



Blind source separation using kurtosis, negentropy and maximum likelihood functions

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Abstract

Independent component analysis (ICA) is a thriving tool in separating blind sources from its determined or over-determined instantaneous mixture signals. FastICA is one of the successful algorithms in ICA. The objective of this paper is to examine various contrast functions using FastICA algorithm, and to find highly performed available contrast function for the application of speech signal analysis in noisy environments. The contrast function is a non-linear function used to measure the independence of the estimated sources from the observed mixture signals in FastICA algorithm. Kurtosis, negentropy and maximum likelihood functions are used as contrast functions in FastICA algorithm. The FastICA algorithm using these contrast functions is tested on the synthetic instantaneous mixtures and real time recorded mixture signals. We evaluate the performance of the contrast functions based on signal to distortion ratio, signal to artifact ratio, signal to interference ratio and computational complexity. The result shows the maximum likelihood function performs better than the other contrast functions in noisy environments.

Keywords Blind source separation · Entropy · Independent component analysis · Maximum likelihood estimation · Speech processing

Abbreviations

BSE	Blind source extraction
BSS	Blind source separation
EVD	Eigen value decomposition
FA	Factor analysis
FFT	Fast Fourier transform;
fMRI	Functional magnetic resonance imaging
HSS	Heart sound signals
ICA	Independent component analysis
LSS	Lung sound signals
NMF	Non-negative matrix factorization
PCA	Principal component analysis
SAR	Signal to artifact ratio
SCA	Sparse component analysis
SDR	Signal to distortion ratio
SIR	Signal to interference ratio

SNR	Signal to noise ratio
TFR	Time-frequency representation

1 Introduction

The objective of Blind source separation (BSS) is to separate the desired sources from their mixture signals without knowing information about the source signals and mixing environment. Hence it is termed as blind source separation.

Figure 1 illustrates general instantaneous BSS or blind source extraction (BSE) problem. $\mathbf{x}(\mathbf{n}) = [x_1(n), x_2(n) \dots x_M(n)]^T$ stands for observation of mixed signals from the sensors. M denotes the number of sensors or number of mixture signals and n denotes the discrete time index. $\mathbf{s}(\mathbf{n}) = [s_1(n), s_2(n) \dots s_N(n)]^T$ are source signals contributing for the mixture signals; where N is the number of sources and it is unknown in many situations. The mixing system is characterized by a matrix \mathbf{A} of order $M \times N$ and is known as mixing matrix. $\mathbf{v}(\mathbf{n})$ designates uncorrelated additive white Gaussian background noise sources and contributes for the mixture signals $\mathbf{x}(\mathbf{n})$ apart from the contribution of $\mathbf{s}(\mathbf{n})$. Inverse of mixing matrix \mathbf{A} is \mathbf{W} of order $N \times M$. \mathbf{W} separates the desired sources from the mixture signals and it is known as separation matrix. $\mathbf{y}(\mathbf{n}) = [y_1(n), y_2(n) \dots y_N(n)]^T$

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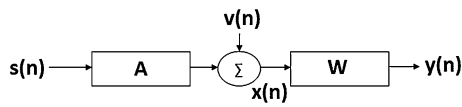


Fig. 1 General instantaneous BSS problem formulation or BSE problem

is estimated version of source signals $\mathbf{s}(\mathbf{n})$ as a result of applying BSS technique to the mixed signals $\mathbf{x}(\mathbf{n})$. The general BSS problem may be stated as an estimation of the source signals $\mathbf{s}(\mathbf{n})$ from the observed mixture signals $\mathbf{x}(\mathbf{n})$ without knowing any information about \mathbf{A} , $\mathbf{s}(\mathbf{n})$ and $\mathbf{v}(\mathbf{n})$. The problem may seem as quite illogical without any available information. Fortunately, statistical independence of the sources is true in most applications and it is useful a priori information for solving the BSS problem. However, we cannot estimate source signals up to their amplitude and their order. These indeterminate states are called as scaling and permutation ambiguity in the BSS respectively (Tong et al. 1991). As the signal structure gives the useful information about the sources rather than the exact amplitude of the signals, scaling ambiguity is not considered as a serious issue in practice. BSS technique used in various fields like speech enhancement, speech recognition, feature extraction, biomedical signal analysis (EEG, fMRI), signal de-noising, telecommunications, data mining and financial time series analysis. Several approaches are used in BSS to extract the original sources from the mixture signal. Few of them are independent component analysis (ICA), Sparse component analysis (SCA) and Non-negative matrix factorization (NMF). The success and efficiency of these approaches remain with the fact that availability of a prior knowledge about the sources, preprocessing of mixture signals, post processing of the recovered sources and suitable model selection. Even though the BSS model may get changed according to specific applications (Amari and Cichocki 1998; Hyvarinen et al. 2001), the general instantaneous BSS model is $\mathbf{x} = \mathbf{A}\mathbf{s} + \mathbf{v}$. Noiseless instantaneous mixing model is the simplest model of BSS and it is expressed as $\mathbf{x} = \mathbf{A}\mathbf{s}$. ICA is one central tool available to solve the BSS problem (Cichocki and Amari 2002). Based on the central limit theorem, mixture of sources is close to Gaussian whereas independent sources are away from Gaussian distributions. So finding a component with maximum non-Gaussianity would result a statistically independent source. To measure non-Gaussianity, a non-linear function is used and it is known as contrast function or cost function. The principle of non-Gaussianity is used in the ICA by Delfosse and Loubaton (1995). In general, ICA algorithm is structured as formation, determination and optimization of the contrast function (Chiu et al. 2008; Oja and Kiviluoto 1999). To test the Gaussianity of the signals, a higher order statistic is proposed and explicit mathematical explanation is discussed in Luo et al. (2015). The limitations and ambiguities of ICA reviewed in Acharya and Panda (2014). ICA and Sparse Component Analysis (SCA) methods are integrated and developed a novel method

to separate the source signals from the mixed signals in Yangi and Jin (2012). A study carried out for separating heart sound signals (HSS) and the lung sound signals (LSS) from bio sound signal in Nersisson and Noel (2016). ICA applied to extract the sources from functional Magnetic Resonance Imaging (fMRI) data by two simple contrast functions (Khaliq et al. 2013). Formulation of the cost function is the significant part in the ICA tool to measure the degree of independence sources. Classical methods are available for the measurement of non-Gaussianity in Comon (1994b). Higher order statistics, information theoretic approach and likelihood functions are exploited to measure the non-Gaussianity. Higher order statistics like kurtosis is used to find the independent components present in the mixture source (Comon 1994a; Donoho 1980; Shalvi and Weinstein 1990; Wiggins 1978). But kurtosis is sensitive to outliers (Hyvarinen and Oja 2000; Meinecke et al. 2004). Negentropy is an information theoretic approach and it is utilized as the contrast function to measure non-Gaussianity. Since the computation of negentropy is difficult, its approximation is used in Hyvarinen (1998). Minimization of mutual information is another information theoretic approach to measure non-Gaussianity. Maximum likelihood approach is a popular contrast function used to separate the sources from mixed signals in Lacoume and Gaeta (1990), Pham (1996) and Pham and Garrat (1997). However mutual information and maximum likelihood approaches are connected to one another and it is proved in Cardoso (1997), Obradovic and Deco (2006) and Pearlmutter and Parra (1997). FastICA algorithm is the popular method in ICA, due to its simplicity, convergence speed and satisfactory results in many applications (Hyvarinen and Oja 1997a; Hyvarinen 1999). FastICA is originally introduced for noise free ICA model. It is an iterative method to find local maxima of the contrast function using Newton's method. Kurtosis maximization is starting point for the FastICA algorithm (Hyvarinen and Oja 1997b). The extension of the FastICA algorithm for complex valued sources proposed in Bingham and Hyvarinen (2000). Apart from FastICA algorithm, Sparse Component Analysis and machine learning techniques are used to separate the sources from mixed signals. CapsNet is used for blind source separation of speech mixture in Kumar and Jayanthi (2019). In Sect. 2, the theoretical framework is described for the FastICA algorithm using various contrast functions. In Sect. 3, the performance of the FastICA algorithm with three different contrast functions and their results is compared. Finally, conclusion is drawn along with future research directions.

2 Materials and methodology

2.1 Central limit theorem

Contrast functions are used to measure the degree of statistical independence. A measure of independence is equivalent to measure of non-Gaussianity. According to the central limit

theorem, sum of non-Gaussian random variables are closer to Gaussian than the individual random variable.

$$y_i(n) = \sum_{j=1}^M w_{ij}x_j(n) \quad i = 1, 2, 3 \dots N \quad (1)$$

where, w_{ij} are elements of separation matrix \mathbf{W} . w_{ij} are chosen such that the linear transformation produces $y_i(n)$ as one of the independent components. Its non-Gaussianity is more than any other combinations of source signal. Because, if $y_i(n)$ are a combination of two or more sources then it would be closer to Gaussian distribution than independent component. So searching for the combination which has maximum non-Gaussianity, then such combination would be originated from a single source rather than more than one source.

2.2 Whitening

Typical preprocessing tools include Principal Component Analysis (PCA), Factor Analysis (FA), whitening, Fast Fourier Transform (FFT), Time Frequency Representation (TFR) and sparsification. Whitening is used as a preprocessing technique in the FastICA algorithms. A zero mean random vector is said to be white, if its elements are uncorrelated and have unit variance. Indeed, there are many possible ways to make the signal white. Given the mixture signals $\mathbf{x}(\mathbf{n})$, find a linear transformation matrix \mathbf{V} so that the mixture signals transformed into white.

$$\mathbf{z}(\mathbf{n}) = \mathbf{V}_{M \times M} \mathbf{x}(\mathbf{n})_{M \times 1} \quad (2)$$

The transformation can be computed from Eigen vectors and Eigenvalues of the covariance matrix of mixture signals $\mathbf{x}(\mathbf{n})$. For obtaining Eigen vectors and Eigen values, Eigen value Decomposition (EVD) is used.

$$\mathbf{C}_{M \times M} = \text{cov}[\mathbf{x}(\mathbf{n})] \quad (3)$$

$$\mathbf{C}_{M \times M} = \mathbf{E}_{M \times M} \mathbf{D}_{M \times M} \mathbf{E}_{M \times M}^T \quad (4)$$

Where \mathbf{E} has Eigen vectors as columns and \mathbf{D} is a diagonal matrix containing Eigenvalues as its diagonal elements.

$$\mathbf{V}_{M \times M} = \mathbf{D}_{M \times M}^{-1/2} \mathbf{E}_{M \times M}^T \quad (5)$$

2.3 Kurtosis

Kurtosis is the name of the fourth order cumulant of a random variable and it is a classical measure of non-Gaussianity. Kurtosis is defined for zero mean random variable as

$$\text{kurt}[\mathbf{y}(\mathbf{n})] = E\{\mathbf{y}^4(\mathbf{n})\} - 3[E\{\mathbf{y}^2(\mathbf{n})\}]^2 \quad (6)$$

Since $\mathbf{y}(\mathbf{n})$ is an estimate of the source signals, $E\{\mathbf{y}^2(\mathbf{n})\}$ is assumed to be unit variance and hence the kurtosis of the source signal is just a normalized version of the fourth moment. Kurtosis is zero for Gaussian random variable and nonzero for most non-Gaussian random variables. Kurtosis is positive for supergaussian random variables and negative for subgaussian random variables. To measure non-Gaussianity of the random variable, the absolute value of the kurtosis is taken as the measure of non-Gaussianity. Consider the estimation of source signals from whitened signals by using separation matrix $\mathbf{W}_{N \times M}$

$$\mathbf{y}(\mathbf{n})_{N \times 1} = \mathbf{W}_{N \times M} \mathbf{z}(\mathbf{n})_{M \times 1} \quad (7)$$

$$\mathbf{y}(\mathbf{n})_{N \times 1} = \mathbf{W}_{N \times M} \mathbf{V}_{M \times M} \mathbf{x}(\mathbf{n})_{M \times 1} \quad (8)$$

$$\mathbf{W}_{N \times M} \mathbf{V}_{M \times M} \mathbf{A}_{M \times M} \mathbf{s}(\mathbf{n})_{N \times 1} = \mathbf{B}_{N \times N} \mathbf{s}(\mathbf{n})_{N \times 1} \quad (9)$$

$$y_i(n) = \sum_{j=1}^N b_{ij}^T s_j(n) = \mathbf{b}_i^T \mathbf{s}(\mathbf{n}) \quad (10)$$

The estimated source signal $y_i(n)$ is equal to one of source signals $\mathbf{s}(\mathbf{n})$ if only one element of matrix \mathbf{B} in each and every row and column is unity. This shows that source signals can be estimated up to permutation. The kurtosis of the estimated source signal is calculated as

$$\text{kurt}[y_i(n)] = \text{kurt}[\mathbf{b}_i^T \mathbf{s}(\mathbf{n})] \quad (11)$$

$$\begin{aligned} \text{kurt}[y_i(n)] &= b_{i1}^4 \text{kurt}[s_1(n)] + b_{i2}^4 \\ &\text{kurt}[s_2(n)] + \dots + b_{iN}^4 \text{kurt}[s_N(n)] \end{aligned} \quad (12)$$

The maximum kurtosis is obtained, if only one source contributes to the estimated signal $y_i(n)$. As the variance of the estimated source signals is assumed as unity,

$$E[y_i^2(n)] = ||\mathbf{w}_i||^2 = 1 \quad (13)$$

Now the problem is simplified to find the separation matrix $\mathbf{W}_{N \times M}$, so as to maximize the kurtosis of the estimated sources. This is constrained numerical optimization problem and it is given as Maximize $\text{kurt}[y_i(n)]$ or $\text{kurt}[\mathbf{w}_i^T \mathbf{z}(\mathbf{n})]$ subject to the constraint $||\mathbf{w}_i||^2 = 1$.

$$\frac{\partial \text{kurt}[\mathbf{w}_i^T \mathbf{z}(\mathbf{n})]}{\partial \mathbf{w}_i} = E\{\mathbf{z}(\mathbf{n})[\mathbf{w}_i^T \mathbf{z}(\mathbf{n})]^3\} \quad (14)$$

