## it uttévariable Calculus

Note Title 2/22/2013

## Lectures 1 and 2:

- · The Space TRn · Convergence un TRn
- $\frac{1}{2} \frac{why}{do} \frac{do}{do} \frac{do}$

Consider the "matrix"

$$V = SSf(x,y)dxdy$$

Is 
$$\int (\int f(x,y)dx)dy =$$

$$\int \left( \int f(x,y) dy \right) dx$$

Consider

$$f(x,y) = e^{xy} - xye^{xy}$$
and
$$\int_{x=0}^{\infty} f(x,y) dx dy$$

Then
$$\int_{0}^{\infty} \left( \int_{0}^{1} f(x,y) dy \right) dx dx = \int_{0}^{\infty} e^{xy} dx = 1$$
and
$$\int_{0}^{\infty} \left( \int_{0}^{1} f(x,y) dx \right) dy dx dy = 0$$

$$\int_{0}^{1} \int_{0}^{1} x e^{xy} dy = 0$$

$$f(x,y) = \frac{x^2}{x^2 + y^2}$$

and lem lem 
$$f(x,y) = 0$$
  
 $y \rightarrow x \rightarrow 0$ 

(d) Partial Derivatives

18

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$$
?

Consider

 $f(x,y) = \left\{\frac{xy^3}{x^2 + y^2}, (x,y) \neq (0,0)\right\}$ 

Then

 $\frac{\partial^2 f(0,0)}{\partial x \partial y} = 1$ 

and

 $\frac{\partial^2 f(0,0)}{\partial y \partial x} = 0$ 

(e) Differentiability:

f(x,y) = (ey, Sin(xy), xy)

18 f defferentiable?

Let f: R TRM

What does it mean to say that

f is differentiable?

Analysis & the Foundation of Calculus

---v --v --

2. Review of analysis in R.

• (R, +, •, 1.1) us an

ordered field.

Lub profesty holds in IR (also known is completeness property) — Convergence of sequence.

