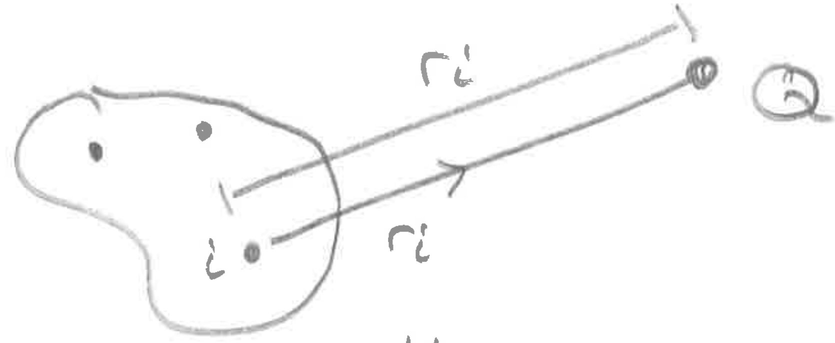
RLC

Circuits

Chapter 1: DC Circuits
and Resistance

Electric Field:



$$\vec{F} = Q\, k_e \sum_{i=1}^{N} \frac{q_i}{r_i^2} \hat{r}_i$$

$\vec{E} = \frac{\vec{F}}{Q}$ is independent of Q.

$$\vec{F} = \vec{E} Q$$

Electric Potential

Electric Force is conservative

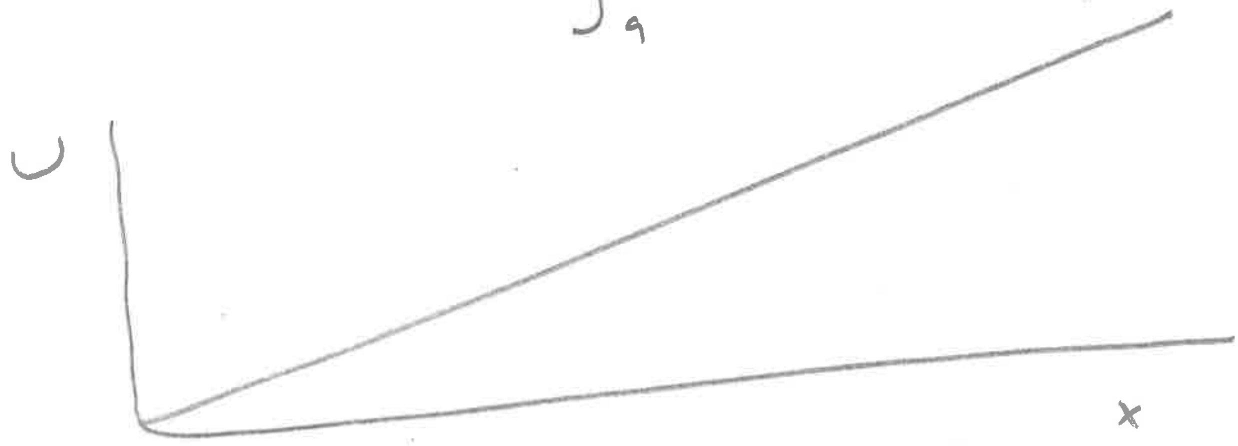
$$U_{ab} = -W_{esv} = -\int_a^b F_x \, dx$$

" Change in potential energy going from $a \to b$. "

$$V_{ab} = \frac{U_{ab}}{Q} = -\int_a^b E_x \, dx$$

" Change in electric potential going from $a \to b$ ".

