1. A spherical surface of radius R and center at origin carries a surface charge $\sigma(\theta,\phi) = \sigma_0 \cos\theta$. Find the electric field at z on z-axis. Treat the case $z < R$ (inside) as well as $z > R$ (outside). [Hint: Be sure to take the positive square root: $\sqrt{R^2 + z^2 - 2Rz} = (R - z)$ if $R > z$, but its $(z - R)$ if $R < z$.]

   Solution: In this problem, we find the electric field due to non-uniform charge distribution by direct integration.

   Let $\hat{\mathbf{r}} = z\hat{\mathbf{z}}$ and $\mathbf{r}' = R\left(\sin\theta' \cos\phi' \hat{\mathbf{x}} + \sin\theta' \sin\phi' \hat{\mathbf{y}} + \cos\theta' \hat{\mathbf{z}}\right)$.
   $dS' = R^2 \sin\theta' \, d\theta' \, d\phi'$
   $\left|\mathbf{r} - \mathbf{r}'\right| = \left(R^2 + z^2 - 2Rz\cos\theta'\right)^{1/2}$

$$
\begin{aligned}
\mathbf{E}(z\hat{\mathbf{z}}) &= \frac{1}{4\pi\epsilon_0}\int_S \frac{\sigma(\mathbf{r}')(\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^3} dS \\
&= \frac{\sigma_0}{4\pi\epsilon_0}\int_S \frac{\cos\theta'(-R\sin\theta'\cos\phi'\hat{\mathbf{x}} - R\sin\theta'\sin\phi'\hat{\mathbf{y}} + (z - R\cos\theta')\hat{\mathbf{z}})}{(R^2+z^2-2Rz\cos\theta')^{3/2}} R^2\sin\theta'\,d\theta'\,d\phi' \\
&= \frac{\sigma_0 R^2}{2\epsilon_0}\hat{\mathbf{z}}\int_{-1}^{1}\frac{u(z-Ru)}{(R^2+z^2-2Rzu)^{3/2}}du \\
&= \frac{2\sigma_0 R^3}{3\epsilon_0 z^3}\hat{\mathbf{z}} \quad z > R \\
&= -\frac{\sigma_0}{3\epsilon_0}\hat{\mathbf{z}} \quad z < R
\end{aligned}
$$

   The $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$ terms vanish because of ϕ' integral. Note: $\mathbf{E}$ is constant inside sphere.

---

2. Suppose the electric field in some region is found to be $\mathbf{E} = 2r\sin\theta\cos\phi\hat{\mathbf{r}} + r\cos\theta\cos\phi\hat{\theta} - r\sin\phi\hat{\phi}$, in spherical coordinates (k is some constant).

   (a) Find the charge density ρ.

   (b) Find the total charge contained in the sphere of radius R, centered at the origin. (Do it two different ways.)

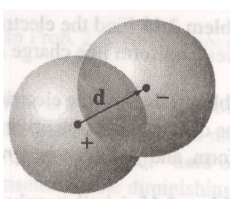
   Solution:

   (a) By Gauss law, the charge density is given by

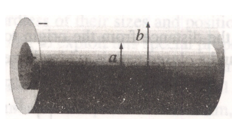
$$
\begin{aligned}
\rho &= \epsilon_0 \nabla\cdot\mathbf{E} \\
&= \epsilon_0\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 E_r\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta E_\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}(E_\phi)\right] \\
&= 4\sin\theta\cos\phi
\end{aligned}
$$

   (b) First method: Integrate ρ

$$
Q = \int \rho\left(r^2\sin\theta dr d\theta d\phi\right) = 0
$$



(a) Problem 3



(b) Problem 4

because of ϕ integral.
Second method:

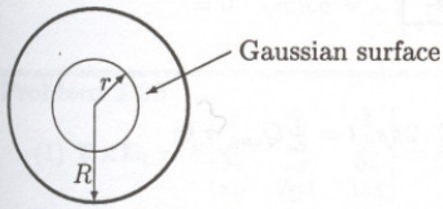
$$
\begin{aligned}
Q &= \epsilon_0 \int \mathbf{E} \cdot d\mathbf{S} \\
&= \epsilon_0 \int \mathbf{E} \cdot \left(\hat{\mathbf{r}} R^2 \sin\theta d\theta d\phi \right) \\
&= \epsilon_0 \int 2R \sin\theta \cos\phi \left(R^2 \sin\theta d\theta d\phi \right) = 0
\end{aligned}
$$

again because of ϕ integral.

