Analysis of Data Collected
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The Game Playing Bots
RandomAI: This bot randomly chooses a move.
VectorAI: This bot optimizes for free moves, then for captures. If there are no free moves or captures, then it randomly chooses a move.
LeftmostAI: This bot always chooses the leftmost legal move.
RightmostAI: This bot always chooses the rightmost legal move.

Data Collection
For each of the below matches data was collected over 2,000 games, except for the matches when Player 1 and Player 2 are the same, then only 1,000 games are played. 1,000 games were played and then the players were switched so that we could see if there was an advantage to being Player 1 or Player 2 for each of the strategy pairings named below.

RandomAI vs RandomAI
VectorAI vs RandomAI
VectorAI vs VectorAI
LeftmostAI vs RandomAI
LeftmostAI vs VectorAI
LeftmostAI vs LeftmostAI
RightmostAI vs RandomAI
RightmostAI vs VectorAI
RightmostAI vs LeftmostAI
RightmostAI vs RightmostAI

SAS 9.4 Enterprise Guide was used to generate the tables and graphs. Any interesting observations about the data collected or further experiments conducted are described at the beginning of each section.

Contents

Analysis of Player Number Compared to Game Winner

Overall Statistics

Over the 16,000 games played, Player 1 wins more often than Player 2 at a rate of 53%. Player 2 wins 39% of the time, and 8% of the time there is a tie.

Game Winner				
Winner	Frequency	Percent	Cumulative Frequency	Cumulative Percent
Tie	1248	7.80	1248	7.80
Player 1	8548	53.43	9796	61.23
Player 2	6204	38.78	16000	100.00
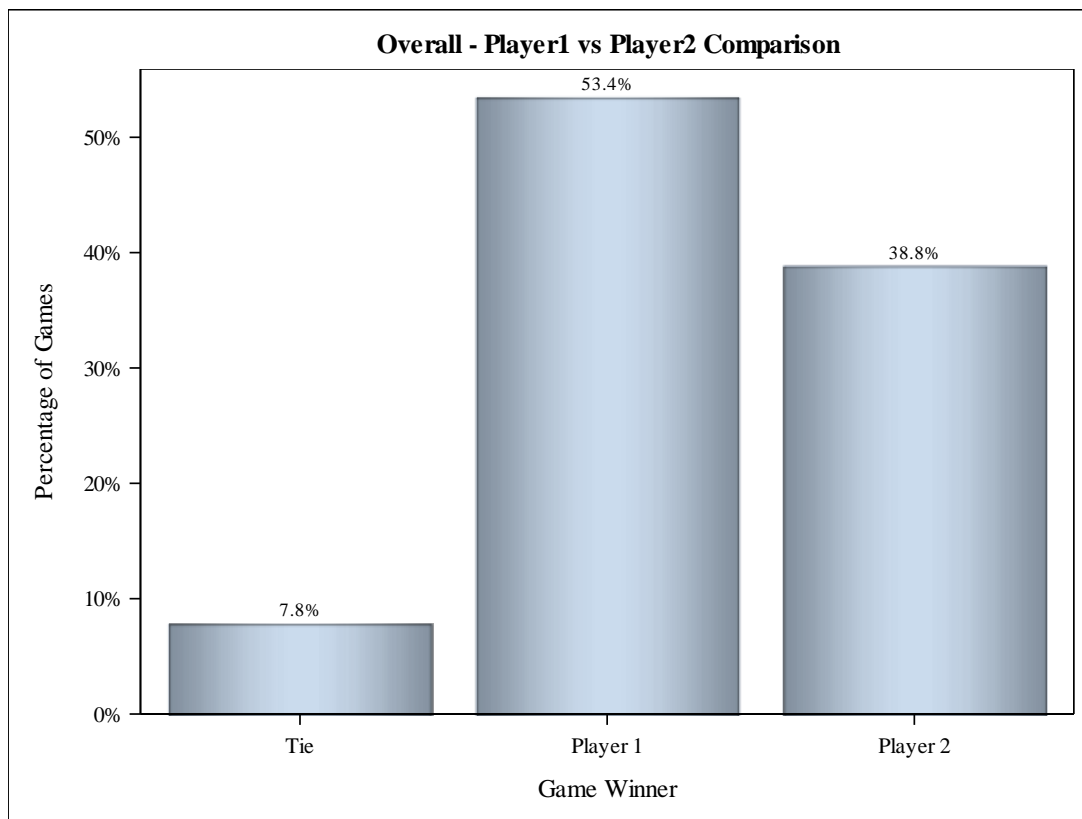
RandomAI vs RandomAI Statistics

The RandomAI are as expected. The player that starts does not hold a significant advantage.

Table of Match by Game Winner			
Match	Game Winner	Frequency	Percent
RandomAI vs RandomAI	Tie	68	6.80
	Player 1	504	50.40
	Player 2	428	42.80
	Total	1000	100.00
Total	Tie	68	6.80
	Player 1	504	50.40
	Player 2	428	42.80
	Total	1000	100.00

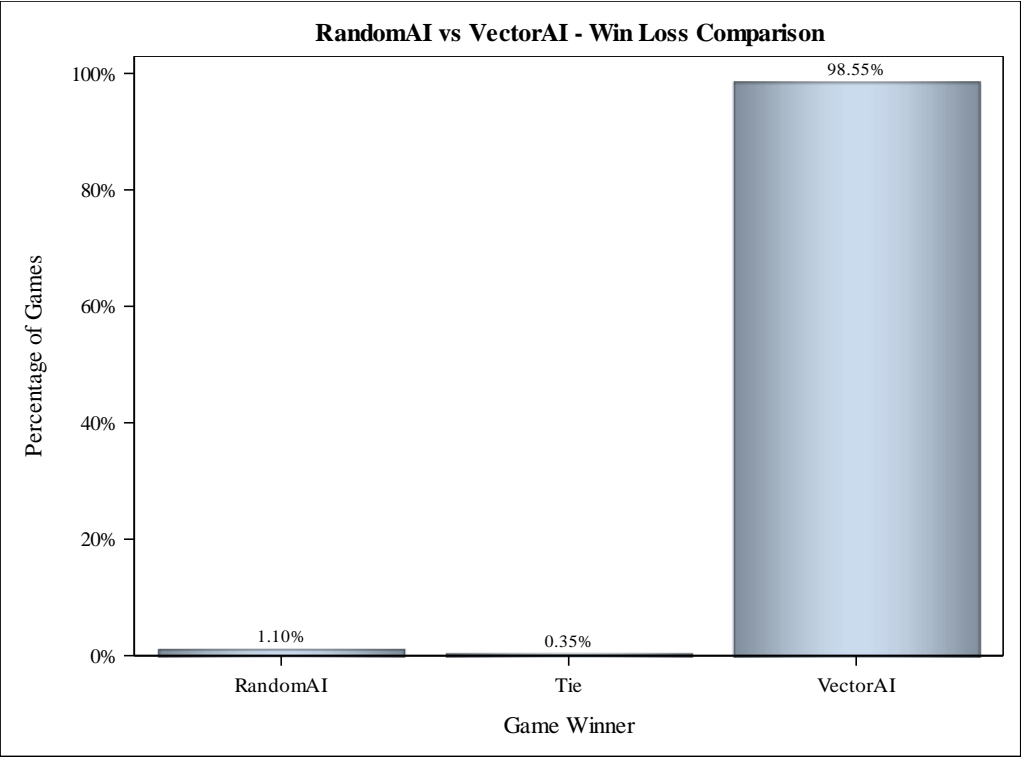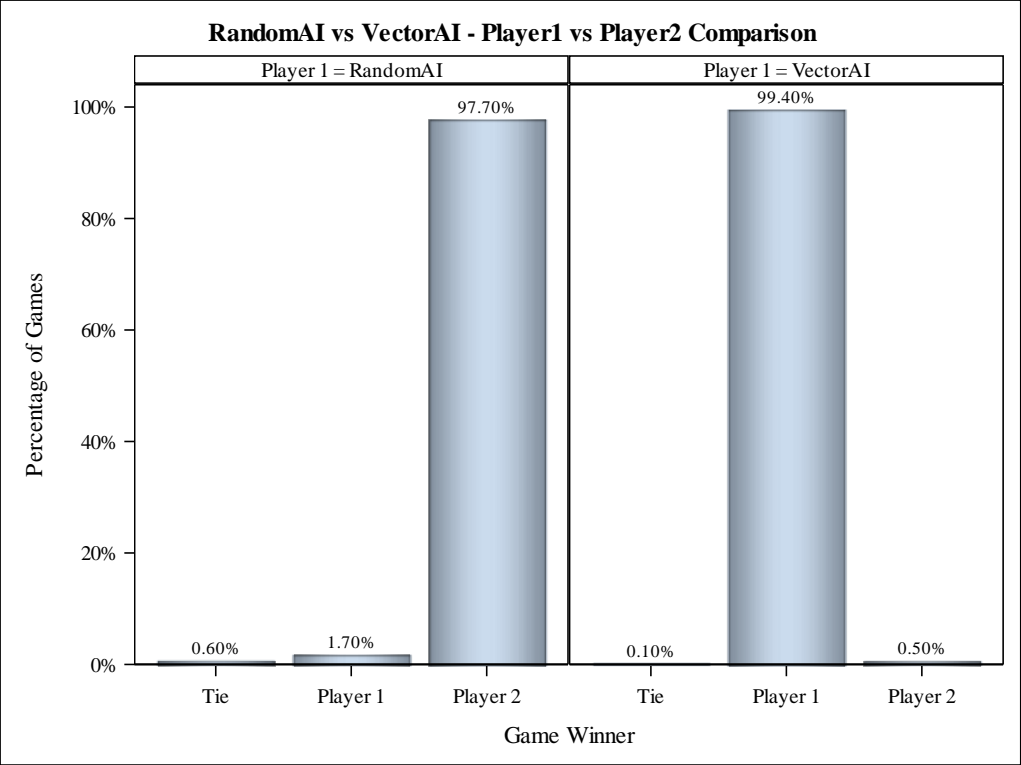RandomAI - Player1 vs Player2 Comparison
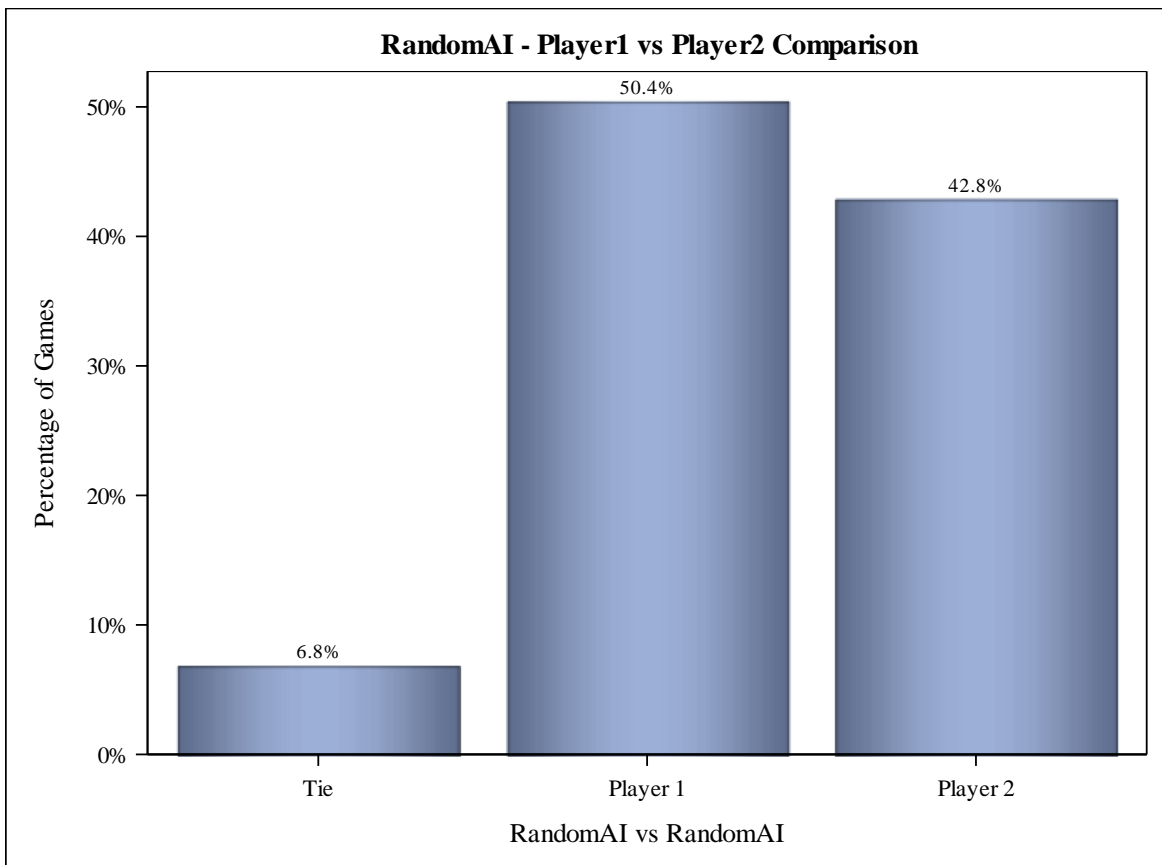
RandomAI vs VectorAI Statistics

The VectorAI almost always wins, regardless of which bot played first. This shows that optimizing for free turns and captures holds a significant advantage over choosing randomly. Interestingly, even though both are unlikely, a tie is more likely if the RandomAI plays first, with a percentage of 0.6% compared to 0.1% when the VectorAI plays first.

