Roll No.: 170108024
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Mukul Ranjan

Dv 20/9/17

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Important Instructions: Read carefully!

- Write your answers in the space provided against each question. If required, you may continue your solutions only on pages 16, 17, 18. Answers written elsewhere will not be evaluated.
- No extra sheets will be provided for writing solutions.
- You can use the supplement(s) for rough work. These sheets will **not** be evaluated.
- No marks will be awarded if you write only the final answer.

1. Check whether the following systems of linear equations are equivalent or not: [3]

System – I	*System – II*
$4x+3y-8z = 47$	$4x+7y+z = 40$
$3x-y+5z = -14$	$x+3y-z = 20$
$5x+3y+4z = 13$	$x+2y+3z = 3$
	$4y+z = 17$

Space only for Q. 1

Ans:

Augmented Matrix of system-I

$$\left[\begin{array}{ccc|c} 4 & 3 & -8 & 47 \\ 3 & -1 & 5 & -14 \\ 5 & 3 & 4 & 13 \end{array}\right] \xrightarrow{R_3 \leftarrow R_3 - R_1} \left[\begin{array}{ccc|c} 4 & 3 & -8 & 47 \\ 3 & -1 & 5 & -14 \\ 1 & 0 & 12 & -34 \end{array}\right]$$

$$\xrightarrow{R_1 \leftrightarrow R_3} \left[\begin{array}{ccc|c} 1 & 0 & 12 & -34 \\ 3 & -1 & 5 & -14 \\ 4 & 3 & -8 & 47 \end{array}\right] \xrightarrow{R_3 \leftarrow R_3 - R_2} \left[\begin{array}{ccc|c} 1 & 0 & 12 & -34 \\ 3 & -1 & 5 & -14 \\ 1 & 4 & -13 & 61 \end{array}\right]$$

$$\xrightarrow[R_3 \leftarrow R_3 - R_1]{R_2 \leftarrow R_2 - 3R_1} \left[\begin{array}{ccc|c} 1 & 0 & 12 & -34 \\ 0 & -1 & -31 & 88 \\ 0 & 4 & -25 & 95 \end{array}\right] \xrightarrow{R_3 \leftarrow R_3 + 4R_2} \left[\begin{array}{ccc|c} 1 & 0 & 12 & -34 \\ 0 & -1 & -31 & 88 \\ 0 & 0 & -149 & 447 \end{array}\right]$$

$$\xrightarrow{R_3 \leftarrow R_3/149} \left[\begin{array}{ccc|c} 1 & 0 & 12 & -34 \\ 0 & -1 & -31 & 88 \\ 0 & 0 & -1 & 3 \end{array}\right]$$

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Space only for Q. 1

$$\xrightarrow{R_2 \leftarrow R_2 - 31R_3} \left[\begin{array}{ccc|c} 1 & 0 & 12 & -34 \\ 0 & -1 & 0 & -5 \\ 0 & 0 & -1 & 3 \end{array}\right] \xrightarrow{R_1 \leftarrow R_1 + 12R_3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & -1 & 0 & -5 \\ 0 & 0 & -1 & 3 \end{array}\right]$$

$x = 2$

$-y = -5 \Rightarrow y = 5$

$-z = 3 \Rightarrow z = 3$

Missing a negative sign here.

Now writing the argumented matrix of System-II

$$\begin{bmatrix} 4 & 7 & 1 & 40 \\ 1 & 3 & -1 & 20 \\ 1 & 2 & 3 & 3 \\ 0 & 4 & 1 & 17 \end{bmatrix} \xrightarrow{R_3 \leftarrow R_3 - R_2} \begin{bmatrix} 4 & 7 & 1 & 40 \\ 1 & 3 & -1 & 20 \\ 0 & -1 & 4 & -17 \\ 0 & 4 & 1 & 17 \end{bmatrix}$$

$$\xrightarrow{R_4 \leftarrow R_4 + R_3} \begin{bmatrix} 4 & 7 & 1 & 40 \\ 1 & 3 & -1 & 20 \\ 0 & -1 & 4 & -17 \\ 0 & 3 & 5 & 0 \end{bmatrix} \xrightarrow{R_1 \leftarrow R_1 - 3R_2} \begin{bmatrix} 1 & -2 & 4 & -20 \\ 1 & 3 & -1 & 20 \\ 0 & -1 & 4 & -17 \\ 0 & 3 & 5 & 0 \end{bmatrix}$$

