfrac{c}{(\mathbf{s}+j2)} + \dfrac{c^*}{(\mathbf{s}-j2)}$

$$a = \left.\frac{(\mathbf{s}+2)}{(\mathbf{s}^2+4)}\right|_{\mathbf{s}=0} = \frac{2}{4} = 0.5$$

$$b = \frac{d}{d\mathbf{s}}\left[\frac{(\mathbf{s}+2)}{(\mathbf{s}^2+4)}\right]_{\mathbf{s}=0} = \left[\frac{(\mathbf{s}^2+4) - 2\mathbf{s}(\mathbf{s}+2)}{(\mathbf{s}^2+4)^2}\right]_{\mathbf{s}=0} = \frac{4}{4^2} = 0.25$$

$$c = \left.\frac{(\mathbf{s}+2)}{\mathbf{s}^2(\mathbf{s}-j2)}\right|_{\mathbf{s}=-j2} = 0.1768\angle -135^\circ \qquad (c^* = 0.1768\angle 135^\circ)$$

so

$$f(t) = 0.5\,t\,u(t) + 0.25\,u(t) + 0.1768\,e^{-j135^\circ}\,e^{-j2t}\,u(t) + 0.1768\,e^{j135^\circ}\,e^{j2t}\,u(t)$$

The last two terms may be combined so that

$$\boxed{f(t) = 0.5\,t\,u(t) + 0.25\,u(t) + 0.3536\cos(2t + 135^\circ)}$$

44. (a) $\mathbf{G}(\mathbf{s})$ is not a rational function, so first we perform polynomial long division (some intermediate steps are not shown):

$$\begin{array}{r} 12\mathbf{s}-36 \\ \left(\mathbf{s}^2+3\mathbf{s}+2\right)\overline{\big)12\mathbf{s}^3} \\ \hline -36\mathbf{s}^2-24\mathbf{s} \end{array} , \quad \text{and} \quad \begin{array}{r} -36 \\ \left(\mathbf{s}^2+3\mathbf{s}+2\right)\overline{\big)-36\mathbf{s}^2-24\mathbf{s}} \\ \hline 84\mathbf{s}+72 \end{array}$$

$$\text{so } \mathbf{G}(\mathbf{s}) = 12\mathbf{s}-36+\frac{84\mathbf{s}+72}{(\mathbf{s}+1)(\mathbf{s}+2)} = 12\mathbf{s}-36-\frac{12}{\mathbf{s}+1}+\frac{96}{\mathbf{s}+2}$$

Hence, $\boxed{g(t) = 12\frac{d}{dt}\delta(t)-36\delta(t)-12e^{-t}u(t)+96e^{-2t}u(t)}$

(b) $\mathbf{G}(\mathbf{s})$ is not a rational function, so first we perform polynomial long division (some intermediate steps are not shown):

$$\begin{array}{r} 12 \\ \left(\mathbf{s}^3+4\mathbf{s}^2+5\mathbf{s}+2\right)\overline{\big)12\mathbf{s}^3} \\ \hline \underline{12\mathbf{s}^3+48\mathbf{s}^2+60\mathbf{s}+24} \\ -48\mathbf{s}^2-60\mathbf{s}-24 \end{array} ,$$

$$\text{so } \mathbf{G}(\mathbf{s}) = 12-\frac{48\mathbf{s}^2+60\mathbf{s}+24}{(\mathbf{s}^2+2\mathbf{s}+1)(\mathbf{s}+2)} = 12+\frac{A}{(\mathbf{s}+1)^2}+\frac{B}{\mathbf{s}+1}+\frac{C}{\mathbf{s}+2}$$

Where A = –12, B = 48 and C = –96.

Hence, $\boxed{g(t) = 12\delta(t)-12te^{-t}u(t)+48e^{-t}u(t)-96e^{-2t}u(t)}$

(c) $\mathbf{G}(\mathbf{s})$ is not a rational function, so first we perform polynomial long division on the second term (some intermediate steps are not shown):

$$\begin{array}{r} 12 \\ \left(\mathbf{s}^3+6\mathbf{s}^2+11\mathbf{s}+6\right)\overline{\big)12\mathbf{s}^3} \\ \hline \underline{12\mathbf{s}^3+72\mathbf{s}^2+132\mathbf{s}+72} \\ -72\mathbf{s}^2-132\mathbf{s}-72 \end{array} ,$$

$$\text{so } \mathbf{G}(\mathbf{s}) = 3\mathbf{s}-12+\frac{72\mathbf{s}^2+132\mathbf{s}+72}{(\mathbf{s}+1)(\mathbf{s}+2)(\mathbf{s}+3)} = 3\mathbf{s}-12+\frac{A}{\mathbf{s}+1}+\frac{B}{\mathbf{s}+2}+\frac{C}{\mathbf{s}+3}$$

Where A = 6, B = –96 and C = 162.

Hence, $\boxed{g(t) = 3\frac{d}{dt}\delta(t)-12\delta(t)+6e^{-t}u(t)-96e^{-2t}u(t)+162e^{-3t}u(t)}$

45. (a) $\mathbf{H(s)} = \dfrac{\mathbf{s}+1}{\mathbf{s}+2} = 1 - \dfrac{1}{\mathbf{s}+2}$, hence $\boxed{h(t) = \delta(t) - e^{-2t}\,u(t)}$

(b) $\mathbf{H(s)} = \dfrac{\mathbf{s}+3}{(\mathbf{s}+1)(\mathbf{s}+2)} = \dfrac{2}{\mathbf{s}+1} - \dfrac{1}{\mathbf{s}+2}$, hence $\boxed{h(t) = \left[2e^{-t} - e^{-2t}\right]u(t)}$

(c) We need to perform long division on the second term prior to applying the method of residues (some intermediate steps are not shown):

$$\begin{array}{r} \mathbf{s}-5 \\ \left(\mathbf{s}^3 + 5\mathbf{s}^2 + 7\mathbf{s} + 3\right)\overline{\smash{\big)}\,\mathbf{s}^4} \\ \hline 18\mathbf{s}^2 + 32\mathbf{s} + 15 \end{array}$$

Thus, $\mathbf{H(s)} = 3\mathbf{s} - \mathbf{s} + 5 - \dfrac{18\mathbf{s}^2 + 32\mathbf{s} + 15}{(\mathbf{s}+1)^2(\mathbf{s}+3)} + 1 = 2\mathbf{s} + 6 + \dfrac{A}{(\mathbf{s}+1)^2} + \dfrac{B}{\mathbf{s}+1} + \dfrac{C}{\mathbf{s}+3}$

where A = –1/2, B = 9/4, and C = –81/4.

Thus, $\boxed{h(t) = 2\dfrac{d}{dt}\delta(t) + 6\delta(t) - \dfrac{1}{2}te^{-t}u(t) + \dfrac{9}{4}e^{-t}u(t) - \dfrac{81}{4}e^{-3t}u(t)}$

46.

(a) $\boxed{5[\mathbf{s}\mathbf{I}(\mathbf{s}) - i(0^-)] - 7[\mathbf{s}^2\mathbf{I}(\mathbf{s}) - \mathbf{s}i(0^-) - i'(0^-)] + 9\mathbf{I}(\mathbf{s}) = \frac{4}{\mathbf{s}}}$

(b) $\boxed{m[\mathbf{s}^2\mathrm{P}(\mathbf{s}) - \mathbf{s}p(0^-) - p'(0^-)] + \mu_f\,[\mathbf{s}\mathbf{P}(\mathbf{s}) - p(0^-)] + k\mathbf{P}(\mathbf{s}) = 0}$

(c) $\boxed{[\mathbf{s}\,\Delta\mathbf{N_p}(\mathbf{s}) - \Delta n_p(0^-)] = -\frac{\Delta\mathbf{N}_p(\mathbf{s})}{\tau} + \frac{G_L}{\mathbf{s}}}$

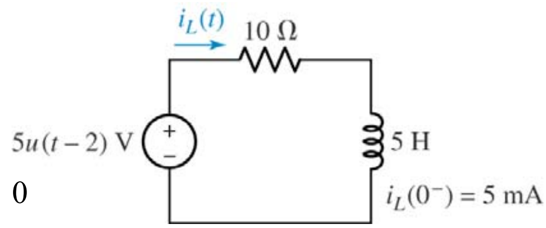
47.

$$15u(t) - 4\delta(t) = 8f(t) + 6f'(t), \quad f(0) = -3$$

$$\therefore \frac{15}{s} - 4 = 8\mathrm{F}(s) + 6s\mathrm{F}(s) + 18 = \frac{15-4s}{s} \quad \therefore \mathrm{F}(s)\ \ (6s+8) = 18 + \frac{15-4s}{s}$$

$$\therefore \mathrm{F}(s) = \frac{-22s+15}{6s(s+4/30)} = \frac{15/8}{s+4/3} \quad \therefore \boxed{f(t) = (1.875 - 5.542e^{-4t/3})\,u(t)}$$

48.



(a) $\boxed{-5\,u(t-2) + 10\,i_L(t) + 5\,\dfrac{di_L}{dt} \;=\; 0}$

(b) $\dfrac{-5}{\mathbf{s}}e^{-2\mathbf{s}} + 10\,\mathbf{I}_L(\mathbf{s}) + 5[\mathbf{s}\mathbf{I}_L(\mathbf{s}) - i_L(0^-)] \;=\; 0$

$$\mathbf{I}_L(\mathbf{s}) = \frac{\frac{5}{\mathbf{s}}e^{-2\mathbf{s}} + 5\;i_L(0^-)}{5\mathbf{s}+10} \;=\; \boxed{\frac{e^{-2\mathbf{s}} \;+\; 5\times10^{-3}\,\mathbf{s}}{\mathbf{s}(\mathbf{s}+2)}}$$

(c) $\mathbf{I}_L(\mathbf{s}) \;=\; e^{-2\mathbf{s}}\left[\dfrac{a}{\mathbf{s}} + \dfrac{b}{\mathbf{s}+2}\right] \;+\; \dfrac{5\times10^{-3}}{\mathbf{s}+2}$

where $a \;=\; \left.\dfrac{1}{\mathbf{s}+2}\right|_{\mathbf{s}=0} = \dfrac{1}{2}$, and $b \;=\; \left.\dfrac{1}{\mathbf{s}}\right|_{\mathbf{s}=-2} = -\dfrac{1}{2}$, so that we may write

$$\mathbf{I}_L(\mathbf{s}) \;=\; \frac{1}{2}e^{-2\mathbf{s}}\left[\frac{1}{\mathbf{s}} - \frac{1}{\mathbf{s}+2}\right] \;+\; \frac{5\times10^{-3}}{\mathbf{s}+2}$$

Thus, $\quad i_L(t) \;=\; \dfrac{1}{2}\left[u(t-2) - e^{-2(t-2)}\,u(t-2)\right] \;+\; 5\times10^{-3}e^{-2t}\;u(t)$

$$\boxed{= \frac{1}{2}\left[1 - e^{-2(t-2)}\right]u(t-2) \;+\; 5\times10^{-3}e^{-2t}\;u(t)}$$

49.

(a) $v_c(0^-) = 50\,\text{V},\ v_c(0^+) = \boxed{50\ \text{V}}$

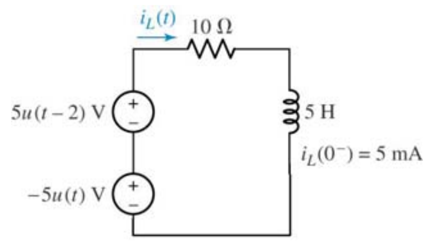
(b) $\boxed{0.1v_c' + 0.2v_c + 0.1(v_c - 20) = 0}$

(c) $\therefore 0.1v_c' + 0.3v_c = 2,\ 0.1s\text{V}_c - 5 + 0.3\text{V}_c = \dfrac{2}{s}$

$$\therefore \text{V}_c(0.1s + 0.3) = 5 + \frac{2}{s} = \frac{5s+2}{s}$$

$$\therefore \text{V}_c(s) = \frac{5s+2}{s(0.1s+0.3)} = \frac{20/3}{s} + \frac{130/3}{s+3} \ \therefore \boxed{v_c(t) = \left(\frac{20}{3} + \frac{130}{3}e^{-3t}\right)u(t)\ \text{V}}$$

50.



(a) $$\boxed{5\,u(t) - 5\,u(t-2) + 10\,i_L(t) + 5\frac{di_L}{dt} = 0}$$

(b) $$\frac{5}{\mathbf{s}} - \frac{5}{\mathbf{s}}e^{-2\mathbf{s}} + 10\,\mathbf{I}_L(\mathbf{s}) + 5[\mathbf{s}\mathbf{I}_L(\mathbf{s}) - i_L(0^-)] = 0$$

$$\mathbf{I}_L(\mathbf{s}) = \frac{\frac{5}{\mathbf{s}}e^{-2\mathbf{s}} - \frac{5}{\mathbf{s}} + 5\,i_L(0^-)}{5\mathbf{s}+10} = \boxed{\frac{e^{-2\mathbf{s}} + 5\times10^{-3}\mathbf{s} - 1}{\mathbf{s}(\mathbf{s}+2)}}$$

(c) $$\mathbf{I}_L(\mathbf{s}) = e^{-2\mathbf{s}}\left[\frac{a}{\mathbf{s}} + \frac{b}{\mathbf{s}+2}\right] + \frac{c}{\mathbf{s}} + \frac{d}{\mathbf{s}+2} \text{ where}$$

$$a = \left.\frac{1}{\mathbf{s}+2}\right|_{\mathbf{s}=0} = \frac{1}{2},\; b = \left.\frac{1}{\mathbf{s}}\right|_{\mathbf{s}=-2} = -\frac{1}{2},\; c = \left.\frac{5\times10^{-3}\mathbf{s}-1}{\mathbf{s}+2}\right|_{\mathbf{s}=0} = -\frac{1}{2}, \text{ and}$$

$$d = \left.\frac{5\times10^{-3}\mathbf{s}-1}{\mathbf{s}}\right|_{\mathbf{s}=-2} = \frac{-10\times10^{-3}-1}{-2} = 0.505,$$

so that we may write

$$\mathbf{I}_L(\mathbf{s}) = \frac{1}{2}e^{-2\mathbf{s}}\left[\frac{1}{\mathbf{s}} - \frac{1}{\mathbf{s}+2}\right] + \frac{0.505}{\mathbf{s}+2} - \left(\frac{1}{2}\right)\frac{1}{\mathbf{s}}$$

Thus, $$i_L(t) = \boxed{\frac{1}{2}\left[u(t-2) - e^{-2(t-2)}\,u(t-2)\right] + 0.505e^{-2t}\,u(t) - \frac{1}{2}\,u(t)}$$

51.

$$12\,u(t) = 20\,f_2'(t) + 3\,f_2(0^-) = 2 \;\therefore\; \frac{12}{s} = 20s\mathrm{F}_2 - 20(2) + 3\mathrm{F}_2$$

$$\therefore \frac{12}{s} + 40 = (20s+3)\;\mathrm{F}_2 = \frac{12+40s}{s} \;\therefore\; \mathrm{F}_2(s) = \frac{2s+0.6}{s(s+0.15)}$$

$$\therefore \mathrm{F}_2(s) = \frac{4}{s} - \frac{2}{s+0.15} \leftrightarrow \boxed{(4-2e^{-0.15t})\,u(t)}$$

52. (a) $f(t) = \boxed{2\,u(t) - 4\delta(t)}$

(b) $f(t) = \boxed{\cos\left(\sqrt{99}\,t\right)}$

(c) $$\mathbf{F(s)} = \frac{1}{\mathbf{s}^2 + 5\mathbf{s} + 6} - 5 = \frac{a}{\mathbf{s}-3} + \frac{b}{\mathbf{s}-2} - 5$$

where $a = \left.\frac{1}{\mathbf{s}-2}\right|_{\mathbf{s}=3} = 1$ and $b = \left.\frac{1}{\mathbf{s}-3}\right|_{\mathbf{s}=2} = -1$

Thus,

$$f(t) = \boxed{e^{-3t}\,u(t) - e^{-2t}\,u(t) - 5\delta(t)}$$

(d) $f(t) = \boxed{\delta'(t)}$ (a "doublet")

(e) $f(t) = \boxed{\delta''(t)}$

53.

$$x' + y = 2u(t),\ \ y' - 2x + 3y = 8u(t),\ \ x(0^-) = 5,\ \ y(0^-) = 8$$

$$sX - 5 + Y = \frac{2}{s},\ sY - 8 - 2X + 3Y = \frac{8}{s}\ \therefore\ X = \frac{1}{s}\left(\frac{2}{s} + 5 - Y\right) = \frac{2}{s^2} + \frac{5}{s} - \frac{Y}{s}$$

$$\therefore sY + 3Y - \frac{4}{s^2} - \frac{10}{s} + \frac{2Y}{s} = 8 + \frac{8}{s}\ \therefore\ Y\left(s + 3 + \frac{2}{s}\right) = \frac{4}{s^2} + \frac{18}{s} + 8$$

$$Y\left(\frac{s^2 + 3s + 2}{s}\right) = \frac{4 + 18s + 8s^2}{s^2},\ \ Y(s) + \frac{8s^2 + 18s + 4}{s(s+1)(s+2)} = \frac{2}{s} + \frac{6}{s+1} + \frac{0}{s+2}$$

$$\therefore \boxed{y(t) = (2 + 6e^{-t})\ u(t);}\ \ x(t) = \frac{1}{2}\ [y' + 3y - 8u(t)] = \frac{1}{2}y' + 1.5y - 4u(t)$$

$$\therefore x(t) = \frac{1}{2}\,[-6e^{-t}u(t)] + 1.5\,[2 + 6e^{-t}]\ u(t) - 4u(t)$$

$$\therefore x(t) = 6e^{-t}u(t) - u(t) = \boxed{(6e^{-t} - 1)\,u(t)}$$

54. (a) $\mathbf{F(s)} = 8\mathbf{s} + 8 + \frac{8}{\mathbf{s}}$, with $f(0^-) = 0$. Thus, we may write:

$$\boxed{f(t) = 8\,\delta(t) + 8\,u(t) + 8\delta'(t)}$$

(b) $\mathbf{F(s)} = \frac{\mathbf{s}^2}{(\mathbf{s}+2)} - \mathbf{s} + 2$.

$$f(t) = \delta'(t) - 2\delta(t) + 4e^{-2t}\,u(t) - \delta'(t) + 2\delta(t) = \boxed{4e^{-2t}\,u(t)}$$

55.

(a) $i_c(0^-) = 0,\ v_c(0) = 100\text{ V},\ \therefore i_c(0^+) = \dfrac{40-100}{100} = \boxed{-0.6\text{ A}}$

(b) $\boxed{40 = 100\,i_c + 50\int_{0^-}^{\infty} i_c\,dt + 100}$

(c) $-\dfrac{60}{s} = 100\ \text{I}_c(s) + \dfrac{50}{s}\ \text{I}_c(s)$

$\therefore \dfrac{6}{s} = \text{I}_c\ \dfrac{10s+5}{s},\ \ \text{I}_c(s) = \dfrac{-6}{10s+5} = \dfrac{-0.6}{s+0.5} \leftrightarrow i_c(t) = \boxed{-0.6e^{-0.5t}u(t)}$

56. (a) $4 \cos 100t \leftrightarrow \boxed{\dfrac{4\mathbf{s}}{\mathbf{s}^2 + 100^2}}$

(b) $2 \sin 10^3 t - 3 \cos 100t \leftrightarrow \boxed{\dfrac{2\times 10^3}{\mathbf{s}^2 + 10^6} - \dfrac{3\mathbf{s}}{\mathbf{s}^2 + 100^2}}$

(c) $14 \cos 8t - 2 \sin 8^\circ \leftrightarrow \boxed{\dfrac{14\mathbf{s}}{\mathbf{s}^2 + 64} - \dfrac{2 \sin 8^\circ}{\mathbf{s}}}$

(d) $\delta(t) + [\sin 6t\,]u(t) \leftrightarrow \boxed{1 + \dfrac{6}{\mathbf{s}^2 + 36}}$

(e) $\cos 5t \ \sin 3t = ½ \sin 8t + ½ \sin(-2t) = ½ (\sin 8t - \sin 2t) \leftrightarrow \boxed{\dfrac{4}{\mathbf{s}^2 + 64} - \dfrac{1}{\mathbf{s}^2 + 4}}$

57. $i_s = 100e^{-5t}u(t)$ A; $i_s = v' + 4v + 3\int_{0^-}^{t} v\,dt$

(a) $i_s = \dfrac{v}{\mathrm{R}} + \mathrm{C}v' + \dfrac{1}{\mathrm{L}}\int_{0^-}^{t} v\,dt$; $\boxed{\mathrm{R} = \dfrac{1}{4}\Omega,\ \ \mathrm{C} = 1\mathrm{F},\ \ \mathrm{L} = \dfrac{1}{3}\ \mathrm{H}}$

(b) $\dfrac{100}{s+5} = s\mathrm{V}(s) + 4\mathrm{V}(s) + \dfrac{3}{s}\ \mathrm{V}(s)$

$\mathrm{V}(s)\left(s + 4 + \dfrac{3}{4}\right) = \mathrm{V}(s)\dfrac{s^2 + 4s + 3}{s} = \dfrac{100}{s+5},\ \ \mathrm{V}(s) = \dfrac{100s}{(s+1)(s+3)(s+5)}$

$\therefore \mathrm{V}(s) = \dfrac{-12.5}{s+1} + \dfrac{75}{s+3} - \dfrac{62.5}{s+5},$ $\boxed{v(t) = (75e^{-3t} - 12.5e^{-t} - 62.5e^{-5t})u(t)\ \mathrm{V}}$

58.

(a) $\mathbf{V}(\mathbf{s}) = \boxed{\dfrac{7}{\mathbf{s}} + \dfrac{e^{-2\mathbf{s}}}{\mathbf{s}}\ \text{V}}$

(b) $\mathbf{V}(\mathbf{s}) = \boxed{\dfrac{e^{-2\mathbf{s}}}{\mathbf{s}+1}\ \text{V}}$

(c) $\mathbf{V}(\mathbf{s}) = \boxed{48e^{-\mathbf{s}}\ \text{V}}$

59.

$$4u(t)+i_c+10\int_{0^-}^{\infty} i_c dt+4\,[i_c-0.5\delta(t)]=0$$

$$\therefore \frac{4}{s}+\mathrm{I}_c+\frac{10}{s}\,\mathrm{I}_c+4\mathrm{I}_c=2,\ \mathrm{I}_c\left(5+\frac{10}{s}\right)=2-\frac{4}{s}+\frac{2s-4}{s}$$

$$\therefore \mathrm{I}_c=\frac{2s-4}{5s+10}=0.4-\frac{1.6}{s+2}$$

$$\therefore \boxed{i_c(t)+0.4\delta(t)-1.6e^{-2t}u(t)\ \mathrm{A}}$$

60.

$$v' + 6v + 9\int_{0^-}^{t} v(z)\ dz = 24(t-2)\ u(t-2),\ v'(0) = 0$$

$$\therefore sV(s) - 0 + 6\ V(s) + \frac{9}{s}\ V(s) = 24e^{-2s}\ \frac{1}{s^2} = V(s)\frac{s^2+6s+9}{s} = V(s)\ \frac{(s+3)^2}{s}$$

$$\therefore\ V(s) = 24e^{-2s}\ \frac{1}{s^2}\ \frac{s}{(s+3)^2} = 24e^{-2s}\left[\frac{1/9}{s} - \frac{1/9}{s+3} - \frac{1/3}{(s+3)^2}\right]$$

$$\therefore\ V(s) = e^{-2s}\left[\frac{8/3}{s} - \frac{8}{s+3} - \frac{8}{(s+3)^2}\right] \leftrightarrow \frac{8}{3}\ [u(t-2) - e^{-3(t-2)}\ u(t-2)]$$

$$-8(t-2)\,e^{-3(t-2)}\,u(t-2)\ \therefore\ \boxed{v(t) = \left[\frac{8}{3} - \frac{8}{3}e^{-3(t-2)} - 8(t-2)\,e^{-3(t-2)}\right]u(t-2)}$$

61. (a) All coefficients of the denominator are positive and non-zero, so we may apply the Routh test:

1	47	
13	35	
44.308	0	[(13)(47) – 35]/13
35		[35(44.308) – 0]/44.308

No sign changes, so STABLE.

(b) All coefficients of the denominator are positive and non-zero, so we may apply the Routh test:

1	1	
13	35	
–1.69	0	[13 – 35]/13

No need to proceed further: we see a sign change, so UNSTABLE.

62. (a) All coefficients of the denominator are positive and non-zero, so we may apply the Routh test:

1	8	
3	0	
8		[(3)(8) – 0]/3

No sign changes, so STABLE.

Verification: roots of **D**(**s**) = $-\frac{3}{2} \pm \sqrt{\left(\frac{3}{2}\right)^2 - 8} = -\frac{3}{2} \pm j\left(\frac{23}{4}\right)^{\frac{1}{2}}$, which have negative real parts, so the function is indeed stable.

(b) All coefficients of the denominator are positive and non-zero, so we may apply the Routh test:

1	1	
2	0	
1		[(2)(1) – 0]/2

No sign changes, so STABLE.

Verification: roots of **D**(**s**) = –1, –1, which have negative real parts, so the function is indeed stable.

63. (a) All coefficients of the denominator are positive and non-zero, so we may apply the Routh test:

1	3	1	
3	3	0	
2	1		[(3)(3) – 3]/3
1.5			[6 – 3]/2

$\boxed{\text{No sign changes, so STABLE.}}$

(b) All coefficients of the denominator are positive and non-zero, so we may apply the Routh test:

1
3

$\boxed{\text{No sign changes, so STABLE.}}$

64. (a) $v(t) = 7u(t) + 8e^{-3t}u(t)$ Therefore

$$\mathbf{V}(\mathbf{s}) = \frac{7}{\mathbf{s}} + \frac{8}{\mathbf{s}+3} = \frac{15\mathbf{s}+21}{\mathbf{s}(\mathbf{s}+3)}.$$

$$\lim_{\mathbf{s}\to\infty} \mathbf{s}\mathbf{V}(\mathbf{s}) = \lim_{\mathbf{s}\to\infty} \frac{15\mathbf{s}+21}{\mathbf{s}+3} = \lim_{\mathbf{s}\to\infty} \frac{15+\frac{21}{\mathbf{s}}}{1+\frac{3}{\mathbf{s}}} = \boxed{15\text{ V}}$$

(b) $v(0) = 7 + 8 = \boxed{15\text{ V}\quad\text{(verified)}}$

65. (a) $v(t) = 7u(t) + 8e^{-3t}u(t)$ Therefore

$$\mathbf{V}(\mathbf{s}) = \frac{7}{\mathbf{s}} + \frac{8}{\mathbf{s}+3} = \frac{15\mathbf{s}+21}{\mathbf{s}(\mathbf{s}+3)}.$$

$$\lim_{s \to 0} \mathbf{sV}(\mathbf{s}) = \lim_{s \to 0} \frac{15\mathbf{s}+21}{\mathbf{s}+3} = \boxed{7 \text{ V}}$$

(b) $v(\infty) = 7 + 0 = \boxed{7 \text{ V} \quad \text{(verified)}}$

66.

(a) $$\mathrm{F}(s) = \frac{5(s^2+1)}{(s^3+1)} \quad \therefore \quad f(0^+) = \lim_{s\to\infty} \frac{5s(s^2+1)}{s^3+1} = \boxed{5}$$

$$f(\infty) = \lim_{s\to 0} \frac{5s(s^2+1)}{s^3+1}, \text{ but 1 pole in RHP} \therefore \text{ indeterminate}$$

(b) $$\mathrm{F}(s) = \frac{5(s^2+1)}{s^3+16} \quad \therefore \quad f(0^+) = \lim_{s\to\infty} \frac{5s(s^2+1)}{s^4+16} = \boxed{0}$$

$f(\infty)$ is indeterminate since poles on $j\omega$ axis

(c) $$\mathrm{F}(s) = \frac{(s+1)(1+e^{-4s})}{s^2+2} \quad \therefore \quad f(0^+) = \lim_{s\to\infty} \frac{s(s+1)(1+e^{-4s})}{s^2+2} = \boxed{1}$$

$f(\infty)$ is indeterminate since poles on $j\omega$ axis

67. (a) $f(0^+) = \lim_{s\to\infty}[\mathbf{s}\,\mathbf{F}(\mathbf{s})] = \lim_{s\to\infty}\left(\frac{2\mathbf{s}^2+6}{\mathbf{s}^2+5\mathbf{s}+2}\right) = \boxed{2}$

$f(\infty) = \lim_{s\to 0}[\mathbf{s}\,\mathbf{F}(\mathbf{s})] = \lim_{s\to 0}\left(\frac{2\mathbf{s}^2+6}{\mathbf{s}^2+5\mathbf{s}+2}\right) = \frac{6}{2} = \boxed{3}$

(b) $f(0^+) = \lim_{s\to\infty}[\mathbf{s}\,\mathbf{F}(\mathbf{s})] = \lim_{s\to\infty}\left(\frac{2\mathbf{s}e^{-\mathbf{s}}}{\mathbf{s}+3}\right) = \boxed{0}$

$f(\infty) = \lim_{s\to 0}[\mathbf{s}\,\mathbf{F}(\mathbf{s})] = \lim_{s\to 0}\left(\frac{2\mathbf{s}e^{-\mathbf{s}}}{\mathbf{s}+3}\right) = \boxed{0}$

(c) $f(0^+) = \lim_{s\to\infty}[\mathbf{s}\,\mathbf{F}(\mathbf{s})] = \lim_{s\to\infty}\left[\frac{\mathbf{s}(\mathbf{s}^2+1)}{\mathbf{s}^2+5}\right] = \boxed{\infty}$

$f(\infty)$: This function has poles on the $j\omega$ axis, so we may not apply the final value theorem to determine $f(\infty)$.

68.

(a) $$\mathrm{F}(s)=\frac{5(s^2+1)}{(s+1)^3}\ \therefore\ f(0^+)=\lim_{s\to\infty}\frac{5s(s^2+1)}{(s+1)^3}=\boxed{5}$$

$$f(\infty)=\lim_{\mathbf{s}\to 0}\left[\mathbf{s}\frac{5(\mathbf{s}^2+1)}{(\mathbf{s}+1)^3}\right]=\boxed{0}\ \text{(pole OK)}$$

(b) $$\mathrm{F}(s)=\frac{5(s^2+1)}{s(s+1)^3}\ \therefore\ f(0^+)=\lim_{s\to\infty}\frac{5(s^2+1)}{(s+1)^3}=\boxed{0}$$

$$f(\infty)=\lim_{s\to 0}\frac{5(s^2+1)}{(s+1)^3}=5\ \text{(pole OK)}$$

(c) $$\mathrm{F}(\mathbf{s})=\frac{(1-e^{-3\mathbf{s}})}{\mathbf{s}^2}\ \therefore\ f(0^+)=\lim_{\mathbf{s}\to\infty}\frac{1-e^{-3\mathbf{s}}}{\mathbf{s}}=\boxed{0}$$

$$f(\infty)=\lim_{\mathbf{s}\to 0}\left[\frac{1-e^{-3\mathbf{s}}}{\mathbf{s}}\right]=\text{(using L'Hospital's rule)}\ \lim_{\mathbf{s}\to 0}\left(3e^{-3\mathbf{s}}\right)=\boxed{3}$$

69.

$$f(t) = \frac{1}{t}(e^{at} - e^{-bt})u(t)$$

(a) Now, $\frac{1}{t}f(t) \leftrightarrow \int_s^\infty \mathrm{F}(s)\,ds \therefore e^{-at}\,u(t) \leftrightarrow \frac{1}{s+a},\ -e^{-bt}u(t) \leftrightarrow -\frac{1}{s+b}$

$$\therefore \frac{1}{t}\,(e^{-at} - e^{-bt}\,u(t) \leftrightarrow \int_s^\infty \left(\frac{1}{s+a} - \frac{1}{s+b}\right)ds = \ell n\,\frac{s+a}{s+b}\bigg|_s^\infty = \ell n\,\frac{s+a}{s+b}\bigg|_s = \boxed{\ell n\frac{s+b}{s+a}}$$

(b) $\lim_{t\to 0^+} \frac{1}{t}(e^{-at} - e^{-bt})\,u(t) = \lim_{t\to 0^+} \frac{1 - at + \ldots - 1 + bt}{t} = \boxed{b-a}$

$$\lim_{s\to\infty} s\ell n\frac{s+b}{s+a} = \lim_{s\to\infty} \frac{\ell n(s+b) - \ell n(s+a)}{1/s}$$

Use ℓ' Hospital. $\therefore \lim_{s\to\infty} s\mathrm{F}(s) = \frac{1/(s+b) - 1/(s+a)}{-1/s^2} = \lim_{s\to\infty}\left[-s^2\,\frac{(a-b)}{(s+b)(s+a)}\right] = \boxed{b-a}$

70.

(a) $$\mathrm{F}(s) = \frac{8s-2}{s^2+6s+10} \quad \therefore \quad f(0^+) = \lim_{s\to\infty} \frac{s(8s-2)}{s^2+6s+10} = \boxed{8}$$

$$f(\infty) = \lim_{s\to 0} \frac{s(8s-2)}{s^2+6s+10} = 0 \left(\text{poles: } s = \frac{-6 \pm \sqrt{36-40}}{2}, \text{ LHP, } \therefore \text{OK} \right)$$

(b) $$\mathrm{F}(s) = \frac{2s^3-s^2-3s-5}{s^3+6s^2+10s} \quad \therefore \quad f(0^+) = \lim_{s\to\infty} \frac{2s^3-s^2-3s-5}{s^2+6s+10} = \boxed{\infty}$$

$$f(\infty) = \lim_{s\to 0} \frac{2s^3-s^2-3s-5}{s^2+6s+10} = -0.5 \text{ (poles OK)}$$

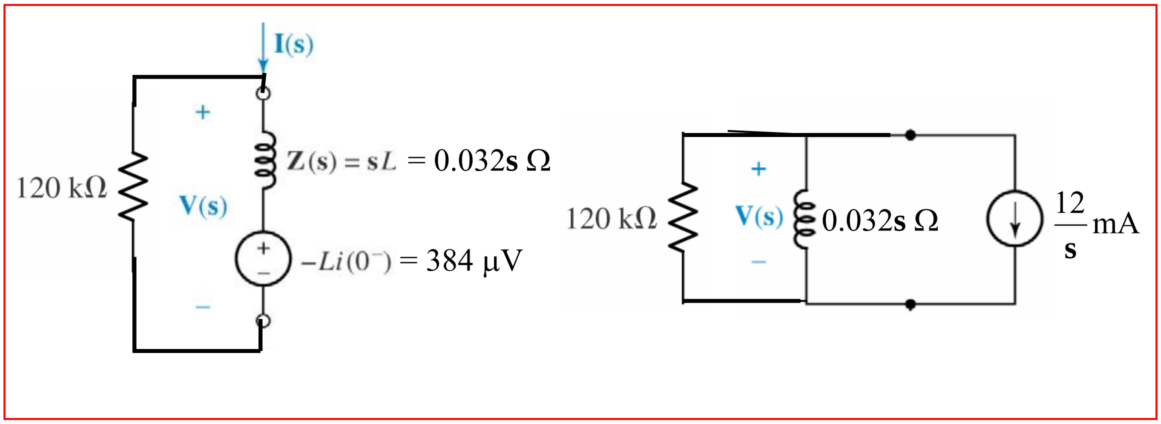
(c) $$\mathrm{F}(s) = \frac{8s-2}{s^2-6s+10} \quad \therefore \quad f(0^+) = \lim_{s\to\infty} \frac{s(8s-2)}{s^2-6s+10} = \boxed{8}$$

$$f(\infty) = \lim_{s\to 0} \frac{s(8s-2)}{s^2-6s+10}, \; s = \frac{6 \pm \sqrt{36-40}}{2} \text{ RHP } \therefore \text{ indeterminate}$$

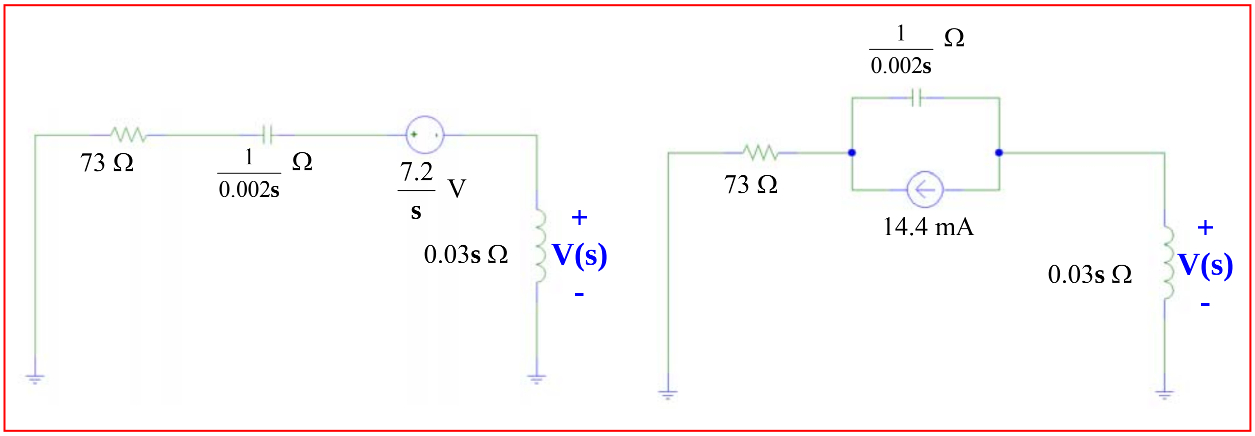
(d) $$\mathbf{F}(\mathbf{s}) = \frac{8\mathbf{s}^2-2}{(\mathbf{s}+2)^2(\mathbf{s}+1)(\mathbf{s}^2+6\mathbf{s}+10)} \quad \therefore \quad f(0^+) = \lim_{\mathbf{s}\to\infty} \mathbf{s}\mathrm{F}(\mathbf{s}) = \boxed{0}$$

$$f(\infty) = \lim_{\mathbf{s}\to 0} \frac{\mathbf{s}(8\mathbf{s}^2-2)}{(\mathbf{s}+2)^2(\mathbf{s}+1)(\mathbf{s}^2+6\mathbf{s}+10)} = \boxed{0} \text{ (pole OK)}$$

1. Note that $i_L(0^+) = 12$ mA. We have two choices for inductor model:



2. $i_L(0^-) = 0$, $v_C(0^+) = 7.2$ V ('+' reference on left). There are two possible circuits, since the inductor is modeled simply as an impedance:



3.

(a) $$\mathbf{Z}_m(\mathbf{s}) = \frac{2\mathbf{s}}{20+0.1\mathbf{s}} + \frac{2000/\mathbf{s}}{2+1000/\mathbf{s}} = \frac{20\mathbf{s}}{\mathbf{s}+200} + \frac{1000}{\mathbf{s}+500}$$
$$= \frac{20\mathbf{s}^2 + 10{,}000\mathbf{s} + 1000\mathbf{s} + 200{,}000}{\mathbf{s}^2 + 700\mathbf{s} + 100{,}000} = \boxed{\frac{20\mathbf{s}^2 + 11{,}000\mathbf{s} + 200{,}000}{\mathbf{s}^2 + 700\mathbf{s} + 100{,}000}}$$

(b) $\mathbf{Z}_{\text{in}}(-80) = \boxed{-10.95\ \Omega}$

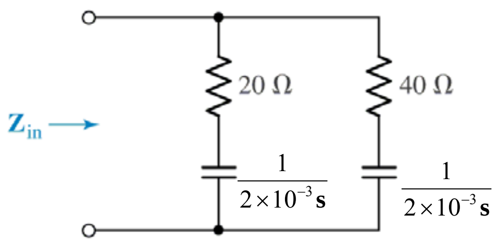
(c) $$\mathbf{Z}_{in}(j80) = \frac{-128{,}000 + j880{,}000 + 200{,}000}{-6400 + j56{,}000 + 100{,}000} = \boxed{8.095\angle 54.43°\ \Omega}$$

(d) $$\mathbf{Y}_{RL} = \frac{1}{20} + \frac{10}{\mathbf{s}} = \boxed{\frac{\mathbf{s}+200}{20\mathbf{s}}}$$

(e) $$\mathbf{Y}_{RC} = \frac{1}{2} + 0.001\mathbf{s} = \boxed{\frac{\mathbf{s}+500}{1000}}$$

(f) $$\boxed{\begin{aligned}\frac{\mathbf{Y}_{RL}+\mathbf{Y}_{RC}}{\mathbf{Y}_{RL}\mathbf{Y}_{RC}} &= \frac{\frac{\mathbf{s}+200}{20s} + 0.5 + 0.001\mathbf{s}}{\frac{(\mathbf{s}+200)}{20\mathbf{s}}(0.001\mathbf{s}+0.5)} = \frac{\mathbf{s}+200+10\mathbf{s}+0.02\mathbf{s}^2}{0.001\mathbf{s}^2 + 0.7\mathbf{s} + 100} \\ &= \frac{20\mathbf{s}^2 + 11{,}000\mathbf{s} + 200{,}000}{\mathbf{s}^2 + 700\mathbf{s} + 100{,}000} = \mathbf{Z}(\mathbf{s})\end{aligned}}$$

4.



$$\mathbf{Z}_{in} = \left(20 + \frac{1}{2\times10^{-3}\mathbf{s}}\right) \parallel \left(40 + \frac{1}{2\times10^{-3}\mathbf{s}}\right) = (20 + 500\mathbf{s}^{-1}) \parallel (40 + 500\mathbf{s}^{-1})$$

$$\boxed{= \frac{80\mathbf{s}^2 + 3000\mathbf{s} + 25000}{6\mathbf{s}^2 + 100\mathbf{s}}}$$

5. (a) $\mathbf{Z}_{in} = \frac{50}{\mathbf{s}} + \frac{16(0.2\mathbf{s})}{16+0.2\mathbf{s}} = \frac{50}{\mathbf{s}} + \frac{16\mathbf{s}}{\mathbf{s}+80} = \boxed{\frac{16\mathbf{s}^2 + 50\mathbf{s} + 4000}{\mathbf{s}^2 + 80\mathbf{s}}}$

(b) $\mathbf{Z}_{in}(j8) = \frac{-1024 + 4000 + j400}{-64 + j640} = \boxed{0.15842 - j4.666\ \Omega}$

(c) $\mathbf{Z}_{in}(-2+j6) = \frac{16(4-36-j24) - 100 + j300 + 4000}{-32 - j24 - 160 + j480} = \boxed{6.850\angle -114.3°\,\Omega}$

(d) $\mathbf{Z}_{in} = \frac{50}{\mathbf{s}} + \frac{0.2\mathbf{s}\mathrm{R}}{\mathrm{R}+0.2\mathbf{s}} = \frac{0.2\mathrm{R}\mathbf{s}^2 + 10\mathbf{s} + 50\mathrm{R}}{0.2\mathbf{s}^2 + \mathrm{R}\mathbf{s}}$,

$\mathbf{Z}_{in}(-5) = \frac{5\mathrm{R} - 50 + 50R}{5 - 5\mathrm{R}}$ $\therefore 55\mathrm{R} = 50,\quad \mathrm{R} = \boxed{0.9091\ \Omega}$

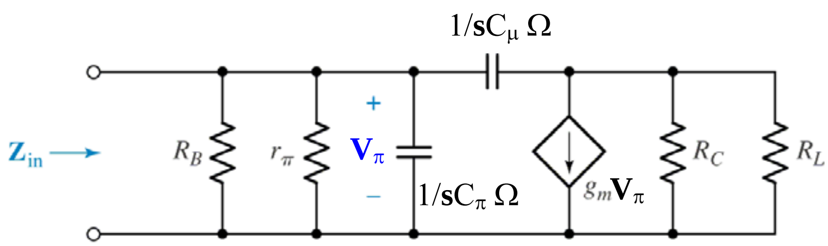
(e) $\mathrm{R} = \boxed{1\Omega}$

6. $2\text{ mF} \rightarrow \dfrac{1}{2\times10^{-3}\mathbf{s}}\,\Omega,\;\; 1\text{ mH} \rightarrow 0.001\mathbf{s}\;\Omega,$

$$\mathbf{Z}_{\text{in}} = (55 + 500/\,\mathbf{s}) \,||\, (100 + \mathbf{s}/\,1000) =$$

$$\frac{\left(55+\dfrac{500}{\mathbf{s}}\right)\left(100+\dfrac{\mathbf{s}}{1000}\right)}{155+\dfrac{500}{\mathbf{s}}+\dfrac{\mathbf{s}}{1000}} = \boxed{\frac{55\mathbf{s}^2 + 5.5005\times10^6\mathbf{s} + 5\times10^7}{\mathbf{s}^2 + 5\times10^5\mathbf{s} + 1.55\times10^5}}$$

7. We convert the circuit to the s-domain:



Defining $\mathbf{Z}_\pi = R_B \,||\, r_\pi \,||\, (1/\mathbf{s}C_\pi) = \dfrac{r_\pi R_B}{r_\pi + R_B + r_\pi R_B C_\pi \mathbf{s}}$ and

$\mathbf{Z}_L = R_C \,||\, R_L = R_C R_L/(R_C + R_L)$, we next connect a 1-A source to the input and write two nodal equations:

$$1 = \mathbf{V}_\pi / \mathbf{Z}_\pi + (\mathbf{V}_\pi - \mathbf{V}_L) C_\mu \mathbf{s} \qquad [1]$$

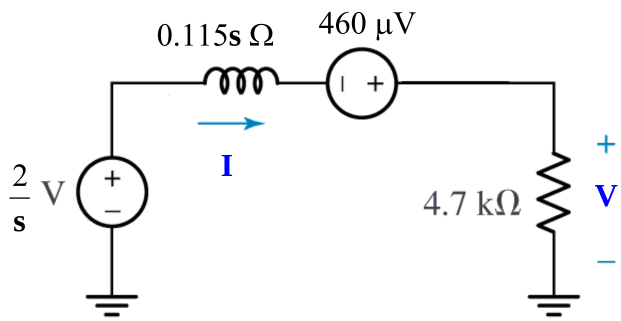
$$-g_m \mathbf{V}_\pi = \mathbf{V}_L / \mathbf{Z}_L + (\mathbf{V}_L - \mathbf{V}_\pi) C_\mu \mathbf{s} \qquad [2]$$

Solving,

$$\mathbf{V}_\pi = \boxed{\frac{r_\pi R_B \left(1 + \mathbf{Z}_L C_\mu \mathbf{s}\right)}{\mathbf{Z}_L r_\pi R_B C_\pi C_\mu \mathbf{s}^2 + (g_m \mathbf{Z}_L r_\pi R_B C_\mu + r_\pi R_B C_\pi + r_\pi R_B C_\mu + \mathbf{Z}_L r_\pi C_\mu + \mathbf{Z}_L R_B C_\mu)\mathbf{s} + r_\pi + R_B}}$$

Since we used a 1-A 'test' source, this is the input impedance. Setting both capacitors to zero results in $r_\pi \,||\, R_B$ as expected.

8.



$$\mathbf{V}(\mathbf{s}) = 4700\frac{\frac{2}{\mathbf{s}} + 460\times 10^{-6}}{4700 + 0.115\mathbf{s}} = \frac{2.162}{0.115\mathbf{s} + 4700} + \frac{9400}{\mathbf{s}\,(0.115\mathbf{s} + 4700)}$$

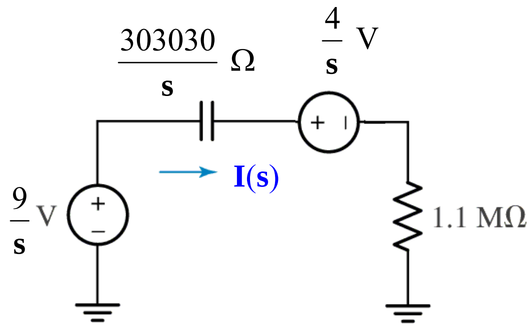
$$= \frac{18.8}{\mathbf{s} + 40870} + \frac{81740}{\mathbf{s}\,(\mathbf{s} + 40870)} = \frac{18.8}{\mathbf{s} + 40870} + \frac{a}{\mathbf{s}} + \frac{b}{\mathbf{s} + 40870}$$

where $a = \left.\frac{81740}{\mathbf{s} + 40870}\right|_{\mathbf{s}=0} = 2$ and $b = \left.\frac{81740}{\mathbf{s}}\right|_{\mathbf{s}=-40870} = -2$

Thus, $\mathbf{V}(\mathbf{s}) = \frac{18.8}{\mathbf{s} + 40870} + \frac{2}{\mathbf{s}} - \frac{2}{\mathbf{s} + 40870}$. Taking the inverse transform of each term,

$$\boxed{v(t) = [16.8\, e^{-40870t} + 2]\, u(t)\ \text{V}}$$

9. $v(0^-) = 4$ V

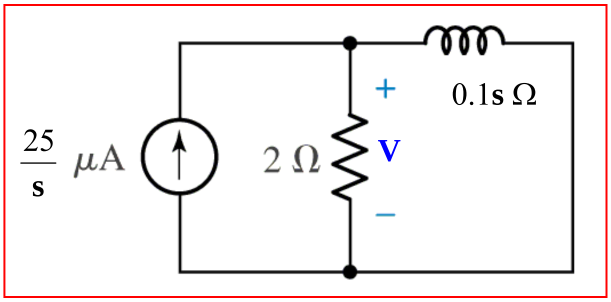


$$\mathbf{I(s)} = \frac{\frac{9}{\mathbf{s}} - \frac{4}{\mathbf{s}}}{\frac{303030}{\mathbf{s}} + 1.1\times10^{6}} = \frac{5}{1.1\times10^{6} + 303030} = \frac{4.545\times10^{-6}}{\mathbf{s}+0.2755}$$

Taking the inverse transform, we find that $i(t) = \boxed{4.545\ e^{-0.2755t}\ u(t)\ \mu\text{A}}$

10. From the information provided, we assume no initial energy stored in the inductor.

(a) Replace the 100 mH inductor with a $0.1\mathbf{s}$-Ω impedance, and the current source with a $\dfrac{25\times10^{-6}}{\mathbf{s}}$ A source.



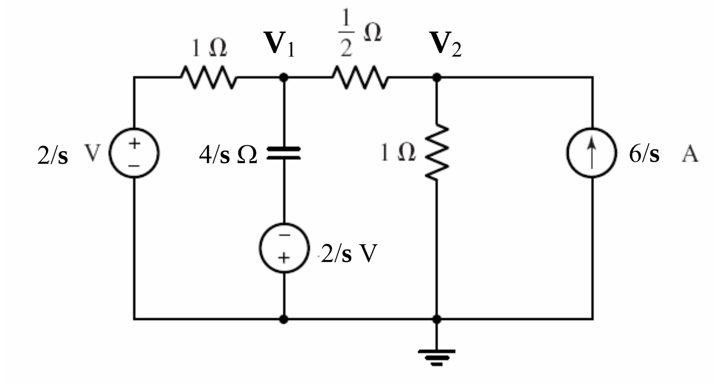
(b) $$\mathbf{V}(\mathbf{s}) = \frac{25\times10^{-6}}{\mathbf{s}}\left[\frac{2\,(0.1\mathbf{s})}{2+0.1\mathbf{s}}\right] = \frac{5\times10^{-6}}{0.1\mathbf{s}+2} = \frac{5\times10^{-5}}{\mathbf{s}+20}\ \text{V}$$

Taking the inverse transform,

$v(t) = 50\ e^{-20t}$ mV

The power absorbed in the resistor R is then $p(t) = 0.5\ v^2(t) = \boxed{1.25\ e^{-40t}\ \text{nW}}$

11. We transform the circuit into the s-domain, noting the initial condition of the capacitor:



Writing our nodal equations,

$$\frac{\mathbf{V}_1 - \frac{2}{\mathbf{s}}}{1} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{\frac{1}{2}} + \frac{\mathbf{V}_1 + \frac{2}{\mathbf{s}}}{\frac{4}{\mathbf{s}}} = 0 \qquad [1]$$

$$\frac{\mathbf{V}_2 - \mathbf{V}_1}{\frac{1}{2}} + \frac{\mathbf{V}_2}{1} = 6 \qquad [2]$$

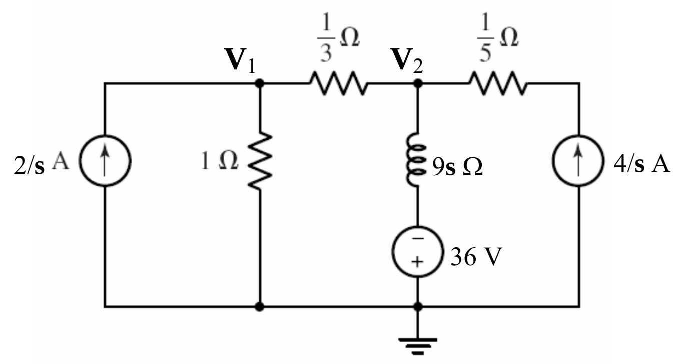
We may solve to obtain $\mathbf{V}_1 = \dfrac{-6(\mathbf{s}-12)}{\mathbf{s}(3\mathbf{s}+20)} = \dfrac{-5.6}{\mathbf{s}+6.67} + \dfrac{3.6}{\mathbf{s}}$

and $\mathbf{V}_1 = \dfrac{2(\mathbf{s}+44)}{\mathbf{s}(3\mathbf{s}+20)} = \dfrac{-3.73}{\mathbf{s}+6.67} + \dfrac{4.4}{\mathbf{s}}$

Taking the inverse transforms, $v_1(t) = -5.6e^{-6.67t} + 3.6 \text{ V}, \ t \geq 0$

and $v_2(t) = -3.73e^{-6.67t} + 4.4 \text{ V}, \ t \geq 0$

12. We transform the circuit into the **s**-domain, noting the initial condition of the inductor:



(a) Writing our nodal equations,

$$4\mathbf{V}_1 - 3\mathbf{V}_2 = \frac{2}{\mathbf{s}} \qquad [1]$$

and

$$-3\mathbf{V}_1 + 3\mathbf{V}_2 + \frac{\mathbf{V}_2 + 36}{9\mathbf{s}} = \frac{4}{\mathbf{s}}$$

or [2]

$$-3\mathbf{V}_1 + \left(3 + \frac{1}{9\mathbf{s}}\right)\mathbf{V}_2 = 0$$

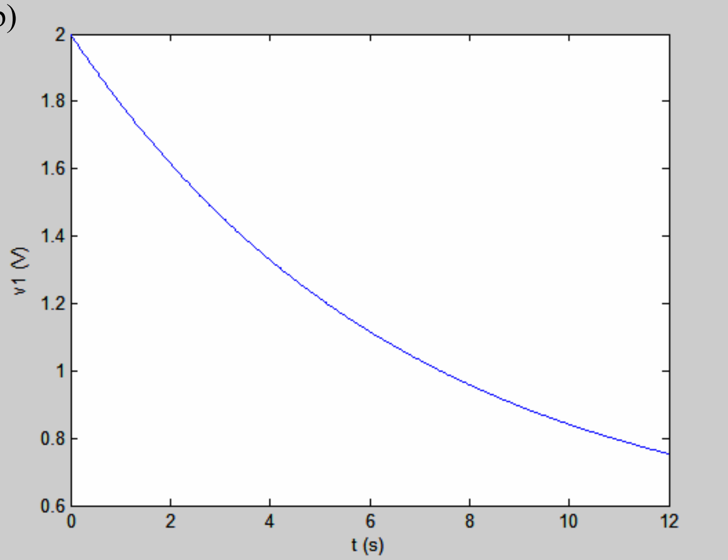
We may solve to obtain $$\mathbf{V}_1 = \frac{2(27\mathbf{s}+1)}{\mathbf{s}(27\mathbf{s}+4)} = \frac{3}{2}\left(\frac{1}{\mathbf{s}+\frac{4}{27}}\right) + \frac{1}{2}\left(\frac{1}{\mathbf{s}}\right)$$

and $$\mathbf{V}_2 = \frac{54}{27\mathbf{s}+4} = \frac{2}{\mathbf{s}+\frac{4}{27}}$$

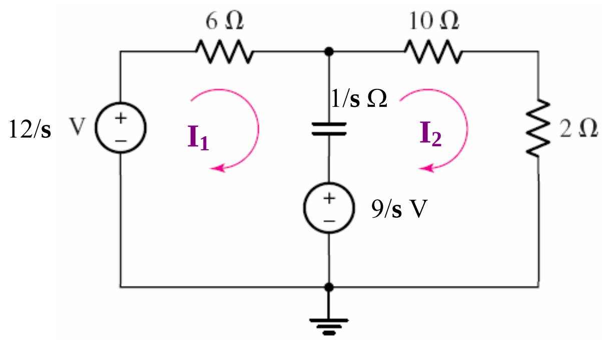
Taking the inverse transforms,
and

$v_1(t) = 1.5^{-0.1481t} + 0.5$ V, $t \geq 0$
$v_2(t) = 2e^{-0.1481t}$ V, $t \geq 0$

(b)



13. (a) We transform the circuit into the **s**-domain, noting the initial condition of the capacitor:



Writing the two required mesh equations:

$$\left(6+\frac{1}{\mathbf{s}}\right)\mathbf{I}_1-\frac{1}{\mathbf{s}}\mathbf{I}_2=\frac{3}{\mathbf{s}} \qquad [1]$$

$$-\frac{1}{\mathbf{s}}\mathbf{I}_1+\left(12+\frac{1}{\mathbf{s}}\right)\mathbf{I}_2=\frac{9}{\mathbf{s}} \qquad [2]$$

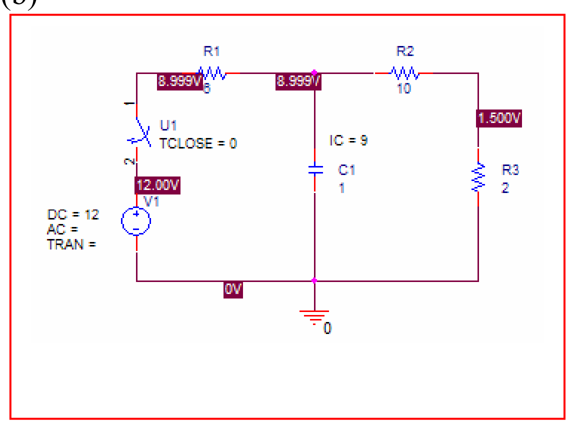
Solving yields $\mathbf{I}_1=\frac{2}{3}\left[\frac{(3\mathbf{s}+1)}{\mathbf{s}(4\mathbf{s}+1)}\right]=\frac{2}{3\mathbf{s}}-\frac{1}{6}\left(\frac{1}{\mathbf{s}+\frac{1}{4}}\right)$

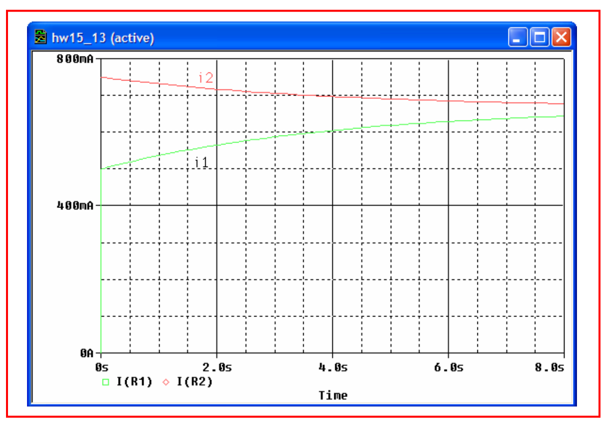
and $\mathbf{I}_2=\frac{1}{3}\left[\frac{(9\mathbf{s}+2)}{\mathbf{s}(4\mathbf{s}+1)}\right]=\frac{2}{3\mathbf{s}}+\frac{1}{12}\left(\frac{1}{\mathbf{s}+\frac{1}{4}}\right)$

Thus, taking the inverse Laplace transform, we obtain

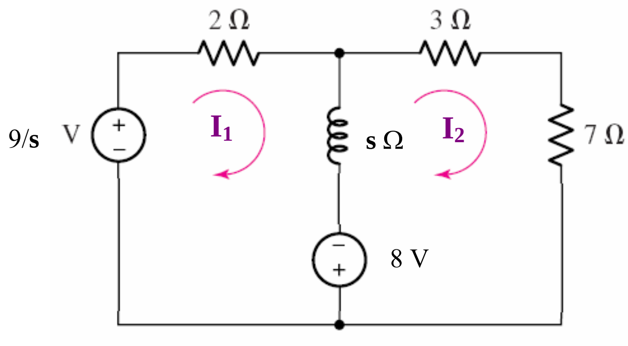
$$\boxed{i_1(t)=\frac{2}{3}-\frac{1}{6}e^{-t/4}\text{ A, } t\geq 0 \qquad \text{and} \qquad i_2(t)=\frac{2}{3}+\frac{1}{12}e^{-t/4}\text{ A, } t\geq 0}$$

(b)





14. (a) We transform the circuit into the **s**-domain, noting the initial condition of the inductor:



Writing the two required mesh equations:

$$(2+\mathbf{s})\mathbf{I}_1 - \mathbf{s}\mathbf{I}_2 = \frac{9}{\mathbf{s}} + 8 \qquad [1]$$

$$-\mathbf{s}\mathbf{I}_1 + (10+\mathbf{s})\mathbf{I}_2 = -8 \qquad [2]$$

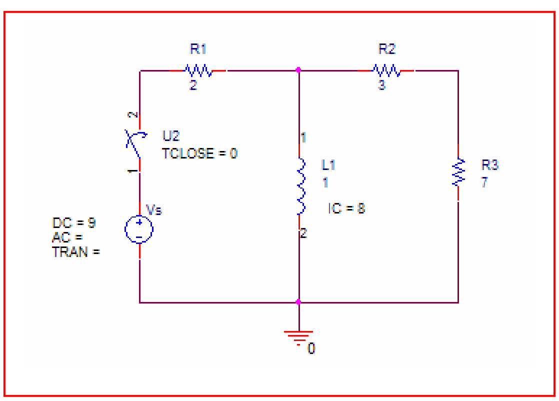
Solving yields $\mathbf{I}_1 = \frac{1}{12}\left[\frac{(89\mathbf{s}+90)}{\mathbf{s}\left(\mathbf{s}+5/3\right)}\right] = \frac{35}{12}\left(\frac{1}{\mathbf{s}+5/3}\right) + \frac{4.5}{\mathbf{s}}$

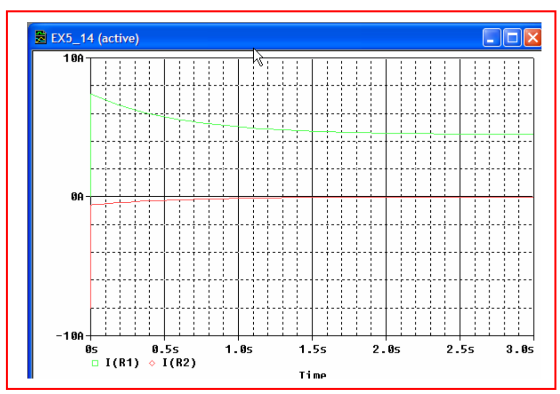
and $\mathbf{I}_2 = \frac{-7}{12}\left[\frac{1}{\left(\mathbf{s}+5/3\right)}\right] = -\frac{7}{12}\left(\frac{1}{\mathbf{s}+5/3}\right)$

Thus, taking the inverse Laplace transform, we obtain

$$\boxed{i_1(t) = \frac{35}{12}e^{-1.667t} + 4.5 \text{ A}, \ t \geq 0 \qquad \text{and} \qquad i_2(t) = -\frac{7}{12}e^{-1.667t} \text{ A}, \ t \geq 0}$$

(b)





15. $v(t) = 10e^{-2t}\cos(10t + 30^{\circ})$ V

$$\cos(10t + 30^{\circ}) \Leftrightarrow \frac{\mathbf{s}\cos 30^{\circ} - 10\sin 30^{\circ}}{\mathbf{s}^2 + 100} = \frac{0.866\mathbf{s} - 5}{\mathbf{s}^2 + 100}$$

$\mathcal{L}\{f(t)e^{-at}\} \Leftrightarrow$ F($\mathbf{s}$ + a), so

$$\mathbf{V}(\mathbf{s}) = 10\,\frac{0.866(\mathbf{s}+2) - 5}{(\mathbf{s}+2)^2 + 100} = \frac{8.66\mathbf{s} - 16.34}{\mathbf{s} + 100}$$

The voltage across the 5-Ω resistor may be found by simple voltage division. We first note that $\mathbf{Z}_{\text{eff}} = (10/\mathbf{s}) \,||\, 5 = \dfrac{50}{5\mathbf{s}+10}\,\Omega$. Thus,

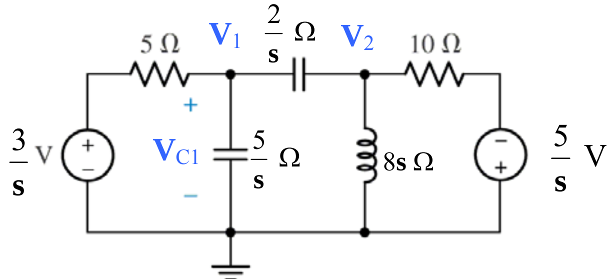
$$\mathbf{V}_{5\Omega} = \frac{\left(\dfrac{50}{5\mathbf{s}+10}\right)\mathbf{V}_s}{0.5\mathbf{s} + 5 + \left(\dfrac{50}{5\mathbf{s}+10}\right)} = \frac{50\,\mathbf{V}_s}{(0.5\mathbf{s}+5)\,(5\mathbf{s}+10) + 50} = \frac{50\,\mathbf{V}_s}{2.5\mathbf{s}^2 + 30\mathbf{s} + 100}$$

(a) $$\mathbf{I}_x = \frac{\mathbf{V}_{\text{eff}}}{5} = 40\,\frac{0.866\mathbf{s} - 3.268}{\left[(\mathbf{s}+2)^2 + 100\right]\left[\mathbf{s}^2 + 12\mathbf{s} + 40\right]} = \boxed{\frac{34.64\mathbf{s} - 130.7}{\left[(\mathbf{s}+2)^2 + 100\right]\left[(\mathbf{s}+6)^2 + 100\right]}}$$

(b) Taking the inverse transform using MATLAB, we find that

$$\boxed{i_x(t) = e^{-6t}\,[0.0915\cos 2t - 1.5245\sin 2t] - e^{-2t}\,[0.0915\cos 10t - 0.3415\sin 10t] \text{ A}}$$

16.



