EE 101
Electrical Sciences

Department of Electronics & Electrical Engineering

Lecture 10-11

3-Phase Systems

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3-phase Systems- Introduction

- A brief history….
- Almost all electric power generation and transmission in the world today is in the form of 3-phase
- Most of the large industrial loads are also 3-phase
- Small loads (such as residential loads) are 1-phase, but this is simply a tap-off from a 3-phase system
- A 3-phase system is an integration of three 1-phase systems
- A 3-phase system consists of 3-phase generators, 3-phase transmission lines and 3-phase loads
- A 3-phase power system is preferred over a 1-phase power system for the following reasons:
    - 3-phase generators, motors and transformers are simpler, cheaper and more efficient
    - 3-phase transmission lines deliver more power for a given cost or for a given weight of conductor
    - Voltage regulation of a 3-phase system is inherently better

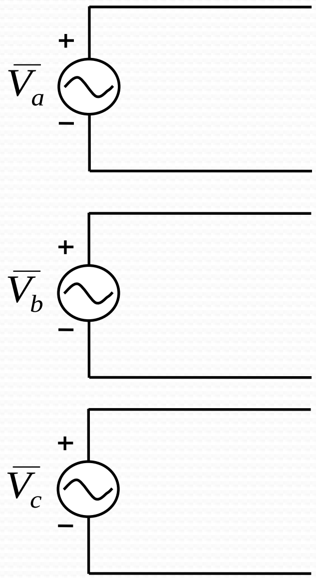
Generation of 3-phase Voltages and Currents

- A 3-phase generator consists of three 1-phase generators
- All three voltages are equal in magnitude but differing in phase angle from others by 120°
- Quantities of three phases (of a 3-phase system) are represented by using subscripts '*a*', '*b*' and '*c*'. They are called phase-*a*-, phase-*b* and phase-*c*

Expression of Three Phase Voltages

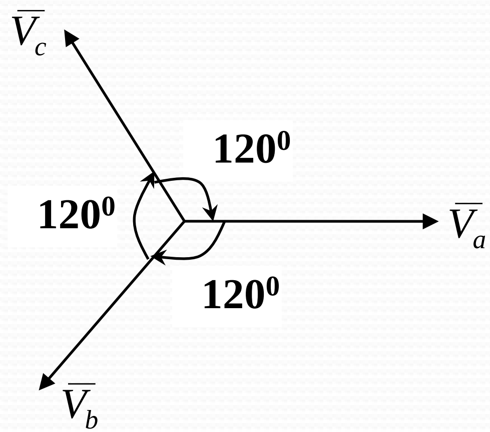
Time domain		Phasor
$v_a(t) = \sqrt{2}V\sin(\omega t)\,V$	$\Leftrightarrow$	$\bar{V}_a = V\angle 0^\circ\,V$
$v_b(t) = \sqrt{2}V\sin(\omega t - 120^\circ)\,V$	$\Leftrightarrow$	$\bar{V}_b = V\angle -120^\circ\,V$
$v_c(t) = \sqrt{2}V\sin(\omega t + 120^\circ)\,V$	$\Leftrightarrow$	$\bar{V}_c = V\angle 120^\circ\,V$

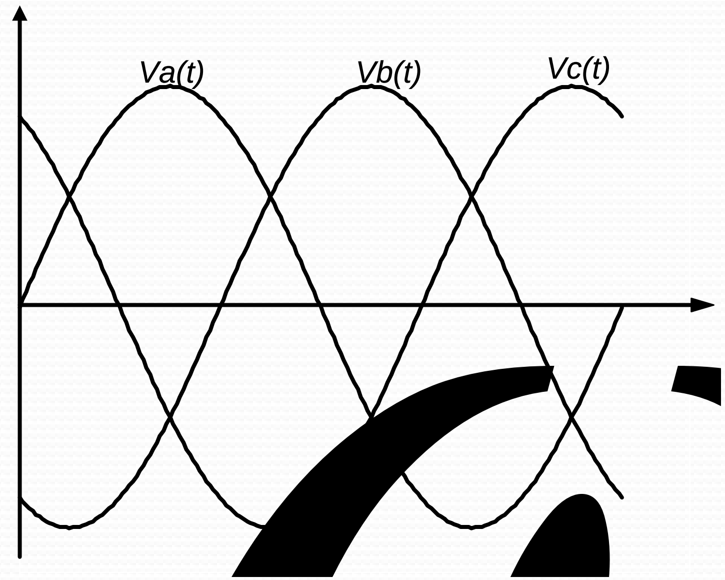
Nature of 3-phase Voltages



Three 1-ph generators

Phasor diagram

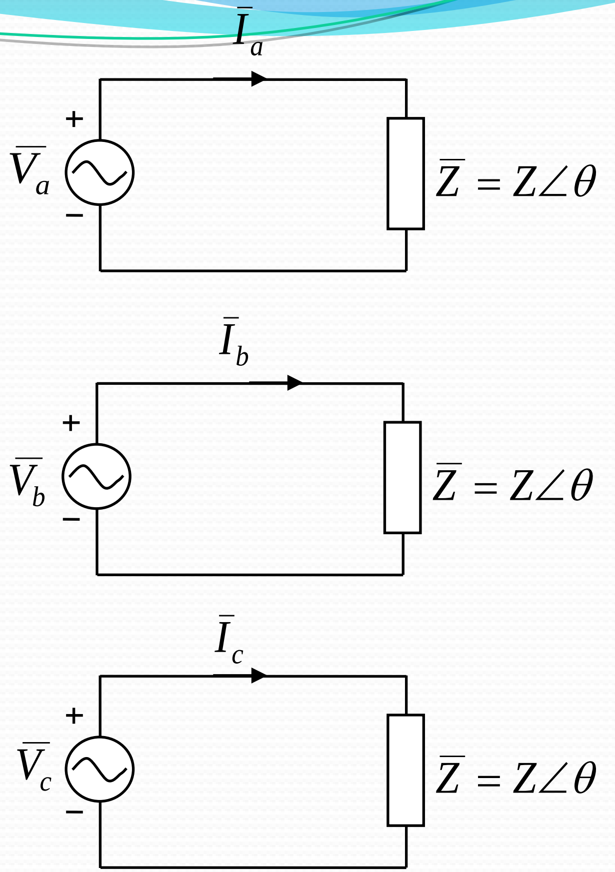




Voltage waveforms

Nature of 3-phase Currents

- Each of these three generators can be connected to one of the three identical loads by a pair of wires.
- The load impedance is considered as:
  $\bar{Z} = Z\angle\theta\,\Omega$
- Such a system is basically three 1-phase circuits which happen to differ in phase angle by 120°



