PH 102 : Electromagnetism **Department of Physics**
Tutorial 4 **Indian Institute of Technology Guwahati**

Please solve the star ($\star$) marked problems first and discuss the rest if time permits.

1. Consider a large flat horizontal sheet with thickness x and volume charge density ρ. This sheet is tangent to a sphere with radius R and volume charge density ρ_0, as shown in Fig. 1. Let A be the point of tangency, and let B the point opposite to A on the top side of the sheet. Show that the net upward electric field (from the sphere plus the sheet) at B is larger than at A if $\rho > \frac{2}{3}\rho_0$ (Assume $x << R$).

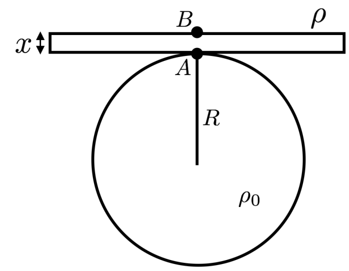

Figure 1

Solution:

We will calculate the electric field at points A and B due to the slab and the sphere using the superposition principle.

Let us assume that the slab has a small thickness $x << R$, we can think of it as a sheet carrying uniform surface charge density $\sigma = \rho x$. The electric field due to a surface with uniform σ can be calculated by Gauss's law as

$$\begin{aligned} \oint \vec{E}.d\vec{a} &= |\vec{E}|2A = \frac{\sigma A}{\epsilon_0} \\ \therefore \vec{E}_{\rm B}^{\rm sheet} &= \frac{\sigma}{2\epsilon_0}\hat{n};\ \vec{E}_{\rm A}^{\rm sheet} = -\frac{\sigma}{2\epsilon_0}\hat{n}. \end{aligned}$$

Where we used the fact that the magnitude of the electric field is same above and below the sheet by the symmetry of the problem.

However, the field due to the sphere at point A will be governed by the spherical Gaussian surface of radius R while that at point B will be due to the spherical Gaussian surface of radius $R + x$. Hence the fields due to the sphere are as follows (which points radially outward, i.e. along $\hat{n}$),

$$\begin{aligned} \vec{E}_{\rm A}^{\rm sphere} &= \frac{1}{4\pi\epsilon_0}\frac{Q_{\rm enc}}{R^2}\hat{n} = \frac{(4/3)\pi R^3\rho_0}{4\pi\epsilon_0 R^2}\hat{n} \\ \vec{E}_{\rm B}^{\rm sphere} &= \frac{1}{4\pi\epsilon_0}\frac{Q_{\rm enc}}{(R+x)^2}\hat{n} = \frac{(4/3)\pi R^3\rho_0}{4\pi\epsilon_0 (R+x)^2}\hat{n}. \end{aligned}$$

Hence the total field

$$\begin{aligned}\vec{E}_{\rm A} &= \frac{(4/3)\pi R^3\rho_0}{4\pi\epsilon_0 R^2}\hat{n} - \frac{\rho x}{2\epsilon_0}\hat{n}\\ \vec{E}_{\rm B} &= \frac{(4/3)\pi R^3\rho_0}{4\pi\epsilon_0 (R+x)^2}\hat{n} + \frac{\rho x}{2\epsilon_0}\hat{n}.\end{aligned}$$

Therefore the field is larger above the sheet at point A if

$$\begin{aligned}\frac{(4/3)\pi R^3\rho_0}{4\pi\epsilon_0 (R+x)^2} + \frac{\rho x}{2\epsilon_0} &> \frac{(4/3)\pi R^3\rho_0}{4\pi\epsilon_0 R^2} - \frac{\rho x}{2\epsilon_0}\\ \implies \rho x &> \frac{R\rho_0}{3}\left(1 - \frac{1}{(1+x/R)^2}\right)\\ \implies \rho x &> \frac{R\rho_0}{3}\left(\frac{2x}{R}\right) \implies \rho > \frac{2}{3}\rho_0\end{aligned}$$

where we have used $\frac{1}{(1+\epsilon)^2} \sim \frac{1}{(1+2\epsilon)} \sim 1 - 2\epsilon$, where $\epsilon = x/R$.