Table of Match by Game Winner					
Match	Game Winner	Frequency	Percent	Row Percent	Column Percent
RandomAI vs VectorAI	Tie	6	0.30	0.60	85.71
	Player 1	17	0.85	1.70	1.68
	Player 2	977	48.85	97.70	99.49
	Total	1000	50.00	100.00	
VectorAI vs RandomAI	Tie	1	0.05	0.10	14.29
	Player 1	994	49.70	99.40	98.32
	Player 2	5	0.25	0.50	0.51
	Total	1000	50.00	100.00	
Total	Tie	7	0.35		100.00
	Player 1	1011	50.55		100.00
	Player 2	982	49.10		100.00
	Total	2000	100.00		

Table of Win Percentages				
Winner	Frequency	Percent	Cumulative Frequency	Cumulative Percent
RandomAI	22	1.10	22	1.10
Tie	7	0.35	29	1.45
VectorAI	1971	98.55	2000	100.00

RandomAI vs VectorAI - Player1 vs Player2 Comparison

Player 1 = RandomAI | Player 1 = VectorAI

Percentage of Games

RandomAI: Tie 0.60%, Player 1 1.70%, Player 2 97.70%
VectorAI: Tie 0.10%, Player 1 99.40%, Player 2 0.50%

Game Winner

RandomAI vs VectorAI - Win Loss Comparison

Percentage of Games

RandomAI 1.10%, Tie 0.35%, VectorAI 98.55%

Game Winner

VectorAI vs VectorAI Statistics

When VectorAI plays against itself, Player 1 holds a significant advantage over Player 2, with a tie being 0.5% more likely than Player 2 winning.

Table of Match by Game Winner			
Match	**Game Winner**	**Frequency**	**Percent**
VectorAI vs VectorAI	**Tie**	15	1.50
	Player 1	975	97.50
	Player 2	10	1.00
	Total	1000	100.00
Total	**Tie**	15	1.50
	Player 1	975	97.50
	Player 2	10	1.00
	Total	1000	100.00

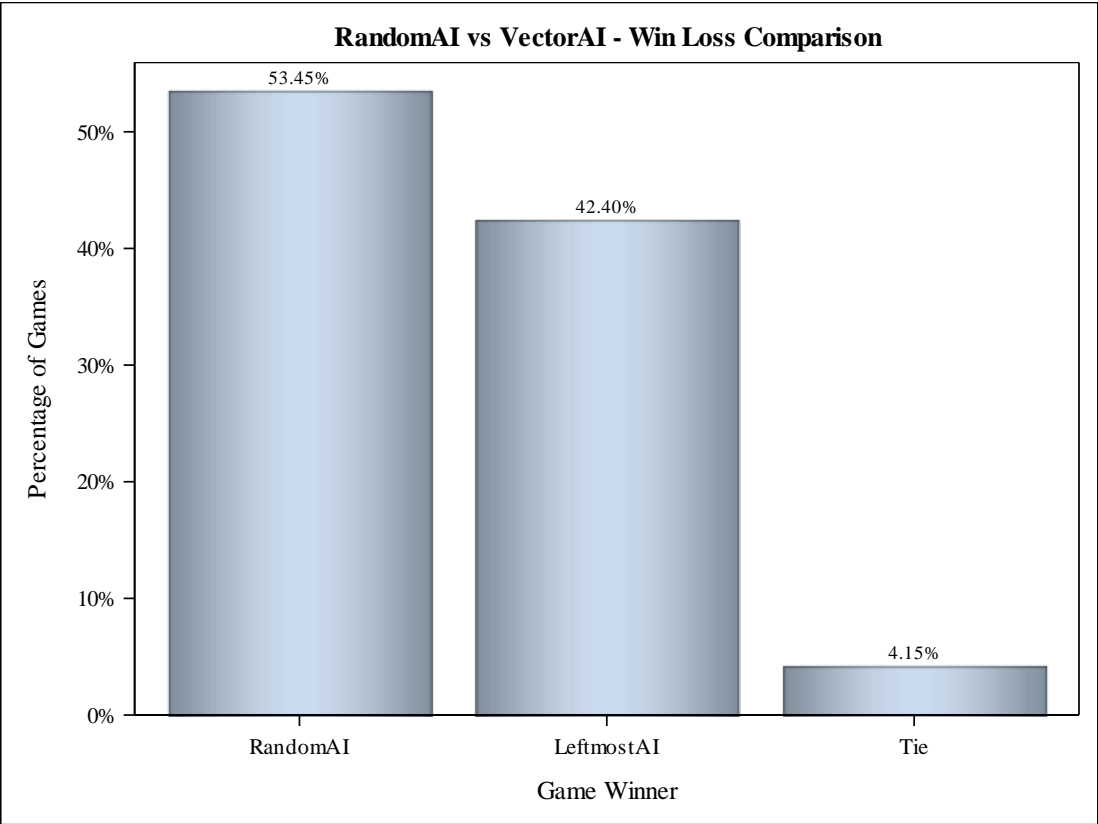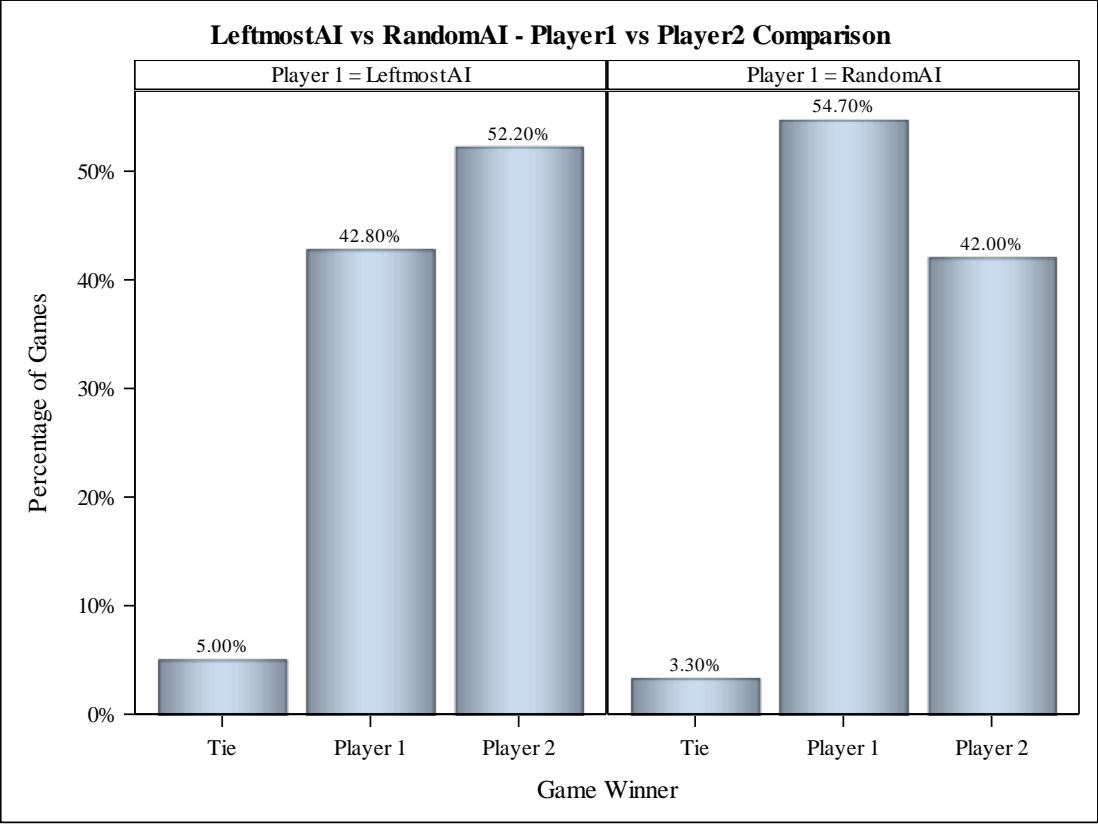VectorAI vs VectorAI - Win Counts and Percentages
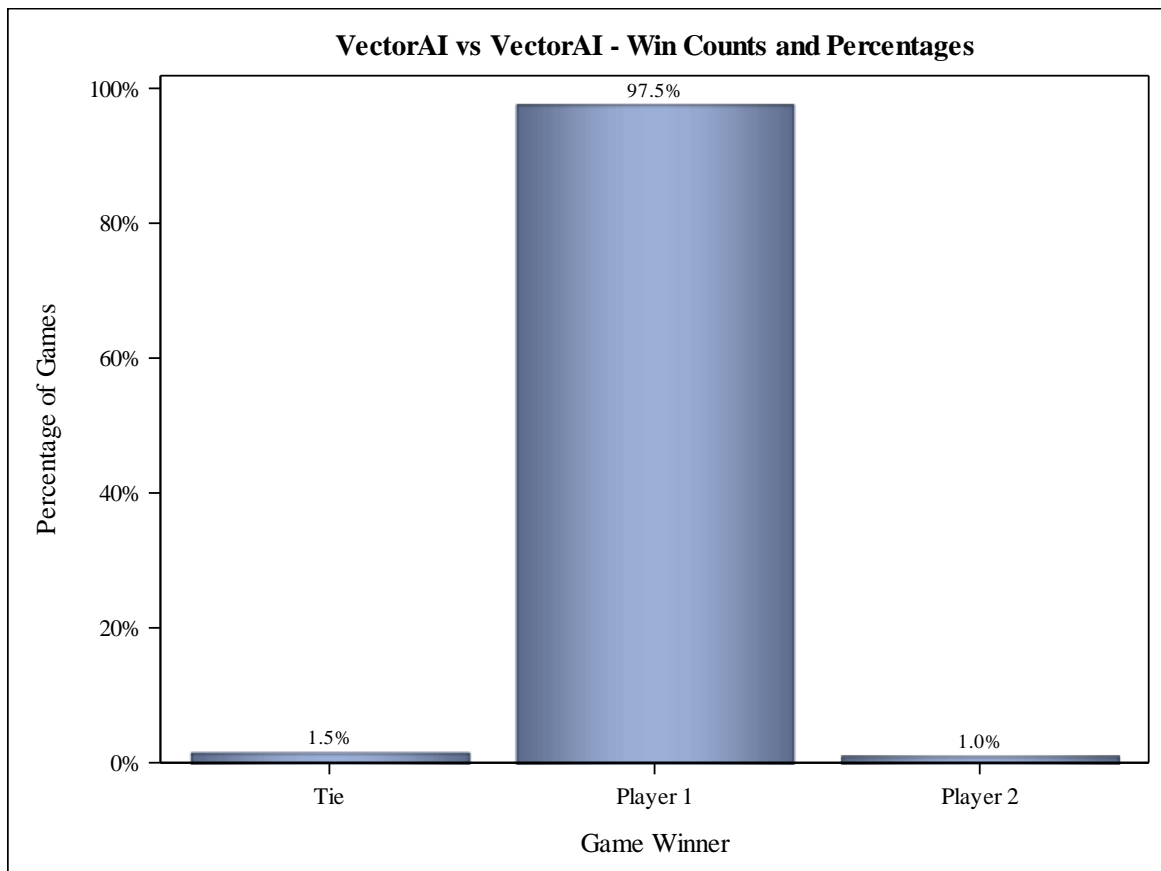
LeftmostAI vs RandomAI Statistics

RandomAI is approximately 10-12% more likely to win when playing against LeftmostAI. A tie is approximately 2% more likely when LeftmostAI is Player 1.

Table of Match by Game Winner					
Match	**Game Winner**	**Frequency**	**Percent**	**Row Percent**	**Column Percent**
LeftmostAI vs RandomAI	**Tie**	50	2.50	5.00	60.24
	Player 1	428	21.40	42.80	43.90
	Player 2	522	26.10	52.20	55.41
	Total	1000	50.00	100.00	
RandomAI vs LeftmostAI	**Tie**	33	1.65	3.30	39.76
	Player 1	547	27.35	54.70	56.10
	Player 2	420	21.00	42.00	44.59
	Total	1000	50.00	100.00	
Total	**Tie**	83	4.15		100.00
	Player 1	975	48.75		100.00
	Player 2	942	47.10		100.00
	Total	2000	100.00		

Table of Win Percentages				
Winner	**Frequency**	**Percent**	**Cumulative Frequency**	**Cumulative Percent**
LeftmostAI	848	42.40	848	42.40
RandomAI	1069	53.45	1917	95.85
Tie	83	4.15	2000	100.00

LeftmostAI vs RandomAI - Player1 vs Player2 Comparison

| Player 1 = LeftmostAI | Player 1 = RandomAI |

Percentage of Games

Player 1 = LeftmostAI:
- Tie: 5.00%
- Player 1: 42.80%
- Player 2: 52.20%

Player 1 = RandomAI:
- Tie: 3.30%
- Player 1: 54.70%
- Player 2: 42.00%

Game Winner

RandomAI vs VectorAI - Win Loss Comparison

Percentage of Games

- RandomAI: 53.45%
- LeftmostAI: 42.40%
- Tie: 4.15%

Game Winner

LeftmostAI vs VectorAI Statistics

The most interesting development is when LeftmostAI is Player 1 it never wins and it ties only 0.2% of the time. When LeftmostAI is Player 2, it wins 5.4% of the time.

