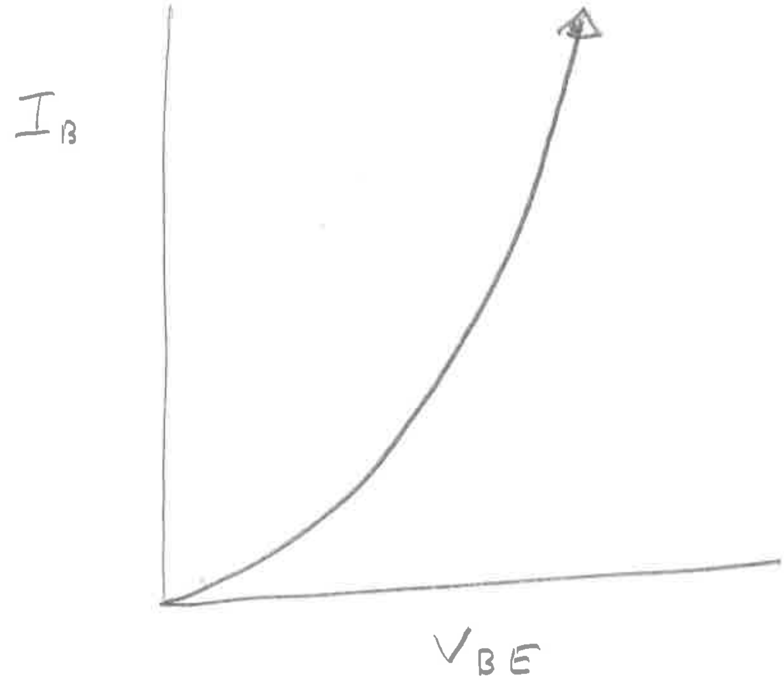
Transconductance

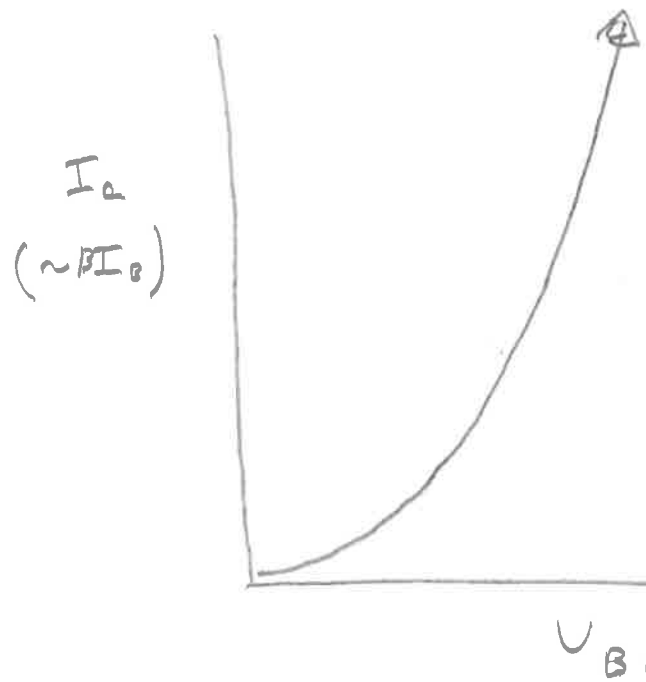
Model

for

Transistor

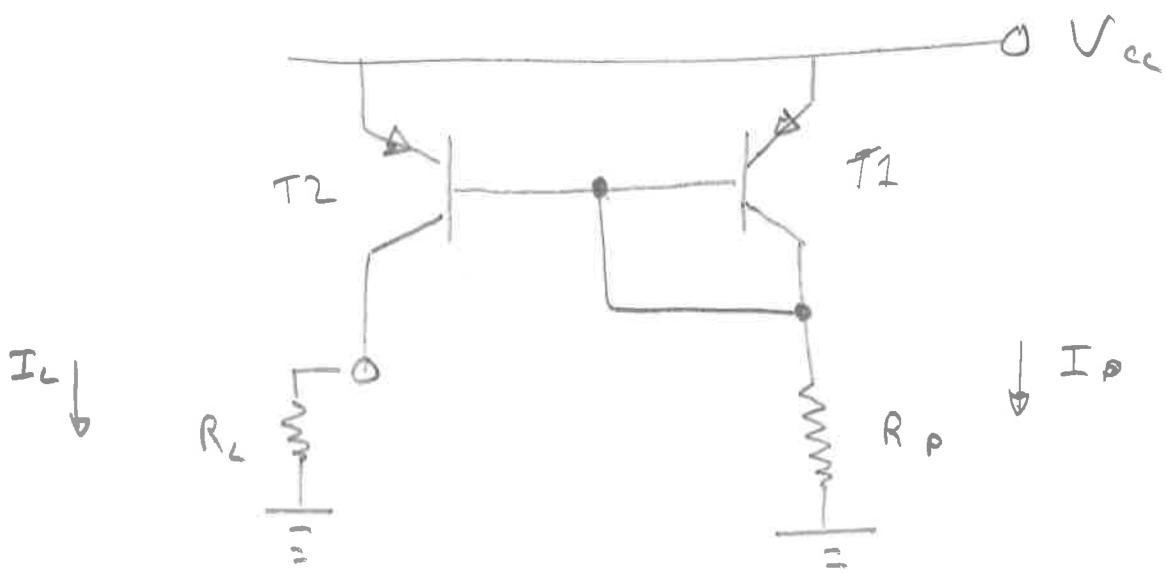
Transistor Curves





$$\sim I_s(t)\ \exp\left(-\frac{V_{BE}}{nV_T}\right)$$

Current Mirror



The short-circuit between E and B keeps V_C^1 about one diode drop below V_{cc}.

So, the "programming resistance" R_p causes current

$$I_p = \frac{V_{cc} - V_D}{R_p}$$ to flow.

E-M relates I_c to V_{BE}, and both T1 and T2 have the <u>same</u> V_{BE}. As a result, a current

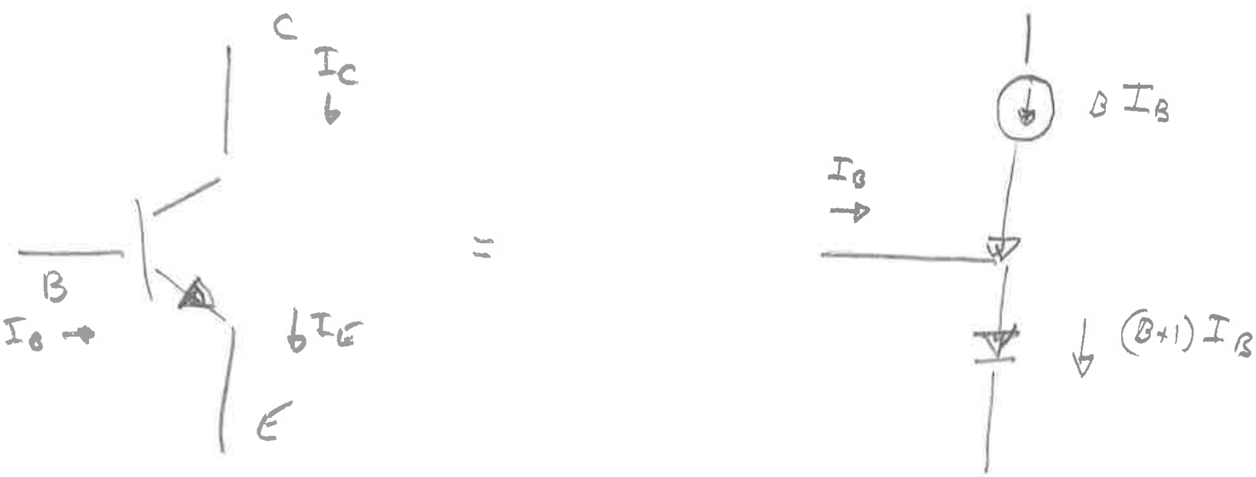
$I_L \sim I_p$ (match T_1 and T_2)

flows *independent* of R_L $(R_L < R_p)$

A <u>current</u> <u>source</u>.

(Note: large resistance <u>hard</u> for current source
easy for voltage source)

Large-Signal Model



(*) In literature, you'll see $\alpha \equiv \dfrac{I_C}{I_E} = \dfrac{\beta}{\beta+1}$

This is known as "Approximate Ebers-Moll" model, and it's sufficient to find DC operating point for circuit in active mode.

The B-E junction is a diode, so

$$I_B = I_0 \left(\exp\left(\frac{U_{BE}}{n V_T} \right) - 1 \right)$$

and the current source (βI_B) results in

$$I_C = \beta I_0 \left(\exp\left(\frac{U_{BE}}{n V_T} \right) - 1 \right)$$

We lump βI_0 into one T-dependent, material dependent factor:

