① Find the extrema of the function $f(x,y) = x^2+2y^2$ on the disk $x^2+y^2 \le 1$.

Soln To find critical points on $x^2+y^2<1$

$(x,y) \in \mathbb{R}^2$ such that $\nabla f(x,y) = (0,0)$

$\Rightarrow (2x, 4y) = (0,0)$

$\Rightarrow (x,y) = (0,0)$

$(0,0)$ is the only critical point in the open disk $x^2+y^2<1$

Now $Hf(0,0) = \begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix}$ and $D = 8>0$, $f_{xx}(0,0) = 2>0$

Hence, local minima attains at $(0,0)$ and minimum value is $f(0,0) = 0$.

Now on the boundary, i.e. $\{(x,y) \in \mathbb{R}^2 \mid x^2+y^2=1\}$, we have to use Lagrange Method.

consider $L(x,y,\lambda) = x^2+2y^2 - \lambda(x^2+y^2-1)$

then Lagrange multiplier equations are

$$\left.\begin{aligned} L_x(x,y,\lambda) &= 2x - 2\lambda x = 0 \\ L_y(x,y,\lambda) &= 4y - 2\lambda y = 0 \\ L_\lambda(x,y,\lambda) &= x^2+y^2-1 = 0 \end{aligned}\right] \qquad \begin{aligned} x(1-\lambda) &= 0 \quad \text{— ①} \\ y(2-\lambda) &= 0 \quad \text{— ②} \\ x^2+y^2-1 &= 0 \quad \text{— ③} \end{aligned}$$

from ① $x=0$ or $\lambda = 1$

case - $x=0$

If $x=0$, from ③ $y=\pm1$

hence. points are $(0,\pm1)$

case - $\lambda=1$

From ② $y=0$

then from ③ $x=\pm1$

So the points are $(\pm1,0)$

Thus $(\pm1,0)$ and $(0,\pm1)$ are eligible solutions for the boundary.

we have $f(\pm1,0) = 1$ and $f(0,\pm1) = 2$

So minimum value is 0 and attended at $(0,0)$ and maximum value is 2, attended at $(0,\pm1)$.

③ Find the maximum and minimum of $f(x,y) = 5x - 3y$ subject to the constraint $x^2+y^2 = 136$.

Soln

consider $L(x,y,\lambda) = 5x - 3y - \lambda(x^2+y^2-136)$

$L_x(x,y,\lambda) = 5 - 2\lambda x = 0$ —— ①

$L_y(x,y,\lambda) = -3 - 2\lambda y = 0$ —— ②

$L_\lambda(x,y,\lambda) = x^2+y^2 - 136 = 0$ —— ③

from ① $\lambda \neq 0$. and $x = \dfrac{5}{2\lambda}$

from ②

$y = \dfrac{-3}{2\lambda}$

putting the value of x and y in ③, we have.

$$\frac{25}{4\lambda^2} + \frac{9}{4\lambda^2} - 136 = 0$$

$\Rightarrow$ $\boxed{\lambda = \pm \frac{1}{2\sqrt{2}}}$ putting the value of λ in ① and ② we have $(5\sqrt{2}, -3\sqrt{2}), (-5\sqrt{2}, 3\sqrt{2})$

$f(5\sqrt{2}, -3\sqrt{2}) = 34\sqrt{2}$

$f(-5\sqrt{2}, 3\sqrt{2}) = -34\sqrt{2}$

So, maximum and minimum of $f(x,y)$, is $34\sqrt{2}$ and $-34\sqrt{2}$ respectively.

④ find the global maximum (also called absolute maximum) of $f(x,y) := xy$ on the unit disk $x^2+y^2 \le 1$.

<u>Soln</u>

To find critical points in $x^2+y^2<1$, we have to solve $(x,y) \in \mathbb{R}^2$ such that $\nabla f(x,y) = (0,0)$

$$\Rightarrow (y,x) = (0,0)$$

$$x=0 \quad y=0.$$

$(0,0)$ is the only critical point in the open disk $x^2+y^2<1$

Now $Hf(0,0) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, $D = -1 < 0$

so, $(0,0)$ is a saddle point.

