Intro to Semiconductors

Time Independent SE in 1-D:

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + U(x)\,\psi_E(x) = E\,\psi_E(x)$$

ψ_E is energy eigenstate with energy E

$U(x)$ is potential energy

Free Particles: $U(x) = 0$

$$-\frac{\hbar^2}{2m}\frac{d^2\psi_E(x)}{dx^2} = E\,\psi_E(x)$$

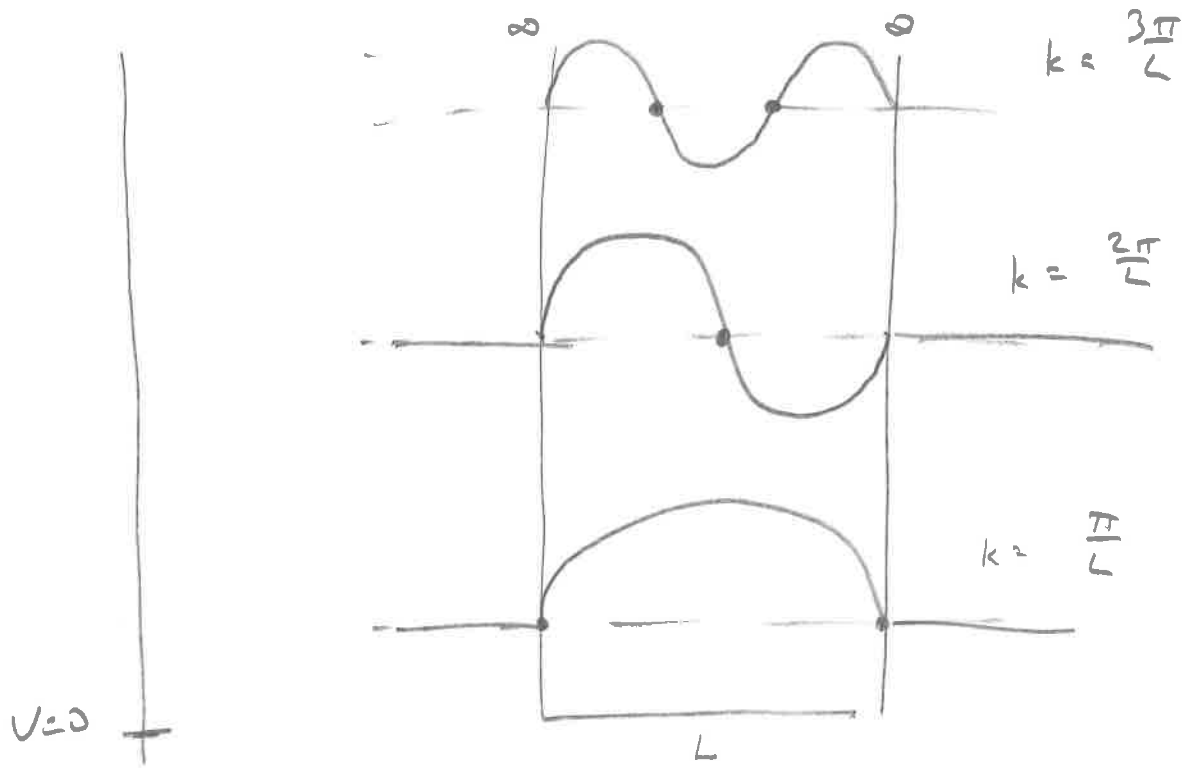
$$\psi_E(x) = e^{ikx}$$

$$-\frac{\hbar^2 (ik)^2}{2m} = E$$

$$\frac{\hbar^2 k^2}{2m} = E = \frac{p^2}{2m} \qquad \boxed{p = \hbar k}$$

$\psi_E \sim e^{ikx}$ is free particle with momentum $p = \hbar k$, k is a continuous variable (no quantization).

Particle In Infinite Well



Wave equation must go to zero at boundary.

Solution inside is free-particle.

Energy can be calculated because in well, $U(x) = 0$ and

$$E = \frac{p^2}{2m} = \frac{\hbar k^2}{2m}$$

*** More <u>nodes</u> = <u>higher</u> <u>energy</u>! ***

General Sketching of Wave Equations

$$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\psi_E(x) = \left[E - U(x)\right]\psi_E(x)$$

Qualitatively

1) ψ_E is continuous everywhere, and goes to 0 as $x \to \pm\infty$

2) Where $E > V(x)$ $\to$ oscillatory function

3) where $E < V(x)$ $\to$ exponential

4) $E \sim \#$ nodes

5) ψ_E has no kinks if potentials are finite

Proof (5):