Newton's method is one of the most efficient methods for optimization. The iteration rule in the Newton's method is

$$\mathbf{w}_i \leftarrow \mathbf{w}_i \pm \left[\frac{\partial^2 J(\mathbf{w}_i)}{\partial \mathbf{w}_i^2} \right]^{-1} \frac{\partial J(\mathbf{w}_i)}{\partial \mathbf{w}_i} \quad (15)$$

where, $J(\mathbf{w}_i)$ is the function to be optimized. First and second order derivatives of the contrast function (kurtosis) yields

$$\Delta \mathbf{w}_i \text{sign}\{kurt[\mathbf{w}_i^T \mathbf{z}(\mathbf{n})]\} E\{\mathbf{z}(\mathbf{n})[\mathbf{w}_i^T \mathbf{z}(\mathbf{n})]\} \quad (16)$$

The constraint of the optimization problem is satisfied by bounding \mathbf{w}_i to unit norm after every iteration.

$$\mathbf{w}_i \leftarrow \mathbf{w}_i / \|\mathbf{w}_i\| \quad (17)$$

Since kurtosis is much sensitive to outliers, it is not being considered as a robust measure of non-Gaussianity.

2.4 Negentropy

In the information theory, negentropy is used as a measure of non-Gaussianity. Negentropy measures the difference in entropy between a given distribution and a Gaussian distribution with the same mean and variance.

$$J[y_i(n)] = H[y_G(n)] - H[y_i(n)] \quad (18)$$

Where, $H(y_G)$ is the entropy of Gaussian random variable with the same mean and variance as $y_i(n)$ and $H[y_i(n)]$ is entropy of a random variable $y_i(n)$. Property of entropy suggests that Gaussian distributed random variable has maximum entropy than any other distributions. So negentropy is always non-negative and it is zero only if the distribution is Gaussian. The advantage of negentropy is that the linear transformation of a random variable does not change its negentropy value, whereas the kurtosis varies with linear transformation of random variable. Since kernel estimator is used to estimate the probability density function and integration of probability density function involves in the negentropy function, it is computationally complex. The efficiency of such density estimation method depends on the correct choice of kernel estimation parameters. Since the estimation of probability density function is difficult, some approximation of negentropy is used in practice. A reliable and flexible approximation proposed in Pearlmutter and Parra (1997).

$$J[y_i(n)] \approx \rho [E\{f_i(y_i)\} - E\{f_i(y_G)\}]^2 \quad (19)$$

where, ρ is constant and $f_i(\cdot)$ is non-quadratic function. If $y_i^4(n)$ is to be considered as that non-quadratic function, then it again leads to kurtosis based measurement of non-Gaussianity. The following choices of non-quadratic functions proved to be robust estimators (Hyvarinen et al. 2001).

$$\begin{aligned} f_i[y_i(n)] &= \log \cosh[y_i(n)] \\ f_i[y_i(n)] &= -\exp[-y_i^2(n)/2] \end{aligned} \quad (20)$$

Now the problem is simplified to find the separation matrix $\mathbf{W}_{N \times M}$, so as to maximize the negentropy of the estimated sources. This is again constrained numerical optimization

problem given as Maximize $J[y_i(n)]$ or $J[\mathbf{w}_i^T \mathbf{z}(\mathbf{n})]$ subject to the constraint $\|\mathbf{w}_i\|^2 = 1$. So now the constrained optimization problem is defined as

$$J[\mathbf{w}_i^T \mathbf{z}(\mathbf{n})] \approx \rho [E\{f_i[\mathbf{w}_i^T \mathbf{z}(\mathbf{n})]\} - E\{f_i(y_G)\}]^2 + \lambda [\|\mathbf{w}_i\|^2 - 1] \quad (21)$$

Deriving the Newton's method for optimization, we get

$$\begin{aligned} \frac{\partial J[\mathbf{w}_i^T \mathbf{z}(\mathbf{n})]}{\partial \mathbf{w}_i} &\alpha \left[E\{\mathbf{z}(\mathbf{n})\} f_i'[\mathbf{w}_i^T \mathbf{z}(\mathbf{n})] \right] + \beta \mathbf{w}_i \\ \frac{\partial^2 J[\mathbf{w}_i^T \mathbf{z}(\mathbf{n})]}{\partial \mathbf{w}_i^2} &\alpha \left[E\{\mathbf{z}(\mathbf{n})\} f_i''[\mathbf{w}_i^T \mathbf{z}(\mathbf{n})] \right] + \beta \end{aligned} \quad (22)$$

2.5 Maximum likelihood estimation

Maximum likelihood is a popular method in the estimation of independent components. Consider the basic ICA model, $\mathbf{x}(\mathbf{n}) = \mathbf{A}\mathbf{s}(\mathbf{n})$. According to basic ICA model, the density of observed mixture signals can be represented as

$$\begin{aligned} p_x[\mathbf{x}(\mathbf{n})] &= \frac{1}{|\det \mathbf{A}|} p_s[\mathbf{s}(\mathbf{n})] \\ &= |\det \mathbf{B}| p_s[\mathbf{B}\mathbf{x}(\mathbf{n})] \end{aligned} \quad (23)$$

