Homework Assignment 2
Review

Practice Problems

These problems are graded on effort only.

Griffiths: P1.5, P1.8

Hint for P1.8: Let $f(x,t)$ be a solution to the S.E.:

$$i\hbar \frac{\partial f}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 f}{\partial x^2} + V f$$

Define:

$$g(x,t) = f(x,t)\,\exp(-\frac{iV_0 t}{\hbar})$$

Then calculate:

$$i\hbar \frac{\partial g}{\partial t}$$

Additional Problems

Problem 1: Consider the discrete probability distribution function $P(n)$ with:

$$\begin{aligned} P(0) &= \frac{1}{6} \\ P(3) &= \frac{1}{3} \\ P(4) &= \frac{1}{2} \end{aligned}$$

and $P(n) = 0$ for all other n.

(a) Is $P(n)$ properly normalized? Show your work.

(b) Find the expectation value $\langle n \rangle$ of the random variable n.

(b) Find the variance σ^2 of the random variable n.

Problem 2: Suppose the wave function for a particle is:

$$\Psi(x,t) = \begin{cases} \sqrt{\dfrac{\pi}{2a}}\, e^{it/t_0} \sqrt{\sin\left(\dfrac{\pi x}{a}\right)} & 0 \le x \le a \\ 0 & \text{otherwise} \end{cases}$$

(a) Plot $|\Psi(x,t)|^2$ as a function of x. Does the time t matter?

(b) Is Ψ properly normalized? Show how you determined this.

(c) For any position b what is the probability that you observe the particle with $x \le b$? (Hint: consider three cases $b < 0$, $0 \le b \le L$, and $b > L$.)

Problem 3: Define $P_{ab}(t)$ as the probability of measuring a particle in the range $a < x < b$, at time t. Show that:

$$\frac{dP_{ab}}{dt} = J(a,t) - J(b,t),$$

where:

$$J(x,t) \equiv \frac{i\hbar}{2m}\left(\Psi\frac{\partial\Psi^*}{\partial x} - \Psi^*\frac{\partial\Psi}{\partial x}\right)$$

Hint: look closely at the section on Normalization in Chapter 1 of the *lecture notes*.

Problem 4 (Worth Double Credit): Suppose we have two wave functions:

$$\Psi_1(x,t) = \begin{cases} \sqrt{\dfrac{2}{a}} \sin\left(\dfrac{2\pi x}{a}\right) \exp\left(-i\omega_1 t\right) & -\dfrac{a}{2} \le x \le \dfrac{a}{2} \\ 0 & \text{otherwise} \end{cases}$$

$$\Psi_2(x,t) = \begin{cases} \sqrt{\dfrac{2}{a}} \sin\left(\dfrac{4\pi x}{a}\right) \exp\left(-i\omega_2 t\right) & -\dfrac{a}{2} \le x \le \dfrac{a}{2} \\ 0 & \text{otherwise} \end{cases}$$

(a) Calculate $|\Psi_1|^2$ for $-a/2 \le x \le a/2$. Is it time dependent?

(b) Calculate $|\Psi_2|^2$ for $-a/2 \le x \le a/2$. Is it time dependent?

Define the wave function Ψ as:

$$\Psi = A \cdot (\Psi_1 + \Psi_2)$$

for some constant A.

(c) Calculate $|\Psi|^2$ for $-a/2 \le x \le a/2$. Is it time dependent?

Hint: Use HW1, Problem 2 to make short work of the remaining steps!

(d) Show that Ψ_1 and Ψ_2 are properly normalized.

(e) Find the positive real value A that properly normalizes Ψ.

(f) Show that $\langle x \rangle$ is zero for Ψ_1 and Ψ_2.

(g) Calculate $\langle x \rangle$ for Ψ.