Table of Match by Game Winner					
Match	Game Winner	Frequency	Percent	Row Percent	Column Percent
LeftmostAI vs VectorAI	Tie	2	0.10	0.20	50.00
	Player 1	54	2.70	5.40	5.13
	Player 2	944	47.20	94.40	100.00
	Total	1000	50.00	100.00	
VectorAI vs LeftmostAI	Tie	2	0.10	0.20	50.00
	Player 1	998	49.90	99.80	94.87
	Player 2	0	0.00	0.00	0.00
	Total	1000	50.00	100.00	
Total	Tie	4	0.20		100.00
	Player 1	1052	52.60		100.00
	Player 2	944	47.20		100.00
	Total	2000	100.00		

Table of Win Percentages				
Winner	Frequency	Percent	Cumulative Frequency	Cumulative Percent
LeftmostAI	54	2.70	54	2.70
Tie	4	0.20	58	2.90
VectorAI	1942	97.10	2000	100.00

LeftmostAI vs VectorAI - Player1 vs Player2 Comparison

LeftmostAI vs VectorAI - Win Loss Comparison
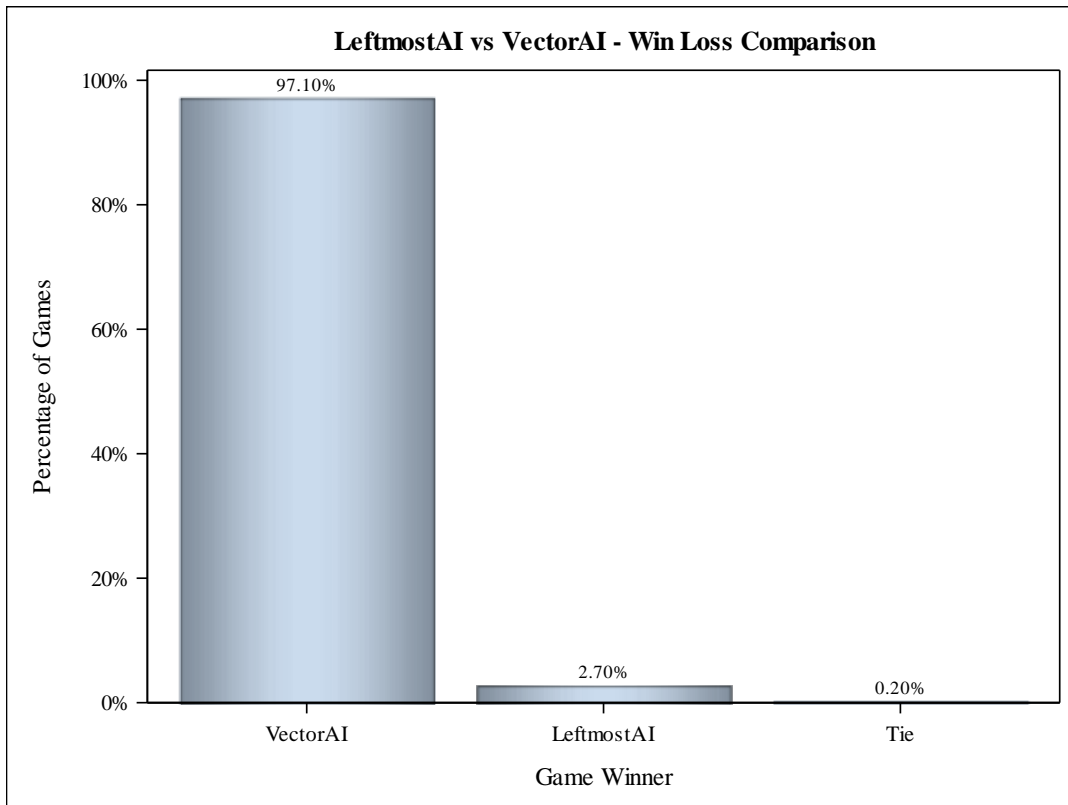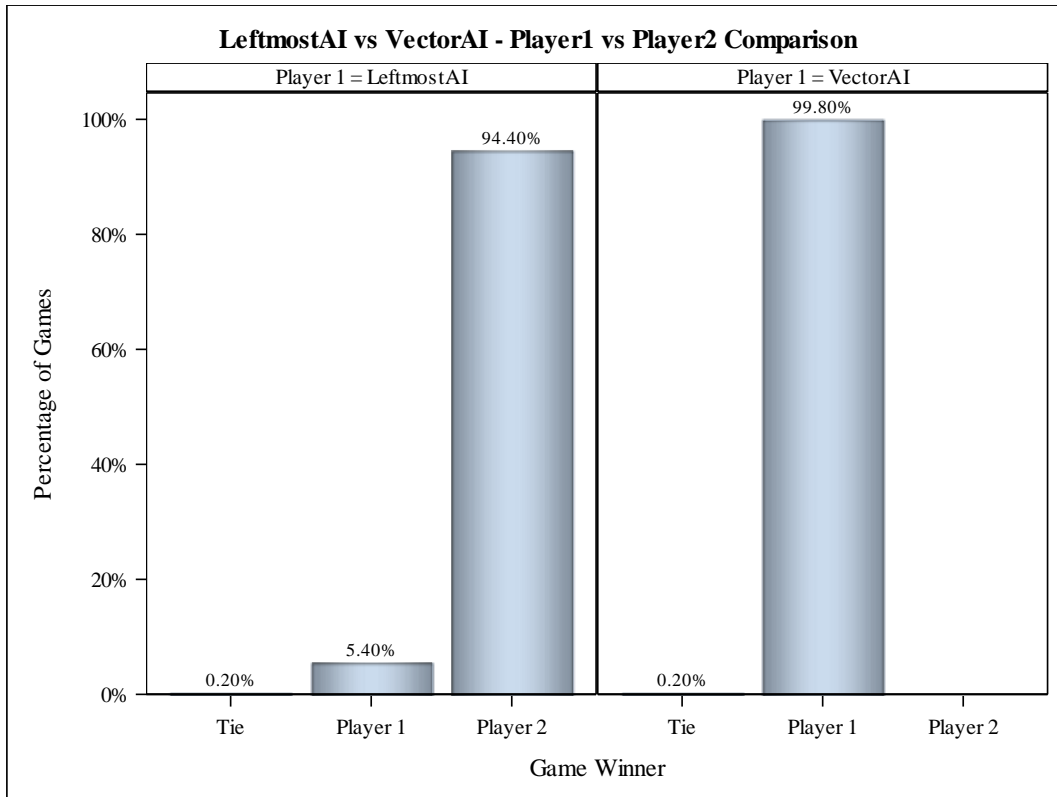
LeftmostAI vs LeftmostAI Statistics

The data in question was puzzling at first glance and required further exploration. It turns out that Player 1 will always run out of moves first and Player 2 will collect the remaining pieces. This is because Player 1 refills Player 2's leftmost pit when choosing the leftmost move, but Player 2 does not refill Player 1's leftmost pit, resulting in Player 1 running out of pieces first and Player 2 collecting the majority of pieces.

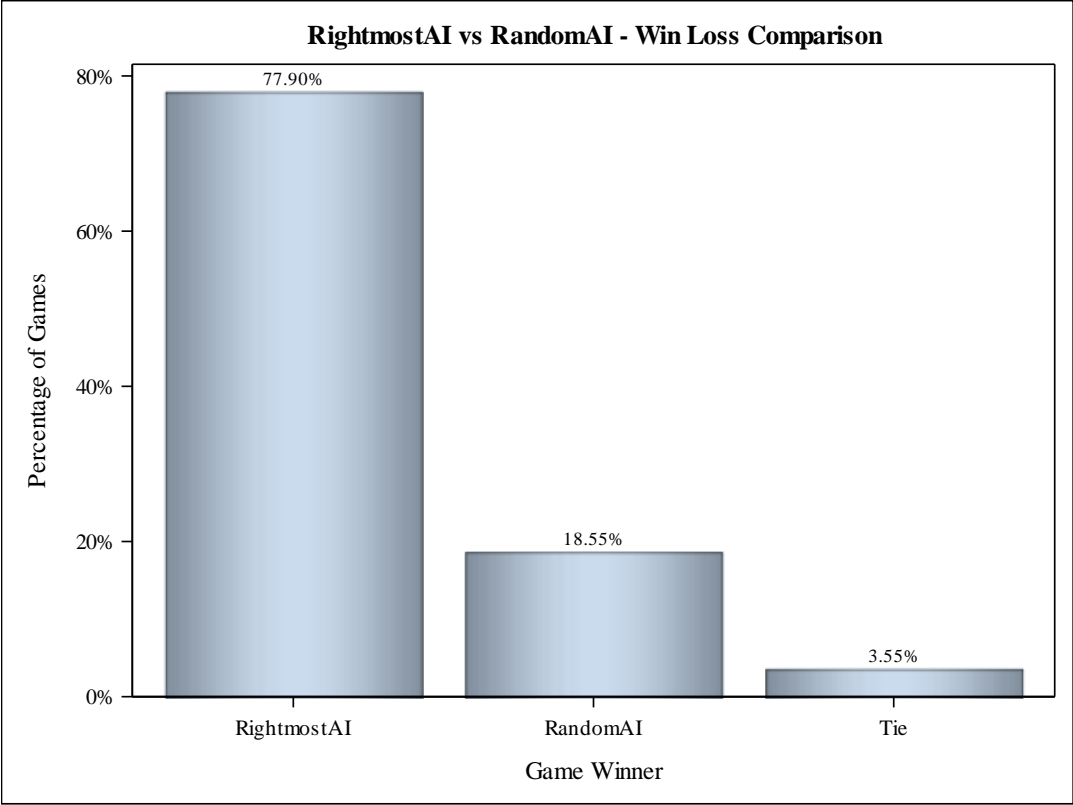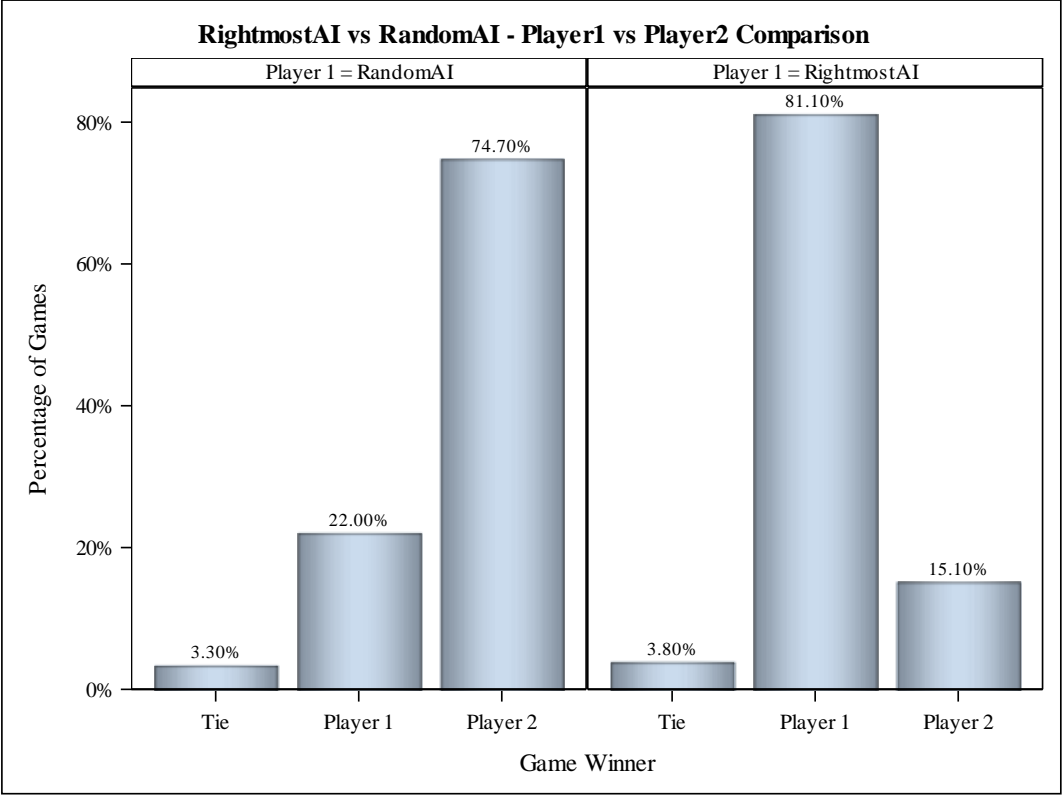| Table of Match by Game Winner			
Match	**Game Winner**	**Frequency**	**Percent**
LeftmostAI vs LeftmostAI	**Player 2**	1000	100.00
	Total	1000	100.00
Total	**Player 2**	1000	100.00
	Total	1000	100.00
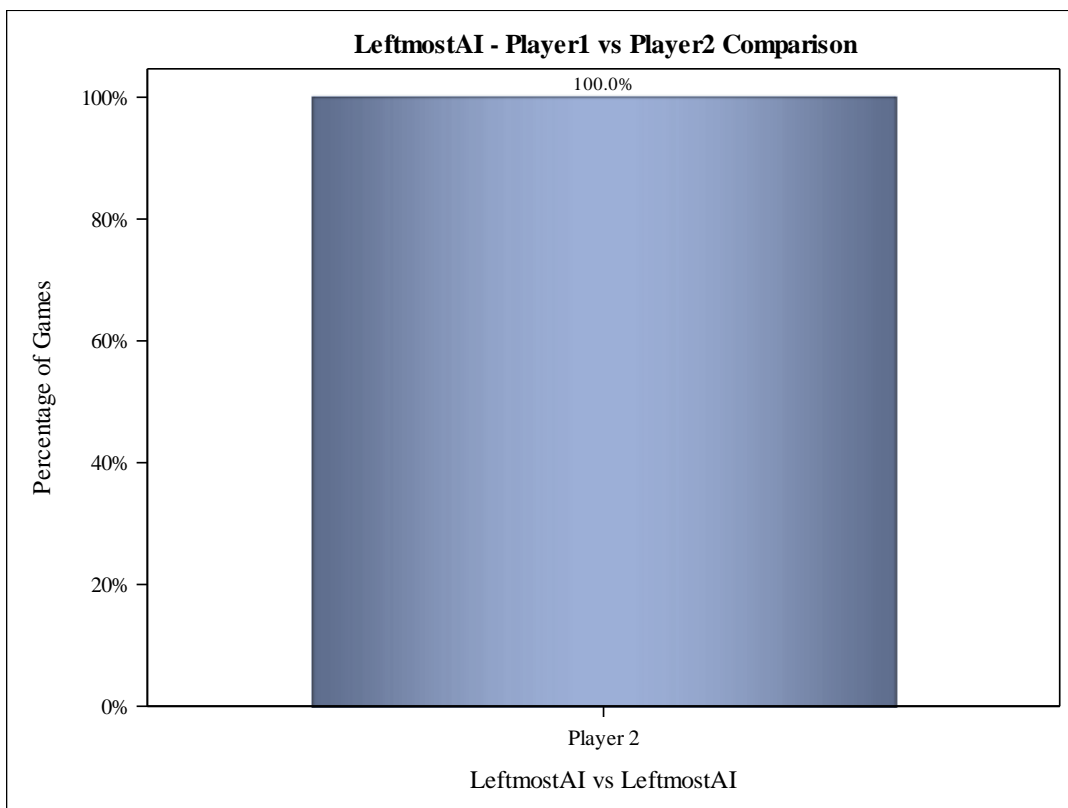
RightmostAI vs RandomAI Statistics

As we can see in the graphs, the RightmostAI plays significantly better than the RandomAI, regardless of whether it is Player 1 or Player 2.

Table of Match by Game Winner					
Match	**Game Winner**	**Frequency**	**Percent**	**Row Percent**	**Column Percent**
RandomAI vs RightmostAI	**Tie**	33	1.65	3.30	46.48
	Player 1	220	11.00	22.00	21.34
	Player 2	747	37.35	74.70	83.18
	Total	1000	50.00	100.00	
RightmostAI vs RandomAI	**Tie**	38	1.90	3.80	53.52
	Player 1	811	40.55	81.10	78.66
	Player 2	151	7.55	15.10	16.82
	Total	1000	50.00	100.00	
Total	**Tie**	71	3.55		100.00
	Player 1	1031	51.55		100.00
	Player 2	898	44.90		100.00
	Total	2000	100.00		

Table of Win Percentages				
Winner	**Frequency**	**Percent**	**Cumulative Frequency**	**Cumulative Percent**
RandomAI	371	18.55	371	18.55
RightmostAI	1558	77.90	1929	96.45
Tie	71	3.55	2000	100.00

RightmostAI vs RandomAI - Player1 vs Player2 Comparison

**RightmostAI vs RandomAI - Win Loss Comparison**

Rightmost vs VectorAI Statistics

The results of playing RightmostAI against VectorAI yield interesting results. Regardless of which bot is Player 1, Player 1 always wins. This phenomenon was explored further by playing each bot against other bots online – playing our bot as the human player online and the online bot as a human player in our project.* Playing two identical games is time consuming, and many times I had to restart one or both games because of an error. Of the games that did not error, the results were approximately 50/50 for RightmostAI and VectorAI winning when playing as Player 1 and Player 2 against the online bot. Unfortunately, the algorithm for the online bot's decision making were not published. We have concluded that there must be a bug in either the game code or the code for either bot. After reviewing the code, the issue is still not apparent and we are unsure of the cause of the observed phenomenon.

* The online bots used were at http://play-mancala.com and https://www.coolmath-games.com/0-mancala.

Table of Match by Game Winner					
Match	Game Winner	Frequency	Percent	Row Percent	Column Percent
RightmostAI vs VectorAI	Player 1	1000	50.00	100.00	50.00
	Total	1000	50.00	100.00	
VectorAI vs RightmostAI	Player 1	1000	50.00	100.00	50.00
	Total	1000	50.00	100.00	
Total	Player 1	2000	100.00		100.00
	Total	2000	100.00		

Table of Win Percentages				
Winner	Frequency	Percent	Cumulative Frequency	Cumulative Percent
RightmostAI	1000	50.00	1000	50.00
VectorAI	1000	50.00	2000	100.00

RightmostAI vs VectorAI - Player1 vs Player2 Comparison

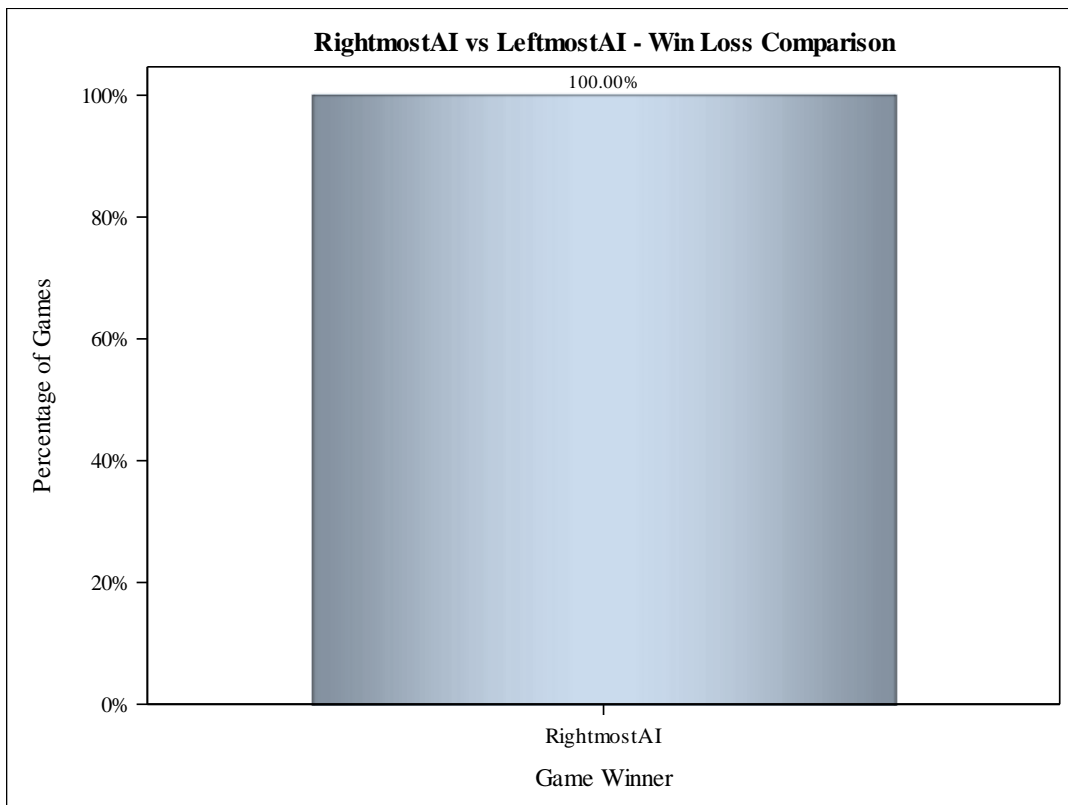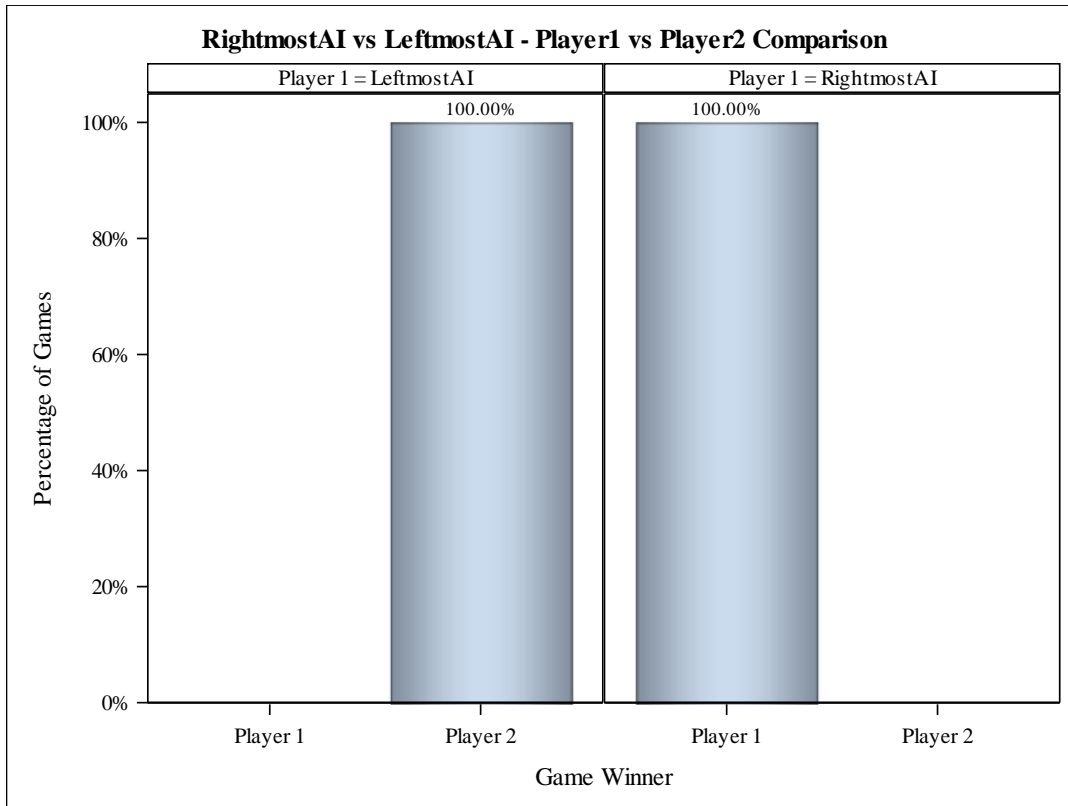RightmostAI vs VectorAI - Win Loss Comparison
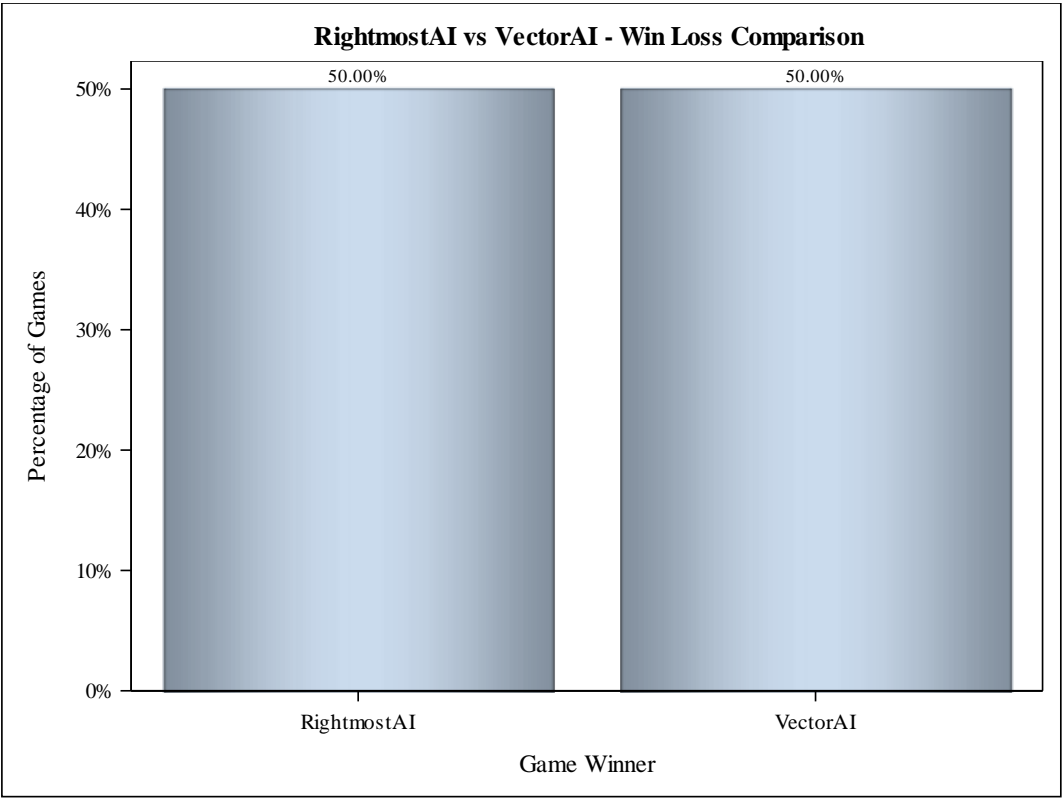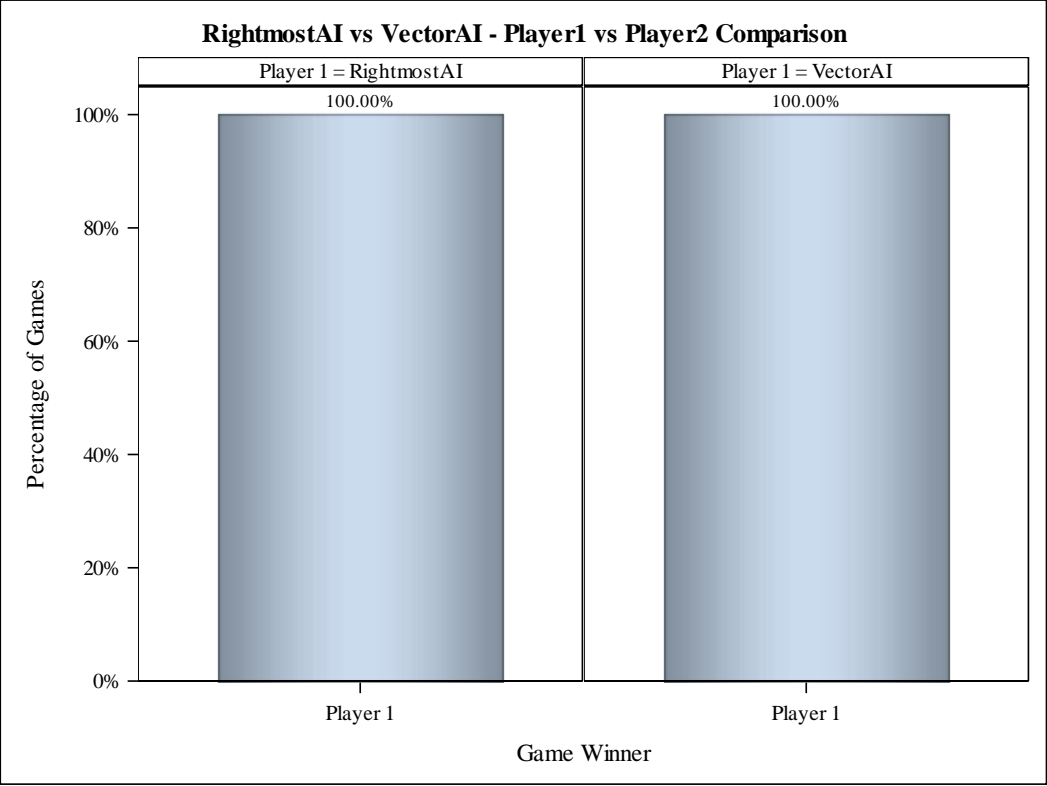
RightmostAI vs LeftmostAI Statistics

