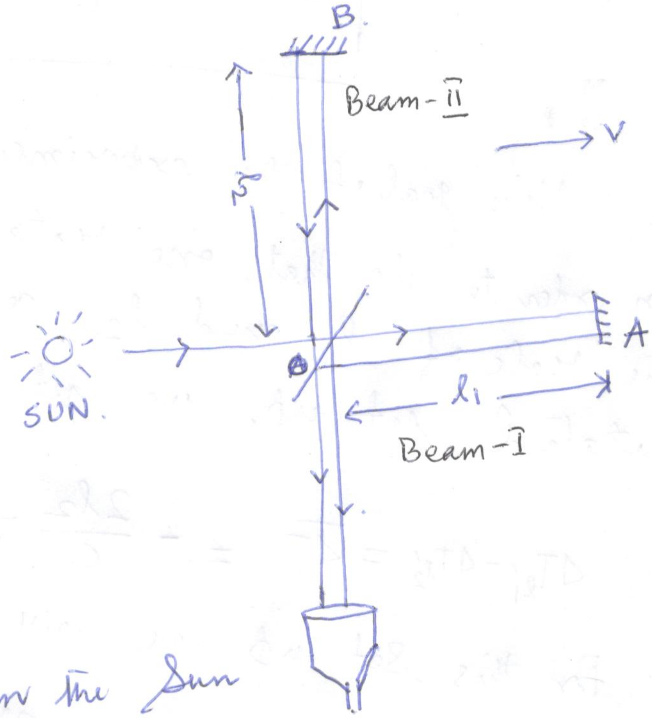
Q1



Ans: The set up is going away from the Sun with a velocity 'V'.

According to Galilean relativity:

a) Time taken by the light starting from beam splitter 'O' to A and again back to 'O' is $(\Delta t_1 + \Delta t_2) = \Delta T_{l_1}$

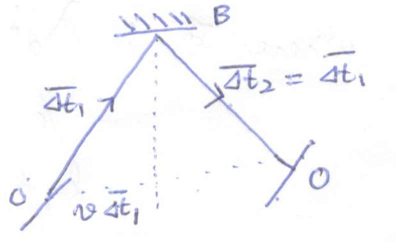
$$\Delta t_1 = \frac{l_1}{c} + \frac{v\Delta t_1}{c} \quad\Rightarrow\quad \Delta t_1 = \frac{l_1}{c}\frac{1}{1-v/c}$$

and $\Delta t_2 = \frac{l_1}{c} - \frac{v\Delta t_2}{c}$ " $\Rightarrow \Delta t_2 = \frac{l_1}{c}\frac{1}{1+v/c}$

Therefore: $$\boxed{\Delta T_{l_1} = \frac{l_1}{c}\left[\frac{1}{1-v/c} + \frac{1}{1+v/c}\right] = \frac{2l_1}{c}\frac{1}{1-v^2/c^2}}$$

b) Time taken by the light starting from beam splitter 'O' to 'B' and back to 'O' is $(\bar{\Delta t_1} + \bar{\Delta t_2}) = 2\bar{\Delta t_1} = \Delta T_{l_2}$

$$\bar{\Delta t_1}^2 c^2 = v^2\bar{\Delta t_1}^2 + l_2^2 \Rightarrow \boxed{\bar{\Delta t_1} = \frac{l_2}{c}\frac{1}{\sqrt{1-v^2/c^2}}}$$



Hence: $$\boxed{\Delta T_{l_2} = \frac{2l_2}{c}\frac{1}{\sqrt{1-v^2/c^2}}}$$

(2)

because of the time difference between ΔT_{l_1} & ΔT_{l_2} interference pattern will be observed.

$$\boxed{\Delta T = \Delta T_{l_1} - \Delta T_{l_2} = \frac{2l_1}{c}\frac{1}{1-v^2/c^2} - \frac{2l_2}{c}\frac{1}{\sqrt{1-v^2/c^2}}}$$

c) Main goal of this experiment is to measure the fringe shift. In order to do that, one rotates the set up by 90°, so that the role of l_1 and l_2 get interchanged. Therefore, for the rotated set up, we get the ~~time~~ new time difference

$$\Delta T_{l_1'} - \Delta T_{l_2'} = \overline{\Delta T} = -\frac{2l_2}{c}\frac{1}{1-v^2/c^2} + \frac{2l_1}{c}\frac{1}{\sqrt{1-v^2/c^2}} \qquad [l_1 \leftrightarrow l_2]$$

For this set up we will have new fringe pattern.

