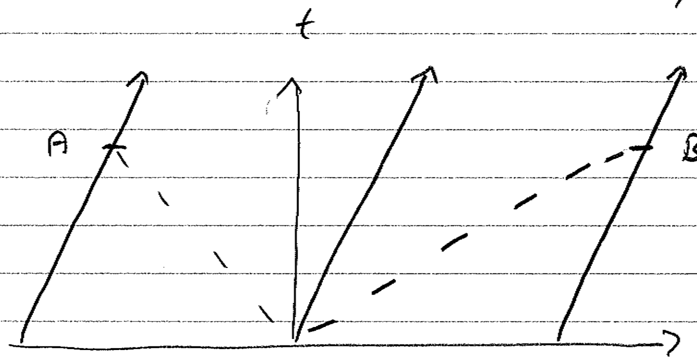
My Time: 5 minutes

R3S.1



In Newtonian view, $t_A = t_B = t'_A = t'_B$.

This requires the two light pulses to travel at different speeds in the Home Frame $(c-\beta$ and $c+\beta)$

My Time: 5 minutes

R3 S. 3

a) 2:17 pm (6:17 pm − 4:00)

b) Event B happens at 7:17 pm by clock at location (mine)
Event A happens at 2:17 pm by clock at Neptune (part (a))
Coordinate time is therefore 5 hours.

c) 4.0 light-hours / 5.0 hours

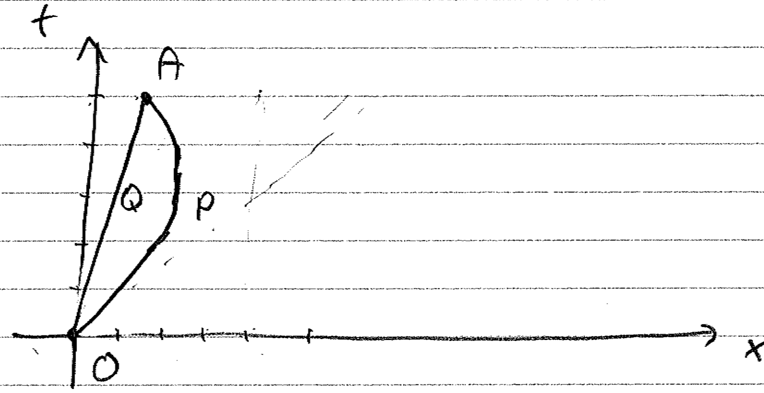
$\boxed{4/5}$

d) A spacetime interval, a special case of both proper and coordinate time, so all three!

My Time: 7 minutes

R3S.4

a)



b) Not necessarily...

1) Q measures a spacetime interval while P measure proper time only.

2) P is non-inertial, so all bets are off!

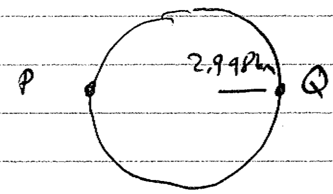
c) Proper-time: P and Q

Space-time interval: Q only

Coordinate time: All three

My Time: 7 minutes

R35.5



$d = \pi \cdot 2{,}998 \times 10^3\,m \,/\, 2{,}998 \times 10^8\,m/s$

$= \pi \cdot 10^{-5}\,s = 31.4\,\mu s$

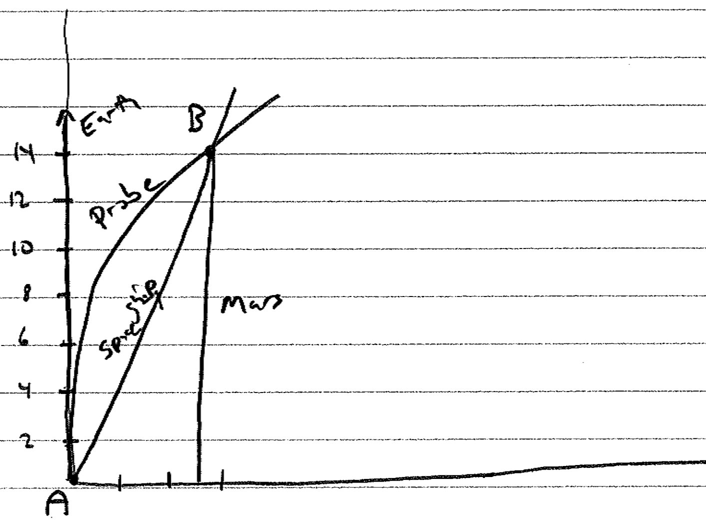
a) $v = 31.4 / 34.9 = 0.9$

b) Coordinate Time

c) Proper Time (and Coordinate Time)
NOT space-time interval because
constant velocity in circle is non-inertial.

R3 S. 6

a)



b) Both probe and spaceship, (as well as synchronized Earth - Mars clocks) measure coordinate time

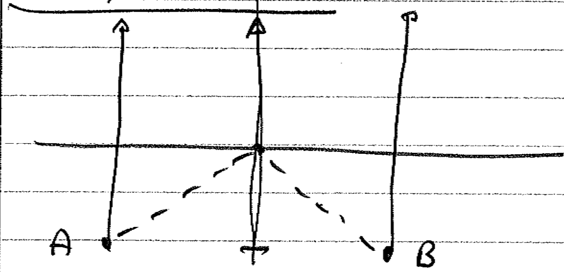
c) Probe and Space ship

d) Space Ship, as it is inertial!

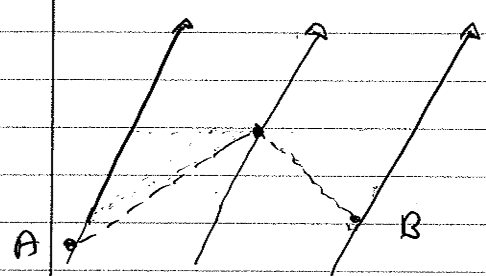
R3R.1

Harry's Frame

Simultaneous



Sally's Frame



R4S.1

You measure $\Delta x = 0.4\ \mu s$ and $\Delta t = 1.0\ \mu s$
so spacetime interval is:

$$\Delta s = \sqrt{(\Delta t)^2 - (\Delta x)^2}$$

$$= \sqrt{1.0^2 - 0.4^2} = 0.91$$

This is shortest possible time interval,
so not possible for friend to measure

$$\Delta t = 0.6\ \mu s < \Delta s$$

(For then

$$\Delta x^2 = (\Delta t)^2 - (\Delta s)^2 < 0 \quad)$$

RYS.2

a) $\Delta x = v \cdot \Delta t = 0.998 \cdot 2.0\ \mu s$

$= 1.996\ \mu s$

b) $\Delta s^2 = \Delta t^2 - \Delta x^2$

$\Delta t = \Delta x / B$

$\Delta s^2 = \Delta x^2 \left(\frac{1}{B^2} - 1 \right)$

$\Delta x^2 = B^2 \Delta s^2 / (1 - B^2)$

$\Delta x = B \Delta s / \sqrt{1 - B^2} = 31\ \mu s$

R4S.3

$$\Delta s^2 = \Delta t^2 - \Delta x^2$$

$$\Delta t = \Delta x / v$$

$$\Delta s^2 = \frac{1}{v^2}\left(1 - v^2\right) \Delta x^2$$

$$\Delta s = \frac{\sqrt{1-v^2}}{v} \Delta x = 6327 \text{ years}$$

R4S.4

$$\Delta s = 32 \text{ ns}$$

a) $\Delta s^2 = \Delta t^2 - \Delta x^2$

$$\Rightarrow \Delta t = \sqrt{\Delta s^2 + \Delta x^2} = \sqrt{(0.32)^2 + (0.45)^2 \text{ ns}^2}$$

$$\Delta t = 0.55 \text{ ns}$$

b) $\beta = \Delta x / \Delta t = 0.45 / 0.55 = 0.82$

R4S.5

R4 5.5

a) $\Delta s = 2.5\,h$

$\Delta x = 5.0\,h$

$$\Delta s^2 = \Delta t^2 - \Delta x^2 \Rightarrow \Delta t^2 = \Delta s^2 + \Delta x^2$$