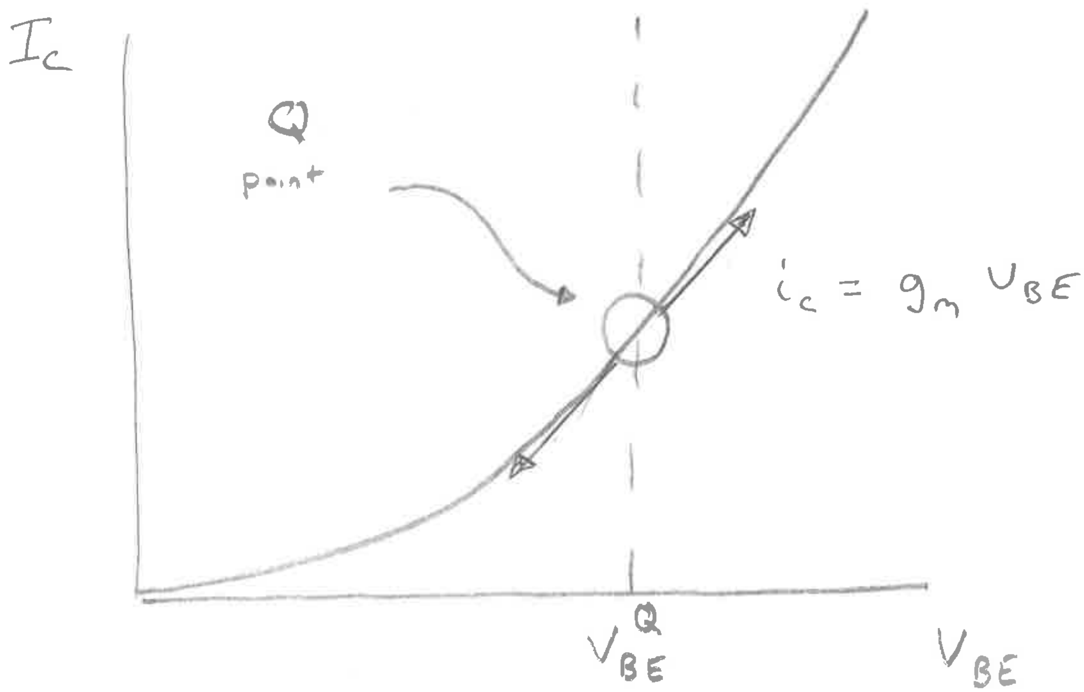
$$\boxed{I_C = I_S(T) \left(\exp\left(\frac{U_{BE}}{n V_T} \right) - 1 \right)}$$

Transconductance

The Eber's Moll result:

$$I_C = I_S(T)\left(\exp\left(\frac{V_{BE}}{n V_T}\right) - 1\right)$$

relates I_C to V_{BE} <u>not</u> I_B ...



So after determing Q-point, we can approximate small AC signal as linear relationship between i_c and v_{BE}

$$i_c = g_m v_{BE} \qquad [g_m] = \left[\frac{1}{R}\right]$$

g_m is called "transconductance":

$$g_m \equiv \left.\frac{d I_C}{d V_{BE}}\right|_Q$$

Transconductance

In active region

$$I_c = I_s(T)\left(\exp\left(\frac{V_{BE}}{n V_T}\right) - 1\right)$$

$$\sim I_s \exp\left(\frac{V_{BE}}{n V_T}\right)$$

So

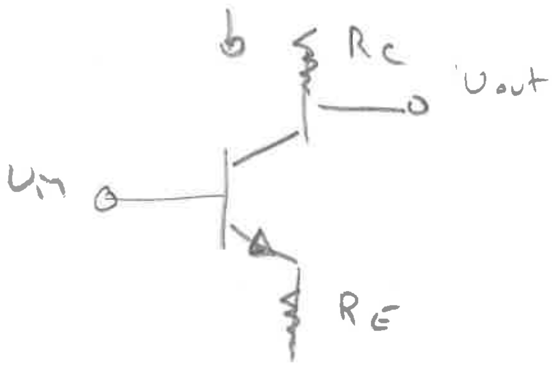
$$g_m \equiv \left.\frac{d I_c}{d V_{BE}}\right|_Q = \left.\frac{1}{n V_T} I_s \exp\left(\frac{V_{BE}}{n V_T}\right)\right|_Q$$

$$= \left.\frac{I_c}{n V_T}\right|_Q = \frac{I_c^Q}{n V_T}$$

Transconductance g_m is "set" by I_c^Q.

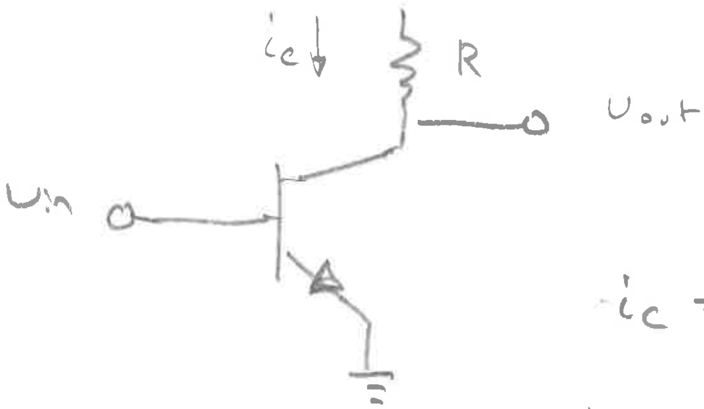
Unlike B, we controll I_c^Q to reasonable accuracy, and so g_m is a "good" parameter.

