Physics II: Electromagnetism (PH102)

Lecture 9

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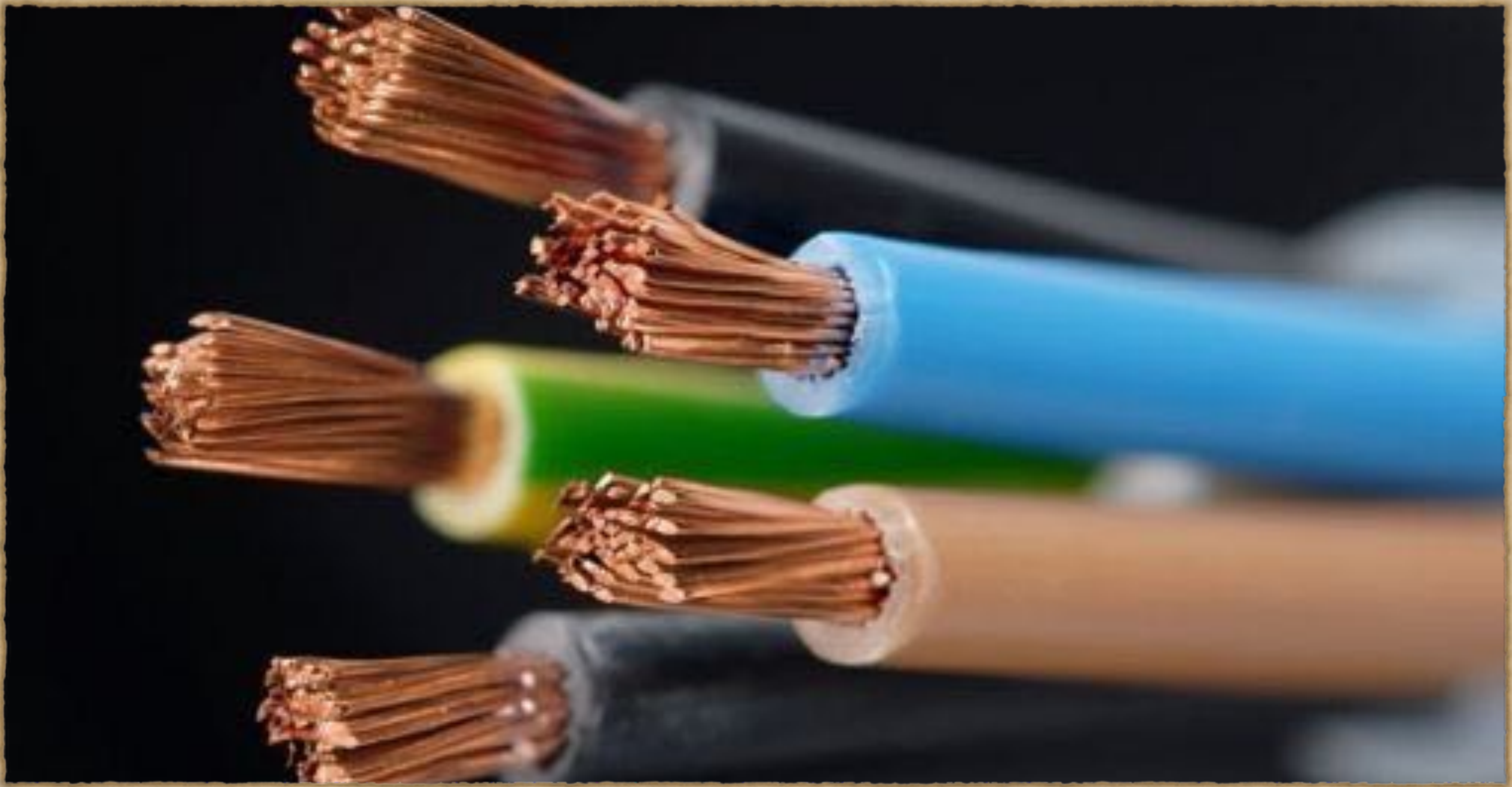
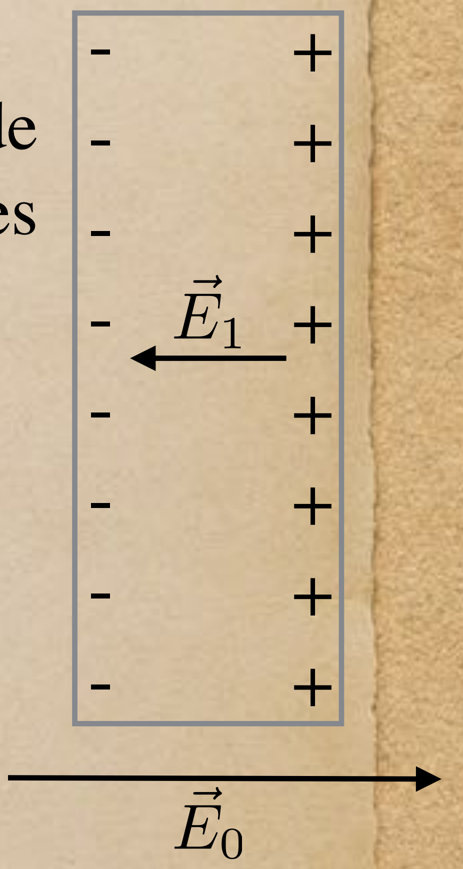


Conductors

What is a conductor ?

An electrical conductor is a solid that contains many free electrons. Electrons can move freely inside, but can not leave the surface.

