PH 101: Quiz II : 29-10-2015 : Time: 45 mins: Max marks : 10

Q.1[1 mark] A rigid body is made of four non-coplanar point masses (of non-zero mass) connected to each other by massless rods. If I choose my axes to be the principal axes of the rigid body with the origin fixed to one of the masses, which of following matrices best describes the moment of inertia of the rigid body? Write down all valid possibilities in the space provided. Marks will be awarded only if all choices are listed and are correct.

$$(A)\begin{pmatrix} 5 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 3 \end{pmatrix} \quad (B)\begin{pmatrix} 5 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 3 \end{pmatrix} \quad (C)\begin{pmatrix} 5 & 0.5 & 0 \\ 0.5 & 4 & 0 \\ 0 & 0 & 3 \end{pmatrix} \quad (D)\begin{pmatrix} 5 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

Ans. Correct answer is (A). Choices (B) and (D) are not possible because the diagonal elements can never be zero or negative. For example, $I_{yy} = \sum_i m_i(x_i^2 + z_i^2)$ cannot be negative. It cannot even be zero, because if it is zero $x_i = z_i = 0$ for all i, which makes the masses lie on a straight line and hence become coplanar. (C) is not possible because in principal axes the matrix is diagonal.

Q.2[4 marks] Three rods of mass m each and length $2L$ each are attached as shown. The center of mass C of the rigid body is fixed and is the origin of the coordinate system.

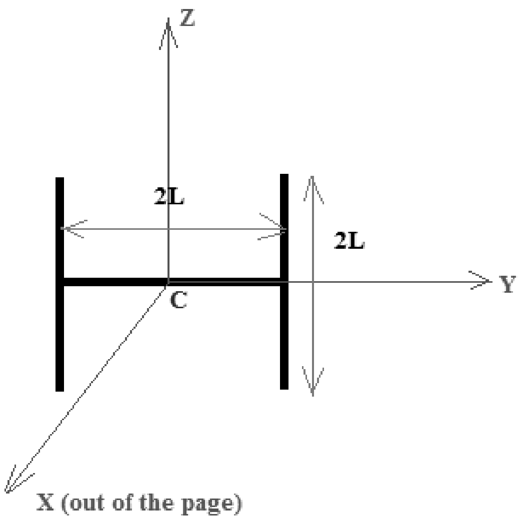

FIG. 1: Three rods

(2.1) Write down the moment of inertia matrix of this rigid body about the axes shown.

Ans. By implication, the axes X,Y and Z are BODY FIXED and rotate along with the rods. This is why the moment of inertia matrix about these axis continues to be diagonal and independent of time even though the axes themselves are spinning.

$$I = \begin{pmatrix} \frac{m(2L)^2}{12} + 2(mL^2 + \frac{m(2L)^2}{12}) & 0 & 0 \\ 0 & 2(\frac{m(2L)^2}{12}) & 0 \\ 0 & 0 & mL^2 + mL^2 + \frac{m(2L)^2}{12} \end{pmatrix} = mL^2 \begin{pmatrix} 3 & 0 & 0 \\ 0 & 2/3 & 0 \\ 0 & 0 & 7/3 \end{pmatrix}$$

(2.2) Write down the angular momentum vector and torque vector of this rigid body about C if the rigid body is spinning with angular velocity vector $\frac{\Omega}{\sqrt{3}}(\hat{i}+\hat{j}+\hat{k})$.