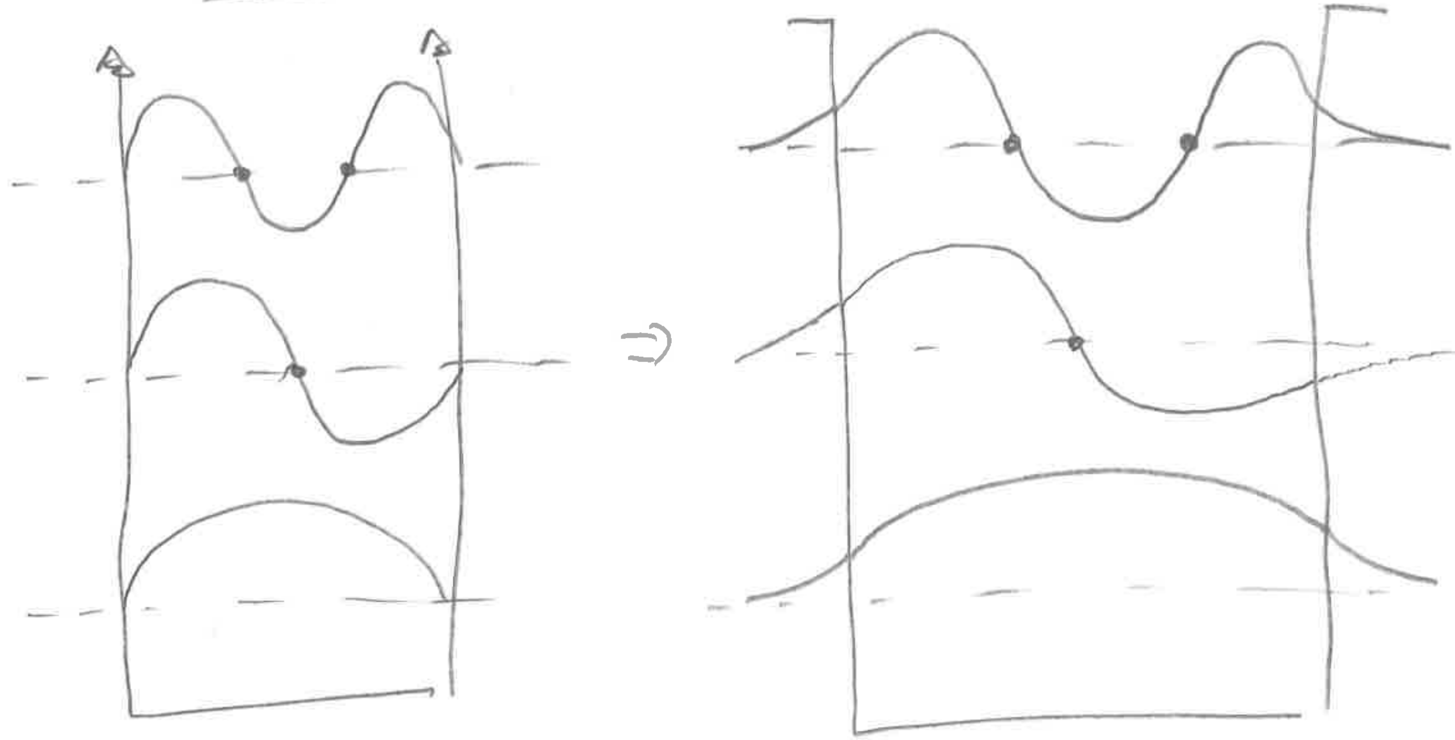
RHS looks at worst

 $= \dfrac{d^2\psi}{dx^2}$

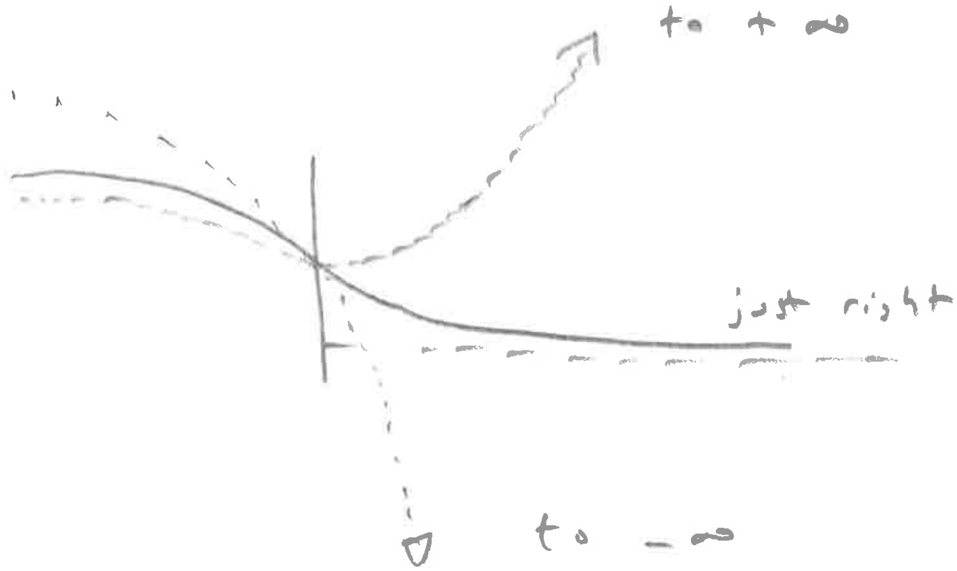
$\Rightarrow \dfrac{d\psi}{dx} =$  $\leftarrow$ kink

$\psi =$  $\leftarrow$ smooth

Particles in Finite Well

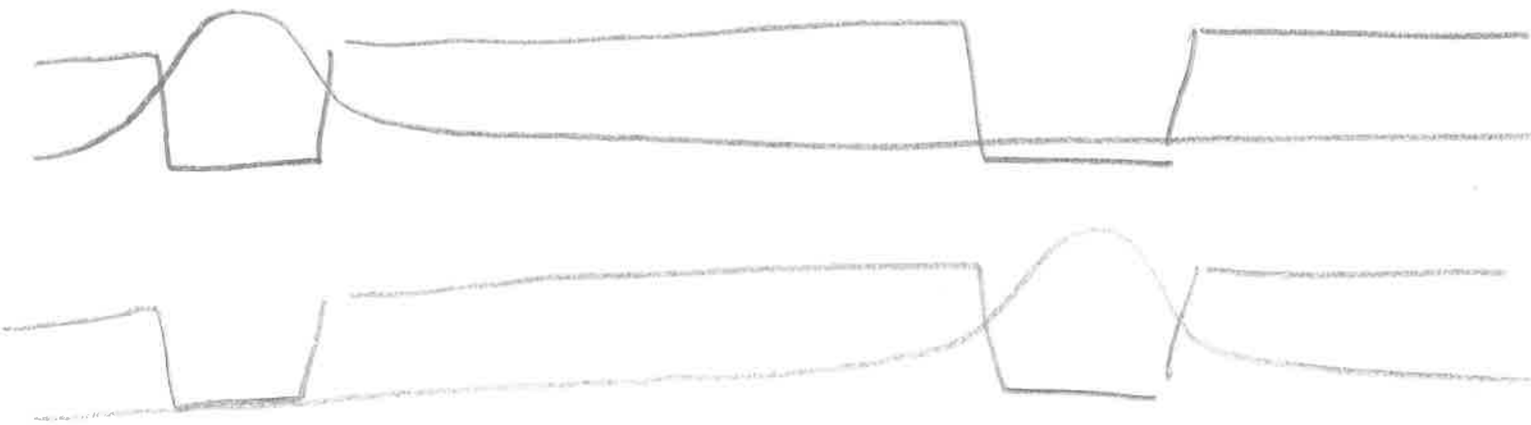


Quantization persists because only certain values of E tie an oscillatory solution to an exponential that doesn't diverge as $x \to \infty$

