Calculating Latency in Satellite Communications

Research Question - To What Extent can Mathematics(Vectors and Trigonometry) be used to calculate the latency in communication between a ground station and a satellite, when given the satellite's NORAD data and the Ground Station's latitude and longitude?

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Subject - Mathematics

Contents

1 Introduction

Determining the latency in communication between a ground station on Earth and a satellite signifies calculating the time taken to send a signal from the ground station to the satellite and then receive the signal back. A ground station facilitates the flow of information that enables satellite data acquisition, control, and the transmission of valuable information [17]. Calculating the latency in communication between a ground station and a satellite is important during the real-time operation of the satellite, especially during orbital maneuvers for collision avoidance. It also ensures that customers are aware of the lag time before purchasing satellite-based internet. [18].

Previous research done concerning this has either calculated the range of the satellite's orbit where it can be communicated with [3], calculated the latency in communication between satellites in orbit [6], or optimized a satellite constellation to minimize latency using genetic algorithms [22]. I will instead be exploring a method to calculate the latency in communication between a singular ground station and satellite based on the satellite's orbital data, the ground station's latitude and longitude, and a specific time at which the latency is being measured.

To do this, the methodology used first entails representing the moving ground station and satellite as vector-valued functions that begin from the center of the earth through trigonometry, the properties of an ellipse, and a coordinate transformation. These two vector functions are then used to determine the vector-valued function that connects the ground station to the satellite. Calculating the position of the satellite in relation to time is then calculated through Kepler's equation and laws of planetary motion [2]. Finally, this method is applied to a real-world example and its accuracy is evaluated.

2 Background Information

For our mathematical approach to calculating latency to be practical, we must ensure that the method we derive must be able to be applied to real-life satellites. NORAD (North American Aerospace Defense Command) is a combined organization of the United States and Canada that is responsible for the aerospace defense of North America [1]. NORAD provides Two-Line Element Sets (TLEs), a data format that reveals the orbital data for satellites. An example TLEs is shown below [4].

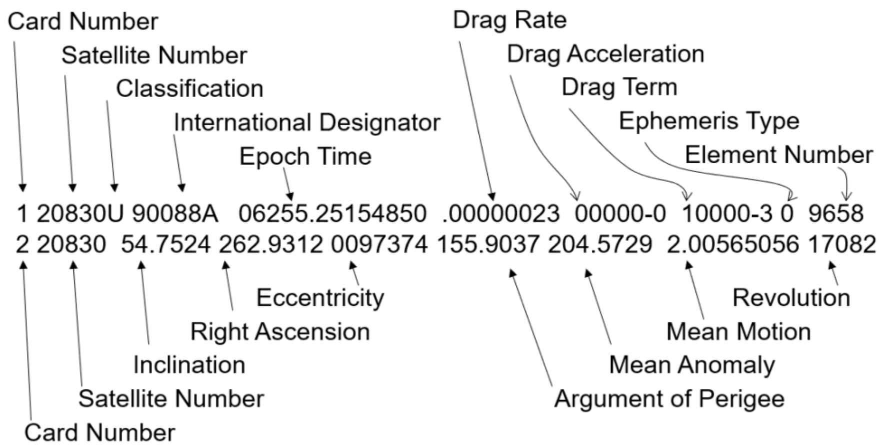

Figure 1: NORAD data

This image was taken from [10].

We will take and define whatever orbital elements are required for the calculation from the TLEs.

3 Vector Equation

The distance between a ground station and a satellite is constantly changing as time passes since the ground station moves with the earth's rotation and the satellite orbits around earth. To take account of this we can represent the distance between the ground station and the satellite as a vector-valued function. A vector-valued function is an expression of the form $\langle f(t), h(t), g(t) \rangle$ and is a function from the real numbers $\mathbb{R}$ to the set of all 3-dimensional vectors [7]. The value of a Vector function changes as its parameter t(time) changes,.

We can represent the vector between a ground station and a satellite as shown below:

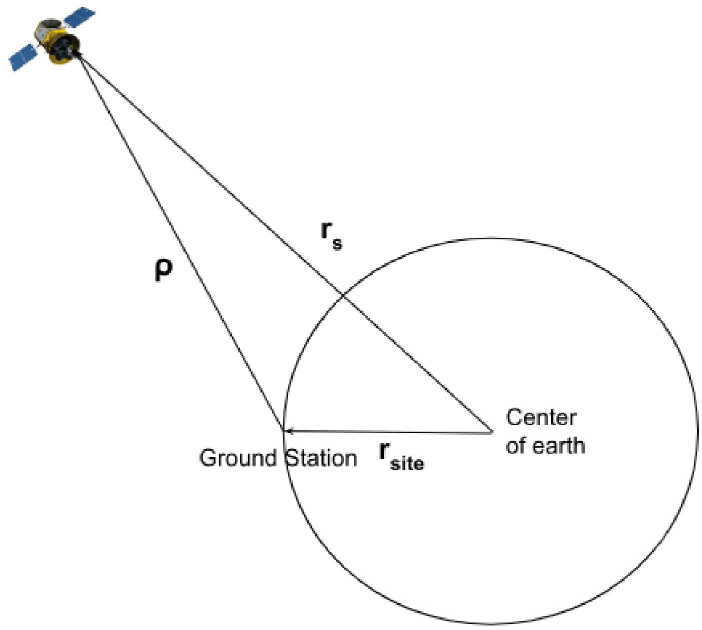

Figure 2: Vector-Valued functions of Satellite and Ground Station

We notice two other vectors in Figure 2. $\vec{r_s}$ and $\vec{r_{site}}$ are the vectors connecting the center of the Earth to the satellite and ground station respectively. Through the laws of vector addition [11]:

$$\vec{r_s} = \vec{r_{site}} + \vec{\rho} \tag{3.1}$$

$$\vec{\rho} = \vec{r_s} - \vec{r_{site}} \tag{3.2}$$

4 Ground station Vector-Valued function derivation

To represent the $\vec{r_{site}}$ vector, we place the earth in an IJK coordinate frame that stems from the center of the Earth and doesn't move with the earth's rotation as shown [15]:

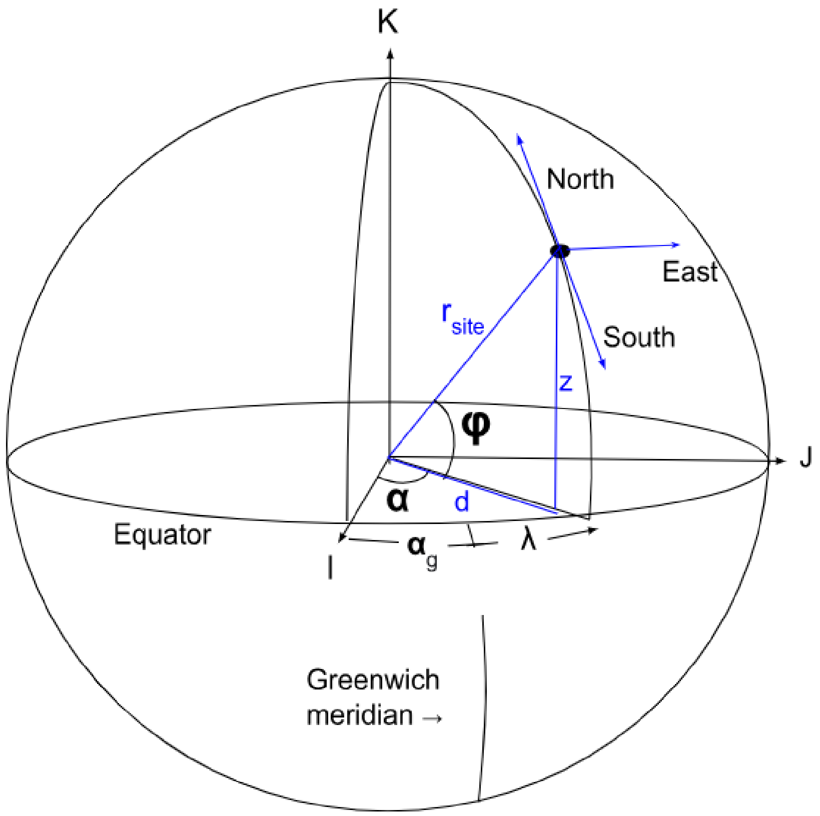

Figure 3: Vector-Valued function of Ground Station

In Figure 3, variables are first assigned to the station's latitude and longitude. Assuming the Earth to be a sphere on a three-dimensional plane, Longitude is defined as degrees east of the Greenwich Meridian and is represented by the symbol Lambda (λ). Similarly, Latitude is defined as degrees north of the equator and is represented by the symbol lowercase Phi (φ).