Where, $\mathbf{B} = \mathbf{A}^{-1}$. As the sources are assumed to be statistically independent and the density of mixture signals is the product of the marginal densities of sources.

$$p_x[\mathbf{x}(\mathbf{n})] = \prod_{i=1}^N p_s[\mathbf{b}_i^T \mathbf{x}(\mathbf{n})] |\det \mathbf{B}| \quad (24)$$

If there are T samples observed in each sensor signals then likelihood of matrix \mathbf{B} is given as

$$L[\mathbf{B}] = p_x[\mathbf{x}(\mathbf{n})|\mathbf{b}_i] = \prod_{n=1}^T \prod_{i=1}^N p_s[\mathbf{b}_i^T \mathbf{x}(\mathbf{n})] |\det \mathbf{B}| \quad (25)$$

Normally log likelihood function is considered rather than likelihood function for computational convenience. So the log likelihood function with respect to the parameter \mathbf{B} is

$$\begin{aligned} \log L[\mathbf{B}] &= T \log |\det \mathbf{B}| + \sum_{n=1}^T \sum_{i=1}^N \log \{p_i[\mathbf{b}_i^T \mathbf{x}(\mathbf{n})]\} \\ \frac{1}{T} \log L[\mathbf{B}] &= \log |\det \mathbf{B}| + E \left\{ \sum_{i=1}^N \log \{p_i[\mathbf{b}_i^T \mathbf{x}(\mathbf{n})]\} \right\} \end{aligned} \quad (26)$$

Here, log likelihood is the function of elements of separation matrix \mathbf{B} and marginal densities of the estimated sources. The estimation of densities of the estimated sources is a non-parametric problem. The non-Gaussianity is used to solve non-parametric problem. Approximation of all densities of

the unknown sources is carried out by using either super Gaussian or sub Gaussian approximation. A theorem in Hyvarinen et al. (2001) (p. 206) states that if $p_i[s_i(n)]$ is assumed density of independent component, then the ML estimator is locally consistent provided

$$E\{s_i(n)g_i[s_i(n)] - g'_i[s_i(n)]\} > 0 \quad (27)$$

Where, $g_i[s_i(n)] = \frac{\partial}{\partial s_i(n)} p_i[s_i(n)]$. Consider the following log densities for super Gaussian and sub Gaussian

$$\begin{aligned} \log p_i[s_i(n)] &= a - \log \cosh[p_i(s_i)] \\ \log p_i[s_i(n)] &= b - \{s_i^2(n)/2 - \log \cosh[p_i(s_i)]\} \end{aligned} \quad (28)$$

Both are opposite sign to each other. The choice between the two functions is made by calculating the values of $E\{s_i(n)g_i[s_i(n)] - g'_i[s_i(n)]\}$. If the value is greater than zero, super Gaussian density can be taken otherwise sub Gaussian function is considered. Similar to negentropy expression, log likelihood function can be maximized by replacing vector notations by matrix.

$$\mathbf{W} \leftarrow \mathbf{W} + \text{diag}(\alpha_i) [\text{diag}(\beta_i) + E\{g(\mathbf{W}\mathbf{z})(\mathbf{W}\mathbf{z})^T\}] \mathbf{W} \quad (29)$$

Where, $\alpha_i = \frac{-1}{E\{g'(\mathbf{w}_i^T \mathbf{z}) + \beta_i\}}$. Infomax is another contrast

function used to estimate the independent components. Infomax principle is equivalent to maximum likelihood approach and it is proved in Cardoso (1997).

2.6 FastICA algorithm

The FastICA algorithm uses any contrast function which maximizes the non-Gaussianity to find the independent components from the observed mixture signals after proper preprocessing. The next step in the FastICA algorithm is solving constrained optimization problem. Here the contrast function is optimized with the constraint that bounds the norm of the separation vector to unit. Newton's method is used for solving constrained optimization problem. FastICA algorithm steps are as follows

1. Center the observed sensor signals to make its mean zero.

$$x_i(n) = x_i(n) - E[x_i(n)] \text{ for } i = 1, 2, \dots, M$$

2. Linearly transform the observed sensor signals so that it is white.

$$\mathbf{z}(\mathbf{n}) = \mathbf{V}\mathbf{x}(\mathbf{n})$$

3. Choose an initial separating matrix randomly.

4. Compute the source signal by using separation matrix and observed sensor signals.

$$\mathbf{y}(\mathbf{n}) = \mathbf{W}\mathbf{z}(\mathbf{n})$$

5. Update separation matrix based on the contrast function calculation.
6. Normalize the separation matrix so that to avoid repeated extraction of independent components

$$\mathbf{W} \leftarrow (\mathbf{W}\mathbf{C}\mathbf{W}^T)^{-1/2} \mathbf{W}$$

7. If separation matrix not converged, then go to step 4.

3 Results and conclusion

3.1 Experimental setup

Noisy speech data sets from Araki and Vincent (2019) is used to evaluate the efficiency of various algorithms. Data sets containing a synthetic instantaneous mixture as well as live recording mixture signals used. Two channel mixtures of two speech sources and real background noises sampled at 16 KHz are considered for testing our algorithm. The mixture and source signals of length 10 s are used. The signals are recorded in office room in the reverberant environments using microphones. The direction of arrival of each speech sources is different in each mixture and the Signal to Noise Ratio (SNR) is drawn randomly between -17 and $+12$ dB. Recording room dimensions for both synthetic mixture and live recordings are $4.45 \times 3.55 \times 2.5$ m.

