Mid Semester Examination (Solutions)

Industrial Organization (ECO 312)

Total points: 40 — Weightage towards final grade: 30%

Date: 21/02/2019 — Time: 9:30 AM - 10:30 AM

- Show all your steps.
- In Part II., if you attempt all three problems, you must state which two you want graded.
- Calculators are allowed.
- No cheat sheets or any other material are permitted.

Part I. Answer the following in brief. $(4 \times 5 = 20)$

1. **Nash Equilibrium.** Consider the following game of rock, paper and scissors:

1 \ 2	Rock	Paper	Scissors
Rock	0,0	-1,1	1,-1
Paper	1,-1	0,0	-1,1
Scissors	-1,1	1,-1	0,0

Find all Nash Equilibria of the game (mixed or pure). Answer without steps will not be accepted.

We can see in the above game that there is no pure strategy Nash Equilibrium. Let us suppose player 1 plays rock with probability p_1, paper with probability p_2 and scissors with probability $1 - p_1 - p_2$. Then, player 2's expected payoff for playing.

$$\text{Rock: } (0)p_1 + (-1)p_2 + (1)(1 - p_1 - p_2)$$

$$\text{Paper: } (1)p_1 + (0)p_2 + (-1)(1 - p_1 - p_2)$$

$$\text{Scissors: } (-1)p_1 + (1)p_2 + (0)(1 - p_1 - p_2)$$

Setting the three equal to each other (to make player 2 indifferent), we get $p_1 = p_2 = 1 - p_1 - p_2$. This yields a mixed strategy of $\sigma_1^* = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$, which gives us the mixed strategy for player 1. Similarly, we find the msNE for player 2 to be $\sigma_2^* = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$. The msNE is

$$\left\{\left(\frac{1}{3}R, \frac{1}{3}P, \frac{1}{3}S\right) . \left(\frac{1}{3}R, \frac{1}{3}P, \frac{1}{3}S\right)\right\}$$

Note: It's okay if you showed lesser steps in this question as it was just for 5 points. As long as you showed the equations for rock paper and scissors and stated the msNE, it's fine.

2. **Bilateral Monopoly with Asymmetric Information.** Consider the following setting of a bilateral monopoly. There is one good to be traded, which is produced by the supplier at a cost c. The valuation of the buyer is given by v. v is common knowledge, but c is known only to the supplier. $f(c) > 0$ on the interval $[\underline{c}, \overline{c}]$. $F(\underline{c}) = 0$, $F(\overline{c}) = 1$. $v \in (\underline{c}, \overline{c})$. Suppose the supplier has all the bargaining power, and makes a "take it or leave it" offer. What is the price the supplier will offer? Is the expected volume of trade here different than it would be in a full information game with equal bargaining power? What about CS and PS?

   Since the seller is making the offer, and has all the bargaining power, the buyer's uncertainty is irrelevant here. The seller will make an offer of $p = v$ is $v \geq c$ and $p > v$ if $v < c$. If the first case, the buyer will accept the take it or leave it offer. In the second case, the buyer will reject. In both cases, the seller appropriates the entire gains from trade. Thus, $CS = 0$ and $PS = v - c$ if trade happens and 0 otherwise. However, the volume of trade is efficient, in that as long as $v - c > 0$, trade takes place. In a **full information game with equal bargaining power**, the solution would be that trade takes place when $v > c$ (same in both cases), but the gains from trade are distributed equally here, with $v - p = p - c$. This implies $CS = PS$.

3. **Vertical Differentiation.** In our model of vertical differentiation in class, the utility function was as follows:

$$U = \begin{cases} \theta s - p & \text{if he buys a good with quality} \\ & s \text{ and price } p, \\ & \\ 0 & \text{if he does not buy.} \end{cases}$$

