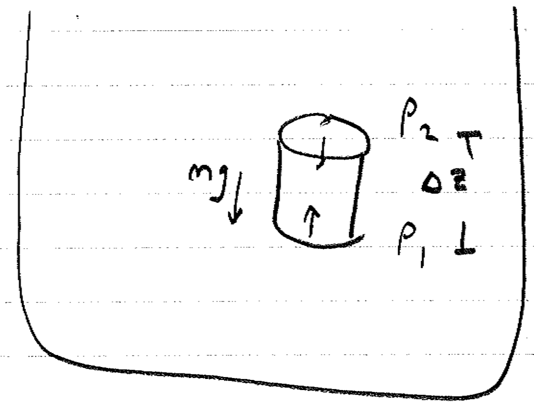
T1 S.3



$$0 = mg + P_2 \cdot A - P_1 \cdot A$$

$$\Rightarrow \quad P_1 - P_2 = \frac{mg}{A}$$

$$= \frac{m}{A \Delta z} \cdot \Delta z \cdot g = \frac{m}{V} \Delta z \, g$$

$$\boxed{P_1 - P_2 = \rho \, \Delta z \, g}$$

TZ 5.3

a) Assume for simplicity $m_i = m$

$$\langle x \rangle = \sum m_i x_i \Big/ \sum m_i = \frac{1}{N}\sum_i x_i$$

$$\frac{d\langle x \rangle}{dt} = 0 \Rightarrow \frac{d}{dt}\sum_i x_i = \frac{1}{N}\sum_i v_i$$

$$\Rightarrow \boxed{\langle v \rangle = 0}$$

b) Simple example: if every particle has a duplicate with $v \to -v$

$$\langle v \rangle = \frac{1}{2N}\left(\sum_i v_i + (-v_i)\right)$$

$$\langle v^2 \rangle = \frac{1}{2N}\sum_i v_i^2 + (-v_i)^2$$

$$> 0$$

T2 3,4

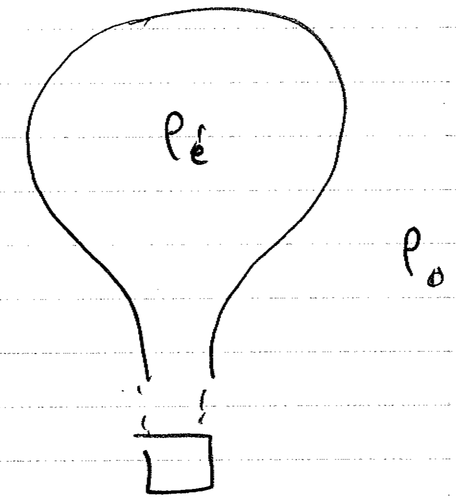
$$\Delta p = 2 m v_0$$

$$N = r \cdot \Delta t \cdot A$$ (defines "r")

$$P = N \frac{\Delta p}{\Delta t} \frac{1}{A} = \left(\frac{N}{\Delta t\, A}\right) \Delta p = \boxed{r\, 2 m v_0}$$

T2 5.5

Boyant Force is



$$F = \rho_o \cdot V \cdot g$$

Gravity is

$$F = -\rho_i V \cdot g - Mg$$

Floating has

$$\rho_o V g + (-\rho_i V g) - Mg = 0$$

$$\Rightarrow \boxed{\rho_o - \rho_i = \frac{M}{V}}$$

b) $$\rho = \frac{N \cdot m}{V} \qquad m = \frac{M_A}{N_A}$$

$$= \frac{N}{N_A} \cdot \frac{M_A}{V} = \frac{n}{V} M_A$$

$$PV = nRT \quad \Rightarrow \quad \frac{n}{V} = \frac{P}{RT}$$

$$\Rightarrow \quad \boxed{\rho = \frac{P\, M_A}{R\, T}}$$

c) $\rho_o - \rho_i = \frac{M}{V}$

$$\Rightarrow V = M / (\rho_o - \rho_i)$$

$$= \frac{M}{\rho_o} \frac{1}{(1 - \rho_i/\rho_o)}$$

From (b) $\rho \sim \frac{1}{T}$

$$V = \frac{M}{\rho_o} \frac{1}{(1 - T_o/T_i)}$$

TY S.8

a) No matter where matches are, total energy will add to number of marbles.
Any $M-1$ matchsticks will divide exactly M unique energies.

b) $(M+q-1) * (M+q-2) * \cdots (1)$

$$= (M+q-1)!$$

$$q \cdot (q-1) \cdot (q-2) \cdots 1$$

Marble q can choose from q positions
marble $q-1$ can choose from $q-1$ position
...

