

# **INTRODUCTORY CLASSICAL MECHANICS**

**\* WITH PROBLEMS AND SOLUTIONS \***

**David Morin**

Copyright © 2003 by David Morin

All rights reserved

# Contents

<b>1</b>	<b>Statics</b>	<b>I-1</b>
1.1	Balancing forces . . . . .	I-1
1.2	Balancing torques . . . . .	I-5
1.3	Problems . . . . .	I-8
1.4	Solutions . . . . .	I-12
<b>2</b>	<b>Using <math>F = ma</math></b>	<b>II-1</b>
2.1	Newton's Laws . . . . .	II-1
2.2	Free-body diagrams . . . . .	II-3
2.3	Solving differential equations . . . . .	II-6
2.4	Projectile motion . . . . .	II-11
2.5	Motion in a plane, polar coordinates . . . . .	II-13
2.6	Exercises . . . . .	II-16
2.7	Problems . . . . .	II-18
2.8	Solutions . . . . .	II-22
<b>3</b>	<b>Oscillations</b>	<b>III-1</b>
3.1	Linear differential equations . . . . .	III-1
3.2	Oscillatory motion . . . . .	III-4
3.2.1	Simple harmonic motion . . . . .	III-4
3.2.2	Damped harmonic motion . . . . .	III-6
3.2.3	Driven (and damped) harmonic motion . . . . .	III-8
3.3	Coupled oscillators . . . . .	III-11
3.4	Exercises . . . . .	III-17
3.5	Problems . . . . .	III-19
3.6	Solutions . . . . .	III-21
<b>4</b>	<b>Conservation of Energy and Momentum</b>	<b>IV-1</b>
4.1	Conservation of energy in 1-D . . . . .	IV-1
4.2	Small Oscillations . . . . .	IV-4
4.3	Conservation of energy in 3-D . . . . .	IV-6
4.3.1	Conservative forces in 3-D . . . . .	IV-6
4.4	Gravity due to a sphere . . . . .	IV-9
4.4.1	Derivation via the potential energy . . . . .	IV-9
4.4.2	Tides . . . . .	IV-11

4.5	Conservation of linear momentum . . . . .	IV-14
4.5.1	Conservation of $\mathbf{p}$ . . . . .	IV-14
4.5.2	Rocket motion . . . . .	IV-16
4.6	The CM frame . . . . .	IV-17
4.6.1	Definition . . . . .	IV-17
4.6.2	Kinetic energy . . . . .	IV-19
4.7	Collisions . . . . .	IV-20
4.7.1	1-D motion . . . . .	IV-20
4.7.2	2-D motion . . . . .	IV-21
4.8	Inherently inelastic processes . . . . .	IV-22
4.9	Exercises . . . . .	IV-25
4.10	Problems . . . . .	IV-29
4.11	Solutions . . . . .	IV-36
<b>5</b>	<b>The Lagrangian Method</b>	<b>V-1</b>
5.1	The Euler-Lagrange equations . . . . .	V-1
5.2	The principle of stationary action . . . . .	V-4
5.3	Forces of constraint . . . . .	V-9
5.4	Change of coordinates . . . . .	V-11
5.5	Conservation Laws . . . . .	V-14
5.5.1	Cyclic coordinates . . . . .	V-14
5.5.2	Energy conservation . . . . .	V-15
5.6	Noether's Theorem . . . . .	V-16
5.7	Small oscillations . . . . .	V-20
5.8	Other applications . . . . .	V-22
5.9	Exercises . . . . .	V-26
5.10	Problems . . . . .	V-27
5.11	Solutions . . . . .	V-33
<b>6</b>	<b>Central Forces</b>	<b>VI-1</b>
6.1	Conservation of angular momentum . . . . .	VI-1
6.2	The effective potential . . . . .	VI-2
6.3	Solving the equations of motion . . . . .	VI-4
6.3.1	Finding $r(t)$ and $\theta(t)$ . . . . .	VI-5
6.3.2	Finding $r(\theta)$ . . . . .	VI-5
6.4	Gravity, Kepler's Laws . . . . .	VI-6
6.4.1	Calculation of $r(\theta)$ . . . . .	VI-6
6.4.2	The orbits . . . . .	VI-8
6.4.3	Proof of conic orbits . . . . .	VI-9
6.4.4	Kepler's Laws . . . . .	VI-10
6.4.5	Reduced mass . . . . .	VI-11
6.5	Problems . . . . .	VI-13
6.6	Solutions . . . . .	VI-14

