Hubble's Law, The Age and The Expansion of the Universe - A Data-driven Exploration Cosmology

By: Mukul Patnaik

TABLE OF CONTENTS

1 INTRODUCTION

In December of 1995, the Hubble Space Telescope took a series of 342 separate exposures of a dark portion of the sky that is approximately the size of the head of a pin. The resulting image changed astronomy, and my life forever. There are about 3000 objects in the image, almost all of which are galaxies. GALAXIES. With quadrillions of celestial bodies of their own, some larger than our milky way galaxy. And just in a minuscule portion of the sky- approximately the size of the head of a pin. That's just how big the universe is. And not only is the universe so incomprehensibly big, but it is also expanding.

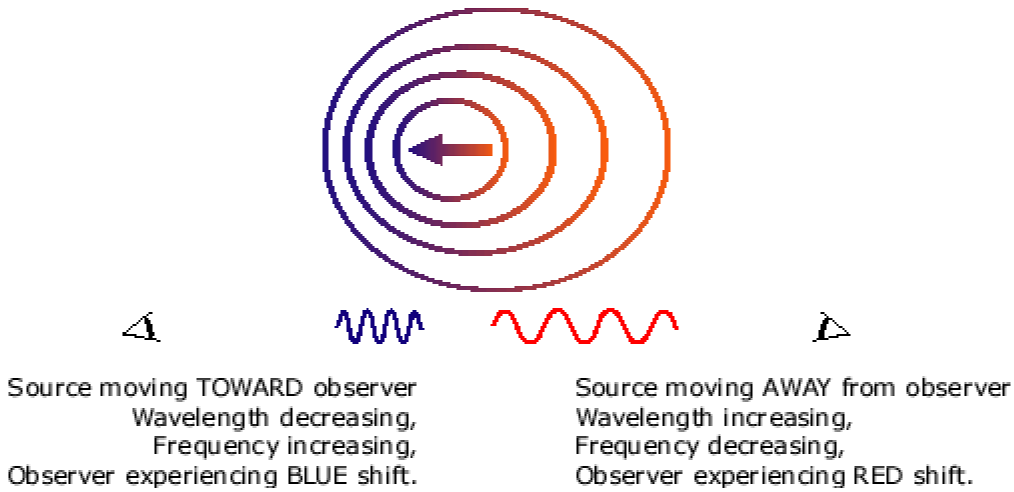
The Hubble Constant is one of the most significant numbers in all of Cosmology because it is an account of the age of the universe. It shows the rate at which the universe is expanding, the velocity originating from the primordial "Big Bang." The Hubble Constant is also used to calculate the quantity of dark matter existing in the Universe, determine the inherent brightness and masses of stars in nearby galaxies, explore those same characteristics in more remote galaxies and galaxy clusters, get the scale size of faraway galaxy clusters and help as an important test used in abstract cosmological models.

2 RESEARCH QUESTION

This exploration aims to determine Hubble's constant from data points of 10 spiral galaxies. It also aims to find the age of the universe from the calculated Hubble's constant and uses it as verification of the experiment's success. Finally, this exploration also tries to find the best fitting statistical model that explains the expansion of the universe. Therefore, the research question is: ***What is the relationship between the distance of a galaxy from Earth and it's recessional velocity? How does this relationship determine the age of the universe?***

3 THEORY

3.1 Doppler effect



([link to source](#))

The Doppler shift is an effect we observe in sources of waves that are moving. Waves are compressed in the direction of motion and are spaced further in the opposite direction. When the Doppler shift is applied to light, it results in objects moving towards an observer appear to have shorter wavelengths (and thus higher frequencies) than they truly do in the real word, shifting the light found nearer to the blue end of the spectrum, called a blueshift. The opposite effect, redshift, occurs when an object moves away from an observer — the waves seem to appear more spread apart, providing the impression of a longer wavelength and lower frequency that is red-shifted.[1][2]

3.2 Emission Spectrum

[1] https://rasmi.github.io/hubbles-law/
[2] https://imagine.gsfc.nasa.gov/features/yba/M31_velocity/spectrum/doppler_more.html

The spectrum of a galaxy will represent the total light coming from those objects that are contributing the most to the light of the galaxy. The spectra given with the data displays the relative intensity of the total light radiated from a galaxy as a function of wavelength. Sharp spikes in the spectrum of a galaxy, means radiation is being emitted. Dips in the spectrum of a galaxy indicate particular wavelengths of radiation are being absorbed. [3]

