

$\gamma \rightarrow n$

$S \rightarrow 4$

$S_{PY}$

$$S_{P_S} = S! \quad n_{P_Y} = \frac{n!}{(n-\gamma)!}$$

$$S_{PY} = \frac{S!}{(S-4)!} = \frac{S!}{1!} = \underline{\underline{120}}$$

A B C D

(1)

D A B C

B C A D (2)

D B C A

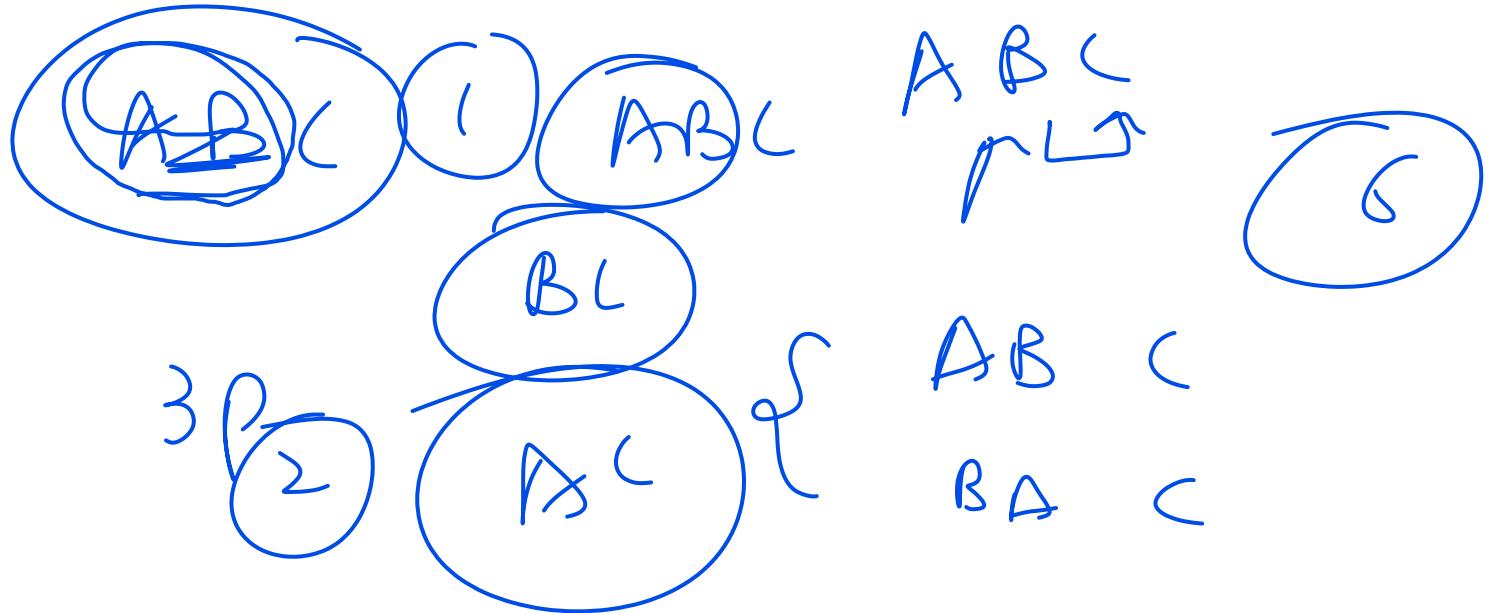
C B A D

D C B A

$$3_{P_K} = \frac{3!}{(3-2)!}$$

= 6      A B C

(2)