   Now, assume that instead of the good being a search good, it is an experience good. Let $\theta = 2$, and s is distributed uniformly from 0 to 1. The cost of production $c = 0$. Write the demand for the good in this case. The first thing to note here is that since the good is an experience good, and not a search good, the consumer has an ex-ante expectation on quality. But he does not know what it is. Since s is uniformly distributed over $[0, 1]$, his initial expectation is $E(s) = 0.5$. In such a case, the price is $2(0.5) = 1$. Seller's will not provide a high quality good where $s > 0.5$ in this setting. However, since consumers know that high quality goods will be completely off the market, their expectations get further updated t o$E(s|p = 1) = 0.25$. This is a case of the lemons problem, where the market eventually converges to a case in which only the worst quality good ($s = 0$) is sold at $p = 0$. **For the exam answer, it is sufficient to say that since it is an experience good with no indication of quality, quality and price will both go to 0 and there will be no market for the good (demand = 0), and it is a case of the lemons problem.**

4. Explain in a couple of lines the intuition behind the following:

   (a) The Coase conjecture. (2.5)

   The intuition behind the Coase conjecture is that when a monpolist enters a durable goods market where consumers have different valuations, he will initially set a high price, but in the second period, he is likely to "flood the market" by selling off the rest of the goods at a lower price. Consumers of intermediate valuation who anticipate this behaviour may hold off on purchasing the product at a higher piece in the initial period, causing the firm to lose some of its monopoly power. Common examples are clothes that are later sold in clearance sales, or video games like FIFA 19.

(b) Learning by doing in dynamic monopolies (show with cost functions). (2.5)

In a dynamic monopoly, the idea behind learning by doing is that as the monopolist produces more in period 1, his costs decrease in period 2 (as workers gain more skill in producing that particular good). In a 2 period case, the idea is that $\frac{\partial q_1}{\partial c_2} < 0$ where q_1 is quantity produced in period 1 and c_2 is costs in period 2.

Part II. Answer the following in detail (Any 2). $(2 \times 10 = 20)$

1. **Multiproduct Monopoly.** Consider two goods with the following demand functions:

$$q_i = a - bp_i + dp_j \text{ where } i, j \in \{1, 2\};\ i \neq j;\ a, b > 0 \text{ and } |d| < b$$

The cost of production of each good is 0.

(a) Are the goods differentiated? Why? (1)

Yes, the goods are differentiated. This is because the demand of one good depends on the price of the other in a continuous way. If they were homogeneous perfects substitutes, demand would switch entirely to the good with the lower price (like in a Betrand game).

(b) Suppose that the two goods are produced by the same firm (a monopoly). Compute optimal prices. Compare the Lerner index and the inverse of the elasticity for each good. (7)

A multi-product monopolist maximizes total profits. Hence, here, here maximizes

$$\max_{p_1,p_2} \pi = p_1(a - bp_1 + dp_2) + p_2(a - bp_2 + dp_1)$$

The FOCs are as follows:

$$\frac{\partial \pi}{\partial p_1} = a - 2bp_1 + 2dp_2 = 0$$

$$\frac{\partial \pi}{\partial p_2} = a - 2bp_2 + 2dp_1 = 0$$

Solving simultaneously yields $p_1 = p_2 = p$. This implies:

$$a - 2(b-d)p = 0 \Rightarrow p = \frac{a}{2(b-d)} \text{ and } q_1 = q_2 = q = a - (b-d)p = \frac{a}{2}$$

The elasticity for good i is given by

$$\epsilon_{ii} = -\frac{\partial q_i}{\partial p_i} \cdot \frac{p_i}{q_i} = -\left[-b \cdot \frac{\frac{a}{2(b-d)}}{\frac{a}{2}}\right] = \frac{b}{b-d}$$

Therefore, $\frac{1}{\epsilon_{ii}} = 1 - \frac{d}{b}$. Since $|d| < b$, $1 - \frac{d}{b} < 1$ if $d < 0$ and $1 - \frac{d}{b} > 1$ if $d > 0$. Since the cost of production $c = 0$, Lerner index $= \frac{p-c}{p} = 1$.

(c) Are the goods complements or substitutes? (2)

The products are substitutes is $d > 0$ and complements if $d < 0$. We can also see the relationship between the Lerner index and $\frac{1}{\epsilon_{ii}}$ depends on the same.

