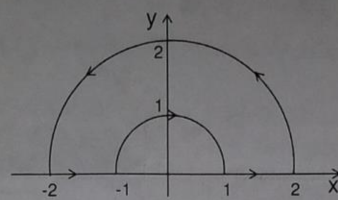
Indian Institute of Technology Guwahati, PH102: Physics-II

Mid Sem Exam   Total marks: 30   Time allotted :2 hours   Date: 22 Feb, 2013

Important instruction: Please keep the answers to all parts of a question in one place.

1. Let $\vec{A} = r\sin\phi\,\hat{r} + r^2\,\hat{\phi}$. (a) Evaluate $\oint \vec{A}.\vec{dl}$ along the closed path shown below. (b) Verify the above result using the Stoke's theorem. [3+3=6]



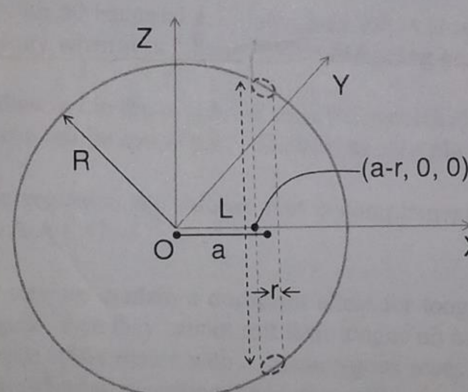
2. A spherical conducting shell of radius 'a', centred at the origin has a potential

$$V(r) = \begin{cases} V_0 & \text{if } r \leq a \\ \frac{V_0 a}{r} & \text{if } r > a \end{cases}$$

with the reference point at infinity. (a) Obtain the expression of stored energy for a volume enclosing all space. (b) What is the net charge Q on the spherical shell? [4+2=6]

3. A cylindrical slot of radius 'r' is removed from a charged sphere of radius 'R' having uniform volume charge density 'ρ'. The axis of the cylindrical slot of mean length 'L', is parallel to the Z axis and is intersecting the X axis at a distance 'a' from the origin, O, as shown in the figure below. Assuming L≫r, find (a) the electric field, and (b) the potential, at the point $(a-r, 0, 0)$ located on the surface of the cylindrical slot. Take the origin O as the reference to describe the potential. [3+3=6]

& L >> a



4. Consider two long concentric circular cylinders of radius r=1 mm and r=20 mm. For the inner cylinder potential, $V = 0$ while for the outer cylinder $V = 150$ V and there is no charge in the region in between. Use Laplace's equation to find (a) the potential, $V(r)$, and (b) the electric field, for the region between the two cylinders (i.e. 1 mm<r< 20 mm). [4+2=6]

5. Two infinite grounded conducting planes are held a distance 'a' apart. A point charge 'q' is placed between them at a distance x from one plane. (a) What is the number of image charges? (b) Obtain the expression of force on q. (c) Show that as $a \to \infty$, the force on 'q' is same as due to a single grounded conducting plane at a distance x. [1+3+2=6]

Useful information: In the case of a curvilinear co-ordinate system

1. $\vec{\nabla} T = \frac{1}{h_1}\frac{\partial T}{\partial u_1}\hat{u}_1 + \frac{1}{h_2}\frac{\partial T}{\partial u_2}\hat{u}_2 + \frac{1}{h_3}\frac{\partial T}{\partial u_3}\hat{u}_3$
2. $\vec{\nabla}\times\vec{A} = \frac{1}{h_2 h_3}\{\frac{\partial}{\partial u_2}(h_3 A_{u_3}) - \frac{\partial}{\partial u_3}(h_2 A_{u_2})\}\hat{u}_1 + \frac{1}{h_3 h_1}\{\frac{\partial}{\partial u_3}(h_1 A_{u_1}) - \frac{\partial}{\partial u_1}(h_3 A_{u_3})\}\hat{u}_2 + \frac{1}{h_1 h_2}\{\frac{\partial}{\partial u_1}(h_2 A_{u_2}) - \frac{\partial}{\partial u_2}(h_1 A_{u_1})\}\hat{u}_3$
3. $\nabla^2 T = \frac{1}{h_1 h_2 h_3}[\frac{\partial}{\partial u_1}(\frac{h_2 h_3}{h_1}\frac{\partial T}{\partial u_1}) + \frac{\partial}{\partial u_2}(\frac{h_3 h_1}{h_2}\frac{\partial T}{\partial u_2}) + \frac{\partial}{\partial u_3}(\frac{h_1 h_2}{h_3}\frac{\partial T}{\partial u_3})]$