---

3. [G 2.12] Use Gauss's law to find the electric field inside a uniformly charged sphere (charge density ρ). [G 2.18] Two spheres each of radius R and carrying uniform charge densities $+\rho$ and $-\rho$, respectively, are placed so that they partially overlap (See Figure). Call the vector from the positive center to the negative center $\mathbf{d}$. Show that the field in the region of overlap is constant, and find its value.

Solution:



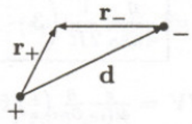
$\oint \mathbf{E} \cdot d\mathbf{a} = E \cdot 4\pi r^2 = \frac{1}{\epsilon_0} Q_{\text{enc}} = \frac{1}{\epsilon_0} \frac{4}{3} \pi r^3 \rho.$ So

$$\boxed{\mathbf{E} = \frac{1}{3\epsilon_0} \rho r \hat{\mathbf{r}}.}$$

Since $Q_{\text{tot}} = \frac{4}{3}\pi R^2 \rho$, $\mathbf{E} = \frac{1}{4\pi\epsilon_0} \frac{Q}{R^3} \hat{\mathbf{r}}$

From Prob. 2.12, the field inside the positive sphere is $\mathbf{E}_+ = \frac{\rho}{3\epsilon_0}\mathbf{r}_+$, where $\mathbf{r}_+$ is the vector from the positive center to the point in question. Likewise, the field of the negative sphere is $-\frac{\rho}{3\epsilon_0}\mathbf{r}_-$. So the *total* field is

$$\mathbf{E} = \frac{\rho}{3\epsilon_0}(\mathbf{r}_+ - \mathbf{r}_-)$$

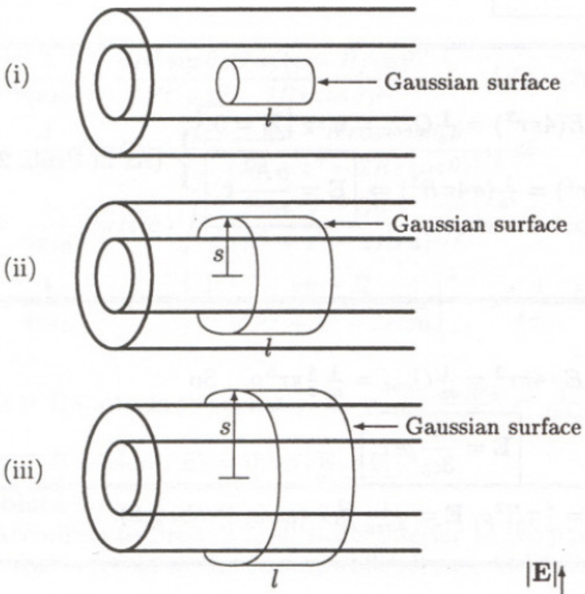


But (see diagram) $\mathbf{r}_+ - \mathbf{r}_- = \mathbf{d}$. So $\boxed{\mathbf{E} = \frac{\rho}{3\epsilon_0}\mathbf{d}.}$

---

4. [G 2.16] A long coaxial cable (see figure) carries a uniform *volume* charge density ρ on the inner cylinder (radius a), and a uniform *surface* charge density on the outer cylindrical shell (radius b). This surface charge is negative and of just the right magnitude so that the cable as a whole is electrically neutral. Find the electric field in each of the three regions: (a) inside the inner cylinder ($s < a$), (b) between the cylinders ($a < s < b$), (c) outside the cable ($s > b$). Plot $|\mathbf{E}|$ as a function of s.