$q!$

Similar for $(M-1)!$

$$\Omega = \frac{(M+q-1)!}{(M-1)!\, q!}$$

T3 S.3

*** Isothermal process for an ideal gas at constant N is has $U = NkT = \text{const} \Rightarrow dU = 0$.

$$\Delta U = 0 \qquad \text{as} \qquad U = \frac{f}{2} NkT \qquad ***$$

$$\Delta U = Q + W$$

$$\Rightarrow \boxed{Q = -W = +NkT \ln \frac{V_f}{V_i}}$$

T3 S. 4

	Q	W	ΔU
$A \to B$	+	+	+
$B \to C$	+	0	+
$C \to A$	−	+	−

$A \to B$:

$$W = -\int P\, dV = -P\Delta V < 0$$

$$PV = NkT \Rightarrow V \sim T$$

$$\Rightarrow T_F > T_I \Rightarrow U_F > U_I \Rightarrow dU > 0$$

Since $\Delta U = Q + W$, and $\Delta U > 0$ but $W < 0$

$$Q > 0$$

$B \to C$

$$W = 0$$

$$PV = NkT \Rightarrow P \sim T$$

$$P_F > P_I \Rightarrow T_F > T_i \Rightarrow U_F > U_i$$

$$\Rightarrow \Delta U > 0$$

$C \to A$

ΔU for the cycle must be 0, so $\Delta U = -$.

W is negative. In fact,

$$|W_{C\to A}| > |W_{A\to B}|$$

Implying that W for the entire cycle is $+$.

Since $\Delta U = 0 = Q + W$ for the whole cycle, Q for the cycle must be negative. But Q for $A\to B$, $B\to C$ is positive, so Q for $C \to A$ must be negative

Alternate:

$P_I V_I > P_F V_F$ for $C \to A$

$\Rightarrow T_I > T_F \Rightarrow \boxed{dU < 0}$

As $dV < 0$, $\boxed{W > 0}$

Since $dU < 0$ and $W > 0$, $\boxed{Q < 0}$

Q	W	ΔU
0	−	−
−	+	0
+	0	+

T3 5.5

A → B

$W = -\int P\,dV < 0$

Adiabatic $\Rightarrow Q = 0$
Hence $\Delta U = W < 0$

Alternative: Since $T_B = T_C$, $T_A > T_B \Rightarrow U_A > U_B$

$\Rightarrow dU < 0$

B → C

Isothermal $\Rightarrow T = \text{const} \Rightarrow U = \text{const} \Rightarrow \Delta U = 0$

$W = -\int P\,dV > 0$

$\Delta U = Q + W = 0 \Rightarrow Q = -W$

C → A

$W = 0$

Since $V = \text{const}$, and $PV = NkT$, $T_A > T_C$
So $U_A > U_C \Rightarrow dU > 0$.

Alternatively: Since ΔU for cycle $= 0$, $\Delta U > 0$,

Since $W = 0$, $\Delta U = Q$, $Q > 0$

T3 5.6 $\quad T\uparrow, \quad$ so $\quad \Delta U > 0$

$V = \text{const} \Rightarrow W = 0, \quad$ so $\quad Q = \Delta U > 0$

$$PV = NkT \Rightarrow P \sim T$$

$$\frac{T_f}{T_i} = \frac{P_f}{P_i} \Rightarrow T_f = \frac{P_f}{P_i} T_i = \frac{50}{30}\, 290\text{ K}$$

From

$$PV = NkT \Rightarrow N = \frac{P_i V_i}{k T_i} \left(= \frac{(30\text{ kPa})\,(1\text{ m}^3)}{k\;\; 290\text{ K}} \right)$$

