MARKING SCHEME FOR Q1

I. Both ref / rref / solution correct & conclusion with proper justification. (3)

$$\text{RREF (SYSTEM-I)} = \left(\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & -3 \end{array}\right)$$

$$\text{RREF (SYSTEM-II)} = \left(\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

II. Only one ref/ rref/ solution correct & conclusion with proper justification (based on computations) (2)

III. Both ref / rref / solution wrong & conclusion with proper justification (based on computations) (1)

IV. Finding solution of one system as (2, 5, −3). Since (2,5,−3) satisfies the other system, it is also a solution of the other system. Not justifying the uniqueness of solution for this other system & concluding equivalence. (1)

No marks for (Common mistakes)

V. Writing ref / rref of the 4x4 augmented matrix of System II as 3X4 matrix.

VI. Finding solution for System-II only using 3 equations from System-II.

VII. Replacing 2 equations from System-II by only 1 equation & then solving the 3 X4 system.

VIII. Substituting z value from the last equation (in II) in only one of the equation (in II) & then solving the 3 X 4 system.

Type-1

Let $T : \mathbb{R}^3 \to \mathbb{R}^4$ be a linear transformation such that

$$T\left(\begin{bmatrix}1\\1\\1\end{bmatrix}\right) = \begin{bmatrix}1\\0\\1\\0\end{bmatrix}, \quad T\left(\begin{bmatrix}1\\1\\0\end{bmatrix}\right) = \begin{bmatrix}1\\2\\1\\2\end{bmatrix}, \quad T\left(\begin{bmatrix}0\\1\\1\end{bmatrix}\right) = \begin{bmatrix}0\\1\\0\\2\end{bmatrix}$$

1. Determine a matrix A such that $T(x) = Ax$ for all $x \in \mathbb{R}^3$.

Solution:

$$Te_1 = T\left(\begin{bmatrix}1\\0\\0\end{bmatrix}\right) = T\left(\begin{bmatrix}1\\1\\1\end{bmatrix} - \begin{bmatrix}0\\1\\1\end{bmatrix}\right) = \begin{bmatrix}1\\-1\\1\\-2\end{bmatrix}$$

$$Te_2 = T\left(\begin{bmatrix}0\\1\\0\end{bmatrix}\right) = T\left(\begin{bmatrix}1\\1\\0\end{bmatrix} - \begin{bmatrix}1\\0\\0\end{bmatrix}\right) = \begin{bmatrix}0\\3\\0\\4\end{bmatrix}$$

$$Te_3 = T\left(\begin{bmatrix}0\\0\\1\end{bmatrix}\right) = T\left(\begin{bmatrix}0\\1\\1\end{bmatrix} - \begin{bmatrix}0\\1\\0\end{bmatrix}\right) = \begin{bmatrix}0\\-2\\0\\-2\end{bmatrix}$$

Then,

$$A = \begin{bmatrix}Te_1 & Te_2 & Te_3\end{bmatrix} = \begin{bmatrix}1 & 0 & 0\\-1 & 3 & -2\\1 & 0 & 0\\-2 & 4 & -2\end{bmatrix}$$

Marking Scheme:

(a) One or two Te_i correct [1 mark]

(b) All three Te_i correct [2 marks]

(c) Writing matrix A correctly [1 mark]

2. Justify whether such a matrix A as above is unique.

Solution: If possible suppose there exists another matrix B such that $T(x) = Bx$ for all $x \in \mathbb{R}^3$

$$Tx = Ax = Bx$$

$$\implies (A - B)x = 0 \qquad \text{for all } x \in \mathbb{R}^3$$

$$\implies null(A - B) = \mathbb{R}^3$$

$$\implies nullity(A - B) = 3$$

$$\implies rank(A - B) = 0 \qquad \text{(Using rank-nullity theorem)}$$

$$\implies A - B = 0 \implies A = B$$

[2 marks]

Hence it shows that such a matrix A is unique

There are no step marks for this part

Type-2

Let $T : \mathbb{R}^3 \to \mathbb{R}^4$ be a linear transformation such that

$$T\left(\begin{bmatrix}1\\1\\1\end{bmatrix}\right) = \begin{bmatrix}1\\0\\1\\0\end{bmatrix}, \qquad T\left(\begin{bmatrix}1\\1\\0\end{bmatrix}\right) = \begin{bmatrix}1\\2\\1\\2\end{bmatrix}, \qquad T\left(\begin{bmatrix}0\\1\\1\end{bmatrix}\right) = \begin{bmatrix}0\\1\\0\\2\end{bmatrix}$$

1. Determine a matrix A such that $T(x) = Ax$ for all $x \in \mathbb{R}^3$.

Solution: Let $A = \begin{bmatrix}a & b & c\\ l & m & n\\ p & q & r\\ x & y & z\end{bmatrix}$ be a general 4×3 matrix such that $Tx = Ax$ for all $x \in \mathbb{R}^3$.