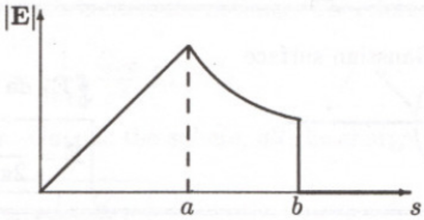
Solution:



(i) $\oint \mathbf{E}\cdot d\mathbf{a} = E\cdot 2\pi s\cdot l = \frac{1}{\epsilon_0}Q_{\text{enc}} = \frac{1}{\epsilon_0}\rho\pi s^2 l;$ $\boxed{\mathbf{E} = \frac{\rho s}{2\epsilon_0}\hat{\mathbf{s}}.}$

(ii) $\oint \mathbf{E}\cdot d\mathbf{a} = E\cdot 2\pi s\cdot l = \frac{1}{\epsilon_0}Q_{\text{enc}} = \frac{1}{\epsilon_0}\rho\pi a^2 l;$ $\boxed{\mathbf{E} = \frac{\rho a^2}{2\epsilon_0 s}\hat{\mathbf{s}}.}$

(iii) $\oint \mathbf{E}\cdot d\mathbf{a} = E\cdot 2\pi s\cdot l = \frac{1}{\epsilon_0}Q_{\text{enc}} = 0;$ $\boxed{\mathbf{E} = 0.}$



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5. [G 2.20] Which of these vector fields can be electrostatic field? If it is, find the corresponding potential, using the *origin* as your reference point. Check your answers by computing ∇V.

   (a) $\mathbf{E} = k\,[xy\hat{\mathbf{x}} + 2yz\hat{\mathbf{y}} + 3xz\hat{\mathbf{z}}]$

   (b) $\mathbf{E} = k\,[y^2\hat{\mathbf{x}} + (2xy + z^2)\hat{\mathbf{y}} + 2yz\hat{\mathbf{z}}]$.

   (c) $\mathbf{E} = 2r\sin\theta\cos\phi\hat{\mathbf{r}} + r\cos\theta\cos\phi\hat{\theta} - r\sin\phi\hat{\phi}$.

Here k is a constant with the appropriate units. [Hint: You must select a specific path to integrate along. It does not matter *what* path you choose, since the answer is path-independent, but you simply cannot integrate unless you have a particular path in mind.]

Solution:

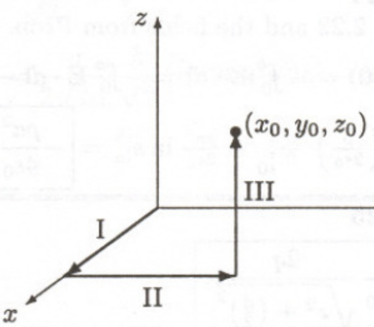
(1) $\nabla\times\mathbf{E}_1 = k\begin{vmatrix}\hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}}\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ xy & 2yz & 3zx\end{vmatrix} = k\,[\hat{\mathbf{x}}(0-2y) + \hat{\mathbf{y}}(0-3z) + \hat{\mathbf{z}}(0-x)] \neq 0,$

so $\mathbf{E}_1$ is an *impossible* electrostatic field.

(2) $\nabla\times\mathbf{E}_2 = k\begin{vmatrix}\hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}}\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ y^2 & 2xy+z^2 & 2yz\end{vmatrix} = k\,[\hat{\mathbf{x}}(2z-2z) + \hat{\mathbf{y}}(0-0) + \hat{\mathbf{z}}(2y-2y)] = 0,$

so $\mathbf{E}_2$ is a *possible* electrostatic field.

Let's go by the indicated path:



$$\mathbf{E}\cdot d\mathbf{l} = (y^2\,dx + (2xy+z^2)dy + 2yz\,dz)k$$

Step I: $y = z = 0$; $dy = dz = 0$. $\mathbf{E}\cdot d\mathbf{l} = ky^2\,dx = 0$.

