PH 102 : Electromagnetism **Department of Physics**
Tutorial 5 **Indian Institute of Technology Guwahati**

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1. Image charge for a grounded conducting spherical shell:
   (a) A point charge $-q$ is located at $x = a$, and a point charge Q is located at $x = A$. Show that the locus of points with $V = 0$ is a circle in the xy plane (and hence a spherical shell in space).
   (b) What must be the relation among q, Q, a, and A, so that the center of the circle is located at $x = 0$?
   (c) Assuming that the relation you found in part (b) holds, what is the radius of the circle in terms of a and A?
   (d) Explain why the previous results imply the following statement: if a charge Q is externally located a distance $A > R$ from the centre of a grounded conducting spherical shell with radius R, then the external field due to the shell is the same as the field of an image point charge $-q = -QR/A$ located a distance $a = R^2/A$ from the centre of the shell; see Fig. 1 (upper panel). The total external field is the sum of this field plus the field from Q. (The internal field is zero, by the uniqueness theorem.)
   (e) Likewise for the following statement: if a charge $-q$ is internally located a distance $a < R$ from the centre of a grounded conducting spherical shell with radius R, then the internal field due to the shell is the same as the field of an image point charge $Q = qR/a$ located a distance $A = R^2/a$ from the centre of the shell; see Fig. 1 (lower panel). The total internal field is the sum of this field plus the field from q. (The external field is zero, because otherwise the shell would not have the same potential as infinity. Evidently a charge $+q$ flows onto the grounded shell.)

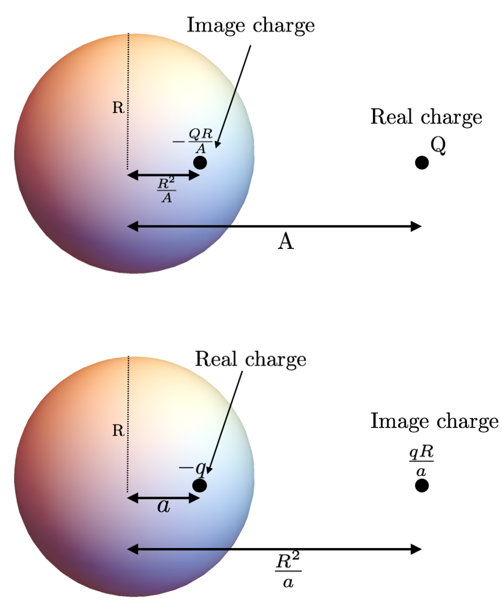

Figure 1

2. Consider a generalisation of the above problem. Suppose that the point charge Q is situated at a distance A from the centre of a conducting sphere of radius R kept at a finite potential. The same basic model discussed in the previous problem will handle the case of a sphere at any finite potential V_0 (relative to infinity) with the addition of a second image charge. What charge should you use, and where should you put it ? Find the force of attraction between a point charge Q and a neutral conducting sphere.

3. Now consider, two similar charges ($+q$) placed at a distance $2b$ apart.

   (a) Find, approximately, the minimum radius a (assume $a << b$) of a grounded conducting sphere placed mid way between them that would neutralise their mutual repulsion (*Hint: Expand the expression of force in terms of* a/b).
   (b) What is the force on each of the two charges, if the same sphere, with the radius determined in part 3(a) above, is now kept at a constant potential V.

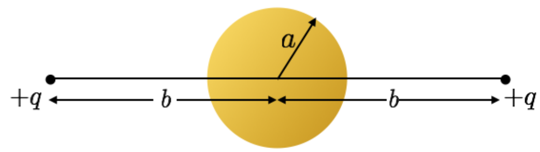

Figure 2

4. Two semi infinite grounded conducting planes meet at right angles. In the region between them, there is a point charge q situated as shown in Fig. Set up the image configuration, and calculate the potential in this region - what charge do you need, and where should they be located ? What is the force on q ? Suppose the planes met at some angle other than 90°; would you still be able to solve the problem by method of images ? If not (in general), for what particular angles does the method work ?

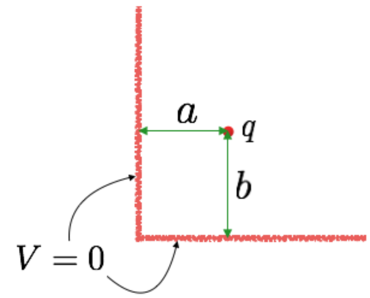

Figure 3

5. A rectangular pipe, running parallel to the z axis (from $-\infty \rightarrow +\infty$), has three grounded metal sides, at $y = 0$, $y = a$, and $x = 0$. The fourth side, at $x = b$, is maintained at a specific potential $V_0(y)$.
   (a) Develop a general formula for the potential within the pipe.
   (b) Find the potential explicitly, for the case $V_0(y) = V_0$ (a constant).

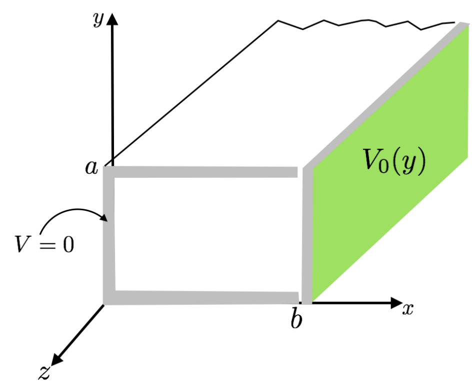

Figure 4

6. Two infinite grounded metal plates lie parallel to the xz plane, one at $y = 0$, the other at $y = a$. The left end, at $x = 0$, consists of two metal strips: one, from $y = 0$ to $y = \frac{a}{2}$, is held at a constant potential V_0, and the other, from $y = \frac{a}{2}$ to $y = a$, is at potential $-V_0$. Find the potential in the infinite slot.

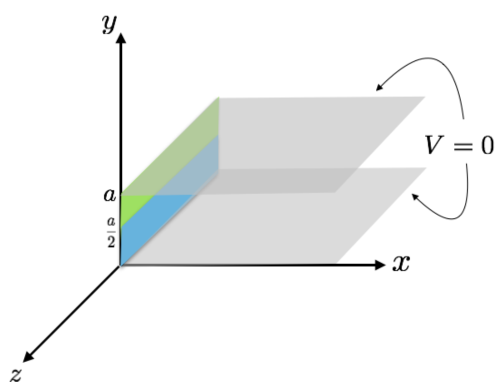

Figure 5