Infinite Gain Problem Resolved



$$G \sim -\frac{R_C}{R_E}$$

$$G \to -\infty \quad \text{as} \quad R_E \to 0$$



$$i_c = g_m V_{in}$$

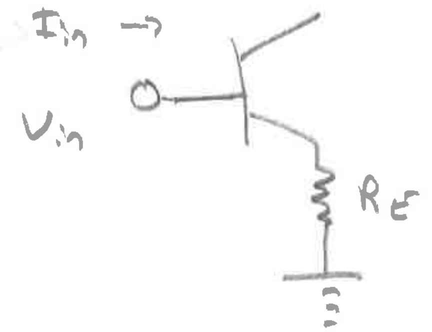
$$V_{out} \sim -\left(g_m R\right) V_{in}$$

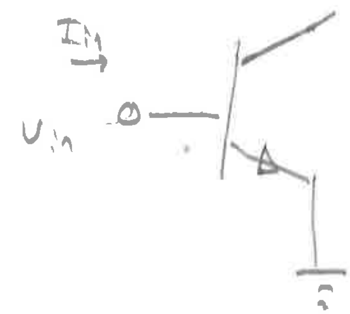
$$G \sim -g_m R$$

g_m takes role of $\frac{1}{R_E}$.

As if R_E has $r_E = \frac{1}{g_m}$ in series.

Input Impedance:





$$R_{in} = \frac{\Delta V_{in}}{\Delta I_{in}} = \beta R_E$$

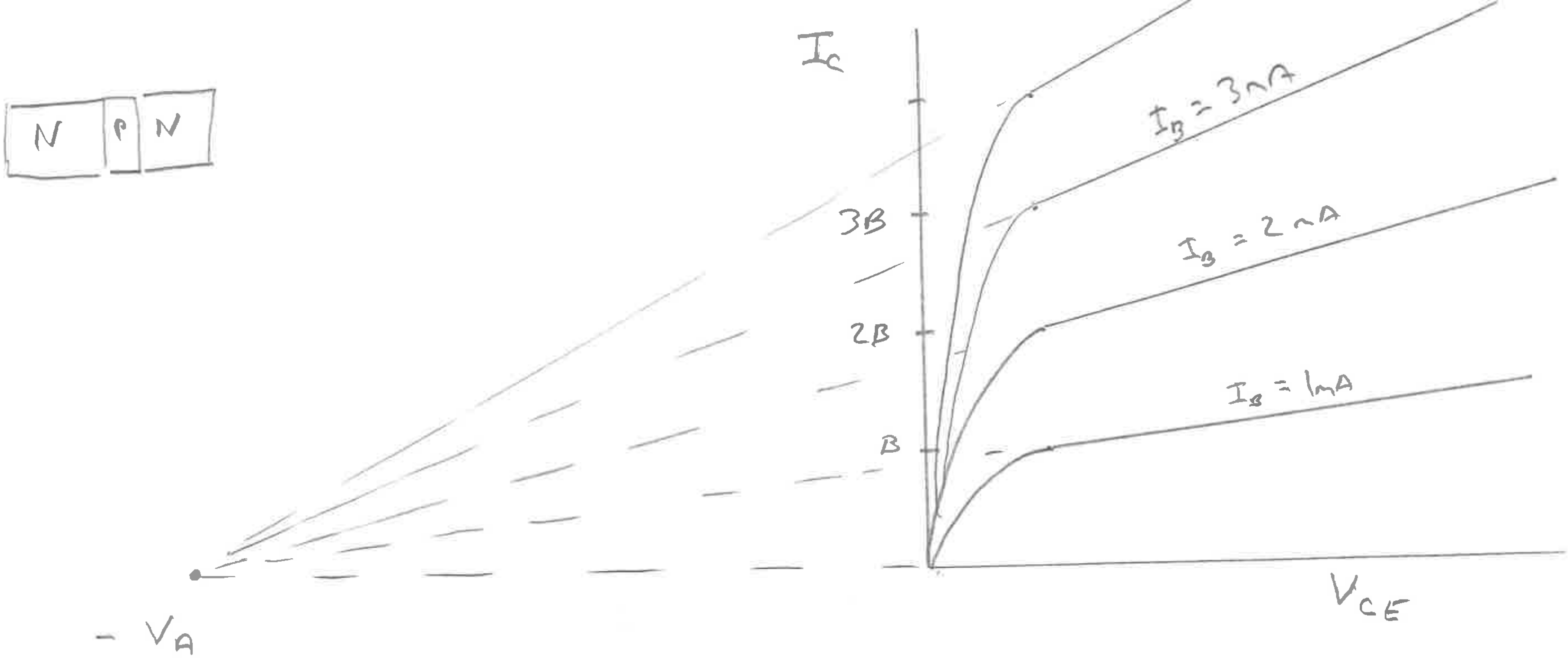
$$R_{in} = \left.\frac{d V_{in}}{d I_{in}}\right|_Q$$

$$= \left.\frac{d V_{BE}}{d I_B}\right|_Q = \beta \left.\frac{d V_{BE}}{d I_c}\right|_Q$$