Currents in the above circuits are:

$$\bar{I}_a = \frac{\bar{V}_a}{\bar{Z}} = \frac{V\angle 0^\circ}{Z\angle\theta} = \frac{V}{Z}\angle -\theta^\circ \quad \Rightarrow \quad I\angle -\theta$$

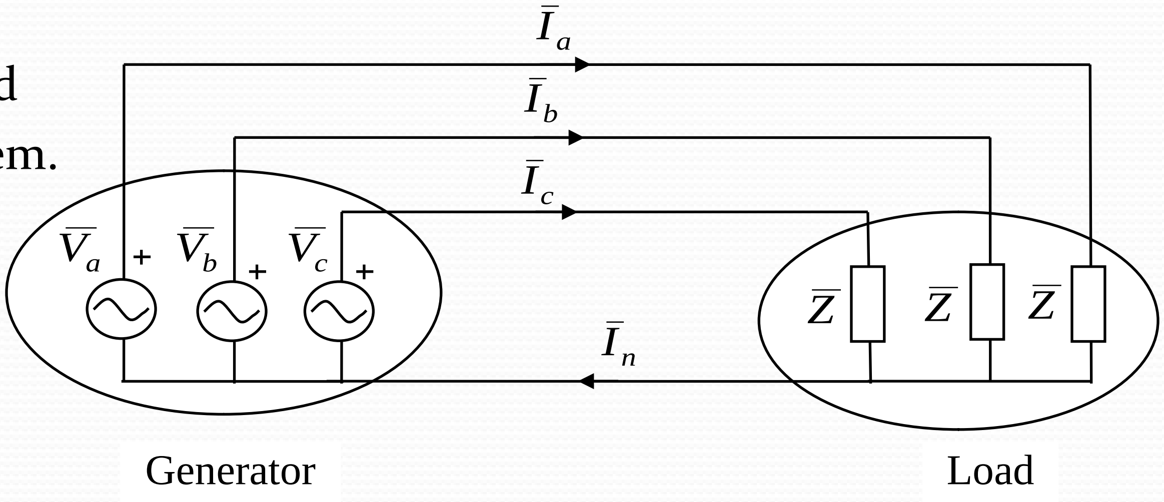
$$\bar{I}_b = \frac{\bar{V}_b}{\bar{Z}} = \frac{V\angle -120^\circ}{Z\angle\theta} = \frac{V}{Z}\angle -120-\theta^\circ \quad \Rightarrow \quad I\angle -120-\theta$$

$$\bar{I}_b = \frac{\bar{V}_c}{\bar{Z}} = \frac{V\angle +120^\circ}{Z\angle\theta} = \frac{V}{Z}\angle 120-\theta^\circ \Rightarrow \quad I\angle 120-\theta$$

All currents have the same magnitude (V/Z) but a phase shift of 120° from others

Integration of Three Generators

- The above three 1-phase systems require six wires (two for each system)
- In fact, three of the six wires are not necessary for the generators to supply power to the loads
- Consider that the three negative ends of the generators are joined together. Similarly, the three lower ends of the loads are also joined together
- Three return wires can now be replaced by a single wire called neutral wire. Current can still return from the loads to the generators as shown in the following figure
- Such a system is called **3-phase, 4-wire** system. It consists of three phase wires and a neutral wire

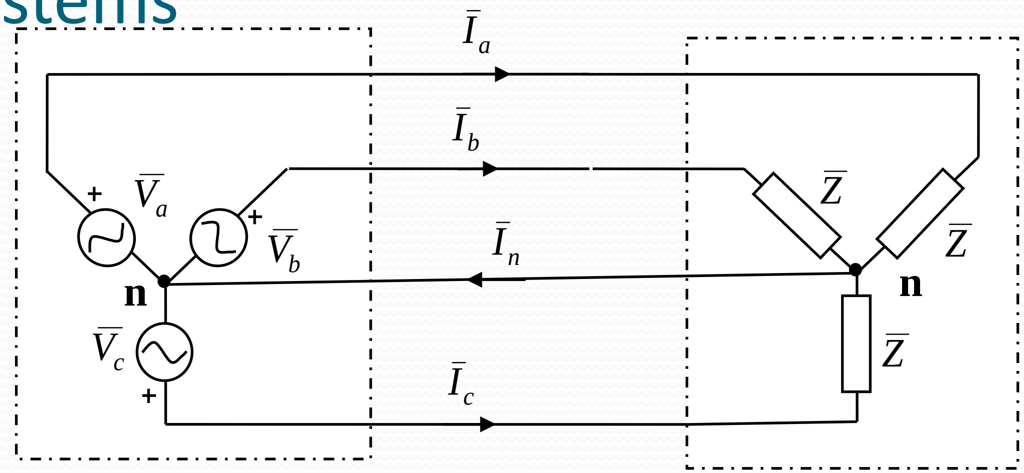


4-wire and 3-wire Systems

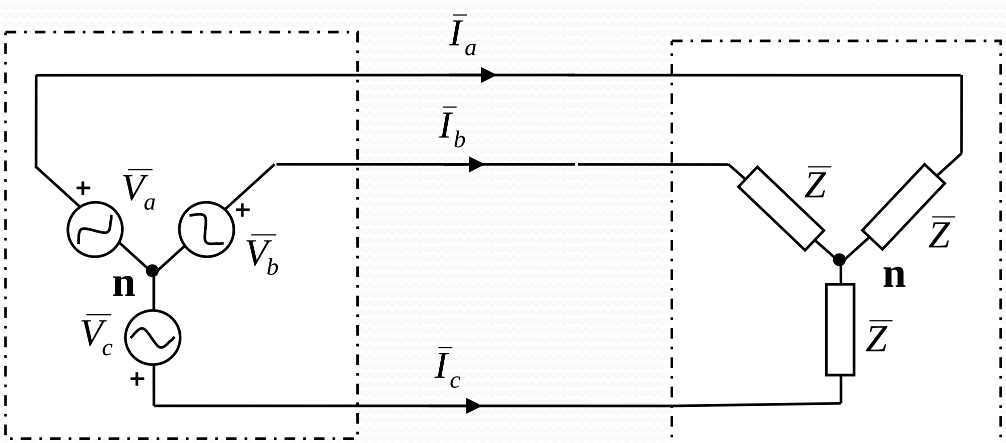
- Current in the neutral wire can be written as (using KCL)

$$\bar{I}_n = \bar{I}_a + \bar{I}_b + \bar{I}_c$$
$$= I\angle -\theta + I\angle(-\theta - 120°)$$
$$+ I\angle(-\theta + 120°) \;=\; 0$$