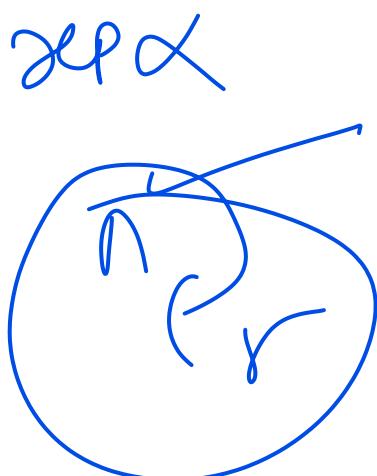


$$\gamma = \frac{\gamma_{Pr}}{\gamma_1}$$

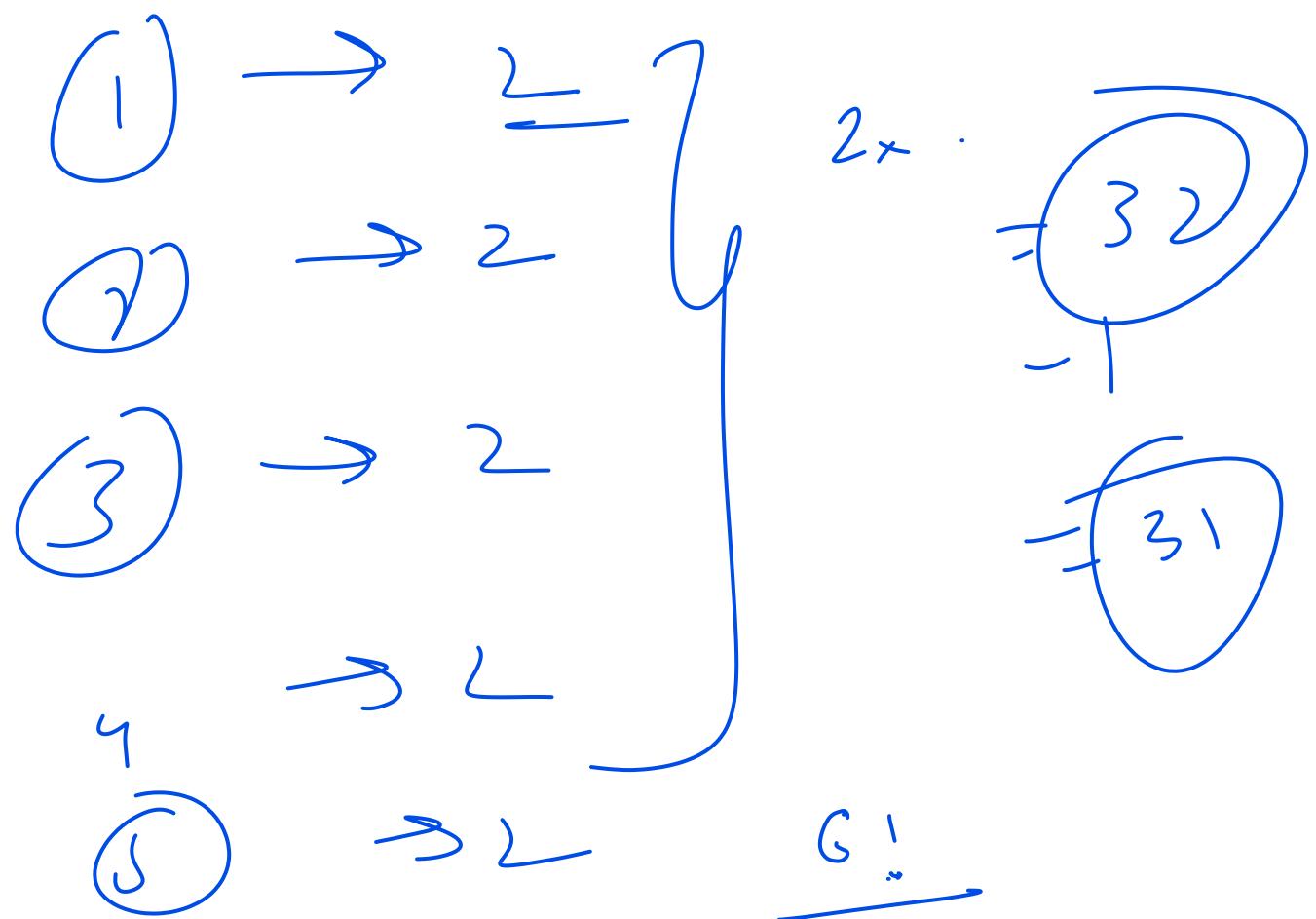
0, 1, 2, 3, 4, 5, 6, 7, 8, 9

How many numbers between 100 and 1000 have 4 in the units place?

