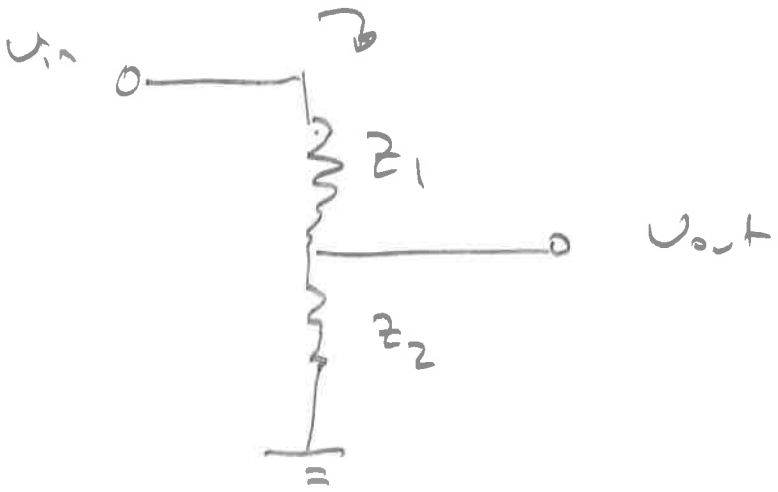
RLC

Filters

(Not used in 2017)

Voltage Dividers



$$I = \frac{V_{in}}{Z_1 + Z_2}$$

$$V_{out} = Z_2 I = \frac{Z_2}{Z_1 + Z_2} V_{in}$$

e.g. $Z_1 = Z_2 = R$

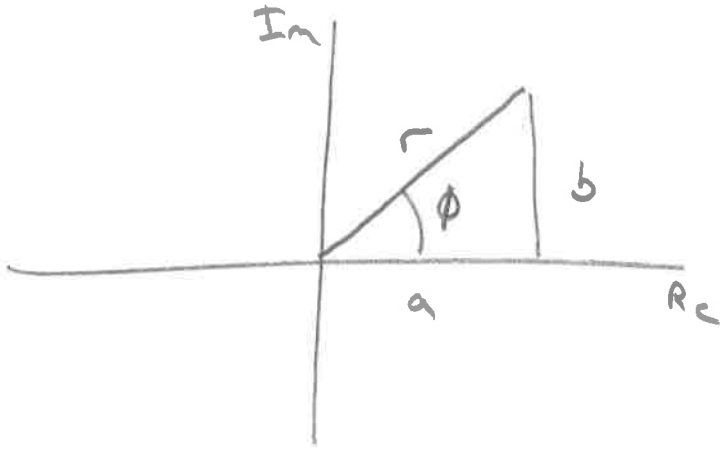
$$V_{out} = \frac{R}{R+R} V_{in} = \frac{1}{2} V_{in}$$