- As long as the three loads are identical, the current in the neutral line is always zero
- Thus, the neutral line can be eliminated (as shown in the following figure) and power can still be transferred from the generators to the loads
- Such a system is called **3-phase, 3-wire** system



With a neutral line



Without a neutral line

Balanced 3-phase Systems

- A 3-phase source is called balanced if it consists of three voltages of equal magnitude with phase shifts of exactly 120° from others

  Example:

  $$\overline{V}_a = V\angle\alpha\, V \qquad \overline{V}_b = V\angle(\alpha - 120°)\, V \qquad \overline{V}_c = V\angle(\alpha + 120°)\, V$$

- A 3-phase source is called unbalanced if the voltage magnitudes are not equal and/or the phase shifts are not exactly 120°
- A 3-phase load is called balanced if all three impedances are identical. Identical means all impedances have the same magnitude and same angle
- A 3-phase transmission system is called balanced if the impedances of all three lines are identical
- A 3-phase system is called balanced if the sources, transmission systems and loads are balanced
- In this subject, we will mainly concentrate on balanced 3-phase systems

Phase Sequence

There are two possibilities for the phase sequence.

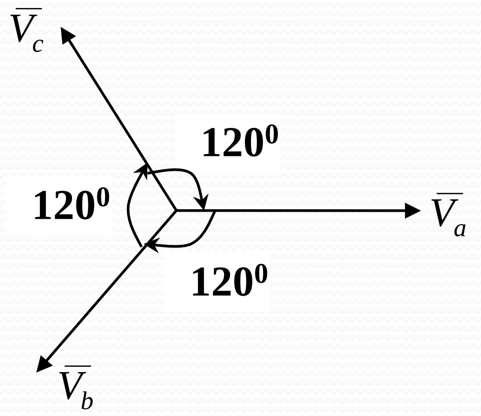
(a) Phase sequence a-b-c

In this case, *a* leads *b*, *b* leads *c* and *c* leads *a* (in the sequence of a-b-c)

$$\bar{V}_a = V\angle 0^\circ\, V$$

$$\bar{V}_b = V\angle -120^\circ\, V$$

$$\bar{V}_c = V\angle 120^\circ\, V$$



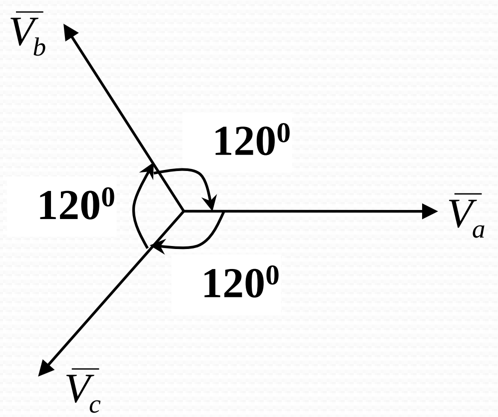
(b) Phase sequence a-c-b

In this case, leads , leads and leads (in the sequence of a-c-b)

$$\bar{V}_a = V\angle 0^\circ\, V$$

$$\bar{V}_c = V\angle -120^\circ\, V$$

$$\bar{V}_b = V\angle 120^\circ\, V$$

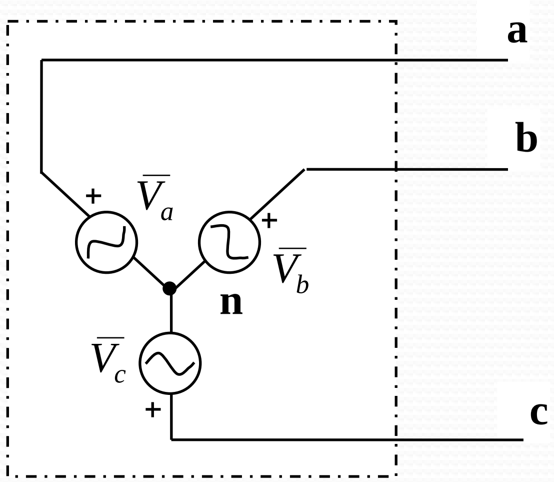


Connections of 3-phase Generators and Loads

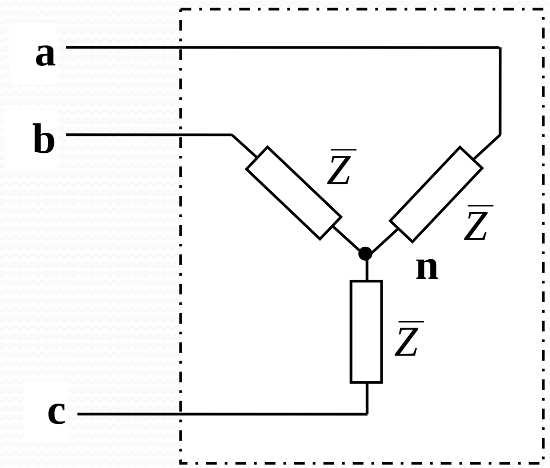
- A 3-phase generator or load may be connected in two ways

1. **Star or Y (Wye) connection**

- In star or Y-connection, one end of each source (or load) is connected to a common point called neutral point. In a balanced system, the voltage at the neutral point is zero and sometimes it is connected to the ground