$$9 \times (10+1) - 90$$

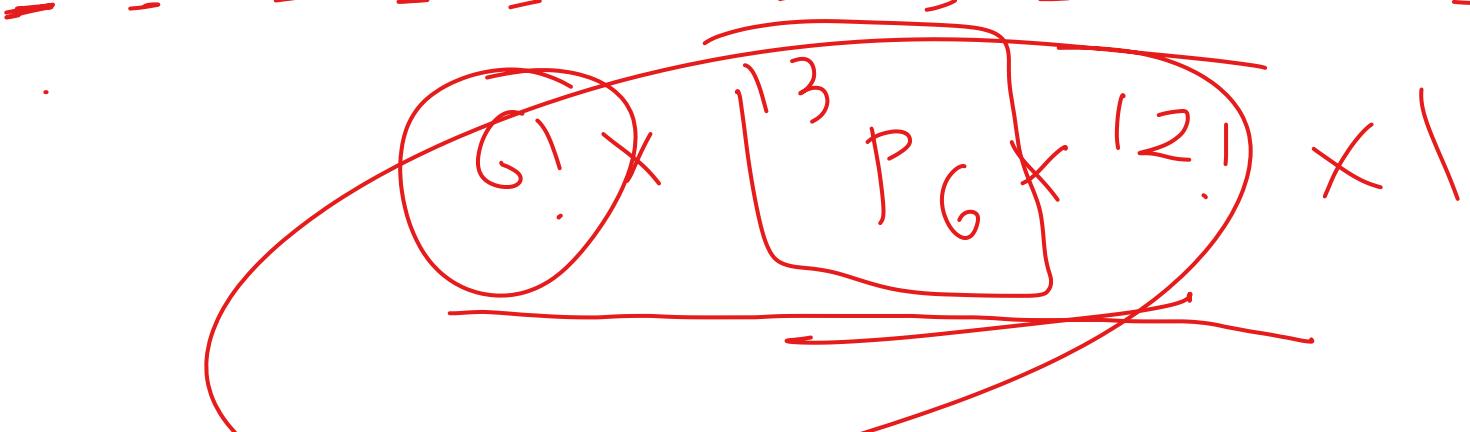


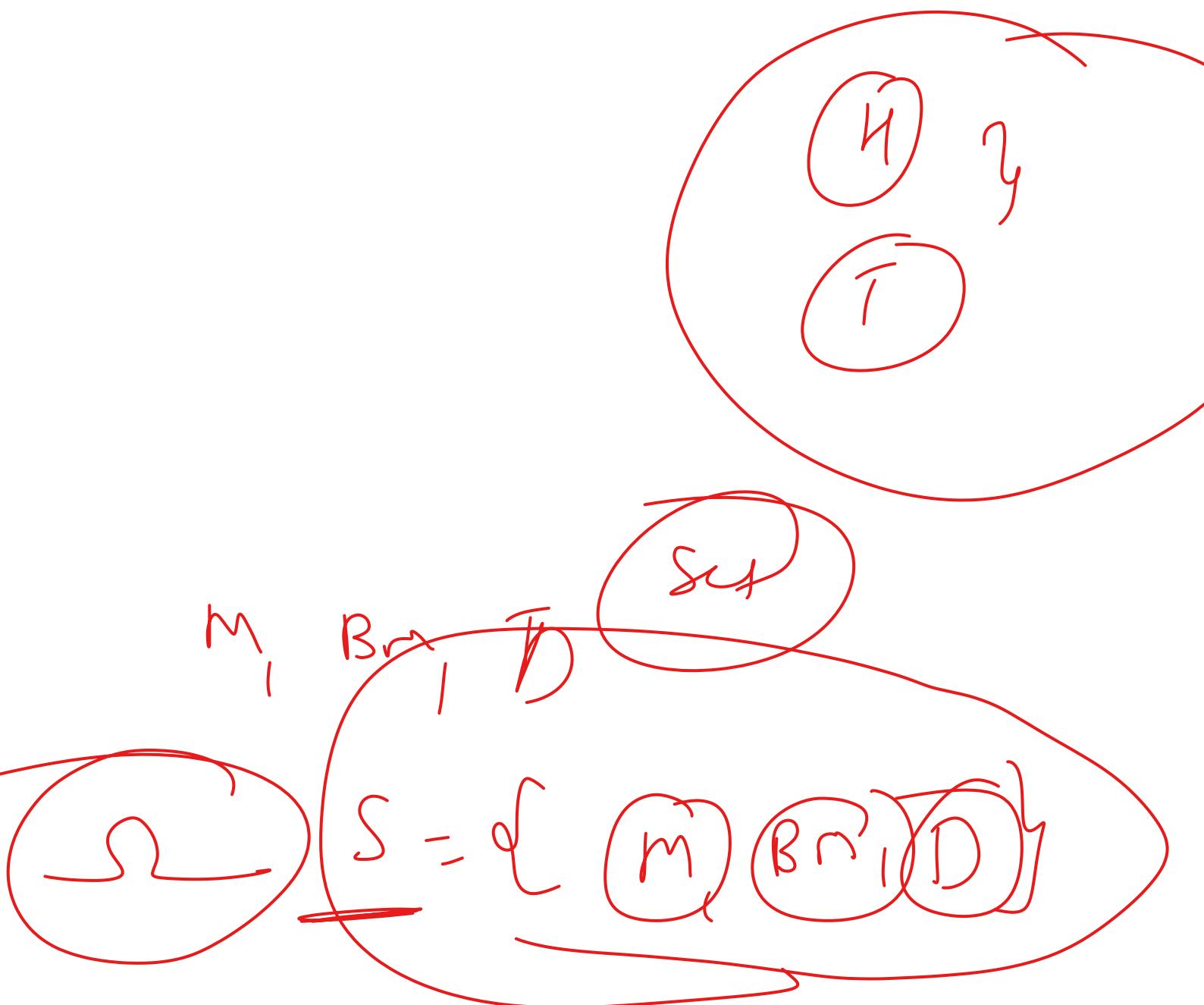
$9$   
 $10$   
 $1$   
 allowed



$B_1 \perp B_2 \perp B_3 \perp B_4 \perp B_5 \perp B_6 \perp$   
 $B_7 \perp B_8 \perp B_9 \perp B_{10} \perp \perp \perp$

$\perp + \underline{2} \underline{2} \underline{3} \underline{3} \underline{4} \underline{4} \underline{5} \underline{5} \underline{6} \underline{6} \underline{7} \underline{7} \underline{8} \underline{8} \underline{9} \underline{9} \underline{10} \underline{10} \perp \perp \perp$





$$n(S) = 3$$

$2^1$  1  $\rightarrow \{ H, T \}$

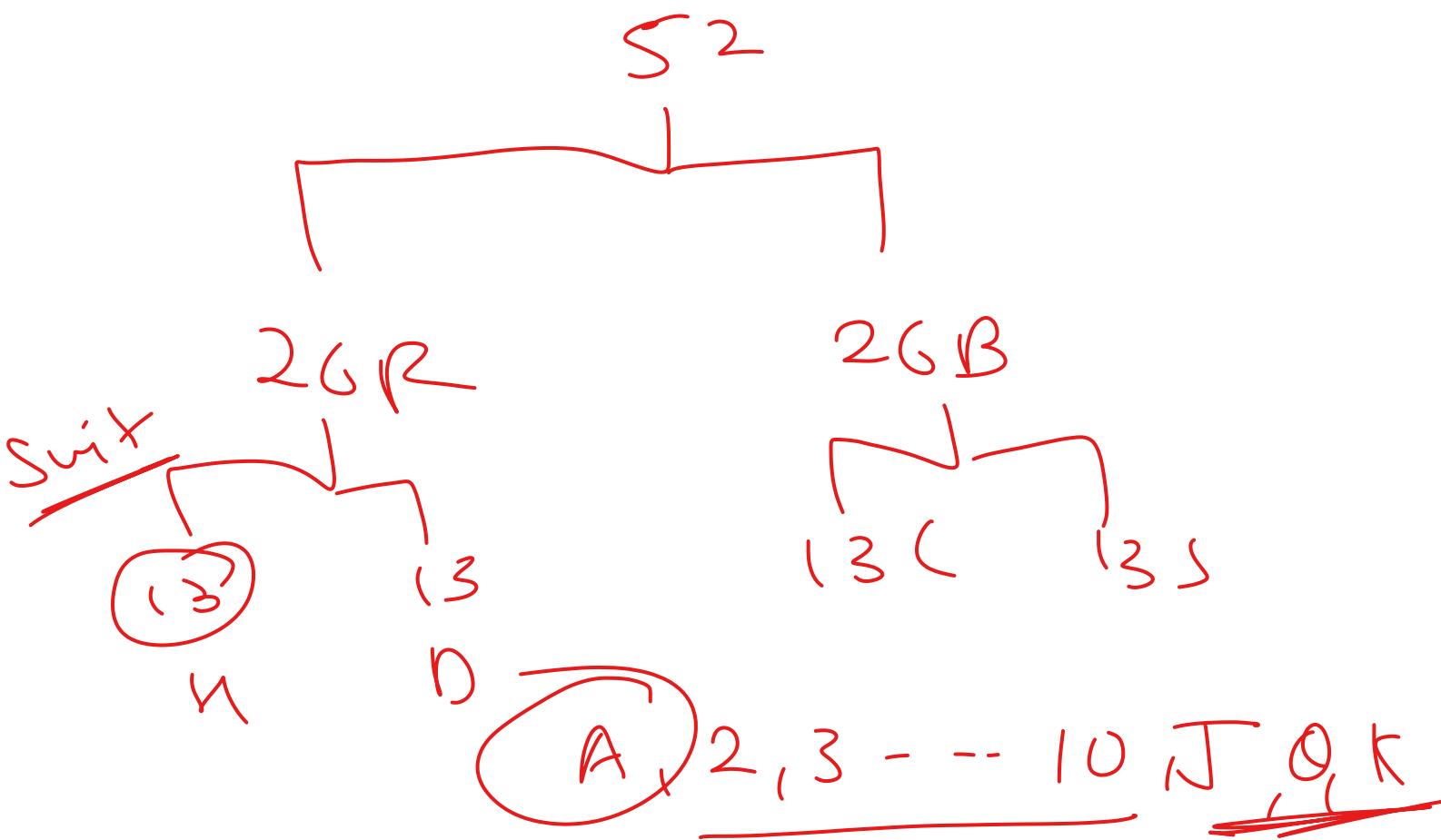
$2^2$   $2 \times 2 \rightarrow \{ HH, HT, TH, TT \}$

