Lecture R1

⇒ Name Tags
⇒ Discuss Syllabus

Overview: of Relativity

1) Exploring the consequenses of a simple idea " The laws of physics are the same inside a lab at rest and a lab moving a constant velocity".

Q: (Example?)

2) Axiomatic → From a few assumptions we come to surprising far-reaching conclusions about the nature of space and time
DON'T BE FOOLED! Experimental verification of these consequences (not the mathematical beauty) is why we believe Relativity to be "true"!

3) We have to be very clear about how we discuss concepts like "lab", by which we make measurements,

Q: What is the most fundamental measurement you can make?

→ motion of a particle, such as an atom, in space

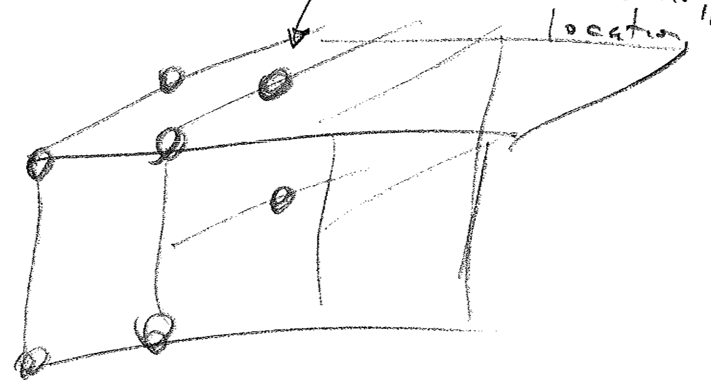
Q: How do we describe motion?

→ Series of position and time measurements.

$\boxed{\text{Event}}$ occurs at a specific place and time (space-time coordinates)

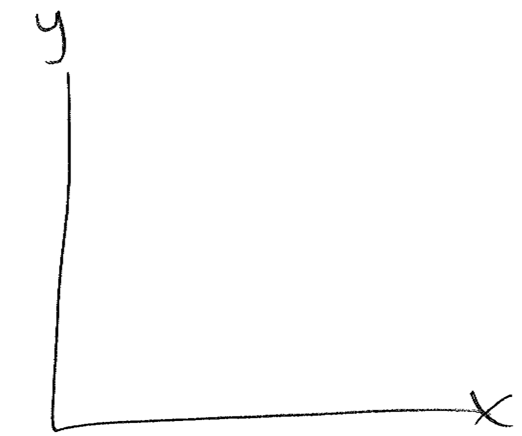
Q: How do we measure "Events"? (Assume money is no object)

$\boxed{\text{Reference Frame}}$

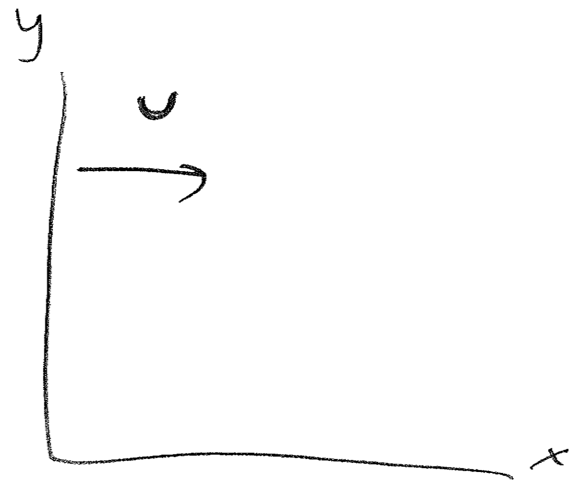


$\boxed{\text{Q:}}$ $\boxed{\text{R1T.2}}$

Convention



Home Frame (F)



Other Frame (F') ← (Hard to resist)

(You can ignore convention, but then have to be careful with formulas)

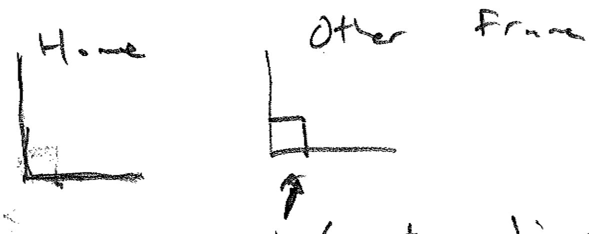
Inertial Reference Frame

→ Newton's First Law holds
(objects w/o net external Force are at rest)

Claim: 1 Any inertial frame will be observed to move at a constant velocity relative to any other frame.

Claim 2: Any frame moving with a constant velocity wrt any other inertial frame will be itself an inertial frame

Q: Proof Claim 2:

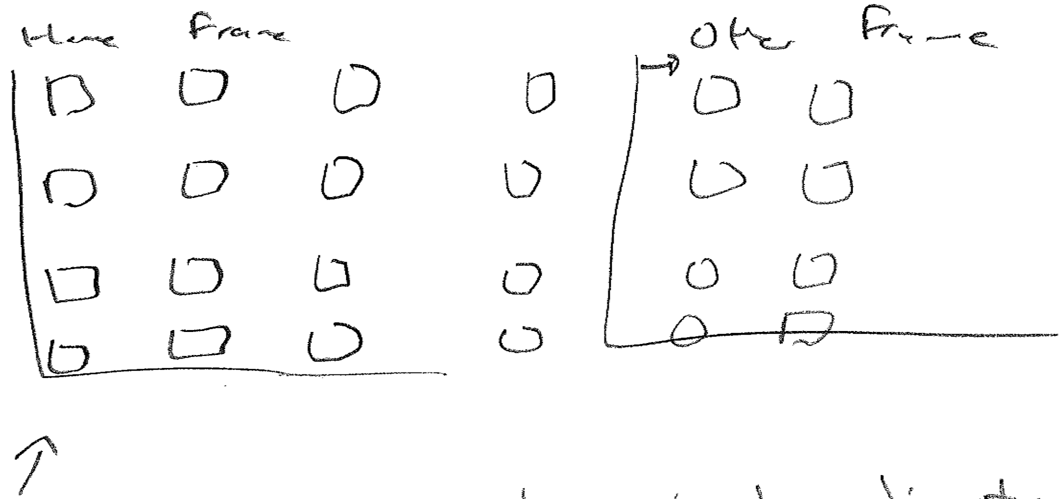


test object at rest in other frame

test object must be moving at constant velocity in Home Frame (else violates 2nd Law!)

Therefore other frame moves at constant velocity.

Q: Proof Claim 2



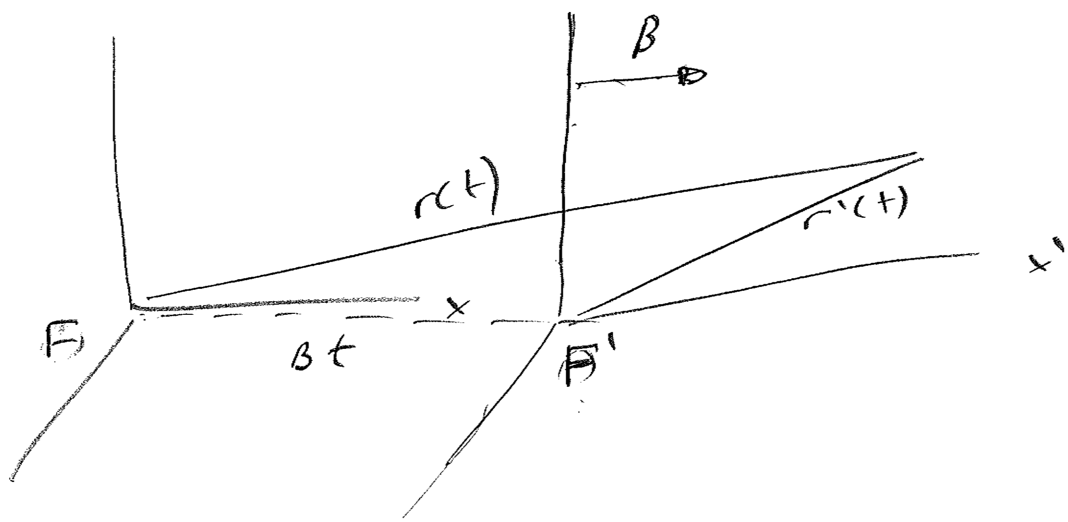
Other frame sees test objects everywhere at constant velocity, i.e. obeying 2nd law.

Newtonian Relativity

" time is absolute and flows equally without regard to anything external",

Any one clock will do, and can simply be carried around as needed...

$$\boxed{t = t'}$$



$$\boxed{\begin{aligned} t' &= t \\ x' &= x - \beta t \\ y' &= y \\ z' &= z \end{aligned}}$$

$\frac{d}{dt}$:

$$\boxed{\begin{aligned} u_x' &= u_x - B \\ u_y' &= u_y \\ u_z' &= u_z \end{aligned}}$$

$\frac{d^2}{dt^2}$

$$\boxed{\begin{aligned} a_x' &= a_x \\ a_y' &= a_y \\ a_z' &= a_z \end{aligned}}$$

Both Frames agree about

<u>acceleration</u> !

Both Frames agree that

Newton's Laws apply, even if story is <u>slightly</u> <u>different</u>.

R1 S7

<u>Q: R1 S9</u>

$\varepsilon_0 = 8{,}854 \times 10^{-12}$ $\cancel{m^{-3}\, kg^{-1}}\, s^{4} A^{2}$

F/m

$\mu_0 = 4\pi \times 10^{-7}$ Us/Am

$F/m = \frac{As}{Vm}$

$\varepsilon_0 = 8{,}854 \times 10^{-12} \; \frac{As}{Vm}$

$\mu_0 = 4\pi \times 10^{-7} \; \frac{Us}{Am}$

$\cancel{\sqrt{\frac{1}{\varepsilon_0 \mu_0}}}$ $\quad \frac{1}{\sqrt{\varepsilon_0 \mu_0}} = \, ? \; m/s$

$\frac{1}{(8{,}854 \cdot 4\pi)^{\frac{1}{2}} \times 10^{-19}}$ $\quad \frac{10^{10}}{88 \cdot 4 \cdot 3} \sim 3 \times 10^{8}$

R2

Electromagnetic Waves

Maxwell (1873) developed theory of classical E+M.

Two important constants:

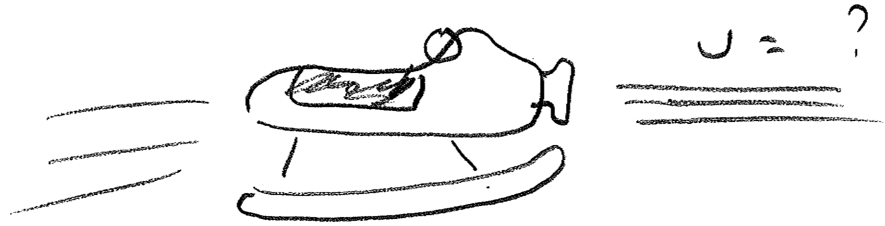
$$\boxed{\begin{aligned} \varepsilon_0 &= 8.854 \times 10^{-12} \ \frac{A \cdot s}{V \cdot m} \\ \mu_0 &= 4\pi \times 10^{-7} \ \frac{V \cdot s}{A \cdot m} \end{aligned}}$$

Q: Calculate $\dfrac{1}{\sqrt{\varepsilon_0 \mu_0}} = \ ?$

As you will see in E+M, Maxwell's Equations predicted new phenomenon of electromagnetic waves, travelling at $c = 3.0 \times 10^8$ m/s.

An important question!

(Flashlight)



(Snow mobile)

In compatible:

Maxwell's Equations ($c = 3.0 \times 10^8$ m/s)

Relativity

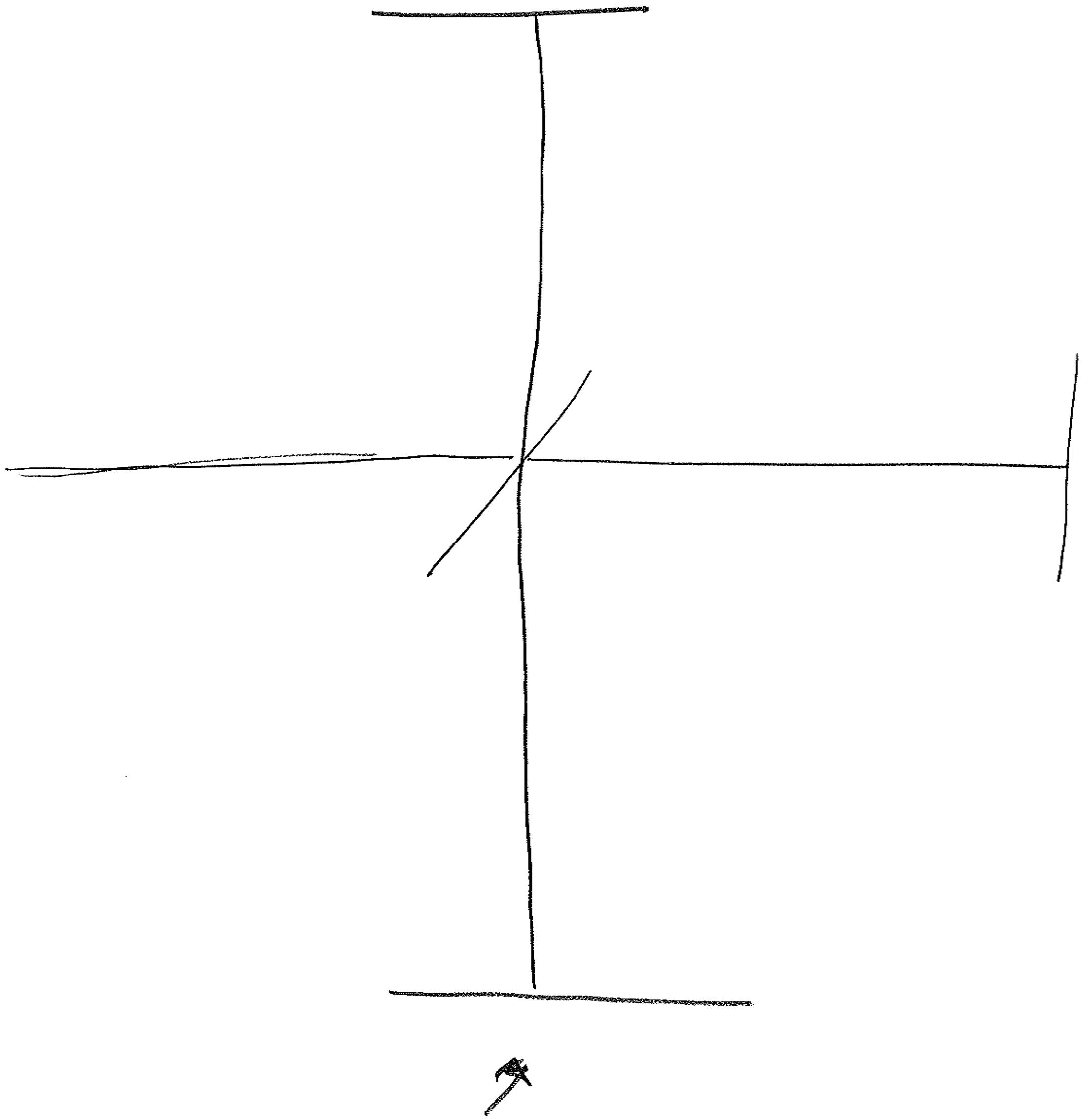
Gallilean Transformation

First approach was to throw out relativity

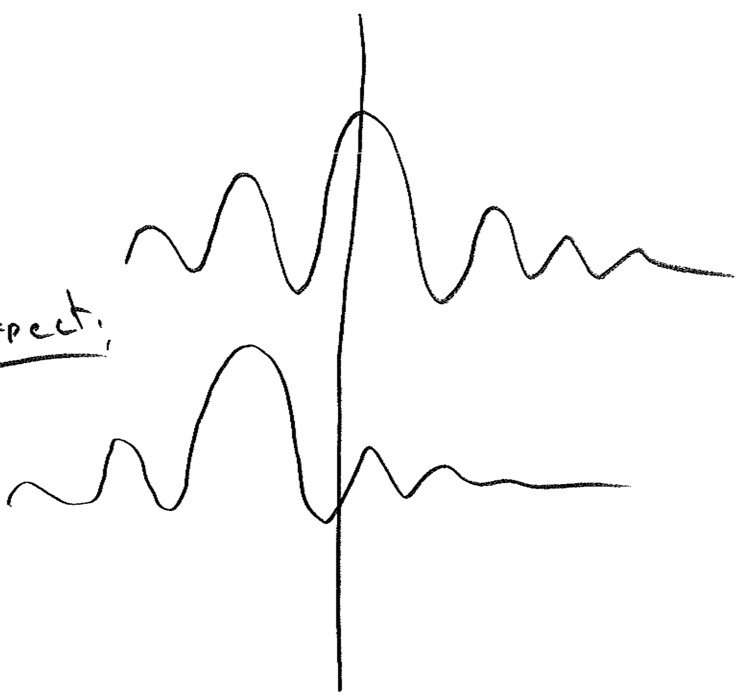
In essence:

Light propogates in ether

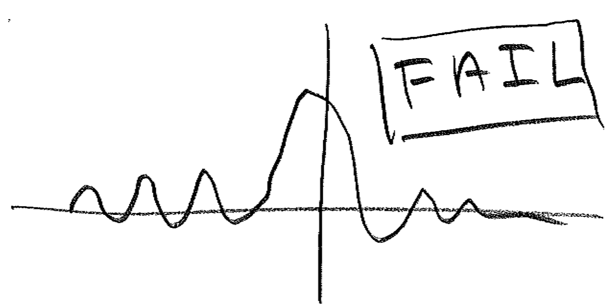
Michelson - Morley experiment



Expect:



Saw



Einstein's Approach

1. Principle of Relativity Holds

2. Maxwell's Equations Hold

   $\Rightarrow$ $c = 3,0$ m/s in every inertial frame

3. Gallilean Transformations are wrong.

Think about how hard that is to except. It seems more a matter of math than physics.

What next? $r \neq \sqrt{x^2 + y^2}$!!!

SR Units

$c = 3{,}0$ m/s is quite central ...

So much so we can define a length by the amount of time it takes to travel:

light-year
light-second

In this class, we even drop the "light".

$L = 1$ s
$L = 1$ yr

Q: A meterstick has length ?

