PH 102 : Electromagnetism | **Department of Physics**
Tutorial 1 | **Indian Institute of Technology Guwahati**

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1. The equation giving a family of ellipsoids is

$$u = \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2},$$

where a, b, c are constants. Find the unit vector normal to each point of the surface of these ellipsoids.

2. (a) Find the gradients of the following scalar functions,

$$\begin{aligned} &(a)\ \ln \mathcal{r}, \\ &(b)\ \mathcal{r}^n, \end{aligned}$$

where $\mathcal{r}$ is the magnitude of the vector joining points (x_0, y_0, z_0) and (x, y, z).

(b) Find the divergences of the following vector functions:

$$\begin{aligned} \vec{V}_1 &= xy\hat{x} + 2yz\hat{y} + 3zx\hat{z}, \\ \vec{V}_2 &= y^2\hat{x} + (2xy + z^2)\hat{y} + 2yz\hat{z}. \end{aligned}$$

(c) Find the curl of the vector functions given in (ii).

3. Prove the following product rules.

$$\begin{aligned} \vec{\nabla}(fg) &= f\vec{\nabla}g + g\vec{\nabla}f \\ \vec{\nabla}.(\vec{A} \times \vec{B}) &= \vec{B}.(\vec{\nabla} \times \vec{A}) - \vec{A}.(\vec{\nabla} \times \vec{B}) \\ \vec{\nabla} \times (f\vec{A}) &= f(\vec{\nabla} \times \vec{A}) - \vec{A} \times (\vec{\nabla}f) \end{aligned}$$

4. For each of the 2 dimensional vector fields shown in Figures (1) to (6), can you try to guess the nature (i.e. comment on their divergence and curl) by looking at them? (Recall that divergence of a vector field implies a net flux into, or out of, a neighbourhood. It is easy to spot in certain patterns in the plots where you may be able to see at once that the divergence is zero. Similarly the curl implies how much a vector swirls around a point, like in case of a fluid flow a non zero curl means a rotational flow.)