2. **Principal-Agent Problem.** Consider a worker (agent) in a firm with a utility function as follows:

$$u(w,e) = \frac{w}{1+e}$$

where wage $w \geq 0$ denotes salary and effort $e \in [0,1]$ denotes effort. The principal randomly checks agents for effort with probability p. If the agent is checked, he receives a wage of we, while if he is not checked, he receives w. The probability of being checked is independent of the worker's effort level.

(a) Set up the worker's maximization problem (2) Here, w is taken as given and the worker chooses e to maximize utility. His problem thus becomes

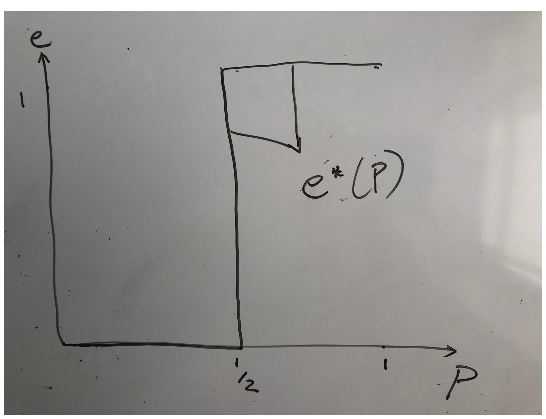
$$\max_{e\in[0,1]} \quad \underbrace{p\left(\frac{we}{1+e}\right)}_{\text{expected payoff from being checked}} \quad + \quad \underbrace{(1-p)\left(\frac{w}{1+e}\right)}_{\text{expected payoff from not being checked}}$$

(b) Determine the optimal effort level. (6)

Taking FOC w.r.t. e yields:

$$\frac{w(2p-1)}{(1+e)^2} = 0 \Rightarrow \frac{(2p-1)}{(1+e)^2} = 0$$

Note that the above assumes $w > 0$. If $w = 0$, the optimal effort level is clearly also 0. It is possible for this expression to actually be equal to 0 only if $p = \frac{1}{2}$. When $p = \frac{1}{2}$, the FOC stands for any effort level. Thus, $e^* \in [0,1]$. When $p < \frac{1}{2}$, we have a corner solution. Since it is more likely that he will not be checked, the solution here is $e^* = 0$. The final case is when $p > \frac{1}{2}$. In such a case, the corner solution is $e^* = 1$.



(c) Is the optimal effort level increasing or decreasing in w and p. (2)

As is evident from the FOC, e^* is independent of wage as long as wage is positive. One could also say effort is weakly increasing in wage. e^* is weakly increasing in profits.

3. **Negative Advertising.** Two soft drink companies, P and CC can choose to run negative ads against each other during a national cricket league. Due to demand from other industries, the organizers have decided to allow a maximum of 2 hours of such advertising during the entire tournament. Given the pair of choices (a_P, a_{CC}), the payoff for firm $i \in \{P, CC\}$ is given by

$$v_i(a_i, a_j) = a_i - 2a_j + a_i a_j - (a_i)^2 \text{ where } j \neq i$$

Both firms make their choice simultaneously.

(a) Find the best response functions. (3)

Each player maximizes $a_i - 2a_j + a_i a_j - (a_i)^2$ w.r.t. a_i, which gives 2 best response functions:

$$a_1(a_2) = \frac{1 + a_2}{2} \quad \text{and} \quad a_2(a_1) = \frac{1 + a_1}{2}$$

(b) Find the optimal level of advertising for each firm. (5)

Solving this simultaneously yields a Nash equilibrium of $a_1 = a_2 = 1$. This gives each party a payoff of $a_i - 2a_j + a_i a_j - (a_i)^2 = 1 - 2(1) + (1)(1) - (1)^2 = -1$. It is clear that negative advertising hurts both firms in Nash equilibrium.

(c) If the companies could sign binding agreement on how much to campaign, what levels would they choose? (2).

Since this is a symmetric game, a binding agreement would equally help both equally if $a_i = a_j = a$. This would yield an individual payoff of $a_i - 2a_j + a_i a_j - (a_i)^2 = a - 2a + (a)(a) - (a)^2 = -a$. Given that $a_i \geq 0$, this is minimized at $a = 0$. Thus, they should choose to not run negative ads.