3.2 Performance evaluation

The performance of blind source separation (BSS) algorithm is evaluated using the following terms which are defined in Araki et al. (2012)

Signal to distortion ratio (SDR)

$$SDR = 10 \log_{10} \frac{\|S_{\text{target}}\|^2}{\|e_{\text{inter}} + e_{\text{noise}} + e_{\text{artif}}\|^2} \quad (30)$$

Signal to interference ratio (SIR)

$$SIR = 10 \log_{10} \frac{\|S_{\text{target}}\|^2}{\|e_{\text{inter}}\|^2} \quad (31)$$

Signal to artifact ratio (SAR)

$$SAR = 10 \log_{10} \frac{\|S_{\text{target}} + e_{\text{inter}} + e_{\text{noise}}\|^2}{\|e_{\text{artif}}\|^2} \quad (32)$$

The computational complexity of BSS is much important in certain applications like speech recognition. So we calculate the computational complexity of the algorithm using number of iterations and computational time requirement for the algorithm. The BSS evaluation tool box proposed by Vincent et al. (2006) used for evaluating the performance parameters.

3.3 Results

Consider two sources without noise interference, one male speech and another female speech. The mixing matrix is randomly assumed as

$$A = \begin{bmatrix} 5 & 2 \\ 3 & 4 \end{bmatrix}$$

and it is instantaneously mixed with the sources. The mixed signals were created by multiplying the sources with randomly chosen matrix A. The estimated sources and original sources are used to evaluate SDR, SIR and SAR values, which are listed in Table 1. Figure 2 illustrates the performance of the algorithm by comparing three different contrast functions with the parameters like Signal to Distortion Ratio (SDR), Signal to Interference Ratio (SIR) and Signal to Artifact Ratio (SAR).

Table 1 Performance measures of synthetic instantaneous mixture and real room recorded mixture signals

Parameter	Type of mixture signals	Source	Kurtosis	Negentropy	Maximum likelihood
SDR (dB)	Synthetic	Source1	49.7015	45.5588	19.4378
		Source2	45.0607	43.0009	19.2976
SIR (dB)		Source1	51.3274	47.4021	19.4392
		Source2	45.5089	43.2810	19.2988
SAR (dB)		Source1	54.7559	54.7559	54.5249
		Source2	55.0454	55.0454	54.7559
SDR (dB)	Real time	Source1	− 13.9451	− 13.9581	− 18.3843
		Source2	− 18.6029	− 18.6478	− 13.94
SIR (dB)		Source1	10.8432	10.6818	4.0665
		Source2	3.3453	3.2067	10.8848
SAR (dB)		Source1	− 13.5870	− 13.5870	− 16.9230
		Source2	− 16.9230	− 16.9230	− 13.5870