---

2. $\star$ A rod placed along the x axis has charge density $\lambda(x) = \frac{\lambda_0 x}{L}$ in the interval $-L < x < L$. Find the electric field at a point $x = x_0 > L$ along the x axis. Examine this result for $x_0 \to \infty$ and show that it falls off like a dipole field $\vec{E} = \hat{x}\frac{\lambda_0 L^2}{3\pi\epsilon_0 x_0^3}$ and find the associated dipole moment. Hint: Expand in a Taylor series to an order that yields a nonzero result.

Solution:

The electric field of the rod at a point x_0 on the x axis is given by

$$\begin{aligned}\vec{E} &= \hat{x}\frac{1}{4\pi\epsilon_0}\int_{-L}^{L}\frac{\lambda_0 x dx}{L(x_0-x)^2}\\ &= \hat{x}\frac{1}{4\pi\epsilon_0}\frac{\lambda_0}{L}\int_{-L}^{L}\left[\frac{(x-x_0)}{(x-x_0)^2} + \frac{x_0}{(x-x_0)^2}\right]dx\\ &= \hat{x}\frac{1}{4\pi\epsilon_0}\frac{\lambda_0}{L}\left[\ln\left(\frac{x_0-L}{x_0+L}\right) + \frac{2x_0 L}{x_0^2 - L^2}\right]\end{aligned}$$

Now note that for very large x_0, the electric field is very close to zero. However, we will show, when we expand the field in terms of $\frac{L}{x_0}$ and keep terms upto third order, we will obtain a non-zero result. Electric field in terms of $\frac{L}{x_0}$

$$\vec{E} = \hat{x}\frac{1}{4\pi\epsilon_0}\frac{\lambda_0}{L}\left[\ln\left(\frac{1-\frac{L}{x_0}}{1+\frac{L}{x_0}}\right) + \frac{2\frac{L}{x_0}}{1-\frac{L^2}{x_0^2}}\right]$$

Next, let us perform a Taylor series expansion in terms of $\frac{L}{x_0}$ around $\frac{L}{x_0} = 0$. The first term above becomes

$$\ln\left(\frac{1-\frac{L}{x_0}}{1+\frac{L}{x_0}}\right) \sim 0 - \frac{2L}{x_0} - \frac{2L^3}{3x_0^3},$$

and the second term

$$\begin{aligned}\frac{2x_0 L}{x_0^2 - L^2} &= 2\frac{L}{x_0}\left[\frac{1}{1-\left(\frac{L}{x_0}\right)^2}\right] \\ &= 2\frac{L}{x_0}\left[1+\left(\frac{L}{x_0}\right)^2\right] = \frac{2L}{x_0} + \frac{2L^3}{x_0^3}\end{aligned}$$

Putting all these together, we arrive at

$$\begin{aligned}\vec{E} &= \hat{x}\frac{1}{4\pi\epsilon_0}\frac{\lambda_0}{L}\left(-\frac{2L}{x_0} - \frac{2L^3}{3x_0^3} + \frac{2L}{x_0} + \frac{2L^3}{x_0^3}\right) \\ &= \hat{x}\frac{1}{4\pi\epsilon_0}\frac{\lambda_0}{L}\frac{4L^3}{3x_0^3} = \hat{x}\frac{\lambda_0 L^2}{3\pi\epsilon_0 x_0^3} \qquad (1)\end{aligned}$$

In lecture 8, the electric field of a dipole was calculated using spherical polar coordinate and the expression for the field was

$$\vec{E} = \frac{p}{4\pi\epsilon_0 r^3}(2\cos\theta\hat{r} + \sin\theta\hat{\theta})$$

Because of the symmetry of the present problem, we had to use Cartesian coordinate system. Therefore, let us write the electric field of a dipole aligned along the x axis from the above general expression: since $\theta = 0$ when the dipole is aligned with its axis (along x direction), $\cos\theta = 1, \sin\theta = 0$ and since $y = z = 0$ on the x axis, $\hat{r} = \hat{x}, r^3 = x^3$. Hence, the electric field at a distance x_0 as

$$\vec{E} = \hat{x}\frac{2p}{4\pi\epsilon_0 x_0^3} \qquad (2)$$

Comparing Eqs. (1) and (2), we find

$$\begin{aligned}\hat{x}\frac{\lambda_0 L^2}{3\pi\epsilon_0 x_0^3} &= \hat{x}\frac{2p}{4\pi\epsilon_0 x_0^3} \\ \Longrightarrow p &= \frac{2\lambda_0 L^2}{3}\end{aligned}$$