Now on the boundary, i.e. $\{(x,y) \in \mathbb{R}^2 \mid x^2+y^2=1\}$, we have to use Lagrange method.

consider $L(x,y,\lambda) = xy - \lambda(x^2+y^2-1)$

Then Lagrange multiplier equations are

$$\left.\begin{aligned} L_x(x,y,\lambda) &= y - \lambda 2x = 0 \\ L_y(x,y,\lambda) &= x - 2\lambda y = 0 \\ L_\lambda(x,y,\lambda) &= x^2+y^2-1 = 0 \end{aligned}\right] \quad \begin{aligned} y &= 2\lambda x \quad — ① \\ x &= 2\lambda y \quad — ② \\ x^2+y^2-1 &= 0 \quad — ③ \end{aligned}$$

putting the value of $y = 2\lambda x$ in ②, we have

$$x - 4\lambda^2 x = 0$$

$$\Rightarrow x(1-4\lambda^2) = 0$$

$$\Rightarrow x = 0 \quad \text{or} \quad \lambda = \pm\tfrac{1}{2}$$

<u>case-1</u> <u>$x=0$</u>

putting $x=0$ in ①, we have $y=0$

but then ~~from ③~~, putting $x=0, y=0$ in ③ contradict ③.

so $x \ne 0$, $y \ne 0$.

<u>case-2</u> $\lambda = \pm\frac{1}{2}$, then from ① $y = \pm x$

Now from ③, $x = \pm\frac{1}{\sqrt{2}} \Rightarrow y = \pm\frac{1}{\sqrt{2}}$

so we have points, $\left(\pm\frac{1}{\sqrt{2}}, \pm\frac{1}{\sqrt{2}}\right)$, $\left(\pm\frac{1}{\sqrt{2}}, \mp\frac{1}{\sqrt{2}}\right)$.

evaluating the functional values at these point, we have

$f\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) = \frac{1}{2} = f\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$. So global maximum attains at $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ and $\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$ and value is $\frac{1}{2}$.

⑤ Assume that among all rectangular boxes with fixed surface area of 10 square meters there is a box of largest possible volume. Find the dimensions of the optimum box.

Soln.: We have to maximize $f(x,y,z) = xyz$, $\underline{x>0,\ y>0,\ z>0}$

subject to condition

$$2(xy + yz + zx) = 10$$

Consider $L(x,y,z) = xyz - \lambda(xy + yz + zx - 5)$

Then Lagrange multiplier equations are

$L_x(x,y,z) = yz - \lambda(y+z) = 0$ —— ①

$L_y(x,y,z) = xz - \lambda(x+z) = 0$ —— ②

$L_z(x,y,z) = xy - \lambda(x+y) = 0$ —— ③

$L_\lambda(x,y,z) = xy + yz + zx - 5 = 0$ —— ④

~~Now adding ①, ②, ③, we have~~

$$\cancel{(yz + xz + xy) - 2\lambda(x+y+z) = 0}$$

$$\cancel{\Rightarrow\ 5 - 2\lambda(x+y+z) = 0}$$

$\cancel{\Rightarrow}$

from ① and ② we have $\dfrac{yz}{y+z} = \dfrac{xz}{x+z}$

$$\Rightarrow \boxed{x = y}$$

Similarly from ① and ③, we have $\boxed{x = z}$

Hence we have, $\boxed{x = y = z}$, putting this in ④

$$x = y = z = \sqrt{\frac{5}{3}}$$

$$f(x,y,z) = \left(\frac{5}{3}\right)^{3/2}$$

Largest possible volume is $\left(\frac{5}{3}\right)^{3/2}$ and dimention of the optimum box is $\left(\sqrt{\frac{5}{3}}, \sqrt{\frac{5}{3}}, \sqrt{\frac{5}{3}}\right)$.

⑥ Consider the equation $e^{2x-y} + \cos(x^2 + xy) - 2 - 2y = 0$ for $(x,y) \in \mathbb{R}^2$. Can the solutions be written as $y = \phi(x)$ and $x = \psi(y)$ in a neighbourhood of 0? If so, compute the derivatives $\phi'(0)$ and $\psi'(0)$.

Sol^n :- Consider $f(x,y) = e^{2x-y} + \cos(x^2+xy) - 2 - 2y$.

then $f(0,0) = 0$

$f_x(x,y) = 2e^{2x-y} + (2x+y)(-\sin(x^2+xy))$

$\Rightarrow f_x(0,0) = 2 \neq 0$.

