Tutorial - 10

Note Title 4/3/2014

1 (a) We know that $\vec{B} = \mu_0 n \vec{I}$

The flux through a single turn would be

$$\phi_1 = \mu_0 n I \pi R^2$$

$\therefore$ In a length l there are nl such turns.

$\therefore$ The total flux is $\mu_0 n^2 \pi R^2 I l$

$\therefore$ The self-inductance is given by $\phi = L\vec{I}$

$\therefore$ The self-inductance of solenoid is

$$L = \mu_0 n^2 \pi R^2 l.$$

Total energy $= E_m = \frac{1}{2} L I^2 = \frac{\mu_0 n^2 I^2 \pi R^2 l}{2}$.

(b) Energy stored in one turn $E_m = \frac{1}{2} \oint (\vec{A} \cdot \vec{I}) dl$

$$\vec{A} \cdot \vec{I} = \frac{\mu_0 n I^2}{2} R$$

$$E_m = \frac{1}{2} \frac{\mu_0 n I^2}{2} R \oint dl$$

$$= \frac{\mu_0}{4} n I^2 R \cdot 2\pi R = \frac{\mu_0}{2} n I^2 \pi R^2$$

Total energy stored $E_m = E_m \cdot nl$

$$= \frac{\mu_0}{2} n^2 I^2 \pi R^2 l$$

Note Title 2/19/2014

(2) we know that $I = \int \vec{J} \cdot d\vec{a}$

$$= \vec{J}(s)\, 2\pi s L$$

$$\Rightarrow \vec{J}(s) = \frac{I}{2\pi s L}$$

and $\vec{J} = \sigma \vec{E} \Rightarrow \vec{E} = \frac{\vec{J}}{\sigma} = \frac{I}{2\pi s \sigma L} = \frac{I}{2\pi K L}$

$$V = -\int_b^a \vec{E} \cdot d\vec{l} = -\frac{I}{2\pi K L}(a-b)$$

$$\therefore R = \frac{b-a}{2\pi K L}$$

(3)

(a) The magnetic field due to loop 1 is

$$\vec{B}_1 = \frac{\mu_0}{4\pi} \frac{1}{r^3} \vec{I}_1 \left[3(a_1 \cdot \hat{r})\hat{r} - a_1 \right]$$

$\because m_1 = I_1 a_1$

The flux through loop 2 would be

$$\phi_2 = \vec{B}_1 \cdot a_2 = \frac{\mu_0}{4\pi} \frac{1}{r^3} \vec{I}_1 \left[3(a_1 \cdot \hat{r})(a_2 \cdot \hat{r}) - a_1 \cdot a_2 \right]$$

$$= M \vec{I}_1 \qquad (M - \text{mutual inductance})$$

where $M = \frac{\mu_0}{4\pi r^3} \left[3(a_1 \cdot \hat{r})(a_2 \cdot \hat{r}) - a_1 \cdot a_2 \right]$

(6) So, the total work done per unit time is

$$\left.\frac{dW}{dt}\right|_1 = -\mathcal{E}_1 \vec{I}_1 = M \vec{I}_1 \frac{dI_2}{dt}$$

since $\mathcal{E}_1 = -M\dfrac{dI_2}{dt}$

which is the work done per unit time against the mutual emf in loop one, hence the -ve sign.

$\therefore$ W1 can be written as $W_1 = M \vec{I}_1 \vec{I}_2$

where $\vec{I}_2$ is the final current in the loop 2.

$\therefore$ The work done against the mutually induced emf is $W = \dfrac{\mu_0}{4\pi r^3}\left[3(\vec{m}_1 \cdot \hat{r})(\vec{m}_2 \cdot r) - \vec{m}_1 \cdot \vec{m}_2\right]$

(4) The displacement current density is

$$\vec{J}_d = \epsilon_0 \frac{\partial \vec{E}}{\partial t} = \frac{\vec{I}}{A} = \frac{I}{\pi a^2}\hat{z}$$

$$\therefore \frac{\partial E}{\partial t} = \frac{\partial}{\partial t}\left(\frac{\sigma}{\epsilon_0}\right) = \frac{\vec{I}}{A\epsilon_0}$$

Draw an amperian loop at radius 's', then

$$\oint \vec{B} \cdot d\vec{l} = \vec{B} \cdot 2\pi s = \mu_0 I_{denc}$$

$$= \mu_0 \frac{\vec{I}}{\pi a^2} \cdot \pi s^2$$

$$= \frac{\mu_0 \vec{I}\, s^2}{2\pi s a^2}$$

$$\therefore \vec{B} = \frac{\mu_0 I s}{2\pi a^2} \hat{\phi}$$

(5) $\vec{F} = \vec{v} \times \vec{B}$ (per unit charge) and $\vec{v} = wa \sin\theta \,\hat{\phi}$

$$\Rightarrow \vec{F} = wa B_0 \sin\theta (\hat{\phi} \times \hat{z})$$

$\therefore$ Emf can be written as $\mathcal{E} = \int \vec{F} \cdot d\vec{l}$ and

$dl = a\, d\theta\, \hat{\theta}$

$$\therefore \mathcal{E} = wa^2 B_0 \int_0^{\pi/2} \sin\theta (\hat{\phi} \times \hat{z}) \cdot \hat{\theta}\, d\theta$$

But $\hat{\theta} \cdot (\hat{\phi} \times \hat{z}) = \hat{z} \cdot (\hat{\theta} \times \hat{\phi}) = \hat{z} \cdot \hat{r} = \cos\theta$

$$\therefore \mathcal{E} = wa^2 B_0 \int_0^{\pi/2} \sin\theta \cos\theta\, d\theta = wa^2 B_0 \left[\frac{\sin^2\theta}{2}\right]_0^{\pi/2}$$

$$= \frac{1}{2} wa^2 B_0$$