---

3. A dipole with electric dipole moment $\vec{p}$ and of length d is at an angle of $\frac{\pi}{6}$ with respect to a uniform electric field $\vec{E} = E\hat{x}$ along the x axis. What work will it take to align it

at an angle π ? If disturbed from the position of stable equilibrium, what will be the angular frequency ω of small oscillations if the dipole has a mass m at each end ?

Solution:

The torque on the dipole will be

$$\vec{N} = \vec{p} \times \vec{E} = -pE\sin\theta\hat{z}$$

The negative sign is because of the fact that the torque is pointing into the page, while the z axis is pointing out of the page.

The energy of a dipole in an electric field is given by

$$U = -\vec{p}.\vec{E}$$

Hence, the work done is:

$$W = U(\pi) - U\left(\frac{\pi}{6}\right) = -pE\cos\pi + pE\cos\left(\frac{\pi}{6}\right) = pE\left(1 + \frac{\sqrt{3}}{2}\right)$$

For the frequency of small oscillations, we can use Newton's second law:

$$\begin{aligned} N &= \frac{dL}{dt} \\ -pE\sin\theta &= I\ddot{\theta} \end{aligned}$$

Here, L is the angular momentum and I is the moment of inertia of the dipole about the center. Now, expanding $\sin\theta$ for small θ,

$$\begin{aligned} -\frac{pE}{I}\theta &= \ddot{\theta} \\ \ddot{\theta} + \omega^2\theta &= 0 \\ \therefore \omega^2 &= \frac{pE}{I} \\ \omega &= \sqrt{\frac{2pE}{md^2}} \end{aligned}$$

where, $I = m(d/2)^2 + m(d/2)^2 = md^2/2$.

---

4. $\star$ A conical surface (an empty ice cream cone) carries a uniform surface charge σ. The height of the cone is a, as is the radius of the top. Find the potential difference between points P (the vertex) and Q (the centre of the top).

Solution:

The potential at a can be calculated due to the charged ring at a distance r' from the vertex of the cone

$$V(a) = \frac{1}{4\pi\epsilon_0}\int_0^{\sqrt{2}h}\left(\frac{\sigma 2\pi r}{r'}\right)dr' = \frac{2\pi\sigma}{4\pi\epsilon_0}\frac{\sqrt{2}h}{\sqrt{2}} = \frac{\sigma h}{2\epsilon_0},$$

where $r = r'/\sqrt{2}$; as the height and the radius of the cone are same.

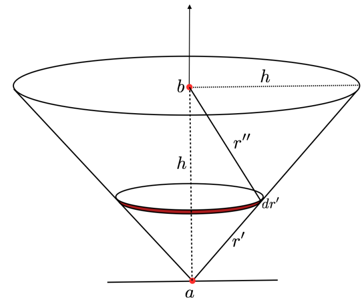

Figure 2

Similarly one can find the potential at point b as follows

$$\begin{aligned}
V(a) &= \frac{1}{4\pi\epsilon_0}\int_0^{\sqrt{2}h}\left(\frac{\sigma 2\pi r}{r''}\right)dr' \\
&= \frac{2\pi\sigma}{4\pi\epsilon_0}\frac{1}{\sqrt{2}}\int_0^{\sqrt{2}h}\frac{r'}{\sqrt{h^2+r'^2-\sqrt{2}hr'}}dr' \\
&= \frac{\sigma}{2\sqrt{2}\epsilon_0}\left[\sqrt{h^2+r'^2-\sqrt{2}hr'}+\frac{h}{\sqrt{2}}\ln(2\sqrt{h^2+r'^2-\sqrt{2}hr'}+2r'-\sqrt{2}h)\right]\Bigg|_0^{\sqrt{2}h} \\
&= \frac{\sigma h}{2\epsilon_0}\ln(1+\sqrt{2}).
\end{aligned}$$