$$V = -\int_a^b E\,dx$$

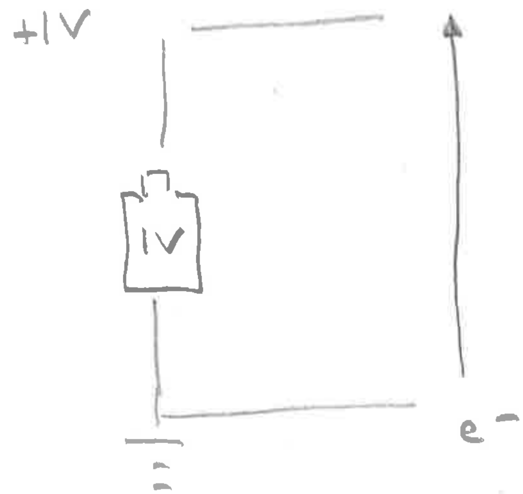


⊖ "Fall" toward high V

⊕ "Fall toward low V

Unit of Electrical Potential is

Volt (V) $1\,V = 1\,J/C$



1.6×10^{-19} C

$K = 1.6 \times 10^{-19}$ J $\equiv$ 1eV

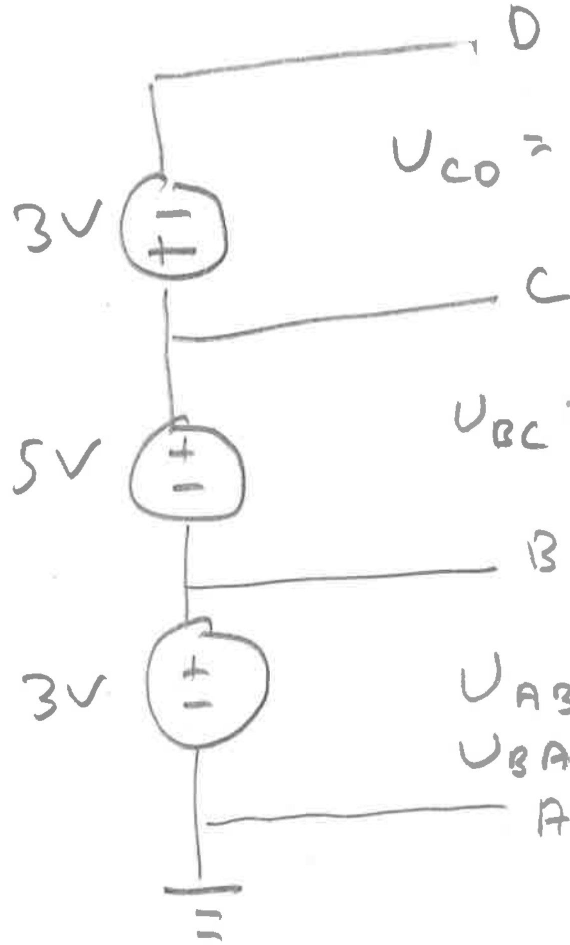
Volt is at useful scale in many contexts.

- your car battery (12), toaster (120V)
- Kinetic energy of photo-electron (~1eV)
- CMB energy density 0.25 eV/cm^3

Circuit Diagrams

Cell | Battery | Voltage Source

e.g.



$U_D = 5$

$U_{CD} = -3V$

$U_C = 8$

$U_{BC} = 5V$

$U_B = 3$

$U_{AB} = 3V$

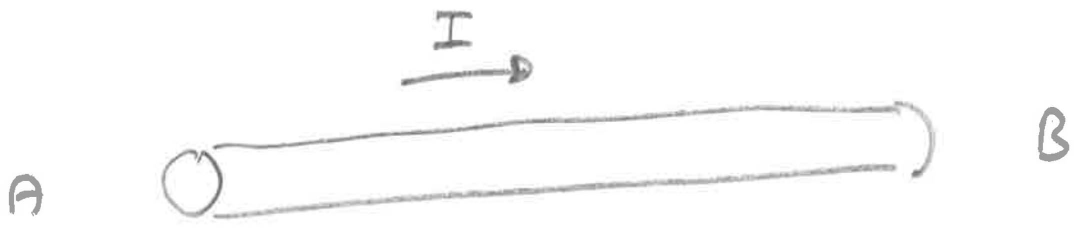
$U_{BA} = -3V$

$U_A = 0$

$U_{AC} = 8V$

$U_{CA} = -8V$

Conductors and Insulators



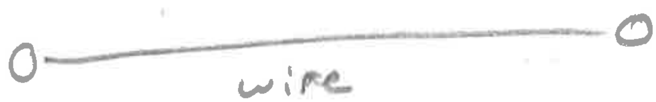
$$I = dQ/dt$$

<u>Conductors</u> have mobile charge carriers which readily conduct.

$\Rightarrow$ $E = 0$ inside conductor (else, charges rearrange)

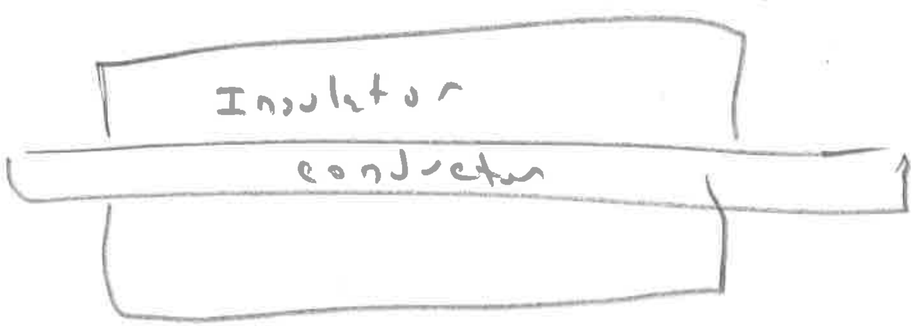
$\Rightarrow$ ΔV across conductor is 0.

$V_{AB} = 0 \Leftrightarrow$ short circuit

symbol: 

* Treat good conductors as if ideal *

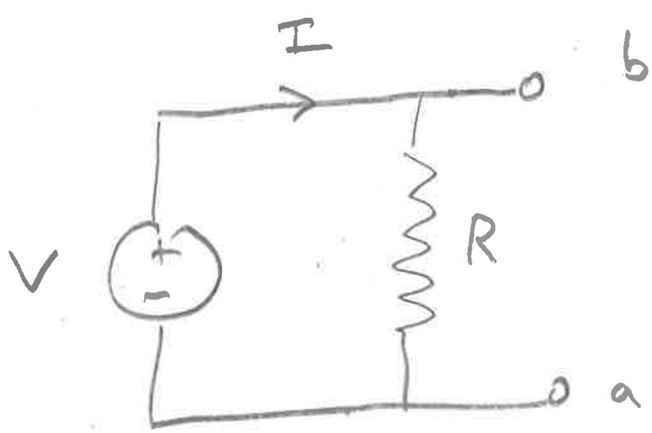
<u>Insulators</u>: Do not readily conduct. Implicit in diagrams.