We notice however that with reference to the IJK coordinate frame, the Greenwich meridian is shifted by an angle of α_g from the I axis. The value of this angle is called the Greenwich Sidereal time and is fixed relative to the stars. It is quite difficult to compute it due to its variability created by the Precession of the Earth: a wobbling motion that the Earth undergoes because it is not a perfect sphere [2]. However, it is known that the Earth completes 1.0027379093 rotations relative to the stars during a day [16], so we can calculate the value of α_g based on an initial value we will call α_{gi} and the number of days passed since α_{gi} which we will label as D, through the formula:

$$\alpha_g = \alpha_{gi} + (360^o \times (1.0027379093 \times D) \tag{4.1}$$

We will obtain our value of α_{gi} from *The Astronomical Almanac* [16], which stores Greenwich Sidereal time values for each day.

Then, as seen in Figure 3, α can be represented as:

$$\alpha = \alpha_g + \lambda \tag{4.2}$$

Now to derive the vector equation, we notice that in Figure 3 the angles (α) and (φ) create two right-angled-triangles. Utilizing the principles of trigonometry, we are able to represent $r_{\vec{site}}$ as:

$$r_{\vec{site}} = d\cos(\alpha)\vec{\mathbf{I}} + d\sin(\alpha)\vec{\mathbf{J}} + z\vec{\mathbf{K}} \tag{4.3}$$

$$d = r_{site}\cos(\varphi) \tag{4.4}$$

$$z = r_{site}\sin(\varphi) \tag{4.5}$$

Through the substitution of equations 4.4 and 4.5 into equation 4.3, $r_{\vec{site}}$ is defined as

$$r_{\vec{site}} = r_e\cos(\varphi)\cos(\alpha)\vec{i} + r_e\sin(\varphi)\sin(\alpha)\vec{j} + r_e\sin(\varphi)\vec{k} \tag{4.6}$$

5 Satellite Vector-Valued function derivation

5.1 Radius of an ellipse

Since we have derived a vector equation for $\vec{r_{site}}$, to solve for $\vec{\rho}$ we must derive a vector function for $\vec{r_s}$ as well. We notice however that it is hard to do this in the IJK frame due to a satellite's elliptical orbit. To get around this we will use the Perifocal Coordinate system, where the fundamental plane is the plane of the satellite's orbit and we will use the letters X, Y, and Z to depict the three axes.

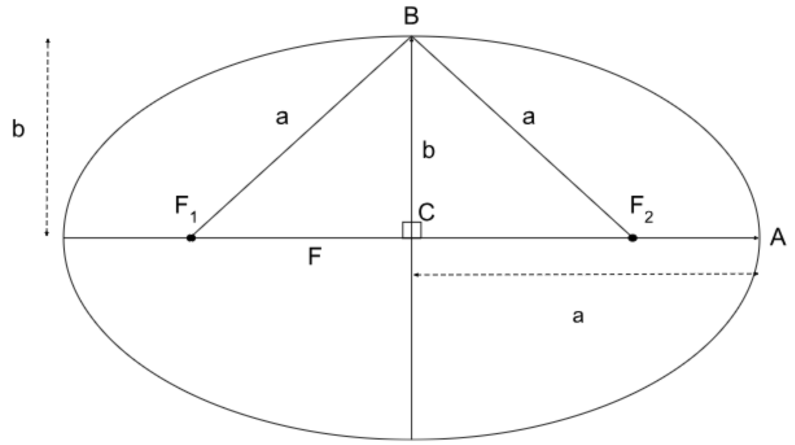

Figure 4: Properties of an ellipse

Before we derive an equation for the $\vec{r_s}$ vector, we must first establish some basic equations that arise through the geometry of an ellipse. By definition, an ellipse is a circle scaled in one direction and with semimajor axis **a** and semiminor axis **b** is defined by the equation [9]:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \tag{5.1}$$

Additionally, an ellipse can be defined as the set of points such that the sum of the distances from two fixed points is a constant length [9]. These fixed points are labeled F_1 and F_2 and are called the foci of the ellipse. The sum of the distances always adds up to $2a$ [9], so when the point is at the top of the semiminor axis **B**, two right-angled isosceles triangles are formed (F_1CB and F_2CB) in Figure 4. Using Pythagoras theorem we derive the following equation:

$$a^2 = F^2 + b^2 \tag{5.2}$$

Finally, a measurement known as the eccentricity of an ellipse is used to measure how far off-center the foci are from the center of the ellipse and is defined by the equation:

$$e = \frac{F}{a} \tag{5.3}$$

Now we have all the tools ready to derive an equation for the orbit of the satellite in the Perifocal System:

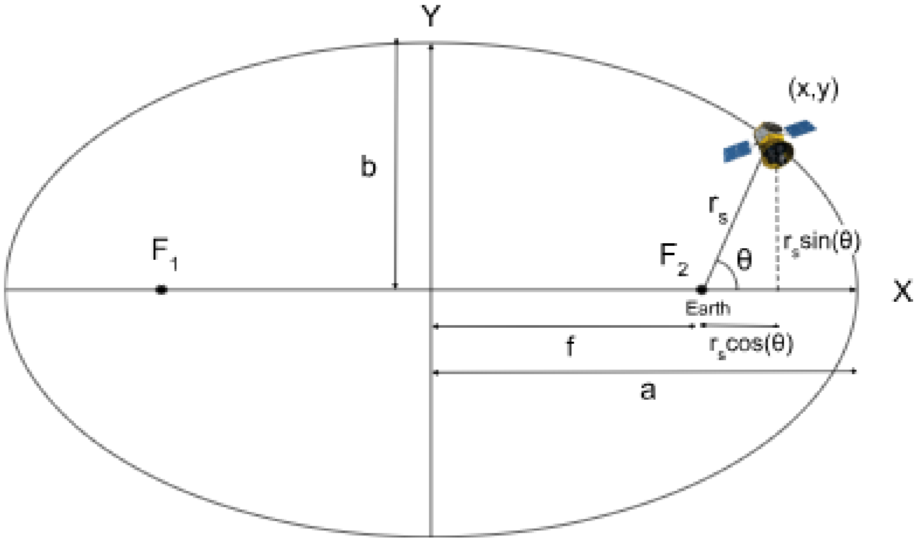

Figure 5: Satellite in Perifocal Coordinate System

In Figure 5, we can see a satellite following an elliptical orbit around Earth which is located at one of the Foci of the ellipse(F_2)[13]. We notice that $\vec{r_s}$ can be expressed as:

$$\vec{r_s} = r_s\cos(\theta)\vec{\mathbf{X}} + r_s\sin(\theta)\vec{\mathbf{Y}} + 0\vec{\mathbf{Z}} \tag{5.4}$$

Now to solve for r_s, we can see that the satellite is at a point on the ellipse labeled (x, y). x and y are defined as:

$$x = f + r_s\cos(\theta)$$

$$y = r_s\sin(\theta)$$

Plugging these equations into equation (5.1) we arrive at:

$$\frac{(f + r\cos(\theta))^2}{a^2} + \frac{(r\sin(\theta))^2}{b^2} = 1$$

Getting rid of the denominator by multiplying by a^2b^2:

$$b^2(f + r\cos(\theta))^2 + a^2((r\sin(\theta))^2) = a^2b^2$$

Simplifying:

$$b^2(f^2 + 2rf\cos(\theta) + r^2\cos^2(\theta)) + a^2(r^2\sin^2(\theta)) = a^2b^2$$

Since $\sin^2(\theta) + \cos^2(\theta) = 1$[11], we substitute $1 - \cos^2(\theta)$ for $\sin^2(\theta)$

$$b^2f^2 + 2b^2rf\cos(\theta) + b^2r^2\cos^2(\theta)) + a^2r^2(1 - \cos^2(\theta)) = a^2b^2$$

$$b^2r^2\cos^2(\theta)) - a^2r^2\cos^2(\theta) + 2b^2rf\cos(\theta) + a^2r^2 = a^2b^2 - b^2f^2$$

$$(b^2 - a^2)r^2\cos^2(\theta) + 2b^2rf\cos(\theta) + a^2r^2 = b^2(a^2 - f^2)$$

Substituting in equation 5.2

$$-f^2r^2\cos^2(\theta) + 2b^2rf\cos(\theta) + a^2r^2 = b^2(b^2)$$

$$a^2r^2 = b^4 - 2b^2rf\cos(\theta) + f^2r^2\cos^2(\theta)$$

$$a^2r^2 = (b^2 - rf\cos(\theta))^2$$

$$ar = b^2 - rf\cos(\theta)$$

$$r(a + f\cos(\theta)) = b^2$$

$$r = \frac{b^2}{a + f\cos(\theta)}$$

$$r = \frac{b^2}{a(1 + \frac{f}{a}\cos(\theta))}$$

Substituting in equation 5.2:

$$r = \frac{a^2 - f^2}{a(1 + \frac{f}{a}\cos(\theta))}$$

Substituting in equation 5.3:

$$r = \frac{a^2 - a^2e^2}{a(1 + e\cos(\theta))}$$

$$r = \frac{\cancel{a}(a - ae^2)}{\cancel{a}(1 + e\cos(\theta))}$$

$$r = \frac{a(1 - e^2)}{(1 + e\cos(\theta))} \tag{5.5}$$

Finally, we substitute this equation into equation 5.4:

$$\vec{r_s} = \frac{a(1 - e^2)}{(1 + e\cos(\theta))}\cos(\theta)\vec{\mathbf{X}} + \frac{a(1 - e^2)}{(1 + e\cos(\theta))}\sin(\theta)\vec{\mathbf{Y}} + 0\vec{\mathbf{Z}} \tag{5.6}$$

