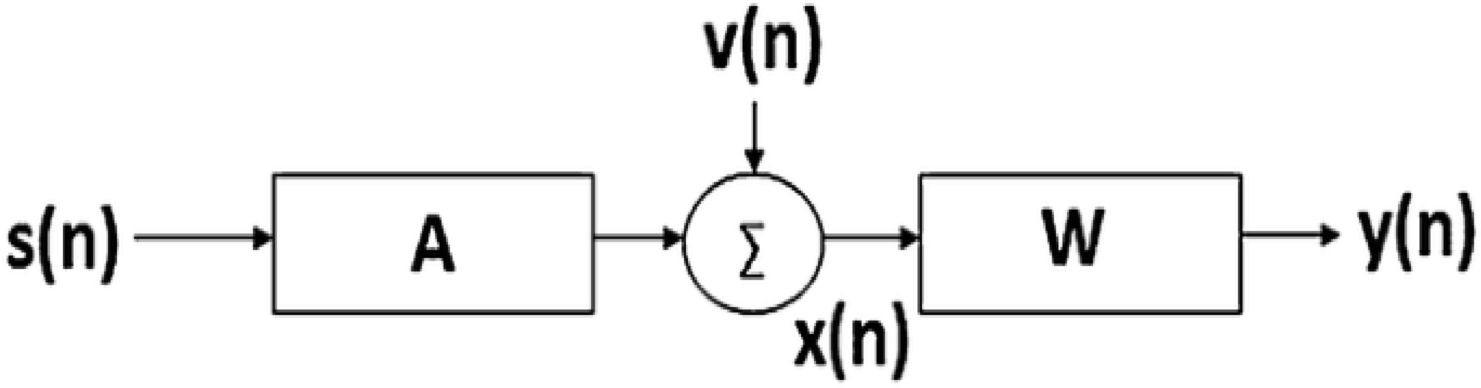
BSS based on Kurtosis,Negentropy and Maximum likelihood Functions

Introduction

- Blind Source Separation (BSS) involves separating blind sources from its determined or over determined mixtures. Independent Component Analysis (ICA) is the state of the art tool used in BSS.
- FastICA Algorithm is one of the successful algorithm in ICA.
- The objective of this paper is to examine various contrast functions using fastICA algorithm.

Definition

- The Objective of BSS is to separate the desired sources from the their mixtures without knowing information about the source signals or the mixing environment.



Contd…

- The above figure illustrates the general instantaneous BSS problem
- $x(n) = [x_1(n), x_2(n) \ldots x_m(n)]^T$ stands for observation of mixed signals from the sensors.
- M denotes the number of sensors or number of mixture signals and n denotes the discrete time index.
- $s(n)=[s_1(n),s_2(n) \ldots s_N(n)]^T$ are source signals contributing for the mixture signals. where N is the number of sources and it is unknown in many situations.

Contd…

- The mixing system is characterized by a matrix A of order $M \times N$ and is known as mixing matrix.
- V designates uncorrelated additive white Gaussian background noise sources and contributes for the mixture signals x(n) apart from the contribution of x(n).
- Inverse of mixing matrix A is W of order $N \times M$. W separates the desired sources from the mixture signals and it is known as separation matrix. $y(n) = [y_1(n), y_2(n) \ldots y_N(n)]^T$ are the estimated source signals.

Equations

$x(n) = A \times s(n) + V$. (BSS problem model)

$y(n) = W \times x(n)$ (mixing matrix)

So BSS problem becomes estimation of

$W = A^{-1}$ (Inverted Mixing matrix)

Literature Survey

- Blind source separation using kurtosis, negentropy and maximum likelihood functions, M. Kumar, V. E. Jayanthi, International Journal of Speech Technology, Volume 23, 2020.https://link.springer.com/article/10.1007/s10772-019-09664-z#citeas (SCOPUS INDEX).
- The Above cited is our base paper for the project which use statistical contrast functions mentioned in the title.

Literature Survey

- As mentioned above, BSS might seem illogical if we estimate sources without prior information. But assuming statistical independence of sources can be a useful prior information.
- However we cannot estimate sources up to their amplitude and order and these indeterminate states are referred to as scaling and permutation ambiguity. (Tong et al. 1991).
- Though BSS may be application specific, but generally it is modelled as : $x=As+v$ (with noise) or $x=As$ (noiseless).

Literature Survey

- According to Delfosse and Loubaton (1995) , Mixed sources are Gaussian while the independent ones are not. Thus, this factor is utilized by using Negentropy as contrast function in ICA. The Negentropy has been estimated using the function given by Hyvarinen (1998) [as its computation is difficult].
- On similar lines, Kurtosis is used for measuring the non-Gaussianity (tailedness). Kurtosis maximization is starting point for the FastICA algorithm (Hyvarinen and Oja 1997b).

Concepts used:

- Central Limit Theorem: Contrast function are used to measure statistical independence, which is equivalent to measure of non-Gaussianity. The theorem states that the mixture of signals is closer to a gaussian distribution even when individual sources are non-gaussian. This property is exploited by the contrast functions to extract source signals from noise by maximising non-gaussianity.

- Whitening: Before ICA, the mixture signals undergo a preprocessing step called whitening. Whitening involves making the mixture samples uncorrelated and unit variance.

Concepts used (Contd)...

- Kurtosis : It is a fourth order cumulant of a random variable and is classical measure of non guassianity.
  $kurt[y(n)] = E\{ y^4(n)\} - 3[E^2\{y(n)\}]^2$
- Negentropy : It is also measure of non guassianity. It measures the difference in entropy between a given distribution and gaussian distribution with same mean and variance.
  $J[y_i(n)] = H[y_G(n) - H[y_i(n)]$

$$J[y_i(n)] = H[y_G(n)] - H[y_i(n)]$$

Concepts used (contd)...

Maximum Likelihood : Estimating the joint distribution of the source signal from the mixtures

$x(n) = A^* s(n)$

$P_x[x(n)] = (1/\det(A))^* p_s(x(n))$

Fast ICA Algorithm:

1. Center the observed sensor signals to make its mean zero. $x_i(n) = x(n) - E[x_i(n)]$ for $i = 1, 2, \ldots M$
2. Linearly transform the observed sensor signals so that it is white. $z(n) = V\, x(n)$
3. Choose an initial separating matrix randomly.
4. Compute the source signal by using separation matrix and observed sensor signals. $y(n) = W\, z(n)$.
5. Update separation matrix based on the contrast function calculation.

Fast ICA Algorithm (Contd.)

6. Normalize the separation matrix to avoid repeated extraction of independent components.
   W ← (WCW')^(-0.5) . W
6. If separation matrix does not converge, then go to step 4.

Results

Run the code

Novel Idea and Future Prospects: