Department of Mathematics, IIT Guwahati

MA 101 - Mathematics – 1

Quiz - 1

1. Consider the function $T : \mathbb{R}^3 \to \mathbb{R}^2$ given by, $T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x - 7y + 3z \\ 4x + 2y + 9z \end{bmatrix}$.

   (a) Show that T is a linear transformation. [1]

   (b) Find the matrix of T with respect to the basis $B = \{(3, 1, 0), (-1, 1, -1), (1, 0, 1)\}$ of $\mathbb{R}^3$. [2]

2. Solve the following system of linear equations using Gauss-Jordan elimination: [3]

$$x + 2y + 3z = 19, \qquad 2x + 4y + z = 13, \qquad x + 2y + z = 9.$$

3. Give a basis and the dimension for the subspace of $\mathbb{R}^3$ spanned by the set of vectors [2]

$$\{(2, 4, 1), (1, 2, 3), (1, 2, 1)\}.$$

4. Consider the matrix $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 1 \\ 1 & 2 & 3 \end{bmatrix}$.

   (a) Give the basis of column space of A among the column vectors of A. [1]

   (b) Express the vector $v = (5, 7, 11)$ as a linear combination of above basis vectors. [1]

   (c) Give a basis of null space and nullity of A. [1]

5. Let A and B be two row equivalent matrices. Show that the columns of A and B have the same dependence relationships. [2]

6. Let p be a prime number and let A be an $m \times n$ matrix of rank r with entries in $\mathbb{Z}_p$. Prove that every consistent system of equations with coefficient matrix A has exactly p^{n-r} solutions over $\mathbb{Z}_p$. [2]