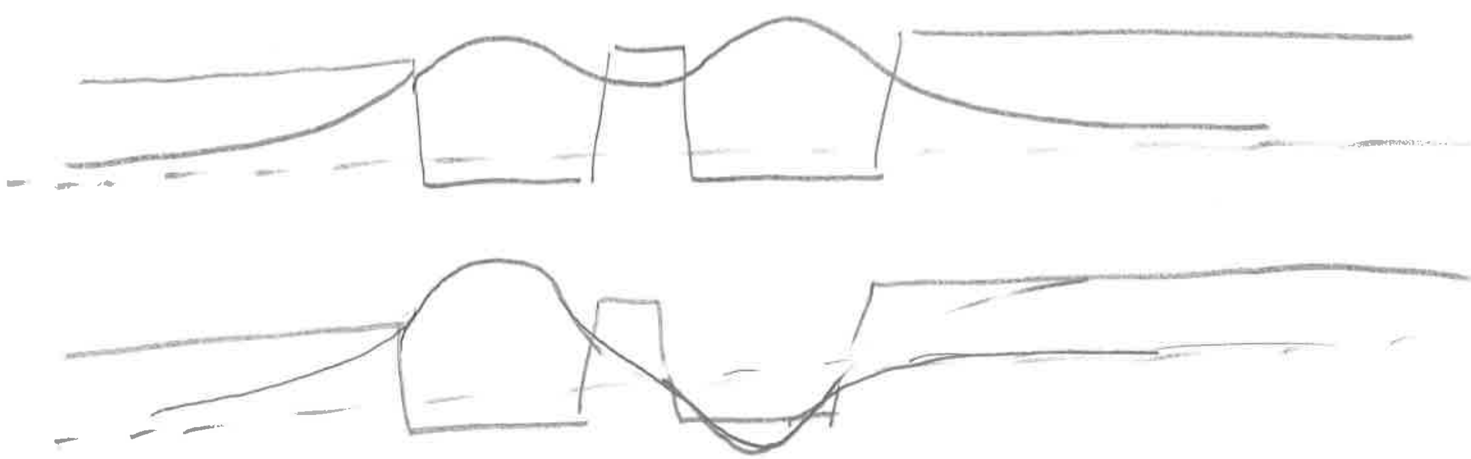


Two particles in two wells:

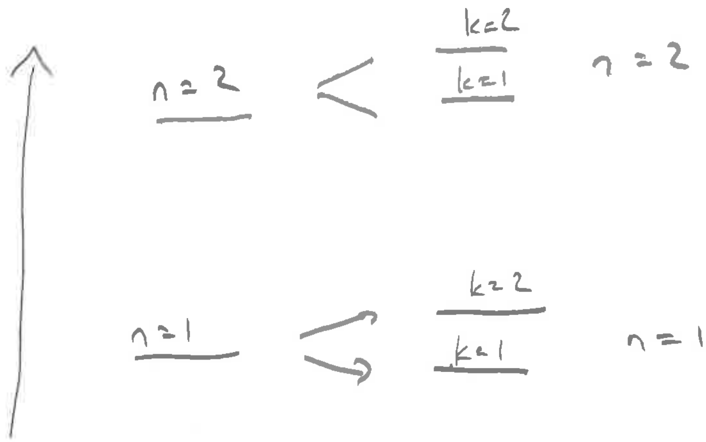
Far Apart



But close together



Must be even or odd, due to symmetry.
Odd solution (assymmetric) has higher energy, due to one more node.



Chemistry

(Energy of covalent bond is $2\Delta E$ ...)

"This is, like, 70% of chemistry" goes over well!

4 wells

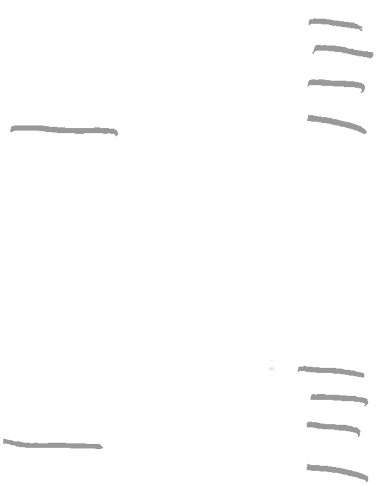
"Envelope"

