Time: 3 hours **PH101/Physics I/End-semester/24.11.2015** **Marks: 50**

♦**INSTRUCTIONS**♦

1. *The following questions have* ***ONLY ONE CORRECT ANSWER each.***
2. *Write the CORRECT OPTION in the ANSWER SHEET ONLY WHICH IS PROVIDED AT THE END OF THIS QUESTION PAPER.*
3. *Use the answer booklet for rough work. SUBMIT ONLY THE ANSWER SHEET to the invigilator.*
4. ***ALL QUESTIONS CARRY 1/2 NEGATIVE MARKS FOR INCORRECT ANSWER***.
5. *USEFUL DATA:* $c = 3.0 \times 10^8$ m/s; $m_e = 9.11 \times 10^{-31}$ kg; *Rest mass energy of an electron,* $m_ec^2 = 0.512$ *MeV.*

1. A particle of rest mass m_0 and kinetic energy $5m_0c^2$ strikes and sticks to a stationary particle of rest mass $3m_0$. The rest mass M_0 of the composite particle is **[3]**

(A) $\sqrt{45}m_0$ (B) m_0 (C) $\sqrt{46}m_0$ (D) $9\ m_0$

2. Two rods of proper length 10 m move towards each other with velocity $0.9c$ along a common axis parallel to their lengths. The length of one rod as seen with respect to the other is **[2]**

(A) 1.1 m (B) 8.7 m (C) 4.4 m (D) 2.2 m

3. For a particle of mass m in an infinite square well potential of $V(x) = 0,\ 0 \le x \le a$; $V(x) = \infty$, otherwise, the wave function at $t = 0$ is $\Psi(x, 0) = A[\psi_1(x) + \psi_2(x) + \psi_3(x) + \psi_4(x)]$. A is a constant, ψ_i is the wave function corresponding to the i^{th} energy state and a is the dimension of the square well. The expectation value of H is **[2]**

(A) $\dfrac{15\pi^2\hbar^2}{ma^2}$ (B) $\dfrac{15\pi^2\hbar^2}{\sqrt{2}ma^2}$ (C) $\dfrac{15\pi^2\hbar^2}{2ma^2}$ (D) $\dfrac{15\pi^2\hbar^2}{4ma^2}$

Paragraph for questions 4-5

A merry-go-round (MGR) fixed on the ground is rotating at an angular velocity of 0.5 rad/sec. A man of weight 50 kg standing at a distance 4 m from the centre of the MGR is running in a circular path drawn on the MGR with an angular velocity of 0.3 rad/sec relative to MGR and in the same direction of motion of the MGR.

4. The centripetal force felt by the man as seen from the ground frame is **[2]**

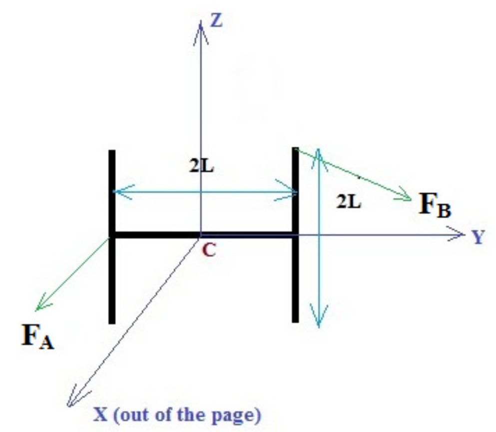
(A) 18 N (B) 128 N (C) 68 N (D) 8 N

5. The magnitude of the coriolis force felt by the man in the MGR frame is **[2]**

(A) 50 N (B) 110 N (C) 60 N (D) 10 N

Paragraph for questions 6-8

Three rods of mass m= 1 kg and length $2L$ = 2 m each are attached as shown. The center of mass C of the rigid body is fixed and is the origin of the coordinate system. X, Y and Z are the time-dependent, body-fixed principal directions that spin along with the rigid body. The unit vectors in these directions are $\hat{e}_1(t),\ \hat{e}_2(t)$ and $\hat{e}_3(t)$, respectively.