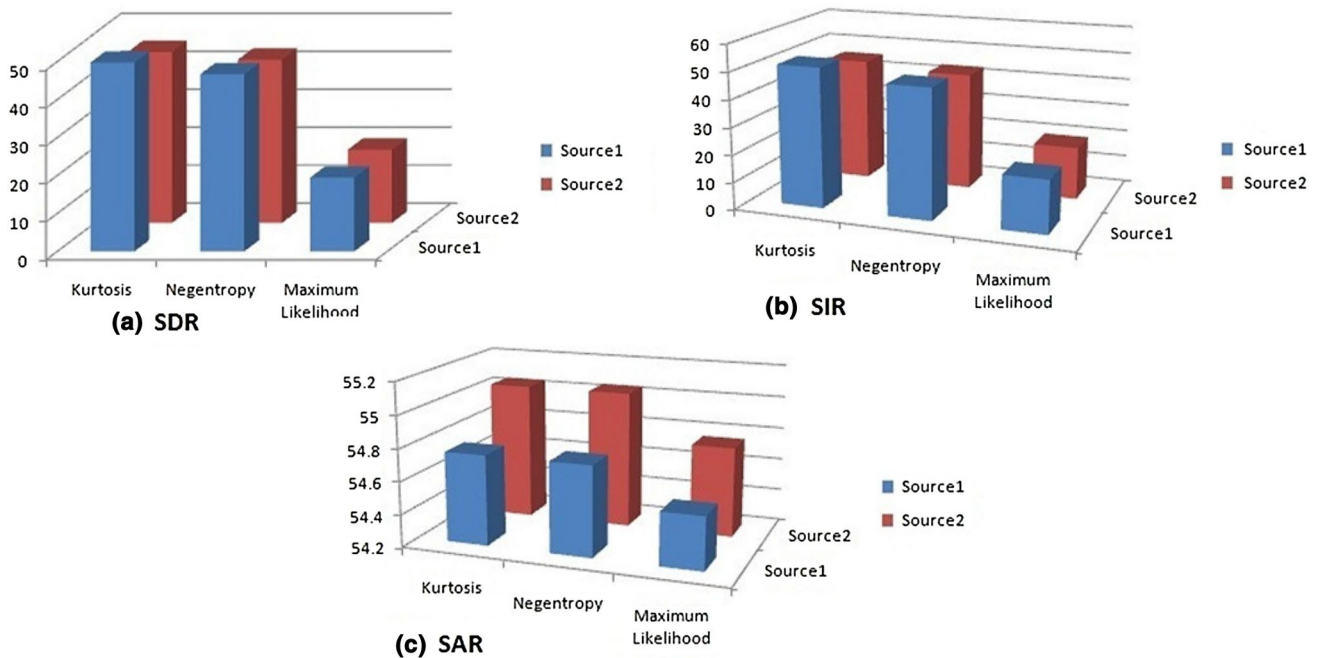


Fig. 2 Performance of FastICA algorithm for synthetic convolutive mixtures

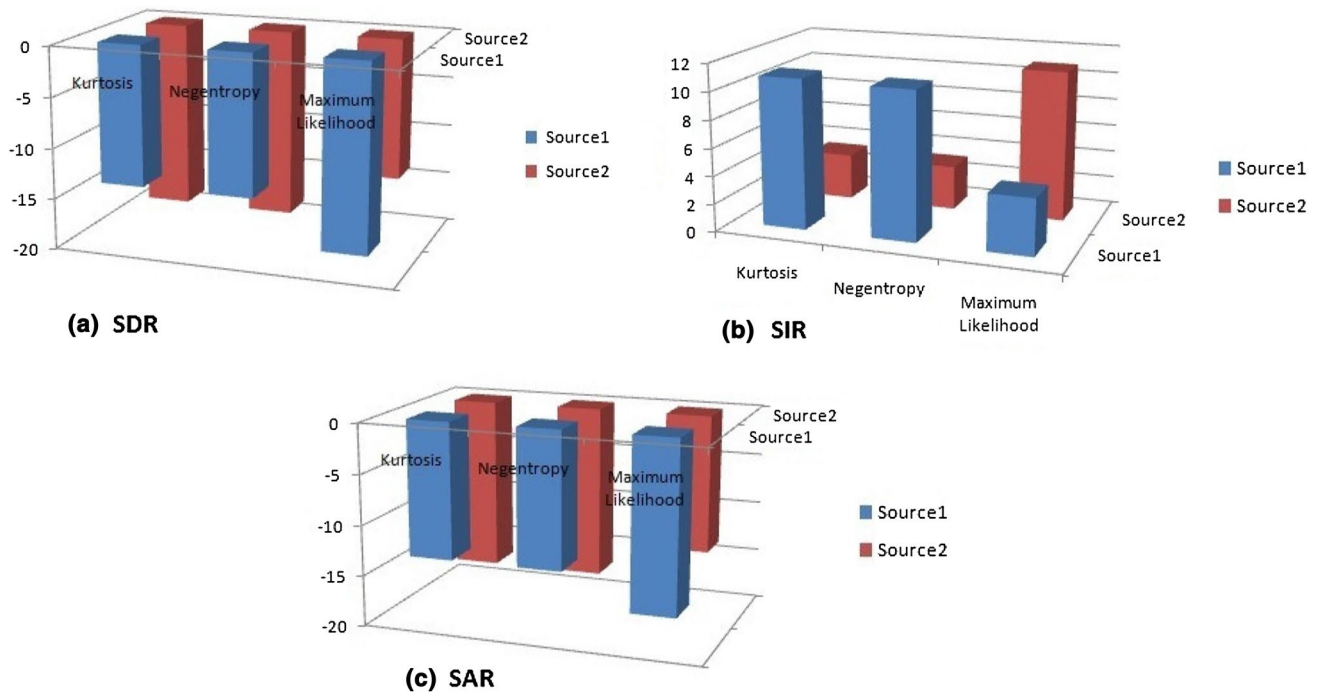


Fig. 3 Performance of FastICA algorithm for real mixtures

The SDR, SIR and SAR of FastICA algorithm using maximum likelihood contrast function are much less as compared with kurtosis and negentropy. However, maximum likelihood function provides better robustness against the outliers. kurtosis and negentropy performs better than the maximum likelihood function, in the noiseless ICA model, but its performance degrades in the presence of background noises. Next, real room recording mixture signal with the background noises are considered for checking the performance of FastICA. The SNR of real recorded mixture signals is ranged from -17 to 12 dB. Figure 3 illustrates the performance of the FastICA algorithm for three different contrast functions with real room recording mixture signals (noisy signal).