Step II: $x = x_0$, $y: 0 \to y_0$, $z = 0$. $dx = dz = 0$.

$\mathbf{E}\cdot d\mathbf{l} = k(2xy + z^2)dy = 2kx_0y\,dy.$

$\int_{II}\mathbf{E}\cdot d\mathbf{l} = 2kx_0\int_0^{y_0} y\,dy = kx_0y_0^2.$

Step III: $x = x_0$, $y = y_0$, $z: 0 \to z_0$; $dx = dy = 0$.

$$\mathbf{E}\cdot d\mathbf{l} = 2kyz\,dz = 2ky_0 z\,dz.$$
$$\int_{III}\mathbf{E}\cdot d\mathbf{l} = 2y_0 k\int_0^{z_0} z\,dz = ky_0 z_0^2.$$

$$V(x_0,y_0,z_0) = -\int_0^{(x_0,y_0,z_0)}\mathbf{E}\cdot d\mathbf{l} = -k(x_0y_0^2 + y_0z_0^2),\ \text{or}\ \boxed{V(x,y,z) = -k(xy^2+yz^2).}$$

Check: $-\boldsymbol{\nabla}V = k\left[\frac{\partial}{\partial x}(xy^2+yz^2)\,\hat{\mathbf{x}} + \frac{\partial}{\partial y}(xy^2+yz^2)\,\hat{\mathbf{y}} + \frac{\partial}{\partial z}(xy^2+yz^2)\,\hat{\mathbf{z}}\right] = k[y^2\,\hat{\mathbf{x}} + (2xy+z^2)\,\hat{\mathbf{y}} + 2yz\,\hat{\mathbf{z}}] = \mathbf{E}.$

(c) Check that $\nabla\times\mathbf{E} = 0$. Now we will choose a path: $(0,0,0)\to(r,\theta,\phi)$ keeping θ and ϕ constant. Then

$$V = -\int_0^r 2r'\sin\theta\cos\phi dr' = r^2\sin\theta\cos\phi$$

Just for the sake of it, if we choose the reference point at $z_0\hat{\mathbf{z}}$ (a point on z axis), then a convenient paths is: $z_0\hat{\mathbf{z}}\to(0,0,0)\to(r,\theta,\phi)$. This parametrization is simple. How about a line directly from $z_0\hat{\mathbf{z}}$ to (r,θ,ϕ)? Can you figure out the parametrization?

---

6. [G 2.21] Find the potential inside and outside a uniformly charged solid sphere whose radius is R and whose total charge is q. Use infinity as your reference point. Compute the gradient of V in each region, and check that it yields the correct field. Sketch $V(r)$.

Solution:

$$V(r) = -\int_\infty^r \mathbf{E}\cdot d\mathbf{l}.\quad \begin{cases}\text{Outside the sphere } (r>R): & \mathbf{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{\mathbf{r}}.\\ \text{Inside the sphere } (r<R): & \mathbf{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{R^3}r\hat{\mathbf{r}}.\end{cases}$$

So for $r>R$: $V(r) = -\int_\infty^r\left(\frac{1}{4\pi\epsilon_0}\frac{q}{\bar{r}^2}\right)d\bar{r} = \frac{1}{4\pi\epsilon_0}q\left(\frac{1}{\bar{r}}\right)\Big|_\infty^r = \boxed{\frac{q}{4\pi\epsilon_0}\frac{1}{r}},$

and for $r<R$: $V(r) = -\int_\infty^R\left(\frac{1}{4\pi\epsilon_0}\frac{q}{\bar{r}^2}\right)d\bar{r} - \int_R^r\left(\frac{1}{4\pi\epsilon_0}\frac{q}{R^3}\bar{r}\right)d\bar{r} = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{R} - \frac{1}{R^3}\left(\frac{r^2-R^2}{2}\right)\right]$

$$= \boxed{\frac{q}{4\pi\epsilon_0}\frac{1}{2R}\left(3-\frac{r^2}{R^2}\right).}$$

