MA 102 (Ordinary Differential Equations)

IIT Guwahati

Tutorial Sheet No. 3 **Date:** March 22, 2018

Undetermined coefficients, Annihilator method, Operator method, variation of parameters.

(1) Let $S = \{f : \mathbb{R} \to \mathbb{R} \,|\, L(f) = 0\}$, where $L(f) := f''' + f'' - 2$. Find the $Ker(L)$. Let $S_0 \subset Ker(L)$ be the subspace of solutions g such that $\lim_{x\to\infty} g(x) = 0$. Find $g \in S_0$ such that $g(0) = 0$ and $g'(0) = 2$.

(2) Find the general solution of the following differential equations.

(a) $\dfrac{d^4y}{dx^4} + y(x) = 0.$

(b) $\dfrac{d^5y}{dx^5} - 2\dfrac{d^4y}{dx^4} + \dfrac{d^3y}{dx^3} = 0.$

(c) $\dfrac{d^3y}{dx^3} - \dfrac{d^2y}{dx^2} + \dfrac{dy}{dx} - y(x) = 0.$

(d) $\dfrac{d^5y}{dx^5} + 5\dfrac{d^4y}{dx^4} + 10\dfrac{d^3y}{dx^3} + 10\dfrac{d^2y}{dx^2} + 5\dfrac{dy}{dx} + y(x) = 0.$

(3) Solve the following initial-value problems:

(a) $y'' - 2y' + y = 2xe^{2x} + 6e^x;\ y(0) = 1,\ y'(0) = 0.$

(b) $y''(x) + y(x) = 3x^2 - 4\sin x,\ y(0) = 0,\ y'(0) = 1.$

(4) Use the method of undermined coefficients to find a particular solution to the following differential equations:

(a) $y'' - 3y' + 2y = 2x^2 + 3e^{2x}.$

(b) $y''(x) - 3y'(x) + 2y(x) = xe^{2x} + \sin x.$

(5) Use the annihilator method to determine the form of a particular solution for the equations:

(a) $y''(x) - 5y'(x) + 6y(x) = \cos(2x) + 1.$

(b) $y''(x) - 5y'(x) + 6y(x) = e^{3x} - x^2.$

(6) Let $P(D) = a_nD^n + \cdots + a_1D + a_0,\ \ a_n \neq 0$, where $D = \frac{d}{dx}$.

(a) If $P(D)y = ce^{ax}$, where c is a constant then a particular solution is given by

$$y_p = \frac{1}{P(D)}(ce^{ax}) = \frac{ce^{ax}}{P(a)},\ \ P(a) \neq 0.$$

(b) If $P(D)y = h(x)e^{ax}$, where $h(x)$ is any function in x, then

$$y_p = \frac{1}{P(D)}(h(x)e^{ax}) = e^{ax}\frac{1}{P(D+a)}h(x).$$

(c) In particular, if $P(D) = (D-a)^rP_1(D)$, where $P_1(a) \neq 0$ then $y_p = \frac{1}{P(D)}(ce^{ax}) = \frac{cx^re^{ax}}{r!P_1(a)}$.

(7) Use operator method to find a particular solution of the following ODEs.

(a) $y''' + y'' + y' + y = x^5 - 2x^2 + x.$

(b) $y''' - 5y'' + 8y' - 4y = 3e^{2x}.$

(c) $y'' - 3y' + 2y = 3\sin 2x.$

(8) Find a particular solution to the following differential equations:

(a) $y'' + 4y = \tan 2x.$

(b) $y'' + y = \tan x + 3x - 1.$

(c) $y'' - 2y' + y = e^x \sin^{-1} x.$

(9) Find a general solution to the differential equation given that the functions $y_1(x)$ and $y_2(x)$ are linearly independent solutions to the corresponding homogeneous equation for $x > 0$.

(a) $(\sin^2 x)y'' - 2\sin x\cos x y' + (\cos^2 x + 1)y = \sin^3 x;\quad y_1(x) = \sin x,\ \ y_2(x) = x\sin x.$
(b) $(x^2 + 2x)y'' - 2(x+1)y' + 2y = (x+2)^2;\quad y_1(x) = x + 1,\ \ y_2(x) = x^2.$

(10) Use the method of variation of parameters to show that

$$y(x) = c_1 \cos x + c_2 \sin x + \int_0^x f(s)\sin(x - s)\, ds$$

is a general solution to the differential equation $y'' + y = f(x)$, where $f(x) \in C(\mathbb{R})$.

(11) A differential equation and a non-trivial solution y_1 are given. Find the general solution.
(a) $x^2y'' + xy' - y = 0,\ \ x \neq 0\ y_1(x) = x.$
(b) $x^2y'' - 2xy' - 4y = 0,\ x > 0;\quad y_1(x) = x^{-1}.$