Then from,

$$T\left(\begin{bmatrix}1\\1\\1\end{bmatrix}\right) = A\left(\begin{bmatrix}1\\1\\1\end{bmatrix}\right) = \begin{bmatrix}1\\0\\1\\0\end{bmatrix}$$

We shall get following three equations,

$$a + b + c = 1 \tag{1}$$

$$l + m + n = 0 \tag{2}$$

$$p + q + r = 1 \tag{3}$$

$$x + y + z = 0 \tag{4}$$

From ,

$$T\left(\begin{bmatrix}1\\1\\0\end{bmatrix}\right) = A\left(\begin{bmatrix}1\\1\\0\end{bmatrix}\right) = \begin{bmatrix}1\\2\\1\\2\end{bmatrix}$$

We shall get following three equations,

$$a + b = 1 \tag{5}$$

$$l + m = 2 \tag{6}$$

$$p + q = 1 \tag{7}$$

$$x + y = 2 \tag{8}$$

From ,

$$T\left(\begin{bmatrix}0\\1\\1\end{bmatrix}\right) = A\left(\begin{bmatrix}0\\1\\1\end{bmatrix}\right) = \begin{bmatrix}0\\1\\0\\2\end{bmatrix}$$

We shall get following three equations,

$$b + c = 0 \tag{9}$$

$$m = n = 1 \tag{10}$$

$$q + r = 0 \tag{11}$$

$$y + z = 2 \tag{12}$$

Solving equations 1, 5 and 9, we get

$$a = 1 \quad b = 0 \quad c = 0$$

Solving equations 2, 6 and 10, we get

$$l = -1 \quad b = 3 \quad c = -2$$

Solving equations 3, 7 and 11, we get

$$l = 1 \quad b = 0 \quad c = 0$$

Solving equations 4, 8 and 12, we get

$$l = -2 \quad b = 4 \quad c = -2$$

Hence matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 3 & -2 \\ 1 & 0 & 0 \\ -2 & 4 & -2 \end{bmatrix}$

Marking Scheme:

(a) One column or two column correct [1 mark]

(b) Writing matrix A correctly [2 mark]

2. Justify whether such a matrix A as above is unique.

Solution: Since in part [a] we took a general matrix and calculated all the entries of matrix and found that value of each entry is coming out to be unique and hence such a matrix is unique. [2 marks]

There are no step marks for this part

Type-3

Let $T : \mathbb{R}^3 \to \mathbb{R}^4$ be a linear transformation such that

$$T\left(\begin{bmatrix}1\\1\\1\end{bmatrix}\right) = \begin{bmatrix}1\\0\\1\\0\end{bmatrix}, \qquad T\left(\begin{bmatrix}1\\1\\0\end{bmatrix}\right) = \begin{bmatrix}1\\2\\1\\2\end{bmatrix}, \qquad T\left(\begin{bmatrix}0\\1\\1\end{bmatrix}\right) = \begin{bmatrix}0\\1\\0\\2\end{bmatrix}$$

1. Determine a matrix A such that $T(x) = Ax$ for all $x \in \mathbb{R}^3$.

Solution:

$$A\begin{bmatrix}1&1&0\\1&1&1\\1&0&1\end{bmatrix} = \begin{bmatrix}A\begin{bmatrix}1\\1\\1\end{bmatrix} & A\begin{bmatrix}1\\1\\0\end{bmatrix} & A\begin{bmatrix}0\\1\\1\end{bmatrix}\end{bmatrix} = \begin{bmatrix}1&1&0\\0&2&1\\1&1&0\\0&2&2\end{bmatrix}$$