3.3 Recessional Velocity

Recessional velocity is the rate at which an extragalactic astronomical object recedes (becomes more distant) from an observer as a result of the expansion of the universe.[4] It is measured using the Doppler effect. The radiation from a moving object is shifted in wavelength, causing a redshift according to the formula:

$$\frac{v}{c} = z = \frac{\lambda_{\text{measured}} - \lambda_{\text{true}}}{\lambda_{\text{true}}}$$

Where v is the recessional velocity of the galaxy and c is the speed of light. The ratio v/c is called the redshift, or z. The wavelength λ_{true} is the wavelength that would be observed from an object at rest, such as in a lab, and λ measured is the wavelength measured from a moving object.[5]

3.4 Distance to each galaxy

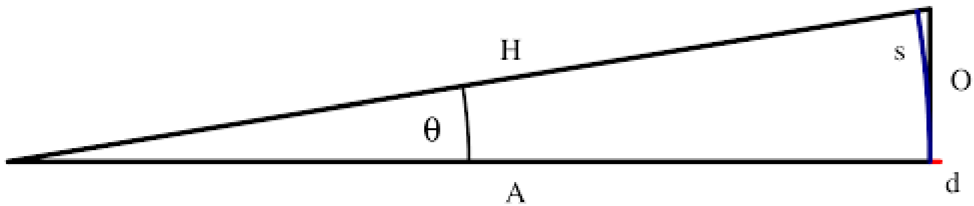
A more complex task is to find the distances to galaxies. Standard candles such as Type I supernovae Cepheid variables are used. However, for very remote galaxies, small-angle approximation is used. A very important assumption for this exploration is that galaxies of similar Hubble types are, in fact, of similar actual size, no matter how far away they are. [6]

---

[3] Emission Spectrum Definition in Science - ThoughtCowww.thoughtco.com › Chemistry › Chemical Laws
[4] http://hosting.astro.cornell.edu/academics/courses/astro201/hubbles_law.htm
[5] https://physics.fullerton.edu/files/Labs/Astr%20101L/HubbleLab(1).pdf

[6] Hubble's Distance - Redshift Relationastro.wku.edu › astr106 › Hubble_intro

The distance and size of galaxies are related using the small-angle approximation:



$$a=s/d$$
$$d=s/a$$

where a is the observed angular size (in radians), d is the distance to the galaxy and s is the galaxy's true size (diameter)

3.5 Hubble's Law and Constant

While studying galaxies, Edwin P. Hubble used Cepheid variable stars to measure the distances to a sample of galaxies. In a research paper, he found if one plots the distance to a galaxy and the velocity of the galaxy, the two quantities are directly correlated. He discovered that distant galaxies were all moving away from the Milky Way.[7] Furthermore, the farther away a star he observed, the faster the they were receding. This is now known as Hubble's Law: *the recessional velocity of a galaxy is proportional to its distance from us*.[8] The equation is:

$$v = H_0 \times d$$

where v is the galaxy's velocity (in km/sec), d is the distance to the galaxy (in megaparsecs; 1 Mpc = 1 million parsecs), and Ho is the proportionality constant, called "The Hubble constant." [9]

[7] https://www.e-education.psu.edu/astro801/content/l10_p3.html
[8] A relation between distance and radial velocity among extra ...www.pnas.org › content
[9] Hubble E (1929) A relation between distance and radial velocity among extra-galactic nebulae

To determine the Hubble Constant, “standard candles” such as a type of galaxy, supernova, or other event with well-understood size and magnitude is used. This study used galaxies approximated to be 22 kiloparsecs across. Assuming galaxies of size 22 kiloparsecs (1 kpc = 3.26 × 103 light-years), trigonometry with the small-angle approximation can be used to calculate the distance of galaxies θ radians in angular size:

$$\sin\theta = opp \acute{} \theta = 0.022\text{Mpc hyp } d$$

Rearranging for distance d, this gives:

$$d = 0.022 \text{ Mpc } \theta \text{ milliradians}$$

4 HYPOTHESIS

The exact value of the Hubble constant is still somewhat uncertain, but is generally believed to be around 65 kilometers per second for every megaparsec in distance. (A megaparsec is given by 1 Mpc = 3 x 10^6 light-years). This means that a galaxy 1 megaparsec away will be moving away from us at a speed of 65 km/sec, while another galaxy 100 megaparsecs away will be receding at 100 times this speed. I hypothesise the value to be near this generally accepted value and for the line of best-fit to pass through the origin.[10]

