• Nilanjan Bag
Question No - 2
Grading Scheme :

i) For $AB = 0$

$AB = 0$

$\Rightarrow Col(B) \subseteq Null(A)$

$\Rightarrow rank(B) \leq Nullity(A)$ — (1)

$rank\,A + Nullity(A) = n$ (from rank-nullity)

$\Rightarrow rank A + rank(B) \leq rank A + Nullity(A) = n$
(using (1))

$\Rightarrow rank A + rank(B) \leq n$ (proved)

(2) 2 marks is given ~~for~~ only for full sol^n
no part marking ~~consi~~ was considered.

~~$\mathrm{Col}(AB) \subseteq \mathrm{Col}\,A$~~
~~$\mathrm{Row}(AB) \subseteq \mathrm{Row}(B)$~~

Mohit Tripathi

2. Let A and B be two $n \times n$ matrices such that $AB = 0$. Prove that $\mathrm{rank}(A) + \mathrm{rank}(B) \leq n$. [2]
(*Hint: you can use the fact that if* U *is a subspace of* W *then* $\dim U \leq \dim W$.)

~~$rk(A) + n(A) = n$~~

$AB = 0 \Rightarrow \mathrm{col}(B) \subseteq \mathrm{null}(A)$

$\Rightarrow rk(B) \leq n(A)$ — (1)

Now,

$rk(A) + n(A) = n$

$\Rightarrow rk(A) + rk(B) \leq rk(A) + n(A) = n$ — (1)

Comments: (i) For writing only $rk(A) + n(A) = n$ or $rk(A) + n(B) = n$ } '0' mark

(ii) For showing $rk(B) \leq n(A)$ with justification (i.e. $\mathrm{col}(B) \leq \mathrm{null}(A)$) — 1 mark

if $BA = 0 \Rightarrow \mathrm{Col}(A) \subseteq \mathrm{Null}(B)$

$\Rightarrow \mathrm{Rank}(A) \leq n(B)$ — (1)

~~$\mathrm{Rank}(A) + n(B) = n$~~

$\mathrm{Rank}(A) + \mathrm{Rank}(B) \leq n(B) + \mathrm{Rank}(B) = n$ — (1)