PaperMan
Cepstrum Initiative
Visit us at:
http://www.iitg.ac.in/scifac/cep/public_html

Department of Physics, IIT Guwahati

PH102: Physics –II; Mid Sem / Date: 24/02/2014 /*Duration: 2hrs* / *Max.Marks:30*

Answer all questions. Answer to all parts of a question including subsections a, b, *etc.* should be written together. For every question start answering from a fresh page. Do not write more than one answer for a single question.

1. Check the divergence theorem for the function $\vec{v} = r^2\hat{r} + k^2 \sin\theta\,\hat{\theta} + k^2 \sin\theta\cos\phi\,\hat{\phi}$ using the volume of one octant of sphere of radius R as shown in Fig.1. Take *k* as a constant and r,θ and φ as spherical polar co-ordinates. [6]

$\int(\nabla\cdot v)\,d\tau = \oint v\cdot da$

2. An inverted hemispherical bowl of radius R carries a uniform surface charge density σ. Find the potential at (a) P_1 (0, 0, z), (b) P_2 (0, 0, R) and (c) P_3 (0, 0, 0) as shown in Fig.2. [6]

3. (a) Two vector functions $\vec{v}_1$ and $\vec{v}_2$ are given. Find out the vector(s) whose line integral(s) $\int \vec{v}.\vec{dl}$ is (are) independent of path. Justify your answer. $\vec{v}_1 = \frac{k_1}{s}\hat{s} + k_2 z\,\hat{z}$ and $\vec{v}_2 = 2xz\hat{x} + (x^2 - k_3 y)\hat{y} + (k_4 z - x^2)\hat{z}$. k_1, k_2, k_3 & k_4 are constants. Other symbols carry the usual meaning in polar and Cartesian co-ordinates. (b) Evaluate the following integrals. $\int_2^6 (3x^2 - 2x - 1)\,\delta(x-3)\,dx$ and $\int_{-1}^{1} 9x^2\delta(3x+1)\,dx$. (c) Write down the boundary conditions for both parallel and perpendicular components of **E** and **D** at the plane surface of a material having free charge density σ_f and polarization **P**. [6]

4. A thick spherical shell (inner radius a, outer radius b) is made of dielectric material with a "frozen - in" polarization $\mathbf{P(r)} = \frac{k}{r}\hat{\mathbf{r}}$ where k is a constant and r is the distance from the center (Fig. 3). (a) Find the electric displacement (**D**) and electric field (**E**) in the regions r < a, a < r < b and r > b. (b) Calculate the surface and volume bound charge densities and (c) the total bound charge. [6]

5. An *infinite* straight wire carrying line charge density λ is kept parallel to the x-axis at a perpendicular distance *d* from the surface of an infinite grounded conducting plane as shown in Fig. 4. Using the method of images find (a) the electric potential at point p above the conducting plane, (b) the induced surface charge density on the conducting plane and (c) the force per unit length experienced by the given line charge. [6]

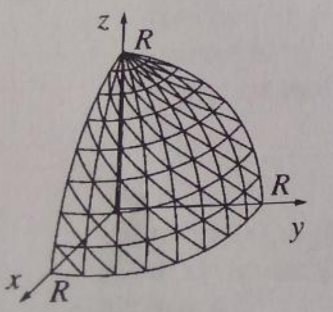

Fig.1

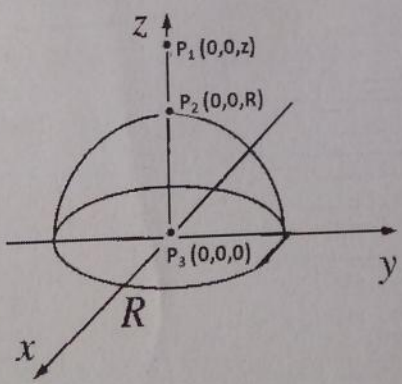

Fig.2

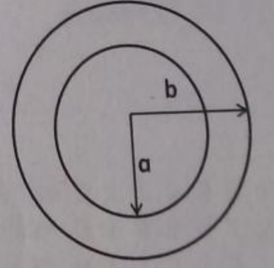

Fig.3

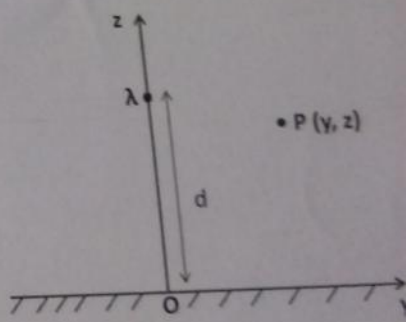

Fig.4

Useful formula:
$$\vec{\nabla}.\vec{v} = \frac{1}{h_1h_2h_3}\left\{\frac{\partial(v_1h_2h_3)}{\partial u_1} + \frac{\partial(v_2h_3h_1)}{\partial u_2} + \frac{\partial(v_3h_1h_2)}{\partial u_3}\right\} \quad \vec{\nabla}\times\vec{v} = \frac{1}{h_1h_2h_3}\begin{vmatrix} \hat{u}_1h_1 & \hat{u}_2h_2 & \hat{u}_3h_3 \\ \frac{\partial}{\partial u_1} & \frac{\partial}{\partial u_2} & \frac{\partial}{\partial u_3} \\ h_1v_1 & h_2v_2 & h_3v_3 \end{vmatrix}$$

---------------------- &----------------------