Y-Connected generator

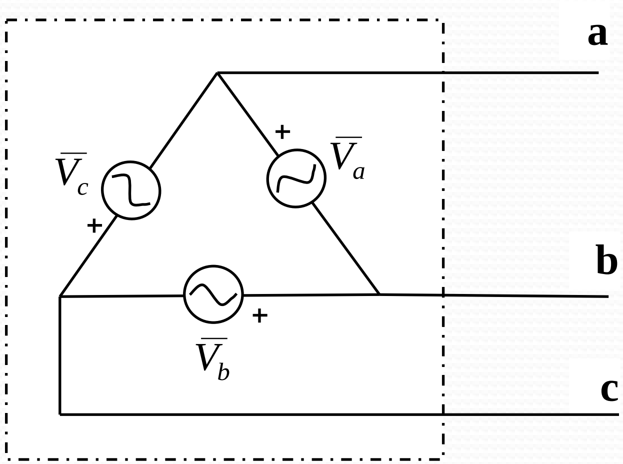


Y-Connected load

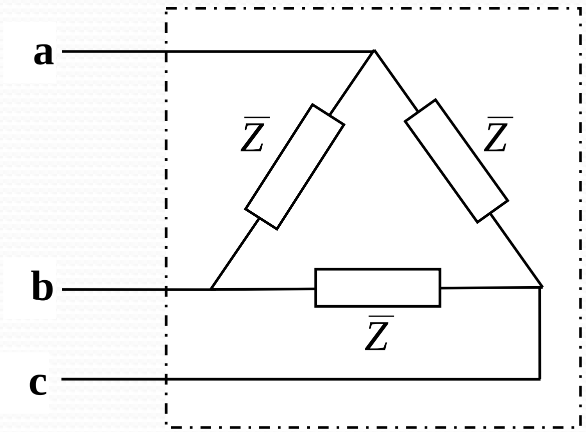
Connections of 3-phase Generators and Loads

2. **Mesh or Δ (Delta) connection**

- In mesh or Δ-connection, three sources (or loads) are connected in Δ (or closed form) as shown below. There is no neutral point in Δ-connection



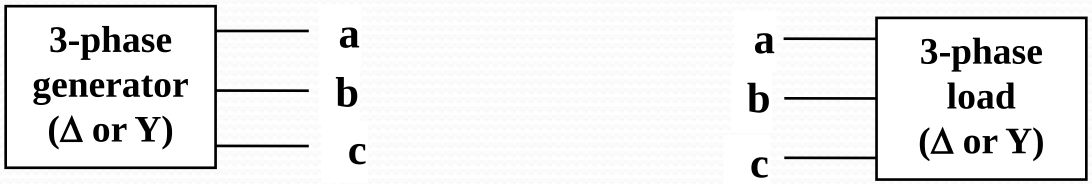
Δ-Connected generator



Δ-Connected load

- Any number of Y and Δ connected generators and loads may be mixed up in a power system

General Representation of 3-phase Generators and Loads



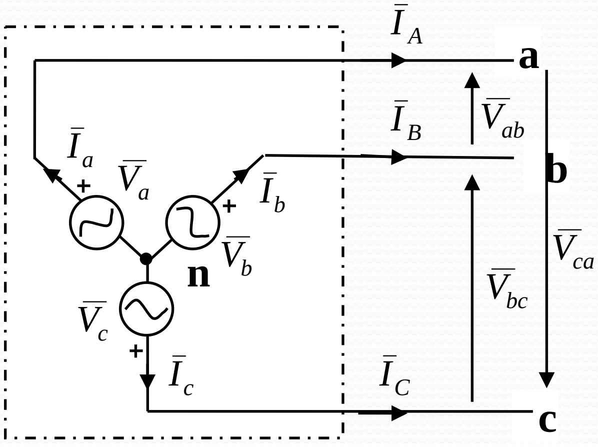
3-phase, 3-wire system

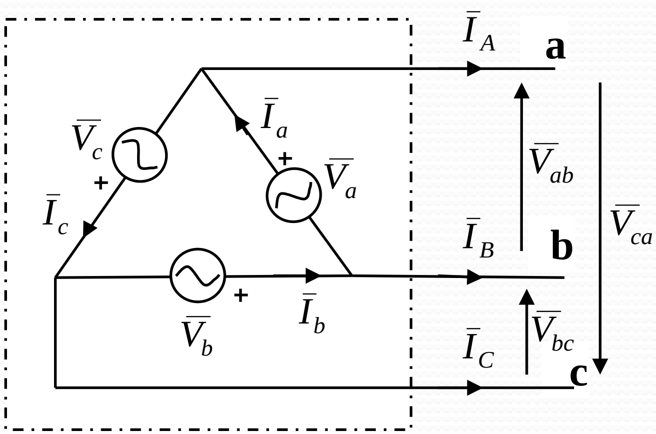
3-phase, 4-wire system

- In Δ-connection, there is no neutral point and thus it is always 3-phase, 3-wire.
- However, a Y-connected system can be 3-phase, 3-wire or 3-phase, 4-wire

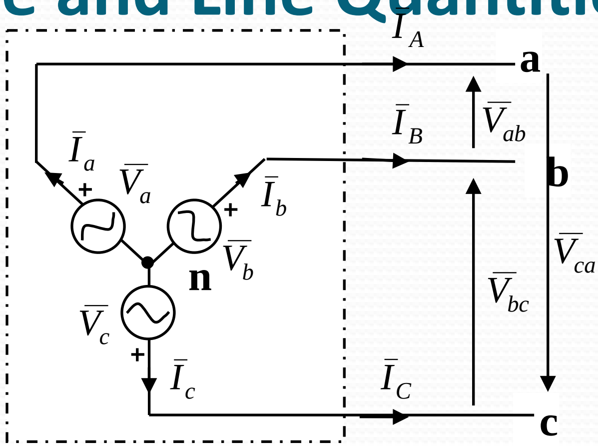
Phase and Line Quantities

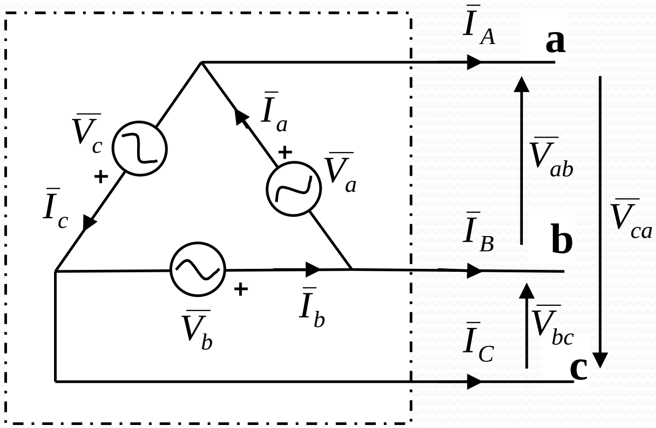
- **In both Y- and Δ-connection**
  - Voltage of each source or across each load impedance (inside the dotted box) is called phase voltage
  - Current of each source or through each load impedance (inside the dotted box) is called phase current
  - Voltage between any two lines (outside the dotted box) is called line voltage. Exclude the neutral wire (if any)
  - Current through any line (outside the dotted box) is called line current. Exclude the neutral wire (if any)





