1. Evaluate $\int \mathbf{A}\cdot\hat{\mathbf{n}}\, ds$, where $\mathbf{A} = 18z\,\hat{\mathbf{x}} - 12\,\hat{\mathbf{y}} + 3y\,\hat{\mathbf{z}}$ and S is that part of the plane $2x+3y+6z = 12$ which is located in the first octant.

2. Evaluate $\int_S \mathbf{A}\cdot\hat{\mathbf{n}}\, ds$, where $A = z\hat{\mathbf{i}} + x\hat{\mathbf{j}} - 3y^2z\hat{\mathbf{k}}$ and S is the surface of the cylinder $x^2+y^2 = 16$ included in the first octant between $z = 0$ and $z = 5$.

3. **[G 1.30]** Calculate the volume integral of the function $T = z^2$ over the tetrahedron with corners at $(0,0,0)$, $(1,0,0)$, $(0,1,0)$ and $(0,0,1)$.

4. **[G 1.31]** Check the fundamental theorem for gradients, using $T = x^2 + 4xy + 2yz^3$, the points $\mathbf{a} = (0,0,0)$, $\mathbf{b} = (1,1,1)$, and the three paths in Fig.:

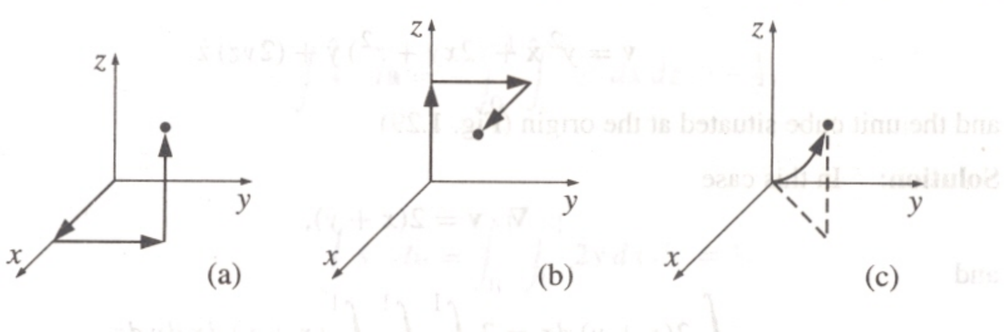


Figure 1: Problem 4

   (a) $(0,0,0) \to (1,0,0) \to (1,1,0) \to (1,1,1)$;
   (b) $(0,0,0) \to (0,0,1) \to (0,1,1) \to (1,1,1)$;
   (c) the parabolic path $z = x^2$; $y = x$.

5. **[G 1.33]** Test Stokes' theorem for the function $\mathbf{v} = (xy)\hat{\mathbf{x}} + (2yz)\hat{\mathbf{y}} + (3zx)\hat{\mathbf{z}}$, using the triangular shaded area of Fig.

6. Consider a vector field $\mathbf{F}$, for which line integral in independent of path between **any** two points. Show that $\nabla \times \mathbf{F} = 0$.

7. **[G 1.39]** Compute the divergence of the function

$$\mathbf{v} = (r\cos\theta)\hat{\mathbf{r}} + (r\sin\theta)\hat{\theta} + (r\sin\theta\cos\phi)\hat{\phi}.$$

   Check the divergence theorem for this function, using as your volume the inverted hemispherical bowl of radius R, resting on the xy plane and centered at the origin (See fig).

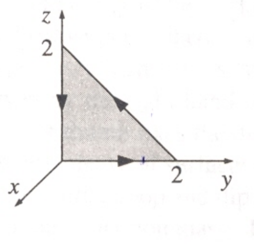
8. **[G 1.41]** Derive the relations for unit vectors of cylindrical coordinate system:

$$\begin{aligned}\hat{\mathbf{s}} &= \cos\phi\hat{\mathbf{x}} + \sin\phi\hat{\mathbf{y}},\\ \hat{\phi} &= -\sin\phi\hat{\mathbf{x}} + \cos\phi\hat{\mathbf{y}},\\ \hat{\mathbf{z}} &= \hat{\mathbf{z}}.\end{aligned}$$

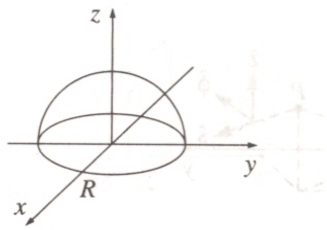
   Invert the formulas to get $\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$, $\hat{\mathbf{z}}$ in terms of $\hat{\mathbf{s}}$, $\hat{\phi}$, $\hat{\mathbf{z}}$ (and ϕ).

9. **[G 1.42]**

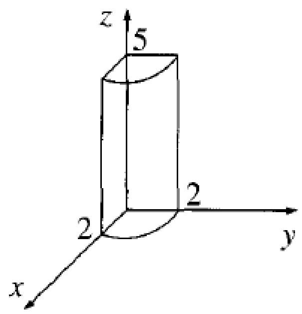
   (a) Find the divergence of the function $\mathbf{v} = s(2+\sin^2\phi)\hat{\mathbf{s}} + s\sin\phi\cos\phi\hat{\phi} + 3z\hat{\mathbf{z}}$.



(a) Problem 5



(b) Problem 7



(c) Problem 9

(b) Test the divergence theorem for this function, using the quarter-cylinder (radius 2, height 5) shown in Fig.

(c) Find the curl of v.

10. **[G 1.44]** Evaluate the following integrals:

(a) $\int_{-2}^{2}(2x+3)\delta(3x)dx$.

(b) $\int_{0}^{2}(x^3+3x+2)\delta(1-x)dx$.

(c) $\int_{-1}^{1} 9x^2\delta(3x+1)dx$.

(d) $\int_{-\infty}^{a} \delta(x-b)dx$.