Rm 140 (190?)

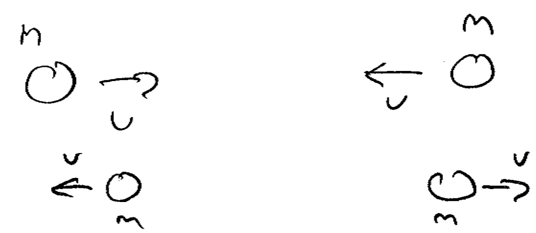
Intro to Elementary Particles

Recall: $d\tau = \frac{dt}{\gamma}$ ← why preferred way? (invariant)

$$\eta^\mu = \frac{dx^\mu}{d\tau} = \gamma \frac{dx^\mu}{dt} = \gamma(c, x, y, z)$$

Define: $p^\mu = m\,\eta^\mu$

Why on Earth should we throw out our old definition?



Clearly $+v \to O$, $-v \to X$

$P_i = x$ $\qquad P_f = x$ ✓

$K_i = \frac{1}{2} m x^2$ $\qquad K_f = \frac{1}{2} m x^2$ ✓

What's the problem?

$$2mu_3 + mu_4 = 0$$

$$\Rightarrow \quad u_4 = -2u_3$$

$$\frac{1}{2}(2m)u_0^2 + \frac{1}{2}m(-2u_0)^2 = \frac{1}{2}(2m)u_3^2 + \frac{1}{2}mu_4^2$$

$$6u_0^2 = 2u_3^2 + u_4^2 = 6u_3^2$$

$$u_3 = \pm u_0$$ (+ solution is trivial)

$$\boxed{\begin{aligned} u_3 &= -u_0 \\ u_4 &= 2u_0 \end{aligned}}$$

Next we need

$$u_x' = \frac{dx}{dt} = \frac{\gamma(dx' - \beta c\,dt')}{\gamma(c\,dt - \beta\,dx')}$$

$$\boxed{u_x' = \frac{u_x - \beta}{1 - \beta u_x}}$$

Take $u_0 = B = \frac{1}{4}$

$u_1 = \frac{1}{4}$

$u_2 = -\frac{1}{2}$

$u_3 = -\frac{1}{4}$

$u_4 = \frac{1}{2}$

$u_1' = 0$

$$u_2' = \frac{-\frac{1}{2} - \frac{1}{4}}{1 + \frac{1}{4}\frac{1}{2}} = -\frac{2}{3}$$

$$u_3' = \frac{-\frac{1}{4} - \frac{1}{4}}{1 + \frac{1}{4}\frac{1}{4}} = \frac{-8}{17}$$

$$u_4' = \frac{\frac{1}{2} - \frac{1}{4}}{1 - \frac{1}{4}\frac{1}{2}} = \frac{2}{7}$$

$$\frac{P_i}{mc} = -\frac{2}{3} \qquad \frac{P_f}{mc} = \frac{-16}{17} + \frac{2}{7}$$

$$\frac{P_i}{mc} = -0.666 \qquad \frac{P_f}{mc} = \frac{+78}{119} \sim 0.655$$

FAIL

O.K. so <u>not</u> $p = mu$.

Why $p^\mu = m\eta^\mu$?

→ Will reduce to $p = mu$ at low velocity.

→ Its a four-vector, so if

p^μ is conserved,

$$p^\mu_i = p^\mu_f$$

$$\Rightarrow \quad p'^\mu_i = p'^\mu_f \quad \text{in any frame}$$

(Just apply LT!)

Turns out it <u>is</u> conserved, in precise experimental tests...

where ?

