Homework Assignment 3
Infinite Square Well Potential and Fourier Series

Practice Problems

These problems are graded on effort only.

Griffiths: P1.17, P2.1bc, P2.3, P2.4

Additional Problems

Problem 1: The axiomatic definition of a vector space V over the real numbers $\mathbb{R}$ is detailed in Table 1.

(A) Show that D1 follows from A1-5 and M1-5. Hint: we already know for the scalars that $0+0=0$

(B) Show that D2 follows from A1-5, M1-5, and D1.

Problem 2: The axiomatic definition of an inner product space H over the real numbers $\mathbb{R}$ is detailed in Table 1.

(A) Show that D3 follows from A1-5,M1-5,D1-2 Hint: we already know for the scalars that $0+0=0$

(B) Show that D4 follows from A1-5,M1-5,D1-3.

Table 1: Here we define the properties of a vector space V and an inner product space H. Note that no complex conjugation appears in these definitions as the scalar field is the real numbers numbers.

Useful Math Symbols:

$\forall\, x \in V$	for all x in V (for any vector x)
$\forall\, \alpha \in \mathbb{R}$	for all α in $\mathbb{R}$ (for any real number α)
$\exists!\, y$	there exists unique y
s.t.	such that

Properties of Addition:

A1	**Closure**	$\forall x, y \in V$	$(x + y) \in V$
A2	**Commutative**	$\forall x, y \in V$	$x + y = y + z$
A3	**Associative**	$\forall x, y, z \in V$	$(x + y) + z = x + (y + z)$
A4	**Zero**	$\exists!\ 0$ s.t. $\forall x \in V$	$x + 0 = x$
A5	**Inverse**	$\forall x \in V \exists!\ (-x) \in V$ s.t.	$x + (-x) = 0$

Properties of Scalar Multiplication:

M1	**Closure**	$\forall x \in V$ and $\forall \alpha \in \mathbb{R}$	$\alpha x \in V$
M2	**Identity**	$\forall x \in V$	$1x = x$
M3	**Associative**	$\forall x \in V$ and $\forall \alpha, \beta \in \mathbb{R}$	$\alpha(\beta x) = (\alpha\beta)x$
M4	**Distributive**	$\forall x, y \in V$and $\forall \alpha \in \mathbb{R}$	$\alpha(x + y) = \alpha x + \alpha y$
M5	**Distributive**	$\forall x \in V$and $\forall \alpha, \beta \in \mathbb{R}$	$(\alpha + \beta)x = \alpha x + \beta x$

Deducible Properties:

D1	$\forall x \in V$	$0x = 0$
D2	$\forall x \in V$	$(-1)x = (-x)$

Properties of Inner Products:

I1	$\forall x, y \in H$	$\langle x\|y\rangle = \langle y\|x\rangle$
I2	$\forall x, y, z \in H$and $\forall \alpha \in \mathbb{R}$	$\langle x\|\alpha y\rangle = \alpha \langle x\|y\rangle$
I3	$\forall x, y, z \in H$	$\langle x + y\|z\rangle = \langle x\|z\rangle + \langle y\|z\rangle$
I4	$\forall x \in H$	$\langle x\|x\rangle \geq 0$
I5	$\forall x \in H$	$\langle x\|x\rangle = 0$ if and only if $x = 0$

Deducible Properties:

D3	$\forall x, y \in H$and $\forall \alpha \in \mathbb{R}$	$\langle \alpha x\|y\rangle = \alpha \langle x\|y\rangle$
D4	$\forall x, y, z \in H$	$\langle x\|y + z\rangle = \langle x\|y\rangle + \langle x\|z\rangle$