0 nodes

1 node

2 nodes

3 nodes



n = 2

n = 1

For periodic potential

Bloch showed

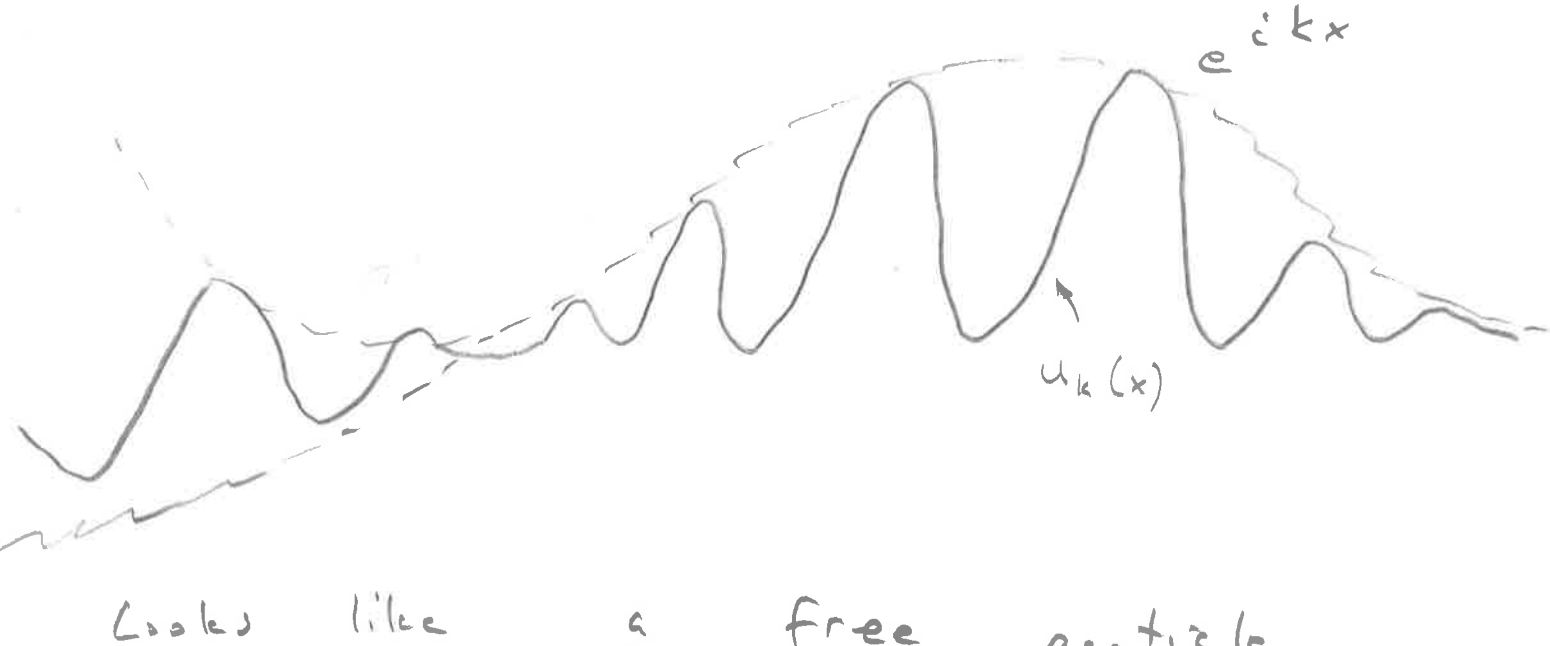
$$\psi(x) = e^{ikx} u_k(x)$$

where:

$$u_k(x) = u_k(x+a)$$

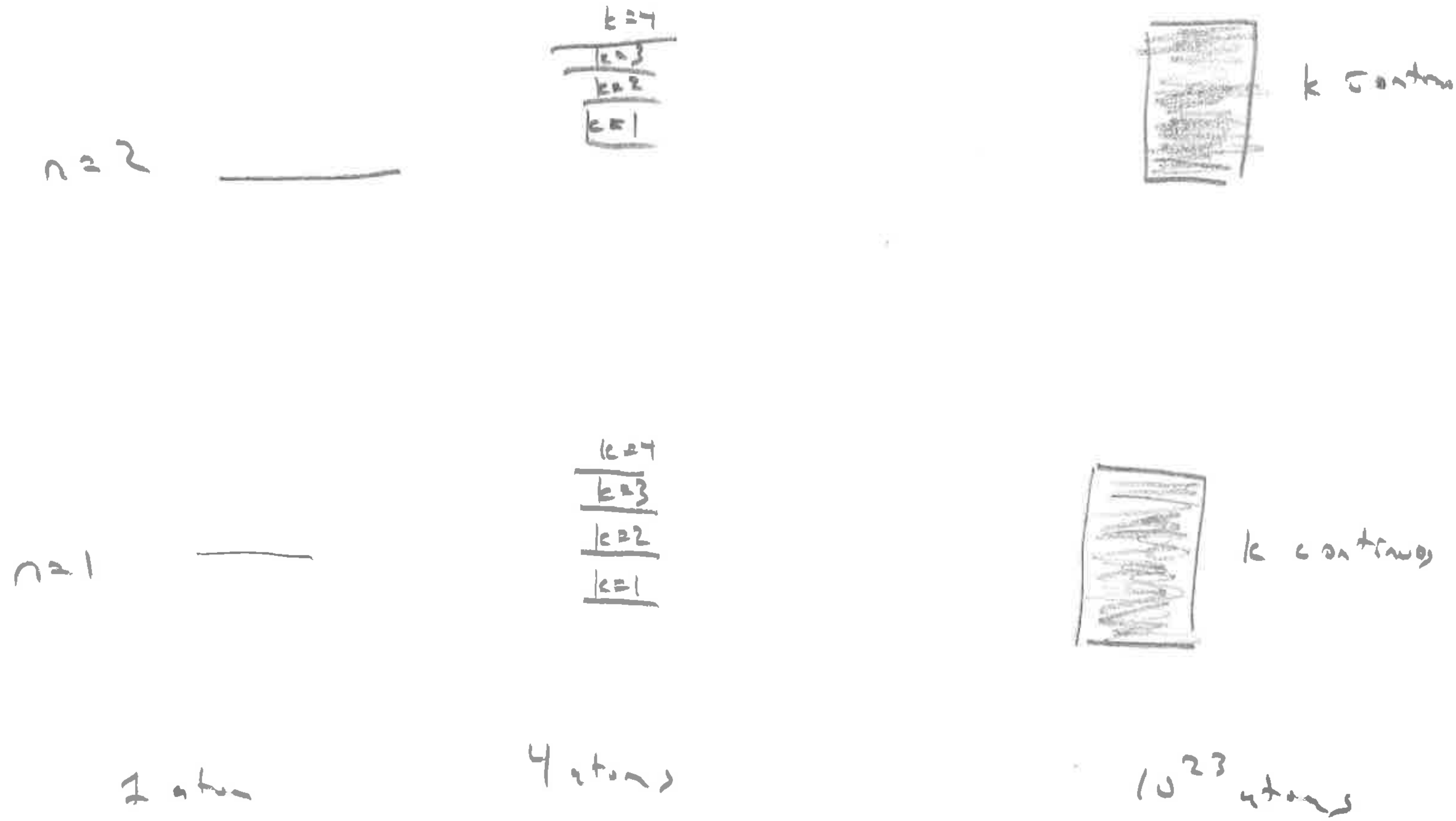
k is continuous;

The "envelope" is continuous



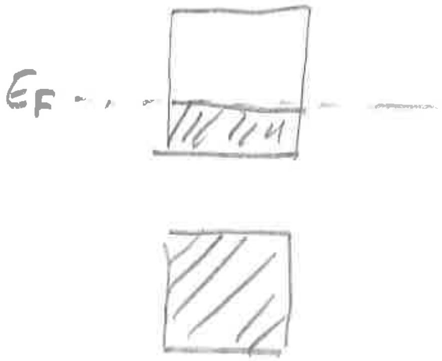
Looks like a free particle

with momentum $p = \hbar k$.



* Within bands, wave equations are similar to free particle wave equations, with continuous value "k".

Conductors:

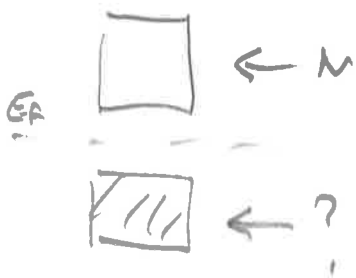


approximately free electrons (wave functions of continuous wave number k!)

(Drude model ~ valid!)

Recall: $\boxed{\sigma \sim n q^2 \tau / m}$

Q: What about a completely <u>full</u> band? Can it conduct?



