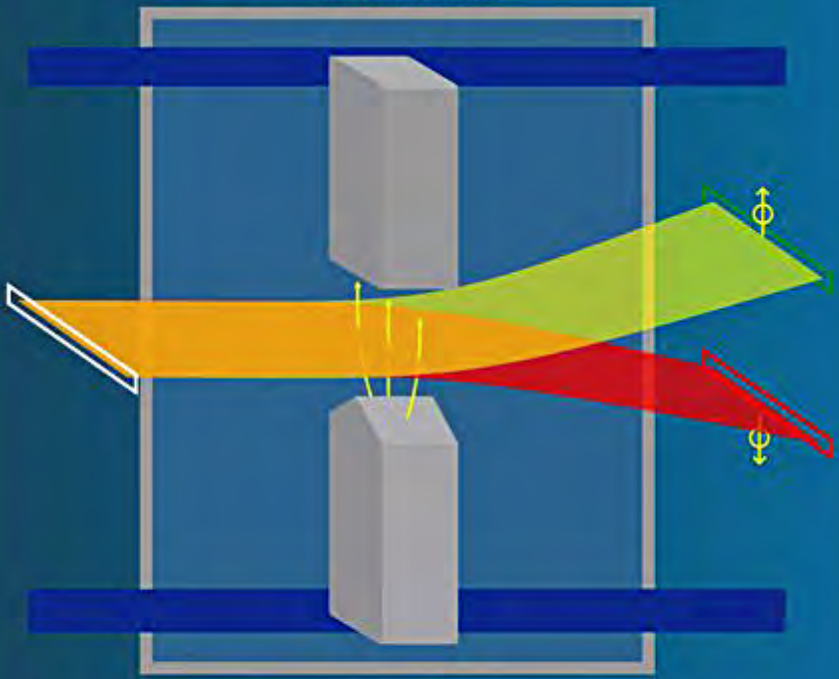
A Modern Approach to QUANTUM MECHANICS

Second Edition



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Production Manager: *Paul C. Anagnostopoulos, Windfall Software*
Copyeditor: *Lee A. Young*
Proofreader: *MaryEllen N. Oliver*
Text Design: *Yvonne Tsang*
Cover Design: *Genette Itoko McGrew*
Illustrator: *Lineworks*
Compositor: *Windfall Software*
Printer & Binder: *Edwards Brothers, Inc.*

This book was set in Times Roman and Gotham and composed with ZzTEX, a macro package for Donald Knuth's TEX typesetting system.

This book is printed on acid-free paper. ∞

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ISBN: 978-1-891389-78-8

Library of Congress Cataloging-in-Publication Data

Townsend, John S.
A modern approach to quantum mechanics / John S. Townsend. — 2nd ed.
p. cm.
Includes index.
ISBN 978-1-891389-78-8 (alk. paper)
1. Quantum theory—Textbooks. I. Title.
QC174.12.T69 2012
530.12—dc23 2011049655

Printed in the United States of America
10 9 8 7 6 5 4 3 2 1

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