

# Homework Assignment 2

## Review

### Practice Problems

These problems are graded on effort only.

**Griffiths: P1.5, P1.8**

**Hint for P1.8:** Let  $f(x, t)$  be a solution to the S.E.:

$$i\hbar \frac{\partial f}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 f}{\partial x^2} + V f$$

Define:

$$g(x, t) = f(x, t) \exp\left(-\frac{iV_0 t}{\hbar}\right)$$

Then calculate:

$$i\hbar \frac{\partial g}{\partial t}$$

### Additional Problems

**Problem 1:** Consider the discrete probability distribution function  $P(n)$  with:

$$P(0) = \frac{1}{6}$$

$$P(3) = \frac{1}{3}$$

$$P(4) = \frac{1}{2}$$

and  $P(n) = 0$  for all other  $n$ .

(a) Is  $P(n)$  properly normalized? Show your work.

(b) Find the expectation value  $\langle n \rangle$  of the random variable  $n$ .

(b) Find the variance  $\sigma^2$  of the random variable  $n$ .

**Problem 2:** Suppose the wave function for a particle is:

$$\Psi(x, t) = \begin{cases} \sqrt{\frac{\pi}{2a}} e^{it/t_0} \sqrt{\sin\left(\frac{\pi x}{a}\right)} & 0 \leq x \leq a \\ 0 & \text{otherwise} \end{cases}$$

- (a) Plot  $|\Psi(x, t)|^2$  as a function of  $x$ . Does the time  $t$  matter?
- (b) Is  $\Psi$  properly normalized? Show how you determined this.
- (c) For any position  $b$  what is the probability that you observe the particle with  $x \leq b$ ? (Hint: consider three cases  $b < 0$ ,  $0 \leq b \leq a$ , and  $b > a$ .)

**Problem 3:** Define  $P_{ab}(t)$  as the probability of measuring a particle in the range  $a < x < b$ , at time  $t$ . Show that:

$$\frac{dP_{ab}}{dt} = J(a, t) - J(b, t),$$

where:

$$J(x, t) \equiv \frac{i\hbar}{2m} \left( \Psi \frac{\partial \Psi^*}{\partial x} - \Psi^* \frac{\partial \Psi}{\partial x} \right)$$

Hint: look closely at the section on Normalization in Chapter 1 of the *lecture notes*.

**Problem 4 (Worth Double Credit):** Suppose we have two wave functions:

$$\Psi_1(x, t) = \begin{cases} \sqrt{\frac{2}{a}} \sin\left(\frac{2\pi x}{a}\right) \exp(-i\omega_1 t) & -\frac{a}{2} \leq x \leq \frac{a}{2} \\ 0 & \text{otherwise} \end{cases}$$

$$\Psi_2(x, t) = \begin{cases} \sqrt{\frac{2}{a}} \sin\left(\frac{4\pi x}{a}\right) \exp(-i\omega_2 t) & -\frac{a}{2} \leq x \leq \frac{a}{2} \\ 0 & \text{otherwise} \end{cases}$$

- (a) Calculate  $|\Psi_1|^2$  for  $-a/2 \leq x \leq a/2$ . Is it time dependent?
- (b) Calculate  $|\Psi_2|^2$  for  $-a/2 \leq x \leq a/2$ . Is it time dependent?

Define the wave function  $\Psi$  as:

$$\Psi = A \cdot (\Psi_1 + \Psi_2)$$

for some constant  $A$ .

- (c) Calculate  $|\Psi|^2$  for  $-a/2 \leq x \leq a/2$ . Is it time dependent?

**Hint:** Use HW1, Problem 2 to make short work of the remaining steps!

- (d) Show that  $\Psi_1$  and  $\Psi_2$  are properly normalized.
- (e) Find the positive real value  $A$  that properly normalizes  $\Psi$ .
- (f) Show that  $\langle x \rangle$  is zero for  $\Psi_1$  and  $\Psi_2$ .
- (g) Calculate  $\langle x \rangle$  for  $\Psi$ .