5.2 Coordinate transformation

In order to subtract $\vec{r_{site}}$ from $\vec{r_s}$ to solve for $\vec{\rho}$, both $\vec{r_{site}}$ and $\vec{r_s}$ need to be in the same coordinate system. We will be calculating $\vec{\rho}$ in the IJK coordinate system so we need to ensure that $\vec{r_s}$ is expressed in the IJK system and not the Perifocal system. This can be done through a coordinate transformation.

A vector may be expressed in any coordinate frame. A coordinate transformation simply changes the coordinate frame in which a vector is situated within without altering the vector's length and direction [2]. In order to figure out what the coordinate transformation between the Perifocal system and the IJK system is, we draw a diagram that relates the Perifocal and IJK coordinate system as shown below.

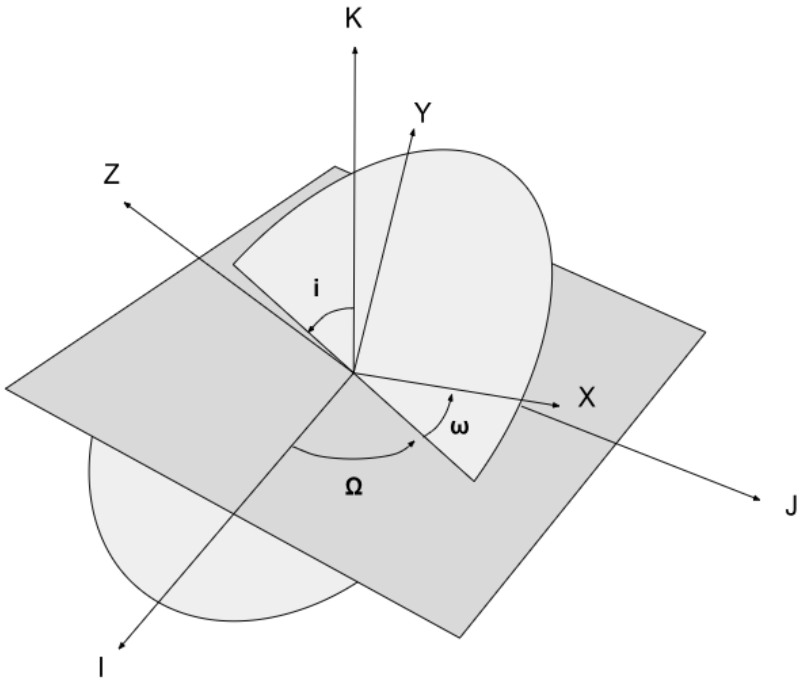

Figure 6: Perifocal and IJK system

The three angles depicted in Figure 6 are known as Euler angles and are the only 3 parameters required to describe the orientation of an orbit in 3 dimensions [8]. The first Euler angle Ω, is called the "Right Ascension" and is the angle between the positive I axis and the node line, where the node line is just the intersection between the orbital plane and the Earth's equatorial plane [8]. The second Euler angle i is labeled as the "inclination" of an orbit and is the smallest angle between the positive K and Z axis of the orbital plane. Finally, the third Euler angle ω is called the argument of perigee and is the angle between the ascending node line and the line of perigee (line connecting the center of the orbital plane to the nearest point on the orbit) and is in the direction of orbital motion [5]. However, since our focus is a coordinate transformation, we instead depict ω as the angle between the ascending node line and the X-axis.

Now to use these angles for the coordinate transformation. From Figure 6, we see that it is easier to visualize $IJK \rightarrow XYZ$ than $XYZ \rightarrow IJK$. We must perform 3 rotations to accomplish $IJK \rightarrow XYZ$:

1. Rotate Ω about the K axis

2. Rotate i about the I axis

3. Rotate ω about the K axis

To visualize how these 3 rotations will transform the IJK frame into the Perifocal, a diagram is shown below:

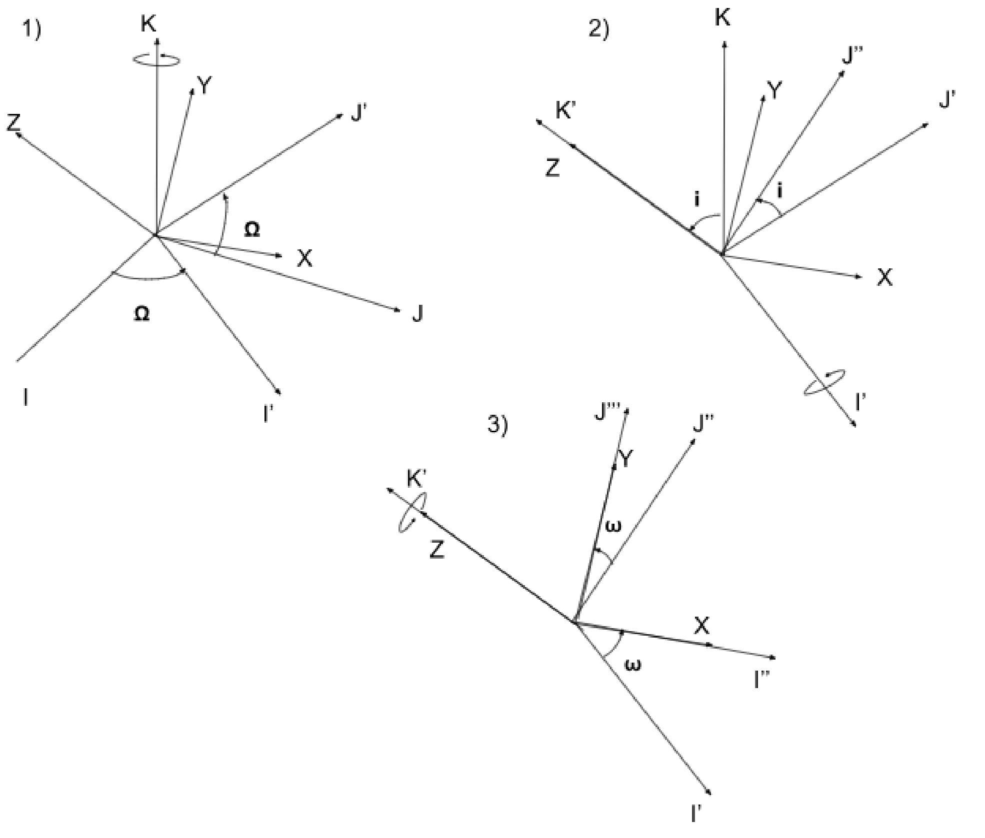

Figure 7: Rotations that transform IJK to Perifocal

We must now describe the 3 transformations through vector notation in order to figure out what the transformation matrices are. To be able to visualize them more accurately, we will look at diagrams of these transformations from a birds-eye view of the vector that is unchanged. For example, for the first rotation, we will look at the rotation through the K axis. All 3 of the rotations are shown in the diagram below:

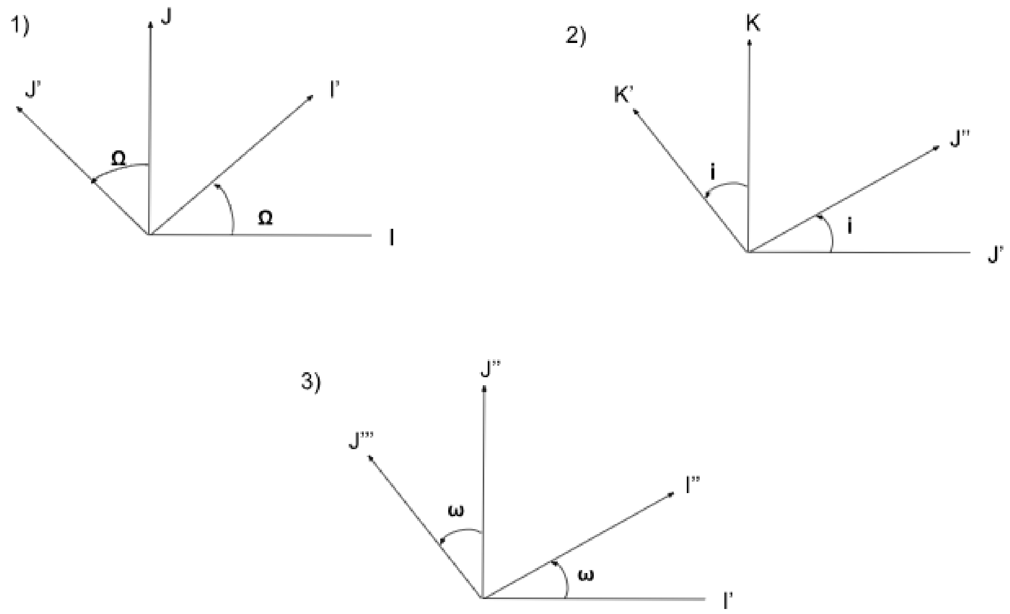

Figure 8: Birds eye view of rotations

If we now represent the rotated vectors in terms of the original vectors through trigonometry, we will derive transformation matrices that describe a coordinate transform from IJK to Perifocal. Our focus however is transforming Perifocal to IJK so we will represent our original vectors in terms of the rotated ones. For example for the first rotation:

$$I = \cos(\Omega)\mathbf{I'} - \sin(\Omega)\mathbf{J'} + 0\mathbf{K} \tag{5.7}$$

$$J = \sin(\Omega)\mathbf{I'} + \cos(\Omega)\mathbf{J'} + 0\mathbf{K} \tag{5.8}$$

$$K = 0\mathbf{I'} + 0\mathbf{J'} + 1\mathbf{K} \tag{5.9}$$

For the second rotation:

$$I' = 1\mathbf{I'} + 0\mathbf{J''} + 0\mathbf{K'} \tag{5.10}$$

$$J' = 0\mathbf{I'} + \cos(i)\mathbf{J''} - \sin(i)\mathbf{K'} \tag{5.11}$$

$$K = 0\mathbf{I'} + \sin(i)\mathbf{J''} + \cos(i)\mathbf{K'} \tag{5.12}$$

For the third rotation:

$$I' = \cos(\omega)\mathbf{I"} - \sin(\omega)\mathbf{J"'} + 0\mathbf{K'} \tag{5.13}$$

$$J'' = \sin(\omega)\mathbf{I"} + \cos(\omega)\mathbf{J"'} + 0\mathbf{K'} \tag{5.14}$$

$$K' = 0\mathbf{I"} + 0\mathbf{J"'} + 1\mathbf{K'} \tag{5.15}$$

We will label the transformation matrices $\mathbf{R}_x(y)$, where x represents the axis the rotation is about and y represents the angle the frame is rotated through. Using the coefficients of equations 5.7-5.15, we arrive at the following transformation matrices:

$$\mathbf{R}_K(\Omega) = \begin{bmatrix} \cos(\Omega) & -\sin(\Omega) & 0 \\ \sin(\Omega) & \cos(\Omega) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{R}_I(i) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(i) & -\sin(i) \\ 0 & \sin(i) & \cos(i) \end{bmatrix}$$

$$\mathbf{R}_K(\omega) = \begin{bmatrix} \cos(\omega) & -\sin(\omega) & 0 \\ \sin(\omega) & \cos(\omega) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Since $R_{XYZ \to IJK} = \mathbf{R}_K(\Omega)\mathbf{R}_I(i)\mathbf{R}_K(\omega)$

$$R_{XYZ \to IJK} = \begin{bmatrix} \cos(\Omega) & -\sin(\Omega) & 0 \\ \sin(\Omega) & \cos(\Omega) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(i) & -\sin(i) \\ 0 & \sin(i) & \cos(i) \end{bmatrix} \begin{bmatrix} \cos(\omega) & -\sin(\omega) & 0 \\ \sin(\omega) & \cos(\omega) & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{5.16}$$

In order to multiply a 3 X 3 matrix with another, we use the following formula [21]. This formula multiplies each row of the first matrix with each column of the second matrix. the rows and columns are labeled 1 to 3 from left to right and top to bottom.

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} j & k & l \\ m & n & o \\ p & q & r \end{bmatrix} = \begin{bmatrix} Row(1)Col(1) & Row(1)Col(2) & Row(1)Col(3) \\ Row(2)Col(1) & Row(2)Col(2) & Row(2)Col(3) \\ Row(3)Col(1) & Row(3)Col(2) & Row(3)Col(3) \end{bmatrix}$$

More specifically:

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} j & k & l \\ m & n & o \\ p & q & r \end{bmatrix} = \begin{bmatrix} (aj+bm+cp) & (ak+bn+cq) & (al+bo+cr) \\ (dj+em+fp) & (dk+en+fq) & (dl+eo+fr) \\ (gj+hm+ip) & (gk+hn+iq) & (gl+ho+ir) \end{bmatrix}$$

Applying this formula to equation 5.16, we multiply the first two rotation matrices as shown:

$$\begin{bmatrix} \cos(\Omega) & -\sin(\Omega) & 0 \\ \sin(\Omega) & \cos(\Omega) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(i) & -\sin(i) \\ 0 & \sin(i) & \cos(i) \end{bmatrix} =$$

$$\begin{bmatrix} (\cos(\Omega)(1) + -\sin(\Omega)(0) + (0)(0) & (\cos(\Omega)(0) + -\sin(\Omega)\cos(i) + (0)\cos(i)) & (\cos(\Omega)(0) + -\sin(\Omega)(-\sin(i)) + (0)\cos(i)) \\ (\sin(\Omega)(1) + \cos(\Omega)(0) + (0)(0) & (\sin(\Omega)(0) + \cos(\Omega)\cos(i) + (0)\sin(i)) & (\sin(\Omega)(0) + \cos(\Omega)(-\sin(i)) + (0)\cos(i)) \\ ((0)(1) + (0)(0) + (1)(0)) & ((0)(0) + (0)\cos(i) + (1)\sin(i)) & ((0)(0) + (0)(-\sin(i)) + (1)\cos(i)) \end{bmatrix} \tag{5.17}$$

$$= \begin{bmatrix} \cos(\Omega) & -\sin(\Omega)\cos(i) & \sin(\Omega)\sin(i)) \\ \sin(\Omega) & \cos(\Omega)\cos(i) & -\cos(\Omega)\sin(i) \\ 0 & \sin(i)) & \cos(i)) \end{bmatrix}$$

Now we must multiply this matrix with the third one in order to get our final transformation matrix:

$$R_{XYZ \to IJK} = \begin{bmatrix} \cos(\Omega) & -\sin(\Omega)\cos(i) & \sin(\Omega)\sin(i) \\ \sin(\Omega) & \cos(\Omega)\cos(i) & -\cos(\Omega)\sin(i) \\ 0 & \sin(i) & \cos(i)) \end{bmatrix} \begin{bmatrix} \cos(\omega) & -\sin(\omega) & 0 \\ \sin(\omega) & \cos(\omega) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_{XYZ \to IJK} = \begin{bmatrix} \cos(\Omega)\cos(\omega) - \sin(\Omega)\cos(i)\sin(\omega) & -\cos(\Omega)\sin(\omega) - \sin(\Omega)\cos(i)\cos(\omega) & \sin(\Omega)\sin(i) \\ \sin(\Omega)\cos(\omega) + \cos(\Omega)\cos(i)\sin(\omega) & -\sin(\Omega)\sin(\omega) + \cos(\Omega)\cos(i)\cos(\omega) & -\cos(\Omega)\sin(i) \\ \sin(i)\sin(\omega) & \sin(i)\cos(\omega) & \cos(i) \end{bmatrix} \tag{5.18}$$

Expressing $\vec{r_s}$ is just a matter of substitution now. we know that:

$$(\vec{r_s})_{IJK} = (R_{XYZ \to IJK})((\vec{r_s})_{XYZ})$$

Substituting equations 5.18 and 5.6:

$$
(\vec{r_s})_{IJK} = \begin{bmatrix} \cos(\Omega)\cos(\omega) - \sin(\Omega)\cos(i)\sin(\omega) & -\cos(\Omega)\sin(\omega) - \sin(\Omega)\cos(i)\cos(\omega) & \sin(\Omega)\sin(i) \\ \sin(\Omega)\cos(\omega) + \cos(\Omega)\cos(i)\sin(\omega) & -\sin(\Omega)\sin(\omega) + \cos(\Omega)\cos(i)\cos(\omega) & -\cos(\Omega)\sin(i) \\ \sin(i)\sin(\omega) & \sin(i)\cos(\omega) & \cos(i) \end{bmatrix} \begin{bmatrix} \frac{a(1-e^2)}{(1+e\cos(\theta))}\cos(\theta) \\ \frac{a(1-e^2)}{(1+e\cos(\theta))}\sin(\theta) \\ 0 \end{bmatrix} \tag{5.19}
$$

However, we still face one issue: we do not have a value for θ in equation 5.19. the angle θ is known as the true anomaly and changes as the satellite orbits around the Earth. We can see that from Figure 1, NORAD Data for satellites gives us the values for a satellite's Right Ascension, Inclination, and Argument of perigee (Ω, i, ω) respectively. The Data, however, does not give us a value for the true anomaly(θ), but it does give us a time-sensitive value called "Epoch time" which we can use to calculate the true anomaly.