Now notice that the matrix $C = \begin{bmatrix}1&1&0\\1&1&1\\1&0&1\end{bmatrix}$ is invertible and hence

$$A = \begin{bmatrix}1&1&0\\0&2&1\\1&1&0\\0&2&2\end{bmatrix} \cdot \begin{bmatrix}1&1&0\\1&1&1\\1&0&1\end{bmatrix}^{-1} = \begin{bmatrix}1&1&0\\0&2&1\\1&1&0\\0&2&2\end{bmatrix} \cdot \begin{bmatrix}1&-1&1\\0&1&-1\\-1&1&0\end{bmatrix}$$

$$\implies A = \begin{bmatrix}1&0&0\\-1&3&-2\\1&0&0\\-2&4&-2\end{bmatrix}$$

Marking Scheme:

(a) If C^{-1} calculated correctly [1 mark]

(b) Doing matrix multiplication and finding A correctly [2 marks]

(c) One column or two column correct [1 mark]

2. Justify whether such a matrix A as above is unique.

Solution: If possible suppose there exists another matrix B such that $T(x) = Bx$ for all $x \in \mathbb{R}^3$. Then,

$$BC = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 1 & 1 & 0 \\ 0 & 2 & 2 \end{bmatrix} \qquad \implies B = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 1 & 1 & 0 \\ 0 & 2 & 2 \end{bmatrix} \cdot C^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 3 & -2 \\ 1 & 0 & 0 \\ -2 & 4 & -2 \end{bmatrix} = A$$

Which shows that $A = B$ and hence there exists unique matrix. [2 marks]

There are no step marks for this part

We have given zero marks for following:

1.

In uniqueness part, writing

$$Ax = Bx \implies (A - B)x = 0$$

So this implies that either $A - B = 0$ or $x = 0$ but $x \neq 0$ and hence $A - B = 0$. This gives that $A = B$ and hence A is unique.

2.

In uniqueness part, writing

$$Ax = Bx \implies (A - B)x = 0 \text{ for all } x \in \mathbb{R}^3$$

This implies that $A - B = 0$ and hence $A = B$. Which gives that A is unique.

Midsem Marking Scheme :: Question No. 3

No partial mark has been awarded for this question

Problem 3. State with proper justification, whether each of the following statements is **True** or **False**.
(a) If A be a nonzero matrix such that $A^{31} = 0$, then all eigenvalues of A are equal to 0 and A is not diagonalizable. $\boxed{1}$

Solution. [Model Answer] **True**

Let λ be an eigenvalue of A. Then there exists a nonzero vector x such that $Ax = \lambda x$. So $A^{31}x = \lambda^{31}x$. Now

$$\begin{aligned} A^{31} = 0 &\Rightarrow \lambda^{31}x = 0 \\ &\Rightarrow \lambda^{31} = 0 \quad (\text{Since } x \neq 0) \\ &\Rightarrow \lambda = 0. \end{aligned}$$

So all eigenvalues of A are zero.

Suppose A is diagonalizable. Then there exists an invertible matrix P such that $P^{-1}AP = D$, where D is a diagonal matrix with diagonal entries as the eigenvalues of A i.e., $D = 0$. Thus $A = 0$, since P is invertible. This is a contradiction to the given condition that $A \neq 0$. Thus A is not diagonalizable.

Hence the given statement is true.

[No Mark is given for this type of answer]

1. Writing 'True' without justification.
2. Not showing one of the following
   (a) All eigenvalues of A are 0.
   (b) A is not diagonalizable.
3. Showing all the eigenvalues of A are 0 with the assumption that A is diagonalizable.
4. Since all eigenvalues of A are 0, so A is not diagonalizable.
5. Since A is not invertible, so A is not diagonalizable.
6. Zero matrix is not an diagonal matrix.
7. Diagonalizable matrix must have n distinct eigenvalues.

Problem 3. (b) Any square matrix is similar to its Reduced Row Echelon Form. $\boxed{1}$

Solution. [Model Answer] **False**

Consider $A = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$. Then $\text{RREF}(A) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. We know that if two matrices are similar then they have same determinant. But $\det(A) = 2 \neq 1 = \det RREF(A)$. So, A and $RREF(A)$ are not similar.

Hence the given statement is false.

[No Mark is given for this type of answer]

1. Writing 'False' without justification.
2. Writing the matrix only.
3. Matrix for counter example is correct, but calculating RREF wrong.
4. Matrix for counter example is correct, RREF is also correct. But wrong argument to show that they are not similar.
5. There exists an invertible matrix E such that $EA = R$, where $R = RREF(A)$. So R is not of the form $P^{-1}AP$.

