PH 102 : Electromagnetism **Department of Physics**
Tutorial 9 **Indian Institute of Technology Guwahati**
20/03/18

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1. Two long coaxial solenoids each carry current I, but in opposite directions, as shown in figure 1. The inner solenoid (radius a) has n_1 turns per unit length, and the outer one (radius b) has n_2. Find $\vec{B}$ in each of the three regions: (i) inside the inner solenoid, (ii) between them, and (iii) outside both.

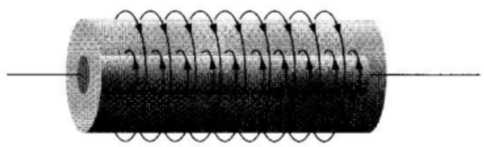

Figure 1: Figure for problem 1.

2. Just as $\vec{\nabla} \cdot \vec{B} = 0$ allows us to express $\vec{B}$ as the curl of a vector potential ($\vec{B} = \vec{\nabla} \times \vec{A}$), so $\vec{\nabla} \cdot \vec{A} = 0$ permits us to write $\vec{A}$ itself as the curl of a higher potential: $\vec{A} = \vec{\nabla} \times \vec{W}$.
(a) Find the general formula for $\vec{W}$ (as an integral over $\vec{B}$), which holds when $\vec{B} \to 0$ at ∞.
(b) Determine $\vec{W}$ for the case of a uniform magnetic field $\vec{B}$.
(c) Find $\vec{W}$ inside and outside an infinite solenoid.

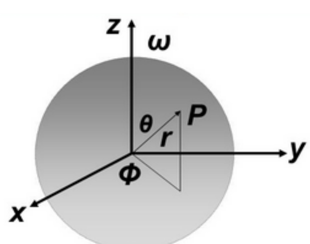

Figure 2: Figure for problem 3.

3. Calculate the magnetic force of attraction between the northern and southern hemispheres of a spinning charged spherical shell, shown in figure 2. Hint: The magnetic force on a surface current $\vec{K}$ is given by

$$\vec{F} = \int (\vec{K} \times \vec{B}_{\text{avg}}) da, \quad \vec{B}_{\text{avg}} = \frac{1}{2}(\vec{B}_{\text{inside}} + \vec{B}_{\text{outside}})$$

4. A uniformly charged solid sphere of radius R carries a total charge Q, and is set spinning with angular velocity ω about the z axis.
(a) What is the magnetic dipole moment of the sphere?
(b) Find the magnetic field at a point (r, θ) inside the sphere.
(c) Using the results of (b) find the average magnetic field within the sphere. Hint: Average magnetic field is defined as

$$\vec{B}_{\text{avg}} = \frac{1}{\frac{4}{3}\pi R^3} \int \vec{B} d\tau$$

Compare this result with the result of (a) and show that the average magnetic field is related to the magnetic dipole moment as

$$\vec{B}_{\text{avg}} = \frac{\mu_0}{4\pi} \frac{2\vec{m}}{R^3}$$

5. A thin uniform donut, carrying charge Q and mass M, rotates about its axis as shown in the figure 3.
(a) Find the gyromagnetic ratio (g), i.e. the ratio of its magnetic dipole moment to its angular momentum.
(b) What is the gyromagnetic ratio a uniform spinning sphere?
(c) According to quantum mechanics, the angular momentum of a spinning electron is $\frac{\hbar}{2}$, where $\hbar$ is Planck's constant. What, then, is the electron's magnetic dipole moment (in units of $A.m^2$)?

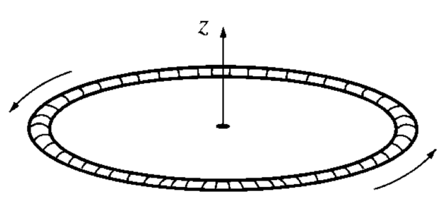

Figure 3: Figure for problem 5.

6. A thin glass rod of radius R and length L carries a uniform charge σ. It is spinning about its axis, at an angular velocity ω. Find the magnetic field at a distance $s \gg R$

from the center of the rod (see figure 4).

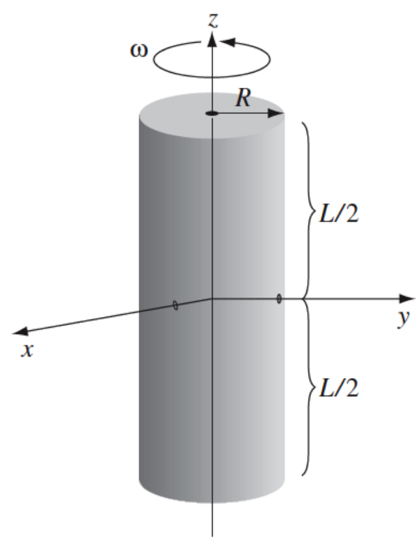

Figure 4: Figure for problem 6.

7. A uniform current density $\vec{J} = J_0\hat{z}$ fills a slab straddling the yz plane as shown in figure 5, from $x = -a$ to $x = +a$. A magnetic dipole $\vec{m} = m_0\hat{x}$ is situated at the origin.
(a) Find the force on the dipole.
(b) Do the same for a dipole pointing in the y direction: $\vec{m} = m_0\hat{y}$.
(c) In the *electrostatic case*, the expressions $\vec{F} = \vec{\nabla}(\vec{p}.\vec{E})$ and $\vec{F} = (\vec{p}.\vec{\nabla})\vec{E}$ are equivalent (prove it), but this is not the case for the magnetic analogs (explain why). As an example, calculate $(\vec{m}.\vec{\nabla})\vec{B}$ for the configurations in (a) and (b).

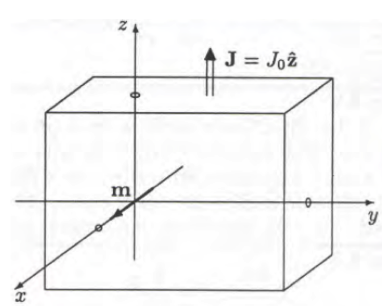

Figure 5: Figure for problem 7.