$2^3$   $3 \times 3 \rightarrow \{ HHH, HHT, HTH, TTT, HTH, THT, TTH, HTT \}$

1 die  $\rightarrow \{ 1, 2, 3, 4, 5, 6 \}$

$$n(S) = 6$$

2 dice  $\rightarrow \{ \}$



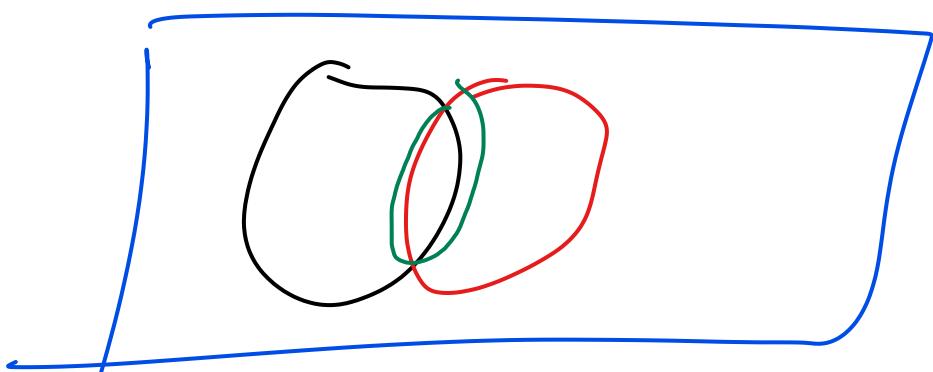
$$A = \{1, 2, 3, 4, 5, 6\}$$

$$B = \{2, 4, 7, 8, 9\}$$

$$A - B = \{1, 3, 5, 6\}$$

$$B - A = \{7, 8, 9\}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$



$$P(A) + P(B) - P(A \cap B)$$

$$P(A \cup B \cup C) = P(A) + P(B) + P(C)$$

$$- P(A \cap B) - P(B \cap C) - P(C \cap A)$$
$$+ P(A \cap B \cap C)$$

$$P(A) - P(A \cap B)$$
$$P(B) - P(A \cap B)$$

$\frac{3}{3}$

$\frac{3}{4}$

$\frac{1}{1}$

$$\frac{3}{3} + \frac{1}{1} = \frac{8}{8}$$

$$\frac{10! - 8! \times 3!}{10!}$$

$$10! - 8! \times 3!$$

$$10!$$

A girl is preparing for National Level Entrance exam and State Level Entrance exam for professional courses. The chances of her cracking National Level exam is 0.42 and that of State Level exam is 0.54. The probability that she clears both the exams is 0.11. Find the probability that (i) She cracks at least one of the two exams (ii) She cracks only one of the two (iii) She cracks none

$$\frac{0.85}{0.34}$$

$$0.15$$

$$P(N) - P(N \cap S)$$

$$+ P(S) - P(N \cap S)$$

The probability that a student will pass in French is 0.64, will pass in Sociology is 0.45 and will pass in both is 0.40. What is the probability that the student will pass in at least one of the two subjects?

$$P(F \cup S) =$$

The probability that a student will solve problem A is  $\frac{2}{3}$ , and the probability that he will not solve problem B is  $\frac{5}{9}$ . If the probability that student solves at least one problem is  $\frac{4}{5}$ , what is the probability that he will solve both the problems?

$$\frac{4}{5} = \frac{2}{3} + \frac{1}{9} - P(A \cap B)$$

$$P(A \cap B) = \frac{2}{3} + \frac{1}{9} - \frac{4}{5}$$

$$= \frac{6+1}{9} - \frac{4}{5}$$

$$= \frac{10}{45} - \frac{4}{5}$$

$$\frac{59}{45} = \frac{50 - 36}{45}$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A|B) = P(A)$$

$$P(A \cap B)$$

$$= P(A) P(B)$$

$$P(A \cap B) = P(A|B) P(B)$$

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

$$P(A \cap B) = P(B|A) P(A)$$

$$P(B|A) = P(B)$$

$$P(A' \cap B') = P(A \cup B)'$$

$$= 1 - P(A \cup B)$$

$$P(A' \cup B') = P(A \cap B)'$$

2 shooters are firing at target. The probability that they hit the target are  $\frac{1}{3}$  and  $\frac{1}{2}$  respectively. If they fire independently find the probability that

- a) both hit the target.  $P(A) = \frac{1}{3}, P(B) = \frac{1}{2}$
- b) Nobody hits the target.  $P(A') = \frac{2}{3}, P(B') = \frac{1}{2}$
- c) At least one hits the target.
- d) Exactly one hits the target.

$$P(A \cap B') + P(A' \cap B)$$

$$= P(A)P(B') + P(A')P(B)$$

=

$$P(A) = 0.4 \quad P(A \cup B') = \underline{0.7}$$

$$P(A \cup B') = P(A) + P(B') - P(A \cap B')$$

$$0.7 = 0.4 + P(B') - P(A)P(B')$$

$$0.7 = \underline{0.4} + \underline{P(B')} - \underline{0.4} \underline{P(B')}$$

$$0.3 = \underline{P(B')} (1 - 0.4) \quad P(B) = \frac{1}{2}$$

$$P(B') = \frac{0.3}{0.6} = \underline{\frac{1}{2}}$$

$$P(A) + P(B) - P(A \cap B) = 0.4 + 0.5 - 0.4 \times \underline{0.5}$$

Three vendors were asked to supply a component. The respective probabilities that the component supplied by them is 'good' are 0.8, 0.7 and 0.5. Each vendor supplies only one component. Find the probability that at least one component is 'good'.

$$\rightarrow P(A' \cap B' \cap C') = P(A \cup B \cup C)^{'} \quad |$$

$$P(A' \cap B' \cap C') = P(A') P(B') P(C') \\ = 0.2 \times 0.3 \times 0.5$$

$$P(A \cup B \cup C) = 1 - P(A \cup B \cup C)^{'} \\ =$$

A box contains 6 white and 4 black balls. One ball is selected at random and its colour is noted. The ball is replaced and two balls of the opposite colour are added and then second ball is selected at random find the probability that both balls are white.