$$\xrightarrow{R_2 \leftarrow R_2 - R_1} \begin{bmatrix} 1 & -2 & 4 & -20 \\ 0 & 5 & -5 & 40 \\ 0 & -1 & 4 & -17 \\ 0 & 3 & 5 & 0 \end{bmatrix} \xrightarrow{R_2 \leftarrow R_2/5} \begin{bmatrix} 1 & -2 & 4 & -20 \\ 0 & 1 & -1 & 8 \\ 0 & -1 & 4 & -17 \\ 0 & 3 & 5 & 0 \end{bmatrix}$$

$$\xrightarrow{R_3 \leftarrow R_3 + R_2} \begin{bmatrix} 1 & -2 & 4 & -20 \\ 0 & 1 & -1 & 8 \\ 0 & 0 & 3 & -9 \\ 0 & 3 & 5 & 0 \end{bmatrix} \xleftarrow{R_3 \leftarrow R_3/3} \begin{bmatrix} 1 & -2 & 4 & -20 \\ 0 & 1 & -1 & 8 \\ 0 & 0 & 1 & -3 \\ 0 & 3 & 5 & 0 \end{bmatrix}$$

$$\xleftarrow{R_4 \leftrightarrow R_3} \begin{bmatrix} 1 & -2 & 4 & -20 \\ 0 & 1 & -1 & 8 \\ 0 & 3 & 5 & 0 \\ 0 & 0 & 1 & -3 \end{bmatrix} \xleftarrow{R_3 \leftarrow R_3 - 3R_2} \begin{bmatrix} 1 & -2 & 4 & -20 \\ 0 & 1 & -1 & 8 \\ 0 & 0 & 8 & -24 \\ 0 & 0 & 1 & -3 \end{bmatrix}$$

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$\boxed{Q.1}$ continue

$$\xrightarrow{R_3 \leftarrow R_3/8} \begin{bmatrix} 1 & -2 & 4 & -20 \\ 0 & 1 & -1 & 8 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 1 & -3 \end{bmatrix} \xrightarrow{R_4 \leftarrow R_4 - R_3} \begin{bmatrix} 1 & -2 & 4 & -20 \\ 0 & 1 & -1 & 8 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\xrightarrow{R_2 \leftarrow R_2 + R_3} \begin{bmatrix} 1 & -2 & 4 & -20 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow{R_1 \leftarrow R_1 + 2R_2} \begin{bmatrix} 1 & 0 & 4 & -10 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\xrightarrow{R_1 \leftarrow R_1 - 4R_3} \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\therefore$ $x = 2$
$y = 5$
$z = -3$

Since the both system has same solution set hence they are equivalent.

continue Question-2(a)

$$\therefore T \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2x_1 - x_3 \\ -x_1 + x_2 + x_3 \\ -x_1 + x_2 + x_3 \\ 2x_2 + 2x_3 \end{bmatrix}$$

$$\therefore A = \begin{bmatrix} 2 & 0 & -1 \\ -1 & 1 & 1 \\ -1 & 1 & 1 \\ 0 & 2 & 2 \end{bmatrix}$$ Ans

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2. Let $T : \mathbb{R}^3 \to \mathbb{R}^4$ be a linear transformation such that

$$T\left(\begin{bmatrix}1\\1\\1\end{bmatrix}\right) = \begin{bmatrix}1\\0\\1\\0\end{bmatrix}, \quad T\left(\begin{bmatrix}1\\1\\0\end{bmatrix}\right) = \begin{bmatrix}1\\2\\1\\2\end{bmatrix}, \quad T\left(\begin{bmatrix}0\\1\\1\end{bmatrix}\right) = \begin{bmatrix}0\\1\\0\\2\end{bmatrix}$$

(a) Determine a matrix A such that $T(x) = Ax$ for all $x \in \mathbb{R}^3$. [3]

Space only for Q. 2 (a)

Let $x = \begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} \in \mathbb{R}^3$ ~~and~~

$$\begin{bmatrix}1 & 1 & 0\\1 & 1 & 1\\1 & 0 & 1\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = \begin{bmatrix}1 & 1 & 0\\0 & 2 & 1\\1 & 1 & 0\\0 & 2 & 2\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}$$

solving this eqn ~~eq~~ using augmented matrix

$$\left[\begin{array}{ccc|ccc}1 & 1 & 0 & 1 & 1 & 0\\1 & 1 & 1 & 0 & 2 & 1\\1 & 0 & 1 & 1 & 1 & 0\\0 & 0 & 0 & 0 & 2 & 2\end{array}\right] \xrightarrow{R_2 \leftarrow R_2 - R_1} \left[\begin{array}{ccc|ccc}1 & 1 & 0 & 1 & 1 & 0\\0 & 0 & 1 & -1 & 1 & 1\\1 & 0 & 1 & 1 & 1 & 0\\0 & 0 & 0 & 0 & 2 & 2\end{array}\right]$$

$$\xrightarrow{R_3 \leftrightarrow R_2} \left[\begin{array}{ccc|ccc}1 & 1 & 0 & 1 & 1 & 0\\1 & 0 & 1 & 1 & 1 & 0\\0 & 0 & 1 & -1 & 1 & 1\\0 & 0 & 0 & 0 & 2 & 2\end{array}\right] \xrightarrow[\cancel{R_1 \leftarrow}]{R_2 \leftarrow R_2 - R_3} \left[\begin{array}{ccc|ccc}1 & 1 & 0 & 1 & 1 & 0\\1 & 0 & 0 & 2 & 0 & -1\\0 & 0 & 1 & -1 & 1 & 1\\0 & 0 & 0 & 0 & 2 & 2\end{array}\right]$$

$$\xrightarrow{R_1 \leftarrow R_1 - R_2} \left[\begin{array}{ccc|ccc}0 & 1 & 0 & -1 & 1 & 1\\1 & 0 & 0 & 2 & 0 & -1\\0 & 0 & 1 & -1 & 1 & 1\\0 & 0 & 0 & 0 & 2 & 2\end{array}\right] \quad \text{— (1)}$$

Thus, we have

$$T\begin{bmatrix}x_2\\x_1\\x_3\end{bmatrix} = \begin{bmatrix}-x_1 + x_2 + x_3\\2x_1 - x_3\\-x_1 + x_2 + x_3\\2x_2 + 2x_3\end{bmatrix}$$

continue on Page 16

(b) Justify whether such a matrix A as above is unique. [2]

Space only for Q. 2 (b)

$$A = \begin{bmatrix} 2 & 0 & -1 \\ -1 & 1 & 1 \\ -1 & 1 & 1 \\ 0 & 2 & 2 \end{bmatrix}$$

Since, the left part of argumented matrix of (1) is in RREF which is unique hence matrix A must be unique.

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$\boxed{Q.1}$ continue

$$\xrightarrow{R_3 \leftarrow R_3/8} \begin{bmatrix} 1 & -2 & 4 & -20 \\ 0 & 1 & -1 & 8 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 1 & -3 \end{bmatrix} \xrightarrow{R_4 \leftarrow R_4 - R_3} \begin{bmatrix} 1 & -2 & 4 & -20 \\ 0 & 1 & -1 & 8 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\xrightarrow{R_2 \leftarrow R_2 + R_3} \begin{bmatrix} 1 & -2 & 4 & -20 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow{R_1 \leftarrow R_1 + 2R_2} \begin{bmatrix} 1 & 0 & 4 & -10 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\xrightarrow{R_1 \leftarrow R_1 - 4R_3} \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad \therefore \begin{aligned} x &= 2 \\ y &= 5 \\ z &= -3 \end{aligned}$$

Since the both system has same solution set hence they are equivalent.

continue Question-2(a)

$$\therefore T\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2x_1 - x_3 \\ -x_1 + x_2 + x_3 \\ -x_1 + x_2 + x_3 \\ 2x_2 + 2x_3 \end{bmatrix}$$

$$\therefore A = \begin{bmatrix} 2 & 0 & -1 \\ -1 & 1 & 1 \\ -1 & 1 & 1 \\ 0 & 2 & 2 \end{bmatrix} \quad \text{Ans}$$

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3. State with proper justification, whether each of the following statements is **True** or **False**.

(a) If A be a nonzero matrix such that $A^{31} = 0$ then all eigenvalues of A are equal to 0 and A is not diagonalizable. [1]

Space only for Q. 3 (a)

Wrong argument. A matrix with all eigenvalue zero may be diagonalizable.

