Planck's Constant

May 27, 2020

1 Introduction

In this lab, we will measure Planck's constant by measuring the V-I curves of three different colored light emitting diodes (LEDs).

An LED is a particular type of diode for which the recombination of electrons and holes produces photons, typically in the visible light spectrum. These diodes have an activation voltage given by:

$$V_{\mathrm{A}} = \phi + \frac{hc}{e}\frac{1}{\lambda} \tag{1}$$

where λ is the wave-length of the light produced by the diode, and ϕ is the contribution to the voltage drop due to other effects in the $p-n$ junctions. The diodes we are using have been chosen to ensure that ϕ is approximately constant across all three diodes.

The quantity

$$\frac{hc}{e}$$

can therefore be determine from the slope of the activation voltage as a function of $1/\lambda$.

The 2018 redefinition of the SI is means that the quantity

$$hc = 1.23984193 \text{ eV}\mu\text{m}$$

is technically now exactly known, because the values h, c, and e are now taken as exact values which define the corresponding SI units. Of course, it is still useful and fun to measure this quantity ourselves in the lab. We'll interpret this measurement as our determination of Planck's constant, using the known values for c and e.

2 Experimental Setup

To determine the activation voltage of our three LEDs, we'll make measurements as shown in Fig. 1. By varying the voltage V_1 and measuring the current and voltage drop across the diode, we can determine the $V-I$ curve for each diode and accurately determine it's activation voltage.

To implement this measurement with your Arduino kit, use the experimental setup shown in Fig. 2. By configuring pin A0 to 5 V and A2 as an output pin set to ground, the variable resistor R_{pot} becomes a potentiometer just as in the LED dimming example of the introductory lab, with a variable voltage at pin A1 controlled by turning the knob on your custom shield. This variable DC voltage will be applied to the device-under-test (one of the three LEDs reserved for this lab in the small plastic bag included with your kit) through the 100 Ω resistor R_{cur} (also reserved in the

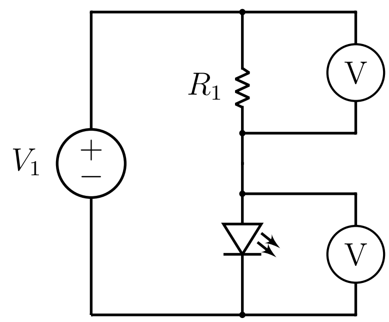

Figure 1: Conceptual setup for the LED measurements

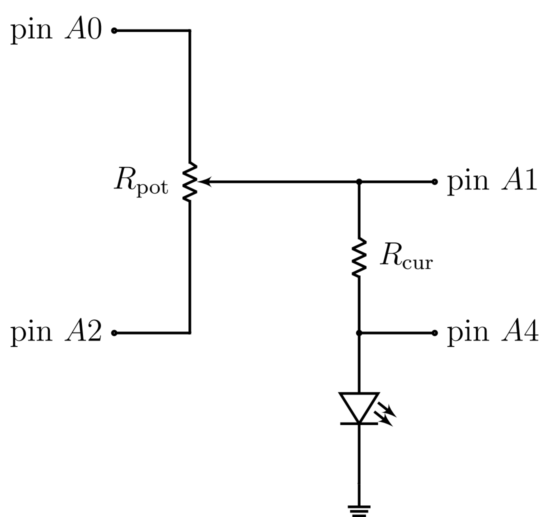

Figure 2: The experimental setup used to measure the V-I response of an LED diode.

small plastic bag). By measuring the voltage at pin A1 and pin A4, you can calculate the voltage across and current through the diode.

To setup the aparatus, recall that the potentiometer is already installed on your custom proto-shield. Use the tiny bread board, small bits of wire, and components included in your kit. If we had all quarter to work together in person, I would have spent as much time as I could teaching you the one real trick that I know: divide and conquer. Here's how I would implement this circuit, broken into multiple, **testable** steps.

Step 1: Create a circuit to test the LED and resistor part of the circuit independent of the potentiometer. Instead of driving the LED from A1, drive it from one of the Arduino's fixed 5 V outputs. Then connect the LED, resistor, and ground as shown in the circuit, leaving pin A4 disconnected for now. Try reversing the polarity of the LED if it does not light when the Arduino is powered. By testing it this way, you can test the hardware in a way that is independent of your software, making testing and debugging this stage that much simpler!

Step 2: Leave your LED circuit alone and setup the potentiometer as for the introductory lab, with the AnalogReadSerial sketch modified to make use of our potentiometer. For example, you need to configure the pins A0 and A2 in the `setup()` function as before:

```
// The potentiometer is installed at A0-A1-A2.
// Set A0 to VCC and A2 to ground
pinMode(A0, OUTPUT);
pinMode(A2, OUTPUT);
digitalWrite(A0, HIGH);
digitalWrite(A2, LOW);
```

Check that you are able to read the output voltage at A1 and display it on the serial port, and that it changes as your turn the knob.

Step 3: Replace the fixed 5 V driving your LED and resistor with the varibale voltage at pin A1. As you adjust the knob on the potentiometer, the LED should brighten or dim.

Step 4: Make the connection to A4. Adjust your sketch to read both A1 and A4, and write the results to the serial port.

Step 5: For convenience when taking lots of data, convert the ADC measurements to voltages and currents directly in your Arduino sketch, but use integer values in units of uA and mV, as it is challenging to display floating point numbers to the Arduino serial port. To convert an ADC value to a voltage in mA, recall that full scale is 1023 with a nominal voltage of 5000 mV, so use an integer calculation like:

$$V = 5000 * ADC/1023;$$

to obtain a current in uA you will have to account for the resistance and subtract two voltage levels. For scale, these LEDs should draw $10 - 30$ mA when fully bright and have a diode drop from

$1000 - 3000$ mV.