Complex Numbers and Phase

$$z = a + bi = re^{i\phi}$$

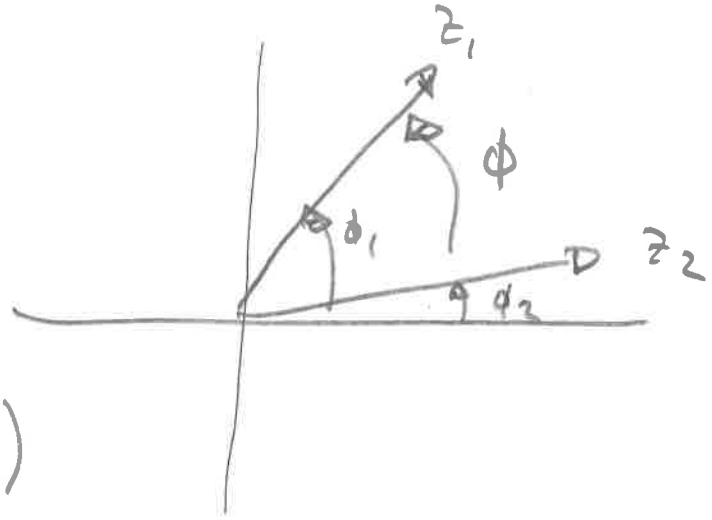
$$\tan\phi = \frac{b}{a}$$

$$r^2 = a^2 + b^2 = |z|^2$$



$$\frac{z_1}{z_2} = \frac{r_1 e^{i\phi_1}}{r_2 e^{i\phi_2}}$$

$$= \frac{r_1}{r_2} e^{i(\phi_1 - \phi_2)}$$



$$\frac{z_1}{z_2} = \left|\frac{z_1}{z_2}\right| e^{i(\phi_1 - \phi_2)}$$

$$\phi\left(\frac{z_1}{z_2}\right) = \phi_1 - \phi_2$$

High - Pass Filter

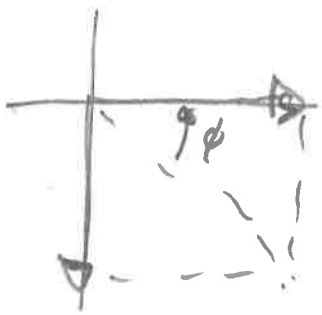
$$\frac{V_{out}}{V_{in}} = \frac{R}{R - j\frac{1}{\omega C}}$$

$\omega \ll \omega_0$

$\phi \to 90$

as $\omega \ll \omega_0$

$\omega = \omega_0$



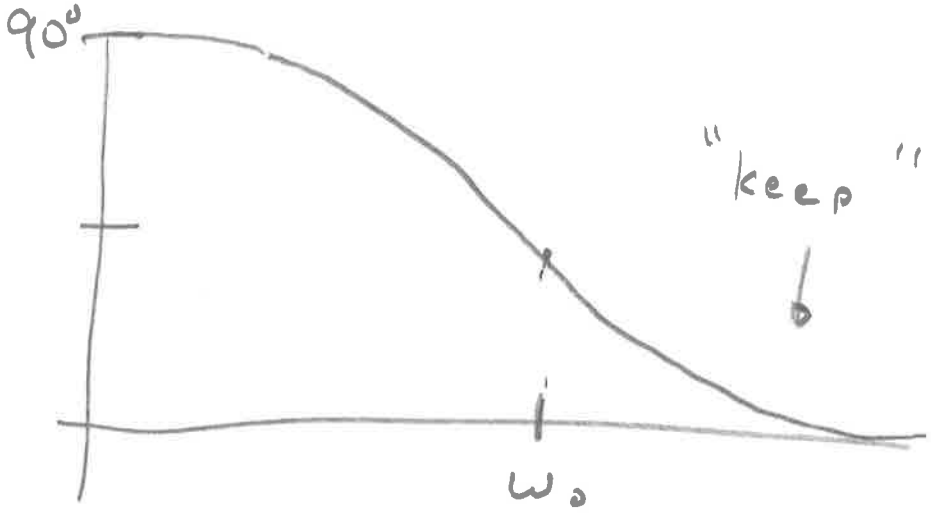
$\phi = 45°$

at $\omega = \omega_0$

$\omega \gg \omega_0$

$\phi \to 0$

as $\omega \gg \omega_0$



More precisely:

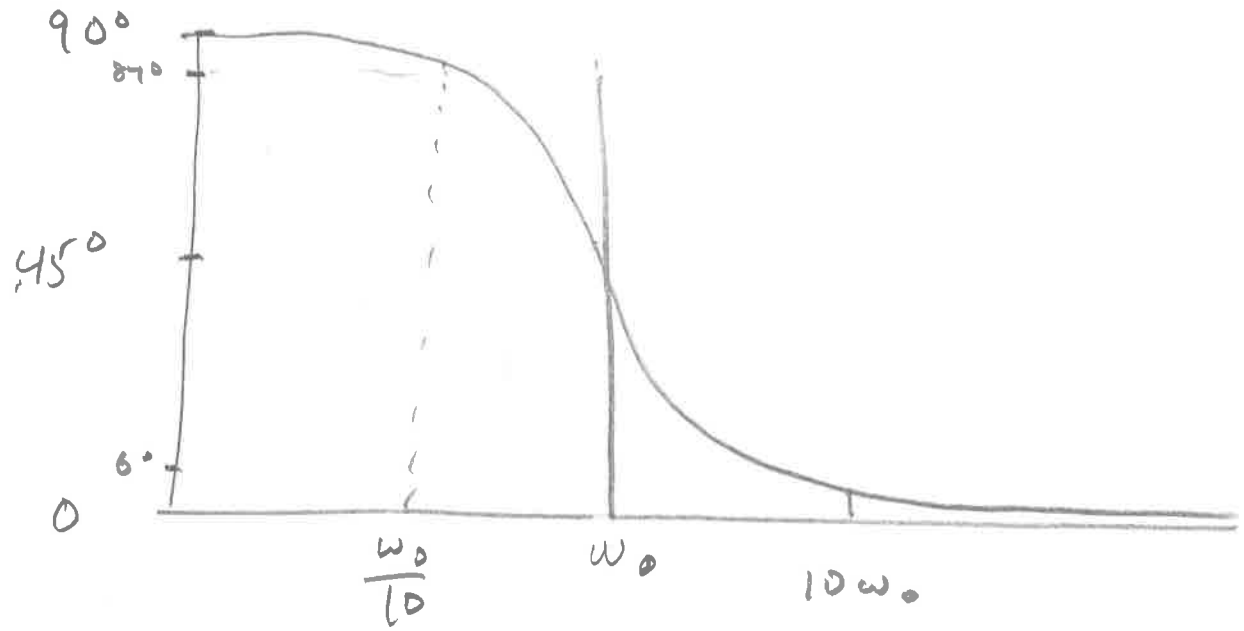
$$\frac{U_{out}}{U_{in}} = \frac{1}{1 - j\frac{\omega_0}{\omega}} = C \cdot 1 + j\frac{\omega_0}{\omega}$$

$$\phi\left(\frac{U_{out}}{U_{in}}\right) = \tan^{-1}\left(\frac{\omega_0}{\omega}\right)$$

$$\tan^{-1}(1) = 45^\circ$$

$$\tan^{-1}\left(\frac{1}{10}\right) \sim 6^\circ$$

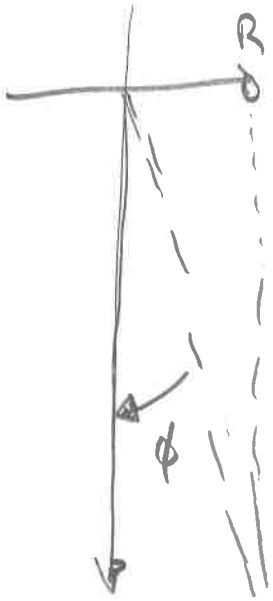
$$\tan^{-1}(10) \sim 90^\circ - 6^\circ$$

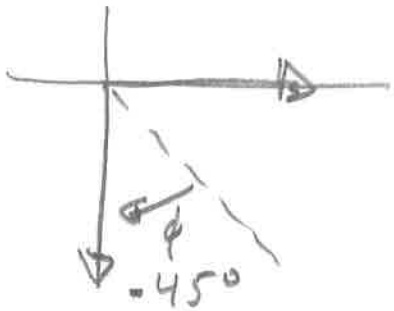


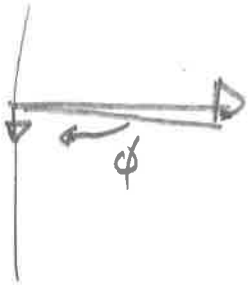
Low-Pass Filter

$$\frac{V_{out}}{V_{in}} = \frac{-j\frac{1}{\omega C}}{R - j\frac{1}{\omega C}}$$