Answer: No . . .

Why?

$$\psi_1 = e^{i(kx - \omega t)}$$

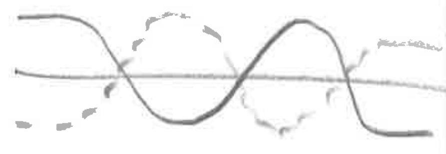
Re ψ_1 

$$\psi_2 = e^{i(-kx - \omega t)}$$

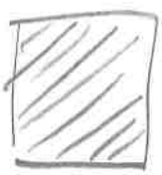
Re ψ_2 

$$\psi_1 + \psi_2 = e^{-i\omega t} \cdot \left(e^{ikx} + e^{-ikx}\right)$$

$$= e^{-i\omega t}\, 2\cos kx$$



<u>Standing wave</u>



In full Energy band, all k-states are full, and wave in one direction exactly cancelled by wave in another direction.

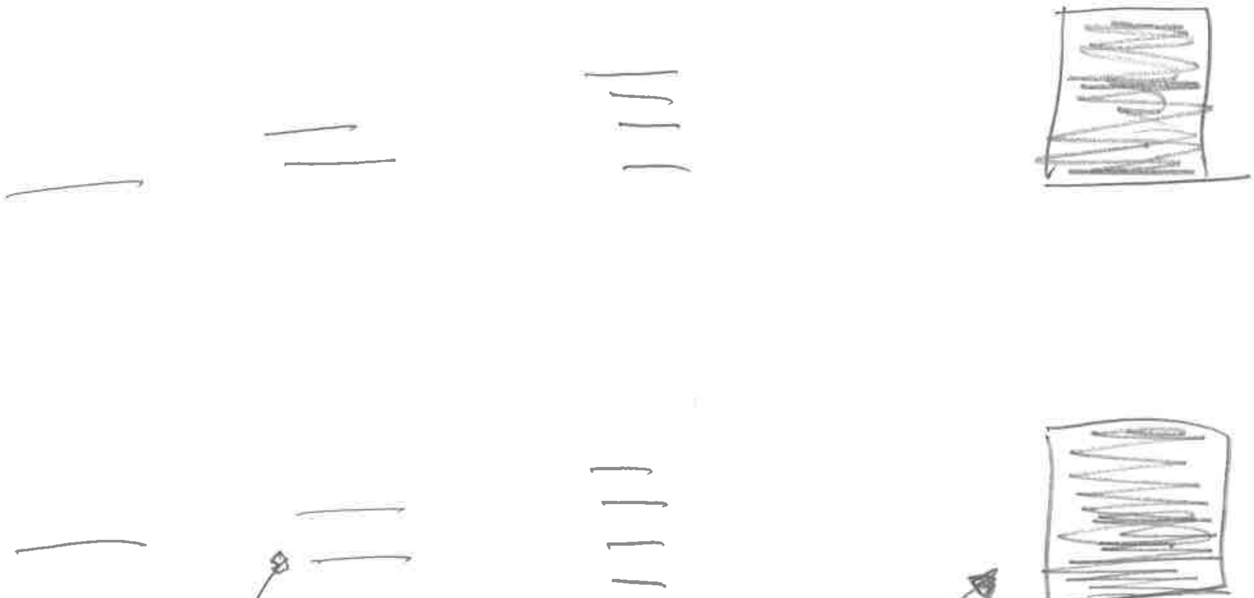
There's no way, with small amount of energy, for total wave function to have net current.....

(End Lecture 1)

Recap:

n=2

n=1



1 atom 2 atoms 4 atoms 10^{23} atoms

(Origin of bands)

Continuous Bands, when full, no conduction possible!