4. (a) Let A be a real $n \times n$ matrix such that A is similar to A^{2017}. Determine all the possible values for the determinant of A. [1]

Space only for Q. 4 (a)

$A^{2017} \sim A$

i.e $A^{2017} = P^{-1}AP$ for some invertible P

$\Rightarrow (\det A)^{2017} = \det(A)$

$\Rightarrow \det A = 0, 1, -1$ as $\det A$ is ~~rea~~ real

— 1 marks

Roll No.: 0
Name: ..

(b) Let v_1 and v_2 be two nonzero eigenvectors corresponding to distinct eigenvalues λ_1 and λ_2 of a 5×5 matrix A. Show that $v_1 - v_2$ can not be an eignvector of A. [2]

Space only for Q. 4

$Av_1 = \lambda_1 v_1$ and $Av_2 = \lambda_2 v_2$
for non-zero v_1, v_2

Let $A(v_1 - v_2) = \lambda_3 (v_1 - v_2)$

i.e $Av_1 - Av_2 = \lambda_3 (v_1 - v_2)$

i.e $\lambda_1 v_1 - \lambda_2 v_2 = \lambda_3 (v_1 - v_2)$

i.e $(\lambda_1 - \lambda_3) v_1 + (\lambda_3 - \lambda_2) v_2 = 0$

— 1 marks.

As v_1 and v_2 are linearly independent
$\lambda_1 - \lambda_3 = 0$ and $\lambda_3 - \lambda_2 = 0$

$\Rightarrow \lambda_1 = \lambda_2 = \lambda_3 \quad \Rightarrow\Leftarrow$ as (λ_1 and λ_2 are distinct)

— 2 marks.

Do not write below this line

5(a) Given that $[A]_{p\times n}, [B]_{q\times n}$

Consider $C = \left[\frac{A}{B}\right]$

$$\text{Rank}(C) \leq \text{Rank}(A) + \text{Rank}(B) < n \quad \text{——— } 1$$

$\Rightarrow$ $\text{nullity}(C) > 0$

$\Rightarrow$ $\exists\, x \neq 0$, s.t $Cx = 0$

$\Rightarrow Ax = 0,\ Bx = 0$ ——— 1

Alternate method —

Since $\text{Rank}(A) + \text{Rank}(B) < n$

$\Rightarrow$ $\text{nullity}(A) + \text{nullity}(B) > n$

we know that

$$\dim(\text{null}(A) \cap \text{Null}(B)) = \text{nullity}(A) + \text{nullity}(B) - \dim(\text{null}(A) + \text{null}(B))$$

$$> n - n = 0$$

$\Rightarrow \dim(\text{null}(A) \cap \text{Null}(B)) > 0$

$\Rightarrow \exists\, x \neq 0$ s.t $Ax = 0,\ B(x) = 0$

——— 2

$\boxed{\text{No Partial marking.}}$

(5) (b) let $S = \{w_1, \ldots, w_l\}$ be a basis of

Span $\{v_1, v_2, \ldots v_k\}$. ——— (1)

To show: $\{Aw_1, \ldots Aw_l\}$ be a basis of

Span $\{Av_1, \ldots Av_k\}$.

Consider: $\alpha_1 Aw_1 + \ldots + \alpha_l Aw_l = 0$ where $\alpha_i \in \mathbb{R}$

$$\Rightarrow A(\alpha_1 w_1 + \ldots \cancel{A}\, \alpha_l w_l) = 0$$

Since A is invertible matrix

$$\Rightarrow \alpha_1 w_1 + \ldots + \alpha_l w_\alpha = 0$$

$$\Rightarrow \alpha_i = 0 \quad \forall\, i = 1, \ldots l$$

$\Rightarrow \{Aw_1, \ldots, Aw_l\}$ are linearly independent.

