MA 102 (Ordinary Differential Equations)

IIT Guwahati

Tutorial Sheet No. 1 **Date:** March 8, 2018

Basics of ODEs, Picard's Theorem.

(1) Determine the *order* and *degree* of the following differential equations. Also, state whether they are *linear* or *nonlinear*.
(a) $\frac{d^4y}{dx^4}+19\left(\frac{dy}{dx}\right)^2 = 11y$; (b) $\frac{d^2y}{dx^2}+x\sin y = 0$; (c) $\frac{d^2y}{dx^2}+y\sin x = 0$; (d) $(1+\frac{dy}{dx})^{\frac{1}{2}} = x\frac{d^2y}{dx^2}$;
(e) $\frac{d^6y}{dx^6} + \left(\frac{d^4y}{dx^4}\right)\left(\frac{d^3y}{dx^3}\right) + y = x$; (f) $x^3\frac{d^3y}{dx^3} + x^2\frac{d^2y}{dx^2} + y = e^x$.

(2) Eliminating the arbitrary constants c_1, c_2, obtain the differential equation satisfied by the following functions.
(a) $y = c_1e^{-x} + c_2e^{2x}$; (b) $x^2 + c_1y^2 = 1$; (c) $y = c_1x - c_1^3$.

(3) Consider the equation $y'(x) = cy(x)$, $0 < x < \infty$, where c is a real constant. Then
(a) Show that if ϕ is any solution and $\psi(x) = \phi(x)e^{-cx}$ then $\psi(x)$ is a constant.
(b) If $c < 0$, show that every solution tends to zero as $x \to \infty$.
(c) If $c > 0$, prove that the magnitude of every non-trivial solution tends to ∞ as $x \to \infty$.
(d) When $c = 0$, what can be said about the magnitude of the solution?

(4) Find all real valued C^1 solutions $y(x)$ of the differential equation $xy'(x) + y(x) = x$, $x \in (1, 2)$.

(5) Discuss the existence and uniqueness of a solution of the following initial value problems (IVP) in the region $R: \ |x| \leq 1 \ |y| \leq 1$.
(a) $\dfrac{dy}{dx} = 3y^{2/3}$, $y(0) = 0$; (b) $\dfrac{dy}{dx} = \sqrt{|y|}$, $y(0) = 0$;
(c) $\dfrac{dy}{dx} = x^2 + y^2$, $y(0) = 0$.

(6) Show that the equation $|y'(x)| + |y(x)| + 1 = 0$ has no real solutions.

(7) A point P is dragged along the xy plane by a string PT of length a. If T starts at the origin and moves along the positive y axis, and if P starts at $(a, 0)$, what is the path of P? Assume here that the string is always tangent to the curve traced by the point P.