Conduction + Valence

Conduction

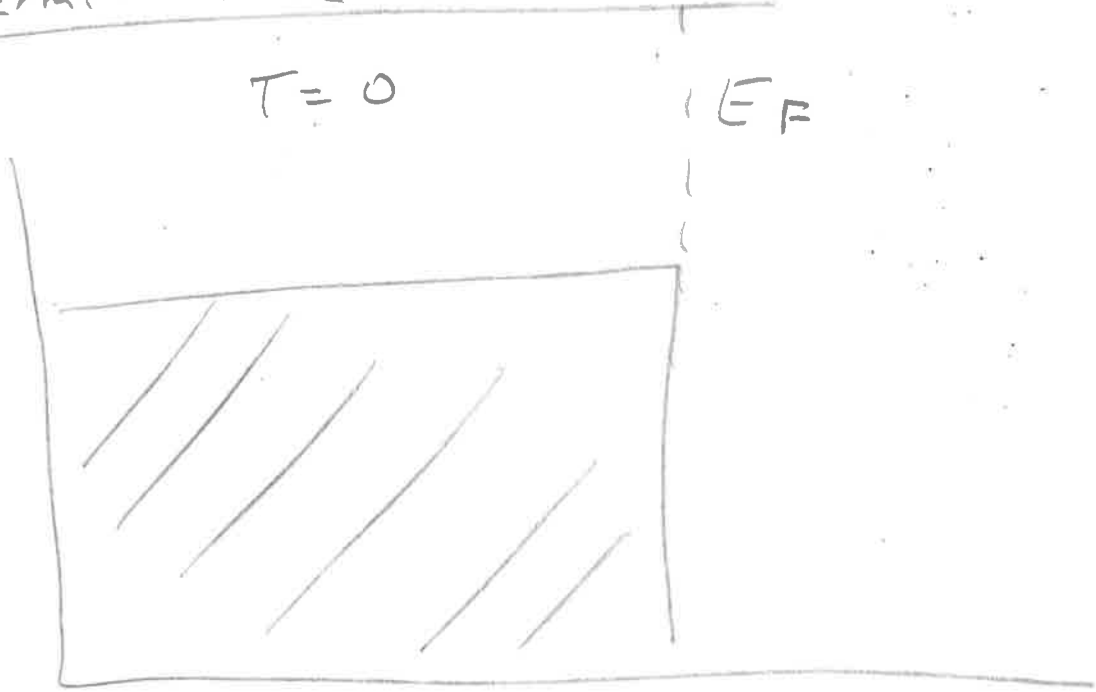
Valence

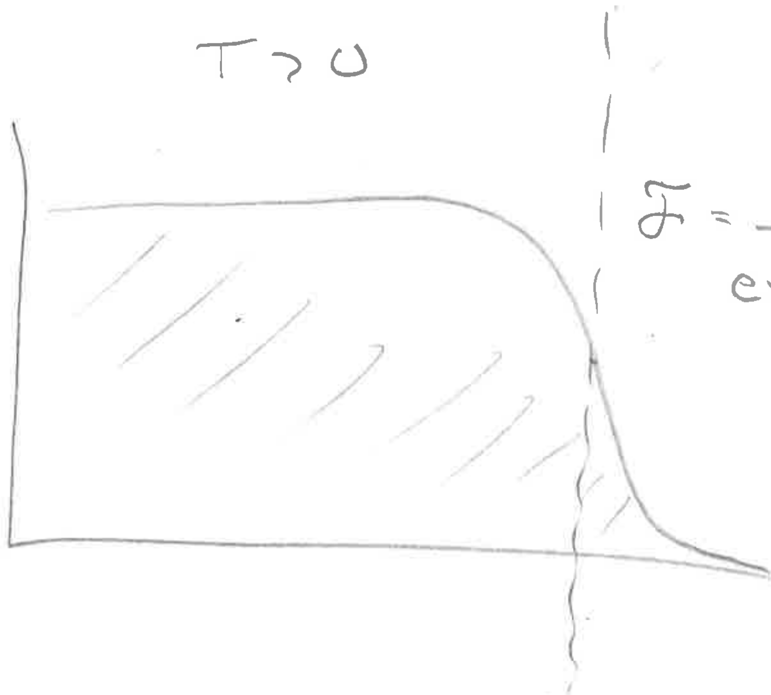
Conductor Insulator

Conduction Band = lowest band not fully occupied (@T=0)

Valence Band = highest band not empty (@T=0)

Fermi - Dirac Distribution





$$\mathcal{F} = \frac{1}{\exp\left(\frac{E - E_F}{kT}\right) + 1}$$

E

At $E \gg E_F$ $\qquad \mathcal{F} \sim \exp\left(-\frac{E - E_F}{kT}\right)$

which looks like a boltzmann factor but relative to the E_F.

Intrinsic Semiconductors: $T > 0$

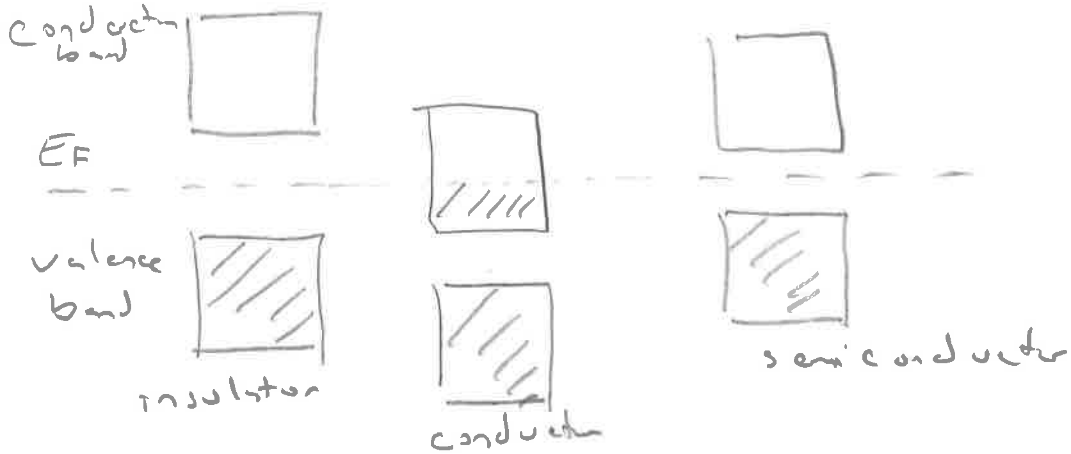
E_F is midway, equal number of electrons in conduction band, and vacant electron states in valence band (holes).

Holes are mobile charge carriers as well as electrons in valence band. *

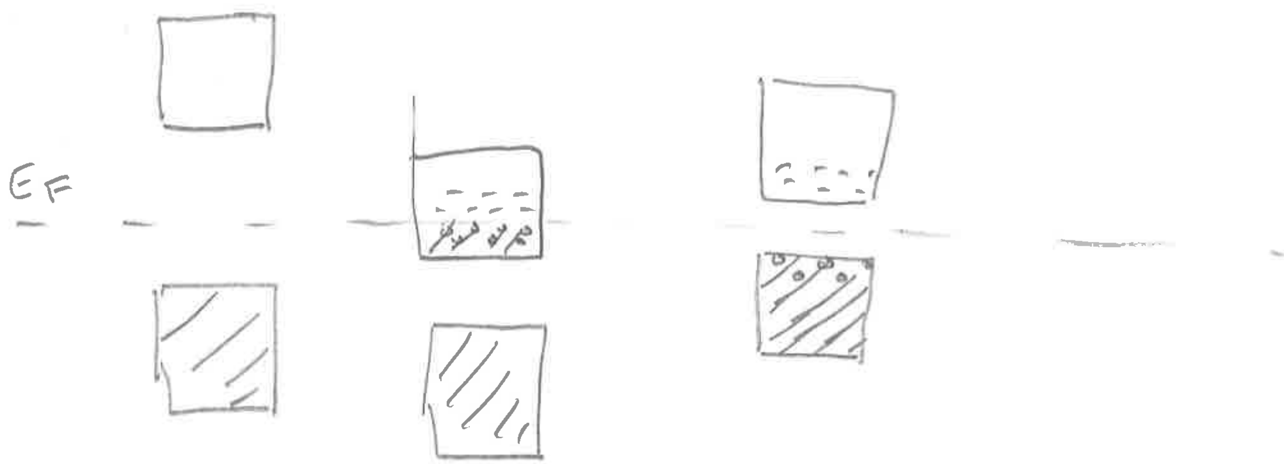
Intrinsic semiconductors have equal #'s: $p = n$

(*) More accurately think of standing wave $\psi_L + \psi_R$ minus one component ψ_L ...

$T = 0$



$T \sim 275$ K



Semiconductors have small band gap (~ 1 eV) such that thermal fluctuations lead to electrons in conduction band (and holes in valence band)

Exercise: Given that band gap is $\sim 1\,eV$, estimate how much less conductive semiconductors are compared to conductors.

