PH 102 : Electromagnetism **Department of Physics**
Tutorial 6 **Indian Institute of Technology Guwahati**

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1. (a) Show that the electric field of a 'pure dipole' can be written as

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{1}{r^3}[3(\vec{p}.\hat{r})\hat{r} - \vec{p}]$$

Note that this form has the advantage of not committing to a particular coordinate system.

(b) Find the force and torque on a dipole in the field of a point charge. Let the charge q be at the origin and the dipole $\vec{p} = p_0(\sin\zeta_0\hat{x} + \cos\zeta_0\hat{z})$ be at the point $(0, 0, z_0)$. Also find the force on q due to the dipole and verify Newton's third law.

2. Energy of a dipole:

(a) Show that the energy of a dipole with dipole moment $\vec{p}$ in an electric field $\vec{E}$

$$U = -\vec{p}.\vec{E}$$

(b) Show that the interaction between two dipoles with dipole moments $\vec{p}_1$ and $\vec{p}_2$ separated by distance $\vec{r}$ is given by:

$$U = \frac{1}{4\pi\epsilon_0}\frac{1}{r^3}[\vec{p}_1.\vec{p}_2 - 3(\vec{p}_1.\hat{r})(\vec{p}_2.\hat{r})]$$

3. Suppose we have limited dielectric material of dielectric constant ϵ_r to only half fill a parallel plate capacitor. Compare the two cases depicted in Figure (a) and (b) commenting on which case the capacitance is more. Assuming a potential difference V between the plates, find $\vec{E}, \vec{D}, \vec{P}$ in each region and free and bound charges on all surfaces in both the cases.

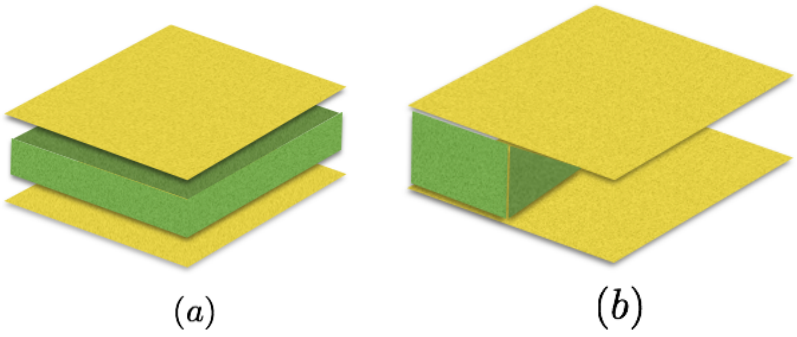

Figure 1

4. Consider a conducting spherical shell with an inner radius a and outer radius c. Let the space between two surfaces be filled with two different dielectric materials of dielectric constant κ_1 between a and b, and κ_2 between b and c. Determine the capacitance of the system.

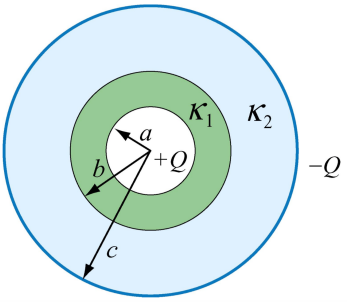

Figure 2

5. Two long coaxial cylindrical metal tubes (inner radius a and outer radius b) stand vertically in a tank of dielectric oil (susceptibility χ_e, mass density ρ). The inner one is maintained at a potential V and the outer one is grounded. To what height does the oil rise in the space between the tubes ?

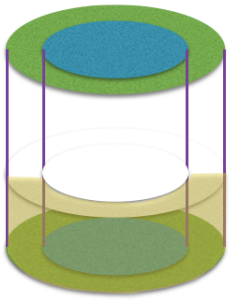

Figure 3

6. A spherical conductor of radius a, carries a charge Q. It is surrounded by linear dielectric material of susceptibility χ_e, out to radius b. Find the energy of this configuration.