——— (1)

Alternate method:

Consider $B = [v_1, v_2 \ldots v_k]$ $\qquad AB = A[v_1, v_2, \ldots, v_k]$

claim; $\text{Rank}(B) = \text{Rank}(AB)$

$$\text{Rank}(AB) \leq \text{Rank}(B)$$

Take $\text{Rank}(B) = \text{Rank}(A^{-1}AB) \leq \text{Rank}(AB)$

$$\Rightarrow \text{Rank}(AB) = \text{Rank}(B)$$

——— (2)

$\boxed{\text{No Partial marking.}}$

5 (b) Alternate method —

Consider $B = [v_1, \ldots, v_k]$ $AB = A[v_1, \ldots, v_k]$

let $x \in Null(B) \Rightarrow Bx = 0$

$\Rightarrow ABx = 0 \Rightarrow x \in Null(AB)$

$\Rightarrow Null(B) \subseteq Null(AB)$

let $x \in Null(AB) \Rightarrow ABx = 0$

Since A is invertible

$Bx = 0 \Rightarrow x \in Null(B)$

$Null(AB) \subseteq Null(B)$

$\Rightarrow Null(AB) = Null(B)$

Hence By Rank-nullity theorem

$Rank(B) = Rank(AB)$ ——— 2

$\boxed{\text{No Partial marking}}$

~~Note:~~ 1

MA 101 Midsem exam

Model answer and Grading scheme

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6. Let A be an $n \times n$ matrix, where $n \geqslant 2$.
(a) Show that $\det(\operatorname{adj}(A)) = \det(A)^{n-1}$.

We know that
$$A \operatorname{adj}(A) = \det(A) I_n. \tag{1}$$
Taking determinant of both sides
$$\begin{aligned} \det(A)\det(\operatorname{adj}(A)) &= \det(A)^n \det(I_n) \\ \det(A)\det(\operatorname{adj}(A)) &= \det(A)^n \end{aligned} \tag{2}$$

Case-1: $\det(A) \neq 0$.

Then it follows from eqn(2) that
$$\det(\operatorname{adj}(A)) = \det(A)^{n-1}. \qquad \text{(1 mark)}$$

Case-2: $\det(A) = 0$.

If $A = 0$, then $\operatorname{adj}(A) = 0$. Hence the proof follows trivially. So let $A \neq 0$.

If possible, let $\operatorname{adj}(A)$ be invertible. Since $\det(A) = 0$, from eqn(1), $A \operatorname{adj}(A) = 0$. This implies that
$$A = A \operatorname{adj}(A)(\operatorname{adj}(A))^{-1} = 0\,(\operatorname{adj}(A))^{-1} = 0.$$
This contradicts the assumption that $A \neq 0$. Thus $\operatorname{adj}(A)$ is not invertible. So we have $\det(A) = 0$ and $\operatorname{adj}(A) = 0$ and hence the proof follows. (1 mark)

Grading scheme:

Q.6(a) has 2 step markings and each of them will be awarded only when they are completely done with correct justification.
The following statements are wrong and the corresponding step mark(s) will be deducted for using them.

(i) $A \operatorname{adj}(A) = 0$ implies $\operatorname{adj}(A) = 0$.

(ii) $rank(A + B) = rank(A) + rank(B)$

(iii) $rank(AB) = rank(A)rank(B)$

(iv) $null(A) = adj A$

(b) If A has rank $n-1$, then show that $\operatorname{adj}(A)$ *has rank* 1.
(You may use the rank-nullity theorem.)

Since rank of A is less than n, $\det(A) = 0$. (1 mark)

Moreover, as $A \operatorname{adj}(A) = \det(A) I_n$, we have

$$\begin{aligned} & A \operatorname{adj}(A) = 0 \\ \Rightarrow & \operatorname{col}(\operatorname{adj}(A)) \subseteq \operatorname{null}(A) \\ \Rightarrow & \operatorname{rank}(\operatorname{adj}(A)) \leqslant \operatorname{nullity}(A). \end{aligned} \quad \text{(1 mark)}$$

Since $\operatorname{rank}(A) = n-1$, $\operatorname{nullity}(A) = 1$. So it follows from the above inequality that $\operatorname{rank}(\operatorname{adj}(A))$ is 0 or 1.

If $\operatorname{rank}(\operatorname{adj}(A)) = 0$, then $\operatorname{adj}(A) = 0$. So all co-factors of A are 0. Hence $\operatorname{rank}(A) \leqslant n-2$, which is a contradiction. Thus $\operatorname{rank}(\operatorname{adj}(A)) = 1$. (1 mark)

Grading scheme:

Q.6(b) has 3 step markings, which will be awarded only when the steps are completely done with correct justification.