5 VARIABLES

5.1 Independent Variable

Distance of the galaxy from the Earth

[10] http://www.astro.wisc.edu/~heinzs/astro113/hubble.pdf

- Distance will be measured in MegaParsecs (Mpc)
- It will be measured using the angular size which is found from coordinates in the images
- There's a component of human error included as it requires the user to select coordinates manually

5.2 Dependent Variable

The recessional velocity of the galaxy from the Earth

- Velocity will be measured in Kilometers per second (km/sec)
- It will be measured by multiplying the red shift calculated from the spectra and the speed of light

5.3 Control Variables

- ***Hubble Classification:*** Galaxies come in many different shapes and sizes and range from 1,000 to 100,000 parsecs in diameter and are usually separated by millions of parsecs. Edwin Hubble created a classification of galaxies and grouped them as: spirals, barred spirals, irregulars and elliptical. *This exploration assumes that galaxies of the same Hubble time are also similar in size.* In order to reduce random error I have selected only spiral galaxies. [11]

- ***Similar Spectral emissions:*** I have also further filtered for galaxies with similar emission and absorption lines on their spectrum. Spiral galaxy spectra are characterized by strong emission lines, due to hot young stars which heat the surrounding gas, and by absorption features due to the older, underlying stellar population. [12]

5.4 Uncontrolled Variables

Galaxies are not spread evenly throughout the known universe, but are typically found in groups or clusters, where they have a significant

[11] https://lco.global/spacebook/galaxy-classification/
[12] http://astronomy.nmsu.edu/nicole/teaching/astr505/lectures/lecture26/slide01.html

gravitational effect on each other. Velocity dispersions of galaxies arising from this gravitational attraction are usually in the hundreds of kilometers per second, but they can rise to over 1000 km/s in rich clusters. This velocity can alter the recessional velocity that would be expected from the Hubble flow and affect the observed redshift of objects. [13][14]

6 DATA COLLECTION AND PROCESSING

6.1 Data

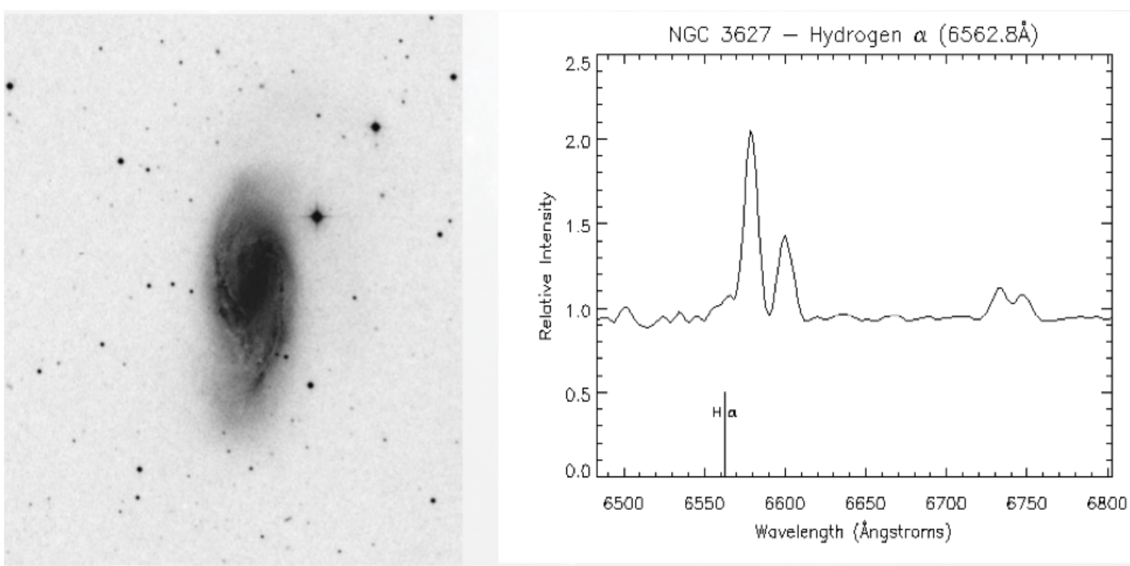
Galaxy	Data	
	Image	Spectra
-NGC 1357	Image	NGC 1357 Spectra
-NGC 1832	Image	NGC 1832 Spectra
-NGC 2276	Image	NGC 2276 Spectra
-NGC 2775	Image	NGC 2775 Spectra
-NGC 2903	Image	NGC 2903 Spectra
-NGC 3034	Image	NGC 3034 Spectra
-NGC 3147	Image	NGC 3147 Spectra
-NGC 3227	Image	NGC 3227 Spectra
NGC 3245	Image	NGC 3245 Spectra
NGC 3310	Image	NGC 3310 Spectra
-NGC 3368	Image	NGC 3368 Spectra
NGC 3471	Image	NGC 3471 Spectra
NGC 3516	Image	NGC 3516 Spectra
-NGC 3623	Image	NGC 3623 Spectra
-NGC 3627	Image	NGC 3627 Spectra
NGC 3941	Image	NGC 3941 Spectra
NGC 4472	Image	NGC 4472 Spectra
NGC 4631	Image	NGC 4631 Spectra
-NGC 4775	Image	NGC 4775 Spectra
-NGC 5248	Image	NGC 5248 Spectra
-NGC 5548	Image	NGC 5548 Spectra
NGC 5866	Image	NGC 5866 Spectra
-NGC 6181	Image	NGC 6181 Spectra
-NGC 6217	Image	NGC 6217 Spectra
-NGC 6643	Image	NGC 6643 Spectra
-NGC 6764	Image	NGC 6764 Spectra
NGC 7469	Image	NGC 7469 Spectra
(The ' - ' indicates galaxies with easily identifiable spectral lines. The others can be used but the lines can be harder to identify.)		