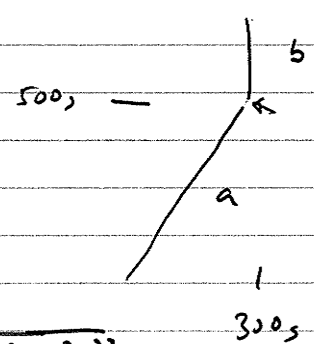
$$\Delta t = \sqrt{\Delta x^2 + \Delta s^2} = \sqrt{(5)^2 + (2.5)^2}$$

$$= 5.5 \text{ hours}$$

b)

$$v = \Delta x / \Delta t = \frac{5.0 \text{ hours}}{5.5 \text{ hours}} = 0.9$$

R4 5.6



Leg a:

$$\Delta s = \sqrt{(500\,s)^2 - (300\,s)^2}$$

$$= 400\,s$$

Leg b:

$$\Delta s = \Delta t = 200\,s$$

a) Proper Time is $400s + 200s = 600s$

b) Spacetime interval is

$$\Delta s = \sqrt{(500s + 200s)^2 - (300s)^2}$$

$$= 635\,s$$

R45.7

(a) In our units

$$1\,\text{km} = (1\,\text{km})\left(\frac{1}{3\times10^8\,\text{m/s}}\right)\left(\frac{1000\,\text{m}}{1\,\text{km}}\right)$$

$$= 3.3\ \mu\text{s}$$

So

$$\Delta t = \Delta x/\beta = 3.3\ \mu\text{s}/0.866$$

$$= 3.84\ \mu\text{s}$$

(b) $\Delta s = \sqrt{\Delta t^2 - \Delta x^2}$

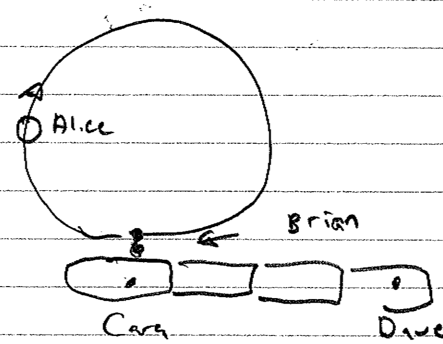
$$= \sqrt{(3.84)^2 - (3.3)^2} = 1.96\ \mu\text{s}$$

About $\frac{1}{2}$

(c) About $\frac{1}{7}$.

RSS.3

a)



- Cara plus Dave measure a coordinate time ($\Delta t_{E,F}$)
- Brian measures the space time interval ($\Delta S_{E,F}$)
- Alice measures a proper-time interval ($\Delta \tau_{E,F}$)

$\beta = 60\,m/s \,/\, 3\times10^{8}\,m/s = 20\times10^{-8} = 2\cdot10^{-7}$

$\Delta\tau = \sqrt{1-\beta^2}\,\Delta t$, so $\Delta\tau_{E,F} \leq \Delta S_{EF}$

From space time metric,

$$\Delta S^2 = \Delta t^2 - \Delta x^2 \Rightarrow \Delta S_{EF} < \Delta t_{E,F}$$

So $\Delta\tau_{E,F} \leq \Delta S_{E,F} < \Delta t_{E,F}$

$$\boxed{t(\text{Alice}) < t(\text{Brain}) < t(\text{Cara + Dave})}$$

b)

$$\Delta\tau = \sqrt{1-\beta^2}\;\Delta t \sim \left(1 - \tfrac{1}{2}\beta^2\right)\Delta t$$

$$\Delta t - \Delta\tau \sim \tfrac{1}{2}\beta^2 \Delta t$$

$$= \tfrac{1}{2}\left(2\cdot 10^{-7}\right)^2 \cdot 100\,s$$

$$= \boxed{2\times 10^{-12}\ s}$$

c) β for train is $\left(30\,m/s \,/\, 3\times 10^8\,m/s\right) = 1\cdot 10^{-7}$

$$\Delta s^2 = \Delta t^2 - \Delta x^2 = \Delta t^2\left(1-\beta^2\right)$$

$$\Delta t = \frac{\Delta s}{\sqrt{1-\beta^2}}$$

$$\Delta t - \Delta s = \tfrac{1}{2}\beta^2 \Delta s = \tfrac{1}{2}\left(10^{-7}\right)^2 \cdot 100\,s$$

$$= 5\cdot 10^{-12}\ s$$

R5 S.4

$$\Delta\tau = 1.52\ \mu s$$

$$\Delta\tau^2 = (1 - B^2)\,\Delta t^2$$

$$\left(\frac{\Delta\tau}{\Delta t}\right)^2 = 1 - B^2$$

$$B^2 = 1 - \left(\frac{\Delta\tau}{\Delta t}\right)^2$$

$$B = \sqrt{1 - \left(\frac{\Delta\tau}{\Delta t}\right)^2} \sim 1 - \frac{1}{2}\left(\frac{\Delta\tau}{\Delta t}\right)^2$$

$$1 - B = \frac{1}{2}\left(\frac{\Delta\tau}{\Delta t}\right)^2 = \frac{1}{2}\left(\frac{1.52 \times 10^{-6}\,s}{0.25\,s}\right)^2$$

$$= 1.8 \times 10^{-11}$$

$$B = 0.999999999982$$

R5 S.5

$$(1+x)^2 = (1+x)(1+x) = 1 + 2x + x^2$$

$$\sim 1 + 2x$$

$$(1+x)^3 = (1+x)^2 \cdot (1+x) = (1+2x)(1+x)$$

$$= 1 + 3x + 2x^2 \sim 1 + 3x$$

$$(1+x)^4 = (1+x)^3\,(1+x)$$

$$= (1+3x)(1+x) = 1 + 4x + 3x^2$$

$$\sim 1 + 4x$$

RSS.6

$$f(x) = (1+x)^a$$

$$\frac{\partial f}{\partial x} = a(1+x)^{a-1}$$

$$f(x) \sim f(0) + \left.\frac{\partial f}{\partial x}\right|_0 \cdot x$$

$$= 1 + a \cdot x$$

R55.8

$$g = \frac{v^2}{r} \Rightarrow v = \sqrt{g \cdot r}$$

$$= \sqrt{9.8 m/s^2 \cdot 6.3 \times 10^6 m} \;/\; 3 \times 10^8 m/s$$

$$= 2.6 \times 10^{-5}$$

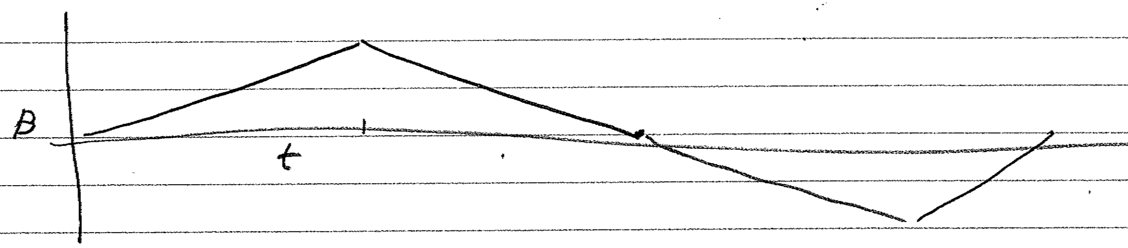
$$\Delta t = \left[(14.24 + 3.45) \text{ hrs} \right] \cdot \frac{60 \cdot 60 s}{1 hr}$$

$$= 1.22 \times 10^6 s$$

Diff: $\frac{1}{2} \beta^2 \cdot \Delta t = \frac{1}{2} (2.6 \times 10^{-5})^2 \cdot 1.22 \times 10^6 s$

$$\boxed{4 \times 10^{-4} s}$$

RSS.9



a) $a = (10\,m/s^2)$

$t = 10^6\,s$

$v_f = 10^7\,m/s$

$B_f = 10^7\,m/s \;/\; 3\times10^8\,m/s = 0.033$

b) $B = at \Rightarrow dB = a\,dt \qquad B_f = a t_f$

$$\tau = \int \sqrt{1-B^2}\,dt \sim \int \left(1 - \tfrac{1}{2}B^2\right) dt$$

$$= t - \frac{1}{2}\int B^2 dt$$

$$t - \tau = \frac{1}{2}\int B^2\,dt = \frac{1}{2a}\int B^2\,dB$$

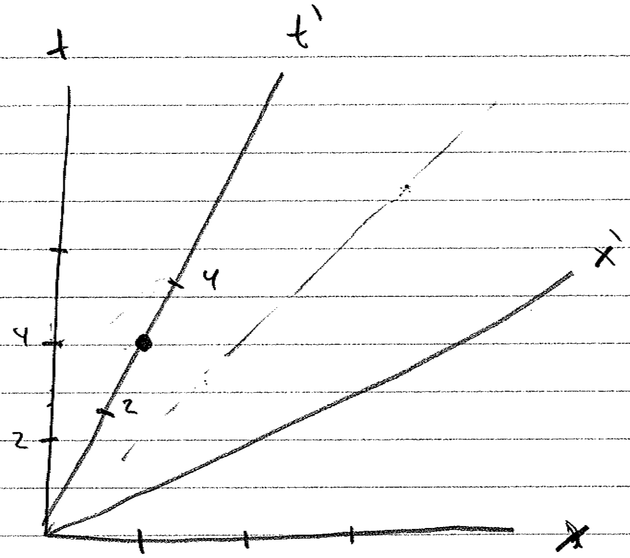
$$\boxed{t - \tau = \frac{B_f^3}{6a}}$$

Total difference:

$$4 \times \frac{B_f^3}{6a} = 4 \times \frac{0.033}{6\cdot(10\,m/s^2)} \frac{(3\times10^8\,m/s)}{1}$$

$$=$$

RGS.1



Using

$t = n \gamma \tau$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-\frac{1}{4}}} = \frac{2}{\sqrt{3}}$$

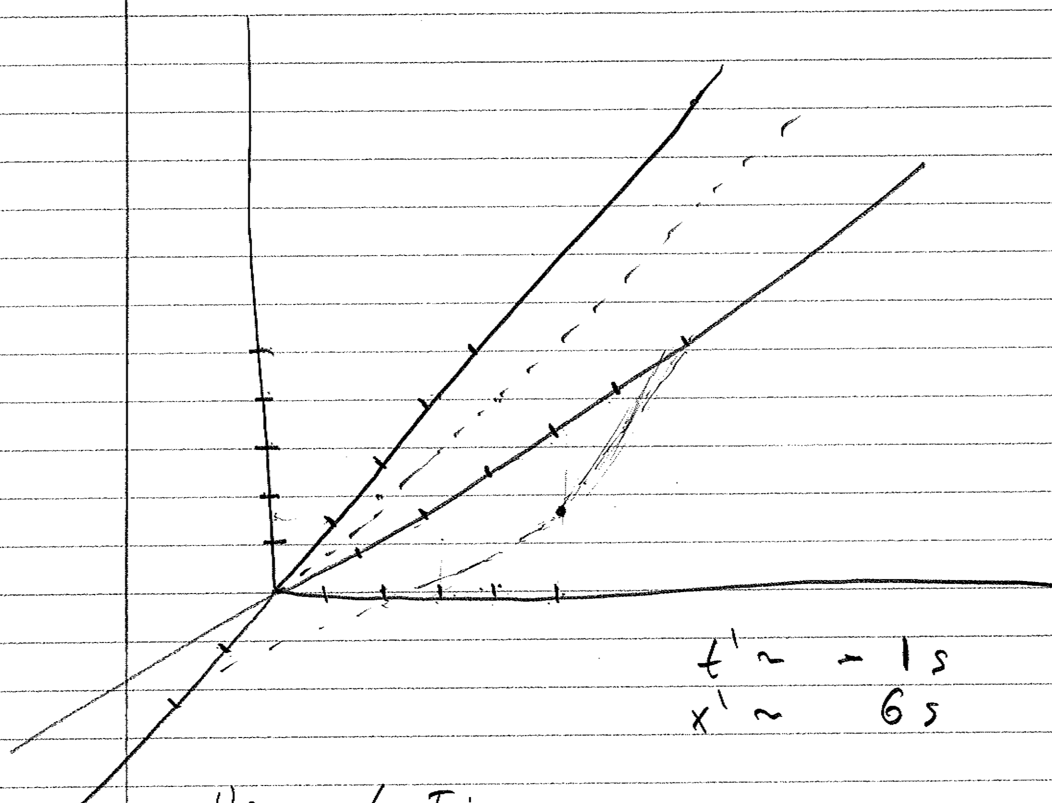
$$\boxed{\text{At} \quad t' = 3 \qquad x' = 0}$$

RGS2 $\quad \beta = \frac{3}{5}$

$$\gamma = \frac{1}{\sqrt{1-(3/5)^2}} = \frac{5}{4}$$

RG 52

$B = \frac{3}{5}$ $\quad \gamma = \frac{1}{\sqrt{1-(3/5)^2}} = \frac{5}{4}$



$t' \sim -1s$

$x' \sim 6s$

Using L-T:

$t' = \gamma(t - Bx)$ $\qquad x' = \gamma(x - Bt)$

$= \frac{5}{4}\left(t - \frac{3}{5}x\right)$ $\qquad = \frac{5}{4}\left(x - \frac{3}{5}t\right)$

$= \frac{5}{4}\left(\frac{3}{2} - 3\right)$ $\qquad = \frac{5}{4}\left(5 - \frac{3}{5}\frac{3}{2}\right)$

$= \frac{5}{4}\left(-\frac{3}{2}\right)$ $\qquad = \frac{5}{4}\left(\frac{50 - 9}{10}\right)$

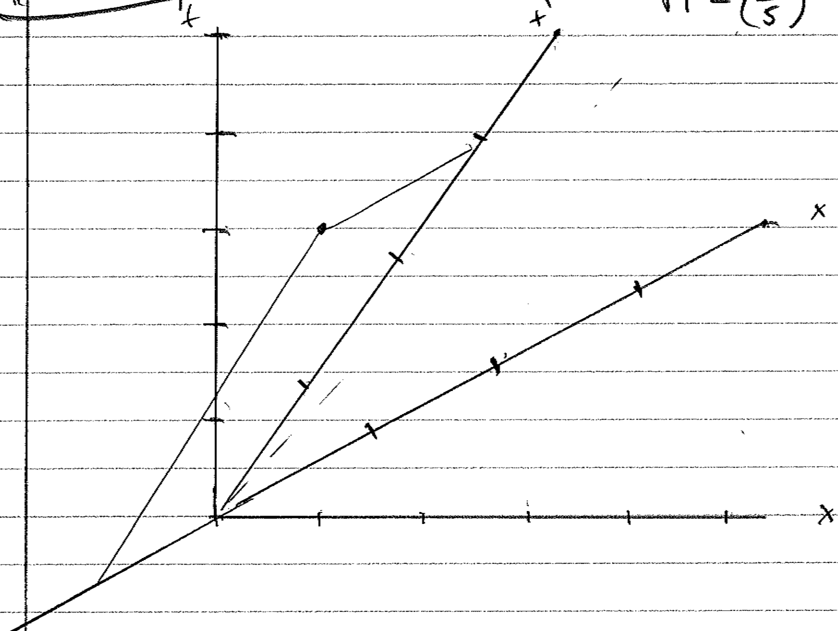
$= -\frac{15}{8}$ $\qquad = \frac{5 \cdot 41}{40}$

$= 5.1$

$\frac{41}{40}$

$\frac{40}{4} + \frac{1}{40}$

$\boxed{\text{RGS.3}}$ $\beta = \frac{3}{5}$ $\gamma = \frac{1}{\sqrt{1-\left(\frac{3}{5}\right)^2}} = \frac{5}{\sqrt{5^2-3^2}} = \frac{5}{4}$



$t = 3s$
$x = 1s$

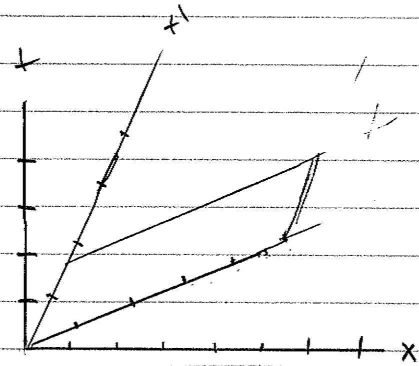
$x' \approx -1s$ $t' \sim 3s$

$$t' = \gamma(t - \beta x)$$
$$t' = \frac{5}{4}\left(3 - \frac{3}{5}\cdot 1\right)$$
$$= \frac{5 \cdot 12}{4 \cdot 5}$$
$$= 3$$

$$x' = \gamma(x - \beta t)$$
$$x' = \frac{5}{4}\left(1 - \frac{3}{5}3\right)$$
$$= \frac{5}{4}\frac{(5-9)}{5}$$
$$= -1s$$

