Homework Assignment 1
Review

Practice Problems

(No Practice Problems)

Additional Problems

Problem 1: You have already covered[1] linear algebra in PHY 104A. Consider the real symmetric matrix:

$$A = \begin{pmatrix} 1 & 3 \\ 3 & 1 \end{pmatrix}$$

(a) Find the eigenvalues of the matrix by finding the roots of the characteristic polynomial:

$$\det (A - \lambda I) = 0$$

(b) Find the two eigenvectors $\mathbf{u}$ and $\mathbf{v}$, and normalize them so that:

$$\mathbf{u} \cdot \mathbf{u} = \mathbf{v} \cdot \mathbf{v} = 1$$

(c) Note that $A = A^T$. Show that the two eigenvectors are orthogonal, i.e.:

$$\mathbf{u} \cdot \mathbf{v} = 0$$

(d) For the vector:

$$x = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

Find the values a and b such that:

$$x = a\mathbf{u} + b\mathbf{v}$$

Next problem on next page...

[1] See Chapter 2, Section 11 of **Mathematical Methods in the Physical Sciences (Third Edition)** by M.L. Boas if you need a review.

Problem 2: Show that:

$$\begin{aligned}
\frac{2}{L}\int_{-\frac{L}{2}}^{\frac{L}{2}} \sin\left(\frac{2\pi n}{L}\,x\right)\sin\left(\frac{2\pi m}{L}\,x\right)\,dx &= \delta_{nm} \\
\frac{2}{L}\int_{-\frac{L}{2}}^{\frac{L}{2}} \cos\left(\frac{2\pi n}{L}\,x\right)\cos\left(\frac{2\pi m}{L}\,x\right)\,dx &= \delta_{nm} \\
\frac{2}{L}\int_{-\frac{L}{2}}^{\frac{L}{2}} \sin\left(\frac{2\pi n}{L}\,x\right)\cos\left(\frac{2\pi m}{L}\,x\right)\,dx &= 0
\end{aligned}$$

for integers n and m where:

$$\delta_{nm} = \begin{cases} 1 & \text{if } n = m \\ 0 & \text{otherwise} \end{cases}$$

You may use the trigonometric identities:

$$\begin{aligned}
\sin\alpha\sin\beta &= \frac{1}{2}\{\cos(\alpha-\beta) - \cos(\alpha+\beta)\} \\
\cos\alpha\cos\beta &= \frac{1}{2}\{\cos(\alpha-\beta) + \cos(\alpha+\beta)\} \\
\cos\alpha\sin\beta &= \frac{1}{2}\{\sin(\alpha+\beta) - \sin(\alpha-\beta)\}.
\end{aligned}$$