MA 102 (Mathematics II)

Department of Mathematics, IIT Guwahati

Tutorial Sheet No. 2

(1) Examine if the limits as $(x,y) \to (0,0)$ exist?

(a) $f(x,y) = \begin{cases} \frac{x^3+y^3}{x^2-y^2} & x \neq \pm y \\ 0 & x = \pm y \end{cases}$ (b) $xy\left(\frac{x^2-y^2}{x^2+y^2}\right)$ (c) $\frac{\sin(xy)}{x^2+y^2}$

(d) $\frac{|x|}{y^2}e^{-|x|/y^2}$ (e) $\frac{1-\cos(x^2+y^2)}{(x^2+y^2)^2}$.

(2) Examine the continuity of $f : \mathbb{R}^2 \to \mathbb{R}$ at $(0,0)$, where for all $(x,y) \in \mathbb{R}^2$,

(a) $f(x,y) := \begin{cases} xy\cos(1/x) & \text{if } x \neq 0, \\ 0 & \text{if } x = 0. \end{cases}$

(b) $f(x,y) := \begin{cases} 1 & \text{if } x > 0 \ \& \ 0 < y < x^2, \\ 0 & \text{otherwise.} \end{cases}$

(c) $f(x,y) := \begin{cases} \frac{x^3}{x^2+y^2} & \text{if } (x,y) \neq (0,0), \\ 0 & \text{if } (x,y) = (0,0). \end{cases}$

(d) $f(x,y) := \begin{cases} \frac{x^3 y}{x^4+y^2} & \text{if } (x,y) \neq (0,0), \\ 0 & \text{if } (x,y) = (0,0). \end{cases}$

(e) $f(x,y) := \begin{cases} \frac{\sin(x+y)}{|x|+|y|} & \text{if } (x,y) \neq (0,0), \\ 0 & \text{if } (x,y) = (0,0). \end{cases}$

(f) $f(x,y) := \begin{cases} xy\ln(x^2+y^2) & \text{if } (x,y) \neq (0,0), \\ 0 & \text{if } (x,y) = (0,0). \end{cases}$

(3) Suppose that $f : \mathbb{R}^2 \to \mathbb{R}$ is a continuous function at $X_0 \in \mathbb{R}^2$ and that $|f(X_0)| > 2$. Show that there is a $\delta > 0$ such that $|f(X)| > 2$ whenever $\|X - X_0\| < \delta$.

(4) $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by $f(x,y) = 0$ if $x \in \mathbb{Q}, y \in \mathbb{Q}$ and $f(x,y) = xy$ otherwise. Find all the points in $\mathbb{R}^2$ where f is continuous.

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