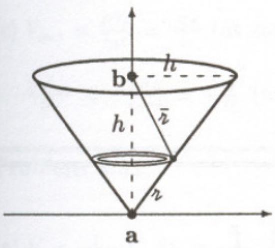
When $r>R$, $\boldsymbol{\nabla}V = \frac{q}{4\pi\epsilon_0}\frac{\partial}{\partial r}\left(\frac{1}{r}\right)\hat{\mathbf{r}} = -\frac{q}{4\pi\epsilon_0}\frac{1}{r^2}\hat{\mathbf{r}}$, so $\mathbf{E} = -\boldsymbol{\nabla}V = \frac{q}{4\pi\epsilon_0}\frac{1}{r^2}\hat{\mathbf{r}}.$

When $r<R$, $\boldsymbol{\nabla}V = \frac{q}{4\pi\epsilon_0}\frac{1}{2R}\frac{\partial}{\partial r}\left(3-\frac{r^2}{R^2}\right)\hat{\mathbf{r}} = \frac{q}{4\pi\epsilon_0}\frac{1}{2R}\left(-\frac{2r}{R^2}\right)\hat{\mathbf{r}} = -\frac{q}{4\pi\epsilon_0}\frac{r}{R^3}\hat{\mathbf{r}}$; so $\mathbf{E} = -\boldsymbol{\nabla}V = \frac{1}{4\pi\epsilon_0}\frac{q}{R^3}r\hat{\mathbf{r}}.$

---

7. [G 2.26] A conical surface (an empty ice-cream cone) carries a uniform surface charge σ. The height of the cone is h, as is the radius of the top. Find the potential difference between points **a** (the vertex) and **b** (the center of the top).

Solution:



$$V(\mathbf{a}) = \frac{1}{4\pi\epsilon_0}\int_0^{\sqrt{2}h}\left(\frac{\sigma 2\pi r}{\mathscr{r}}\right)d\mathscr{r} = \frac{2\pi\sigma}{4\pi\epsilon_0}\frac{1}{\sqrt{2}}(\sqrt{2}h) = \frac{\sigma h}{2\epsilon_0}.$$
(where $r = \mathscr{r}/\sqrt{2}$)

$$V(\mathbf{b}) = \frac{1}{4\pi\epsilon_0}\int_0^{\sqrt{2}h}\left(\frac{\sigma 2\pi r}{\bar{\mathscr{r}}}\right)d\mathscr{r},\quad \text{where } \bar{\mathscr{r}} = \sqrt{h^2+\mathscr{r}^2-\sqrt{2}h\mathscr{r}}.$$

$$= \frac{2\pi\sigma}{4\pi\epsilon_0}\frac{1}{\sqrt{2}}\int_0^{\sqrt{2}h}\frac{\mathscr{r}}{\sqrt{h^2+\mathscr{r}^2-\sqrt{2}h\mathscr{r}}}d\mathscr{r}$$

$$= \frac{\sigma}{2\sqrt{2}\epsilon_0}\left[\sqrt{h^2+\mathscr{r}^2-\sqrt{2}h\mathscr{r}} + \frac{h}{\sqrt{2}}\ln(2\sqrt{h^2+\mathscr{r}^2-\sqrt{2}h\mathscr{r}}+2\mathscr{r}-\sqrt{2}h)\right]_0^{\sqrt{2}h}$$

$$= \frac{\sigma}{2\sqrt{2}\epsilon_0}\left[h + \frac{h}{\sqrt{2}}\ln(2h+2\sqrt{2}h-\sqrt{2}h) - h - \frac{h}{\sqrt{2}}\ln(2h-\sqrt{2}h)\right] = \frac{\sigma}{2\sqrt{2}\epsilon_0}\frac{h}{\sqrt{2}}\left[\ln(2h+\sqrt{2}h) - \ln(2h-\sqrt{2}h)\right]$$