R6 5.4 $\beta = 0.40 = \frac{2}{5}$ $\gamma = \frac{1}{\sqrt{1-\left(\frac{2}{5}\right)^2}} = \frac{5}{\sqrt{21}}$

~ 1.1



$t' = 1.5$

$x' = 5\,s$

$\boxed{t = 4\,s \qquad x = 6\,s}$

$x = \gamma(x' + \beta t)$

$= 1.1\left(5 + \frac{2}{5}1.5\right)$

$= 6.1\,s$

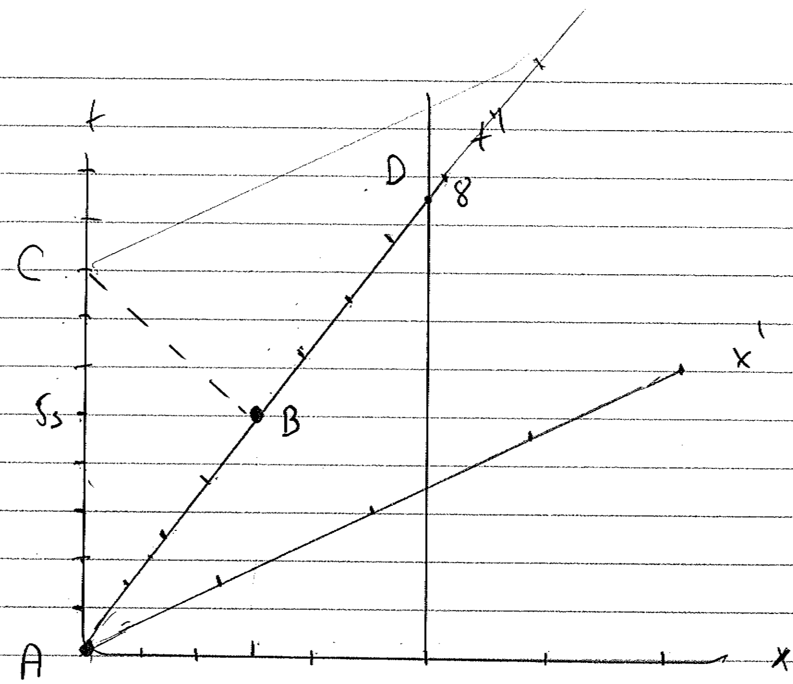
$t = \gamma(t' + \beta x)$

$= 1.1\left(1.5 + \frac{2}{5}5\right)$

$= 3.85\,s$

RGS 5

a)



$\beta = \frac{3}{5}$ $\gamma = \frac{4}{5}$

b) Enter Klingon territory at $t' \sim 7.5$ min

Blast hits cruiser at $t' \sim 10$ min

c) Yes ...

R7 S.1

$$\Delta\tau = \sqrt{1-\beta^2}\,\Delta t$$

$$1-\beta^2 = \left(\Delta\tau/\tau\right)^2 \Rightarrow \beta^2 = 1-\left(\frac{\Delta\tau}{\tau}\right)^2$$

$$\Rightarrow \beta = \sqrt{1-\left(\frac{\Delta\tau}{\tau}\right)^2} \sim 1-\frac{1}{2}\left(\frac{\Delta\tau}{\tau}\right)^2$$

$$\Rightarrow \cancel{1-\beta} \quad 1-\beta = \frac{1}{2}\left(\frac{\Delta\tau}{\tau}\right)^2$$

$$= \frac{1}{2}\left(\frac{100y}{100,000y}\right)^2$$

$$= \frac{1}{2}10^{-6}$$

$(1-\beta)(1+\beta)$

$(1+\beta)\ [2+(1-\beta)]$

$2(1+$

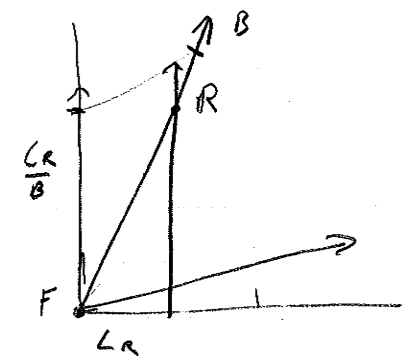
$$L = L_R/\gamma = \sqrt{1-\beta^2}\cdot L_R$$

$$\sim \sqrt{2}\cdot\sqrt{1-\beta}\cdot L_R$$

$$\sim 10^{-3}\cdot L_R$$

$$L = 100 \text{ lyr}$$

R75.7



a) L_R/B :

b) L_R

c) $\Delta s^2 = \Delta t'^2 - \Delta x'^2$

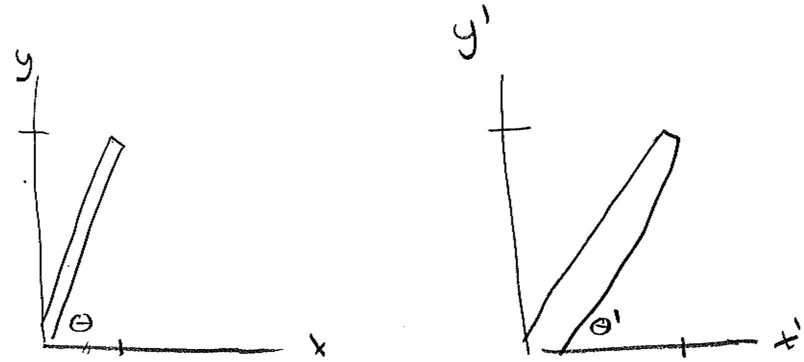
$$= \left(\frac{L_R}{B}\right)^2 - (L_R)^2 = L_R^2\left(\frac{1}{B^2} - 1\right)$$

$$L \equiv B\Delta t' \Rightarrow \Delta t'^2 = \frac{L^2}{B^2}$$

$$\Delta s^2 = \left(\frac{L}{B}\right)^2 = L_R\left(\frac{1}{B^2} - 1\right) = \left(\frac{L_R}{B}\right)^2(1 - B^2)$$

$$\boxed{L = \frac{1}{\gamma} L_R}$$

R7S.9



a) $y = y'$

$$x = \frac{1}{\gamma} x'$$

$$\tan\theta = y/x$$

$$\tan\theta' = y'/x' = y/\gamma x = \frac{1}{\gamma}\tan\theta$$

$$\boxed{\tan\theta = \gamma \tan\theta'}$$

b) $L_R^2 = x'^2 + y'^2 = (\cos^2\theta' + \sin^2\theta') L_R^2$

$$x^2 = \frac{1}{\gamma} x' = \frac{1}{\gamma^2}\cos^2\theta' \, L_R^2$$

$$y^2 = \sin^2\theta' \, L_R^2$$

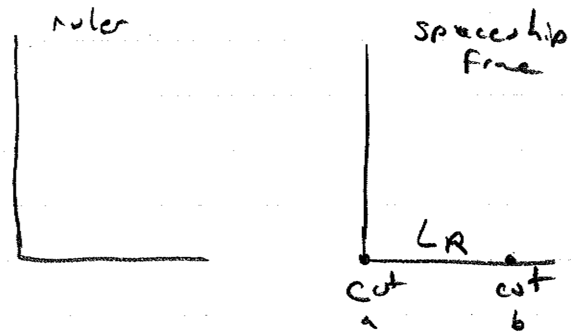
$$L = \sqrt{\left[\left(\frac{\cos\theta}{\gamma}\right)^2 + \sin^2\theta\right]} \, L_R$$

R7 R3

$\beta = \frac{4}{5}$ $\qquad \gamma = \frac{1}{\sqrt{1-\left(\frac{4}{5}\right)^2}} = \frac{1}{\sqrt{\frac{25}{25} - \frac{16}{25}}} = \frac{5}{3}$

a) L_R

b) $\frac{3}{5} L_R$



c)