Node 1: $0 = 0.2\,(\mathbf{V}_1 - 3/\,\mathbf{s}) + 0.2\,\mathbf{V}_1\,\mathbf{s} + 0.5\,(\mathbf{V}_1 - \mathbf{V}_2)\,\mathbf{s}$

Node 2: $0 = 0.5\,(\mathbf{V}_2 - \mathbf{V}_1)\,\mathbf{s} + 0.125\,\mathbf{V}_2\,\mathbf{s} + 0.1\,(\mathbf{V}_2 + 5/\,\mathbf{s})$

Rewriting, $(3.5\,\mathbf{s}^2 + \mathbf{s})\,\mathbf{V}_1 + 2.5\,\mathbf{s}^2\,\mathbf{V}_2 = 3$ [1]

$-4\,\mathbf{s}^2\,\mathbf{V}_1 + (4\,\mathbf{s}^2 + 0.8\,\mathbf{s} + 1)\,\mathbf{V}_2 = -4$ [2]

Solving using MATLAB or substitution, we find that

$$\mathbf{V}_1(\mathbf{s}) = \frac{-20\mathbf{s}^2 + 16\mathbf{s} + 20}{40\mathbf{s}^4 + 68\mathbf{s}^3 + 43\mathbf{s}^2 + 10\mathbf{s}}$$

$$= \left(\frac{1}{40}\right)\frac{-20\mathbf{s}^2 + 16\mathbf{s} + 20}{\mathbf{s}(\mathbf{s} + 0.5457 - j0.3361)(\mathbf{s} + 0.5457 + j0.3361)(\mathbf{s} + 0.6086)}$$

which can be expanded:

$$\mathbf{V}_1(\mathbf{s}) = \frac{a}{\mathbf{s}} + \frac{b}{\mathbf{s} + 0.5457 - j0.3361} + \frac{b^*}{\mathbf{s} + 0.5457 + j0.3361} + \frac{c}{\mathbf{s} + 0.6086}$$

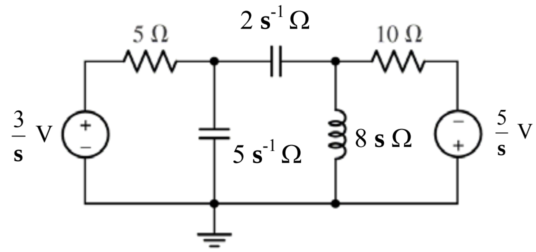
Using the method of residues, we find that

$a = 2$, $b = 2.511\,\angle 101.5^\circ$, $b^* = 2.511\angle{-101.5^\circ}$ and $c = -1.003$.

Thus,taking the inverse transform,

$$\boxed{v_1(t) = [2 - 1.003\,e^{-0.6086t} + 5.022\,e^{-0.5457t}\cos(0.3361t - 101.5^\circ)]\,u(t)\text{ V}}$$

17. With zero initial energy, we may draw the following circuit:



Define three clockwise mesh currents $\mathbf{I}_1$, $\mathbf{I}_2$, and $\mathbf{I}_3$ in the left, centre and right meshes, respectively.

Mesh 1: $-3/\mathbf{s} + 5\mathbf{I}_1 + (5/\mathbf{s})\mathbf{I}_1 - (5/\mathbf{s})\mathbf{I}_2 = 0$
Mesh 2: $-(5/\mathbf{s})\mathbf{I}_1 + (8\mathbf{s} + 7/\mathbf{s})\mathbf{I}_2 - 8\mathbf{s}\,\mathbf{I}_3 = 0$
Mesh 3: $-8\mathbf{s}\mathbf{I}_2 + (8\mathbf{s} + 10)\mathbf{I}_3 - 5/\mathbf{s} = 0$

Rewriting,

$$(5\mathbf{s} + 5)\,\mathbf{I}_1 - 5\,\mathbf{I}_2 = 3 \qquad [1]$$
$$-5\,\mathbf{I}_1 + (8\mathbf{s}^2 + 7)\,\mathbf{I}_2 - 8\mathbf{s}^2\,\mathbf{I}_3 = 0 \qquad [2]$$
$$-8\mathbf{s}^2\,\mathbf{I}_2 + (8\mathbf{s}^2 + 10\mathbf{s})\,\mathbf{I}_3 = 5 \qquad [3]$$

Solving, we find that

$$\mathbf{I}_2(\mathbf{s}) = \frac{20\mathbf{s}^2 + 32\mathbf{s} + 15}{40\mathbf{s}^3 + 68\mathbf{s}^2 + 43\mathbf{s} + 10} = \left(\frac{1}{40}\right)\frac{20\mathbf{s}^2 + 32\mathbf{s} + 15}{(\mathbf{s} + 0.6086)(\mathbf{s} + 0.5457 - j0.3361)(\mathbf{s} + 0.5457 + j0.3361)}$$

$$= \frac{a}{(\mathbf{s} + 0.6086)} + \frac{b}{(\mathbf{s} + 0.5457 - j0.3361)} + \frac{b^*}{(\mathbf{s} + 0.5457 + j0.3361)}$$

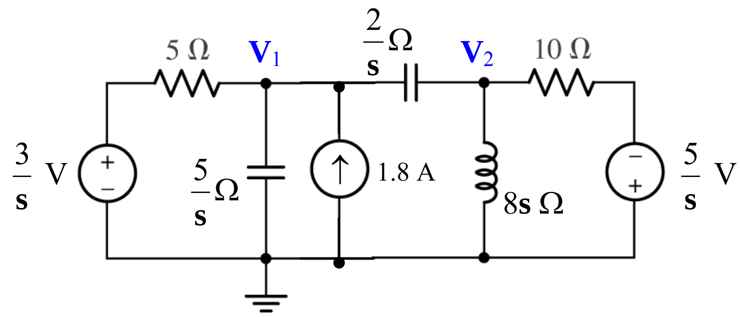
where $a = 0.6269$, $b = 0.3953\angle{-99.25^\circ}$, and $b^* = 0.3955\angle{+99.25^\circ}$

Taking the inverse tranform, we find that

$$i_2(t) = [0.6271e^{-0.6086t} + 0.3953e^{-j99.25^\circ}\, e^{(-0.5457 + j0.3361)t} + 0.3953e^{j99.25^\circ}\, e^{(-0.5457 - j0.3361)t}]u(t)$$

$$\boxed{= [0.6271e^{-0.6086t} + 0.7906\, e^{-0.5457t}\cos(0.3361t + 99.25^\circ)]\, u(t)}$$

18. We choose to represent the initial energy stored in the capacitor with a current source:



Node 1: $$1.8 = \frac{\mathbf{V}_1 - \frac{3}{\mathbf{s}}}{5} + \frac{\mathbf{s}}{5}\mathbf{V}_1 + \frac{\mathbf{s}}{2}(\mathbf{V}_1 - \mathbf{V}_2)$$

Node 2: $$0 = \frac{\mathbf{s}}{2}(\mathbf{V}_2 - \mathbf{V}_1) + \frac{1}{8\mathbf{s}}\mathbf{V}_2 + \frac{\mathbf{V}_2 + \frac{5}{\mathbf{s}}}{10}$$

Rewriting,
$$(5\mathbf{s}^2 + 4\mathbf{s})\,\mathbf{V}_1 - 5\mathbf{s}^2\,\mathbf{V}_2 = 18\mathbf{s} + 6 \qquad [1]$$
$$-4\mathbf{s}^2\,\mathbf{V}_1 + (4\mathbf{s}^2 + 0.8\mathbf{s} + 1)\mathbf{V}_2 = -4 \qquad [2]$$

Solving, we find that $$\mathbf{V}_1(\mathbf{s}) = \frac{360\mathbf{s}^3 + 92\mathbf{s}^2 + 114\mathbf{s} + 30}{\mathbf{s}(40\mathbf{s}^3 + 68\mathbf{s}^2 + 43\mathbf{s} + 10)}$$

$$= \frac{a}{\mathbf{s}} + \frac{b}{\mathbf{s} + 0.6086} + \frac{c}{\mathbf{s} + 0.5457 - j0.3361} + \frac{c^*}{\mathbf{s} + 0.5457 + j0.3361}$$

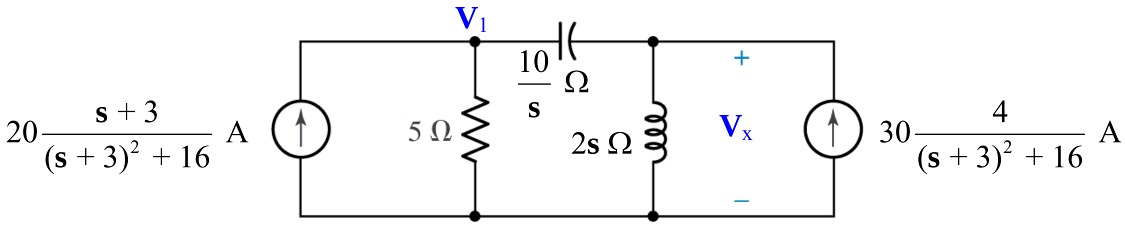
where a = 3, b = 30.37, c = 16.84 $\angle 136.3^\circ$ and c^* = 16.84 $\angle -136.3^\circ$

Taking the inverse transform, we find that

$$v_1(t) = [3 + 30.37e^{-0.6086t} + 16.84\,e^{j136.3^\circ}\,e^{-0.5457t}\,e^{j0.3361t} + 16.84\,e^{-j136.3^\circ}\,e^{-0.5457t}\,e^{-j0.3361t}]u(t) \text{ V}$$

$$= \boxed{[3 + 30.37e^{-0.6086t} + 33.68e^{-0.5457t}\cos(0.3361t + 136.3^\circ]u(t) \text{ V}}$$

19. We begin by assuming no initial energy in the circuit and transforming to the s-domain:



(a) via nodal analysis, we write:

$$\frac{20\mathbf{s}+60}{(\mathbf{s}+3)^2+16} = \frac{\mathbf{s}}{10}(\mathbf{V}_1 - \mathbf{V}_x) + \frac{\mathbf{V}_1}{5} \qquad [1] \quad \text{and}$$

$$\frac{120}{(\mathbf{s}+3)^2+16} = \frac{\mathbf{V}_x}{2\mathbf{s}} + \frac{\mathbf{s}}{10}(\mathbf{V}_x - \mathbf{V}_1) \qquad [2]$$

Collecting terms and solving for $\mathbf{V}_x(\mathbf{s})$, we find that

$$\mathbf{V}_x(\mathbf{s}) = \frac{200\mathbf{s}(\mathbf{s}^2+9\mathbf{s}+12)}{2\mathbf{s}^4+17\mathbf{s}^3+90\mathbf{s}^2+185\mathbf{s}+250}$$

$$= \boxed{\frac{200\mathbf{s}(\mathbf{s}^2+9\mathbf{s}+12)}{(\mathbf{s}+3-j4)(\mathbf{s}+3+j4)(\mathbf{s}+1.25-j1.854)(\mathbf{s}+1.25+j1.854)}}$$

(b) Using the method of residues, this function may be rewritten as

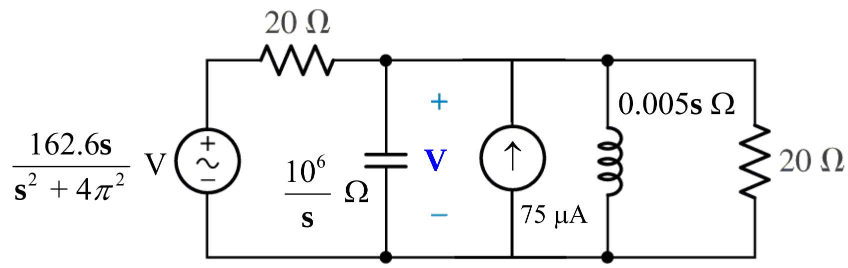
$$\frac{a}{(\mathbf{s}+3-j4)} + \frac{a^*}{(\mathbf{s}+3+j4)} + \frac{b}{(\mathbf{s}+1.25-j1.854)} + \frac{b^*}{(\mathbf{s}+1.25+j1.854)}$$

with $a = 92.57 \angle -47.58^\circ$, $a^* = 92.57 \angle 47.58^\circ$, $b = 43.14 \angle 106.8^\circ$, $b^* = 43.14 \angle -106.8^\circ$
Taking the inverse transform, then, yields

$$v_x(t) = [92.57\, e^{-j47.58^\circ} e^{-3t} e^{j4t} + 92.57\, e^{j47.58^\circ} e^{-3t} e^{-j4t} + 43.14 e^{j106.8^\circ} e^{-1.25t} e^{j1.854t} + 43.14 e^{-j106.8^\circ} e^{-1.25t} e^{-j1.854t}]\, u(t)$$

$$= \boxed{[185.1\, e^{-3t} \cos(4t - 47.58^\circ) + 86.28\, e^{-1.25t} \cos(1.854t + 106.8^\circ)]\, u(t)}$$

20. We model the initial energy in the capacitor as a 75-μA independent current source:



First, define $\mathbf{Z}_{\text{eff}} = 10^6/\mathbf{s} \parallel 0.005\mathbf{s} \parallel 20 = \dfrac{\mathbf{s}}{10^{-6}\mathbf{s}^2 + 0.005\mathbf{s} + 200}\ \Omega$

Then, writing a single KCL equation, $75\times10^{-6} = \dfrac{\mathbf{V}(\text{s})}{\mathbf{Z}_{\text{eff}}} + \dfrac{1}{20}\left(\mathbf{V}(\mathbf{s}) - \dfrac{162.6\mathbf{s}}{\mathbf{s}^2 + 4\pi^2}\right)$

which may be solved for $\mathbf{V}(\mathbf{s})$:

$$\mathbf{V}(\mathbf{s}) = \frac{75\mathbf{s}\left(\mathbf{s}^2 + 1.084\times10^5\mathbf{s} + 39.48\right)}{\mathbf{s}^4 + 5.5\times10^4\mathbf{s}^3 + 2\times10^8\mathbf{s}^2 + 2.171\times10^6\mathbf{s} + 7.896\times10^9}$$

$$= \frac{75\mathbf{s}\left(\mathbf{s}^2 + 1.084\times10^5\mathbf{s} + 12.57\right)}{(\mathbf{s}+51085)(\mathbf{s}+3915)(\mathbf{s}-j6.283)(\mathbf{s}+j6.283)}$$

(NOTE: factored with higher-precision denominator coefficients using MATLAB to obtain accurate complex poles: otherwise, numerical error led to an exponentially growing pole i.e. real part of the pole was positive)

$$= \frac{a}{(\mathbf{s}+51085)} + \frac{b}{(\mathbf{s}+3915)} + \frac{c}{(\mathbf{s}-j2\pi)} + \frac{c^*}{(\mathbf{s}+j2\pi)}$$

where a = -91.13, b = 166.1, c = $0.1277\angle 89.91^\circ$ and $c^* = 0.1277\angle{-89.91^\circ}$.
Thus, consolidating the complex exponential terms (the imaginary components cancel),

$$\boxed{v(t) = [-91.13e^{-51085t} + 166.1e^{-3915t} + 0.2554\cos(2\pi t + 89.91^\circ)]\,u(t)\ \text{V}}$$

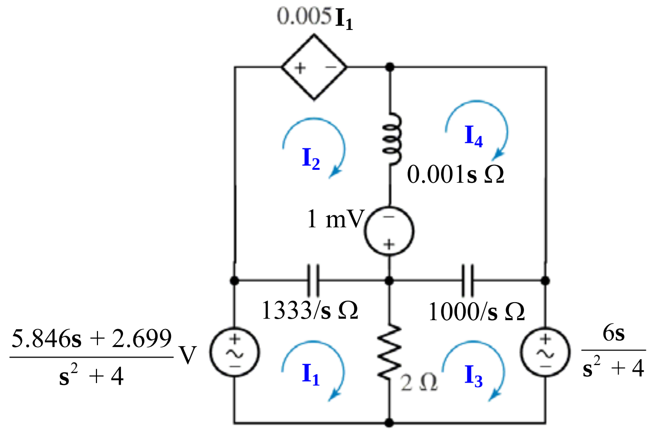
(b) The steady-state voltage across the capacitor is $\mathbf{V} = [255.4\cos(2\pi t + 89.91^\circ)]$ mV This can be written in phasor notation as $0.2554\ \angle 89.91^\circ$ V. The impedance across which this appears is $\mathbf{Z}_{\text{eff}} = [j\omega C + 1/j\omega L + 1/20]^{-1} = 0.03142\ \angle 89.91^\circ\ \Omega$, so $\mathbf{I}_{\text{source}} = \mathbf{V}/\mathbf{Z}_{\text{eff}} = 8.129\angle{-89.91^\circ}$ A.

Thus, $i_{\text{source}} = \boxed{8.129\cos 2\pi t\ \text{A}.}$

(c) By phasor analysis, we can use simple voltage division to find the voltage division to find the capacitor voltage:

$$\mathbf{V}_C(j\omega) = \frac{(162.6\angle 0)(0.03142\angle 89.91^\circ)}{20 + 0.03142\angle 89.91^\circ} = 0.2554\angle 89.92^\circ\ \text{V}$$ which agrees with our answer to (a), assuming steady state. Dividing by $0.03142\ \angle 89.91^\circ\ \Omega$, we find $i_{\text{source}} = 8.129\cos 2\pi t$ A.

21. Only the inductor appears to have initial energy, so we model that with a voltage source:



Mesh 1: $$\frac{5.846\mathbf{s}+2.699}{\mathbf{s}^2+4} = \left(2+\frac{1333}{\mathbf{s}}\right)\mathbf{I}_1 - \frac{1333}{\mathbf{s}}\mathbf{I}_2 - 2\mathbf{I}_3$$

Mesh 2: $0 = 0.005\mathbf{I}_1 - 0.001 + (0.001\mathbf{s} + 1333/\mathbf{s})\,\mathbf{I}_2 - (1333/\mathbf{s})\mathbf{I}_1 - 0.001\mathbf{s}\mathbf{I}_4$

Mesh 3: $$0 = (2 + 1000/\mathbf{s})\mathbf{I}_3 - 2\mathbf{I}_1 - (1000/\mathbf{s})\mathbf{I}_4 + \frac{6\mathbf{s}}{\mathbf{s}^2+4}$$

Mesh 4: $0 = (0.001\mathbf{s} + 1000/\mathbf{s})\,\mathbf{I}_4 - 0.001\mathbf{s}\mathbf{I}_2 - (1000/\mathbf{s})\mathbf{I}_3 + 0.001$

Solving, we find that $\mathbf{I}_1 = -0.2\dfrac{154\mathbf{s}-2699}{\mathbf{s}^2+4}$ and

$$\mathbf{I}_2 = 0.001\frac{154\mathbf{s}^4 - 7.378\times10^7\mathbf{s}^3 - 1.912\times10^{10}\mathbf{s}^2 - 4.07\times10^{13}\mathbf{s} + 7.196\times10^{14}}{2333\mathbf{s}^4 + 6.665\times10^5\mathbf{s}^3 + 1.333\times10^9\mathbf{s}^2 + 5.332\times10^9}$$

$$= \frac{0.4328\angle-166.6^\circ}{\mathbf{s}+142.8+j742} + \frac{0.4328\angle+166.6^\circ}{\mathbf{s}+142.8-j742}$$

$$+ \frac{135.9\angle-96.51^\circ}{\mathbf{s}-j2} + \frac{135.9\angle+96.51^\circ}{\mathbf{s}+j2} + 6.6\times10^{-5}$$

Taking the inverse transform of each,

$i_1(t) = 271.7\cos(2t - 96.51^\circ)$ A and

$i_2(t) = 0.8656\, e^{-142.8t}\cos(742.3t + 166.6^\circ) + 271.8\cos(2t - 96.51^\circ) + 6.6\times10^{-5}\,\delta(t)$ A

Verifying via phasor analysis, we again write four mesh equations:

$6\angle-13^\circ = (2 - j666.7)\mathbf{I}_1 + j667\mathbf{I}_2 - 2\mathbf{I}_3$

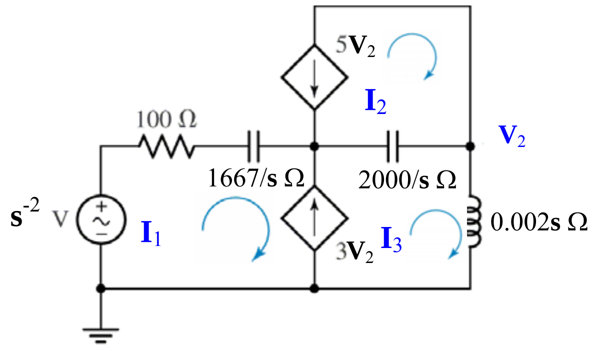
$0 = (0.005 + j666.7)\mathbf{I}_1 + (j2\text{x}10^{-3} - j666.7)\mathbf{I}_2 - j2\times10^{-3}\mathbf{I}_4$

$-6\angle0 = -2\mathbf{I}_1 + (2 - j500)\mathbf{I}_3 + j500\mathbf{I}_4$

$0 = -j2\times10^{-3}\mathbf{I}_2 + j500\mathbf{I}_3 + (j2\times10^{-3} - j500)\mathbf{I}_4$

Solving, we find $\mathbf{I}_1 = 271.7\angle-96.5^\circ$ A and $\mathbf{I}_2 = 272\angle-96.5^\circ$ A. From the Laplace analysis, we see that this agrees with our expression for $i_1(t)$, and as $t \to \infty$, our expression for $i_2(t)$ $\to 272\cos(2t - 96.5^\circ)$ in agreement with the phasor analysis.

22. With no initial energy storage, we simply convert the circuit to the s-domain:



Writing a supermesh equation,

$$\frac{1}{\mathbf{s}^2} = 100\mathbf{I}_1 + \frac{1}{6\times10^{-4}\mathbf{s}}\mathbf{I}_1 + \frac{2000}{\mathbf{s}}\mathbf{I}_3 + 0.002\mathbf{s}\mathbf{I}_3 - \frac{2000}{\mathbf{s}}\mathbf{I}_2$$

we next note that $\mathbf{I}_2 = -5\mathbf{V}_2 = -5(0.002\mathbf{s})\mathbf{I}_3 = -0.01\mathbf{s}\mathbf{I}_3$
and $\mathbf{I}_3 - \mathbf{I}_1 = 3\mathbf{V}_2 = 0.006\mathbf{s}\mathbf{I}_3$, or $\mathbf{I}_1 = (1 - 0.006\mathbf{s})\mathbf{I}_3$, we may write

$$\mathbf{I}_3 = \frac{1}{-0.598\mathbf{s}^3 + 110\mathbf{s}^2 + 3666\mathbf{s}}$$

$$\mathbf{V}_2(\mathbf{s}) = \mathbf{I}_3/\,0.002\mathbf{s} = \frac{1}{-0.0012\mathbf{s}^4 + 0.22\mathbf{s}^3 + 7.332\mathbf{s}^2}$$

$$= -\frac{7.645\times10^{-5}}{\mathbf{s} - 212.8} + \frac{4.167\times10^{-3}}{\mathbf{s} + 28.82} - \frac{4.091\times10^{-3}}{\mathbf{s}} + \frac{0.1364}{\mathbf{s}^2}$$

Taking the inverse transform,

$$v_2(t) = -7.645\times10^{-5}\,e^{212.8t} + 4.167\times10^{-3}\,e^{-28.82t} - 4.091\times10^{-3} + 0.1364\,t]\,u(t)\text{ V}$$

(a) $v_2(1\text{ ms}) =$ -5.58×10^{-7} V
(b) $v_2(100\text{ ms}) =$ -1.334×10^{5} V
(c) $v_2(10\text{ s}) =$ -1.154×10^{920} V. This is pretty big- best to start running.

23. We need to write three mesh equations:

Mesh 1: $$\frac{5.846\mathbf{s} + 2.699}{\mathbf{s}^2 + 4} = \left(2 + \frac{1333}{\mathbf{s}}\right)\mathbf{I}_1 - 2\mathbf{I}_3$$

Mesh 3: $$0 = (2 + 1000/\mathbf{s})\mathbf{I}_3 - 2\mathbf{I}_1 - (1000/\mathbf{s})\mathbf{I}_4 + \frac{6\mathbf{s}}{\mathbf{s}^2 + 4}$$

Mesh 4: $$0 = (0.001\mathbf{s} + 1000/\mathbf{s})\,\mathbf{I}_4 - (1000/\mathbf{s})\mathbf{I}_3 + 10^{-6}$$

Solving,

$$\mathbf{I}_1 = -0.001\mathbf{s}\frac{\left(154\mathbf{s}^3 - 2.925\times10^6\mathbf{s}^2 + 1.527\times10^8\mathbf{s} - 2.699\times10^9\right)}{2333\mathbf{s}^4 + 6.665\times10^5\mathbf{s}^3 + 1.333\times109\mathbf{s}^2 + 2.666\times10^6\mathbf{s} + 5.332\times10^9}$$

$$= \frac{0.6507\angle12.54^\circ}{\mathbf{s} + 142.8 - j742.3} + \frac{0.6507\angle-12.54^\circ}{\mathbf{s} + 142.8 + j742.3}$$

$$+ \frac{0.00101\angle-6.538^\circ}{\mathbf{s} - j2} + \frac{0.00101\angle6.538^\circ}{\mathbf{s} + j2} - 6.601\times10^{-5}$$

which corresponds to
$i_1(t) = 1.301\ e^{-142.8t}\cos(742.3t + 12.54^\circ) + 0.00202\cos(2t - 6.538^\circ) - 6.601\times10^{-5}\ \delta(t)$ A

and

$$\mathbf{I}_3 = -0.001\frac{\left(154\mathbf{s}^4 + 3.997\times10^6\mathbf{s}^3 + 1.547\times10^8\mathbf{s}^2 + 3.996\times10^{12}\mathbf{s} - 2.667\times10^6\right)}{\left(\mathbf{s}^2 + 4\right)\left(2333\mathbf{s}^2 + 6.665\times10^5\mathbf{s} + 1.333\times10^9\right)}$$

$$= \frac{0.7821\angle-33.56^\circ}{\mathbf{s} + 142.8 - j742.3} + \frac{0.7821\angle33.56^\circ}{\mathbf{s} + 142.8 + j742.3}$$

$$+ \frac{1.499\angle179.9^\circ}{\mathbf{s} - j2} + \frac{1.499\angle-179.9^\circ}{\mathbf{s} + j2}$$

which corresponds to
$i_3(t) = 1.564\ e^{-142.8t}\cos(742.3t - 33.56^\circ) + 2.998\cos(2t + 179.9^\circ)$ A

The power absorbed by the 2-Ω resistor, then, is $2\left[i_1(t) - i_3(t)\right]^2$ or

$$p(t) = \boxed{\begin{array}{l} 2[1.301\ e^{-142.8t}\cos(742.3t + 12.54^\circ) + 0.00202\cos(2t - 6.538^\circ) - 6.601\times10^{-5}\ \delta(t) \\ \quad - 1.564\ e^{-142.8t}\cos(742.3t - 33.56^\circ) - 2.998\cos(2t + 179.9^\circ)]^2\ \text{W} \end{array}}$$

24. (a) We first define $\mathbf{Z}_{eff} = R_B \parallel r_\pi \parallel (1/\mathbf{s}C_\pi) = \dfrac{r_\pi R_B}{r_\pi + R_B + r_\pi R_B C_\pi \mathbf{s}}$. Writing two nodal equations, then, we obtain:

$$0 = (\mathbf{V}_\pi - \mathbf{V}_S)/R_S + \mathbf{V}_\pi (r_\pi + R_B + r_\pi R_B C_\pi \mathbf{s})/r_\pi R_B + (\mathbf{V}_\pi - \mathbf{V}_o)C_\mu \mathbf{s}$$

and

$$-g_m \mathbf{V}_\pi = \mathbf{V}_o(R_C + R_L)/R_C R_L + (\mathbf{V}_o - \mathbf{V}_p) C_\mu \mathbf{s}$$

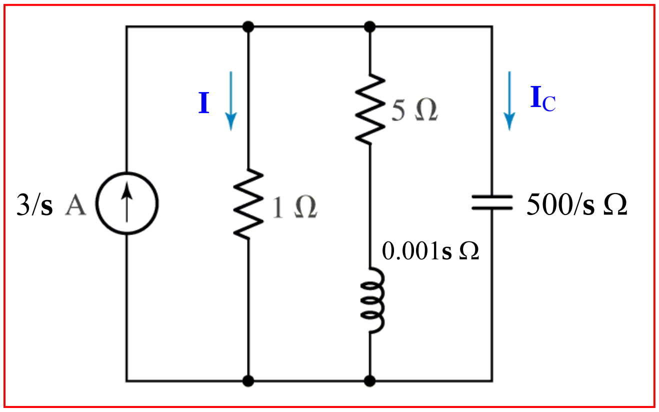
Solving using MATLAB, we find that

$$\boxed{\begin{aligned}\frac{\mathbf{V}_o}{\mathbf{V}_s} = {} & r_\pi R_B R_C R_L(-g_m + C_\mu \mathbf{s})\,[R_s r_\pi R_B R_C R_L C_\pi C_\mu \mathbf{s}^2 + (R_s r_\pi R_B R_C C_\pi + R_s r_\pi R_B R_C C_\mu \\ & + R_s r_\pi R_B R_L C_\pi + R_s r_\pi R_B R_L C_\mu + r_\pi R_B R_C R_L C_\mu + R_s r_\pi R_C R_L C_\mu \\ & + R_s R_B R_C R_L C_\mu + g_m R_s r_\pi R_B R_C R_L C_\mu)\mathbf{s} \\ & + r_\pi R_B R_C + R_s r_\pi R_C + R_s R_B R_C + r_\pi R_B R_L + R_s r_\pi R_L + R_s R_B R_L]^{-1}\end{aligned}}$$

(b) Since we have only two energy storage elements in the circuit, the maximum number of poles would be two. The capacitors cannot be combined (either series or in parallel), so we expect a second-order denominator polynomial, which is what we found in part (a).

25.

(a)



(b) $\mathbf{Z}_{TH} = (5 + 0.001\mathbf{s}) \,||\, (500/\,\mathbf{s}) = \boxed{\dfrac{2500\mathbf{s} + 0.5}{0.001\mathbf{s}^2 + 5\mathbf{s} + 500}\ \Omega}$

$$\mathbf{V}_{TH} = (3/\,\mathbf{s})\mathbf{Z}_{TH} = \boxed{\frac{7.5\times10^6\mathbf{s} + 1500}{\mathbf{s}\left(\mathbf{s}^2 + 5000\mathbf{s} + 5\times10^5\right)}\ \text{V}}$$

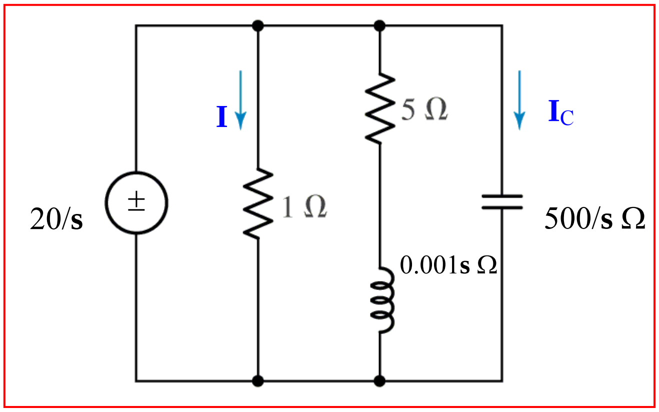
(c) $$\mathbf{V}_{1\Omega} = \mathbf{V}_{TH}\frac{1}{1+\mathbf{Z}_{TH}} = \frac{7.5\times10^6\mathbf{s} + 1500}{\mathbf{s}\left(\mathbf{s}^2 + 505000\right)\left(1 + \dfrac{2500\mathbf{s} + 0.5}{0.001\mathbf{s}^2 + 5\mathbf{s} + 500}\right)}$$

$$= -\frac{2.988}{\mathbf{s} + 2.505\times10^6} + \frac{10.53\angle{-89.92^\circ}}{\mathbf{s} + j710.6} + \frac{10.53\angle{+89.92^\circ}}{\mathbf{s} - j710.6} + \frac{2.956}{\mathbf{s} + 0.1998} + \frac{2.967\times10^{-3}}{\mathbf{s}}$$

Thus, $i_{1\Omega} = v_{1\Omega}(t) = \boxed{[-2.988\, e^{-2.505\times10^6 t} + 2.956\, e^{-0.1998t} + 2.967\times10^{-3} + 21.06\cos(710.6t + 89.92^\circ)]\, u(t)}$

26.

(a)



(b) $\boxed{\mathbf{Z}_{TH} = 0}$, $\mathbf{V}_{TH} = 20/\mathbf{s}$ V so $\boxed{\mathbf{I}_N = \infty}$

(c) $\mathbf{I}_C = \dfrac{\left(\dfrac{20}{\mathbf{s}}\right)}{\left(\dfrac{500}{\mathbf{s}}\right)} = 0.04 \text{ A}$. Taking the inverse transform, we obtain a delta function:

$\boxed{i_C(t) = 40\delta(t) \text{ mA.}}$

This "unphysical" solution arises from the circuit above attempting to force the voltage across the capacitor to change in zero time.

27. $$\mathbf{V}_{TH} = \left(\frac{7}{\mathbf{s}}\right)\left(\frac{\left(3+\frac{1}{\mathbf{s}}\right)\|10\mathbf{s}}{3+\left(3+\frac{1}{\mathbf{s}}\right)\|10\mathbf{s}}\right)\left(\frac{\frac{1}{\mathbf{s}}}{3+\frac{1}{\mathbf{s}}}\right) = \boxed{\frac{70}{60\mathbf{s}^2+19\mathbf{s}+3}\ \text{V}}$$

$$\mathbf{Z}_{TH} = \left(\frac{1}{\mathbf{s}}\right)\|\left(3+\frac{30\mathbf{s}}{3+10\mathbf{s}}\right) = \frac{\left(\frac{1}{\mathbf{s}}\right)\left(\frac{9+60\mathbf{s}}{3+10\mathbf{s}}\right)}{\frac{1}{\mathbf{s}}+\frac{9+60\mathbf{s}}{3+10\mathbf{s}}} = \frac{9+60\mathbf{s}}{60\mathbf{s}^2+19\mathbf{s}+3}\ \Omega$$

$$\mathbf{Z}_{Total} = \frac{9+60\mathbf{s}}{60\mathbf{s}^2+19\mathbf{s}+3}+7\mathbf{s}^2 = \boxed{\frac{420\mathbf{s}^4+133\mathbf{s}^3+21\mathbf{s}^2+60\mathbf{s}+9}{60\mathbf{s}^2+19\mathbf{s}+3}\ \Omega}$$

Thus,

$$\mathbf{I}(\mathbf{s}) = \left(\frac{70}{60\mathbf{s}^2+19\mathbf{s}+3}\right)\left(\frac{60\mathbf{s}^2+19\mathbf{s}+3}{420\mathbf{s}^4+133\mathbf{s}^3+21\mathbf{s}^2+60\mathbf{s}+9}\right)\ \text{A}$$

$$\boxed{= \frac{70}{420\mathbf{s}^4+133\mathbf{s}^3+21\mathbf{s}^2+60\mathbf{s}+9}\ \text{A}}$$

28. We begin by noting that the source is not really a dependent source – it's value is not based on a voltage or current parameter. Therefore, we should treat it as an independent source.

$$\mathbf{Z}_{th} = \frac{2}{\mathbf{s}} \,||\, (2\mathbf{s}+10) = \frac{\frac{2}{\mathbf{s}}(2\mathbf{s}+10)}{\frac{2}{\mathbf{s}}+(2\mathbf{s}+10)} = \boxed{\frac{2\mathbf{s}+10}{\mathbf{s}^2+5\mathbf{s}+1}\ \Omega}$$

$$\mathbf{V}_{th} = \left(\frac{\frac{2}{\mathbf{s}}}{10+2\mathbf{s}+\frac{2}{\mathbf{s}}}\right)\left[\left(\frac{9}{\mathbf{s}}\right)(10)\right] = \boxed{\frac{90}{\mathbf{s}(\mathbf{s}^2+5\mathbf{s}+1)}\ \text{V}}$$

29. Beginning with the source on the left (10/**s** V) we write two nodal equations:

$$\left(\mathbf{V}_1' - \frac{10}{\mathbf{s}}\right)\frac{1}{47000} + \frac{\mathbf{s}}{30303}\mathbf{V}_1' + \frac{\mathbf{V}_1' - \mathbf{V}_2'}{56 + 336\times10^{-6}\mathbf{s}} = 0$$

$$\frac{\mathbf{V}_2'}{47000} + \frac{\mathbf{s}}{10870}\mathbf{V}_2' + \frac{\mathbf{V}_2' - \mathbf{V}_1'}{56 + 336\times10^{-6}\mathbf{s}} = 0$$

Solving,

$$\mathbf{V}_1' = \frac{303030(0.3197\times10^{13} + 0.1645\times10^{11}\mathbf{s} + 98700\mathbf{s}^2)}{\mathbf{s}(0.4639\times10^{10}\mathbf{s}^3 + 0.7732\times10^{15}\mathbf{s}^2 + 0.5691\times10^{18}\mathbf{s} + 0.1936\times10^{18})}$$

$$\mathbf{V}_2' = \frac{0.9676\times10^{18}}{\mathbf{s}(0.4639\times10^{10}\mathbf{s}^3 + 0.7732\times10^{15}\mathbf{s}^2 + 0.5691\times10^{18}\mathbf{s} + 0.1936\times10^{18})}$$

Shorting out the left source and activating the right-hand source (5 – 3/**s**) V:

$$\frac{1}{47000}\mathbf{V}_1'' + \frac{\mathbf{s}}{30303}\mathbf{V}_1'' + \frac{\mathbf{V}_1'' - \mathbf{V}_2''}{56 + 336\times10^{-6}\mathbf{s}} = 0$$

$$\frac{\mathbf{V}_2'' - 5 + \frac{3}{\mathbf{s}}}{47000} + \frac{\mathbf{s}}{10870}\mathbf{V}_2'' + \frac{\mathbf{V}_2'' - \mathbf{V}_1''}{56 + 336\times10^{-6}\mathbf{s}} = 0$$

Solving,

$$\mathbf{V}_1'' = \frac{0.9676\times10^{17}(5\mathbf{s} - 3)}{\mathbf{s}(0.4639\times10^{10}\mathbf{s}^3 + 0.7732\times10^{15}\mathbf{s}^2 + 0.5691\times10^{18}\mathbf{s} + 0.1936\times10^{18})}$$

$$\mathbf{V}_2'' = \frac{7609(705000\mathbf{s}^3 + 0.1175\times10^{12}\mathbf{s}^2 + 0.6359\times10^{14}\mathbf{s} - 0.3819\times10^{14})}{\mathbf{s}(0.4639\times10^{10}\mathbf{s}^3 + 0.7732\times10^{15}\mathbf{s}^2 + 0.5691\times10^{18}\mathbf{s} + 0.1936\times10^{18})}$$

Adding, we find that

$$\mathbf{V}_1 = \frac{30303(0.2239\times10^{13} + 0.1613\times10^{13}\mathbf{s} + 98700\mathbf{s}^2)}{\mathbf{s}(0.4639\times10^{10}\mathbf{s}^3 + 0.7732\times10^{15}\mathbf{s}^2 + 0.5691\times10^{18}\mathbf{s} + 0.1936\times10^{18})}$$

$$\mathbf{V}_2 = \frac{7609(705000\mathbf{s}^3 + 0.1175\times10^{12}\mathbf{s}^2 + 0.6359\times10^{14}\mathbf{s} + 0.8897\times10^{14})}{\mathbf{s}(0.4639\times10^{10}\mathbf{s}^3 + 0.7732\times10^{15}\mathbf{s}^2 + 0.5691\times10^{18}\mathbf{s} + 0.1936\times10^{18})}$$

(b) Using the *ilaplace*() routine in MATLAB, we take the inverse transform of each:

$$v_1(t) = [3.504 + 0.3805\times10^{-2}\,e^{-165928t} - 0.8618\,e^{-739t} - 2.646\,e^{-0.3404t}]\,u(t)\ \text{V}$$

$$v_2(t) = [3.496 - 0.1365\times10^{-2}\,e^{-165928t} + 0.309\,e^{-739t} - 2.647\,e^{-0.3404t}]\,u(t)\ \text{V}$$

30. $(10/\mathbf{s})(1/47000) = 2.128\times10^{-4}/\mathbf{s}$ A
$(5 - 3/\mathbf{s})/47000 = (1.064 - 0.6383/\mathbf{s})\times10^{-4}$ A

$$\mathbf{Z}_L = 47000 \parallel (30303/\mathbf{s}) = \frac{1.424\times10^9}{47000\mathbf{s} + 30303}\,\Omega$$

$$\mathbf{Z}_R = 47000 \parallel (10870/\mathbf{s}) = \frac{5.109\times10^8}{47000\mathbf{s} + 10870}\,\Omega$$

Convert these back to voltage sources, one on the left ($\mathbf{V}_L$) and one on the right ($\mathbf{V}_R$):

$$\mathbf{V}_L = (2.128\times10^{-4}/\mathbf{s})\left(\frac{1.424\times10^9}{47000\mathbf{s} + 30303}\right) = \frac{3.0303\times10^5}{\mathbf{s}(47000\mathbf{s} + 30303)}\text{ V}$$

$$\mathbf{V}_R = (1.064 - 0.6383/\mathbf{s})\times10^{-4}\left(\frac{5.109\times10^8}{47000\mathbf{s} + 10870}\right)$$

$$= \frac{54360}{47000\mathbf{s} + 10870} - \frac{32611}{\mathbf{s}(47000\mathbf{s} + 10870)}$$

Then, $\mathbf{I}_{56\Omega} = \dfrac{\mathbf{V}_L - \mathbf{V}_R}{\mathbf{Z}_L + \mathbf{Z}_R + 336\times10^{-6}\mathbf{s} + 56}$

$$= -6250\frac{2.555\times10^9\mathbf{s}^2 - 1.413\times10^{10}\mathbf{s} - 4.282\times10^9}{\mathbf{s}\left(4.639\times10^9\mathbf{s}^3 + 7.732\times10^{14}\mathbf{s}^2 + 5.691\times10^{17}\mathbf{s} + 1.936\times10^{17}\right)}$$

$$= \frac{0.208}{\mathbf{s} + 1.659\times10^5} - \frac{0.0210}{\mathbf{s} + 739} - \frac{1.533\times10^{-18}}{\mathbf{s} + 0.6447}$$
$$+ \frac{2.658\times10^{-5}}{\mathbf{s} + 0.3404} + \frac{2.755\times10^{-18}}{\mathbf{s} + 0.2313} + \frac{1.382\times10^{-4}}{\mathbf{s}}$$

Thus,

$i_{56\Omega}(t)$ = $[0.208 \exp(-1.659\times10^5 t) - 0.0210 \exp(-739t) - 1.533\times10^{-18} \exp(-.06447t)$
$+ 2.658\times10^{-5} \exp(-0.3404t) + 2.755\times10^{-18} \exp(-0.2313t) + 1.382\times10^{-4}]\,u(t)$ A.

The power absorbed in the 56-Ω resistor is simply $56\,[i_{56\Omega}(t)]^2$ or

$$\boxed{\begin{aligned}&56\,[0.208 \exp(-1.659\times10^5 t) - 0.0210 \exp(-739t) - 1.533\times10^{-18} \exp(-.06447t)\\&+ 2.658\times10^{-5} \exp(-0.3404t) + 2.755\times10^{-18} \exp(-0.2313t) + 1.382\times10^{-4}]^2 \text{ W}\end{aligned}}$$

31. (a) Begin by finding $\mathbf{Z}_{TH} = \mathbf{Z}_N$:

$$\mathbf{Z}_{TH} = 47000 + (30303/\,\mathbf{s}) \,||\, [336\times10^{-6}\,\mathbf{s} + 56 + (10870/\,\mathbf{s}) \,||\, 47000]$$

$$= \boxed{\frac{4.639\times109\mathbf{s}^3 + 7.732\times10^{14}\mathbf{s}^2 + 5.691\times10^{17}\mathbf{s} + 1.936\times10^{17}}{98700\mathbf{s}^3 + 1.645\times10^{10}\mathbf{s}^2 + 1.21\times10^{13}\mathbf{s} + 2.059\times10^{12}}\ \Omega}$$

To find the Norton source value, define three clockwise mesh currents $\mathbf{I}_1$, $\mathbf{I}_2$ and $\mathbf{I}_3$ in the left, centre and right hand meshes, such that $\mathbf{I}_N(\mathbf{s}) = -\mathbf{I}_1(\mathbf{s})$ and the 10/**s** source is replaced by a short circuit.

$$(47000 + 30303/\,\mathbf{s})\,\mathbf{I}_1 - (30303/\,\mathbf{s})\,\mathbf{I}_2 = 0$$
$$(10870/\,\mathbf{s} + 56 + 336\times10^{-6}\,\mathbf{s} + 30303/\,\mathbf{s})\,\mathbf{I}_2 - (30303/\,\mathbf{s})\,\mathbf{I}_1 - (10870/\,\mathbf{s})\mathbf{I}_3 = 0$$
$$(47000 + 10870/\,\mathbf{s})\,\mathbf{I}_3 - (10870/\,\mathbf{s})\mathbf{I}_2 = -5 + 3/\,\mathbf{s}$$

Solving,

$$\mathbf{I}_N = -\mathbf{I}_1 = \boxed{\frac{2.059\times10^{12}(5\mathbf{s} - 3)}{\mathbf{s}(4.639\times10^{9}\mathbf{s}^3 + 7.732\times10^{14}\mathbf{s}^2 + 5.691\times10^{17}\mathbf{s} + 1.936\times10^{17})}}$$

(b) $\mathbf{I}_{source} = (10/\,\mathbf{s})\,(1/\,\mathbf{Z}_{TH}) - \mathbf{I}_N(\mathbf{s})$

$$= 0.001(0.4579\times10^{13}\mathbf{s}^6 + 0.1526\times10^{19}\mathbf{s}^5 + 0.1283\times10^{24}\mathbf{s}^4 + 0.1792\times10^{27}\mathbf{s}^3$$
$$+ 0.6306\times10^{29}\mathbf{s}^2 + 0.3667\times10^{29}\mathbf{s} + 0.5183\times10^{28})[\mathbf{s}(4639\mathbf{s}^3 + 0.7732\times10^{9}\mathbf{s}^2$$
$$+ 0.5691\times1012\mathbf{s} + 0.1936\times10^{12})(0.4639\times10^{10}\mathbf{s}^3 + 0.7732\times10^{15}\mathbf{s}^2 + 0.5691\times10^{18}\mathbf{s}$$
$$+ 0.1936\times10^{18})]^{-1}$$

Taking the inverse transform using the MATLAB *ilaplace*() routine, we find that

$$i_{source}(t) = 0.1382\times10^{-3} + 0.8607\times10^{-8}\exp(-165930t) + 0.8723\times10^{-7}\exp(-739t)$$
$$+ 0.1063\times10^{-3}\exp(-0.3403t) - 0.8096\times10^{-7}\exp(-165930t)$$
$$+ 0.1820\times10^{-4}\exp(-739t) - 0.5\times10^{-4}\exp(-0.3404t)$$

$$i_{source}(1.5\text{ ms}) = 2.0055\times10^{-4}\text{ A} = \boxed{200.6\ \mu\text{A}}$$

32. We begin by shorting the 7 cos 4t source, and replacing the 5 cos 2t source with $\dfrac{5\mathbf{s}}{\mathbf{s}^2+4}$.

(a) Define four clockwise mesh currents $\mathbf{I}_1$, $\mathbf{I}_2$, $\mathbf{I}_3$ and $\mathbf{I}_x$ in the top left, top right, bottom left and bottom right meshes, respectively. Then,

$$\frac{5\mathbf{s}}{\mathbf{s}^2+4} = (12 + 1/2\mathbf{s})\,\mathbf{I}_3 - 7\,\mathbf{I}_1 - (1/2\mathbf{s})\,\mathbf{I}_x \qquad [1]$$
$$0 = -4\,\mathbf{I}_x + (9.5 + \mathbf{s})\,\mathbf{I}_1 - \mathbf{s}\,\mathbf{I}_2 - 7\,\mathbf{I}_3 \qquad [2]$$
$$0 = (3 + \mathbf{s} + 2/\mathbf{s})\,\mathbf{I}_2 - \mathbf{s}\,\mathbf{I}_1 - 3\,\mathbf{I}_x \qquad [3]$$
$$0 = (4 + 3\mathbf{s} + 1/2\mathbf{s})\,\mathbf{I}_x - 3\,\mathbf{I}_2 - (1/2\mathbf{s})\,\mathbf{I}_3 \qquad [4]$$
$$\mathbf{V}_1' = (\mathbf{I}_3 - \mathbf{I}_x)\,(2\mathbf{s}) \qquad [5]$$

Solving all five equations simultaneously using MATLAB, we find that

$$\mathbf{V}_1' = \frac{20\mathbf{s}^3(75\mathbf{s}^3 + 199\mathbf{s}^2 + 187\mathbf{s} + 152)}{1212\mathbf{s}^6 + 3311\mathbf{s}^5 + 7875\mathbf{s}^4 + 15780\mathbf{s}^3 + 12408\mathbf{s}^2 + 10148\mathbf{s} + 1200}$$

Next we short the 5 cos 2t source, and replace the 7 cos 4t source with $\dfrac{7\mathbf{s}}{\mathbf{s}^2+16}$.

Define four clockwise mesh currents $\mathbf{I}_1$, $\mathbf{I}_2$, $\mathbf{I}_3$ and $\mathbf{I}_x$ in the bottom left, top left, top right and bottom right meshes, respectively (*note order changed from above*). Then,

$$0 = (12 + 1/2\mathbf{s})\,\mathbf{I}_1 - 7\,\mathbf{I}_2 - (1/2\mathbf{s})\,\mathbf{I}_x \qquad [1]$$
$$0 = -4\,\mathbf{I}_x + (9.5 + \mathbf{s})\,\mathbf{I}_2 - \mathbf{s}\,\mathbf{I}_3 - 7\,\mathbf{I}_1 \qquad [2]$$
$$-\frac{7\mathbf{s}}{\mathbf{s}^2+16} = (3 + \mathbf{s} + 2/\mathbf{s})\,\mathbf{I}_3 - \mathbf{s}\,\mathbf{I}_2 - 3\,\mathbf{I}_x \qquad [3]$$
$$0 = (4 + 3\mathbf{s} + 1/2\mathbf{s})\,\mathbf{I}_x - 3\,\mathbf{I}_3 - (1/2\mathbf{s})\,\mathbf{I}_1 \qquad [4]$$
$$\mathbf{V}_1'' = (\mathbf{I}_1 - \mathbf{I}_x)\,(2\mathbf{s}) \qquad [5]$$

Solving all five equations simultaneously using MATLAB, we find that

$$\mathbf{V}_1'' = \frac{-56\mathbf{s}^4(21\mathbf{s}^2 - 8\mathbf{s} - 111)}{\left(1212\mathbf{s}^6 + 3311\mathbf{s}^5 + 22420\mathbf{s}^4 + 55513\mathbf{s}^3 + 48730\mathbf{s}^2 + 40590\mathbf{s} + 4800\right)}$$

The next step is to form the sum $\mathbf{V}_1(\mathbf{s}) = \mathbf{V}_1' + \mathbf{V}_1''$, which is accomplished in MATLAB using the function *symadd*(): V1 = symadd(V1prime, V1doubleprime);

$$\mathbf{V}_1(\mathbf{s}) = \frac{4\mathbf{s}^3(81\mathbf{s}^5 + 1107\mathbf{s}^4 + 7313\mathbf{s}^3 + 17130\mathbf{s}^2 + 21180\mathbf{s} + 12160)}{\left(\mathbf{s}^2+4\right)\left(1212\mathbf{s}^6 + 3311\mathbf{s}^5 + 22420\mathbf{s}^4 + 55513\mathbf{s}^3 + 48730\mathbf{s}^2 + 40590\mathbf{s} + 4800\right)}$$

(b) Using the *ilaplace*() routine from MATLAB, we find that

$$\boxed{\begin{aligned} v_1(t) = [&0.2673\,\delta(t) + 6.903\times10^{-3}\cos 2t - 2.403\sin 2t - 0.1167\,e^{-1.971t} \\ &- 0.1948\,e^{-0.3315t}\cos 0.903t + 0.1611\,e^{-0.3115t}\sin 0.903t - 0.823\times10^{-3}\,e^{-0.1376t} \\ &+ 3.229\cos 4t + 3.626\sin 4t]\,u(t)\text{ V}\end{aligned}}$$

33. (a) We can combine the two sinusoidal sources in the time domain as they have the same frequency. Thus, there is really no need to invoke source transformation as such to find the current.

$$65 \cos 10^3 t \Leftrightarrow \frac{65\mathbf{s}}{\mathbf{s}^2 + 10^6}, \text{ and } 13 \text{ mH} \rightarrow 0.013\mathbf{s}\ \Omega$$

We may therefore write

$$\mathbf{I}(\mathbf{s}) = \left(\frac{65\mathbf{s}}{\mathbf{s}^2 + 10^6}\right)\left(\frac{1}{83 + 0.013\mathbf{s}}\right) = \boxed{\frac{5000\mathbf{s}}{\left(\mathbf{s}^2 + 10^6\right)\left(\mathbf{s} + 6385\right)}}$$

$$= -\frac{0.7643}{\left(\mathbf{s} + 6385\right)} + \frac{0.3869\angle -8.907^\circ}{\left(\mathbf{s} - j10^3\right)} + \frac{0.3869\angle 8.907^\circ}{\left(\mathbf{s} + j10^3\right)}$$

(b) Taking the inverse transform,

$$i(t) = \boxed{[-0.7643\ e^{-6385t} + 0.7738 \cos (10^3 t - 8.907^\circ)]\ u(t) \text{ A}}$$

(c) The steady-state value of $i(t)$ is simply $\boxed{0.7738 \cos (10^3 t - 8.907^\circ) \text{ A.}}$

34. (a)

$$\frac{7\mathbf{s}}{\mathbf{s}\left(3\mathbf{s}^2-9\mathbf{s}+4\right)}=\frac{7}{3\left(\mathbf{s}^2-3\mathbf{s}+\frac{4}{3}\right)}=\frac{\frac{7}{3}}{\left(\mathbf{s}-\frac{3}{2}+\sqrt{\frac{11}{12}}\right)\left(\mathbf{s}-\frac{3}{2}-\sqrt{\frac{11}{12}}\right)}$$

$\boxed{\text{Poles at } \frac{3}{2}\pm\sqrt{\frac{11}{12}}\text{, double zero at } \infty.}$

(b) $$\frac{\mathbf{s}^2-1}{\left(\mathbf{s}^2+2\mathbf{s}+4\right)\left(\mathbf{s}^2+1\right)}=\frac{(\mathbf{s}+1)(\mathbf{s}-1)}{\left(\mathbf{s}+1+j\sqrt{3}\right)\left(\mathbf{s}+1-j\sqrt{3}\right)(\mathbf{s}+j)(\mathbf{s}-j)}$$

$$\boxed{\begin{array}{l}\text{Zeroes at } \mathbf{s}=-1,\ +1,\ \infty \\ \text{Poles at } -1+j\sqrt{3},\ -1-j\sqrt{3},\ \pm j\end{array}}$$

35. (a)

$$\frac{3\mathbf{s}^2}{\mathbf{s}(\mathbf{s}^2+4)(\mathbf{s}-1)} = \frac{3\mathbf{s}}{(\mathbf{s}+j2)(\mathbf{s}-j2)(\mathbf{s}-1)}$$

$\boxed{\text{Poles at } \pm j2,\ 1;\ \text{zeroes at } \mathbf{s} = 0, \infty.}$

(b) $$\frac{\mathbf{s}^2+2\mathbf{s}-1}{\mathbf{s}^2(4\mathbf{s}^2+2\mathbf{s}+1)(\mathbf{s}^2-1)} = \frac{(\mathbf{s}+1+\sqrt{2})(\mathbf{s}+1-\sqrt{2})}{\mathbf{s}^2\left(\mathbf{s}+\frac{1}{4}+j\frac{\sqrt{3}}{4}\right)\left(\mathbf{s}+\frac{1}{4}-j\frac{\sqrt{3}}{4}\right)(\mathbf{s}+1)(\mathbf{s}-1)}$$

$$\boxed{\begin{array}{l}\text{Poles at } \mathbf{s} = \pm 1,\ -\frac{1}{4} \pm j\frac{\sqrt{3}}{4},\ \text{double at } \mathbf{s} = 0 \\ \text{Zeroes at } -1 \pm j\sqrt{2},\ \infty\end{array}}$$

36.

(a) $$\mathbf{Z}_{in} = \frac{\left(5+\frac{5}{\mathbf{s}}\right)(2+5\mathbf{s})}{5\mathbf{s}+7+5/\mathbf{s}} = \frac{(5\mathbf{s}+5)(2+5\mathbf{s})}{5\mathbf{s}^2+7\mathbf{s}+5} = \frac{25\mathbf{s}^2+35\mathbf{s}+10}{5\mathbf{s}^2+7\mathbf{s}+5}$$

$$\therefore \boxed{\mathbf{Y}_{in}(\mathbf{s}) = \frac{5\mathbf{s}^2+7\mathbf{s}+5}{25\mathbf{s}^2+35\mathbf{s}+10}}$$

(b) Poles: $\mathbf{s}^2+1.4\mathbf{s}+0.2=0,\ \mathbf{s}=\dfrac{-1.4\pm\sqrt{1.96-1.6}}{2} = \boxed{-1, -0.4\,\mathrm{s}^{-1}}$

Zeros: $\mathbf{s}^2+1.4\mathbf{s}+1=0,\ \mathbf{s}=\dfrac{-1.4\pm\sqrt{1.96-4}}{2} = \boxed{-0.7\pm j0.7141\,\mathrm{s}^{-1}}$

(c) Poles: same; $\mathbf{s} = \boxed{-1, -0.4\ \mathrm{s}^{-1}}$

(d) Zeros: same; $\mathbf{s} = \boxed{-0.7\pm j0.7141\ \mathrm{s}^{-1}}$

37. (a) Regarding the circuit of Fig. 15.45, we replace each 2-mF capacitor with a 500/ **s** Ω impedance. Then,

$$\mathbf{Z}_{in}(\mathbf{s}) = \frac{\left(20 + \frac{500}{\mathbf{s}}\right)\left(40 + \frac{500}{\mathbf{s}}\right)}{60 + \frac{100}{\mathbf{s}}} = 13.33\frac{(\mathbf{s}+25)(\mathbf{s}+12.5)}{\mathbf{s}(\mathbf{s}+1.667)}$$

Reading from the transfer function, we have

zeros at **s** = -25 and -12.5 s^{-1}, and
poles at **s** = 0 and **s** = -1.667 s^{-1}.

(b) Regarding the circuit of Fig. 15.47, we replace the 2-mF capacitor with a 500/ **s** Ω impedance and the 1-mH inductor with a 0.001**s**-Ω impedance. Then,

$$\mathbf{Z}_{in}(\mathbf{s}) = \frac{\left(55 + \frac{500}{\mathbf{s}}\right)(100 + 0.001\mathbf{s})}{155 + \frac{500}{\mathbf{s}} + 0.001\mathbf{s}} = 55\frac{(\mathbf{s} + \frac{500}{55})(\mathbf{s} + 10^5)}{\left(\mathbf{s} + 1.55\times10^5\right)(\mathbf{s}+3.226)}$$

Reading from the transfer function, we have

zeros at **s** = -9.091 and -10^5 s^{-1}, and
poles at **s** = -1.55×10^5 and **s** = -3.226 s^{-1}.

38. $\mathbf{Y}(\mathbf{s})$: zeros at $\mathbf{s} = 0; -10;$ *poles* at $\mathbf{s} = -5, -20 \text{ s}^{-1}$; $\mathbf{Y}(\mathbf{s}) \to 12 \text{ S}$ as $\mathbf{s} \to \infty$

(a) $$\mathbf{Y}(\mathbf{s}) = \frac{\text{K}\mathbf{s}(\mathbf{s}+10)}{(\mathbf{s}+5)(\mathbf{s}+20)}, \text{ K} = 12 \therefore$$

$$\mathbf{Y}(\mathbf{s}) = \frac{12\mathbf{s}(\mathbf{s}+10)}{(\mathbf{s}+5)(\mathbf{s}+20)} = \frac{12\mathbf{s}^2 + 120\mathbf{s}}{\mathbf{s}^2 + 25\mathbf{s} + 100}$$

$$\therefore \mathbf{Y}(j10) = \frac{-1200 + j1200}{-100 + j250 + 100} = 4.800 + j4.800 = \boxed{6.788\angle 45^\circ \text{ S}}$$

(b) $$\mathbf{Y}(-j10) = \boxed{6.788\angle -45^\circ \text{ S}}$$

(c) $$\mathbf{Y}(-15) = \frac{12(-15)(-5)}{(-10)5} = \boxed{-18 \text{ S}}$$

(d) $$5 + \mathbf{Y}(\mathbf{s}) = 5 + \frac{12\mathbf{s}^2 + 120\mathbf{s}}{\mathbf{s}^2 + 25\mathbf{s} + 100} = \frac{17\mathbf{s}^2 + 245\mathbf{s} + 500}{(\mathbf{s}+5)(\mathbf{s}+20)}, \mathbf{s} = \frac{-245 \pm \sqrt{245^2 - 68(500)}}{34}$$

Zeros: $\mathbf{s} = \boxed{-2.461 \text{ and } -11.951 \text{ s}^{-1}}$; Poles: $\mathbf{s} = \boxed{-5, -20 \text{ s}^{-1}}$

39.