Example: Metal



- An external field $\vec{E}_0$ will separate the positive and negative charges inside the metal and they pile up on two opposite sides. These induced charges produce a field of their own $\vec{E}_1$, which is in opposite direction to $\vec{E}_0$

- Field of the induced charges tends to cancel the original field. Charges will continue to flow until the resultant field inside the conductor is precisely zero. The whole process is practically instantaneous. The only electrostatic situation is that the field is zero everywhere inside. $\vec{E} = 0$

- $\rho = 0$ inside a conductor: Follows from Gauss's law $\vec{\nabla}.\vec{E} = \rho/\epsilon_0$

What about potential inside a conductor ? It must be constant -> Equipotential so that the field is zero everywhere.

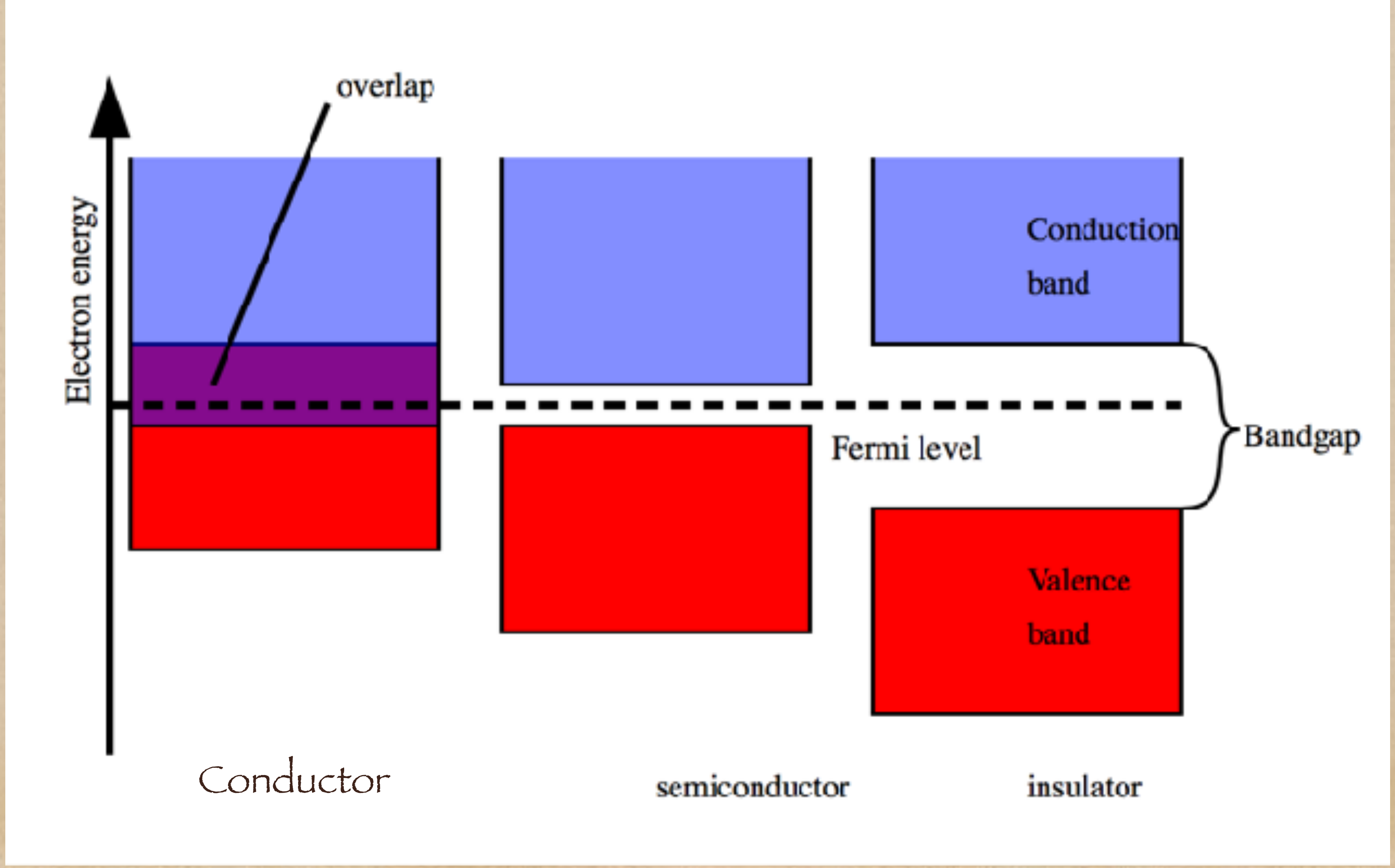
$$V(b) - V(a) = \int_a^b \vec{E}.d\vec{l} = 0$$

$$V_a = V_b$$

Why conductors are conductors ?

Quantum Mechanics successfully describes theory of solids through concepts of valence band and conduction band.

Electrical conductivity depends on the capability to flow electrons from valence band to conduction band.