~~So, by Implicit function theorem (IFT), there exist a function $\phi \in C'$, U a neighbourhood of 0. and $\phi : U \to \mathbb{R}$ s.t. $x = \phi(y)$, $\forall y \in U$, and $f(\phi(y), y) = 0 \ \forall y \in U$.~~

~~Now $f_y(0,0) = -3 \neq 0$.~~

~~Similarly,~~

so, by Implicit function theorem (IFT), $\exists$ a function $\psi \in C'$, U a nbd. of 0 and $\psi : U \to \mathbb{R}$ s.t. $x = \psi(y) \ \forall y \in U$ and $f(\psi(y), y) = 0 \ \forall y \in U$.

Now, $f_y(0,0) = -3 \neq 0$.

so by IFT, $\exists$ a function $\phi \in C'$, V a nbd of 0 and $\phi : V \to \mathbb{R}$ s.t. $y = \phi(x) \ \forall x \in V$ and $f(x, \phi(x)) = 0 \ \forall x \in V$.

Now, $$\phi'(0) = \frac{-f_x(0,0)}{f_y(0,0)} = \frac{2}{3}$$

$$\psi'(0) = -\frac{f_y(0,0)}{f_x(0,0)} = \frac{3}{2}$$

(7) $F(x,y,z) = xy - z\log y + e^{xz} - 1$

$F(0,1,1) = 0$

$$\frac{\partial F}{\partial y} = x - \frac{z}{y}$$

$$\left.\frac{\partial F}{\partial y}\right|_{(0,1,1)} = -1 \neq 0$$

and therefore from Implicit function Theorem, $\exists$ f such that

$$y = f(x,z)$$

again $\frac{\partial F}{\partial x} = y + z e^{xz}$ and $\frac{\partial F}{\partial z} = -\log y + x e^{xz}$

$F_x(0,1,1) = 2$ and $F_z(0,1,1) = 0$

Now $F(x,f,z) = 0$

Differentiating partially with respect to x & z respectfully

$F_x + F_y f_x = 0$ and $F_z + F_y f_z = 0$

$$\Rightarrow \quad f_x(0,1) = -\frac{F_x(0,1,1)}{F_y(0,1,1)} \quad \text{and} \quad f_z(0,1) = -\frac{F_z(0,1,1)}{F_y(0,1,1)}$$

$F_x(0,1) = 2$ and $f_z(0,1) = 0$

9 (a) $F(x,y) = \left(x + x^2 + e^{x^2y^2},\ -x+y+\sin xy\right)$

$$J_F(x,y) = \begin{bmatrix} 1+2x+2xy^2e^{x^2y^2} & 2ye^{x^2y^2} \\ -1+y\cos xy & 1+x\cos xy \end{bmatrix}$$

$$J_F(0,0) = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$$

$$\det\left(J_F(0,0)\right) = 1 \neq 0$$

and Therefore from inverse function Theorem function is locally invertible at $(0,0)$.

again $J_{F^{-1}}(0,0) = \left(J_F(0,0)\right)^{-1} = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}^{-1}$

$$= \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

(b) $F(x,y) = \left(e^{x+y},\ e^{x-y}\right)$

$$J_F(x,y) = \begin{bmatrix} e^{x+y} & e^{x+y} \\ e^{x-y} & -e^{x-y} \end{bmatrix}$$

$$J_F(0,0) = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$\det\left(J_F(0,0)\right) = -2 \neq 0$$

and therefore from inverse function Theorem function is locally invertible at $(0,0)$.

again $J_{F^{-1}}(0,0) = \left(J_F(0,0)\right)^{-1} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}^{-1}$

$= \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

(11) (a)

$$\frac{\partial}{\partial x_i}(fg) = f\frac{\partial g}{\partial x_i} + g\frac{\partial f}{\partial x_i}$$