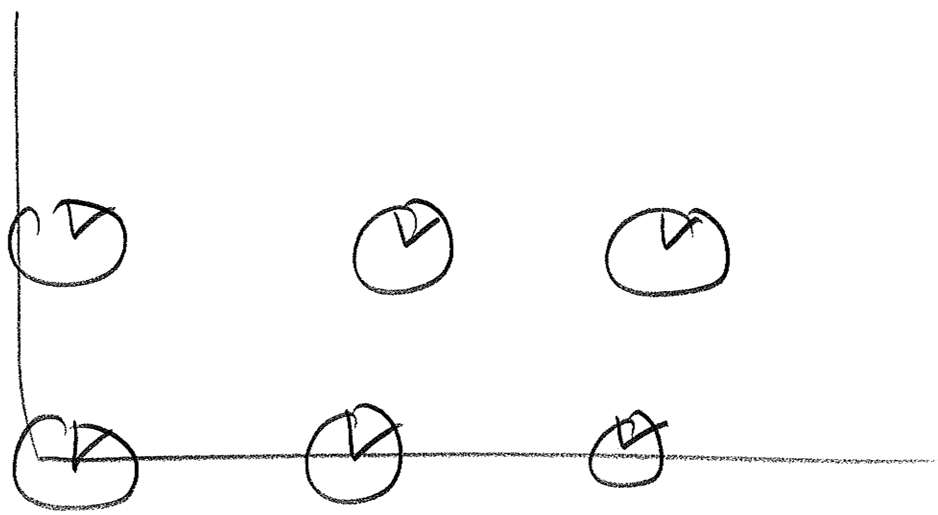
$\boxed{3.3 \text{ ns}}$

m	10^{-3}
μ	10^{-6}
n	10^{-9}

Not a meter stick, it's a (3.3 ns stick)

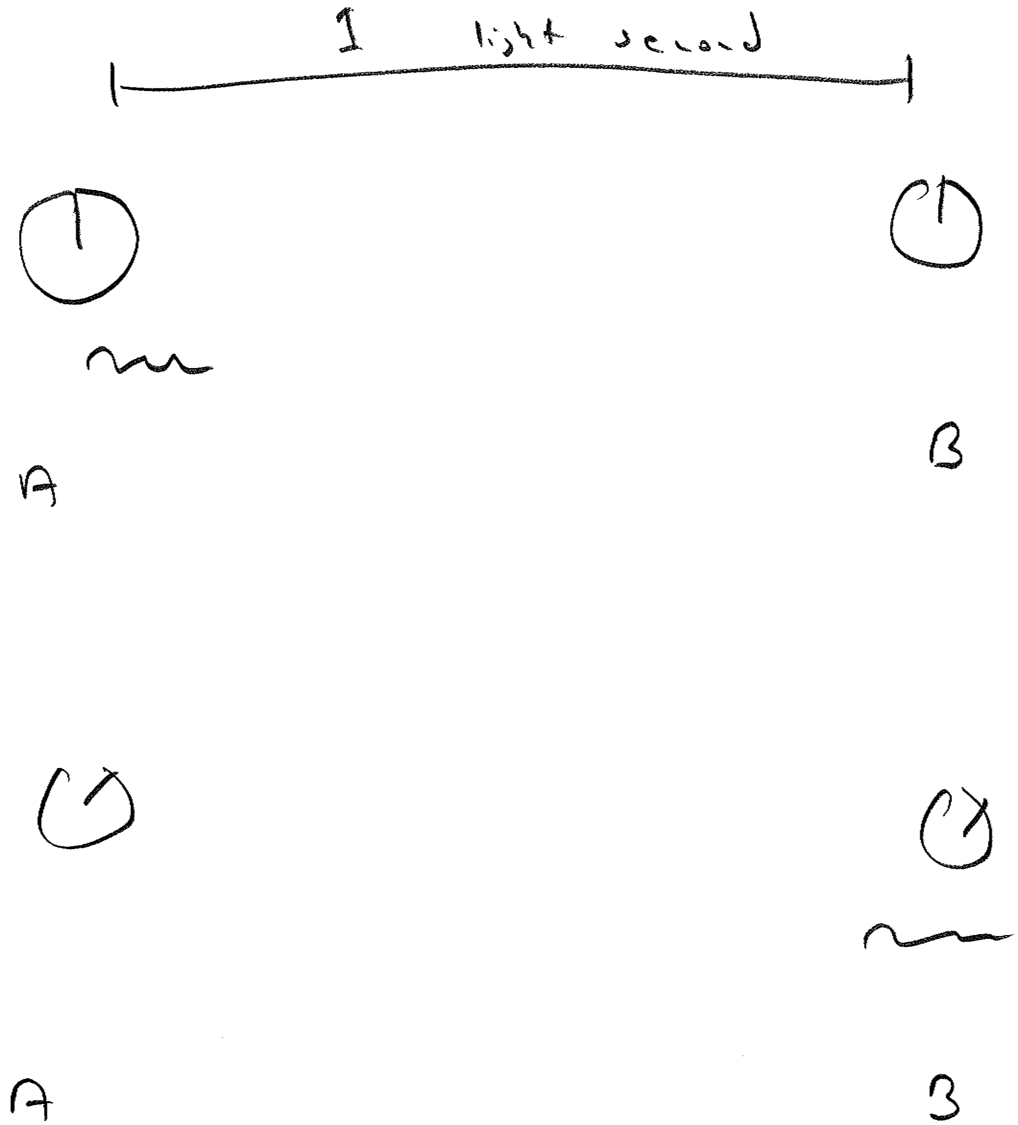
Clock Synchronization

We reject the gallilean transformation which is "universal time" plus math. So we reject "universal time". Have to be VERY CAREFUL going forward



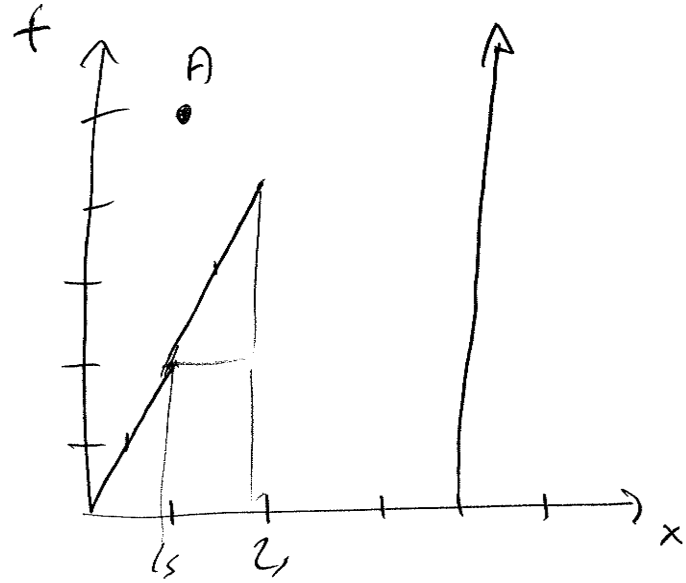
Q: How can we synchronize clocks in a way consistent with relativity + Maxwell's Equation?

A: Use light pulses



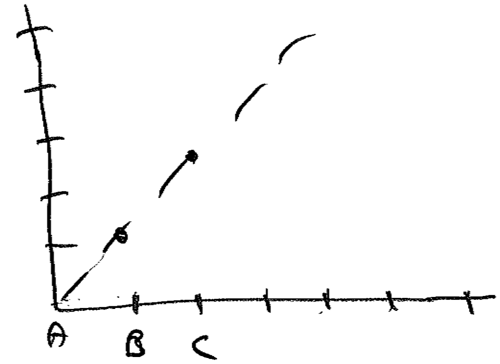
Clock's are synchronized when time interval is <u>equal</u> to space interval.

ST Diagrams

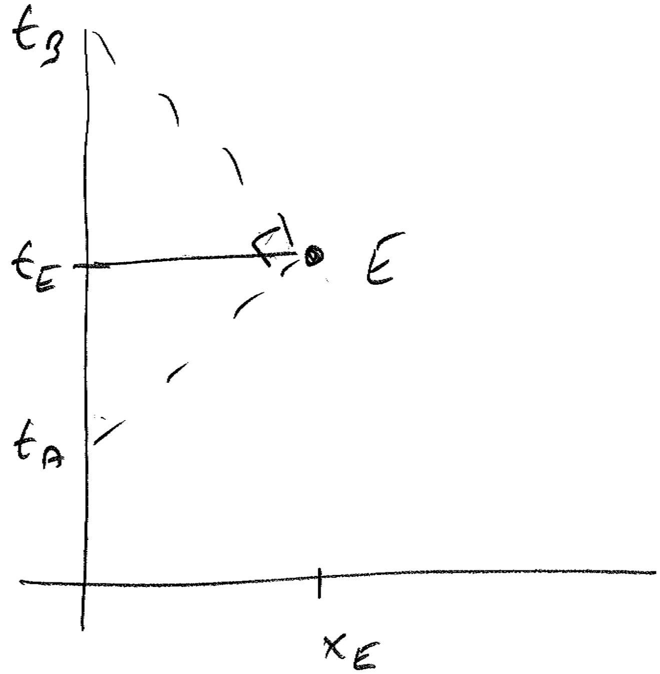


- Events
- World lines
- $\frac{1}{m}$ = Velocity
- ~~horizontal~~ vertical line is particle at rest
- light has slope "1".

⟹ that's synchronization method



Radar method:



$$t_E = \frac{1}{2}(t_A + t_B)$$ (average)

$$x_E = \frac{1}{2}(t_B - t_A)$$ ($\frac{1}{2}$ the time interval)

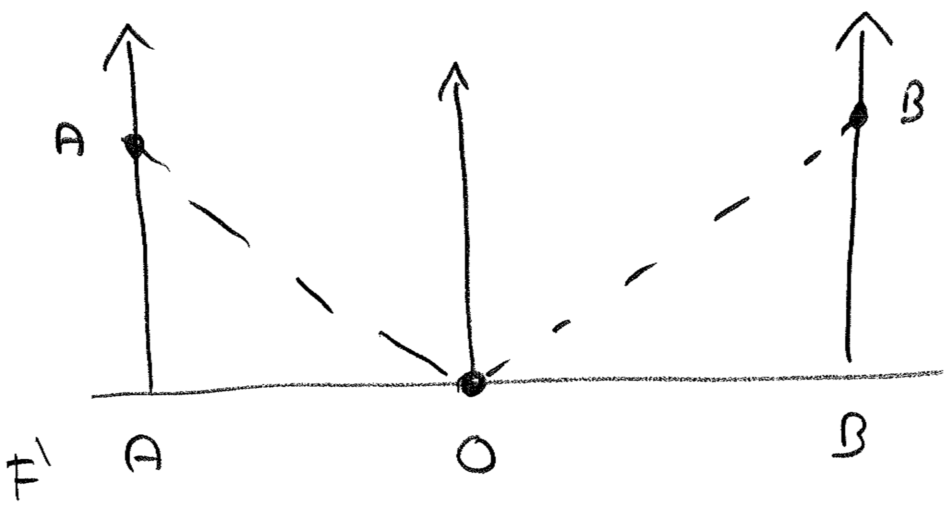
(Or Geometric)

Coordinate-Time Interval:

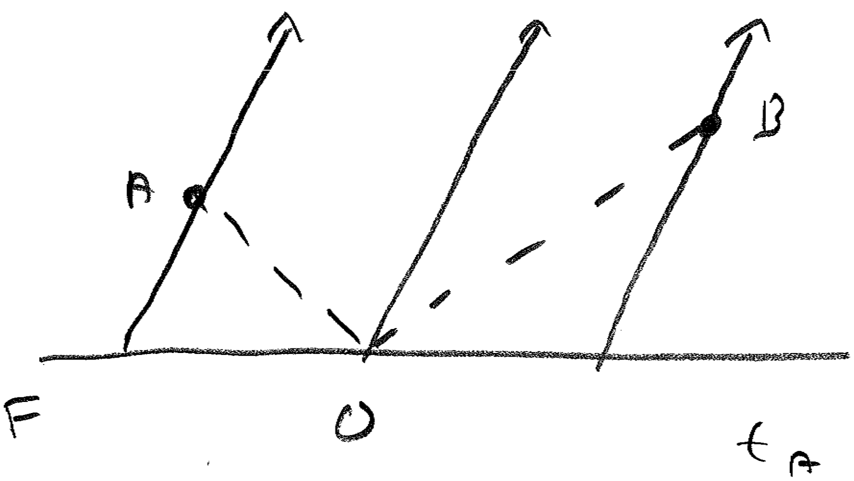
Δt between two events measured in an inertial frame.

Time to face a difficult fact:

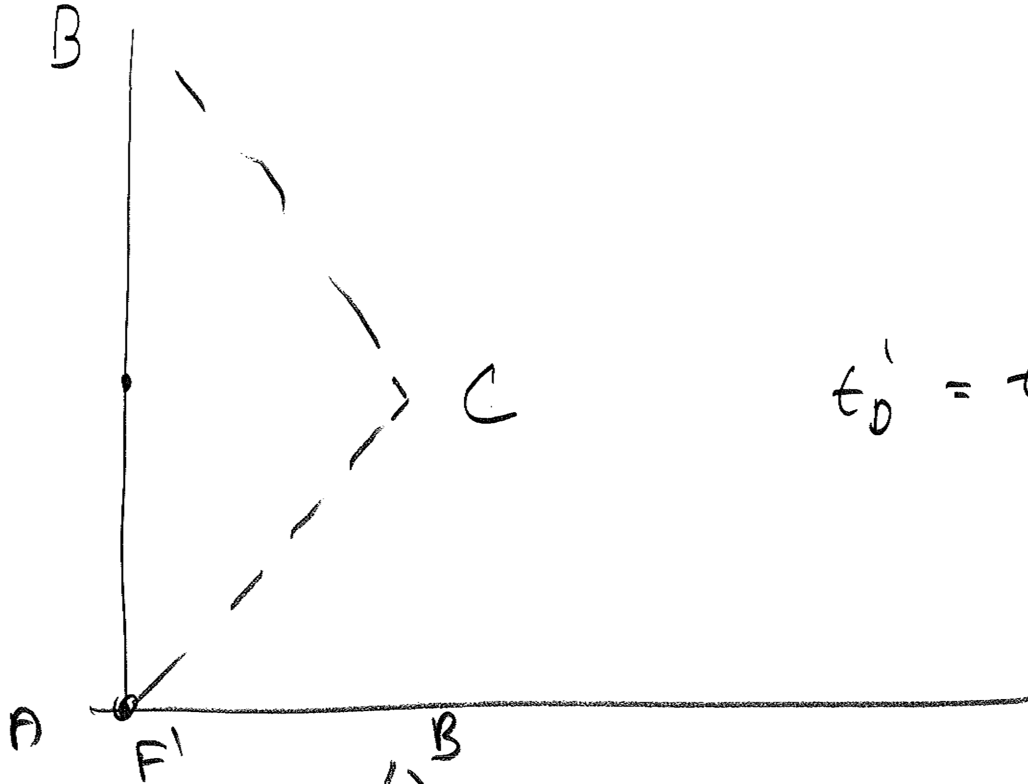
Coordinate Time is Frame-Dependent



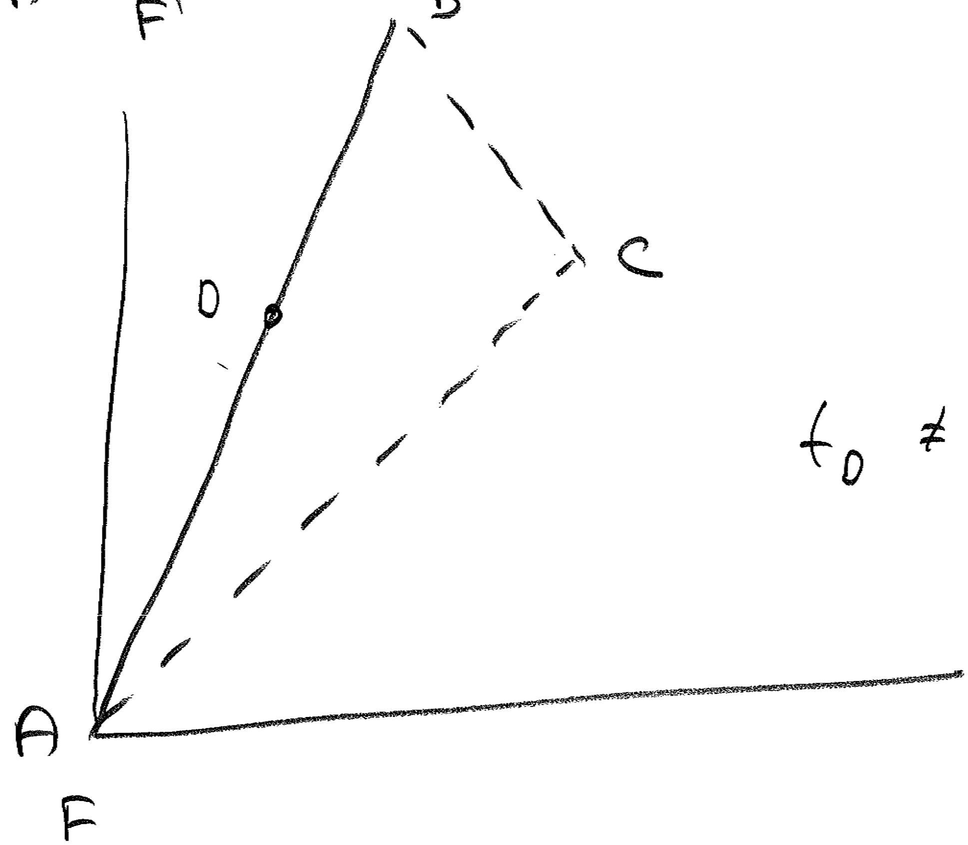
Events A and B occur at same time (synchronous) $(t_A' = t_B')$



$\boxed{t_A \neq t_B}$!!!



