PHYS 3142 Spring 2019

Computational Methods in Physics

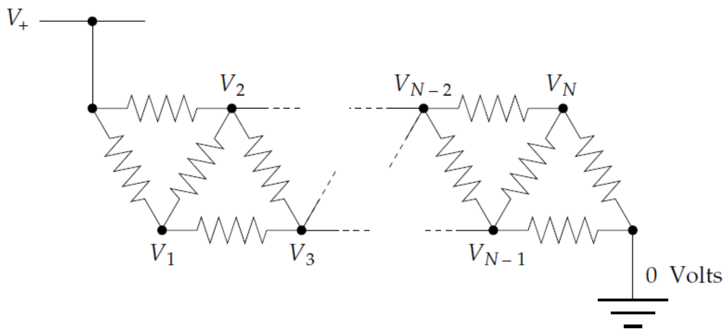
Assignment 3

Due: 18 Mar 2019

Before you submit your assignment, do remember:

1. the due day
2. submit a report which contains your figures and results along with your code
3. make sure your code can run
4. do not upload a compressed file(e.g. rar, zip, . . .)

1. Consider a long chain of resistors wired up like this:
All the resistors have the same resistance R. The power rail at the top is at voltage $V_+ = 5$V.
The problem is to find the voltages $V_1 \ldots V_N$ at the internal points in the circuit.



(a) Using Ohm's law and the Kirchhoff current law, which says that the total net current flow out of (or into) any junction in a circuit must be zero, show that when N = 3, the voltages V_1, V_2, V_3 satisfy the equations

$$3V_1 - V_2 - V_3 = V_+$$
$$4V_2 - V_1 - V_3 = V_+$$
$$3V_3 - V_1 - V_2 = 0$$

Express these equations in vector form $\boldsymbol{A}\boldsymbol{v} = \boldsymbol{w}$ and find the values of the matrix $\boldsymbol{A}$ and the vector $\boldsymbol{w}$. Factorize the matrix A analytically using the LU decomposition. Then solve the simultaneous equations by either using the Gaussian elimination or LU decomposition. Also write a simple program and use numpy.linalg to check your result.
(Answer: V_1=3.125V, V_2=2.5V, V_3=1.875V)

(b) If there are N internal junctions, show that the voltages $V_1 \ldots V_N$ satisfy the equations:

$3V_1 - V_2 - V_3 = V_+$,

$-V_1 + 4V_2 - V_3 - V_4 = V_+$,

$-V_1 - V_2 + 4V_3 - V_4 - V_5 = 0$,

...

$-V_{i-2} - V_{i-1} + 4V_i - V_{i+1} - V_{i+2} = 0$,

...

$-V_{N-3} - V_{N-2} + 4V_{N-1} - V_N = 0$,

$-V_{N-2} - V_{N-1} + 3V_N = 0$.

Express these equations in vector form $\boldsymbol{A}\boldsymbol{v} = \boldsymbol{w}$ and find the values of the matrix $\boldsymbol{A}$ and the vector $\boldsymbol{w}$.

(c) Write a program to use solve in numpy.linalg to solve for the values of the V_i when there are $N = 6$ internal junctions with unknown voltages. (Hint: All the values of V_i should lie between zero and 5V. If they don't, something is wrong.) (Answer: V_1 = 3.7254902 V, V_2 = 3.43137255 V, V_3 = 2.74509804 V, V_4 = 2.25490196 V, V_5 = 1.56862745 V, V_6 = 1.2745098 V)

(d) Now repeat your calculation for the case where there are N = 10 000 internal junctions.

This part is not possible using standard tools like the solve function. You need to make use of the fact that the matrix $\boldsymbol{A}$ is banded and use the banded function from the file banded.py, which can be downloaded from the link:

http://www-personal.umich.edu/~mejn/cp/programs/banded.py

Hint: By writing the matrix for N=6, you can easily see that on each non-vanishing (sub-)diagonal, most of the elements are identical. And you should use this fact to construct the rectangular matrix used in banded.py Please also refer to the lecture note (linear algebra.pdf) we have discussed in the class.

(Answer: V_1 = 4.99888… V, V_2 = 4.99861… V, V_3 = 4.99802… V ..., V_{9998} = 0.00197… V, V_{9999} = 0.00138… V, V_{10000} = 0.00111… V)