Now $\nabla = \sum_{i=1}^{n} e_i \frac{\partial}{\partial x_i}$

$$\Rightarrow \nabla(fg) = \sum_{i=1}^{n} e_i \frac{\partial}{\partial x_i}(fg)$$

$$= \sum e_i f \frac{\partial g}{\partial x_i} + \sum e_i g \frac{\partial f}{\partial x_i}$$

$$= f\sum e_i \frac{\partial g}{\partial x_i} + g\sum e_i \frac{\partial f}{\partial x_i}$$

$$\Rightarrow \nabla fg = f\nabla g + g\nabla f$$

(b)

$$\frac{\partial}{\partial x_i}(f^m) = m f^{m-1}\frac{\partial f}{\partial x_i}$$

$$\nabla f^m = \sum e_i \frac{\partial}{\partial x_i}(f^m) = \sum e_i m f^{m-1}\frac{\partial f}{\partial x_i}$$

$$= m f^{m-1}\sum e_i \frac{\partial f}{\partial x_i}$$

$$\nabla f^m = m f^{m-1}\nabla f$$

(c) $$\frac{\partial}{\partial x_i}\left(\frac{f}{g}\right) = \frac{g\,\frac{\partial f}{\partial x_i} - f\,\frac{\partial g}{\partial x_i}}{g^2}$$

(Quotient rule for diff)

$$\nabla\left(\frac{f}{g}\right) = \sum e_i \frac{\partial}{\partial x_i}\left(\frac{f}{g}\right)$$

$$= \frac{1}{g^2}\left(g \sum e_i \frac{\partial f}{\partial x_i} - f \sum e_i \frac{\partial g}{\partial x_i}\right)$$

$$\nabla\left(\frac{f}{g}\right) = \frac{g\,\nabla f - f\,\nabla g}{g^2}$$

2 (a)

$$\operatorname{div}(\bar{F}+\bar{G}) = \nabla\cdot(\bar{F}+\bar{G}) = \nabla\cdot\bar{F} + \nabla\cdot\bar{G}$$

($\cdot$ is distributive over $+$)

$$= \operatorname{div}\bar{F} + \operatorname{div}\bar{G}$$

$$\operatorname{Curl}(\bar{F}+\bar{G}) = \nabla\times(\bar{F}+\bar{G}) = \nabla\times\bar{F} + \nabla\times\bar{G}$$

($\times$ is distributive over $+$)

$$= \operatorname{Curl}\bar{F} + \operatorname{Curl}\bar{G}$$

(b)

Suppose $\bar{G} = G_1\hat{i} + G_2\hat{j} + G_3\hat{k}$

$$\operatorname{div}(f\bar{G}) = \nabla\cdot(f\bar{G}) = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)\cdot\left(\hat{i}fG_1 + \hat{j}fG_2 + \hat{k}fG_3\right)$$

$$= \frac{\partial}{\partial x}(fG_1) + \frac{\partial}{\partial y}(fG_2) + \frac{\partial}{\partial z}(fG_3)$$

$$= \frac{\partial f}{\partial x}G_1 + f\frac{\partial G_1}{\partial x} + \frac{\partial f}{\partial y}G_2 + f\frac{\partial G_2}{\partial y} + \frac{\partial f}{\partial z}G_3 + f\frac{\partial G_3}{\partial z}$$

$$= \left(\frac{\partial f}{\partial x_1}G_1 + \frac{\partial f}{\partial y}G_2 + \frac{\partial f}{\partial z}G_3\right) + f\left(\frac{\partial G_1}{\partial x} + \frac{\partial G_2}{\partial y} + \frac{\partial G_3}{\partial z}\right)$$

$$= \bar{G}\cdot\nabla f + f\operatorname{div}\bar{G}$$

$$\operatorname{Curl}(f\bar{G}) = \nabla \times (f\bar{G})$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ fG_1 & fG_2 & fG_3 \end{vmatrix}$$

$$= \hat{i}\left(\frac{\partial(fG_3)}{\partial y} - \frac{\partial(fG_2)}{\partial z}\right) - \hat{j}\left(\frac{\partial(fG_3)}{\partial x} - \frac{\partial(fG_1)}{\partial z}\right) + \hat{k}\left(\frac{\partial(fG_2)}{\partial x} - \frac{\partial}{\partial y}(fG_1)\right)$$

$$= f\left(\hat{i}\left(\frac{\partial G_3}{\partial y} - \frac{\partial G_2}{\partial z}\right) - \hat{j}\left(\frac{\partial G_3}{\partial x} - \frac{\partial G_1}{\partial z}\right) + \hat{k}\left(\frac{\partial G_2}{\partial x} - \frac{\partial G_1}{\partial y}\right)\right)$$
$$+ \hat{i}\left(\frac{\partial f}{\partial y}G_3 - \frac{\partial f}{\partial z}G_2\right) - \hat{j}\left(\frac{\partial f}{\partial x}G_3 - \frac{\partial f}{\partial z}G_1\right) + \hat{k}\left(\frac{\partial f}{\partial x}G_2 - \frac{\partial f}{\partial y}G_1\right)$$

$$\operatorname{Curl}(f\bar{G}) = f \operatorname{curl}\bar{G} + \nabla f \times G$$

