MA 102 (Ordinary Differential Equations)

IIT Guwahati

Tutorial Sheet No. 2 **Date:** March 15, 2018

Exact differential equations; Integrating Factors; Higher-order linear IVPs; Wronskian.

(1) Under what conditions, the following differential equations are exact?
(a) $(ax+by)dx+(kx+ly)dy=0$; (b) $[f(x)+g(y)]dx+[h(x)+l(y)]dy=0$;
(c) $(x^3+xy^2)dx+(ax^2y+bxy^2)dy=0$.

(2) Are the following equations exact? If exact, obtain the general solution.
(a) $(2xy-\sec^2 x)dx+(x^2+2y)dy=0$. (b) $(x-2xy+e^y)dx+(y-x^2+xe^y)dy=0$.

(3) In each case find an integrating factor and solve:
(a) $y'-(2/x)y=x^2\cos x$, (b) $ydx+(x^2y-x)dy=0$, (c) $y(2x^2y^3+3)dx+x(x^2y^3-1)dy=0$

(4) Show that if $(N_x-M_y)/(xM-yM)=g(xy)$ then the equation $M(x,y)dx+N(x,y)dy=0$ has an integrating factor of the form $\mu(xy)$, where $\mu(u)=\exp\left(\int g(u)du\right)$.

(5) Find the particular solution of
(a) $xy'+3y=\frac{\sin x}{x^2}$, $x\neq 0$, $y(\pi/2)=1$.
(b) $y'+y=f(x)$, $y(0)=0$, where $f(x)=\begin{cases}2, & 0\le x<1,\\ 0, & x\ge 1.\end{cases}$
(c) $x^2y'+xy=\frac{y^3}{x}$, $y(1)=1$, $x\neq 0$.

(6) Given that $y_1(x)=x$ is a solution of $\dfrac{dy}{dx}=-y^2+xy+1$, obtain the general solution.

(7) Find the value of n such that the curves $x^n+y^n=c_1$ are the orthogonal trajectories of the family $y=\dfrac{x}{1-c_2x}$, where c_1 and c_2 are arbitrary constants.

(8) Determine the largest interval (a,b) in which the given IVP is certain to have a unique solution:
(a) $e^xy''-\frac{y'}{x-3}+3y=\ln x$, $y(1)=3$, $y'(1)=2$.
(b) $(1-x)y''-3xy'+3y=\sin x$, $y(0)=1$, $y'(0)=1$.
(c) $x^2y''+4y=\cos x$, $y(1)=0$, $y'(1)=-1$.

(9) Let y_1 and y_2 be two solutions of $y''(x)+p(x)y'(x)+q(x)y=0$ defined in the interval $[a,b]$. Show that if their Wronskian $W(y_1,y_2)=0$ at least one point in $[a,b]$ then $W(y_1,y_2)=0$ for all $x\in[a,b]$.

(10) If y_1 and y_2 are linearly independent solutions of $xy''+2y'+xe^xy=0$ and if $W(y_1,y_2)(1)=2$, find the value of $W(y_1,y_2)(5)$.

(11) (a) Verify that the functions $y_1(x)=x^3$ and $y_2(x)=x^2|x|$ are linearly independent solutions of the differential equation $x^2y''-4xy'+6y=0$ on $(-\infty,\infty)$; (b) Show that y_1 and y_2 are linearly dependent on $(-\infty,0)$, but are linearly independent on $(-\infty,\infty)$; (c) Although y_1 and y_2 are linearly independent, show that $W(y_1,y_2)=0$ for all $x\in(-\infty,\infty)$. Does this violate the fact that $W(y_1,y_2)=0$ for every $x\in(-\infty,\infty)$ implies y_1 and y_2 are linearly dependent?

(12) Let $p(x),q(x)\in C(I)$. Assume that the functions y_1, $y_2\in C^2(I)$ are solutions of the differential equations $y''+p(x)y'+q(x)y=0$ on an open interval I. Prove that (a) if y_1 and y_2 are zero at the same point in I, then they cannot be a fundamental set of solutions on that interval; (b) if y_1 and y_2 have a common point of inflection x_0 in I, then they cannot be a fundamental set of solutions on that interval.