Autocovariance function

PRACTICAL TIME SERIES ANALYSIS

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Objectives

- recall random variables and covariance of two random variables
- characterize time series as a realization of a stochastic process
- define autocovariance function

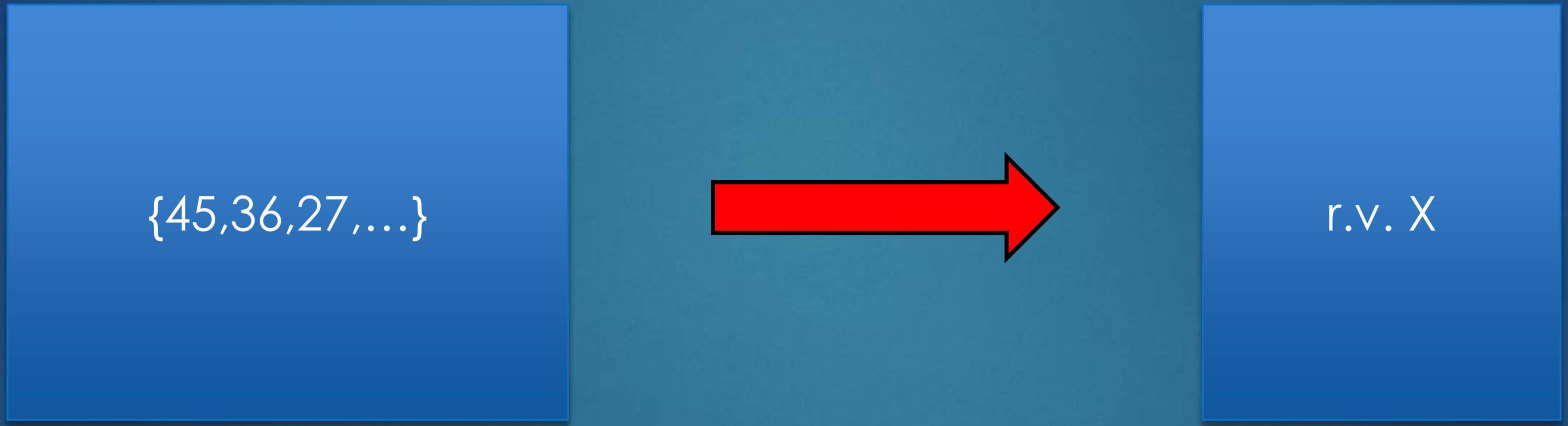
Random variables

- Random variable is defined

$$X : S \to \mathbb{R}$$

where S is the sample space of the experiment.

From data to a model



Discrete vs. Continuous r.v.

X= {20, 37, 57, …}

VS.

Y in (10, 60)

- 20 is a realization of r.v. X
- 30.29 is a realization of a r.v. Y

Covariance

- X, Y are two random variables.
- Measures the <u>linear</u> dependence between two random variables

$$CoV(X,Y) = E[(X - \mu_X)(Y - \mu_Y)] = Cov(Y,X)$$

Stochastic Processes

Collection of a random variables

$$X_1, X_2, X_3, \dots$$

$$X_t \sim distribution\,(\mu, \sigma^2)$$

Time series as a realization of a stochastic process

$$X_1, X_2, X_3, \ldots$$

$$30, 29, 57, \ldots$$

Autocovariance function

$$\gamma(s,t) = Cov(X_s, X_t) = E[(X\_s - \mu_s)(X_t - \mu_t)]$$

$$\gamma(t,t) = E[(X_t - \mu_t)^2] = Var(X_t) = \sigma_t^2$$

Autocovariance function cont.

$$\gamma_k = \gamma(t, t + k) \approx c_k$$

What We've Learned

- the definition of a stochastic processes
- how to characterize time series as realization of a stochastic process
- how to define autocovariance function of a time series