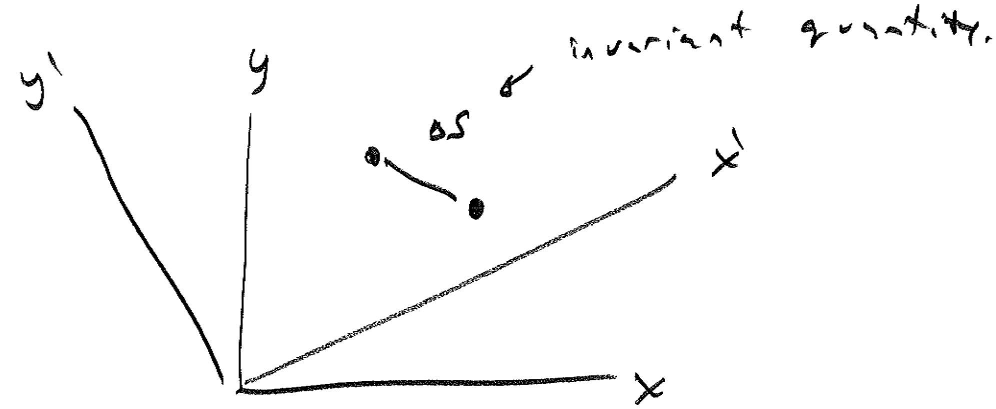
$t_D' = t_C'$



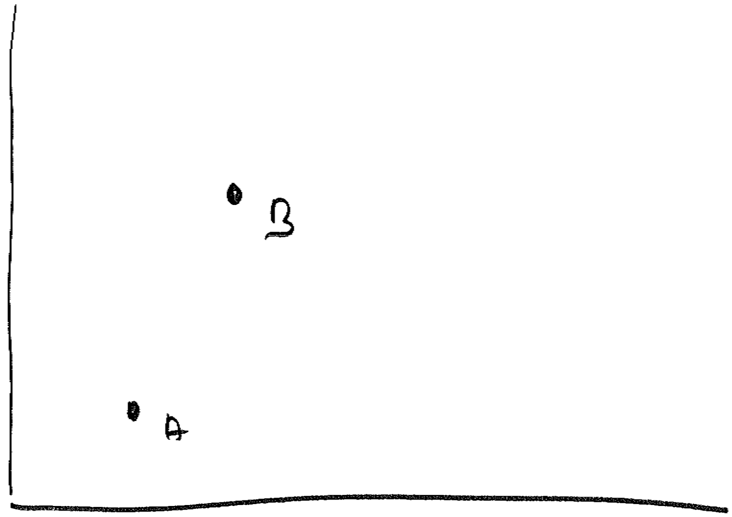
$t_D \neq t_C$

Geometric Analogy

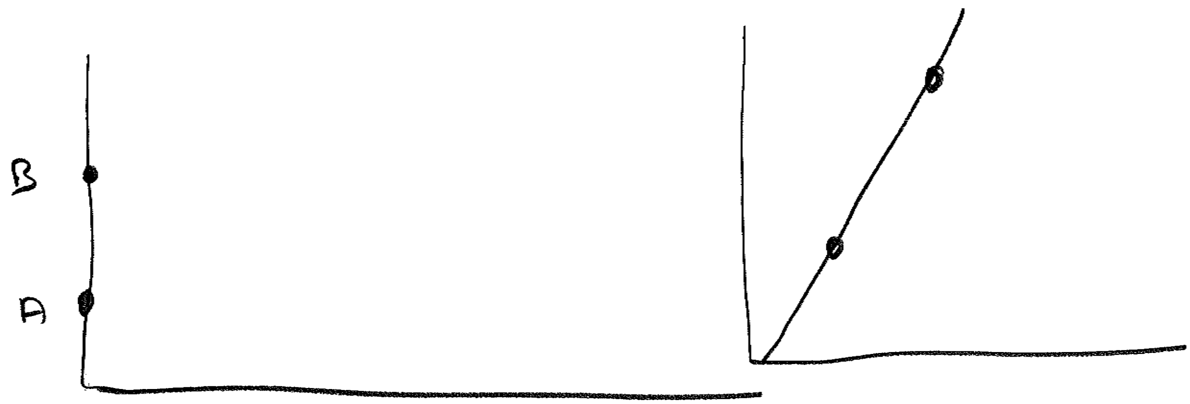
→ We will see that SR is very much about geometry, but I understand if this makes you skeptical as written in book.



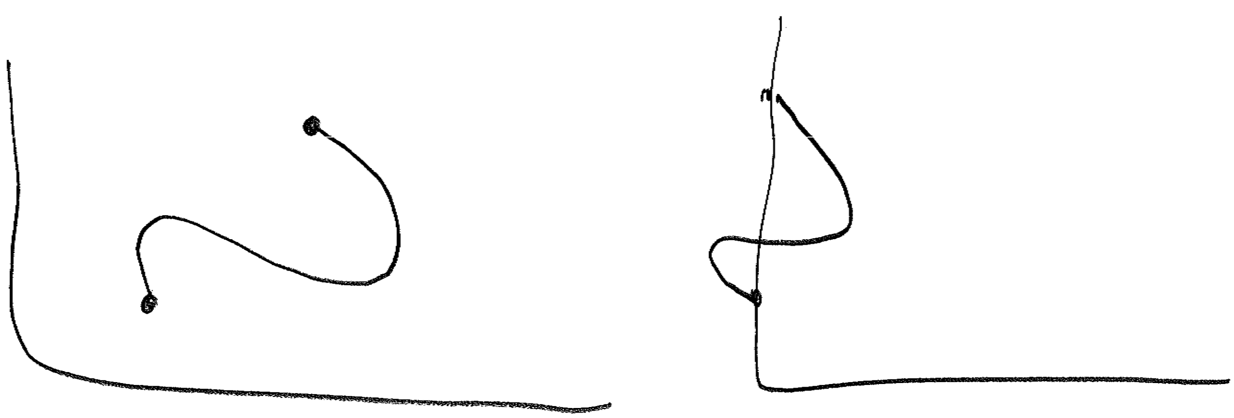
For now, we'll see that there is an invariant quantity



Coordinate Time



Space-Time Interval



Proper-Time

R3 S 4

Problems:

R3 S.1

R3S. 2

Take roll of roomate

R3 S 4

Try to illicit:

1) Relativity not about perception

2) Relativity not about being wrong / mistaken

3) Surprising consequences may seem contradictory, e.g. non-simultaneity could lead to impossible scenarios, but when rephrased as an experiment no contradiction is seen.

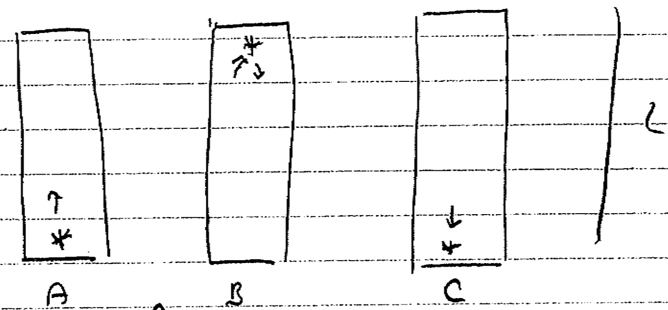
CORRECTION:

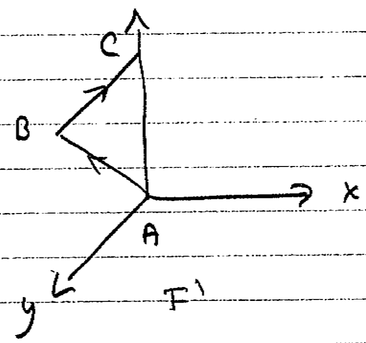
* 3 D also true: argument fails for faster than light.

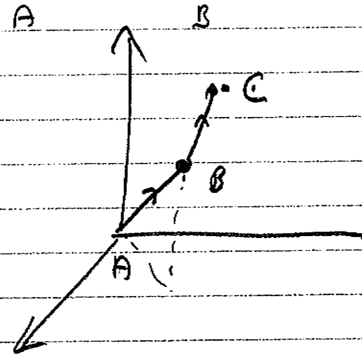
R4

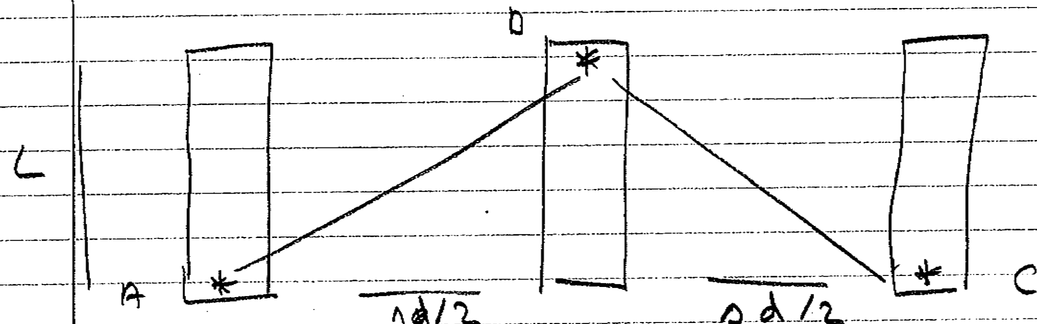
The Metric Equation

Light Clock









In inertial frame of clock (F'), clock reads a spacetime interval

$$\boxed{\Delta s = 2L}$$

In frame F where clock moves a distance Δd between events A and C, the light travels a distance

$$2\sqrt{L^2+\left(\frac{\Delta d}{2}\right)^2} = \sqrt{(2L)^2+\Delta d^2}$$

Since speed of light is 1

we must have

$$\Delta t = \sqrt{(2L)^2 + \Delta d^2}$$

But $2L = \Delta S$ so then

$$\Delta S^2 = (\Delta d)^2 - (\Delta t)^2$$

We can devise light clocks of any size, speed and orientation (in principle) so this equation is quite general!

To measure a space-time interval for two events with coordinates

$$(\vec{x}_1, t_1), (\vec{x}_2, t_2)$$

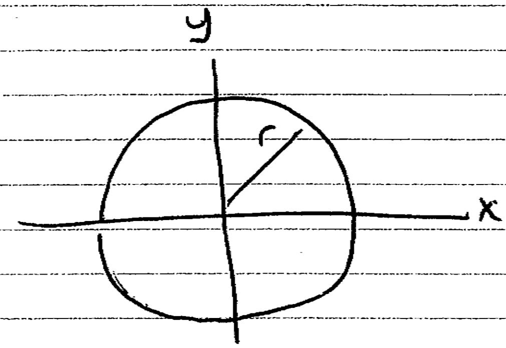
we merely calculate

$$\Delta S = (t_1 - t_2)^2 - (x_1 - x_2)^2 - (y_1 - y_2)^2 - (z_1 - z_2)^2$$

Q: This equation uses x, y, and z, but our derivation only uses one direction. Is this valid?

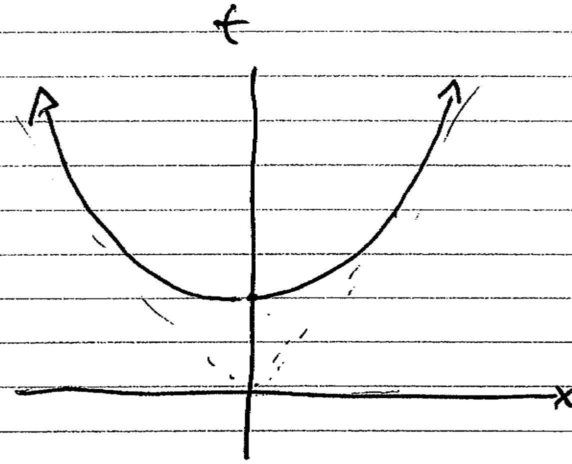
Euclidean Metric:

$$\Delta r^2 = \Delta x^2 + \Delta y^2$$



S.R.:

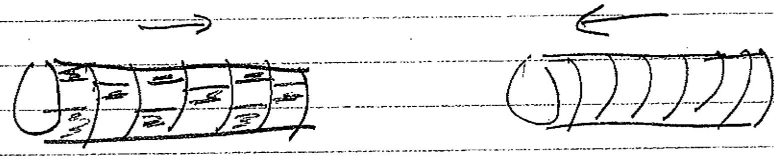
$$\Delta s^2 = \Delta t^2 - \Delta x^2$$

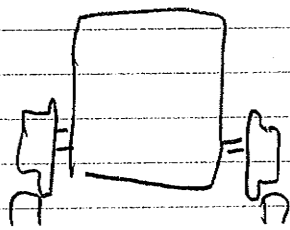


$$\Rightarrow \boxed{\Delta t^2 \geq \Delta s^2} \quad \Rightarrow \Delta t \geq \Delta s$$

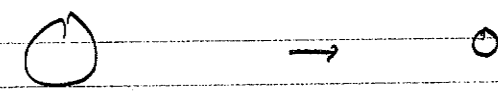
(for $\Delta t > 0$, $\Delta s > 0$)

Alternative Proofs of $\perp$ non-contraction

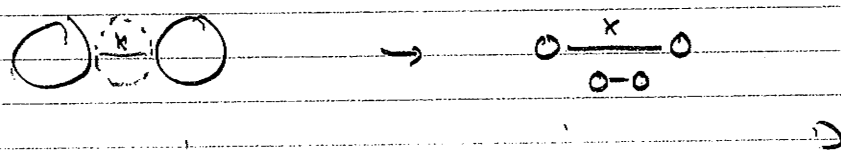
(1) 

(2) 

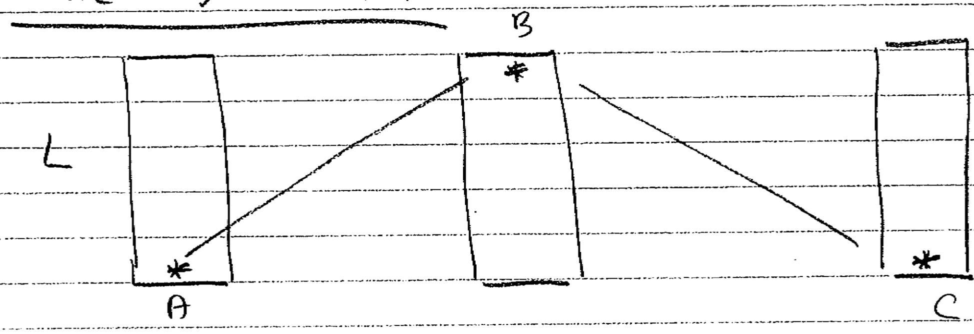
(3) If



what does



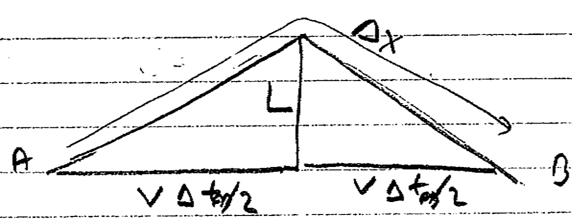
More Light Clocks



In light-clock frame, proper-time (= spacetime interval) is

$$\Delta\tau_{AB} \; (= \Delta s_{AB}) = 2L$$

In frame where clock is moving at constant velocity v:



Light travels a distance:

$$\Delta x = 2\sqrt{L^2 + (v\Delta t/2)^2} = \sqrt{(2L)^2 + (v\Delta t)^2}$$

$$\therefore \Delta x^2 = \Delta\tau_{AB} + (v\Delta t)^2$$

But light travels at speed 1, so

$$\Delta x = \Delta t, \quad \Delta t^2 = \Delta\tau_{AB} + (v\Delta t)^2$$

$$\Delta\tau_{AB} = (1-v^2)\,\Delta t^2$$

$$\boxed{\Delta\tau_{AB} = \sqrt{1-v^2}\;\Delta t} \quad !!!$$

!!! Visualize This !!!

Problems :

R4 B.1

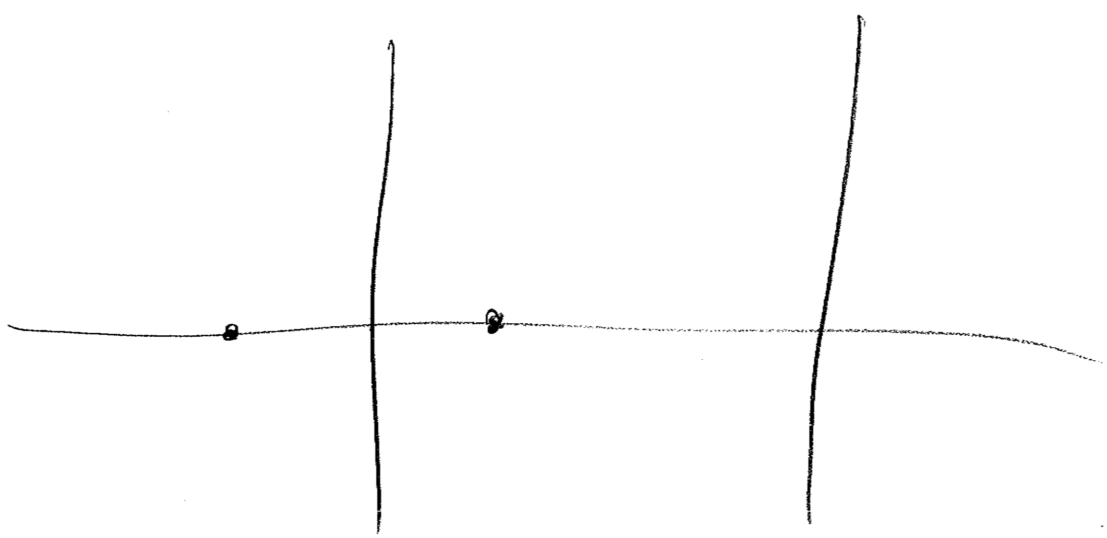
R4 B.4 *

R4 B.5

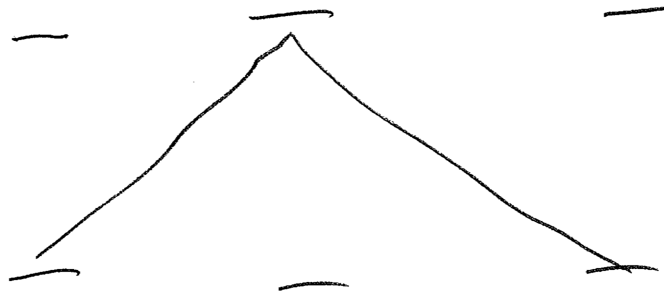
$\boxed{\text{R4 S1}}$

$\boxed{\text{R4 R2}}$

Recap :



$\boxed{\text{R5}}$ (RECAP)



light clocks

$$\Delta s^2 = \Delta t^2 - \Delta d^2 = \Delta t^2 - \Delta x^2 - \Delta y^2 - \Delta z^2$$

$$\Delta \tau = \sqrt{1-\beta^2}\,\Delta t$$

Organization:

→ READING QUIZZES ON SMART SITE

→ LEAVE H.W. IN GRADER'S BOX.

