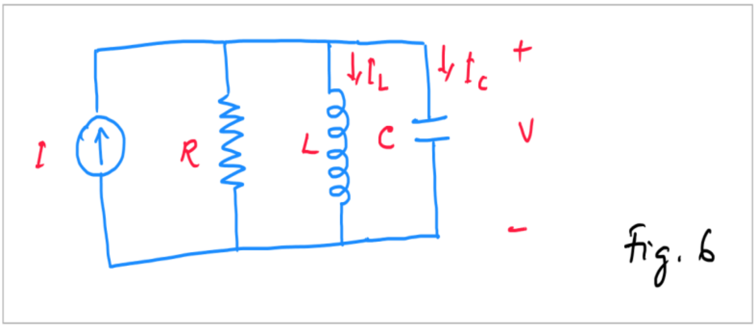
Resonance

In some applications, electrical circuits are designed to emphasize or amplify certain frequency components or the frequency response be chosen deliberately to supress or reject some frequency components. Such characteristics are the requirement of tuned circuits or resonant circuits.

Resonance is a condition that exists in all the physical systems. In resonance, a fixed amplitude sinusoidal forcing function produces a response of maximum amplitude. In an electrical network with at least one inductor and one capacitor, resonance is the condition when the input impedance of the network is purely resistive***. In resonance, the voltage and the current at the input terminals are in phase.*** Fig. 6 shows a parallel RLC circuit with a sinusoidal current source.



The admittance offered to the ideal current source is

$$\boldsymbol{Y = \frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)}$$

For the voltage and current to be in phase, the admittance must be real. The necessary condition for this is given by

$$\boldsymbol{\omega C - \frac{1}{\omega L} = 0}$$

This condition can be achieved by adjusting ***L, C*** or $\boldsymbol{\omega}$. Assuming $\boldsymbol{\omega}$.as the variable, the resonant frequency $\boldsymbol{\omega_o}$

$$\boldsymbol{\omega_0 = \frac{1}{\sqrt{LC}} rad/s}$$

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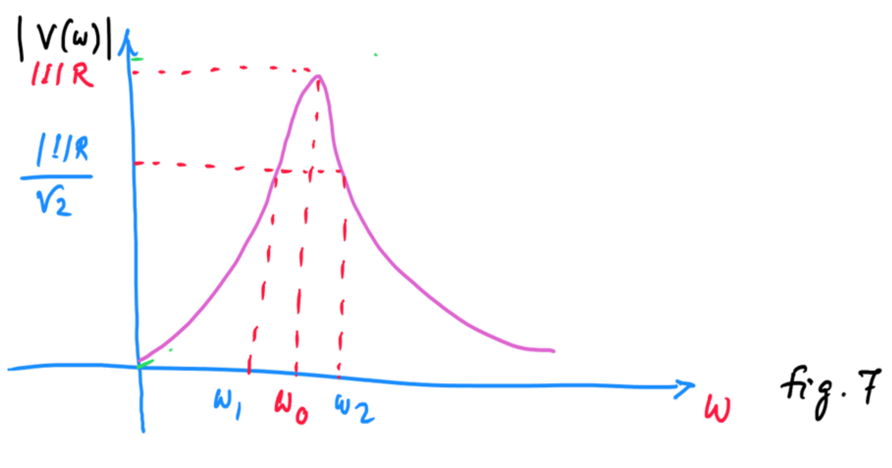
$$\boldsymbol{f_0 = \frac{1}{2\pi\sqrt{LC}}\text{Hz}}$$

The admittance is minimum at the resonant frequency. The voltage response will be maximum at the resonant frequency if the input is a constant amplitude current source. Fig. 7 shows the voltage response of the parallel RLC network.



At resonance the currents through the inductor and the capacitor are

$$I_{LO} = \frac{V}{j\omega_0 L} = \frac{IR}{j\omega_0 L} = -jIR/\omega_0 L$$
$$I_{CO} = Vj\omega_0 C = IRj\omega_0 C$$
$$w_0 C = \frac{1}{\omega_0 L}$$
$$I_{LO} = -jIR\omega_0 C = -I_{CO}$$
$$\Rightarrow I_{LO} + I_{CO} = 0$$

This shows that the currents in the inductor and the capacitor are equal in amplitude and their sum is zero. The input current ***I*** flows through ***R***. At resonance the inductive impedance is equal to the capacitive impedance.

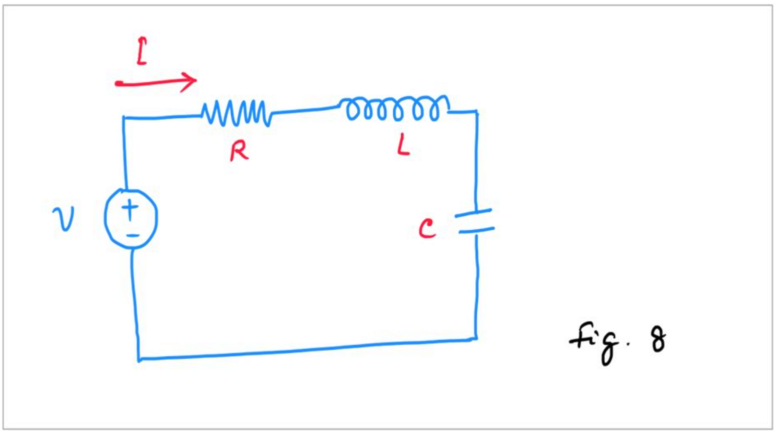
Series Resonance

Fig. 8 shows a series RLC circuit. Its impedance at the source

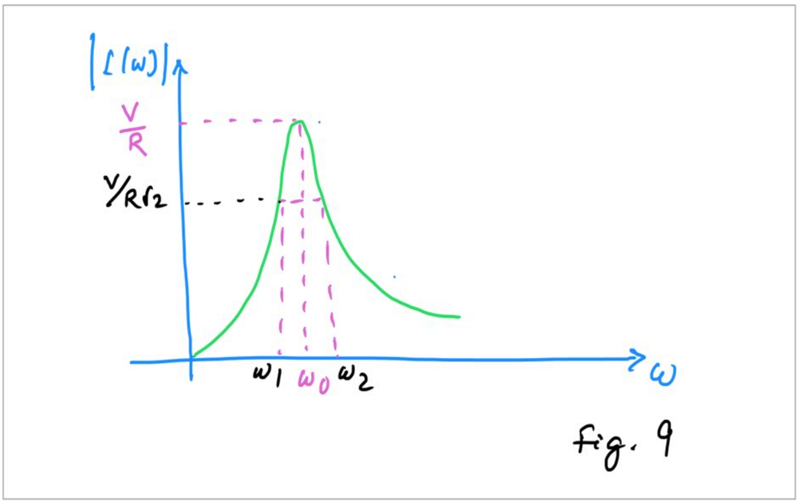
$$z = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

For the voltage and the current to be in phase, the impedance must be resistive. If ω_O is the resonant frequency, then

$$\omega_0 L - \frac{1}{\omega_0 C} = 0$$
$$\Rightarrow \omega_0 = \frac{1}{\sqrt{LC}}$$



The current response for the series resonant circuit is shown in Fig. 9



Maximum current flows at resonance as the impedance has the lowest value. The capacitive voltage drop is equal to the inductive voltage drop. These two voltage drops are opposite in signs. Hence the complete input voltage is dropped across the resistor.