The SIR of the FastICA algorithm shows the superiority in separating interferences for both synthetic and real time mixtures. The SDR, SIR and SAR values proves that the FastICA algorithms reduce the interference of speech sources effectively but it needs a separate approach for background noises. The SDR and SAR of the algorithm is almost equal

which also interprets that the interferences is effectively removed from the mixed signals. The FastICA algorithm takes care of separating the interference speech sources and hence the SIR is better than SDR and SAR whereas it needs separate approaches to treat both artifacts and noises. Computational complexity of the FastICA algorithm is analyzed by taking the number of iterations and computational time requirement for the execution of an algorithm which are given in Table 2. The Fig. 4 shows the number of iterations and computational time for various contrast functions. The computational time and number of iterations is lesser for maximum likelihood function than kurtosis and negentropy. The FastICA algorithm performs well for instantaneous mixed signals. But When the background noises are added to the mixed signals in the real environment, the performance is degraded considerably.

Table 2 Computational complexity of the FastICA algorithm

Computational complexity	Type of mixture signals	Kurtosis	Negentropy	Maximum likelihood
Number of iterations required	Synthetic instantaneous mixture signal	3	3	2
Computation time (s)		0.6	0.106	0.143
Number of iterations required	Real room recorded mixture signal	4	5	2
Computation time (s)		0.61331	0.122	0.1003

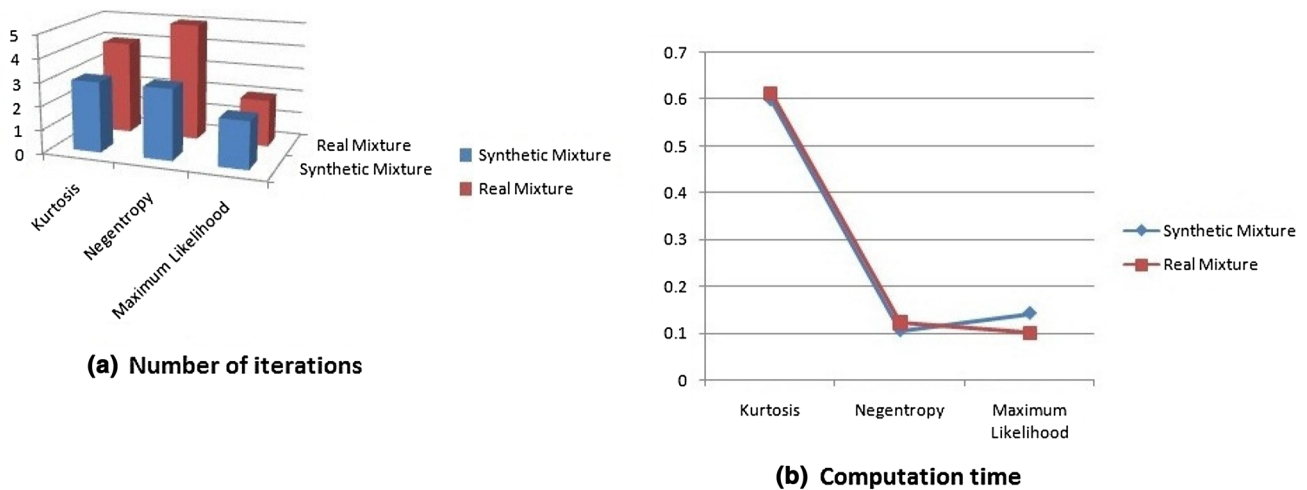


Fig. 4 Computational complexity of the FastICA algorithm

4 Conclusion

In this work, the performance of FastICA algorithms for the blind source separation problem using three different contrast functions such as kurtosis, negentropy and maximum likelihood are reviewed and tested. Central limit theorem plays an important role in the ICA. Based on central limit theorem, the independent sources are away from Gaussian distribution whereas the mixture signal tends more towards Gaussian distribution. So, the sources are separated by maximizing non-Gaussianity with respect to the separation matrix. The statistical quantity of kurtosis, information quantity of negentropy and maximum likelihood function are used as measure of non-Gaussianity. The optimization of such contrast functions are carried out using Newton's method with respect to the separation matrix. Synthetic instantaneous mixture and real room recording mixture signals are considered for testing FastICA algorithms. The performance of the algorithms compared in terms of SDR, SIR, SAR and computational load. Since FastICA algorithm is originally developed for noiseless instantaneous BSS, all the contrast functions produced satisfactory results for synthetic instantaneous mixture signals without any outliers or background noises. But its performance degraded considerably when it is tested on real room recorded mixture signals with background noises. However, maximum likelihood function provides better robustness than kurtosis and negentropy at the cost of reduced SDR. Maximum likelihood functions take less number of iterations and less computational time.

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Compliance with ethical standards

Conflict of interest The authors declare that they have no competing interests.

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