*WORK SOME R4 Examples TODAY
(→ Have until Tuesday 11 AM)

→ [(Check Smart site)]

Lecture 5 Plan

$\Rightarrow$ $\Delta t \geq \Delta s \geq \Delta \tau$ Lecture

$\Rightarrow$ Binomial ~~Theorem~~ Approximation

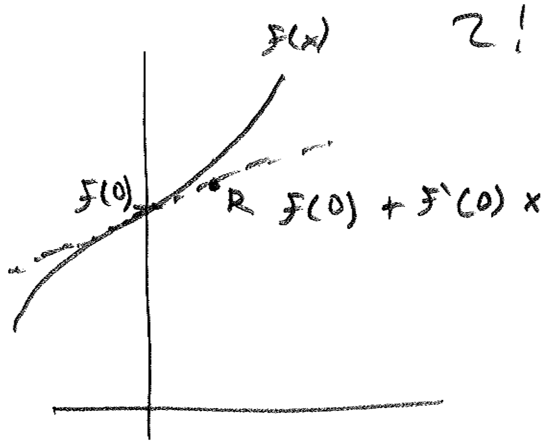
$$(1+x)^a \approx 1 + ax$$

Q: $\frac{1}{\sqrt{1-\beta^2}}$ $\left(\sim 1 + \frac{1}{2}\beta^2\right)$

$\Rightarrow$ Twin Paradox

Binomial Approximation

$$f(x) = f(0) + f'(0)\,x + \frac{f''(0)}{2!}x^2 + \cdots$$



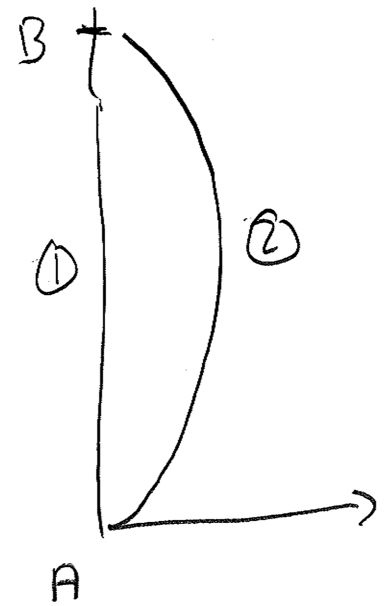
$$f(x) = (1+x)^a$$

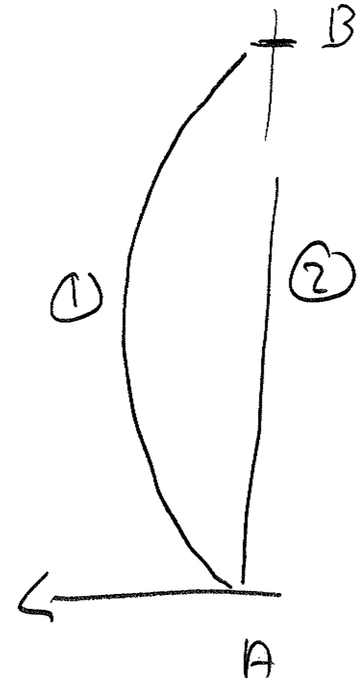
$$f(x) = (1+0)^a + a(1+x)^{a-1}\Big|_{x=0} \cdot x$$

$$= 1 + ax$$

$$\therefore \quad \frac{1}{\gamma} = \sqrt{1-\beta^2} = 1 + \frac{1}{2}(-\beta^2) = 1 - \frac{1}{2}\beta^2$$

Twin - Paradox





Both twin 1 and twin 2 measure a proper-time interval...

Twin 1 can calculate Twin 2's measurement by

$$\Delta \tau_{AB}^{(2)} = \int_0^{\tau_{AB}^{(1)}} dt \sqrt{1 - \beta^2(t)} < \int_0^{\tau_{AB}^{(1)}} dt = \tau_{AB}^{(1)}$$

Twin 1 concludes: $\Delta \tau_{AB}^{(2)} < \Delta \tau_{AB}^{(1)}$

Twin 2 calculates

$$\Delta \tau_{AB}^{(1)} = \int_0^{\tau_{AB}^{(2)}} dt \sqrt{1 - \beta^2(t)}$$

$$< \tau_{AB}^{(2)} \Rightarrow \Delta \tau_{AB}^{(1)} < \Delta \tau_{AB}^{(2)}$$

Q:
What happens when they meet???

RS

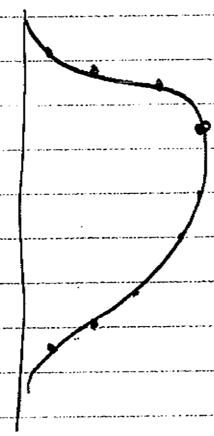
Time Dilation:

We have a way to calculate space time interval from coordinate time.

$$\Delta S = \Delta t^2 - \Delta x^2 - \Delta y^2 + \Delta z^2$$

This is only valid for an inertial frame (by definition of space-time interval).

We'd like to calculate proper-time for arbitrary space-time trajectories



Trick: Break trajectory into pieces of approximately constant velocity.

$$d\tau^2 \approx ds^2 = dt^2 - dx^2 - dy^2 - dz^2$$

$$d\tau = \sqrt{1 - \left(\frac{dx}{dt}\right)^2 - \left(\frac{dy}{dt}\right)^2 - \left(\frac{dz}{dt}\right)^2}\ dt$$

$$= \sqrt{1 - v^2}\ dt$$

$$\Delta \tau_{AB} \int_{t_A}^{t_B} (1 - v^2)^{1/2}\, dt$$

Time Dilation!

For constant velocity v

$$\Delta \tau_{AB} \ (= \Delta S_{AB}) = (1 - v^2)^{1/2}\ t_A - t_B$$

$$\Rightarrow \boxed{t_A - t_B = \Delta \tau_A / (1 - v^2)^{1/2}}$$

*** Already saw this factor with light clocks: now purely geometric argument!!!

$$\Delta s^2 = \Delta t^2 - \Delta x^2 - \Delta y^2 - \Delta z^2$$

$$\boxed{\text{Assume } \Delta s^2 > 0 \quad (\Rightarrow) \quad \Delta t^2 > 0}$$

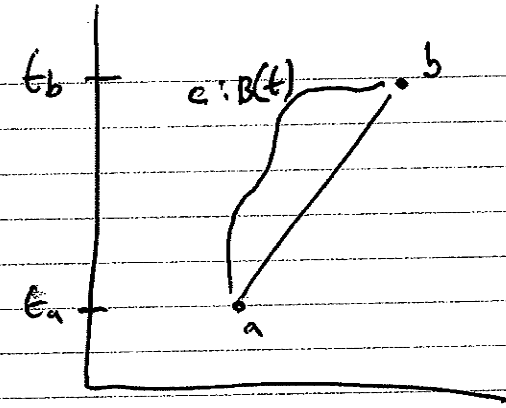
$$\Delta s^2 \geqslant \Delta t^2$$

$$\Delta s^2 = \Delta t^2 \quad \Rightarrow \quad \Delta x^2 + \Delta y^2 + \Delta z^2 = 0$$

$$\Rightarrow \quad \Delta x = \Delta y = \Delta z = 0$$

* Definition of proper - time (as $\Delta d = 0$)
falls out of geometry too *

$$\Delta \tau = \int_{t_1}^{t_2} \sqrt{1 - B^2} \; dt$$



Consider this in frame F', travelling at constant B between a and b.

$$\Delta S \equiv \int_{t_a'}^{t_b'} dt' \qquad \text{(by definition of } \Delta S\text{!)}$$

$$\Delta \tau_c = \int_{t_a'}^{t_b'} \sqrt{1 - B'(t')^2}\, dt'$$

$$\leq \int_{t_a'}^{t_b'} dt' = \Delta S$$

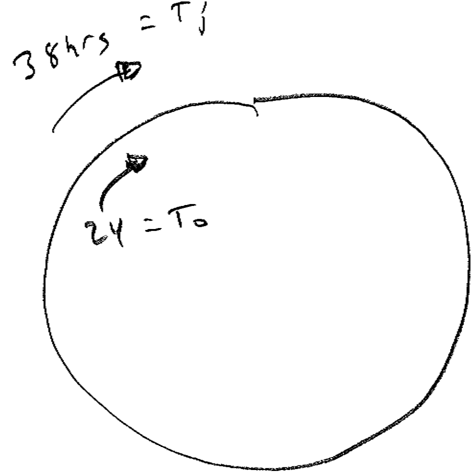
$$\Rightarrow \quad \boxed{\Delta \tau \leq \Delta S}$$

Finally:

$$\Delta t \geq \Delta S \geq \Delta \tau$$

Trick to remember

$\Delta \tau = 0$ when connected by light cones.



Light circles earth in

$$T_c = \frac{2\pi R}{c} = 0.4\ \text{s}$$

$$\sqrt{1-B^2} = 1 - \frac{1}{2} B_1^2$$

$$\tau_1 = \left(1 - \frac{1}{2} B_1^2\right) T_j$$

$$\tau_2 = \left(1 - \frac{1}{2} B_2^2\right) T_j'$$

$$\Delta\tau = \left|\tau_2 - \tau_1\right| = \frac{T_j}{2} \cdot \left|B_1^2 - B_2^2\right|$$

$$B = \frac{2\pi R / T}{c} = \frac{2\pi R / c}{T} = \frac{T_c}{T}$$

$$B_1 = \frac{T_c}{T_0} \qquad B_2 = \frac{T_c}{T_j'} + \frac{T_c}{T_0}$$

$$\Delta\tau = \frac{T_j}{2}\left[\left(\frac{T_c}{T_0}\right)^2 - \left(\frac{T_c}{T_0} + \frac{T_c}{T_j}\right)^2\right]$$

$$= \frac{T_j}{2}\left[-2\frac{T_c^2}{T_0 T_j} + \frac{T_c^2}{T_j^2}\right]$$

$$= T_c^2\left[\frac{1}{2}\frac{1}{T_j} - \frac{1}{T_0}\right]$$

$$= (0.4\,s)^2 \left[\frac{1}{2} \frac{1}{38\,hrs} - \frac{1}{24\,hrs} \right] \left(\frac{1\,hr}{60 \cdot 60\,s} \right)$$

$$= -1.26 \times 10^{-6}\ s$$

$$\sim 1\,\mu s$$

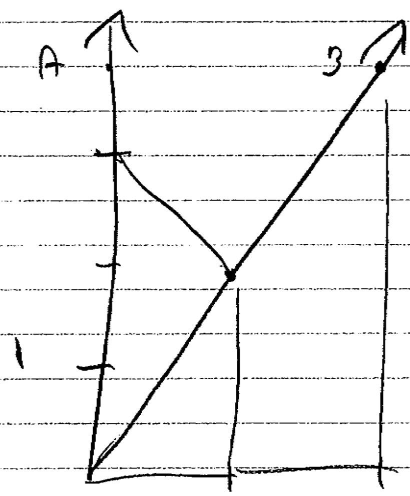
Atomic Clocks precise to 10^{-9} s / day

→ 1000 x's needed precision!

FUN PROBLEM

IDEAS

→ Stay Away Twin Non Paradox



$$\beta = \frac{1}{2}$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-\frac{1}{2}^2}} = \frac{2}{\sqrt{3}}$$

Twin B arranges to send a birthday message to twin A:

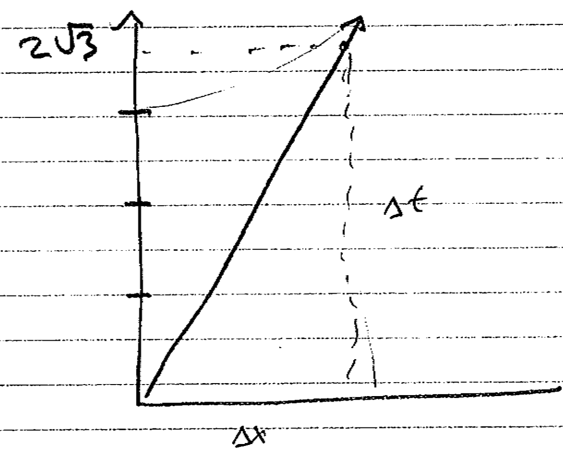
Happy 3rd birthday I am ?.

Solve for ?:

$$?^2 = \Delta s^2 = \Delta t^2 - \Delta x^2 = 2^2 - 1 = 3$$

$$\Rightarrow\ ? = \sqrt{3}$$

Twin A wants to arrange similar message, to arrive on B's 3rd birthday



$\therefore \Delta t = \gamma \cdot 3 = \frac{2}{\sqrt{3}} \cdot 3 = 2 \cdot \sqrt{3}$

$\Delta x = \beta \Delta t = \sqrt{3}$

To arrive at $\Delta t = 2\sqrt{3}$ $(= \Delta t' = 3)$

need to send at $2\sqrt{3} - \sqrt{3} = \sqrt{3}$

Happy birthday 3, I am $\sqrt{3}$!

RG 5.3

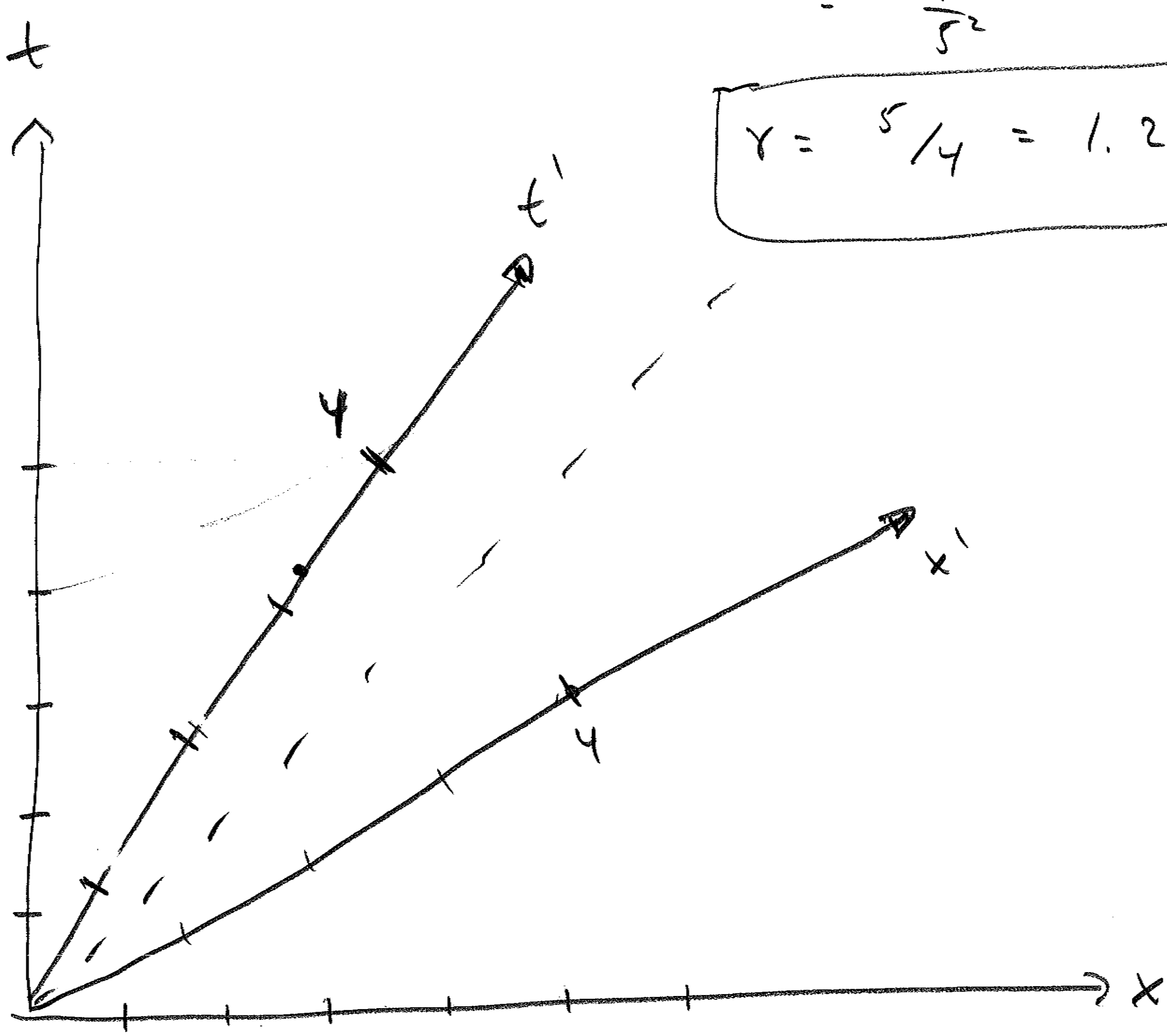
$$\beta = 0.60 = \frac{3}{5}$$

$$1/\gamma^2 = 1 - \beta^2$$

$$= 1 - \frac{3^2}{5^2}$$

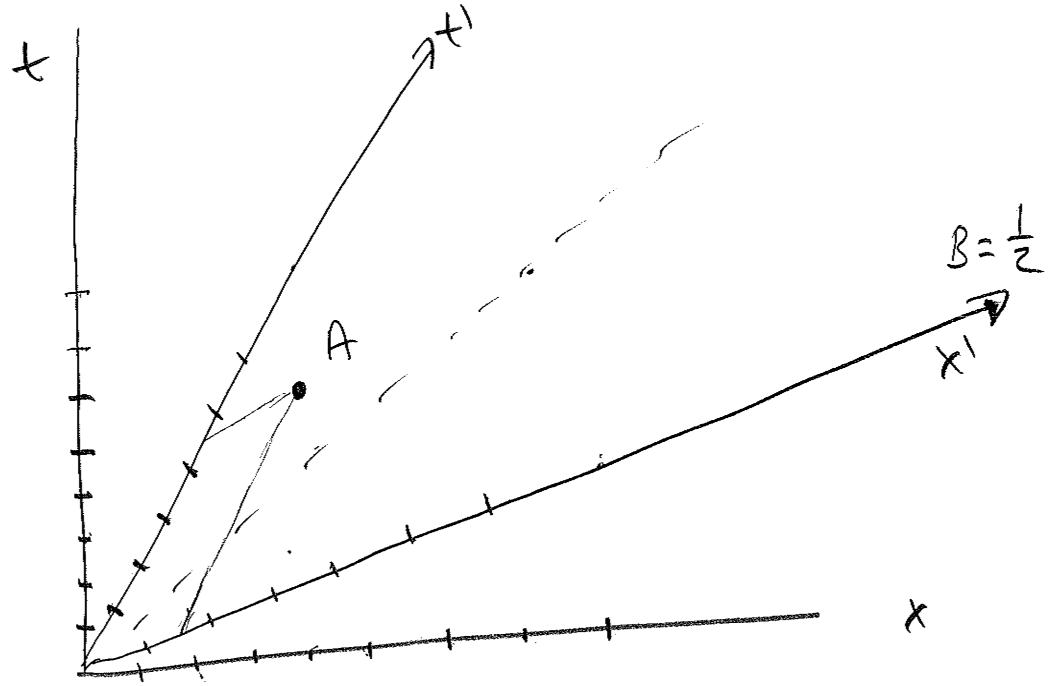
$$= \frac{4^2}{5^2}$$

$$\boxed{\gamma = 5/4 = 1.25}$$



$$\boxed{\begin{aligned} 1 \cdot 1.25 &= 1.25 \\ 2 \cdot 1.25 &= 2.50 \\ 3 \cdot 1.25 &= 3.75 \\ 4 \cdot 1.25 &= 5 \end{aligned}}$$