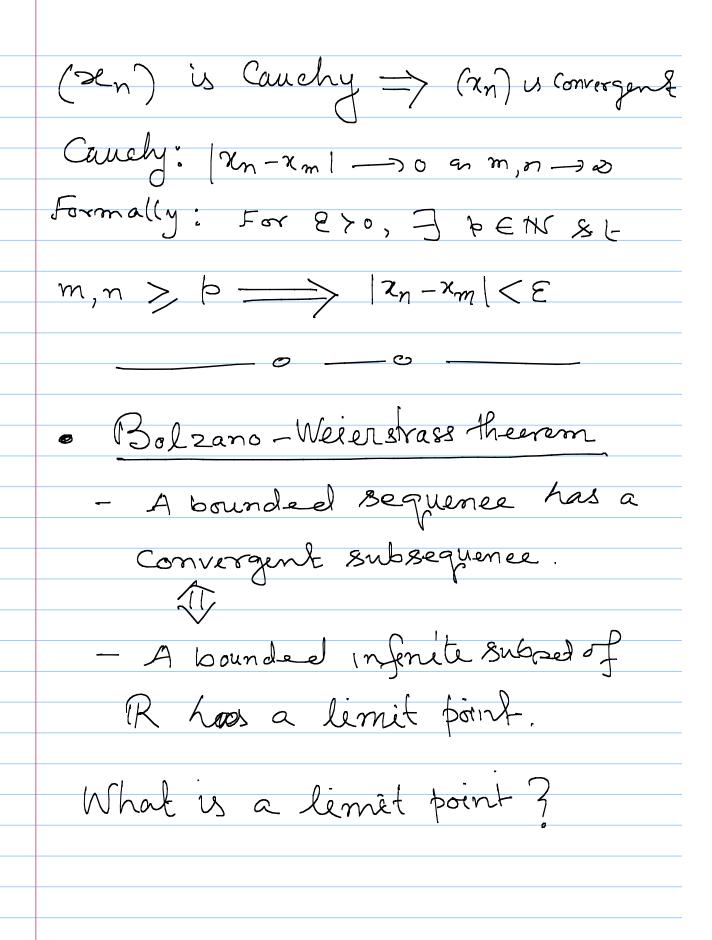
Monotone Convergence profesty

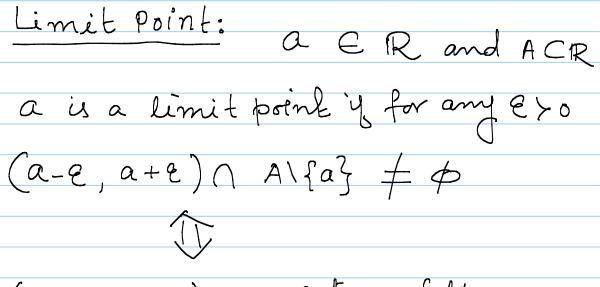
(Xn) 1:  $x_1 \leq x_2 \leq \cdots \leq x_n \leq \cdots$ 

(xn) . x, / x2/ -- - - .

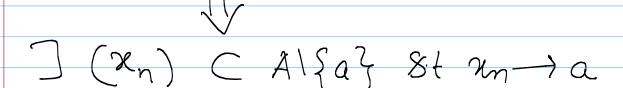
- Bounded + Monotone = Convergence

Cauchy criterion holds in TR (R is complete)





(a-e, b+e) (A centains infinitely many elements of A.



- Heine-Borel theorem Closed + Bounded = Compact

- Compact: ACR is Compact of (2n) (A) then  $3(xn_x)$   $(x_n)$ 8. t  $xn_x \rightarrow x$  in R and  $x \in A$ 

- Q. Con these results se generalised to Rn?
- 3. The Euclidean Space Rn

R":= RXRX---XR
n times

 $= \left\{ \left( \chi_{1, \chi_{2, ---}} \chi_{n} \right) : \chi_{i} \in \mathbb{R} \right\}$ 

•  $\mathcal{X} = (\mathcal{X}_1, \mathcal{X}_2 - \mathcal{X}_n), \mathcal{J} = (\mathcal{Y}_1, \mathcal{Y}_2 - \mathcal{Y}_n) \in \mathbb{R}^n$ 

 $\mathcal{X} + \mathcal{Y} := (\mathcal{X}_1 + \mathcal{Y}_1, \dots, \mathcal{X}_n + \mathcal{Y}_n)$ 

· dl := (x2, . . ., x2n), x ER

- (Rn, +, e) is a vicolor spalle over R.

. Fundamental propostes:

11 sell := 
$$(x_1^2 + x_2^2 + \dots + x_n^2)^2$$

Jundamental properties.

(e) 
$$|| \propto x || = |\alpha| || 1| \propto || + \alpha \in \mathbb{R}, \; \gamma \in \mathbb{R}^n$$

11 oll -> Euclidean norm

$$\frac{d(x,y)}{=} \frac{11x-y11}{\sum_{i=1}^{n} (x_i-y_i)^2}$$

is the Euclidean distance setuen & and y.

· Inner product / dot product.

$$\langle , \rangle : \mathbb{R}^n \times \mathbb{R}^n \longrightarrow \mathbb{R}$$

· Cauchy - Schwarz inequality
[⟨x, y⟩ | ≤ ||x|| || y1|

Angle between vcotors:

$$-1 \leq 22, \% \leq 1$$
 $|1x|| 1|y||$ 

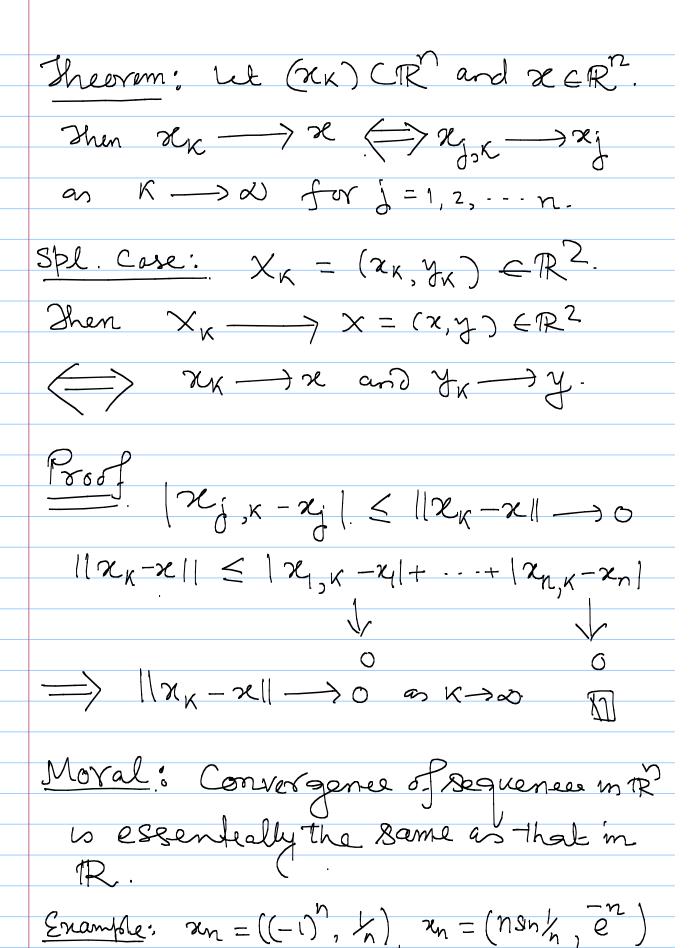
Junique  $\theta \in [0, \%] \otimes E$ 
 $Cos\theta = \langle x, \% \rangle \times (x \neq 0, \%) = 0$ 
 $||x|| ||y|| = 0$ 
 $\langle x, \% \rangle = ||x|| ||y|| ||cos\theta||$ 

Orthogonality:

Orthogonality: <re></re>
< = 0 then sely.</p>

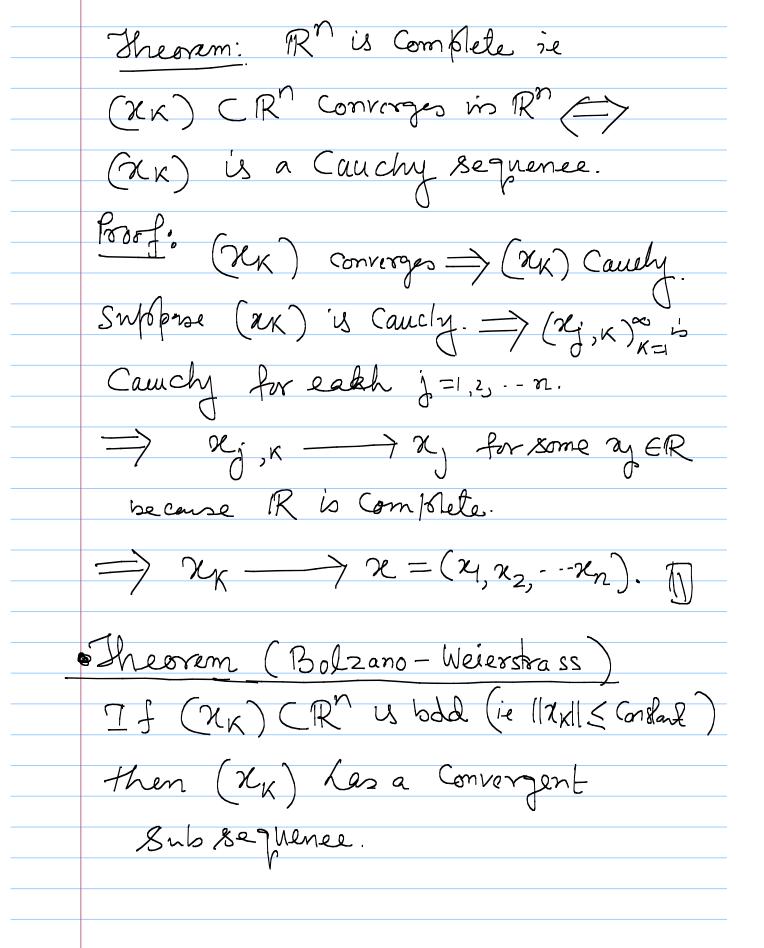
Convergence in R A function N — R, K in the six is called a sequence, written as (rek), (rek) = {2k} f 2k} x = 1