$A \neq 0$ ~~True~~

$Ax = \lambda x$ let λ be the eigen value of A

Since we know that eigen value of A^n is λ^n.

hence, $A^{31} x = \lambda^{31} x$

$Ax = \lambda x$

$A \cdot Ax = A\lambda x = \lambda (Ax)$

$\Rightarrow A^2 x = \lambda(\lambda x) = \lambda^2 x$ $\therefore A^{31} x = \lambda^{31} x$

$A^{31} = 0$

$A^{31} x = 0 \Rightarrow \lambda^{31} x = 0$

$x \neq 0$

$\lambda^{31} = 0$ $\therefore \lambda = 0$

Thus all the eigen value of $A = 0$ ad hence A is not diagonisable.

Hence, the statement is true.

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(b) Any square matrix is similar to its Reduced Row Echelon Form. [1]

Space only for Q. 3 (b)

Let R be the R.R.E.F of any square matrix A.

R = R.R.EF(A)

- we know ~~that~~ for two matrix A, B, ~~to~~ to be similar

i) det (A) = det (B)

~~ii) they should have same set of eig~~

But we know that det (A) ≠ det R

thus, The ~~given~~ any square matrix is not similar to its R.R.E.F.

- similar matrix have same set of eigenvalues but we know that Row transformation does preserve eigen values hence eigen values of R.R.E and A are different thus, they ~~need not~~ are not similar.

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To show a statement is false you are required to give a counter example with proper justification.

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Why $\det(A) \neq \det(R)$? Give an example.

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4. (a) Let A be a real $n \times n$ matrix such that A is similar to A^{2017}. Determine all the possible values for the determinant of A. [1]

Space only for Q. 4 (a)

we know the determinant of similar matrices are equal ; as $AP = PB$

$$\det(AP) = \det(PB)$$

$$\det A \cdot \det P = \det P \cdot \det A$$

$$\boxed{\therefore \det A = \det B}$$

Now,

$$\det A = \det(A^{2017})$$

we know that $\det(A^n) = (\det A)^n$

$$\Rightarrow \det A = (\det A)^{2017}$$

$$\Rightarrow (\det A)^{2017} - \det A = 0$$

$$\det A\left[(\det A)^{2016} - 1\right] = 0$$

$$\det A = 0 \quad \text{or} \quad (\det A)^{2016} = 1$$

$$\det A = \pm 1$$

Thus, $\boxed{\det A = 0, ~~or~~ 1, -1}$ Ans

(b) Let v_1 and v_2 be two nonzero eigenvectors corresponding to distinct eigenvalues λ_1 and λ_2 of a 5×5 matrix A. Show that $v_1 - v_2$ can not be an eignvector of A. [2]

Space only for Q. 4

~~A~~ we have λ_1 and λ_2 as a corresponding eigen values of ~~A~~ eigenvector v_1 and v_2 of a vector

we know that,

$$A v_1 = \lambda_1 v_1 \quad -(i)$$

and $$A v_2 = \lambda_2 v_2 \quad -(ii)$$

subtracting (i) and (ii)

$$A v_1 - A v_2 = \lambda_1 v_1 - \lambda_2 v_2$$

$$\Rightarrow \boxed{A(v_1 - v_2) = \lambda_1 v_1 - \lambda_2 v_2} \quad -(iii)$$

From eqn (iii) we ~~k~~ can see that

$v_1 - v_2$ will be the eigen ~~value of A~~ vector of A iff $\lambda_1 = \lambda_2$ but $\lambda_1 \neq \lambda_2$

hence $(v_1 - v_2)$ can not be an eigenvector of.

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need proper justification

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• Incorrect justification.

• Need to justify why $v_1 - v_2$ will be an eigen vector iff $\lambda_1 = \lambda_2$.

5. (a) Let A be a $p \times n$ matrix and B be a $q \times n$ matrix such that $\text{rank}(A) + \text{rank}(B) < n$. Show that there exists a nonzero vector $x \in \mathbb{R}^n$ such that $Ax = 0$ and $Bx = 0$. [2]

Space only for Q. 5 (a)

a) Since, rank A + rank B $<$ n

rank(A) $<$ n - rank(B)

rank (A) $<$ nullity (B)

Row (A) $\subseteq$ null (B)

we have
rank (A) + nullity (A) = n
rank (B) + nullity (B) = n

thus, dim(null (B)) $\neq 0$ therefore there must be some vector $x \in \mathbb{R}^n$ such that

$\boxed{Bx = 0}$

Similarly

rank B $<$ n - rank (A)

rank B $<$ nullity A

Row (B) $\subseteq$ null (A)

$\therefore$ dim(null (A)) $\neq 0$ hence there must exist some x such that

$\boxed{Ax = 0}$

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Why common solution exixts

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your aim is to show that vector x is comman , such that Ax=0, Bx=0. in your case vector may be different.