[13] Kaiser, N. (1987). "Clustering in real space and in redshift space". *Monthly Notices of the Royal Astronomical Society*. **227** (1): 1–21. Bibcode:1987MNRAS.227....1K. doi:10.1093/mnras/227.1.1

[14] Girardi, M.; Biviano, A.; Giuricin, G.; Mardirossian, F.; Mezzetti, M. (1993). "Velocity dispersions in galaxy clusters". *The Astrophysical Journal*. **404**: 38–50. Bibcode:1993ApJ...404...38G. doi:10.1086/172256

All data used in this exploration can be found at this University of Washington Department of Astronomy’s page: https://depts.washington.edu/astroed/HubbleLaw/galaxies.html. The galaxy spectra were obtained by Robert C. Kennicutt Jr. of the University of Arizona, and are published in The Astrophysical Journal Supplement Series, volume 79, pages 255-284, 1992. The digital images of the galaxies have been extracted from the CD-ROM version of the Palomar Observatory Sky Survey, which was produced under NASA contract by the Space Telescope Science Institute, operated by AURA, Inc. I gratefully acknowledge the various copyrights for that work.[15]

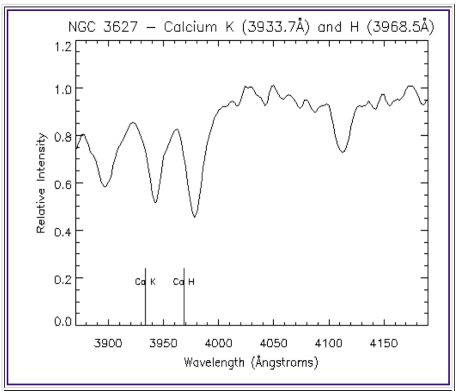
6.2 Collection

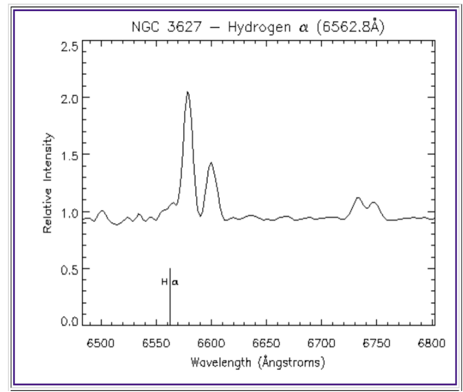


As specified in the control variable section, 10 galaxies of similar hubble classification and type are selected from all the available galaxies. The angular size of the galaxies is obtained from their images. To measure the size, opposite ends of the galaxy are clicked, along its longest part and the angular size is shown. The velocity of the galaxy is determined by measuring the redshift of spectral lines in the galaxy's spectrum. The full

[15] https://depts.washington.edu/astroed/HubbleLaw/galaxies.html

spectrum of the galaxy is shown at the top of the webpage. Below it are enlarged portions of the same spectrum, containing some common spectral features: the Calcium H & K lines and the Hydrogen transitions Balmer-alpha (abbreviated H-alpha) . [16]





6.3 Processing

Using the raw data collected from the database, processed values that will better help in the analysis are found. Distance from the Earth is calculated using the small-angle approximation. We know the distance to the Andromeda galaxy through observations of the Cepheid variables in it, and can thus use its angular size to determine that the Andromeda galaxy is 22 kpc (1 kiloparsec = 1000 pc) acrossc producing an angular size in milliradians or mrad:

distance (Mpc) = size (kpc) / angle (mrad)

with the assumed size of each galaxy being 22 kpc.