6. This rigid body is seen to spin with an angular velocity given by $\vec{\omega}(t) = \dfrac{2\pi}{\sqrt{3}}\left(\hat{e}_1(t) + \hat{e}_2(t) + \hat{e}_3(t)\right)\ rad/\sec$. At time $t = 0$ it is given that these directions are the laboratory-fixed, Cartesian directions $\hat{e}_1(t=0) = \hat{x},\ \hat{e}_2(t=0) = \hat{y}$ and $\hat{e}_3(t=0) = \hat{z}$. At time t = 0.5 sec, $\hat{e}_3(t=0.5)$ is, **[4]**

(A) $\dfrac{1}{3}(2\hat{x} + 2\hat{y} - \hat{z})$ (B) $\dfrac{1}{\sqrt{3}}(\hat{x} + \hat{y} + \hat{z})$ (C) $\dfrac{1}{3}(2\hat{x} - \hat{y} + 2\hat{z})$ (D) $\hat{z}$

7. A force $\mathbf{F_A}$ parallel to the body-fixed X-axis acts at the location shown in the figure and another force $\mathbf{F_B}$ perpendicular to the body-fixed Z axis acts at the location shown. The magnitude of $\mathbf{F_A}$ (in newtons) so that the body spins as shown in Q6 i.e. with angular velocity $\vec{\omega}(t) = \dfrac{2\pi}{\sqrt{3}}\left(\hat{e}_1(t) + \hat{e}_2(t) + \hat{e}_3(t)\right)\ rad/\sec$ is [*Hint: The torque about C when calculated comes out to be* $\vec{\tau}(t) = \dfrac{(2\pi)^2}{9}\left(5\hat{e}_1(t) + 2\hat{e}_2(t) - 7\hat{e}_3(t)\right)\ Nm$], **[3]**

(A) $(2\pi)^2$ (B) $\dfrac{2}{9}(2\pi)^2$ (C) $\dfrac{7}{9}(2\pi)^2$ (D) $\dfrac{5}{9}(2\pi)^2$

8. If instead, this rigid body spins with an angular velocity that is fixed both in magnitude and direction given by $\vec{\omega} = 2\pi\,\hat{z}\ \ rad/\sec$ and the initial directions of the body-fixed axes are $\hat{e}_1(t=0) = \hat{y},\ \hat{e}_2(t=0) = \hat{z}$ and $\hat{e}_3(t=0) = \hat{x}$ then at time t = 0.5 sec, $\hat{e}_1(t=0.5)$ is, **[3]**

(A) $\hat{z}$ (B) $-\hat{x}$ (C) $-\hat{y}$ (D) $(\hat{y} + \hat{z})/\sqrt{2}$

Paragraph for questions 9-11

A stationary state of a quantum particle of mass m is described by a wave-function $\varphi(x) = R/\sqrt{x^2+a^2}$ where a, $R > 0$

9. The normalization of $\varphi(x)$ gives the value of R as **[3]**

(A) $1/\sqrt{2\pi a}$ (B) $\sqrt{a/\pi}$ (C) $\sqrt{2\pi a}$ (D) $\sqrt{\pi/a}$

10. If this particle were classical, the position of stable equilibrium would be at [*Hint: find the potential function first*] **[3]**

(A) $x=\sqrt{2}a$ (B) $x=-\sqrt{2}a$ (C) $x=0$ (D) $x=\infty$

11. A measurement of the position of the quantum particle yields a value corresponding to the point of stable equilibrium in Q10. After some time has elapsed, I measure the position again. This position can only be, **[2]**

(A) $x=\sqrt{2}a$ (B) $x=-\sqrt{2}a$ (C) $x=0$ (D) *Anywhere between* $x=-\infty$ *and* $x=+\infty$

Paragraph for questions 12-15

A particle of mass m slides freely on a frictionless groove, fixed in a horizontal plane, described by $r = r_0 e^{a\theta}$ where a is a positive constant. Initially at t = 0 the speed is v_0 and $\theta = 0$.