(c)

$$\bar{F} = F_1\hat{i} + F_2\hat{j} + F_3\hat{k} \quad \text{and} \quad \bar{G} = G_1\hat{i} + G_2\hat{j} + G_3\hat{k}$$

$$\bar{F}\times\bar{G} = (F_2G_3 - F_3G_2)\hat{i} - (F_1G_3 - F_3G_1)\hat{j} + (F_1G_2 - F_2G_1)\hat{k}$$

$$\operatorname{div}(\bar{F}\times\bar{G}) = \frac{\partial}{\partial x}(F_2G_3 - F_3G_2) - \frac{\partial}{\partial y}(F_1G_3 - F_3G_1) + \frac{\partial}{\partial z}(F_1G_2 - F_2G_1)$$

$$= F_1\left(\frac{\partial G_2}{\partial z} - \frac{\partial G_3}{\partial y}\right) + F_2\left(\frac{\partial G_3}{\partial x} - \frac{\partial G_1}{\partial z}\right) + F_3\left(\frac{\partial G_1}{\partial y} - \frac{\partial G_2}{\partial x}\right) -$$
$$\left[\left(\frac{\partial F_2}{\partial z} - \frac{\partial F_3}{\partial y}\right)G_1 + G_2\left(\frac{\partial F_3}{\partial x} - \frac{\partial F_1}{\partial z}\right) + G_3\left(\frac{\partial F_1}{\partial y} - \frac{\partial F_2}{\partial x}\right)\right]$$

$$= \bar{F}\cdot\operatorname{Curl}\bar{G} - \bar{G}\cdot\operatorname{Curl}\bar{F}$$

$$\text{Curl Curl } \bar{F} = \text{curl}(\nabla \times \bar{F})$$
$$= \nabla \times (\nabla \times \bar{F})$$
$$= \nabla (\bar{\nabla} \cdot \bar{F}) - (\nabla \cdot \nabla)\bar{F}$$
$$\text{Curl } \bar{F} = \nabla \,\text{div}\, \bar{F} - \nabla^2 \bar{F}$$

13 (a) We have $\frac{\partial}{\partial x}\left(\sqrt{x^2+y^2+z^2}\right) = \frac{x}{r}$

Using the symmetry of r with respect to x, y & z we get

$$\frac{\partial r}{\partial y} = \frac{y}{r}, \quad \frac{\partial r}{\partial z} = \frac{z}{r}$$

So $\nabla r = \hat{i}\frac{\partial r}{\partial x} + \hat{j}\frac{\partial r}{\partial y} + \hat{k}\frac{\partial r}{\partial z}$

$$= \frac{x\hat{i} + y\hat{j} + z\hat{k}}{r} = \frac{\bar{r}}{r}$$

(b)

$$r^m \bar{r} = r^m x\,\hat{i} + r^m y\,\hat{j} + r^m z\,\hat{k}$$

$$\text{div}(r^m \bar{r}) = \nabla \cdot r^m \bar{r}$$
$$= \frac{\partial}{\partial x}(r^m x) + \frac{\partial}{\partial y}(r^m y) + \frac{\partial}{\partial z}(r^m z)$$
$$= x\frac{\partial r^m}{\partial x} + r^m + y\frac{\partial r^m}{\partial y} + r^m + z\frac{\partial r^m}{\partial z} + r^m \quad (*)$$

$$\frac{\partial r^m}{\partial x} = \frac{\partial}{\partial x}\left(\sqrt{x^2+y^2+z^2}\right)^m = 2x \cdot \frac{m}{2}\left(x^2+y^2+z^2\right)^{\frac{m}{2}-1}$$
$$= m x\, r^{m-2}$$

Similarly $\frac{\partial r^m}{\partial y} = m y\, r^{m-2}$ and $\frac{\partial r^m}{\partial z} = m z\, r^{m-2}$

So from (*), we have

$$\operatorname{div}(r^m \bar{r}) = 3r^m + (mx^2 + my^2 + mz^2)\, r^{m-2}$$

$$= (3+m)\, r^m$$

(c)

$$\operatorname{Curl}(r^m \bar{r}) = r^m \operatorname{curl} \bar{r} + \nabla r^m \times \bar{r}$$