(a) $$\mathbf{Y}_{in} = \frac{1}{4+\mathbf{s}} + \frac{1}{5+5\mathbf{s}} = \frac{0.2(6\mathbf{s}+9)}{(4+\mathbf{s})(1+\mathbf{s})} \quad \therefore \boxed{\mathbf{Z}_{in} = \frac{5(\mathbf{s}+1)(\mathbf{s}+4)}{6(\mathbf{s}+1.5)}}$$

(b) Poles: $\mathbf{s} = \boxed{-1.5, \infty}$; Zeros: $\mathbf{s} = \boxed{-1,\ -4\ \text{s}^{-1}}$

40. $\mathbf{H(s)} = \dfrac{\mathbf{s}+2}{(\mathbf{s}+5)(\mathbf{s}^2+6\mathbf{s}+25)}$

(a) $\delta(t) \Leftrightarrow 1$, so the output is $\boxed{\dfrac{\mathbf{s}+2}{(\mathbf{s}+5)(\mathbf{s}^2+6\mathbf{s}+25)}}$

(b) $e^{-4t}\,u(t) \Leftrightarrow 1\,/\,(\mathbf{s}+4)$, so the output is $\boxed{\dfrac{\mathbf{s}+2}{(\mathbf{s}+4)(\mathbf{s}+5)(\mathbf{s}^2+6\mathbf{s}+25)}}$

(c) $2\cos 15t\,u(t) \Leftrightarrow \dfrac{2\mathbf{s}}{\mathbf{s}^2+225}$, so the output is $\boxed{\dfrac{2\mathbf{s}(\mathbf{s}+2)}{(\mathbf{s}^2+225)(\mathbf{s}+5)(\mathbf{s}^2+6\mathbf{s}+25)}}$

(d) $t\,e^{-t}\,u(t) \Leftrightarrow 1/\,(\mathbf{s}+1)$, so the output is $\boxed{\dfrac{\mathbf{s}+2}{(\mathbf{s}+1)(\mathbf{s}+5)(\mathbf{s}^2+6\mathbf{s}+25)}}$

(e) poles and zeros of each:

(a): zero at $\mathbf{s} = -2$, poles at $\mathbf{s} = -5, -3 \pm j4$
(b): zero at $\mathbf{s} = -2$, poles at $\mathbf{s} = -4, -5, -3 \pm j4$
(c): zeros at $\mathbf{s} = 0, -2$, poles at $\mathbf{s} = \pm j15, -5, -3 \pm j4$
(d): zero at $\mathbf{s} = -2$, poles at $\mathbf{s} = -1, -5, -3 \pm j4$

41. $h(t) = 5\,[\mathrm{u}(t) - \mathrm{u}(t-1)]\,\sin \pi t \qquad x(t) = 2[\mathrm{u}(t) - \mathrm{u}(t-2)]$

$$y(t) = \int_{0^-}^{\infty} h(\lambda)x(t-\lambda)\,d\lambda$$

$$t < 0:\; y(t) = \boxed{0}$$

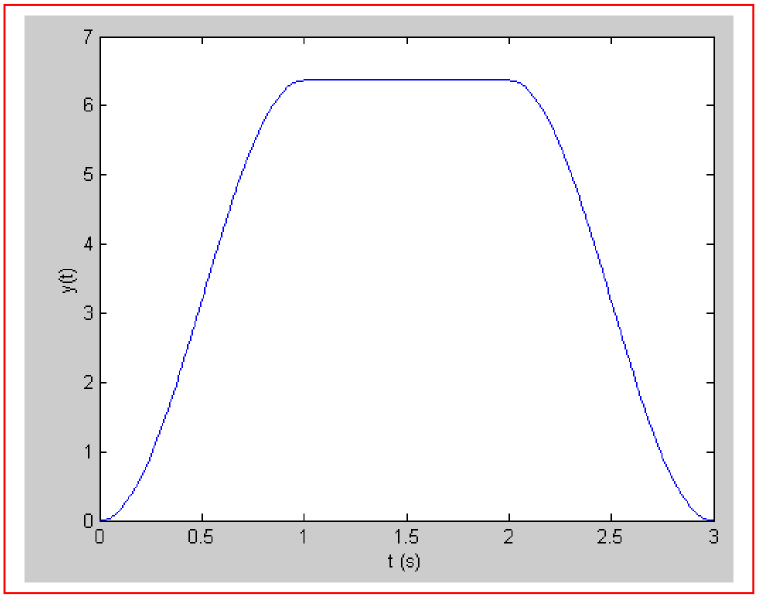
$$0 < t < 1:\; y(t) = \int_0^t 10\sin\pi\lambda\,d\lambda = -\frac{10}{\lambda}\cos\pi\lambda\Big|_0^t = \boxed{\frac{10}{\pi}(1-\cos\pi t)}$$

$$1 < t < 2:\; y(t) = \int_0^1 10\sin\pi\lambda\,d\lambda = \boxed{\frac{20}{\pi}}$$

$$2 < t < 3:\; y(t) = \int_{t-2}^1 10\sin\pi\lambda\,d\lambda = -\frac{10}{\pi}\cos\pi\lambda\Big|_{t-2}^1 = -\frac{10}{\pi}[-1-\cos(\pi t - 2\pi)]$$

$$= \boxed{(10/\pi)\,(1 + \cos \pi t)}$$

$$t > 3:\; y(t) = \boxed{0}$$



42. $f_1(t) = e^{-5t}\,u(t)$, $f_2(t) = (1 - e^{-2t})\,u(t)$

(a) $f_1 * f_2 = \int_{0^-}^{\infty} f_1(\lambda)\, f_2(t-\lambda)\, d\lambda$

$t < 0$: $f_1 * f_2 = 0$

$$t > 0:\ f_1 * f_2 = \int_{0^-}^{t} e^{-5\lambda}\left(1 - e^{2\lambda-2t}\right) d\lambda = \int_{0^-}^{t}\left(e^{-5\lambda} - e^{-2t}e^{-3\lambda}\right) d\lambda$$

$$= -\frac{1}{5}e^{-5\lambda}\Big|_0^t + \frac{1}{3}e^{-2t}e^{-3\lambda}\Big|_0^t = \boxed{\left(\frac{1}{5}+\frac{2}{15}e^{-5t}-\frac{1}{3}e^{-2t}\right)u(t)}$$

(b) $\mathbf{F}_1(\mathbf{s}) = 1/(\mathbf{s}+5)$, $\mathbf{F}_2(\mathbf{s}) = 1/\mathbf{s} - 1/(\mathbf{s}+2)$

$$\mathbf{F}_1(\mathbf{s})\,\mathrm{F}_2(\mathbf{s}) = \frac{1}{\mathbf{s}(\mathbf{s}+5)} - \frac{1}{(\mathbf{s}+5)(\mathbf{s}+2)} = \frac{\mathrm{a}}{\mathbf{s}} + \frac{\mathrm{b}}{\mathbf{s}+2} + \frac{\mathrm{c}}{\mathbf{s}+5}$$

Where a = 0.2, b = -1/3, and c = -1/5 + 1/3 = 2/15.

Taking the inverse transform, we find that $f_1 * f_2 = \boxed{\left(\frac{1}{5}+\frac{2}{15}e^{-5t}-\frac{1}{3}e^{-2t}\right)u(t)}$

43. The impulse response is $v_o(t) = 4u(t) - 4u(t-2)$ V,
so we know that $h(t) = 4u(t) - 4u(t-2)$. $v_i(t) = 2u(t - 1)$, and $v_o(t) = h(t) * v_i(t)$.

Thus, $v_o(t) = \int_0^{\infty} h(\lambda) v_i(t-\lambda)\, d\lambda = 8\int_0^{\infty} [u(\lambda) - u(\lambda-2)]u(t-\lambda-1)\, d\lambda$

or $v_o(t) = 8\int_0^{\infty} [1 - u(\lambda-2)]u(t-\lambda-1)\, d\lambda$. [1]

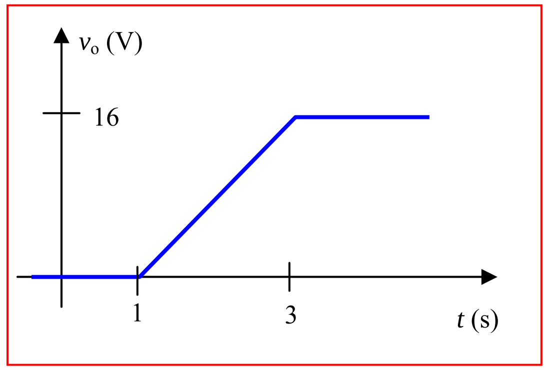
For $\lambda > 2$, this integral is zero.
Also, the second step function results in a zero value for the integral except when $t - \lambda - 1 > 0$, or $\lambda < t - 1$.

With a lower limit of $\lambda = 0$, this means that $t > 1$. When $t > 3$, however, we do not must be careful to constrain λ to less than 2, so we split the integration into two parts:

$1 < t < 3$: $v_o = \int_0^{t-1} 8\, d\lambda = \boxed{8t - 8 \text{ V}}$

$t > 3$: $v_o = \int_0^{2} 8\, d\lambda = \boxed{16 \text{ V}}$

and, of course, for $t < 1$, the output is $\boxed{\text{zero.}}$



44. $h(t) = 2e^{-3t}\, u(t),\, x(t) = u(t) - \delta(t)$

(a) $y(t) = \int_{0^-}^{\infty} h(\lambda)x(t-\lambda)d\lambda$

$$t < 0: y(t) = 0$$

$$t > 0: y(t) = 2\int_{0^-}^{t} e^{-3\lambda}\left[1-\delta(t-\lambda)\right]d\lambda \;=\; 2\left[-\frac{1}{3}e^{-3\lambda}u(t)\Big|_0^t \;-\; e^{-3t}u(t)\right]$$

$$= \frac{2}{3}(1-e^{-3t})u(t) \;-\; 2e^{-3t}u(t) \;=\; \boxed{\left(\frac{2}{3}-\frac{8}{3}e^{-3t}\right)u(t)}$$

(b) $\mathbf{H(s)} = \dfrac{2}{\mathbf{s}+3}$ $\quad\quad \mathbf{X(s)} = \dfrac{1}{\mathbf{s}} - 1$

thus, $\mathbf{Y(s)} = \dfrac{2(1-\mathbf{s})}{\mathbf{s}(\mathbf{s}+3)} = \dfrac{2}{3}\left(\dfrac{1}{\mathbf{s}}\right) - \dfrac{8}{3}\left(\dfrac{1}{\mathbf{s}+3}\right)$

Taking the inverse transform, we find that $y(t) = \boxed{\dfrac{2}{3}u(t) \;-\; \dfrac{8}{3}e^{-3t}u(t)}$

45. $h(t) = 5\,u(t) - 5\,u(t-2)$, so $\mathbf{H(s)} = \dfrac{5}{\mathbf{s}} - 5e^{-2s}$

(a) $v_{in}(t) = 3\delta(t)$, so $\mathbf{V}_{in}(\mathbf{s}) = 3$

$$\mathbf{V}_{out}(\mathbf{s}) = \mathbf{V}_{in}(\mathbf{s})\,\mathbf{H}(\mathbf{s}) = \boxed{\frac{15}{\mathbf{s}} - 15e^{-2s}}.\quad v_{out}(t) = \mathcal{L}^{-1}\{\mathbf{V}_{out}(\mathbf{s})\} = \boxed{15\,u(t) - 15\,u(t-2)}$$

(b) $v_{in}(t) = 3u(t)$, so $\mathbf{V}_{in}(\mathbf{s}) = \dfrac{3}{\mathbf{s}}$

$$\mathbf{V}_{out}(\mathbf{s}) = \mathbf{V}_{in}(\mathbf{s})\,\mathbf{H}(\mathbf{s}) = \left(\frac{3}{\mathbf{s}}\right)\left(\frac{5}{\mathbf{s}} - 5e^{-2s}\right) = \boxed{\frac{15}{\mathbf{s}^2} - \frac{15}{\mathbf{s}}e^{-2s}}.$$

$$v_{out}(t) = \mathcal{L}^{-1}\{\mathbf{V}_{out}(\mathbf{s})\} = 15\,t\,u(t) - 15\,u^2(t-2) = \boxed{15\,t\,u(t) - 15\,u\,(t-2)}$$

(c) $v_{in}(t) = 3u(t) - 3u(t-2)$, so $\mathbf{V}_{in}(\mathbf{s}) = \dfrac{3}{\mathbf{s}} - 3e^{-2s}$

$$\mathbf{V}_{out}(\mathbf{s}) = \mathbf{V}_{in}(\mathbf{s})\,\mathbf{H}(\mathbf{s}) = \left(\frac{3}{\mathbf{s}} - 3e^{-2s}\right)\left(\frac{5}{\mathbf{s}} - 5e^{-2s}\right) = \boxed{\frac{15}{\mathbf{s}^2} - \frac{30}{\mathbf{s}}e^{-2s} + 15e^{-4s}}.$$

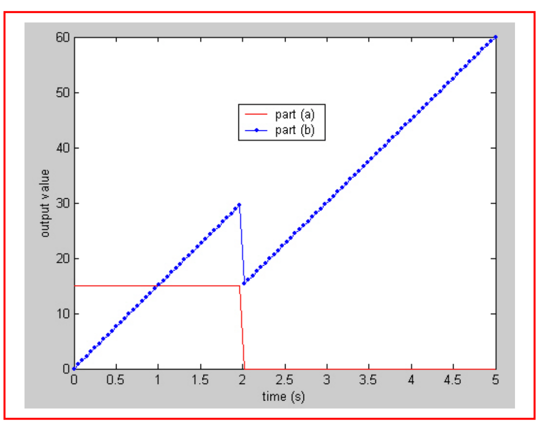
$$v_{out}(t) = \mathcal{L}^{-1}\{\mathbf{V}_{out}(\mathbf{s})\} = 15\,t\,u(t) - 30\,u^2(t-2) + 15\,u^2\,(t-4)$$

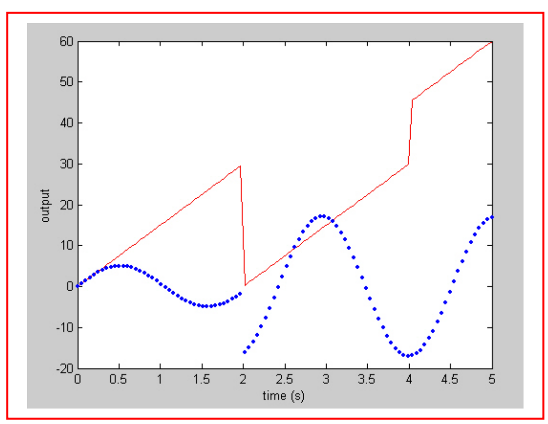
$$= \boxed{15\,t\,u(t) - 30\,u\,(t-2) + 15\,u(t)}$$

(d) $v_{in}(t) = 3\cos 3t$, so $\mathbf{V}_{in}(\mathbf{s}) = \dfrac{3\mathbf{s}}{\mathbf{s}^2 + 9}$

$$\mathbf{V}_{out}(\mathbf{s}) = \mathbf{V}_{in}(\mathbf{s})\,\mathbf{H}(\mathbf{s}) = \boxed{\frac{15}{\mathbf{s}^2 + 9} - \frac{15\mathbf{s}}{\mathbf{s}^2 + 9}e^{-2s}}.$$

$$v_{out}(t) = \mathcal{L}^{-1}\{\mathbf{V}_{out}(\mathbf{s})\} = \boxed{5\sin 3t\,u(t) - 15\cos\,[3(t-2)]\,u(t-2)}$$





46. (a) Since $v_o(t) = v_{in}(t)$, $\mathbf{H}(\mathbf{s}) = 1$. Thus, $\boxed{h(t) = \delta(t).}$

(b) $v_o(t) = \int_{-\infty}^{\infty} v_{in}(x)h(t-x)dx = \int_{-\infty}^{\infty} v_{in}(x)\delta(t-x)dx = v_{in}(t) = \boxed{8u(t)\ \text{V}}$

47. (a) Since $v_o(t) = v_{in}(t)$, $\mathbf{H}(\mathbf{s}) = 1$. Thus, $\boxed{h(t) = \delta(t)}$.

(b) $v_o(t) = \int_{-\infty}^{\infty} v_{in}(x)h(t-x)dx = \int_{-\infty}^{\infty} v_{in}(x)\delta(t-x)dx = v_{in}(t) = \boxed{8e^{-t}u(t)\text{ V}}$

48.

$$\mathbf{I}_{in} = \frac{\mathbf{V}_{in}}{\frac{10}{\mathbf{s}} + 20 \| 20\left(20 + \frac{10}{\mathbf{s}}\right)} = \frac{\mathbf{V}_{in}}{\frac{10}{\mathbf{s}} + \frac{20(20 + 10/\mathbf{s})}{40 + 10/\mathbf{s}}}$$

$$= \frac{\mathbf{V}_{in}}{\frac{10}{\mathbf{s}} + \frac{40\mathbf{s} + 20}{4\mathbf{s} + 1}} = \frac{\mathbf{V}_{in}}{\frac{40\mathbf{s}^2 + 60\mathbf{s} + 10}{4\mathbf{s}^2 + \mathbf{s}}} = \mathbf{V}_{in}\frac{40\mathbf{s}^2 + \mathbf{s}}{40\mathbf{s}^2 + 60\mathbf{s} + 10}$$

$$\therefore \mathbf{I}_{top} = \mathbf{I}_{in}\frac{20}{40 + \frac{10}{s}} = \mathbf{I}_{in}\frac{2\mathbf{s}}{4\mathbf{s} + 1} = \mathbf{V}_{in}\frac{2\mathbf{s}^2}{40\mathbf{s}^2 + 60\mathbf{s} + 10};$$

$$\mathbf{V}_{out} = \frac{10}{s}\mathbf{I}_{in} + 20\mathbf{I}_{top} = \mathbf{V}_{in}\left[\frac{4\mathbf{s} + 1}{4\mathbf{s}^2 + 6\mathbf{s} + 1} + \frac{4\mathbf{s}^2}{4\mathbf{s}^2 + 6\mathbf{s} + 1}\right] \therefore$$

$$\mathbf{H}(\mathbf{s}) = \frac{\mathbf{V}_{out}}{\mathbf{V}_{in}} = \frac{4\mathbf{s}^2 + 4\mathbf{s} + 1}{4\mathbf{s}^2 + 6\mathbf{s} + 1} = \frac{\mathbf{s}^2 + \mathbf{s} + 0.25}{\mathbf{s}^2 + 1.5\mathbf{s} + 0.25} = \frac{(\mathbf{s} + 0.5)^2}{(\mathbf{s} + 0.19098)(\mathbf{s} + 1.3090)} \therefore$$

$$\boxed{\text{zeros: } \mathbf{s} = -0.5, s = -0.5; \text{ poles: } \mathbf{s} = -1.3090, -0.19098}$$

49.

(a)

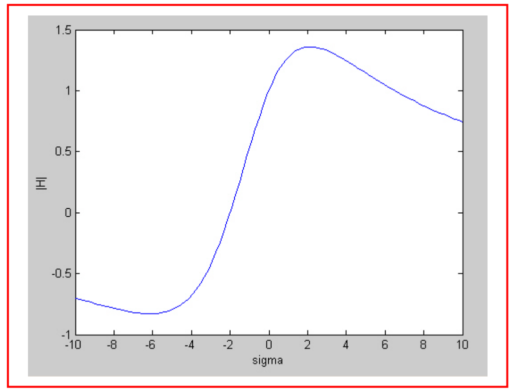
$$\mathbf{H}(\mathbf{s}) = \mathbf{V}_2(\mathbf{s})/\mathbf{V}_1(\mathbf{s}),\ \mathbf{H}(0) = 1$$

$$\therefore \mathbf{H}(\mathbf{s}) = \frac{\mathrm{K}(\mathbf{s}+2)}{(\mathbf{s}+1+j4)(\mathbf{s}+1-j4)} = \frac{\mathrm{K}(\mathbf{s}+2)}{\mathbf{s}^2+2\mathbf{s}+17}$$

$$1 = 2\frac{\mathrm{K}}{17},\ \text{so}\ \ \mathrm{K}=8.5$$

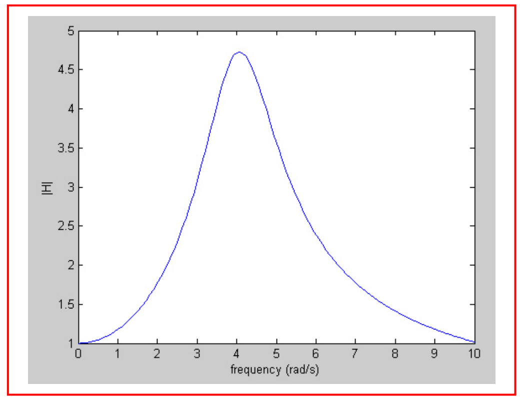
$$\textit{Thus},\ \ \mathbf{H}(\mathbf{s}) = \frac{8.5(\mathbf{s}+2)}{\mathbf{s}^2+2\mathbf{s}+17}$$

$$\text{Let } \omega = 0\ \ \therefore \mathbf{H}(\sigma) = \frac{8.5(\sigma+2)}{\sigma^2+2\sigma+17}$$



(b)

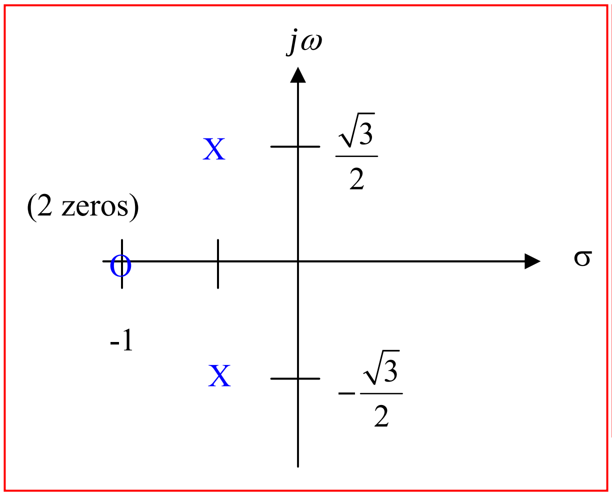
$$|H(j\omega)| = 8.5\sqrt{\frac{\omega^2+4}{(17-\omega^2)^2+4\omega^2}}$$

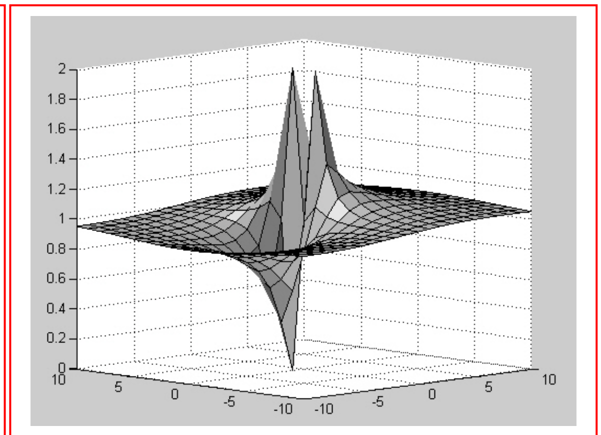


(c)

By trial & error: $|\mathbf{H}(j\omega)|_{\max} = \boxed{4.729 \text{ at } \omega = 4.07 \text{ rad/s}}$

50. (a) pole-zero constellation (b) elastic-sheet model

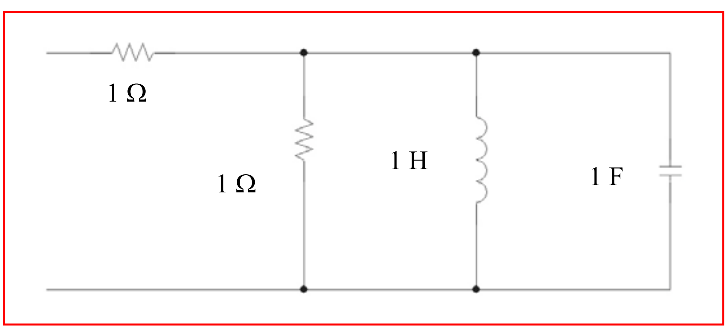




(c) $$\mathbf{H}(\mathbf{s}) = \frac{(\mathbf{s}+1)^2}{\left(\mathbf{s}+0.5+j\frac{\sqrt{3}}{2}\right)\left(\mathbf{s}+0.5-j\frac{\sqrt{3}}{2}\right)} = \frac{(\mathbf{s}+1)^2}{\mathbf{s}^2+\mathbf{s}+1}$$

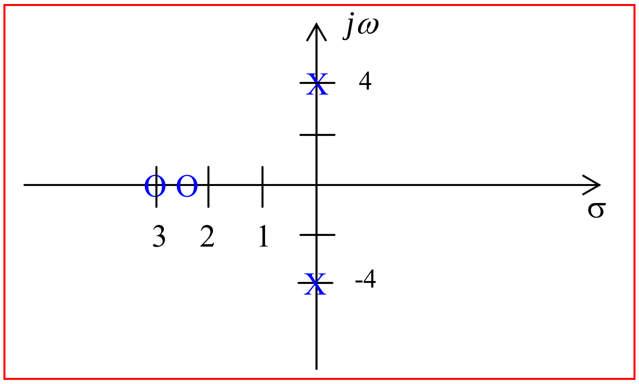
$$= \frac{\mathbf{s}^2+2\mathbf{s}+1}{\mathbf{s}^2+\mathbf{s}+1} = 1 + \frac{\mathbf{s}}{\mathbf{s}^2+\mathbf{s}+1}$$

We can implement this with a 1-Ω resistor in series with a network having the impedance given by the second term. There are two energy storage elements in that network (the denominator is order 2). That network impedance can be rewritten as $\frac{\mathbf{s}}{\mathbf{s}^2+\mathbf{s}+1} = \frac{1}{\mathbf{s}+1+\frac{1}{\mathbf{s}}}$, which can be seen to be equal to the parallel combination of a 1-Ω resistor, a 1-H inductor, and a 1-F capacitor.



51. $\mathbf{H}(\mathbf{s}) = (10\mathbf{s}^2 + 55\mathbf{s} + 75)/(\mathbf{s}^2 + 16)$

(a) $\mathbf{H}(\mathbf{s}) = 10\dfrac{(\mathbf{s}+3)(\mathbf{s}+2.5)}{(\mathbf{s}+j4)(\mathbf{s}-j4)}$. Critical frequencies: zeros at –3, -2.5; poles at $\pm j4$.

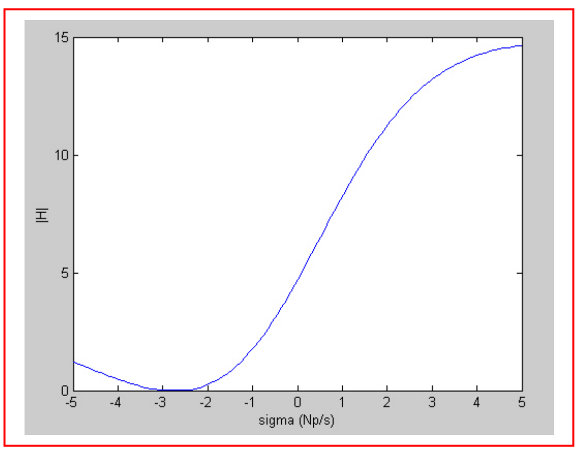


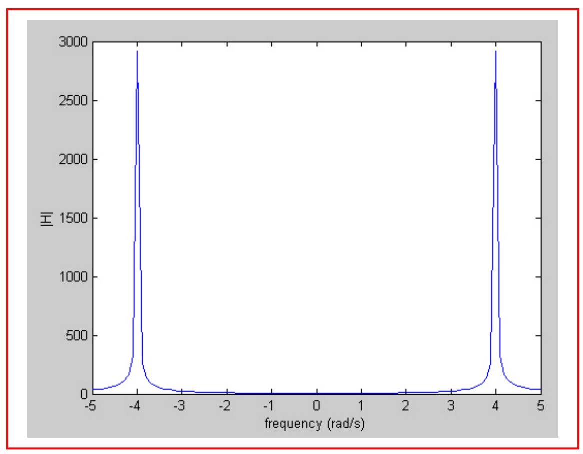
(b) $\mathbf{H}(0) = \dfrac{75}{16} = \boxed{4.688,}\ \mathbf{H}(\infty) = \boxed{10}$

(c) $\mathbf{H}(0) = 4.679\ \mathrm{K} = 3, \text{ so } \mathrm{K} = 0.64$

$\therefore \mathbf{H}(j3) = 0.64\left|\dfrac{-90+75+j165}{7}\right| = \dfrac{0.64}{7}\left|-15+j165\right| = \boxed{15.15\ \ cm}$

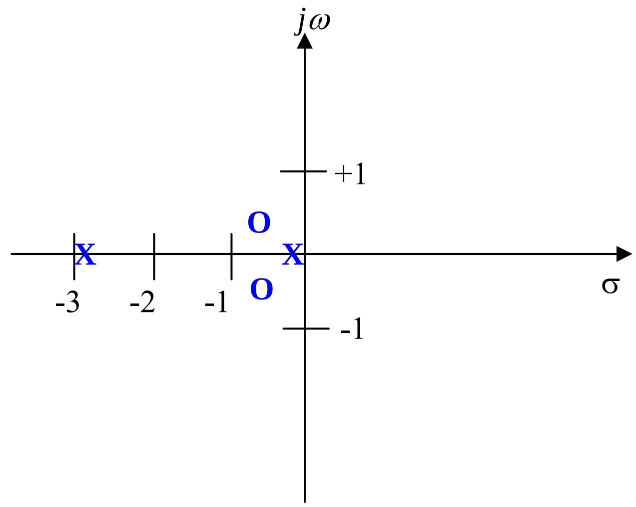
(d)



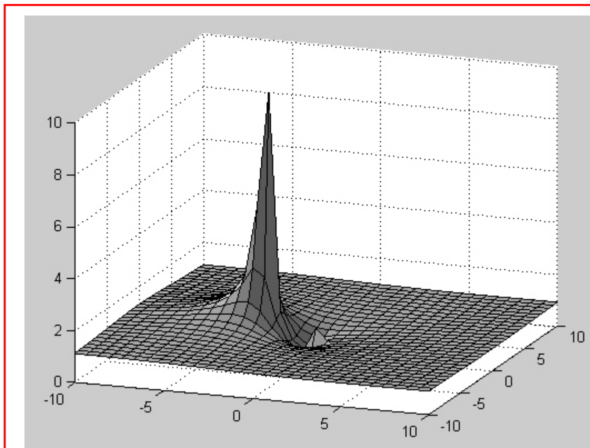


52. (a) $\mathbf{Y}(\mathbf{s}) = \dfrac{5\mathbf{s}^2 + 5\mathbf{s} + 2}{5\mathbf{s}^2 + 15\mathbf{s} + 2} = \dfrac{(\mathbf{s} + 0.5 + j0.3873)(\mathbf{s} + 0.5 - j0.3873)}{(\mathbf{s} + 2.86)(\mathbf{s} + 0.1399)}$

Zeros: $\mathbf{s} = -0.5 \pm j0.3873$
Poles: $\mathbf{s} = -2.86$, $\mathbf{s} = -0.1399$



(b) elastic sheet model



(c) lattitude 5°5’2”, longitude 5°15’2” puts it a little off the coast of Timbuktu.

53. $\mathbf{H}(\mathbf{s}) = \dfrac{\mathbf{I}_0}{\mathbf{I}_M}$; $\mathbf{H}(-2) = 6$

(a) $$\mathbf{H}(\mathbf{s}) = \mathrm{K}\frac{(\mathbf{s}-1)(\mathbf{s}+1)(\mathbf{s}+3)}{(\mathbf{s}+3+j2)(\mathbf{s}+3-j2)}$$

$$\mathbf{H}(-2) = 6 = \frac{(-3)(-1)\mathrm{K}}{(1+j2)(1-j2)} = \frac{3\mathrm{K}}{5} \quad \therefore \mathrm{K} = 10,$$

$$Thus,\quad \mathbf{H}(\mathbf{s}) = 10\frac{(\mathbf{s}^2-1)(\mathbf{s}+3)}{\mathbf{s}^2+6\mathbf{s}+13} = \boxed{\frac{10\mathbf{s}^3+30\mathbf{s}^2-10\mathbf{s}-30}{\mathbf{s}^2+6\mathbf{s}+13}}$$

(b) $$\mathbf{H}(0) = -\frac{30}{13} = \boxed{-2.308,}\quad \mathbf{H}(\infty) = \boxed{\infty}$$

(c) $$1: (s-1) = (j2-1) = \boxed{2.236\angle 116.57^\circ}$$

$$-1: (s+1) = (j2+1) = \boxed{2.236\angle 63.43^\circ}$$

$$-3: (s+3) = j2+3 = \boxed{3.606\angle 33.69^\circ}$$

$$-3-j2:\ j2+3+j2 = \boxed{5.000\angle 53.13^\circ}$$

$$-3+j2:\ j2+3-j2 = \boxed{3\angle 0^\circ}$$

54.

$\mathbf{Z}_A$: zero at $\mathbf{s} = -10 + j0$; $\mathbf{Z}_A + 20$: zero at $\mathbf{s} = -3.6 + j0$

$$\therefore \mathbf{Z}_A = 5 + \frac{\text{R}/\mathbf{s}\text{C}}{\text{R} + 1/\text{SC}} = 5 + \frac{\text{R}}{\mathbf{s}\text{CR} + 1} = 5 + \frac{1/\text{C}}{\mathbf{s} + 1/\text{RC}} = \frac{5\mathbf{s} + 5/\text{RC} + 1/\text{C}}{\mathbf{s} + 1/\text{RC}}$$

$$\therefore \mathbf{Z}_A = \frac{5(\mathbf{s} + 1/\text{RC} + 1/5\text{C})}{\mathbf{s} + 1/\text{RC}}$$

Thus, using the fact that $\mathbf{Z}_A = 0$ at $\mathbf{s} = -10$, we may write $\frac{1}{\text{RC}} + \frac{1}{5\text{C}} = 10$

$$\text{Also, } \mathbf{Z}_B = 25 + \frac{1/\text{C}}{\mathbf{s} + 1/\text{RC}} = \frac{25\mathbf{s} + \frac{25}{\text{RC}} + \frac{1}{\text{C}}}{\mathbf{s} + 1/\text{RC}} = \frac{25\left(\mathbf{s} + \frac{1}{\text{RC}} + \frac{1}{25\text{C}}\right)}{\mathbf{s} + \frac{1}{\text{RC}}}$$

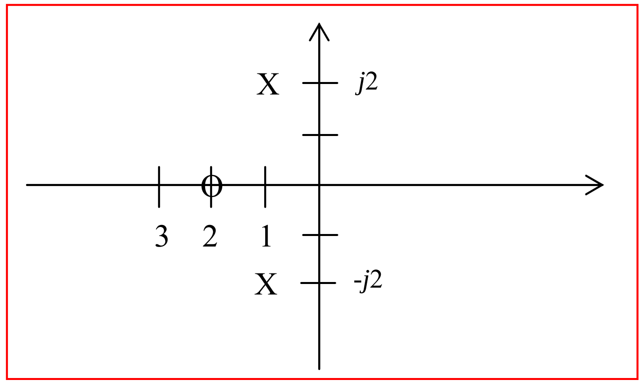
$$\therefore \frac{1}{\text{RC}} + \frac{1}{25\text{C}} = 3.6 \quad \text{or} \quad \frac{4}{25\text{C}} = 6.4,$$

$$\text{C} = \frac{1}{40} = \boxed{25 \text{ mF},}$$

$$\frac{40}{\text{R}} + \frac{40}{5} = 10, \ \frac{40}{\text{R}} = 2, \quad \text{so} \quad \text{R} = \boxed{20 \ \Omega}$$

55. $\mathbf{H}(\mathbf{s}) = 100(\mathbf{s}+2)/(\mathbf{s}^2 + 2\mathbf{s} + 5)$

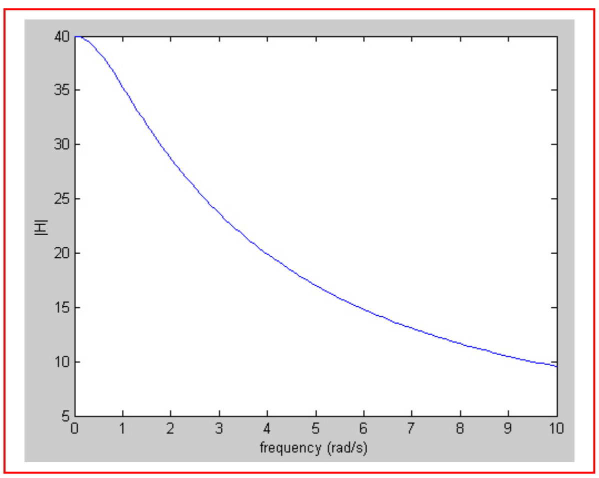
(a) zero at **s** = -2, poles at $\mathbf{s} = \dfrac{-2 \pm \sqrt{4-20}}{2} = -1 \pm j2$



(b) $\mathbf{H}(j\omega) = \boxed{\dfrac{100(2+j\omega)}{(5-\omega^2)+j2\omega}}$

(c) $|\mathbf{H}(j\omega)| = \boxed{100\sqrt{\dfrac{\omega^2+4}{\omega^4 - 6\omega^2 + 25}}}$

(d)



(e) $\dfrac{|\mathbf{H}(j\omega)|^2}{10\ 000} = \dfrac{\omega^2+4}{\omega^4 - 6\omega^2 + 25},\ \dfrac{d\,|\mathbf{H}(j\omega)|^2}{d\omega} = \dfrac{(\omega^4 - 6\omega^2 + 25)2\omega - (\omega^2+4)(4\omega^3 - 12\omega)}{etc}$

$\therefore \omega^4 - 6\omega^2 + 25 = (\omega^2+4)(2\omega^2 - 6),\ \omega^4 - 6\omega^2 + 25 = 2\omega^4 + 2\omega^2 - 24,\ \omega^4 + 8\omega^2 - 49 = 0$

$\therefore \omega^2 = \dfrac{-8 \pm \sqrt{64+196}}{2} = 4.062\ \therefore \omega_{mar} = \boxed{2.016\,\text{rad/s}},\ |\text{H}(j2.016)| = \boxed{68.61}$

56. $\mathbf{Z}_{in}(\mathbf{s}) = \dfrac{5\mathbf{s}+20}{\mathbf{s}+2}\ \Omega$

(a) $v_{ab}(0) = 25\text{ V};\ \mathbf{Z}_{in}(\mathbf{s}) = \dfrac{5(\mathbf{s}+4)}{\mathbf{s}+2},\ \mathbf{V}_{ab} = \mathbf{Z}_{in}\mathbf{I}_{in}$

$\therefore \mathbf{H}(\mathbf{s}) = \dfrac{5(\mathbf{s}+4)}{\mathbf{s}+2},\quad \text{single pole at } \mathbf{s} = -2\ \therefore v_{ab}(t) = \boxed{25e^{-2t}\text{ V},\ t > 0}$

(b) $i_{ab}(0) = 3\text{ A}\ \therefore \mathbf{I}_{ab} = \dfrac{\mathbf{V}_s}{\mathbf{Z}_{in}}\ \therefore \mathbf{H}(\mathbf{s}) = \dfrac{\mathbf{I}_{ab}}{\mathbf{V}_{in}} = \dfrac{1}{\mathbf{Z}_{in}} = \dfrac{\mathbf{s}+2}{5(\mathbf{s}+4)}\ \text{single pole at } \mathbf{s} = -4$

$\therefore i_{ab}(t) = \boxed{3e^{-4t}\text{ A},\ t > 0}$

57. $\mathbf{Z}_{in}(\mathbf{s}) = 5(\mathbf{s}^2 + 4\mathbf{s} + 20)/(\mathbf{s}+1)$

(a) $v_{ab} = 160e^{-6t}\,\text{V} \;\therefore \mathbf{V}_{ab} = 160\,\text{V},\ \mathbf{s} = -6$

$$\mathbf{I}_a = \frac{\mathbf{V}_{ab}}{\mathbf{Z}_{in}} = \frac{160(\mathbf{s}+1)}{5(\mathbf{s}^2 + 4\mathbf{s} + 20)} = \frac{32(-5)}{3b - 24 + 20} = -5\,\text{A} \;\therefore i_a(t) = \boxed{-5e^{-6t}\,\text{A (all}\,t)}$$

(b) $v_{ab} = 160e^{-6t}u(t),\ i_a(0) = 0, i_a'(0) = 32\,\text{A/s} \;\therefore \mathbf{H}(\mathbf{s}) = \dfrac{\mathbf{I}_a}{\mathbf{V}_s} = \dfrac{1}{\mathbf{Z}_{in}} = \dfrac{\mathbf{s}+1}{5(\mathbf{s}^2 + 4\mathbf{s} + 20)}$

$$\mathbf{s} = \frac{-4 \pm \sqrt{16-80}}{2} = -2 \pm j4 \;\therefore i_a(t) = -5e^{-6t} + e^{-2t}(\text{A}\cos 4t + \text{B}\sin 4t) \;\therefore 0 = -5 + \text{A},\ \text{A} = 5$$

$$i_a'(0) = 32 = 30 - 10 + 4\text{B} \;\therefore \text{B} = 3 \;\therefore i_a(t) = \boxed{[-5e^{-6t} + e^{-2t}(5\cos 4t + 3\sin 4t)]u(t)\,\text{A}}$$

58.

(a) $\mathbf{H(s)} = \mathbf{I}_c / \mathbf{I}_s = \dfrac{0.5}{0.5 + 0.002\mathbf{s} + 500/\mathbf{s}} = \boxed{\dfrac{250\mathbf{s}}{\mathbf{s}^2 + 250\mathbf{s} + 25\ 000}}$

(b) $\mathbf{s} = \dfrac{1}{2}(-250 \pm \sqrt{62\ 500 - 10^6}) = \boxed{-125 \pm j484.1\text{s}^{-1}}$

(c) $\alpha = \dfrac{\text{R}}{2\text{L}} = \dfrac{0.5}{0.004} = \boxed{125\ \text{s}^{-1}}$, $\omega_o = \sqrt{10^6/4} = \boxed{500\ \text{s}^{-1}}$, $\omega_d = \sqrt{25\times10^4 - 15{,}625} = \boxed{484.1\ \text{s}^{-1}}$

(d) $\mathbf{I}_s = 1,\ \mathbf{s} = 0\ \therefore \text{I}_c = 0\ \therefore i_{cf} = \boxed{0}$

(e) $i_{c,n} = \boxed{e^{-125t}(A\cos 484t + \text{B}\sin 484t)}$

(f) $i_L(0) = 0\ \therefore i_c(0^+) = 0,\ v_c(0) = 0\ \therefore 1\times\dfrac{1}{2} = 2\times10^{-3} i(0^+) + 0\ \therefore i(0^+) = \boxed{250\ \text{A/s}}$

(g) $\therefore \text{A} = 0,\ 484\text{B} = 250,\ \text{B} = 0.5164\ \therefore i_c(t) = \boxed{(0.5164 e^{-125t} \sin 484.1t)\ u(t)\ \text{A}}$

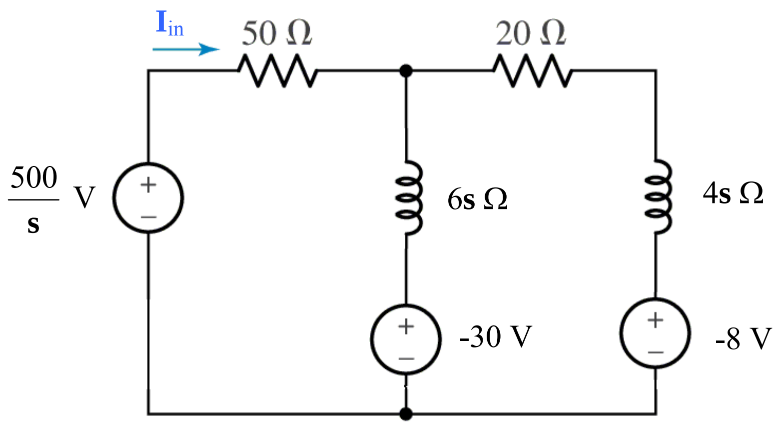
59.

(a)

$$\mathbf{H(s)} = \mathbf{I}_{in} / \mathbf{V}_{in} = \frac{1}{\mathbf{Z}_{in}} = \frac{1}{50 + \dfrac{6\mathbf{s}(4\mathbf{s}+20)}{10\mathbf{s}+20}} = \frac{10\mathbf{s}+20}{24\mathbf{s}^2 + 620\mathbf{s} + 1000}$$

$$\therefore \mathbf{s} = \frac{1}{48}(-620 \pm \sqrt{620^2 - 96{,}000}) = \boxed{-1.729 \text{ and } -24.10 \text{ s}^{-1}}$$

(b) Note that the element labeled 6 H should be an inductor, as is suggested by the context of the text (i.e. initial condition provided). Convert to s-domain and define a clockwise mesh current $\mathbf{I}_2$ in the right-hand mesh.



Mesh 1: $0 = -500/\mathbf{s} + (50 + 6\mathbf{s})\,\mathbf{I}_{in} - 30 - 6\mathbf{s}\,\mathbf{I}_2$ [1]

Mesh 2: $0 = 30 + (20 + 10\mathbf{s})\,\mathbf{I}_2 - 6\mathbf{s}\,\mathbf{I}_{in} - 8$ [2]

Solving, we find that

$$\mathbf{I}_{in} = \frac{42\mathbf{s}^2 + 1400\mathbf{s} + 2500}{\mathbf{s}\left(6\mathbf{s}^2 + 155\mathbf{s} + 250\right)} = \frac{7\mathbf{s}^2 + 233.3\mathbf{s} + 416.7}{\mathbf{s}\left(\mathbf{s} + 24.10\right)\left(\mathbf{s} + 1.729\right)}$$

$$= \frac{a}{\mathbf{s}} + \frac{b}{\left(\mathbf{s} + 24.10\right)} + \frac{c}{\left(\mathbf{s} + 1.729\right)}$$

where a = 10, b = -2.115 and c = -0.8855. Thus, we may write

$$\boxed{i_{in}(t) = [10 - 2.115\, e^{-24.10t} - 0.885\, e^{-1.729t}]\, u(t) \text{ A}}$$

60.

(a) $$\mathbf{H}(\mathbf{s}) = \frac{\mathbf{V}}{\mathbf{I}_s} = \frac{50(1000/\mathbf{s})}{50+(1000/\mathbf{s})} = \boxed{\frac{1000}{\mathbf{s}+20}}$$

(b)

$$\mathbf{I}_s = \frac{2}{\mathbf{s}} \text{ so } \mathbf{V}(\mathbf{s}) = \left(\frac{2}{\mathbf{s}}\right)\left(\frac{1000}{(\mathbf{s}+20)}\right) = \frac{2000}{\mathbf{s}(\mathbf{s}+20)} = \frac{a}{s} + \frac{\text{b}}{\mathbf{s}+20}$$

$$a = \left.\frac{2000}{(\mathbf{s}+20)}\right|_{\mathbf{s}=0} = 100; \text{ b } = a = \left.\frac{2000}{(\mathbf{s})}\right|_{\mathbf{s}=-20} = -100$$

$$\textit{Thus}, \ \mathbf{V}(\mathbf{s}) = \frac{100}{s} - \frac{100}{\mathbf{s}+20} \text{ and } \boxed{v(t) = 100\left[1 - e^{-20t}\right]u(t) \text{ V}}$$

(c) This function as written is technically valid for all time (although that can't be possible physically). Therefore, we can't use the one-sided Laplace technique we've been studying. We can, however, use simple s-domain/ complex frequency analysis:

$$i_s = 4e^{-10t} \text{ A } \therefore \mathbf{I}_s = 4 \text{ A}, \ \mathbf{s} = 10 \ \therefore \mathbf{V} = 4\mathbf{H}(-10) = 4 \times \frac{1000}{10} = 400 \text{ V } \therefore$$

$$\boxed{v(t) = 400e^{-10t} \text{ V (all } t)}$$

(d) $$4e^{-10t}\,u(t) \Leftrightarrow \frac{4}{\mathbf{s}+10}, \text{ so } \mathbf{V}(\mathbf{s}) = \left(\frac{4}{\mathbf{s}+10}\right)\left(\frac{1000}{\mathbf{s}+20}\right) = \frac{\text{a}}{\mathbf{s}+10} + \frac{\text{b}}{\mathbf{s}+20}$$

a = 400 and b = -400, so $v(t) = \boxed{400\,[e^{-10t} - e^{-20t}]\,u(t) \text{ V}}$

61.

(a)

$$\mathbf{H(s)} = \frac{\mathbf{V}_{c2}}{\mathbf{V}_s} = \frac{\dfrac{100}{\mathbf{s}}}{20+\dfrac{100}{\mathbf{s}}} \times \frac{\left(20+\dfrac{100}{\mathbf{s}}\right)\dfrac{25}{\mathbf{s}}}{20+\dfrac{125}{\mathbf{s}}} / \left[50+\frac{(20\mathbf{s}+100)25}{\mathbf{s}(20\mathbf{s}+125)}\right]$$

$$\therefore \mathbf{H(s)} = \frac{2500}{\mathbf{s}(20\mathbf{s}+125)}\,\frac{\mathbf{s}(20\mathbf{s}+125)}{1000\mathbf{s}^2+6250\mathbf{s}+500\mathbf{s}+2500}$$

$$\therefore \mathbf{H(s)} = \boxed{\frac{2.5}{\mathbf{s}^2+6.75\mathbf{s}+2.5}}$$

(b) No initial energy stored in either capacitor. With $v_s = u(t)$, $\mathbf{V}_s(\mathbf{s}) = \dfrac{1}{\mathbf{s}}$, so

$$\mathbf{V}_{C2} = \frac{2.5}{\mathbf{s}(\mathbf{s}+6.357)(\mathbf{s}+0.3933)} = \frac{a}{\mathbf{s}} + \frac{b}{\mathbf{s}+6.357} + \frac{c}{\mathbf{s}+0.3933}$$

Where a = 1, b = 0.06594 and c = -1.066. Thus,

$$v_{C2}(t) = \boxed{[1 + 0.06594\, e^{-6.357t} - 1.066\, e^{-0.3933t}]\, u(t) \text{ V}}$$

62.

$$\mathbf{Z}_{in}(\mathbf{s}) = \frac{1}{0.1 + 0.025\mathbf{s} + \dfrac{1}{20 + (80/\mathbf{s})}} = \frac{1}{0.1 + 0.025\mathbf{s} + \dfrac{0.05\mathbf{s}}{\mathbf{s}+4}}$$

$$= \frac{\mathbf{s}+4}{0.025\mathbf{s}^2 + 0.25\mathbf{s} + 0.4} = \frac{40(\mathbf{s}+4)}{\mathbf{s}^2 + 10\mathbf{s} + 16} = \frac{40(\mathbf{s}+4)}{(\mathbf{s}+2)(\mathbf{s}+8)}\,\Omega$$

$20u(t) \Leftrightarrow \dfrac{20}{\mathbf{s}}$, so $\mathbf{V}_{\text{in}}(\mathbf{s}) = \left(\dfrac{20}{\mathbf{s}}\right)\left[\dfrac{40(\mathbf{s}+4)}{(\mathbf{s}+2)(\mathbf{s}+8)}\right] = \dfrac{a}{\mathbf{s}} + \dfrac{b}{\mathbf{s}+2} + \dfrac{c}{\mathbf{s}+8}$

a = 200, b = -133.3 and c = -66.67, so $\boxed{v_{\text{in}}(t) = [200 - 133.3\, e^{-2t} - 66.67\, e^{-8t}]\, u(t) \text{ V}}$

63.

$$\mathbf{H}(\mathbf{s}) = -\frac{\mathbf{Z}_f}{\mathbf{Z}_1}$$

(a) $$\mathbf{Z}_1 = 10^3 + \frac{10^8}{\mathbf{s}},\ \mathbf{Z}_f = 5000\ \therefore \mathbf{H}(\mathbf{s}) = -\frac{5000}{1000 + (10^8/\mathbf{s})} = -\frac{5000\mathbf{s}}{1000\mathbf{s} + 10^8}$$

$$\therefore \boxed{\mathbf{H}(s) = \frac{-5\mathbf{s}}{\mathbf{s} + 10^5}}$$

(b) $$\mathbf{Z}_1 = 5000,\ \mathbf{Z}_f = 10^3 + 10^8/\mathbf{s}\ \therefore \mathbf{H}(\mathbf{s}) = -\frac{10^3 + 10^8/\mathbf{s}}{5000} = -\frac{1000\mathbf{s} + 10^8}{5000\mathbf{s}} = \boxed{-\frac{R + 10^5}{5\mathbf{s}}}$$

(c) $$\mathbf{Z}_1 = 10^3 + 10^8/\mathbf{s},\ \mathbf{Z}_f = 10^4 + 10^8/\mathbf{s}\ \therefore \mathbf{H}(\mathbf{s}) = -\frac{10^4 + 10^8/\mathbf{s}}{1000 + 10^8/\mathbf{s}} = -\frac{10^4\mathbf{s} + 10^8}{1000\mathbf{s} + 10^8} = \boxed{-\frac{10\mathbf{s} + 10^5}{\mathbf{s} + 10^5}}$$

64.

$$R_f = 20\ \text{k}\Omega,\ \mathbf{H}(\mathbf{s}) = \frac{\mathbf{V}_{out}}{\mathbf{V}_{in}} = -R_f C_1\left(\mathbf{s} + \frac{1}{R_1C_1}\right)$$

$$\therefore \mathbf{H}(\mathbf{s}) = \ -2\times10^4 C_1\left(\mathbf{s} + \frac{1}{R_1C_1}\right)$$

(a) $\mathbf{H}(\mathbf{s}) = -50 \therefore \boxed{C_1 = 0,}\ \frac{2\times10^4}{R_1} = 50,\quad R_1 = \boxed{400\,\Omega}$

(b) $\mathbf{H}(\mathbf{s}) = -10^{-3}(\mathbf{s}+10^4) = -2\times10^4\ C_1\left(\mathbf{s} + \frac{1}{R_1C_1}\right)\ \therefore 2\times10^4 C_1 = 10^{-3}$

$\therefore \boxed{C_1 = 50\,\text{nF};}\quad \frac{1}{50\times10^{-9}R_1} = 10^4$, so $\boxed{R_1 = 2\ \text{k}\,\Omega}$

(c) $\mathbf{H}(\mathbf{s}) = -10^{-4}(\mathbf{s}+1000) = -2\times10^4 C_1\left(\mathbf{s} + \frac{1}{R_1C_1}\right)\ \therefore 2\times10^4 C_1 = 10^{-4},\ \boxed{C_1 = 5\,\text{nF}}$

$\frac{1}{R_1C_1} = 10^3 \quad \therefore\ R_1 = \frac{1}{\left(5\times10^{-9}\right)\left(10^3\right)} = \boxed{200\ \text{k}\Omega}$

(d) Stage 1: Need a simple inverting amplifier with gain of –1, so select $\boxed{C_1 = 0,}$ and $R_1 = R_f = \boxed{20\ \text{k}\Omega.}$

Stage 2: $-10^3 = -2\ \ 10^4 C_1 \qquad C_1 = \frac{10^3}{2\times10^4} = \boxed{50\ \text{mF}}$

$\frac{1}{R_1C_1} = 10^5 \qquad \therefore\ R_1 = \frac{1}{\left(50\times10^{-3}\right)\left(10^5\right)} = \boxed{200\ \mu\Omega}$

65.

(a)

$$\mathbf{H(s)} = \frac{\mathbf{V}_{out}}{\mathbf{V}_{in}} = -50,$$

$$\mathbf{H(s)} = \frac{-1/\mathrm{R}_1\mathrm{C}_f}{\mathbf{s}+1/\mathrm{R}_f\mathrm{C}_f},\ \mathrm{R}_f = 20\ \mathrm{k}\Omega$$

$$\text{set } \boxed{\mathrm{C}_f = 0}\ \therefore -50 = -\frac{\mathrm{R}_f}{\mathrm{R}_1}\ \therefore \mathrm{R}_1 = \frac{20\times 10^3}{50} = \boxed{400\ \Omega}$$

(b)

$$\mathbf{H(s)} = -\frac{1000}{\mathbf{s}+10\ 000} = \frac{1/\mathrm{R}_1\mathrm{C}_f}{\mathbf{s}+1/20\ 000\mathrm{C}_f}\ \therefore 10\ 000 = \frac{1}{20\ 000\mathrm{C}_f}$$

$$\mathrm{C}_f = \frac{1}{2\times 10^8} = \boxed{5\ \mathrm{nF}}\ \text{ We may then find } \mathrm{R}_1\text{: } 1000 = \frac{1}{5\times 10^{-9}\mathrm{R}_1}\ \therefore \boxed{\mathrm{R}_1 = 200\ \mathrm{k}\Omega}$$

(c)

$$\mathbf{H(s)} = -\frac{10\ 000}{\mathbf{s}+1000} = \frac{1/\mathrm{R}_1\mathrm{C}_f}{\mathbf{s}+1/20\ 000\mathrm{C}_f}\ \therefore 1000 = \frac{1}{20\ 000 C_f}\ \mathrm{C}_f = \boxed{50\ \mathrm{nF}}$$

$$\frac{1}{5\times 10^{-9}\mathrm{R}_1} = 1000,\ \mathrm{R}_1 = \boxed{200\ \mathrm{k}\Omega}$$

(d)

$$\mathbf{H(s)} = \frac{\mathbf{V}_{out}}{\mathbf{V}_{in}} = \frac{100}{\mathbf{s} + 10^5}$$

$$= \left[-\frac{1/\mathrm{R}_{1A}\mathrm{C}_{fA}}{\mathrm{s} + 1/\mathrm{R}_{fA}\mathrm{C}_{fA}}\right]\left[-\frac{1/\mathrm{R}_{1B}\mathrm{C}_{fB}}{\mathrm{s} + 1/\mathrm{R}_{fB}\mathrm{C}_{fB}}\right] = \left[-\frac{1/\mathrm{R}_{1A}\mathrm{C}_{fA}}{\mathrm{s} + 1/\mathrm{R}_{fA}\mathrm{C}_{fA}}\right]\left[-\frac{\mathrm{R}_{fB}}{\mathrm{R}_{1B}}\right]$$

We may therefore set $\dfrac{\mathrm{R}_{fB}}{\mathrm{R}_{1B}\mathrm{R}_{1A}\mathrm{C}_{fA}} = 100$

and $1/\mathrm{R}_{fA}\mathrm{C}_{fA} = 10^5$. Arbitrarily choosing $\boxed{\mathrm{R}_{fA} = 1\ \mathrm{k}\Omega}$, we find that $\boxed{\mathrm{C}_{fA} = 10\ \mathrm{nF}}$.

Arbitrarily selecting $\boxed{\mathrm{R}_{fB} = 100\ \Omega}$, we may complete the design by choosing

$\boxed{\mathrm{R}_{1B} = \mathrm{R}_{1A} = 10\ \mathrm{k}\Omega}$

66.

$$\mathbf{H}(\mathbf{s}) = \frac{-10^{-4}\mathbf{s}(\mathbf{s}+100)}{\mathbf{s}+1000} = \frac{[-\mathrm{K}_A\mathbf{s}][-\mathrm{K}_B(\mathbf{s}+100)]}{\left(-\dfrac{\mathrm{K}_C}{\mathbf{s}+1000}\right)}$$

Let $\mathbf{H}_A(\mathbf{s}) = -\mathrm{K}_A\mathbf{s}$. Choose inverting op amp with parallel RC network at inverting input.

$$0 = \frac{-\mathrm{V_i}}{\mathrm{R_{1A}}}\left(1+\mathbf{s}\mathrm{C_{1A}}\right) - \frac{\mathrm{V_o}}{\mathrm{R_{fa}}}$$

$$\therefore \mathbf{H}_A(\mathbf{s}) = -\frac{\mathrm{R}_{fA}}{R_{1A}}\left(1+\mathbf{s}\mathrm{R}_{1A}\mathrm{C}_{1A}\right) = -\frac{\mathrm{R}_{fA}}{R_{1A}} - \mathbf{s}\mathrm{R}_{fA}\mathrm{C}_{1A} = -\mathrm{K}_A\mathbf{s}. \quad \text{Set } \boxed{\mathrm{R}_{1A} = \infty.} \quad \text{Then}$$

$$-\mathrm{R}_{fA}\mathrm{C}_{1A}\mathbf{s} = -10^4\mathrm{C}_{1A}\mathbf{s}$$

Same configuration for $\mathbf{H}_B(\mathbf{s})$ $\therefore \mathbf{H}_B(\mathbf{s}) = -\mathrm{K}_B(\mathbf{s}+100) = -\dfrac{\mathrm{R}_{fB}}{R_{1B}}\left(1+\mathbf{s}\mathrm{R}_{1B}\mathrm{C}_{1B}\right)$

For the last stage, choose an inverting op amp circuit with a parallel RC circuit in the feedback loop.

Let $\mathbf{H}_C(\mathbf{s}) = -\mathrm{K}_C\dfrac{1}{\mathbf{s}+1000} = -\dfrac{\mathrm{R}_{fC}}{R_{1C}}\dfrac{1}{\left(1+\mathbf{s}\mathrm{R}_{FC}\mathrm{C}_{FC}\right)}$

Cascading these three tranfer functions, we find that

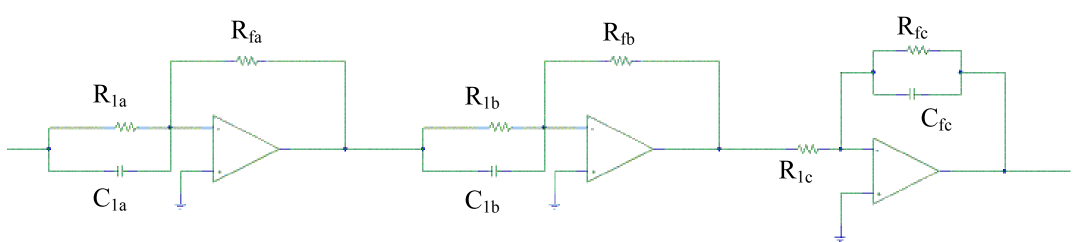
$$\mathbf{H}_\mathrm{A}\,\mathbf{H}_\mathrm{B}\,\mathbf{H}_\mathrm{C} = \left[-\mathbf{s}\mathrm{R}_{fA}\mathrm{C}_{1A}\right]\left[-\left(\mathrm{R_{fB}C_{1B}}\mathbf{s} + \frac{\mathrm{R_{fB}}}{\mathrm{R_{1B}}}\right)\right]\left[-\left(\frac{\mathrm{R_{fC}}}{\mathrm{R}_{1C}}\right)\frac{1}{\mathrm{R_{fc}C_{fc}}\mathbf{s}+1}\right]$$

Choosing $\boxed{\text{all remaining resistors to be 10 k}\Omega}$, we compare this to our desired transfer function.

$(\mathrm{R_{fc}\,C_{fc}})^{-1} = 1000$ so $\boxed{\mathrm{C_{fc}} = 100\text{ nF}}$

Next, $\dfrac{\mathrm{R_{fB}}}{\mathrm{R_{1B}\ R_{fB}\ C_{1B}}} = 100$ so $\boxed{\mathrm{C_{1B}} = 1\ \mu\mathrm{F}.}$

Finally, $\mathrm{R_{fA}C_{1A}R_{fB}C_{1B}R_{fC}}\ (\mathrm{R_{1C}R_{fC}C_{fC}}) = 10^{-4}$, so $\boxed{\mathrm{C_{1A}} = 1\text{ nF}}$



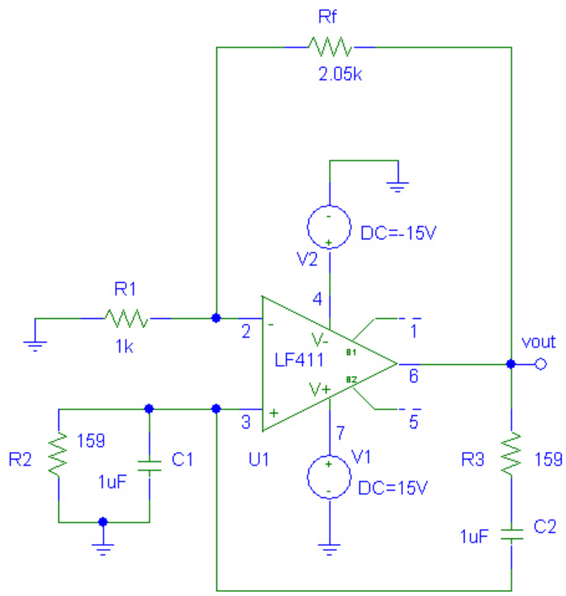
67. Design a Wien-bridge oscillator for operation at 1 kHz, using only standard resistor values. One possible solution:

$$\omega = 2\pi f = 1/RC, \text{ so set } (2\pi RC)^{-1} = 1000$$

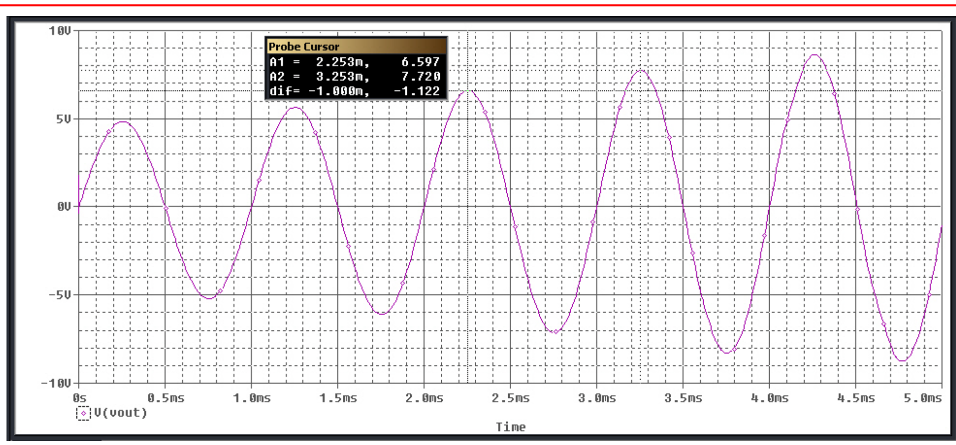
If we use a 1-μF capacitor, then R = 159 Ω. To construct this using standard resistor values, connect a 100-Ω, 56-Ω and 3-Ω in series.