3 LED Model

For the purpose of this experiment, we will model the LED as an ideal diode with voltage drop equal to the activation voltage V_A of Equation 1 plus a series resistance R_LED. As shown in Fig. 3, the effect of this resistance is to replace the vertical line at the activation voltage V_A with a line of slope $1/R_\mathrm{LED}$.

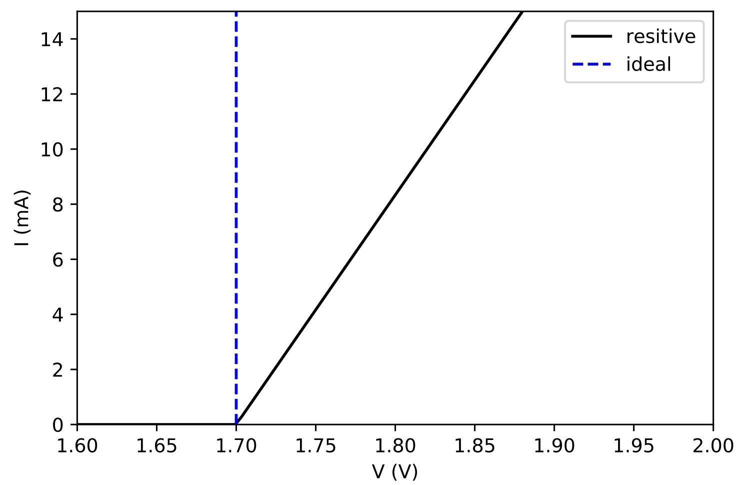

Figure 3: The LED model for $V_A = 1.7$ V and $R_\mathrm{LED} = 12\ \Omega$

4 Data Collection

For each of your three reseved LEDs (red, yellow, and green) take a series of measurements with the applied voltage adjusted so that $I = 5, 10, 15, 20$ mA and the maximum current (should be just above 20 mA). You will find that it is difficult to precisely set the current. Often when collecting data, you can measure more precisely than you can set, so always get as close to your target as you can conveniently reach, and then simply record the actual value. The target values simply give you data that is appropriately spaced across the dynamic range.

Record the color of each LED or simply assign each LED a letter (A,B,C) for now and determine the color from your data later. Example instructor data for the red LED is shown in the table.

5 Analysis

The V-I response for a red diode as measured by the instructor is plotted in Fig 4. Notice that at these currents the VI response is linear, indicating that it is dominated by internal resistance of the diode, and a simple linear fit will suffice to determine the activation voltage as the intercept with the voltage axis. An example fit for the instructor data is shown in Fig. 4. You can assume the uncertainty on each current measurement is 1 mA. For each LED, fit the V versus I data to a

Table 1: Instructor data for a red LED.

target I (mA)	I (uA)	V_D (mV)
5	5088	1661
10	10948	1705
15	16031	1730
20	21749	1759
MAX	25415	1779

Table 2: LEDs used in this experiment.

color	part no.	λ (nm)
green	LTL-4238	565
yellow	LTL-4256N	587
red	LTL-4268-H3	620

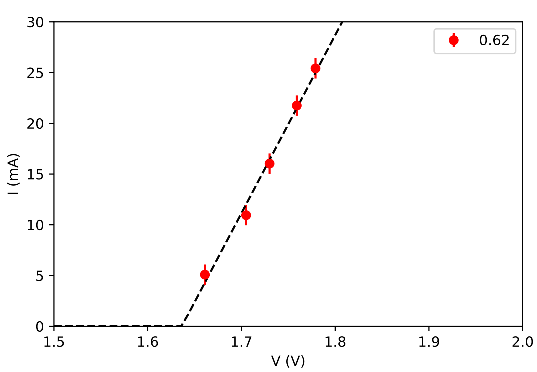

Figure 4: Instructor fit for the red LED.

linear function, determine the best fit resistance and activation voltage, and the uncertainty from the fit. The red LED will have the lowest activation energy, and the green LED the highest.

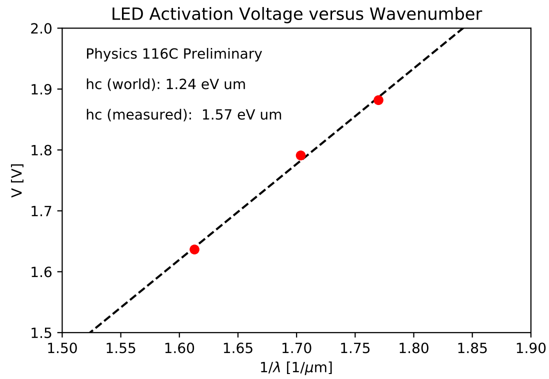

Figure 5: Instructor plot for determination of hc.

Using the best-fit values of V_A determined from the previous plots, plot V_A of each diode versus $1/\lambda$, using the wavelengths determined from the device specs. Determine the slope (hc/e) and its uncertainty from a linear fit. The instructor plot is shown in Fig. 5. Recall that 1 eV is the change in potential energy of one electron passing through 1 V of potential energy, allowing you to conveniently convert the slope in Vμm to eVμm for comparison with the established value $hc = 1.240$ eVμm. This measurement is dominated by systematic effects, notably our assumption that the constant ϕ is common to all devices, which I estimate to be about 20%.

6 Design Improvements

For fun and extra credit, you can automate the data taking process. Instead of using the potentiometer output as the adjustable voltage level, use PWM output at digital pin 5, filtered through the on-device RC filter, as in the function generator and digital scope exercise. This will give you digital control of the applied voltage level, which you can use to automatically scan the VI curve for the device-under-test.