PH 102 : Electromagnetism **Department of Physics**
Tutorial 3 **Indian Institute of Technology Guwahati**

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1. Compute the line integral of $\vec{F} = r\cos^2\theta\hat{r} - r\cos\theta\sin\theta\hat{\theta} + 3r\hat{\phi}$ around the path shown in Figure 5. Do it either in cylindrical or in spherical coordinates. Check your answer using Stokes' theorem.

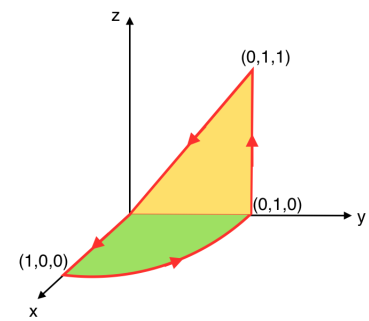

Figure 1: The path

2. Calculate the flux of $\vec{F} = x\hat{x}+y\hat{y}+z\hat{z}$ through the surface defined by a cone $z = \sqrt{x^2+y^2}$ and the plane $z = 1$.

3. Evaluate the following integrals:
   (a) $\int_{-1}^{2}[\sin x\delta(x+2) - \cos x\delta(x)]dx$.
   (b) $\int_{-3}^{2}(x^3 - 2x^2 + 3x + 1)\delta(x+2)dx$
   (c) $\int_{\mathcal{V}} \vec{r}.(\vec{d} - \vec{r})\delta^3(\vec{e} - \vec{r})d\tau$, where $\vec{d} = (1,2,3)$, $\vec{e} = (3,2,1)$ and $\mathcal{V}$ is the volume of a sphere of radius 1.5 units centred at $(2,2,2)$.
   (d) Show that $x\frac{d}{dx}\delta(x) = -\delta(x)$

4. Imagine four unit charges nailed to four corners of a square of side 2 units, with the North-East corner being at $(x = 1, y = 1)$. Draw pictures whenever appropriate.
   (a) Show that a charge -1 unit placed at the origin is in equilibrium, i.e., has no net force on it using symmetry arguments.
   (b) Now consider the stability of this equilibrium by lifting the charge slightly out of the plane by a tiny amount δ. Show that there is a restoring force $-k\delta$ and find k. (Use Taylor series. Since you need the force only for small displacement, drop any thing in the formula that goes like (displacement)^2 or higher.)
   (c) Find the angular frequency (ω) of small oscillations if the charge has mass m.
   (d) With what speed will it cross the origin if released from $z = \delta$?
   (e) Establish next the instability under displacements in the plane by choosing δ to be along the x-axis and showing $\kappa = -1/(4\pi\epsilon_0\sqrt{2})$.

5. A semi-infinite rod extending from the origin up the y-axis carries a line charge density λ. Find the field at the point $x = a$, $y = 0$. Show how you could relate the x-component of the field to the result for an infinite rod.

6. A charge q is at the center of a unit cube. What is the flux through one of its faces? Now consider the charge to be at one corner of the cube, as shown. What is the flux through the shaded side?

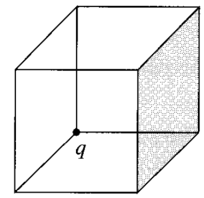

Figure 2: Charge at the corner

7. A solid sphere of radius R has uniform charge density ρ. A hole of radius $R/2$ is scooped out of it as shown in Figure. Show that the field inside the hole is uniform and along the x-axis and of magnitude $\rho R/6\epsilon_0$.

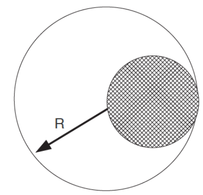

Figure 3: Sphere with a hole