Here we have used $r'' = \sqrt{h^2+r'^2-\sqrt{2}hr'}$ since the angle between r' and h is $\frac{\pi}{4}$. Hence

$$V(a) - V(b) = \frac{\sigma h}{2\epsilon_0}\left[1 - \ln(1+\sqrt{2})\right]$$

---

5. ⋆ Two infinitely long wires running parallel to x axis carry line charge densities $+\lambda$ and $-\lambda$ as shown in Fig 2.
   (a) Find the potential at any point (x, y, z), using origin as your reference.
   (b) Show that the equipotential surfaces are circular cylinders, and locate the axis and radius of the cylinder corresponding to a given potential V_0.

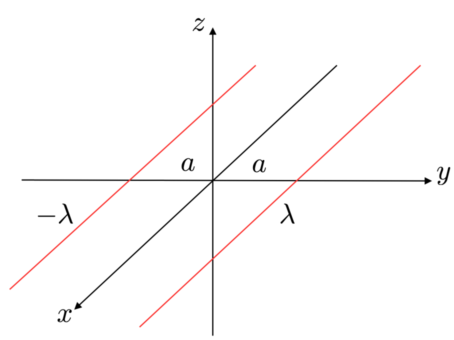

Figure 3

Solution:

(a) Potential due to $+\lambda$ line charge is $V_+ = -\frac{\lambda}{2\pi\epsilon_0}\ln\left(\frac{s_+}{a}\right)$, where s_+ is the distance of the test point (x, y, z) from $+\lambda$ charged wire and we have chosen our reference at $s = a$. We have already shown this in the class. Similarly potential due to $-\lambda$ line charge is $V_- = \frac{\lambda}{2\pi\epsilon_0}\ln\left(\frac{s_-}{a}\right)$, where s_- is the distance of the test point (x, y, z) from $-\lambda$ charged wire. Hence using superposition principle, the total potential at (x, y, z) is

$$V(x, y, z) = V_+ + V_- = \frac{\lambda}{2\pi\epsilon_0}\ln\left(\frac{s_-}{s_+}\right).$$

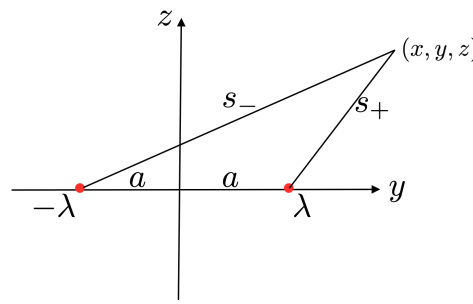

Figure 4: Cross sectional view

Since the wires are parallel to x-axis, the field and potential will only depend on y, z. Now, using $s_\pm = \sqrt{(y \mp a)^2 + z^2}$, we get

$$\begin{aligned} V(x, y, z) &= \frac{\lambda}{2\pi\epsilon_0}\ln\left(\frac{\sqrt{(y+a)^2+z^2}}{\sqrt{(y-a)^2+z^2}}\right) \\ &= \frac{\lambda}{4\pi\epsilon_0}\ln\left(\frac{(y+a)^2+z^2}{(y-a)^2+z^2}\right) \end{aligned}$$

(b) The equipotential surfaces will be given by $V(x, y, z) = \text{constant} = V_0$. Hence

$$\frac{(y+a)^2+z^2}{(y-a)^2+z^2} = e^{(4\pi\epsilon_0 V_0/\lambda)} = \kappa \implies y^2 + z^2 + a^2 - 2ay$$

The above equation represents a circle of the form $(y - y_0)^2 + z^2 = R^2$, where $R = \frac{2a\sqrt{\kappa}}{|\kappa-1|}$ and $y_0 = a\left(\frac{\kappa+1}{\kappa-1}\right)$. One can replace $\kappa = e^{(4\pi\epsilon_0 V_0/\lambda)}$ to finally arrive at

$$\begin{aligned} y_0 &= a\coth\left(\frac{2\pi\epsilon_0 V_0}{\lambda}\right) \\ R &= a\ \text{cosech}\left(\frac{2\pi\epsilon_0 V_0}{\lambda}\right). \end{aligned}$$