To complete the design, select R_f = 2 kΩ and R_1 = 1 kΩ.

PSpice verification:



The feedback resistor was set to 2.05 kΩ to initiate oscillations in the simulation. The output waveform shown below exhibits a frequency of 1 kHz as desired.



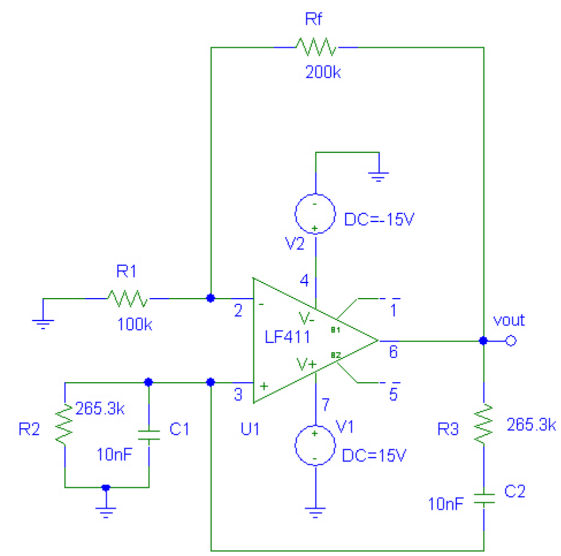
68. Design a Wien-bridge oscillator for operation at 60 Hz. One possible solution:

$$\omega = 2\pi f = 1/RC, \text{ so set } (2\pi RC)^{-1} = 60$$

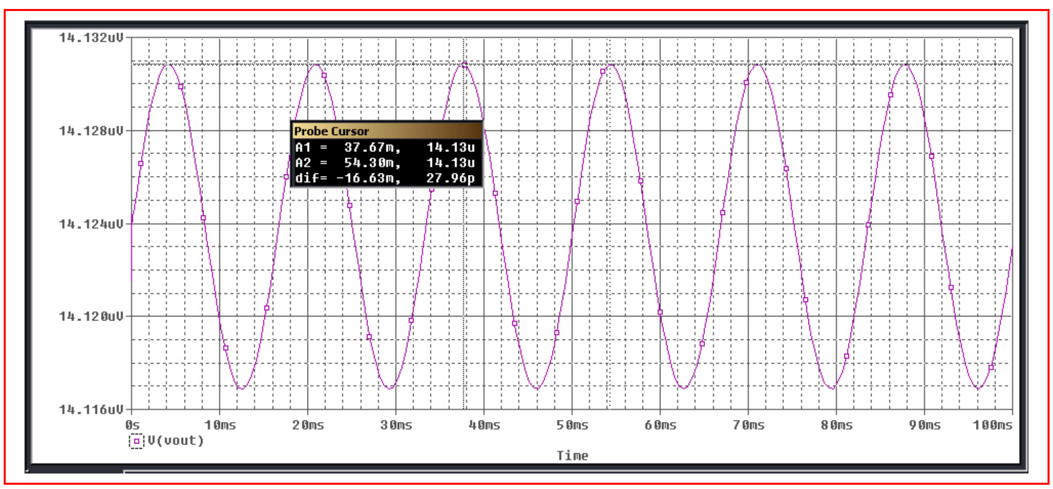
If we use 10-nF capacitors, then R = 265.3 kΩ.

To complete the design, select R_f = 200 kΩ and R_1 = 100 kΩ.

PSpice verification:



The simulated output of the circuit shows a sinusoidal waveform having period 54.3 ms – 37.67 ms = 0.01663 ms, which corresponds to a frequency of 60.13 Hz, as desired.



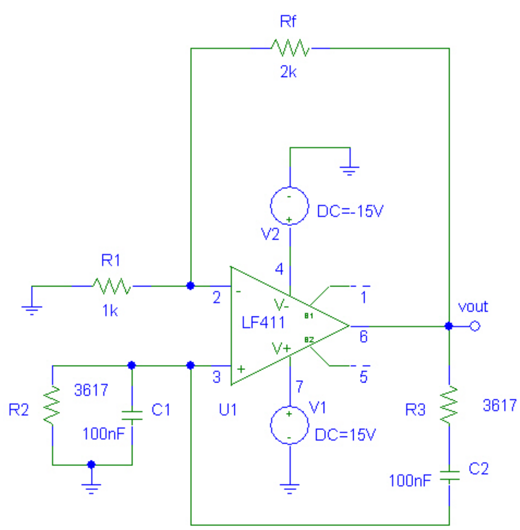
69. Design a Wien-bridge oscillator for operation at 440 Hz, using only standard resistor values. One possible solution:

$$\omega = 2\pi f = 1/RC, \text{ so set } (2\pi RC)^{-1} = 440$$

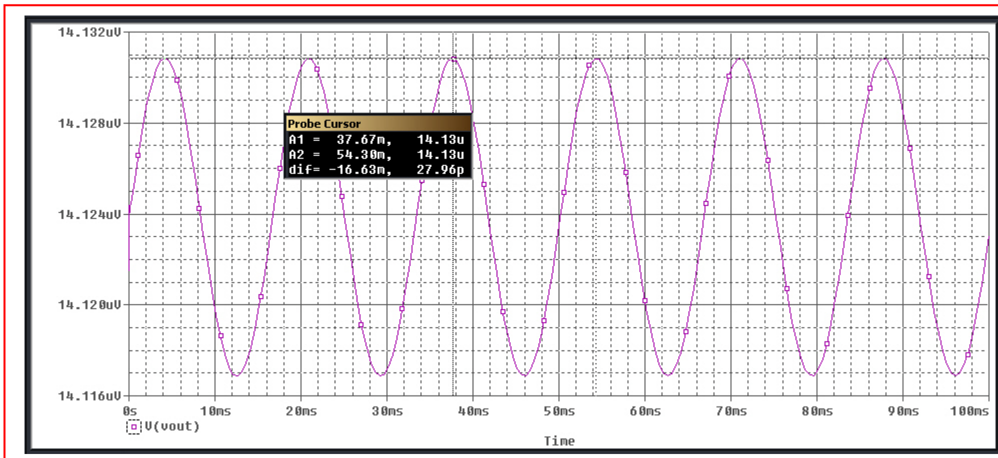
If we use 100-nF capacitors, then R = 3.167 kΩ. To construct this using standard resistor values, connect a 3.6-kΩ, 16-Ω and 1-Ω in series. (May not need the 1-Ω, as we're using 5% tolerance resistors!). This circuit will produce the musical note, 'A.'

To complete the design, select R_f = 2 kΩ and R_1 = 1 kΩ.

PSpice verification:



Simulation results show a sinusoidal output having a period of approximately 5.128 – 2.864 = 2.264 ms, or a frequency of approximately 442 Hz. The error is likely to uncertainty in cursor placement; a higher-resolution time simulation would enable greater precision.



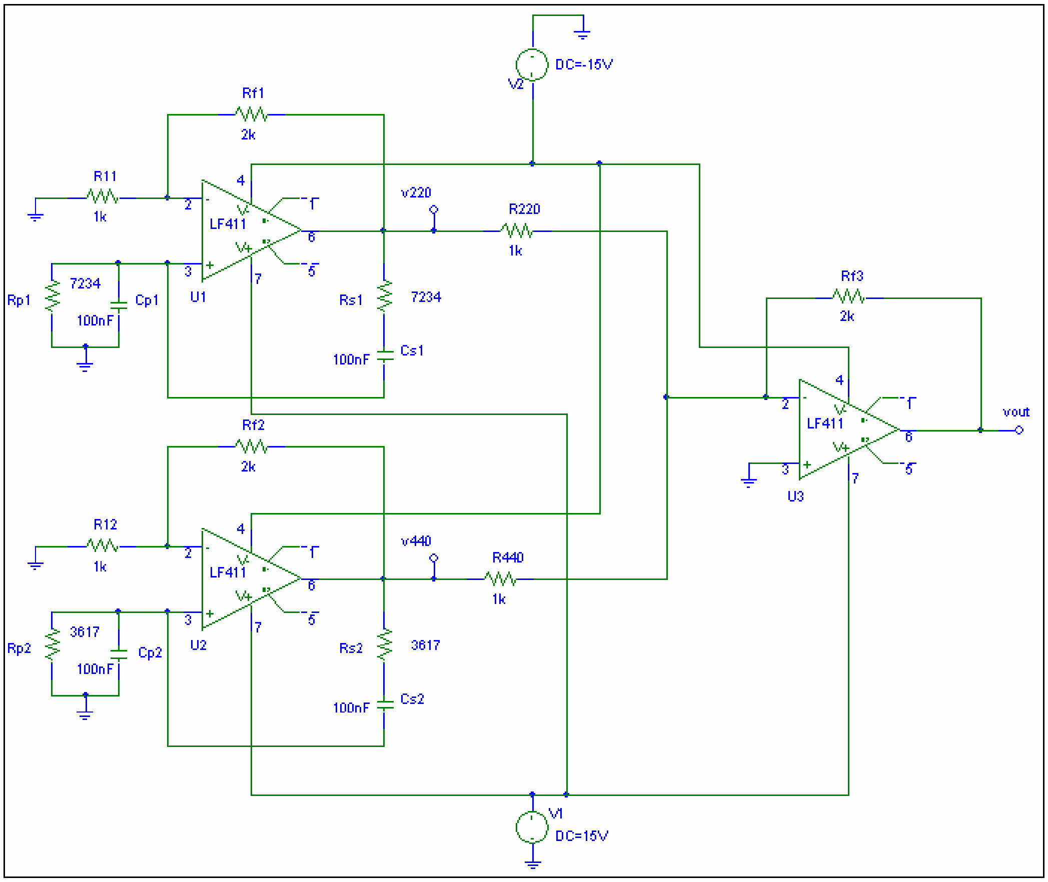
70. Design a Wien-bridge oscillator for 440 Hz: $\omega = 2\pi f = 1/RC$, so set $(2\pi RC)^{-1} = 440$

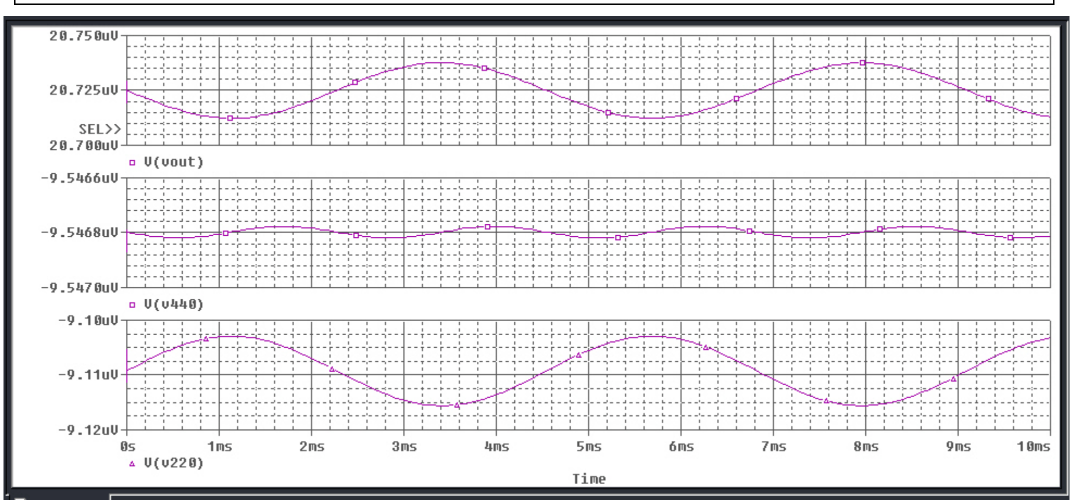
$\boxed{\text{If we use 100-nF capacitors, then R = 3.167 k}\Omega\text{.}}$

Design a Wien-bridge oscillator for 220 Hz: $\omega = 2\pi f = 1/RC$, so set $(2\pi RC)^{-1} = 220$

$\boxed{\text{If we use 100-nF capacitors, then R = 7.234 k}\Omega\text{.}}$

Using a summing stage to add the two waveforms together:



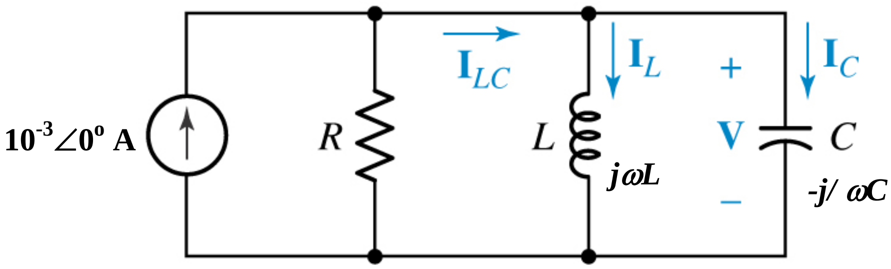


1. We have a parallel RLC with R = 1 kΩ, C = 47 μF and L = 11 mH.

(a) $Q_o = R(C/L)^{½} = \boxed{65.37}$

(b) $f_o = \omega_o / 2\pi = (LC)^{-½} / 2\pi = \boxed{221.3\text{ Hz}}$

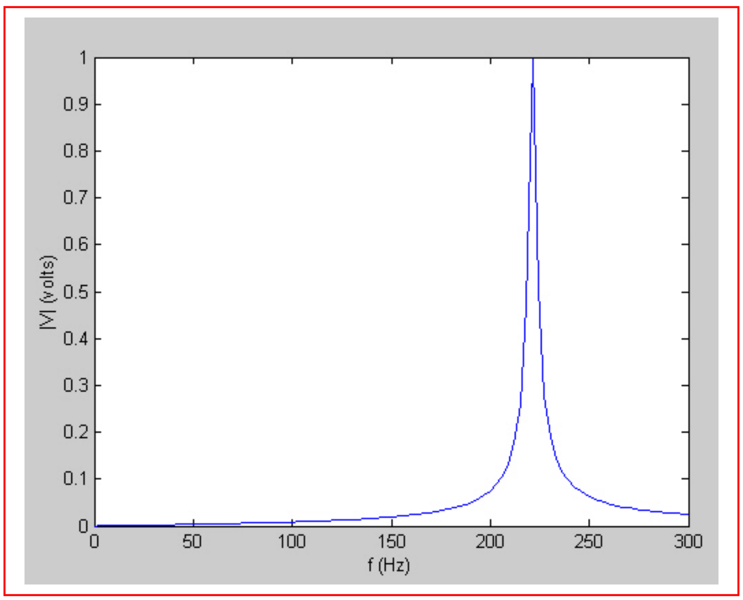
(c) The circuit is excited by a steady-state 1-mA sinusoidal source:



The admittance $\mathbf{Y(s)}$ facing the source is $\mathbf{Y(s)} = 1/R + 1/\mathbf{s}L + \mathbf{s}C$

$= C(\mathbf{s}^2 + \mathbf{s}/RC + 1/LC)/\,\mathbf{s}$ so $\mathbf{Z(s)} = (\mathbf{s}/C) / (\mathbf{s}^2 + \mathbf{s}/RC + 1/LC)$ and
$\mathbf{Z}(j\omega) = (1/C)\,(j\omega) / (1/LC - \omega^2 + j\omega/RC)$.

Since $\mathbf{V} = 10^{-3}\,\mathbf{Z}$, we note that $|\mathbf{V}| > 0$ as $\omega \rightarrow 0$ and also as $\omega \rightarrow \infty$.



2. (a) R = 1000 Ω and C = 1 μF.

$Q_o = R(C/L)^{½} = 200$ so $L = C(R/\ Q_o)^2 = \boxed{25\ \mu\text{H}}$

(b) L = 12 fH and C = 2.4 nF

$R = Q_o\ (L/\ C)^{½} = \boxed{447.2\ \text{m}\Omega}$

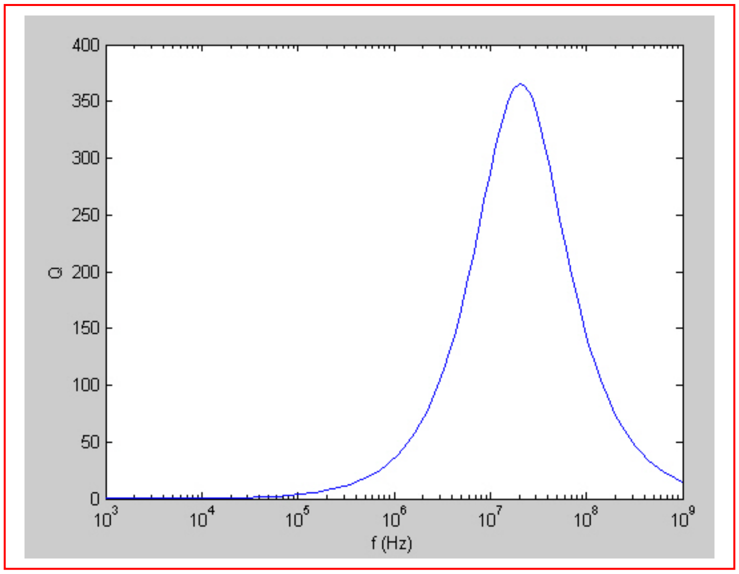
(c) R = 121.7 kΩ and L = 100 pH

$C = (Q_o\ /\ R)^2\ L = \boxed{270\ \text{aF}}$

3. We take the approximate expression for Q of a varactor to be

$$Q \approx \omega C_j R_p / (1 + \omega^2 C_j^2 R_p R_s)$$

(a) $C_j = 3.77$ pF, $R_p = 1.5$ MΩ, $R_s = 2.8$ Ω



(b) $dQ/d\omega = [(1 + \omega^2 C_j^2 R_p R_s)(C_j R_p) - \omega C_j R_p (2\omega C_j^2 R_p R_s)] / (1 + \omega^2 C_j^2 R_p R_s)$

Setting this equal to zero, we may subsequently write

$$C_j R_p (1 + \omega^2 C_j^2 R_p R_s) - \omega C_j R_p (2\omega C_j^2 R_p R_s) = 0$$

Or $1 - \omega^2 C_j^2 R_p R_s = 0$. Thus, $\omega_o = (C_j^2 R_p R_s)^{-½} = 129.4$ Mrad/s $= \boxed{21.00 \text{ MHz}}$

$Q_o = Q(\omega = \omega_o) = \boxed{366.0}$

4. Determine Q for (*dropping onto a smooth concrete floor*):

(a) A ping pong ball: Dropped twice from 121.1 cm (arbitrarily chosen). Both times, it bounced to a height of 61.65 cm.

$$Q = 2\pi h_1 / (h_1 - h_2) = \boxed{12.82}$$

(b) A quarter (25 ¢). Dropped three times from 121.1 cm.
Trial 1: bounced to 13.18 cm
Trial 2: bounced to 32.70 cm
Trial 3: bounced to 16.03 cm. *Quite a bit of variation, depending on how it struck.*

Average bounce height = 20.64 cm, so

$$Q_{avg} = 2\pi h_1 / (h_1 - h_2) = \boxed{7.574}$$

(c) Textbook. Dropped once from 121.1 cm. Didn't bounce much at all- only 2.223 cm. Since the book bounced differently depending on angle of incidence, only one trial was performed.

$$Q = 2\pi h_1 / (h_1 - h_2) = \boxed{6.4}$$

All three items were dropped from the same height for comparison purposes. An interesting experiment would be to repeat the above, but from several different heights, preferrably ranging several orders of magnitude (*e.g.* 1 cm, 10 cm, 100 cm, 1000 cm).

5.

$$\alpha = 80\,\text{Np/s},\ \omega_d = 1200\ \text{rad/s},\ \left|\bar{Z}(-2\alpha + j\omega_d)\right| = 400\,\Omega$$

$$\omega_o = \sqrt{1200^2 + 80^2} = 1202.66\ \text{rad/s} \quad \therefore Q_o = \frac{\omega_o}{2\alpha} = 7.517$$

$$\text{Now, } \bar{Y}(s) = C\frac{(s+\alpha - j\omega_d)(s+\alpha + j\omega_d)}{s} \ \therefore \bar{Y}(-2\alpha + j\omega_d) = C\frac{(-\alpha)(-\alpha + j2\omega_d)}{-2\alpha + j\omega_d}$$

$$\therefore \bar{Y}(-160 + j1200) = C\frac{-80(-80 + j2400)}{-160 + j1200} \ \therefore \left|\bar{Y}(-160 + j1200)\right| = \frac{1}{400} = 80C\left|\frac{-1 + j30}{-2 + j15}\right|$$

$$\therefore C = \frac{1}{32,000}\sqrt{\frac{229}{901}} = \boxed{15.775^{-}\ \mu\text{F};}\ L = \frac{1}{\omega_o^2 C} = \boxed{43.88\,\text{mH};}\ R = \frac{1}{2\alpha C} = \boxed{396.7\,\Omega}$$

6.

$$\bar{Y}_{in} = \frac{1}{2 + j0.1\omega} + 0.2 + \frac{1}{1 + 1000 / j\omega} = \frac{2 - j0.1\omega}{4 + 0.01\omega^2} + 0.2 + \frac{j\omega}{1000 + j10}$$

$$= \frac{2 - j0.1\omega}{4 + 0.1\omega^2} + 0.2 + \frac{\omega^2 + j1000\omega}{10^6 + \omega^2} \ \therefore \frac{-0.1\omega}{4 + 0.01\omega^2} + \frac{1000\omega}{\omega^2 + 10^6} = 0$$

$$\therefore 0.1\omega^3 + 10^5\omega = 4000\omega + 10\omega^3 \ \therefore 9.9\omega^2 = 96,000 \ \therefore \omega = \boxed{98.47 \text{ rad/s}}$$

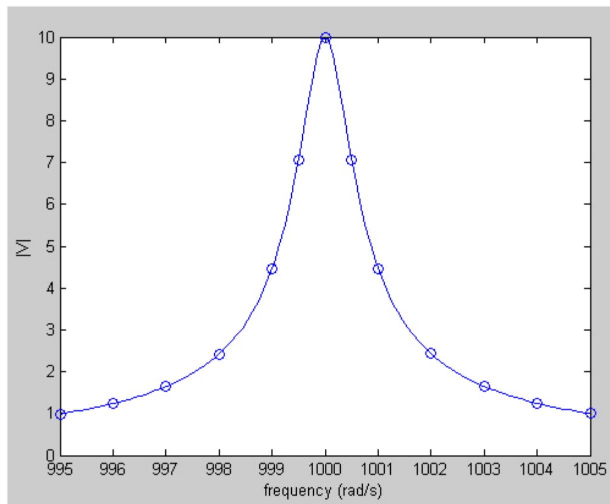
7. Parallel: $R = 10^6,\ L = 1,\ C = 10^{-6},\ \bar{I}_s = 10\angle 0°\ \mu A$

(a) $\omega_o = \dfrac{1}{\sqrt{LC}} = \boxed{1000 \text{ rad/s;}}\ Q_o = \omega_o RC = 10^{3+6-6} = \boxed{1000}$

(b) $\bar{Y} = 10^{-6} + j\left(10^{-6} - \dfrac{1}{\omega}\right),\ \bar{V} = \dfrac{\bar{I}}{\bar{Y}} = 10^{-5} / 10^{-3}\left[10^{-3} + j\left(\dfrac{\omega}{1000} - \dfrac{1000}{\omega}\right)\right]$

$$\therefore \bar{V} = \frac{10^{-2}}{0.001 + j\left(\dfrac{\omega}{1000} - \dfrac{1000}{\omega}\right)},\ |\bar{V}| = \frac{10^{-2}}{\sqrt{10^{-6} + \left(\dfrac{\omega}{1000} - \dfrac{1000}{\omega}\right)^2}}$$

ω	$\lvert\bar{V}\rvert$
995	0.993
996	1.238
997	1.642
998	2.423
999	4.47
1000	10.0
1001	4.47
1002	2.428
1003	1.646
1004	1.243
1005	0.997
999.5	7.070
1000.5	7.072



8.

(a) $$\bar{Z}_{in} = \frac{5(100/j\omega)}{5+(100/j\omega)} + 2 + \frac{j0.1\omega}{10+j0.01\omega}$$

$$= \frac{500}{100+j5\omega} + 2 + \frac{j10\omega}{1000+j\omega} = \frac{100}{20+j\omega} + 2 + \frac{j10\omega}{1000+j\omega} = \frac{100(20-j\omega)}{\omega^2+400} + 2 + \frac{j10\omega(1000-j)}{\omega^2+10^6}$$

$$\therefore \frac{-100\omega}{\omega^2+400} + \frac{10^4\omega}{\omega^2+10^6} = 0 \;\; \therefore \omega^2+10^6 = 100\omega^2+40,000,\; 99\omega^2 = 960,000$$

$$\therefore \omega_o = \sqrt{960,000/99} = \boxed{98.47 \text{ rad/s}}$$

(b) $$\bar{Z}_{in}(\omega_o) = \frac{2000}{\omega_o^2+400} + 2 + \frac{10\omega_o^2}{\omega_o^2+10^6} = \boxed{2.294\,\Omega}$$

9.

(a) $\alpha = 50\,s^{-1}, \omega_d = 1000 s^{-1} \;\therefore \omega_o^2 = \alpha^2 + \omega_d^2 = 1,002,500 \;\therefore \omega_o = 1001.249$

$$\mathrm{L} = \frac{1}{\omega_o^2 \mathrm{C}} = \frac{10^6}{1,002,500} = \boxed{0.9975^+\,\mathrm{H};}\;\; \mathrm{R} = \frac{1}{2\alpha \mathrm{C}} = \frac{10^6}{100} = \boxed{10\,k\Omega}$$

(b) $$\overline{\mathrm{Y}} = 10^{-4} + j\left(10^{-6}\omega - \frac{1}{0.9975\omega}\right),\; \omega = 1000 \;\therefore \overline{\mathrm{Z}} = \frac{1}{\mathrm{Y}} = \boxed{9997\angle 1.4321°\,\Omega}$$

10.

$f_{\min} = 535\,\text{kHz},\ f_{\max} = 1605\,\text{kHz},\ \text{Q}_o = 45$ at one end and
$\text{Q}_o \le 45$ for $535 \le f \le 1605$ kHz

$$f_o = 1/2\pi\sqrt{\text{LC}} \quad \therefore 535\times10^3 = \frac{1}{2\pi\sqrt{\text{L}_{\max}\text{C}}}, 1605\times10^3 = \frac{1}{2\pi\sqrt{\text{L}_{\min}\text{C}}}$$

$$\therefore \sqrt{\text{L}_{\max}/\text{L}_{\min}} = 3;\ \text{L}_{\max}\text{C} = \left(\frac{1}{2\pi\times535\times10^3}\right)^2 = 8.8498\times10^{-14}$$

$$\omega_o\text{RC} \le 45, 535\times10^3 \le \frac{\omega_o}{2\pi} \le 1605\times10^3.\ \text{Use } \omega_{o\max}$$

$$\therefore 2\pi\times1605\times10^3\times20\times10^3\text{C} = 45 \ \therefore \boxed{\text{C} = 223.1\text{pF}}$$

$$\therefore \text{L}_{\max} = \frac{8.8498\times10^{-14}}{223.1\times10^{-12}} = \boxed{397.6\ \mu\text{H}},\ \text{L}_{\min} = \frac{\text{L}_{\max}}{9} = \boxed{44.08\,\mu\text{H}}$$

11.

(a) Apply ± 1V. $\therefore \bar{I}_R = -10^{-4}$ A

$$\therefore \bar{Y}_{in} = \bar{I}_{in} = \frac{1}{4.4\times 10^{-3} s} + 10^{-4} + (1 - [10^5(-10^{-4})])10^{-8} s$$

$$\therefore \bar{Y}_{in} = \frac{1000}{4.4s} + 10^{-4} + 11\times 10^{-8} s = \frac{48.4\times 10^{-8} s^2 + 4.4\times 10^{-4} s + 1000}{4.4s}$$

$$\therefore \boxed{\bar{Y}_{in}(j\omega) = \frac{1000 - 48.4\times 10^{-8}\omega^2 + j4.4\times 10^{-4}\omega}{j4.4\omega}}$$

(b) At $\omega = \omega_o$, $1000 = 48.4\times 10^{-8}\omega_o^2$, $\omega_o = \boxed{45.45^{-}\ \text{krad/s}}$

$$\bar{Z}_{in}(j\omega_o) = \left(\frac{j4.4\times 10^{-4}\omega_o}{j4.4\omega_o}\right)^{-1} = \boxed{10\, k\Omega}$$

12. $\omega_0 = \dfrac{1}{\sqrt{LC}} = \sqrt{24} =$ 4.9 rad/s or $f_0 = \dfrac{\omega_0}{2\pi} = \boxed{780 \text{ mHz}}$

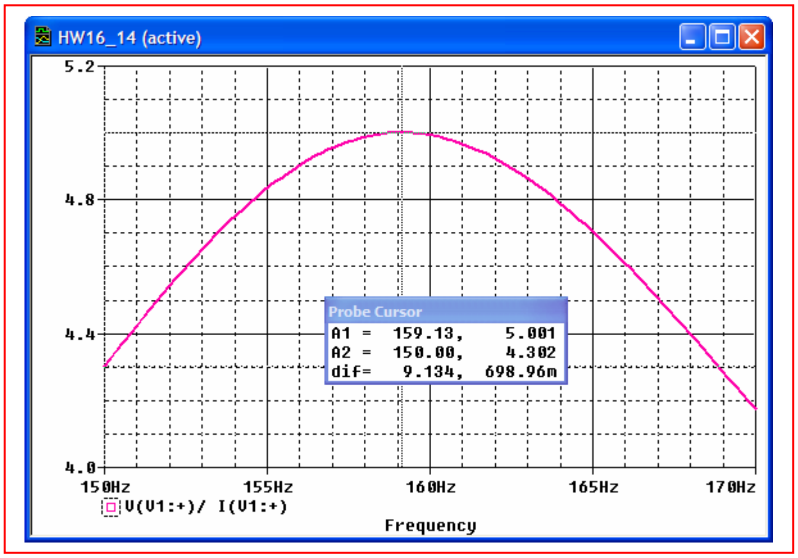
13. $\omega_0 = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{\dfrac{1}{1.01}\left(25\times10^{-6}\right)}} = $ 200 rad/s or $f_0 = \dfrac{\omega_0}{2\pi} = \boxed{31.99 \text{ Hz}}$

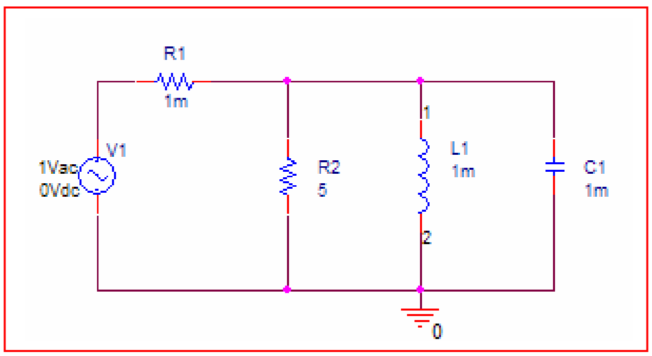
14. (a) $\alpha = \dfrac{1}{2RC} \quad \therefore \quad R = \dfrac{1}{2\alpha C} = \dfrac{10^3}{200} = 5\ \Omega$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10^{-6}}} = 1000 \text{ rad/s or } f_0 = \frac{\omega_0}{2\pi} = \boxed{159.2 \text{ Hz}}$$

$$\mathbf{Z}_{in}(\omega_0) = R = \boxed{5\ \Omega}$$

(b) We see from the simulation result that the ratio of the test source voltage to its current is 5 Ω at the resonant frequency; the small error is due to the series resistance PSpice required.





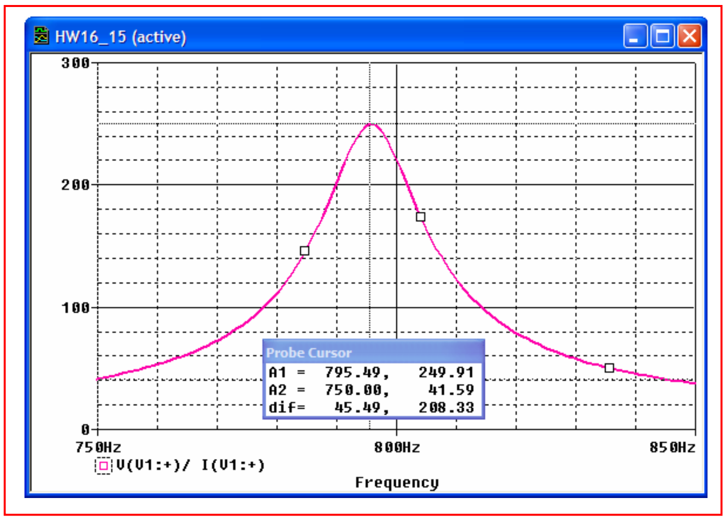
15. (a) $\alpha = \dfrac{1}{2RC} = 50 \text{ s}^{-1}$ and $\omega_d = \sqrt{\omega_0^2 - \alpha^2} = 5000$ rad/s

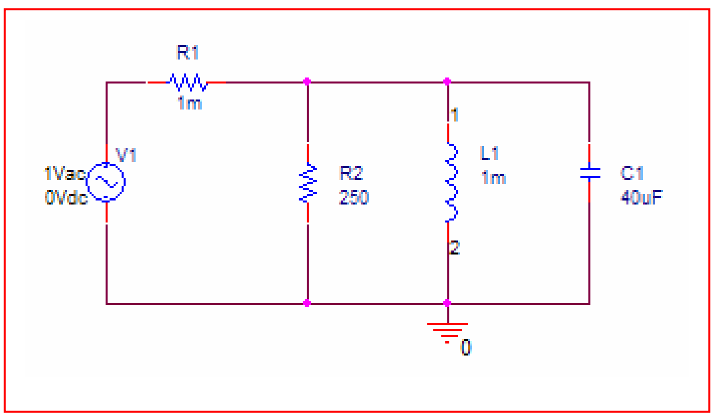
$\mathbf{Z}_{\text{in}}(\omega_0) = R$ so find R.

$$C = \frac{1}{\omega_0^2 L} = \frac{1}{\left(\omega_d^2 + \alpha^2\right)L} = \boxed{40\ \mu\text{F}}. \quad R = \frac{1}{2\alpha C} = \frac{L\left(\omega_d^2 + \alpha^2\right)}{2(50)} = \boxed{250\ \Omega}$$

(b) The resonant frequency is $\dfrac{1}{\sqrt{LC}} = 5000$ rad/s or $f_0 = 795.8$ Hz.

We see from the simulation result that the ratio of the test source voltage to its current is 250 Ω at the resonant frequency; the small error is due to the series resistance PSpice required.





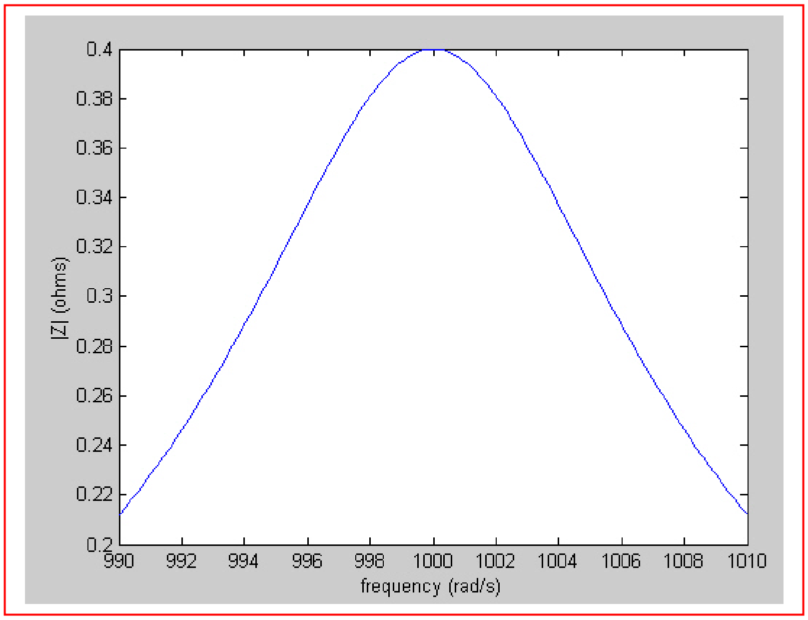
16. $\omega_o = 1000$ rad/s, $Q_o = 80$, $C = 0.2\,\mu$F

(a) $L = \dfrac{1}{\omega_o^2 C} = \dfrac{10^6}{0.2\times 10^6} = \boxed{5\,\text{H},}\; Q_o = \omega_o RC \;\therefore R = \dfrac{80}{10^3 \times 0.2\times 10^{-6}} = \boxed{400\,k\Omega}$

(b) $B = \omega_o / Q_o = 1000/80 = 12.5$

$\therefore \frac{1}{2}B = 6.25$ rad/s

$\therefore \left|\bar{Z}\right| = R \Big/ \left|1 + j\dfrac{\omega - \omega_o}{B/2}\right| = 400\times 10^3 \Big/ \sqrt{1 + \left(\dfrac{\omega - \omega_o}{6.25}\right)^2}$



17.

$$\omega_1 = 103\,\text{rad/s},\ \omega_2 = 118,$$

$$\left|\bar{Z}(j105)\right| = 10\,\Omega$$

$$\omega_o^2 = \omega_1\omega_2 = 103\times 118$$

$$\therefore \omega_o = 110.245^+,\ \text{B} = 118 - 103 = 15\ \text{rad/s},\ \text{Q}_o = \frac{\omega_o}{\text{B}} = \frac{110.245^+}{15} = 7.350$$

$$\therefore 7.350 = \omega_o\text{RC}\quad \therefore \text{RC} = \frac{7.350}{110.245_1^+} = 66.67\times 10^{-3},\ \text{LC} = \frac{1}{\omega_o^2} = \frac{1}{12{,}154}$$

$$\left|\bar{Y}(j105)\right| = 0.1 = \left|\frac{1}{\text{R}} + j\left(105\text{C} - \frac{1}{105\text{L}}\right)\right| = \left|15\text{C} + j\left(105\text{C} - \frac{12{,}154}{105}\text{C}\right)\right| = 18.456\,\text{C}$$

$$\therefore \text{C} = \frac{0.1}{18.456} = \boxed{5.418\,\text{mF},}\ \text{R} = \frac{1}{15}\text{C} = \boxed{12.304\,\Omega,}\ \text{L} = \frac{1}{12{,}154\text{C}} = \boxed{15.185^-\,\text{mH}}$$

18. $\omega_o = 30$ krad/s, $Q_o = 10$, $R = 600\,\Omega$,

(a) $B = \dfrac{\omega_o}{Q_o} = \boxed{3 \text{ krad/s}}$

(b) $N = \dfrac{\omega - \omega_o}{B/2} = \dfrac{28-30}{1.5} = \boxed{-1.3333}$

(c) $\mathbf{Z}_{in}(j28\,000) = 600 / (1 - j1.333) = \boxed{360 \angle 53.13^o\ \Omega}$

(d) $\bar{Z}_{in}(j28{,}000) = \left[\dfrac{1}{600} + j28{,}000C - j\dfrac{1}{28{,}000L}\right]^{-1}, C = \dfrac{Q_o}{\omega_o R} = \dfrac{10}{30{,}000 \times 600}$

$L = \dfrac{R}{\omega_o Q_o} = \dfrac{600}{30{,}000 \times 10}, \dfrac{1}{L} = \dfrac{30{,}000 \times 10}{600} \quad \therefore \bar{Z}_{in} = \left[\dfrac{1}{600} + j\left(\dfrac{28}{30} \times \dfrac{10}{600} - \dfrac{30}{28}\,\dfrac{10}{600}\right)\right]^{-1}$

$\bar{Z}_{in} = \dfrac{600}{1 + j10\left(\dfrac{28}{30} - \dfrac{30}{28}\right)} = \boxed{351.906\angle 54.0903°\,\Omega}$

(e) magnitude: $100\%\dfrac{\text{approx-true}}{\text{true}} = 100\%\dfrac{360 - 351.906}{351.906} = \boxed{2.300\%}$

angle: $100\%\dfrac{53.1301° - 54.0903°}{54.0903°} = \boxed{-1.7752\%}$

19. $f_o = 400\,\text{Hz},\ Q_o = 8,\ R = 500\,\Omega,\ \bar{I}_S = 2\times10^{-3}\,\text{A}\ \ \therefore B = 50\,\text{Hz}$

(a) $\left|\bar{V}\right| = 2\times10^{-3}\times500/\sqrt{1+N^2} = 0.5\ \ \therefore 1+N^2 = 4,\ N = \pm\sqrt{3} = \dfrac{f-400}{50/2}$

$\therefore f = 400\pm25\sqrt{3} = \boxed{443.3 \text{ and } 356.7\,\text{Hz}}$

(b) $\left|\bar{I}_R\right| = \dfrac{|v|}{R} = \dfrac{1}{\sqrt{1+N^2}}\times\dfrac{1}{500} = 0.5\times10^{-3}\ \ \therefore \sqrt{1+N^2} = 4,\ N^2 = 15,\ N = \pm\sqrt{15}$

$\therefore f = 400\pm25\sqrt{15} = \boxed{496.8 \text{ and } 303.2\text{ Hz}}$

20. $\omega_o = 10^6,\ Q_o = 10,\ R = 5\times10^3,\ p.r.$

(a) $Q_o = \dfrac{R}{\omega_o L}\quad \therefore L = \dfrac{5\times10^3}{10\times10^6} = \boxed{0.5\,\text{mH}}$

(b) Approx: $2 = 5/\sqrt{1+N^2}\quad \therefore N = 2.291 = \dfrac{\omega - 10^6}{10^6/20}\quad \therefore \omega = \boxed{1.1146 \text{ Mrad/S}}$

Exact: $\bar{Y} = \dfrac{1}{R}\left[1 + jQ_o\left(\dfrac{\omega}{\omega_o} - \dfrac{\omega_o}{\omega}\right)\right]\ \therefore 0.5 = 0.2\sqrt{1+100\left(\omega - \dfrac{1}{\omega}\right)^2}$ (ω in Mrad/S)

$\therefore 6.25 = 1 + 100(\omega^2 - 2 + 1/\omega^2),\ \omega^2 - 2 + \dfrac{1}{\omega^2} = 0.0525,\ \omega^2 + \dfrac{1}{\omega^2} = 2.0525$

$\omega^4 - 2.0525\omega^2 + 1 = 0,\ \omega^2 = \dfrac{1}{2}\left(2.0525 + \sqrt{2.0525^2 - 4}\right) = 1.2569, \omega = \boxed{1.1211 \text{ Mrad/s}}$

(c) Approx: $\angle Y = 30°\quad \therefore \tan^{-1} N = 30°,\ N = 0.5774 = \dfrac{\omega - 1}{1/20},\ \omega = \boxed{1.0289 \text{ Mrad/s}}$

Exact: $\bar{Y} = \dfrac{1}{5000}\left[1 + j10\left(\omega - \dfrac{1}{\omega}\right)\right]$ (in Mrad/s) $\therefore \tan 30° = 0.5774 = 10\left(\omega - \dfrac{1}{\omega}\right)$

$\therefore \omega - \dfrac{1}{\omega} = 0.05774,\ \omega^2 - 0.05774\omega - 1 = 0, \omega = \dfrac{0.05774 + \sqrt{0.05774^2 + 4}}{2} = \boxed{1.0293 \text{ Mrad/s}}$

21.

(a) $C = 3 + 7 = 10\,\text{nF} \quad \therefore \omega_o = \dfrac{1}{\sqrt{10^{-4}10^{-8}}} = \boxed{10^6 \text{ rad/s}}$

(b) $Q_o = \omega_o CR = 10^6 10^{-8} 5\ 5\times10^3 = 50$

$B = \omega_o / Q_o = 20 \text{ krad/s}$

Parallel current source is $\dfrac{1\angle 0°}{\bar{Z}_3} = j\omega 3\times10^{-9}$ At ω_o, $I_s = j10^{6-9}\times 3$

$\therefore V_{1,0} = j3\times10^{-3}\times5\times10^3 = \boxed{15\angle 90°\text{ V}}$

(c) $\omega - \omega_o = 15\times10^3 \quad \therefore N = \dfrac{15\times10^3}{10\times10^3} = 1.5 \quad \therefore \bar{V}_1 = \dfrac{15\angle 90°}{1 + j1.5} = \boxed{8.321\angle 33.69°\text{ V}}$

22.

(a) $$\bar{Z}_{in}(s) = \frac{(5+0.01s)(5+10^6/s)}{10+0.01s+10^6/s} = \frac{(5+0.01s)(5s+10^6)}{0.01s^2+10s+10^6}$$

$$\bar{Z}_{in}(s) = \frac{0.05s^2+25s+10^4 s+5\times10^6}{0.01s^2+10s+10^6}$$

$$\therefore \bar{Z}_{in}(j\omega) = \frac{5\times10^6-0.05\omega^2+j10,025\omega}{10^6-0.01\omega^2+j10\omega}$$

At $\omega = \omega_o$, $\dfrac{10,025\omega_o}{5\times10^6-0.05\omega_o^2} = \dfrac{10\omega_o}{10^6-0.01\omega_o^2}$, $10.025\times10^9 - 100.25\omega_o^2 = 5\times10^7 - 0.5\omega_o^2$

$$\therefore 99.75\omega_o^2 = 9.975\times10^9, \omega_o = \boxed{10,000 \text{ rad/s}}$$

(b) $$\bar{Z}_{in}(j\omega_o) = (5+j100)\|(5-j100) = \frac{25+10,000}{10} = \boxed{1002.5\,\Omega}$$

23. $f_o = 1000\,\text{Hz},\ \text{Q}_o = 40, \left|\overline{\text{Z}}_{in}(j\omega_o)\right| = 2k\Omega \quad \therefore \text{B} = 25\,\text{Hz}$

(a) $\mathbf{Z}_{\text{in}}(j\omega) = \dfrac{2000}{1 + j\text{N}}, \ \text{N} = \dfrac{f - 1000}{12.5}, \ f = 1010, \ \therefore \text{N} = 0.8$

$\mathbf{Z}_{\text{in}} = 2000 / (1 + j0.8) = \boxed{1562 \angle -38.66^{\text{o}}\ \Omega}$

(b) $0.9 f_o < f < 1.1 f_o \quad \boxed{\therefore 900 < f < 1100\ \text{Hz}}$

24. Taking $2^{-½} = 0.7$, we read from

Fig. 16.48*a*: 1.7 kHz – 0.6 kHz = $\boxed{\text{1.1 kHz}}$

Fig. 16.48*b*: 2×10^7 Hz – 900 Hz = $\boxed{\text{20 MHz}}$

25. Bandwidth = $2\pi f_0 = 2\pi 10^6 = \omega_2 - \omega_1$, where $\omega_1 = 2\pi(5.5)10^3$.

(a) $\omega_2 = \omega_1 + B$, therefore f_2 = 5.5 + 10^3 kHz = $\boxed{1.0055 \text{ MHz}}$

(b) $f_0 = \sqrt{f_1 f_2} = \sqrt{(5.5)(1005.5)} = \boxed{74.37 \text{ kHz}}$

(c) $Q_0 = \dfrac{f_0}{B} = \dfrac{74.37\times 10^3}{10^6} = \boxed{0.074}$

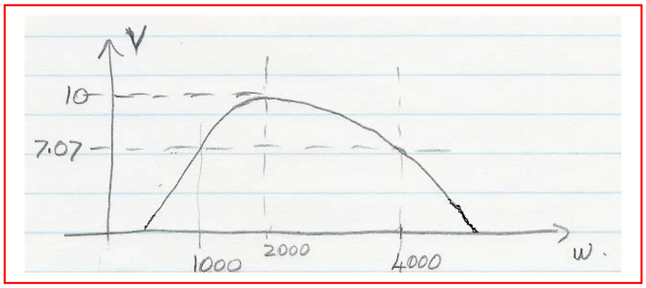
26. Bandwidth = 10^9 Hz = $f_2 - f_1$, where $f_1 = 75.3\times10^6$ Hz.

(a) $f_2 = f_1 + B$, therefore $\boxed{f_2 = 1.0753 \text{ GHz}}$

(b) $f_0 = \sqrt{f_1 f_2} = \sqrt{\left(75.3\times10^6\right)\left(1.0753\times10^9\right)} = \boxed{284.6 \text{ MHz}}$

(c) $Q_0 = \dfrac{f_0}{B} = \dfrac{284.6\times10^6}{10^9} = \boxed{0.2846}$

27. (a) To complete the sketch, we need to first find ω_0, which we obtain in part (b).



(b) $\omega_0 = \sqrt{\omega_1 \omega_2} = 2000$ rad/s or $\boxed{f_0 = 318.3 \text{ Hz}}$

(c) B = $\omega_2 - \omega_1 = 3000$ rad/s or $\boxed{477.5 \text{ Hz}}$

(d) $Q = \dfrac{\omega_0}{\omega_2 - \omega_1} = \dfrac{2000}{3000} = \boxed{0.667}$

28. (a) We begin by labelling the series string with the capacitor as string 1, and the other as string 2. We next find the parallel equivalent of each, and determine the frequency where $X_{p1} + X_{p2} = 0$.

Then $X_{p_1} = \dfrac{R_1^2 + X_1^2}{X_1}$, and similarly $X_{p_2} = \dfrac{R_2^2 + X_2^2}{X_2}$.

For $X_{p_1} + X_{p_2} = 0$ we have $\dfrac{R_1^2 + X_1^2}{X_1} + \dfrac{R_2^2 + X_2^2}{X_2} = 0$ [1]

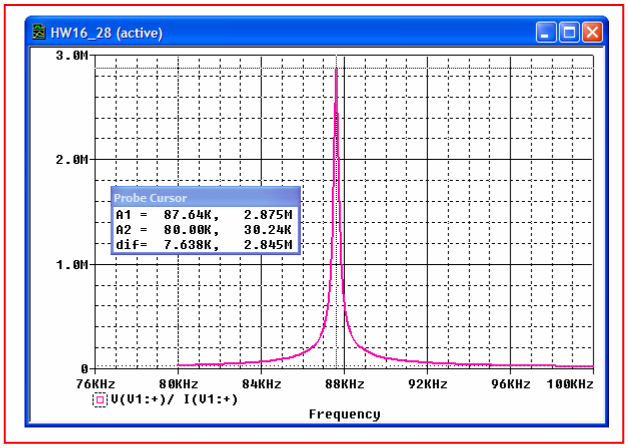
At ω_0, $X_1 = -\dfrac{1}{\omega_0 C}$ $\therefore$ $\dfrac{R_1^2 + X_1^2}{X_1} = \dfrac{5^2 + \dfrac{10^{24}}{\omega_0^2 (330)^2}}{\dfrac{-10^{12}}{330\omega_0}}$.

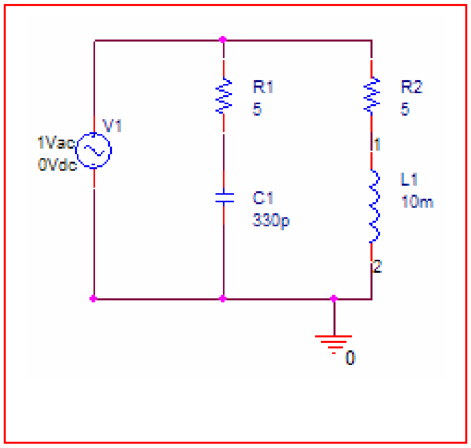
At ω_0, $X_2 = \omega_0 L$ $\therefore$ $\dfrac{R_2^2 + X_2^2}{X_2} = \dfrac{5^2 + 10^{-4}\omega_0^2}{10^{-2}\omega_0}$.

Enforcing Eq. [1], then, leads to $\omega_0 = \sqrt{\dfrac{10^{22} - (25)(330)10^{12}}{(330)10^8 - 25(33)^2}} = 550.5$ krad/s

or $\boxed{f_0 = 87.61 \text{ kHz.}}$

(b) We see the simulation result agrees reasonably, with a resonant frequency of 87.6 kHz





29. (a) We design for a bandwidth of 5.5 kHz, a low-frequency cut-off of 500 Hz, and a resonant impedance of 1 kΩ (no value was specified). Thus, we need to specify values for R, L, and C.

$$f_2 = f_1 + B = 6 \text{ kHz}$$

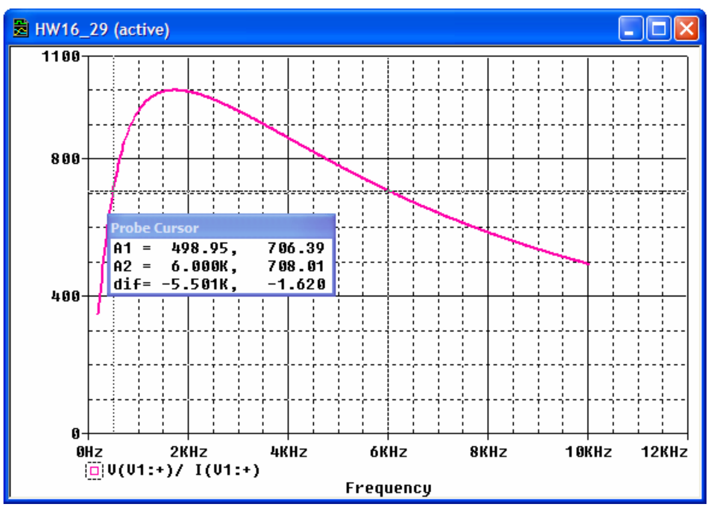
$$f_0 = \sqrt{f_1 f_2} = \sqrt{(0.5)(6)} = \sqrt{3} \text{ kHz}$$

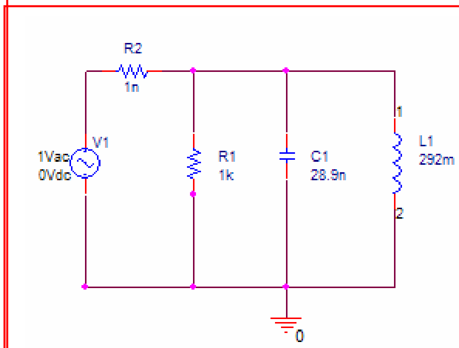
$$Q_0 = \frac{f_0}{B} = \frac{\sqrt{3}\times 10^3}{5.5\times 10^3}$$

$$Q_0 = \omega_0 RC \quad \text{so} \quad C = \frac{Q_0}{\omega_0 R} = \frac{1}{\left(5.5\times10^3\right)(2\pi)10^3} = \boxed{28.9 \text{ nF}}$$

$$\text{L} = \frac{1}{\omega_0^2 C} = \frac{\left(5.5\times10^3\right)10^3}{2\pi\left(3\times10^6\right)} = \boxed{292 \text{ mH}} \qquad \text{and, of course, } \boxed{\text{R} = 1 \text{ k}\Omega}$$

(b) From the simulation, we observe a bandwidth of 5.5 kHz, a lower frequency cutoff of approximately 500 Hz, and a peak impedance of 1000 Ω, as desired.





30. (a) $f_0 = \frac{1}{2\pi}\frac{1}{\sqrt{LC}} = \frac{1}{2\pi}\frac{1}{\sqrt{\left(400\times10^{-6}\right)\left(3.3\times10^{-6}\right)}} = \boxed{4.38\text{ kHz}}$

(b) $Q_0 = \frac{\omega_0 L}{R} = \frac{1}{\sqrt{LC}}\frac{L}{R} = \frac{1}{10}\sqrt{\frac{400}{3.3}} = \boxed{1.10}$

(c) **Z** at resonance = $\boxed{R = 10\ \Omega}$

(d) **Z** at 0.438 kHz =

$$10 + j\left[2\pi(438)\left(400\times10^{-6}\right) - \frac{1}{2\pi(438)\left(3.3\times10^{-6}\right)}\right] = \boxed{10 - j109.01\ \Omega}$$

(e) **Z** at 43.8 kHz =

$$10 + j\left[2\pi(438)\left(400\times10^{-4}\right) - \frac{1}{2\pi(438)\left(3.3\times10^{-4}\right)}\right] = \boxed{10 - j108.98\ \Omega}$$

31. Bandwidth = 3 MHz, f_1 = 17 kHz.

(a) $f_2 = f_1 + B = \boxed{3.017 \text{ MHz}}$

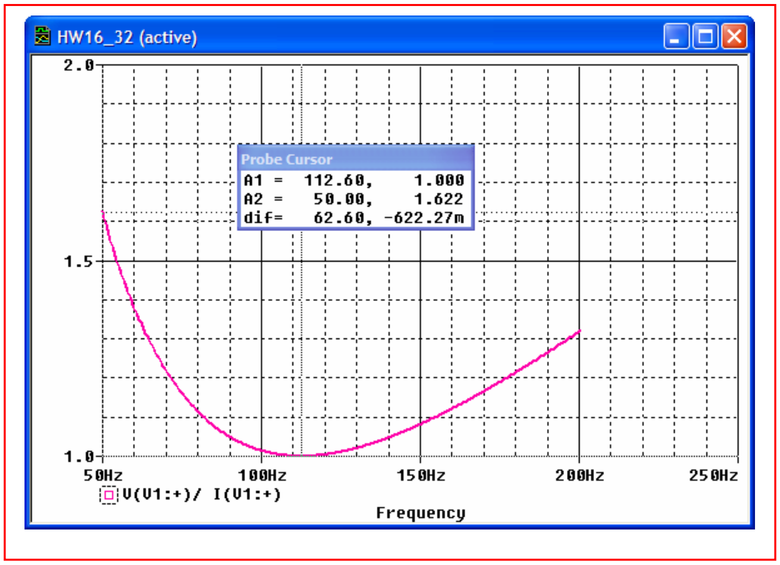
(b) $f_0 = \sqrt{f_1 f_2} = \boxed{226.5 \text{ kHz}}$

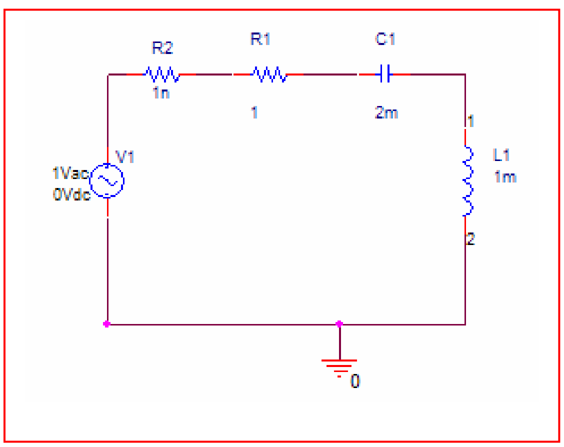
(c) $Q_0 = \dfrac{f_0}{B} = \boxed{0.0755}$

32. (a) $\boxed{\mathbf{Z}_0 = 1\ \Omega \quad \text{by definition}}$

(b) $\omega_0 = \dfrac{1}{\sqrt{LC}} = \dfrac{10^3}{\sqrt{2}} = 707$ rad/s $\quad = \boxed{112.5 \text{ Hz}}$

(c) PSpice simulation verifies an impedance of 1 Ω at f = 112.6 Hz.

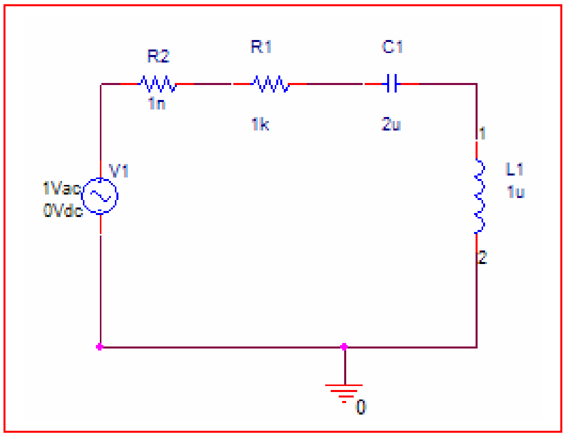


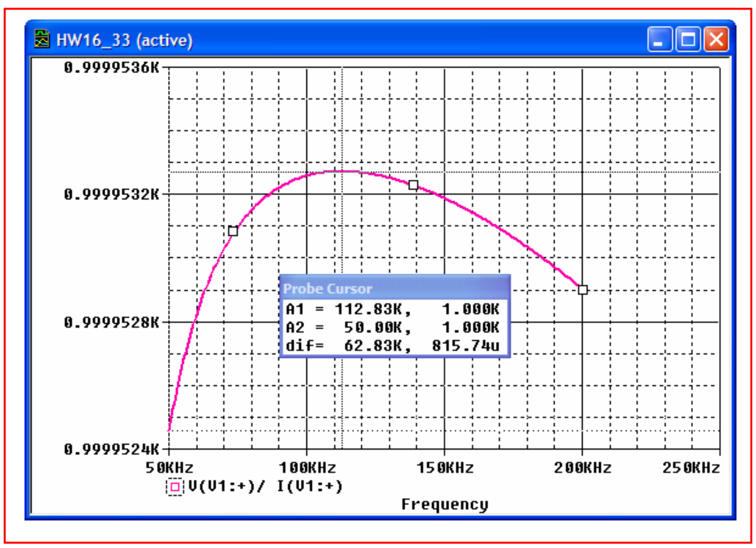


33. (a) $\boxed{\mathbf{Z}_0 = 1\ \text{k}\Omega \quad \text{by definition}}$

(b) $\omega_0 = \dfrac{1}{\sqrt{LC}} = \dfrac{10^6}{\sqrt{2}} = 707$ krad/s $= \boxed{112.5\ \text{kHz}}$

(c) PSpice simulation verifies an impedance of 1 kΩ at f = 112.8 kHz.





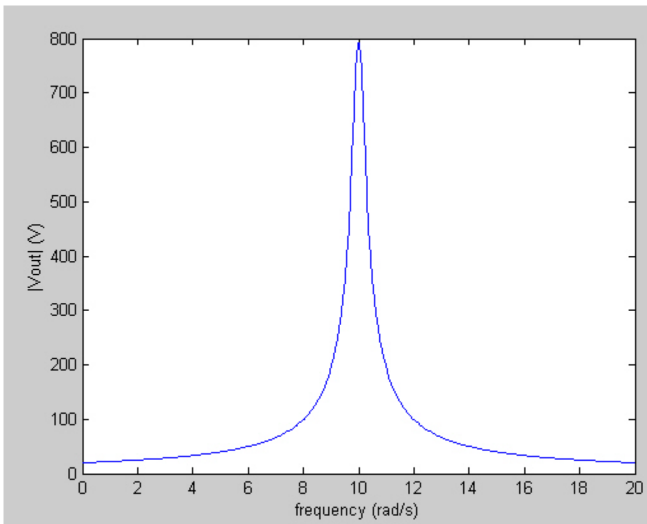
34.