$\rightarrow 6W \ 4B$

$SW \ 6B$

$$P(W_I) = \frac{6}{10} \quad P(B) = \frac{4}{10}$$

$$P(W_{II}/\cancel{W_I}) = \frac{5}{11}$$

$$\rightarrow P(W, \cap W_{II}) = P(W_I) P(W_{II}/\cancel{W_I})$$

$$= \frac{6}{10} \times \frac{5}{11}$$

$$= \cancel{3}/11 \cancel{\phi} = \cancel{3}/11$$

# BAYES · THM

FR 84

8R 11G

$$P(I) = \frac{1}{2}$$

$$P(II) = \frac{1}{2}$$

$$P(R|I) = \frac{7}{15}$$

$$P(R|II) = \frac{8}{19}$$

$$P(G|I) = \frac{8}{15} \quad P(G|II) = \frac{11}{19}$$

$$P(I|R) = \frac{P(I \cap R)}{P(R)}$$

$$= \frac{P(I)P(R|I)}{P(I)P(R|I) + P(II)P(R|II)}$$

In a bolt factory, three machine P, Q and R produce 25%, 35% and 40% of the total output respectively. It is found that in their production, respectively 5%, 4% and 2% are defective bolts. If a bolt is selected at random and found defective, find the probability that it is produced by machine Q.

$$P(P) = 0.25, P(Q) = 0.35, P(R) = 0.4$$

$$P(D|P) = 0.05, P(D|Q) = 0.04$$

$$P(D|R) = 0.02$$

$$P(Q|D) = \frac{P(Q)P(D|Q)}{P(P)P(D|P) + P(Q)P(D|Q) + P(R)P(D|R)}$$

$$= \frac{0.35 \cdot 0.04}{0.25 \cdot 0.05 + 0.35 \cdot 0.04 + 0.4 \cdot 0.02}$$

7

A shop has equal number of LED bulbs of two different types. The probability that the life of an LED bulb is more than 100 hours given that it is of type-1 is 0.7 and given that it is of type-2 is 0.4. If an LED bulb is selected at random, find the probability that the life of the bulb is more than 100 hours.

$$P(I) = 0.5 \quad P(\bar{I}) = 0.5$$

$$P(m|I) = 0.7 \quad P(m|\bar{I}) = 0.4$$

$$P(m) = P(I)P(m|I) + \underbrace{P(\bar{I})P(m|\bar{I})}$$

$$P(I|m) = \underline{P(I)p(m|I)}$$

The chance of a student passing a test is 20%. The chance of student passing the test and getting above 90% marks is 5%. Given that a student passes the test, find the probability that the student gets above 90% marks.

$$\rightarrow P(P) = 0.2 \checkmark$$

$$P(P \cap N) = \underline{0.05}$$

$$P(N|P) = \frac{P(P \cap N)}{P(P)}$$

$$= \frac{0.05}{0.2}$$

A certain test for a particular cancer is known to be 95% accurate. A person submits to the test and the results are positive. Suppose that the person comes from a population of 1,00,000 where 2,000 people suffer from that disease. What can we conclude about the probability that the person under test has cancer

$$P(C) = 0.02 \quad | P(C/P)$$

$$P(NC) = 0.98 \quad | = 0.95 \times 0.02$$

$$P(P/C) = 0.95 \quad \nearrow$$

$$P(P/NC) = 0.05$$

$$P(P) = \frac{0.95 \times 0.02 +}{0.98 \times 0.05}$$

$$\frac{P(A)}{P(A^c)} \quad \text{Favor} \rightarrow a:b$$
$$O \rightarrow \frac{a}{a+b}$$

$$No \rightarrow \frac{b}{a+b}$$

Against  $c:d$

$$O \rightarrow \frac{d}{c+d}, No \rightarrow \frac{c}{c+d}$$

In a single toss of a fair die, what are the odds against the event that number 3 or 4 turns up?

$$P(A) = \frac{2}{6} = \cancel{\frac{1}{3}}$$

$$P(A') = \cancel{\frac{2}{3}}$$

$$P(A'): P(A) = \cancel{\frac{1}{3}} : \cancel{\frac{1}{3}}$$

~~2 : 1~~

The odds against John solving a problem are 4 to 3 and the odds in favor of Rafi solving the same problem are 7 to 5. What is the chance that the problem is solved when both try it?

$$P(J) = \frac{3}{7} \quad P(J') = \frac{4}{7}$$

$$P(R) = \cancel{\frac{7}{12}} \quad P(R') = \cancel{\frac{5}{12}}$$

$$\begin{aligned} P(JS) &= P(J \cap R') + P(J \cap R) \\ &\quad + P(J \cap R) \\ &= P(J) P(R') + P(J') P(R) \\ &\quad + P(J) P(R) \end{aligned}$$

The odds against a husband who is 60 years old, living till he is 85 are 7:5. The odds against his wife who is now 56, living till she is 81 are 5:3. Find the probability that

a) at least one of them will be alive 25 years hence

$$P(H \cup W) = P(H) + P(W) - P(H \cap W) =$$

b) exactly one of them will be alive 25 years hence

$$P(H) = \frac{5}{12} \quad P(W) = \frac{7}{12}$$

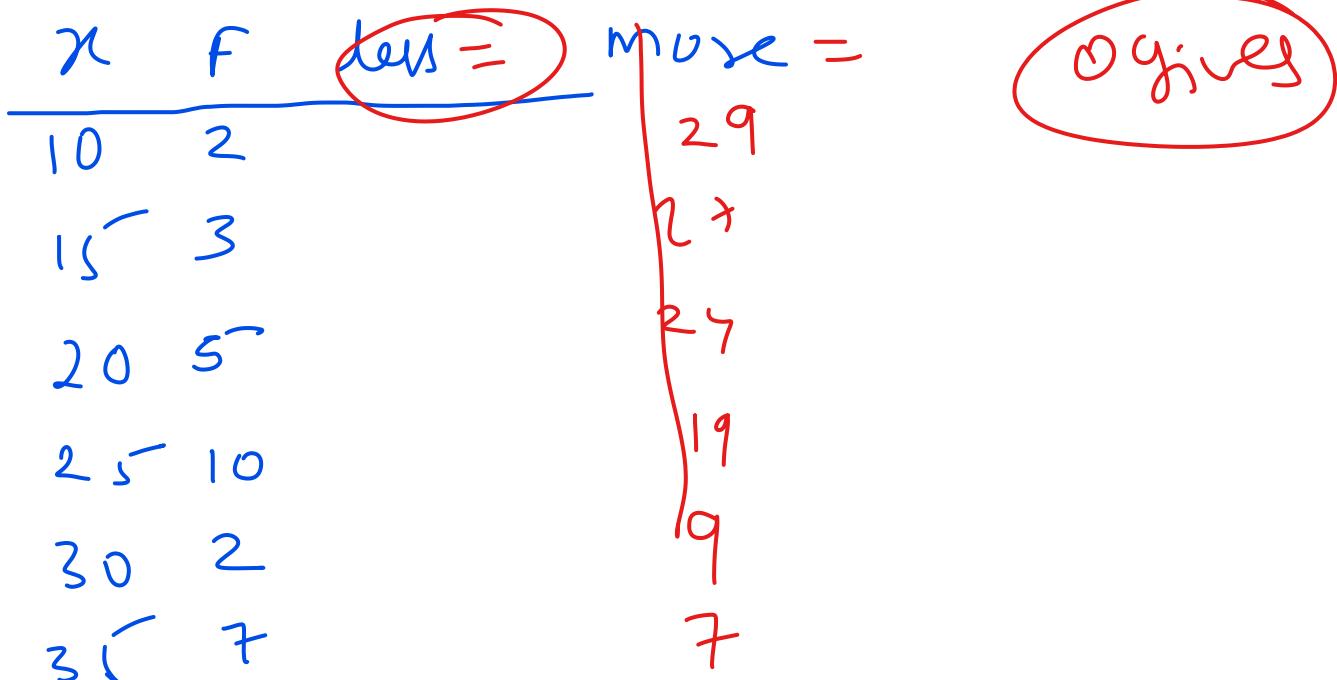
$$P(H) = \frac{5}{12} \quad P(W) = \frac{3}{8}$$