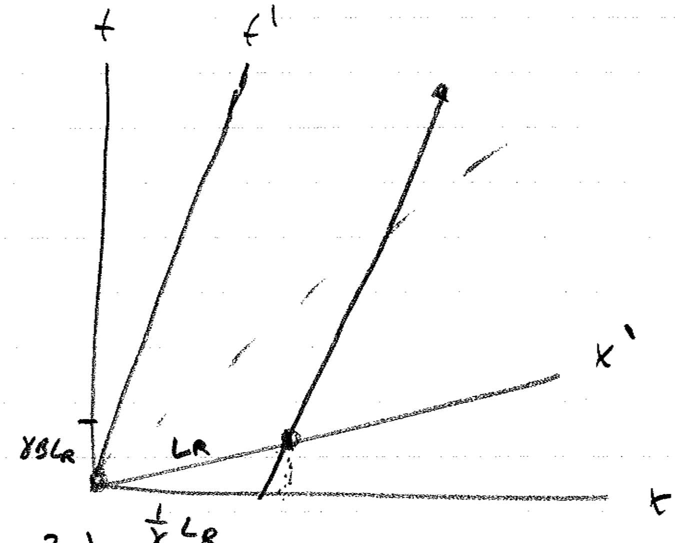
$x = \gamma(x' + \beta t')$

$x = \frac{5}{3} L_R$

d)

$t = \gamma(t' + \beta x')$

$= \gamma \cdot \beta L_R$



e) In ruler rest frame 2nd cut happens later when ship has traveled addition distance.

$$\text{size} = \frac{1}{\gamma} L_R + \beta \cdot (\gamma \beta) L_R$$

$$= \left(\frac{1}{\gamma} + \beta^2 \gamma\right) L_R$$

$$= \gamma\left(\frac{1}{\gamma^2} + \beta^2\right) L_R = \gamma L_R$$

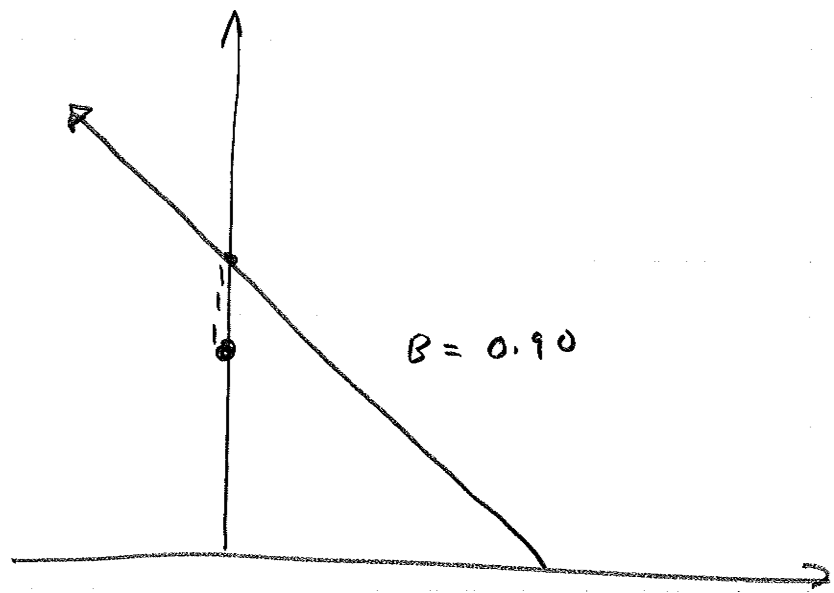
R8R.1

$\beta = \frac{4}{5} \qquad \gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1 - \frac{16}{25}}} = \frac{1}{\sqrt{\frac{25}{25} - \frac{16}{25}}} = \frac{5}{3}$

$\beta = \frac{4}{5} \qquad \gamma = \frac{5}{3}$

R88A

RS R.2



$$\frac{300\text{ km}}{3\times10^5\text{ km/s}} = \frac{3\times10^2}{3\times10^5} = 10^{-3}\text{ s} = 1\text{ ms}$$

B_* light second $- B \cdot t$ hours

R8 5.11

$$u' = \frac{1 - B}{1 - B} = 1c$$

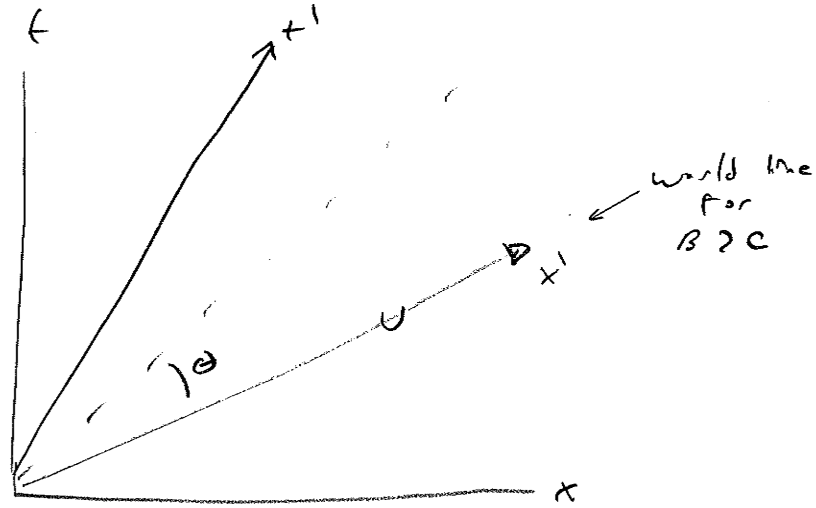
$$\frac{\Delta x'}{\Delta t'} << 1$$

$$\Delta x' << \Delta t'$$

$$\Delta t = \gamma\left(\Delta t' + \frac{B \Delta x'}{c}\right)$$

$$\Delta x = \gamma\left(\Delta x' + Bc\Delta t'\right)$$

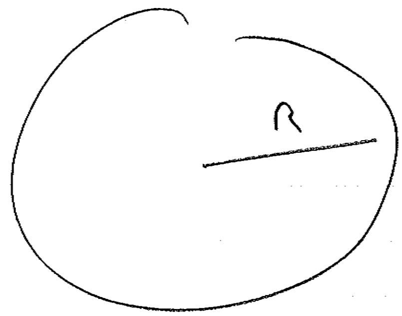
R8 5.1



$$\beta = \frac{1}{v}$$

R8 5.3

$$\omega = \frac{2\pi}{T}$$



$$v = \omega R > c$$

$$\Rightarrow \quad R > \frac{c}{\omega^2} = \frac{c}{2\pi} T$$

R8 S.7

$$\Delta u = 0.1$$

$$U_2 = \frac{(\Delta u) + u_1}{1 + (\Delta u)(u_1)} \qquad \frac{2\Delta u}{1+(\Delta u)^2}$$

$$2\Delta u \frac{1}{1+(\Delta u)^2}$$

$$0$$

$$\Delta u$$

$$2\Delta u \frac{1}{1+(\Delta u)^2}$$

$$3\Delta u \frac{1+\frac{\Delta u^3}{3}}{(1+3\Delta u^2)}$$

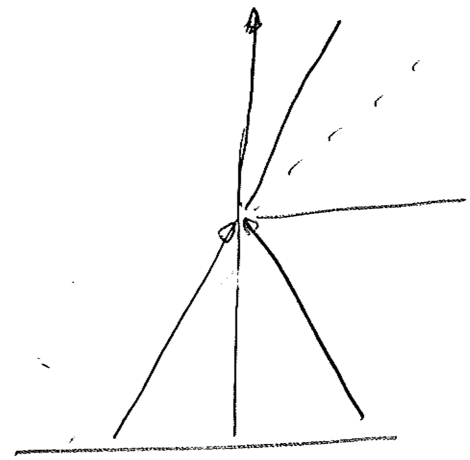
$$U_3 = \frac{\Delta u + u_1}{1 + \Delta u\, u_1}$$

$$= \frac{\Delta u + 2\Delta u\left(\frac{1}{1+(\Delta u)^2}\right)}{1 + \Delta u\, 2\Delta u \frac{1}{1+(\Delta u)^2}}$$

$$\frac{\Delta u\left(1+(\Delta u)^2\right) + 2\Delta u}{1+\Delta u^2 + 2\Delta u^2}$$

$$\frac{3\Delta u + \Delta u^3}{1+\Delta u^2 + 2\Delta u^2} = 3\Delta u\left(\frac{1+\frac{\Delta u^3}{3}}{\cancel{1+\Delta u^2+2\Delta u^2}\ 1+3\Delta u^2}\right)$$