12. The angle β the velocity vector makes with position vector is **[2]**

(A) $\beta = \tan^{-1}\left(1/\sqrt{1+a^2}\right)$ (B) $\beta = \tan^{-1}(1/a)$ (C) $\beta = \sin^{-1}\left(1/\sqrt{1+a^2}\right)$ (D) $\beta = \sin^{-1}(1/a)$

13. The angular momentum L at any point is **[3]**

(A) $\dfrac{mv_0r_0}{\sqrt{a^2+1}}e^{a\theta}$ (B) mv_0r_0 (C) $\dfrac{mv_0r_0}{\sqrt{a^2+1}}e^{2a\theta}$ (D) $mv_0r_0e^{2a\theta}$

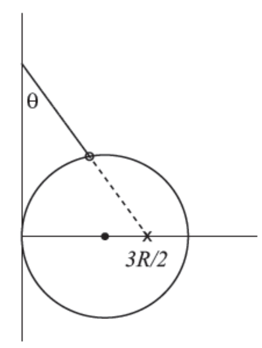
14. Which is the correct unit normal for any one surface of the groove? **[2]**

(A) $\dfrac{\hat{r}-a\hat{\theta}}{\sqrt{a^2+1}}$ (B) $-\dfrac{\hat{r}+a\hat{\theta}}{\sqrt{a^2+1}}$ (C) $\hat{r}$ (D) $-\hat{r}$

15. The normal force N exerted by the groove corresponding to surface of Q14 at any point is **[3]**

(A) $\dfrac{mv_0^2(a^2-1)}{r_0\sqrt{a^2+1}}e^{a\theta}$ (B) $\dfrac{mv_0^2}{r_0\sqrt{a^2+1}}e^{a\theta}$ (C) $\dfrac{2mv_0^2}{r_0\sqrt{a^2+1}}e^{a\theta}$ (D) $\dfrac{mv_0^2}{r_0}e^{a\theta}$

16. A sphere of radius R and mass M is supported by a rope attached to a wall as shown in the Fig. The rope makes an angle θ with respect to the wall. The point where the rope is attached to the surface of ball is such that if the line (dotted line in fig.) of the rope is extended it crosses the horizontal line through the centre of the ball at a distance $3R/2$ from the wall. The coefficient of friction between the wall and the ball is μ_s. For the system to be in static equilibrium at $\theta = 30^0$ the minimum value of the coefficient of static friction μ_s is **[4]**



(A) $1/\sqrt{3}$ (B) $\sqrt{3}$ (C) $\sqrt{3}/2$ (D) $1/(2\sqrt{3})$

17. The kinetic energy of an electron is 6 times its rest energy m_ec^2. The momentum of the electron is **[2]**

(A) 3.74 MeV/c (B) 3.55 MeV/c (C) 3.07 MeV/c (D) 1.79 MeV/c

18. In Michelson interferometer, the vertical and horizontal path lengths are 20 m and 15 m (in S' frame), respectively. If the velocity of the interferometer is 0.6c along the horizontal path length, then the difference between the total travel time between two path lengths in S frame is **[2]**

(A) 1×10^{-8} sec (B) 8×10^{-8} sec (C) 4×10^{-8} sec (D) 5×10^{-8} sec

19. Two trains, each measuring 300 m in their own rest frames, pass by each other traveling in opposite directions. Clocks on train A determine that the front end of train B requires 5.00×10^{-5} sec to traverse the full length of A. A clock in the front end of B reads exactly one o'clock as it passes by the front end of A. How much time is elapsed as it passes by the rear end of A? **[3]**

(A) 9.98×10^{-5} sec (B) 1.99×10^{-5} sec (C) 2.49×10^{-5} sec (D) 4.99×10^{-5} sec