$$= \frac{\sigma h}{4\epsilon_0}\ln\left(\frac{2+\sqrt{2}}{2-\sqrt{2}}\right) = \frac{\sigma h}{4\epsilon_0}\ln\left(\frac{(2+\sqrt{2})^2}{2}\right) = \frac{\sigma h}{2\epsilon_0}\ln(1+\sqrt{2}).$$

$$\therefore \boxed{V(\mathbf{a}) - V(\mathbf{b}) = \frac{\sigma h}{2\epsilon_0}\left[1-\ln(1+\sqrt{2})\right].}$$

8. **[G 2.34]** Consider two concentric spherical shells, of radii a and b. Suppose the inner one carries a charge q, and the outer one a charge $-q$ (both of them uniformly distributed over the surface). Calculate the energy of this configuration, (a) using $W = \frac{\epsilon_0}{2}\int E^2 d\tau$, and (b) using $W_{tot} = W_1 + W_2 + \epsilon_0 \int \mathbf{E}_1 \cdot \mathbf{E}_2 d\tau$.

Solution:

Problem 2.34

(a) $W = \frac{\epsilon_0}{2}\int E^2\, d\tau$. $\quad \mathbf{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$ $(a < r < b)$, zero elsewhere.

$$W = \frac{\epsilon_0}{2}\left(\frac{q}{4\pi\epsilon_0}\right)^2 \int_a^b \left(\frac{1}{r^2}\right)^2 4\pi r^2 dr = \frac{q^2}{8\pi\epsilon_0}\int_a^b \frac{1}{r^2} = \boxed{\frac{q^2}{8\pi\epsilon_0}\left(\frac{1}{a}-\frac{1}{b}\right)}.$$

(b) $W_1 = \frac{1}{8\pi\epsilon_0}\frac{q^2}{a}$, $W_2 = \frac{1}{8\pi\epsilon_0}\frac{q^2}{b}$, $\mathbf{E}_1 = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{\mathbf{r}}$ $(r > a)$, $\mathbf{E}_2 = \frac{1}{4\pi\epsilon_0}\frac{-q}{r^2}\hat{\mathbf{r}}$ $(r > b)$. So $\mathbf{E}_1 \cdot \mathbf{E}_2 = \left(\frac{1}{4\pi\epsilon_0}\right)^2 \frac{-q^2}{r^4}$, $(r > b)$, and hence $\int \mathbf{E}_1 \cdot \mathbf{E}_2\, d\tau = -\left(\frac{1}{4\pi\epsilon_0}\right)^2 q^2 \int_b^\infty \frac{1}{r^4} 4\pi r^2 dr = -\frac{q^2}{4\pi\epsilon_0 b}$.

$W_{tot} = W_1 + W_2 + \epsilon_0 \int \mathbf{E}_1 \cdot \mathbf{E}_2\, d\tau = \frac{1}{8\pi\epsilon_0} q^2 \left(\frac{1}{a} + \frac{1}{b} - \frac{2}{b}\right) = \frac{q^2}{8\pi\epsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right)$. ✓

---

9. **[G 2.45]** A sphere of radius R carries a charge density $\rho(r) = kr$ (where k is a constant). Find the energy of the configuration. Check your answer by calculating it in at least two different ways.

Solution:

First let's determine the electric field inside and outside the sphere, using Gauss's law:

$$\epsilon_0 \oint \mathbf{E} \cdot d\mathbf{a} = \epsilon_0 4\pi r^2 E = Q_{enc} = \int \rho\, d\tau = \int (k\bar{r})\bar{r}^2 \sin\theta\, d\bar{r}\, d\theta\, d\phi = 4\pi k \int_0^r \bar{r}^3 d\bar{r} = \begin{cases} \pi k r^4 & (r < R), \\ \pi k R^4 & (r > R). \end{cases}$$

So $\mathbf{E} = \frac{k}{4\epsilon_0} r^2 \hat{\mathbf{r}}$ $(r < R)$; $\quad \mathbf{E} = \frac{kR^4}{4\epsilon_0 r^2}\hat{\mathbf{r}}$ $(r > R)$.