R8 S.8 + R8 S.9



a)

$$u_1' = \frac{u_1 - B}{1 - Bu_1} = \frac{B - B}{1 - Bu_1} = 0$$

$$u_2' = \frac{-B - B}{1 - BB} = \frac{-2B}{1 - B^2}$$

$$= -2\gamma^2 B$$

b) In F $\quad P_i = mB + m(-B) = 0$

$P_f = 0$

In F' $\quad P_i = -\frac{2mB}{1-B^2}$

$P_f = -2mB$

R9B.5

$P = [P_t, P_x, P_y, P_z] = [5.0, 4.0, 0, 0]$ kg

$m^2 = 5^2 - 4^2 = 3^2 \Rightarrow m = 3$

$E = \gamma m \Rightarrow \gamma = E/m = 5/3$ $\qquad \beta^2 = 1 - \frac{1}{\gamma^2} = 1 - \left(\frac{3}{5}\right)^2 = \left(\frac{4}{5}\right)^2$

a) $\beta_x = p_x / \gamma m = p_x / E = 4.0/5.0$

b) $m^2 = E^2 - p^2 = 5^2 - 4^2 \text{kg}^2 = 3^2 \text{kg}^2 \Rightarrow \boxed{m = 3 \text{ kg}}$

c) $K = E - m = 5.0 - 3.0$ kg $= 2.0$ kg

d) $p = 4.0$ kg

R95.1

$$|P| = \sqrt{P_t^2 - P_x^2 - P_y^2 - P_z^2}$$

$$= \sqrt{(\gamma m)^2 - (\gamma m \beta_x)^2 - (\gamma m \beta_y)^2 - (\gamma m \beta_z)^2}$$

$$= \gamma m \sqrt{1 - \beta_x^2 - \beta_y^2 - \beta_z^2} = \gamma m \sqrt{1 - \beta^2}$$

$$= m$$

R9 S. 2 Approach 1:

$$E = \gamma m = \frac{m}{\sqrt{1-\beta^2}}$$

For $\beta \to 1$, $\varepsilon \to 0$ if $\beta = 1 - \varepsilon$

$$E = \frac{m}{\sqrt{(1+\beta)(1-\beta)}} = \frac{m}{\sqrt{2\varepsilon}} \frac{1}{\sqrt{1+\varepsilon/2}}$$

For $\varepsilon \to 0$, $\frac{1}{\sqrt{1+\varepsilon/2}} \sim 1 - \frac{\varepsilon}{4}$

$$E = \frac{m}{\sqrt{2\varepsilon}}\left(1 - \frac{\varepsilon}{4}\right)$$, so $E \to \frac{m}{\sqrt{2\varepsilon}}$ as $\beta \to 1$

$$\gamma\beta = \frac{1-\varepsilon}{\sqrt{\varepsilon(2+\varepsilon)}} = \frac{1}{\sqrt{2\varepsilon}} \frac{1-\varepsilon}{\sqrt{1+\varepsilon/2}}$$

$$= \frac{1}{\sqrt{2\varepsilon}}(1-\varepsilon)\left(1 - \frac{\varepsilon}{4}\right)$$

$p = m\gamma\beta$, so $p \to \frac{m}{\sqrt{2\varepsilon}}$ as $\beta \to 1$

Hence $E \to p$ as $\beta \to 1$

Approach 2:

$$E^2 = m^2 + p^2$$

But $p^2 = (\gamma\beta m)^2$

As $\beta \to 1$ $\gamma\beta >> 1$

so $m^2 + (\gamma\beta m)^2 \to (\gamma\beta m)^2 = p^2$

ie. $E \to p$

R9S.3

1) Units inconsistent

2) $P_t = E < 0$

3) $m < 0$

R95.4 $m = 2\,\mu g$

$$(1\ kg)(c^2) = (3\times10^8)^2\ kg\ m^2/s^2$$

$$= 9\times10^{16}\ J \qquad SI \qquad\qquad SR$$

$$m = 2\,\mu g \Rightarrow m = 2\times10^{-9}\ kg \quad \Big| \quad 1.8\times10^8\ J$$

$$\beta = \frac{4}{5} \qquad \gamma = \frac{1}{\sqrt{1-(4/5)^2}} = \frac{1}{\sqrt{\frac{25-16}{5^2}}} = \frac{5}{3}$$

a) $E = \gamma m = \frac{5}{3}\cdot 1.8\times10^8\ J =$

b) $p = \gamma\beta m = \frac{4}{3}\ 1.8\times10^8\ J =$

d) $K = (\gamma-1)\,m = \frac{2}{3}\cdot 1.8\times10^8\ J =$

c) $p_x = \cos 30^\circ \left(\frac{4}{3}\right)(1.8\times10^8\ J) =$

$p_y = \sin 30^\circ \left(\frac{4}{3}\right)(1.8\times10^8\ J) =$

$p_z = 0$

R9S.5 $\quad m = 1.0\ \mu g \qquad K = 2.0\ \mu g$

$E = m + K = 3.0\ \mu g$

$E^2 - m^2 = p^2 = 3^2 - 1^2\ \mu g^2 = 9 - 1\ \mu g^2 = 8\ \mu g^2$

$p = \sqrt{8}\ \mu g = 2\sqrt{2}\ \mu g$

$\mathbf{p} = (3.0,\ 0,\ 2\sqrt{2},\ 0)\ \mu g$

R95.6

General Solution

$$p/m = (G,\ Gu_x,\ Gu_y,\ Gu_z)$$

$$\frac{p'}{m} = \left(\gamma(G - Gu_x B),\ \gamma(Gu_x - GB),\ Gu_y,\ Gu_z\right)$$

$$= (G',\ G'u_x',\ G'u_y',\ G'u_z')$$

$$G' = \gamma G(1 - u_x B)$$

$$u_x' = \frac{\gamma G(u_x - B)}{\gamma G(1 - u_x B)} = \frac{u_x - B}{1 - u_x B}$$

$$u_y' = \frac{G u_y}{\gamma G(1 - u_x B)} = \frac{u_y \sqrt{1 - B^2}}{1 - u_x B}$$

$$u_z' = \frac{G u_z}{\gamma G(1 - u_y B)} = \frac{u_z \sqrt{1 - B^2}}{1 - u_y B}$$

R9 S.7 $\quad kg\, c^2 = 9\times10^{16}\, J$

$m \sim 100\ kg$

$$mc^2 = 100 \times 9\times10^{16}\ J$$

$$= 9\times10^{18}\ J$$

$$P = E\cdot c = \left(9\times10^{18}\,J\right)\times\left(\frac{\$0.03}{10^{6}\,J}\right)$$

$$= \$2.7 \times 10^{18+2-6+1}$$

$$\boxed{P = \$2.7\times10^{15}}$$

R9 S.8 $\quad E = \gamma m \Rightarrow \gamma = E/m$

$$\gamma^2 = \frac{1}{1-\beta^2} \Rightarrow 1-\beta^2 = \frac{1}{\gamma^2} = \left(\frac{m}{E}\right)^2$$

$$\beta^2 = 1-\left(\frac{m}{E}\right)^2$$

$\Delta E = T/C \qquad T =$ total assets $\qquad c =$ cost

Since Energy is conserved

$$E_{TOT} = m + \Delta E = m + T/C$$

$$\beta^2 = 1-\left(\frac{m}{m+\Delta E}\right)^2 = 1-\left(\frac{1}{1+\Delta E/m}\right)^2$$

R958 Cont...

$$B^2 = 1 - \left(\frac{1}{1 + \Delta E / m} \right)^2$$

$$\frac{\Delta E}{m} = \frac{T}{m\,C} = \frac{\$\,1.5 \times 10^6}{(1.0 \times 10^{-3}\ \text{kg}c^2)(9 \times 10^{16}\ \text{J/kg}c^2)(\$0.04 / 10^6\ \text{J})}$$

$$= \frac{3}{2} \cdot \frac{1}{9} \cdot \frac{1}{4} \times 10^{6+3-16+2+6}$$

$$= \frac{10}{24}$$

$$B^2 = 1 - \left(\frac{1}{1 + 10/24} \right)^2$$

$$= 1 - \left(\frac{24}{34} \right)^2$$

$$= \frac{34^2 - 24^2}{34^2}$$

$$B = \frac{\sqrt{34^2 - 24^2}}{34} = 0.70$$

Not to impressive!