Resistors

Mix graphite (conductor) and ceramic dust (insulator) held together with resin, build

resistor → mobile charges that do not readily conduct



Ohm's Law

$$V = I R$$

$$1 \Omega = 1 \frac{V}{A}$$

Note sign:

$$V_{ab} = I R \qquad a \to b \text{ faces current}$$

$$V_{ba} = -I R \qquad b \to a \text{ with current}$$

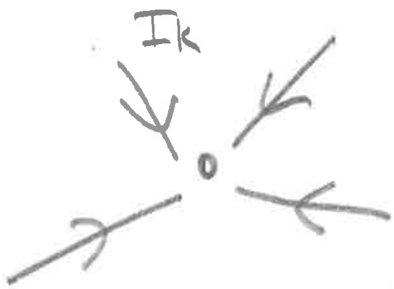
Conductors $R \sim 0$

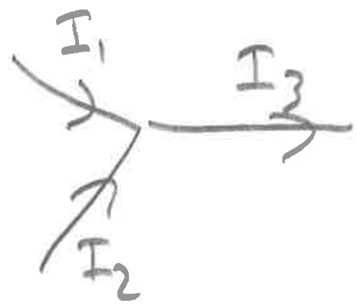
Insulators $R \to \infty$

All materials break down and conduct at some V.

Kirchhoffs Laws

Conservation of charge (KCL)



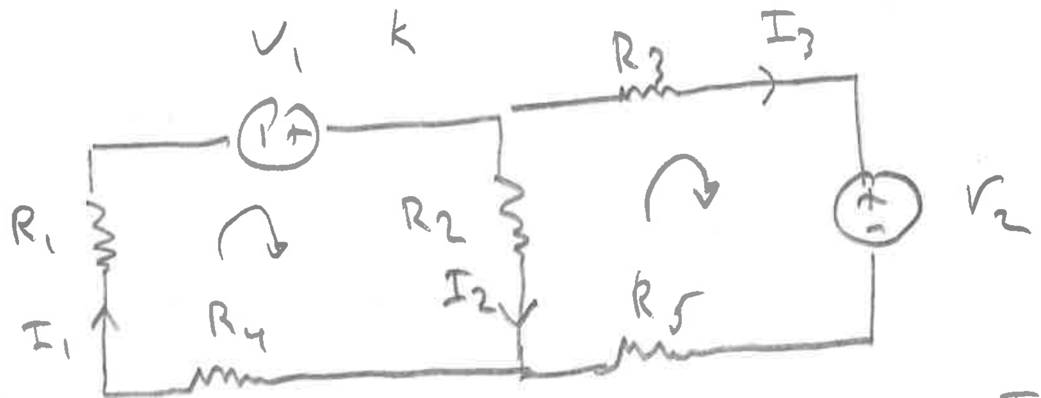


$$\sum_k I_k = 0$$

$$I_1 + I_2 = I_3$$

Electric Field in conservative (KVL).

$$\sum_k V_k = 0$$



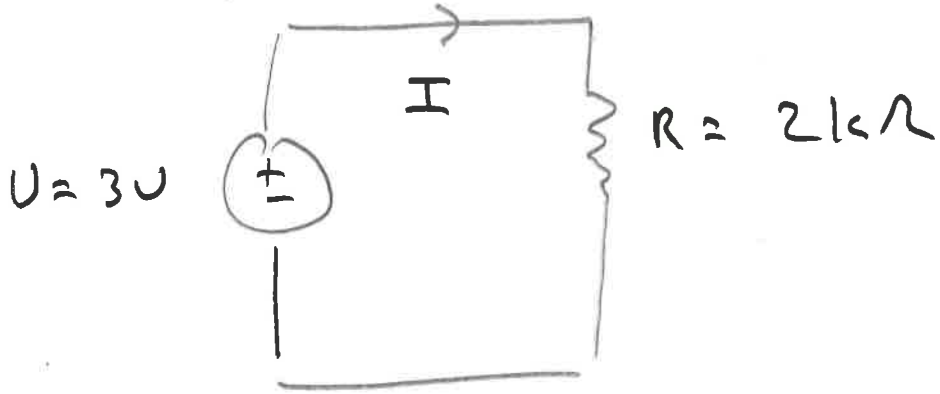
$$I_1 = I_2 + I_3$$

$$0 = -I_1 R_1 + V_1 - I_2 R_2 - I_1 R_4$$

$$0 = R_2 I_2 - R_3 I_3 - V_2 - I_3 R_5$$

$$0 = -I_1 R_1 + V_1 - I_3 R_3 - V_2 - I_3 R_5 - I_1 R_4$$

Exercise 1



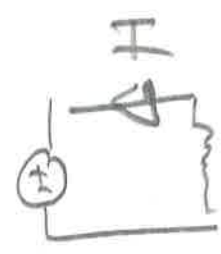
$$0 = U - IR$$

$$I = U/R = \frac{3V}{2k\Omega} = \boxed{\frac{3}{2} mA}$$

Loop in other direction

$$0 = (-U) + IR \Rightarrow I = V/R = \frac{3}{2} mA$$

"I" in other direction

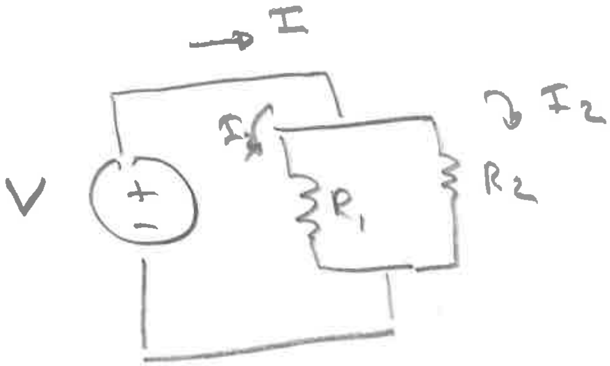


$$0 = U + IR$$

$$I = -\frac{U}{R} = \boxed{-\frac{3}{2} mA}$$

* Always draw current directions and note the signs *

Exercise 2:



$$I = I_1 + I_2$$

$$U = I_1 R_1$$

$$U = I_2 R_2$$

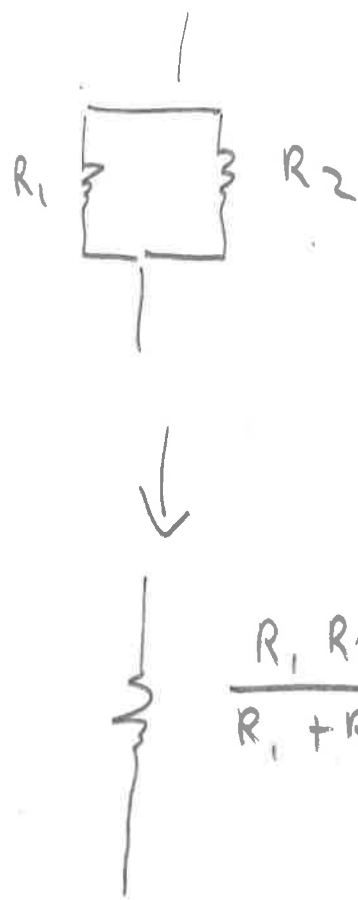
$$I = \frac{U}{R_1} + \frac{U}{R_2}$$

$$I = \left(\frac{1}{R_1} + \frac{1}{R_2}\right) V$$

As if $\quad I = \dfrac{1}{R_{eq}} U$

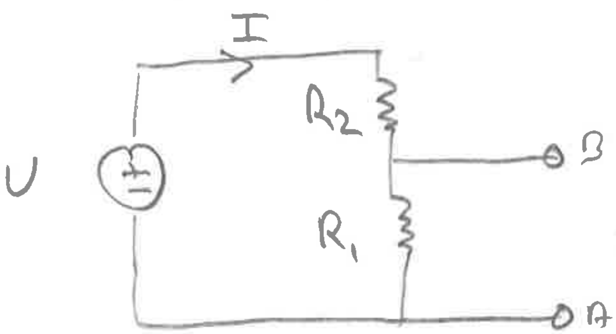
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$



$$\frac{R_1 R_2}{R_1 + R_2}$$

Example 3



$I = ?$

$U_{AB} = ?$

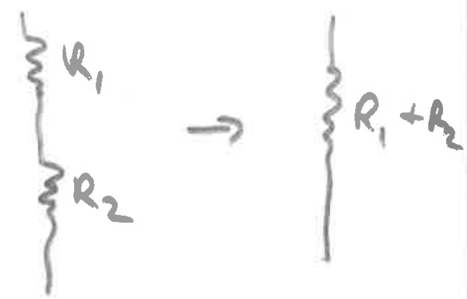
$$0 = U - IR_2 - IR_2$$

$$U = IR_2 + IR_1$$

$$= (R_1 + R_2)\, I$$

$$I = \frac{U}{R_1 + R_2} = \frac{U}{R_{eq}}$$

$$R_{eq} = R_1 + R_2$$



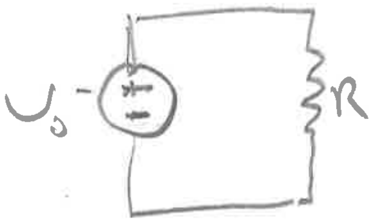
$$U_{AB} = IR_1 = \frac{R_1}{R_1 + R_2} U$$

" Takes a share R_1 from the voltage across $R_1 + R_2$. "

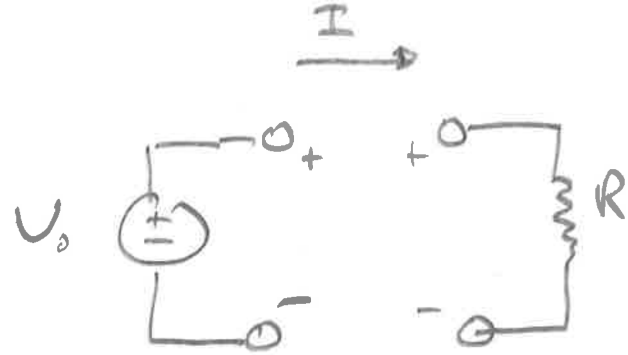
** Ready for DC circuits Lab **

Sources and Loads

Start simple:



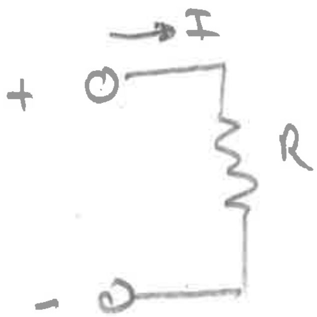
closed circuit

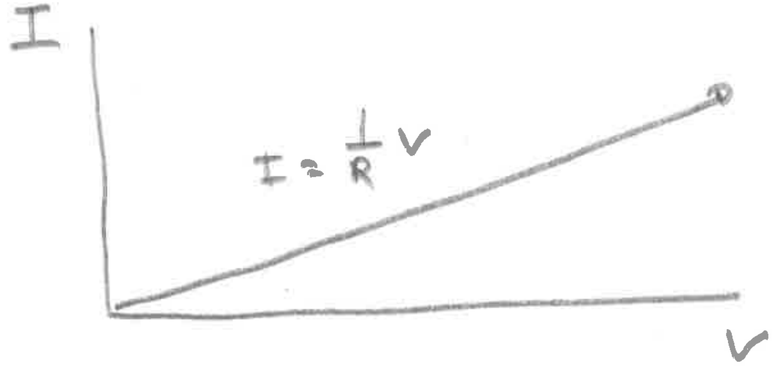


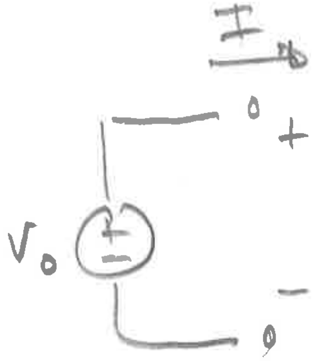
source network (open circuit)

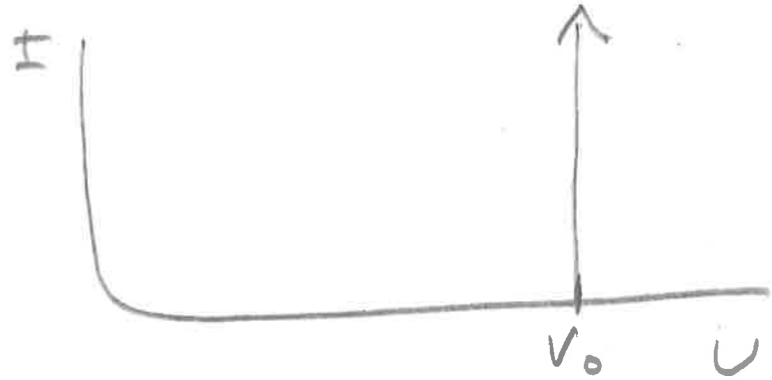
load network (open circuit)

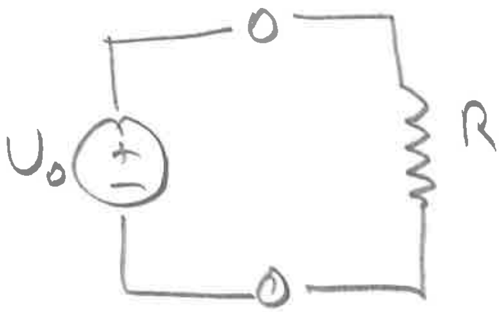
Source / Load differ only in direction of current: source +I is out of positive terminal, load +I is into positive terminal (Ohm's Law defined for R in load).

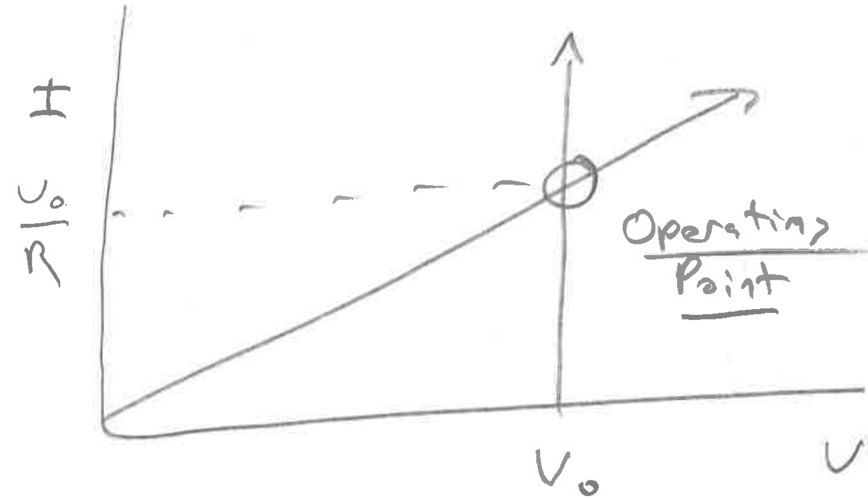












Rower :