R 6.S 1



$$\gamma = \sqrt{\frac{1}{1 - \beta^2}}$$

$$= \frac{1/\beta}{\sqrt{\frac{1}{\beta^2} - 1}} = \frac{2}{\sqrt{4 - 1}}$$

$$= \frac{2}{\sqrt{3}} \sim 1.15$$

$\gamma \sim 1.15$

$\Rightarrow$ 4 tick will be

$4 \cdot 1.15 = 4.6$

$3 \cdot 1.15 = 3.45$

$2 \cdot 1.15 = 2.3$

$1 \cdot 1.15 = 1.15$

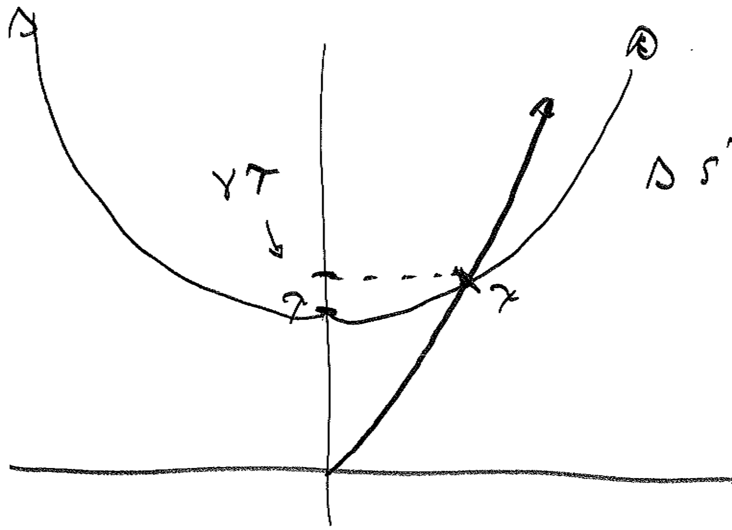
$6 \cdot 1.15 = 6.9$

R7

Recap:

* Derived time-dilation: from constant speed of light

* Also can see it from space-time interval



$\Delta s^2 = \text{const}$

* Now add one more:

Proper-Time from L.T.

$$\Delta t = \gamma(\Delta t' + \beta \Delta x')$$

A proper-time interval is a coordinate time interval made at the same location

i.e. $\Delta t' = \Delta \tau$ when $\Delta x' = 0$

So $\Delta t = \gamma \Delta \tau$

$$\Delta \tau = \Delta t / \gamma$$

$$\Delta \tau = \sqrt{1-\beta^2}\ \Delta t$$

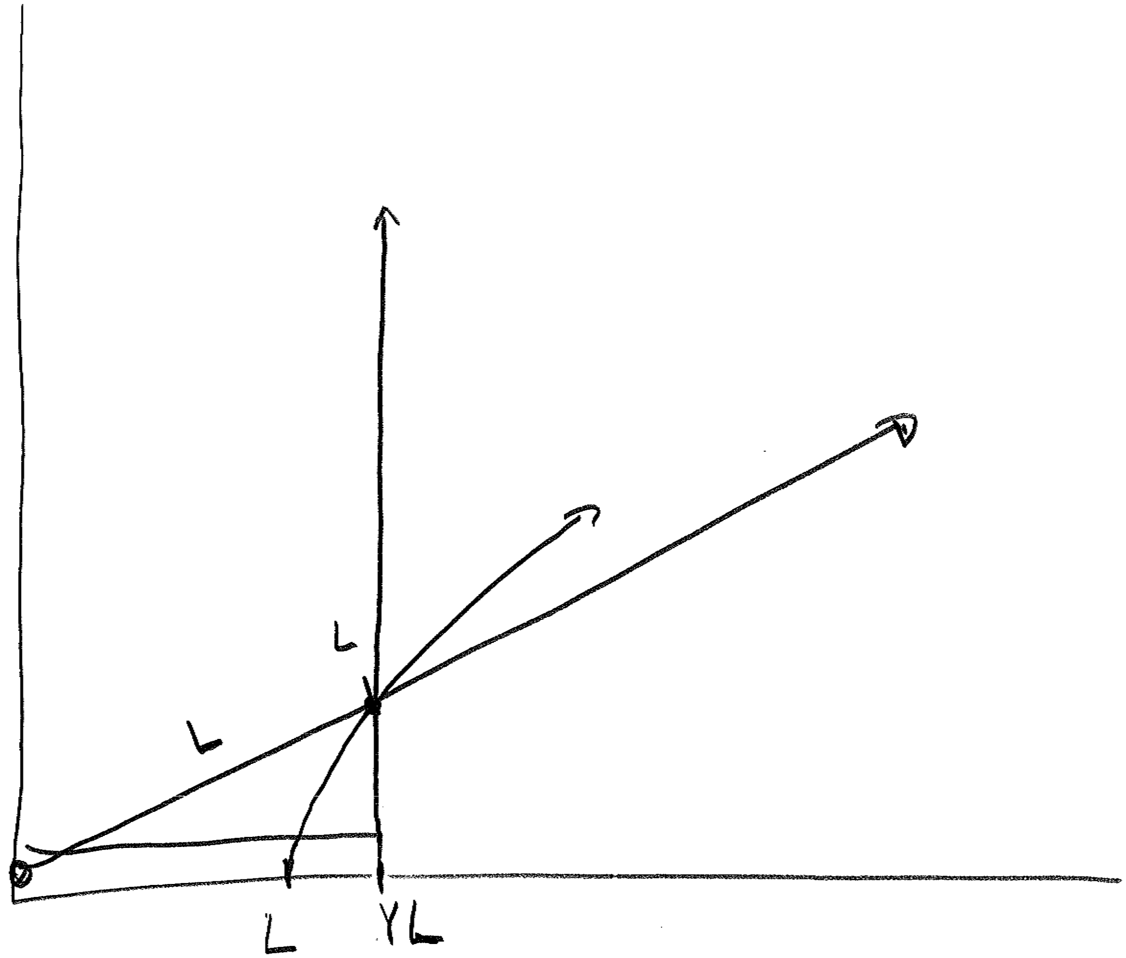
Q: If time and space are "symmetric" why do we have

$\Delta s =$

proper $\rightarrow$ $\Delta \tau = \sqrt{1-\beta^2}\ \Delta t$

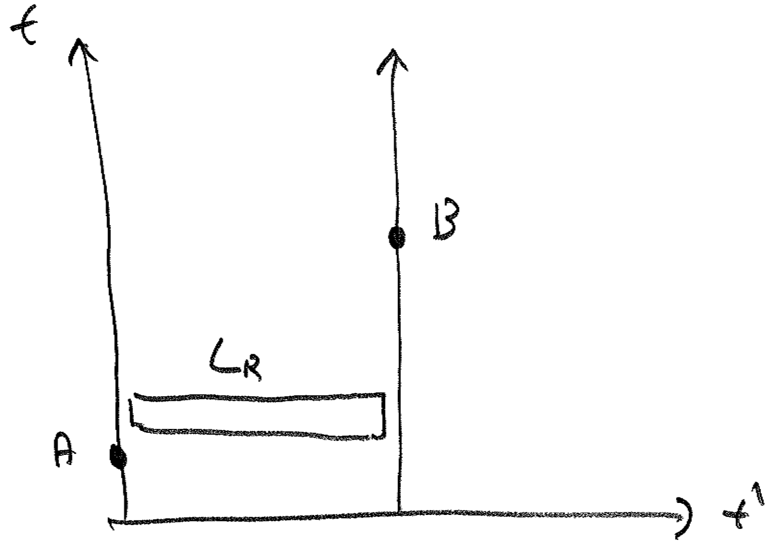
$L = \sqrt{1-\beta^2}\ L_R$ $\leftarrow$ "proper"

Length Contraction from 2-body Diagram



$$\boxed{L_R = \gamma L}$$

Length Contraction from L.T.



Define rest length as frame in which world lines for the ends of object are vertical lines (object is at rest).

CRUCIAL OBSERVATION:

Even when $\Delta t_{AB} \neq 0$, $\Delta x_{AB} = L_R$

because object is at rest.

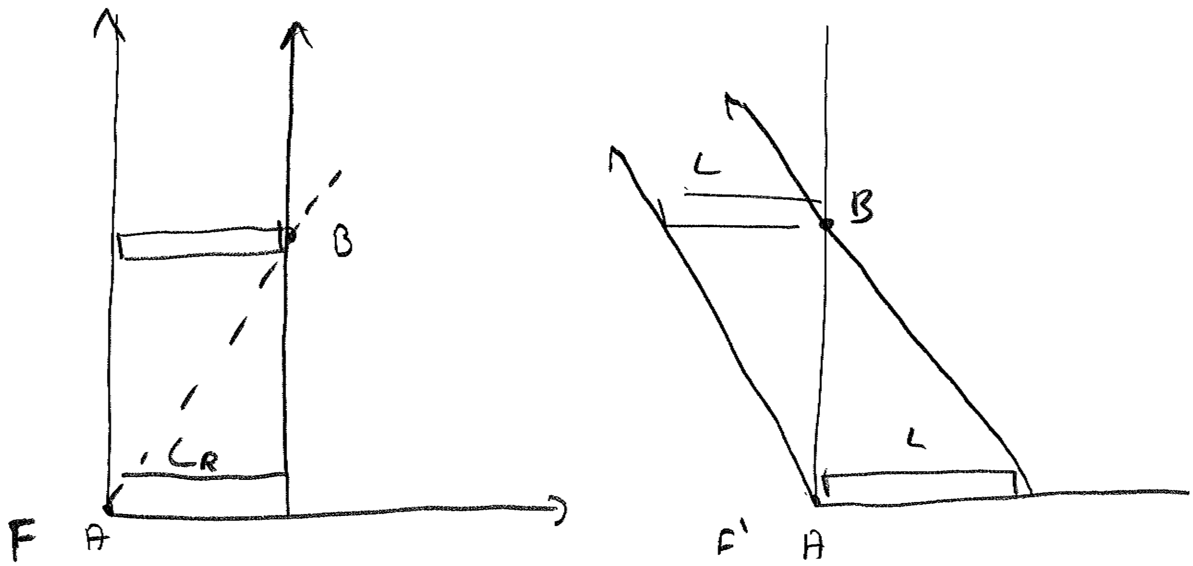
From a moving frame, we measure the length of the object as a simultaneous ($\Delta t'_{AB} = 0$) measurement of the distance between the ends $L = \Delta x'_{AB}$

From L.T.

$$L_R = \Delta x_{AB} = \gamma(\beta \Delta t'_{AB} + \Delta x'_{AB})$$
$$= \gamma(0 + L)$$

$$\boxed{L = L_R/\gamma = \sqrt{1-\beta^2}\, L_R}$$

Length Contraction from Time-Dilation



F measures $L_R = \beta \, \Delta t_{AB}$

F' measures $L = \beta \, \Delta t'_{AB}$

But $\Delta x'_{AB} = 0$ (i.e. $\Delta t'_{AB} = \Delta \tau_{AB}$)

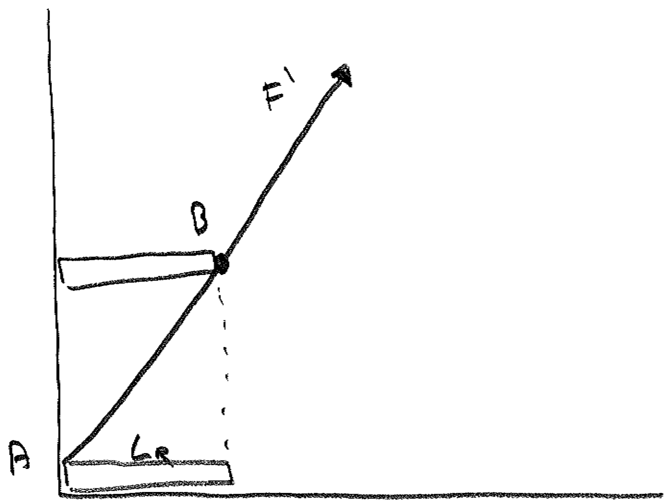
$$\frac{L_R}{L} = \frac{\beta \, \Delta t_{AB}}{\beta \, \Delta \tau_{AB}} = \frac{\Delta t_{AB}}{\Delta \tau_{AB}} = \gamma$$

$$L = \sqrt{1-\beta^2} \, L_R \quad \text{(Length contraction)}$$

$$\Delta \tau_{AB} = \sqrt{1-\beta^2} \, \Delta t_{AB} \quad \text{(time dilation)}$$

Length Contraction from ST Interval

(I)



F

$$\Delta S_{AB}^2 = (\Delta t'_{AB})^2 = (\Delta t_{AB})^2 - (\Delta x_{AB})^2$$

[F measures the proper length L_R
F' measures a length L]

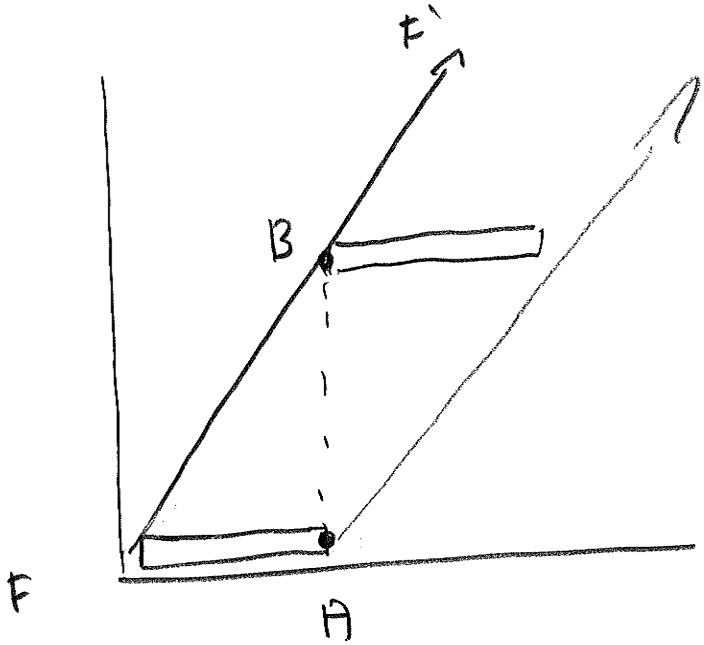
$$\Delta S_{AB}^2 = \left(\frac{L}{\beta}\right)^2 = \left(\frac{L_R}{\beta}\right)^2 - (L_R)^2$$

$$L^2 = (1-\beta^2) L_R^2$$

$$\boxed{L = \sqrt{1-\beta^2}\; L_R}$$

(Keep on Board)

(II) Same frames but now imagine a ruler is at rest in F' ...



$\boxed{\text{Clean Up}}$

$$\Delta s^2 = \Delta t_{AB}^{2} = \Delta t'^{2}_{AB} - \Delta x'^{2}_{AB}$$

Q: Note this is last equation with F and F' reversed.... so does moving ruler get longer now?

A: No! Now F' measures the proper length, exactly as required for relativistic invariance!

$$\left(\frac{L}{B}\right)^2 = \left(\frac{L_R}{B}\right)^2 - \left(L_R\right)$$

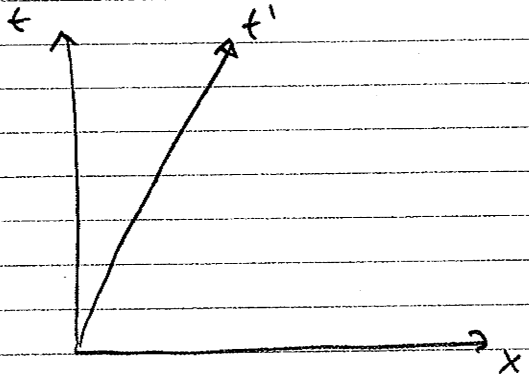
$$\boxed{L = \sqrt{1-B^2}\, L_R}$$

R6

Two Observer Diagram

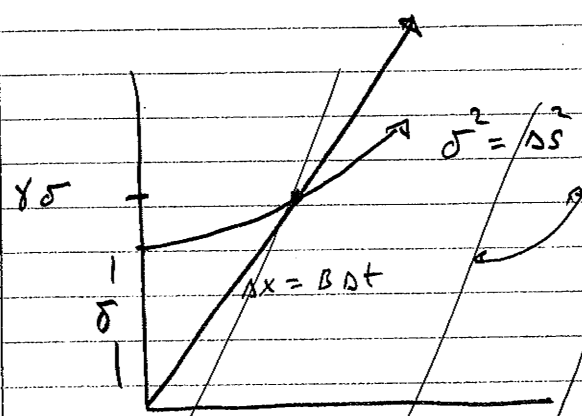
1) Draw the t' axis

→ along worldline of $x' = 0$!



→ slope is $1/\beta$!!!

2) Calibrate t' axis: (See Alt)



$\sigma^2 = \Delta s^2 = \Delta t^2 - \Delta x^2$

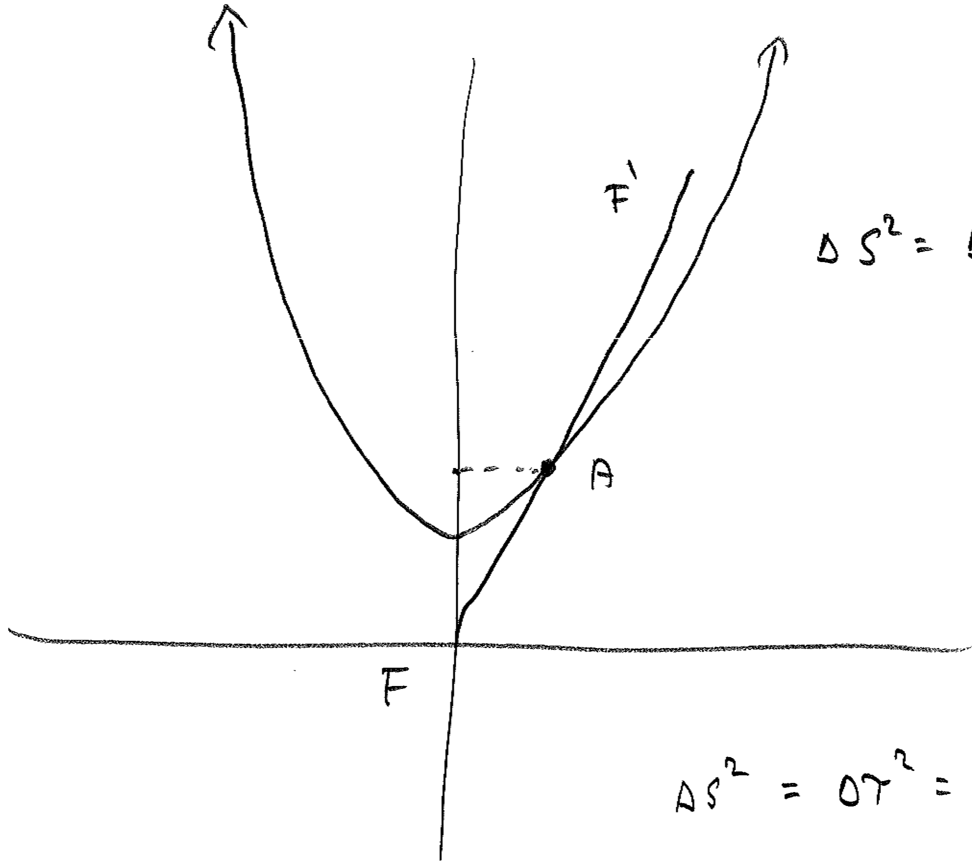
$\Rightarrow \sigma^2 = (1-\beta^2)\,\Delta t^2$

$\Delta t = \frac{1}{\sqrt{1-\beta^2}}\,\sigma = \gamma\sigma$

A: Measurements of t along an axis are always space-time intervals. Why?