Phase and Line Quantities





In the Above Diagrams

- V_a, V_b, and V_c are phase voltages and V_{ab}, V_{bc}, and V_{ca} are line voltages
- I_a, I_b, and I_c are phase currents and I_A, I_B, and I_C are line currents
- Phase quantities are inside the dotted box (which may not be accessible) and line quantities are outside the dotted box (which are always available)
- **By Inspection**

For Y-connection	Line current = phase current
	Line voltage ≠ phase voltage
For Δ-connection	Line voltage = phase voltage
	Line current ≠ phase current

Relationship between V_L and V_p of Y-connection

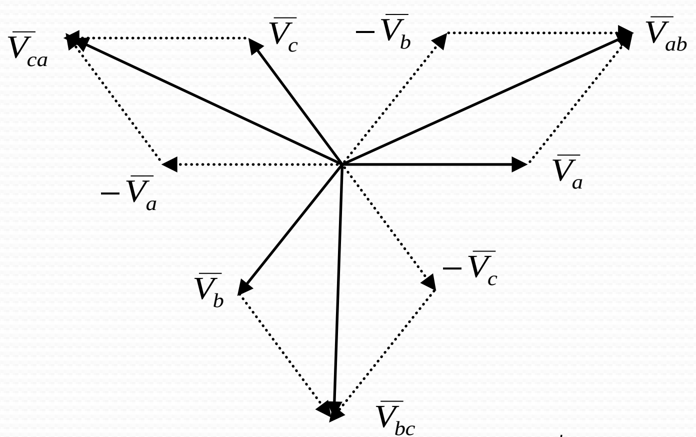
- Let the phase voltages be (for phase sequence a-b-c) be:

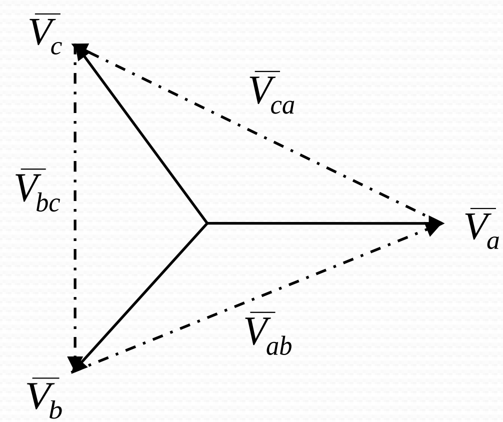
$$\bar{V}_a = V\angle 0°, \qquad \bar{V}_b = V\angle -120°, \qquad \bar{V}_c = V\angle 120°$$

- Then, the line voltages are:

$$V_{ab} = V_a - V_b = V\angle 0° - V\angle -120° = \sqrt{3}V\angle 30°$$
$$V_{bc} = V_b - V_c = V\angle -120° - V\angle 120° = \sqrt{3}V\angle -90°$$
$$V_{ca} = V_c - V_a = V\angle 120° - V\angle 0° = \sqrt{3}V\angle 150°$$

- The above voltages are shown in the following phasor diagrams





- Magnitude of line voltage = $\sqrt{3}\times$magnitude of phase voltage, and the line voltages are shifted by 30° from the phase voltages.

Relationship between I_L and I_p of Δ-connection

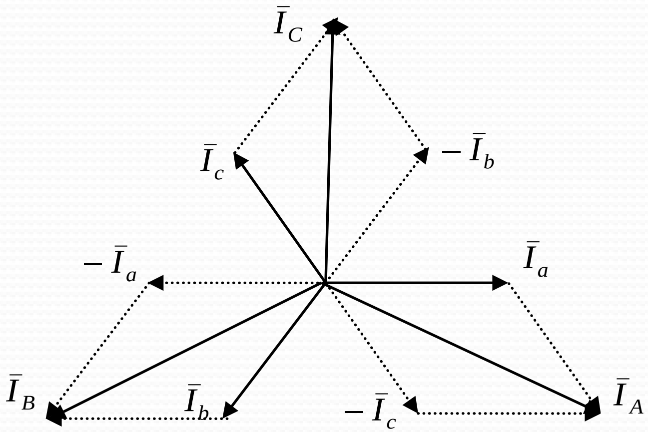
- Let the phase currents (for phase sequence a-b-c) be:

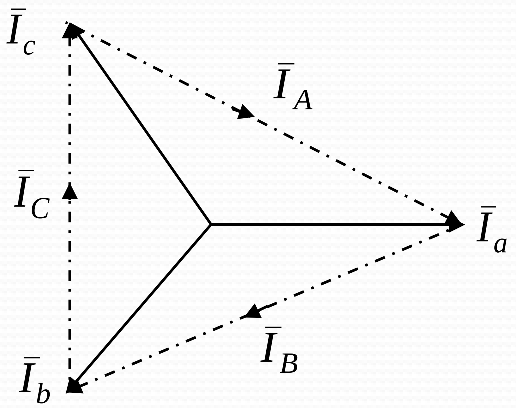
$$\bar{I}_a = I\angle 0°, \qquad \bar{I}_b = I\angle -120°, \qquad \bar{I}_c = I\angle 120°$$

- Then, the line currents (using KCL) are:

$$I_A = I_a - I_c = I\angle 0° - I\angle 120° = \sqrt{3}I\angle -30°$$
$$I_B = I_b - I_a = I\angle -120° - I\angle 0° = \sqrt{3}I\angle -150°$$
$$\bar{I}_C = I_c - I_b = I\angle 120° - I\angle -120° = \sqrt{3}I\angle 90°$$

- The above currents are shown in the following phasor diagrams



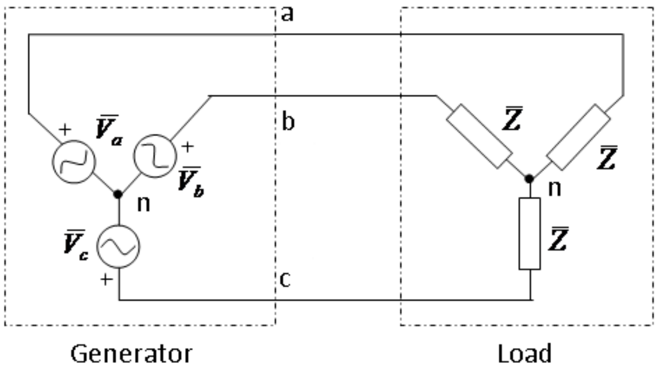


- Magnitude of line current = √3×magnitude of phase current, and the line currents are shifted by 30° from the phase currents.