6 Calculating a Satellite's true anomaly based on Epoch Time

The Epoch time is given in Figure 1 and is just the time at which all the other orbital elements in Figure 1 were recorded. It is given in UTC (Coordinated Universal Time), where the first two digits indicate the year, the next 3 digits indicate the day of the year (001 to 365), and the remaining digits after the decimal indicate the fraction of the day that has passed [1].

For example, the Epoch time given in Figure 1 is

$$06255.25154850$$

This means the epoch's year was "06" or 2006. The epoch's day is "255", so the 255th day of the year which is September 12th. Finally, "0.25154850" of the epoch's day has passed, which is $0.25154850 \times 24 = 6.037164$ hours or rather 6 hours and 2 seconds, giving us a final UTC time of 06:00:02 U.T.C. September 12, 2006.

We will calculate the time that has passed since the Epoch time and then use this to determine the satellite's change in position during this time, allowing us to determine the satellite's current true anomaly. However, predicting a satellite's change in position based on a specific input of time that has passed is quite tricky. It is called the Kepler Prediction problem and Kepler derived an equation known as "Kepler's equation" which provides us with a method to solve the problem.

In order to derive Kepler's equation, Kepler's second and third laws of planetary motion are used, so we must first understand and derive these laws.

6.1 Kepler's Second Law derivation

Kepler's second law states "an orbiting body sweeps out equal areas in equal intervals of time" [2]. This is depicted in the figure below.

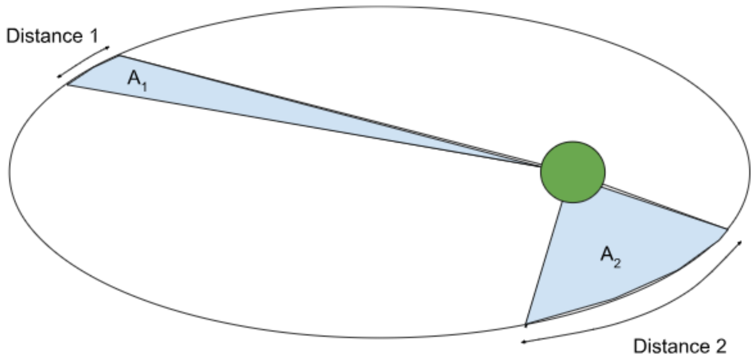

Figure 9: Kepler's Second Law

Kepler's second law is just another way of saying that if the time taken for an orbiting object to cross distance 1 is equal to the time it takes to cross distance 2, Area A_1 = Area A_2. A small proof is shown [20]:

Consider an extremely small wedge traced out by the satellite in time $d\theta$.

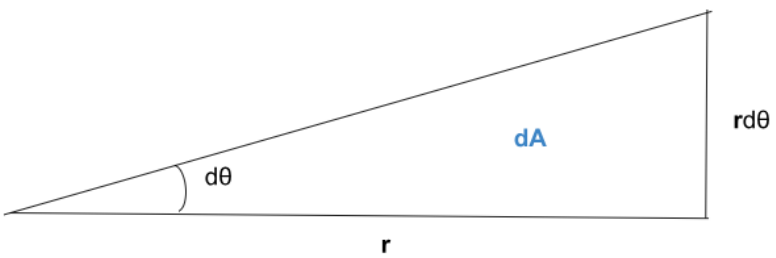

Figure 10: Small Wedge of Orbiting Body's Motion

The height of the triangle (distance object has traveled) is $r\theta$ since:

$$\tan(d\theta) = \frac{y}{r} \rightarrow y = r\tan(d\theta)$$

For very small angles $\tan(\theta) \approx \theta$ so

$$y = rd\theta \tag{6.1}$$

The Area is:

$$dA = \frac{1}{2}(r)(rd\theta) \tag{6.2}$$

If the body orbiting has a tangential velocity of v, we know that the distance traveled in this time will be equal to vdt. Equating this to equation 6.1:

$$vdt = rd\theta$$

Substituting into equation 6.2:

$$dA = \frac{1}{2}(r)(vdt) \tag{6.3}$$

Dividing by dt to find the rate or area swept by the orbiting body:

$$\frac{dA}{dt} = \frac{(r)(v\cancel{dt})}{2\cancel{dt}}$$

$$\frac{dA}{dt} = \frac{(r)(v)}{2} \tag{6.4}$$

To simplify this, we know that the angular momentum of a rotating object is known to be constant and defined by [20]:

$$L = mrv \tag{6.5}$$

Substituting this into 6.4:

$$\frac{dA}{dt} = \frac{L}{2m} \tag{6.6}$$

Since $\frac{L}{2m}$ is a constant, we have proved that the rate of area swept will always be constant meaning equal areas will be swept at equal times.

6.2 Kepler's Third Law derivation

Kepler's third law can be derived quite easily by equating Newton's law of gravitation to the formula for centripetal force [14]:

$$\frac{GMm}{r^2} = \frac{mv^2}{r} \tag{6.7}$$

$$\frac{GM\cancel{m}}{r^{\cancel{2}}} = \frac{\cancel{m}v^2}{\cancel{r}}$$

$$v^2 = \frac{GM}{r^2} \tag{6.8}$$

Since $v = \dfrac{2\pi r}{T}$ where T is the orbital period of the satellite:

$$\frac{GM}{r} = \frac{4\pi^2 r^2}{T^2} \rightarrow T = 2\pi\sqrt{\frac{r^3}{GM}}$$

So Kepler's third law states:

$$T = 2\pi\sqrt{\frac{r^3}{\mu}} \tag{6.9}$$

where $\mu = GM$, G is the gravitational constant 6.67420×10^{-11} and M is the mass of the heavier body.

6.3 Kepler's Equation derivation

Now back to the problem, since a satellite does not move at a constant speed, its true anomaly doesn't change at a constant rate making it hard to calculate for based on a change in time. To get around this, we inscribe the ellipse inside a circle with a radius equal to the ellipse's semimajor axis (a), to introduce a new angle (E) as shown below:

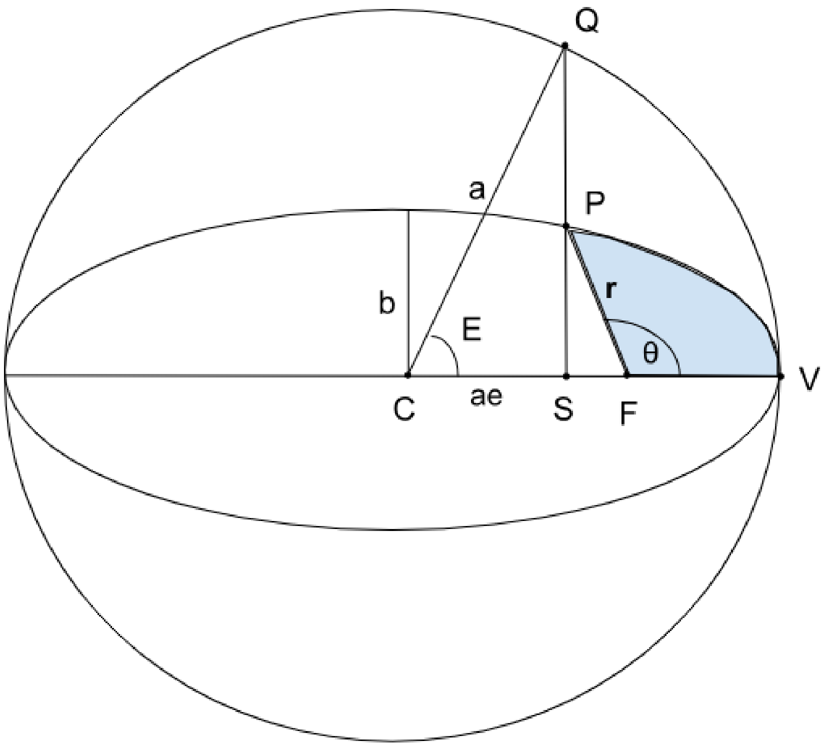

Figure 11: Satellite's Orbit inscribed inside a Circle

The new angle is known as the eccentric anomaly (E) and is just the angle between the ellipse's horizontal line and the line connecting the center of the ellipse (C) to the satellite's position projected onto the circumscribing circle (Q). Before proceeding further, we must derive some sort of relation between the circle and the ellipse. This is done using their functions below:

$$Ellipse: \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \tag{6.10}$$

$$Circle: \frac{x^2}{a^2} + \frac{y^2}{a^2} = 1 \tag{6.11}$$

$$Ellipse: y^2 = b^2 - \frac{x^2b^2}{a^2}$$

$$y^2a^2 = b^2a^2 - x^2b^2$$

$$y = \sqrt{\frac{b^2a^2 - x^2b^2}{a^2}}$$

$$y = \sqrt{\frac{b^2}{a^2}(a^2 - x^2)}$$

$$y = \frac{b}{a}\sqrt{a^2 - x^2} \tag{6.12}$$

$$Circle: y^2 = a^2 - x^2$$

$$y = \sqrt{a^2 - x^2} \tag{6.13}$$

Therefore:

$$\frac{y_{ellipse}}{y_{circle}} = \frac{b}{a} \tag{6.14}$$

We can use this ratio to derive a formula for the area of an ellipse. We have established that each coordinate on the ellipse is $\frac{b}{a}$ times the coordinate on the circumscribing circle. Therefore:

$$(Area)_{ellipse} = \frac{b}{a}(Area)_{circle}$$

$$(Area)_{ellipse} = \frac{b}{a}(\pi a^2)$$

$$(Area)_{ellipse} = \pi ab \tag{6.15}$$

Now back to the main derivation. From Kepler's second law, we can state that:

$$\frac{t_f - t_i}{A} = \frac{T}{\pi ab} \tag{6.16}$$

Where $t_f - t_i$ is the time taken for the orbiting object to get to point P, and T is the period (time taken to complete the whole orbit). We now solve for the unknown area A. From Figure 11 we can see that

$$A = PSV - PSF \tag{6.17}$$

We will solve for the area of PSF first, we express its area as:

$$PSF = \frac{1}{2}(SF)(PS) \tag{6.18}$$

Where:

$$SF = CF - CS \tag{6.19}$$

Through trigonometry and substitution of equation 5.3 ($e = \frac{F}{a}$)

$$SF = ae - a\cos(E) \tag{6.20}$$

Substituting this back into equation 6.18:

$$PSF = \frac{1}{2}(a(e - \cos(E)))(PS) \tag{6.21}$$

Now to solve for PS, from the ratio derived in equation 6.14 we can express PS as:

$$PS = \frac{b}{a}QS$$

$$PS = \frac{b}{\cancel{a}}\cancel{a}\sin(E)$$

$$PS = b\sin(E) \tag{6.22}$$

Substituting this into equation 6.21 and simplifying we get:

$$PSF = \frac{1}{2}a(e - \cos(E)(b\sin(E))$$

$$PSF = \frac{ab}{2}(e\sin(E) - \cos(E)\sin(E)) \tag{6.23}$$

Now we solve for PSV, again from the ratio derived in equation 6.14, we see that:

$$PSV = \frac{b}{a}QSV \tag{6.24}$$

Where:

$$QSV = QCV - QCS \tag{6.25}$$

Through the area of an arc formula of $A = \frac{1}{2}\theta r^2$ [11], assuming E is expressed in radians we can express QCV as:

$$QCV = \frac{1}{2}Ea^2 \tag{6.26}$$

Using trigonometry we can also express QCS as:

$$QCS = \frac{1}{2}(a\cos(E))(a\sin(E)) \tag{6.27}$$

Substituting these (6.26 and 6.27) into equation 6.25 and simplifying:

$$QSV = \frac{1}{2}Ea^2 - \frac{1}{2}(a\cos(E))(a\sin(E))$$

$$QSV = \frac{a^2}{2}(E - \cos(E)\sin(E)) \tag{6.28}$$

Substituting this into equation 6.24:

$$PSV = \frac{b}{\cancel{a}}\frac{a^{\cancel{2}}}{2}(E - \cos(E)\sin(E))$$

$$PSV = \frac{ab}{2}(E - \cos(E)\sin(E)) \tag{6.29}$$

Now that we have derived both QSV and PSV, we can substitute this back into equation 6.17 and express A as:

$$A = \frac{ab}{2}(E - \cos(E)\sin(E)) - \frac{ab}{2}(e\sin(E) - \cos(E)\sin(E)) \tag{6.30}$$

Simplifying this:

$$A = \frac{ab}{2}E - \cancel{\frac{ab}{2}\cos(E)\sin(E)} - \frac{ab}{2}e\sin(E) + \cancel{\frac{ab}{2}\cos(E)\sin(E)}$$

$$A = \frac{ab}{2}(E - e\sin(E)) \tag{6.31}$$

Plugging this back into our original equation 6.16 we get:

$$\frac{2(t_f - t_i)}{ab(E - e\sin(E))} = \frac{T}{\pi ab}$$

Simplifying this:

$$\frac{t_f - t_i}{\cancel{ab}(E - e\sin(E))} = \frac{T}{2\pi\cancel{ab}}$$

$$t_f - t_i = \frac{T}{2\pi}(E - e\sin(E))$$

Substituting equation 6.9 from Kepler's third law:

$$t_f - t_i = \frac{\cancel{2\pi}\sqrt{\frac{a^3}{\mu}}}{\cancel{2\pi}}(E - e\sin(E))$$

$$\sqrt{\frac{\mu}{a^3}}(t_f - t_i) = E - e\sin(E) \tag{6.32}$$

This is known as Kepler's equation and he made it simpler by introducing two new terms[2]:

$$Mean\ Motion : n = \sqrt{\frac{\mu}{a^3}} \tag{6.33}$$

$$Mean\ Anomaly : M = n(t_f - t_i) \tag{6.34}$$

Finally giving:

$$M = E - e\sin(E) \tag{6.35}$$

6.4 Method to calculate true anomaly

Since Mean motion is a constant for an orbiting object, we notice that an object's Mean Anomaly linearly changes as time progresses, making it much easier to calculate for instead of the irregularly changing Eccentric anomaly and true anomaly. Hence, we will calculate the Mean anomaly at a specific time and then use this to calculate the eccentric anomaly and true anomaly. This process is outlined below:

Let $t_f - t_i = \Delta t$ Since the Mean Anomaly updates linearly with time:

$$M_f = M_i + n\Delta t \tag{6.36}$$

We calculate the eccentric anomaly(E) at M_f through equation 6.35. As to a formula that calculates true anomaly from eccentric anomaly, rearranging equation 6.20:

$$a\cos(E) = ae - SF \tag{6.37}$$

Since $\angle PFS = \cos(180 - \theta)$, we use trigonometry to solve for SF in Figure 11:

$$SF = -\mathbf{r}\cos(\theta) \tag{6.38}$$

Substituting this into equation 6.37:

$$a\cos(E) = ae + \mathbf{r}\cos(\theta)$$

Plugging in equation 5.5 for **r**:

$$a\cos(E) = ae + \frac{a(1-e^2)\cos(\theta)}{1+e\cos(\theta)}$$

Simplifying:

$$a\cos(E) = \frac{ae(1+e\cos(\theta)) + a(1-e^2)\cos(\theta)}{1+e\cos(\theta)}$$

$$\cancel{a}\cos(E) = \frac{\cancel{a}(e(1+e\cos(\theta)) + (1-e^2)\cos(\theta))}{1+e\cos(\theta)}$$

$$\cos(E) = \frac{e + \cancel{e^2\cos(\theta)} + \cos(\theta) - \cancel{e^2\cos(\theta)}}{1+e\cos(\theta)}$$

$$\cos(E) = \frac{e+\cos(\theta)}{1+e\cos(\theta)} \tag{6.39}$$

Solving for $\cos(\theta)$ we obtain:

$$\cos(E)(1+e\cos(\theta)) = e + \cos(\theta)$$

$$\cos(E) + e\cos(E)\cos(\theta) - \cos(\theta) = e$$

$$\cos(E) + \cos(\theta)(e\cos(E) - 1) = e$$

$$\cos(\theta) = \frac{e - \cos(E)}{e\cos(E) - 1} \tag{6.40}$$

Now that we have all the required formulas, We can summarize calculating the true anomaly at a specific point in time as shown:

1. First obtain M_i and Δt from the NORAD data
2. Calculate M_f from M_i and Δt through the formula $M_f = M_i + n\Delta t - 2k\pi$
3. Calculate the Eccentric anomaly from M_f through the formula $M = E - e\sin(E)$
4. Finally calculate the true anomaly(θ) from E through the formula

$$\cos(\theta) = \frac{e - \cos(E)}{e\cos(E) - 1} \tag{6.41}$$

6.5 Approximating solution to Eccentric Anomaly

While trying to solve for E in step 3 of our method, we notice that we are unable to solve this equation simply through algebraic means. This equation is known as a **transcendental** equation and can be solved numerically through the Newton-Raphson method [8].