At first glance, it seems odd that the RightmostAI always wins. After further exploration, we discovered that this is because the LeftmostAI will always run out of pieces first, and the RightmostAI will collect the pieces remaining on the board, leading the RightmostAI to win.

Table of Match by Game Winner					
Match	Game Winner	Frequency	Percent	Row Percent	Column Percent
LeftmostAI vs RightmostAI	Player 1	0	0.00	0.00	0.00
	Player 2	1000	50.00	100.00	100.00
	Total	1000	50.00	100.00	
RightmostAI vs LeftmostAI	Player 1	1000	50.00	100.00	100.00
	Player 2	0	0.00	0.00	0.00
	Total	1000	50.00	100.00	
Total	Player 1	1000	50.00		100.00
	Player 2	1000	50.00		100.00
	Total	2000	100.00		

Table of Win Percentages				
Winner	Frequency	Percent	Cumulative Frequency	Cumulative Percent
RightmostAI	2000	100.00	2000	100.00

RightmostAI vs LeftmostAI - Player1 vs Player2 Comparison

RightmostAI vs LeftmostAI - Win Loss Comparison

RightmostAI vs RightmostAI Statistics

At first, it was puzzling that RightmostAI would always tie when playing itself. After playing through a few games by hand using RightmostAI's decision making method, this result makes sense. The player's moves mirror each other, resulting in the same number of captures and free moves and thus a tie.

Table of Match by Game Winner			
Match	**Game Winner**	**Frequency**	**Percent**
RightmostAI vs RightmostAI	**Tie**	1000	100.00
	Total	1000	100.00
Total	**Tie**	1000	100.00
	Total	1000	100.00
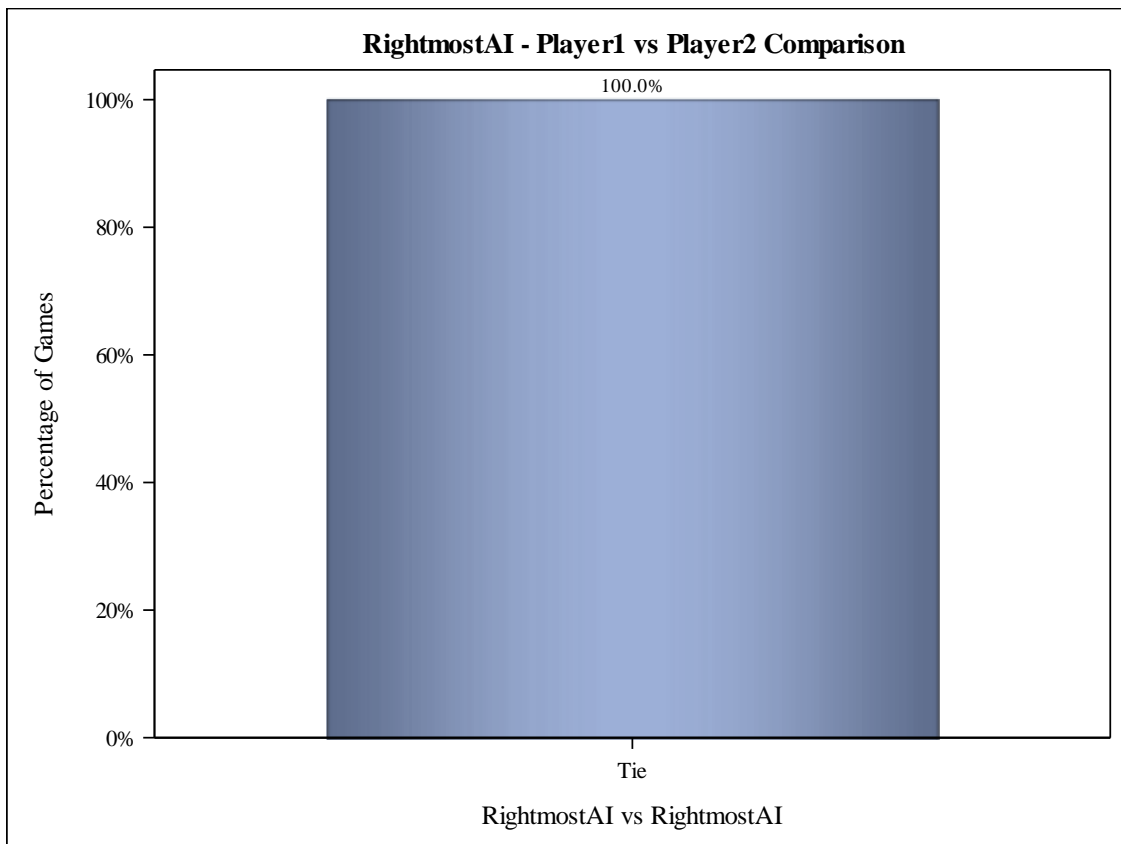
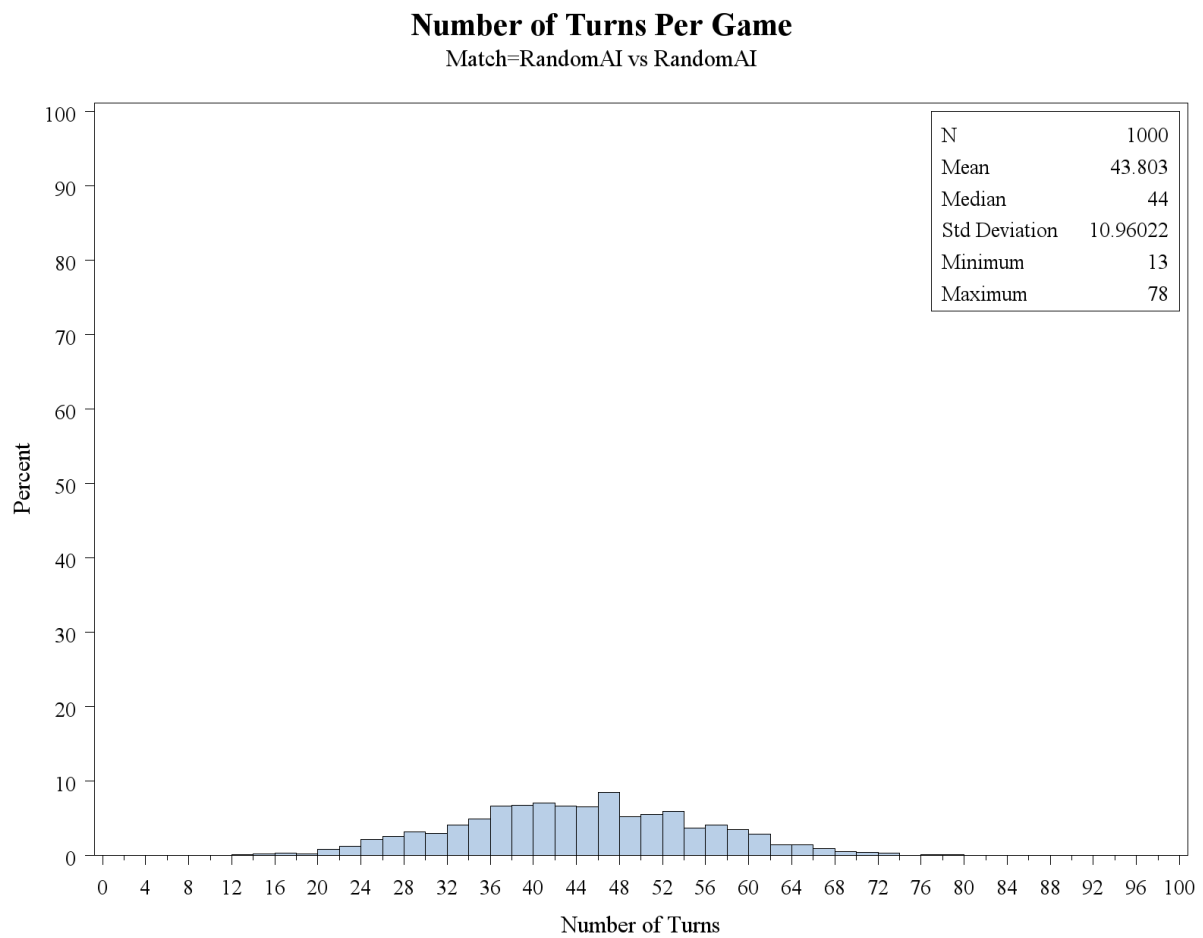<u>Analysis of Number of Turns Per Game</u>