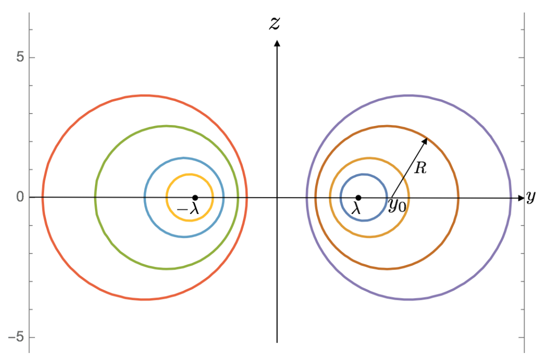

Figure 5: Equipotentials

We show the equipotential lines in Fig. 5 where different coloured circles indicate different choices of constant potential V_0.

Note Added :

Please note that the student may not be familiar with the hyperbolic sines and cosines, a discussion towards this may be necessary.

$$y_0 = a\frac{e^{4\pi\epsilon_0 V_0/\lambda}+1}{e^{4\pi\epsilon_0 V_0/\lambda}-1} = a\frac{e^{2\pi\epsilon_0 V_0/\lambda}+e^{-2\pi\epsilon_0 V_0/\lambda}}{e^{2\pi\epsilon_0 V_0/\lambda}-e^{-2\pi\epsilon_0 V_0/\lambda}} = a\coth\left(\frac{2\pi\epsilon_0 V_0}{\lambda}\right)$$

where, the hyperbolic sines and cosines are defined as $\sinh x = \frac{e^x - e^{-x}}{2}$ and $\cosh x =$

$\frac{e^x+e^{-x}}{2}$, the other hyperbolic functions follow from these two. Similarly,

$$R = \frac{2a\sqrt{\kappa}}{|\kappa - 1|} = 2a\frac{e^{2\pi\epsilon_0 V_0/\lambda}}{e^{4\pi\epsilon_0 V_0/\lambda} - 1} = a\frac{2}{e^{2\pi\epsilon_0 V_0/\lambda} - e^{-2\pi\epsilon_0 V_0/\lambda}} = \frac{a}{\sinh(2\pi\epsilon_0 V_0/\lambda)}$$

---

6. ⋆ A sphere of radius R carries a charge density $\rho(r) = kr$ (where k is a constant). Find the energy of the configuration and check your answer at least in two different ways.

Solution:

In order to calculate the electrostatic energy of the configuration we need to determine the electric field inside and outside the sphere for the given charge density $\rho = kr$. Using Gauss's law,

$$\epsilon_0 \oint \vec{E}.d\vec{a} = \epsilon_0 4\pi r^2 E = Q_{\text{enc}} = \int \rho d\tau = \int\int\int kr'r'^2 \sin\theta dr' d\theta d\phi = 4\pi k \int r'^3 dr'.$$

Hence, the field inside

$$\begin{aligned} E_{(r<R)} &= \frac{1}{4\pi\epsilon_0 r^2} 4\pi k \int_0^r r'^3 dr' = \frac{kr^2}{4\epsilon_0} \implies \vec{E}_{(r<R)} = \frac{kr^2}{4\epsilon_0}\hat{r} \\ E_{(r>R)} &= \frac{1}{4\pi\epsilon_0 r^2} 4\pi k \int_0^R r'^3 dr' = \frac{kR^4}{4\epsilon_0 r^2} \implies \vec{E}_{(r>R)} = \frac{kR^4}{4\epsilon_0 r^2}\hat{r} \end{aligned}$$

Hence, we can find the electrostatic energy as

$$\begin{aligned} W = \frac{\epsilon_0}{2}\int E^2 d\tau &= \frac{\epsilon_0}{2}\int_0^R \left(\frac{kr^2}{4\epsilon_0}\right)^2 4\pi r^2 dr + \frac{\epsilon_0}{2}\int_R^\infty \left(\frac{kR^4}{4\epsilon_0 r^2}\right)^2 4\pi r^2 dr \\ &= \frac{\pi k^2 R^7}{7\epsilon_0} \end{aligned}$$