The Newton-Raphson method provides increasingly more accurate approximations of a function f(x) through an initial guess x_n and is represented mathematically as:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \tag{6.42}$$

The method can be understood geometrically through the following figure:

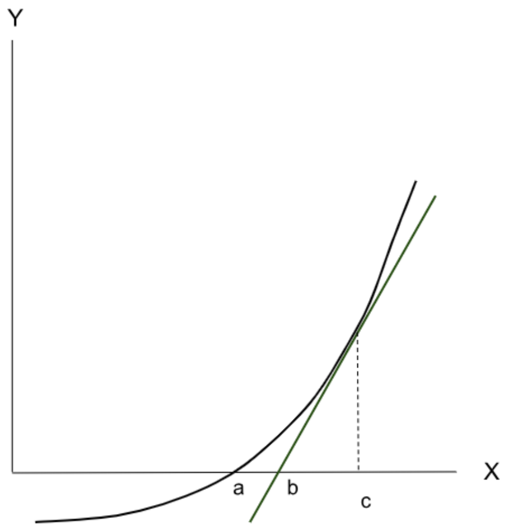

Figure 12: Newton-Raphson iterative method

We see that the curve f(x) has its root at a. In order to figure out the value of a, let c be the current estimate for a. We can define the tangent at the point $(c, f(c))$, using point-slope form [11], as:

$$y - f(c) = f'(c)(x - c) \tag{6.43}$$

Now to solve for b we plug in the point (b,0) into equation 6.43:

$$\begin{aligned} -f(x) &= f'(c)(b - c) \\ b &= c - \frac{f(x)}{f'(x)} \end{aligned} \tag{6.44}$$

We can see that b is a better approximation of a than c in Figure 12 so as we keep repeating this process, we approximate the root better. In our case, we move all the variables of equation 6.35 and define the function f(x) to be:

$$f(E) = E - e\sin(E) - M \tag{6.45}$$

Through differentiation:

$$f'(E) = 1 - e\cos(E) \tag{6.46}$$

By substituting equations 6.45 and 6.46 into equation 6.42, our formula that approximates E becomes:

$$E_{n+1} = E_n - \frac{E_n - e\sin(E_n) - M}{1 - e\cos(E_n)} \tag{6.47}$$

7 Application to Real-World Example

Now we can finally apply our derived formulas to a real-world example. We will be analyzing the latency in communication with the Hughesnet satellite *JUPITER 3*. Hughesnet is a US-based satellite internet provider that boasts an average latency of around 650 ms(milliseconds) [19]. The satellite it uses to provide internet is called *JUPITER 3* and has the following NORAD data [4]:

```
1 57479U 23108A   24020.93160126 -.00000146  00000-0  00000-0 0  9994
2 57479   0.0132  63.4038 0001971 255.8347  40.7219  1.00271748  1831
```

From Figure 1, we can determine the following orbital elements for JUPITER 3 through its NORAD data:

$$\text{Inclination}(i) = 0.0132$$

$$\text{Right Ascension}(\Omega) = 63.4038$$

$$\text{Argument of Perigee}(\omega) = 255.8347$$

$$\text{Mean Anomaly} = 40.7219$$

$$\text{Epoch time} = 24020.93160126$$

$$\text{Mean Motion} = 1.00271748 \text{ revolutions/day}$$

$$\text{Eccentricity}(e) = 0.0001971$$

$$\text{Semi-major axis}(a) = 42164km$$

We will define our ground station to be the HughesNet National Network Operation Center in Germantown, Maryland. This has latitude and longitude coordinates of 39.1732° N, and -77.2717° E respectively [19]. At the time of writing our current time is 18:00:00 U.T.C Jan 21, 2024, which is an Epoch time of 24021.25.

First, we determine the true anomaly using the algorithm 6.41:

1.

$$M_i = 40.7219$$

$$\text{Mean Motion}(n) = \frac{1.00271748}{86400} = 1.16 \times 10^{-5} \text{ revs/s}$$

$$\Delta t = 24021.25 - 24020.93160126 = 0.3184 = 7.6416 \text{ hours} = 27509.76 \text{ seconds}$$

2. Calculating M_f with equation 6.36

$$M_f = 40.7219 + (1.16 \times 10^{-5} \times 27509.76)$$

$$M_f = 41.04$$

3. Substituting into equation 6.35:

$$41.04 = E - 0.0001971 \sin(E)$$

Solving for E through equation 6.42. We will do 2 iterations and take our initial guess of E as M_f (41.04).

$$E_1 = 41.04$$

$$E_2 = 41.04 - \frac{41.04 - 0.0001971 \sin(41.04) - 41.04}{1 - 0.0001971 \cos(41.04)} = 41.0401$$

$$E_3 = 41.0401 - \frac{41.0401 - 0.0001971 \sin(41.0401) - 41.04}{1 - 0.0001971 \cos(41.0401)} = 41.0401$$

$$E = 41.0401$$

4. Calculating θ through equation 6.41:

$$\theta = \cos^{-1}(\frac{0.0001971 - \cos(41.0401)}{0.0001971 \cos(41.0401) - 1})$$

$$\theta = 41.048$$

We can see that due to the low eccentricity of the orbit, the ellipse closely resembles a circle making the three angles quite similar. Now that we have the value of θ, we substitute the orbital elements of JUPITER 3 into the transformation matrix $R_{XYZ \to IJK}$ (5.18):

$$\begin{bmatrix} \cos 63.4 \cos 255.8 - \sin 63.4 \cos 0.0132 \sin 255.8 & -\cos 63.4 \sin 255.8 - \sin 63.4 \cos 0.013 \cos 255.8 & \sin 63.4 \sin 0.013 \\ \sin 63.4 \cos 255.8 + \cos 63.4 \cos 0.013 \sin 255.8 & -\sin 63.4 \sin 255.8 + \cos 63.4 \cos 0.013 \cos 255.8 & -\cos 63.4 \sin 0.013 \\ \sin 0.013 \sin 255.8 & \sin 0.013 \cos 255.8 & \cos 0.013 \end{bmatrix} \tag{7.1}$$

Which simplifies to:

$$R_{XYZ\to IJK} = \begin{bmatrix} 0.757 & 0.653 & 0.000203 \\ -0.653 & 0.757 & -0.000102 \\ -0.00022 & -0.0000557 & 0.1 \end{bmatrix}$$

Plugging this and θ and the orbital elements into $\vec{r_s}$ (5.19):

$$(\vec{r_s})_{IJK} = \begin{bmatrix} 0.757 & 0.653 & 0.000203 \\ -0.653 & 0.757 & -0.000102 \\ -0.00022 & -0.0000557 & 0.1 \end{bmatrix} \begin{bmatrix} \frac{42164(1-(0.0001971)^2)}{(1+0.0001971\cos(41.048))}\cos(41.048) \\ \frac{42164(1-(0.0001971)^2)}{(1+0.0001971\cos(41.048))}\sin(41.048) \\ 0 \end{bmatrix}$$

$$(\vec{r_s})_{IJK} = \begin{bmatrix} 0.757 & 0.653 & 0.000203 \\ -0.653 & 0.757 & -0.000102 \\ -0.00022 & -0.0000557 & 0.1 \end{bmatrix} \begin{bmatrix} 31778.2 \\ 27684.61 \\ 0 \end{bmatrix}$$

Through matrix multiplication [21], $\vec{r_s}_{IJK}$ simplifies to:

$$(\vec{r_s})_{IJK} = \begin{bmatrix} (0.757\times 31778.2) + (0.653\times 27684.61) + (0.000203\times 0) \\ (-0.653\times 31778.2) + (0.757\times 27684.61) + (-0.000102\times 0) \\ (-0.00022\times 31778.2) + (-0.0000557\times 27684.61) + (0.1\times 0) \end{bmatrix}$$

$$\vec{(r_s)}_{IJK} = \begin{bmatrix} 42134.15 \\ 206.09 \\ -8.53 \end{bmatrix}$$

$$\vec{(r_s)}_{IJK} = 42134.15\vec{\mathbf{I}} + 206.09\vec{\mathbf{J}} - 8.53\vec{\mathbf{K}}$$

Now we find $r_{\vec{site}}$, first we obtain α_g from the *Astronomical Almanac* [16]. On Jan 21, 2019 α_g is listed as 8:00:16.7646 UTC. This translates to 0.3335 of the day so $\alpha_g = 0.3335 * 360 = 120.07^o$. 1,826 days have passed since then so we calculate the current α_g through equation 4.1.