<u>Overall Statistics</u>

Below is a histogram of the number of turns per game over all of the matches played. The mean number of turns per game is 33. The shortest games were only ten turns, while the longest game lasted 92 turns.
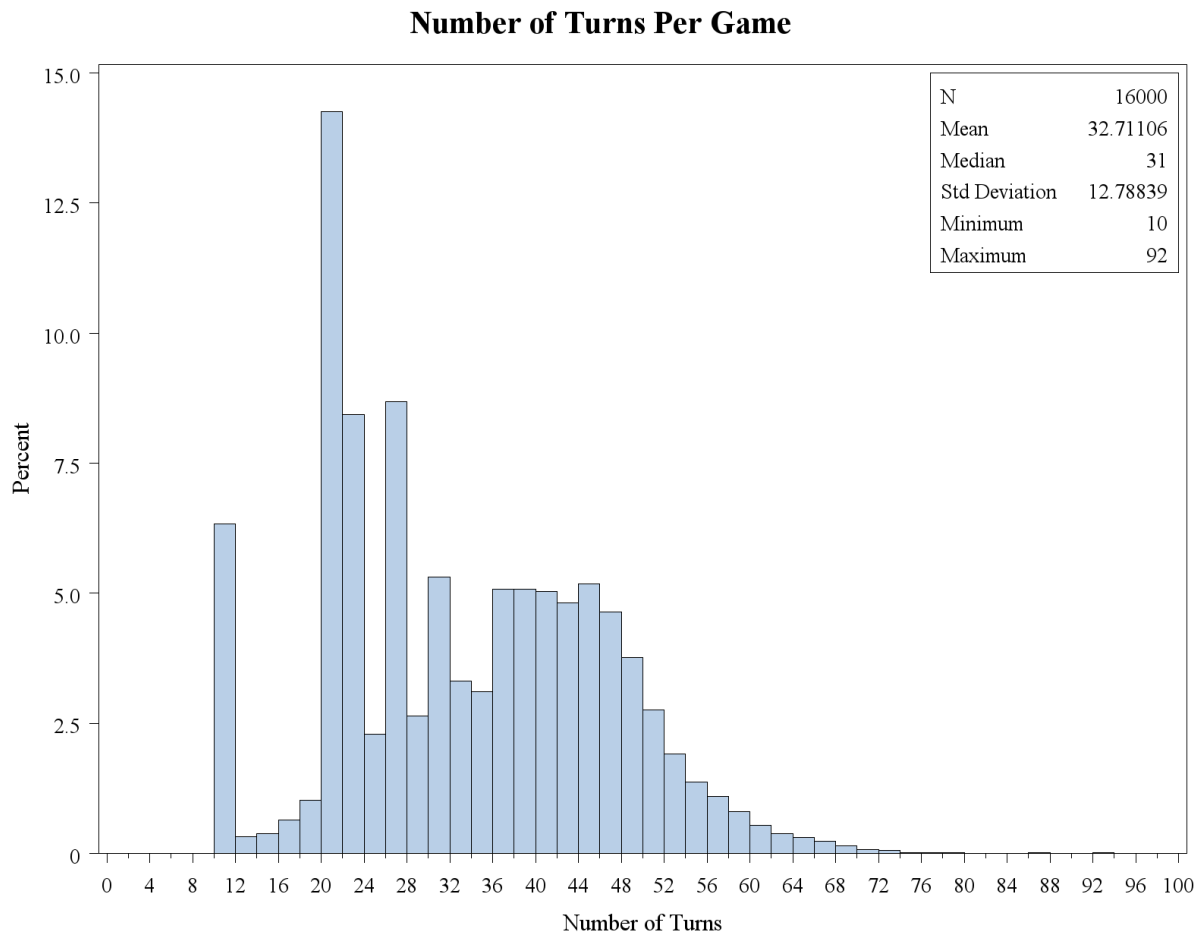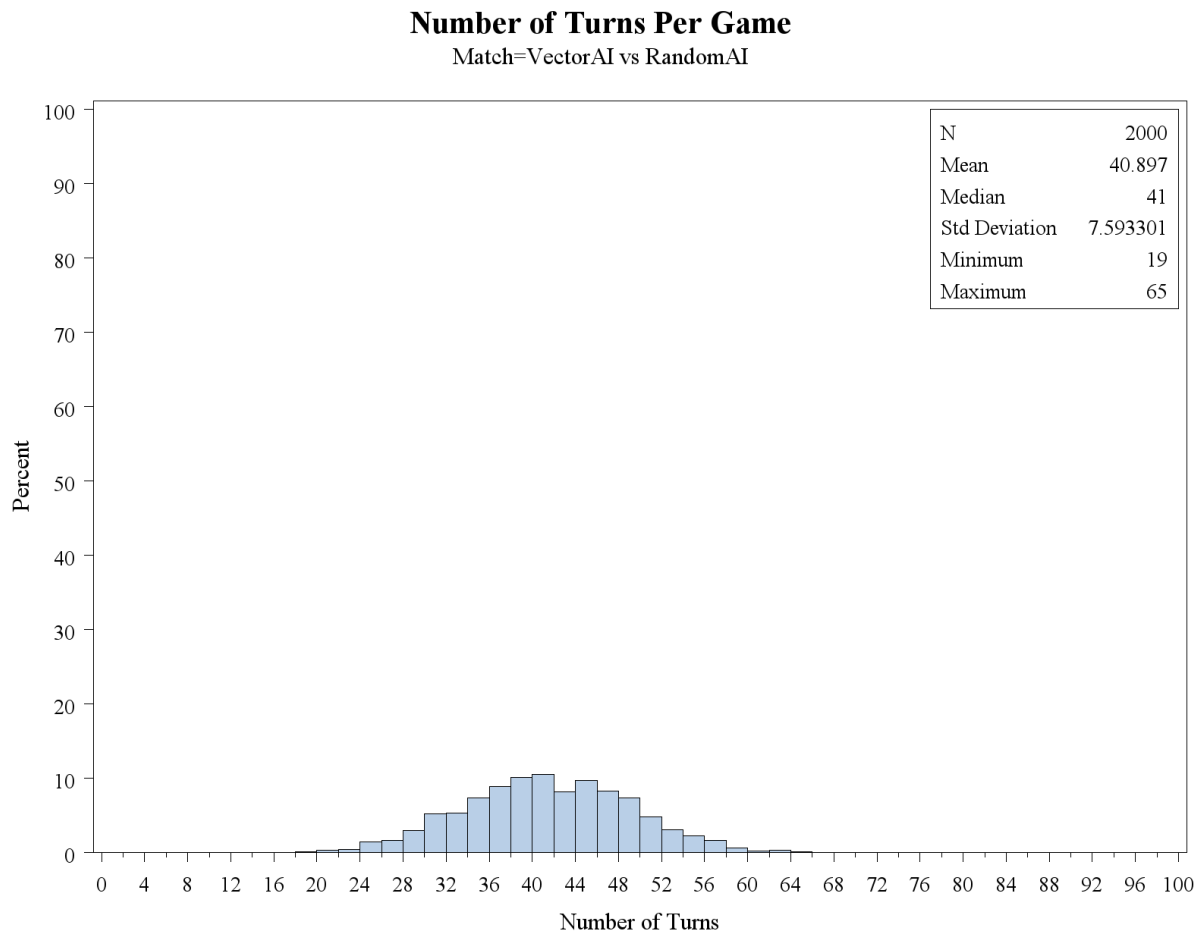
Number of Turns Per Game

N	16000
Mean	32.71106
Median	31
Std Deviation	12.78839
Minimum	10
Maximum	92

RandomAI - Number of Turns Per Game

The number of turns per game is a nice normal curve with an average game length of 44 turns.

Number of Turns Per Game
Match=RandomAI vs RandomAI

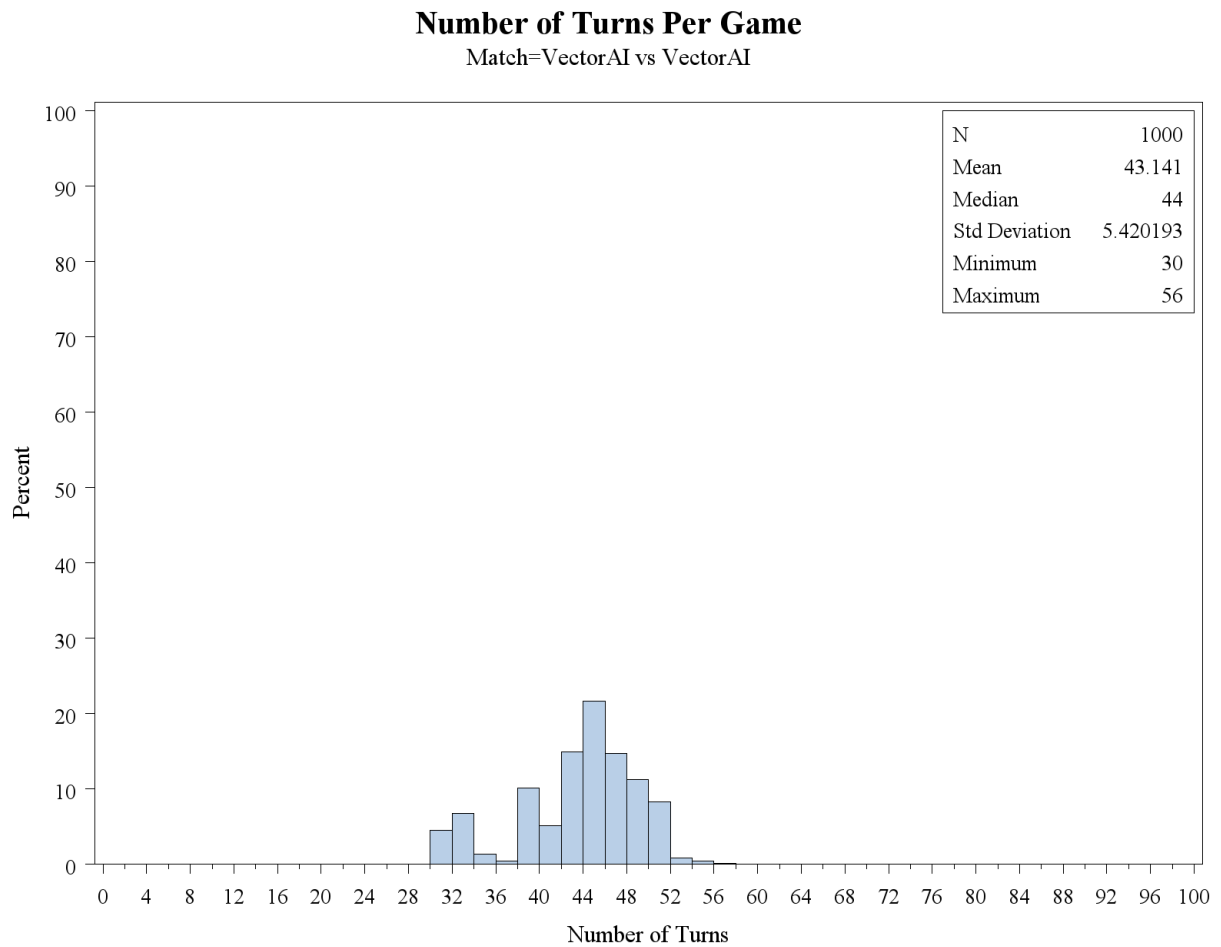| N	1000
Mean	43.803
Median	44
Std Deviation	10.96022
Minimum	13
Maximum	78

VectorAI vs RandomAI - Number of Turns Per Game

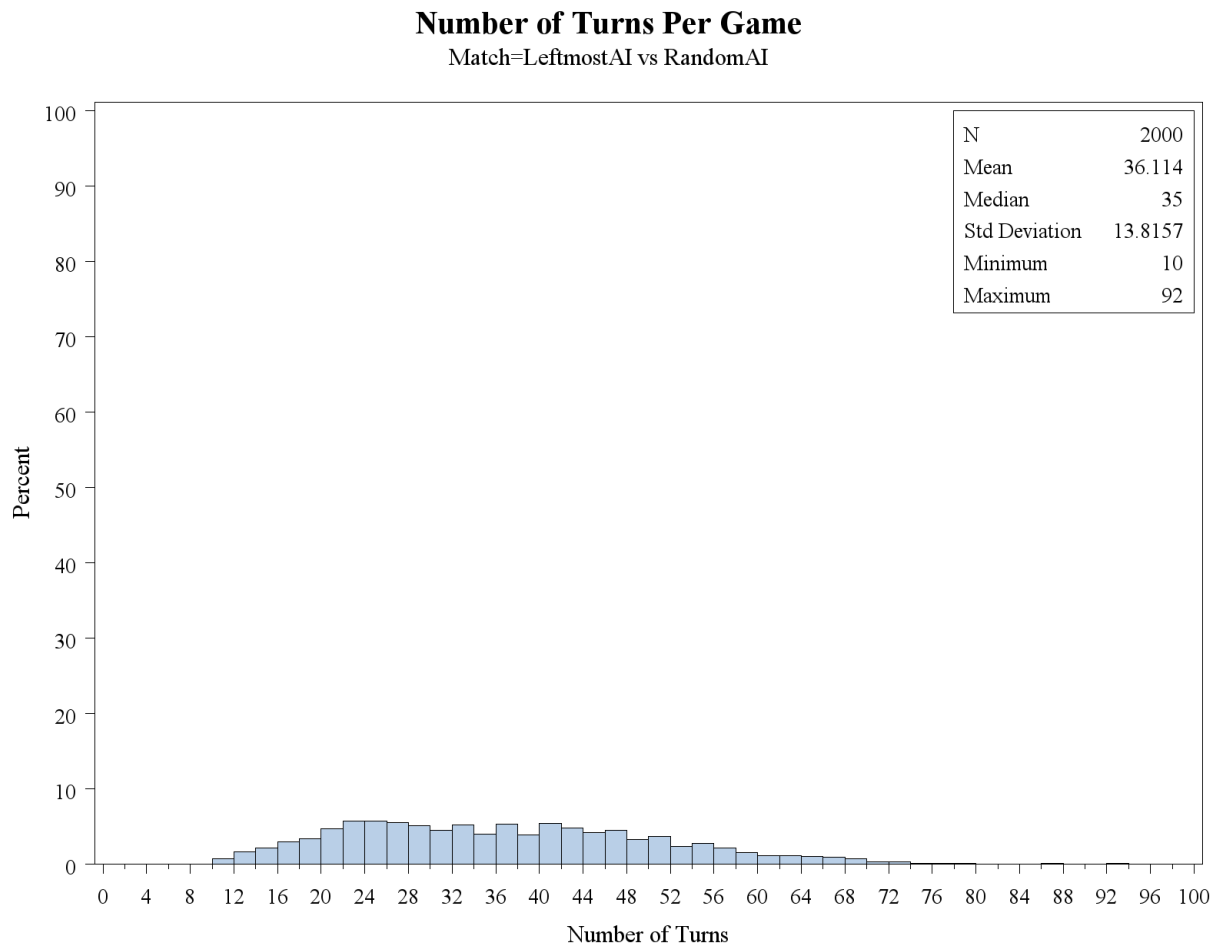The number of turns per game is a nice normal curve with an average game length of 40 turns.

