Homework Assignment 9
“Brave and adept from this day on”

Practice Problems

These problems are graded on effort only.

Problem 1: Reproduce the derivation of generalized Ehrenfest theorem (Eq. 3.73).

Griffiths: P3.18, P3.19, P3.25, P3.26

Additional Problems

Problem 1: Suppose that the state vector is:

$$|\Psi\rangle = \frac{i}{\sqrt{3}} |\lambda_1\rangle + k |\lambda_2\rangle$$

where $|\lambda_n\rangle$ is a properly normalized eigenvector of an observable $\hat{Q}$ with eigenvalue λ_n. Assume $\hat{Q}$ has a non-degenerate spectrum. Hint: I am not trying to trick you here. If any of the answers below seem easy to you, then good for you!

(A) What is the probability of a measurement yielding the value λ_1?

(B) What is the probability of a measurement yielding the value λ_2? (Give a numerical answer, such as 1/10, not something that depends on k).

(C) What is expectation value of the observable $\hat{Q}$?

(D) What is $\langle\lambda_1|\lambda_1\rangle$?

(E) What is $\langle\lambda_1|\lambda_2\rangle$?

(F) What is $\langle\lambda_1|\Psi\rangle$?

(G) What is $\langle\Psi|\lambda_1\rangle$?

(H) What is $|\langle\lambda_2|\Psi\rangle|$? (Give a numerical answer)

Problem 2: The Parseval-Plancherel identity:

$$\int_{-\infty}^{+\infty} |\Psi(x)|^2 \, dx \;=\; \int_{-\infty}^{+\infty} |\widetilde{\Psi}(p)|^2 \, dp$$

shows that if a position-space wave function $\Psi(x)$ is normalized, then so is it's momentum-space wave function $\widetilde{\Psi}(p)$.

(A) Derive the Parseval-Plancherel identity by explicit integration. **Hint:** start with the LHS and write out $\Psi(x)$ and $\Psi^*(x)$ in terms of their Fourier transforms. Make sure you use two different dummy variables for the integral over momentum (e.g. dq and dp) . At this point, you should have **three** integrals: dx,dp, and dq. Look carefully and note that only two factors involve x. Move the integral dx to include just those factors, and use the orthogonality of the complex exponentials:

$$\frac{1}{2\pi\hbar} \int_{-\infty}^{+\infty} \exp((q-p)x/\hbar) \, dx = \delta(q-p)$$

This kills the integral over dx, then use the resulting δ-function to kill the integral of dq.

(B) Derive Parseval-Plancherel identity using the Dirac notation. Start with:

$$\langle \Psi | \Psi \rangle$$

then insert the identity:

$$\int dx \, |x\rangle \langle x| = 1$$

and identify:

$$\Psi(x) = \langle x | \Psi \rangle$$

Next, do the same thing but using the identity:

$$\int dp \, |p\rangle \langle p| = 1$$

and identify:

$$\widetilde{\Psi}(p) = \langle p | \Psi \rangle$$