The redshift of each galaxy is determined by examining 3 common absorption and emission lines in the spectrum of light coming from the respective galaxy and comparing them to the known stationary wavelengths of some strong absorption and emission lines. These are the calcium "K" and

[16] http://www.cnrt.scsu.edu/~dms/cosmology/hubbleLawLab/lab_text.shtml.html

"H" absorption lines and the Hydrogen α emission line. This is done with the formula:

To find the average redshift, the three redshifts are added and divided by three. Then multiplied by 300,000 km/sec to find the **recessional velocity**.

7 DATA TABLES

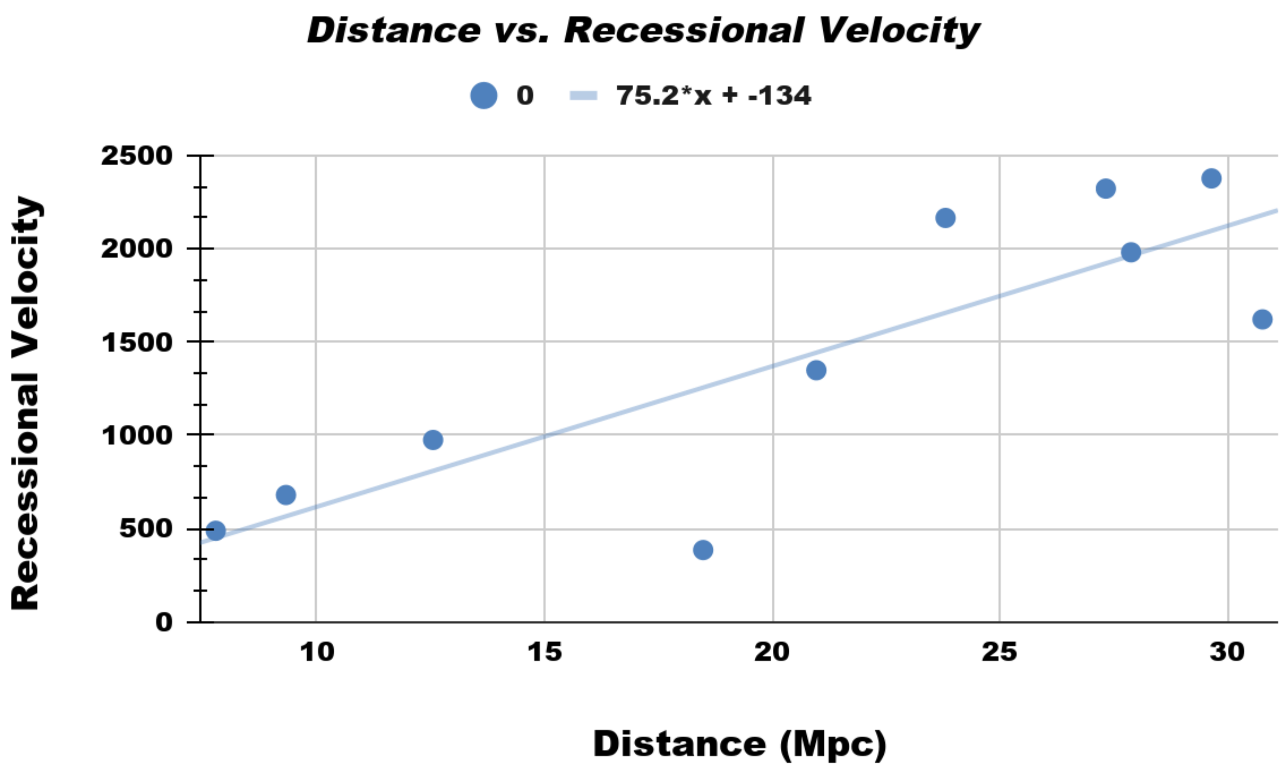
7.1 Raw

Galaxy							
NGC 1357	0.9240	0.0073	0.0072	0.0071	0.00721	2164	24
NGC 1832	0.7890	0.0066	0.0066	0.0066	0.00660	1980	28
NGC 2276	0.7420	0.0082	0.0077	0.0079	0.00792	2376	30
NGC 2903	1.1900	0.0011	0.0011	0.0016	0.00129	387	18
NGC 3623	2.8260	0.0015	0.0017	0.0017	0.00163	490	8
NGC 3627	2.3590	0.0021	0.0025	0.0022	0.00227	681	9
NGC 4775	0.7150	0.0061	0.0052	0.0049	0.00540	1621	31
NGC 5248	1.7520	0.0031	0.0033	0.0034	0.00325	976	13
NGC 6181	0.8050	0.0082	0.0080	0.0070	0.00774	2321	27
NGC 6643	1.0490	0.0047	0.0043	0.0046	0.00450	1349	21

7.2 Processed

Galaxy	Recessional Velocity	Distance
NGC 1357	2164	24
NGC 1832	1980	28
NGC 2276	2376	30
NGC 2903	387	18
NGC 3623	490	8
NGC 3627	681	9
NGC 4775	1621	31
NGC 5248	976	13
NGC 6181	2321	27
NGC 6643	1349	21

8 Results



8.1 Hubble's Constant

The above graph shows the final values of Recessional Velocity plotted over distance- showing us the relationship between the two. There are many outliers and I have used a line of best fit to show the overall trend. As shown on the graph, solving for the equation of the line gives 75.2x - 134. According to this line, the slope is 75.2 which also is the calculated Hubble's constant in this experiment. The uncertainty in Hubble's constant is calculated using the minimum-maximum slope method.

Maximum slope: (2376-490)/(27-8) = 99.2631
Minimum slope: (1621- 490)/(31 - 8) = 49.1739

Half of the difference between the two slopes is the estimated error in the value of H_0.

$$\frac{1}{2}(99.2631 - 49.1739) = 25.0446$$