$$dU = \frac{3}{2} N k \, dT$$

$$= \frac{3}{2} \frac{P_i V}{T_i} \left(T_f - T_i\right)$$

T3 S.7

$$T V^{\gamma-1} = \text{const}$$
$$p V^{\gamma} = \text{const}$$

$$P_i V_i^{\gamma} = P_f V_f^{\gamma}$$

$$\frac{V_i^{\gamma}}{V_f^{\gamma}} = \frac{P_f}{P_i}$$

$$\frac{V_i}{V_f} = \left(\frac{P_f}{P_i}\right)^{\frac{1}{\gamma}}$$

$$T_i V_i^{\gamma-1} = T_f V_f^{\gamma-1}$$

$$\Rightarrow \quad T_f = \left(\frac{V_i}{V_f}\right)^{\gamma-1} \cdot T_i$$

$$= \left(\frac{P_f}{P_i}\right)^{\frac{\gamma-1}{\gamma}} \cdot T_i$$

$$= \left(\frac{1}{3}\right)^{\frac{\gamma-1}{\gamma}} T_i$$

For air $\gamma \sim 1.4$

T3 5.8 $\gamma \sim 1.4$

$$T V^{\gamma-1} = \text{const}$$
$$P V^{\gamma} = \text{const}$$

We are given $P_f / P_i \sim 0.045$

$$P_i V_i^{\gamma} = P_f V_f^{\gamma}$$
$$\Rightarrow V_f / V_i = \left(P_i / P_f \right)^{1/\gamma}$$

$$T_i V_i^{\gamma-1} = T_f V_f^{\gamma-1}$$
$$T_f = T_i \left(\frac{V_i}{V_f} \right)^{\gamma-1}$$
$$= T_i \left(\frac{P_f}{P_i} \right)^{\left(\frac{\gamma-1}{\gamma}\right)}$$

T3 S.9 For adiabatic process:

$$TV^{\gamma-1} = \text{const}$$
$$PV^{\gamma} = \text{const} \Rightarrow P = P_I V_I^{\gamma} V^{-\gamma}$$

$$W = -\int P\,dV = -P_I V_I^{\gamma} \int_{V_I}^{V_F} V^{-\gamma}\,dV$$
$$= \frac{-P_I V_I^{\gamma}}{1-\gamma} V^{1-\gamma} \Big|_{V_I}^{V_F}$$
$$= \frac{P_I V_I^{\gamma}}{\gamma-1} \left(V_F^{1-\gamma} - V_I^{1-\gamma}\right)$$

!!!

$$= \frac{P_I V_I}{\gamma-1} \left(\frac{V_I^{\gamma-1}}{V_F^{\gamma-1}} - 1\right)$$
$$= \frac{NkT_I}{\gamma-1} \left(\frac{T_F}{T_I} - 1\right)$$
$$= \left(\frac{1}{\gamma-1}\right) Nk\,(T_F - T_I)$$
$$= \frac{f}{2} Nk\,dT$$

!!!

T3 S.10

a) $PV = NkT$

$$P\frac{dV}{dT} + V\frac{dP}{dT} = Nk$$

$$\boxed{PdV + VdP = Nk\,dT}$$

b) For adiabatic process, $Q=0$, so

$$\Delta U = Q + W \Rightarrow \Delta U = W$$

But $dW = -PdV$

As $dU = \frac{f}{2} Nk\,dT$

$$\Rightarrow PdV = -\frac{f}{2} Nk\,dT$$

c) $Nk\,dT = -\frac{2}{f} PdV$

$$\Rightarrow PdV + VdP = -\frac{2}{f} PdV$$

$$VdP + P\left(1+\frac{2}{f}\right)dV = 0$$

$$VdP + \gamma P\,dV = 0 \qquad \gamma \equiv 1+\frac{2}{f}$$

$$\boxed{\frac{dP}{dV} + \gamma\frac{P}{V} = 0}$$

d) $\frac{d}{dV} V^{\gamma} = \gamma V^{\gamma-1}$

So, as we have a term

$$\gamma \frac{p}{V} \underset{(\times V^{\gamma})}{\rightarrow} \gamma V^{\gamma-1} \cdot p \rightarrow p \frac{d}{dV}(V^{\gamma})$$

So multiply above by V^{γ}

$$V^{\gamma} \frac{dp}{dV} + \gamma p V^{\gamma-1} = 0$$

$$V^{\gamma} \frac{dp}{dV} + p \frac{d}{dV} V^{\gamma} = 0$$

$$\frac{d}{dV}\left(p V^{\gamma}\right) = 0$$

$$\Rightarrow \boxed{p V^{\gamma} = \text{const}}$$

T4S.1

Ω_A for $U_A = 4\varepsilon$, $N = 3$

1	2	3
4	0	0
0	4	0
0	0	4
3	1	0
3	0	1
0	3	1
1	3	0
1	0	3
0	1	3
2	2	0
2	0	2
0	2	2
2	1	1
1	2	1
1	1	2