$\Delta x' = 0$

R6



$$\Delta s^2 = \Delta t^2 - \Delta d^2$$

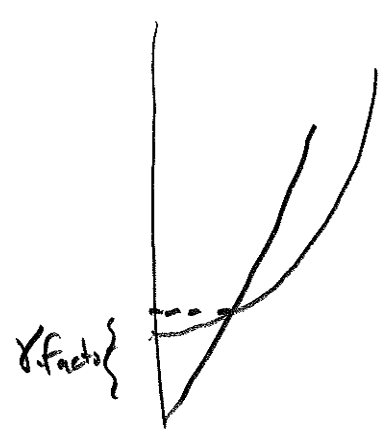
$$\Delta s^2 = \Delta \tau^2 = \left(1 - \beta^2\right) \Delta t^2$$

$$\Delta t = \frac{\Delta s}{\left(1 - \beta^2\right)} \equiv \gamma \Delta s$$

Q: Measurements of t along an 'inertial' axis are always spacetime intervals. Why?

A: $\Delta x' = 0$ and inertial

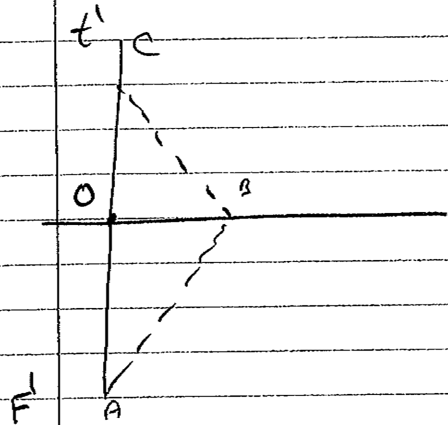
Stress:



When you see this, think γ!

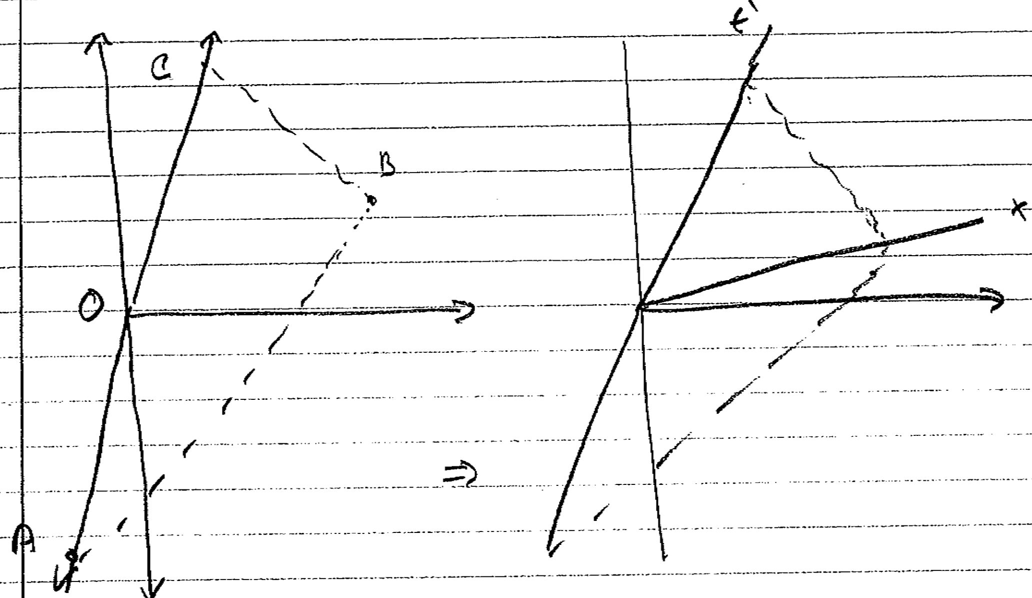
3) Draw the x' axis

⇒ Connects all simultaneous' events

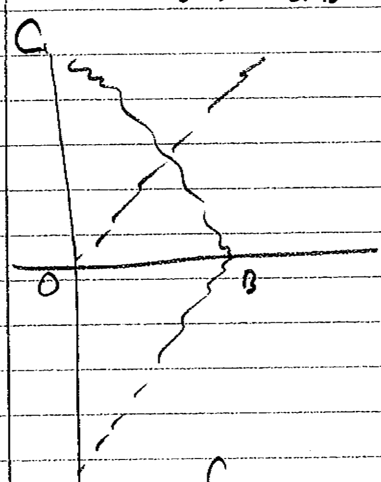


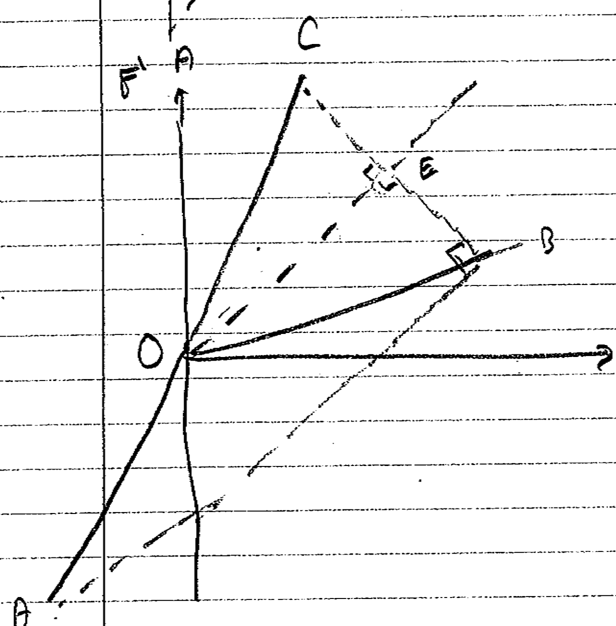
(KEEP ON BOARD)

What does this look like in F frame



Now add another pulse





$\Delta ABC \sim \Delta OEC$

$OC = \frac{1}{2} AC$

$\Rightarrow CE = EB$

$\boxed{\angle COE = \angle BOE}$

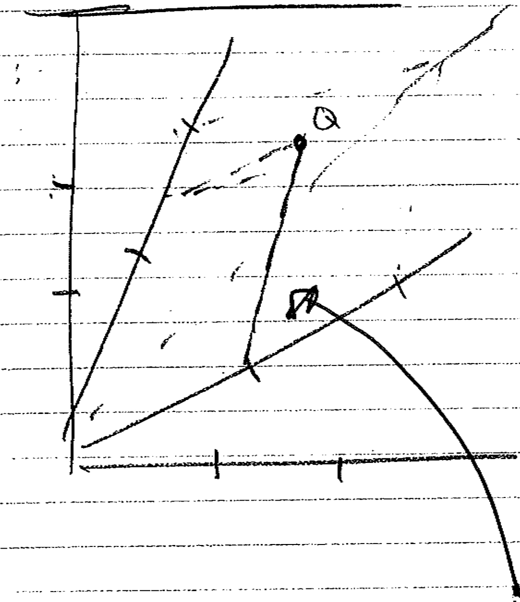
$\boxed{CO = OB}$

Same scale factor

Q: Why does $CO = OB$ lead to same scale factor

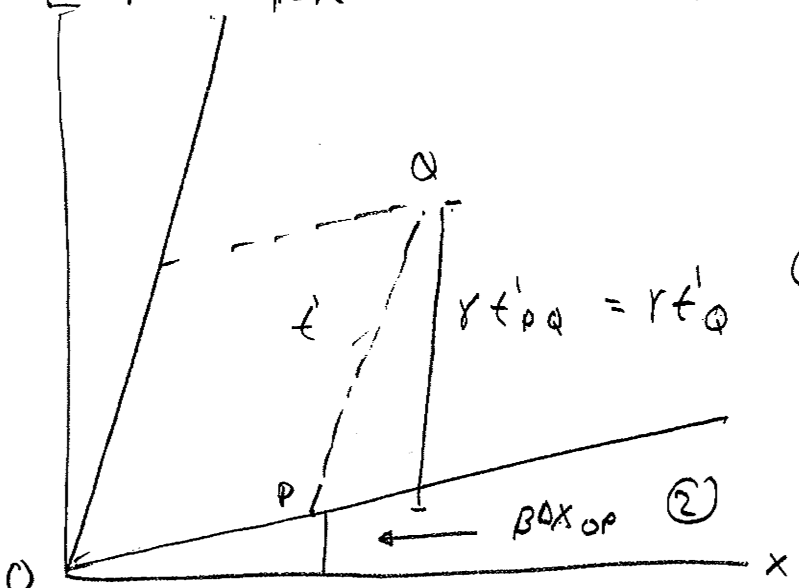
$O \Rightarrow B$ Lorentz [illegible] diagram in F'

Reading Events



parallel to y axis

Deriving LT from 2-Observer Diagrams

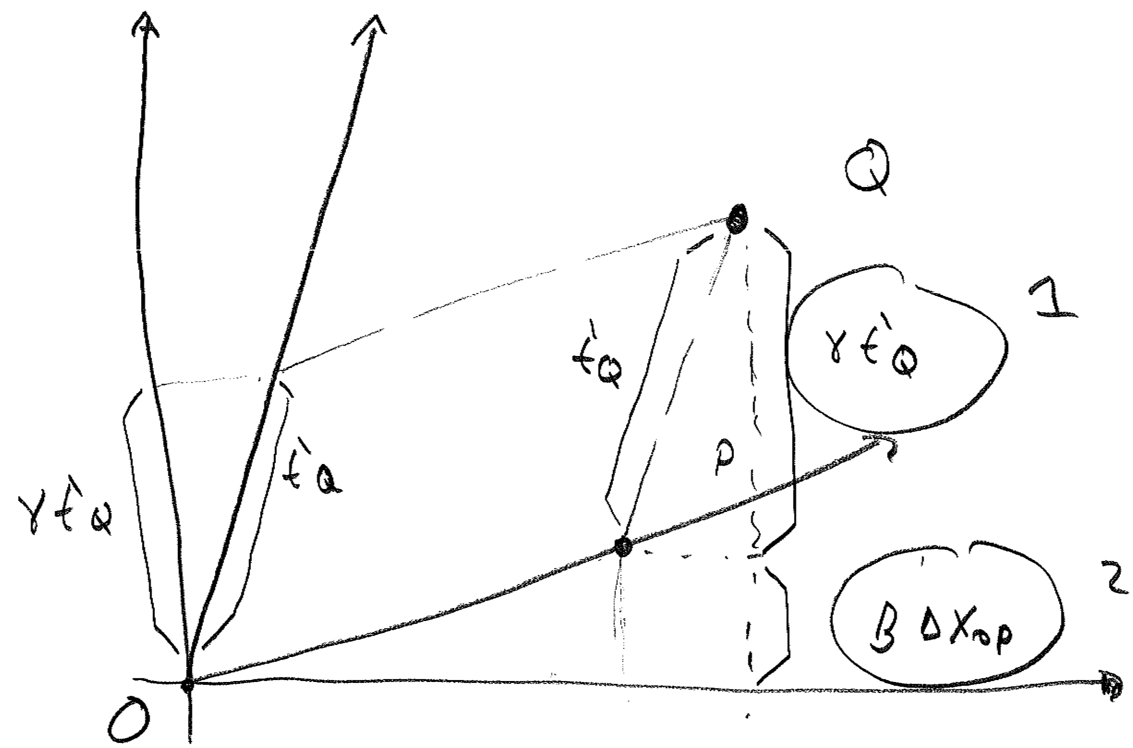


③ $\Delta x_{OP} = \gamma \Delta x'_{OP} = \gamma x'_Q$

$$\Rightarrow t_Q = \gamma (t'_Q + \beta x'_Q)$$

But x is just reflection of t across $x = t$

$$\Rightarrow \quad x_Q = \gamma (x'_Q + \beta t'_Q)$$



(2) But $\Delta x_{op} = \gamma x'_{op}$ and $x'_{op} = x'_Q$

So $B \Delta x_{op} = \gamma B x'_Q$

Finally $t_Q = \gamma t'_Q + \gamma B x'_Q$

(Talk about active learning here ...)

Purely geometric so

$$x_Q = \gamma x'_Q + \gamma B t'_Q$$

Q is any arbitrary point so:

L.T.

$$t = \gamma(t' + \beta x')$$
$$x = \gamma(x' + \beta t')$$
$$y = y'$$
$$z = z'$$

$\Longleftrightarrow$

$$t' = \gamma(t - \beta x)$$
$$x' = \gamma(x - \beta t)$$
$$y' = y$$
$$z' = z$$

1. Algebra or
2. $\beta \rightarrow -\beta$

How to remember:

$$x = x' + \beta t \quad \leftarrow \text{ Gallilean}$$

$$x = \gamma(x' + \beta t) \quad \leftarrow \text{ Add Gamma Factor}$$
$$t = \gamma(t' + \beta x) \quad \leftarrow \text{ Similar for } t$$
$$y = y'$$
$$z = z'$$
} Trivial

Low velocity limit of L.T.

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} = 1 + \tfrac{1}{2}\beta^2 \sim 1$$

$$x = x' + \beta t \quad ✓$$

$$t = t' + \beta x \quad ?$$

Why can we neglect βx but not βt?

1. $\beta \ll c \Rightarrow \frac{\Delta x}{\Delta t} \ll 1 \qquad \Delta x \ll \Delta t$

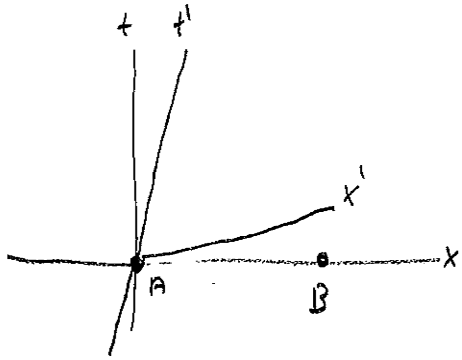
2. light hour vs hour

astronomical vs everyday units

Problems R6 S1, S2, S3

HW "gimme".

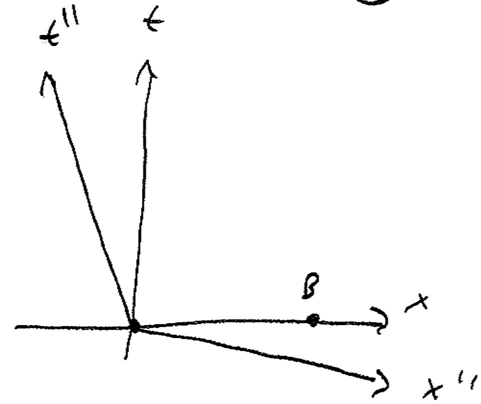
Lecture R8



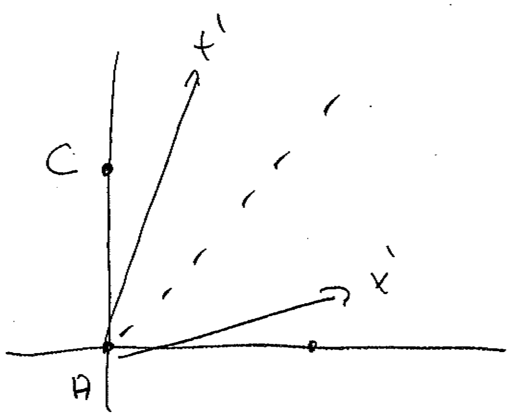
$t_A = t_B$

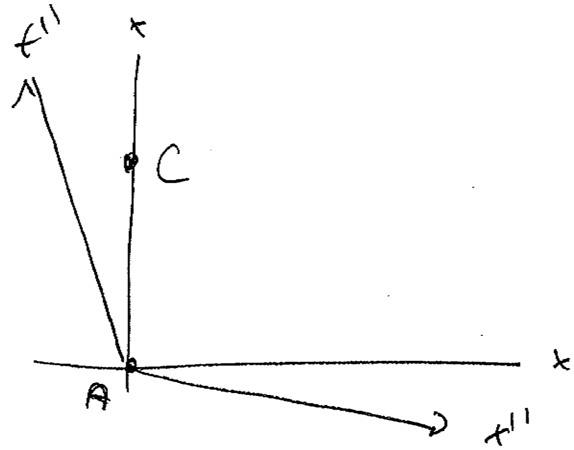
$t'_A > t'_B$

$t''_A < t''_B$



Q: Can A cause B?





$t_A < t_C$

$t'_A < t'_C$

$t''_A < t''_C$

Q: Can A cause C?

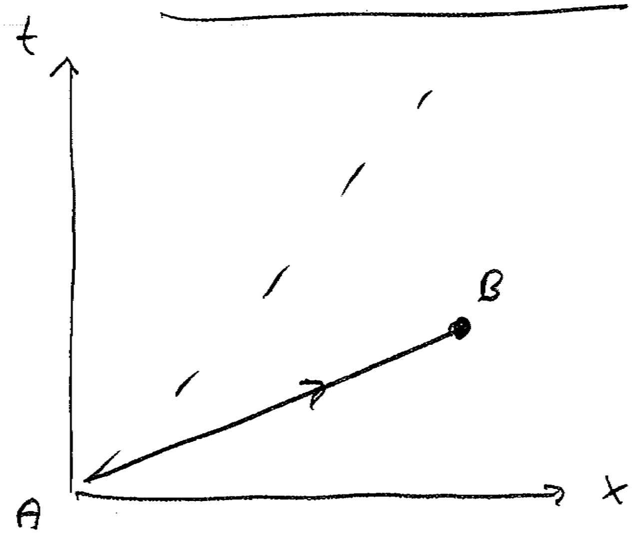
Yes.