Soln:

Recall $\sigma = \dfrac{n q^2 \tau}{m}$

Assume τ is comparable and only difference in σ is from n, density of mobile charge carriers (which will assume are just electrons in conduction band)

$$\frac{\sigma_{sc}}{\sigma_c} = \frac{n_{sc}}{n_c} = \exp\left(-\frac{E-E_F}{kT}\right)$$

$$= \exp\left(-\frac{E_G/2}{kT}\right)$$

$E_G = 1\,eV$ $\quad$ kT at $275 \sim \frac{1}{40}\,eV$

$$\frac{\sigma_{sc}}{\sigma_c} \simeq \exp(-40/2) = \exp(-20)$$

$$= 10^{-9}$$

$\sigma(Cu) = 10^{7}\ S/m$

$\sigma(Si) \quad 10^{-3}\ S/m$

(Not For Lecture)

Aside: why is E_F defined at midway between highest occupied and lowest unoccupied? (Or, why is such a qty the chem potential)

Ans: E_F is the "chemical potential" for the fermions, i.e. the energy at which it is at equilibrium,

So consider

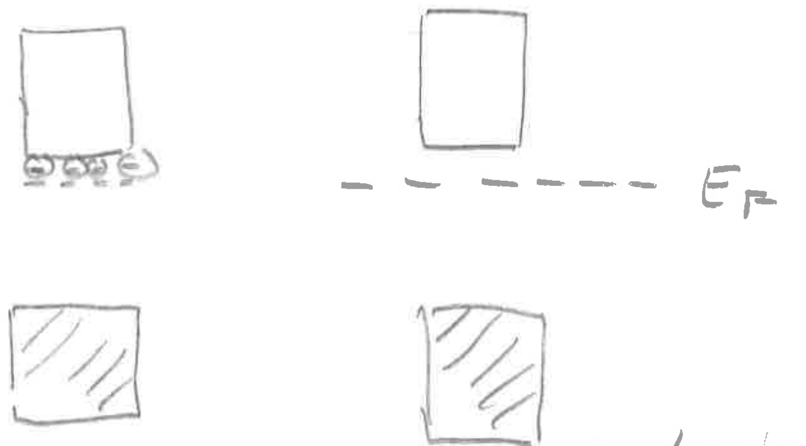
S1 is not in equilibrium with S2, since S1 can donate to S2 for $\Delta U_{TOT} = 0$. But S2 can donate to S1 only at $\Delta U_{TOT} = E_G$.

S1 and S3 are in equilibrium,

S1 → S3 cost $E_G/2$

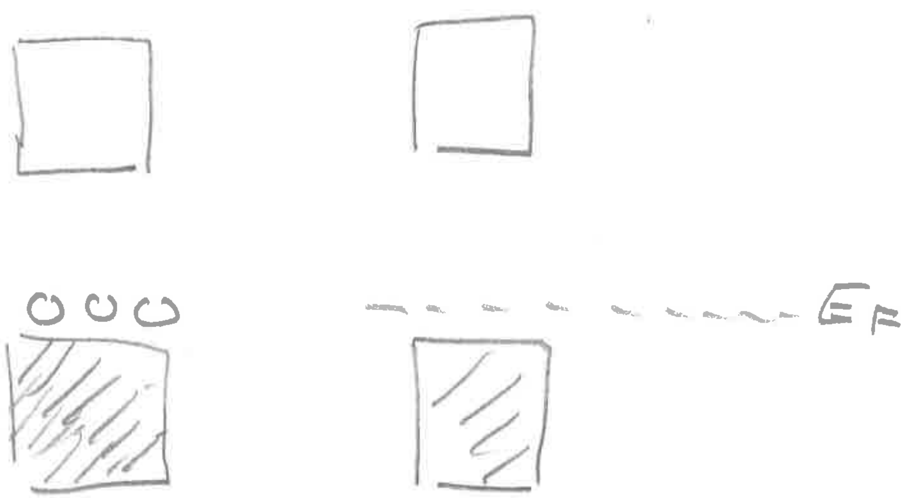
S3 → S1 costs $E_G/2$!

By doping semiconductor, artificial electron absorber or donors can be placed near valence or conduction bands