R95.10

$$E^2 = m^2 + p^2$$

$$E = m + K$$

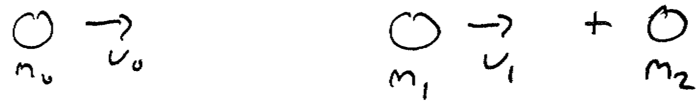
$$m^2 + p^2 = (m + K)^2$$

$$p^2 = m^2 + 2mK + K^2 - m^2$$

$$= K(2m + K)$$

$$p = \sqrt{K(K + 2m)}$$

R9S.11



a) $m_0 = m_1 + m_2$ $\qquad \left(v_0 = \frac{3}{5} \qquad v_1 = \frac{4}{5} \right)$

$$m_0 v_0 = m_1 v_1$$

$$m_0 v_0 = m_1 v_1$$

$$\Rightarrow \quad m_1 = m_0 \frac{v_0}{v_1}$$

$$m_2 = m_0 - m_1 = m_0 \left(1 - \frac{v_0}{v_1}\right)$$

$$v_0 / v_1 = \frac{3}{5} \cdot \frac{5}{4} = \frac{3}{4}$$

$$\Rightarrow \quad m_1 = \frac{3}{4} m_0 \qquad m_2 = \frac{1}{4} m_0$$

b) $E_i = \gamma_0 m_0 \qquad p_i = \gamma_0 m_0 v_0$

$E_f = \gamma_1 m_1 + m_2 \qquad p_f = \gamma_1 m_1 v_1$

$$\gamma_0 m_0 = \gamma_1 m_1 + m_2 \qquad \gamma_0 m_0 v_0 = \gamma_1 m_1 v_1$$

$$m_1 = \frac{\gamma_0 v_0}{\gamma_1 v_1} m_0$$

$$m_2 = \gamma_0 m_0 - \gamma_1 m_1 = \gamma_0 m_0 - \frac{\gamma_0 v_0}{v_1} m_0$$

$$= \gamma_0 \left(1 - \frac{v_0}{v_1}\right) m_0$$

Recall: $u_0 = \frac{3}{5}$ $u_1 = \frac{4}{5}$

$\Rightarrow$ $\gamma_0 = \frac{5}{4}$ $\gamma_1 = \frac{5}{3}$

$\Rightarrow$ $\gamma_0 u_0 = \frac{3}{4}$ $\gamma_1 u_1 = \frac{4}{3}$ $u_0/u_1 = \frac{3}{4}$

$$m_1 = \left(\frac{3}{4}\right)^2 m_0 = \frac{9}{16} m_0$$

$$m_2 = \frac{5}{4}\left(1 - \frac{3}{4}\right) = \frac{5}{16} m_0$$

Note $m_1 + m_2 = \frac{14}{16} m_0 < m_0$

c) $\underset{m_0}{\bigcirc} \ \underset{v_0}{\rightarrow} \qquad \underset{m_1}{\bigcirc} \ \underset{v_1}{\rightarrow} \ + \ m_2$

In F' moving at v_0;

$$v_0' = 0 \qquad v_2' = -v_0 = -\frac{3}{5}$$

$$v_1' = \frac{v_1 - v_0}{1 - v_1 v_0} = \frac{\frac{4}{5} - \frac{3}{5}}{1 - \frac{4}{5}\cdot\frac{3}{5}}$$

$$= \frac{\frac{1}{5}\cdot 25}{25 - 12} = \frac{5}{13}$$

d) $p_i = 0$

$$p_f = m_1 v_1 + m_2 v_2$$

$$= \left(\frac{3}{4}\right)\frac{5}{13} m_0 + \left(\frac{1}{4}\right)\left(-\frac{3}{5}\right) m_0$$

$$= \frac{3\cdot 5\cdot 5 - 13\cdot 3}{5\cdot 4\cdot 13} m_0 \quad \underline{\neq 0}$$

e) $p_i = 0$

$p_f = \gamma_1' u_1' m_1 + \gamma_2' u_2' m_2$

$u_1' = \frac{5}{13}$ $\qquad u_2' = -\frac{3}{5}$

$\gamma_1' = \frac{13}{\sqrt{13^2 - 5^2}} = \frac{13}{12}$ $\qquad \gamma_2' = \frac{5}{4}$

$p_f = \frac{\cancel{13}}{\cancel{12}_4} \cdot \frac{5}{\cancel{13}} \cdot \frac{\cancel{9}^3}{16} m_0 + \frac{5}{4}\left(-\frac{3}{5}\right) \cdot \frac{5}{16} m_0$

$= \left(\frac{15}{43} - \frac{15}{43}\right) m_0 = 0$

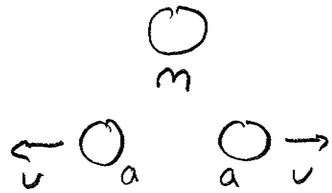
$E_f = \gamma_1' m_1 + \gamma_2' m_2$

$= \left(\frac{13}{\cancel{12}_4} \frac{\cancel{9}^3}{16} + \frac{5}{4} \frac{5}{16}\right) m_0$

$\frac{39 + 25}{64} m_0 = m_0$

$E_i = m_0$

R105.1



13
13
39
130
169
144
25

$$p_i = (m, 0)$$

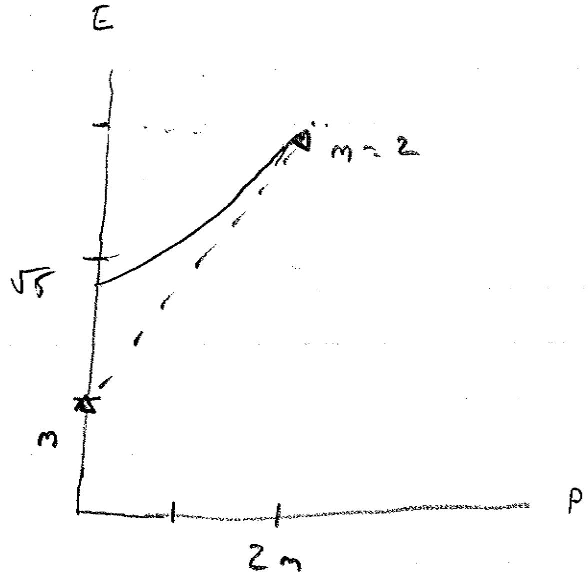
$$p_f = (\gamma a, \gamma v a) + (\gamma a, -\gamma v a)$$

$$= (2\gamma a, 0)$$

$$v = \frac{12}{13} \qquad \gamma = \frac{1}{\sqrt{1-v^2}} = \frac{13}{\sqrt{13^2 - 12^2}} = \frac{13}{5}$$

$$m = \frac{26}{5} a \Rightarrow a = \boxed{\frac{5}{26} m}$$

R10S.2



$\beta = \frac{2}{3}$ $\qquad \gamma = \frac{3}{\sqrt{9-4}} = \frac{3}{\sqrt{5}}$