(a) $20\text{A}\|6\Omega,\ 3\|6=2,\ 40\,\text{V in series with } 2+1=3\Omega$

$$\omega_o=\frac{1}{\sqrt{\text{LC}}}=10\text{ rad/s, Q}_o=\frac{\omega_o\text{L}}{\text{R}}=\frac{60}{3}=20\,\Omega$$

$$\text{B}=\frac{10}{20}=0.5,\frac{1}{2}\text{B}=0.25,\left|\overline{\text{V}}_{out}(j\omega_o)\right|=40\text{Q}_o=800\,\text{V}$$

$$\therefore\left|\overline{\text{V}}_{out}(j\omega)\right|=800\Big/\sqrt{1+\left(\frac{\omega-10}{0.25}\right)^2}$$



(b) $\omega=9$ rad/s

$$\text{(Approx: }\left|\overline{\text{V}}_{out}(j9)\right|=\frac{800}{\sqrt{17}}=194.03\,\text{V}$$

$$\text{Exact: }\overline{\text{V}}_{out}=\frac{40}{3+j(6\omega-600/\omega)}\times\frac{600}{j\omega}$$

$$\therefore\overline{\text{V}}_{out}(j9)=\frac{24,000}{9[3+j(54-66.67)]}=\boxed{204.86\angle-13.325^\circ\text{ V}}$$

35. Series: $R = 50\,\Omega,\ L = 4\,\text{mH},\ C = 10^{-7}$

(a) $\omega_o = 1/\sqrt{4\times 10^{-3-7}} = \boxed{50\ \text{krad/s}}$

(b) $f_o = 50\times 10^3 / 2\pi = \boxed{7.958\,\text{kHz}}$

(c) $Q_o = \dfrac{\omega_o L}{R} = \dfrac{50\times 10^3 \times 4\times 10^{-3}}{50} = \boxed{4}$

(d) $B = \omega_o / Q_o = 50\times 10^3 / 4 = \boxed{12.5\ \text{krad/s}}$

(e) $\omega_1 = \omega_o\left[\sqrt{1+(1/2Q_o)^2} - 1/2Q_o\right] = 50\left[\sqrt{1+1/64} - 1/8\right] = \boxed{44.14\ \text{krad/s}}$

(f) $\omega_2 = 50\left[\sqrt{65/64} + 1/8\right] = \boxed{56.64\ \text{krad/s}}$

(g) $\bar{Z}_{in}(j45{,}000) = 50 + j(180 - 10^{7-3}/45) = 50 - j42.22 = \boxed{65.44\angle -40.18°\,\Omega}$

(h) $\left|\bar{Z}_c / \bar{Z}_R\right|_{45{,}000} = \left|10^7 / j45{,}000\times 50\right| = \boxed{4.444}$

36. Apply 1 A, in at top. $\therefore \overline{\mathrm{V}}_R = 10\,\mathrm{V}$

(a) $\overline{\mathrm{V}}_{in} = \overline{\mathrm{Z}}_{in} = 10^{-3}s + 10 + \dfrac{10^8}{5s}(0.5\times 10 + 1) = 10^{-3}s + 10 + \dfrac{1.2\times 10^8}{s}$

$\overline{\mathrm{Z}}_{in}(j\omega) = 10 + j(10^{-3}\omega - 1.2\times 10^8/\omega)\ \ \therefore 10^{-3}\omega_o = 1.2\times 10^8/\omega_o$

$\therefore \omega_o^2 = 1.2\times 10^{11},\ \omega_o = \boxed{346.4\text{ krad/s}}$

(b) $\mathrm{Q}_o = \dfrac{\omega_o \mathrm{L}}{\mathrm{R}} = \dfrac{346.4\times 10^{3-3}}{10} = \boxed{34.64}$

37. Find the Thévenin equivalent seen by the inductor-capacitor combination:

$$\text{SC}: 1.5 = \bar{\text{V}}_1 + 10\left(\frac{\text{V}_1}{125} - 0.105\,\text{V}_1\right) \therefore \bar{\text{V}}_1 = 50\,\text{V}$$

$$\therefore \downarrow \bar{\text{I}}_{SC} = \frac{50}{125} = 0.4\,\text{A}$$

$$\text{OC}: \bar{\text{V}}_1 = 0 \;\therefore \bar{\text{V}}_{OC} = 1.5\,\text{V} \;\;\therefore \text{R}_{th} = \frac{1.5}{0.4} = 3.75\,\Omega$$

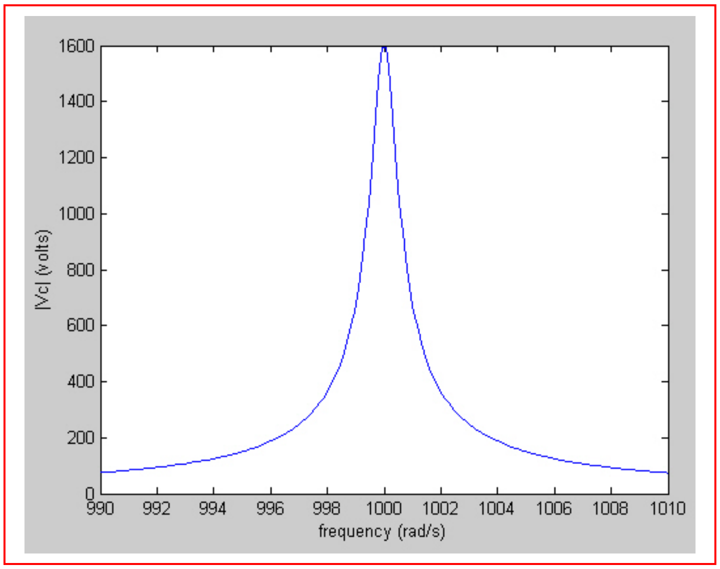
$$\therefore \omega_o = 1/\sqrt{4 \times 0.25 \times 10^{-6}} = 1000, \text{Q}_o = \frac{1000 \times 4}{3.75} = 1066.7$$

$$\text{B} = \omega_o / \text{Q}_o = \frac{1000}{1066.7} = 0.9375, \frac{1}{2}\text{B} = 0.4688 \text{ rad/s}$$

$$\left|\bar{\text{V}}_C\right|_{\max} = \text{Q}_o \text{V}_{th} = 1066.7 \times 1.5 = 1600\,\text{V}$$

Therefore, keep your hands off!

To generate a plot of $|\mathbf{V}_C|$ vs. frequency, note that $\mathbf{V}_C(j\omega) = 1.5\dfrac{-\dfrac{j}{\omega C}}{3.75 + j\omega L - \dfrac{j}{\omega C}}$



38. Series, $f_o = 500\,\text{Hz},\ Q_o = 10,\ X_{L,0} = 500\,\Omega$

(a) $500 = \omega_o L = 2\pi(500)L \ \therefore \boxed{L = 0.15915^+\ \text{H}},\ C = \dfrac{1}{\omega_o^2 L} = \dfrac{2\pi}{(2\pi\times 500)^2} = \boxed{0.6366\,\mu\text{F}}$

$Q_o = 10 = \dfrac{X_{L,0}}{R} = \dfrac{500}{R} \ \therefore \boxed{R = 50\,\Omega}$

(b) $1 = \bar{I}\left(50 + j2\pi f \times \dfrac{1}{2\pi} - j\dfrac{10^6 \times 0.5\pi}{2\pi f}\right) = \bar{I}\left(50 + jf - j\dfrac{250{,}000}{f}\right)$

$\therefore \bar{I} = 1/50 + j(f - 250{,}000/f),\ \bar{V}_c = \dfrac{10^6 \times 0.5\pi}{j2\pi f}\bar{I}$

$\bar{V}_C = \dfrac{-j250{,}000/f}{50 + j(f - 250{,}000/f)} \quad \therefore \left|\bar{V}_c(2\pi\times 450)\right| = \boxed{4.757\,\text{V}}$

$\left|\bar{V}_c(2\pi\times 500)\right| = \boxed{10{,}000\,\text{V}}\ \left|\bar{V}_c(2\pi\times 550)\right| = \boxed{4.218\,\text{V}}$

39.

$$\text{X}: s = 0, \infty, 0: s = -20,000 \pm j80,000\,s^{-1},\ \overline{\text{Z}}_{in}(-10^4) = -20 + j0\,\Omega \quad \therefore \text{SERIES}$$

$$\alpha = 20,000,\ \omega_d = 80,000 \therefore \omega_o = \sqrt{(64+4)10^8} = 82,462 \text{ rad/s}, \frac{1}{\text{LC}} = \omega_o^2 = 68\times10^8$$

$$\frac{\text{R}}{2\text{L}} = \alpha = 20,000 \therefore \frac{\text{R}}{\text{L}_1} = 40,000, \frac{1}{\text{LC}} \times \frac{\text{L}}{\text{R}} = \frac{68\times10^8}{40,000} = 170,000;\ \text{Z}(\sigma) = \text{R} + \sigma\text{L} + \frac{1}{\sigma\text{C}}$$

$$\therefore -20 = \text{R} - 10,000\text{L} - \frac{1}{10,000\text{C}} = \text{R} - \frac{1}{4}\text{R} - \frac{170,000}{10,000}\text{R} \therefore \boxed{\text{R} = 1.2308\,\Omega}$$

$$\therefore \text{L} = \frac{1.2308}{40,000} = \boxed{30.77\ \mu\text{H},}\ \text{C} = \frac{1}{170,000\times1.2308} = \boxed{4.779\ \mu\text{F}}$$

40.

$$\omega_o \; 1/\sqrt{10^{-3-7}} = \boxed{10^5 \text{ rad/s,}}\; Q_L = \frac{10^{5-3}}{1} = 100,\; R_{PL} = 10{,}000\,\Omega$$

$$Q_c = \frac{1}{10^{5-7} \times 0.2} = 500,\; R_{PC} = 500^2 \times 0.2 = 50{,}000\,\Omega$$

$$50 \| 10 = 8.333\,k\Omega \quad \therefore Q_o = \omega_o CR = 10^{5-7} \times 8333 = \boxed{83.33}$$

$$B = \frac{100{,}000}{83.33} = \boxed{1200 \text{ rad/s,}}\; \overline{Z}_{in}(j\omega_o) = \boxed{8333\Omega}$$

$$\omega = 99{,}000 \;\; \therefore N = \frac{(99-100)10^3}{600} = -1.6667,\; \overline{Z}_{in}(j99{,}000) = \frac{8.333}{1-j1.667}$$

$$= \boxed{4.287 \angle 59.04^{o} \text{ k}\Omega}$$

41. $R_{eq} = Q_o / \omega_o C = 50 / 10^{5-7} = 5000\ \Omega$.
Thus, we may write $1/5000 = 1/8333 + 1/R_x$ so that

$R_x = \boxed{12.5\ \text{k}\Omega.}$

42.

$$3\,\text{mH}\,\|\,1.5\,\text{mH} = 1\,\text{mH},\ 2\,\mu\text{F} + 8\,\mu\text{F} = 10\,\mu\text{F},\ \therefore \omega_o = \frac{1}{\sqrt{10^{-3-5}}} = 10\ \text{krad/s}$$

$$Q = \frac{3\times10^{-3}\times10^{4}}{0.3} = 100,\ R_p = 100^2\times0.3 = 3\,k\Omega$$

$$Q = \frac{1.5\times10^{-3}\times10^{4}}{0.25} = 60,\ R_p = 60\times0.25 = 900\,\Omega$$

$$900\,\|\,3000 = 692.3\,\Omega\ \ \therefore Q_L = \frac{692.3}{10^{4-3}} = 69.23$$

$$\therefore R_{LS} = \frac{692.3}{69.23^2} = 0.14444\,\Omega$$

$$Q = \frac{10^6}{10^4\times0.1\times8} = 125,\ R_{pc} = 125^2\times0.1 = 1562.5\ \Omega\,\|\,10\,\mu\text{F}$$

$$\therefore Q_c = 10^4\times10^{-5}\times15625 = 156.25\ \ \therefore R_{SC} = \frac{1562.5}{(156.25)^2} = 0.064\,\Omega$$

$$\therefore R_{S,tot} = 0.14444 + 0.064 = \boxed{0.2084\,\Omega} = \left|\overline{Z}_{in}\right|_{\min},\ \omega_o = \boxed{10\ \text{krad/s}}$$

43.

(a)

$$\omega_o \cong 1/\sqrt{2\times 0.2\times 10^{-3}} = 50 \text{ rad/s}$$

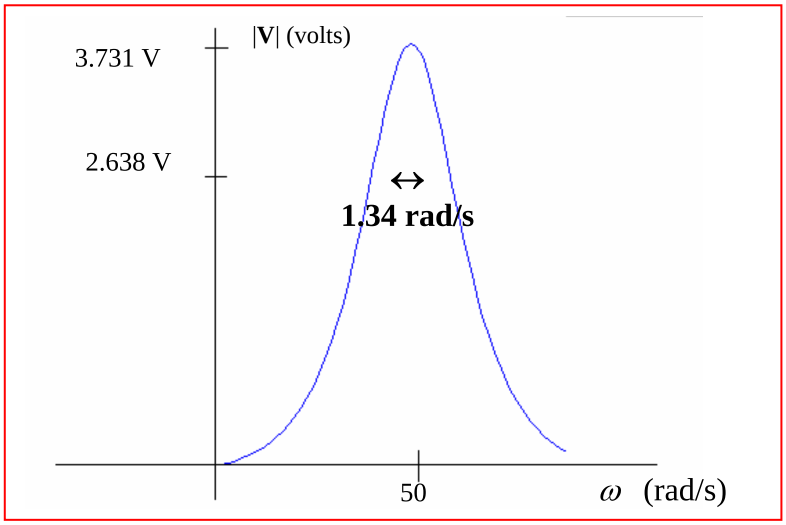
$$Q_{leftL} = 50\times 2.5/2 = 62.5,\ 2\times 62.5^2 = 7812.5\,\Omega$$

$$Q_{rightL} = \frac{50\times 10}{10} = 50,\ 10\times 50^2 = 25\,k\Omega$$

$$Q_c = \frac{1000}{50\times 0.2\times 1} = 100,\ 100^2\times 1 = 10\,k\Omega,\ R_p = 7.8125\|25\|10 = 3731\,\Omega$$

$$Q_o = 50\times 3731\times 0.2\times 10^{-3} = 37.31;\ B = \frac{50}{37.31} = 1.3400,\ \frac{1}{2}B = 0.6700$$

$$\therefore |V|_o = 10^{-3}\times 3731 = \boxed{3.731\,\text{V}}$$



(b)

$$V = 10^{-3}[(2+j125)\|(10+j500)\|(1-j100)]$$

$$= \frac{10^{-3}}{\frac{1}{2+j125}+\frac{1}{10+j500}+\frac{1}{1-j100}} = \boxed{3.7321\angle -0.3950^{+\circ}\,\text{V}}$$

44.

(a)

$$\omega_o \approx \frac{1000}{\sqrt{0.25}} = 2000 \text{ rad/s, } Q_c = 2000 \times 2 \times 10^{-6} \times 25 \times 10^3 = 100$$

$$\therefore R_{C,S} = 25,000/100^2 = 2.5\Omega;\ Q_L = \frac{R}{\omega_o L} = \frac{20 \times 10^4}{2000 \times 0.25} = 40$$

$$\therefore R_{L,S} = \frac{20,000}{1600} = 12.5\Omega \ \ \therefore R_{tot} = 12.5 + 2.5 = 15\Omega$$

$$\therefore Q_o = \frac{2000 \times 0.25}{15} = 33.33 \ \ \therefore |\bar{V}_x| = 1 \times 33.33 \times \frac{1}{2} = \boxed{16.667 \text{ V}}$$

(b)

$$20,000 \| j500 = \frac{20,000 \times j500}{20,000 + j500} = 12,4922 + j499.688\Omega$$

$$25,000 \| -j250 = \frac{25,000(-j250)}{25,000 - j250} = 2.4998 - j249.975$$

$$\therefore \bar{Z}_{in} = 12.4922 + 2.4998 + j499.688 - j250 - j249.975 = 14.9920 - j0.2870\,\Omega$$

$$\therefore |\bar{I}| = 1/|14.9920 - j0.2870| = 66.6902 \text{ mA} \ \ \therefore |\bar{V}_x| = 250 \times 66.6902 \times 10^{-3} = \boxed{16.6726 \text{ V}}$$

45. $Q = \omega CR, \quad R_S = \dfrac{R_P}{1+Q^2}$, and $X_S = \dfrac{Q^2 X_P}{1+Q^2}$

$$X_S = -\frac{1}{\omega C_S},\ X_P = -\frac{1}{\omega C_P} \therefore C_S = C_P \frac{1+Q^2}{Q^2}$$

(a) $\omega = 10^3$ rad/s, Q = 5

Therefore, $R_S = 5/26 = \boxed{192\ \Omega,}$ $C_S = 26/25\ \mu F = \boxed{1.06\ \mu F}$

(b) $\omega = 10^4$ rad/s, Q = 50

Therefore, $R_S = 5/2501 = \boxed{2\ \Omega,}$ $C_S = 2501/2500\ \mu F = \boxed{1.0004\ \mu F}$

(c) $\omega = 10^5$ rad/s, Q = 500

Therefore, $R_S = 5000/250001 = \boxed{20\ m\Omega,}$ $C_S = 250001/250000\ \mu F = \boxed{1.0\ \mu F}$

46. $R_P = R_S\left(1+Q^2\right)$, and $X_P = X_S \dfrac{1+Q^2}{Q^2}$

$$C_P = C_S \frac{Q^2}{1+Q^2}$$

(a) $\omega = 10^3$ rad/s, Q = 0.2

Therefore, $R_P = 5(1 + 0.04) = \boxed{5.2\ \text{k}\Omega}$, $\boxed{C_P = 38.5\ \text{nF}}$

(b) $\omega = 10^4$ rad/s, Q = 50

Therefore, $R_P = 5(1 + 0.0004) = \boxed{5.002\ \text{k}\Omega,}$ $\boxed{C_P = 400\ \text{pF}}$

(c) $\omega = 10^5$ rad/s, Q = 500

Therefore, $R_P = 5(1 + 4\times10^{-6}) = \boxed{5\ \text{k}\Omega,\quad C_P = 4\ \text{pF}}$

47. $Q = \dfrac{R}{\omega L}$, $R_S = \dfrac{R_P}{1+Q^2}$, and $X_S = \dfrac{Q^2 X_P}{1+Q^2}$. $L_S = L_P \dfrac{Q^2}{1+Q^2}$

(a) $\omega = 10^3$ rad/s, Q = 142.4×10^3

Therefore, $R_S = 470/(1 + Q^2) =$ $\boxed{23.2 \text{ n}\Omega,}$ $\boxed{L_S = 3.3\ \mu\text{H}}$

(b) $\omega = 10^4$ rad/s, Q = 14.24×10^3

Therefore, $R_S = 470/(1 + Q^2) =$ $\boxed{23.2\ \mu\Omega,}$ $\boxed{L_S = 3.3\ \mu\text{H}}$

(c) $\omega = 10^5$ rad/s, Q = 1.424×10^3

Therefore, $R_S = 470/(1 + Q^2) =$ $\boxed{232\ \mu\Omega, \quad L_S = 3.3\ \mu\text{H}}$

48. $R_P = R_S\left(1+Q^2\right)$, and $X_P = X_S\left(\dfrac{1+Q^2}{Q^2}\right)$

$$L_P = L_S\left(\frac{1+Q^2}{Q^2}\right)$$

(a) $\omega = 10^3$ rad/s, Q = 7.02×10^{-6}

Therefore, $R_P = 470(1 + Q^2) = \boxed{470\ \Omega,\quad L_P = 67\text{ mF}}$

(b) $\omega = 10^4$ rad/s, Q = 50

Therefore, $R_P = 470(1 + Q^2) = \boxed{470\ \Omega,\quad L_P = 670\ \mu\text{F}}$

(c) $\omega = 10^5$ rad/s, Q = 500

Therefore, $R_P = 470(1 + Q^2) = \boxed{470\ \Omega,\quad L_P = 6.70\ \mu\text{F}}$

49. (a) For the left parallel circuit, $Q = \frac{R}{\omega L} \approx \frac{470}{10^7 10^{-6}} = 47$. Since Q > 5, the series equivalent is a 10/47 Ω resistor in series with 1 μH.

For the right parallel circuit, $Q = \omega CR \approx 10^7 10^{-8}(200) = 20$. Again, Q > 5, so the series equivalent is

a 10/20 Ω = 500 mΩ resistor in series with 10 nF.

We may therefore approximate the network as a 700 mΩ resistor in series with a 10 nF capacitor, in series with a 1 μH inductor, in series with the 10 μH inductor of interest.

At the resonant frequency the network connected in series with the inductor has an impedance of 700 mΩ. The inductor present an impedance of 100 Ω. Thus, $|\mathbf{V}_x| = 1$ V.

(b) $\mathbf{Z}_L = \frac{(470)(j10^7 10^{-6})}{470 + j10} = 0.213 + j9.995\ \Omega$. $\mathbf{Z}_L = \frac{R_2 \frac{1}{j\omega C_2}}{R_2 + \frac{1}{j\omega C_2}} = 0.499 - j9.975\ \Omega$

$\mathbf{Z}_3 = j100\ \Omega$.

Thus, $\mathbf{V}_x = \frac{\mathbf{Z}_3}{\mathbf{Z}_1 + \mathbf{Z}_L + \mathbf{Z}_3}(1\angle 0) = \frac{j100}{0.714 + j0.02} = 0.99745 + j0.0071$ V

So that $|\mathbf{V}_x| = 0.99977$ V. Our approximation was pretty accurate, at least at this frequency.

50.

(a) $K_m = \frac{50}{100} = 0.5 \;\; K_f = \frac{20\times 10^3}{10^6} = 0.02$

$\therefore 9.82\,\mu\text{H} \rightarrow 0.5\times 9.82\times \frac{1}{0.02} = 24.55\,\mu\text{H},\; 31.8\,\mu\text{H} \rightarrow \frac{0.5}{0.02}\times 31.8 = 795\,\mu\text{H}$

$2.57\,\text{nF} \rightarrow \frac{2.57}{0.5\times 0.02} = \boxed{257\,\text{nF}}$

(b) same ordinate; divide numbers on abscissa by $\boxed{50}$

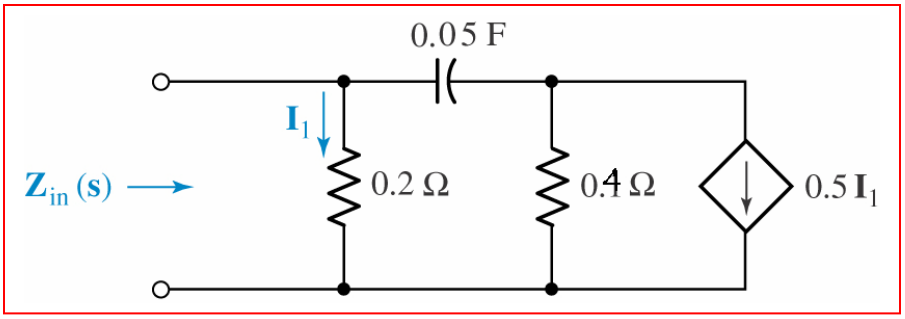
51.

(a) Apply 1 V $\therefore \bar{I}_1 = 10\text{A} \quad \therefore 0.5\,\bar{I}_1 = 5\text{A} \downarrow$; $5\text{A} \| 0.2\ \Omega$ can be replaced by 1 V in series with 0.2 Ω

$$\therefore \bar{I}_{in} \rightarrow = 10 + \frac{1-(-1)}{0.2+2/s} = 10 + \frac{2s}{0.2s+2} = \frac{4s+20}{0.2s+2} = \frac{20(s+5)}{s+10} \quad \therefore \boxed{\bar{Z}_{in}(s) = \frac{s+10}{20(s+5)}}$$

(b) $$\text{K}_m = 2,\ \text{K}_f = 5 \ \therefore \bar{Z}_{in}(s) \rightarrow \frac{2(s/5+10)}{20(s/5+5)} = \boxed{\frac{0.1(s+50)}{s+25}}$$

(c) $\boxed{0.1\Omega \rightarrow 0.2\Omega,\ 0.2\Omega \rightarrow 0.4\Omega,\ 0.5\text{F} \rightarrow 0.05\text{F},\ 0.5\bar{I}_1 \rightarrow 0.5\bar{I}_1}$



52.

(a) $\omega_o \approx 1/\sqrt{(2+8)10^{-3}10^{-6}} = \boxed{10^4 \text{ rad/s}}$

$Q_{L,8} = 10^4 / 8\times10^{-3}10^4 = 125 \ \therefore R_{L,S} = \dfrac{10^4}{125^2} = 0.64\,\Omega$

$2+8 = 10\,\text{mH} \ \ \therefore Q_L = \dfrac{10^4\times10\times10^{-3}}{0.64} = 156.25$

$\therefore R_{L,P} = 0.64\times156.25^2 = 15.625\,k\Omega;\ Q_C = \dfrac{1}{10^4\times10^{-6}} = 100,\ R_{C,P} = 100^2\times1 = 10\,k\Omega$

$\therefore R_P = 20\|15.625\|10 = 4.673\,k\Omega \ \therefore Q_o = 10^4\times10^{-6}\times4.673\times10^3 = \boxed{46.73}$

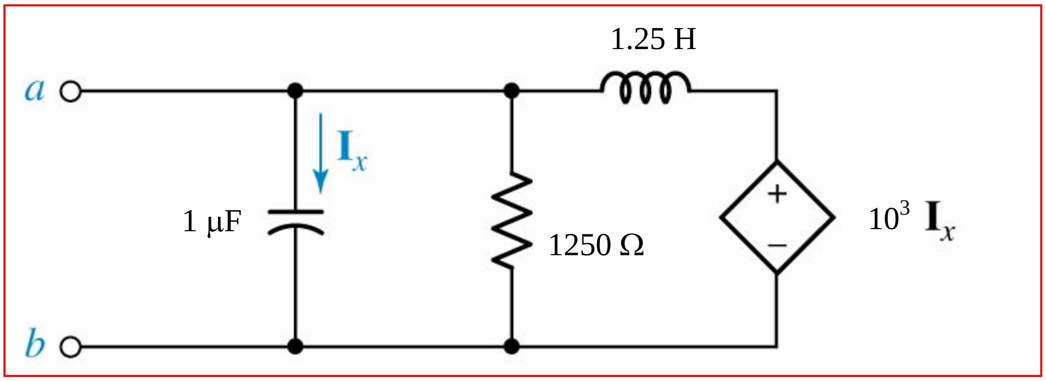
(b) $K_f = 10^6/10^4 = 100,\ K_m = 1 \ \therefore \boxed{\text{R's stay the same; 2 mH} \rightarrow 20\,\mu\text{H, } 8\,\text{mH} \rightarrow 80\,\mu\text{H}, 1\,\mu\text{F} \rightarrow 10\,\text{nF}}$

(c) $\omega_o = \boxed{10^6 \text{ rad/s,}}\ Q_o$ stays the same, $\therefore B = \dfrac{10^6}{46.73} = \boxed{21.40 \text{ krad/s}}$

53.

(a) $\quad K_m = 250,\ K_f = 400 \ \therefore 0.1\text{F} \to \dfrac{0.1}{250 \times 400} = 1\mu\text{F}$

$5\Omega \to 1250\,\Omega,\ 2\text{H} \to \dfrac{2 \times 250}{400} = 1.25\,\text{H},\ 4\bar{\text{I}}_x \to 10^3\ \bar{\text{I}}_x$



(b) $\quad \omega = 10^3.$ Apply 1 V $\ \therefore \text{I}_x = 10^{-6}s,\ \downarrow \text{I}_{1250} = \dfrac{1}{1250}$

$\therefore 1000\ \text{I}_x = 10^{-3}s \quad \therefore \to \text{I}_L = \dfrac{1 - 10^{-3}s}{1.25s}$

$\therefore \text{I}_{in} = 10^{-6}s + \dfrac{1}{1250} + \dfrac{0.8}{s}(1 - 10^{-3}s) = 10^{-6}s + \dfrac{0.8}{s};\ s = j10^3$

$\therefore \text{I}_{in} = j10^{-3} + \dfrac{0.8 \times 10^{-3}}{j} = j0.2 \times 10^{-3} \ \therefore \text{Z}_{th} = \dfrac{1}{\text{I}_{in}} = \dfrac{1000}{j0.2} = \boxed{-j5\ k\Omega\ \bar{\text{V}}_{oc} = 0}$

54.

(a) $\bar{\mathrm{I}}_s = 2\angle 0°\ \mathrm{A},\ \omega = 50\ \ \therefore\ \bar{\mathrm{V}}_{out} = \boxed{60\angle 25°\ \mathrm{V}}$

(b) $\bar{\mathrm{I}}_s = 2\angle 40°\ \mathrm{A},\ \omega = 50\ \ \therefore\ \bar{\mathrm{V}}_{out} = \boxed{60\angle 65°\ \mathrm{V}}$

(c) $\bar{\mathrm{I}}_s = 2\angle 40°\ \mathrm{A},\ \omega = 200,\ \ \therefore \boxed{\mathrm{OTSK}}$

(d) $\mathrm{K}_m = 30,\ \bar{\mathrm{I}}_S = 2\angle 40°\ \mathrm{A},\ \omega = 50\ \ \therefore\ \bar{\mathrm{V}}_{out} = \boxed{1800\angle 65°\ \mathrm{V}}$

(e) $\mathrm{K}_m = 30,\ \mathrm{K}_f = 4,\ \bar{\mathrm{I}}_s = 2\angle 40°\ \mathrm{A},\ \omega = 200\ \ \therefore\ \bar{\mathrm{V}}_{out} = \boxed{1800\angle 65°\ \mathrm{V}}$

55.

(a) $\bar{\mathrm{H}}/(s) = 0.2 \quad \therefore \mathrm{H}_{dB} = 20 \log 0.2 = \boxed{-13.979\,\mathrm{dB}}$

(b) $\bar{\mathrm{H}}(s) = 50 \quad \therefore \mathrm{H}_{dB} = 20 \log 50 = \boxed{33.98\,\mathrm{dB}}$

(c) $\bar{\mathrm{H}}(j10) = \dfrac{12}{2+j10} + \dfrac{26}{20+j10} \quad \therefore \mathrm{H}_{dB} = 20 \log \left|\dfrac{6}{1+j5} + \dfrac{13}{10+j5}\right| = 20 \log \left|\dfrac{292+j380}{-60+j220}\right| = \boxed{6.451\mathrm{dB}}$

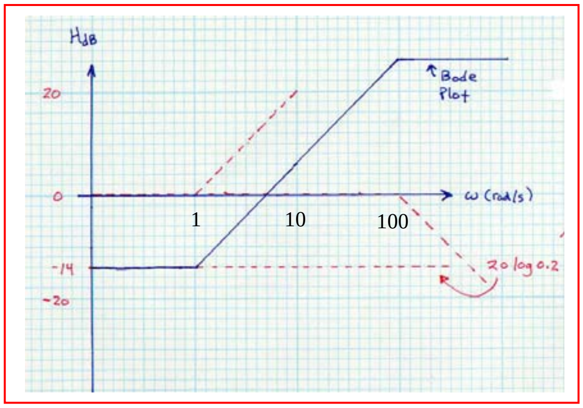
(d) $\mathrm{H}_{dB} = 37.6\,\mathrm{dB} \quad \therefore \left|\bar{\mathrm{H}}(s)\right| = 10^{37.6/20} = \boxed{75.86}$

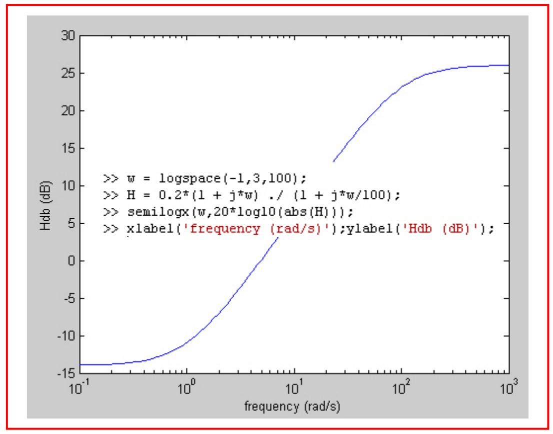
(e) $\mathrm{H}_{dB} = -8\,\mathrm{dB} \quad \therefore \left|\bar{\mathrm{H}}(s)\right| = 10^{-8/20} = \boxed{0.3981}$

(f) $\mathrm{H}_{dB} = 0.01\mathrm{dB} \quad \therefore \left|\bar{\mathrm{H}}(s)\right| = 10^{0.01/20} = \boxed{1.0012}$

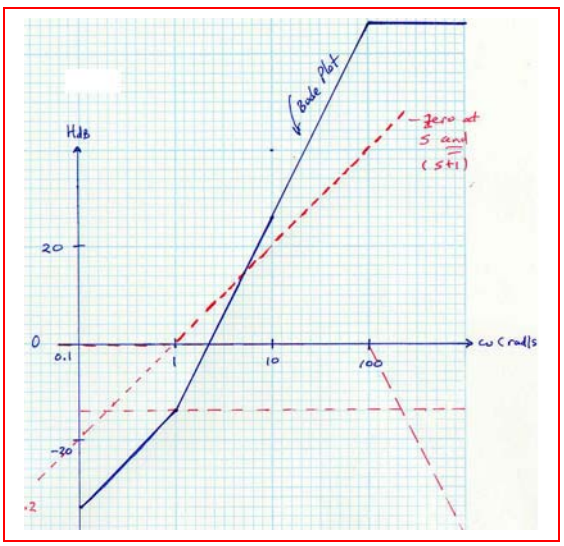
56. (d) MATLAB verification- shown adjacent to Bode plots below.

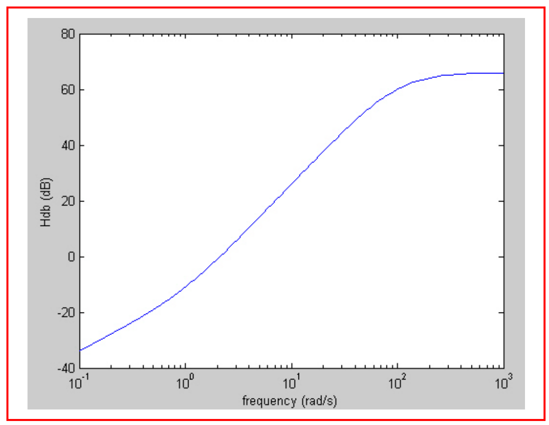
(a) $\bar{\mathrm{H}}(s) = \dfrac{20(s+1)}{s+100} = \dfrac{0.2(1+s)}{1+s/100}$, $0.2 \to -14\,\text{dB}$



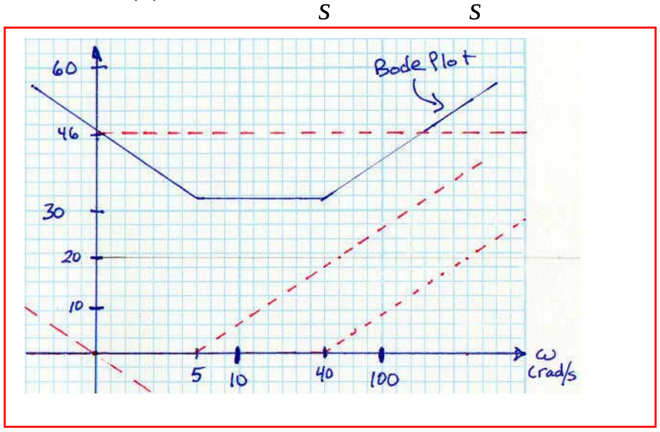


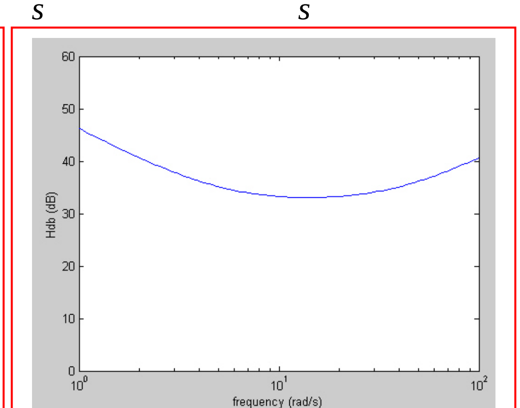
(b) $\bar{\mathrm{H}}(s) = \dfrac{2000(s+1)s}{(s+100)^2} = \dfrac{0.2s(1+s)}{(1+s/100)^2}$, $0.2 \to -14\,\text{dB}$





(c) $\bar{\mathrm{H}}(s) = s + 45 + \dfrac{200}{s} = \dfrac{s^2 + 45s + 200}{s} = \dfrac{(s+5)(s+40)}{s} = \dfrac{200(1+s/5)(1+s/40)}{s}$, $200 \to 46\,\text{dB}$



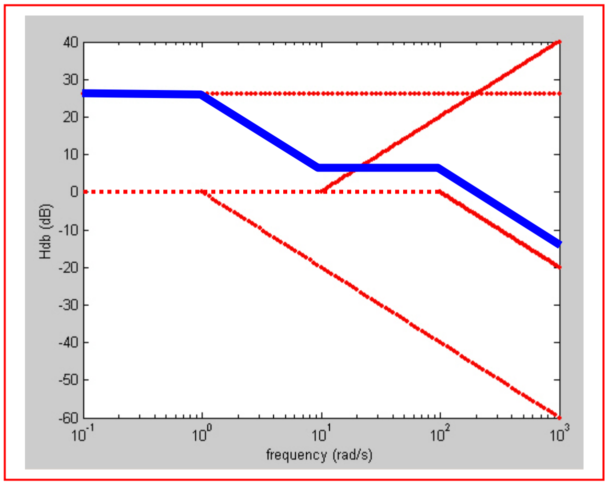


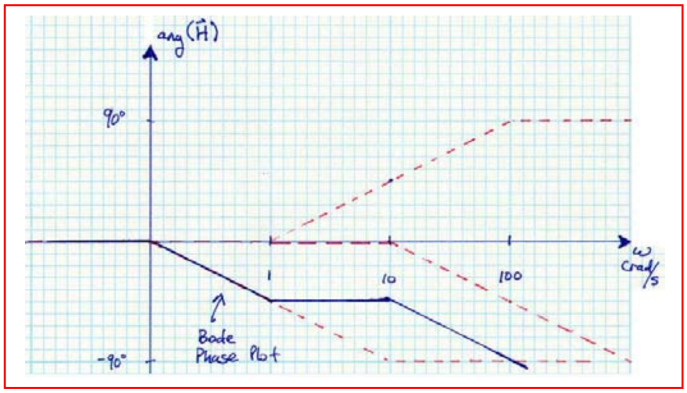
57.

$$\bar{\mathrm{H}}(s) = \frac{\mathrm{V}_C}{I_R} = \frac{(20+2s)(182+200/s)}{202+2s+200/s} \times \frac{200/s}{182+200/s}$$

$$= \frac{400(s+10)}{2(s^2+101s+100)} = \frac{200(10+s)}{(1+s)(100+s)}$$

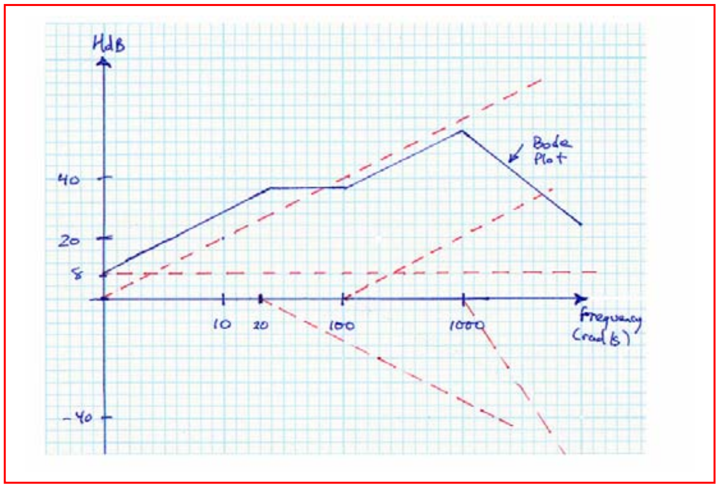
$$\bar{\mathrm{H}}(s) = \frac{20(1+s/10)}{(1+s)(1+s/100)},\ 20 \rightarrow 26\,\mathrm{dB}$$





58.

(a) $\bar{\text{H}}(s) = \dfrac{5\times10^8 s(s+100)}{(s+20)(s+1000)^3} = \dfrac{2.5s(1+s/100)}{(1+s/20)(1+s/1000)^3}$, $2.5 \rightarrow 8\,\text{dB}$



(b) Corners: $\omega = \boxed{20,\ 34\,\text{dB};}$

$\omega = \boxed{100,\ 34\,\text{dB};}$

$\omega = \boxed{1000,\ 54\,\text{dB}}$

Intercepts: $0\,\text{dB},\ 2.5\omega = 1,\ \omega = \boxed{0.4}$

$$\omega = 1,\ 8\,\text{dB};\ 0\,\text{dB}, \frac{2.5\omega(\omega/100)}{(\omega/20)(\omega/1000)^3} = \frac{2.5\omega^2(20)10^9}{100\omega\omega^3} = 1 \quad \therefore\ \omega = \boxed{22,360 \text{ rad/s}}$$

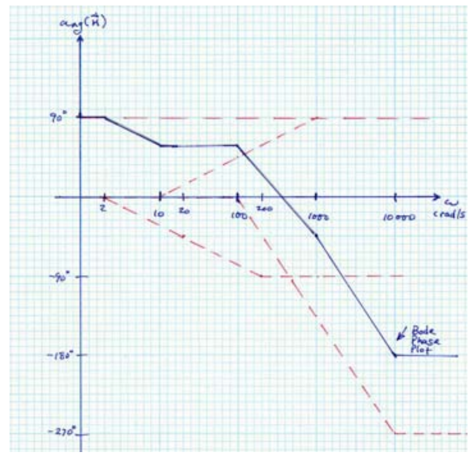
(c) Corners: $\omega = 20, \boxed{31.13\,\text{dB}}$

$$\omega = 100, \boxed{36.69\,\text{dB}} \quad \text{H}_{dB} = 20\log 2.5\omega \frac{\sqrt{1+(\omega/100)^2}}{\sqrt{[1+(\omega/20)^2][1+(\omega/1000)^2]^3}}$$

$\omega = 1000, \boxed{44.99\,\text{dB}}$

59.

(a) $\bar{H}(s) = \dfrac{5\times10^8 s(s+100)}{(s+20)(s+1000)^3} = \dfrac{2.5s(1+s/100)}{(1+s/20)(1+s/1000)^3}$,



(b) $\omega = 2: \angle = \boxed{90°}$

$\omega = 10: \angle = 90° - \left(45° + 45° \log\dfrac{10}{20}\right) = \boxed{58.5°}$

$\omega = 100: \angle = 90° - \left(45° + 45° \log\dfrac{100}{20}\right) + \left(45° + 45° \log\dfrac{100}{100}\right) = \boxed{58.5°}$

$\omega = 200: \angle = 90° - 90° + \left(45° + 45° \log\dfrac{200}{100}\right) - 3\left(45° + 45° \log\dfrac{200}{100}\right) = \boxed{17.9°}$

$\omega = 1000: \angle = 90° - 90° + 90° - 3\left(45° + 45° \log\dfrac{1000}{1000}\right) = \boxed{-45°}$

$\omega = 10{,}000: \angle = 90° - 90° + 90° - 3\times90° = \boxed{-180°}$

(c) $\omega = 2: \angle = 90° + \tan^{-1}0.02 - \tan^{-1}0.1 - 3\tan^{-1}0.002 = \boxed{85.09°}$

$\omega = 10: \angle = 90° + \tan^{-1}0.1 - \tan^{-1}0.5 - 3\tan^{-1}0.01 = \boxed{67.43°}$

$\omega = 100: \angle = 90° + \tan^{-1}1 - \tan^{-1}5 - 3\tan^{-1}0.1 = \boxed{39.18°}$

$\omega = 200: \angle = 90° + \tan^{-1}2 - \tan^{-1}10 - 3\tan^{-1}0.2 = \boxed{35.22°}$

$\omega = 1000: \angle = 90° + \tan^{-1}10 - \tan^{-1}50 - 3\tan^{-1}1 = \boxed{-49.56°}$

$\omega = 10{,}000: \angle = 90° + \tan^{-1}100 - \tan^{-1}500 - 3\tan^{-1}10 = \boxed{-163.33°}$

60.

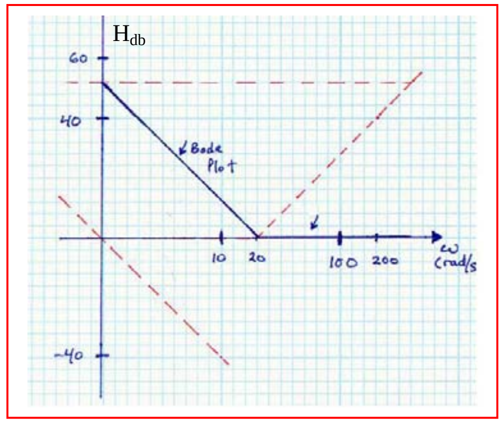
(a) $$\mathrm{H}(s) = 1 + \frac{20}{s} + \frac{400}{s^2} = \frac{s^2 + 20s + 400}{s^2}$$

$$= 400\frac{1 + 2\times 0.5(s/20) + (s/20)^2}{s^2}$$

$$\therefore \omega_o = 20,\ \zeta = 0.5$$

$$20 \log 400 = 52\text{dB}$$

Correction at ω_o is $20 \log 2\,\zeta = 0$ dB



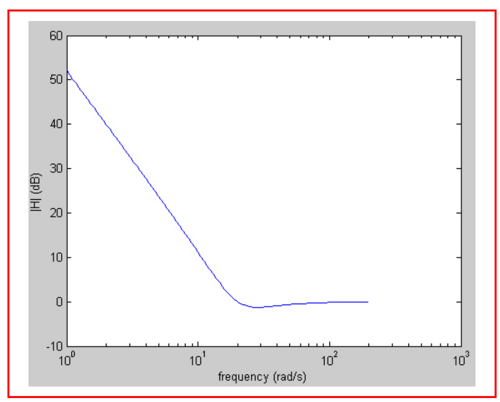
(b) $\omega = 5: \mathrm{H}_{dB} = 52 - 2\times 20 \log 5 = \boxed{24.0\,\text{dB}}$ (plot)

$\mathrm{H}_{dB} = 20 \log |1 - 16 + j4| = \boxed{23.8\,\text{dB}}$ (exact)

$\omega = 100: \mathrm{H}_{dB} = \boxed{0\,\text{dB}}$ (plot)

$\mathrm{H}_{dB} = 20 \log |1 - 0.04 + j0.2| = \boxed{-0.170\text{ dB}}$ (exact)

(c)

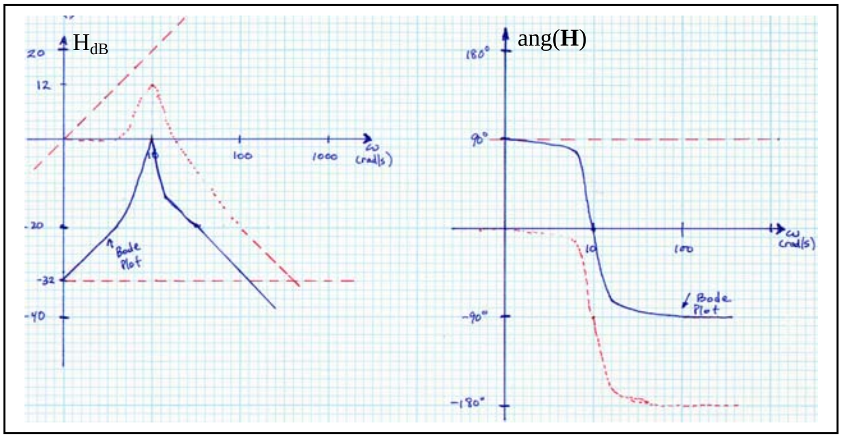


61.

(a) $$\bar{H}(s) = \frac{V_R}{V_5} = \frac{25}{10s + 25 + 1000/s} = \frac{25s}{10s^2 + 25s + 1000} = \boxed{\frac{0.025s}{1 + 2\left(\frac{1}{8}\right)\left(\frac{s}{10}\right) + \left(\frac{s}{10}\right)^2}}$$

(b) $$\therefore \omega_o = 10,\ \zeta = 1/8 \quad \therefore \text{correction} = -20\log\left(2 \times \frac{1}{8}\right) = 12 \text{ dB}$$

$0.025 \rightarrow -32$ dB



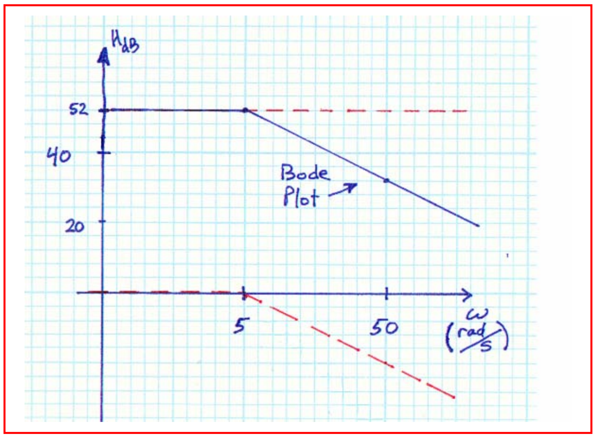
(c) $$\omega = 20,\ \bar{H}(j20) = \frac{j0.5}{1 - 4 + j0.5} \quad \therefore H_{dB} = \boxed{-15.68 \text{ dB}} \quad \angle H(j20) = \boxed{-80.54°}$$

62.

1st two stages, $\overline{\mathrm{H}}_1(s) = \overline{\mathrm{H}}_2(s) = -10;\ \overline{\mathrm{H}}_3(s) = \dfrac{-1/(50\times10^3\times10^{-6})}{s+1/(200\times10^3\times10^{-6})} = \dfrac{-20}{s+5}$

$\therefore \overline{\mathrm{H}}(s) = (-10)(-10)\left(\dfrac{-20}{s+5}\right) = \boxed{\dfrac{-400}{1+s/5}}$

$-400 \rightarrow 52$ dB



63.

(a) 1st stage: $C_{1A} = 1\ \mu F,\ R_{1A} = \infty,\ R_{fA} = 10^5 \quad \therefore \bar{H}_A(S) = -R_{fA}C_{1A}s = -0.1s$

2nd stage: $R_{1B} = 10^5,\ R_{fB} = 10^5,\ C_{fB} = 1\ \mu F \quad \therefore \bar{H}_B(s) = \dfrac{-1/R_{1B}C_{fB}}{s + 1/R_{fB}C_{fB}}$

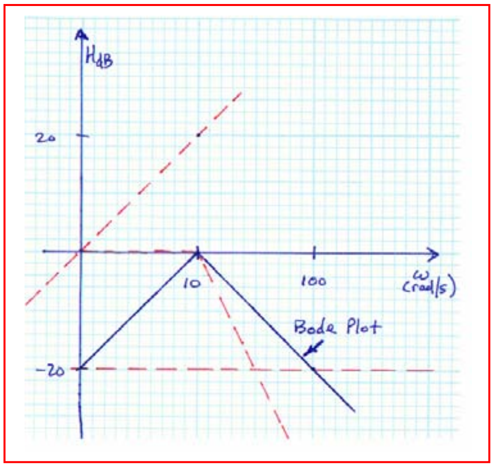
$$\therefore \bar{H}_B(s) = \frac{1/(10^5 \times 10^{-6})}{s + 1/(10^5 \times 10^{-6})} = -\frac{10}{s+10}$$

3rd stage: same as 2nd

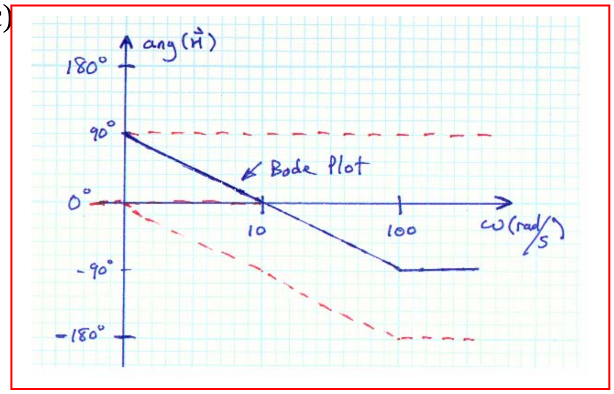
$$\therefore \bar{H}(s) = (-0.1s)\left(\frac{-10}{s+10}\right)\left(\frac{-10}{s+10}\right) = \boxed{-\frac{0.1s}{(1+s/10)^2}}$$

$20\log_{10}(0.1) = -20$ dB

(b)



(c)



64. An amplifier that rejects high-frequency signals is required. There is some ambiguity in the requirements, as social conversations may include frequencies up to 50 kHz, and echolocation sounds, which we are asked to filter out, may begin below this value. Without further information, we decide to set the filter cutoff frequency at 50 kHz to ensure we do not lose information. However, we note that *this decision is not necessarily the only correct one*.

Our input source is a microphone modeled as a sinusoidal voltage source having a peak amplitude of 15 mV in series with a 1-Ω resistor. Our output device is an earphone modeled as a 1-kΩ resistor. A voltage of 15 mV from the microphone should correspond to about 1 V at the earphone according to the specifications, requiring a gain of 1000/15 = 66.7.

If we select a non-inverting op amp topology, we then need $\frac{R_f}{R_1} = 66.7 - 1 = 65.7$

Arbitrarily choosing $R_1 = 1$ kΩ, we then need $R_f = 65.7$ kΩ. This completes the amplification part. Next, we need to filter out frequencies greater than 50 kHz.

Placing a capacitor across the microphone terminals will "short out" high frequencies. We design for $\omega_c = 2\pi f_c = 2\pi(50\times10^3) = \frac{1}{R_{mic}C_{filter}}$. Since $R_{mic} = 1\ \Omega$, we require

$C_{filter} = 3.183\ \mu$F.

65. We choose a simple series RLC circuit. It was shown in the text that the "gain" of the circuit with the output taken across the resistor is $|A_V| \;=\; \dfrac{\omega RC}{\left[(1-\omega^2 LC)^2 + \omega^2 R^2 C^2\right]^{1/2}}$.

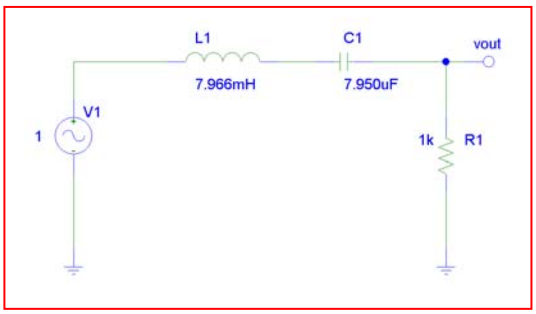
This results in a bandpass filter with corner frequencies at

$$\omega_{c_L} \;=\; \frac{-RC + \sqrt{R^2C^2 + 4LC}}{2LC} \quad \text{and} \quad \omega_{c_H} \;=\; \frac{RC + \sqrt{R^2C^2 + 4LC}}{2LC}$$

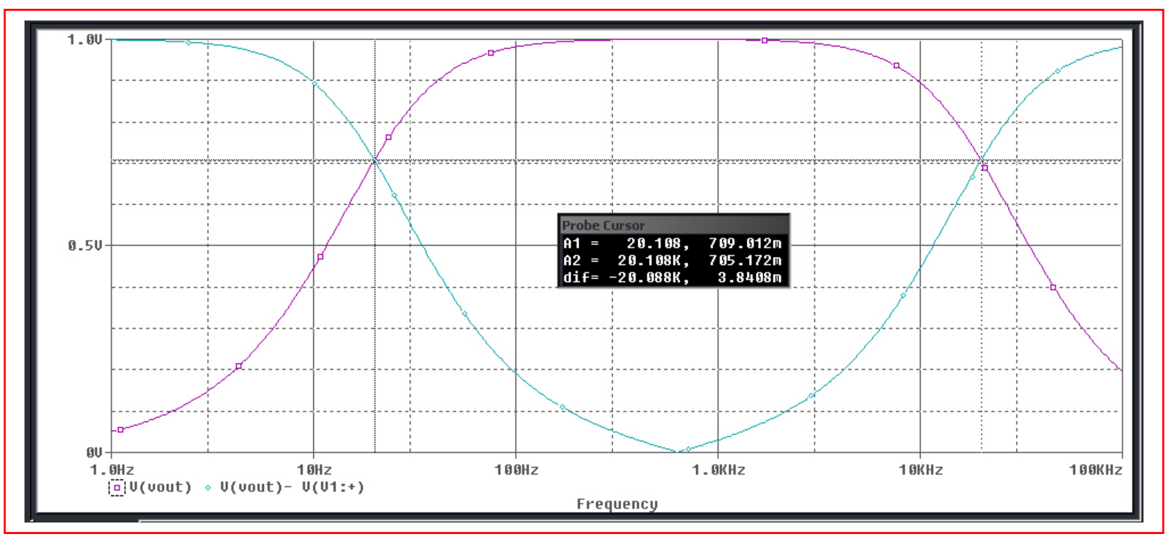
If we take our output across the inductor-capacitor combination instead, we obtain the opposite curve- *i.e.* a bandstop filter with the same cutoff frequencies. Thus, we want

$$2\pi(20) = \frac{-RC + \sqrt{R^2C^2 + 4LC}}{2LC} \text{ and } 2\pi(20\times10^3) \;=\; \frac{RC + \sqrt{R^2C^2 + 4LC}}{2LC}$$

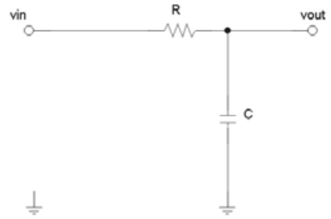
Noting that $\omega_{c_H} - \omega_{c_L}$ = R/L = 125.5 krad/s, we arbitrarily select R = 1 kΩ, so that L = 7.966 mH. Returning to either cutoff frequency expression, we then find C = 7.950 μF



PSpice verification. The circuit performs as required, with a lower corner frequency of about 20 Hz and an upper corner frequency of about 20 kHz.



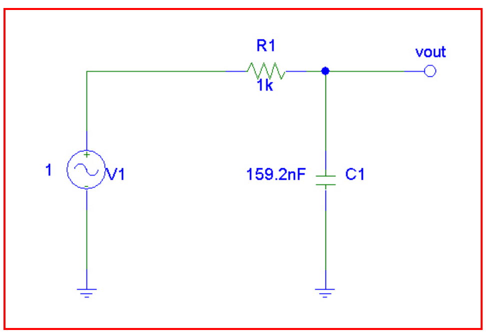
66. We choose a simple RC filter topology:

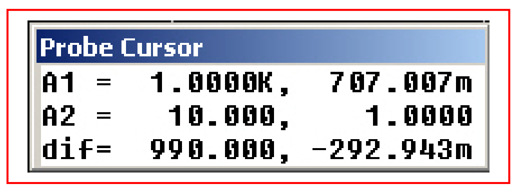


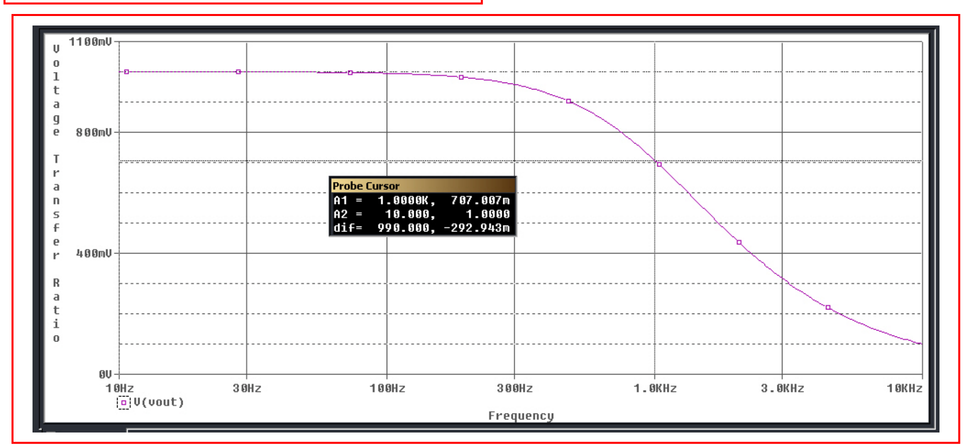
Where $\dfrac{V_{out}}{V_{in}} = \dfrac{1}{1 + j\omega RC}$ and hence $\left|\dfrac{V_{out}}{V_{in}}\right| = \dfrac{1}{\sqrt{1+(\omega RC)^2}}$. We desire a cutoff frequency of 1 kHz, and note that this circuit does indeed act as a low-pass filter (higher frequency signals lead to the capacitor appearing more and more as a short circuit). Thus,

$$= \frac{1}{\sqrt{1+(\omega_c RC)^2}} \quad = \quad \frac{1}{\sqrt{2}}$$ where $\omega_c = 2\pi f_c = 2000\pi$ rad/s.

A small amount of algebra yields $1 + [2\pi(1000)RC]^2 = 2$ or $2000\pi RC = 1$. Arbitrarily setting R = 1 kΩ, we then find that C = 159.2 nF. The operation of the filter is verified in the PSpice simulation below:

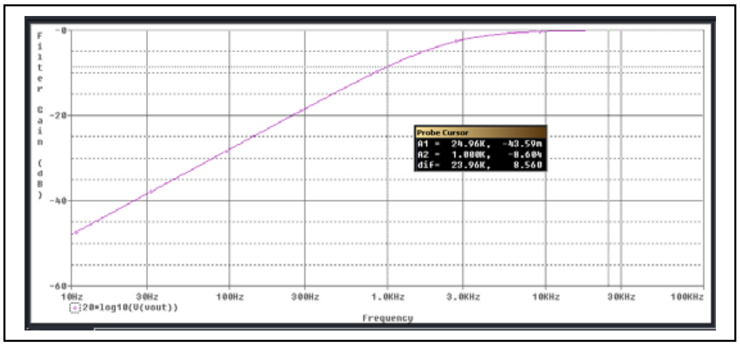


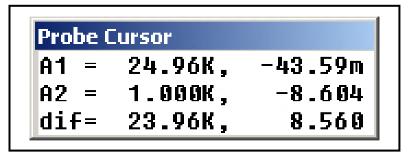




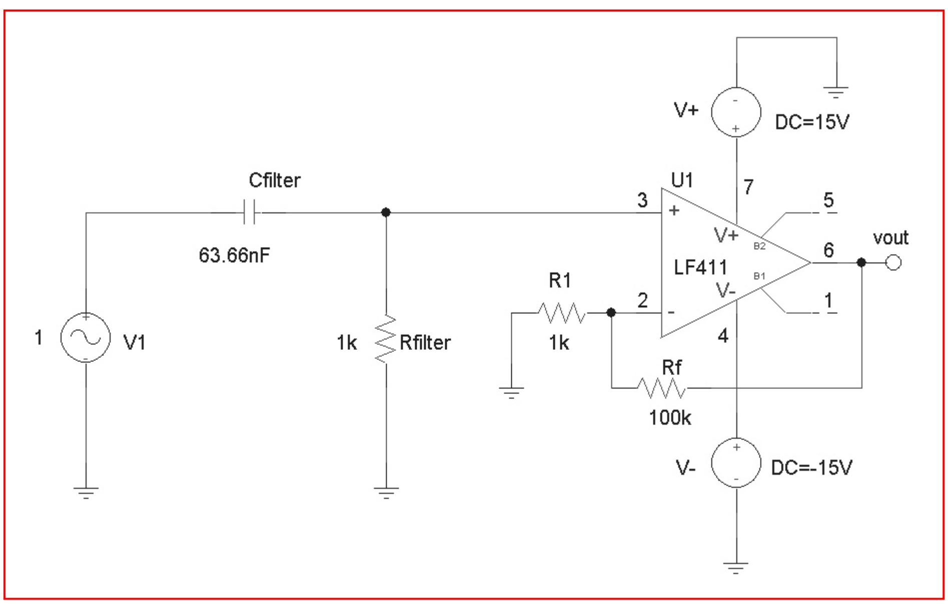
67. We are not provided with the actual spectral shape of the noise signal, although the reduction to 1% of its peak value (a drop of 40 dB) by 1 kHz is useful to know. If we place a simple high-pass RC filter at the input of an op amp stage, designing for a pole at 2.5 kHz should ensure an essentially flat response above 25 kHz, and a 3 dB reduction at 2.5 kHz. If greater tolerance is required, the 40 dB reduction at 1 kHz allows the pole to be moved to a frequency even closer to 1 kHz. The PSpice simulation below shows a filter with R = 1 kΩ (arbitrarily chosen) and C = $\dfrac{1}{2\pi(2.5\times10^{3})(1000)} = 63.66\ \text{nF}$.