Number of Turns Per Game

Match=VectorAI vs RandomAI

N	2000
Mean	40.897
Median	41
Std Deviation	7.593301
Minimum	19
Maximum	65

<u>VectorAI - Number of Turns Per Game</u>

The number of turns per game is a more tightly grouped normal curve than seen in previous graphs.

Number of Turns Per Game

Match=VectorAI vs VectorAI

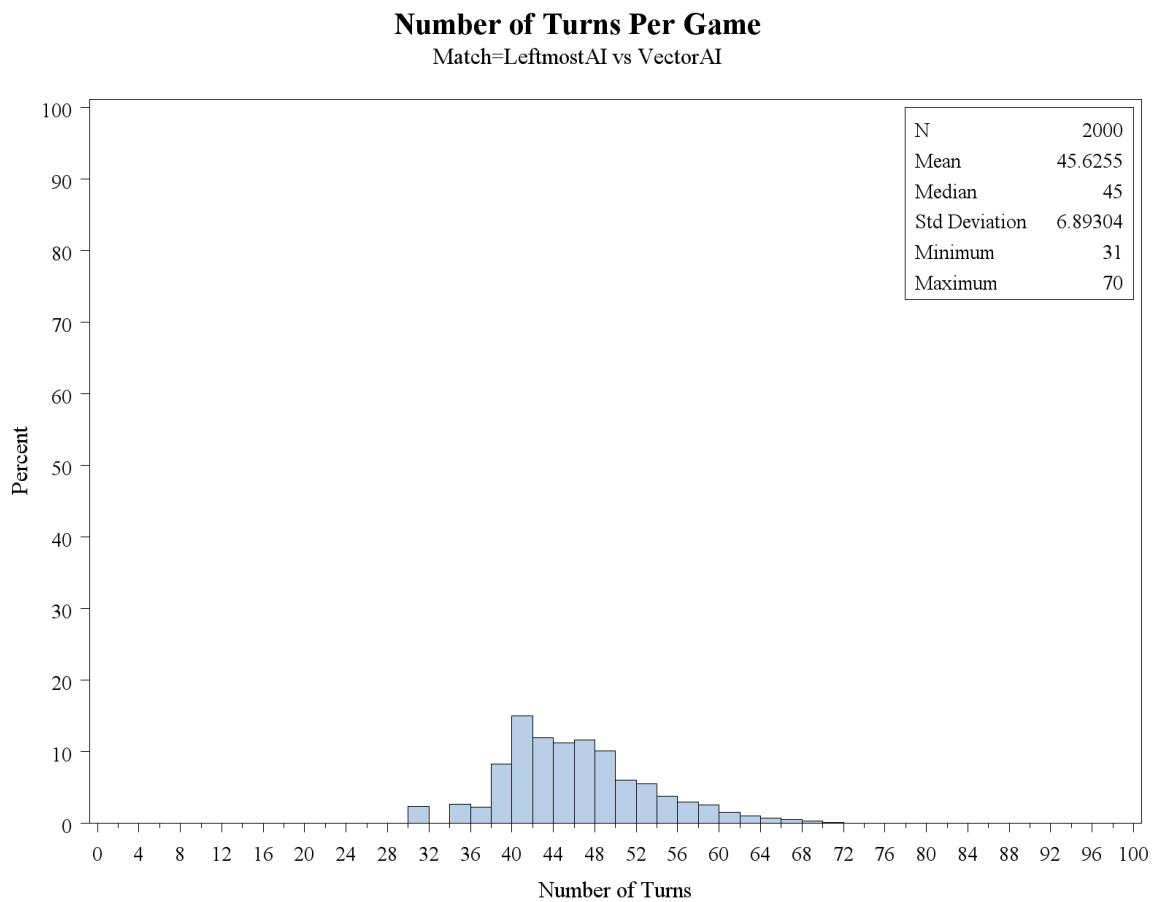| N	1000
Mean	43.141
Median	44
Std Deviation	5.420193
Minimum	30
Maximum	56

LeftmostAI vs RandomAI - Number of Turns Per Game

The number of turns per game is the most widespread in these matches when compared to other matches.
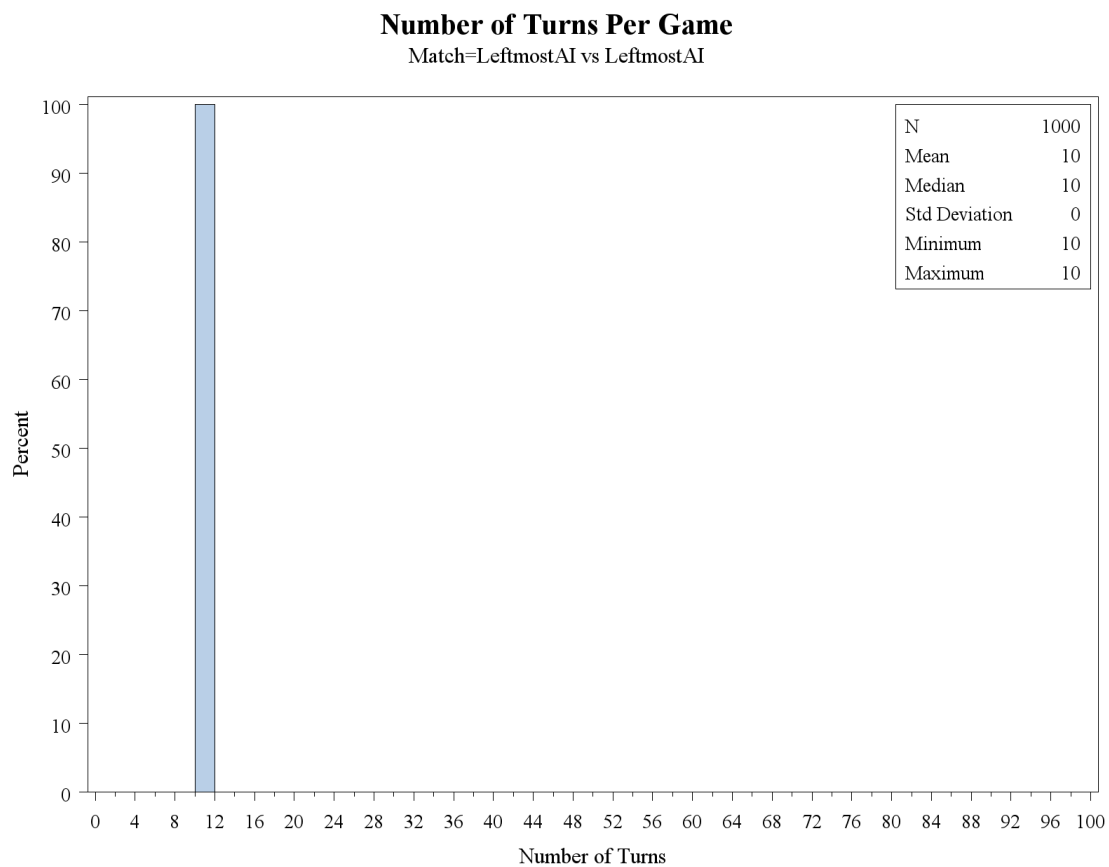
Number of Turns Per Game

Match=LeftmostAI vs RandomAI

N	2000
Mean	36.114
Median	35
Std Deviation	13.8157
Minimum	10
Maximum	92

The number of turns per game is skewed to the right in this group of matches.

Number of Turns Per Game

Match=LeftmostAI vs VectorAI

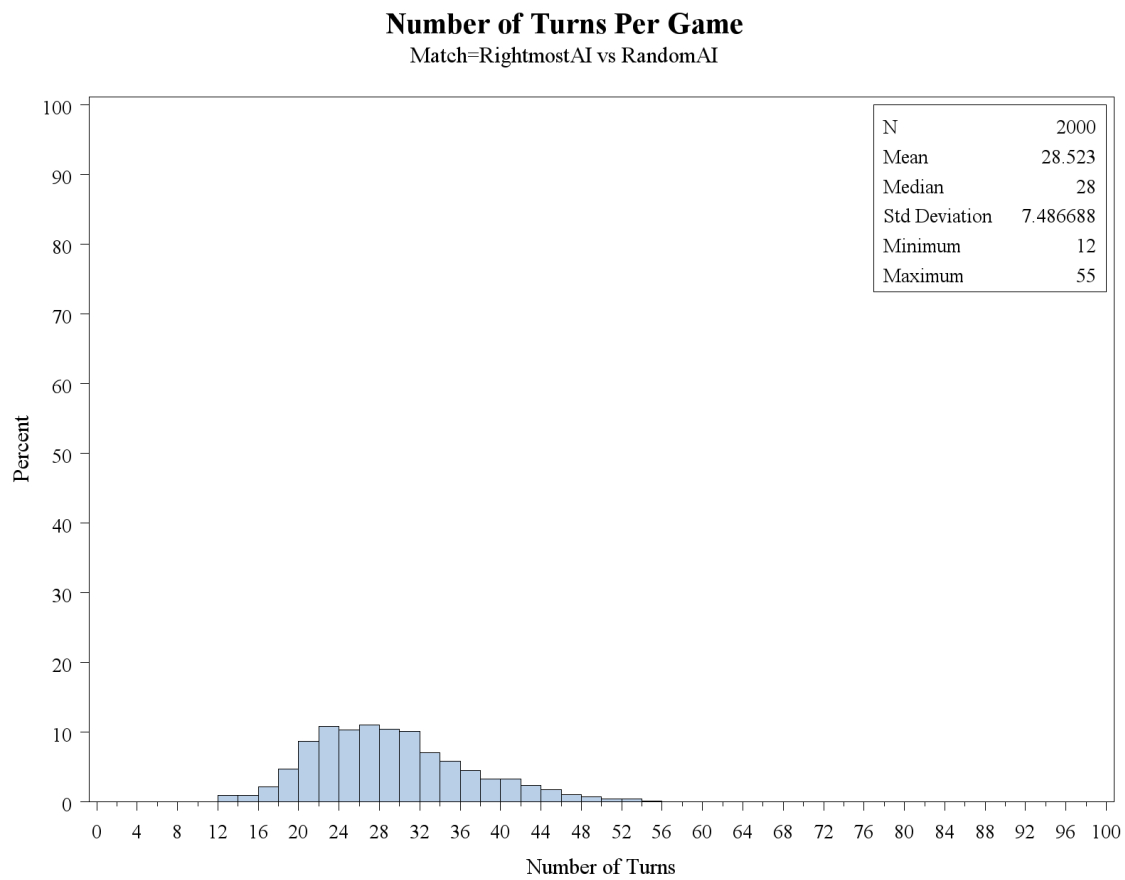| N	2000
Mean	45.6255
Median	45
Std Deviation	6.89304
Minimum	31
Maximum	70

LeftmostAI vs LeftmostAI - Number of Turns Per Game

The number of turns per game is always 10. As explained previously, when both players always choose the leftmost legal move the game is always the same. There is no randomness or prioritizing by either player, so the outcome is predetermined. In this case, it always takes 10 turns for Player 1 to run out of pieces.
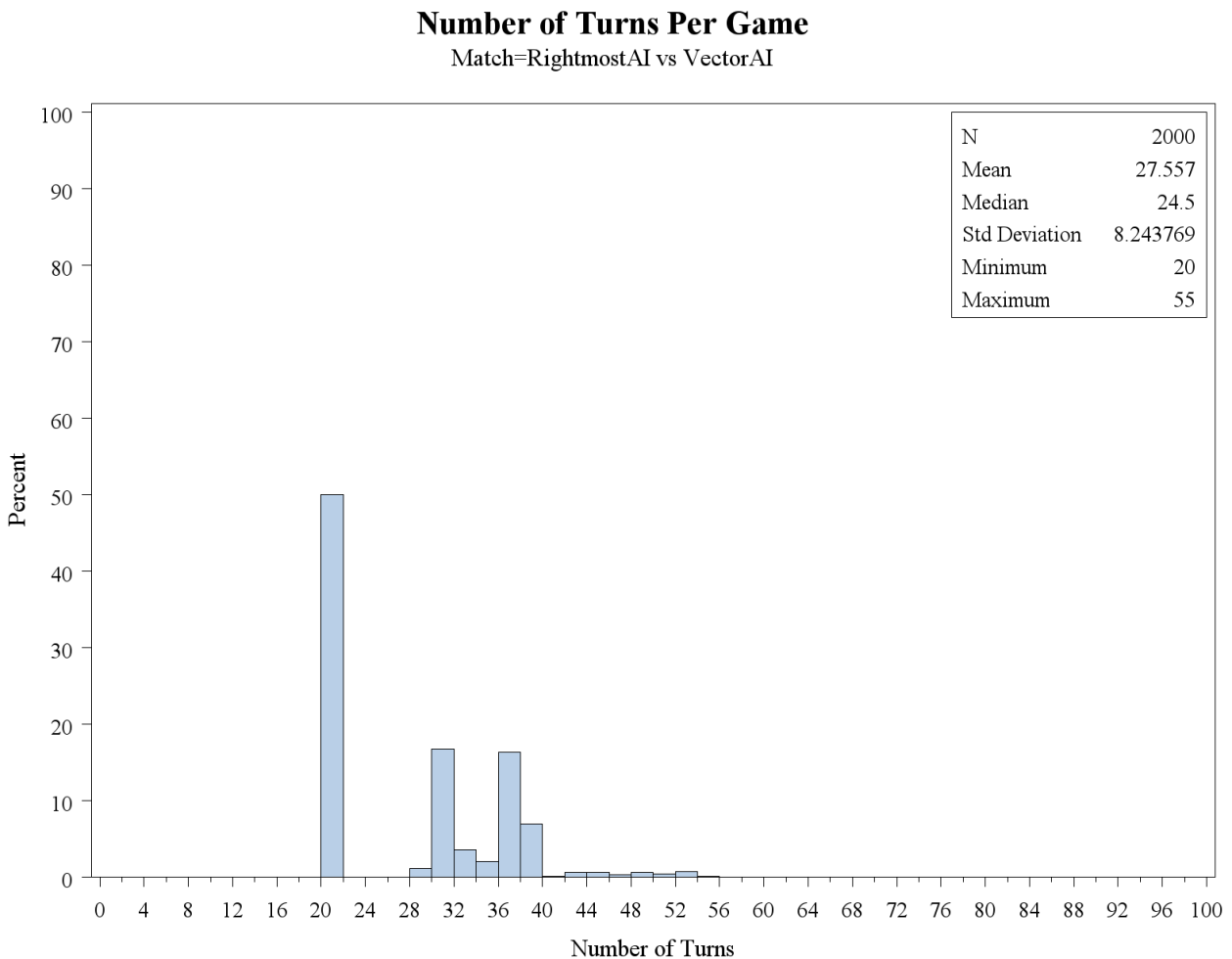
Number of Turns Per Game
Match=LeftmostAI vs LeftmostAI

N	1000
Mean	10
Median	10
Std Deviation	0
Minimum	10
Maximum	10

RightmostAI vs RandomAI - Number of Turns Per Game

The number of turns per game is slightly skewed to the right in this group of matches.

