Plan

- Schur Unitary Triangularization
- Spectral Theorem
- Cayley-Hamilton Theorem

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The rest follows from: 'if A, λ are real then $\exists x \neq 0$ real, s.t. $Ax = \lambda x$'. ■

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So $\sum \lambda_i = \sum t_{ii} = \text{TR}(T) = \text{TR}(U^*AU) = \text{TR}(AUU^*) = \text{TR}(A)$.

Remark. In SUT, as $U^*AU = T$, we have

$$\{\lambda_1, \ldots, \lambda_n\} = \sigma(A) = \sigma(T) = \{t_{11}, \ldots, t_{nn}\}.$$

Further, we can get the λ_is in the diagonal of T in any prescribed order. Υ

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And $\prod \lambda_i = \prod t_{ii} = \mathrm{DET}\, T = \mathrm{DET}\, U^*AU = \mathrm{DET}\, A$. ■

- By DIAG A we denote the diagonal $\{a_{11}, \ldots, a_{nn}\}$ of A. By DIAG v we denote the diagonal matrix A with $a_{11} = v_1, \ldots, a_{nn} = v_n$.

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Th[spectral Theorem]. Let A be normal ($AA^* = A^*A$).

Remark. In SUT, as $U^*AU = T$, we have

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So $\sum \lambda_i = \sum t_{ii} = \text{TR}(T) = \text{TR}(U^*AU) = \text{TR}(AUU^*) = \text{TR}(A)$.

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Remark. In SUT, as $U^*AU = T$, we have

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Th[spectral Theorem]. Let A be normal ($AA^* = A^*A$). Then $\exists U$ unitary, s.t. $U^*AU = D$ is diagonal. Further, DIAG $D = \sigma(A)$.

Remark. In SUT, as $U^*AU = T$, we have

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Cor. Let $\sigma(A) = \{\lambda_1, \ldots, \lambda_n\}$. Then $\textsc{det}\, A = \prod \lambda_i$ and $\textsc{tr}\, A = \sum \lambda_i$.

Pr. By SUT, $\exists U$ unitary, s.t. $U^*AU = T$ is upper triangular. So $\sigma(T) = \sigma(A)$.

So $\sum \lambda_i = \sum t_{ii} = \textsc{tr}(T) = \textsc{tr}(U^*AU) = \textsc{tr}(AUU^*) = \textsc{tr}(A)$.

And $\prod \lambda_i = \prod t_{ii} = \textsc{det}\, T = \textsc{det}\, U^*AU = \textsc{det}\, A$. ∎

- By $\textsc{diag}\, A$ we denote the diagonal $\{a_{11}, \ldots, a_{nn}\}$ of A. By $\textsc{diag}\, v$ we denote the diagonal matrix A with $a_{11} = v_1, \ldots, a_{nn} = v_n$.

Th[spectral Theorem]. Let A be normal ($AA^* = A^*A$). Then $\exists U$ unitary, s.t. $U^*AU = D$ is diagonal. Further, $\textsc{diag}\, D = \sigma(A)$.

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Further, we can get the λ_is in the diagonal of T in any prescribed order. Υ

Cor. Let $\sigma(A) = \{\lambda_1, \ldots, \lambda_n\}$. Then DET $A = \prod \lambda_i$ and TR $A = \sum \lambda_i$.

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So $\sum \lambda_i = \sum t_{ii} = \text{TR}(T) = \text{TR}(U^*AU) = \text{TR}(AUU^*) = \text{TR}(A)$.

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Remark. In SUT, as $U^*AU = T$, we have

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Further, we can get the λ_is in the diagonal of T in any prescribed order. Υ

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So $\sum \lambda_i = \sum t_{ii} = \text{TR}(T) = \text{TR}(U^*AU) = \text{TR}(AUU^*) = \text{TR}(A)$.

And $\prod \lambda_i = \prod t_{ii} = \text{DET}\, T = \text{DET}\, U^*AU = \text{DET}\, A$. ■

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Pr. By SUT, $\exists U$ unitary, s.t $U^*AU = T$ is upper triangular. As $A^*A = AA^*$, we get $T^*T = TT^*$. Note: $\sum |t_{1i}|^2 = (TT^*)_{11} = (T^*T)_{11} = |t_{11}|^2$. So $t_{12} = \cdots = t_{1n} = 0$.

Remark. In SUT, as $U^*AU = T$, we have

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Further, we can get the λ_is in the diagonal of T in any prescribed order. Υ

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And $\prod \lambda_i = \prod t_{ii} = \textsc{det}\, T = \textsc{det}\, U^*AU = \textsc{det}\, A$. ■

- By $\textsc{diag}\, A$ we denote the diagonal $\{a_{11}, \ldots, a_{nn}\}$ of A. By $\textsc{diag}\, v$ we denote the diagonal matrix A with $a_{11} = v_1, \ldots, a_{nn} = v_n$.

Th[spectral Theorem]. Let A be normal ($AA^* = A^*A$). Then $\exists U$ unitary, s.t. $U^*AU = D$ is diagonal. Further, $\textsc{diag}\, D = \sigma(A)$.

Pr. By SUT, $\exists U$ unitary, s.t $U^*AU = T$ is upper triangular. As $A^*A = AA^*$, we get $T^*T = TT^*$. Note: $\sum |t_{1i}|^2 = (TT^*)_{11} = (T^*T)_{11} = |t_{11}|^2$. So $t_{12} = \cdots = t_{1n} = 0$. Repeating the process, T is diagonal. ■

Cor. Let A be Hermitian.

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Further, if A is real, then U can be chosen real orthogonal.

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Further, if A is real, then U can be chosen real orthogonal.

Pr. By SUT, $\exists U$ unitary, s.t. $U^*AU = T$ is upper triangular. As $A^* = A$, we get $T^* = T$.

Cor. Let A be Hermitian. Then $\exists U$ unitary, s.t. $U^*AU = D$ is real diagonal. Further, if A is real, then U can be chosen real orthogonal.

Pr. By SUT, $\exists U$ unitary, s.t. $U^*AU = T$ is upper triangular. As $A^* = A$, we get $T^* = T$. So T is real diagonal. ■

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Pr. By SUT, $\exists U$ unitary, s.t. $U^*AU = T$ is upper triangular. As $A^* = A$, we get $T^* = T$. So T is real diagonal. ∎

- Multiply $\begin{bmatrix} 0 & * & * & * \\ 0 & * & * & * \\ 0 & 0 & * & * \\ 0 & 0 & 0 & * \end{bmatrix} \begin{bmatrix} * & * & * & * \\ 0 & 0 & * & * \\ 0 & 0 & * & * \\ 0 & 0 & 0 & * \end{bmatrix}$

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Further, if A is real, then U can be chosen real orthogonal.

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- Multiply $$\begin{bmatrix} 0 & 0 & 0 & * \\ 0 & 0 & 0 & * \\ 0 & 0 & 0 & * \\ 0 & 0 & 0 & * \end{bmatrix} \begin{bmatrix} * & * & * & * \\ 0 & * & * & * \\ 0 & 0 & * & * \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

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