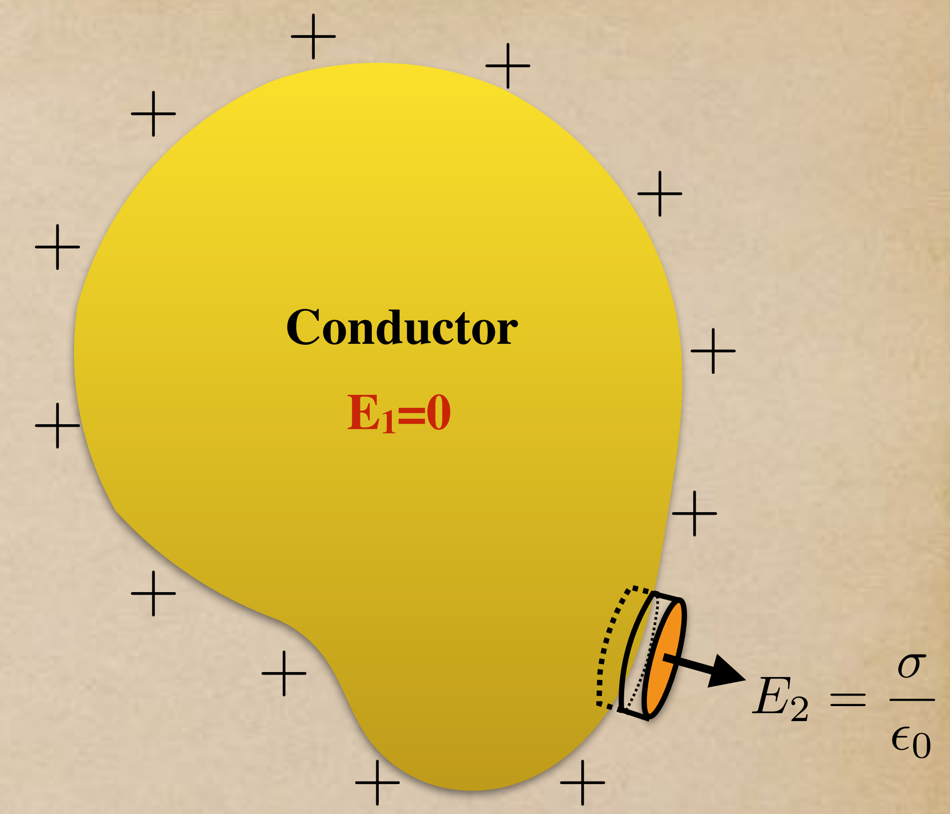
Conductors: charges on the surface

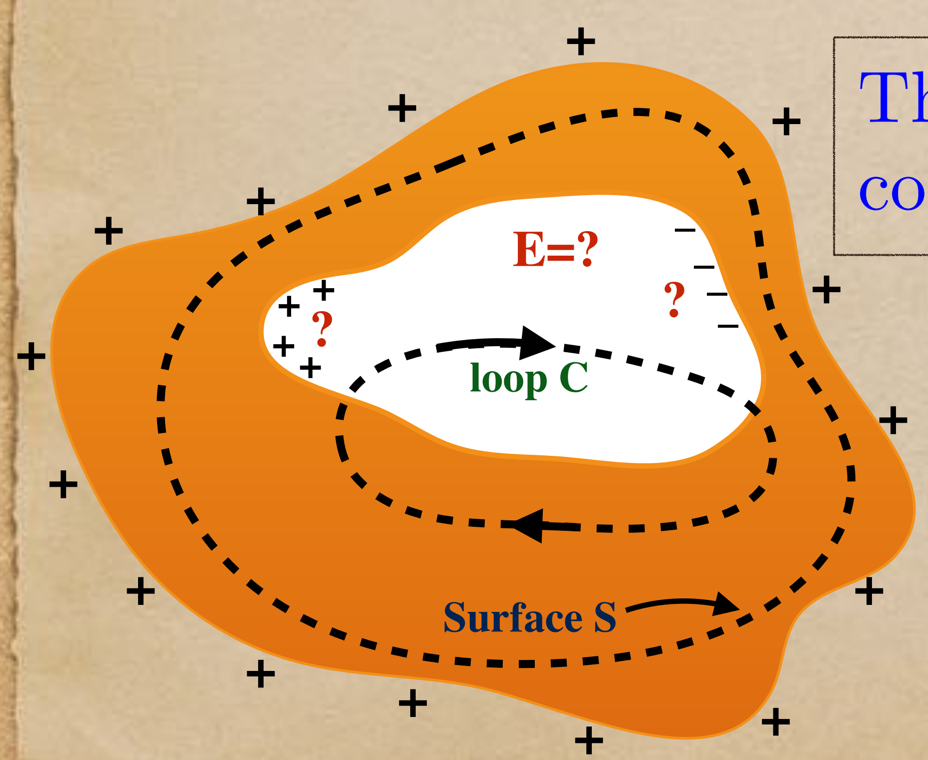
What is the electric field outside the conductor ?

Recall, $\vec{E}_{\text{above}} - \vec{E}_{\text{below}} = \frac{\sigma}{\epsilon_0}\hat{n}$

But for conductors : $\vec{E}_{\text{below}} = 0$

Therefore, $\vec{E}_{\text{above}} = \frac{\sigma}{\epsilon_0}\hat{n}$





There can not be any field inside a cavity within a conductor, nor any charges on the inside surface.

$$\oint \vec{E}.d\vec{l} = 0 \rightarrow \vec{E}_{cavity} = 0$$

$$\oint_{sur} \vec{E}.d\vec{a} = \frac{Q_{\text{enc}}}{\epsilon_0} = 0 \text{ as } \vec{E} = 0 \quad \therefore Q = 0$$