At a frequency of 25 kHz, the filter shows minimal gain reduction, but at 1 kHz any signal is reduced by more than 8 dB.





We therefore design a simple non-inverting op amp circuit such as the one below, which with R_f = 100 kΩ and R_1 = 1 kΩ, has a gain of 100 V/V. In simulating the circuit, a gain of approximately 40 dB at 25 kHz was noted, although the gain dropped at higher frequencies, reaching 37 dB around 80 kHz. Thus, to completely assess the suitability of design, more information regarding the frequency spectrum of the "failure" signals would be required.



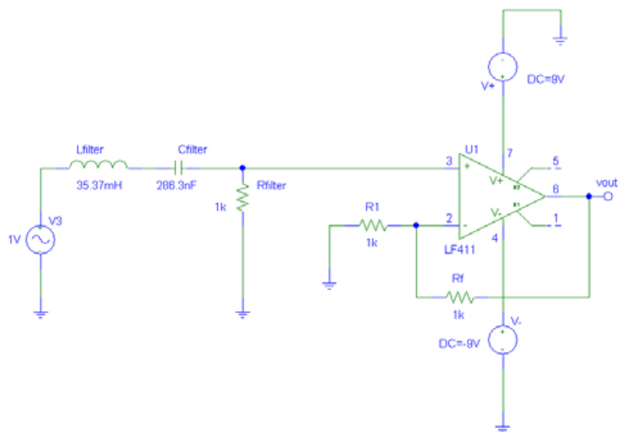
68. We select a simple series RLC circuit with the output taken across the resistor to serve as a bandpass filter with 500 Hz and 5000 Hz cutoff frequencies. From Example 16.12, we know that

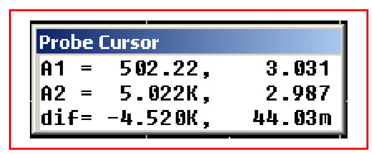
$$\omega_{c_L} = -\frac{R}{2L} + \frac{1}{2LC}\sqrt{R^2C^2 + 4LC} = 2\pi(500)$$

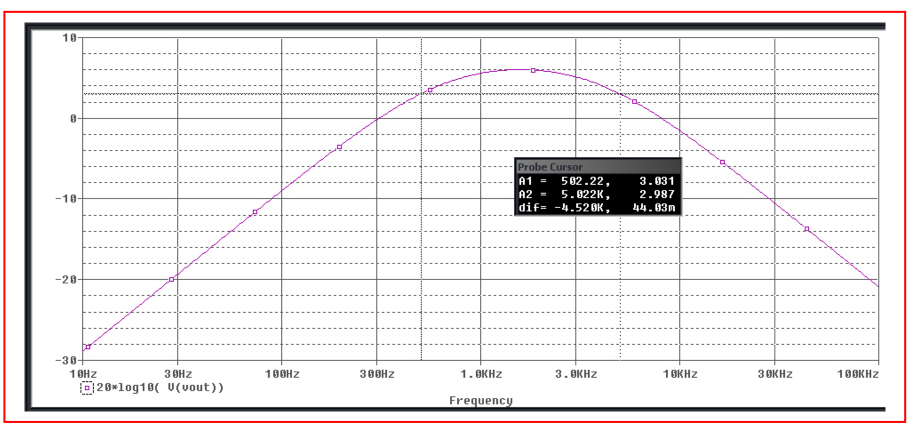
and

$$\omega_{c_H} = \frac{R}{2L} + \frac{1}{2LC}\sqrt{R^2C^2 + 4LC} = 2\pi(5000)$$

With ω_{c_H} - ω_{c_L} = 2p(5000 – 500) = R/L, we (arbitrarily) select R = 1 kΩ, so that L = 35.37 mH. Substituting these two values into the equation for the high-frequency cutoff, we find that C = 286.3 nF. We complete the design by selecting R_1 = 1 kΩ and R_f = 1 kΩ for a gain of 2 (no value of gain was specified). As seen in the PSpice simulation results shown below, the circuit performs as specified at maximum gain (6 dB or 2 V/V), with cutoff frequencies of approximately 500 and 5000 KHz and a peak gain of 6 dB.



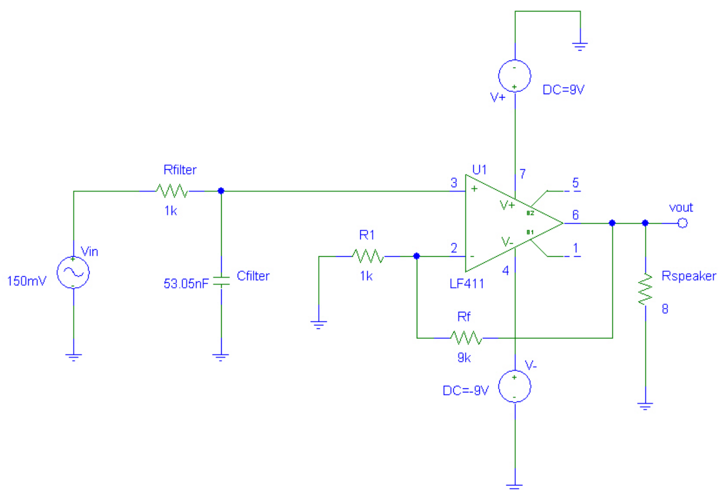


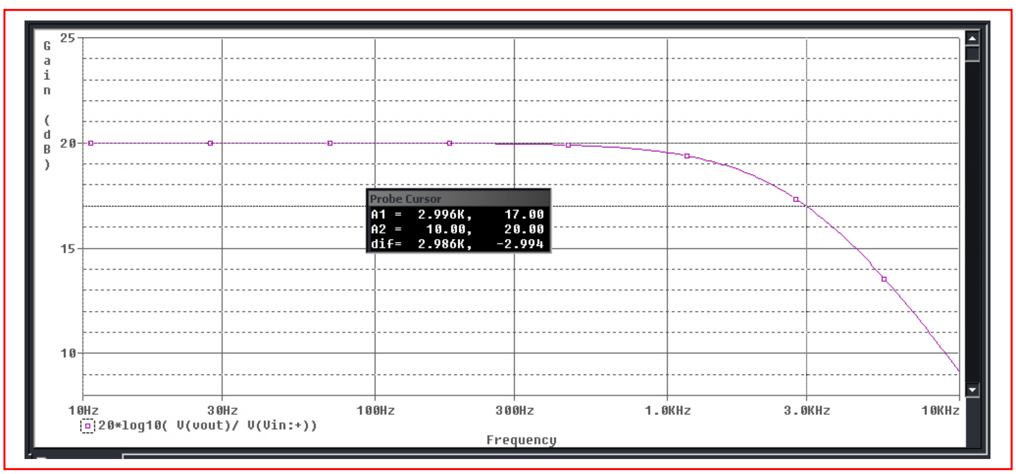


69. For this circuit, we simply need to connect a low-pass filter to the input of a non-inverting op amp having $R_f/R_1 = 9$ (for a gain of 10). If we use a simple RC filter, the cutoff frequency is

$$\omega_c = \frac{1}{RC} = 2\pi(3000)$$

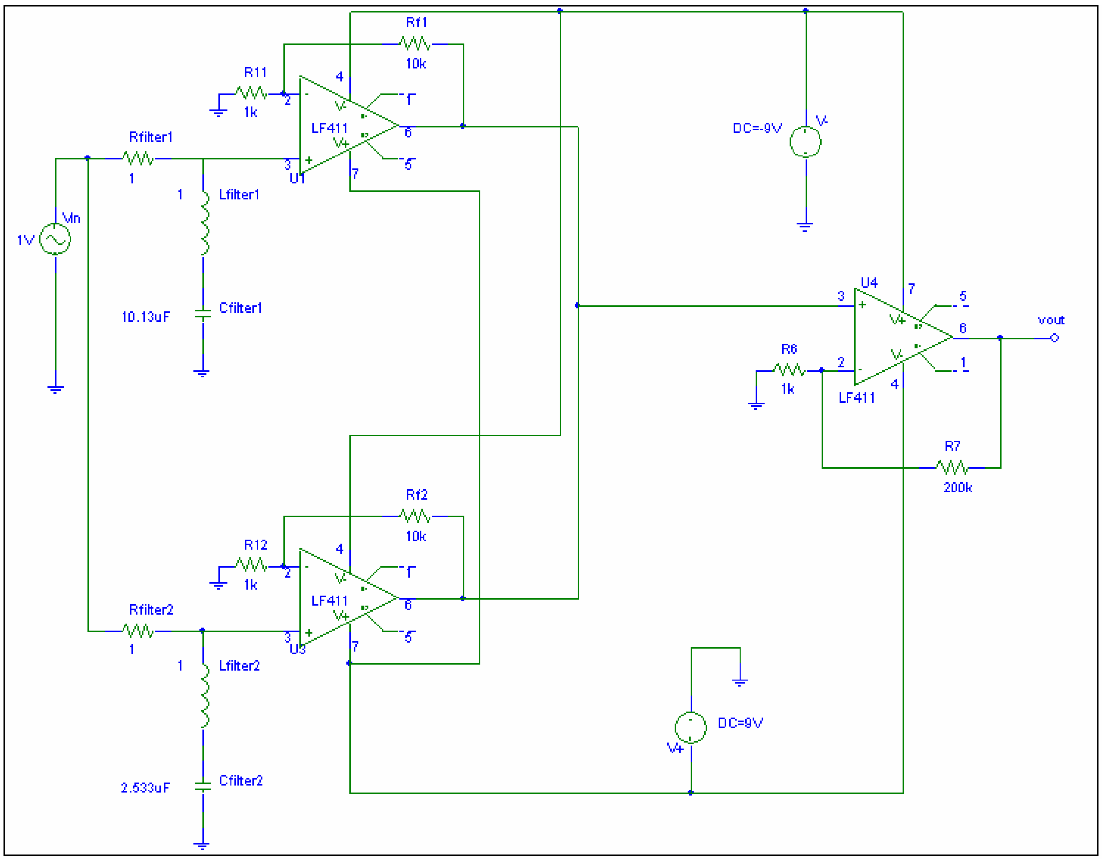
Selecting (arbitrarily) R = 1 kΩ, we find C = 53.05 nF. The PSpice simulation below shows that our design does indeed have a bandwidth of 3 kHz and a peak gain of 10 V/V (20 dB).

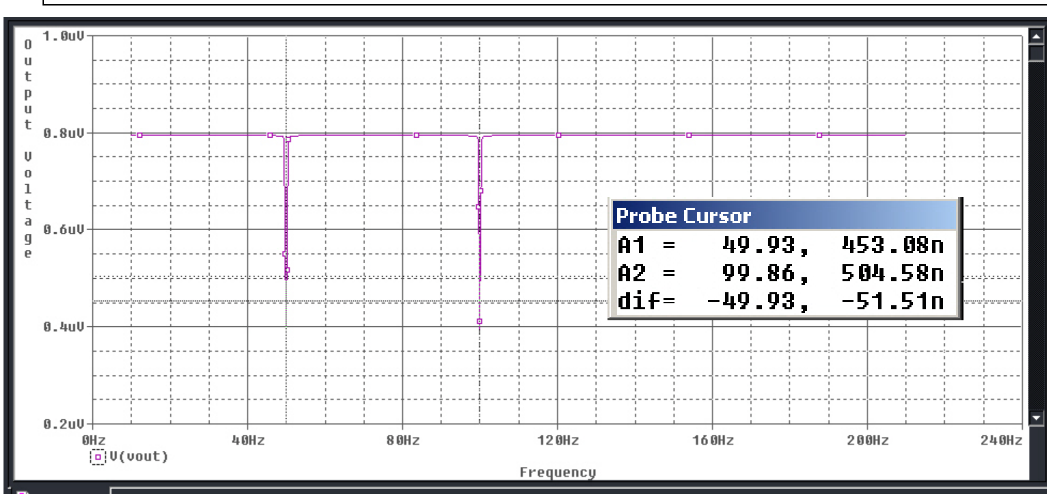




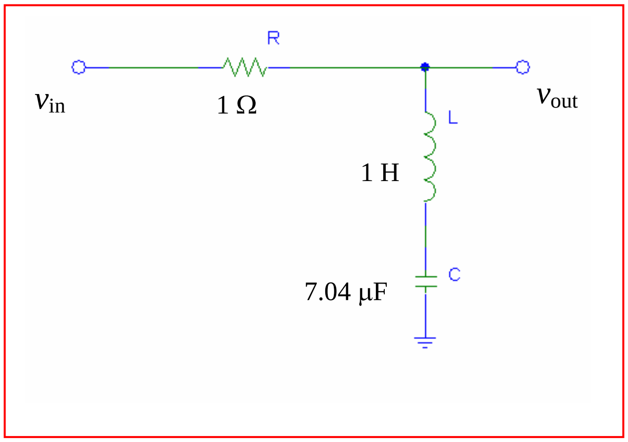
70. We require four filter stages, and choose to implement the circuit using op amps to isolate each filter sub-circuit. Selecting a bandwidth of 1 rad/s (no specification was given) and a simple RLC filter as suggested in the problem statement, a resistance value of 1 Ω leads to an inductor value of 1 H (bandwidth for this type of filter = $\omega_H - \omega_L$ = R/L). The capacitance is found by designing each filter's respective resonant frequency ($1/\sqrt{LC}$) at the desired "notch" frequency. Thus, we require C_{F1} = 10.13 μF, C_{F2} = 2.533 μF, C_{F3} = 1.126 μF and C_{F4} = 633.3 nF.

The Student Version of PSpice® will not permit more than 64 nodes, so that the total solution must be simulated in two parts. The half with the filters for notching out 50 and 100 Hz components is shown below; an additional two op amp stages are required to complete the design.





71. Using the series RLC circuit suggested, we decide to design for a bandwidth of 1 rad/ s (as no specification was provided). With $\omega_H - \omega_L$ = R/ L, we arbitrarily select R = 1 Ω so that L = 1 H. The capacitance required is obtained by setting the resonant frequency of the circuit ($1/\sqrt{LC}$) equal to 60 Hz (120π rad/s). This yields C = 7.04 μF.



1. (a) $$\boxed{\begin{bmatrix} 4 & -8 & 9 \\ 5 & 0 & -7 \\ 7 & 3 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \\ \mathbf{I}_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 4 \\ 0 \end{bmatrix}}$$

(b) $$\Delta_z = \begin{vmatrix} 4 & -8 & 9 \\ 5 & 0 & -7 \\ 7 & 3 & 1 \end{vmatrix} = 4(21) + 8(5+49) + 9(15) = \boxed{651}$$

(c) $$\Delta_{11} = \begin{vmatrix} 0 & -7 \\ 3 & 1 \end{vmatrix} = \boxed{21}$$

(d) $$\mathbf{I}_1 = \frac{\begin{vmatrix} 12 & -8 & 9 \\ 4 & 0 & -7 \\ 0 & 3 & 1 \end{vmatrix}}{\Delta_z} = \frac{(12)(21) + 8(4) + 9(12)}{651} = \boxed{0.602 \text{ A}}$$

(e) $$\mathbf{I}_1 = \frac{\begin{vmatrix} 4 & -8 & 12 \\ 5 & 0 & 4 \\ 7 & 3 & 0 \end{vmatrix}}{\Delta_z} = \frac{4(-12) + 8(-28) + 12(15)}{651} = \boxed{-0.141 \text{ A}}$$

2.

$$\Delta_Z = \begin{vmatrix} 17 & -8 & -3 \\ -8 & 17 & -4 \\ -3 & -4 & 17 \end{vmatrix} = 17(273) + 8(-148) - 3(83) = 3208\,\Omega^3$$

(a) $$Z_{in1} = \frac{\Delta_Z}{\Delta_{11}} = \frac{3208}{273} = 11.751\,\Omega \quad \therefore P_1 = \frac{100^2}{11.751} = \boxed{851.0\text{ W}}$$

(b) $$Z_{in2} = \frac{\Delta_Z}{\Delta_{22}} = \frac{3208}{280} = 11.457\,\Omega \quad \therefore P_2 = \frac{100^2}{11.457} = \boxed{872.8\text{ W}}$$

(c) $$Z_{in3} = \frac{\Delta_Z}{\Delta_{33}} = \frac{3208}{225} = 14.258\,\Omega \quad \therefore P_3 = \frac{100^2}{14.258} = \boxed{701.4\,\Omega}$$

3.

$$\Delta_Y = \begin{vmatrix} 0.35 & -0.1 & -0.2 \\ -0.1 & 0.5 & -0.15 \\ -0.2 & -0.15 & 0.75 \end{vmatrix} = 0.35(0.3525) + 0.1(-0.105) - 0.2(0.115) = 0.089875 \text{ S}^3$$

(a) $$Y_{in1} = \frac{\Delta_Y}{\Delta_{11}} = \frac{0.089875}{0.3525} = 0.254965 \quad \therefore P_1 = \frac{10^2}{0.254965} = \boxed{392.2 \text{ W}}$$

(b) $$Y_{in2} = \frac{\Delta_Y}{\Delta_{22}} = \frac{0.089875}{0.2225} = 0.403933 \quad \therefore P_2 = \frac{10^2}{0.403933} = \boxed{247.6 \text{ W}}$$

(c) $$Y_{in3} = \frac{0.089875}{0.165} = 0.544697 \text{ S} \quad \therefore P_3 = \frac{100}{0.544697} = \boxed{183.59 \text{ W}}$$

4.

$$[\mathrm{R}] = \begin{bmatrix} 3 & -1 & -2 & 0 \\ -1 & 4 & 1 & 3 \\ -2 & 2 & 5 & 2 \\ 0 & -3 & -2 & 6 \end{bmatrix} (\Omega) = 3\begin{vmatrix} 4 & 1 & 3 \\ 2 & 5 & 2 \\ -3 & -2 & 6 \end{vmatrix} + 1\begin{vmatrix} -1 & -2 & 0 \\ 2 & 5 & 2 \\ -3 & -2 & 6 \end{vmatrix} - 2\begin{vmatrix} -1 & -2 & 0 \\ 4 & 1 & 3 \\ -3 & -2 & 6 \end{vmatrix}$$

$$= 3[4(34) - 2(12) - 3(-13)] + [-1(34) - 2(-12) - 3(-4)] = 2[-1(12) - 4(-12) - 3(-6)]$$

$$= 3(73) + (-22) - 2(18) = 161\ \Omega^4 \quad \therefore \mathrm{R}_{in} = \frac{\Delta_R}{\Delta_{11}} = \frac{161}{73} = \boxed{2.205^+\ \Omega}$$

5. Define a counter-clockwise current $\mathbf{I}_2$ in the left-most mesh, and a counter-clockwise current $\mathbf{I}_1$ flowing in the right-most mesh. Then,

$$\overline{V}_1 = 4\overline{I}_2 \quad \therefore 0.2\overline{V}_1 = 0.8\overline{I}_2$$

$$\overline{V}_{in} = \overline{I}_1 s + 5(\overline{I}_1 + 0.8\overline{I}_2 - \overline{I}_2) = (s+5)\overline{I}_1 - \overline{I}_2$$

$$\text{Also, } \overline{I}_2(2s+4) - 5(\overline{I}_1 + 0.8\overline{I}_2 - \overline{I}_2) = 0$$

$$\text{or } 0 = -5\overline{I}_1 + (5+2s)\overline{I}_2$$

$$\therefore \Delta_Z = (s+5)(5+2s) - 5 = 2s^2 + 15s + 20,\ \Delta_{11} = 5 + 2s$$

$$\therefore \boxed{Z_{th} = \frac{2s^2 + 15s + 20}{2s+5}}$$

6. Define a clockwise mesh current $\mathbf{I}_1$ flowing in the bottom left mesh, a clockwise mesh current $\mathbf{I}_2$ flowing in the top mesh, and a clockwise mesh current $\mathbf{I}_3$ flowing in the bottom right mesh. Then,

(a)

$$\bar{V}_{in} = 10(\bar{I}_1 - \bar{I}_2) - 0.6\times 8\bar{I}_2 = 10\bar{I}_1 - 14.8\bar{I}_2$$

$$0 = 50\bar{I}_2 - 10\bar{I}_1 - 12\bar{I}_3 = -10\bar{I}_1 + 50\bar{I}_2 - 12\bar{I}_3$$

$$0 = 4.8\bar{I}_2 + 17\bar{I}_3 - 12\bar{I}_2 = -7.2\bar{I}_2 + 17\bar{I}_3$$

$$\therefore \Delta_Z = \begin{vmatrix} 10 & -14.8 & 0 \\ -10 & 50 & -12 \\ 0 & -7.2 & 17 \end{vmatrix} = 10(763.6) + 10(-251.6) = 5120 \;\therefore Z_{in} = \frac{5120}{763.6} = \boxed{6.705^+\ \Omega}$$

(b)

$$\bar{I}_{in} = \frac{\bar{V}_1 - \bar{V}_2}{28} + \frac{\bar{V}_1 - 0.6\bar{V}_x}{10} = 0.13571\,\bar{V}_1 - 0.03571\bar{V}_2 - 0.06\bar{V}_x$$

$$0 = \frac{\bar{V}_2 - \bar{V}_1}{28} + \frac{\bar{V}_2 - 0.6\bar{V}_x}{12} + \frac{\bar{V}_2}{5} = -0.03571\bar{V}_1 + 0.31905\bar{V}_2 - 0.05\bar{V}_x$$

$$0 = -\frac{\bar{V}_x}{8} + \frac{\bar{V}_2 - \bar{V}_x - V_1}{20} = -0.05\bar{V}_1 + 0.05\bar{V}_2 - 0.175\bar{V}_x$$

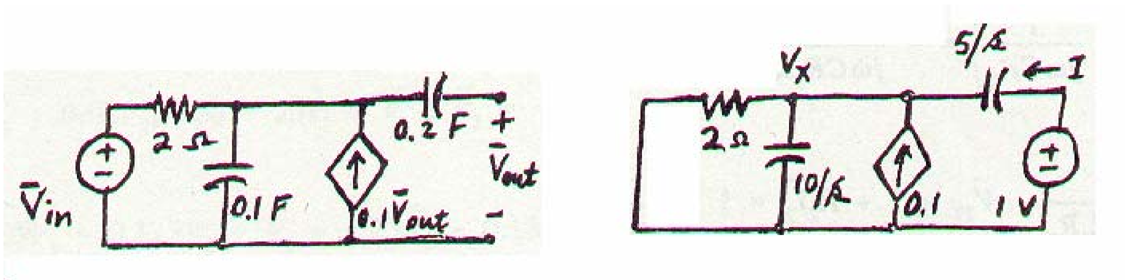
$$\therefore \Delta_y = \begin{vmatrix} 0.13571 & -0.03571 & -0.06 \\ -0.03571 & 0.31905^- & -0.05 \\ -0.05 & 0.05 & -0.175 \end{vmatrix} = 0.13571(-0.05583 + 0.0025) + 0.03571(0.00625 + 0.003)$$

$$-0.05(0.00179 + 0.01914) = -0.00724$$

$$\therefore \Delta_y = 0.007954,\ \Delta_{11} = -0.05333 \;\therefore \bar{Y}_m = \frac{\Delta_y}{\Delta_{11}} = \frac{-0.007954}{-0.05333} = 0.14926\ \text{S}$$

$$\therefore \bar{Z}_{in} = \frac{1}{0.14926} = \boxed{6.705^+\ \Omega}$$

7.



$$\frac{\bar{V}_x}{2} + \frac{s\bar{V}_x}{10} - 0.1 + \frac{s}{5}(\bar{V}_x - 1) = 0$$

$$\therefore \bar{V}_x(0.5 + 0.3s) = 0.1 + 0.2s$$

$$\therefore \bar{V}_x = \frac{0.2s + 0.1}{0.3s + 0.5}$$

$$\therefore \bar{I} = (1 - \bar{V}_x)\frac{s}{5} = \left(1 - \frac{0.2s + 0.1}{0.3s + 0.5}\right)0.2s = 0.2s\frac{0.1s + 0.4}{0.3s + 0.5}$$

$$\therefore \bar{Y}_{out} = \bar{I} = \frac{s(0.1s + 0.4)}{1.5s + 2.5},\ \bar{Z}_{out} = \frac{1.5s + 2.5}{s(0.1s + 0.4)} = \boxed{\frac{15s + 25}{s(s + 4)}}$$

8.

$$\overline{\mathrm{V}}_{in} = 1\ \mathrm{V},\ \overline{\mathrm{V}}_{i} = 0\ \ \therefore \overline{\mathrm{V}}_{x} + \overline{\mathrm{V}}_{in} = 0,\ \overline{\mathrm{V}}_{x} = -1\ \mathrm{V}$$

$$\overline{\mathrm{I}}_{x} = \frac{\overline{\mathrm{V}}_{x}}{\mathrm{R}_{x}} = -\frac{1}{\mathrm{R}_{x}};\ 2\times10^{4}\,\mathrm{I}_{in} + 2\times10^{4}\,\mathrm{I}_{x} = 0$$

$$\therefore \overline{\mathrm{I}}_{in} = -\mathrm{I}_{x} = \frac{1}{\mathrm{R}_{x}}\ \ \therefore \mathrm{R}_{in} = -\mathrm{V}_{in}/\mathrm{I}_{in} = \boxed{-R_{x}}$$

9.

(a) Assume 1 V at input. Since $V_i = 0$ at each op-amp input, 1 V is present between R_2 and R_3, and also C and R_4.

$$\therefore \bar{V}_4 = \frac{1}{R_4}\left(R_4 + \frac{1}{j\omega C}\right) = 1 + \frac{1}{j\omega CR_4}$$

$$\therefore \bar{I}_3 = \frac{1}{R_3}\left(1 - 1 - \frac{1}{j\omega CR_4}\right) = -\frac{1}{j\omega CR_3R_4}$$

$$\therefore \bar{I}_2 = \bar{I}_3 = -\frac{1}{j\omega CR_3R_4} \quad \therefore \bar{V}_{12} = 1 + R_2\bar{I}_2 = 1 - \frac{R_2}{j\omega CR_3R_4}$$

$$\bar{I}_1 = \frac{1 - \bar{V}_{12}}{R_1} = \frac{R_2}{j\omega CR_1R_3R_4} = \bar{I}_{in} \quad \therefore \bar{Z}_{in} = \frac{1}{I_{in}} = \boxed{j\omega C\frac{R_1R_3R_4}{R_2}}$$

(b) $R_1 = 4\times10^3,\ R_2 = 10\times10^3,\ R_3 = 10\times10^3,\ R_4 = 10^3,\ C = 2\times10^{-10}$

$$\therefore \bar{Z}_{in} = j\omega 2\times10^{-10}\frac{4\times10\times1}{10}\times10^6 = \boxed{j\omega 0.8\times10^3\,\Omega\ (L_{in} = 0.8\text{ mH})}$$

10. (a) $[y] = \begin{bmatrix} 0.01 & 0.3 \\ 0.3 & -0.02 \end{bmatrix}$, $\mathrm{V} = \begin{bmatrix} 9 \\ -3.5 \end{bmatrix}$

$[\mathbf{I}] = [y][\mathbf{V}]$

$I_2 = (0.3)(9) + (0.02)(3.5) = 2.77$ A

(b) $\mathbf{V} = [y]^{-1}[\mathbf{I}]$

$[y] = \begin{bmatrix} -0.1 & 0.15 \\ 0.15 & 0.8 \end{bmatrix}$ and $[\mathbf{I}] = \begin{bmatrix} 0.001 \\ 0.02 \end{bmatrix}$

$$\text{Thus, } \mathrm{V}_1 = \frac{\begin{vmatrix} 0.001 & 0.15 \\ 0.02 & 0.8 \end{vmatrix}}{\begin{vmatrix} -0.1 & 0.15 \\ 0.15 & 0.8 \end{vmatrix}} = \frac{0.0008 - 0.003}{-0.08 - 0.0225} = \boxed{0.0215 \text{ V}}$$

11. Define a clockwise mesh current $\mathbf{I}_1$ in the left-most mesh, a clockwise mesh current $\mathbf{I}_x$ in the center mesh, and a counter-clockwise mesh current $\mathbf{I}_2$ in the right-most mesh. Then,

$$\bar{V}_1 = 13\bar{I}_1 - 10\bar{I}_2$$

$$0 = -10\bar{I}_1 + 35\bar{I}_x + 20\bar{I}_2 \quad \therefore \bar{I}_1 = \frac{\begin{vmatrix} \bar{V}_1 & -10 & 0 \\ 0 & 35 & 20 \\ \bar{V}_2 & 20 & 22 \end{vmatrix}}{\begin{vmatrix} 13 & -10 & 0 \\ -10 & 35 & 20 \\ 0 & 20 & 22 \end{vmatrix}}$$

$$\bar{V}_2 = 20\bar{I}_x + 22\bar{I}_2$$

$$\therefore \bar{I}_1 = \frac{\bar{V}_1(370) + \bar{V}_2(-200)}{13(370) + 10(-220)} = \frac{37}{261}\,\bar{V}_1 - \frac{20}{261}\,\bar{V}_2$$

$$\therefore \bar{y}_{11} = \frac{37}{261} = \boxed{141.76 \text{ mS},}\ \bar{y}_{12} = \frac{-20}{261} = \boxed{-76.63 \text{ mS}}$$

12.

$$[y] = \begin{bmatrix} 10 & -5 \\ 50 & 20 \end{bmatrix} \text{ (mS)} \quad \therefore \bar{I}_1 = 0.01\bar{V}_1 - 0.005\bar{V}_2,$$

$$\bar{I}_2 = 0.05\bar{V}_1 + 0.02\bar{V}_2,\ 100 = 25\bar{I}_1 + \bar{V}_1,\ \bar{V}_2 = -100\bar{I}_2$$

$$\therefore 100 = 0.25\bar{V}_1 - 0.125\bar{V}_2 + \bar{V}_1 = 1.25\bar{V}_1 - 0.125\bar{V}_2$$

$$\bar{I}_2 = -0.01\bar{V}_2 = 0.05\bar{V}_1 + 0.02\bar{V}_2 \quad \therefore -0.03\bar{V}_2 = 0.05\bar{V}_1 \quad \therefore \bar{V}_2 = -\frac{5}{3}\ \bar{V}_1$$

$$\therefore 100 = 1.25\bar{V}_1 + \frac{0.625}{3}\ \bar{V}_1 = \frac{4.375}{2}\ \bar{V}_1 \quad \therefore \bar{V}_1 = \frac{300}{4.375} = \boxed{68.57\text{ V},}\ \bar{V}_2 = -\frac{5}{3}\ \bar{V}_1 = \boxed{-114.29\text{ V}}$$

13.

$$\bar{I}_1 = \frac{\bar{V}_1 - \bar{V}_2}{25} = 0.04\bar{V}_1 - 0.04\bar{V}_2$$

$$\bar{I}_2 = 2I_1 + \frac{\bar{V}_2}{100} - \bar{I}_1 = \bar{I}_1 + 0.01\bar{V}_2 = 0.04\bar{V}_1 - 0.03\bar{V}_2$$

$$\therefore\ \bar{y}_{11} = \boxed{0.04\,\text{S},}\ \bar{y}_{12} = \boxed{-0.04\,\text{S}},\ \bar{y}_{21} = \boxed{0.04\,\text{S},\ }\bar{y}_{22} = \boxed{-0.03\ \text{S}}$$

14.

$$\therefore \bar{V}_1 = 100(\bar{I}_1 - 0.5\bar{I}_1) = 50\bar{I} \quad \therefore \bar{I}_1 = 0.02\,\bar{V}_1$$

$$\bar{V}_2 = 300\bar{I}_2 + 200(\bar{I}_2 + 0.5\bar{I}_1) = 100\bar{I}_1 + 500\bar{I}_2$$

$$\therefore \bar{V}_2 = 2\bar{V}_1 + 500\bar{I}_2,\ \bar{I}_2 = -0.004\bar{V}_1 + 0.002\bar{V}_2$$

$$\therefore \boxed{[\bar{y}] = \begin{bmatrix} 0.02 & 0 \\ -0.004 & 0.002 \end{bmatrix} \text{(S)}}$$

15.

$$[y] = \begin{bmatrix} 0.1 & -0.0025 \\ -8 & 0.05 \end{bmatrix} \text{ (S)}$$

(a)

$$\bar{I}_1 = 0.1\bar{V}_1 - 0.0025\bar{V}_2,\ \bar{I}_2 = -8\bar{V}_1 + 0.05\bar{V}_2$$

$$1 = 2\bar{I}_1 + \bar{V},\ \bar{V}_2 = -5\bar{I}_2$$

$$\therefore \bar{I}_2 = -0.2\bar{V}_2 = -8\bar{V}_1 + 0.05\bar{V}_2 \quad \therefore 0.25\bar{V}_2 = 8\bar{V}_1,\ \bar{V}_2 / \bar{V}_1 = 32$$

$$\bar{I}_2 = -8\bar{V}_1 + 0.05 \times 32\bar{V}_1,\ \bar{I}_1 = 0.1\bar{V}_1 - 0.0025 \times 32\bar{V}_1 \ \therefore \bar{I}_2 = -6.4\bar{V}_1,\ \bar{I}_1 = 0.02\bar{V}_1$$

$$\therefore \bar{I}_2 / \bar{I}_1 = \frac{-6.4}{0.02} = -320,\ \bar{V}_1 / \bar{I}_1 = \boxed{50\Omega}$$

(b)

$$\bar{V}_1 = -2\bar{I}_1,\ \bar{I}_1 = 0.1\bar{V}_1 - 0.0025V_2,\ \bar{I}_2 = -8\bar{V}_1 + 0.05\bar{V}_2$$

$$\therefore \bar{I}_1 = -0.5\bar{V}_1 = 0.1\bar{V}_1 - 0.0025\bar{V}_2 \therefore 0.6\bar{V}_1 = 0.0025\bar{V}_2$$

$$\therefore \bar{V}_1 = \bar{V}_2 / 240,\ \bar{I}_2 = -8 \times \bar{V}_2 / 240 + \frac{1}{20}\ \bar{V}_2 = \frac{1}{60}\bar{V}_2$$

$$\therefore \frac{\bar{V}_2}{\bar{I}_2} = \boxed{60\,\Omega}$$

16. $[\bar{y}] = \begin{bmatrix} 10 & -5 \\ -20 & 2 \end{bmatrix}$ (mS)

(a)

$$\bar{I}_1 = 0.01\bar{V}_1 - 0.005\bar{V}_2, \ \bar{I}_2 = -0.02\bar{V}_1 + 0.002\bar{V}_2$$

$$\bar{V}_1' = 100\bar{I}_1 + \bar{V}_1$$

$$\therefore \bar{V}_1 = \bar{V}1 - 100\bar{I}_1 \ \therefore \bar{I}_1 = 0.01\bar{V}1 - \bar{I}_1 - 0.005\bar{V}_2 \quad \therefore \bar{I}_1 = 0.005\bar{V}1 - 0.0025\bar{V}_2$$

$$\bar{I}_2 = -0.02\bar{V}1 + 2\bar{I}_1 + 0.002\bar{V}_2 = -0.02\bar{V}1 + 0.01\bar{V}1 - 0.005\bar{V}_2 + 0.002\bar{V}_2 = -0.01\bar{V}1 - 0.003\bar{V}_2$$

$$\therefore \boxed{[y]_{new} = \begin{bmatrix} 0.005 & -0.0025 \\ -0.01 & -0.003 \end{bmatrix} \text{(S)}}$$

(b)

$$\bar{V}2 = 100\bar{I}_2 + \bar{V}_2, \ \therefore \bar{V}_2 = \bar{V}2 - 100\bar{I}_2$$

$$\therefore \bar{I}_2 = -0.02\bar{V}_1 + 0.002\bar{V}2 - 0.2\bar{I}_2$$

$$\therefore 1.2\bar{I}_2 = -0.02\bar{V}_1 + 0.002\bar{V}2 \ \therefore \bar{I}_2 = -\frac{1}{60}\bar{V}_1 + \frac{1}{600}\bar{V}2$$

$$\bar{I}_1 = 0.01\bar{V}_1 - 0.005(\bar{V}2 - 100\bar{I}_2) = 0.01\bar{V}_1 - 0.005\bar{V}2 + 0.5\left(-\frac{1}{60}\bar{V}_1 + \frac{1}{600}\bar{V}2\right)$$

$$\therefore \bar{I}_1 = \left(\frac{1}{100} - \frac{1}{120}\right)\bar{V}_1 - \left(\frac{1}{200} - \frac{1}{1200}\right)\bar{V}_2' = \frac{1}{600}\bar{V}_1 - \frac{1}{240}\bar{V}_2'$$

$$\therefore \boxed{[\bar{y}]_{new} = \begin{bmatrix} 1/600 & -1/240 \\ -1/60 & 1/600 \end{bmatrix} \text{(S)}}$$

17.

	$\overline{V}_{S1}$	$\overline{V}_{S2}$	$\overline{I}_1$	$\overline{I}_2$
Exp #1	100 V	50 V	5 A	-32.5 A
Exp #2	50	110	-20	-5
Exp #3	20	0	4	-8
Exp #4	-8.333	-22.22	5	0
Exp #5	-58.33	-55.56	5	15

$$\overline{I}_1 = \overline{y}_{11}\overline{V}_1 + \overline{y}_{12}\overline{V}_2$$
$$\overline{I}_2 = \overline{y}_{21}V_1 + y_{22}\overline{V}_2$$

Use 1st 2 rows to find y's

$\therefore 5 = 100\overline{y}_{11} + 50\overline{y}_{12}, -32.5 = 100\overline{y}_{21} + 50\overline{y}_{22}$

$-20 = 50\overline{y}_{11} + 100\overline{y}_{12,}\ -5 = 50\overline{y}_{21} + 100\overline{y}_{22} \rightarrow \therefore -10 = 100\overline{y}_{21} + 200\overline{y}_{22}$

$\therefore -40 = 100\overline{y}_{11} + 200\overline{y}_{12}$ Substracting, $150\overline{y}_{12} = -45\ \therefore \boxed{\overline{y}_{12} = -0.3\text{ S}}$

$\therefore 5 = 100\overline{y}_{11} - 15\ \therefore \boxed{\overline{y}_{11} = 0.2\text{ S}}$ Subtracting $22.5 = 150\overline{y}_{22}$

$\therefore \boxed{\overline{y}_{22} = 0.15\text{ S}}\ \therefore -32.5 = 100\overline{y}_{21} + 7.5\ \therefore \boxed{\overline{y}_{21} = -0.4\text{ S}}\ \therefore [\overline{y}] = \begin{bmatrix} 0.2 & -0.3 \\ -0.4 & 0.15 \end{bmatrix}$ (S)

Completing row 3: $\overline{I}_1 = 0.2 \times 20 = 4$ A, $\overline{I}_2 = -0.4 \times 20 = -8$ A

Completing row 4: $5 = 0.2\overline{V}_{S1} - 0.3\overline{V}_{S2},\ 0 = -0.4\overline{V}_{S1} + 0.15\overline{V}_{S2}\ \therefore \overline{V}_{S2} = \dfrac{8}{3}\overline{V}_{S1}$

$\therefore 5 = 0.2\overline{V}_{S1} - 0.8\overline{V}_{S1} = -0.6\overline{V}_{S1}\ \therefore \overline{V}_{S1} = -\dfrac{50}{6} = -8.333\text{ V},\ \overline{V}_{S2} = -22.22\text{ V}$

Completing row 5: $5 = 0.2\overline{V}_{S1} - 0.3\overline{V}_{S2},\ 15 = -0.4\overline{V}_{S1} + 0.15\overline{V}_{S2}$

$$\therefore \overline{V}_{S1} = \frac{\begin{vmatrix} 5 & -0.3 \\ 15 & 0.15 \end{vmatrix}}{\begin{vmatrix} 0.2 & -0.3 \\ -0.4 & 0.15 \end{vmatrix}} = \frac{0.75 + 4.5}{0.03 - 0.12} = \frac{5.25}{-0.09} = -58.33\text{ V},\ \overline{V}_{S2} = \frac{\begin{vmatrix} 0.2 & 5 \\ -0.4 & 15 \end{vmatrix}}{-0.09} = -55.56\text{ V}$$

18. (a) $[\mathbf{I}] = [y][\mathbf{V}] = \begin{bmatrix} 10^{-3} & j0.01 \\ j0.01 & -j0.005 \end{bmatrix} \begin{bmatrix} 12\angle 43^\circ \\ 2\angle 0^\circ \end{bmatrix}$

$$\mathrm{I}_2 = (j0.01)(12\angle 43^\circ) - (j0.005)(2\angle 0) = \boxed{-0.0818 + j0.0778}$$

(b) $[\mathbf{V}] = [y]^{-1}[\mathbf{I}]$

$$[y] = \begin{bmatrix} -j5 & 10 \\ 4 & j10 \end{bmatrix} \text{ and } \mathbf{I} = \begin{bmatrix} 120\angle 30^\circ \\ 88\angle 45^\circ \end{bmatrix}$$

$$\mathrm{V}_2 = \frac{\begin{vmatrix} -j5 & 120\angle 30^\circ \\ 4 & 88\angle 45^\circ \end{vmatrix}}{\begin{vmatrix} -j5 & 10 \\ 4 & j10 \end{vmatrix}} = \frac{(-j5)\left(88\angle 45^\circ\right) - 480\angle 30^\circ}{50-40} = \boxed{-10 - j55.13 \text{ V}}$$

19. (a) $\boxed{\text{Input is applied between g-s and output taken from d-s.}}$

(b) $I_g = y_{is}V_{gs} + y_{rs}V_{ds}$
$I_d = y_{fs}V_{gs} + y_{os}V_{ds}$

$$y_{is} = \left.\frac{I_g}{V_{gs}}\right|_{V_{ds}=0} = \boxed{j\omega\left(C_{gs} + C_{gd}\right)}$$

$$y_{rs} = \left.\frac{I_g}{V_{ds}}\right|_{V_{gs}=0} = \boxed{-j\omega C_{gd}}$$

$$y_{fs} = \left.\frac{I_d}{V_{gs}}\right|_{V_{ds}=0} = \boxed{g_m - j\omega C_{gd}}$$

$$y_{os} = \left.\frac{I_d}{V_{ds}}\right|_{V_{gs}=0} = \boxed{\frac{1}{r_d} + j\omega\left(C_{gs} + C_{gd}\right)}$$

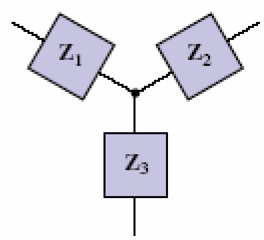
(c) $y_{is} = j\omega(3.4 + 1.4)\times 10^{-12} = \boxed{j4.8\omega \text{ pS}}$

$$y_{rs} = -j\omega(1.4)\times 10^{-12} = \boxed{-j1.4\omega \text{ pS}}$$

$$y_{fs} = \boxed{4.7\times 10^{-3} - j\omega(1.4)\times 10^{-12} \text{ S}}$$

$$y_{os} = \boxed{10^{-4} + j\omega(0.4 + 1.4)\times 10^{-12} \text{ S}}$$

20.



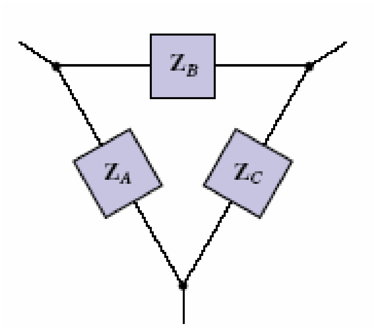
$$\sum R = 7.9 \text{ k}\Omega$$

R_1 = 4.7/7.9 = 595 Ω

R_2 = 2.2/7.9 = 278 Ω

R_3 = (4.7)(2.2)/7.9 = 1.309 kΩ

21.



$$\sum R_i R_j = (470)(100) + (470)(220) + (100)(220)$$
$$= 172400$$

$$\boxed{\begin{aligned} R_A &= 172400 / 220 = 783.6\ \Omega \\ R_B &= 172400 / 100 = 1.724\ \text{k}\Omega \\ R_C &= 172400 / 470 = 366.8\ \Omega \end{aligned}}$$

22.

$$\Delta_1: 1+6+3=10\,\Omega \rightarrow \frac{6\times 1}{10}=0.6,\ \frac{6\times 3}{10}=1.8,\ \frac{3\times 1}{10}=0.3$$

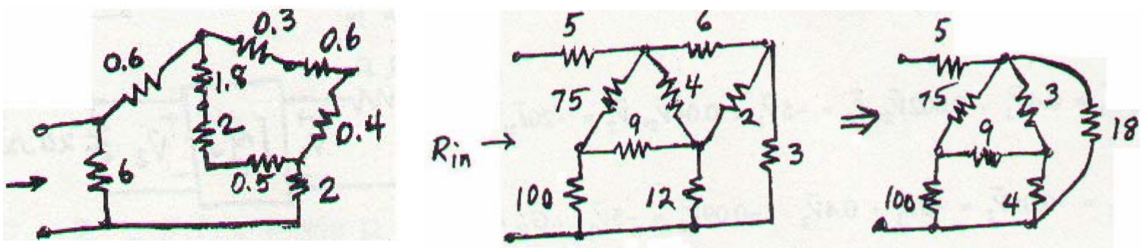
$$\Delta_2: 5+1+4=10\,\Omega \rightarrow \frac{5\times 1}{10}=0.5,\ \frac{1\times 4}{10}=0.4,\ \frac{5\times 4}{10}=2$$

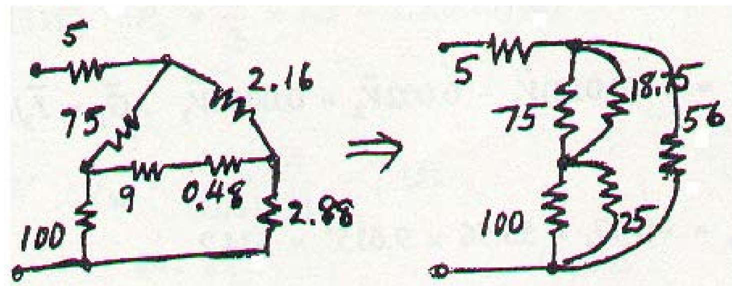
$$1.8+2+0.5=4.3\Omega,\ 0.3+0.6+0.4=1.3\Omega$$

$$1.3\|4.3=0.99821\Omega,\ 0.9982+0.6+2=3.598\,\Omega$$

$$3.598\|6=\boxed{2.249\,\Omega}$$

23.





$$6\times 2+2\times 3+3\times 6=36\Omega^2$$

$$36/6=6,\ 36/2=18,\ 36/3=12$$

$$12\|4=3,\ 6\|12=4$$

$$4+3+18=25\Omega$$

$$3\times 18/25=2.16,\ 4\times 18/25=2.88,\frac{4\times 3}{25}=0.48$$

$$9.48\times 2.16+9.48\times 2.88+2.88\times 2.16=54\Omega^2$$

$$\frac{54}{2.88}=18.75,\ \frac{54}{2.16}=25,\ \frac{54}{9.48}=5.6962,\ 75\|18.75=15,\ 100\|25=20$$

$$(15+20)\|5.696=4.899\ \therefore R_{in}=5+4.899=\boxed{9.899\Omega}$$

24.

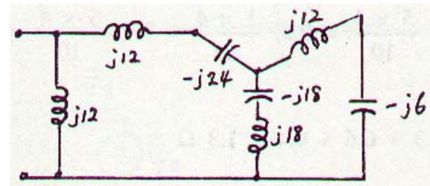
$\Delta: -j6 + j4 + j3 = j1$

$\frac{24}{j1} = -j24, \frac{-12}{j1} = j12, \frac{18}{j1} = -j18, j18 - j18 = 0 \text{(S.C)}$

$\therefore \text{ignore } j12, -j6$

$-j24 + j12 = -j12$

$-j12 \| j12 = \infty \therefore \boxed{\overline{Z}_{in} = \infty}$



25. $[\bar{y}] = \begin{bmatrix} 0.4 & -0.002 \\ -5 & 0.04 \end{bmatrix}$ (S)

(a) $\bar{I}_1 = 0.4\bar{V}_1 - 0.002\bar{V}_2,\ \bar{I}_2 = -5\bar{V}_1 + 0.04\bar{V}_2, \bar{V}_2 = -20\bar{I}_2,\ \bar{V}_S = \bar{V}_1 + 2\bar{I}_1$

$$\bar{I}_2 = -0.05\bar{V}_2 = -5V_1 + 0.4\bar{V}_2 \ \therefore -0.09\bar{V}_2 = -5\bar{V}_1 \ \therefore \bar{G}_V = \bar{V}_2 / \bar{V}_1 = \frac{500}{9} = \boxed{55.56}$$

(b) $$\bar{I}_1 = 0.4(0.018)\bar{V}_2 - 0.002\bar{V}_2 = 0.0052\bar{V}_2 \ \therefore \bar{G}_I = \bar{I}_2 / \bar{I}_1 = \frac{-0.05V_2}{0.0052V_2} = \boxed{-9.615^+}$$

(c) $$G_p = -G_V G_I = 55.56 \times 9.615^+ = \boxed{534.2}$$

(d) $$\bar{I}_1 = 0.0052\bar{V}_2 = 0.0052 \times 55.56\bar{V}_1 \ \therefore \bar{Z}_{in} = \bar{V}_1 / \bar{I}_1 = \frac{1}{0.0052 \times 55.56} = \boxed{3.462\,\Omega}$$

(e) $$\bar{V}_1 = -2\bar{I}_1,\ \bar{V}_S = 0 \ \therefore \bar{I}_1 = -0.5\bar{V}_1 = 0.4\bar{V}_1 - 0.002\bar{V}_2 \ \therefore \bar{V}_1 = \frac{0.002}{0.9}\bar{V}_2$$

$$\bar{I}_2 = -5\left(\frac{0.002}{0.9}\right)\bar{V}_2 + 0.04\bar{V}_2 = 0.02889\bar{V}_2 \ \therefore \bar{Z}_{out} = \bar{V}_2 / \bar{I}_2 = \boxed{34.62\,\Omega}$$

26. $[\bar{y}] = \begin{bmatrix} 0.1 & -0.05 \\ -0.5 & 0.2 \end{bmatrix}$ (S)

(a) $\bar{I}_1 = 0.1\bar{V}_1 - 0.05\bar{V}_2$

$\bar{I}_2 = -0.5\bar{V}_1 + 0.2\bar{V}_2,\ 1 = 10\bar{I}_1 + \bar{V}_1,\ \bar{I}_2 = -0.2\bar{V}_2$

$\therefore -0.2\bar{V}_2 = -0.5\bar{V}_1 + 0.2\bar{V}_2 \quad \therefore \bar{G}_V = \bar{V}_2 / \bar{V}_1 = \boxed{1.25}$

(b) $\bar{G}_I = \bar{I}_2 / \bar{I}_1 = \dfrac{(-0.5 + 0.2 \times 1.25)\bar{V}_1}{(0.1 - 0.005 \times 1.25)\bar{V}_1} = \boxed{-6.667}$

(c) $G_P = 1.25 \times 6.667 = \boxed{8.333}$

(d) $\bar{I}_1 = (0.1 - 0.05 \times 1.25)\bar{V}_1 \quad \therefore \bar{Z}_{in} = \bar{V}_1 / \bar{I}_1 = \boxed{26.67\,\Omega}$

(e) $\bar{V}_S = 0,\ \bar{V}_1 = -10\bar{I}_1 \ \therefore \bar{I}_1 = -0.1\bar{V}_1 = 0.1\bar{V}_1 - 0.05\bar{V}_2$

$\therefore \bar{V}_1 = 0.25\bar{V}_2, \ \therefore \bar{I}_2 = -0.05(0.25\bar{V}_2) + 0.2\bar{V}_2 = 0.075\bar{V}_2 \ \therefore \bar{Z}_{out} = \bar{V}_2 / \bar{I}_2 + 13.333\Omega$

(f) $\bar{G}_{V,rev} = \bar{V}_1 / \bar{V}_2 = \boxed{0.25}$

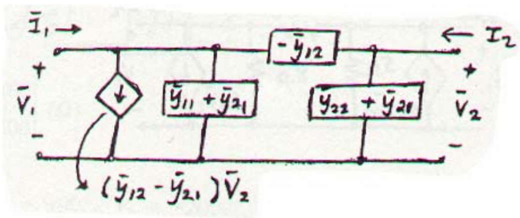
(g) With 2 port: $1 = 10\bar{I}_1 + 26.67I_1$

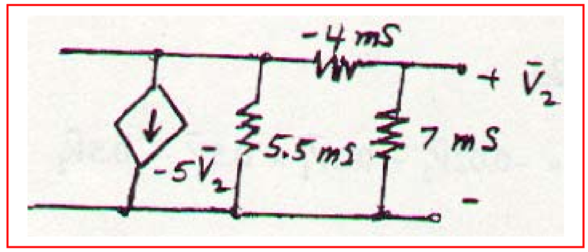
$\therefore 1 = 36.67\bar{I}_1,\ \bar{I}_1 = 1/36.67 \quad \therefore \bar{I}_2 = \dfrac{-6.667}{36.67} = -0.15182 \quad \therefore P_L = \dfrac{1}{2} \times I_2^2 5 = 2.5(0.15182)^2 = 0.08264$ W

Without 2 port: $P_L = \dfrac{1}{2}\left(\dfrac{1}{15}\right)^2 \times 5 = 0.011111$ W $\quad \therefore G_{ins} = \dfrac{0.08264}{0.011111} = \boxed{7.438}$

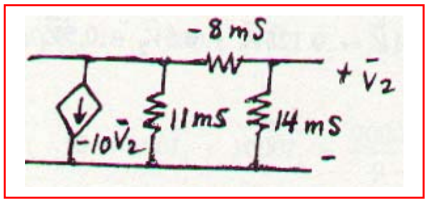
27.

(a)


$\bar{I}_1 \rightarrow$
$- \bar{y}_{12}$
$\leftarrow I_2$
$\bar{V}_1$
$\bar{y}_{11} + \bar{y}_{21}$
$\bar{y}_{22} + \bar{y}_{21}$
$\bar{V}_2$
$(\bar{y}_{12} - \bar{y}_{21})\bar{V}_2$

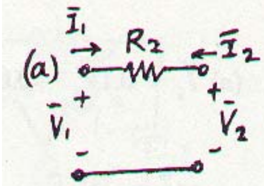

-4 mS
+ $\bar{V}_2$
$-5\bar{V}_2$
5.5 mS
7 mS

(b) 2 in ‖:

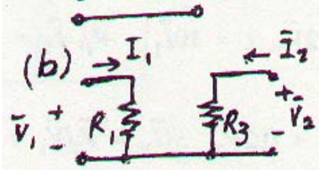

-8 mS
+ $\bar{V}_2$
$-10\bar{V}_2$
11mS
14 mS

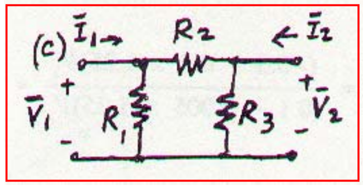
28.

(a) $\bar{I}_1 = \dfrac{\bar{V}_1 - \bar{V}_2}{R_2},\ \bar{I}_2 = \dfrac{\bar{V}_2 - V_1}{R_2}$ $\boxed{[\bar{y}]_a = \begin{bmatrix} 1/R_2 & -1/R_2 \\ -1/R_2 & 1/R_2 \end{bmatrix}}$



(b) $\bar{I}_1 = \bar{V}_1 / R_1,\ \bar{I}_2 = \bar{V}_2 / R_3 \quad \therefore \boxed{[\bar{y}]_B = \begin{bmatrix} 1/R_1 & 0 \\ 0 & 1/R_3 \end{bmatrix}}$





(c) $\bar{I}_1 = \dfrac{\bar{V}_1}{R_1} + \dfrac{\bar{V}_1 - \bar{V}_2}{R_2} \quad \therefore [\bar{y}] = \begin{bmatrix} 1/R_1 + 1/R_2 & -1/R_2 \\ -1/R_2 & 1/R_3 + 1/R_2 \end{bmatrix}$

$\bar{I}_2 = \dfrac{\bar{V}_2}{R_3} + \dfrac{\bar{V}_2 - V_1}{R_2},\ [\bar{y}]_a + [\bar{y}]_b = \begin{bmatrix} 1/R_1 + 1/R_2 & -1/R_2 \\ -1/R_2 & 1/R_3 + 1/R_2 \end{bmatrix}$

29. (a) $[\mathbf{V}] = [z][\mathbf{I}]$ where $[z] = \begin{bmatrix} 4.7 & 2.2 \\ 2.2 & 3.3 \end{bmatrix}$ kΩ and $[\mathbf{I}] = \begin{bmatrix} 1.5 \\ -2.5 \end{bmatrix}$ mA

Thus, $V_1 = (4.7)(1.5) - (2.2)(2.5) = \boxed{1.55 \text{ V}}$

(b) $[\mathbf{I}] = [z]^{-1}[\mathbf{V}]$ where $[z] = \begin{bmatrix} -10 & 15 \\ 15 & 6 \end{bmatrix}$ kΩ and $[\mathbf{V}] = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$ V

$$\text{Thus, } I_2 = \frac{\begin{vmatrix} -10 & 1 \\ 15 & -2 \end{vmatrix}}{\begin{vmatrix} -10 & 15 \\ 15 & 6 \end{vmatrix}} = \frac{20-15}{-60-225} = \boxed{-17.54\ \mu\text{A}}$$

30. (a) $[\mathbf{V}] = [z][\mathbf{I}] = \begin{bmatrix} 5 & j \\ j & -j2 \end{bmatrix} \begin{bmatrix} 2\angle 20^\circ \\ 2\angle 0^\circ \end{bmatrix}$

Thus, $V_2 = j2\angle 20^\circ - j2(2\angle 0) = \boxed{-0.684 - j2.121 \text{ V}}$

(b) $[\mathbf{I}] = [z]^{-1}[\mathbf{V}]$ where $[z] = \begin{bmatrix} -j & 2 \\ 4 & j4 \end{bmatrix}$ kΩ and $[\mathbf{V}] = \begin{bmatrix} 137\angle 30^\circ \\ 105\angle 45^\circ \end{bmatrix}$ V

Thus, $I_1 = \dfrac{\begin{vmatrix} 137\angle 30^\circ & 2 \\ 105\angle 45^\circ & j4 \end{vmatrix}}{\begin{vmatrix} -j & 2 \\ 4 & j4 \end{vmatrix}} = \dfrac{j4\left(137\angle 30^\circ\right) - 210\angle 45^\circ}{4-8} = \boxed{105.6 - j81.52 \text{ A}}$

31.

$$\bar{V}_1 = 8\bar{I}_1 + 0.1\bar{V}_2 \quad \therefore \bar{V}_2 = 10\bar{V}_1 - 80\bar{I}_1$$

$$\bar{I}_2 = \bar{V}_2 / 12 + 0.05\bar{V}_1 \quad \therefore \bar{I}_2 = \frac{1}{12}(10\bar{V}_1 - 80\bar{I}_1) + 0.05\bar{V}_1$$

$$\therefore \bar{I}_2 = \left(\frac{5}{6} + \frac{1}{20}\right)\bar{V}_1 - \frac{20}{3}\bar{I}_1 = \frac{53}{60}\bar{V}_1 - \frac{20}{3}\bar{I}_1$$

$$\therefore \bar{V}_1 = \frac{60}{53}\left(\frac{20}{3}\bar{I}_1 + \bar{I}_2\right) = \frac{400}{53}\bar{I}_1 + \frac{60}{53}\bar{I}_2 \quad \therefore \bar{V}_2 = \frac{4000}{53}\bar{I}_1 + \frac{600}{53}\bar{I}_2 - 80\bar{I}_1$$

$$\therefore \bar{V}_2 = -\frac{240}{53}\bar{I}_1 + \frac{600}{53}\bar{I}_2 \quad \therefore \boxed{[\bar{z}] = \begin{bmatrix} 7.547 & 1.1321 \\ -4.528 & 11.321 \end{bmatrix} (\Omega)}$$

32.

(a) $\overline{\mathrm{I}}_1 = -0.02\overline{\mathrm{V}}_2 + 0.2\overline{\mathrm{V}}_1 + 0.5\overline{\mathrm{V}}_1 - 0.5\overline{\mathrm{V}}_2$

$\therefore \overline{\mathrm{I}}_1 = 0.7\overline{\mathrm{V}}_1 - 0.52\overline{\mathrm{V}}_2 \quad \overline{\mathrm{I}}_2 = 0.1\overline{\mathrm{V}}_1 + 0.125\overline{\mathrm{V}}_2 + 0.5\overline{\mathrm{V}}_2 - 0.5\overline{\mathrm{V}}_1$

$\therefore \overline{\mathrm{I}}_2 = -0.4\overline{\mathrm{V}}_1 + 0.625\overline{\mathrm{V}}_2$

$$\therefore \overline{\mathrm{V}}_1 = \frac{\begin{vmatrix} \overline{\mathrm{I}}_1 & -0.52 \\ \overline{\mathrm{I}}_2 & 0.625 \end{vmatrix}}{\begin{vmatrix} 0.7 & -0.52 \\ -0.4 & 0.625 \end{vmatrix}} = \frac{0.625\overline{\mathrm{I}}_1 + 0.52\overline{\mathrm{I}}_2}{0.2295} = 2.723\overline{\mathrm{I}}_1 + 2.266\overline{\mathrm{I}}_2,\ \overline{\mathrm{V}}_2 = \frac{\begin{vmatrix} 0.7 & \overline{\mathrm{I}}_1 \\ -0.4 & \overline{\mathrm{I}}_2 \end{vmatrix}}{0.2295}$$

$$\therefore \overline{\mathrm{V}}_2 = \frac{0.4\overline{\mathrm{I}}_1 + 0.7\overline{\mathrm{I}}_2}{0.2295} = 1.7429\overline{\mathrm{I}}_1 + 3.050\overline{\mathrm{I}}_2 \quad \therefore \boxed{[\overline{z}] = \begin{bmatrix} 2.723 & 2.266 \\ 1.7429 & 3.050 \end{bmatrix} (\Omega)}$$

(b) $\overline{\mathrm{I}}_1 = \overline{\mathrm{I}}_2 = 1\ \mathrm{A} \quad \therefore \dfrac{\overline{\mathrm{V}}_2}{\overline{\mathrm{V}}_1} = \dfrac{1.7429 + 3.050}{2.723 + 2.266} = \boxed{0.9607}$

33. $[\overline{z}] = \begin{bmatrix} 4 & 1.5 \\ 10 & 3 \end{bmatrix} (\Omega),\ R_S = 5\Omega,\ R_L = 2\Omega$

(a) $\overline{V}_1 = 4\overline{I}_1 + 1.5\overline{I}_2,\ \overline{V}_2 = 10\overline{I}_1 + 3\overline{I}_2,\ \overline{V}_2 = -2\overline{I}_2 = 10\overline{I} + 3\overline{I}_2\ \therefore \overline{G}_1 = \overline{I}_2 / \overline{I}_1 = \boxed{-2}$

(b) $\overline{G}_v = \overline{V}_2 / \overline{V}_1 = \dfrac{10I_1 - 6\overline{I}_1}{4\overline{I} - 3\overline{I}_1} = \boxed{4}$

(c) $G_p = -\overline{G}_V \overline{G}_I = \boxed{8}$

(d) $\overline{V}_1 = 4\overline{I}_1 - 3\overline{I}_1 = \overline{I}_1\ \ \therefore \overline{Z}_{in} = \dfrac{\overline{V}_1}{\overline{I}_1} = \boxed{1\ \Omega}$

(e) $\overline{V}_1 = -5\overline{I}_1 = 4\overline{I}_1 + 1.5\overline{I}_2\ \ \therefore \overline{I}_1 = -\dfrac{1}{6}\ \overline{I}_2\ \ \therefore \overline{V}_2 = -\dfrac{10}{6}\ \overline{I}_2 + 3\overline{I}_2 = \dfrac{8}{6}\ \overline{I}_2\ \ \therefore \overline{Z}_{out} = \boxed{1.3333\ \Omega}$

34. $[\bar{z}] = \begin{bmatrix} 1000 & 100 \\ -2000 & 400 \end{bmatrix} (\Omega)$

(a) $\bar{V}_1 = 1000\bar{I}_1 + 100\bar{I}_2,\ \bar{V}_2 = -2000\bar{I}_1 + 400\bar{I}_2,\ 10 = 200\bar{I}_1 + \bar{V}_1, \bar{V}_2 = -500\bar{I}_2$

$\therefore -500\bar{I}_2 = -2000\bar{I}_1 + 400\bar{I}_2,\ \bar{I}_2 = \frac{20}{9}\ \bar{I}_1;\ \therefore 10 = 200\bar{I}_1 + 1000\bar{I}_1 + \frac{2000}{9}\ \bar{I}_1$

$\therefore \bar{I}_1 = 7.031 \text{ mA},\quad \therefore \bar{I}_2 = \frac{20}{9}\ \bar{I}_1 = 15.625 \text{ mA} \quad \therefore P_{200} = 7.031^2 \times 200 \times 10^{-6} = \boxed{9.888 \text{ mW}}$

(b) $P_{500} = 15.625^2 \times 500 \times 10^{-6} = \boxed{122.07 \text{ mW}}$

(c) $P_S = 10I_1 = 70.31 \text{ mW(gen)} \quad \therefore P_{2\,port} = P_S - P_{200} - P_{500} = 70.31 - 9.89 - 122.07 \ \therefore$

$\boxed{P_{2\,port} = -61.65^{-}\ \text{mW}}$

35.

