# MA 102 (Mathematics II)

## Department of Mathematics, IIT Guwahati

Tutorial Sheet No. 7 March 07, 2016

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**Vector fields, Line integrals, Double integrals, Green's Theorem**

(1) Determine which of the following vector fields $F$ in $\mathbb{R}^2$ is conservative and find a scalar potential when it exists.
  (a) $F(x,y) = (\cos(xy) - xy\sin(xy), x^2\sin(xy))$.
  (b) $F(x,y) = (xy, xy)$.

(2) Let $S = \mathbb{R}^2 \setminus \{(0,0)\}$. Let $F(x,y) = (\frac{-y}{x^2+y^2}, \frac{x}{x^2+y^2}) =: (P(x,y), Q(x,y))$. Show that $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$ on $S$ while $F$ is not a gradient of a scalar field on $S$.

(3) Evaluate the line integral $\int_\Gamma F \bullet \mathbf{dr}$ of the vector field $F$ given below.
  (a) $F(x,y) := (x^2 + 2xy,\ y^2 - 2xy)$ from $(-1,1)$ to $(1,1)$ along $y = x^2$.
  (b) $F(x,y) := (x^2 - y^2,\ x - y)$ and $\Gamma : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ in the counterclockwise direction.

(4) Evaluate the line integral $\displaystyle\int_\Gamma \frac{(x+y)dx - (x-y)dy}{x^2+y^2}$ along $\Gamma : x^2 + y^2 = a^2$ traversed once in the counter clockwise direction.

(5) Evaluate the line integral $\displaystyle\int_\Gamma \frac{x^2y dx - x^3 dy}{(x^2+y^2)^2}$, where $\Gamma$ is the square with vertices $(\pm 1, \pm 1)$ oriented in the counterclockwise direction.

(6) Evaluate the line integral $\displaystyle\int_\Gamma \frac{dx + dy}{|x| + |y|}$, where $\Gamma$ is the square with vertices $(1,0), (0,1)$, $(-1,0)$ and $(0,-1)$ oriented in the counterclockwise direction.

(7) Evaluate the double integral $\iint_R f(x,y)\ dxdy$ for $f$ and $R$ given below.
  (a) $f(x,y) := (1-x)$ and $R$ is square $[0,1] \times [0,1]$.
  (b) $f(x,y) := x^2 + y^2$ and $R = [-1,1] \times [0,1]$.
  (c) $f(x,y) := x^2 + y$ and $R$ is the square $[0,1] \times [0,1]$.
  (d) $f(x,y) := \sin(x+y)$ and $R$ is the square $[0,\pi] \times [0,\pi]$.

(8) Find the volume of the solid enclosed between the graph of $f(x,y) = x^2 + y^2$ and the planes $x = 0, x = 3, y = -1, y = 1$.

(9) Verify Green's theorem in each of the following cases:
  (a) $f(x,y) := -xy^2$; $g(x,y) := x^2y$; the region $R$ is given by $x \geq 0, 0 \leq y \leq 1 - x^2$.
  (b) $f(x,y) := 2xy$; $g(x,y) := e^x + x^2$; the region $R$ is the triangle with vertices $(0,0), (1,0)$ and $(1,1)$.

(10) Evaluate $\int_\Gamma (y^2 dx + x dy)$ using Green's theorem, where $\Gamma$ is boundary of $R$ and
  (a) $R$ is the square with vertices $(0,0), (0,2), (2,2), (2,0)$.
  (b) $R$ is the square with vertices $(\pm 1, \pm 1)$.
  (c) $R$ is the disc of radius 2 and center $(0,0)$.

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