$$= \beta \left(\frac{1}{g_m}\right)$$

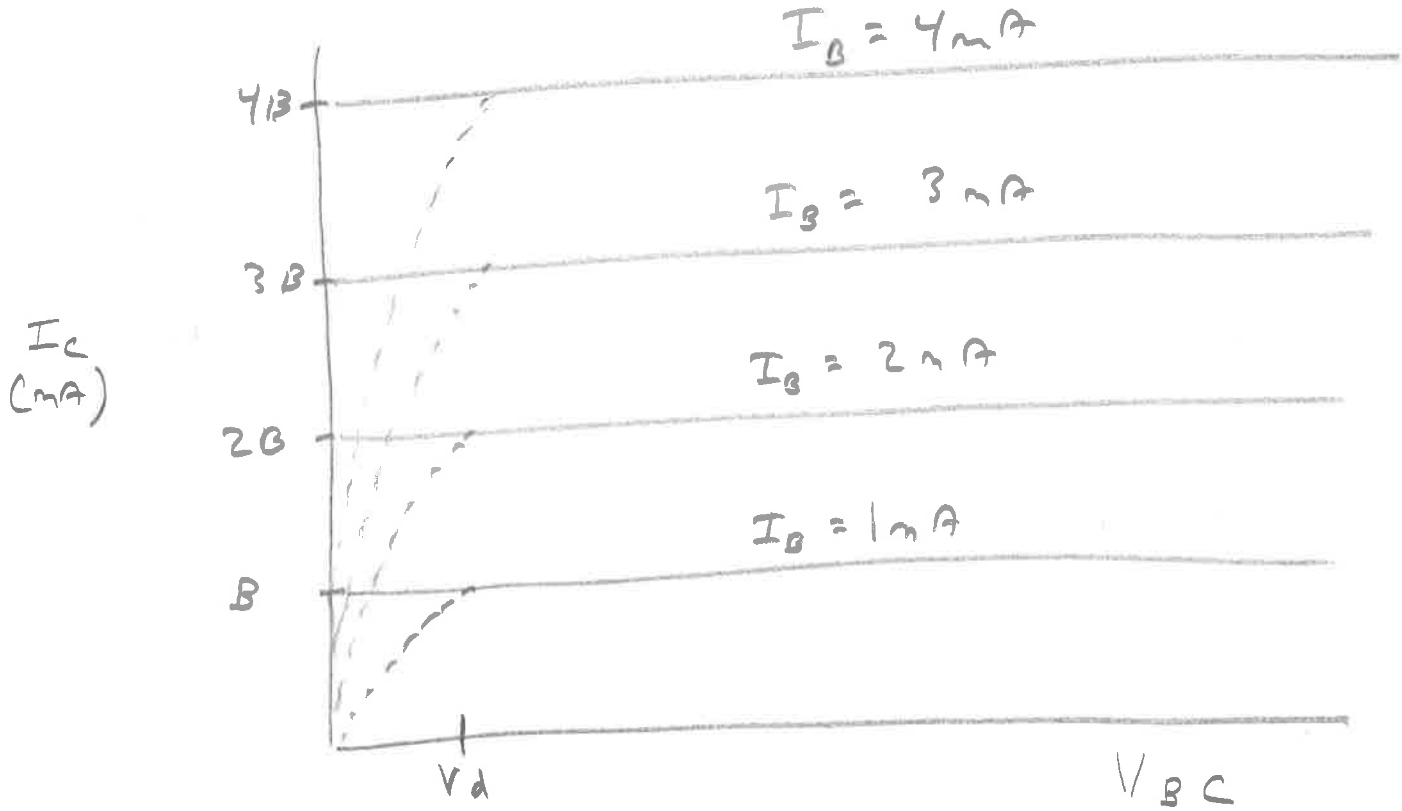
$$= \beta \, r_E$$

Again, as if R_E has small resistance r_E in series....



Increasing V_{BC} widens depletion zone, making "thin" p layer effectively much thinner ... more current!

$$R_o = \frac{\Delta V}{\Delta I} = \frac{V_A + V_{CE}^Q}{I_c^Q}$$



Transistor "B" Model.

