PH 102 : Electromagnetism | **Department of Physics**
Tutorial 4 | **Indian Institute of Technology Guwahati**

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1. Consider a large flat horizontal sheet with thickness x and volume charge density ρ. This sheet is tangent to a sphere with radius R and volume charge density ρ_0, as shown in Fig. 1. Let A be the point of tangency, and let B the point opposite to A on the top side of the sheet. Show that the net upward electric field (from the sphere plus the sheet) at B is larger than at A if $\rho > \frac{2}{3}\rho_0$ (Assume $x << R$).

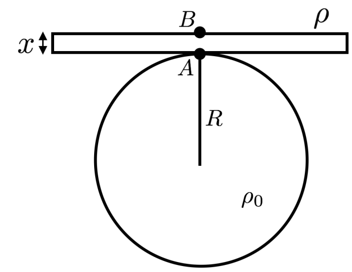

Figure 1

2. A rod placed along the x axis has charge density $\lambda(x) = \frac{\lambda_0 x}{L}$ in the interval $-L < x < L$. Find the electric field at a point $x = x_0 > L$ along the x axis. Examine this result for $x_0 \to \infty$ and show that it falls off like a dipole field $\vec{E} = \hat{x}\frac{\lambda_0 L^2}{3\pi\epsilon_0 x_0^3}$ and find the associated dipole moment. Hint: Expand in a Taylor series to an order that yields a nonzero result.

3. A dipole with electric dipole moment $\vec{p}$ and of length d is at an angle of $\frac{\pi}{6}$ with respect to a uniform electric field $\vec{E} = E\hat{x}$ along the x axis. What work will it take to align it at an angle π ? If disturbed from the position of stable equilibrium, what will be the angular frequency ω of small oscillations if the dipole has a mass m at each end ?

4. A conical surface (an empty ice cream cone) carries a uniform surface charge σ. The height of the cone is a, as is the radius of the top. Find the potential difference between points P (the vertex) and Q (the centre of the top).

5. Two infinitely long wires running parallel to x axis carry line charge densities $+\lambda$ and $-\lambda$ as shown in Fig 2.
   (a) Find the potential at any point (x, y, z), using origin as your reference.
   (b) Show that the equipotential surfaces are circular cylinders, and locate the axis and radius of the cylinder corresponding to a given potential V_0.

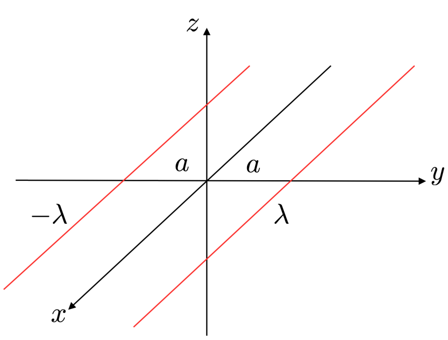

Figure 2

6. A sphere of radius R carries a charge density $\rho(r) = kr$ (where k is a constant). Find the energy of the configuration and check your answer at least in two different ways.

7. Consider the electric field of two protons (charge q) at a distance d apart, the potential energy of the system ought to be given by

$$W = \frac{\epsilon_0}{2}\int \vec{E}_1^2 d\tau + \frac{\epsilon_0}{2}\int \vec{E}_2^2 d\tau + \epsilon_0 \int \vec{E}_1.\vec{E}_2 d\tau$$

where $\vec{E}_1$ is the field of one proton alone and $\vec{E}_2$ is that of other. Show that the third integral yields $\frac{e^2}{4\pi\epsilon_0 b}$, which we already know to be the work required to bring the two protons in from an infinite distance to positions a distance b apart.

8. Two conducting spheres of radius r_1 and r_2 carry charges q_1 and q_2 ($q_1 > q_2$) respectively. They are very far apart so that the charge distribution of one sphere does not affect the potential of the other sphere. What is the potential difference between them? If they are connected by a conducting wire, what will be the final charges on each (assume no charge is accumulated on the wire and the final charges on the spheres are uniformly distributed)?

9. A solid nonconducting sphere of uniform charge density and total charge $-Q$ and radius $r = a$ is surrounded by a concentric conducting spherical shell of inner radius $r = b$ and outer radius $r = c$ with $c > b > a$. The outer shell has charge $2Q$. Use Gauss's law to find the field for all r. Show with a sketch where the charges reside and draw some field lines.