Therefore any fringe shift will be proportional to

$$dt = \overline{\Delta T} - \Delta T = \frac{2(l_2+l_1)}{c\,(1-v^2/c^2)} - \frac{2(l_1+l_2)}{c\sqrt{1-v^2/c^2}}$$

$$= \frac{2(l_1+l_2)}{c}\left[\frac{1}{1-v^2/c^2} - \frac{1}{\sqrt{1-v^2/c^2}}\right]$$

taking leading order in (v^2/c^2) $\Rightarrow$

$$\boxed{dt \simeq \frac{l_1+l_2}{c}\frac{v^2}{c^2}}$$

However Michelson-Morley experiment does not show this fringe shift

For instructor/ you can explain it if you want

T_0: Time period of light.

λ_0: Wave length

Fringe shift $\quad \Delta N = \frac{dt}{T_0} = \frac{dt}{\lambda_0/c} = \frac{c\,dt}{\lambda_0} = \frac{l_1+l_2}{\lambda_0}\left(\frac{v^2}{c^2}\right)$

where λ_0: 5.5×10^{-7} m

v/c: 10^{-4}

$l_1 + l_2 \sim 20$ m

$$\boxed{\Delta N = 0.4}$$

d) So far all the calculations were done with respect to the rest frame. As long as ~~classical~~ Galilean relativity is applied time measurement we have done so far are perfectly acceptable. However, relativity will play important role in this analysis.

The observers for this experiment are at rest with respect to the set up. Therefore, all the measurement of time and length should be proper time and proper length.

We know $T = \frac{T_0}{\sqrt{1-v^2/c^2}}$, $L = L_0\sqrt{1-v^2/c^2}$

Formula derived

$$\boxed{\Delta T_{l_1} = \frac{2l_1}{c}\frac{1}{1-v^2/c^2}}$$

Since the motion is along l_1 - direction $l_1 = l_1^0\sqrt{1-v^2/c^2}$

$$\Downarrow$$

$$\boxed{\frac{\Delta T_{l}^0}{\sqrt{1-v^2/c^2}} = \frac{2l_1^0\sqrt{1-v^2/c^2}}{c\left(1-v^2/c^2\right)}} \Rightarrow \boxed{\Delta T_{l_1}^0 = \frac{2l_1^0}{c}}$$

Similarly

$$\boxed{\Delta T_{l_2} = \frac{2l_2}{c}\frac{1}{\sqrt{1-v^2/c^2}}}$$

Since the motion is along l_1 - direction; $l_2 = l_2^0$

$$\Downarrow$$

$$\boxed{\frac{\Delta T_{l_2}^0}{\sqrt{1-v^2/c^2}} = \frac{2l_2^0}{c}\frac{1}{\sqrt{1-v^2/c^2}}} \Rightarrow \boxed{\Delta T_{l_2}^0 = \frac{2l_2^0}{c}}$$

Therefore: $\Delta T_0 = \Delta T_{l_1}^0 + \Delta T_{l_2}^0 = \frac{2(l_1^0 + l_2^0)}{c}$

After 90^0 rotation: $\overline{\Delta T_0} = \frac{2(l_1^0 + l_2^0)}{c}$ Similar computation

Hence $\boxed{dt = \Delta T_0 - \overline{\Delta T_0} = 0}$ [No fringes shift]

(2) An event occurs in S at $x = 6\times10^8$ m, and in S' at $x' = 6\times10^8$ m & $t' = 4$ s. Find the relative velocity of the systems.

Ans: S : rest frame, S' : moving frame,

$x = 6\times10^8$ m, $x' = 6\times10^8$ m ; $t' = 4$ s.