Recall: in active region,

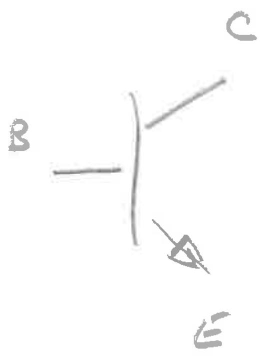
B-E is forward biased

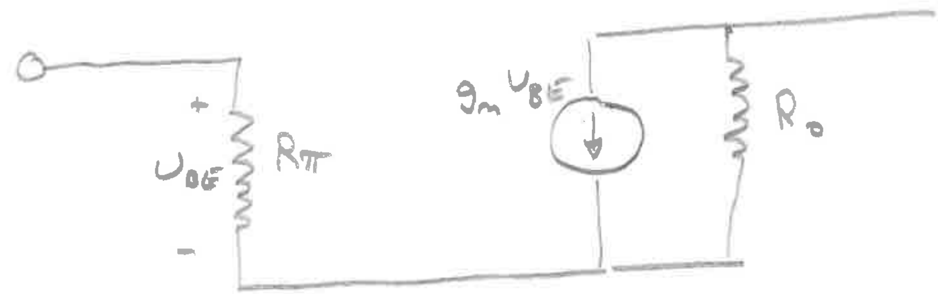
B-C is reverse-biased ... which is "good" because amplified current is due to minority carriers.

When $V_{BC} \lesssim V_D$, B-C is no longer reverse biased, becoming forward biased ... halts minority carriers ... current I_c goes to zero!

Hybrid - Pi Model

Small change v_{BE} from Q-point:





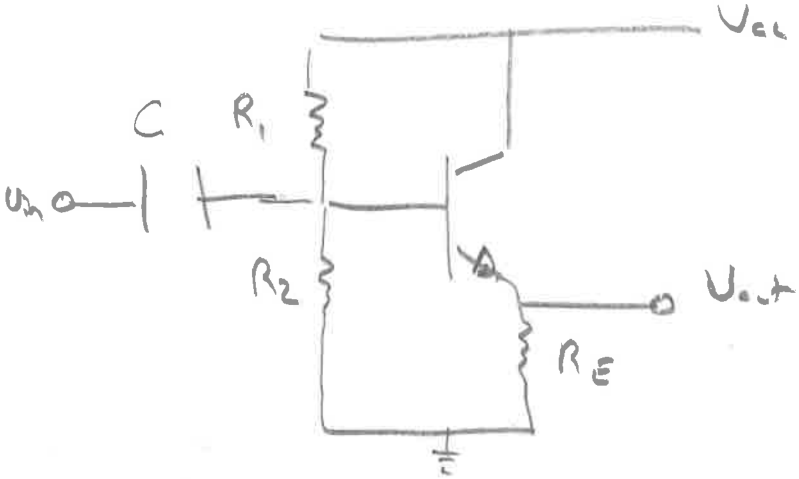
$$\frac{1}{r_E} \equiv g_m \equiv \left.\frac{d I_c}{d V_{BE}}\right|_Q = \frac{I_c^Q}{n V_T}$$