<b>7</b>	<b>Angular Momentum, Part I (Constant <math>\hat{L}</math>)</b>	<b>VII-1</b>
7.1	Pancake object in $x$ - $y$ plane . . . . .	VII-2
7.1.1	Rotation about the $z$ -axis . . . . .	VII-2
7.1.2	General motion in $x$ - $y$ plane . . . . .	VII-3
7.1.3	The parallel-axis theorem . . . . .	VII-5
7.1.4	The perpendicular-axis theorem . . . . .	VII-6
7.2	Calculating moments of inertia . . . . .	VII-6
7.2.1	Lots of examples . . . . .	VII-6
7.2.2	A neat trick . . . . .	VII-9
7.3	Torque . . . . .	VII-10
7.3.1	Point mass, fixed origin . . . . .	VII-10
7.3.2	Extended mass, fixed origin . . . . .	VII-11
7.3.3	Extended mass, non-fixed origin . . . . .	VII-12
7.4	Angular impulse . . . . .	VII-13
7.5	Exercises . . . . .	VII-15
7.6	Problems . . . . .	VII-16
7.7	Solutions . . . . .	VII-21
<b>8</b>	<b>Angular Momentum, Part II</b>	<b>VIII-1</b>
8.1	Preliminaries concerning rotations . . . . .	VIII-1
8.1.1	The form of general motion . . . . .	VIII-1
8.1.2	The angular velocity vector . . . . .	VIII-2
8.2	The inertia tensor . . . . .	VIII-5
8.2.1	Rotation about an axis through the origin . . . . .	VIII-5
8.2.2	General motion . . . . .	VIII-8
8.2.3	The parallel-axis theorem . . . . .	VIII-10
8.3	Principal axes . . . . .	VIII-10
8.4	Two basic types of problems . . . . .	VIII-15
8.4.1	Motion after an impulsive blow . . . . .	VIII-15
8.4.2	Frequency of motion due to a torque . . . . .	VIII-18
8.5	Euler's equations . . . . .	VIII-20
8.6	Free symmetric top . . . . .	VIII-22
8.6.1	View from body frame . . . . .	VIII-23
8.6.2	View from fixed frame . . . . .	VIII-24
8.7	Heavy symmetric top . . . . .	VIII-25
8.7.1	Euler angles . . . . .	VIII-25
8.7.2	Digression on the components of $\vec{\omega}$ . . . . .	VIII-26
8.7.3	Torque method . . . . .	VIII-29
8.7.4	Lagrangian method . . . . .	VIII-30
8.7.5	Gyroscope with $\dot{\theta} = 0$ . . . . .	VIII-31
8.7.6	Nutation . . . . .	VIII-33
8.8	Exercises . . . . .	VIII-36
8.9	Problems . . . . .	VIII-38
8.10	Solutions . . . . .	VIII-44