(using 12(b))

We know $\operatorname{curl} \bar{r} = 0$

and $\nabla r^m = m r^{m-2} \bar{r}$ ——— (**)

putting above,

$$\operatorname{curl}(r^m \bar{r}) = 0 + m r^{m-2} (\bar{r} \times \bar{r})$$

$$= 0$$

so $\operatorname{curl}(r^m \bar{r}) = 0$

$$\nabla\left(\frac{1}{r}\right) = \nabla(r^{-1}) = -r^{-3}\, \bar{r}$$ from (**)

$$\operatorname{div}\left(\nabla \frac{1}{r}\right) = -\operatorname{div}\left(\frac{\bar{r}}{r^3}\right)$$

The x- component of $\frac{\bar{r}}{r^3}$ is $\frac{x}{\sqrt{(x^2+y^2+z^2)^3}}$

$$\frac{\partial}{\partial x}\left(\frac{x}{\sqrt{(x^2+y^2+z^2)^3}}\right) = \left(\frac{r^2-3x^2}{r^5}\right)$$

Similarly, $\frac{\partial}{\partial y}\left(\frac{y}{\sqrt{(x^2+y^2+z^2)^3}}\right) = \left(\frac{r^2-3y^2}{r^5}\right)$

$$\frac{\partial}{\partial z}\left(\frac{z}{\sqrt{(x^2+y^2+z^2)^3}}\right) = \frac{r^2-3z^2}{r^5}$$

and therefore $\operatorname{div}\left(\frac{\bar{r}}{r^3}\right) = \frac{3r^2-3(x^2+y^2+z^2)}{r^5}$

$$= 0$$

So $\operatorname{div}\left(\nabla\left(\frac{1}{r}\right)\right) = 0$

(14) $r(t) := (t, t^2)$

$r'(t) = (1, 2t)$

$\|r'(t)\| = \sqrt{1+4t^2}$

and therefore arclength $= \int_0^4 \sqrt{1+4t^2}\, dt$

$$= \frac{1}{4}\left(8\sqrt{65} + \sinh^{-1}(8)\right)$$

(15)

$$\gamma(t) := (t - \sin t,\ 1 - \cos t)$$

$$\gamma'(t) = (1 - \cos t,\ \sin t)$$

$$\|\gamma'(t)\| = \sqrt{2 - 2\cos t}$$

and therefore arc length $= \displaystyle\int_0^{2\pi} \sqrt{2-2\cos t}\, dt$

$= 8$

(16)

$$\gamma(t) = (|t|,\ |t - \tfrac{1}{2}|,\ 0)$$

Breaking it into three parts, we have

on $[-1, 0]$. $\gamma_1(t) = (-t,\ \frac{1}{2} - t,\ 0)$

$$\gamma_1'(t) = (-1, -1, 0)$$

on $[0, \frac{1}{2}]$ $\gamma_2(t) = (t,\ \frac{1}{2} - t,\ 0)$

$$\gamma_2'(t) = (1, -1, 0)$$

on $[\frac{1}{2}, 0]$ $\gamma_3(t) = (t,\ t - \frac{1}{2},\ 0)$

$$\gamma_3'(t) = (1, 1, 0)$$

and therefore arc length $l = \displaystyle\int_{-1}^{0} \|\gamma_1'(t)\| dt + \int_0^{\frac{1}{2}} \|\gamma_2'(t)\| dt + \int_{\frac{1}{2}}^{0} \|\gamma_3'(t)\| dt$

$$= \int_{-1}^{0} \sqrt{2}\, dt + \int_0^{\frac{1}{2}} \sqrt{2}\, dt + \int_{\frac{1}{2}}^{0} \sqrt{2}\, dt = 2\sqrt{2}$$

(17)

$$r(t) = (t,\ t\sin t,\ t\cos t)$$

$$r'(t) = (1,\ \sin t + t\cos t,\ \cos t - t\sin t)$$

$$\|r'(t)\| = \sqrt{2+t^2}$$

and therefore arclength $= \int_0^{\pi} \sqrt{2+t^2}\, dt$

$$= \frac{1}{2}\pi\sqrt{2+\pi^2} + \sinh^{-1}\left(\frac{\pi}{\sqrt{2}}\right)$$