(b) Let $v_1, \ldots, v_k$ be vectors in $\mathbb{R}^n$ and A be an invertible $n \times n$ matrix. Then show that [2]

$$\dim(\text{Span}(\{v_1, \ldots, v_k\})) = \dim(\text{Span}(\{Av_1, \ldots, Av_k\}))$$

Space only for Q. 5 (b)

Let $A = [c_1 \ c_2 \ \cdots \ c_n]$

~~Since A is an invertible nxn matrix, hence~~

~~Col(A) must be span $\mathbb{R}^n$ as A is nxn matrix.~~

~~thus,~~ ~~dim (Col A) = n~~

span $(Av_1 \cdots Av_k) = \alpha_1 Av_1 + \alpha_2 Av_2 + \cdots + \alpha_k Av_k$

Since A is invertible matrix number of elements ~~vector elements~~ in the span $\{v_1, \ldots, v_k\}$ must be equal to number of elements in span $\{Av_1, Av_2 \cdots Av_k\}$ as any of the row of A is not zero, hence it does not affect or nullify any element of span $(v_1, v_2, \ldots v$

$\therefore \dim(\text{span}(\{v_1, \ldots, v_k\})) = \dim(\text{span}(\{Av_1, \ldots Av_k$

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wrong

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INCORRECT

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6. Let A be an $n \times n$ matrix, where $n \geq 2$.

(a) Show that $\det(\text{adj}(A)) = \det(A)^{n-1}$. [2]

Space only for Q. 6 (a)

Case I

when $\det A \neq 0$

we know that

$$A(\text{Adj} A) = (\text{Adj} A) A = (\det A) I_n$$

$$\therefore A(\text{Adj} A) = (\det A) I_n$$

taking determinant both side -

$$\det(A(\text{Adj} A)) = \det[(\det A) I_n]$$

we have, $\det(AB) = \det A \cdot \det B$

$\det(a I_n) = a^n \det I_n = a^n$

$$\det A \cdot \det(\text{Adj} A) = (\det A)^n$$

$$\boxed{\det(\text{Adj} A) = (\det(A))^{n-1}}$$

as; $(\det A)^n = \det A^n$

$$\boxed{\therefore \det(\text{Adj} A) = \det(A)^{n-1}}$$

Proved

Case II $\det A = 0$ ~~let us assume that Adj~~

then we have to prove that, $\det(\text{Adj} A) = 0$

Let's assume $\det(\text{Adj} A) \neq 0$ thus, Adj A is invertible.

$$(\text{Adj} A) \cdot A = (\det A) I_n = 0$$

$\Rightarrow$ Adj A

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(b) If A has rank $n-1$ then show that adj(A) has rank 1. [3]
(*You may use the rank-nullity theorem.*)

Space only for Q. 6 (b)

~~$(\text{Adj} A)\cdot A = (\det A)\cdot I_n$~~ ~~If A is invertible matrix~~

~~$\text{Adj}(A) = (\det A)\cdot(A^{-1})$~~

Let A be $n\times n$ matrix

we have,

$\text{Rank}(A) + \text{nullity}(A) = n$

$\text{Rank } A = n - \text{nullity}(A)$

$\text{Rank}(A) = n-1$ $\therefore \text{nullity}(A) = 1$

thus matrix A contain 1 zero rows.

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ a_{n-1\,1} & n_{n-1\,2} & & a_{n-1\,n} \\ 0 & \cdots & \cdots & 0 \end{bmatrix}$$

considering last row we can see that $c_{in} \neq$

$$\text{Adj} A = \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1n} \\ \vdots & \ddots & & \vdots \\ c_{n1} & \cdots & \cdots & c_{nn} \end{bmatrix}^T$$

~~thus~~ c_{ij} is the cofactor matrix of element $(a_{ij}$.

