Indian Institute of Technology Guwahati
Department of Mathematics

MA 101 – Mathematics-I | **Date:** 10-Aug-2015
Tutorial Sheet-2 | **Time:** 08:00 – 09:00

Linear Algebra

Topics Covered:
Reduced Row-Echelon form (RREF), Gauss-Jordan elimination, Homogeneous systems, Rank of a matrix,. Inverse of a matrix, Vector space $\mathbb{R}^n$, Spanning set, Linear independance

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1. Compute the rank of the following matrices

$$A_2 = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \qquad A_3 = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix} \qquad A_4 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{pmatrix}$$

Can you generalize this and guess the rank of A_n for $n \geq 2$?

2. Using Gauss-Jordan method, check whether the following matrix is invertible or not! If yes, compute the inverse. Can you write down A as a product of elementary matrices?

$$A = \begin{pmatrix} 1 & 0 & -2 \\ 3 & 1 & -2 \\ -5 & -2 & 3 \end{pmatrix}$$

3. In the following cases find out the conditions on b_i's so that the system is consistent / inconsistent.

(a) $A = \begin{pmatrix} 1 & -3 & -4 \\ -3 & 2 & 6 \\ 5 & -1 & -8 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

(b)

$$\begin{aligned} x_1 + 2x_2 + 3x_3 + 5x_4 &= b_1 \\ 2x_1 + 4x_2 + 8x_3 + 12x_4 &= b_2 \\ 3x_1 + 6x_2 + 7x_3 + 13x_4 &= b_3 \end{aligned}$$

4. Determine if the vector $\mathbf{b}$ is a linear combination of the vectors $\mathbf{a}_1, \mathbf{a}_2$ and $\mathbf{a}_3$, where

$$\mathbf{a}_1 = [1, -2, 0]^T \qquad \mathbf{a}_2 = [0, 1, 2]^T \qquad \mathbf{a}_3 = [5, -6, 8]^T \qquad \mathbf{b} = [2, -1, 6]^T$$

5. State TRUE or FALSE. Give a brief justification.

(a) If the columns of an $m \times n$ matrix A span $\mathbb{R}^m$, then the equation $Ax = b$ is consistent for each $b \in \mathbb{R}^m$.

(b) Every homogeneous system has infinitely many solutions.

(c) If the RREF of a 5×5 matrix A has the third column as $[1, 2, 0, 0, 0]^T$ then $[-1, -2, 1, 0, 0]^T$ is a solution of the homogeneous system $AX = 0$.

(d) For an $n \times n$ matrix A, the systems $AX = 0$ and $A^T X = 0$ are equivalent.

(e) Let A be a 4×3 matrix with $rank(A) = 3$, then there exists another 3×4 matrix B such that $BA = I_3$.

(f) Let A and B be two matrices of the same order having the same rank then they are row equivalent.

6. Does there exists a 2×2 matrix such that

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \text{ and } A^{-1} = \begin{pmatrix} \frac{1}{a} & \frac{1}{b} \\ \frac{1}{c} & \frac{1}{d} \end{pmatrix}$$

Justify your argument.

7. Give an example of a subset $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\} \subset \mathbb{R}^3$ which is linearly dependent but any two of these are linearly independent.