Indian Institute of Technology Guwahati

End Semester Examination, PH101: Physics I

Date: Nov 23, 2012; Time: 1:00 – 4:00 PM

Total Marks: 50

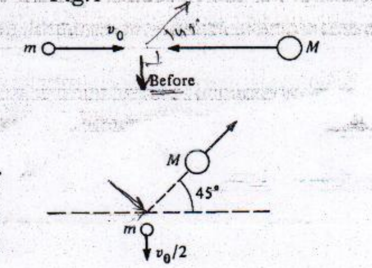
General Instructions: Answer all the questions. Please be sure to keep all parts of a question together and circle your final answer.

1. (a) A particle moves outward along a spiral. Its trajectory is given by $r = A\theta$, where A is a constant. $A = 1/\pi$ m/rad. θ increases in time according to $\theta = \alpha t^2/2$, where α is a constant.

   i. Show that the radial acceleration is zero when $\theta = 1/\sqrt{2}$ rad.

   ii. At what angles do the radial and tangential accelerations have equal magnitude?

Fig.1



(b) A particle of mass m and initial velocity v_0 collides elastically with a particle of unknown mass M coming from the opposite direction as shown in Fig.1. After the collision m has velocity $v_0/2$ at right angles to the incident direction and M moves off in the direction shown in the sketch. Find the ratio M/m

[3+4]

2. An idealized Atwood machine (massless pulley and string) connected to two blocks of masses M and 2M sits initially at rest on a flat horizontal table. The coefficient of static and kinetic friction (assumed equal) between the block and table surfaces is μ. The pulley is accelerated to the left with magnitude of acceleration A as shown in Fig.2. Assume that gravity acts with constant acceleration g down through the plane of the table.

   (a) Find the distances each of the two blocks travel from their initial resting points as a function of time.

   (b) What is the maximum acceleration A for which the block of mass 2M will remain stationary?

[5+2]

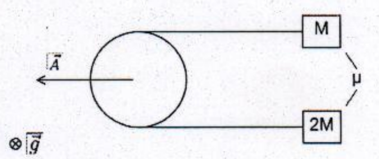


Fig.2

3. A thin plank of mass M and length l is pivoted at one end (see Fig.3). The plank is released at 60° from the vertical. What is the magnitude and direction of the force on the pivot when the plank is horizontal?

[7]

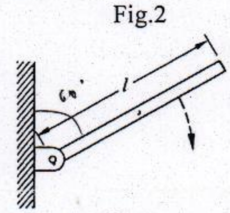


Fig.3

4. A stick of proper length l_0 sits at rest in S, lying in the x-y plane at an angle of $\theta = \tan^{-1}(3/4)$ with the x-axis. A frame S' moves with velocity $\vec{v} = v\hat{x}$ with respect to S. In S', the stick is angled at 45° with respect to the x' axis.

   (a) What is v?

   (b) What is the length l' of the rod as measured in S'? **[4+3]**

5. (a) An event occurs in S at x = 6×10^8 m, and in S' at x' = 6×10^8 m, t' = 4 s. Find the relative velocity of the S' frame with respect to S frame in the unit of c.

(b) A relativistic particles quadruples (4 times) its momentum when its speed doubles. What was the initial speed in the unit of c? **[4+3]**

6. Consider the harmonic oscillator with potential energy $V(x) = \frac{1}{2}m\omega^2x^2$

The wave function for one of the excited states is $\psi(x) = Axe^{-m\omega x^2/2\hbar}$.

   i. Calculate A to normalize the wavefunction.

   ii. Show that it obays $H\psi = E\psi$ and find the energy of the state. Which energy state is it? **[3+4]**

7. (a) Calculate $\langle x\rangle, \langle x^2\rangle, \langle p\rangle, \langle p^2\rangle$ for the wave function $\psi_n(x) = \sqrt{\frac{2}{a}}\sin\left(\frac{n\pi}{a}x\right)$ representing n[th] stationary states of infinite square well of lateral dimension a.

(b) Hence calculate Δx and Δp and check that uncertainty principle is satisfied.

$E = \hbar\omega$

[6+2]

Useful Equations:

1. Moment of inertia of a rod of mass M and length l about one edge: I = 1/3 (ML²)
2. $\int x\sin^2(ax)dx = \frac{x^2}{4} - \frac{x}{4a}\sin(2ax) - \frac{\cos(2ax)}{8a^2}$
3. $\int x^2\sin^2(ax)dx = \frac{x3}{6} - \left(\frac{x^2}{4a} - \frac{1}{8a^3}\right)\sin(2ax) - \frac{x\cos(2ax)}{4a^2}$
4. $\Delta x = \sqrt{\langle x^2\rangle - \langle x\rangle^2}$
5. $\Delta p = \sqrt{\langle p^2\rangle - \langle p\rangle^2}$
6. $\int_{-\infty}^{+\infty} x^2 e^{-ax^2}dx = \frac{1}{2a}\sqrt{\frac{\pi}{a}}$