PHYS 3142 Sping 2019
Computational Methods in Physics
Assignment 7
Due: 7th May 2019

Before you submit your assignment, do remember:

1. the due day

2. submit a report which contains your figures and results

3. make sure your code can run

4. do not upload a compressed file(e.g. rar, zip, . . .)

1. Monte Carlo method for integral Weierstrass function is an example of a pathological real-valued function on the real line. The function has the property of being continuous everywhere but differentiable nowhere. It is named after its discoverer Karl Weierstrass. In Weierstrass's original paper, the function was defined as a Fourier series:

$$f(x) = \sum_{n=0}^{\infty} a^n cos(b^n \pi x)$$

where $0 < a < 1$, b is a positive odd integral, and $ab > 1 + \frac{3}{2}\pi$

1. In practice, the higher of the Fourier series' order, the more accurate result of $f(x)$ we get. And when we consider the accuracy at a specific region of x, the definition of the avarage error as the following :

$$\epsilon = \frac{1}{N_{num}} \sum_{i}^{N_{num}} (f^{n+1}(x_i) - f^n(x_i))^2$$

where N_{num} is the total number of x we calculate, $f^n(x)$ is the value of $f(x)$ with the order of n. Please write a program to plot this function with the parameters $a = 0.5; b = 13; x \in [0, 3.3]; N_{num} = 10^3$. And find the proper order (n) where the avarage error is smaller then 10^{-8}. Please plot the relationship of $(n$ and $avarage$ $error(\epsilon))$ and $(x$ and $f(x))$ with the proper n.

2. With the proper n in the last question, please first write a program to calculate the integral of Weierstrass function$(x \in [0, 3.3])$ by using the Monte Carlo method and find the proper number of samples and total points in every sample with the accuracy of 10^{-4}. (Here, you may use the results from analytic integral and if 10^{-4} is too time comsuming 10^{-3} is also ok)

2. Buffon's needle for Pi Given a needle of length l dropped on a plane ruled with parallel lines t units apart. Here $t > l$ and we count the times(m) with the needle crossing the parallel lines. Please choose the proper total number (M) for every sampling and write a program to calculate the value of Pi with accuracy 10^{-4}.