Number of Turns Per Game
Match=RightmostAI vs RandomAI

| N	2000
Mean	28.523
Median	28
Std Deviation	7.486688
Minimum	12
Maximum	55

RightmostAI vs VectorAI - Number of Turns Per Game

The layout of this histogram is unusual. After further exploration, it turns out that the 50% of games that end in 20 turns are all from when RightmostAI is Player 1. However, further exploration did not uncover why this is the case.

Number of Turns Per Game
Match=RightmostAI vs VectorAI

| N	2000
Mean	27.557
Median	24.5
Std Deviation	8.243769
Minimum	20
Maximum	55

RightmostAI vs LeftmostAI - Number of Turns Per Game

The layout of this histogram is unusual. The 50/50 split of the number of turns, after further exploration, comes from when the AIs take turns as Player 1. There is no randomness or prioritizing of moves on the part of either AI, so the games in each portion of the data are identical, with RightmostAI always winning.

Number of Turns Per Game

Match=RightmostAI vs LeftmostAI

N	2000
Mean	24.5
Median	24.5
Std Deviation	1.500375
Minimum	23
Maximum	26
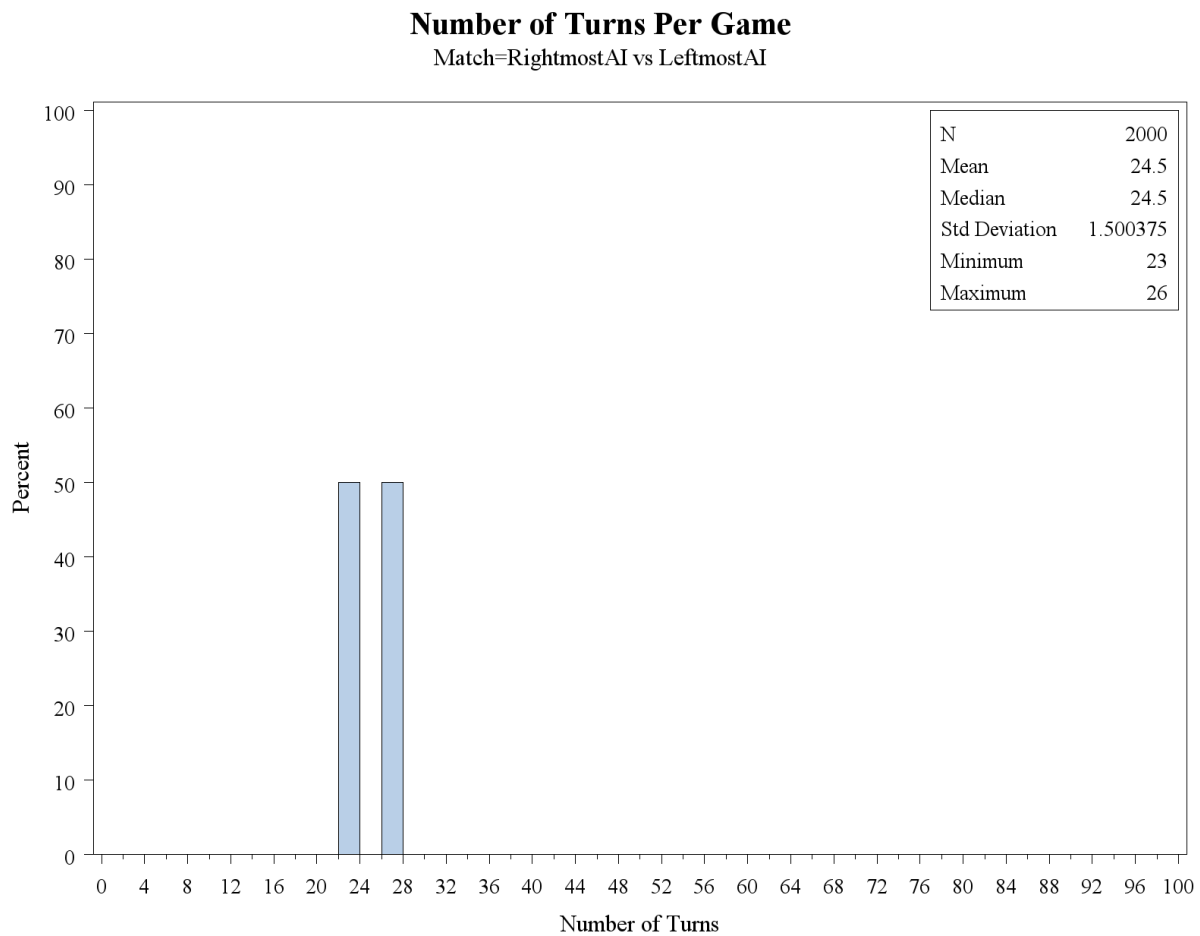
RightmostAI - Number of Turns Per Game

The number of turns per game is always 20. As explained previously, when both players always choose the rightmost legal move the game is always the same. There is no randomness or prioritizing by either player, so the outcome is predetermined. In this case, it always takes 20 turns to tie.

Number of Turns Per Game

Match=RightmostAI vs RightmostAI

N	1000
Mean	20
Median	20
Std Deviation	0
Minimum	20
Maximum	20
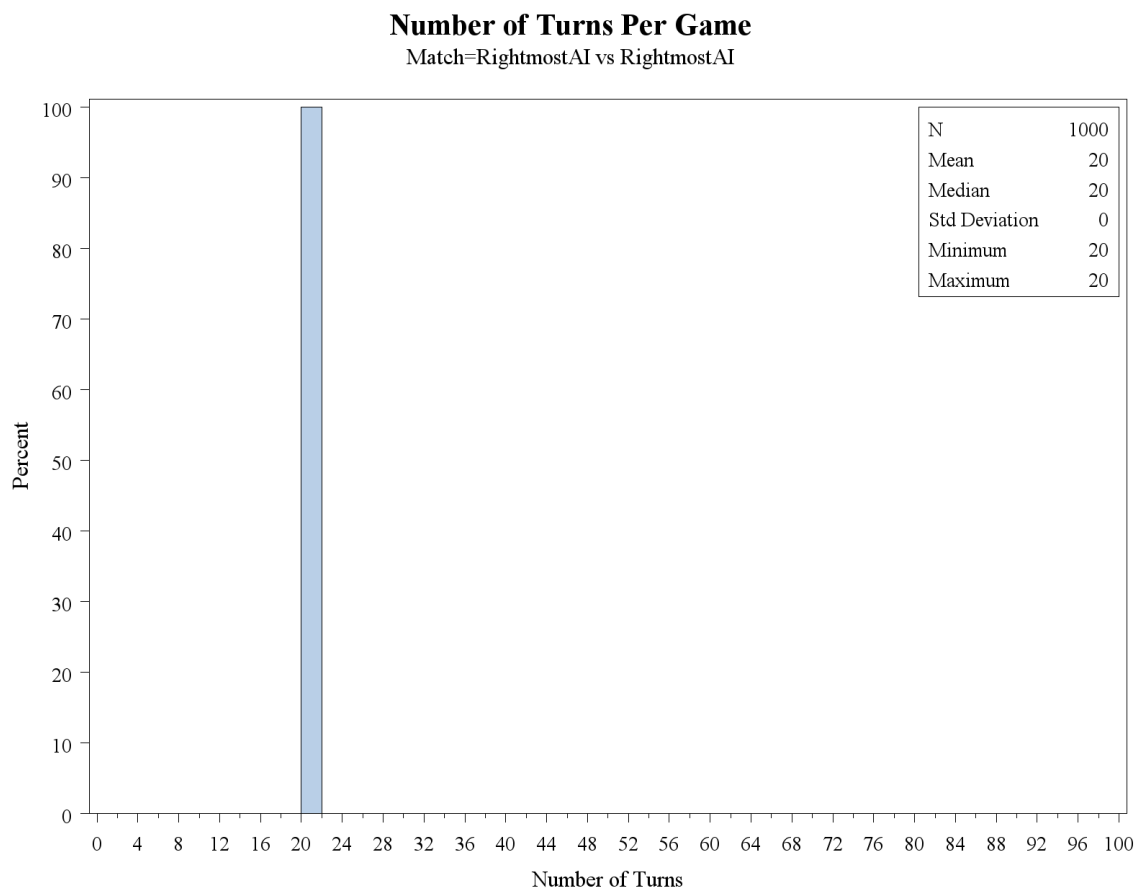
Analysis of the Player that Finished First Compared to the Player that Won

Overall - Finished first vs Winner Comparison

Interestingly enough, when Player 2 finished first (ran out of moves) Player 1 usually won and when Player 1 finished first Player 2 was more likely to win.

Table of Finished First by Game Winner			
Finished First	**Game Winner**	**Frequency**	**Percent**
Player 1	**Tie**	1123	7.02
	Player 1	1772	11.08
	Player 2	5454	34.09
	Total	8349	52.18
Player 2	**Tie**	125	0.78
	Player 1	6776	42.35
	Player 2	750	4.69
	Total	7651	47.82
Total	**Tie**	1248	7.80
	Player 1	8548	53.43
	Player 2	6204	38.78
	Total	16000	100.00

Finished First vs Winner Comparison
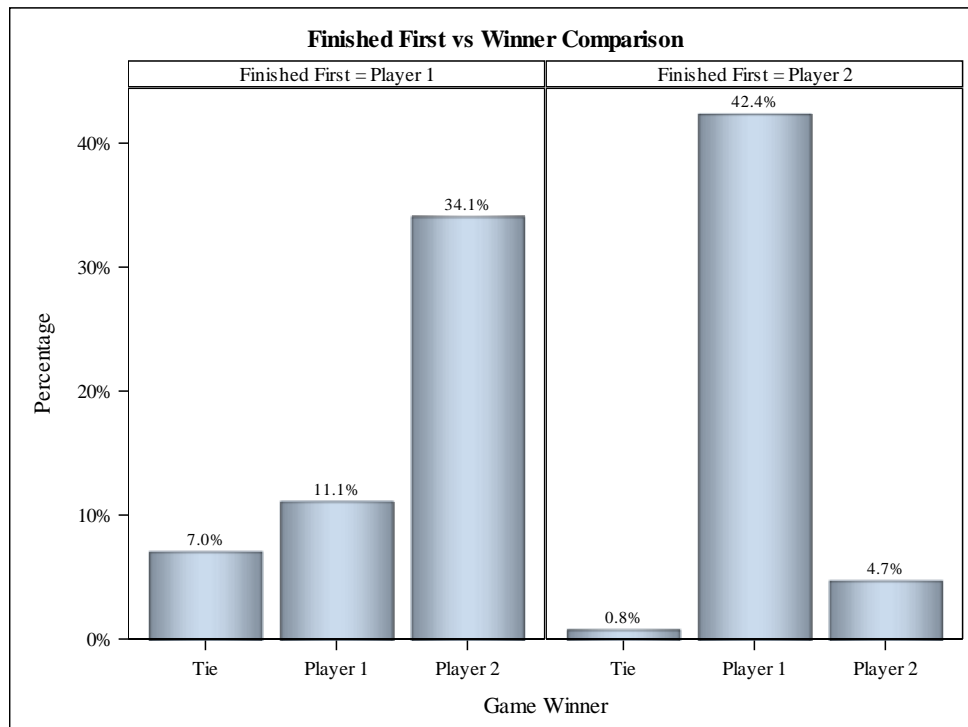
Analysis of the Wins, Losses, and Ties Per Bot

Below are the bar charts depicting the wins, losses, and ties percentages over all games played by that bot.