$$\alpha_g = 120.07^o + (360^o \times (1.0027379093 \times 1826))$$

$$\alpha_g = 659279.6621^o$$

$\frac{659279.6621}{360}$ is 1831 so we subtract 360 times 1831 from α_g

$$\alpha_g = 659279.6621^o - (1831 \times 360) = 119.6621^o$$

Substituting this and our longitude into equation 4.2:

$$\alpha = 119.6621 + (-77.2717) = 42.3904^o$$

We substitute this into equation 4.6 and take the radius of the earth to be 6271 km [13].

$$\vec{r_{site}} = 6271\cos(39.1732)\cos(42.3904)\vec{\mathbf{I}} + 6271\sin(39.1732)\sin(42.3904)\vec{\mathbf{J}} + 6271\sin(39.1732)\vec{\mathbf{K}}$$

$$\vec{r_{site}} = 3590.57\vec{\mathbf{I}} + 2670.54\vec{\mathbf{J}} + 3961.18\vec{\mathbf{K}}$$

Finally, we subtract $\vec{r_{site}}$ from $\vec{r_s}$ to obtain the latency vector $\vec{\rho}$ (3.2):

$$\vec{\rho} = (42134.15 - 3590.57)\vec{\mathbf{I}} + (206.9 - 2670.54)\vec{\mathbf{J}} + (-8.53 - 3961.18)\vec{\mathbf{K}}$$

$$\vec{\rho} = 38543.58\vec{\mathbf{I}} - 2463.64\vec{\mathbf{J}} - 3969.71\vec{\mathbf{K}}$$

Using the magnitude of a vector formula [11]:

$$\rho = \sqrt{(38543.58)^2 + (-2463.64)^2 + (-3969.71)^2}$$

$$\rho = 38825.709 \text{ km} = 38825709 \text{ meters}$$

Therefore latency is:

$$t = \frac{38825709}{3 \times 10^8} = 0.1294 \text{ seconds} = 129.4 \text{ milliseconds}$$

Multiplying by 2 since the signal has to travel to the satellite and back:

$$t = 129.4 \times 2 = 258.8\ ms$$

8 Evaluation

The method outlined in this paper underestimated the real-time latency of communication by 391.2ms. This, however, does not mean that the method proposed was inaccurate, but rather limited by the number of variables it could take into play. For example, one limitation is that the earth was taken to be as spherical when in reality it is an oblate spheroid. Additionally, multiplying the latency by 2 to calculate the round-trip time was a generalization since the satellite and ground station would have moved in the 129.4 ms taken for the signal to reach the satellite, meaning the time taken for the signal to reach the ground station from the satellite would be different, albeit very minorly. The method also did not take into account the Signal-Processing time that the receivers and demodulators on the ground station carry out to access the signal's data which would greatly increase the latency [12]. Additionally, this research could be improved through access to more recent data, For example, I was only able to access the Astronomical Almanac 2019 edition instead of 2024, so my calculated value for α_g would have been slightly off. Another limitation is that the method's reliance on NORAD data meant that it only applies to satellites in the US and Canada and not to satellites whose orbital data is kept private.

9 Conclusion

The Investigation led in this paper successfully explored the research question by providing a method to calculate latency that while did not precisely reflect the real-time latency of satellite connection, provided us with a portion of the connection delay in the overall latency. Further extensions of this research could be using Matlab to simulate equation (5.19) for all 360^o of the true anomaly and locating the minimum latency value generated to determine the optimal time to communicate with a satellite. Additionally, applying this method to Elon Musk's new Starlink satellite constellation would be interesting as his satellites require precise synchronization and orbit much closer to Earth compared to the Hughesnet Satellite JUPITER 3.

10 Citations

References

[1] Defense Media Activity. *North American Aerospace Defense Command*. URL: https://www.norad.mil/. (accessed: 7.5.2023).

[2] Roger R. Bate and Jerry E. White. *Fundamentals of Astrodynamics*. Dover Books on Aeronautical Engineering. Dover Publications Inc., 1972. ISBN: 978-0486600611.

[3] Shkelzen Cakaj et al. "The Range and Horizon Plane Simulation for Ground Stations of Low Earth Orbiting (LEO) Satellites." In: *Int. J. Commun. Netw. Syst. Sci.* 4.9 (2011), pp. 585–589.

[4] Dr. T.S. Kelso CelesTrak. *NORAD GP Element Sets Current Data*. URL: https://celestrak.org/NORAD/elements/. (accessed: 7.5.2023).

[5] Edward P. Chatters et al. *AU-18 Space Primer*. Tech. rep. Air University Press, 2009, pp. 89–114. URL: http://www.jstor.org/stable/resrep13939.13 (visited on 07/16/2023).

[6] Aizaz U Chaudhry and Halim Yanikomeroglu. "When to crossover from earth to space for lower latency data communications?" In: *IEEE Transactions on Aerospace and Electronic Systems* 58.5 (2022), pp. 3962–3978.

[7] Whitman College. *Vector Functions*. URL: https://www.whitman.edu/mathematics/multivariable/multivariable_13_Vector_Functions.pdf. (accessed: 12.11.2023).

[8] Howard D.Curtis. *Orbital Mechanics for Engineering Students*. Aerospace Engineering. Butterworth-Heinemann Ltd, 2013. ISBN: 978-0080977478.

[9] Michael Fowler. *14. Mathematics for Orbits: Ellipses, Parabolas, Hyperbolas*. URL: https://galileoandeinstein.phys.virginia.edu/7010/CM_14_Math_for_Orbits.html#:~:text=The%20semi%2Dlatus%20rectum%2C%20as,(e2%E2%88%921).. (accessed: 9.5.2023).

[10] lynnane George. *Introduction to Orbital Mechanics*. Lyanne Groge books on Orbital mechanics. PressBooks, 2017.

[11] Michael Haese. *Mathematics: Analysis and Approaches HL*. IB HL Mathematics Textbooks. Haese Mathematics, 2019. ISBN: 9781925489590.

[12] Inmarsat. *Space Explained: What is a satellite ground station?* URL: https://www.inmarsat.com/en/insights/corporate/2023/space-explained-satellite-ground-station.html#:~:text=Once%20the%20downlink%20signal%20has,data%20encoded%20in%20the%20signal.. (accessed: 9.11.2023).

[13] Craig A. Kluever. *Encyclopedia of Physical Science and Technology, Third edition*. Encyclopedia of Physical Science and Technology. Robert A. Meyers, 2001. ISBN: 9780122274107.

[14] LumenLearning. *5.6: Kepler's Laws*. URL: https://phys.libretexts.org/Bookshelves/University_Physics/Physics_(Boundless)/5%3A_Uniform_Circular_Motion_and_Gravitation/5.6%3A_Keplers_Laws#:~:text=Kepler's%20third%20law%20can%20be,2a3GM.. (accessed: 11.8.2023).

[15] MatLab. *MathWorks Coordinate System Navigation*. URL: https://in.mathworks.com/help/aerotbx/ug/coordinate-systems-for-navigation.html. (accessed: 12.5.2023).

[16] Government Publishing Office. *Astronomical Almanac for the Year 2019*. U.S. Government Printing Office, 2018. ISBN: 9780707741925. URL: https://books.google.co.in/books?id=-mlZwQEACAAJ.

[17] Radio2Space. *How does a ground station for space communication work?* URL: https://www.radio2space.com/how-does-a-ground-station-for-space-communication-work/#:~:text=In%20addition%20to%20real%2Dtime,%2C%20scientific%20data%2C%20and%20imagery.. (accessed: 9.8.2023).

[18] TELESAT. *Real-Time Latency Rethink Possibilities with Remote Networks*. URL: https://www.telesat.com/wp-content/uploads/2022/11/Real-Time-Latency.pdf#:~:text=%E2%96%B2%20Latency%20is%20the%20time,GEO)%20used%20for%20satellite%20communications.. (accessed: 9.8.2023).

[19] American TV. *What Is HughesNet Latency?* URL: https://www.americantv.com/what-is-hughesnet-latency.php#:~:text=Latency%20isn't%20something%20most,'%20latency%20between%2020%2D80ms.. (accessed: 9.11.2023).

[20] Case Western Reserve University. *Keplers Laws*. URL: http://burro.case.edu/Academics/Astr221/Gravity/kep2rev.htm. (accessed: 11.8.2023).

[21] Unknown. *Vedantu*. URL: https://www.cuemath.com/algebra/multiplication-of-matrices/. (accessed: 12.11.2023).

[22] Peng Zong and Saeid Kohani. "Optimal satellite LEO constellation design based on global coverage in one revisit time". In: *International Journal of Aerospace Engineering* 2019 (2019), pp. 1–12.