MA 102 (Ordinary Differential Equations)

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Tutorial Sheet No. 11 **Date:** April 04, 2016

Operator method, variation of parameters and Cauchy-Euler equations.

(1) Let $P(D) = a_n D^n + \cdots + a_1 D + a_0,\ \ a_n \neq 0$, where $D = \frac{d}{dx}$.
   (a) If $P(D)y = ce^{ax}$, where c is a constant then a particular solution is given by
$$y_p = \frac{1}{P(D)}(ce^{ax}) = \frac{ce^{ax}}{P(a)},\ \ P(a) \neq 0.$$
   (b) If $P(D)y = h(x)e^{ax}$, where $h(x)$ is any function in x, then
$$y_p = \frac{1}{P(D)}(h(x)e^{ax}) = e^{ax}\frac{1}{P(D+a)}h(x).$$
   (c) In particular, if $P(D) = (D-a)^r P_1(D)$, where $P_1(a) \neq 0$ then $y_p = \frac{1}{P(D)}(ce^{ax}) = \frac{cx^r e^{ax}}{r! P_1(a)}$.

(2) Use operator method to find a particular solution of the following ODEs.
   (a) $y''' + y'' + y' + y = x^5 - 2x^2 + x$.
   (b) $y''' - 5y'' + 8y' - 4y = 3e^{2x}$.
   (c) $y'' - 3y' + 2y = 3\sin 2x$.

(3) Find a particular solution to the following differential equations:
   (a) $y'' + 4y = \tan 2x$.
   (b) $y'' + y = \tan x + 3x - 1$.
   (c) $y'' - 2y' + y = e^x \sin^{-1} x$.

(4) Find a general solution to the differential equation given that the functions $y_1(x)$ and $y_2(x)$ are linearly independent solutions to the corresponding homogeneous equation for $x > 0$.
   (a) $(\sin^2 x)y'' - 2\sin x\cos x y' + (\cos^2 x + 1)y = \sin^3 x;\quad y_1(x) = \sin x,\ \ y_2(x) = x\sin x$.
   (b) $(x^2 + 2x)y'' - 2(x+1)y' + 2y = (x+2)^2;\quad y_1(x) = x + 1,\ \ y_2(x) = x^2$.

(5) Use the method of variation of parameters to show that
$$y(x) = c_1\cos x + c_2\sin x + \int_0^x f(s)\sin(x-s)\,ds$$
is a general solution to the differential equation $y'' + y = f(x)$, where $f(x) \in C(\mathbb{R})$.

(6) A differential equation and a non-trivial solution y_1 are given. Find the general solution.
   (a) $x^2y'' + xy' - y = 0,\ \ x \neq 0\ y_1(x) = x$.
   (b) $x^2y'' - 2xy' - 4y = 0,\ x > 0;\quad y_1(x) = x^{-1}$.

(7) Find a general solution to the given equation for $x > 0$.
   (a) $x^3y''' - 3x^2y'' + 6xy' - 6y = 0$.
   (b) $x^2y'' - 5xy' + 8y = 2x^3$.

(8) Given that $y = x$ is a solution of $x^2y'' + xy' - y = 0,\ x \neq 0$, find the general solution of $x^2y'' + xy' - y = x,\ x \neq 0$.