$$P = \frac{dU}{dt} = \frac{V\,dQ}{dt} = V\,\frac{dQ}{dT} = VI$$

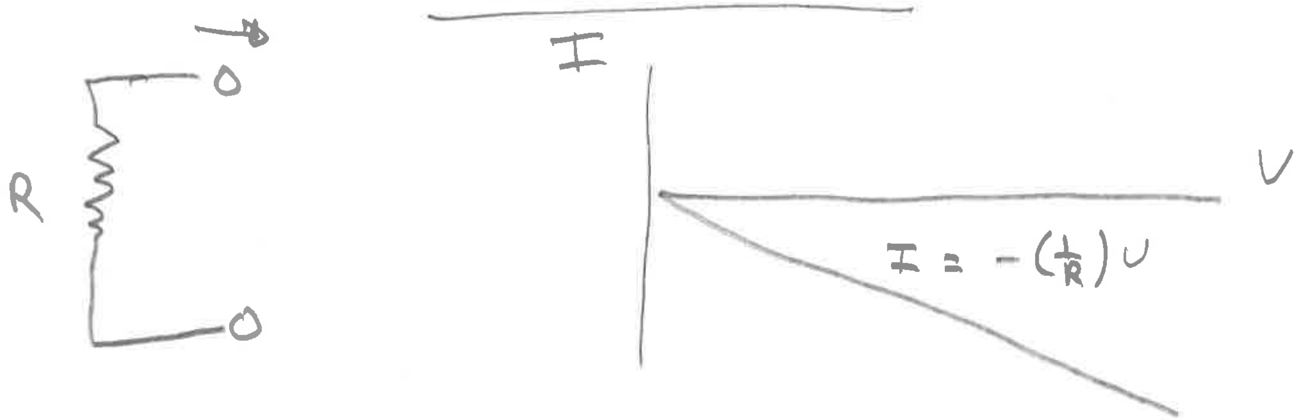
Source provides power UI

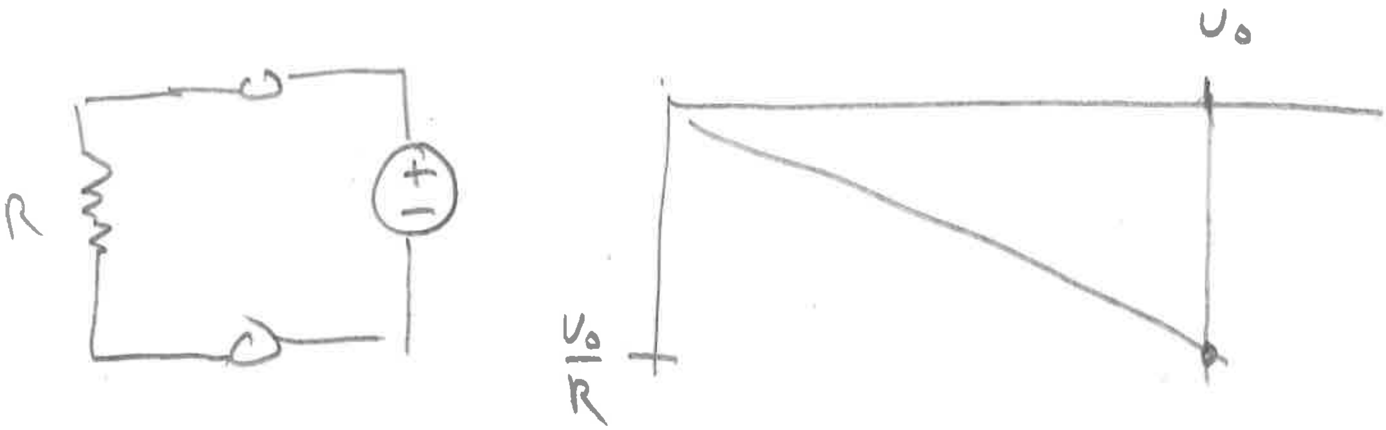
Load consumes power UI

(In this case $P = UI = U_0\left(\frac{U_0}{R}\right)$

$$P = \frac{U_0^2}{R}$$)

Resistor Source





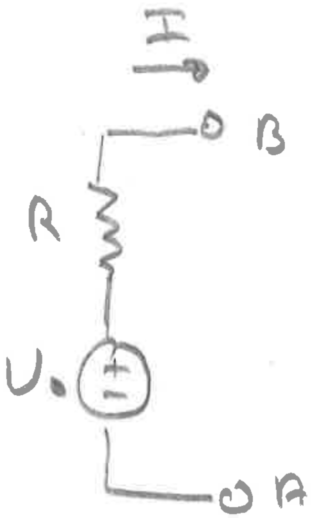
$$P = -\frac{(U_0)^2}{R}$$

source provides negative power
load consumes negative power.

(Maths polite way of saying you choose source and load wrong.)