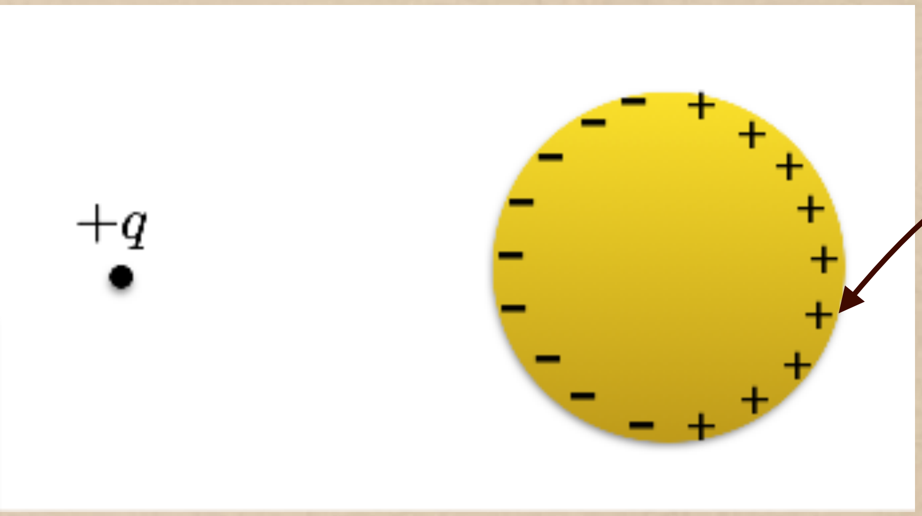
Principle of shielding electrical equipment

Induced charge on conductors

- What happens when you hold a +q charge near a conductor ?

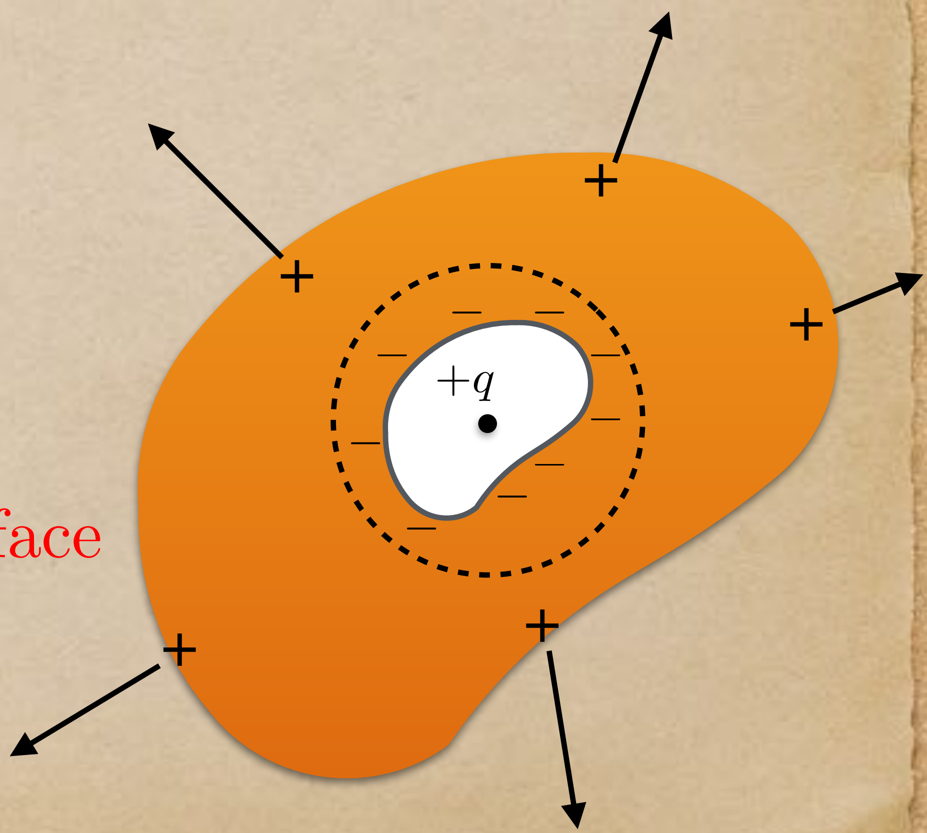
They attract each other

Opposite charges get induced to nearby surface.



induced charges

What happens when you put a +q charge inside a cavity of a conductor ?



For the Gaussian surface inside the conductor
$\oint \vec{E}.d\vec{a} = 0 \implies Q_{\text{enc}} = 0.$

But $Q_{\text{enc}} = q + q_{\text{induced}} \implies q_{\text{induced}} = -q$

Negative charges $(-q)$ are induced on the inner surface and positive charges $(+q)$ go to the outer surface

This is how the induced charge on the surface indicates the presence of a charge inside the cavity