Therefore, the value of H_0 through this exploration, corrected for uncertainties, is 75.2 ± 25.0446. This is a surprisingly accurate value considering the small number of datapoints considered and other errors made. The exact value of the Hubble constant is somewhat uncertain, but is generally believed to be around 70 kilometers per second for every megaparsec in distance, km/sec/Mpc. This means that a galaxy 1 megaparsec away will be moving away from us at a speed of about 70 km/sec, while another galaxy 100 megaparsecs away will be receding at 100 times this speed.[17]

8.2 THE AGE OF THE UNIVERSE

We see all of the galaxies moving away from the Milky Way galaxy at a speed proportional to their distance. Distance is velocity multiplied by the time spent traveling. Galaxies that have traveled further from each other, must have had to be closer together back in time. How long has it taken the galaxies to go that distance? Considering we already know the distances of galaxies and the rate at which they are receding, we can calculate how long they have been moving after that point in time, i.e. the age of the universe - would be galactic distance divided by galactic velocity. Hubble's constant is galactic velocity divided by galactic distance, so the universe's age is 1/Ho. (Assuming the universe expands at a constant rate.) [18]

Convert my H from km/sec/Mpc to inverse seconds (s-1) = 75.2 / 3.1 x 10^19

Using our value of Ho we find that the age of the universe is 12812500000 years.

[17] http://astro.berkeley.edu/~mwhite/darkmatter/hubble.html
[18] http://docplayer.net/43464058-Astronomy-1-introductory-astronomy-spring-2014.html

The current best estimate of the age of the universe is from the Planck satellite observations of relic radiation from the hot early universe: 13.82 billion years. Planck data suggests a Hubble Constant of 67.3 ± 1.4 km/s/Mpc. However, recent estimates of the Hubble constant from supernovae measurements are 74.2 ± 3.6 km/s/Mpc.; we now know (from looking at even more distant galaxies) that the speed of the universe expansion has changed over its history.[19]

9 ERROR ANALYSIS

The main error in this exploration is clearly evident from just one glance at the plotted graph. The line of best-fit does not pass through the origin. Hubble's Law describes a relationship between us and the universe and everything in it. Having a non-zero y-intercept would imply that there was something, a distance in space between two objects at the very start of the universe. This is inconsistent with our current knowledge of the beginning of the universe known as The Big Bang theory. The vertical translation of 490 in the graph is indicative of a systematic error in the data collecting process.