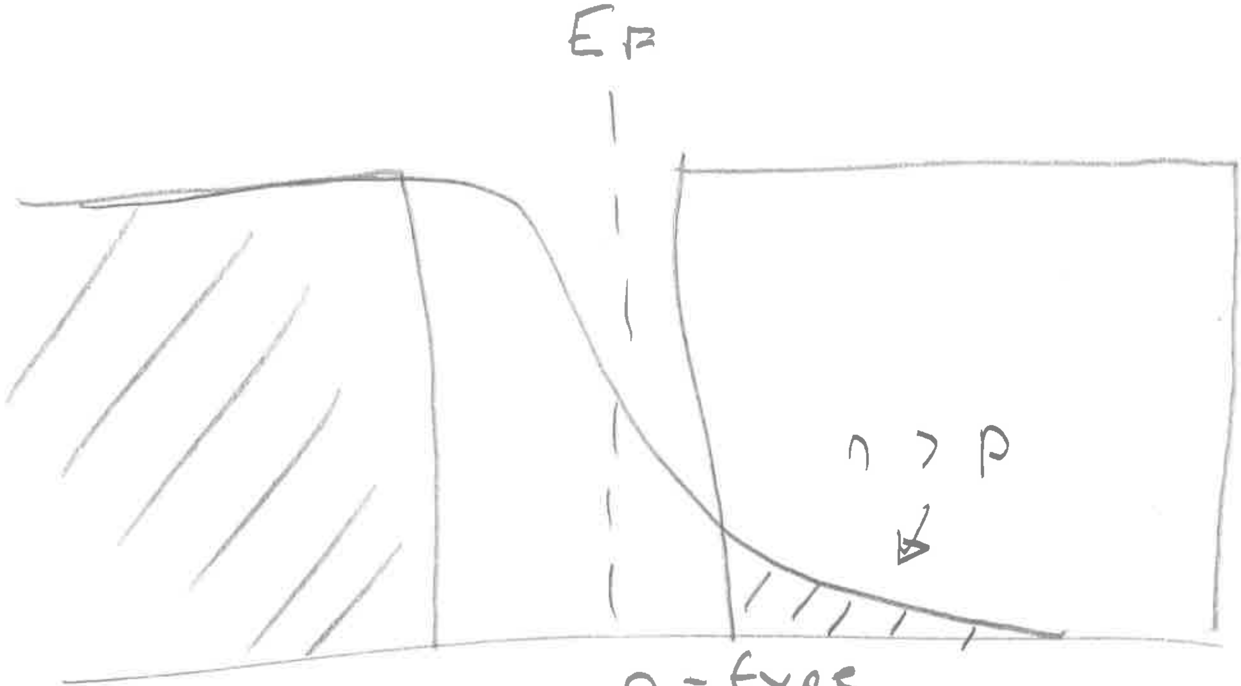


* shifts E_F near conductor

n-type



p-type



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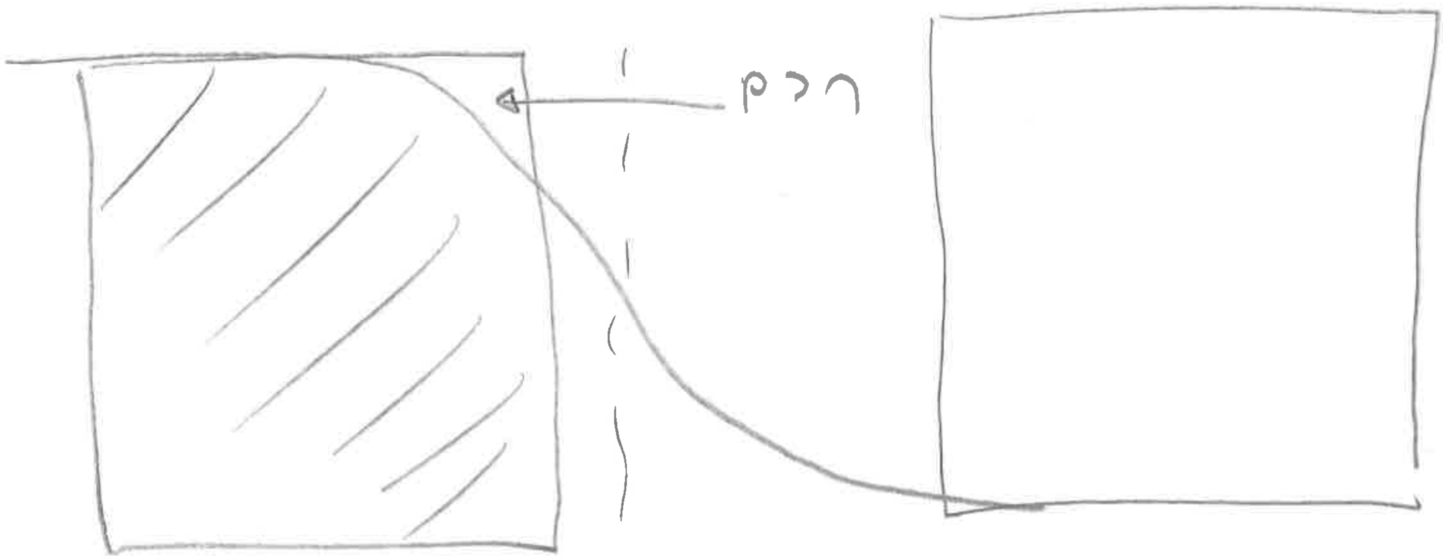
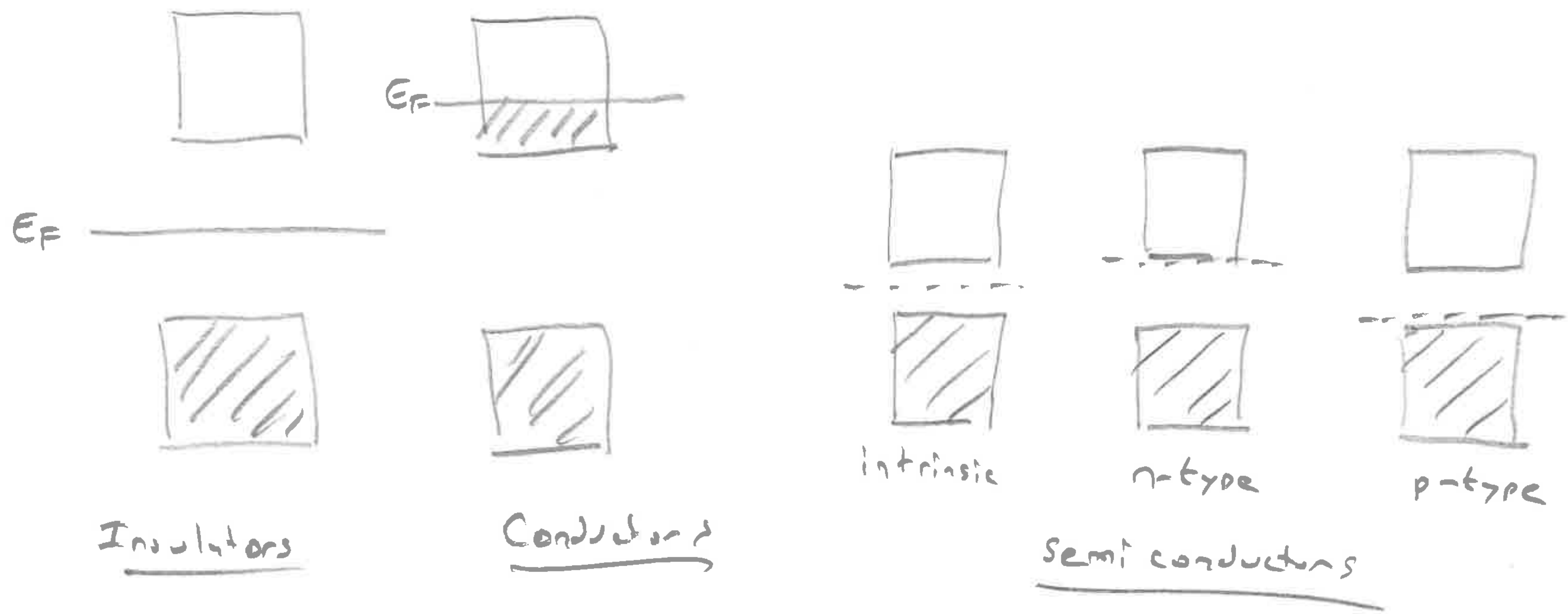


p-type



E_F
E_F
Insulators
Conductors
intrinsic
n-type
p-type
Semi conductors