<b>9 Accelerated Frames of Reference</b>	<b>IX-1</b>
9.1 Relating the coordinates . . . . .	IX-2
9.2 The fictitious forces . . . . .	IX-4
9.2.1 Translation force: $-md^2\mathbf{R}/dt^2$ . . . . .	IX-5
9.2.2 Centrifugal force: $-m\vec{\omega} \times (\vec{\omega} \times \mathbf{r})$ . . . . .	IX-5
9.2.3 Coriolis force: $-2m\vec{\omega} \times \mathbf{v}$ . . . . .	IX-6
9.2.4 Azimuthal force: $-m(d\vec{\omega}/dt) \times \mathbf{r}$ . . . . .	IX-10
9.3 Exercises . . . . .	IX-12
9.4 Problems . . . . .	IX-14
9.5 Solutions . . . . .	IX-16
<b>10 Relativity (Kinematics)</b>	<b>X-1</b>
10.1 The postulates . . . . .	X-2
10.2 The fundamental effects . . . . .	X-4
10.2.1 Loss of Simultaneity . . . . .	X-4
10.2.2 Time dilation . . . . .	X-6
10.2.3 Length contraction . . . . .	X-10
10.3 The Lorentz transformations . . . . .	X-12
10.3.1 The derivation . . . . .	X-12
10.3.2 The fundamental effects . . . . .	X-16
10.3.3 Velocity addition . . . . .	X-18
10.4 The spacetime interval . . . . .	X-21
10.5 Minkowski diagrams . . . . .	X-24
10.6 The Doppler effect . . . . .	X-26
10.6.1 Longitudinal Doppler effect . . . . .	X-26
10.6.2 Transverse Doppler effect . . . . .	X-27
10.7 Rapidity . . . . .	X-29
10.8 Relativity without $c$ . . . . .	X-32
10.9 Exercises . . . . .	X-36
10.10 Problems . . . . .	X-39
10.11 Solutions . . . . .	X-45
<b>11 Relativity (Dynamics)</b>	<b>XI-1</b>
11.1 Energy and momentum . . . . .	XI-1
11.1.1 Momentum . . . . .	XI-2
11.1.2 Energy . . . . .	XI-3
11.2 Transformations of $E$ and $\vec{p}$ . . . . .	XI-6
11.3 Collisions and decays . . . . .	XI-8
11.4 Particle-physics units . . . . .	XI-11
11.5 Force . . . . .	XI-13
11.5.1 Force in one dimension . . . . .	XI-13
11.5.2 Force in two dimensions . . . . .	XI-14
11.5.3 Transformation of forces . . . . .	XI-15
11.6 Rocket motion . . . . .	XI-17
11.7 Relativistic strings . . . . .	XI-20

11.8	Mass . . . . .	XI-22
11.9	Exercises . . . . .	XI-24
11.10	Problems . . . . .	XI-25
11.11	Solutions . . . . .	XI-29
<b>12</b>	<b>4-vectors</b>	<b>XII-1</b>
12.1	Definition of 4-vectors . . . . .	XII-1
12.2	Examples . . . . .	XII-2
12.3	Properties of 4-vectors . . . . .	XII-4
12.4	Energy, momentum . . . . .	XII-6
12.4.1	Norm . . . . .	XII-6
12.4.2	Transformation of $E, p$ . . . . .	XII-6
12.5	Force and acceleration . . . . .	XII-7
12.5.1	Transformation of forces . . . . .	XII-7
12.5.2	Transformation of accelerations . . . . .	XII-8
12.6	The form of physical laws . . . . .	XII-9
12.7	Problems . . . . .	XII-11
12.8	Solutions . . . . .	XII-12
<b>13</b>	<b>General Relativity</b>	<b>XIII-1</b>
13.1	The Equivalence Principle . . . . .	XIII-1
13.1.1	Statement of the principle . . . . .	XIII-1
13.1.2	Time dilation . . . . .	XIII-2
13.2	Uniformly accelerated frame . . . . .	XIII-4
13.2.1	Uniformly accelerated point particle . . . . .	XIII-4
13.2.2	Uniformly accelerated frame . . . . .	XIII-6
13.3	Maximal-proper-time principle . . . . .	XIII-7
13.4	Twin paradox revisited . . . . .	XIII-9
13.5	Exercises . . . . .	XIII-11
13.6	Problems . . . . .	XIII-13
13.7	Solutions . . . . .	XIII-16
<b>14</b>	<b>Appendices</b>	<b>XIV-1</b>
14.1	Appendix A: Useful formulas . . . . .	XIV-1
14.1.1	Taylor series . . . . .	XIV-1
14.1.2	Nice formulas . . . . .	XIV-2
14.1.3	Integrals . . . . .	XIV-2
14.2	Appendix B: Units, dimensional analysis . . . . .	XIV-4
14.2.1	Problems . . . . .	XIV-6
14.2.2	Solutions . . . . .	XIV-6
14.3	Appendix C: Approximations, limiting cases . . . . .	XIV-8
14.4	Appendix D: Solving differential equations numerically . . . . .	XIV-11
14.5	Appendix E: $F = ma$ vs. $F = dp/dt$ . . . . .	XIV-13
14.6	Appendix F: Existence of principal axes . . . . .	XIV-15
14.7	Appendix G: Diagonalizing matrices . . . . .	XIV-18

14.8 Appendix H: Qualitative Relativity Questions . . . . .	XIV-20
14.9 Appendix I: Lorentz transformations . . . . .	XIV-24
14.10Appendix J: Physical constants and data . . . . .	XIV-27