Backward shift operator

PRACTICAL TIME SERIES ANALYSIS

THISTLETON AND SADIGOV

Objectives

- Define and utilize backward shift operator

Definition

- $X_1, X_2, X_3, \ldots$
- Backward shift operator is defined as

$$BX_t = X_{t-1}$$

- $B^2 X_t = BBX_t = BX_{t-1} = X_{t-2}$
- $B^k X_t = X_{t-k}$

Example – Random Walk

$$X_t = X_{t-1} + Z_t$$

$$X_t = BX_t + Z_t$$

$$(1 - B)X_t = Z_t$$

$$\phi(B)X_t = Z_t$$

where

$$\phi(B) = 1 - B$$

Example – MA(2) process

$$X_t = Z_t + 0.2Z_{t-1} + 0.04Z_{t-2}$$

$$X_t = Z_t + 0.2BZ_t + 0.04B^2Z_t$$

$$X_t = (1 + 0.2B + 0.04B^2)\ Z_t$$

$$X_t = \beta(B)Z_t$$

where

$$\beta(B) = 1 + 0.2B + 0.04B^2$$

Example – AR(2) process

$$X_t = 0.2X_{t-1} + 0.3X_{t-2} + Z_t$$

$$X_t = 0.2BX_t + 0.3B^2X_t + Z_t$$

$$(1 - 0.2B - 0.3B^2)\ X_t = Z\_t$$

$$\phi(B)X_t = Z_t$$

where

$$\phi(B) = 1 - 0.2B - 0.3B^2$$

MA(q) process (with a drift)

$$X_t = \mu + \beta_0 Z_t + \beta_1 Z_{t-1} + \cdots + \beta_q Z_{t-q},$$

Then,

$$X_t = \mu + \beta_0 Z_t + \beta_1 B^1 Z_t + \cdots + \beta_q B^q Z_t,$$

$$X_t - \mu = \beta(B) Z_t,$$

where

$$\beta(B) = \beta_0 + \beta_1 B + \cdots + \beta_q B^q.$$

AR(p) process

$$X_t = \phi_1 X_{t-1} + \phi_2 X_{t-2} + \cdots + \phi_p X_{t-p} + Z_t$$

Then,

$$X_t - \phi_1 X_{t-1} - \phi_2 X_{t-2} - \cdots - \phi_p X_{t-p} = Z_t$$

$$X_t - \phi_1 B X_t - \phi_2 B^2 X_t - \cdots - \phi_p B^p X_t = Z_t$$

$$\phi\,(B) X_t = Z_t,$$

Where

$$\phi\,(B) = 1 - \phi_1 B - \phi_2 B^2 - \cdots - \phi_p B^p.$$

What We've Learned

- The definition of the Backward shift operator
- How to utilize backward shift operator to write MA(q) and AR(p) processes