$$P(H \cap W) = \frac{5}{12} \times \frac{3}{8} = \frac{15}{96}$$

The mean of 10 observations was found to be 20. Later on it was discovered that the observations 24 and 34 were wrongly noted as 42 and 54. Find the corrected mean.

$$\bar{x} = \frac{\sum x}{n}$$

$$\begin{aligned} \sum x &= 200 - 42 - 54 \\ &\quad + 24 + 34 \\ &= \frac{162}{10} = 16.2 \end{aligned}$$



```
x = c(10, 15, 25, 40, 60, 75)
```

```
f = c(3, 4, 6, 17, 12, 7)
```

```
cumsum(f)
```

```
a = data.frame(X = x, F = f, lt =  
cumsum(f))
```

```
N = a[6,3]
```

```
obs = (N+1)/2
```

```
for (i in a$lt) {
```

```
  if (i >= obs) {
```

```
    print(i)
```

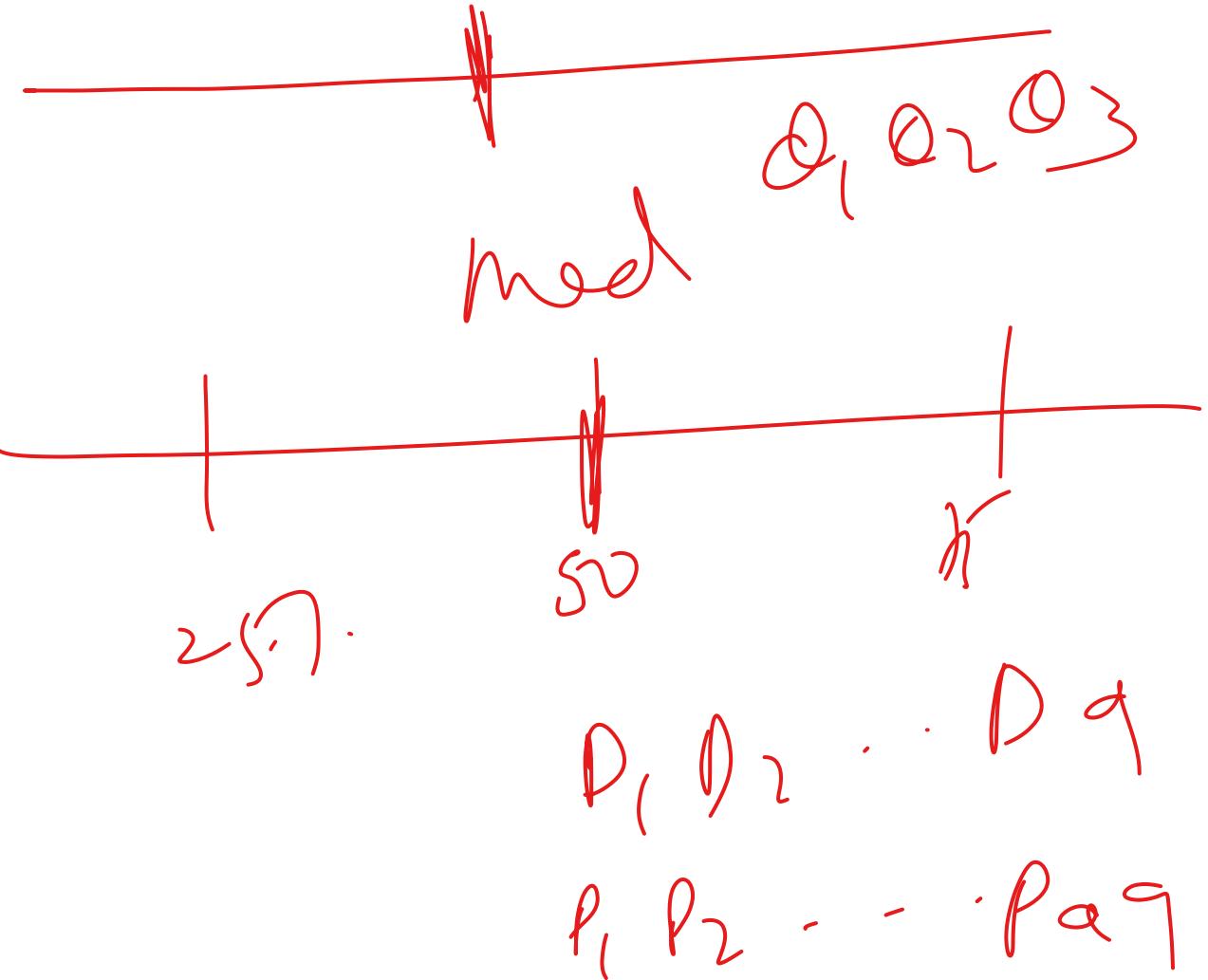
```
    break()
```

```
}
```

```
}
```

```
i
```