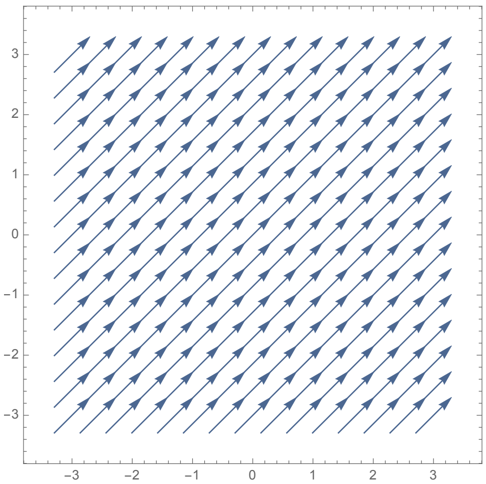

Figure 1

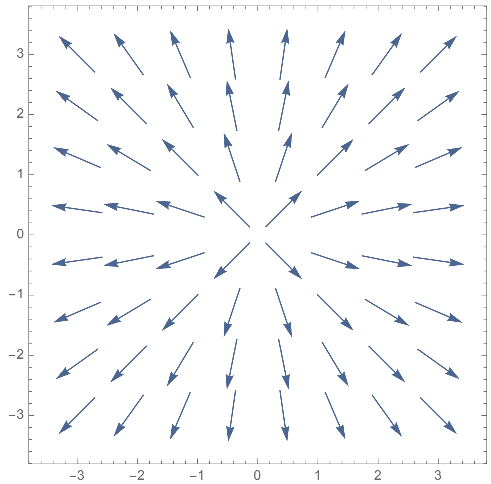

Figure 2

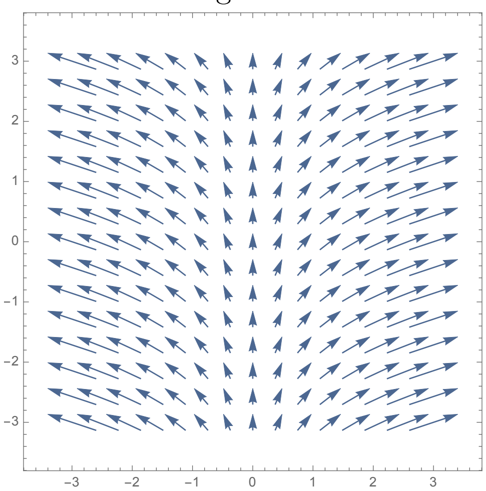

Figure 3

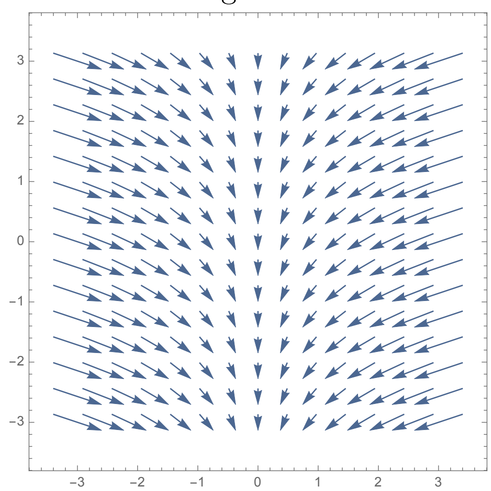

Figure 4

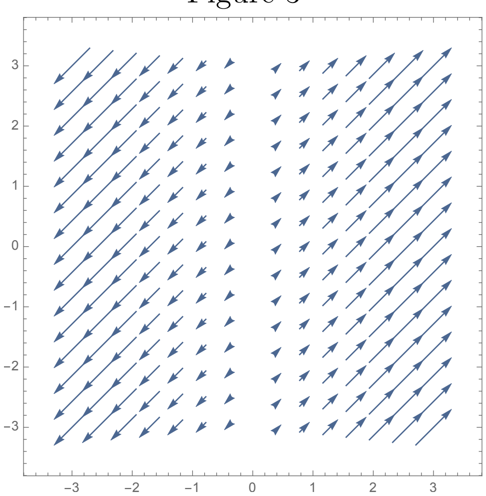

Figure 5

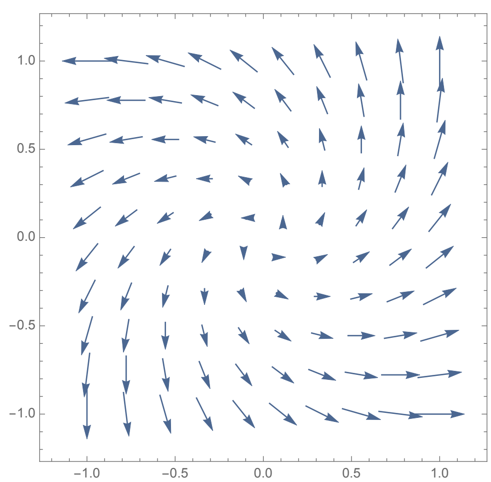

Figure 6

5. Find the gradient of the scalar potential $\phi(x, y, z) = \alpha xy$. Provide a clear sketch of the

$\phi =$ constant lines in the $x - y$ plane and a representation of its gradient field. Such a field is known as a radial quadrupole field and is used in focussing charged particles.

6. Find the total work done in moving a particle under the force field given by $\vec{F} = z\hat{x} + z\hat{y} + x\hat{z}$ along the helix C given by $x = \cos t,\ y = \sin t,\ z = t$ from $t = 0$ to $t = \frac{\pi}{2}$.

7. Show that the work done for a particle moving under the force field $\vec{F} = (2xy + z^3)\hat{x} + x^2\hat{y} + 3xz^2\hat{z}$ from point $a = (1, 1, 0)$ to $b = (2, 2, 0)$ as shown in Figure 1 following path (1) and path (2) is equal. Show that the curl of the force field $\vec{F}$ is zero. Such a force field is called conservative. Hence evaluate the scalar potential.

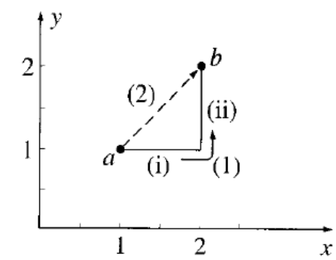

Figure 7: Paths (1) & (2)