⇒ There are events along space-axis
→ can't be causally connected

⇒ Events along time axis
→ can be causally connected

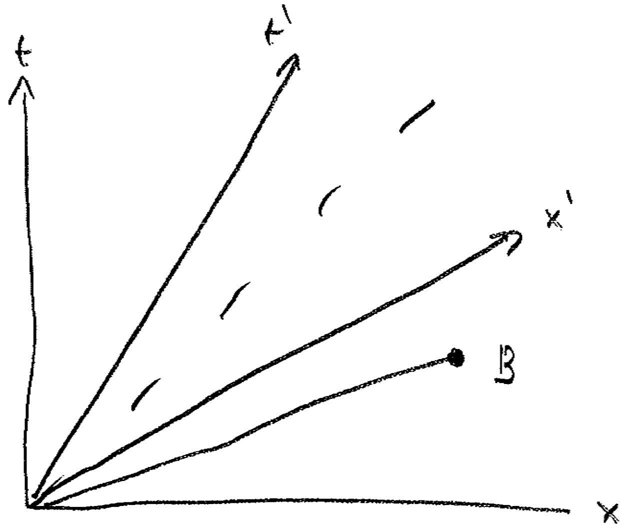
Q: Can we generalize?

Lecture R8



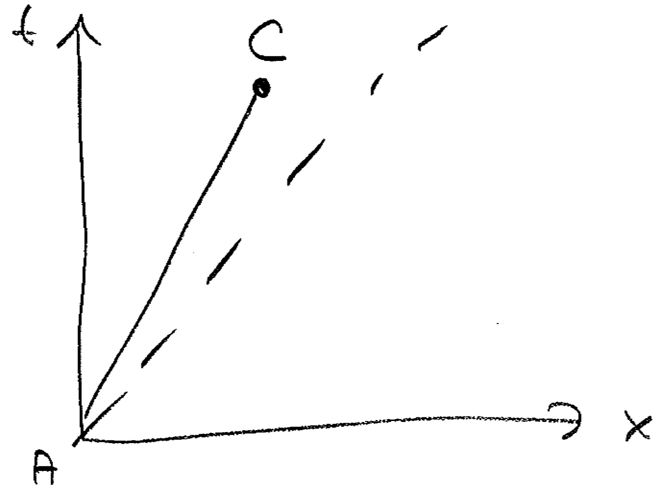
⇒ A causes B at faster than "c";

Q: Can you find a frame where B happens before A?

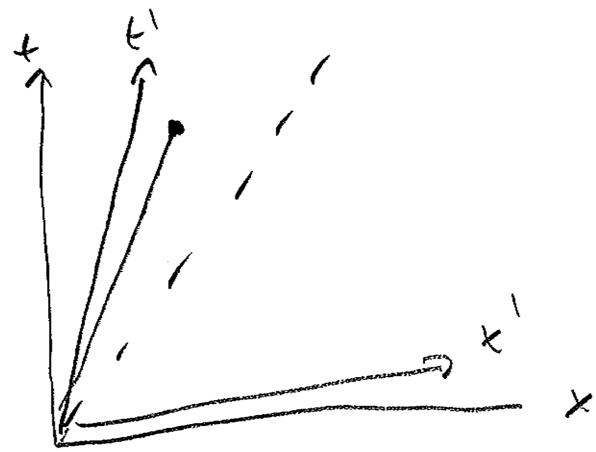


In this frame, B happens <u>before</u> its cause... impossible.

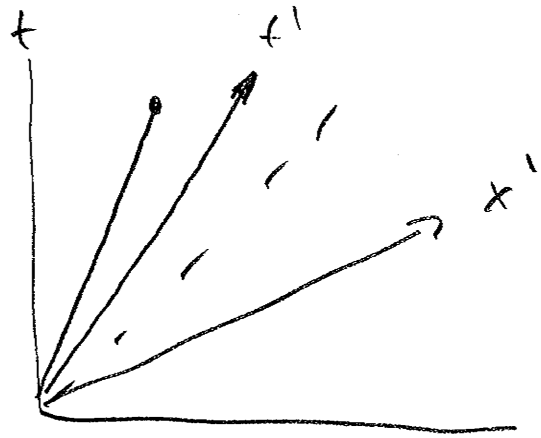
What about event with causal influence less than "c"?



Find a frame where C is before A?

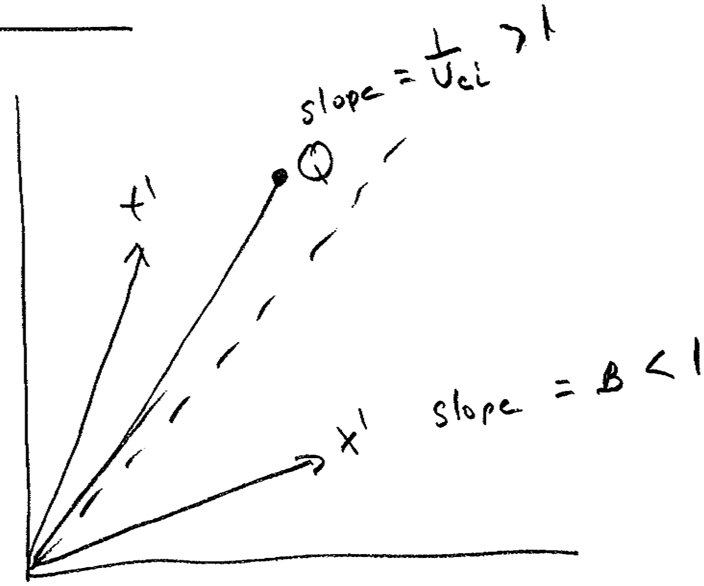


Nope.



Nope

Figure R8.4



$$B < 1 < \frac{1}{U_{ci}'} \Rightarrow t_Q' > 0$$

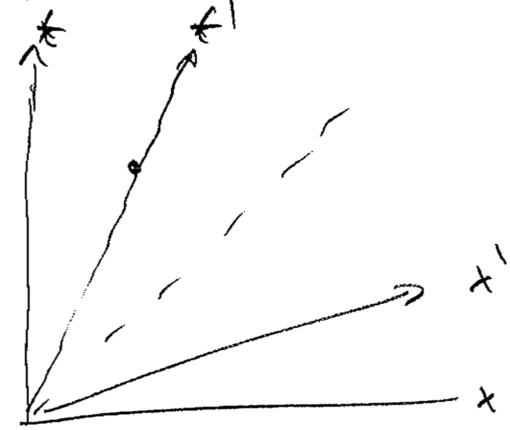
We define 3 categories:

$\Delta s^2 > 0$ time-like

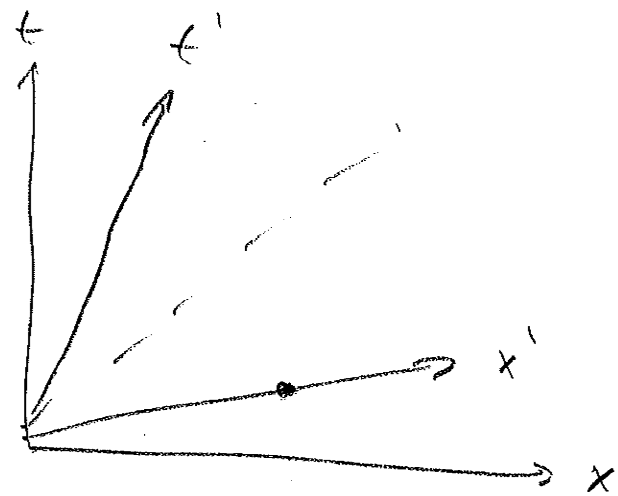
$\Delta s^2 = 0$ light-like

$\Delta s^2 < 0$ space-like

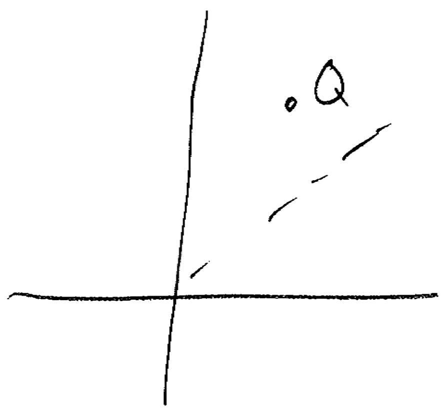
Q: Given a time-like interval, draw frame where they occur at same space position



Q: Give a space-like interval ...



So, how can we define class of events that can be causally connected, in terms of x and t?



$$\frac{|\Delta x_Q|}{|\Delta t_Q|} < 1$$

$$|\Delta x_Q| < |\Delta t_Q|$$

Q: Can two frames differ as to whether events O and Q are causally connected?

No?

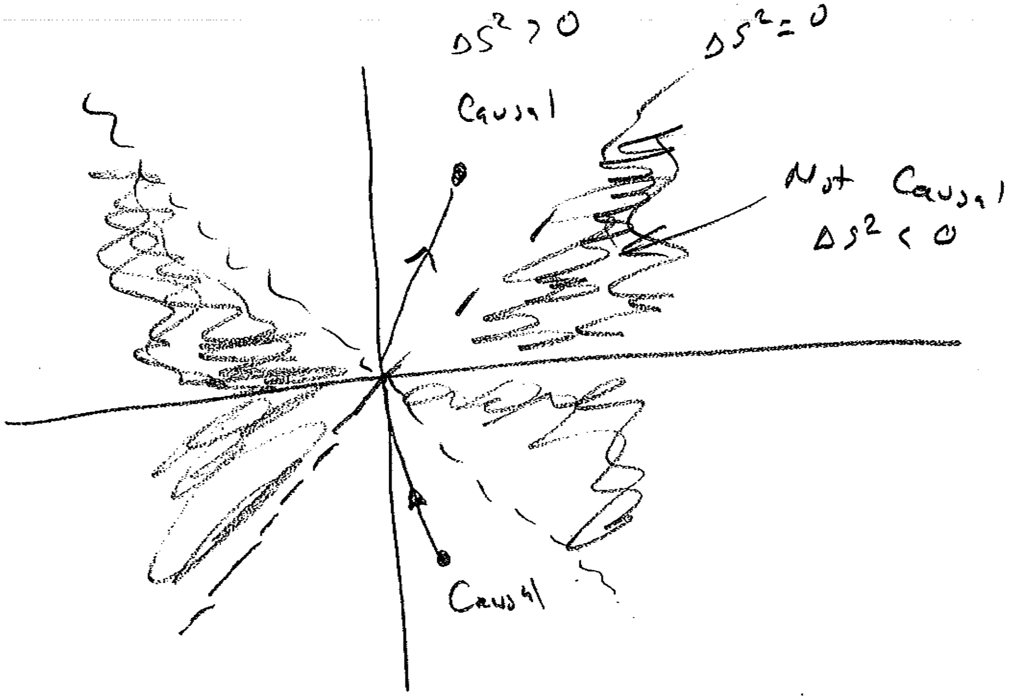
Must be frame invariant quantity

$$\Delta x_Q^2 < \Delta t_Q^2$$

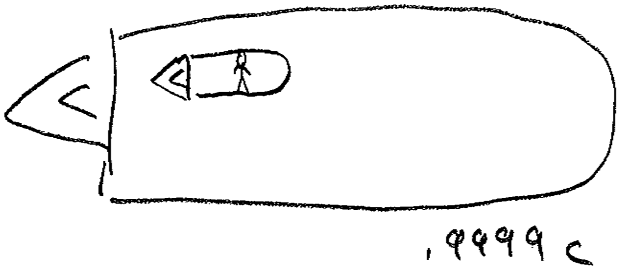
$$\Rightarrow \quad 0 < \Delta t_Q^2 - \Delta x_Q^2 = \Delta S_Q^2$$

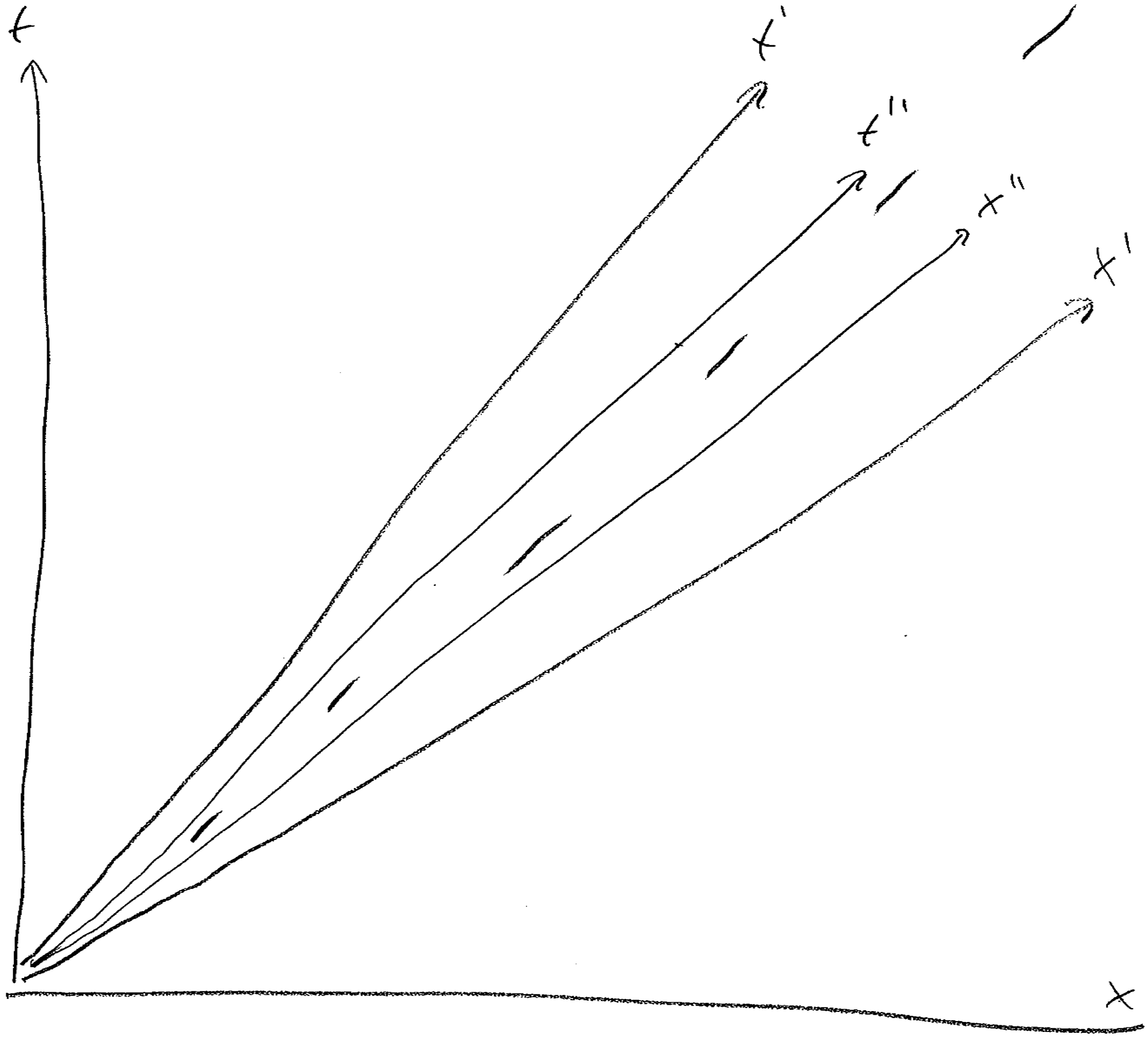
To be causally connected

$$0 < \Delta S^2 \qquad (\underline{\text{time-like}})$$


$\Delta s^2 > 0$
Causal
$\Delta s^2 = 0$
Not Causal
$\Delta s^2 < 0$
Causal

Maybe we can still go faster than "c"!





You can't fight it!

$$U_x \equiv \frac{dx}{dt} = \frac{\gamma\,(dx' + \beta\,dt')}{\gamma\,(dt' + \beta\,dx')}$$

$$= \frac{\beta + U_x'}{1 + \beta\,U_x'}$$

$$U_y = \frac{dy}{dt} = \frac{dy'}{\gamma(dt' + \beta\,dx')} = \frac{U_y'\sqrt{1-\beta^2}}{1 + \beta\,U_x'^{\,2}}$$

$$U_z = \frac{U_z'\sqrt{1-\beta^2}}{1 + \beta\,U_x'^{\,2}}$$

---

e.g., $\beta = 1-\delta \qquad U_x' = 1-\delta$

$$U_x = \frac{(1-\delta) + (1-\delta)}{1 + (1-\delta)(1-\delta)}$$

$$= \frac{2\,(1-\delta)}{1 + 1 - 2\delta + \delta^2} = \frac{2\,(1-\delta)}{2(1-\delta) + \delta^2} < 1$$