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$U_A = 7\varepsilon$

1	2	3
7	0	0
0	7	0
0	0	7
6	1	0
6	0	1
0	6	1
1	6	0
1	0	6
0	1	6
5	2	0
5	0	2
0	5	2
2	5	0
2	0	5
0	2	5
5	1	1
1	5	1
1	1	5
4	3	0
4	0	3
0	4	3
3	4	0
3	0	4
0	3	4

3	3	1
3	1	3
1	3	3
3	2	2
2	3	2
2	2	3
4	2	1
4	1	2
1	4	2
2	4	1
2	1	4
1	2	4

T4 S.2

a)

1	2	3	4	5	6	
2	0	0	0	0	0	
		...				6
0	0	0	0	0	2	
1	1	0	0	0	0	
		...				5
1	0	0	0	0	1	
0	1	1	0	0	0	
		...				4
0	1	0	0	0	1	

$$\Omega_B = 6 + 5 + 4 + 3 + 2 + 1$$

$$= 21$$

b) $$\Omega(N,U) = \frac{(q+3N-1)!}{q!\,(3N-1)!}$$

$U_B = 6\varepsilon \qquad U_B = 9\varepsilon \qquad 3N = 6$

$$\frac{(6+6-1)!}{6!\,5!} \qquad \frac{(9+6-1)!}{9!\,5!}$$

$$\frac{11!}{6!\,5!} \qquad \frac{14!}{9!\,5!}$$

$$\frac{11\cdot\cancel{10}^{2}\cdot\cancel{9}^{3}\cdot\cancel{8}\cdot 7}{\cancel{5}\cdot\cancel{4}\cdot\cancel{3}\cdot\cancel{2}\cdot 1} \qquad \frac{14\cdot 13\cdot\cancel{12}\cdot 11\cdot\cancel{10}}{\cancel{5}\;\cancel{4}\;\cancel{3}\;\cancel{2}}$$

$$66\cdot 7 = \boxed{462}$$

$$\begin{array}{r} {}^{1}\\ 14 \\ 13 \\ \hline 42 \\ 140 \\ \hline 182 \\ 11 \\ \hline 182 \\ 182 \\ \hline \end{array} \quad \boxed{2002}$$

T4 5.3

a)

(1) U_A	(2) U_B	Ω_A	Ω_B	Ω_{AB}
6	0	28	1	28
5	1	21	6	126
4	2	15	21	315
3	3	10	56	560
2	4	6	126	756
1	5	3	252	756
0	6	1	462	462

b) $U_A = 1 \quad U_B = 5 \qquad \frac{U_A}{N_A} = 1 \quad \frac{U_B}{N_B} = 2.5$

$U_A = 2 \quad U_B = 4 \qquad \frac{U_A}{N_A} = 2 \quad \frac{U_B}{N_B} = 2$

T4S.4

U_A	U_B	Ω_A	Ω_B	Ω_{AB}
9	0	55	1	55
8	1	45	6	270
7	2	36	21	756
6	3	28	56	1568
5	4	21	126	2646
4	5	15	252	3780
3	6	10	462	4620
2	7	6	792	4752
1	8	3	1287	3861
0	9	1	2002	2002

$$\frac{U_A}{N_A} = 2 \qquad \frac{U_B}{N_B} = \frac{7}{2} = 3.5$$

T4 5.5

a)

U_A	U_B	Ω_A	Ω_B	Ω_{AB}
9	0	2002	1	2002
8	1	1287	6	7722
7	2	792	21	16632
6	3	462	56	25872
5	4	252	126	31752
4	5	126	252	31752
3	6	56	462	25872
2	7	21	792	16632
1	8	6	1287	7722
0	9	1	2002	2002

b) $\frac{5}{2} = 2,5 \qquad \frac{4}{2} = 2$

T4 5.6

a) (1) 2.828×10^{13}
(2) 10, 10 3.847×10^{12}
(3) 2 and 2
(4) 0.3 ~ 0.7

b) (1) 3×10^{144}
(2) 100, 100 1.3×10^{143}
(3) 2 2
(4) 0.45 – 0.55

c) # states increases,
states become more concentrated toward

T4 S.7

a) $U_A / N_A = U_B / N_B$

Not Always, ... eg $^{39}/_{20}$

b) Hold true.

