PHYS 3142 Spring 2019
Computational Methods in Physics
Assignment 2
Due: 27th Feb 2019

Before you submit your assignment, do remember:

1. the due day
2. submit a report which contains your figures and results along with your code
3. make sure your code can run
4. do not upload a compressed file(e.g. rar, zip, ...)

1. Implement Numerical Integral Algorithms
Calculate the following integral numerically:

$$I = \int_0^1 (\sin(x) + \frac{\cos(x) - 1}{\sqrt{x}})\, \mathrm{dx}$$

using 5 different methods. Including:

1. **Rectangle′s rule**
2. **Adaptive Trapezoidal**
3. **Adaptive Simpson**
4. **Romberg**
5. **Gaussian Quadrature**

And the requirement is following:

- For **Adaptive Trapezoidal** and **Adaptive Simpson**, you can estimate the error in the iteration. For other methods, you can compare with the accurate value you get from other method to calculate the error.
- Achieve the accuracy of 10^{-8}, i.e. the error is at the order of 10^{-8}. See how many slices you used for each method.

- Make a plot like the one in Lec 5(e.g. page 18. the integral value v.s. number of slices used) and see how the numerical integral converge.

2. Non-linear Oscillator
Calculate the period of a non-linear oscillator described by:

$$\frac{d^2\theta}{dt^2} = -\sin(\theta)$$

by numerically integrating:

$$\sqrt{8}\int_0^{\theta_m} \frac{d\theta}{\sqrt{\cos(\theta) - \cos(\theta_m)}}$$

for several values of the maximum angle θ_m, using the trapezoidal rule, Simpson's rule and the Gaussian quadrature. Plot how the period change with the maximum θ.

HINTS

- For the first question, you may separate the function into two parts and calculate one of them analytically. e.g.:

  $$f(x) = \sin(x) - \frac{1}{\sqrt{x}} \quad and \quad g(x) = \frac{\cos(x)}{\sqrt{x}}$$

  $f(x)$ can be calculated analytically and $g(x)$ can be calculated numerically.

- When encounters singularities of function, you can change the integral limit a little bit to avoid divergence of integral(e.g. minus a very small number).

- You can find a file named "gaussxw.py" on Canvas which you will use when you do Gaussian quadrature.

- Use integrate functions from "Scipy" to check your results.