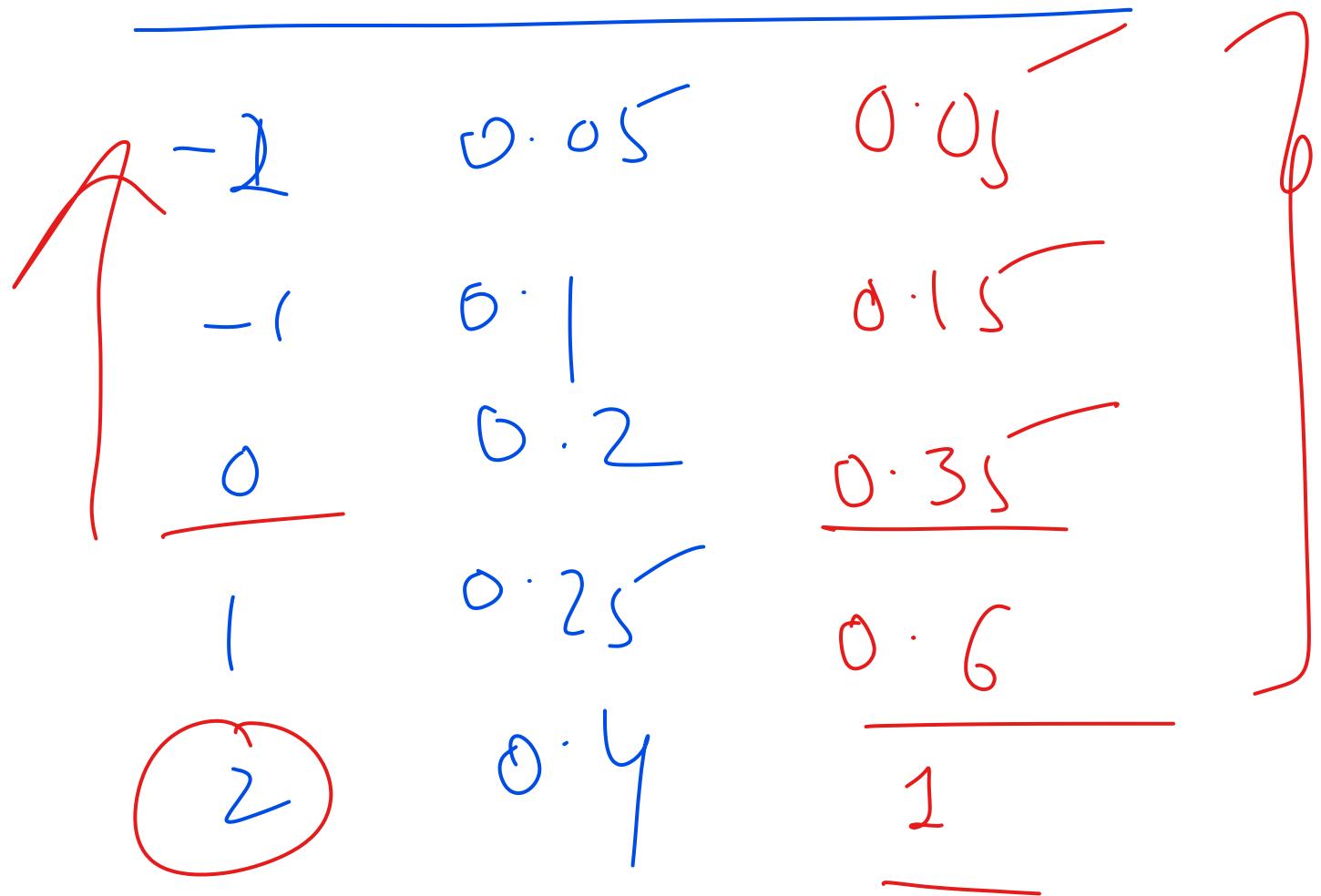
```
a$X[which(a$lt == i)]
```

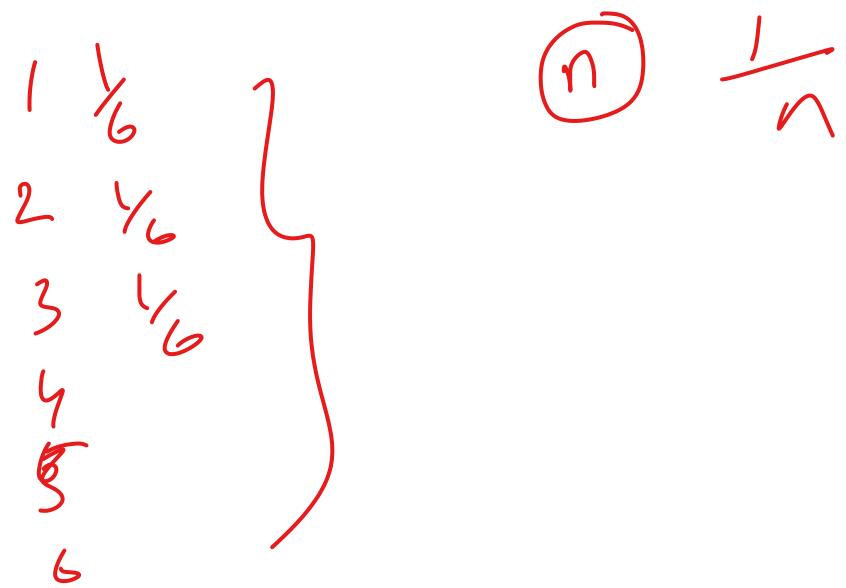


mean - mode

$$= 3 (\text{mean} - \text{median})$$

$\times \quad P(+)$   $F(x)$





$$E(x) = \sum x p(x)$$

$$= \frac{10}{4} \times P(x) \quad x P(x) \quad x^2 P(x)$$

$$E(x^2) = \frac{30}{4}$$

$$\left\{ \begin{array}{l} 1 \\ 2 \\ 3 \\ 4 \end{array} \right. \quad \left. \begin{array}{l} \frac{1}{4} \\ \frac{1}{4} \\ \frac{1}{4} \\ \frac{1}{4} \end{array} \right\} \quad \left. \begin{array}{l} \frac{1}{4} \\ \frac{2}{4} \\ \frac{3}{4} \\ \frac{4}{4} \end{array} \right\} \quad \left. \begin{array}{l} \frac{1}{4} \\ \frac{4}{4} \\ \frac{9}{4} \\ \frac{16}{4} \end{array} \right\}$$

$$V(x) = E(x^2) - (E(x))^2$$

$$= \frac{30}{4} - \left( \frac{10}{4} \right)^2 = \frac{30}{4} - \frac{100}{16} = \frac{120 - 100}{16} = \frac{20}{16} = \frac{5}{4}$$

$x \sim U(a, b)$

$x \sim P(y)$

$a + 1$

$a + 2$

$\vdots$

$b$

$$\begin{array}{c}
 P(X=5) = 0 \quad X \sim U(5,7) \\
 \hline
 \begin{matrix} (1,2) \\ -\infty \end{matrix} \quad \begin{matrix} (0) \\ +\infty \end{matrix} \\
 P(X=1) = 0 \quad P(X>5) \\
 \cancel{\frac{1}{\infty}} = 0 \quad P(X \leq 5)
 \end{array}$$