Ans

$$\mathbf{L} = \begin{pmatrix} 3mL^2 & 0 & 0 \\ 0 & \frac{2}{3}mL^2 & 0 \\ 0 & 0 & \frac{7}{3}mL^2 \end{pmatrix} \frac{\Omega}{\sqrt{3}} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

or,

$$\mathbf{L} = mL^2 \frac{\Omega}{\sqrt{3}}(3\hat{i} + \frac{2}{3}\hat{j} + \frac{7}{3}\hat{k})$$

Since the components of the angular momentum are constant in the rotating frame the torque is just $\vec{\tau} = \vec{\omega} \times \mathbf{L}$. If this was not the case you would have to add further terms which have angular acceleration in them. Thus the torque is

$$\vec{\tau} = \vec{\omega} \times \mathbf{L} = \frac{\Omega}{\sqrt{3}} mL^2 \frac{\Omega}{\sqrt{3}} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 3 & \frac{2}{3} & \frac{7}{3} \end{vmatrix} = \frac{\Omega}{\sqrt{3}} mL^2 \frac{\Omega}{\sqrt{3}} (\frac{5}{3}\hat{i} + \frac{2}{3}\hat{j} - \frac{7}{3}\hat{k})$$

(2.3) If this rigid body spins about any one of its principal axes, about which principal axis is the spinning unstable?

Ans $\hat{k}$ is unstable since this is the axis with intermediate moment of inertia.

Q.3[2 marks] A rigid body is spinning with an angular speed $\vec{\omega} = 100\ rad/sec\ \hat{j}$. A point belonging to the rigid body with position vector $\mathbf{r}_1 = 0.5\ meter\ \hat{j}$ is seen to be moving with velocity $\mathbf{v}_1 = 5.0\ meter/sec\ \hat{i}$ direction. Write down the velocity vector (magnitude and direction) of a point with $\mathbf{r}_2 = 1.5\ meter\ \hat{k}$ which also belongs to the rigid body.

Ans. $\mathbf{v}_1 = \mathbf{v} + \vec{\omega} \times \mathbf{r}_1$ or, $5.0\ meter/sec\ \hat{i} = \mathbf{v} + 100\ rad/sec\ \hat{j} \times 0.5\ meter\ \hat{j}$. This means $\mathbf{v} = 5.0\ meter/sec\ \hat{i}$. But we also know $\mathbf{v}_2 = \mathbf{v} + \vec{\omega} \times \mathbf{r}_2 = 5.0\ meter/sec\ \hat{i} + 100\ rad/sec\ \hat{j} \times 1.5\ meter\ \hat{k} = 155\ \hat{i}\ m/s$. THREE IMPORTANT POINTS - value = 155, direction = $\hat{i}$ and units = m/s. ALWAYS KEEP THESE IN MIND WHILE WRITING ANSWERS

Q.4 [2 marks] **Ans.** The apparent lifetime of this meson would be longer due to time dilation. Note that this problem is about time dilation and NOT length contraction. Many students made this mistake. Length contraction is when you measure the length of an object that is moving. Here you are measuring the time elapsed in a moving object. Length contracts, time dilates (expands). β is the standard symbol in relativity for v/c. If the real (proper) lifetime is $2.5 \times 10^{-8} sec$ then a meson moving with speed $v = c\beta = 0.73c$ will have a lifetime that seems longer which is equal to $2.5 \times 10^{-8} sec/\sqrt{1-\beta^2} = 3.66 \times 10^{-8} sec$. This means, relativistically, the distance traveled in one lifetime is $0.73 \times 3 \times 10^8 m/s \times 3.66 \times 10^{-8} sec = 8.0154m$. Nonrelativistically, the lifetime continues to be the proper one so that the distance travelled is less $2.5 \times 10^{-8} sec \times 0.73 \times 3 \times 10^8 m/s = 5.475m$.

Q.5 [1 mark] **Ans.** Since the galaxy is moving away, the light is red shifted this means the apparent wavelength is longer, $\lambda = (513 + 12)nm = 525nm$. The intrinsic wavelength of the light is $\lambda_0 = 513nm$. These two are related by the Doppler formula,

$$\lambda' = \lambda_0 \sqrt{\frac{1 + v/c}{1 - v/c}}$$

This means

$$\frac{v}{c} = \frac{\lambda'^2 - \lambda_0^2}{\lambda'^2 + \lambda_0^2} = 0.023$$

or

$$v = 6.93 \times 10^6 m/s$$