One possible systematic error in this experiment is the process of measuring a galaxy's angular size to determine its distance. In the data collection, a galaxy's distance was found by selecting two specific points on the galaxy image far on the faint outer edges as shown. When looking closely at the image of the galaxy, there appears to be faded fuzzy dust at the edge of the galaxy. This became a major problem in measuring the galaxy angular size due to the unknown faded shades as part of the galaxy's spiral making it extremely difficult to accurately determine the galaxy's angular size distance at both end edges of the galaxy's diameter. Another systematic error in this experiment is measuring the wavelength for the Calcium (Cal) K, Calcium (Cal) H, and Hydrogen (Hα) to determine the redshift of the galaxy. For the

[19] https://physics.fullerton.edu/files/Labs/Astr%20101L/HubbleLab(1).pdf

Calcium k and H emission line, selecting the precise wavelength is determining the lowest possible peak of the galaxy's spectra line for both lines. However, clicking one precise point on the spectra line becomes difficult due to the size of the spectra line being small to determine the true measure wavelength for both emission lines. The same goes for the Hydrogen (Hα) absorption line when determining the highest possible peaks in the galaxy's spectra.

Another area where there could be a weakness in the exploration is the fact that the measurement of the distance to each galaxy is done by using the brightness of a "standard candle" in a galaxy. If one knows the luminosity of the standard candle, then its observed brightness gives us its distance through the inverse square law. However, there are very little standard candles whose luminosities are accurately and confidently known. As a result, the distances measured for galaxies are far less accurately known than their recession velocities. This is responsible for the large scatter in Hubble's original publications figure and is still the primary uncertainty in Hubble Diagrams today.[20]

A big unknown in this experiment was the peculiar velocity of a galaxy, which is its velocity relative to the motion due to the isotropic expansion of the universe as described by the Hubble Flow. The total velocity of a galaxy is the sum of the velocity due to the Hubble flow and the local motion of the galaxy within its cluster or group environment due to local gravitational effects. The deviations from a pure Hubble flow is referred to as a peculiar velocity.[21] What it means to ``measure a galaxy's distance'' depends on whether one is interested in studying peculiar velocities or determining the value of the Hubble constant. A galaxy's peculiar velocity may be estimated given its ``distance'' in km s^{-1} - the part of its radial velocity due solely to the Hubble expansion. The same object provides an estimate of H_0 only if one can measure its distance in metric units such as megaparsecs. What this means in practice is that accurate peculiar velocity studies may be carried

[20] http://docplayer.net/43464058-Astronomy-1-introductory-astronomy-spring-2014.html
[21] "Peculiar Velocity | COSMOS." *Peculiar Velocity*, Swinburne University of Technology, astronomy.swin.edu.au/cosmos/P/Peculiar+Velocity.

out today, despite the fact that H_0 remains undetermined at the ~ 20% level. Another basic distinction between velocity analysis and the search for H_0 concerns the distance regimes in which they are optimally conducted. Peculiar velocity surveys are best carried out in the ``nearby'' universe, where peculiar velocity errors are comparable to or less than the peculiar velocities themselves. The characteristic amplitude of the radial peculiar velocity is a few hundred km s^{-1} at all distances, whereas the errors we make in estimating peculiar velocity grows linearly with distance. It turns out that the ``break-even'' point occurs at distances of ~ 5000 km s^{-1}. [22]

The key assumption for this exploration is that galaxies of similar Hubble types are, in fact, of similar actual size, no matter how far away they are. This is known as "the standard ruler" assumption. Another assumption made is that all galaxies have the same luminosities, that the entire galaxy is a standard candle.[23] As a result, the distance to a galaxy can be easily calculated from its apparent magnitude. The quality of results highly depends on the efficacy of the presumption that all galaxies are precisely the same. Another leading cause for the errors detected is possibly the very small sample of 10 galaxies.[24]

11 EVALUATION AND CONCLUSION

Hubble's law, says that a galaxy's velocity is directly proportional to its distance, and is also simultaneously saying something very deep about the state of the universe. If the universe is static, there would be no relationship between velocity and distance. However, in an expanding universe, we expect a correlation between distance and velocity. Virtually all the galaxies in the Universe (with the exception of a few nearby ones) are moving away from our galaxy, the Milky Way.[25] Are we the special center of the universe?