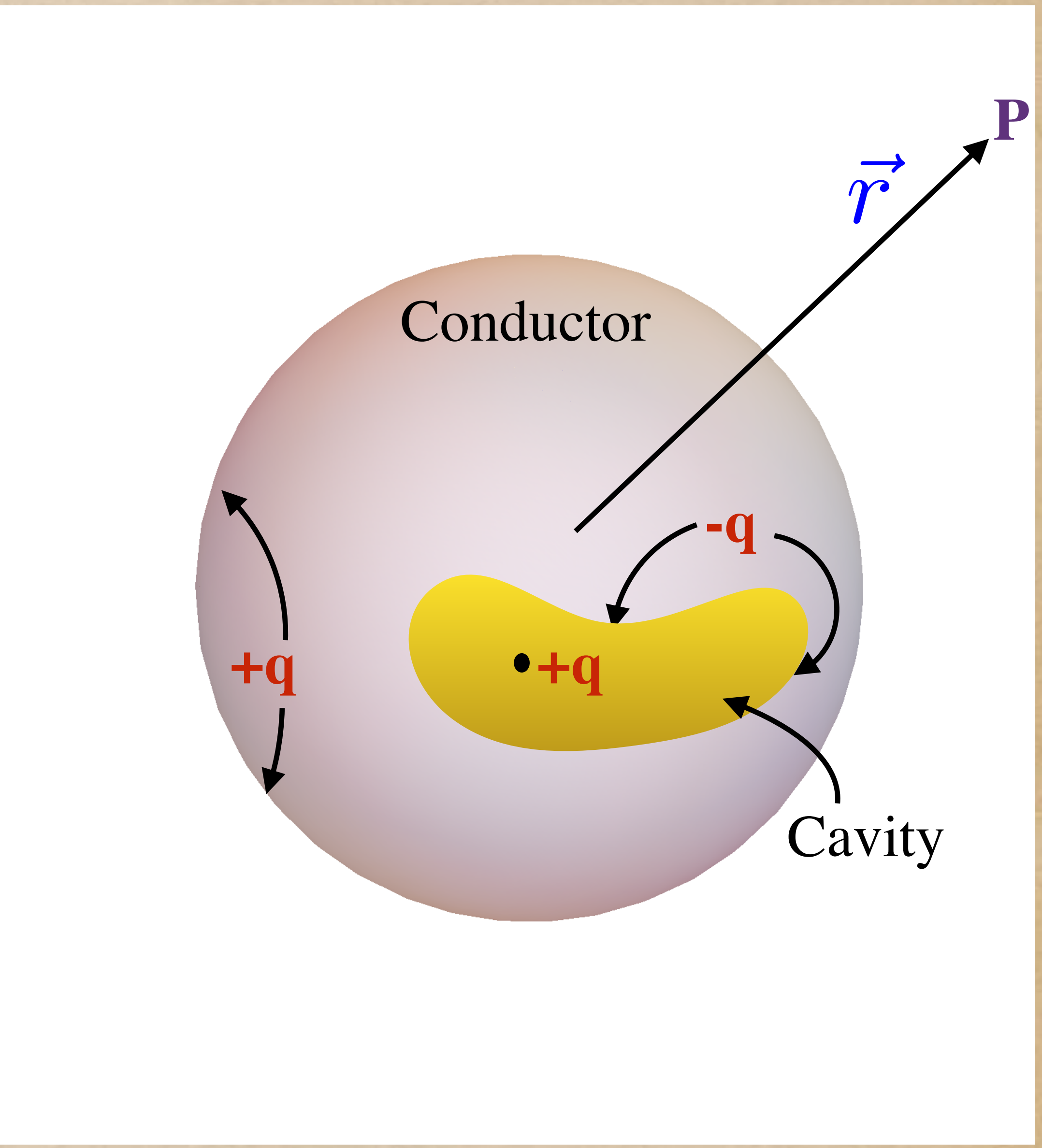
Charge inside the cavity of conductor

Let us assume that there is a point charge +q in a cavity of wired shape and size inside a conductor

- What is the field outside ?

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}$$

Due to the induced surface charge +q that accumulates at the boundary



The conductor conceals from us all the info concerning the nature of the cavity, revealing only total charge it contains.

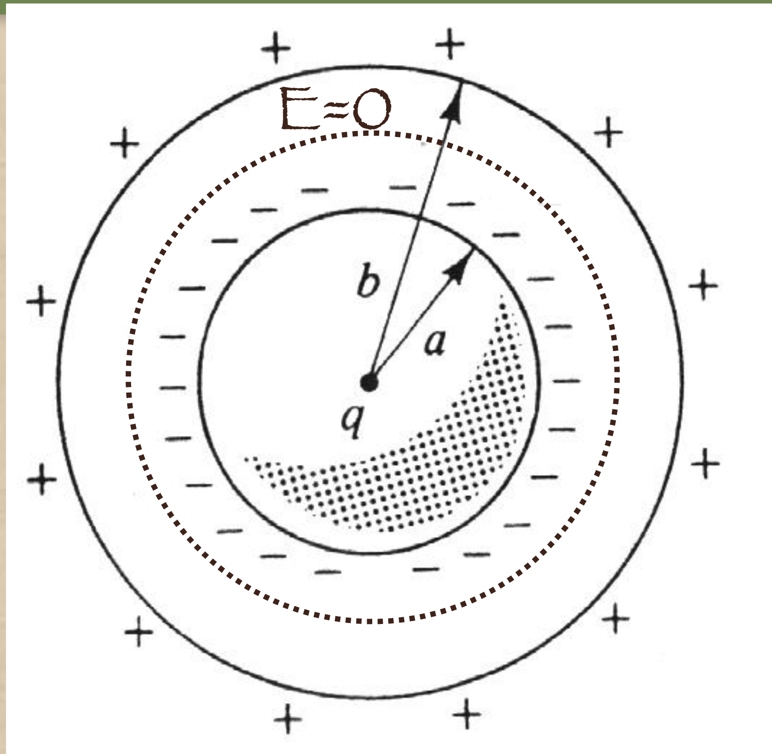
However, this electric field is evaluated at a large distance outside. What happens to immediate above the surface will be discussed soon.

Example: Spherical shell

A point charge q is placed at the centre of a spherical conducting shell. How much induced charge will accumulate ?