Since $c_{in} \neq 0$ as we have to cancel ~~those~~ ~~those~~ coloum and rows ~~to calculate cof~~ containing a_{ij} to calculate cofactor hence. ~~det~~ $\boxed{\text{Rank}(\text{Adj} A) = n}$

Proved

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Incorrect reasoning. $rank(A) = n-1$ implies that REF of A has exactly one zero row, but A need not have a zero row. See grading scheme for an example.
You had to prove that $rank(adj(A)) = n-1$

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Incorrect solution.

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Quest. 6.(a)

$$(Adj\,A) \cdot A = 0$$

$$\Rightarrow (Adj\,A)^{-1} (Adj\,A) \cdot A = 0$$

$$A = 0$$

$\Rightarrow Adj(A) = 0$ $\Rightarrow$ det $(Adj\,A) = 0$

we contradic our assumption

hence

$\boxed{\det (Adj\,A) = 0}$ Proved.

Question-8 :

$$W = \left\{ \begin{bmatrix} 1 \\ -1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 2 \\ -3 \end{bmatrix} \right\}$$

$$w = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

Calculating $W^{\perp}$:

$v_1 = x_1 = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 1 \end{bmatrix}$ and $x_2 = \begin{bmatrix} 1 \\ 1 \\ 2 \\ -3 \end{bmatrix}$

$$v_2 = x_2 - proj_{v_1} x_2$$

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7. Let A be an $n \times n$ $(n \geq 3)$ matrix with entries a_{ij} as follows:

$$a_{ij} = \begin{cases} 3 & \text{if } i = j \\ 2 & \text{if } i \neq j \end{cases}$$

(a) Without computing the characteristic polynomial, determine all the eigenvalues of A, and a basis for each eigenspace of A. (You need to justify why there is no eigenvalue other than those you have found.) [4]

(b) Find an invertible matrix P and a diagonal matrix D such that $P^{-1}AP = D$. [1]

(Answering it only for special cases like $n = 3$ or 4 will be awarded zero marks.)

Space only for Q. 7

a)

$$A = \begin{bmatrix} 3 & \cdots & & & 2 \\ 2 & 3 & 2 & \cdots & \vdots \\ 2 & 2 & 3 & & \vdots \\ \vdots & & & \ddots & \vdots \\ 2 & \cdots & & \cdots & 3 \end{bmatrix}$$

Since A is symmetric matrix

$A^T = A$

Space only for Q. 7

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8. Let

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 & 2 \\ -1 & 1 & 3 & 7 & 0 \\ 0 & 2 & 4 & 8 & 2 \\ 1 & -3 & -7 & -15 & -2 \end{bmatrix}, \quad W = \operatorname{col}(A) \quad \text{and} \quad w = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}.$$

Determine the orthogonal projection of w onto $W^{\perp}$. [3]

Space only for Q. 8

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 & 2 \\ -1 & 1 & 3 & 7 & 0 \\ 0 & 2 & 4 & 8 & 2 \\ 1 & -3 & -7 & -15 & -2 \end{bmatrix} \xrightarrow[R_4 \leftarrow R_4 - R_1]{R_2 \leftarrow R_2 + R_1} \begin{bmatrix} 1 & 1 & 1 & 1 & 2 \\ 0 & 2 & 4 & 8 & 2 \\ 0 & 2 & 4 & 8 & 2 \\ 0 & -4 & -8 & -16 & -4 \end{bmatrix}$$

$$\xrightarrow[\substack{R_4 \leftarrow R_4/_{-4} \\ R_2 \leftarrow R_2/2}]{R_3 \leftarrow R_3 - R_2} \begin{bmatrix} 1 & 1 & 1 & 1 & 2 \\ 0 & 1 & 2 & 4 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 4 & 1 \end{bmatrix} \xrightarrow{R_4 \leftarrow R_4 - R_2} \begin{bmatrix} 1 & 1 & 1 & 1 & 2 \\ 0 & 1 & 2 & 4 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\xrightarrow{R_1 \leftarrow R_1 - R_2} \begin{bmatrix} 1 & 0 & -1 & -3 & 1 \\ 0 & 1 & 2 & 4 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad - R$$

$-C_1 + 2C_2 = C_3$

$-3C_1 + 4C_2 = C_4$

$C_1 + C_2 = C_5$

thus, column 1 and 2 of R.R.EF (A) is linearly independent.

$$\therefore \operatorname{Col}(A) = \operatorname{Span}\left\{ \begin{bmatrix} 1 \\ -1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 2 \\ -3 \end{bmatrix} \right\}$$

Since relation between coloumn does not loose in Row transformation thus, col. 1 and col 2 in A is also independent.