---

[22] http://nedwww.ipac.caltech.edu/level5/Willick/Willick1_1.html
[23] https://physics.fullerton.edu/files/Labs/Astr%20101L/HubbleLab(1).pdf
[24] http://www.astro.wisc.edu/~heinzs/astro113/hubble.pdf
[25] http://www.gsgis.k12.va.us/facultyweb/hhoughton/WebPages/AstroWeb/Galaxies%20and%20Cosmology/+Lab-Hubble.doc

No. The distances between the very space between all galaxies in the Universe are expanding significantly, an observer on any galaxy, not just our own, would see all the other galaxies traveling away, with the furthest galaxies traveling the fastest.[26]

Hubble's discovery initiated the field of observational cosmology and opened up a magnificent vast universe to be explored. A universe that contains 25% exotic non-baryonic dark matter, 70% dark energy and only 25% exotic non-baryonic dark matter. The dark energy is what causes the current expansion rate of the universe to accelerate, and that the expansion rate has been accelerating in the past 6 billion years. The characteristics of the strange dark energy that generates this acceleration are not yet known.[27]

We can observe its effects on galaxies and the expansion of the universe, but we have yet to identify the underlying source.[28] This is one of the biggest mysteries and challenges that faces humanity to this day. It is fascinating to look how far we have come and yet still see how much we do not know. The Universe is a strange and wonderful place.

12 BIBLIOGRAPHY

[26] https://www.e-education.psu.edu/astro801/book/export/html/1967
[27] https://mafiadoc.com/hubbles-law-and-the-expanding-universe-pnas_5c54d448097c472b208b45a5.html
[28] https://hubblesite.org/contents/articles/dark-energy

https://rasmi.github.io/hubbles-law/

https://imagine.gsfc.nasa.gov/features/yba/M31_velocity/spectrum/doppler_more.html

Emission Spectrum Definition in Science - ThoughtCowww.thoughtco.com › Chemistry › Chemical Laws

http://hosting.astro.cornell.edu/academics/courses/astro201/hubbles_law.htm

Hubble's Distance - Redshift Relationastro.wku.edu › astr106 › Hubble_intro

https://www.e-education.psu.edu/astro801/content/l10_p3.html

A relation between distance and radial velocity among extra ...www.pnas.org › content

http://www.astro.wisc.edu/~heinzs/astro113/hubble.pdf

Kaiser, N. (1987). "Clustering in real space and in redshift space". *Monthly Notices of the Royal Astronomical Society*. 227 (1): 1–21. Bibcode:1987MNRAS.227....1K. doi:10.1093/mnras/227.1.1

Girardi, M.; Biviano, A.; Giuricin, G.; Mardirossian, F.; Mezzetti, M. (1993). "Velocity dispersions in galaxy clusters". *The Astrophysical Journal*. 404: 38–50. Bibcode:1993ApJ...404...38G. doi:10.1086/172256

Kaiser, N. (1987). "Clustering in real space and in redshift space". *Monthly Notices of the Royal Astronomical Society*. 227 (1): 1–21. Bibcode:1987MNRAS.227....1K. doi:10.1093/mnras/227.1.1

http://astronomy.nmsu.edu/nicole/teaching/astr505/lectures/lecture26/slide01.html

https://lco.global/spacebook/galaxy-classification/

https://depts.washington.edu/astroed/HubbleLaw/galaxies.html

http://astro.berkeley.edu/~mwhite/darkmatter/hubble.html

http://docplayer.net/43464058-Astronomy-1-introductory-astronomy-spring-2014.html

https://physics.fullerton.edu/files/Labs/Astr%20101L/HubbleLab

"Peculiar Velocity | COSMOS." *Peculiar Velocity*, Swinburne University of Technology, astronomy.swin.edu.au/cosmos/P/Peculiar+Velocity

http://docplayer.net/43464058-Astronomy-1-introductory-astronomy-spring-2014.html

http://www.gsgis.k12.va.us/facultyweb/hhoughton/WebPages/AstroWeb/Galaxies%20and%20Cosmology/+Lab-Hubble.doc

http://www.astro.wisc.edu/~heinzs/astro113/hubble.pdf

https://physics.fullerton.edu/files/Labs/Astr%20101L/HubbleLab(1).pdf

http://www.gsgis.k12.va.us/facultyweb/hhoughton/WebPages/AstroWeb/Galaxies%20and%20Cosmology/+Lab-Hubble.doc

http://www.astro.wisc.edu/~heinzs/astro113/hubble.pdf

https://hubblesite.org/contents/articles/dark-energy

https://mafiadoc.com/hubbles-law-and-the-expanding-universe-pnas_5c54d448097c472b208b45a5.html

https://www.e-education.psu.edu/astro801/book/export/html/1967