Inside the shell : $\vec{E} = 0 \rightarrow Q_{enc} = 0$

$$4\pi a^2 \sigma_a = -q \longrightarrow \boxed{\sigma_a = -\frac{1}{4\pi}\frac{q}{a^2}}$$



Charge conservation of conductor

$$4\pi b^2 \cdot \sigma_b = -4\pi a^2 \sigma_a$$

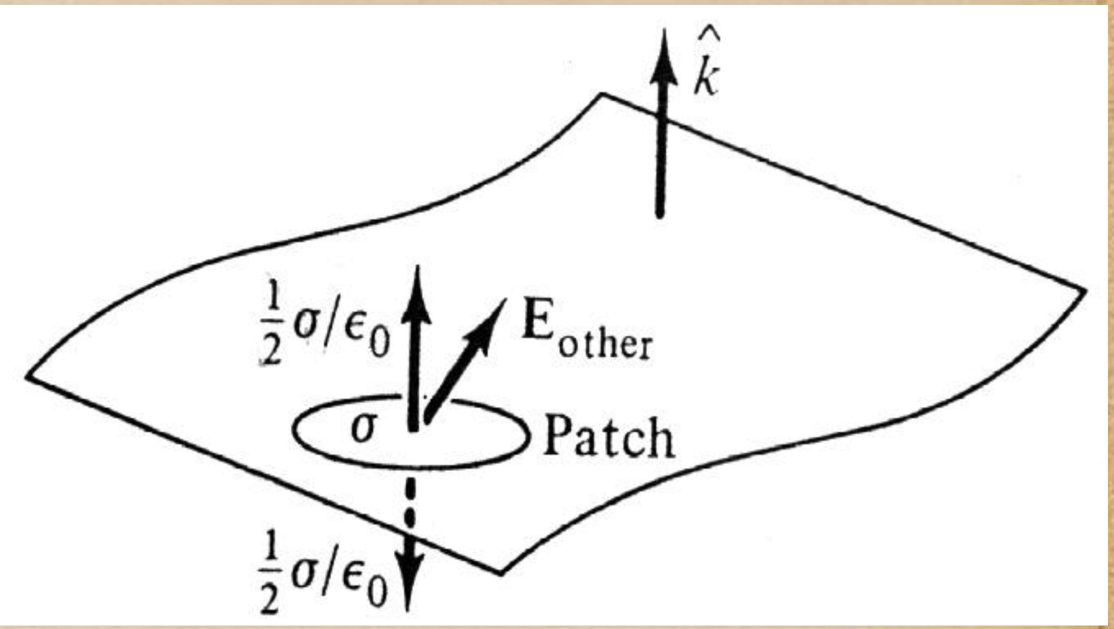
$$\boxed{\sigma_b = \frac{1}{4\pi}\frac{q}{b^2}}$$

Force on surface charges

Recall, that for a surface charge

$$\boxed{\vec{E}_{\text{above}} - \vec{E}_{\text{below}} = \frac{\sigma}{\epsilon_0}\hat{n}}$$

However, for conductors : $\vec{E}_{\text{below}} = 0$



$$\vec{E}_{\text{above}} = \frac{\sigma}{\epsilon_0}\hat{n} \longrightarrow \sigma = -\epsilon_0 \frac{\partial V}{\partial n}$$

In presence of electric field the surface charge density will feel a force

The force per unit area/pressure $\vec{f} = \sigma\vec{E} = \frac{1}{2}\sigma(\vec{E}_{\text{above}} + \vec{E}_{\text{below}})$

On a small patch of surface

$$\vec{E}_{\text{above}} = \vec{E}_{\text{other}} + \frac{\sigma}{2\epsilon_0}\hat{n}$$

$$\vec{E}_{\text{below}} = \vec{E}_{\text{other}} - \frac{\sigma}{2\epsilon_0}\hat{n}$$

$$\longrightarrow \vec{E}_{\text{other}} = \frac{1}{2}(\vec{E}_{\text{above}} + \vec{E}_{\text{below}}) = \vec{E}_{\text{average}}$$

For conductors : $\vec{f} = \sigma\vec{E} = \frac{1}{2}\sigma\vec{E}_{\text{above}} = \frac{\sigma^2}{2\epsilon_0}\hat{n}$ $\xrightarrow{\text{Electrostatic pressure}}$ $P = \frac{\epsilon_0}{2}E^2$

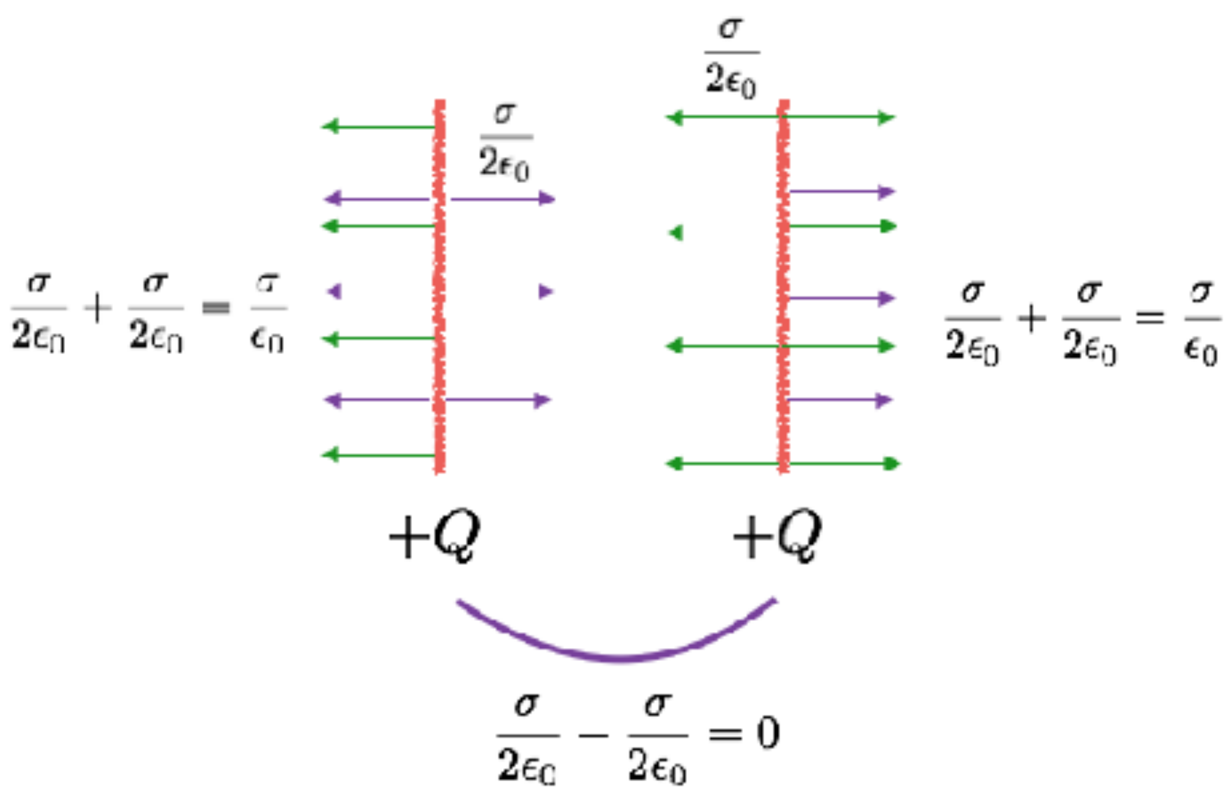
Note that the field is always perpendicular to the surface and hence the charges do not move along the surface even if they feel a pressure.