TS 5.5

a) $S = k \log \Omega$

$\Omega = 77 \times 10^{6}$

$S = k \log(77 \times 10^{6})$

$= k(\log 77 + 6 \log 10)$

b) $\Omega = 3.6 \times 10^{71}$

$S = k \cdot \log \Omega = k \log(3.6 \times 10^{71})$

$$\boxed{S = k(\log 3.6 + 71 \log 10)}$$

TS 5.6

$$S = k \log \Omega$$

$$S_1 = k \log \Omega_1$$
$$S_2 = k \log \Omega_2$$

$$S_1 - S_2 = k \log \Omega_1 / \Omega_2$$

$$\boxed{\frac{\Omega_1}{\Omega_2} = \exp\left(\frac{S_1 - S_2}{k}\right)}$$

TS 5.7

$$\frac{\Omega_1}{\Omega_2} = \exp(X)$$

X is too large to directly calculate

Find $y = X / \log(10)$

$$\exp(X) = \exp(y \log 10)$$
$$= 10^y$$

Say $y = N + \Delta$

$$\exp(X) = (10^\Delta) \times 10^N$$

$\boxed{\text{TS 5.7}}$

The trick:

$$\boxed{e^x = 10^{x/\log 10}}$$

Proof:

Let y be s.t. $x = y \log 10 = \log 10^y$

$$e^x = e^{\log 10^y} = 10^y = 10^{x/\log 10}$$

TS 5.8

$$\frac{\Omega_1}{\Omega_2} = \exp\left(\frac{S_1 - S_2}{k}\right)$$

$$\frac{S_1 - S_2}{k} = \frac{-1 \times 10^{8}}{1.38 \times 10^{-23}} = -7 \times 10^{14}$$

$$= \exp\left(7 \times 10^{14}\right) = ?$$

$$= 10^{7 \times 10^{14} / \log 10} = 10^{.00\overbrace{0000}^{14}}$$

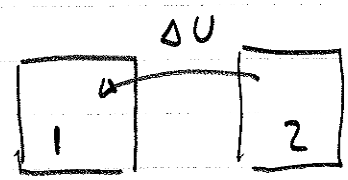
T6S,1

a) $T_{new} = \frac{\partial S}{\partial U}$

$$[T_{new}] = \left[\frac{J/K}{J}\right] = \frac{1}{K}$$

(also $T_{new} = \frac{1}{T} \Rightarrow [T_{new}] = \frac{1}{K}$)

b) By new definition



$T = \frac{\partial S}{\partial U}$

$\Delta S_1 = T_1 \Delta U \qquad \Delta S_2 = T_2 (-\Delta U)$

$\Delta S = \Delta S_1 + \Delta S_2 = (T_1 - T_2)\,\Delta U$

For $\Delta S > 0$ and $\Delta U = Q > 0$

$T_1 - T_2 > 0$

$T_1 > T_2$

ie, Energy flows toward object with <u>higher</u> T_{new} !

c) $T_{new} = \frac{1}{T} \Rightarrow$

T_{new}	T_{old}
∞	0
0	∞

T6S.2 $\quad S = Nk \log U$

$$\frac{dS}{dU} = Nk \frac{1}{U} = \frac{1}{T}$$

$$\Rightarrow \quad U = NkT$$

T6S.3 $\quad \Omega = C V^N U^{3N/2}$

$$S = k \log \Omega = k \log C V^N U^{3N/2}$$

$$= C' + (3N/2) \cdot k \cdot \log U$$

$$\frac{1}{T} = \frac{dS}{dU} = \frac{3}{2} N \frac{1}{U} k$$

$$\Rightarrow \quad \boxed{U = \frac{3}{2} NkT} \qquad !$$

TGS.4

$$\Omega = N e^{\sqrt{NU/\varepsilon}}$$

$$S = k \log \Omega = k \log\left(N e^{\sqrt{NU/\varepsilon}}\right)$$

$$= k \log N + k\sqrt{NU/\varepsilon}$$

$$\frac{dS}{dU} = k\sqrt{\frac{N}{\varepsilon}} \cdot \frac{1}{2}\frac{1}{\sqrt{U}} = 1/T$$

$$\sqrt{U} = \frac{1}{2} kT \sqrt{N/\varepsilon}$$

$$U = \frac{1}{4} k^2 T^2 N/\varepsilon$$

U still increaseses with T...