$\omega = 10^8,\ \bar{I}_1 = 10^{-5}\bar{V}_1 + j5\times10^{-4}\bar{V}_1 + j10^{-4}(\bar{V}_1 - \bar{V}_2)$

$\therefore \bar{I}_1 = (10^{-5} + j6\times10^{-4})\ \bar{V}_1 - j10^{-4}\bar{V}_2$

$\bar{I}_2 = 10^{-4}\bar{V}_2 + 0.01\bar{V}_1 + j10^{-4}\ (\bar{V}_2 - \bar{V}_1)$

$\therefore \bar{I}_2 = (0.01 - j10^{-4})\ \bar{V}_1 + (10^{-4} + j10^{-4})\bar{V}_2$

$$\therefore \bar{V}_1 = \frac{\begin{vmatrix} \bar{I}_1 & -j10^{-4} \\ \bar{I}_2 & 10^{-4} + j10^{-4} \end{vmatrix}}{\begin{vmatrix} 10^{-5} + j6\times10^{-4} & -j10^{-4} \\ 10^{-2} - j10^{-4} & 10^{-4} + 10^{-4} \end{vmatrix}} = \frac{(10^{-4} + j10^{-4})\bar{I}_1 + j10^{-4}\bar{I}_2}{1.0621\times10^{-6}\angle 92.640} \therefore \boxed{\begin{array}{l} \bar{z}_{11} = 133.15^{-}\angle -47.64^\circ\,\Omega \\ \bar{z}_{12} = 94.15^{+}\angle -2.642^\circ\,\Omega \end{array}}$$

$$\bar{V}_2 = \frac{\begin{vmatrix} 10^{-5} + j6\times10^{-4} & \bar{I}_1 \\ 10^{-2} - j10^{-4} & \bar{I}_2 \end{vmatrix}}{1.0621\times10^{-6}\angle 92.64^\circ} \boxed{\begin{array}{l} \therefore \bar{z}_{21} = 9416\angle 86.78^\circ\,\Omega \\ \bar{z}_{22} = 565.0\angle -3.60^\circ\,\Omega \end{array}}$$

36.

$$[\bar{z}] = \begin{bmatrix} 20 & 2 \\ 40 & 10 \end{bmatrix} (\Omega),\ \bar{V}_S = 100\angle 0°\ \text{V, } R_S = 5\ \Omega,\ R_L = 25\ \Omega$$

$$100 = 5\bar{I}_1 + \bar{V}_1,\ \bar{V}_1 = 20\bar{I}_1 + 2\bar{I}_2\ \ \therefore 100 = 25\bar{I}_1 + 2\bar{I}_2$$

$$\bar{V}_2 = 40\bar{I}_1 + 10\bar{I}_2\ \ \therefore \bar{I}_1 = \frac{1}{40}\bar{V}_2 - \frac{1}{4}\bar{I}_2\ \ \therefore 100 = \frac{25}{40}\bar{V}_2 - \frac{25}{4}\bar{I}_2 + 2\bar{I}_2$$

$$\therefore 100 = \frac{5}{8}\bar{V}_2 - \frac{17}{4}\bar{I}_2\ \ \therefore \bar{V}_2 = 160 + \frac{8}{5}\times\frac{17}{4}\bar{I}_2 = 160 + 6.8\bar{I}_2$$

$$\boxed{\therefore \bar{V}_{th} = 160\ \text{V, } R_{th} = 6.8\Omega}$$

37. $[\bar{h}] = \begin{bmatrix} 9\Omega & -2 \\ 20 & 0.2\text{ S} \end{bmatrix}$

(a) $\bar{V}_1 = 9I_1 - 2\bar{V}_2,\ \bar{I}_2 = 20\bar{I}_1 + 0.2\bar{V}_2,\ \bar{V}_1{}' = 1\bar{I}_1 + \bar{V}_1$ Eliminate $\bar{V}_1$

$\therefore \bar{V}_1 = \bar{V}_1{}' - \bar{I}_1\ \ \therefore \bar{V}_1{}' - \bar{I}_1 = 9\bar{I}_1 - 2\bar{V}_2,\ \bar{V}_1{}' = 10\bar{I} - 2\bar{V}_2\ \ \therefore \boxed{[\bar{h}]_{new} = \begin{bmatrix} 10\Omega & -2 \\ 20 & 0.2\text{ S} \end{bmatrix}}$

(b) $\bar{V}_1 = 9\bar{I}_1 - 2\bar{V}_2,\ \bar{I}_2 = 20\bar{I}_1 + 0.2\bar{V}_2,\ \bar{V}_2{}' = 1\bar{I}_2 + \bar{V}_2$

Eliminate $\bar{V}_2\ \ \therefore \bar{V}_2 = \bar{V}_2{}' - \bar{I}_2$

$\bar{V}_1 = 9\bar{I}_1 - 2\bar{V}_2 + 2\bar{I}_2,\ \bar{I}_2 = 20\bar{I}_1 + 0.2\bar{V}_2{}' - 0.2\bar{I}_2\ \ \therefore 1.2\bar{I}_2 = 20\bar{I}_1 + 0.2\bar{V}_2{}'$

$\therefore \bar{I}_2 = 16.667\bar{I}_1 + 0.16667\bar{V}_2{}'\ \ \bar{V}_1 = 9\bar{I}_1 - 2\bar{V}_2{}' + 2(16.667\bar{I}_1 + 0.1667\ \bar{V}_2{}')$

$\therefore \bar{V}_1 = 42.38\ \bar{I}_1 - 1.6667\bar{V}_2{}'\ \ \ \therefore \boxed{[h]_{new} = \begin{bmatrix} 42.33\Omega & -1.6667 \\ 16.667 & 0.16667\text{ S} \end{bmatrix}}$

38.

$$R_S = 100\,\Omega,\ R_L = 500\,\Omega\ [\bar{h}] = \begin{bmatrix} 100\Omega & 0.01 \\ 20 & 1\text{ mS} \end{bmatrix}$$

$$\bar{Z}_{in}:\ \bar{V}_1 = 100\bar{I}_1 + 0.01\bar{V}_2,\ \bar{I}_2 = 20\bar{I}_1 + 0.001\bar{V}_2 = 20\bar{I}_1 - 0.5\bar{I}_2 \quad \therefore 1.5\bar{I}_2 = 20\bar{I}_1$$

$$\therefore \bar{V}_1 = 100\bar{I}_1 + 0.01(-500)\frac{20}{1.5}\ \bar{I}_1 = 33.33\bar{I}_1 \quad \therefore \boxed{\bar{Z}_{in} = 33.33\Omega}$$

$$\bar{Z}_{out}:\ \bar{V}_1 = -100\bar{I}_1 = 100\bar{I}_1 + 0.01\bar{V}_2 \quad \therefore \bar{I}_1 = \frac{0.01}{-200}\ \bar{V}_2$$

$$\bar{I}_2 = 20\left(\frac{0.01}{-200}\ \bar{V}_2\right) + 0.001\ \bar{I}_2 = 0 \quad \therefore \boxed{\bar{Z}_{out} = \infty}$$

39.

(a) $\bar{h}_{12} = \bar{V}_1 / \bar{V}_2 |_{I_1=0}$ Let $\bar{V}_2 = 1$ V

$\therefore \bar{I}_{10} \downarrow = 0.1$ A, $\bar{I}_1 = 0 \quad \therefore \bar{I}_{4\Omega} \leftarrow = 0.2\bar{I}_2$

$\therefore 0.1 = \bar{I}_2 - 0.2\bar{I}_2 = 0.8\bar{I}_2, \ \bar{I}_2 = 0.125$ A

$\therefore \bar{V}_1 = 0.3 - 4(0.2)(0.125) + 1 = 1.2$ V $\therefore \boxed{\bar{h}_{12} = 1.2}$

(b) $\bar{z}_{12} = \left.\dfrac{\bar{V}_1}{\bar{I}_2}\right|_{I_1=0}$ From above, $\bar{z}_{12} = \dfrac{1.2}{0.125} = \boxed{9.6\,\Omega}$

(c) $\bar{y}_{12} = \bar{I}_1 / \bar{V}_2 \big|_{V_1=0}$ SC input Let $\bar{V}_2 = 1$ V

$\bar{I}_2 = 0.1 + \dfrac{1.3}{4} = 0.425$ A, $\bar{I}_1 = 0.2(0.425) - \dfrac{1.3}{4}$

$\therefore \bar{I}_1 = -0.24$ A $\quad \therefore \boxed{\bar{y}_{12} = 0.24 \text{ S}}$

40. $[\bar{h}] = \begin{bmatrix} 1000\Omega & -1 \\ 4 & 500\mu\text{S} \end{bmatrix}$

(a)
$$100 = 200\,\bar{\text{I}}_1 + 1000\bar{\text{I}}_1 - \bar{\text{V}}_2 = 1200\bar{\text{I}}_1 - \bar{\text{V}}_2$$
$$\bar{\text{I}}_2 = 4\bar{\text{I}}_1 + 5\times10^{-4}\,\bar{\text{V}}_2 = -10^{-3}\bar{\text{V}}_2 \quad \therefore 4\bar{\text{I}}_1 = -1.5\times10^{-3}\,\bar{\text{V}}_2$$
$$\therefore \bar{\text{V}}_2 = -\frac{4000}{1.5}\,\bar{\text{I}}_1 \quad \therefore 100 = 1200\bar{\text{I}}_1 + \frac{4000}{1.5}\,\bar{\text{I}}_1 \quad \therefore \bar{\text{I}}_1 = 25.86 \text{ mA}$$
$$\therefore \text{P}_{200} = 25.86^2 \times 10^{-6} \times 200 = \boxed{133.77 \text{ mW}}$$

(b)
$$\bar{\text{V}}_2 = \frac{4000}{1.5} \times 25.86 \times 10^{-3} = 68.97 \text{ V} \quad \therefore \text{P}_{1K} = \frac{68.97^2}{1000} = \boxed{4.756 \text{ W}}$$

(c)
$$\text{P}_S = 100 \times 25.86 \times 10^{-3} = 2.586 \text{ W (gen)}$$
$$\therefore \text{P}_{2\,port} = 2.586 - 0.1338 - 4.756 = \boxed{-2.304\text{W}}$$

41.

(a)

$$\bar{V}_1 = 1000\left(\bar{I}_1 + 10^{-5}\bar{V}_2\right) = 1000\bar{I}_1 + 0.01\bar{V}_2$$

$$\bar{V}_2 = 10^4\bar{I}_2 - 100\bar{V}_1 \quad \therefore \bar{I}_2 = 10^{-4}(100\bar{V}_1 + \bar{V}_2)$$

$$\therefore \bar{I}_2 = 10^{-2}(1000\bar{I}_1 + 0.01\bar{V}_2) + 10^{-4}\bar{V}_2$$

$$\therefore \bar{I}_2 = 10\bar{I}_1 + 2\times10^{-4}\bar{V}_2 \quad \therefore \boxed{[\bar{h}] = \begin{bmatrix} 1000\Omega & 0.01 \\ 10 & 2\times10^{-4}\text{S} \end{bmatrix}}$$

(b)

$$\bar{V}_1 = -200\bar{I}_1 = 1000\bar{I}_1 + 0.01\bar{V}_2$$

$$\therefore \bar{I}_1 = \frac{-1}{12,000}\bar{V}_2 \quad \therefore \bar{I}_2 = 10\bar{I}_1 + 2\times10^{-4}\bar{V}_2 = \frac{-1}{12,000}\bar{V}_2 + \frac{1}{5000}\bar{V}_2 + 116.67\times10^{-6}\bar{V}_2$$

$$\therefore \bar{Z}_{out} = \bar{V}_2 / \bar{I}_2 = 10^6/116.67 = \boxed{8.571k\Omega}$$

42.
(a)

$$\bar{V}_1 = \bar{I}_1 R + \bar{V}_2 \qquad \therefore\ \bar{I}_1 = \frac{\bar{V}_1}{R} - \frac{\bar{V}_2}{R} \qquad [\bar{y}] = \begin{bmatrix} 1/R & -1/R \\ -1/R & 1/R \end{bmatrix}$$

$$\bar{I}_1 = -\bar{I}_2 \qquad \bar{I}_2 = -\frac{\bar{V}_1}{R} + \frac{\bar{V}_2}{R}$$

$[z]$ parameters are all ∞

$$\bar{V}_1 = \bar{I}_1 R + \bar{V}_2 \qquad \therefore \boxed{[\bar{h}] = \begin{bmatrix} R & 1 \\ -1 & 0 \end{bmatrix}}$$

$$\bar{I}_2 = -\bar{I}_1$$

(b) $[\bar{y}]$ parameters are ∞

$$\bar{V}_1 = \bar{V}_2 \qquad \bar{V}_1 = R\bar{I}_1 + R\bar{I}_2 \qquad \therefore [\bar{z}] = \begin{bmatrix} R & R \\ R & R \end{bmatrix}$$

$$\bar{I}_1 = \frac{\bar{V}_1}{R} - \bar{I}_2 \qquad \bar{V}_2 = R\bar{I}_1 + R\bar{I}_2$$

$$\bar{V}_1 = \bar{V}_2$$

$$\bar{I}_2 = -\bar{I}_1 + \frac{\bar{V}_2}{R} \qquad \therefore \boxed{[\bar{h}] = \begin{bmatrix} 0 & 1 \\ -1 & 1/R \end{bmatrix}}$$

43.

$$V_{BE} = h_{ie} I_B + h_{re} V_{CE}$$
$$I_C = h_{fe} I_B + h_{oe} V_{CE}$$

(a) $h_{oe} = \left.\dfrac{I_C}{V_{CE}}\right|_{I_B=0}$ $\qquad v_\pi = \dfrac{\dfrac{r_\pi}{1+j\omega r_\pi C_\pi}}{\dfrac{1}{j\omega C_\mu} + \dfrac{r_\pi}{1+j\omega r_\pi C_\pi}} V_{CE}$

$$I_C = \frac{V_{CE}}{\left(\dfrac{1}{j\omega C_\mu} + \dfrac{r_\pi}{1+j\omega r_\pi C_\pi}\right)} + g_m v_\pi + \frac{1}{r_d} V_{CE}$$

Thus, $\boxed{h_{oe} = \dfrac{(j\omega C_\mu)(1+j\omega r_\pi C_\pi)}{1+j\omega r_\pi (C_\pi + C_\mu)} + g_m \dfrac{j\omega r_\pi C_\mu}{1+j\omega r_\pi (C_\pi + C_\mu)} + \dfrac{1}{r_d}}$

(b) $h_{fe} = \left.\dfrac{I_C}{I_B}\right|_{V_{CE}=0}$

$$I_C = (g_m - j\omega C_\mu) v_\pi \quad \text{and} \quad I_B = \left[\frac{1}{r_\pi} g_m + j\omega (C_\pi + C_\mu)\right] v_\pi$$

Thus, $\boxed{h_{fe} = \dfrac{(g_m - j\omega C_\mu) r_\pi}{1+j\omega r_\pi (C_\pi + C_\mu)}}$

(c) $h_{ie} = \left.\dfrac{V_{BE}}{I_B}\right|_{V_{CE}=0}$

$$\boxed{h_{ie} = r_x + \frac{r_\pi}{1+j\omega r_\pi (C_\pi + C_\mu)}}$$

(d) $h_{re} = \left.\dfrac{V_{BE}}{V_{CE}}\right|_{I_B=0}$

$$V_{BE} = \frac{\dfrac{r_\pi}{1+j\omega r_\pi C_\pi}}{\dfrac{r_\pi}{1+j\omega r_\pi C_\pi} + \dfrac{1}{j\omega C_\mu}} V_{CE}$$

Thus, $\boxed{h_{re} = \dfrac{j\omega C_\mu r_\pi}{1+j\omega r_\pi (C_\pi + C_\mu)}}$

44. $[\bar{y}] = \begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix}$, $[\bar{b}] = \begin{bmatrix} 4 & 6 \\ -1 & 5 \end{bmatrix}$, $[\bar{c}] = \begin{bmatrix} 3 & 2 & 4 & -1 \\ -2 & 3 & 5 & 0 \end{bmatrix}$, $[\bar{d}] = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 5 \\ -2 & -3 & 1 \\ 4 & -4 & 2 \end{bmatrix}$

(a) $$[\bar{y}][\bar{b}] = \begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 4 & 6 \\ -1 & 5 \end{bmatrix} = \boxed{\begin{bmatrix} 6 & -4 \\ 8 & 38 \end{bmatrix}}$$

(b) $$[\bar{b}][\bar{y}] = \begin{bmatrix} 4 & 6 \\ -1 & 5 \end{bmatrix}\begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix} = \boxed{\begin{bmatrix} 22 & 16 \\ 14 & 22 \end{bmatrix}}$$

(c) $$[\bar{b}][\bar{c}] = \begin{bmatrix} 4 & 6 \\ -1 & 5 \end{bmatrix}\begin{bmatrix} 3 & 2 & 4 & -1 \\ -2 & 3 & 5 & 0 \end{bmatrix} = \boxed{\begin{bmatrix} 0 & 26 & 46 & -4 \\ -13 & 13 & 21 & 1 \end{bmatrix}}$$

(d) $$[\bar{c}][\bar{d}] = \begin{bmatrix} 3 & 2 & 4 & -1 \\ -2 & 3 & 5 & 0 \end{bmatrix}\begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 5 \\ -2 & -3 & 1 \\ 4 & -4 & 2 \end{bmatrix} = \boxed{\begin{bmatrix} -3 & -2 & 9 \\ -3 & -19 & 22 \end{bmatrix}}$$

(e) $$[\bar{y}][\bar{b}][\bar{c}][\bar{d}] = \begin{bmatrix} 6 & -4 \\ 8 & 38 \end{bmatrix}\begin{bmatrix} -3 & -2 & 9 \\ -3 & -19 & 22 \end{bmatrix} = \boxed{\begin{bmatrix} -6 & 64 & -34 \\ -138 & -738 & -908 \end{bmatrix}}$$

45.

(a)

$$\bar{V}_1 = \bar{t}_{11}\bar{V}_2 - \bar{t}_{12}\bar{I}_2,\ \bar{I}_1 = \bar{t}_{21}\bar{V}_2 - \bar{t}_{22}\bar{I}_2$$

$$\bar{V}_1 = 10\bar{I}_1 + \bar{V}_2 - 1.5\bar{V}_1,\ \bar{I}_2 = \frac{\bar{V}_2}{20} + \frac{\bar{V}_2 - 1.5\bar{V}_1}{25} + \frac{\bar{V}_2 - 1.5\bar{V}_1 - \bar{V}_1}{10}$$

$$\therefore\ \bar{I}_2 = 0.19\bar{V}_2 - 0.31\bar{V}_1,\ \bar{V}_1 = \frac{0.19}{0.31}\ \bar{V}_2 - \frac{1}{0.31}\ \bar{I}_2$$

$$\therefore\ \bar{V}_1 = 0.6129\bar{V}_2 - 3.226\ \bar{I}_2$$

Then, $10\bar{I}_1 = \bar{V}_1 - (\bar{V}_2 - 1.5\bar{V}_1) = 2.5(0.6129\bar{V}_2 - 3.226\bar{I}_2) - \bar{V}_2$

$$\therefore\ \bar{I}_1 = 0.05323\bar{V}_2 - 0.8065^{-}\ \bar{I}_2 \qquad \therefore [\bar{t}\,] = \boxed{\begin{bmatrix} 0.6129 & 3.226\Omega \\ 0.05323\text{S} & 0.8065^{-} \end{bmatrix}}$$

(b)

Let $R_S = 15\ \Omega$

$$\therefore\ \bar{V}_1 = 0.06129\bar{V}_2 - 3.226\bar{I}_2,\ \bar{I}_1 = 0.05323\bar{V}_2 - 0.8065\bar{I}_2,\ \bar{V}_1 = -15\bar{I}_1$$

$$\therefore\ -15\bar{I}_1 = -15(0.05323\bar{V}_2 - 0.8065^{-}\ \bar{I}_2) = 0.6129\bar{V}_2 - 3.226\bar{I}_2$$

$$\therefore 1.4114\bar{V}_2 = 15.324\bar{I}_2 \quad \therefore\ \bar{Z}_{out} = \bar{V}_2 / \bar{I}_2 = \boxed{10.857\,\Omega}$$

46.

$\bar{V}_1 = 5\bar{I}_1 - 0.3\bar{V}_1 + \bar{V}_2 \quad \therefore 1.3\bar{V}_1 = 5\bar{I}_1 + \bar{V}_2$

$\bar{I}_1 = 0.1\bar{V}_2 + \bar{V}_2 / 4 - \bar{I}_2 \quad \therefore \bar{I}_1 = 0.35\bar{V}_2 - \bar{I}_2$

$\therefore 1.3\bar{V}_1 = 5(0.35\bar{V}_2 - \bar{I}_2) + \bar{V}_2 = 2.75\ \bar{V}_2 - 5\bar{I}_2$

$\therefore \bar{V}_1 = 2.115^+ \bar{V}_2 - 3.846\bar{I}_2 \quad \therefore \boxed{[\bar{t}\,] = \begin{bmatrix} 2.115^+ & 3.846\Omega \\ 0.35\,\text{S} & 1 \end{bmatrix}}$

47.

(a)

$$\bar{V}_1 = 2\bar{I}_1 + \bar{V}_2 \quad \therefore \bar{I}_1 = 0.2\bar{V}_2 - \bar{I}_2$$

$$\bar{I}_2 = 0.2\bar{V}_2 - \bar{I}_1 \quad \therefore \bar{V}_1 = 1.4\bar{V}_2 - 2\bar{I}_2$$

$$\bar{V}_1 = 3\bar{I}_1 + \bar{V}_2 \quad \therefore \bar{I}_1 = \frac{1}{6}\bar{V}_2 - \bar{I}_2 \quad \therefore \boxed{[\bar{t}]_A = \begin{bmatrix} 1.4 & 2\Omega \\ 0.2\text{ S} & 1 \end{bmatrix}}$$

$$\bar{I}_2 = \frac{1}{6}\bar{V}_2 - \bar{I}_1 \quad \therefore \bar{V}_1 = 1.5\bar{V}_2 - 3\bar{I}_2 \quad \therefore \boxed{[\bar{t}]_B = \begin{bmatrix} 1.5 & 3\Omega \\ \frac{1}{6}\text{ S} & 1 \end{bmatrix}}$$

$$\bar{V}_1 = 4\bar{I}_1 + \bar{V}_2 \quad \therefore \bar{I}_1 = \frac{1}{7}\bar{V}_2 - \bar{I}_2 \quad \boxed{[\bar{t}]_C = \begin{bmatrix} 11/7 & 4\Omega \\ 1/7\text{ S} & 1 \end{bmatrix}}$$

$$\bar{I}_R = \frac{1}{7}\bar{V}_2 - \bar{I}_1 \quad \bar{V}_1 = \frac{11}{7}\bar{V}_2 - 4\bar{I}_2$$

(b)

$$[\bar{t}] = [\bar{t}]_A[\bar{t}]_B[\bar{t}]_C = \begin{bmatrix} 1.4 & 2 \\ 0.2 & 1 \end{bmatrix}\begin{bmatrix} 1.5 & 3 \\ 1/6 & 1 \end{bmatrix}\begin{bmatrix} 11/7 & 4 \\ 1/7 & 1 \end{bmatrix} = \begin{bmatrix} 2.433 & 6.2 \\ 0.4667 & 1.6 \end{bmatrix}\begin{bmatrix} 11/7 & 4 \\ 1/7 & 1 \end{bmatrix}$$

$$\boxed{\therefore [\bar{t}] = \begin{bmatrix} 4.710 & 15.933\Omega \\ 0.9619\text{ S} & 3.467 \end{bmatrix}}$$

48.

(a)

$$\bar{V}_1 = 2\bar{I}_1 + \bar{V}_2 = -2\bar{I}_2 + \bar{V}_2 = \bar{V}_2 - 2\bar{I}_2 \quad \therefore \boxed{[\bar{t}\,]_A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}}$$

$$\bar{I}_1 = -\bar{I}_2$$

(b)

$$\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 8 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 8 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 10\Omega \\ 0 & 1 \end{bmatrix} = \left(\begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix}\right)^5 \quad \text{Also, } 10 \rightarrow \begin{bmatrix} 1 & 10 \\ 0 & 1 \end{bmatrix}$$

49.

(a) $\bar{V}_1 = \bar{V}_2 \quad \therefore \boxed{[\bar{t}\,]_a = \begin{bmatrix} 1 & 0 \\ 1/R & 1 \end{bmatrix}}$

$\bar{I}_1 = \bar{V}_2 / R - \bar{I}_2$

$\bar{V}_1 = \bar{V}_2 - R\bar{I}_2 \quad \therefore \boxed{[\bar{t}\,]_b = \begin{bmatrix} 1 & R \\ 0 & 1 \end{bmatrix}}$

$\bar{I}_1 = -\bar{I}_2$

$\bar{V}_1 = \bar{V}_2 / a \quad \therefore \boxed{[\bar{t}\,]_c = \begin{bmatrix} 1/a & 0 \\ 0 & a \end{bmatrix}}$

$\bar{I}_1 = -a\bar{I}_2$

(b) $[\bar{t}\,] = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0.1 & 1 \end{bmatrix} \begin{bmatrix} 0.25 & 0 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} 1 & 20 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0.02 & 1 \end{bmatrix}$

$\therefore [\bar{t}\,] = \begin{bmatrix} 1.2 & 2 \\ 0.1 & 1 \end{bmatrix} \begin{bmatrix} 0.25 & 5 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0.02 & 1 \end{bmatrix} = \begin{bmatrix} 0.3 & 14 \\ 0.025 & 4.5 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0.02 & 1 \end{bmatrix} = \boxed{\begin{bmatrix} 0.58 & 14\,\Omega \\ 0.115\text{ S} & 4.5 \end{bmatrix}}$

50.

(a)

$$\bar{I}_1 = 0.1\bar{V}_x,\ -0.1\bar{V}_x + 0.02(\bar{V}_1 - \bar{V}_x) + 0.2(\bar{V}_1 - \bar{V}_x - \bar{V}_2) = 0$$

$$\bar{I}_2 = 0.08\bar{V}_x + 0.2(\bar{V}_2 - \bar{V}_1 + \bar{V}_x)$$

$$\therefore 0.32\bar{V}_x = 0.22\bar{V}_1 - 0.2\ \bar{V}_2 \quad \therefore \bar{V}_x = \frac{11}{16}\ \bar{V}_1 - \frac{5}{8}\ \bar{V}_2$$

$$\therefore \bar{I}_1 = \frac{11}{160}\ \bar{V}_1 - \frac{1}{16}\ \bar{V}_2 \quad \text{Also, } \bar{I}_2 = 0.28\left(\frac{11}{16}\ \bar{V}_1 - \frac{5}{8}\ \bar{V}_2\right) + 0.2\bar{V}_2 - 0.2\ \bar{V}_1$$

$$\therefore \bar{I}_2 = -\frac{3}{400}\ \bar{V}_1 + \frac{1}{40}\ \bar{V}_2 \quad \therefore \bar{V}_1 = \frac{10}{3}\ \bar{V}_2 - \frac{400}{3}\ \bar{I}_2 \quad [\bar{t}\,] = \begin{bmatrix} 3.333 & 133.33\Omega \\ 0.16667\text{S} & 9.17 \end{bmatrix}$$

$$\therefore \bar{I}_1 = \frac{11}{160}\left(\frac{10}{3}\ \bar{V}_2 - \frac{400}{3}\ \bar{I}_2\right) - \frac{1}{16}\ \bar{V}_2 = \frac{1}{6}\ \bar{V}_2 - \frac{55}{6}\bar{I}_2 \quad \therefore [\bar{t}\,] = \boxed{\begin{bmatrix} 3.333 & 133.33\Omega \\ 0.16667\text{S} & 9.167 \end{bmatrix}}$$

(b)

$$\begin{bmatrix} 1 & 0 \\ 0.05 & 1 \end{bmatrix} \quad \therefore [\bar{t}\,]_{new} = \begin{bmatrix} 10/3 & 400/3 \\ 1/6 & 55/6 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0.05 & 1 \end{bmatrix} = \boxed{\begin{bmatrix} 10 & 133.33\Omega \\ 0.625\ \text{S} & 9.167 \end{bmatrix}}$$

1. (a) $\boxed{5, 10, 15, 20, 25 \quad \text{(all rad/s)}}$

   (b) $\boxed{5, 10, 15, 20, 25 \quad \text{(all rad/s)}}$

   (c) $\boxed{90, 180, 270, 360, 450 \quad \text{(all rad/s)}}$

2. (a) $\omega_o = 2\pi$ rad/s, $\boxed{f = 1 \text{ Hz}, \quad \text{therefore } T = 1 \text{ s.}}$

(b) $\omega_o = 5.95$ rad/s $= 2\pi f$ rad/s, $\boxed{f = 0.947 \text{ Hz}, \quad \text{therefore } T = 1.056 \text{ s.}}$

(c)) $\omega_o = 1$ rad/s $= 2\pi f$ rad/s, $\boxed{f = 1/2\pi \text{ Hz}, \quad \text{therefore } T = 2\pi \text{ s.}}$

3.

$v(t) = 3 - 3\cos(100\pi t - 40°) + 4\sin(200\pi t - 10°) + 2.5\cos 300\pi t$ V

(a) $V_{av} = 3 - 0 + 0 + 0 = \boxed{3.000 \text{ V}}$

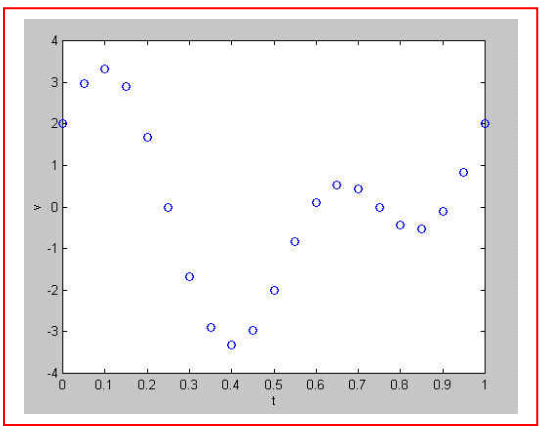
(b) $V_{eff} = \sqrt{3^2 + \frac{1}{2}(3^2 + 4^2 + 2.5^2)} = \boxed{4.962 \text{ V}}$

(c) $T = \dfrac{2\pi}{\omega_o} = \dfrac{2\pi}{100\pi} = \boxed{0.02\, s}$

(d) $v(18ms) = 3 - 3\cos(-33.52°) + 4\sin(2.960°) + 2.5\cos(19.440°) = \boxed{-2.459 \text{ V}}$

4. (a)

t	v	t	v
0	2	0.55	-0.844
0.05	2.96	0.6	0.094
0.1	3.33	0.65	0.536
0.15	2.89	0.7	0.440
0.2	1.676	0.75	0
0.25	0	0.8	-0.440
0.3	-1.676	0.85	-0.536
0.35	-2.89	0.9	-0.094
0.4	-3.33	0.95	0.844
0.45	-2.96	1	2
0.5	-2		



(b) $v' = -4\pi \sin 2\pi t + 7.2\pi \cos 4\pi t = 0$

$\therefore 4\sin 2\pi t = 7.2(\cos^2 2\pi t - \sin^2 2\pi t)$

$\therefore 4\sin 2\pi t = 7.2(1-2\sin^2 2\pi t) \;\; \therefore x = \dfrac{-4 \pm \sqrt{16+414.72}}{28.8} = 0.5817, -0.8595 = \sin 2\pi t$

$\therefore t = 0.09881, 0.83539 \;\; \therefore v_{max} = \boxed{3.330}$(0.5593 for smaller max)

(c) $|v_{min}| = \boxed{3.330}$

5. (a) $a_0 = 0$

(b) $a_0 = 0$

(c) $a_0 = 5$

(d) $a_0 = 5$

6. (a) $a_0 = 0$

(b) $a_0 = 0$

(c) $a_0 = 100$

(d) $a_0 = 100$

7. (a) $a_0 = 3$, $a_1 = 0$, $a_2 = 0$, $b_1 = 0$, $b_2 = 0$

(b) $a_0 = 3$, $a_1 = 3$, $a_2 = 0$, $b_1 = 0$, $b_2 = 0$

(c) $a_0 = 0$, $a_1 = 0$, $a_2 = 0$, $b_1 = 3$, $b_2 = 3$

(d) $3\cos(3t - 10^\circ) = 3\cos 3t \cos 10^\circ + 3\sin 3t \sin 10^\circ$

$a_0 = 0$, $a_1 = 3\cos 10^\circ = 2.954$, $a_2 = 0$, $b_1 = 3\sin 10^\circ = 0.521$, $b_2 = 0$

8. $\boxed{a_o = \frac{1}{T}\int_0^T f(t)dt = 2.5. \quad a_1 = a_2 = 0}$ since function has odd symmetry

$$b_1 = \frac{2}{T}\int_0^T f(t)\sin\omega_0 t dt = \frac{2}{2-0}\int_1^2 5\sin\pi t dt = -\frac{5}{2\pi}\cos\pi t\Big|_1^2 = \boxed{-\frac{10}{\pi}}$$

$$b_2 = \frac{2}{T}\int_0^T f(t)\sin 2\omega_0 t dt = \frac{2}{2-0}\int_1^2 5\sin 2\pi t dt = -\frac{5}{2\pi}\cos 2\pi t\Big|_1^2 = \boxed{0}$$

9. $$a_o = \frac{1}{T}\int_0^T f(t)dt = \frac{1}{3}\int_0^2 2dt = \frac{2}{3}t\Big|_0^2 = \boxed{\frac{4}{3}}.$$

$$a_1 = \frac{2}{T}\int_0^T f(t)\cos\omega_0 tdt = \frac{2}{3}\int_0^2 2\cos\left(\frac{2\pi}{3}t\right)dt = \frac{4}{3}\left(\frac{3}{2\pi}\right)\sin\left(\frac{2\pi}{3}t\right)\Big|_0^2 = \boxed{-0.551}$$

$$a_2 = \frac{2}{T}\int_0^T f(t)\cos 2\omega_0 tdt = \frac{2}{3}\int_0^2 2\cos\left(\frac{4\pi}{3}t\right)dt = \frac{4}{3}\left(\frac{3}{4\pi}\right)\sin\left(\frac{4\pi}{3}t\right)\Big|_0^2 = \boxed{0.276}$$

$$a_3 = \frac{2}{T}\int_0^T f(t)\cos 3\omega_0 tdt = \frac{2}{3}\int_0^2 2\cos\left(\frac{6\pi}{3}t\right)dt = \frac{4}{3}\left(\frac{3}{6\pi}\right)\sin\left(\frac{6\pi}{3}t\right)\Big|_0^2 = \boxed{0}$$

$$b_1 = \frac{2}{T}\int_0^T f(t)\sin\omega_0 tdt = \frac{2}{3}\int_0^2 2\sin\left(\frac{2\pi}{3}t\right)dt = -\left(\frac{4}{3}\right)\frac{3}{2\pi}\cos\left(\frac{2\pi}{3}t\right)\Big|_0^2 = \boxed{0.955}$$

$$b_2 = \frac{2}{T}\int_0^T f(t)\sin 2\omega_0 tdt = \frac{2}{3}\int_0^2 2\sin\left(\frac{4\pi}{3}t\right)dt = -\left(\frac{4}{3}\right)\frac{3}{4\pi}\cos\left(\frac{4\pi}{3}t\right)\Big|_0^2 = \boxed{0.477}$$

10. $h(t) = -3 + 8\sin \pi t + f(t)$

Use linearity and superposition. T = 2 s.

$a_o = -3 + \frac{1}{T}\int_0^T f(t)dt = -3 + \frac{1}{2} = \boxed{-2.5}.$

$\boxed{a_2 = 0}$

$b_1 = 8 + \frac{2}{T}\int_0^T f(t)\sin\omega_0 t dt = 8 + \frac{2}{2}\int_0^2 (1)\sin\pi t dt = 8 - \frac{2}{\pi} = \boxed{7.36}$

$b_2 = \frac{2}{T}\int_0^T f(t)\sin 2\omega_0 t dt = \frac{2}{2}\int_1^2 (1)\sin 2\pi t dt = \boxed{0}$

11.

(a) $\mathrm{T} = 10\ s,\ \mathrm{F}_{av} = a_o = 0.1(2\times 4 + 2\times 2) = \boxed{1.200}$

(b) $$\mathrm{F}_{eff} = \sqrt{\frac{1}{5}\int_0^2 (4-t)^2\,dt} = \sqrt{0.2\int_0^2 (16 - 8t + t^2)\,dt}$$

$$= \sqrt{0.2\left[16t\Big|_0^2 - 4t^2\Big|_0^2 + \frac{1}{3}t^3\Big|_0^2\right]} = \sqrt{0.2\left(32 - 16 + \frac{8}{3}\right)} = \boxed{1.9322}$$

(c) $$a_3 = \frac{2}{10}\times 2\int_0^2 (4-t)\cos 3\times\frac{2\pi t}{10}\,dt = 0.4\int_0^2 4\cos 0.6\pi t\ dt - 0.4\int_0^2 t\cos 0.6\pi t\ dt$$

$$= 1.6\frac{1}{0.6\pi}\sin 0.6\pi t\Big|_0^2 - 0.4\left(\frac{1}{0.36\pi^2}\cos 0.6\pi t + \frac{t}{0.6\pi}\sin 0.6\pi t\right)_0^2$$

$$= \frac{8}{3\pi}\sin 1.2\pi - \frac{10}{9\pi^2}(\cos 1.2\pi - 1) - \frac{4}{3\pi}\sin 1.2\pi = \boxed{-0.04581}$$

12.

(a) T = 8 – 2 = $\boxed{6 \text{ s}}$

(b) $f_o = \boxed{\frac{1}{6}\ \text{Hz}}$

(c) $\omega_o = 2\pi f_o = \boxed{\frac{\pi}{3}\ \text{rad/s}}$

(d) $a_o = \frac{1}{6}(10\times1+5\times1) = \boxed{2.5}$

(e)
$$b_2 = \frac{2}{6}\left[\int_2^3 10\sin\frac{2\pi t}{3}dt + \int_3^4 5\sin\frac{2\pi t}{3}dt\right]$$
$$= \frac{1}{3}\left[-\frac{30}{2\pi}\cos\frac{2\pi t}{3}\bigg|_2^3 - \frac{15}{2\pi}\cos\frac{2\pi t}{3}\bigg|_3^4\right]$$
$$\therefore b_2 = \frac{1}{3}\left[-\frac{15}{\pi}\left(\cos 2\pi - \cos\frac{4\pi}{3}\right) - \frac{7.5}{\pi}\left(\cos\frac{8\pi}{3} - \cos 2\pi\right)\right] = \frac{1}{3}\left[-\frac{15}{\pi}(1.5) - \frac{7.5}{\pi}(-1.5)\right] = \boxed{-1.1937}$$

13.

$$a_3 = \frac{2}{6}\left[\int_2^3 10\cos\frac{6\pi t}{6}\,dt + \int_3^4 5\cos\frac{6\pi t}{6}\,dt\right] = \frac{1}{3}\left[\frac{10}{\pi}\sin\pi t\Big|_2^3 - \frac{5}{\pi}\sin\pi t\Big|_3^4\right]$$

$$= \frac{10}{3\pi}\left(\sin 3\pi - \sin 2\pi + \frac{1}{2}\sin 4\pi - \frac{1}{2}\sin 3\pi\right)\boxed{= 0}$$

$$b_3 = \frac{1}{3}\left[\int_2^3 10\sin\pi t dt + \int_3^4 5\sin\pi t\,dt\right] = \frac{1}{3}\left[-\frac{10}{\pi}\cos\pi t\Big|_2^3 - \frac{5}{\pi}\cos\pi t\Big|_3^4\right]$$

$$= -\frac{10}{3\pi}\left(\cos 3\pi - \cos 2\pi + \frac{1}{2}\cos 4\pi - \frac{1}{2}\cos 3\pi\right) = -\frac{10}{3\pi}(-1) = \boxed{1.0610}$$

$$\sqrt{a_3^2 + b_3^2} = \boxed{1.0610}$$

14.

(a) $3.8\cos^2 80\pi t = 1.9 + 1.9\cos 160\pi t,\ \mathrm{T} = \dfrac{2\pi}{160\pi} = 12.5\ \text{ms, ave value} = \boxed{1.9}$

(b) $3.8\cos^3 80\pi t = (3.8\cos 80\pi t)(0.5 + 0.5\cos 160\pi t)$

$= 1.9\cos 80\pi t + 0.95\cos 240\pi t + 0.95\cos 80\pi t = 2.85\cos 80\pi t + 0.95\cos 240\pi t$

$\mathrm{T} = \dfrac{2\pi}{80\pi} = 25\,\text{ms, ave value } = \boxed{0}$

(c) $3.8\cos 70\pi t - 3.8\sin 80\pi t;\ \omega_o t = \pi t,\ \omega_o = \pi,\ \mathrm{T} = \dfrac{2\pi}{\pi} = 2s;\ \text{ave value } = \boxed{0}$

15. T = 2 s

(a) $$b_4 = \frac{2}{2}\int_0^{t_1} \sin\frac{4\times 2\pi t}{2}\,dt = -\frac{1}{4\pi}\cos 4\pi t\Big|_0^{t_1}$$

$$\therefore b_4 = \frac{1}{4\pi}\left(1-\cos 4\pi t_1\right)$$

max when $4\pi t_1 = \frac{\pi}{2}$, $t_1 = \boxed{0.125\,s}$

(b) $$\boxed{b_4 = \frac{1}{4\pi}}$$

16.

$$g(t) = 5 + 8\cos 10t - 5\cos 15t + 3\cos 20t - 8\sin 10t - 4\sin 15t + 2\sin 20t$$

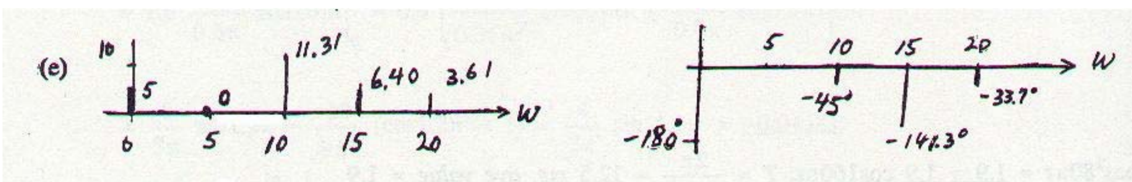
(a) $\omega_o = 5 \quad \therefore \mathrm{T} = \dfrac{2\pi}{5} = \boxed{1.2566\,s}$

(b) $f_o = \dfrac{5}{2\pi}\beta = 4\,f_o = \dfrac{10}{\pi} = \boxed{3.183\text{ Hz}}$

(c) $\mathrm{G}_{av} = -5$

(d) $\mathrm{G}_{eff} = \sqrt{(-5)^2 + \dfrac{1}{2}(8^2 + 5^2 + 3^2 + 8^2 + 4^2 + 2^2)} = \sqrt{116} = \boxed{10.770}$

(e)



17.

$$\mathrm{T} = 0.2,\ f(t) = \mathrm{V}_m \cos 5\pi t, -0.1 < t < 0.1$$

$$a_n = \frac{2}{0.2}\int_{-0.1}^{0.1} \mathrm{V}_m \cos 5\pi t \cos 10\,n\pi t\,dt = 5\mathrm{V}_m \int_{-0.1}^{0.1}\left[\cos(5\pi + 10n\pi)t + \cos(10n\pi - 5\pi)t\right]dt$$

$$= 5\mathrm{V}_m\left[\frac{1}{10n\pi + 5\pi}\sin(10n\pi + 5\pi)\,t + \frac{1}{10n\pi - 5\pi}\ \sin(10n\pi - 5\pi)t\right]_{-0.1}^{0.1}$$

$$= \frac{\mathrm{V}_m}{\pi}\left[\frac{2}{2n+1}\sin(10n\pi + 5\pi)\,0.1 + \frac{2}{2n-1}\sin(10n\pi - 5\pi)\,0.1\right]$$

$$= \frac{\mathrm{V}_m}{\pi}\left[\frac{2}{2n+1}\sin(n\pi + 0.5\pi) + \frac{2}{2n-1}\sin(n\pi - 0.5\pi)\right]$$

$$= \frac{\mathrm{V}_m}{\pi}\left[\frac{2}{2n+1}\cos n\pi + \frac{2}{2n-1}(-\cos n\pi)\right] = \frac{2\mathrm{V}_m}{\pi}\cos n\pi\left(\frac{1}{2n+1} - \frac{1}{2n-1}\right)$$

$$= \frac{2\mathrm{V}_m}{\pi}\cos n\pi\,\frac{2n-1-2n-1}{4n^2-1} = -\frac{4\mathrm{V}_m}{\pi}\,\frac{\cos n\pi}{4n^2-1}$$

$$a_o = \frac{1}{0.2}\int_{-0.1}^{0.1}\mathrm{V}_m \cos 5\pi t\,dt = 5\mathrm{V}_m\,\frac{1}{5\pi}\left[\sin\frac{\pi}{2} - \sin\left(-\frac{\pi}{2}\right)\right] = \frac{2\mathrm{V}_m}{\pi}$$

$$\therefore \boxed{v(t) = \frac{2\mathrm{V}_m}{\pi} + \frac{4\mathrm{V}_m}{3\pi}\cos 10\pi t - \frac{4\mathrm{V}_m}{15\pi}\cos 20\pi t + \frac{4\mathrm{V}_m}{35\pi}\cos 30\pi t - \frac{4\mathrm{V}_m}{63\pi}\cos 40\pi t + \ldots}$$

18.

(a) $\boxed{\text{even, } \frac{1}{2}-\text{wave}}$

(b) $\boxed{b_n = 0 \text{ for all } n;\ a_{even} = 0;\ a_o = 0}$

(c) $\boxed{b_1 = b_2 = b_3 = 0,\ a_2 = 0}$

$$a_n = \frac{8}{12}\int_1^2 5\cos\frac{n\pi t}{6}\,dt = \frac{10}{3}\,\frac{6}{n\pi}\sin\frac{n\pi t}{6}\Big|_1^2 = \frac{20}{n\pi}\left(\sin\frac{n\pi}{3} - \sin\frac{n\pi}{6}\right)$$

$$\therefore a_1 = \frac{20}{\pi}\left(\sin\frac{\pi}{3} - \sin\frac{\pi}{6}\right) = \boxed{2.330},\ a_3 = \frac{20}{3\pi}\left(\sin\pi - \sin\frac{\pi}{2}\right) = -\frac{20}{3\pi} = \boxed{-2.122}$$

19.

(a) $a_o = a_n = 0$

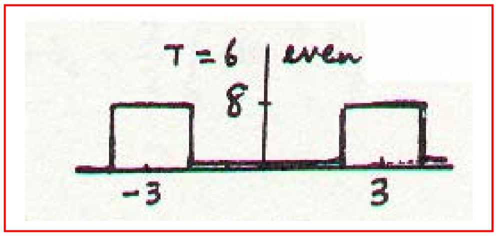
$\therefore \boxed{y(t) = 0.2\sin 1000\pi t + 0.6\sin 2000\pi t + 0.4\sin 3000\pi t}$

(b) $\mathrm{Y}_{eff} = \sqrt{0.5(0.2^2 + 0.6^2 + 0.4^2)} = \sqrt{0.5(0.56)} = \boxed{0.5292}$

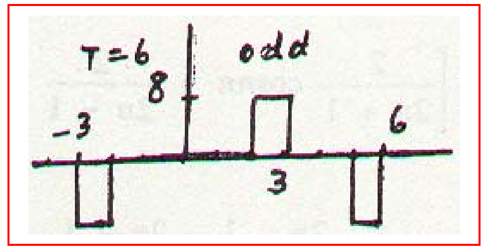
(c) $y(2\text{ms}) = 0.2\sin\ 0.2\pi + 0.6\sin 0.4\pi + 0.4\sin 0.6\pi = \boxed{1.0686}$

20.

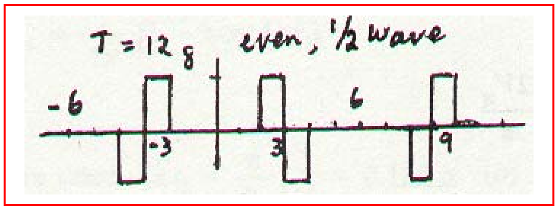
(a)



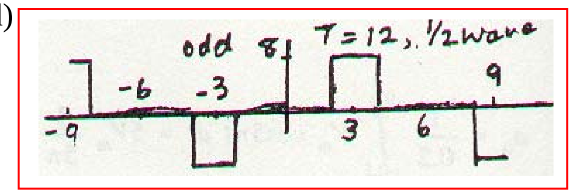
(b)



(c)



(d)



(e)

$$[a] b_5 = 0,\ a_5 = \frac{4}{6}\int_2^3 8\cos\frac{2\pi 5t}{6}\,dt = \frac{32}{6}\,\frac{6}{10\pi}\sin\frac{5\pi t}{3}\Big|_2^3 = \frac{3.2}{\pi}\left(\sin\frac{15\pi}{3} - \sin\frac{10\pi}{3}\right) = \boxed{0.8821}$$

$$[b] a_5 = 0,\ b_5 = \frac{4}{6}\int_2^3 8\sin\frac{2\pi 5t}{6}\,dt = \frac{32}{6}\left(\frac{-6}{10\pi}\right)\left(\cos\frac{15\pi}{3} - \cos\frac{10\pi}{3}\right) = -\frac{3.2}{\pi}(-0.5) = \boxed{0.5093}$$

$$[c] b_5 = 0,\ a_5 = \frac{8}{12}\int_2^3 8\cos\frac{2\pi 5t}{12}\,dt = \frac{64}{12}\,\frac{12}{10\pi}\left(\sin\frac{15\pi}{6} - \sin\frac{10\pi}{6}\right) = \boxed{3.801}$$

$$[d] a_5 = 0,\ b_5 = \frac{8}{12}\int_2^3 8\sin\frac{10\pi t}{12}\,dt = \frac{64}{12}\left(-\frac{12}{10\pi}\right)\left(\cos\frac{15\pi}{6} - \cos\frac{10\pi}{6}\right) = \boxed{1.0186}$$

21.
T = 4 ms

(a) $$a_o = \frac{1000}{4}\int_0^{0.004} 8\sin 125\pi t\ dt = \frac{250\times 8}{-125\pi}\cos 125\pi t\Big|_0^{0.004}$$

$$= -\frac{16}{\pi}\left(\cos\frac{\pi}{2}-1\right) = \frac{16}{\pi} = \boxed{5.093}$$

(b) $$a_1 = 4000\int_0^{0.004} \sin 125\pi t\cos\frac{2\pi t}{0.004}dt$$

$$\therefore a_1 = 4000\int_0^{0.004} \sin 125\pi t\cos 500\pi t\ dt = 2000\int_0^{0.004}(\sin 625\pi t - \sin 375\pi t)\,dt$$

$$= 2000\left(-\frac{\cos 625\pi t}{625\pi}+\frac{\cos 375\pi t}{375\pi}\right)_0^{0.004} = \frac{3.2}{\pi}(1-\cos 2.5\pi) - \frac{5.333}{\pi}(1-\cos 1.5\pi) = \boxed{-0.6791}$$

$$b_1 = 4000\int_0^{0.004} \sin 125\pi t\sin 500\pi t\ dt = 2000\int_0^{0.004}(\cos 375\pi t - \cos 625\pi t)\ dt$$

$$= 2000\left[\frac{1}{375\pi}(\sin 1.5\pi) - \frac{1}{625\pi}(\sin 2.5\pi)\right] = 2000\left(\frac{-1}{375\pi}-\frac{1}{625\pi}\right) = \boxed{-2.716}$$

(c) $-4 < t < 0 : \boxed{8\sin 125\pi t}$

(d) $$b_1 = 0,\ a_1 = \frac{4000}{8}\int_0^{0.004} 8\sin 125\pi t\ \cos 250\pi t\ dt$$

$$\therefore a_1 = 2000\int_0^{0.004}\left[\sin 375\pi t - \sin 125\pi t\right]dt = 2000\left[-\frac{\cos 375\pi t}{375\pi}+\frac{\cos 125\pi t}{125\pi}\right]_0^{0.004}$$

$$= \frac{5.333}{\pi}(1-\cos 1.5\pi) + \frac{16}{\pi}\left(\cos\frac{\pi}{2}-1\right) = \boxed{-3.395^{+}}$$

22.

$$\text{odd and } \frac{1}{2}-\text{wave} \quad \therefore a_o = 0,\ a_n = 0,\ b_{even} = 0$$

$$\text{T} = 10ms = 0.01s$$

$$b_{odd} = \frac{8}{0.01}\left[\int_0^{0.001} 10\sin 200n\pi t\ dt\right] = 8000\left(\frac{-1}{200n\pi}\right)\cos 200n\pi t\Big|_0^{0.001}$$

$$\therefore b_{odd} = -\frac{40}{n\pi}(\cos 0.2n\pi - 1) = \frac{40}{n\pi}(1-\cos 0.2n\pi)$$

$$\therefore \boxed{b_1 = 2.432,\ b_3 = 5.556,\ b_5 = 5.093,\ b_7 = 2.381,\ b_9 = 0.2702}$$

23.

odd and $\frac{1}{2}$ – wave, $T = 8\,ms$ $\therefore b_n = \frac{8}{T}\int_0^{T/4} f(t)\sin n\omega_o t\, dt$

$\omega_o = \frac{2\pi}{T} = 250\pi$ $\therefore b_n = 1000\int_0^{0.001} 1000t\sin 250\pi nt\ dt$

Now, $\int x\sin ax\ dx = \frac{1}{a^2}\left(\sin a_x - ax\cos ax\right),\ a = 250\,n\pi$

$f(t) = 10^3 t$ $\therefore b_n = \frac{10^6}{250^2 n^2\pi^2}\left(\sin 250n\pi t - 250n\pi t\cos 250n\pi t\right)_0^{0.001}$

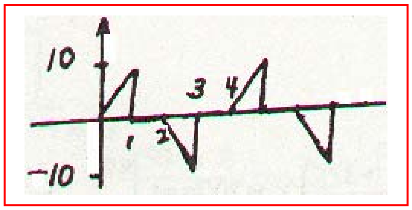
$\therefore b_n = \frac{16}{n^2\pi^2}\left(\sin\frac{n\pi}{4} - 0 - \frac{n\pi}{4}\cos\frac{n\pi}{4} + 0\right)$ $\therefore b_1 = \frac{16}{\pi^2}\left(\sin\frac{\pi}{4} - \frac{\pi}{4}\cos\frac{\pi}{4}\right) = \boxed{0.2460}$

$b_3 = \frac{16}{9\pi^2}\left(\sin\frac{3\pi}{4} - \frac{3\pi}{4}\cos\frac{3\pi}{4}\right) = 0.4275^-$; $b_5 = \frac{16}{25\pi^2}\left(\sin\frac{5\pi}{4} - \frac{5\pi}{4}\cos\frac{5\pi}{4}\right) = \boxed{0.13421}$

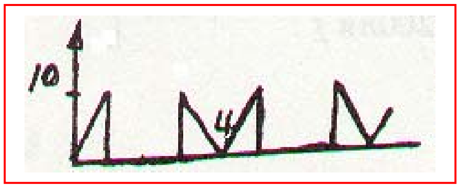
$b_{even} = \boxed{0}$

24.

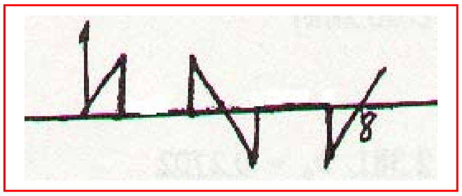
(a) odd, T = 4



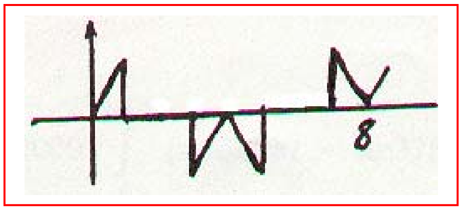
(b) even, T = 4:



(c) odd, $\frac{1}{2}-$wave: $\mathrm{T}=8$



(d) even, $\frac{1}{2}-$wave, $\mathrm{T}=8$:



25.

(a) $v_s = 5 + \frac{20}{\pi}\sum_{1,odd}^{\infty}\frac{1}{n}\sin\frac{2\pi nt}{0.4\pi} \quad \therefore v_{sn} = \frac{20}{n\pi}\sin 5nt,\ \bar{\mathrm{V}}_{sn} = \frac{20}{n\pi}(-j1)$

$$\mathrm{Z}_n = 4 + j5n2 = 4 + j10n,\ \bar{\mathrm{I}}_{fn} = \frac{\bar{\mathrm{V}}_{sn}}{\mathrm{Z}_n} = \frac{-j20}{n\pi(4+j10n)} = -\frac{j5}{1+j2.5n}$$

$$\therefore \bar{\mathrm{I}}_{fn} = -\frac{j5}{n\pi}\frac{1-j2.5n}{1+6.25n^2} = -\frac{12.5+j5}{n\pi(1+6.25n^2)}$$

$$\therefore i_{fn} = -\frac{12.5}{\pi}\frac{1}{1+6.25n^2}\cos 5nt + \frac{5}{n\pi}\frac{1}{1+6.25n^2}\sin 5nt$$

$$\therefore \boxed{i_f = 1.25 + \sum_{1,odd}^{\infty}\frac{1}{1+6.25n^2}\left[-\frac{12.5}{\pi}\cos 5nt + \frac{5}{n\pi}\sin 5nt\right]}$$

(b) $i_n = \mathrm{A}e^{-2t},\ i = i_f + i_n,\ i(0) = 0,\ i_f(0) = 1.25 + \sum_{1,odd}^{\infty}\frac{1}{1+6.25n^2}\left(-\frac{12.5}{\pi}\right)$

$$\therefore i_f(0) = 1.25 - \frac{2}{\pi}\sum_{1,odd}^{\infty}\frac{1}{n^2+0.16} = 1.25 - \frac{2}{\pi}\frac{\pi}{4\times 0.4}\tanh 0.2\pi = 0.55388$$

$$\therefore \mathrm{A} = -0.55388,\ \boxed{i = -0.55388e^{-2t} + 1.25 + \sum_{1,odd}^{\infty}\frac{1}{1+6.25n^2}\left[-\frac{12.5}{\pi}\cos 5nt + \frac{5}{n\pi}\sin 5nt\right]}$$

26.

(a) $0 < t < 0.2\pi : i = 2.5(1 - e^{-2t}) \;\; \therefore i(0.2\pi) = 2.5(1 - e^{-0.4\pi}) = \boxed{1.78848 \text{ A}}$

(b) $0.2\pi < t < 0.4\pi : \;\; i = 1.78848 e^{-2(t-0.2\pi)} \quad \therefore i(0.4\pi) = \boxed{0.50902 \text{ A}}$

(c) $0.4\pi < t < 0.6\pi : i = 2.5 - (2.5 - 0.50902)e^{-2(t-0.4\pi)}, \; i(0.6\pi) = \boxed{1.9335^{-}}$

27.

(a) $$v_s = 5 + \frac{20}{\pi}\sum_{1,odd}^{\infty}\frac{1}{n}\sin 5nt$$

$$v_{sn} = \frac{20}{n\pi}\sin 5nt$$

$$\bar{V}_{sn} = -j\frac{20}{n\pi}$$

$$\bar{Z}_n = 2 + \frac{1}{j5n2} = 2 + \frac{1}{j10n} \quad \therefore \bar{V}_{cn} = \frac{-j20/n\pi}{2+1/j10n}\times\frac{1}{j10n} = \frac{-j20/n\pi}{1+j20n}\times\frac{1-j20n}{1-j20n}$$

$$\therefore \bar{V}_{cn} = \frac{-20n-j1}{1+400n^2}\times\frac{20}{n\pi},\ v_{cn} = \frac{20}{n\pi}\frac{1}{1+400n^2}(-20n\cos 5nt + \sin 5nt)$$

$$\therefore \boxed{v_{cf} = 5 + \frac{20}{\pi}\sum_{1,odd}^{\infty}\frac{1}{1+400n^2}\left(\frac{1}{n}\sin 5nt - 20\cos 5nt\right)}$$

(b) $$\boxed{v_n = \mathrm{A}e^{-t/4}}$$

(c) $$v_c(0) = \mathrm{A} + 5 + \frac{20}{\pi}\sum_{1,odd}^{\infty}\frac{-20}{1+400n^2} = \mathrm{A} + 5 - \frac{1}{\pi}\sum_{1,odd}^{\infty}\frac{1}{n^2+(1/20)^2}$$

$$\sum_{1,odd}^{\infty}\frac{1}{n^2+(1/20)^2} = \frac{\pi}{4(1/20)}\tanh\frac{\pi}{20\times 2} = 5\pi\tanh\frac{\pi}{40} = 1.23117$$

$$\therefore \mathrm{A} = 0 - 5 + \frac{1}{\pi}\times 1.23117 = -4.60811$$

$$\therefore \boxed{v_c(t) = -4.60811e^{-t/4} + 5 + \frac{20}{\pi}\sum_{1,odd}^{\infty}\frac{1}{1+400n^2}\left(\frac{1}{n}\sin 5nt - 20\cos 5nt\right)}$$

28. At the frequency $\omega = 10n\pi$

$$\mathbf{Z}_n = \frac{10\left[10 + j10n\pi\left(5\times10^{-3}\right)\right]}{20 + j10n\pi\left(5\times10^{-3}\right)}\ \Omega \quad \text{and} \quad \mathbf{I}_{Sn} = \frac{8}{\pi n}(-j)$$

Therefore $\quad \mathbf{V}_n = \dfrac{80}{n\pi}(-j)\left[\dfrac{10 + j0.05n\pi}{20 + j0.05n\pi}\right].$

In the time domain, this becomes

$$\boxed{v_1(t) = \sum_{n=1\ (odd)}^{\infty}\left(\frac{40}{n\pi}\right)\frac{\sqrt{1+(0.005n\pi\pi)^2}}{\sqrt{1+(0.0025n\pi\pi)^2}}\cos\left(10n\pi - 90^{\circ} + \tan^{-1}0.005n\pi - \tan^{-1}0.0025n\pi\right)}$$

29. At the frequency $\omega = \text{n}\pi$

$$\mathbf{I}_{Ln} = \frac{10}{20 + jn\pi\left(5\times 10^{-3}\right)}\mathbf{I}_{Sn} \quad \text{and} \quad \mathbf{I}_{Sn} = -j\frac{32}{\left(\pi n\right)^2}\left(-1\right)^{\frac{n-1}{2}}$$

Thus, in the time domain, we can write

$$\boxed{i_L(t) = \sum_{n=1\text{ (odd)}}^{\infty}\left[\frac{320}{\left(\text{n}\pi\right)^2}\left(-1\right)^{\frac{n-1}{2}}\right]\frac{1}{20\sqrt{1+\left(0.00025n\pi\right)^2}}\cos\left(n\pi t - 90^\circ - \tan^{-1}0.00025n\pi\right)}$$

30.

$$c_3 = \frac{10^3}{6}\left[\int_0^{0.001} 100e^{-j3\times2\pi t/6\times10^{-3}} - \int_{0.003}^{0.005} 100e^{-j100\pi t}\right]$$

$$= \frac{10^5}{6}\left[\frac{-1}{j1000\pi}e^{-j1000\pi t}\Big|_0^{0.001} + \frac{1}{j1000\pi}e^{-j1000\pi t}\Big|_{0.003}^{0.005}\right]$$

$$= \frac{100}{j6\pi}\left(e^{-j\pi}+1+e^{-j5\pi}-e^{-j3\pi}\right) = \frac{100}{j6\pi}(1+1-1+1) = \boxed{-j10.610}$$

$$\therefore \boxed{c_{-3} = j10.610;\ |c_3| = 10.610}$$

$$a_3 = \frac{2\times10^3}{6}\left[\int_0^{0.001} 100\cos 100\pi t\ dt - \int_{0.003}^{0.005} 100\cos 1000\pi t\ dt\right]$$

$$= \frac{2\times10^5}{6}\ \frac{1}{1000\pi}\left(\sin\pi - 0 - \sin 5\pi + \sin 3\pi\right) = \boxed{0}$$

$$c_3 = \frac{1}{2}(a_3 - jb_3) = -j\frac{1}{2}b_3 \quad \therefore \boxed{b_3 = 21.22} \text{ and } \sqrt{a_3^2+b_3^2} = \boxed{21.22}$$

31.