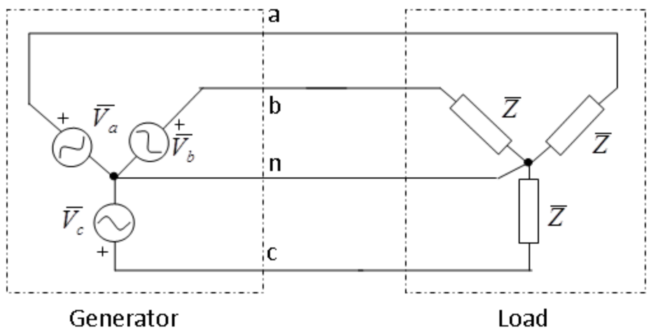
Analysis of a Balanced 3-phase Y-Y System

- Consider a 3-phase Y-connected generator supplying power to a 3-phase Y-connected load as shown in the figure.

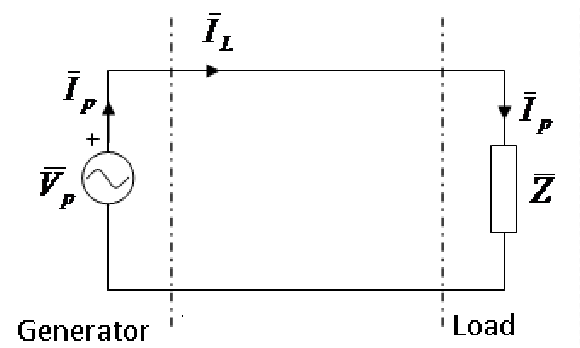
  Let, $\bar{V}_a = V\angle\alpha \, V$, $\bar{V}_b = V\angle(\alpha - 120°)\, V$,
  $\bar{V}_c = V\angle(\alpha + 120°)\, V$, and
  $\bar{Z} = Z\angle\theta$



- Add a fictitious neutral line between the generator neutral point and the load neutral point as shown in the following. Note that addition of the neutral line has no effect because it does not carry any current (under balanced condition).



- Thus, the 3-phase circuit can be analyzed as three single-phase circuits, each consisting of one phase and the neutral line as shown.

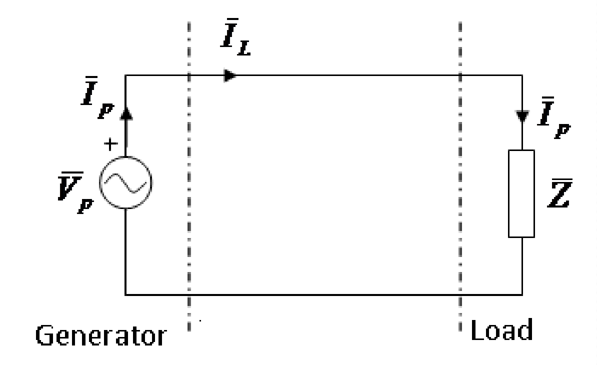


Analysis of YY system:
Single Phase Equivalent Circuit

- All three phases are identical except for a phase shift of 120° in the voltages.
- Therefore, the results of all the phases will be the same, except for the proper phase shift of 120°.
- For example, for phase *a*, $\bar{I}_a = \dfrac{V\angle 0°}{Z\angle\theta} = \dfrac{V}{Z}\angle -\theta$

  And complex power of phase *a* is:

  $$\bar{S}_a = \bar{V}\ \bar{I}_a{}^* = V_a I_a \angle\theta$$



- Then, it can be easily seen that:

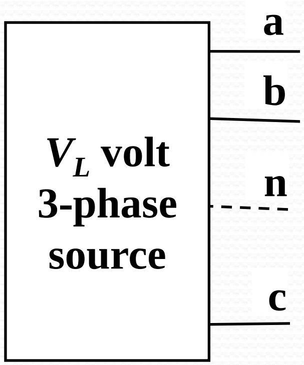
  $$\bar{I}_b = \frac{V}{Z}\angle -\theta - 120°, \qquad \bar{I}_c = \frac{V}{Z}\angle -\theta + 120°$$

  And, $\bar{S}_b = \bar{S}_c = \bar{S}_a = \bar{V}\ \bar{I}_a{}^* = V_a I_a \angle\theta$
- The complex power of all three phases: $\bar{S}_{3ph} = 3V_{ph} I_{ph} \angle\theta$
- Thus one phase of the 3-phase Y-Y system adequately describes all three phases, and it is called the Single Phase Equivalent Circuit of the 3-phase system.

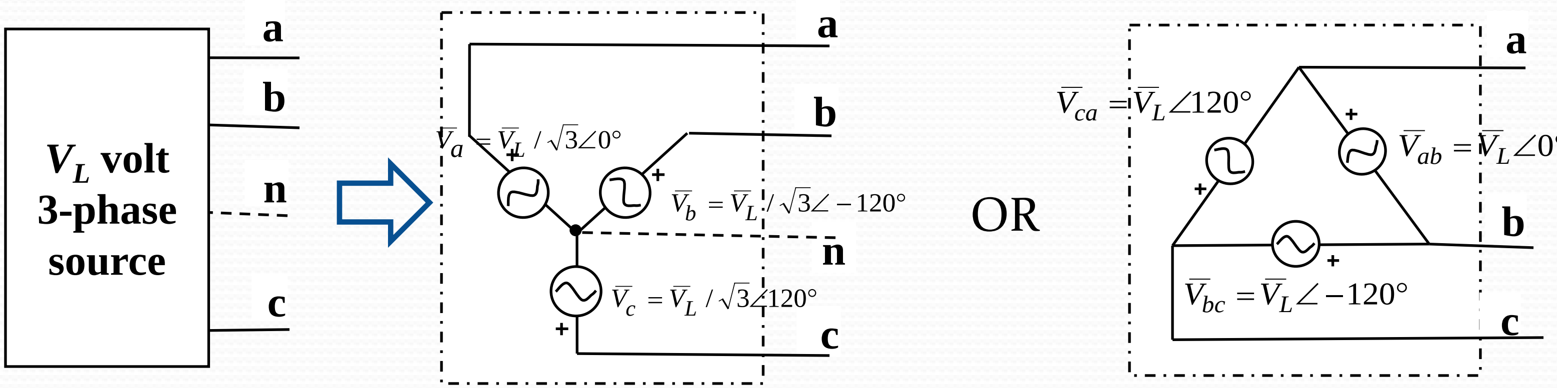
3-Phase Conventions

- In practice, the three phase supply comes from a large number of 3-phase sources, many could be in Star configuration and many in Delta.