TG 5.6

$E \to E + E_0$

$$P_r(E) = \frac{1}{Z} e^{-(E_i + E_0)/k_B T}$$

$$Z = \sum e^{-(E_i + E_0)/k_B T}$$

In both cases $e^{-(E_i+E_0)/k_B T} = \underline{e^{-E_0/k_B T}} \cdot e^{-E_0/k_B T}$

$$P_r = \frac{1}{\cancel{e^{-E_0/k_B T}} Z} \cdot \cancel{e^{-E_0/k_B T}} \cdot e^{-E_0/k_B T}$$

T6.S.7

Define: $e^{-\hbar\omega/kT} \equiv \alpha$ =

e.g. $e^{-\frac{1}{2}\hbar\omega/kT} = \alpha^{\frac{1}{2}}$

$$P(E_n) = \alpha^{\frac{1}{2}}\, \alpha^n \cdot \frac{1}{Z}$$

$$Z = \alpha^{\frac{1}{2}} \left(1 + \alpha + \alpha^2 + \alpha^3 + \dots\right)$$

$$= \alpha^{\frac{1}{2}} \left(\frac{1}{1-\alpha}\right)$$

Geometric Series

$$P(E_n) = \alpha^{\frac{1}{2}} \alpha^n \left(\frac{1-\alpha}{\alpha^{\frac{1}{2}}}\right)$$

$$= (1-\alpha) \cdot \alpha^n$$

$$P(E_0) = (1-\alpha) =$$

$$P(E_1) = (1-\alpha)\alpha =$$

$$P(E_2) = (1-\alpha)\alpha^2 =$$

T6 S.8

a) $\omega = 2\pi c/\lambda$

$E = \hbar\omega = hc/\lambda$

b) $\dfrac{p(\uparrow)}{p(\downarrow)} = \dfrac{\exp(-E_\uparrow/kT)}{\exp(-E_\downarrow/kT)}$

$\dfrac{N_\uparrow}{N_\downarrow} = \exp\left((E_\downarrow - E_\uparrow)/kT\right)$

T6 S.9

$$\frac{P_r(P)}{P_r(N)} = \frac{\exp(-E_P/k_B T)}{\exp(-E_N/k_B T)}$$

$$\boxed{\frac{P}{N} = \exp\left((E_N - E_P)/k_B T\right)}$$

T6 R1

As in other problems

$$R = \exp\left(\Delta E / kT\right)$$

$$\log R = \Delta E / kT$$

$$T = \frac{\Delta E}{k} \cdot \frac{1}{\log R} =$$

T6 R.2

$\Omega = a N U / \varepsilon$

$S = k \log \Omega = k \log (a N U / \varepsilon)$
$\quad = \text{const} + k \log(U/\varepsilon)$

a) Yes

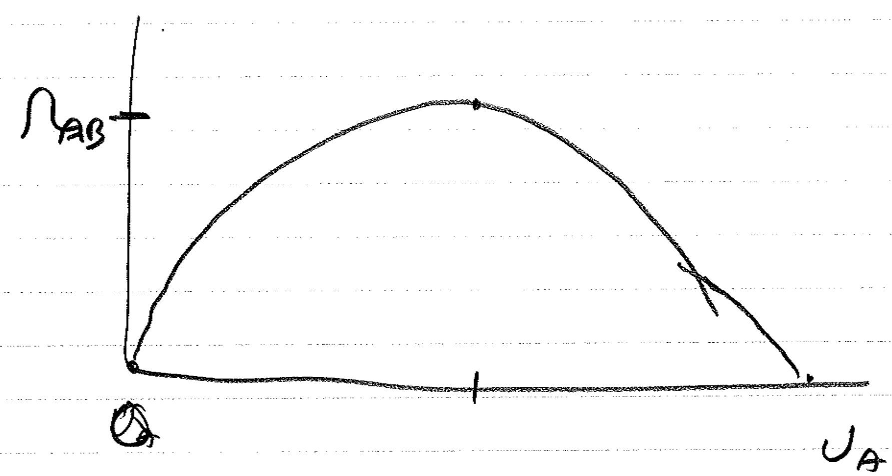
$$\frac{1}{T} = \frac{\partial S}{\partial U} = k \frac{1}{U}$$

$$U = kT$$

b) $\Omega_a = a N U_a / \varepsilon$
$\Omega_B = a N U_b / \varepsilon$

$$\Omega_A \Omega_B = \left(\frac{aN}{\varepsilon}\right)^2 U_a \cdot U_b$$

$$= C \cdot U_a (U - U_a)$$



Yes. . .

c) Very Large fluctuations

d) If you slice in 2, Temperature
of both drops!