$$R_\pi = \beta r_E$$

$$R_o = \frac{V_A + V_{CE}^Q}{I_c^Q}$$

Hybrid Pi Example

Follower



Total

$V_{out}^Q = \frac{1}{2} V_{CC}$

$R_1 // R_2 << B R_E$

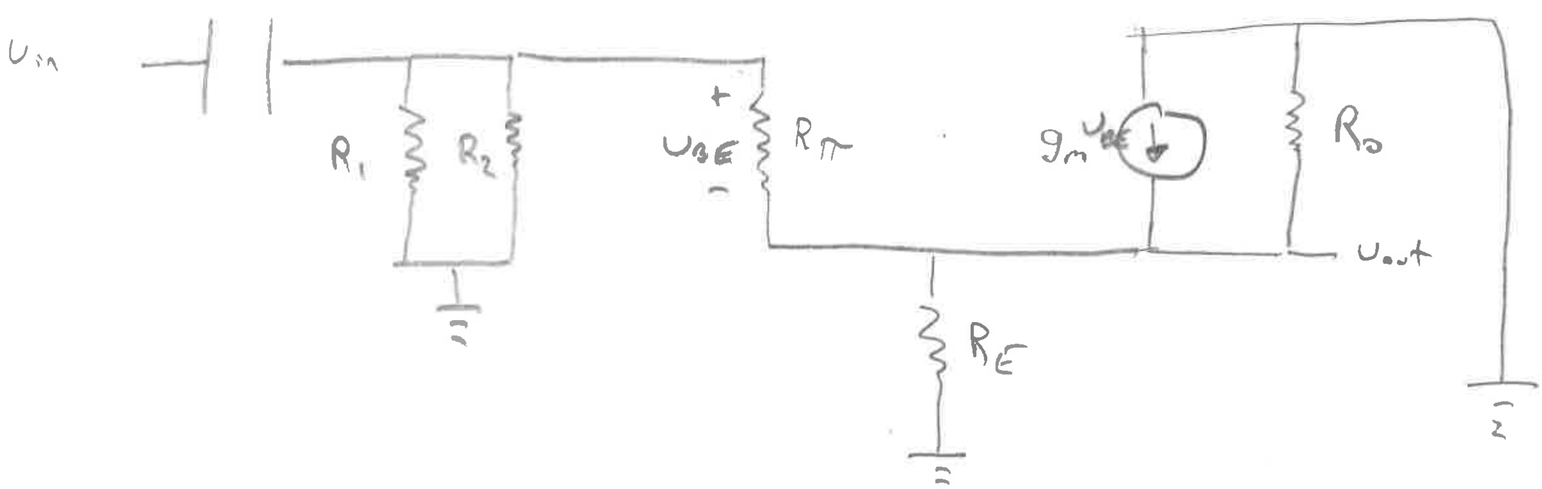
$B = 100$

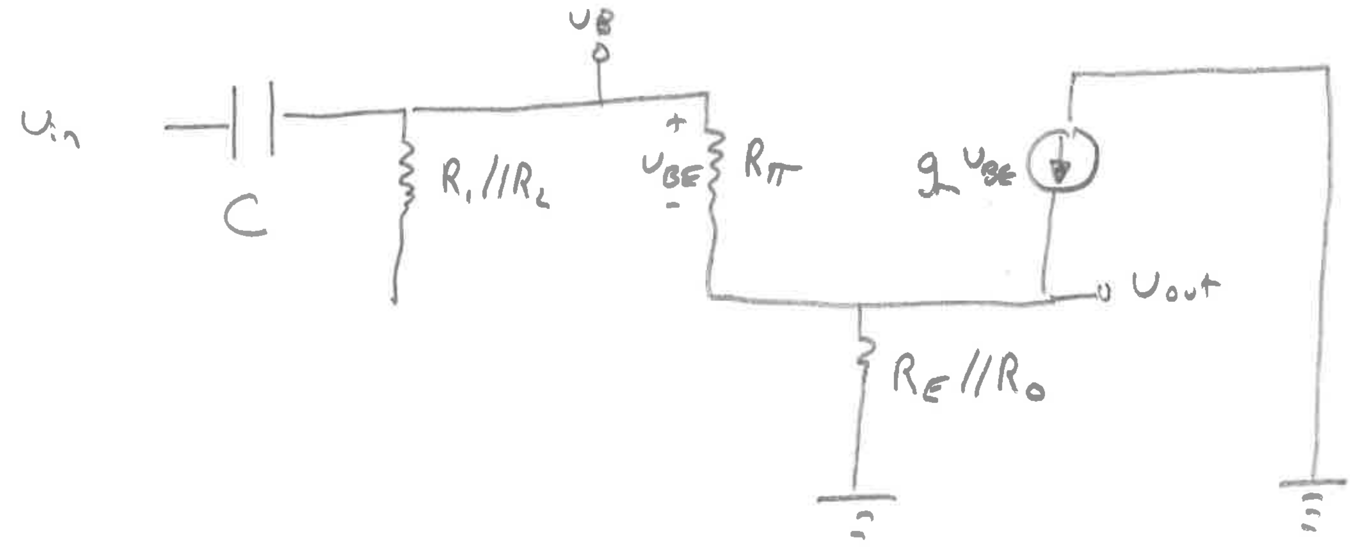
$I_C^Q = \frac{1}{2} V_{CC} / R_E$ ✓

$g_m = \dfrac{I_C^Q}{n V_T}$ ✓

$R_\pi = B \dfrac{1}{g_m}$ ✓

$R_o = \dfrac{V_A + \frac{1}{2} V_{CC}}{I_C^Q}$ ✓





$$V_{out} = (g_m v_{BE}) \; R_E // R_o$$

$$V_{BE} = v_B - V_{out}$$

$$V_{out} = g_m (v_B - V_{out}) \; R_E // R_o$$

$$V_{out} \left(1 + g_m (R_E // R_o)\right) = g_m R_E // R_o \; v_B$$

$$V_{out} = \left(\frac{g_m R_E // R_o}{1 + g_m R_E // R_o}\right) v_B$$

$$V_{out} = \left(\frac{A}{1 + A}\right) v_B \qquad (\sim v_B)$$

By design, $R_1 // R_2 \ll (R_\pi + \beta R_E // R_o)$

so:

$$v_B = \frac{R_1 // R_2}{\frac{j}{\omega C} + R_1 // R_2}$$

$$= \frac{1}{1 + j\frac{\omega_0}{\omega}} \qquad \omega_0 = \frac{1}{C \cdot (R_1 // R_2)}$$

$$v_{out} = \underbrace{\left(\frac{1}{1 + j\frac{\omega_0}{\omega}}\right)}_{\substack{\to 1 \\ \text{as } C \to \infty}} \underbrace{\left(\frac{A}{1 + A}\right)}_{\substack{\to 1 \\ \text{as } A \to \infty}} v_{in}$$

Amplifier

Told:

$R_o \to \infty$

$C \to \infty$

$V_{out}^{Q} = \frac{1}{2} V_{cc}$

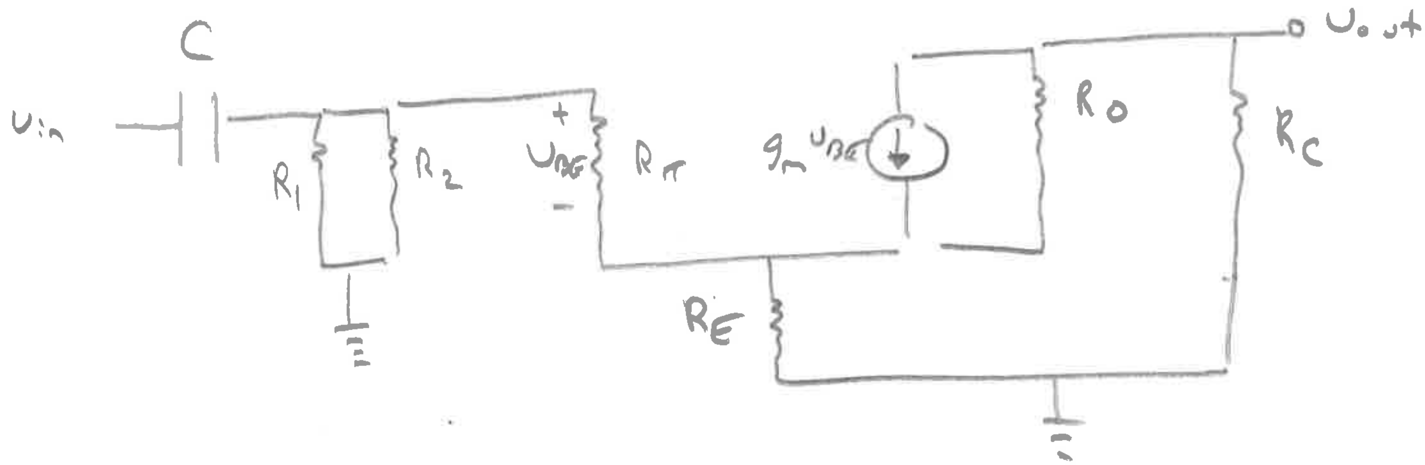
$\beta = 100$

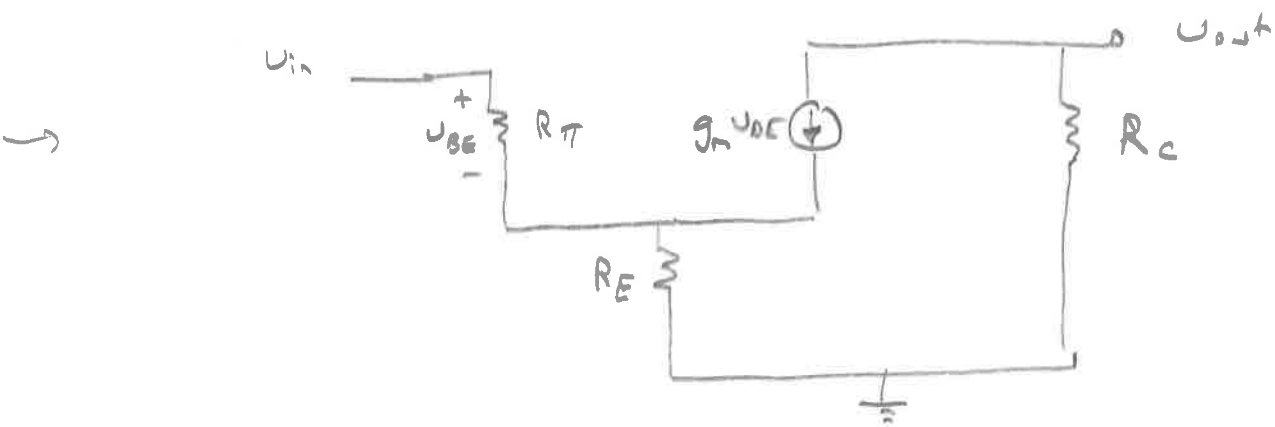
$R_1 // R_2 \ll \beta R_E$

$$I_c^Q = \frac{\frac{1}{2} V_{cc}}{R_c}$$

$$g_m = \frac{I_c^Q}{n V_T}$$

$$R_\pi = \beta \frac{1}{g_m}$$





$$v_{out} = -R_c\, g_m\, v_{BE}$$

$$v_{BE} = v_B - v_E$$
$$= v_{in} - v_E$$
$$= v_{in} - (R_E\, g_m)\, v_{BE}$$

$$v_{BE} = \frac{v_{in}}{1 + R_E g_m} \qquad (v_{BE} \to 0\ !)$$

$$v_{out} = -R_c\, g_m \left(\frac{v_{in}}{1 + R_E g_m} \right)$$

$$= -\frac{R_c}{R_E} \left(\frac{g_m R_E}{1 + g_m R_E} \right) v_{in}$$

$$\boxed{v_{out} = -\frac{R_c}{R_E} \left(\frac{A}{1 + A} \right) v_{in}}$$

$$\uparrow$$

$\longrightarrow 1$

as $\alpha \to \infty$