

## Table of Laplace Transforms

### 0.1 Table Of Laplace Transforms

$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$	$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$
1. 1	$\frac{1}{s}$	2. $e^{at}$	$\frac{1}{s-a}$
3. $t^n, n = 1, 2, 3, \dots$	$\frac{n!}{s^{n+1}}$	4. $t^p, p > -1$	$\frac{\Gamma(p+1)}{s^{p+1}}$
5. $\sqrt{t}$	$\frac{\sqrt{\pi}}{2s^{\frac{3}{2}}}$	6. $t^{n-\frac{1}{2}}, n = 1, 2, 3, \dots$	$\frac{1 \cdot 3 \cdot 5 \cdots (2n-1)\sqrt{\pi}}{2^n s^{n+\frac{1}{2}}}$
7. $\sin(at)$	$\frac{a}{s^2 + a^2}$	8. $\cos(at)$	$\frac{s}{s^2 + a^2}$
9. $t \sin(at)$	$\frac{2as}{(s^2 + a^2)^2}$	10. $t \cos(at)$	$\frac{s^2 - a^2}{(s^2 + a^2)^2}$
11. $\sin(at) - at \cos(at)$	$\frac{2a^3}{(s^2 + a^2)^2}$	12. $\sin(at) + at \cos(at)$	$\frac{2as^2}{(s^2 + a^2)^2}$
13. $\cos(at) - at \sin(at)$	$\frac{s(s^2 - a^2)}{(s^2 + a^2)^2}$	14. $\cos(at) + at \sin(at)$	$\frac{s(s^2 + 3a^2)}{(s^2 + a^2)^2}$
15. $\sin(at + b)$	$\frac{s \sin(b) + a \cos(b)}{s^2 + a^2}$	16. $\cos(at + b)$	$\frac{s \cos(b) - a \sin(b)}{s^2 + a^2}$
17. $\sinh(at)$	$\frac{a}{s^2 - a^2}$	18. $\cosh(at)$	$\frac{s}{s^2 - a^2}$
19. $e^{at} \sin(bt)$	$\frac{b}{(s-a)^2 + b^2}$	20. $e^{at} \cos(bt)$	$\frac{s-a}{(s-a)^2 + b^2}$
21. $e^{at} \sinh(bt)$	$\frac{b}{(s-a)^2 - b^2}$	22. $e^{at} \cosh(bt)$	$\frac{s-a}{(s-a)^2 - b^2}$
23. $t^n e^{at}, n = 1, 2, 3, \dots$	$\frac{n!}{(s-a)^{n+1}}$	24. $f(ct)$	$\frac{1}{c} F\left(\frac{s}{c}\right)$
25. $u_c(t) = u(t-c)$	$\frac{e^{-cs}}{s}$	26. $\delta(t-c)$	$e^{-cs}$
27. $u_c(t)f(t-c)$	$e^{-cs}F(s)$	28. $u_c(t)g(t)$	$e^{-cs}\mathcal{L}\{g(t+c)\}$
29. $e^{ct}f(t)$	$F(s-c)$	30. $t^n f(t), n = 1, 2, 3, \dots$	$(-1)^n F^{(n)}(s)$
31. $\frac{1}{t}f(t)$	$\int_s^\infty F(u) du$	32. $\int_0^t f(v) dv$	$\frac{F(s)}{s}$
33. $\int_0^t f(t-\tau)g(\tau) d\tau$	$F(s)G(s)$	34. $f(t+T) = f(t)$	$\frac{\int_0^T e^{-st}f(t) dt}{1 - e^{-sT}}$
35. $f'(t)$	$sF(s) - f(0)$	36. $f''(t)$	$s^2F(s) - sf(0) - f'(0)$
37. $f^{(n)}(t)$	$s^n F(s) - s^{n-1}f(0) - s^{n-2}f'(0) - \dots - sf^{(n-2)}(0) - f^{(n-1)}(0)$		

**Table Notes**

1. This list is not a complete listing of Laplace transforms and only contains some of the more commonly used Laplace transforms and formulas.

Recall the definition of hyperbolic functions.

$$\cosh(t) = \frac{e^t + e^{-t}}{2} \quad \sinh(t) = \frac{e^t - e^{-t}}{2}$$

2. Be careful when using “normal” trig function vs. hyperbolic functions. The only difference in the formulas is the “ $+a^2$ ” for the “normal” trig functions becomes a “ $-a^2$ ” for the hyperbolic functions!
3. Formula #4 uses the Gamma function which is defined as

$$\Gamma(t) = \int_0^{\infty} e^{-x} x^{t-1} dx$$

If  $n$  is a positive integer then,

$$\Gamma(n+1) = n!$$

The Gamma function is an extension of the normal factorial function. Here are a couple of quick facts for the Gamma function

$$\begin{aligned} \Gamma(p+1) &= p\Gamma(p) \\ p(p+1)(p+2)\cdots(p+n-1) &= \frac{\Gamma(p+n)}{\Gamma(p)} \\ \Gamma\left(\frac{1}{2}\right) &= \sqrt{\pi} \end{aligned}$$