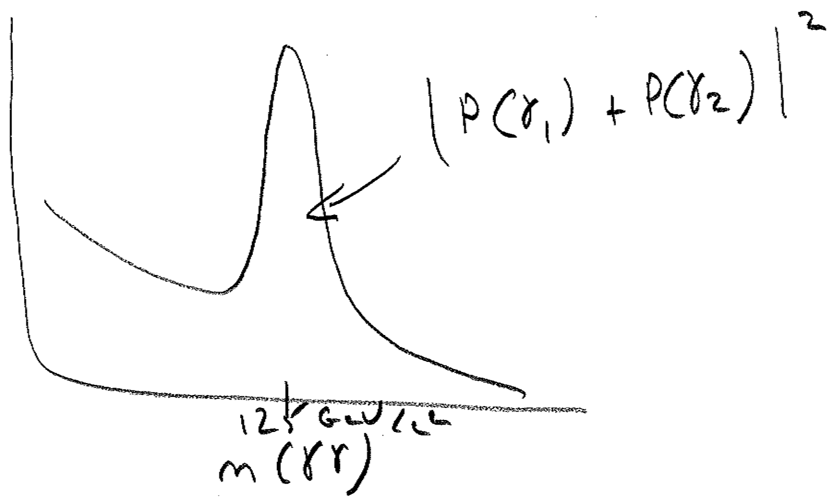
Mass-less Particles

Definition Fails $\quad p = \gamma m v = \frac{0}{0} = ?$

for $m=0$ $v=c$ $\quad E = \gamma m = \frac{0}{0} = ?$

$v = c \qquad E = |p| c$

Why should you believe this?



Classic Collisions

1) Mass is conserved
2) Momentum is conserved
3) K.E. may or may not be c.

EG
Clay lumps, Explosion

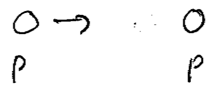
Relativistic Collisions

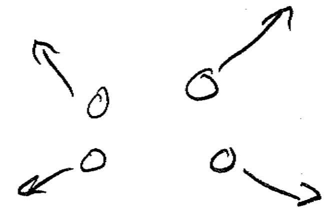
1) Energy is conserved
2) Momentum is conserved
3) K.E. may or may not

*) Mass not conserved in inelastic collisions,
$\pi^0 \to \gamma\gamma$

Bevatron

3.4





LAB: $p^{\mu}_{TOT} = \left(\frac{E+mc^2}{c}, |p|, 0, 0\right)$

$p^{\mu}_{TOT} = \left(4mc, 0, 0, 0\right)$

$$(4mc)^2 = \left(\frac{E+mc^2}{c}\right)^2 - p^2$$

$$= \frac{E^2}{c^2} + 2Em + m^2c^2 - p^2$$

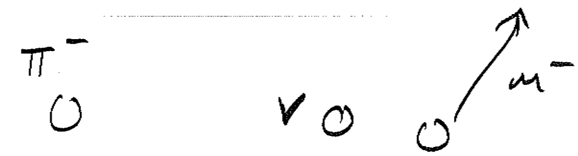
$$= \left(\frac{E^2}{c^2} - p^2\right) + 2Em$$

$$16m^2c^2 = m^2c^2 + 2Em + m^2c^2$$

$$14m^2c^2 = 2Em$$

$$\Rightarrow \quad E = 7mc^2 > 4mc^2$$

3.3)



$$E_\pi = E_\nu + E_\mu$$

$$\vec{p}_\pi = 0 = \vec{p}_\nu + \vec{p}_\mu \Rightarrow \vec{p}_\nu = -\vec{p}_\mu$$

<u>IDEA 1</u>: $\vec{p}_\nu = -\vec{p}_\mu$

$$\Rightarrow \frac{1}{\sqrt{1-\beta_\nu^2}} m_\nu c \beta_\nu = \frac{1}{\sqrt{1-\beta_\mu^2}} m_\mu c \beta_\mu$$

$\rightarrow \quad \beta_\nu = F(\beta_\mu)$

$\rightarrow$ Plug into $E_\pi = E_\nu + E_\mu$
solve for β_ν and β_μ ... DONE???

Velocity is a <u>BAD CHOICE</u>. why?

$$\frac{1}{\sqrt{1-\beta^2}} m \beta$$

IDEA 2 Use $E^2 = m^2c^4 + p^2c^2$

$$\vec{p_\mu} = -\vec{p_\nu}$$

$$E_\pi = m_\pi c^2$$

$$E_\mu = c\sqrt{m_\mu^2 c^2 + p_\mu^2}$$

$$E_\nu = |p_\nu| c = |p_\mu| c$$

$$E_\pi = E_\mu + E_\nu$$

$$m_\pi c^2 = c\sqrt{m_\mu^2 c^2 + p_\mu^2} + p_\mu c$$

.... Solve for E and p

Want v? $\boxed{v = p/E}$