We can also calculate the electrostatic energy using

$$W = \frac{1}{2}\int \rho V d\tau$$

But, we need to calculate the potential within the sphere ($r < R$) first as ρ is zero outside ($r > R$). Hence,

$$\begin{aligned} V(r) = -\int_\infty^r \vec{E}.d\vec{l} &= -\int_\infty^R \left(\frac{kR^4}{4\epsilon_0 r^2}\right) dr - \int_R^r \left(\frac{kr^2}{4\epsilon_0}\right) dr \\ &= \frac{k}{3\epsilon_0}\left(R^3 - \frac{r^3}{4}\right) \end{aligned}$$

Therefore

$$\begin{aligned} W &= \frac{1}{2}\int_0^R (kr)\left[\frac{k}{3\epsilon_0}\left(R^3 - \frac{r^3}{4}\right)\right]4\pi r^2 dr \\ &= \frac{\pi k^2 R^7}{7\epsilon_0} \end{aligned}$$

---

7. $\star$ Consider the electric field of two protons (charge q) at a distance d apart, the potential energy of the system ought to be given by

$$W = \frac{\epsilon_0}{2}\int \vec{E}_1^2 d\tau + \frac{\epsilon_0}{2}\int \vec{E}_2^2 d\tau + \epsilon_0 \int \vec{E}_1.\vec{E}_2 d\tau$$

where $\vec{E}_1$ is the field of one proton alone and $\vec{E}_2$ is that of other. Show that the third integral yields $\frac{e^2}{4\pi\epsilon_0 b}$, which we already know to be the work required to bring the two protons in from an infinite distance to positions a distance b apart.

Solution:

The expression in the problem has already been derived in the class. We will just focus on the cross-term. Let us assume that the proton in the left in Figure is placed at the origin and the angle θ is measure to the horizontal. The fields at the position shown are

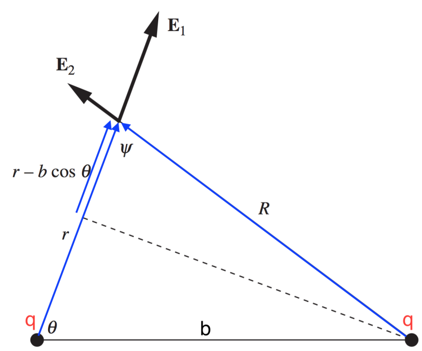

Figure 6

$$E_1 = \frac{q}{4\pi\epsilon_0 r^2}; \quad E_2 = \frac{q}{4\pi\epsilon_0 R^2}$$

where $R = \sqrt{r^2 + b^2 - 2br\cos\theta}$.

Hence,

$$\begin{aligned} \epsilon_0 \int \vec{E}_1.\vec{E}_2 d\tau &= \epsilon_0 \int E_1 E_2 \cos\psi d\tau \\ &= \epsilon_0 \int_0^{2\pi}\int_0^{\pi}\int_0^{\infty} \frac{q}{4\pi\epsilon_0 r^2}\frac{q}{4\pi\epsilon_0 R^2}\frac{r - b\cos\theta}{R} r^2 \sin\theta d\theta d\phi dr \\ &= \frac{2\pi q^2}{16\pi^2\epsilon_0}\int_0^{\pi}\int_0^{\infty}\frac{r - b\cos\theta}{(r^2 + b^2 - 2br\cos\theta)^{3/2}}\sin\theta d\theta dr \\ &= \frac{q^2}{8\pi\epsilon_0}\int_0^{\pi}\left[-\frac{1}{(r^2+b^2-2br\cos\theta)^{1/2}}\right]_{r=0}^{\infty}\sin\theta d\theta \\ &= \frac{q^2}{4\pi\epsilon_0 b} \end{aligned}$$

which is exactly the potential energy of the system of two charges. Note here that the first two terms indicate self energy of the charges themselves as already been discussed in the class. In doing the integration, we have used $r^2 + b^2 - 2br\cos\theta = y^2$. If we have n charges, each of the cross terms of the form $\vec{E}_i.\vec{E}_j$ can be handled exactly in the same way as above, yielding $\frac{q^2}{4\pi\epsilon_0 r_{ij}}$ where r_{ij} is the separation between particles i and j.