$m \sim 2 \qquad p \sim \sqrt{5}\, m \qquad E = 3m$

$$P_{TOT}' = (m, 0) + (2m, 2m)$$
$$= (3m, 2m)$$

$$M^2 = (9-4)\, m^2 = 5m^2$$

$$\boxed{\begin{aligned} M &= \sqrt{5}\, m \\ P &= 2m \\ E &= m \end{aligned}}$$

R10 S.3

$m_1 = 8$ kg $\qquad m_2 = 12$ kg

$u_1 = \frac{15}{17} \qquad u_2 = -\frac{5}{13}$

$\gamma_1 = \frac{17}{\sqrt{17^2 - 15^2}} = \frac{17}{8}$

$\gamma_2 = \frac{13}{12}$

4
17
17
119
170
289

15
15
75
150
225

289
225
64

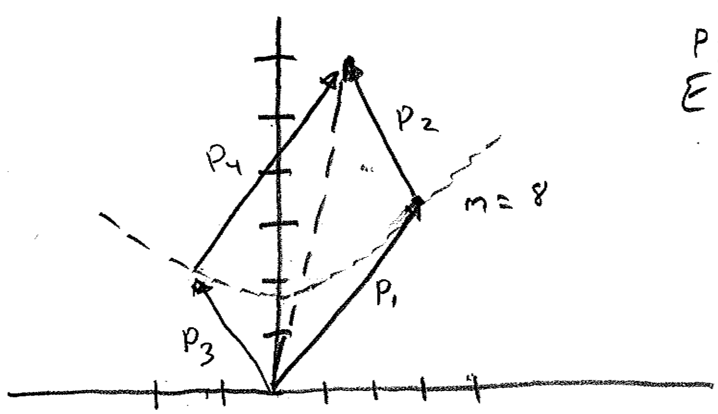
11
11
11
11

After $\quad u_3 = -\frac{3}{5} \qquad \gamma_3 = \frac{5}{4}$

a) $p_1 = \frac{15}{17}\frac{17}{8} 8 = 15$ kg $\qquad E_1 = 17$ kg

$p_2 = \frac{13}{12}\left(-\frac{5}{13}\right) 12 = -5 \qquad E_2 = 13$ kg

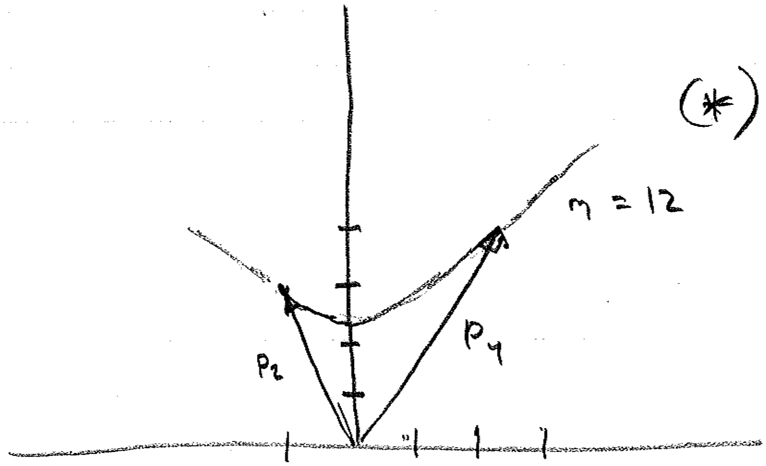
63
3
279
7.9.3



$p_3 = u_3 \gamma_3 m_1 = -6$

$E_3 = \frac{5}{4} \cdot 8 = 10$

$B \sim \frac{11}{20}$



$m_2 \sim$ same

R10 5.3 b)

b) $P_1 = (17, 15)$
$P_2 = (13, -5)$
$P_3 = (10, -6$

$P_1 + P_2 - P_3 = (20, 16)$

$m = 20^2 - 16^2 = 400 - \quad = 384$

$= 12$

400

$$\begin{array}{r} 16 \\ 16 \\ \hline 96 \\ 160 \\ \hline 256 \end{array}$$

$$\begin{array}{r} 400 \\ -256 \\ \hline 144 \end{array}$$

R10 S.4

$$x = \gamma(x' + \beta t') \qquad t = \gamma(t' + \beta x')$$
$$p = \gamma(p' + \beta E') \qquad E = \gamma(E' + \beta p')$$

For light $p = E$, so

$$p = \gamma(p' + \beta p') = \sqrt{\frac{1+\beta}{1-\beta}} \cdot p'$$

a) $$p = \sqrt{\frac{(1-\frac{4}{5})}{(1+\frac{4}{5})}}\, p' = \sqrt{\frac{1}{9}} = p'/3$$

$$p = \frac{3}{4} \cdot \frac{1}{3} m_0 = \frac{m_0}{4}$$

b) $\beta = \frac{4}{5}$ $\gamma = \frac{5}{3}$ $\beta\gamma = \frac{4}{3}$

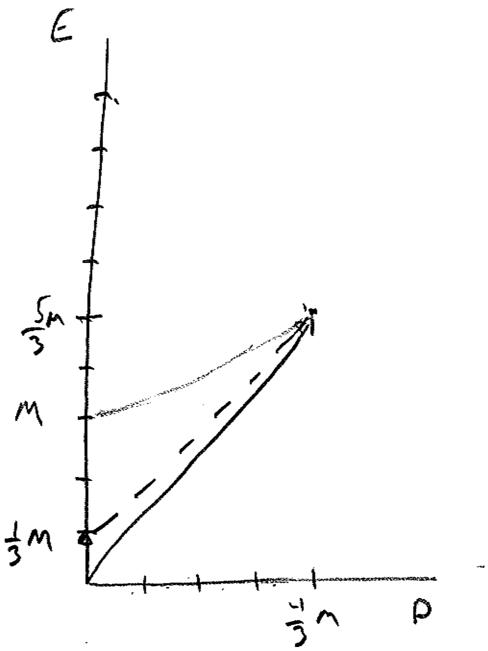
$$p_i = \left(\frac{5}{3} m_0, \frac{4}{3} m_0\right) + \left(\frac{m_0}{4}, -\frac{m_0}{4}\right)$$
$$= \left(\frac{23}{12}, \frac{13}{12}\right) m_0$$

$$\beta = \frac{13}{23}$$

R10 S.5 M $\beta = \frac{4}{5} \Rightarrow \gamma = \frac{5}{3}$ $\gamma\beta = \frac{4}{3}$

$E = \frac{5}{3} m$ $p = \frac{4}{3} m$

a)



$\frac{2}{3} M$ must be converted

$$P_i = (E_0, P_0)$$

$$P_f = (\tilde{m}, 0) + (k, k)$$

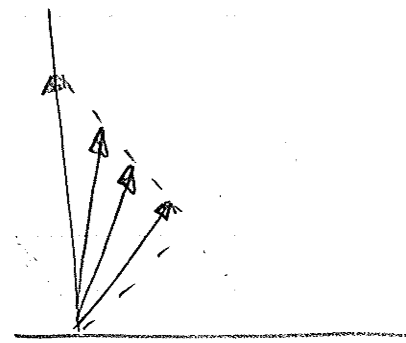
$$k = P_0$$
$$\tilde{m} + k = E_0$$
$$\tilde{m} = E_0 - P_0 = \left(\frac{5}{3} - \frac{4}{3}\right) m = \frac{1}{3} M$$

$\Rightarrow \frac{2}{3}$ must be converted to light.

R10 S.6

$P_i = (M, 0)$

$P_f = (\gamma m, \ \gamma m \beta) + (|k|, -k)$



$k = \gamma m \beta$

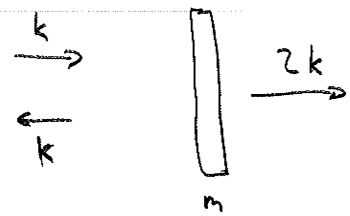
$M = \gamma m + |k| = \gamma(1+|\beta|)\, m$

$$M = \sqrt{\frac{1+|\beta|}{1-|\beta|}}\, m = \sqrt{\frac{1+.95}{1-.95}} \cdot 25\, t$$

$$= \sqrt{39} \cdot 25\, t$$

$$\begin{array}{r} 39 \\ 5\overline{)195} \\ 15 \end{array}$$

R10 5.7



$$\frac{\Delta p}{\Delta t} = m a$$

$$\Delta p = 2\Delta k = m a \Delta t$$

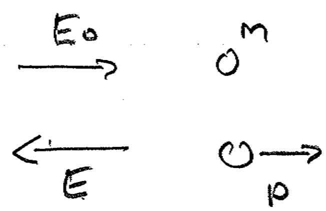
$$\Delta k = \Delta E$$

$$\Rightarrow P = \frac{\Delta E}{\Delta t} = \frac{1}{2} m a$$

?

Not clear what approx is needed this is non-physical

RIO S. 9



$$p_i = (E_0, E_0) + (m, 0)$$

$$p_f = (E, -E) + (\sqrt{m^2+p^2}, p)$$

$$E_0 = p - E \quad \Rightarrow \quad p = E + E_0$$

$$E_0 + m = E + \sqrt{m^2 + p^2}$$

$$= E + \sqrt{m^2 + (E+E_0)^2}$$

$$2E_0 + m = (E + E_0) + \sqrt{m^2 + (E+E_0)^2}$$

$$x = E + E_0 \qquad \alpha = 2E_0 + m$$

$$\alpha = x + \sqrt{m^2 + x^2}$$

$$(\alpha - x)^2 = m^2 + x^2$$

$$\alpha^2 - 2\alpha x \cancel{+ x^2} = m^2 \cancel{+ x^2}$$

$$x = \frac{\alpha^2 - m^2}{2\alpha} = \frac{(2E_0+m)^2 - m^2}{2(2E_0+m)}$$

$$E = \frac{4E_0^2 + 4E_0 m}{4E_0 + 2m} - E_0 \frac{(\qquad)}{(\qquad)}$$

$$= \frac{2E_0 m}{4E_0 + 2m} = \frac{E_0}{1 + 2\frac{E_0}{m}}$$

Check!