* R in source has negative slope *

Realististic Sources



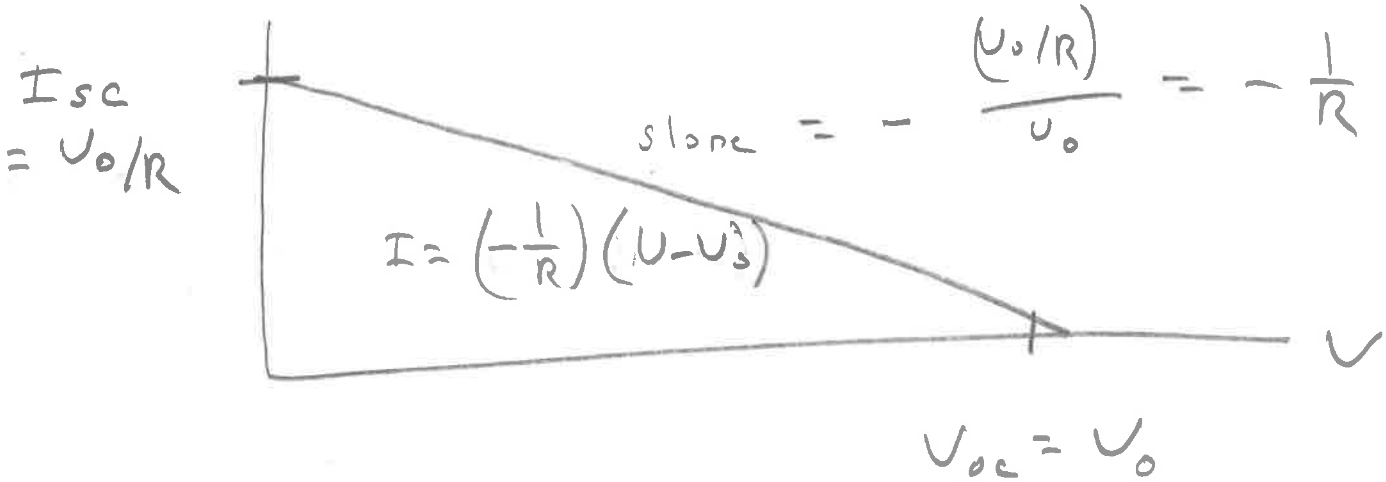
* Expect slope $-\frac{1}{R}$. *

Leave AB open!

$I = 0$ $\qquad V_{AB} = V_0 = V_{oc}$

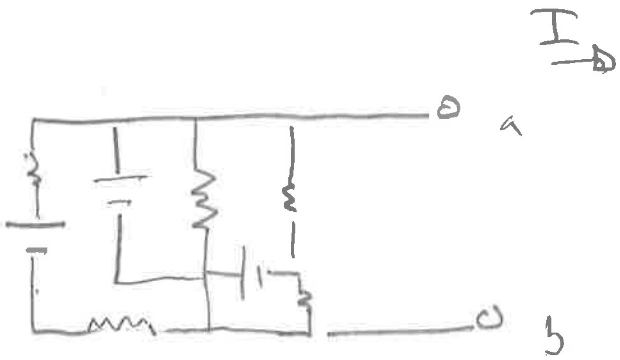
Short AB:

$V_{AB} = 0$ $\qquad I_{sc} = \frac{V_0}{R}$



Thevenin Equivalent Circuits

Because of superposition principle of E+M; combining linear components always has a linear response... all circuits consisting of R, V, and I-sources have a linear I V curve



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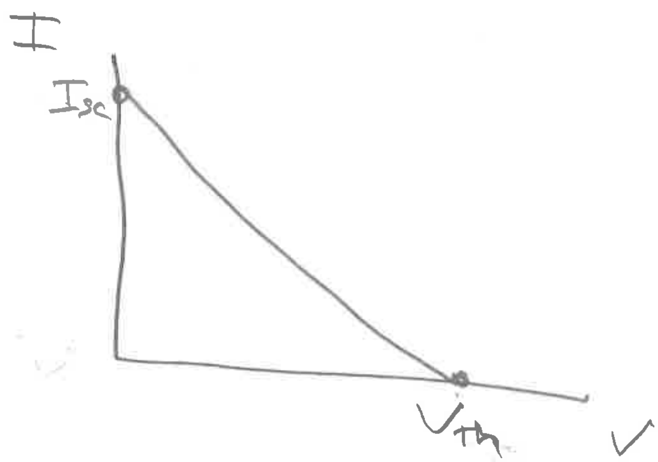

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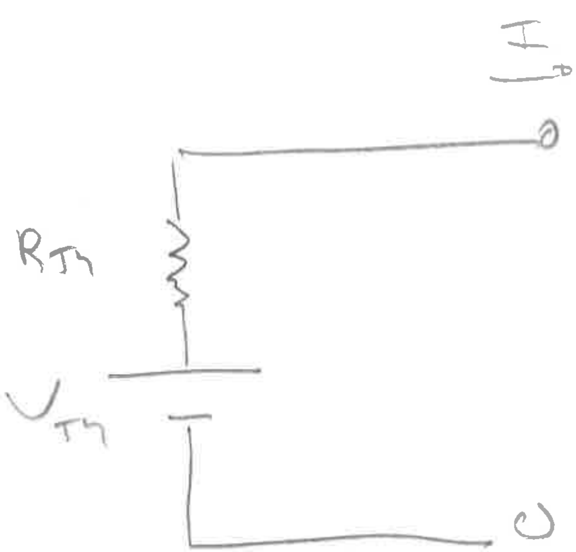
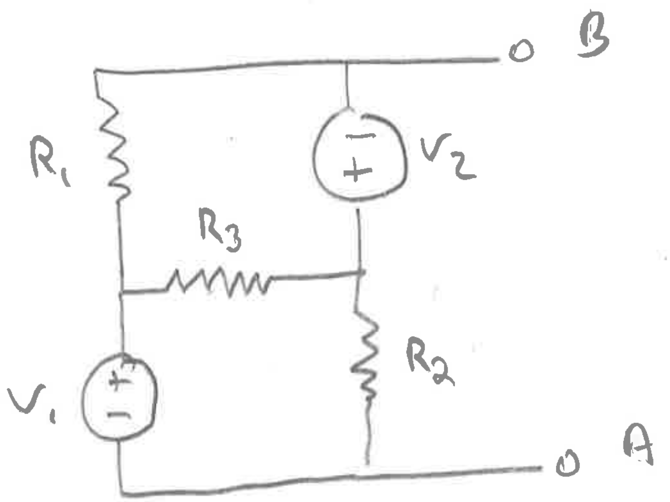