Example..

- Two large metal plates (each of area A) are held a distance d apart. Suppose we put a charge Q on each plate; what is the electrostatic pressure on the plates?

Between the plates, $E = 0$; outside the plates $E = \sigma/\epsilon_0 = Q/\epsilon_0 A$. So

$$P = \frac{\epsilon_0}{2}E^2 = \frac{\epsilon_0}{2}\frac{Q^2}{\epsilon_0^2 A^2} = \boxed{\frac{Q^2}{2\epsilon_0 A^2}}.$$



$$\boxed{\vec{E}_{\text{above}} - \vec{E}_{\text{below}} = \frac{\sigma}{\epsilon_0}\hat{n}}$$

$$\vec{E}_{\text{above}} = -\vec{E}_{\text{below}}$$

$$|\vec{E}_{\text{above}}| = |\vec{E}_{\text{below}}| = \frac{\sigma}{2\epsilon_0}$$

The electric fields cancel in between the plates and adds up outside.

Capacitors

Two conductors with +Q and -Q charges

Potential difference between them:

$$V = V_+ - V_- = -\int_{(-)}^{(+)} \mathbf{E} \cdot d\mathbf{l}.$$

$$\vec{E}(P) = \frac{1}{4\pi\epsilon_0} \int_{vol} \frac{\hat{\boldsymbol{\mathscr{r}}}}{\mathscr{r}^2} \rho \, d\tau$$

Electric field is proportional to charge and hence the potential also. One can define a constant of proportionality