Consider a local area network with ten stations. Assume that at a given moment, each node can be active with  $p = 0.6$ . What is the probability that -

- i) one station is active
- ii) five stations are active
- iii) all ten stations are active

$$\begin{aligned}
 X &\sim \text{Bin}(n=10, p=0.6) \\
 &\text{Choose } (5,1)
 \end{aligned}$$

$$P(X=1) = {}^{10}C_1 0.6^1 0.4^9 = 0.055$$

$$P(X=5) = {}^{10}C_5 0.6^5 0.4^5 = 0.20$$

$$P(X=10) = {}^{10}C_{10} 0.6^{10} 0.4^0 = 0.006$$

$\text{choose}(10,1)*0.6^1*0.4^9$   
 $\text{choose}(10,5)*0.6^5*0.4^5$   
 $\text{choose}(10,10)*0.6^{10}*0.4^0$

$\text{dbinom}(1,10,0.6)$   
 $\text{dbinom}(5,10,0.6)$   
 $\text{dbinom}(10,10,0.6)$

A producer of pins realized that on a normal 5% of his items are faulty. He offers pin a parcel of 100 and insurances that not more than 4 pins will be flawed. What is the likelihood that a bundle will meet the ensured quality?

$$\lambda = 100 * 0.05 = 5$$

$$0.1738$$

$$\frac{e^{-5}}{5!}$$

$$RP = P(0) + P(1) + P(2) + P(3) + P(4)$$

$$= \frac{e^{-5} 5^0}{0!} + \frac{e^{-5} 5^1}{1!} + \frac{e^{-5} 5^2}{2!} + \frac{e^{-5} 5^3}{3!} + \frac{e^{-5} 5^4}{4!}$$

$$e^{-5} \underbrace{d\text{Pois}(0, 5)}_{\text{exp}} +$$

$$\text{PPois}(4, 5) \quad \lambda = 0, 1, 2, 3, \dots, \infty$$

$$\leftarrow \circled{5}$$

$$P(X > 5)$$

$$(F) \quad P(X \leq 10)$$

Expl(1)  
red

Expl(5)

e<sup>c</sup>

There are 20 marbles, 10 blue marbles and 5 white marbles in a jar. Select a marble without looking, note the colour and then replace the marble in a jar. Find the probability that 5 marbles are selected before selecting first red marble.

$$P(\text{red}) = \frac{20}{35} = \frac{4}{7} \quad q = \frac{3}{7}$$

$$P(x = n) = \underbrace{(1-p)^{n-1}}_{\text{Diagram: A horizontal line with a point labeled } p \text{ on it. Above the line, there is a curved arrow pointing from left to right, labeled } n-1.} \cdot p$$
$$P(x = 6) = \left(\frac{3}{7}\right)^5 \cdot \frac{4}{7} =$$

d glom

$$X \sim \text{Bin}(n=10, p=0.5)$$

$$P(X \leq 5)$$

$$= P(X=0) + P(X=1) + P(X=2) +$$

$$+ P(X=3) + P(X=4) +$$

$$P(x = i) = \frac{P_{\text{binom}}(S, 10, 0.5)^i}{\text{sample}(a)}$$

set.seed(100)  
 rbinom(100, 10, 0.6)

$$(n = 10, P = 0.6)$$

0 → 0  
 1 → 1  
 2 → 2

3 → 6

4 → 10

5 → 20

6 → 24

7 → 15

8 → 13

9 → 18

10 → 0

A radioactive source emits 4 particles during a five second period

i) Calculate the prob that it emits 3 particles during 5 second period

$$P(X=3) = \text{Pois}(3, 4)$$

ii) it emits at least one particle during 5 second period

iii) during a ten second period, what is the probability that 6 particles are emitted

$$P(X \geq 1) = 1 - P(X < 1)$$

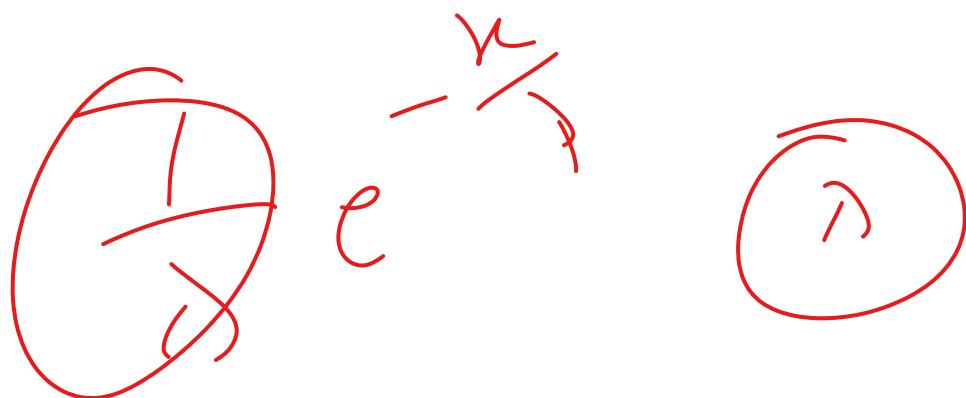
$$= 1 - P(X=0)$$

A baseball player has a 30% Chance of getting a hit on any given pitch. Ignoring balls what is the probability that the player earns a hit before he strikes out.

$$P + qP + q^2P$$

$$\lambda e^{-\lambda t} \quad \text{rate} \rightarrow \lambda$$

.       $\sim \rightarrow \lambda$



1000 students at college level were graded according to IQ and economic conditions use chi square test to find out is there any association between economic conditions and IQ

|               |      | I Q level |     |
|---------------|------|-----------|-----|
|               |      | high      | low |
| ECO Condition | Rich | 460       | 140 |
|               | Poor | 240       | 160 |

Values = c(460, 140, 240, 160)

dimnames = list(c("high", "low"), c("rich", "poor"))

m = matrix(Values, nrow = 2, ncol = 2, byrow = T, dimnames = dimnames); m

t = as.table(m)

a = chisq.test(m)

a\$p.value

format(a\$p.value, scientific = F)

Can Vaccination be regarded as a preventive measure of smallpox as evidenced by following data of 1490 persons exposed to a smallpox in a locality 310 in all were attacked of these 1490 persons 350 were vaccinated and of these 40 were attacked.

|       | Vacc | N V  | TV   |
|-------|------|------|------|
| A     | 40   | 270  | 310  |
| NA    | 310  | 870  | 1180 |
| Total | 350  | 1140 | 1490 |









































































































































































