Q3. (a) At what speed should a meter stick move if its length as observed by a stationary observer to shrink to 0.5m? [2]

(b) A rod of proper length 60 cm is travelling along its length in the positive x-direction with speed $0.6c$ as seen from a stationary frame S. In the same frame S, a particle is moving in the direction opposite to the rod with a speed $0.6c$. c is the speed of light in vacuum. In the frame of the particle, (*i*) what is the speed of the rod, (*ii*) what is the length of the rod and (*iii*) how much time the particle would take to cross the rod? (*iv*) In the S frame, what is the time required for the particle to cross the rod? [2+1+1+2=6]

$E = K + m_0c^2$ K 0 e^- e^+

Q4. An electron e^- with kinetic energy K makes a head-on collision with a positron e^+ at rest. Electron and positron have the same rest mass m_0. In the collision, the two particles annihilate each other and are replaced by two photons. The emitted photons travel making equal angles θ with the electron's direction of motion. Determine (*i*) the energy E_{ph}, (*ii*) momentum p_{ph} and (*iii*) angle of emission θ of each photon in terms of K, m_0 and c, the speed of light in vacuum. (*iv*) If $K = 1$ MeV and rest mass energy of electron (or positron) is 0.5 MeV, what is the value of θ? [3+1+3+1=8]

Q5. The n-th eigenstate of a particle of mass m in a one dimensional infinite square well potential extending from $x = 0$ to $x = a$ is $\psi_n(x) = \sqrt{2/a}\,\sin(n\pi x/a)$ with energy eigenvalue $E_n = n^2\pi^2\hbar^2/(2ma^2)$, $n = 1,2,3,\cdots$. The initial ($t = 0$) wavefunction of the particle is given by

$$\Psi(x,0) = A[2\psi_1(x) + 3\psi_2(x)]$$

where A is a constant. (a) Normalize $\Psi(x,0)$. (b) What is the wave function $\Psi(x,t)$ at an arbitrary time t? (c) Find the lowest possible value of time $T\ (\neq 0)$ for which $\Psi(x,0) = \Psi(x,T)$. (d) What is the average energy of the particle? [4x2=8]

$T = \frac{ma^2}{3\pi\hbar}$

Q6. The n-th eigenstate $\psi_n(x)$ of a particle of mass m in a potential $V(x) = m\omega^2x^2/2$ can be obtained from the (n+1)-th eigenstate following $\psi_n(x) = A_n a_- \psi_{n+1}(x)$ where A_n is a constant and $a_- = (i\hat{p} + m\omega x)/\sqrt{2\hbar m\omega}$. (a) Given the second excited state

$$\psi_2(x) = \frac{1}{\sqrt{2}}\left(\frac{m\omega}{\pi\hbar}\right)^{1/4}\left(\frac{2m\omega x^2}{\hbar} - 1\right)e^{-m\omega x^2/(2\hbar)},$$

determine the first excited state $\psi_1(x)$. (b) Normalize $\psi_1(x)$. (c) Calculate average potential energy $\langle V\rangle$ of the particle in the state $\psi_1(x)$. (d) If the energy of the n-th eigenstate is $E_n = \left(n + \frac{1}{2}\right)\hbar\omega$, $n = 0,1,2,\cdots$, find the average kinetic energy $\langle T\rangle$ of the particle in the state $\psi_1(x)$, without explicit calculation of $\langle p^2\rangle$. [3+2+2+1=8]

Useful integrals: $\int_0^\infty x^2 e^{-\alpha x^2}dx = \frac{1}{4\alpha}\sqrt{\frac{\pi}{\alpha}}$ and $\int_0^\infty x^4 e^{-\alpha x^2}dx = \frac{3}{8\alpha^2}\sqrt{\frac{\pi}{\alpha}}$

$B_1\phi_1 - B_2\phi_2$