1. Forces on the bug: $\vec{F}_{bug} = \vec{F}_W + \vec{F}_{fict}$

$$\vec{F}_{fict} = -2m\,\vec{\omega}\times\vec{v} - m\,\vec{\omega}\times(\vec{\omega}\times\vec{r}) \qquad \Big| \quad \vec{F}_W = -mg\,\hat{k}$$
$$= \underbrace{-2mv_0\omega\,\hat{j}}_{[1]} \underbrace{+\,m\omega^2 x\,\hat{i}}_{[1]}$$

$$\left[\ \vec{\omega} = \omega\,\hat{k}\ ;\ \vec{v} = v_0\,\hat{i}\ ;\ \vec{r} = x\,\hat{i}\ \right]$$

Forces on the bug by MGR are the reaction forces. Hence

$$\vec{F}_{MGR,bug} = \underbrace{mg\,\hat{k}}_{[\frac{1}{2}]} \boxed{+\,2mv_0\omega\,\hat{j} - m\omega^2 x\,\hat{i}}\ \ [\tfrac{1}{2}]$$

[using polar coord: $\vec{r} = x\hat{r}$,
$\vec{F}_{MGR,bug} = mg\hat{k} + 2mv_0\omega\,\hat{\theta} - m\omega^2 x\,\hat{r}$]

2. $$\overleftrightarrow{I} = \begin{pmatrix} \frac{3}{4}ml^2 & -\frac{\sqrt{3}}{4}ml^2 & 0 \\ -\frac{\sqrt{3}}{4}ml^2 & \frac{5}{4}ml^2 & 0 \\ 0 & 0 & 2ml^2 \end{pmatrix}$$

- Each non-zero element carries $\frac{1}{2}$ mark and $\frac{1}{2}$ mark for all 4 zero-entries together.
- In prod. of inertia, if sign is wrong in both the cases, award ½ mark.

3. a) $l_B = l_0\sqrt{1-v_B^2/c^2} = \frac{\sqrt{3}}{2} l_0$ [$\frac{1}{2}$]

$l_A = 2l_0\sqrt{1-\frac{v_A^2}{c^2}}$ [$\frac{1}{2}$]

$l_A = l_B$ [$\frac{1}{2}$] $\Rightarrow \therefore v_A = \frac{\sqrt{13}}{4}c.$ [$\frac{1}{2}$]

b) $E = K + m_0c^2 = (n+1)m_0c^2$ [$\frac{1}{2}$]

$E = mc^2 = m_0\gamma c^2$ [$\frac{1}{2}$]

$\gamma = n+1 \Rightarrow v = \frac{\sqrt{n(n+2)}}{(n+1)}c$ [1].

4 a) $|A|^2\int_{-\infty}^{\infty} e^{-\frac{amx^2}{\hbar}}\,dx = 1$ [$\frac{1}{2}$]

$\Rightarrow |A|^2 \cdot \sqrt{\frac{\pi\hbar}{am}} = 1$, $A = \left(\frac{am}{\pi\hbar}\right)^{1/4}$ [$\frac{1}{2}$]

b) $i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V(x)\psi$

$\frac{\partial\psi}{\partial t} = -\frac{ia}{2}\psi$ [$\frac{1}{2}$]

$\frac{\partial^2\psi}{\partial x^2} = -\frac{am}{\hbar}\psi + \left(\frac{am}{\hbar}\right)^2 x^2\psi.$ [1]

• $\frac{a\hbar}{2}\psi = \frac{a\hbar}{2}\psi - \frac{\hbar^2}{2m}\left(\frac{am}{\hbar}\right)^2 x^2\psi + V\psi$

$V = \frac{1}{2}ma^2x^2$ [$\frac{1}{2}$]