MA 102 (Mathematics II)

Department of Mathematics, IIT Guwahati

Tutorial Sheet No. 4

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(1) Let $f : \mathbb{R}^2 \to \mathbb{R}$ be given by $f(0,0) = 0$ and

$$f(x,y) = (x^2+y^2)\sin\frac{1}{x^2+y^2} \qquad \text{for } (x,y) \neq (0,0).$$

(a) Find f_x and f_y at every $(x,y) \in \mathbb{R}^2$.

(b) Show that the partial derivatives of f are not bounded in any disc (howsoever small) around $(0,0)$.

(c) Examine the differentiability at every point (x,y).

(2) Examine the differentiability of the following function at $(0,0)$:

$$f(x,y) = \begin{cases} \frac{x^3-y^3}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0). \end{cases}$$

(3) Let $f : \mathbb{R}^2 \to \mathbb{R}$. If $f_x(x,y) = 0 = f_y(x,y)$ for all $(x,y) \in \mathbb{R}^2$ then show that f is a constant function.

(4) Let $g : \mathbb{R}^2 \to \mathbb{R}$ be given by $g(0,0) = 0$ and, for $(x,y) \neq (0,0)$,

$$g(x,y) = \frac{\sin^2(x+y)}{|x|+|y|}.$$

Examine the existence of partial and directional derivatives of g at $(0,0)$.

Also, examine the differentiability of g at $(0,0)$.

(5) Find the directional derivative of $f(x,y) = y^3 - 2x^2 + 3$ at the point $(1,2)$ in the direction of $U := \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$. Also, find the directional derivative of $f(x,y) = \log(x^2+y^2)$ at $(1,-3)$ in the direction of $V := (2,-3)$.

(6) Find the directional derivative of $f(x,\ y) = x^2 - 3xy$ along the parabola $y = x^2 - x + 2$ (That is, in the parametric form $x(t) = t$ and $y(t) = t^2 - t + 2$) at the point $(1,\ 2)$. (Note: When a direction is given in terms of a curve, then one must take the direction as the (unit) tangent vector to the curve at that point).

(7) Discuss the differentiability of the following functions at $(0,0)$.

$$(a) f(x,y) = \begin{cases} \frac{xy}{\sqrt{x^2+y^2}} & x^2+y^2 \neq 0, \\ 0 & x = y = 0 \end{cases} \qquad (b)\ g(x,y) = \begin{cases} \frac{x^6-2y^4}{x^2+y^2}, & x^2+y^2 \neq 0 \\ 0, & x = 0, y = 0 \end{cases}$$

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