  Therefore 3-phase sources are commonly shown and specified as depicted in the figure.



The source may be represented as a Star or a Delta as needed.



3-Phase Conventions

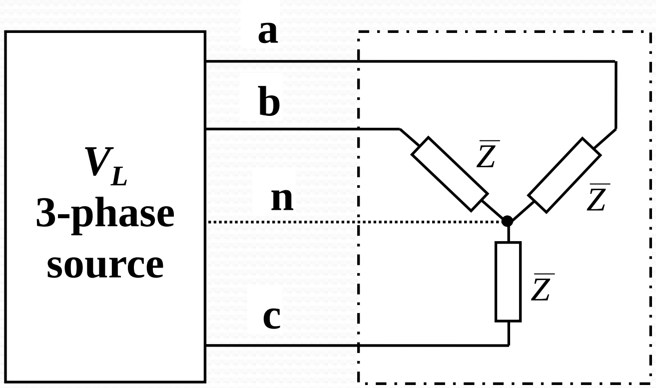
- Unless specified otherwise, the following convention is understood in 3-phase systems:
  - Voltage of a 3-phase system is always specified as line voltage (only magnitude or rms value)
  - Current of a 3-phase system is always specified as line current (only magnitude or rms value)
  - Power (P, Q or S) of a 3-phase system is always specified as total power
  - However, the impedance of a 3-phase system can always specified as phase impedance.

Example

- A 415 V, 3-phase source supplies power to a balanced 3-phase Y-connected load having an impedance of (16+j12) Ω per phase. Determine the current and complex power absorbed by the load.
- **Solution**

  Since the load is in Y, it would be proper to represent the source in Y also.

  Here V_L = 415 V

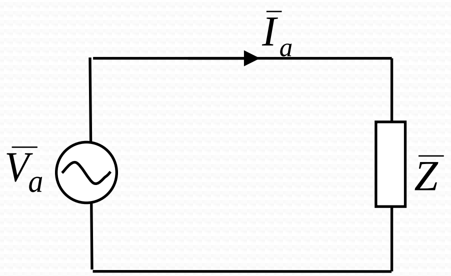


$$\Rightarrow \quad V_p = \frac{V_L}{\sqrt{3}} = \frac{415}{\sqrt{3}} = 239.6 \quad \text{V/phase}$$

- Consider V_a as the reference

$$\Rightarrow \quad \bar{V}_a = 239.6\angle 0° \quad \text{V}$$

$$\therefore \quad \bar{I}_a = \frac{\bar{V}_a}{\bar{Z}} = \frac{239.6\angle 0°}{(16+j12)} = 11.98\angle -36.87° \text{ A}$$



And, $\bar{S}_a = \bar{V}_a \bar{I}_a^* = 239.6\angle 0° \times 11.98\angle 36.87°$

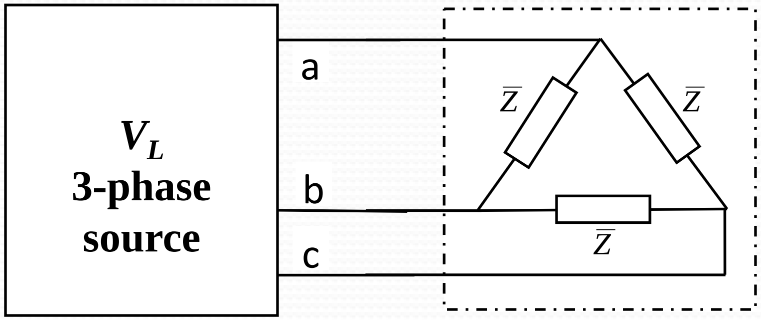
$$= 2870.4\angle 36.87 = (2296.3 + j1722.2)\, VA$$

Finally, I_p = 11.98 A ⇔ $I_L = I_p$ = 11.98 A, and

$\bar{S}_p = 2870.4\angle 36.87°$ VA ⇔ $\bar{S}_T = 3\bar{S}_p$ VA

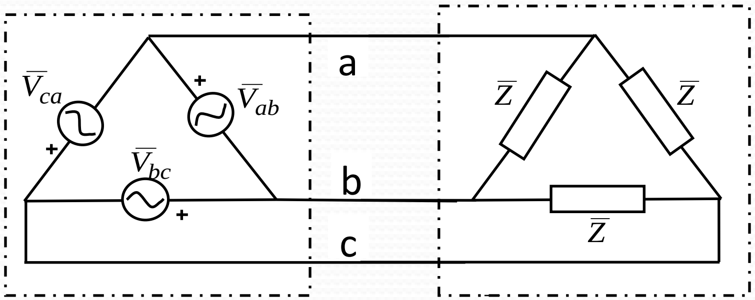
Example

- A 415 V, 3-phase source supplies power to a balanced 3-phase Δ -connected load having an impedance of (16+j12) Ω per phase. Determine the current and complex power absorbed by the load.
- **Solution**
  Since the load is in Δ, it is possible to represent the source in Δ also, and directly find the phase currents.



Here V_L = 415 V

$$\Rightarrow \quad V_p = V_L = 415 \ \text{V/phase}$$

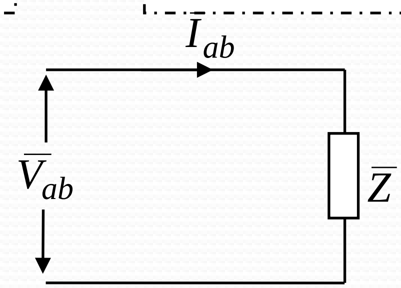


- Consider V_{ab} as the reference

$$\Rightarrow \quad \bar{V}_{ab} = 415\angle 0^\circ \ \text{V} \qquad \therefore \quad \bar{I}_{ab} = \frac{\bar{V}_{ab}}{\bar{Z}}$$

However, if you like to find the line current I_a,

Then, it should be noted that $\bar{I}_a = \bar{I}_{ab} + \bar{I}_{ac} = \bar{I}_{ab} - \bar{I}_{ca}$, etc.