---

8. Two conducting spheres of radius r_1 and r_2 carry charges q_1 and q_2 ($q_1 > q_2$) respectively. They are very far apart so that the charge distribution of one sphere does not affect the potential of the other sphere. What is the potential difference between them? If they are connected by a conducting wire, what will be the final charges on each (assume no charge is accumulated on the wire and the final charges on the spheres are uniformly distributed)?

Solution:

The potential difference between the two spheres which are far apart and not affected by each other is given by

$$V_1 - V_2 = \frac{q_1}{4\pi\epsilon_0 r_1} - \frac{q_2}{4\pi\epsilon_0 r_2}$$

When we connect the two charged spheres by a conducting wire, they become equipotential. Let us assume that charge q moves from sphere 1 to sphere 2 to make potential difference between the two spheres zero. This gives the following condition:

$$\begin{aligned} \frac{q_1 - q}{4\pi\epsilon_0 r_1} &= \frac{q_2 + q}{4\pi\epsilon_0 r_2} \\ r_2(q_1 - q) &= r_1(q_2 + q) \\ q &= \frac{r_2 q_1 - r_1 q_2}{r_1 + r_2} \end{aligned}$$

Therefore the final charge on sphere 1 is: $q_1' = q_1 - q = \frac{r_1(q_1+q_2)}{r_1+r_2}$ and on sphere 2: $q_2' = q_2 + q = \frac{r_2(q_1+q_2)}{r_1+r_2}$.

---

9. $\star$ A solid nonconducting sphere of uniform charge density and total charge $-Q$ and radius $r = a$ is surrounded by a concentric conducting spherical shell of inner radius $r = b$ and outer radius $r = c$ with $c > b > a$. The outer shell has charge $2Q$. Use Gauss's law to find the field for all r. Show with a sketch where the charges reside and draw some field lines.

Solution:

The electric field due to spherical symmetry can easily be evaluated using Gauss's law.

$$\oint \vec{E}.d\vec{a} = 4\pi r^2 E = \frac{Q_{\text{enc}}}{\epsilon_0}$$
$$\implies \vec{E} = \frac{1}{4\pi\epsilon_0}\frac{Q_{\text{enc}}}{r^2}\hat{r}$$

For $r < a$: the charge enclosed is

$$Q_{\text{enc}} = (4/3)\pi r^3\rho = -(4/3)\pi r^3\frac{Q}{(4/3)\pi a^3} = -Q\frac{r^3}{a^3}$$
$$\therefore \quad \vec{E}(r < a) = -\frac{1}{4\pi\epsilon_0}\frac{Qr}{a^3}\hat{r}$$

For $a \leq r < b$: charge enclosed is again $-Q$ and hence the electric field is

$$\vec{E}(a \leq r < b) = -\frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}\hat{r}$$

For $b \leq r < c$: the points are within the conducting material and hence the electric field is identically zero.

$$\vec{E}(b \leq r < c) = 0; \quad \implies Q_{\text{enc}} = 0$$

Due to the requirement of having zero electric field within conductor, the charge enclosed is required to be zero. Hence the induced charge on the interior surface ($r = b$) of the conductor shell will be $+Q$ to nullify the $-Q$ charge of the enclosed non conducting sphere. Hence, the remaining free charges those are distributed on the outer surface of the conducting shell will be $2Q - Q = +Q$. Hence, the electric field outside ($r > c$) is simply

$$\vec{E}(r > c) = +\frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}\hat{r}$$

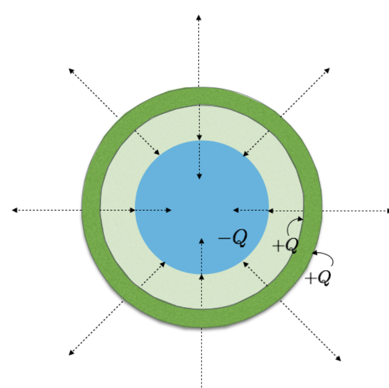

Figure 7: Field lines

Electric field lines are shown in Fig. 7. No field lines appear within conductor ($b \leq r < c$).