Method I:

$$W = \frac{\epsilon_0}{2}\int E^2 d\tau \text{ (Eq. 2.45)} = \frac{\epsilon_0}{2}\int_0^R \left(\frac{kr^2}{4\epsilon_0}\right)^2 4\pi r^2 dr + \frac{\epsilon_0}{2}\int_R^\infty \left(\frac{kR^4}{4\epsilon_0 r^2}\right)^2 4\pi r^2 dr$$

$$= 4\pi\frac{\epsilon_0}{2}\left(\frac{k}{4\epsilon_0}\right)^2 \left\{\int_0^R r^6 dr + R^8 \int_R^\infty \frac{1}{r^2} dr\right\} = \frac{\pi k^2}{8\epsilon_0}\left\{\frac{R^7}{7} + R^8\left(-\frac{1}{r}\right)\Big|_R^\infty\right\} = \frac{\pi k^2}{8\epsilon_0}\left(\frac{R^7}{7} + R^7\right)$$

$$= \boxed{\frac{\pi k^2 R^7}{7\epsilon_0}}.$$

Method II:

$$W = \frac{1}{2}\int \rho V\, d\tau \quad \text{(Eq. 2.43)}.$$

For $r < R$,

$$V(r) = -\int_\infty^r \mathbf{E} \cdot d\mathbf{l} = -\int_\infty^R \left(\frac{kR^4}{4\epsilon_0 r^2}\right) dr - \int_R^r \left(\frac{kr^2}{4\epsilon_0}\right) dr = -\frac{k}{4\epsilon_0}\left\{R^4\left(-\frac{1}{r}\right)\Big|_\infty^R + \frac{r^3}{3}\Big|_R^r\right\}$$

$$= -\frac{k}{4\epsilon_0}\left(-R^3 + \frac{r^3}{3} - \frac{R^3}{3}\right) = \frac{k}{3\epsilon_0}\left(R^3 - \frac{r^3}{4}\right).$$

$$\therefore W = \frac{1}{2}\int_0^R (kr)\left[\frac{k}{3\epsilon_0}\left(R^3 - \frac{r^3}{4}\right)\right] 4\pi r^2 dr = \frac{2\pi k^2}{3\epsilon_0}\int_0^R \left(R^3 r^3 - \frac{1}{4}r^6\right) dr$$

$$= \frac{2\pi k^2}{3\epsilon_0}\left\{R^3 \frac{R^4}{4} - \frac{1}{4}\frac{R^7}{7}\right\} = \frac{\pi k^2 R^7}{2 \cdot 3\epsilon_0}\left(\frac{6}{7}\right) = \frac{\pi k^2 R^7}{7\epsilon_0}.$$

---

10. If the potential $V(s, \phi, z) = s^2 z \sin(\phi)$ in cylindrical coordinates s, ϕ and z, calculate the energy within the region defined by $1 < s < 4$, $-2 < z < 2$, $0 < \phi < \pi/3$.

Solution:

First, we will calculate the electric field.

$$\mathbf{E} = -\nabla V = -2sz\sin\phi\hat{\mathbf{s}} - sz\cos\phi\hat{\phi} - s^2\sin\phi\hat{\mathbf{z}}$$

Then

$$|\mathbf{E}|^2 = 4s^2z^2\sin^2\phi + s^2z^2\cos^2\phi + s^4\sin^4\phi$$

Now, the energy stored in the given region is given by

$$\begin{aligned} W &= \frac{\epsilon_0}{2}\int_1^4 ds \int_0^{\pi/3} s d\phi \int_{-2}^{2} dz\, |\mathbf{E}|^2 \\ &= \frac{\epsilon_0}{2} 1508. \end{aligned}$$