Continue of Page-17

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Extra space for answers, NOT rough work. Doing rough work here or not mentioning the Q. No. clearly will invite **NEGATIVE** marking.

Quest. 6.(a)

$$(\mathrm{Adj}\,A)\cdot A = 0$$

$$\Rightarrow (\mathrm{Adj}\,A)^{-1}(\mathrm{Adj}\,A)\cdot A = 0$$

$$A = 0$$

$\Rightarrow \mathrm{Adj}(A) = 0$ $\Rightarrow \det(\mathrm{Adj}\,A) = 0$

we contradic our assumption

hence

$$\boxed{\det(\mathrm{Adj}\,A) = 0}$$ Proved.

Question-8 :

$$W = \left\{ \begin{bmatrix} 1 \\ -1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 2 \\ -3 \end{bmatrix} \right\}$$

$$w = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

Calculating $W^{\perp}$:

$v_1 = x_1 = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 1 \end{bmatrix}$ and $x_2 = \begin{bmatrix} 1 \\ 1 \\ 2 \\ -3 \end{bmatrix}$

$$v_2 = x_2 - \mathrm{proj}_{v_1} x_2$$

Extra space for answers, NOT rough work. Doing rough work here or not mentioning the Q. No. clearly will invite **NEGATIVE** marking.

$$v_2 = \begin{bmatrix} 1 \\ 1 \\ 2 \\ -3 \end{bmatrix} - \frac{v_1 \cdot x_2}{v_1 \cdot v_1} \cdot v_1$$

$$v_1 = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 1 \end{bmatrix}$$

$v_1 \cdot x_2 = -3$

$v_1 \cdot v_1 = 3$

$$x_2 = \begin{bmatrix} 1 \\ 1 \\ 2 \\ -3 \end{bmatrix}$$

$$\frac{v_1 \cdot x_2}{v_1 \cdot v_1} = -1$$

$$v_2 = \begin{bmatrix} 1 \\ 1 \\ 2 \\ -3 \end{bmatrix} + \begin{bmatrix} 1 \\ -1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 2 \\ -2 \end{bmatrix}$$

$$W^\perp = \left\{ \begin{bmatrix} 1 \\ -1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 2 \\ -2 \end{bmatrix} \right\} \qquad w = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\text{proj}_{W_1^\perp} w = \frac{w \cdot v_1}{v_1 \cdot v_1} v_1 + \frac{w \cdot v_2}{v_2 \cdot v_2} \cdot v_2$$

$w \cdot v_1 = 2$

$v_1 \cdot v_1 = 3$

$w \cdot v_2 = 2$

$v_2 \cdot v_2 = 6$

$$= \frac{2}{3} \begin{bmatrix} 1 \\ -1 \\ 0 \\ 1 \end{bmatrix} + \frac{\cancel{2}}{3\cancel{6}} \begin{bmatrix} 2 \\ 0 \\ 2 \\ -2 \end{bmatrix}$$

$$= \begin{bmatrix} 2/3 \\ -2/3 \\ 0 \\ 2/3 \end{bmatrix} + \begin{bmatrix} 2/3 \\ 0 \\ 2/3 \\ -2/3 \end{bmatrix} = \begin{bmatrix} 4/3 \\ -2/3 \\ 2/3 \\ 0 \end{bmatrix}$$

Ans

DO NOT WRITE BELOW THIS LINE

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The basis obtained by youis the basis for W not for $W^\perp$. You did mistake.

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The basis obtained by you is not for $W^\perp$. This is for W.

MA101:2017-18:Odd:Midsem

Mathematics-I

Graded Answer Script of 170108024:Mukul Ranjan;T7;2203;8;38

Question	YourScore	ClassAverage	ClassMax	ClassMin
1	3	1.02	3	0
2	0	2.49	5	0
3	0	.32	2	0
4	1	1.32	3	0
5	0	.27	4	0
6	2	1.02	5	0
7	0	.29	5	0
8	2	.50	3	0
TOTAL	8	7.27	26	0