PH 102 : Electromagnetism **Department of Physics**
Tutorial 2 **Indian Institute of Technology Guwahati**

1. Calculate the flux of the vector $\vec{F} = x\hat{x} + y\hat{y} + z\hat{z}$ over the surface of a right circular cylinder of radius R bounded by the surfaces $z = 0$ and $z = h$ with the centre of the base of the cylinder situated at origin. Calculate it directly as well as by use of the divergence theorem.

2. Let $\vec{F} = 2xz\hat{x} - x\hat{y} + y^2\hat{z}$. Evaluate $\int_{\mathcal{V}} \vec{F} d\tau$ where $\mathcal{V}$ is the region bounded by the surfaces $x = 0,\ y = 0,\ y = 6,\ z = x^2,\ z = 4$, as pictured in Figure 1.

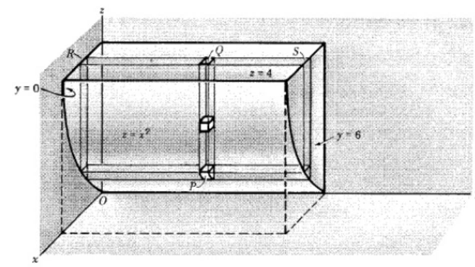

Figure 1: Volume element

3. Check the fundamental theorem for gradients using $T = x^2 + 4xy + 2yz^3$ joining the points $a = (0, 0, 0),\ b = (1, 1, 1)$ following three different paths as shown in Figure 2.

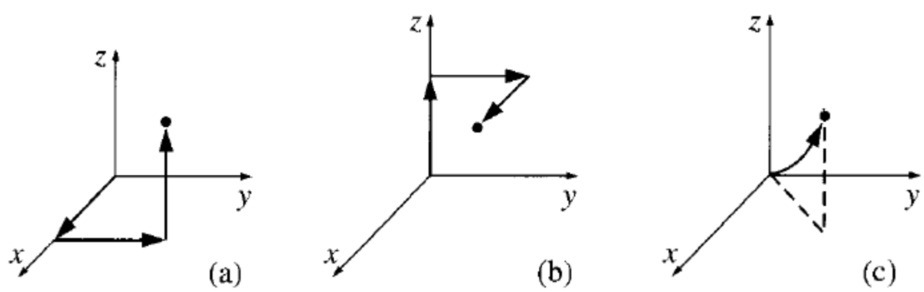

Figure 2: Three different paths

(a) $(0, 0, 0) \to (1, 0, 0) \to (1, 1, 0) \to (1, 1, 1)$
(b) $(0, 0, 0) \to (0, 0, 1) \to (0, 1, 1) \to (1, 1, 1)$
(c) The parabolic path $z = x^2, x = y$.

4. (a) Using Stoke's theorem calculate the line integral of $\vec{F} = 2z\hat{x} + x\hat{y} + y\hat{z}$ over a circle of radius R in the xy plane centered at the origin. Take the open surface to be a hemisphere in $z > 0$ (Fig. 3).
(b) Calculate the same using Divergence theorem imagining the hemispherical surface as well as the disc on the $x - y$ plane to form a closed surface.

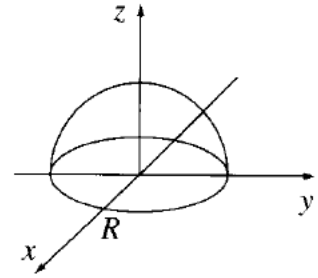

Figure 3: Hemisphere

5. Prove that the cylindrical coordinate system is orthogonal and express velocity and acceleration of a particle in cylindrical polar coordinates.

6. In PH 101 you encountered the momentum operator in quantum mechanics. Recall that the momentum operator had the form $p = \frac{\hbar}{i}\frac{d}{dx}$ in one dimension. Now that we have discussed everything in general in three dimensions, $\vec{p} = \frac{\hbar}{i}\vec{\nabla}$. Hence the angular momentum operator $\vec{L} = \vec{r} \times \vec{p} = \frac{\hbar}{i}(\vec{r} \times \vec{\nabla})$. Show that the angular momentum operator in spherical polar coordinate is of the form

$$\vec{L} = \frac{\hbar}{i}\left(-\hat{\theta}\frac{1}{\sin\theta}\frac{\partial}{\partial\phi} + \hat{\phi}\frac{\partial}{\partial\theta}\right)$$