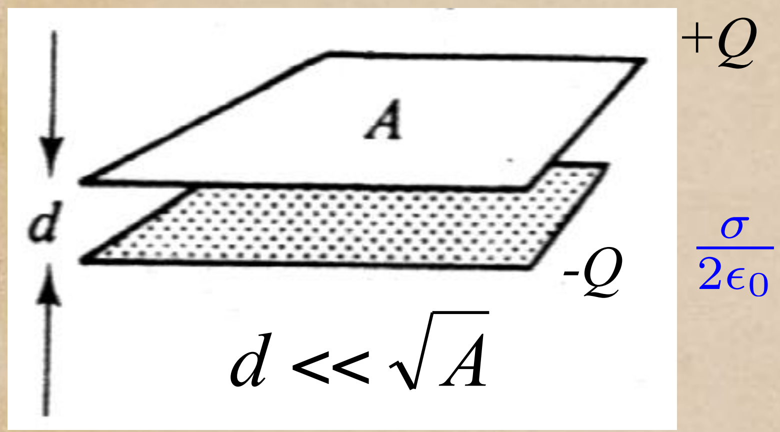
$$C = \frac{Q}{V}$$

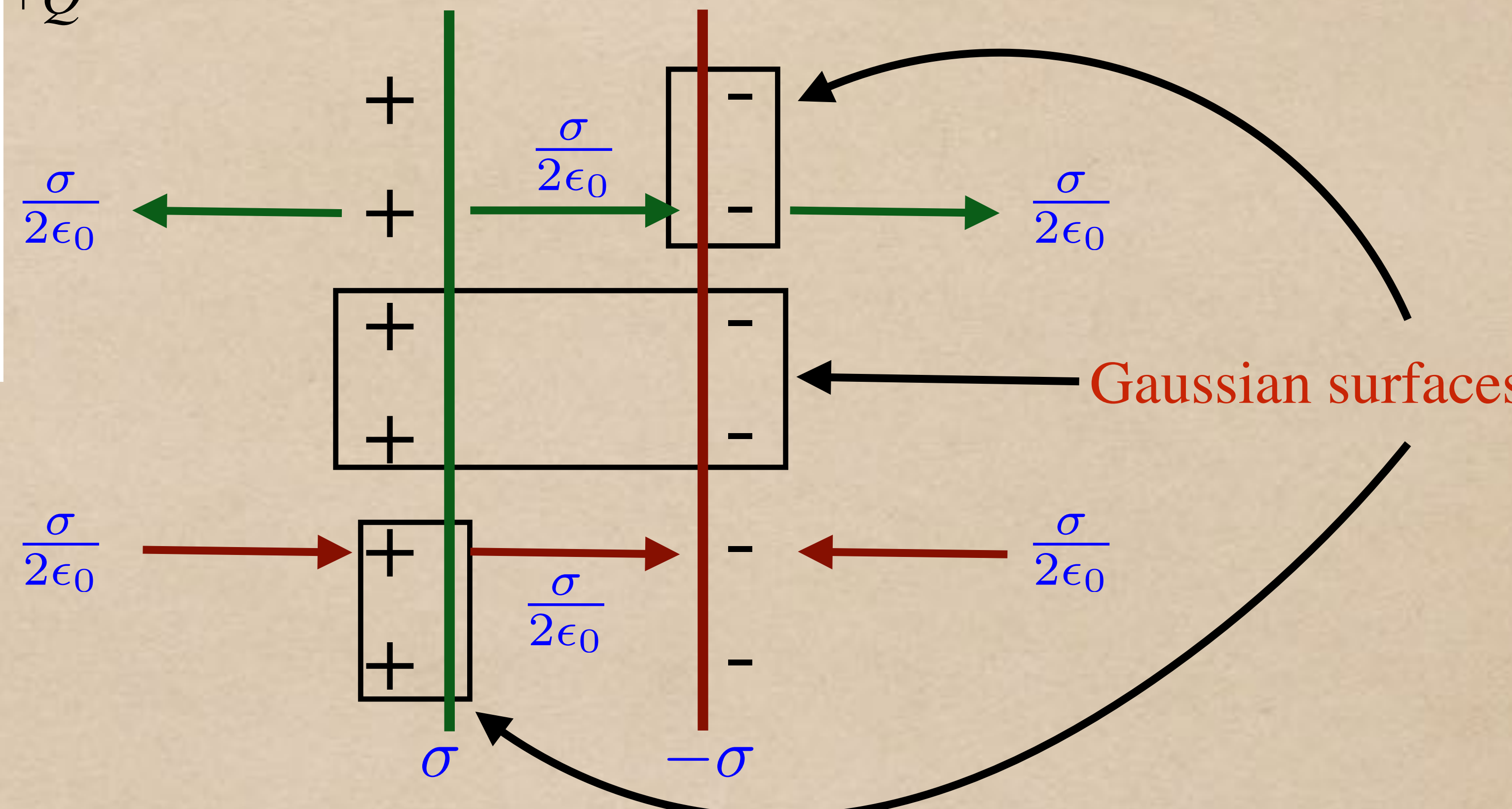
Capacitance → a geometrical quantity

depends on size and shape of the capacitor

Units : 1 Farad (F) = 1 Coulomb/Volt (SI)

Parallel plate capacitor





Therefore

$$\boxed{E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0}}$$

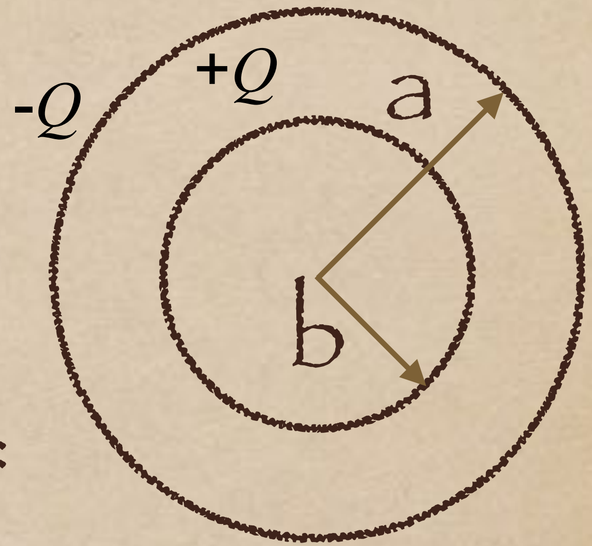
Only non-zero within the plates. Zero elsewhere

Potential difference between the plates : $\boxed{V = E \cdot d = \frac{Q}{A\varepsilon_0} d}$

Capacitance: $C = \frac{Q}{V}$ $\longrightarrow$ $\boxed{C = \frac{A\varepsilon_0}{d}}$

Example: Spherical capacitor

Find capacitance of two concentric spherical shells with radii a and b .



Field in the annular region : $\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} \hat{r}$

Potential difference between the spherical plates :

$$V = -\int_a^b \vec{E}.d\vec{l} = -\frac{Q}{4\pi\epsilon_0} \int_a^b \frac{1}{r^2} dr$$
$$= \frac{Q}{4\pi\epsilon_0} (\frac{1}{a} - \frac{1}{b})$$

Note: As the electric field is same at all the points of the surface, potential is also uniform over there

Capacitance: $C = \frac{Q}{V} = 4\pi\varepsilon_0 \frac{ab}{(b-a)}$

Energy required to charge a capacitor

To charge up a capacitor, one has to remove negative charges electrons from the +Q plate to the negative plate

How much charge does it take to charge a capacitor to charge +Q ?

Upto an intermediate charge q: $\boxed{dW = Vdq = (\frac{q}{C})dq}$

Total work done : $W = \int_0^Q (\frac{q}{C})dq = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}QV = \frac{1}{2}CV^2$

- Note: V is the final potential of the capacitor plate

In summary….

- Electric field and charge density inside the conductor is zero. Induced charges only appear at the surface of the conductor.
- The cavity inside the conductor is safe from outside electric field: Principle of electric shielding.
- The charge induced on the surface of a conductor is equal to the point charge kept inside the cavity and is independent of the shape and size of the cavity.
- Electric Pressure on the surface charge density is exerted in perpendicular direction due to the electric field.
- Capacitor comprises of two equal and oppositely charged conductors. Capacitance is defined by the ratio of Electric Charge to the potential difference and is a geometric quantity.