(a) $$\mathrm{T} = 5\,ms\; c_m = \frac{1}{0.005}\left[\int_0^{0.001} 10^5 t e^{-j400\pi nt}\,dt + \int_{0.001}^{0.002} 100 e^{-j400\pi nt} dt\right]$$

$$\therefore c_n = 20,000\left[\int_0^{0.001} 1000t\, e^{-j400\pi nt} dt + \int_{0.001}^{0.002} e^{-j400\pi nt} dt\right]$$

$$\therefore c_n = 20,000\left[\frac{e^{-j400\pi nt}}{160n^2\pi^2}(j400\pi nt+1)_0^{0.001} + \frac{1}{-j400\pi n} e^{-j400\pi nt}\Big|_{0.001}^{0.002}\right]$$

(b) $$\therefore c_o = a_o = (50\times 10^{-3} + 100\times 10^{-3})\frac{1}{0.005} = 0.15\times 200 = \boxed{30}$$

$$c_1 = 20,000\left[\frac{1}{160\pi^2} e^{-j0.4\pi}(1+j0.4\pi) - \frac{1}{160\pi^2} - \frac{1}{j400\pi}\left(e^{-j0.8\pi} - e^{-j0.4\pi}\right)\right]$$

$$= \frac{125}{\pi^2}(1\angle -72°)(1.60597\angle 51.488°) - 12.66515 + 15.91548\angle 90°(1\angle -144° - 1\angle -72°)$$

$$= 12.665(1\angle -72°)(1+j1.2566) - 12.665 + j15.915(1\angle -144° - 1\angle -72°)$$

$$= 20.339\angle -20.513° - 12.665 + 18.709\angle -108° = \boxed{24.93\angle -88.61°}$$

$$c_2 = 3.16625\angle -144°(1+j2.5133) - 3.16625 + j7.9575(1\angle -288° - 1\angle -144°)$$

$$= 8.5645\angle -75.697° - 3.16625 + 15.1361\angle 144° = \boxed{13.309\angle 177.43°}$$

32.

Fig. 17-8a: $\mathrm{V}_o = 8\ \mathrm{V},\ \tau = 0.2\mu s,\ f_o = 6000\, pps$

(a) $\mathrm{T} = \dfrac{1}{6000},\ f_o = 6000, \tau = 0.2\ \mu s\ \ \therefore\ f = \dfrac{1}{\tau} = \boxed{5\ \mathrm{MHz}}$

(b) $\boxed{f_o = 6000\ \mathrm{Hz}}$

(c) $6000 \times 3 = 18,000$ (closest) $\quad \therefore |c_3| = \dfrac{8 \times 0.2 \times 10^{-6}}{1/6000} \left| \dfrac{\sin(1/2 \times 3 \times 12,000\pi \times 0.2 \times 10^{-6}}{0.0036\pi} \right|$

$\therefore |c_3| = \boxed{9.5998\ \mathrm{mV}}$

(d) $\dfrac{2 \times 10^6}{6 \times 10^3} = 333.3\ \ \therefore |c_{333}| = \dfrac{8 \times 0.2 \times 10^{-6}}{1/6000} \left| \dfrac{\sin(1/2 \times 333 \times 12,000\pi \times 0.2 \times 10^{-6}}{1/2 \times 333 \times 12,000\pi \times 0.2 \times 10^{-6}} \right| = \boxed{7.270\ \mathrm{mV}}$

(e) $\beta = 1/\tau = \boxed{5\ \mathrm{MHz}}$

(f) $2 < \omega < 2.2$ Mrad/s $\quad \therefore \dfrac{2000}{2\pi} < f < \dfrac{2200}{2\pi}$ kHz or $318.3 < f < 350.1$ kHz

$f_o = 6$ kHz $\quad \therefore f = 6 \times 53 = 318;\ 324, 330, 336, 342, 348\,\mathrm{kHz} \quad \therefore \boxed{n = 5}$

(g) $|c_{227}| = \dfrac{8 \times 0.2 \times 10^{-6}}{1/6000} \left| \dfrac{\sin(1/2 \times 227 \times 12,000\pi \times 0.2 \times 10^{-6}}{('')} \right| = \boxed{8.470\ \mathrm{mV}}$

$f = 227 \times 6 = \boxed{1362\ \mathrm{kHz}}$

33.

$\mathrm{T} = 5\,ms;\ \bar{c}_o = 1,\ \bar{c}_1 = 0.2 - j0.2,\ \bar{c}_2 = 0.5 + j0.25,\ \bar{c}_3 = -1 - j2,\ \bar{c}_n = 0,\ |n| \ge 4$

(a) $a_n = -jb_n = 2\bar{c}_n \ \therefore a_o = \bar{c}_o = 1,\ a_1 - jb_1 = 0.4 - jb_1 = 0.4 - j0.4,\ a_2 - jb_2 = 1 + j0.5,\ a_3 - jb_3 = -2$

$\therefore v(t) = \boxed{1 + 0.4\cos 400\pi t + \cos 800\pi t - 2\cos 1200\pi t + 0.4\sin 400\pi t - 0.5\sin 800\pi t + 4\sin 1200\pi t}$

(b) $v(1\,ms) = 1 + 0.4\cos 72° + \cos 144° - 2\cos 216° + 0.4\sin 72° - 0.5\sin 144° + 4\sin 216°$

$\therefore \boxed{v(1\,ms) = -0.332\mathrm{V}}$

34.

(a) $\mathrm{T} = 5\ \mu s \ \therefore \overline{c}_n = \frac{10^6}{5} \times 2 \int_{0.4\times10^{-6}}^{0.6\times10^{-6}} 1\cos 2\pi n \frac{t}{5\times10^{-6}} dt$

$\therefore \overline{c}_n = 4\times10^5 \frac{5\times10^{-6}}{2\pi n}\left(\sin 43.2°n - \sin 28.8°n\right)$

$\therefore \boxed{\overline{c}_n = \frac{1}{n\pi}\left(\sin 43.2°n - \sin 28.8°n\right)}$

(b) $\overline{c}_4 = \frac{1}{4\pi}(\sin 172.8° - \sin 115.2°) = \boxed{-0.06203}$

(c) $\overline{c}_o = a_o = \frac{0.2\times10^{-6} + 0.2\times10^{-6}}{5\times10^{-6}} = \boxed{0.08}$

(d) a little testing shows $|c_o|$ is max $\therefore |\overline{c}_{\max}| = \boxed{0.08}$

(e) $0.01\times0.08 = 0.8\times10^{-3} \ \therefore \left|\frac{1}{n\pi}\left(\sin 43.2°n - \sin 28.8°n\right)\right| \le 0.8\times10^{-3}$

$\therefore \left|\frac{125}{n\pi}\left(\sin 43.2°n - \sin 28.8°n\right)\right| \le 1$

$\boxed{\text{ok for } n > 740}$

(f) $\beta = 740 f_o = \frac{740\times10^6}{5} = \boxed{148 \text{ MHz}}$

35.

$\text{T} = 1/16,\ \omega_o = 32\pi$

(a) $$\bar{c}_3 = 16\int_0^{1/96} 40e^{-j96\pi t}\,dt - \frac{16\times 40}{-j96\pi}\,e^{-j96\pi}\Big|_0^{1/96}$$

$$\therefore \bar{c}_3 = j\frac{20}{3\pi}(e^{-j\pi}-1) = -j\frac{40}{3\pi} = \boxed{-j4.244\text{ V}}$$

(b) Near harmonics are $2f_o = 32$ Hz, $3f_o = 48$ Hz

Only 32 and 48 Hz pass filter $a_n - jb_n = 2\bar{c}_n$

$a_3 - jb_3 = 2\bar{c}_3 = -j8.488 \quad \therefore a_3 = 0,\ b_3 = 8.488$ V

$$\bar{\text{I}}_3 = \frac{8.488}{5 + j0.01\times 96\pi} = 1.4536\ \angle -31.10^\circ\text{ A};\ \text{P}_3 = \frac{1}{2}\times 1.4536^2\times 5 = 5.283\text{ W}$$

$$\bar{c}_2 = \frac{1}{1/16}\int_0^{1/96} 40e^{-j64\pi t}\,dt = \frac{640}{-j64\pi}(e^{-j64\pi/96}-1) = 2.7566 - j4.7746\text{ V}$$

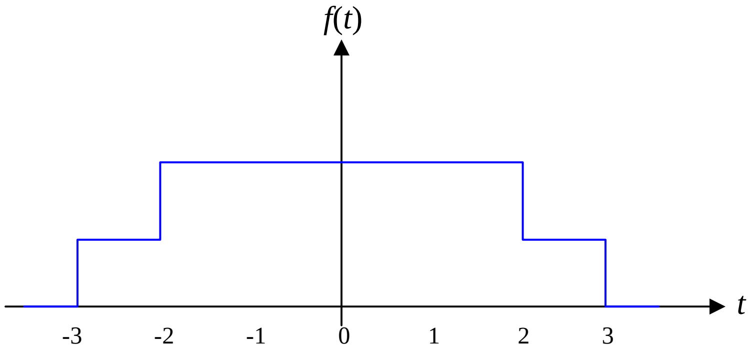
$$a_2 - b_2 = 2\bar{c}_2 = 5.5132 - j9.5492 = 11.026\ \angle -60^\circ$$

$$\therefore \bar{\text{I}}_2 = \frac{11.026\angle -60^\circ}{5 + j0.01\times 64\pi} = 2.046\angle -65.39^\circ\text{ A}$$

$$\therefore \text{P}_2 = \frac{1}{2}\times 2.046^2\times 5 = 10.465\text{ W} \quad \therefore \text{P}_{tot} = \boxed{15.748\text{ W}}$$

36. $f(t) = 5[u(t+3) + u(t+2) - u(t-2) - u(t-3)]$

(a)



(b) $F(j\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t}\, dt$

$$\therefore F(j\omega) = \int_{-3}^{-2} 5e^{-j\omega t}\, dt + \int_{-2}^{2} 10e^{-j\omega t}\, dt + \int_{2}^{3} 5e^{-j\omega t}\, dt$$

$$\therefore F(j\omega) = \frac{5}{-j\omega}(e^{j2\omega} - e^{j3\omega}) + \frac{10}{-j\omega}(e^{-j2\omega} - e^{j2\omega}) + \frac{5}{-j\omega}(e^{-j3\omega} - e^{-j2\omega})$$

$$= \frac{5}{-j\omega}(-e^{j3\omega} + e^{-j3\omega}) + \frac{5}{-j\omega}(e^{j2\omega} - e^{-j2\omega}) + \frac{10}{-j\omega}(-e^{j2\omega} + e^{-j2\omega})$$

$$= \frac{5}{-j\omega}(-j2)\sin 3\omega + \frac{5}{-j\omega}(j2)\sin 2\omega + \frac{10}{-j\omega}(-j2)\sin 2\omega$$

$$\therefore \boxed{F(j\omega) = \frac{10}{\omega}\sin 3\omega - \frac{10}{\omega}\sin 2\omega + \frac{20}{\omega}\sin 2\omega = \frac{10}{\omega}(\sin 3\omega + \sin 2\omega)}$$

37.

(a)

$$f(t) = e^{-at}\, u(t),\ a > 0 \quad \therefore \mathrm{F}(j\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t}\, dt = \int_{0}^{\infty} e^{-at} e^{-j\omega t}\, dt$$

$$\therefore \mathrm{F}(j\omega) = \frac{-1}{a + j\omega} e^{-(a+j\omega)t} \Big|_{0}^{\infty} = \boxed{\frac{1}{a + j\omega}}$$

(b)

$$f(t) = e^{at_6} e^{-at}\, u(t - t_o),\ a > 0 \quad \therefore \mathrm{F}(j\omega) = e^{at_o} \int_{t_o}^{\infty} e^{-(a+j\omega)t}\, dt$$

$$\therefore \mathrm{F}(j\omega) = e^{at_o} \frac{-1}{a + j\omega} e^{-(a+j\omega)\, t} \Big|_{t_o}^{\infty} = e^{at_o} \frac{-1}{a + j\omega} \left[-e^{-(a+j\omega)t_o} \right] = \boxed{\frac{1}{a + j\omega} e^{-j\omega t_o}}$$

(c)

$$f(t) = te^{-at} u(t),\ a > 0 \quad \therefore \mathrm{F}(j\omega) = \int_{0}^{\infty} te^{-(a+j\omega)t} dt$$

$$\therefore \mathrm{F}(j\omega) = \frac{e^{-(a+j\omega)t}}{(a + j\omega)^2} \left[-(a + j\omega)t - 1 \right]_0^{\infty} = 0 - \frac{1}{(a + j\omega)^2} [-1] = \boxed{\frac{1}{(a + j\omega)^2}}$$

38.

$$-4 < t < 0: \ f(t) = 2.5(t+4); \ 0 < t < 4: \ f(t) = 2.5(4-t)$$

$$\therefore \mathrm{F}(j\omega) = \int_{-4}^{0} 2.5(t+4)\, e^{-j\omega t}\, dt + \int_{0}^{4} 2.5(4-5)e^{-j\omega t}\, dt$$

$$\ln 1^{st}, \ \text{let } t = \tau \quad \therefore \mathrm{I}_1 = \int_{4}^{0} 2.5(4-\tau)e^{j\omega\tau}(-d\tau)$$

$$\therefore \mathrm{I}_1 = \int_{0}^{4} 2.5(4-\tau)e^{j\omega\tau} d\tau \ \ \therefore \mathrm{F}(j\omega) = 2.5\int_{0}^{4} (4-t)(e^{j\omega t} + e^{-j\omega t})\, dt$$

$$\therefore \mathrm{F}(j\omega) = 5\int_{0}^{4} (4-t)\cos\,\omega t\ dt = 20 \times \frac{1}{\omega}\sin\omega t\Big|_{0}^{4} - 5\int_{0}^{4} \cos\,\omega t\ dt$$

$$\therefore \mathrm{F}(j\omega) = \frac{20}{\omega}\sin 4\omega - \frac{5}{\omega^2}(\cos\omega t + \omega t\ \sin\ \omega t)_0^4$$

$$= \frac{20}{\omega}\sin 4\omega - \frac{5}{\omega^2}\ (\cos 4\omega - 1) - \frac{5}{\omega^2}\ 4\omega\sin 4\omega = \frac{5}{\omega^2}(1-\cos 4\omega)$$

$$\text{or, } \mathrm{F}(j\omega) = \frac{2\times 5}{\omega^2}\ \sin^2 2\omega = \boxed{10\left(\frac{\sin 2\omega}{\omega}\right)^2}$$

39.

$$f(t) = 5\sin t, -\pi < t < \pi \quad \therefore \mathrm{F}(j\omega) = \int_{-\pi}^{\pi} 5\sin t \; e^{-j\omega t}\, dt$$

$$\therefore \mathrm{F}(j\omega) = \frac{5}{j2}\int_{-\pi}^{\pi} (e^{jt} - e^{-jt})\; e^{-j\omega t}\, dt$$

$$= \frac{5}{j2}\int_{-\pi}^{\pi} [e^{jt(1-\omega)} - e^{-jt(1+\omega)}]\, dt$$

$$\mathrm{F}(j\omega) = \frac{5}{j2}\left[\frac{1}{j(1-\omega)}(e^{j\pi(1-\omega)} - e^{-j\pi(1-\omega)}) - \frac{1}{-j(1+\omega)}(e^{-j\pi(1+\omega)} - e^{j\pi(1+\omega)})\right]$$

$$= \frac{-2.5}{1-\omega}(-e^{-j\pi\omega} + e^{j\pi\omega}) - \frac{2.5}{1+\omega}(-e^{-j\pi\omega} + e^{j\pi\omega})$$

$$= \frac{-2.5}{1-\omega}(j2\sin\pi\omega) - \frac{2.5}{1+\omega}(j2\sin\pi\omega) = j5\sin\pi\omega\left(-\frac{1}{1-\omega} - \frac{1}{1+\omega}\right)$$

$$= j5\sin\pi\omega(-1)\left(\frac{1+\omega+1-\omega}{1-\omega^2}\right) = -\frac{j10\sin\pi\omega}{1-\omega^2} = \boxed{\frac{j10\sin\pi\omega}{\omega^2-1}}$$

40.

$$f(t) = 8\cos t[u(t+0.5\pi) - u(t-0.5\pi)]$$

$$\therefore \mathrm{F}(j\omega) = \int_{-\pi/2}^{\pi/2} 8\cos t e^{-j\omega t}\, dt = 4\int_{-\pi/2}^{\pi/2} (e^{jt} + e^{-jt}) e^{-j\omega t}\, dt$$

$$= 4\int_{-\pi/2}^{\pi/2} \left[e^{jt(1-\omega)} + e^{-jt(1+\omega)}\right] dt$$

$$= 4\left\{\frac{1}{j(1-\omega)} e^{jt}\ e^{-j\omega t}\Big|_{-\pi/2}^{\pi/2} - \frac{1}{j(1+\omega)} e^{-jt} e^{-j\omega t}\Big|_{-\pi/2}^{\pi/2}\right\}$$

$$= 4\left\{\frac{1}{j(1-\omega)}\left[je^{-j\pi\omega/2} - (-j)e^{j\pi\omega/2}\right] - \frac{1}{j(1+\omega)}\left[-je^{-j\pi\omega/2} - je^{j\pi\omega/2}\right]\right\}$$

$$= 4\left\{\frac{1}{1-\omega} \times 2\cos\frac{\pi\omega}{2} + \frac{1}{1+\omega} \times 2\cos\frac{\pi\omega}{2}\right\} = 8\cos\frac{\pi\omega}{2}\left(\frac{1}{1-\omega} + \frac{1}{1+\omega}\right)$$

$$= 8\cos\frac{\pi\omega}{2}\,\frac{2}{1-\omega^2} = 16\frac{\cos \pi\omega/2}{1-\omega^2}$$

(a) $\omega = 0 \quad \therefore \mathrm{F}(j0) = \boxed{16}$

(b) $\omega = 0.8,\ \mathrm{F}(j0.8) = \dfrac{16\cos 72°}{0.36} = \boxed{13.734}$

(c) $\omega = 3.1,\ \mathrm{F}(j3.1) = \dfrac{16\cos(3.1\times 90°)}{1-3.12} = \boxed{-0.2907}$

41.

(a) $$\mathrm{F}(j\omega) = 4\left[u(\omega+2) - \omega(\omega-2)\right] \;\therefore\; f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{j\omega t}\;\mathrm{F}(j\omega)d\omega$$

$$\therefore\; f(t) = \frac{4}{2\pi}\int_{-2}^{2} e^{j\omega t}\,d\omega = \frac{2}{\pi}\,\frac{1}{jt}e^{j\omega t}\Big|_{-2}^{2} = \frac{2}{j\pi t}\left(e^{j2t} - e^{-j2t}\right)$$

$$\therefore\; f(t) = \frac{2}{2\pi t}\,j2\sin 2t = \frac{4}{\pi t}\sin 2t \;\therefore\; f(0.8) = \frac{5}{\pi}\sin 1.6^{rad} = \boxed{1.5909}$$

(b) $$\mathrm{F}(j\omega) = 4e^{-2|\omega|} \;\therefore\; f(t) = \frac{4}{2\pi}\int_{-\infty}^{\infty} e^{-2|\omega| + j\omega t}\,d\omega$$

$$\therefore\; f(t) = \frac{2}{\pi}\int_{-\infty}^{0} e^{(2+jt)\omega}\,d\omega + \frac{2}{\pi}\int_{0}^{\infty} e^{(-2+j\omega)t}\,d\omega$$

$$= \frac{2}{\pi}\left[\frac{1}{2+jt}(1-0) + \frac{1}{-2+jt}(0-1)\right] = \frac{2}{\pi}\left(\frac{1}{2+jt} + \frac{1}{2-jt}\right) = \frac{2}{\pi}\,\frac{4}{4+t^2}$$

$$\therefore\; f(t) = \frac{8}{\pi(4+t^2)} \;\therefore\; f(0.8) = \frac{8}{\pi\times 4.64} = \boxed{0.5488}$$

(c) $$\mathrm{F}(j\omega) = 4\cos\pi\omega\left[u(\omega+0.5) - u(\omega-0.5)\right]$$

$$\therefore\; f(t) = \frac{4}{2\pi}\int_{-0.5}^{0.5}\cos\pi\omega\times e^{j\omega t}\,d\omega = \frac{2}{\pi}\int_{-0.5}^{0.5}\frac{1}{2}\left(e^{j\pi\omega} + e^{-j\pi\omega}\right)e^{j\omega t}\,d\omega$$

$$= \frac{1}{\pi}\int_{-0.5}^{0.5}\left[e^{(j\pi+jt)\omega} + e^{(-j0.5\pi - j0.5t)\omega}\right]d\omega$$

$$= \frac{1}{\pi}\left[\frac{1}{j(\pi+t)}\left(e^{j0.5\pi + j0.5t} - e^{-j0.5\pi - j0.5t}\right) + \frac{1}{j(-\pi+t)}\left(e^{-j0.5\pi + j0.5t} - e^{j0.5\pi - j0.5t}\right)\right]$$

$$= \frac{1}{\pi}\left[\frac{1}{j(\pi+t)}\left(je^{j0.5t} + je^{-j0.5t}\right) + \frac{1}{j(-\pi+t)}\left(-je^{j0.5t} - je^{-j0.5t}\right)\right]$$

$$= \frac{1}{\pi}\left[\frac{1}{\pi+t}2\cos 0.5t - \frac{1}{-\pi+t}2\cos 0.5t\right] = \frac{2\cos 0.5t}{\pi}\left(\frac{1}{\pi+t} - \frac{1}{-\pi+t}\right)$$

$$= 2\cos 0.5t\left(\frac{-2}{t^2-\pi^2}\right) = \frac{4}{\pi^2 - t^2}\cos 0.5t \;\therefore\; f(0.8) = \boxed{0.3992}$$

42. $v(t) = 20e^{1.5t}\ u(-t-2)$ V

(a) $$\mathrm{F}_v(j\omega) = \int_{-\infty}^{\infty} 20e^{1.5t}\ u(-t-2)e^{-j\omega t}\ dt = \int_{-\infty}^{-2} 20e^{1.5t - j\omega t}\ dt$$

$$= \frac{20}{1.5 - j\omega} e^{(1.5 - j\omega)t}\Big|_{-\infty}^{-2} = \frac{20}{1.5 - j\omega} e^{-3 + j2\omega}\ \therefore \mathrm{F}_v(j0) = \frac{20}{1.5}\ e^{-3} = \boxed{0.6638}$$

(b) $$\mathrm{F}_v(j\omega) = \mathrm{A}_v(\omega) + \mathrm{B}_v(\omega) = \frac{20}{1.5 - j\omega}\ e^{-3}e^{j2\omega}$$

$$\therefore \mathrm{F}_v(j2) = \frac{20}{1.5 - j2} e^{-3}\ e^{j4} = 0.39830\ \angle 282.31° = 0.08494 - j0.38913$$

$$\therefore \mathrm{A}_v(2) = \boxed{0.08494}$$

(c) $$\mathrm{B}_v(2) = \boxed{-0.3891}$$

(d) $$|\mathrm{F}_v(j2)| = \boxed{0.3983}$$

(e) $$\boxed{\phi_V(j2)\ =\ 282.3^{\circ}\ \text{ or }\ -77.69^{\circ}}$$

43. $|\mathrm{I}(j\omega)| = 3\cos 10\omega\,[u(\omega + 0.05\pi) - u(\omega - 0.05\pi)]$

(a)
$$W = 4 \times \frac{1}{2\pi}\int_{-\infty}^{\infty} |\mathrm{I}(j\omega)|^2\, d\omega = \frac{2}{\pi}\int_{-0.05\pi}^{0.05\pi} 9\cos^2 10\omega\, d\omega$$
$$= \frac{18}{\pi}\int_{-\pi/20}^{\pi/20}\left(\frac{1}{2} + \frac{1}{2}\cos 20\,\omega\right) d\omega = \frac{9}{\pi}\times 0.1\pi + \frac{9}{\pi}\,\frac{1}{20}\sin 20\omega\Big|_{-\pi/20}^{\pi/20} \boxed{= 0.9 \text{ J}}$$

(b)
$$\frac{9}{\pi}\int_{-\omega_x}^{\omega_x}(1 + \cos 20\omega)\, d\omega = 0.45 = \frac{9}{\pi}\left[2\omega_x + \frac{1}{20}\times 2\sin 20\,\omega_x\right]$$
$$\therefore 0.05\pi = 2\omega_x + 0.1\sin 20\omega_x, \boxed{\omega_x = 0.04159 \text{ rad/s}}$$

44. $f(t) = 10te^{-4t}\, u(t)$

(a) $$W_{1\Omega} = \int_0^\infty f^2(t)\,dt = \int_0^\infty 100t^2 e^{-8t}\,dt = 100 \times \frac{e^{-8t}}{(-512)}(64t^2 + 16t + 2)\Big|_0^\infty$$

$$= \frac{100}{512} \times 2 = \boxed{0.3906 \text{ J}}$$

(b) $$\mathrm{F}(j\omega) = \mathcal{F}\{10te^{-4t}u(t)\} = 10\int_0^\infty t\ e^{-(4+j\omega)t}dt = \frac{10e^{-(4+j\omega)t}}{(4+j\omega)^2}[-(4+j\omega)t - 1\Big|_0^\infty$$

$$= \frac{10}{(4+j\omega)^2} \quad \therefore \boxed{|\mathrm{F}(j\omega)| = \frac{10}{\omega^2 + 16}}$$

(c) $$|F(j\omega)|^2 = \frac{100}{\left(\omega^2 + 16\right)^2}$$

$$\boxed{|F(j\omega)|^2_{\omega=0} = 390.6 \text{ mJ/Hz}, \ |F(j\omega)|^2_{\omega=4} = 97.66 \text{ mJ/Hz}}$$

45. $v(t) = 8e^{-2|t|}$ V

(a) $$W_{1\Omega} = \int_{-\infty}^{\infty} v^2(t)\, dt = 2 \times 64 \int_{0}^{\infty} e^{-4t}\, dt = \boxed{32 \text{ J}}$$

(b) $$F_v(j\omega) = \int_{-\infty}^{\infty} e^{-j\omega t}\, v(t)\, dt = 8 \int_{-\infty}^{\infty} e^{-2|t|} e^{-j\omega t}\, dt$$

$$\therefore F_v(j\omega) = 8 \int_{-\infty}^{0} e^{(2-j\omega)t}\, dt + 8\int_{0}^{\infty} e^{-(2+j\omega)t}\, dt$$

$$= \frac{8}{2-j\omega} e^{(2-j\omega)t}\Big|_{-\infty}^{0} - \frac{8}{2+j\omega} e^{-(2+j\omega)t}\Big|_{0}^{\infty} = \frac{8}{2-j\omega} + \frac{8}{2+j\omega} = \boxed{\frac{32}{4+\omega^2} = |F_v(j\omega)|}$$

(c) $$0.9 \times 32 = \frac{1}{2\pi} \int_{-\omega_1}^{\omega_1} \frac{32^2}{(\omega^2+4)^2}\, d\omega = \frac{32^2}{2\pi}\left[\frac{\omega}{8(\omega_1^2+4)} + \frac{1}{16}\tan^{-1}\frac{\omega_1}{2}\right]$$

$$\therefore 0.9 = \frac{16}{\pi} \times 2\left[\frac{\omega_1}{8(\omega_1^2+4)} + \frac{1}{16}\,\frac{\omega_1}{2}\right] = \frac{2}{\pi}\left[\frac{2\omega_1}{\omega_1^2+4} + \tan^{-1}\frac{\omega_1}{2}\right]$$

$$\therefore 0.45\pi = \frac{2\omega_1}{\omega_1^2+4} + \tan^{-1}\frac{\omega_1}{2} \quad \therefore \boxed{\omega_1 = 2.7174 \text{ rad/s}} \text{ (by SOLVE)}$$

46.

(a) Prove: $\mathcal{F}\{f(t-t_o)\} = e^{-j\omega t_o}\mathcal{F}\{f(t)\} = \int_{-\infty}^{\infty} f(t-t_o)e^{-j\omega t}\,dt$ Let $t - t_o = \tau$

$\therefore \mathcal{F}\{f(t-t_o)\} = \int_{-\infty}^{\infty} f(\tau)e^{-j\omega\tau}e^{-j\omega t_o}\,dt = e^{-j\omega t_o}\mathcal{F}\{f(t)\}$

(b) Prove: $\mathcal{F}\{f(t)\} = j\omega\mathcal{F}\{f(t)\} = \int_{-\infty}^{\infty} e^{-j\omega t}\frac{df}{dt}\,dt$ Let $u = e^{-j\omega t}$, $du = -j\omega e^{-j\omega t}$,

$dv = df$, $v = f$ $\therefore \mathcal{F}\{f(t)\} = f(t)e^{-j\omega t}\Big|_{-\infty}^{\infty} + \int_{-\infty}^{\infty} j\omega f(t)e^{-j\omega t}\,dt$

We assume $f(\pm\infty) = 0$ $\therefore \mathcal{F}\{f(t)\} = j\omega\mathcal{F}\{f(t)\}$

(c) Prove: $\mathcal{F}\{f(kt)\} = \frac{1}{|k|}\mathrm{F}\left(\frac{j\omega}{k}\right) = \int_{-\infty}^{\infty} f(kt)e^{-j\omega t}\,dt$ Let $\tau = kt,\ k > 0$

$\therefore \mathcal{F}\{f(kt)\} = \int_{-\infty}^{\infty} f(\tau)e^{-j\omega\tau/k}\frac{1}{k}\,d\tau = \frac{1}{k}\mathrm{F}\left(\frac{j\omega}{k}\right)$

If $k < 0$, limits are interchanged and we get: $-\frac{1}{k}\mathrm{F}\left(\frac{j\omega}{k}\right)$

$\therefore \mathcal{F}\{f(kt)\} = \frac{1}{|k|}\mathrm{F}\left(\frac{j\omega}{k}\right)$

(d) Prove: $\mathcal{F}\{f(-t)\} = \mathrm{F}(-j\omega)$ Let $k = 1$ in (c) above

(e) Prove: $\mathcal{F}\{tf(t)\} = j\frac{d}{d\omega}\mathrm{F}(j\omega)$ Now, $\mathrm{F}(j\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}\,dt$

$\therefore \frac{d\mathrm{F}(j\omega)}{d\omega} = \int_{-\infty}^{\infty} f(t)(-jt)e^{-j\omega t}\,dt = -j\,\mathcal{F}\{tf(t)\}$ $\therefore \mathcal{F}\{tf(f)\} = j\omega\mathcal{F}f(t)\}$

47.

(a) $f(t) = 4[\mathrm{sgn}(t)\delta(t-1)] \quad \therefore \mathcal{F}\{4[\mathrm{sgn}(t)\delta(t-1)] = \mathcal{F}\{4\,\mathrm{sgn}(1)\ \delta(t-1)\} = \mathcal{F}\{4\delta(t-1)\} = \boxed{4e^{-j\omega}}$

(b) $f(t) = 4[\mathrm{sgn}(t-1)\ \ \delta(t)] \quad \therefore \mathcal{F}\{4\,\mathrm{sgn}(-1)\delta(t)\} = \mathcal{F}\{-4\delta(t)\} = \boxed{-4}$

(c) $f(t) = 4\sin(10t-30°) \quad \therefore \mathcal{F}\{4\sin(10t-30°) = \mathcal{F}\left\{\frac{4}{j2}\left[e^{j(10t-30°)} - e^{-j(10t-30°)}\right]\right\} =$

$$\mathcal{F}\{-j2e^{-j30°}e^{j10t} + j2e^{j30°}e^{-j10t}\} = -j2e^{-j\pi/6}\,2\pi\delta(\omega-10) + j2e^{j\pi/6}\,2\pi\delta(\omega+10)$$

$$\boxed{= -j4\pi\,[e^{-j\pi/6}\delta(\omega-10) - e^{j\pi/6}\delta(\omega+10)]}$$

48.

(a) $f(t) = \text{A}\cos(\omega_o t + \phi) \quad \therefore \text{F}(j\omega) = \mathcal{F}\{\text{A}\cos\phi\cos\omega_o t - \text{A}\sin\phi\sin\omega_o t\} =$

$$\text{A}\cos\phi\{\pi[\delta(\omega+\omega_o)+\delta(\omega-\omega_o)]\} - \text{A}\sin\phi\left\{\frac{\pi}{j}[\delta(\omega-\omega_o)-\delta(\omega+\omega_o)]\right\} =$$

$$\pi\text{A}\{\cos\phi[\delta(\omega+\omega_o)+\delta(\omega-\omega_o)] + j\sin\phi[\delta(\omega-\omega_o)-\delta(\omega+\omega_o)]\}$$

$$\therefore \boxed{\text{F}(j\omega) = \pi\text{A}[e^{j\phi}\,\delta(\omega-\omega_o) + e^{-j\phi}\,\delta(\omega+\omega_o)]}$$

(b) $f(t) = 3\,\text{sgn}(t-2) - 2\delta(t) - u(t-1) \quad \therefore \text{F}(j\omega) = e^{-j2\omega} \times 3 \times \frac{2}{j\omega} - 2 - e^{-j\omega}\left[\pi\delta(\omega) + \frac{1}{j\omega}\right]$

$$\therefore \boxed{\text{F}(j\omega) = -j\frac{6}{\omega}e^{-j2\omega} - 2 - e^{-j\omega}\left[\pi\delta(\omega) - j\frac{1}{\omega}\right]}$$

(c) $f(t) = \sinh kt\ u(t) \quad \therefore \text{F}(j\omega) = \mathcal{F}\left\{\frac{1}{2}[e^{kt} - e^{-kt}]u(t)\right\}$

$$\therefore \boxed{\text{F}(j\omega) = \frac{1}{2}\,\frac{1}{-k+j\omega} - \frac{1}{2}\,\frac{1}{k+j\omega} = \frac{k+j\omega+k-j\omega}{2(-k^2-\omega^2)} = \frac{-k}{\omega^2+k^2}}$$

49.

(a) $$\mathrm{F}(j\omega) = 3u(\omega+3) - 3u(\omega-1) \;\therefore\; f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} [3u(\omega+3) - 3u(\omega-1)]\, e^{j\omega t}\, d\omega$$

$$\therefore\; f(t) = \frac{3}{2\pi}\int_{-3}^{1} e^{j\omega t}\, dt = \frac{3}{2\pi}\,\frac{1}{jt}\, e^{j\omega t}\Big|_{-3}^{1} = \frac{3}{j2\pi t}(e^{+jt} - e^{-j3t})$$

$$\therefore\; f(5) = -j\frac{3}{10\pi}\,(1\angle 5^{rad} - 1\angle -15^{rad}) \boxed{= 0.10390\ \angle -106.48°}$$

(b) $$\mathrm{F}(j\omega) = 3u(-3-\omega) + 3u(\omega-1) \rightarrow$$

$$\therefore \mathrm{F}(j\omega) = 3 - \mathrm{F}_a(j\omega)$$

$$f(t) = 3\delta(t) - \frac{3}{j2\pi t}(e^{jt} - e^{-j3t}) \;\;\therefore\; f(5) = 0 - 0.10390\ \angle -106.48°$$

so $\boxed{f(5) = 0.1039\angle 73.52^{\circ}}$

(c) $$\mathrm{F}(j\omega) = 2\delta(\omega) + 3u(-3-\omega) + 3u(\omega-1) \;\; \text{Now, } \mathcal{F}\{2\delta(\omega)\} = \frac{2}{2\pi} = \frac{1}{\pi}$$

$$\therefore\; f(t) = \frac{1}{\pi} + \left[-\frac{3}{j2\pi t}(e^{jt} - e^{-j3t})\right] \;\therefore\; f(5) = \frac{1}{\pi} - 0.10390\ \angle -106.48° = \boxed{0.3618\ \angle 15.985^{+\circ}}$$

50.

(a) $$F(j\omega) = \frac{3}{1+j\omega} + \frac{3}{j\omega} + 3 + 3\delta(\omega - 1)$$

$$\therefore \boxed{f(t) = 3e^{-t}u(t) + 1.5\,\text{sgn}(t) + 3\delta(t) + \frac{1.5}{\pi}e^{jt}}$$

(b) $$F(j\omega) = \frac{1}{\omega}5\sin 4\omega = 8\frac{\sin \omega 8/2}{\omega 8/2} \times 2.5$$

$$\therefore \boxed{f(t) = 2.5[u(t+4) - u(t-4)]}$$

(c) $$F(j\omega) = \frac{6(3+j\omega)}{(3+j\omega)^2 + 4} = \frac{6(3+j\omega)}{(3+j\omega)^2 + 2^2} \quad \therefore \boxed{f(t) = 3^{-3t}\cos 2t\,u(t)}$$

51.

$T = 4$, periodic; find exp′l form

$$\therefore c_n = \frac{1}{4}\int_{-1}^{1} 10te^{-jn\pi t/2}\,dt$$

$$\therefore c_n = 2.5\left[e^{-jn\pi t/2}\left(\frac{t}{-jn\pi/2} - \frac{1}{-n^2\pi^2/4}\right)\right]_{-1}^{1}$$

$$\therefore c_n = 2.5\left[e^{-jn\pi/2}\left(\frac{1}{-jn\pi 2} + \frac{1}{n^2\pi^2/4}\right) - e^{jn\pi/2}\left(\frac{1}{jn\pi/2} + \frac{1}{n^2\pi^2/4}\right)\right]$$

$$= 2.5\left[\frac{1}{jn\pi/2}(-e^{-jn\pi/2} - e^{jn\pi/2}) + \frac{4}{n^2\pi^2}(e^{-jn\pi/2} - e^{jn\pi/2})\right]$$

$$= \frac{j5}{n\pi} \times 2\cos\frac{n\pi}{2} + \frac{10}{n^2\pi^2}\left(-j2\sin\frac{n\pi}{2}\right)$$

$$\therefore f(t) = \sum_{-\infty}^{\infty}\left[\frac{j10}{n\pi}\cos\frac{n\pi}{2} - j\frac{20}{n^2\pi^2}\sin\frac{n\pi}{2}\right]e^{jn\pi t/2}$$

$$\boxed{\therefore \mathrm{F}(j\omega) = \sum_{-\infty}^{\infty}\left[\frac{j10}{n\pi}\cos\frac{n\pi}{2} - j\frac{20}{n^2\pi^2}\sin\frac{n\pi}{2}\right]2\pi\delta\left(\omega - \frac{n\pi}{2}\right)}$$

53.

$$\mathrm{F}(j\omega) = 20\sum_{-\infty}^{\infty}\frac{1}{|n|!+1}\delta(\omega-20n)$$

$$= 20\left[\frac{1}{1+1}\delta(\omega)+\frac{1}{1+1}\delta(\omega+20)+\frac{1}{1+1}\delta(\omega-20)+\frac{1}{2+1}\delta(\omega+40)+\frac{1}{3}\delta(\omega-40)\right.$$

$$\left.+\frac{1}{7}\delta(\omega+60)+\frac{1}{7}\delta(\omega-60)+\ldots\right]$$

$$=10\delta(\omega)+\frac{20}{2\pi}[\pi\delta(\omega+20)+\pi\delta(\omega-20)]+\frac{20}{3\pi}[\pi\delta(\omega+40)+\pi\delta(\omega-40)]+$$

$$\frac{20}{7\pi}[\pi\delta(\omega+60)+\pi\delta(\omega-60)+\frac{20}{25\pi}[\pi\delta(\omega+80)+\pi\delta(\omega-80)]+\ldots$$

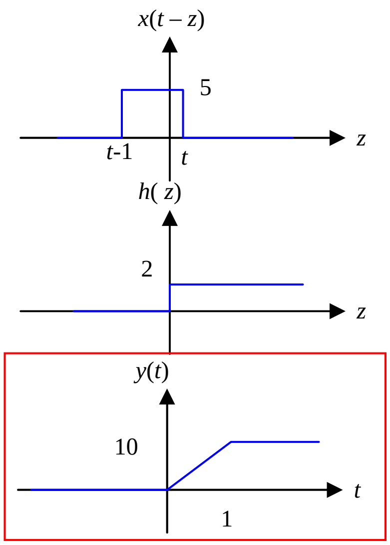
$$\therefore\ f(t)=\frac{10}{2\pi}+\frac{20}{2\pi}\cos 20t+\frac{20}{3\pi}\cos 40t+\frac{20}{7\pi}\cos 60t+\frac{20}{25\pi}\cos 80t+\ldots$$

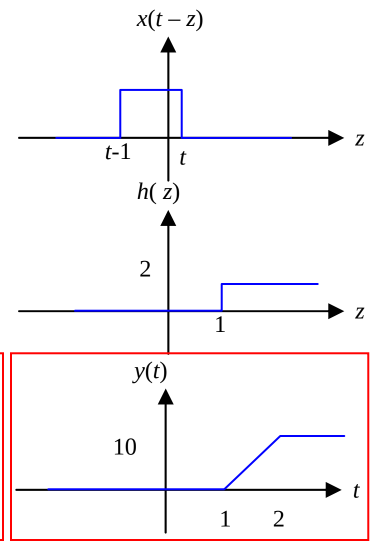
$$=\frac{20}{\pi}\left[0.25+\frac{1}{2}\cos 20t+\frac{1}{3}\cos 40t+\frac{1}{7}\cos 60t+\frac{1}{25}\cos 80t+\ldots\right.$$

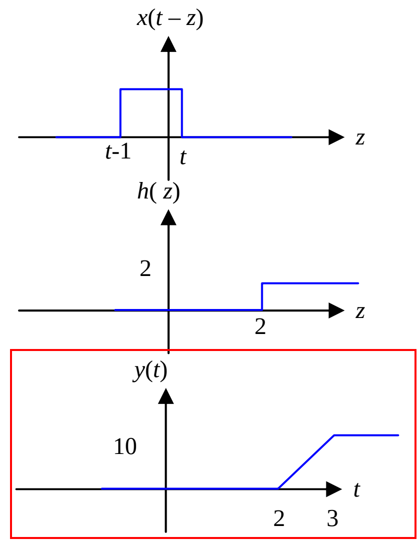
$$\therefore\ f(0.05)=\frac{20}{\pi}\left[0.25+\frac{1}{2}\cos 1^{rad}+\frac{1}{3}\cos 2+\frac{1}{7}\cos 3+\frac{1}{25}\cos 4+\ldots\right]=\boxed{1.3858}$$

54. Input $= x(t) = 5[u(t) - u(t-1)]$ $\quad y(t) = \int_{-\infty}^{t} x(z)\, h(t-z)\, dz$

(a) $h(t) = 2u(t)$ (b) $h(t) = 2u(t-1)$ (c) $h(t) = 2u(t-2)$







(a)

$t < 0$:
$y(t) = 0$

$0 < t < 1$:

$$y(t) = \int_0^t 10dz = 10t$$

$t > 1$:

$$y(t) = \int_{t\text{-}1}^{t} 10dz = 10$$

(b)

$t < 1$: $y(t) = 0$

$1 < t < 2$:

$$y(t) = \int_1^t 10dz = 10(t-1)$$

$t > 2$:

$$y(t) = \int_{t\text{-}1}^{t} 10dz = 10$$

(c)

$t < 2$: $y(t) = 0$

$2 < t < 3$:

$$y(t) = \int_2^t 10dz = 10(t-2)$$

$t > 3$:

$$y(t) = \int_{t\text{-}1}^{t} 10dz = 10$$

55. $x(t) = 5[u(t) - u(t-2)];\ h(t) = 2[u(t-1) - u(t-2)]$

$$y(t) = \int_{-\infty}^{t} x(z)\ h(t-z)\, dz$$

$$t < 1:\ y(t) = 0$$

$$1 < t < 2:\quad y(t) = \int_{0}^{t-1} 10\, dz = 10(t-1)$$

$$2 < t < 3:\ y(t) = 10$$

$$3 < t < 4:\ y(t) = \int_{t-2}^{2} 10\, dz = 10(2-t+2) = 10(4-t)$$

$$t > 4:\ y(t) = 0$$

$$\boxed{\begin{array}{l} \therefore\ y(-0.4) = 0;\quad y(0.4) = 0;\quad y(1.4) = 4 \\ y(2.4) = 10;\ \ y(3.4) = 6;\ \ y(4.4) = 0 \end{array}}$$

or.... $y(t) = \int_{0}^{\infty} x(t-z)\ h(z)\ dz$

$$t < 1:\quad y(t) = 0$$

$$1 < t < 2:\quad y(t) = \int_{1}^{t} 10\, dz = 10(t-1)$$

$$2 < t < 3:\quad y(t) = 10$$

$$3 < t < 4:\quad y(t) = \int_{t-2}^{2} 10\, dz = 10(2-t+2) = 10(4-t)$$

$$t > 4:\quad y(t) = 0$$

$\boxed{\text{same answers as above}}$

56.

$$h(t) = 3[e^{-t} - e^{-2t}],\ x(t) = u(t)$$

$$y(t) = \int_{-\infty}^{t} x(z)h(t-z)\,dz$$

$$= \int_{0}^{t} 3[e^{-(t-z)} - e^{-2(t-z)}]\,dz$$

$$= 3e^{-t}[e^{z}]_0^t - 3e^{-2t}\left[\frac{1}{2}e^{2Z}\right]_0^t$$

$$= 3e^{-t}(e^{t}-1) - 1.5e^{-2t}(e^{2t}-1)$$

$$\therefore\ y(t) = 3(1-e^{-t}) - 1.5(1-e^{-2t}) = \boxed{1.5 - 3e^{-t} + 1.5e^{-2t},\ t > 0}$$

57.

$$y(t) = \int_0^{\infty} x(t-2)h(z)dz$$

$$h(t) = \frac{2}{3}(5-t),\ 2 < t < 5$$

(a) $$y(t) = \int_2^5 10 \times \frac{2}{3}(5-z)\ dz = \boxed{\frac{20}{3}\int_2^5 (5-z)\,dz}$$

Note: $h(z)$ is in window for $4 < t < 6$

(b) $$y(t) = \frac{20}{3}\left(-\frac{1}{2}\right)(5-z)^2\Big|_2^5$$

$$= -\frac{10}{3}(0-9) = \boxed{30 \text{ at } t = 5}$$

58. $x(t) = 5e^{-(t-2)}\,u(t-2),\ h(t) = (4t-16)\,[u(t-4) - u(t-7)],\ y(t) = \int_0^\infty x(t-z)\,h(z)\,dz$

(a) $t < 6:\ \ y(t) = 0 \quad \therefore \boxed{y(5) = 0}$

(b) $t = 8:\ \ y(8) = \int_4^6 5e^{-(8-z-2)}\,(4z-16)\,dz$

$$\therefore y(8) = 20e^{-6}\int_4^6 z\,e^z\,dz - 80e^{-6}\int_4^6 e^z\,dz$$

$$= 20e^{-6}\left[\frac{e^z}{1}(z-1)\right]_4^6 - 80e^{-6}(e^6 - e^4)$$

$$= 20e^{-6}(5e^6 - 3e^4) - 80 + 80e^{-2} = 20 + 80e^{-2} - 60e^{-2}$$

$$= 20\,(1 + e^{-2}) = \boxed{22.71}$$

(c) $t = 10:\ \ y(10) = \int_4^7 5e^{-(10-z-2)}\,(4z-16)\,dz$

$$\therefore y(10) = \int_4^7 20e^{-8}e^z\,(z-4)dz$$

$$\therefore y(10) = 20e^{-8}\int_4^7 ze^z\,dz - 80e^{-8}\int_4^7 e^z\,dz = 20e^{-8}[e^z(z-1)]_4^7 - 80e^{-8}(e^7 - e^4)$$

$$= 20e^{-8}(6e^7 - 3e^4) - 80(e^{-1} - e^{-4}) = 40e^{-1} + 20e^{-4} = \boxed{15.081}$$

59.

$h(t) = \sin t,\ 0 < t < \pi;\ \ 0 \text{ elsewhere},\ \ \text{Let } x(t) = e^{-t}u(t)$

$$y(t) = \int_0^\infty x(t-z)\ h(z)\,dz$$

$t < 0:\ \ y(t) = 0$

$$0 < t < \pi:\ \ y(t) = \int_0^t \sin\ z \times e^{-t+z}\ dz = e^{-t}\int_0^t e^z \sin z\ dz$$

$$\therefore y(t) = e^{-t}\left[\frac{1}{2}e^z(\sin z - \cos z)\right]_0^t$$

$$= \frac{1}{2}e^{-t}[e^t(\sin\ t - \cos\ t) + 1]$$

$$= \frac{1}{2}(\sin\ t - \cos\ t + e^{-t})$$

(a) $y(1) = \boxed{0.3345^+}$

(b) $y(2.5) = \boxed{0.7409}$

(c) $y > \pi:\ y(t) = e^{-t}\int_0^\pi e^z \sin z\,dz$

$$y > \pi:\ \ y(t) = e^{-t}\left[\frac{1}{2}e^z\ (\sin z - \cos z)\right]_0^\pi = \frac{1}{2}e^{-t}(e^\pi + 1) = 12.070e^{-t}$$

$\therefore y(4) = \boxed{0.2211}$

60.

$$x(t) = 0.8(t-1)[u(t-1) - u(t-3)],$$
$$h(t) = 0.2\ (t-2)[u(t-2) - u(t-3)]$$
$$y(t) = \int_0^{\infty} x(t-z)\ h(z)\,dz,$$
$$t < 3:\ \ y(t) = 0$$

(a) $$3 < t < 4:\ \ y(t) = \int_2^{t-1} 0.8(t-z-1)\,0.2(z-2)\,dz$$

$$\therefore\ y(t) = 0.16\int_2^{t-1} (tz - 2t - z^2 + 2z - z + 2)\,dz$$
$$= 0.16\int_2^{t-1}[-z^2 + (t+1)z + 2 - 2t]\,dz = 0.16\left[-\frac{1}{3}z^3 + \frac{1}{2}(t+1)z^2 + (2-2t)z\right]_2^{t-1}$$
$$= 0.16\left[-\frac{1}{3}(t-1)^3 + \frac{8}{3} + \frac{1}{2}(t+1)(t-1)^2 - \frac{1}{2}(t+1)\,4 + (2-2t)(t-1-2)\right]$$
$$\therefore\ y(t) = 0.16\left[-\frac{1}{3}t^3 + t^2 - t + \frac{1}{3} + \frac{8}{3} + \frac{1}{2}(t^2-1)(t-1) - 2t - 2 + 2t - 6 - 2t^2 + 6t\right]$$
$$= 0.16\left[\frac{1}{6}t^3 + t^2\left(1 - \frac{1}{2} - 2\right) + t\left(-1 - \frac{1}{2} + 6\right) + 3 + \frac{1}{2} - 8\right] = 0.16\left(\frac{1}{6}t^3 - \frac{3}{2}t^2 + \frac{9}{2}t - \frac{9}{2}\right)$$

$$\therefore\ \boxed{y(3.8) = 13.653\times 10^{-3}}$$

(b) $$4 < t < 5:\ \ y(t) = \int_2^{3} 0.16(t-z-1)(z-2)\,dz = 0.16\left[-\frac{1}{3}z^3 + \frac{1}{2}(t+1)z^2 + (2-2t)z\right]_2^{3}$$

$$\therefore\ y(t) = 0.16\left[-\frac{1}{3}(27-8) + \frac{1}{2}(t+1)\,5 + (2-2t)1\right]$$
$$= 0.16\left[-\frac{19}{3} + 2.5t + 2.5 + 2 - 2t\right] = 0.16\left(0.5t - \frac{11}{6}\right)$$

$$\therefore\ \boxed{y(4.8) = 90.67\times 10^{-3}}$$

61.

$$x(t) = 10e^{-2t}u(t),\ h(t) = 10e^{-2t}u(t)$$

$$y(t) = \int_0^{\infty} x(t-z)\ h(z)\ dz$$

$$\therefore\ y(t) = \int_0^{t} 10e^{-2(t-z)}\, 10e^{-2z}\ dz$$

$$= 100e^{-2t}\int_0^{t} dz = 100\, e^{-2t} \times t$$

$$\therefore\ \boxed{y(t) = 100t\, e^{-2t}u(t)}$$

62. $h(t) = 5e^{-4t}\,u(t)$

(a) $$W_{1\Omega} = 25\int_{0.1}^{0.8} e^{-8t}\,dt = \frac{25}{8}(e^{-0.8} - e^{-6.4}) = 1.3990 \text{ J}$$

$$\therefore \% = 1.3990/\left(\frac{25}{8}\right)\times 100\% = \boxed{44.77\%}$$

(b) $$H(j\omega) = \frac{5}{j\omega + 4} \quad \therefore W_{1\Omega} = \frac{1}{\pi}\int_0^2 \frac{25}{\omega^2 + 16}\,d\omega = \frac{25}{\pi}\,\frac{1}{4}\tan^{-1}\frac{\omega}{4}\Bigg|_0^2$$

$$\therefore W_{1\Omega} = \frac{25}{4\pi}\tan^{-1}\frac{1}{2} = 0.9224 \text{ J} \quad \therefore \% = \frac{0.9224}{25/8}\times 100\% = \boxed{29.52\%}$$

63.

$$\mathrm{F}(j\omega) = \frac{2}{(1+j\omega)(2+j\omega)} = \frac{2}{1+j\omega} - \frac{2}{2+j\omega} \;\; \therefore \; f(t) = (2e^{-t} - 2e^{-2t})u(t)$$

(a) $$\mathrm{W}_{1\Omega} = \int_0^{\infty} (4e^{-2t} - 8e^{-3t} + 4e^{-4t})\, dt = \frac{4}{2} - \frac{8}{3} + \frac{4}{4} = \boxed{\frac{1}{3}\ \mathrm{J}}$$

(b) $$f(t) = -2e^{-t} + 4e^{-2t} = 0, \; -2 + 4e^{-t} = 0, \; e^{t} = 2, \; t = 0.69315$$

$$\therefore f_{\max} = 2(e^{-0.69315} - e^{-2\times 0.69315}) = \boxed{0.5}$$

64.

(a)

$$F(j\omega) = \frac{1}{j\omega(2+j\omega)(3+j\omega)} = \frac{1/6}{j\omega} - \frac{1/2}{2+j\omega} + \frac{1/3}{3+j\omega}$$

$$\therefore \boxed{f(t) = \frac{1}{12}\text{sgn}(t) - \frac{1}{2}e^{-2t}u(t) + \frac{1}{3}e^{-3t}u(t)}$$

(b)

$$F(j\omega) = \frac{1+j\omega}{j\omega(2+j\omega)(3+j\omega)} = \frac{1/6}{j\omega} + \frac{1/2}{2+j\omega} - \frac{2/3}{3+j\omega}$$

$$\therefore \boxed{f(t) = \frac{1}{12}\text{sgn}(t) + \frac{1}{2}e^{-2t}u(t) - \frac{2}{3}e^{-3t}u(t)}$$

(c)

$$F(j\omega) = \frac{(1+j\omega)^2}{j\omega(2+j\omega)(3+j\omega)} = \frac{1/6}{j\omega} - \frac{1/2}{2+j\omega} + \frac{4/3}{3+j\omega}$$

$$\therefore \boxed{f(t) = \frac{1}{12}\text{sgn}(t) - \frac{1}{2}e^{-2t}u(t) + \frac{4}{3}e^{-3t}u(t)}$$

(d)

$$F(j\omega) = \frac{(1+j\omega)^3}{j\omega(2+j\omega)(3+j\omega)} = 1 + \frac{1/6}{j\omega} + \frac{1/2}{2+j\omega} - \frac{8/3}{3+j\omega}$$

$$\therefore \boxed{f(t) = \delta(t) + \frac{1}{12}\text{sgn}(t) + \frac{1}{2}e^{-2t}u(t) - \frac{8}{3}e^{-3t}u(t)}$$

65. $h(t) = 2e^{-t}u(t)$

(a) $\mathrm{H}(j\omega) = 2 \times \dfrac{1}{1+j\omega} = \boxed{\dfrac{2}{1+j\omega}}$

(b) $\dfrac{1}{2}\,\mathrm{H}(j\omega) = \dfrac{1}{1+j\omega} = \dfrac{1}{2}\,\dfrac{\mathrm{V}_o}{\mathrm{V}_i} = \boxed{\dfrac{1/j\omega}{1+1/j\omega}}$

(c) $\boxed{\text{Gain} = 2}$

66.

$$\mathrm{V}_o(j\omega) = \frac{\frac{1}{2}j\omega + \frac{1}{j\omega}}{1 + \frac{1}{2}j\omega + \frac{1}{j\omega}} = \frac{(j\omega)^2 + 2}{(j\omega)^2 + 2(j\omega) + 2}$$

$$\therefore \mathrm{V}_o(j\omega) = \frac{(j\omega)^2 + 2(j\omega) + 2 - 2(j\omega)}{(j\omega)^2 + 2(j\omega) + 2} = 1 + \frac{-2(j\omega)}{(j\omega)^2 + 2(j\omega) + 2}$$

$$\text{Let } j\omega = x \quad \therefore \mathrm{V}_o(x) = 1 - \frac{2x}{x + 2x + 2}; \quad x = \frac{-2 \pm \sqrt{4-8}}{2} = -1 \pm j1$$

$$\therefore \mathrm{V}_o(x) = 1 + \frac{\mathrm{A}}{x+1+j1} + \frac{\mathrm{B}}{x+1-j1} = \square \quad \text{Let } x = 0 \ \therefore \frac{\mathrm{A}}{1+j1} + \frac{\mathrm{B}}{1-j1} = 0$$

$$\text{Let } x = -1 \ \therefore \frac{\mathrm{A}}{j1} + \frac{\mathrm{B}}{-j1} = 2 \ \therefore \mathrm{A} - \mathrm{B} = j2, \ \mathrm{A} = \mathrm{B} + j2 \ \ \therefore \frac{\mathrm{B}+j2}{1+j1} + \frac{\mathrm{B}}{1-j1} = 0$$

$$\therefore \mathrm{B} - j\mathrm{B} + j2 + 2 + \mathrm{B} + j\mathrm{B} = 0 \quad \therefore \mathrm{B} = -1 - j1 \ \ \therefore \mathrm{A} = -1 + j1$$

$$\therefore \mathrm{V}_o(x) = 1 + \frac{-1+j1}{x+1+j1} + \frac{-1-j1}{x+1-j1}, \ \mathrm{V}_o(j\omega) = 1 - \frac{1-j1}{(j\omega)+1+j1} - \frac{1+j1}{(j\omega)+1-j1}$$

$$\therefore v_o(t) = \delta(t) - (1-j1)e^{(-1-j1)t}u(t) - (1+j1)e^{(-1+j1)t}u(t)$$

$$= \delta(t) - \sqrt{2}\,e^{-j45^\circ - jt - t}u(t) - \sqrt{2}\,e^{j45^\circ + jt - t}u(t)$$

$$= \boxed{\delta(t) - 2\sqrt{2}\,e^{-t}\cos(t + 45^\circ)\,u(t)}$$

67.

$$\mathrm{V}_c(j\omega) = 10\frac{5/j\omega}{5/j\omega + 35 + 30(j\omega)} = \frac{10/j\omega}{1/j\omega + 7 + 6(j\omega)}$$

$$\therefore \mathrm{V}_c(j\omega) = \frac{10}{6(j\omega)^2 + 7(j\omega) + 1} = \frac{10/6}{(j\omega)^2 + \frac{7}{6}(j\omega) + \frac{1}{6}}$$

$$\therefore\ j\omega = \left(-7/6 \pm \sqrt{\frac{49}{36} - \frac{24}{36}}\right)/2 = -\frac{1}{6},\ -1\ \therefore \mathrm{V}_c(j\omega) = \frac{10/6}{(j\omega+1/6)(j\omega+1)} = \frac{2}{j\omega+1/6} - \frac{2}{j\omega+1}$$

$$\therefore \boxed{v_c(t) = 2(e^{-t/6} - e^{-t})u(t)}$$

68. $f(t) = 5e^{-2t}u(t), g(t) = 4e^{-3t}u(t)$

(a) $f * g = \int_0^\infty f(t-z)\, g(z)\, dz$

$$= \int_0^t 5e^{-2t}\, e^{2z}\, 4e^{-3z}\, dz = 20e^{-2t}\int_0^t e^{-z}\, dz$$

$$= -20\, e^{-2t}(e^t - 1)\ \text{V}$$

$$\therefore \boxed{f * g = (e^{-2t} - e^{-3t})\ u(t)}$$

(b) $\text{F}(j\omega) = \dfrac{5}{j\omega+2},\ \text{G}(j\omega) = \dfrac{4}{j\omega+3}\quad \therefore \text{F}(j\omega)\text{G}(j\omega) = \dfrac{20}{(j\omega+2)(j\omega+3)}$

$$\therefore \text{F}(j\omega)\text{G}(j\omega) = \frac{20}{j\omega+2} - \frac{20}{j\omega+3}\quad \therefore \boxed{f * g = 20(e^{-2t} - 2^{-3t})\ u(t)}$$

69. $\mathbf{H}(j\omega) = \dfrac{j2\omega}{4 + j2\omega}$

$\mathbf{V}_i(j\omega) = \dfrac{24}{j\omega}$ from Table 18.2

Therefore $\mathbf{V}_o(j\omega) = \left[\dfrac{j2\omega}{4 + j2\omega}\right]\left(\dfrac{24}{j\omega}\right) = \dfrac{24}{2 + j\omega}$

In the time domain, then, we find

$\boxed{v_o(t) = 24e^{-2t}u(t) \text{ V}}$

70. $h(t) = 2e^{-t}\cos 4t$ so fromTable 18.2,

$\mathbf{H}(j\omega) = \dfrac{2(1+j\omega)}{(1+j\omega)^2+16}$. Define output function *f*(*t*).

(a) $\mathbf{I}(j\omega) = 4\pi\delta(\omega)$

Therefore $\mathbf{F}(\omega) = \left[\dfrac{8\pi(1+j\omega)}{(1+j\omega)^2+16}\right]\delta(\omega) = \dfrac{8\pi}{17}\delta(\omega)$.

The time domain output is then given by *f*(*t*) = $\boxed{4/17.}$

(b) $\mathbf{I}(j\omega) = 2e^{-j\omega}$

Therefore $\mathbf{F}(\omega) = \left[\dfrac{4(1+j\omega)}{(1+j\omega)^2+16}\right]e^{-j\omega}$.

The time domain output is then given by *f*(*t*) = $\boxed{4e^{-(t-1)}\cos\left[4(t-1)\right]u(t-1)}$

(c) We find the response due to a unit step $u(t)$ and treat $i(t)$ as two unit steps, each shifted appropriately.

$$\mathbf{R}(j\omega) = \frac{2(1+j\omega)}{(1+j\omega)^2+16}\left[\pi\delta(\omega) + \frac{1}{j\omega}\right]$$

$$r(t) = \frac{1}{17} + \frac{1}{17}\mathrm{sgn}(t) - 2\frac{e^{-t}}{17}\left[\cos 4t - 4\sin 4t\right]u(t)$$

Therefore the system response is

$$\boxed{\begin{aligned}&\frac{2}{17}\left[1-e^{-(t+0.25)}\left\{\cos 4(t+0.25) - 4\sin 4(t+0.25)\right\}\right]u(t+0.25)\\ &-\frac{2}{17}\left[1-e^{-(t-0.25)}\left\{\cos 4(t-0.25) - 4\sin 4(t-0.25)\right\}\right]u(t-0.25)\end{aligned}}$$