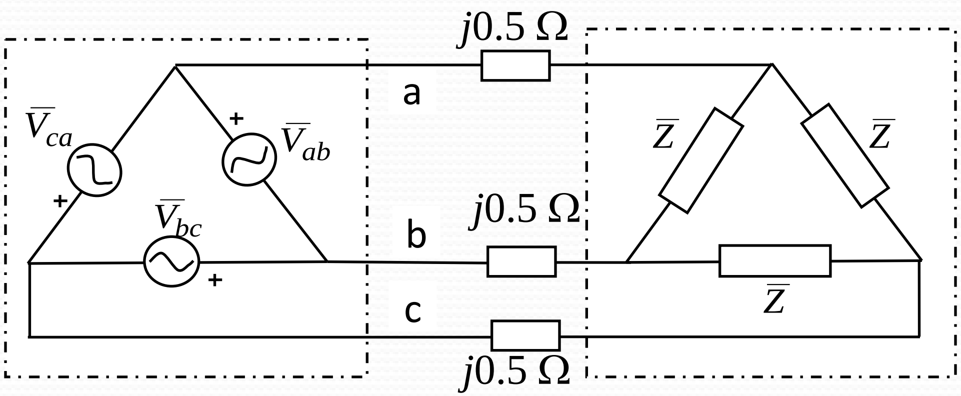
Example

- A 415 V, 3-phase source supplies power to a balanced 3-phase Δ -connected load having an impedance of (16+j12) Ω per phase. Determine the current and complex power absorbed by the load.

  Consider that the line connecting the source to the load has an impedance of $j0.5\ \Omega$ per phase

- **Solution**

  In the presence of tr. line impedance, the Δ Configuration is not easy to analyze.



So, conversion to Y-Y system is preferable.

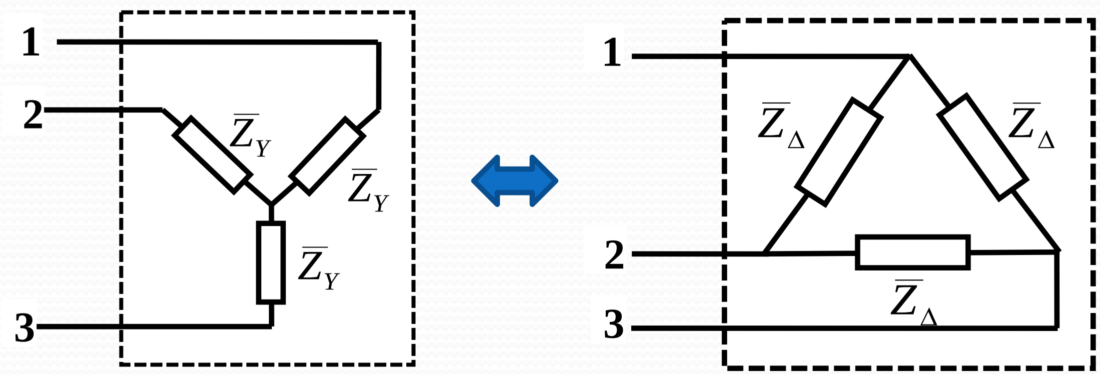
The general approach is to adopt the Y-Y configuration converting all Δs to Y whenever required.

Delta Star Conversion for Balanced Systems

The equivalence between star and delta connected networks were shown to be:

$$Z_{23} = Z_2 + Z_3 + \frac{Z_2 Z_3}{Z_1}, \qquad Z_{31} = Z_3 + Z_1 + \frac{Z_3 Z_1}{Z_2}, \qquad Z_{12} = Z_1 + Z_2 + \frac{Z_1 Z_2}{Z_3}$$

$$Z_1 = \frac{Z_{12} Z_{13}}{Z_{12} + Z_{13} + Z_{23}}, \qquad Z_2 = \frac{Z_{21} Z_{23}}{Z_{12} + Z_{13} + Z_{23}}, \qquad Z_3 = \frac{Z_{31} Z_{32}}{Z_{12} + Z_{13} + Z_{23}}$$



If the networks are balanced, it can be easily observed that,

$$Z_\Delta = 3Z_Y \qquad \text{and,} \qquad Z_Y = \frac{Z_\Delta}{3}$$

Analysis of a Balanced 3-phase System

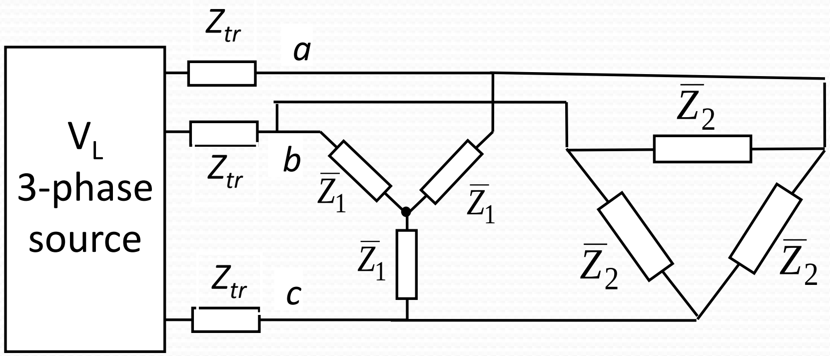
- The general approach to solve balanced 3-phase systems is outlined below:

Convert 3-ph system to Y-Y configuration → **Draw the 1-ph equivalent circuit** → **Solve the 1-ph equivalent circuit** → **Get the 3-ph results**

- Convert the system to Y-Y configuration converting all Δs to Y whenever required.
- Obtain the phase quantities to construct the 1-phase equivalent circuit
- The balanced 3-phase system can be analyzed from its single-phase equivalent circuit
- At the end, results of the 3-phase system can be obtained from the results of the 1-phase equivalent circuit using the standard relationships

Example

- The general approach to solve balanced 3-phase systems is outlined below using the system shown.:



- Represent the source in Y, and get the reference phase voltage.
- The transmission line impedances are already in Y configuration
- The load impedance Z_1 is already in Y configuration.
- Convert the load impedance Z_2 to Y configuration ($Z_{2Y} = Z_2/3$)

Given: V_L=433 V, Z_{tr}= j0.1 Ω, Z_1=1.4+j4.6 Ω, and Z_2=4.2+j13.8 Ω
Find the current output of the source,
The phase currents in load 1, and load 2
And, find whatever you want, say pf and power output of source, etc.

Ans: 100 A, 100 A, 57.7 A, etc.

Thanks !