If 'v' is the relative velocity between the two frame,

we know $x = \gamma(x' + vt')$; $\gamma = \dfrac{1}{\sqrt{1-v^2/c^2}}$

$$\Rightarrow 6\times10^8 = \gamma(6\times10^8 + 4v)$$

$$\Rightarrow (6\times10^8)^2(1 - v^2/c^2) = (6\times10^8 + 4v)^2$$

$$\Rightarrow -(6\times10^8)^2\frac{v^2}{c^2} = 48\times10^8 v + 16v^2$$

$$\Rightarrow v\left(16 + \frac{(6\times10^8)^2}{c^2}\right) + 48\times10^8 = 0$$

$$\Rightarrow v = -\frac{48\times10^8}{16 + \left(\frac{16\times10^8}{c}\right)^2} = -\frac{48\times10^8}{16 + \left(\frac{16}{3}\right)^2}$$

$$= -1.08\times10^8 \text{ m/s}$$

(3) Any quantity which is left unchanged by the Lorentz tranformation is called a Lorentz invariant. Show that Δs is a Lorentz invariant, ~~where~~ which is

$$\Delta s^2 = -c^2 dt^2 + dx^2 + dy^2 + dz^2,$$

where 'dt' is the time interval between two events and $(dx^2 + dy^2 + dz^2)^{\frac{1}{2}}$ is the distance between them in the same inertial frame.

Ans: We know the Lorentz transformation

$$\begin{cases} t = \gamma\left(t' + \frac{v x'}{c^2}\right) \\ x = \gamma(x' + vt') \\ y = y' \\ z = z' \end{cases} \Rightarrow \begin{array}{l} dt = \gamma\left(dt' + \frac{v}{c^2} dx'\right) \\ dx = \gamma(dx' + v dt') \\ dy = dy' \\ dz = dz' \end{array}$$

$$\Delta s^2 = -c^2\left[\gamma dt' + \gamma \frac{v}{c^2} dx'\right]^2 + \gamma^2\left[dx' + v dt'\right]^2 + dy'^2 + dz'^2$$

$$= dt'^2\left[-c^2\gamma^2 + \gamma^2 v^2\right] + dt' dx'\left[\frac{-2c^2\gamma^2 v^2}{c^2} + 2\gamma^2 v^2\right]$$
$$+ dx'^2\left[-\frac{c^2\gamma^2 v^2}{c^4} + \gamma^2\right] + dy'^2 + dz'^2$$

$$= -c^2(1 - v^2/c^2)\gamma^2 dt'^2 + dx'^2 \gamma^2\left(1 - \frac{v^2}{c^2}\right) + dy'^2 + dz'^2$$

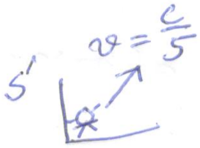
$$= -c^2 dt'^2 + dx'^2 + dy'^2 + dz'^2$$

where $\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$

$$= \Delta s'^2$$

where $\Delta s'$ is the space-time distance measured by the an another inertial observer who is moving with constant velocity v with respect to the rest observer.

(4) A young man voyages to the nearest star, 'α' Centauri, 4.3 light years away. He travels in a spaceship at a velocity of $\frac{c}{5}$. When he returns to earth, how much younger is he than his twin brother who stayed home?



<u>Ans</u>: Assuming T_0 is the time taken by the brother S' to come back to S. T_0 is measured with respect to S'.

Now if 'T' is the time measured by brother S, We know according to time dilation

$$\boxed{\begin{array}{l} L = \text{One light year} \\ c = 3 \times 10^8 \, m \end{array}}$$

$$T = \frac{T_0}{\sqrt{1 - v^2/c^2}} = \frac{T_0}{\sqrt{1 - \left(\frac{1}{5}\right)^2}}$$

Now according to S $\quad T = \frac{4.3 \times 2 \times L}{c/5} = \frac{8.6 \times 5}{c} = 43 \text{ years}$

Hence

$$\frac{4.3 \times 2}{c/5} = \frac{T_0}{\sqrt{1 - (1/5)^2}}$$

$$T_0 = \frac{8.6 \times 5}{c} \times \frac{\sqrt{24}}{5} = \frac{8.6 \times \sqrt{24} \times L}{c}$$

Therefore:

$$T - T_0 = \frac{8.6}{c}\left(5 - \sqrt{24}\right) \times L \approx 0.86 \text{ years.}$$

$$\boxed{L = \text{One light year} = 9.46 \times 10^{15} \, m}$$