V_{th} is open circuit voltage

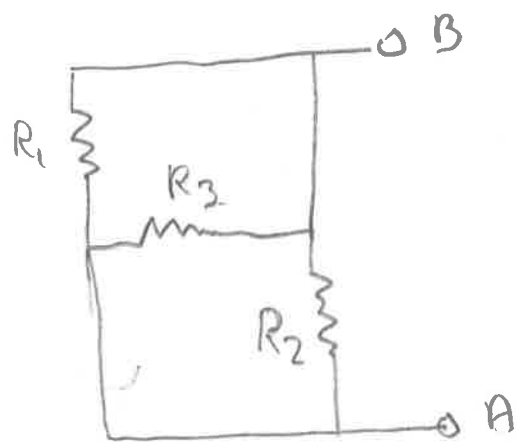
I_{sc} is short circuit current

$R_{Th} = V_{th} / I_{sc}$

Original Circuit: (Thevenin Lab)



For R_{Th} set $V_1 = V_2 = 0$



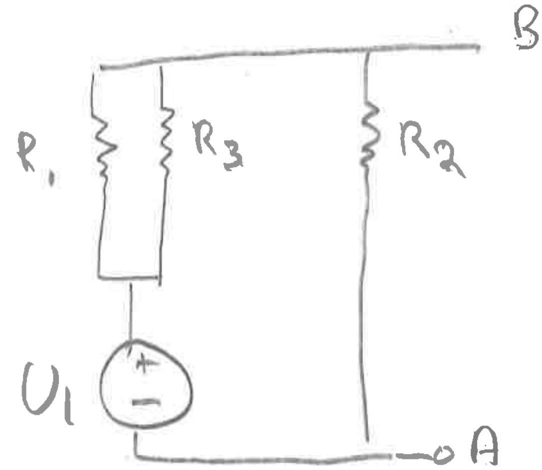
Looks complicated but don't be fooled! Each resistor connects to point a and b directly

$$R_{Th} = R_1 \parallel R_2 \parallel R_3$$

$$\boxed{R_{Th} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}}$$

For V_{Th}, use superposition. First set

$U_2 = 0$



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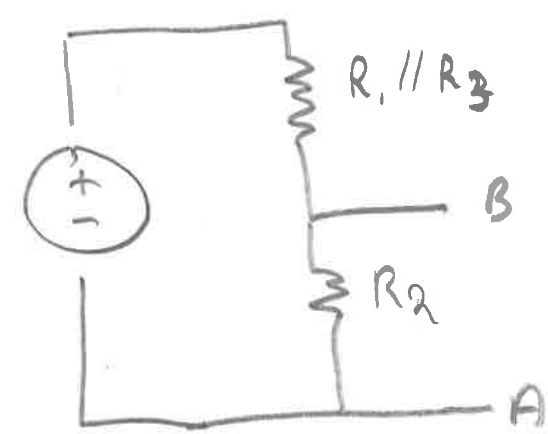
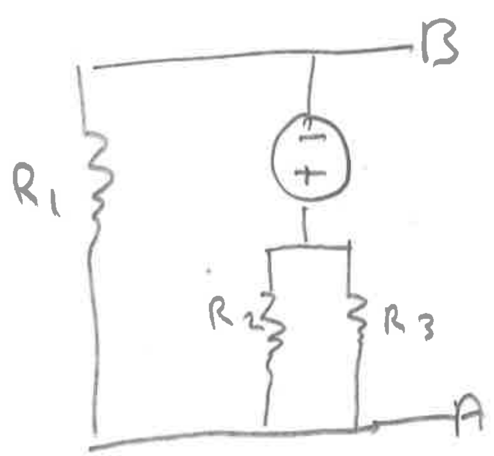


$$U_{Th}^{(1)} = U_{AB} = \frac{R_2}{R_2 + R_1 // R_3} U_1$$

$U_1 = 0$



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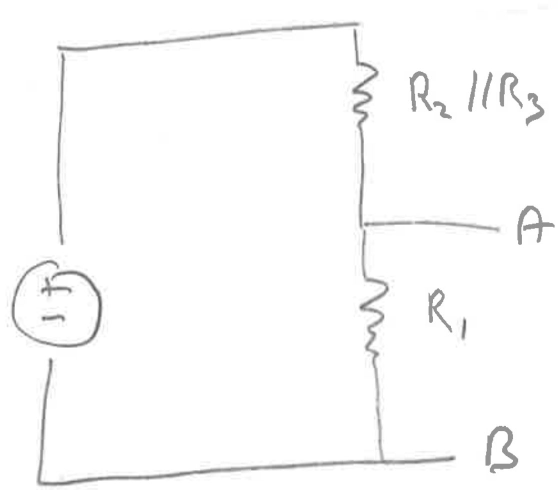

$$U_{th}^{(2)} = U_{AB} = -U_{BA} = -\left(\frac{R_1}{R_1 + R_2 // R_3}\right) U_2$$

* Ready for Thevenin Lab *

$$U_{Th} = U_{Th}^{(1)} + U_{Th}^{(2)}$$

$$= \frac{R_2}{R_2 + R_1 // R_3} U_1 - \frac{R_1}{R_1 + R_2 // R_3} U_2$$

$$= \frac{R_2}{R_2 + \frac{R_1 R_3}{R_1 + R_3}} U_1 - \frac{R_1}{R_1 + \frac{R_2 R_3}{R_2 + R_3}} U_2$$

$$\boxed{R_1 = 3{,}3\,k\Omega, \quad R_2 = 3{,}9\,k\Omega, \quad R_3 = 4{,}7\,k\Omega}$$