(i) The following statements are wrong and the corresponding step mark(s) will be deducted for using them.

$$nullity(adj(A)) = rank(A) \tag{3}$$
$$rank(adj(A)) = nullity(A) \tag{4}$$
$$rank(A) + rank(adj(A)) = n \tag{5}$$
$$nullity(A) + nullity(adj(A)) = n \tag{6}$$

(ii) The statement "$\det(A) = 0$ implies that A has a zero row" is wrong. For example, consider $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$. No marks will be awarded if this statement is used.

(iii) The following statements are wrong and the corresponding step mark(s) will be deducted for using them.

(a) $A \operatorname{adj}(A) = 0$ implies $\operatorname{adj}(A) = 0$.

(b) $rank(A + B) = rank(A) + rank(B)$

(c) $rank(AB) = rank(A)rank(B)$

(d) $null(A) = adj A$

Solution of Q7.

(a) $A = \begin{bmatrix} 3 & 2 & \dots & 2 \\ 2 & 3 & \dots & 2 \\ \vdots & \vdots & \ddots & \vdots \\ 2 & 2 & \dots & 3 \end{bmatrix}$

$\Rightarrow A - I = \begin{bmatrix} 2 & 2 & \dots & 2 \\ 2 & 2 & \dots & 2 \\ \vdots & \vdots & \ddots & \vdots \\ 2 & 2 & \dots & 2 \end{bmatrix}$ which has rank 1 as all rows are same.

Hence nullity$(A - I) = n - 1$, $\therefore$ 1 is a eigenvalue of A.

{1 mark for showing 1 is an eigenvalue of A}

Eigenspace corresponding to $1 = \text{null}(A - I) = \text{null} \begin{bmatrix} 1 & 1 & \dots & 1 \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 \end{bmatrix} = \text{span} \left\{ \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \dots, \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \\ -1 \end{bmatrix} \right\}$.

{1 mark for finding eigenspace corresponding to 1}

Again, $A \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} = (2n+1) \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}$.

{1 mark for showing $(2n + 1)$ is an eigenvalue of A with proper argument}

Now geometric multiplicity of 1 is $n - 1$ and geometric multiplicity of $(2n + 1)$ is 1.

Geometric multiplicity add upto n. So, A can't have any other eigenvalue except 1 and $(2n + 1)$.

{1 mark for the justification why A don't have any other eigenvalue}

(b) D is a diagonal matrix whose diagonal elements are the eigenvalues of A and P is the matrix whose columns are the corresponding eigenvectors.

$D = \begin{bmatrix} 1 & & & & \\ & 1 & & & \\ & & \ddots & & \\ & & & 1 & \\ & & & & 2n+1 \end{bmatrix}$ and $P = \begin{bmatrix} 1 & 1 & \dots & 1 & 1 \\ -1 & 0 & \dots & 0 & 1 \\ 0 & -1 & \dots & 0 & 1 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & -1 & 1 \end{bmatrix}$

{1 mark for finding D and P}

N.B. No mark for finding the REF of A and saying that diagonal elements of REF of A are the eigenvalues of A.

1. Marking Scheme of Q.8

Steps to solve the Q.8 :

- First find the rank of given matrix A or A^T.
- Second find the basis of Column space of $A = W$ or Null space of $A^{\perp} = W^{\perp}$.
- Third normalize the respective basis because projection formula is applicable only for orthogonal basis.
- Fourth write the correct formula for getting the projection of vector w onto subspace $W^{\perp}$.

Marking Steps :

- No mark if rank is not correct. No marks if Basis set is not correct.
- No mark if the solution is stopped up to the third steps.
- Only one mark if upto third steps is correct and further he carry on to solve.
- Only one mark if question is solve without orthogonal basis. But method and formula (for getting the projection of vector w onto subspace $W^{\perp}$)is correct.
- One mark is detected for calculation mistake if by this mistake solution comes very away from actual solution.
- One mark is detected if REF form does not give correct rank. Means if one student is finding the rank of A by REF form. Suppose his final REF form gives rank three but he write rank two and solve the question in correct way even he get correct ANS of the question.
- Full marks is given for following the correct procedure and getting the correct answer.

Q. 8. <u>Marking Scheme</u>

- For finding a basis of col(A) or a basis of null(A^T) + writing the formula for orthogonal projection. ①

- For finding an orthogonal basis of col(A) or null(A^T) + writing the formula for orthogonal projection. ②

- Full correct. ③