10

$B = ?$ $\quad U_x' = 1$

$$U_x = \frac{B + 1}{1 + B} = 1$$

R8S.4

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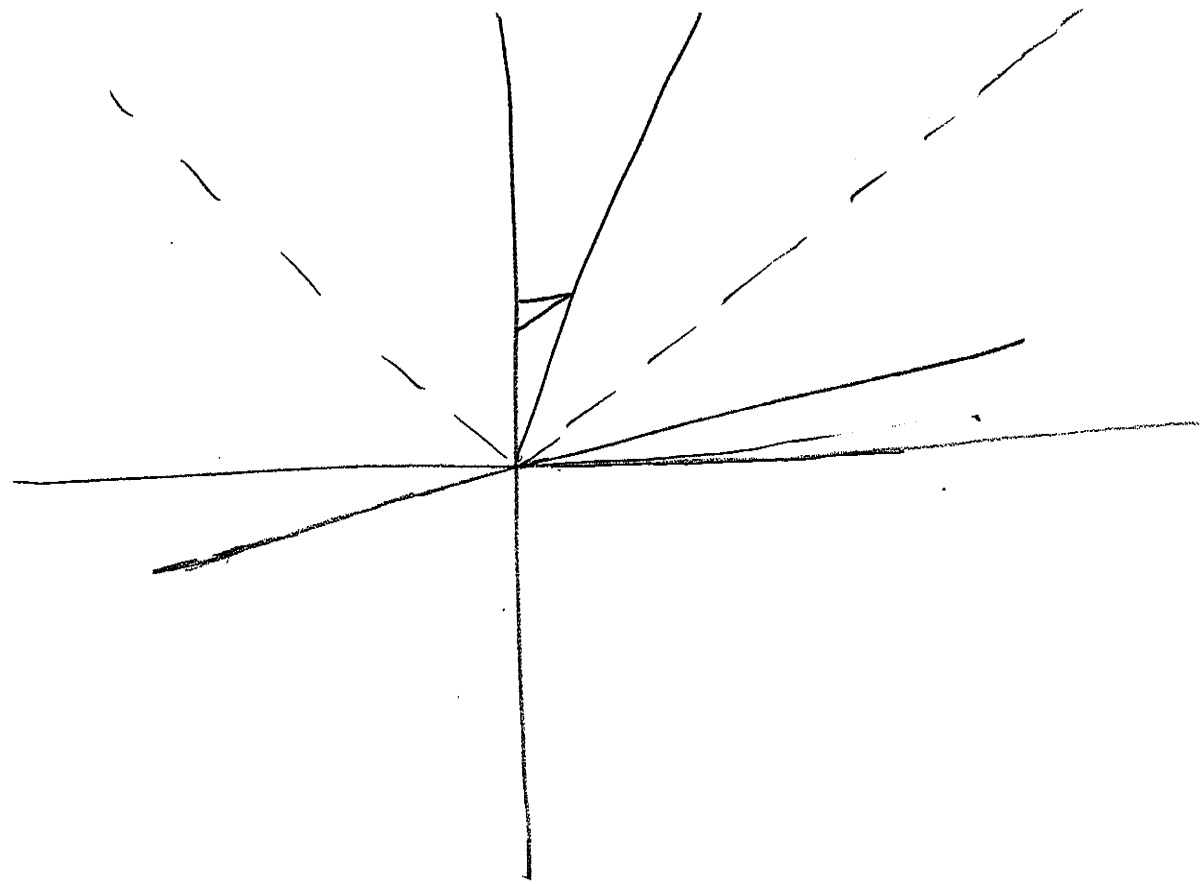
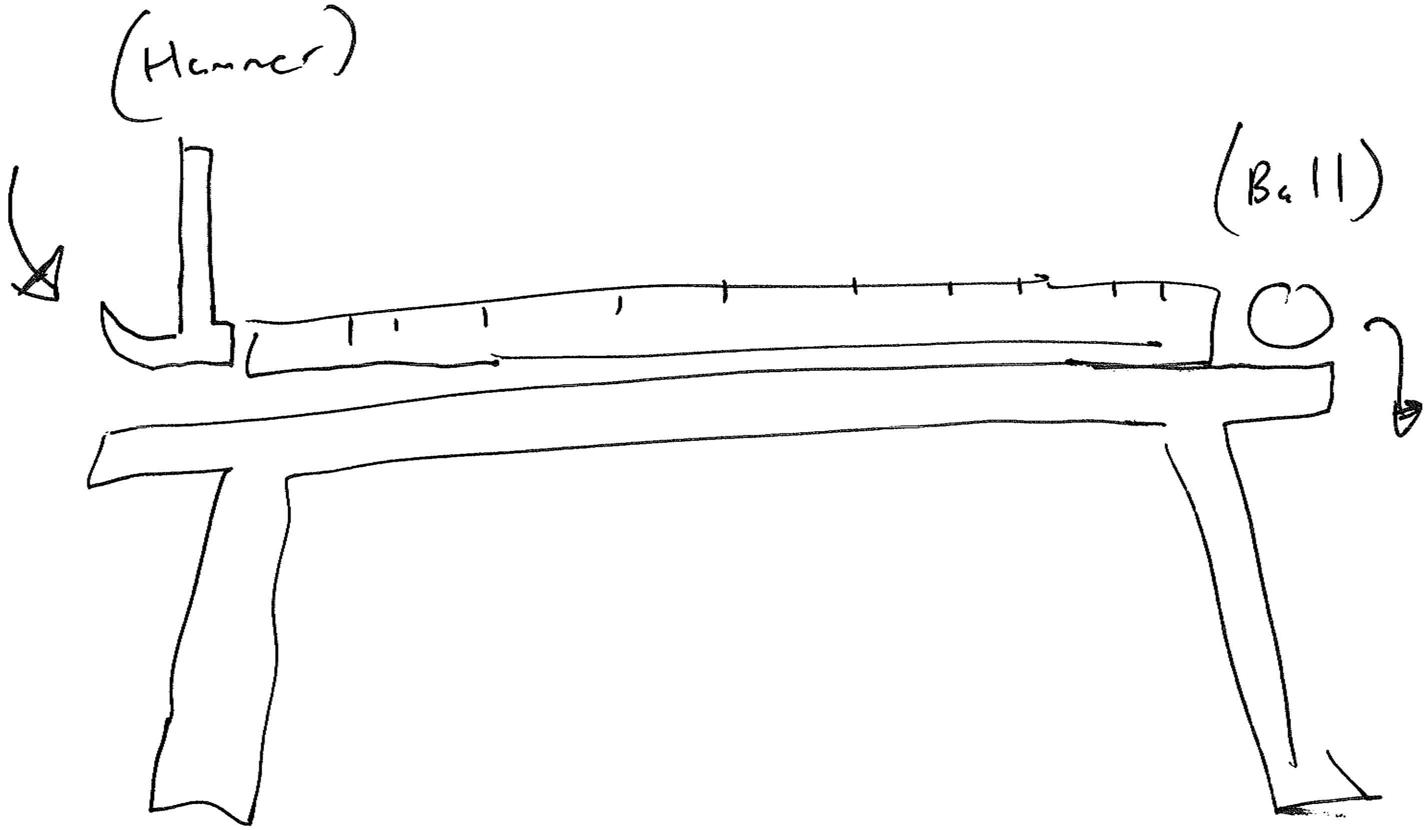


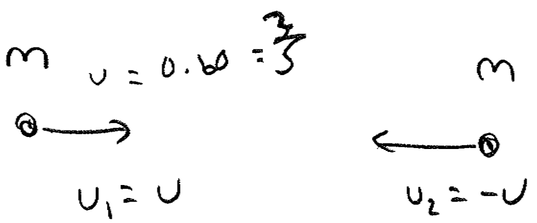
Near ∞ speed, follow the x' axis

(R8 Paradox Idea)



$\boxed{\text{R8S8 - R8S9}}$

$\beta = \frac{3}{5}$



$\beta = v$

$$u_1' = \frac{u - v}{1 - u_1 v} = 0 \qquad u_2' = \frac{-u - v}{1 - (-u)v} = -\frac{2v}{1 + v^2}$$

$\boxed{\text{Motivation for R9}}$

Lecture R9

1) Conservation of classical momentum is <u>not</u> a frame invariant law of nature <u>OR</u> S.R. is wrong.

Why?

$$(p_1 + p_2)' \neq p_1' + p_2'$$

Is same true for spatial four-vectors?

$$(x_1 + x_2)' \stackrel{?}{=} x_1' + x_2'$$

$$\begin{aligned}(x_1 + x_2)' &= \gamma\left((x_1 + x_2) - B(t_1 + t_2)\right) \\ &= \gamma(x_1 - Bt_1) + \gamma(x_2 - Bt_2) \\ &= x_1' + x_2' \qquad !!!\end{aligned}$$

(The sum of 4-vectors is itself a four vector.)

<u>Problem:</u> Classical momentum is <u>not</u> a four-vector

Why isn't p a four-vector???

$$p_x = m\frac{dx}{dt} = m\frac{(\gamma(dx' + \beta dt'))}{(\gamma(dt' + \beta dx'))}$$

p_x does not transform like x ...

<u>What if?</u>

$p_x = \square\, dx$ where $\square = \square'$

$$p_x' = \square'\, dx' = \square(\gamma(dx - \beta dt))$$
$$= \gamma(\underbrace{\square\, dx}_{p_x} - \beta\, \underbrace{\square\, dt}_{p_0})$$

So do we have something frame invariant like dt?

$$p_x = m\frac{dx}{d\tau}, \quad p_y = m\frac{dy}{d\tau}, \quad p_z = m\frac{dz}{d\tau}$$
$$p_0 = m\frac{dt}{d\tau}$$

$$p_x' = m\frac{dx'}{d\tau} = m\frac{\gamma(dx - \beta\, dt)}{d\tau}$$

$$= \gamma\left(m\frac{dx}{d\tau} - \beta\, m\frac{dt}{d\tau}\right)$$

$$= \gamma\left(p_x - \beta\, p_0\right) \quad !!!$$

$$p_y' = m\frac{dy'}{d\tau} = m\frac{dy}{d\tau} = p_y$$

$$p_z' = p_z$$

$$p_0' = m\frac{dt''}{d\tau} = m\frac{\gamma(dt - \beta\, dx)}{d\tau}$$

$$= \gamma\left(m\frac{dt}{d\tau} - \beta m\frac{dx}{d\tau}\right)$$

$$= \gamma\left(p_0 - \beta\, p_x\right) \quad !!!$$

$$p \equiv m\frac{d}{d\tau}(t, x, y, z)$$

Now

$$p = m \frac{d}{d\tau}(t, x, y, z)$$

is a 4-vector ...

Q: Does $(P_1 + P_2)' = P_1' + P_2'$?

Yes ...

Q: $\frac{dx}{d\tau}$, $\frac{dt}{d\tau}$, ... etc are not convenient...

not in terms of easily accessible frame dependent quantities we are familiar with.
How to calculate $d\tau$ in terms of frame-dep quantities?

$$d\tau = \sqrt{1 - v^2}\, dt = \frac{1}{\gamma} dt$$

$$\Rightarrow \quad p = m \frac{d}{d\tau}(x, y, z, t)$$

$$= \frac{m}{(1-v^2)^{1/2}} \frac{d}{dt}(x, y, z, t)$$

$$\boxed{p = \left(\frac{m v_x}{(1-v^2)^{1/2}}, \frac{m v_y}{(1-v^2)^{1/2}}, \frac{m v_z}{(1-v^2)^{1/2}}, \frac{m}{(1-v^2)^{1/2}} \right)}$$

Q: Why not write $\boxed{p = \gamma m v\ ?}$ $\quad \gamma \equiv \frac{1}{\sqrt{1-\beta^2}}$

γ_v ?

To recap:

What do we like about new definition of "p"?

1) It's a 4-vector, know how to transform it.

2) From (1), $(P_1 + P_2)' = P_1' + P_2'$

so conservation of momentum in one frame implies c.o.m. in all inertial frames

3) What if at low v, reproduces classical p?

Note: Haven't proven that relativistic momentum as defined

At low v, $v \ll 1$:

$$p_x = \frac{m v_x}{\sqrt{1 - v^2}}$$

$$\frac{1}{\sqrt{1-v^2}} \sim 1 + \frac{1}{2} v^2$$

$$p_x = m v_x + \mathcal{O}(v^3)$$

Nice! At low v, approaches classical definition.

Q: We have lots of low v data showing conservation of classical momentum. What does this tell us about conservation of relativist momentum at <u>low</u> <u>speeds</u>?

$$P_0 = \frac{m}{\sqrt{1-u^2}}$$

$$= \underbrace{m}_{} + \underbrace{\tfrac{1}{2} m u^2}_{\text{Kinetic Energy}} + \cdots$$

? $\{ \boxed{E = mc^2}$, rest energy, ... $\}$

$$E \equiv \frac{m}{\sqrt{1-u^2}} \sim m + \frac{1}{2} m u^2 + \cdots$$

$$K \equiv E - m = \left(\frac{1}{\sqrt{1-u^2}} - 1 \right) m$$

$$\sim \frac{1}{2} m u^2$$

(In classical mechanics mass is conserved, so adding $m + E_{classical}$ is immaterial.)

<u>Q:</u> Why is this identification $P_0 = E$ a big deal?

RELATIVITY UNIFIES CONSERVATION OF MOMENTUM AND ENERGY

Important Trick:

Don't parameterize in β, γ ...

Use E, and p, then calculate

$$\beta = E/p, \quad \gamma = E/m, \quad \ldots$$

Energy Units

$$\boxed{1\ \text{GeV} = 1.6\times10^{-10}\ \text{J} = 1.8\times10^{-27}\ \text{kg}}$$

Why GeV? Mass of proton : 0.938 GeV

$$(E, P_x, P_y, P_z) = (3, 2, 0, 0)\ \text{GeV}$$

What is $\beta_x = 2/3$

$$m^2 = 3^2 - 2^2 = 9 - 4 = 5$$

$$m = \sqrt{5}\ \text{GeV}$$

ANSWER FORMATS

No need for calculator...

Leave your answer in form of proper fraction:

$$\frac{2\sqrt{3}}{3}, \quad \frac{5}{7}, \quad \sqrt{5}$$

Recap:

$$[E, p_x, p_y, p_z] = m\left[\frac{dt}{d\tau}, \frac{dx}{d\tau}, \frac{dy}{d\tau}, \frac{dz}{d\tau}\right]$$

$$= \frac{m}{\sqrt{1-v^2}}[1, v_x, v_y, v_z]$$

For low momentum:

$$\vec{p} \to m\vec{v}$$

$$E \to \underbrace{m}_{\text{Rest Energy}} + \underbrace{\tfrac{1}{2}mv^2}_{\text{Newtonian Kinetic}} + \underbrace{\cdots}_{\text{Relativistic H.O.T.}}$$

$K \equiv E - m$ (Kinetic Energy)

Tools:

$$\boxed{m^2 = E^2 - p^2} \sim \Delta s^2 = \Delta t^2 - \Delta x^2$$

$$\frac{1}{\sqrt{1-v^2}} = E/m$$

$$v_x = p_x/E, \quad v_y = p_y/E, \quad v_z = p_z/E$$

Trick: work in E, p, m (need 2)

NOT in $v, m,$ (need 2)

Mass of System of Particles

Q: What is mass of combined system?

$2m$?

$p_1 = \left(\sqrt{m^2+p^2},\ p\right)$ $\qquad p_2 = (m,\ 0)$

$p_1 + p_2 = \left(m + \sqrt{m^2+p^2},\ p\right)$

$(p_1 + p_2)^2 = \left(m + \sqrt{m^2+p^2}\right)^2 - p^2$

$m^2 + 2m\sqrt{m^2+p^2} + m^2 + p^2 - p^2$

$= 2m^2 + 2m\sqrt{m+p}$

$M^2 = 2m\left(m + \sqrt{m^2+p^2}\right)$

$p = 2 \qquad M = 2m$

Light

Arg 1

photons travel at speed of light ... $v = 1$

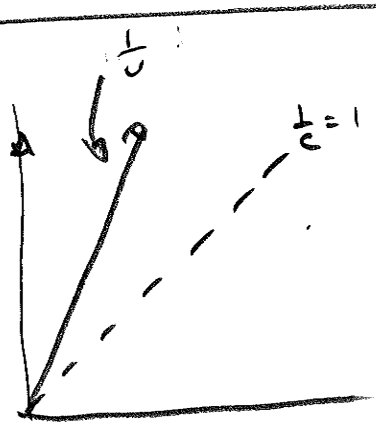
$$P = \frac{1}{\sqrt{1-v^2}}\, m v \qquad E = \frac{1}{\sqrt{1-v^2}}\, m$$

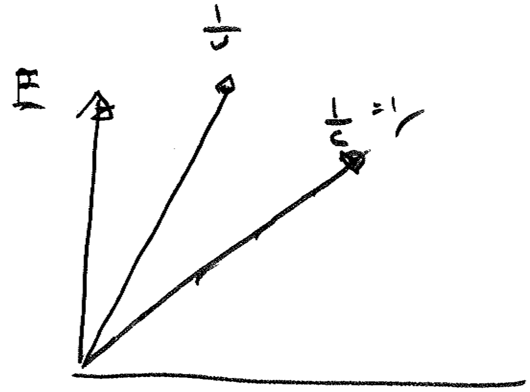
As $v \to 1$ $\qquad p \to E$

If m were positive, $\quad p \to E \to \infty$

But $p \to E$ suggests $\quad m^2 = p^2 - E^2 \Rightarrow 0$

Arg 2





Light has $\Delta s^2 = \Delta t^2 - \Delta x^2 = 0$ <u>AND</u> $\Delta m^2 = \Delta E^2 - \Delta p^2 = 0$

<u>For</u> <u>light</u> $\boxed{p = E}$

Relativistic Mass

$$\Delta s^2 = \Delta t^2 - \Delta x^2 \qquad \text{(frame invariant)}$$

Since momentum is a 4-vector, this quantity

$$E^2 - p^2$$

must also be frame invariant ...

Use $E = \dfrac{m}{\sqrt{1-v^2}}$ $\quad p_x = \dfrac{m v_x}{\sqrt{1-v^2}}$, $p_y = \dfrac{m v_y}{\sqrt{1-v^2}}$, ...

$$E^2 = \frac{m^2}{1-v^2} \qquad p^2 = \frac{m^2 \left(v_x^2 + v_y^2 + v_z^2\right)}{1-v^2}$$

$$= \frac{m^2 v^2}{1-v^2}$$

$$\boxed{E^2 - p^2 = m^2}$$

R10 S1

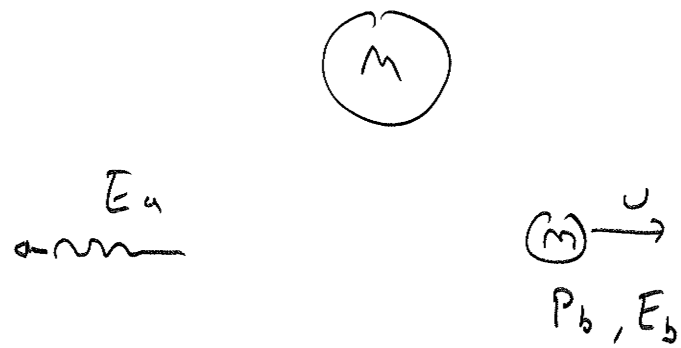
$$(m, 0, 0, 0) = (E_a, p_a, 0, 0) + (E_b, p_b, 0, 0)$$

$$p_a = -p_b \quad , \quad E_a = E_b$$

$$m = E_a + E_b = 2E_a = 2\gamma m_a$$

$$\boxed{m_a = \frac{1}{2} m \sqrt{1-v^2}}$$

R1056 *



$$P_b = \gamma m_b u$$
$$E_b = m_b$$

$$(M, 0, 0, 0) = (E_b, P_b, 0, 0) + (E_a, -E_a, 0, 0)$$

$$M = E_b + E_a$$
$$0 = P_b - E_a$$

① $$\boxed{M = E_b + P_b}$$

② $$\boxed{M = \frac{1}{\sqrt{1-u^2}} m (1 + u)}$$

$$(E, p, 0, 0) = (E_a, -E_a, 0, 0) + (E_b, +E_b, 0, 0)$$

$$E = E_a + E_b$$

$$p = E_b - E_a$$

$$E^2 - p^2 = (E_a + E_b)^2 - (E_b - E_a)^2$$

$$= E_a^2 + 2E_aE_b + E_b^2 - (E_a^2 - 2E_aE_b + E_b^2)$$

$$\boxed{E^2 - p^2 = 4E_aE_b}$$

$\longrightarrow$ $\boxed{\text{Higgs Riff}}$