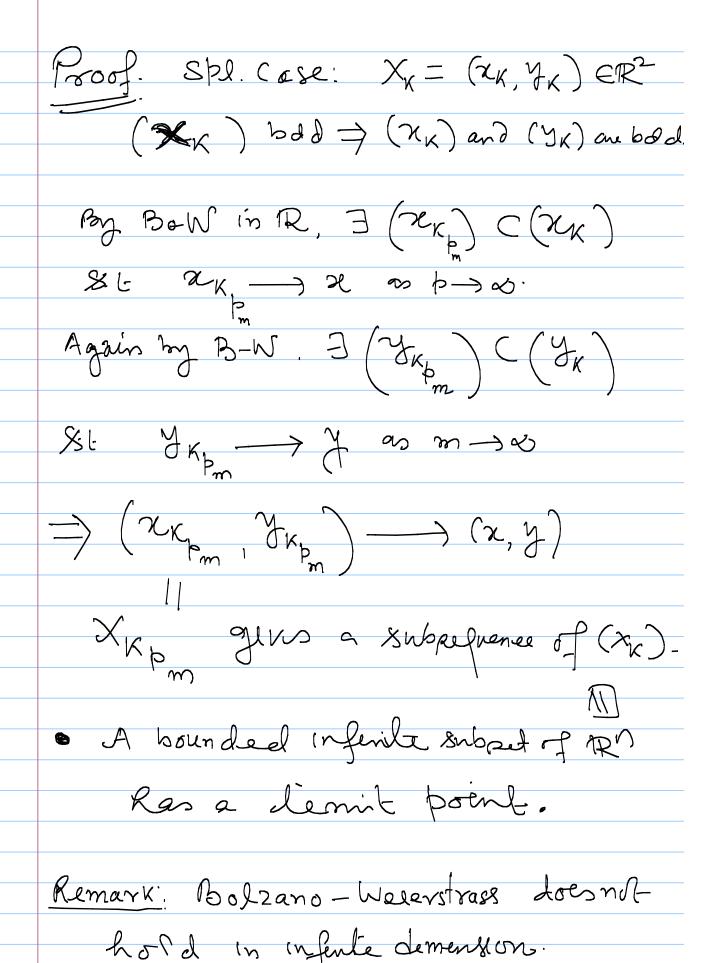
Kemark: Each term 2x of (Xx) is a vector in Rnie  $\mathcal{X}_{\mathcal{K}} = (\mathcal{X}_{1,\mathcal{K}} \mathcal{X}_{2,\mathcal{K}}, - \mathcal{X}_{n,\mathcal{K}}) \in \mathbb{R}^{n}$ Convergence: Let (xx) CRn and se ERT. Shen, Ky->2 Y Informal: 112/x -x11 -> 0 as K-> a Formal: For €>0, 3 Þ∈W s.t  $K > p \implies |x_K - x|| < \varepsilon$ Same old defn !!!  $2l_{K} = (2l_{1,K}, 2l_{2,K}, ..., 2l_{n,K})$  $\chi = (\chi, \chi, \ldots, \chi)$ In deed, 12 - 20, x/ \langle 1/2-2x/ ->0



| · Completeness of Rn                                      |
|---|
|   |
| Obviously Lub. Prof. Mc Prof. Rave                        |
|   |
| Cauchy's Criterion is amenable to<br>generalization in RA |
| glneralization in it.                                     |
| - Cauchy sequence: (RK) CRn                               |
| Informal:<br>112k-2pll-30 as, K, b-30                     |
| Formal: For EYO, 3 m ENSE                                 |
| $ K, b\rangle M \longrightarrow   2k-2p   < \varepsilon$  |
| Same old defn !!!   |
| Fact. (Xx)=(xx, yx) ER is Cauchy.                         |
| (xx) and Cyx) are Calledy.                                |
|   |

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| Def: ACR is said to be bounded is J M>0 such that $1 \times  1 \leq M + \times \in A$ .  |
|--|
| Sheorom (Heine-Borel).   |
| C(osed + bounded = Compact   |
| Proof: A CTR Compact > A to closed and bounded (Why?)  |
| Suppose A is closed and bold.  Let $(\chi_K) \subset A \xrightarrow{B-W} \exists (\chi_K) \subset (\chi_K)$ Sit $\chi_k \longrightarrow \chi \in \mathbb{R}^n$ |
| _  |
| Since A is closed, REA   |
| => A us compact.   |
| — End —  |
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