$\omega \ll \omega_0$



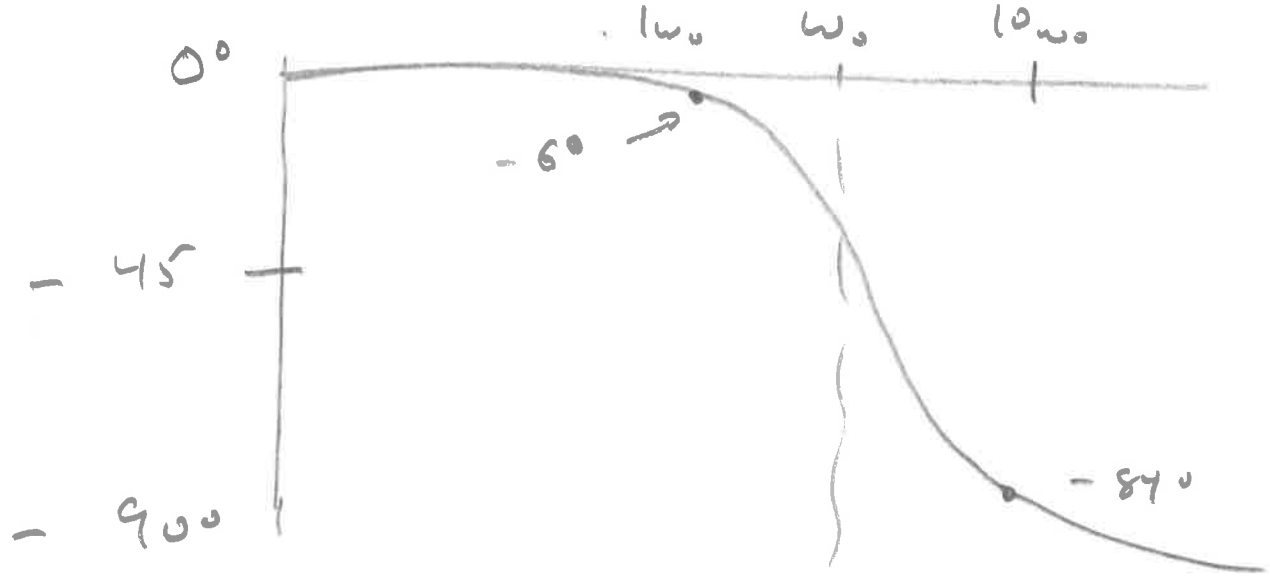




$\phi \to 0$ as $\omega \ll \omega_0$

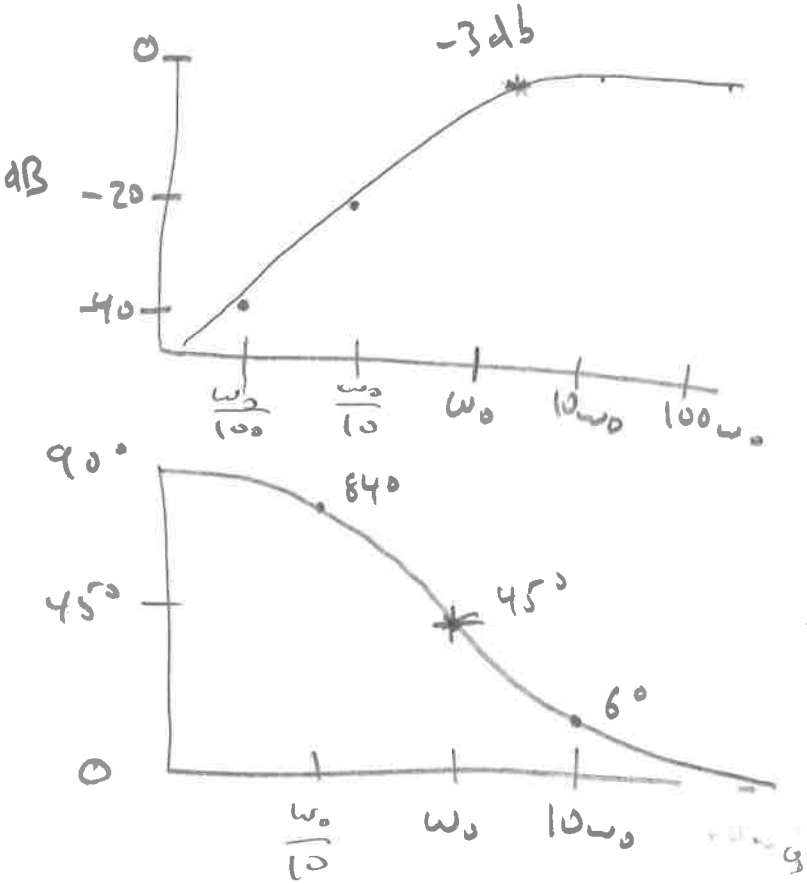
$\phi \to -45°$ at $\omega = \omega_0$

$\phi \to -90°$ as $\omega \gg \omega_0$

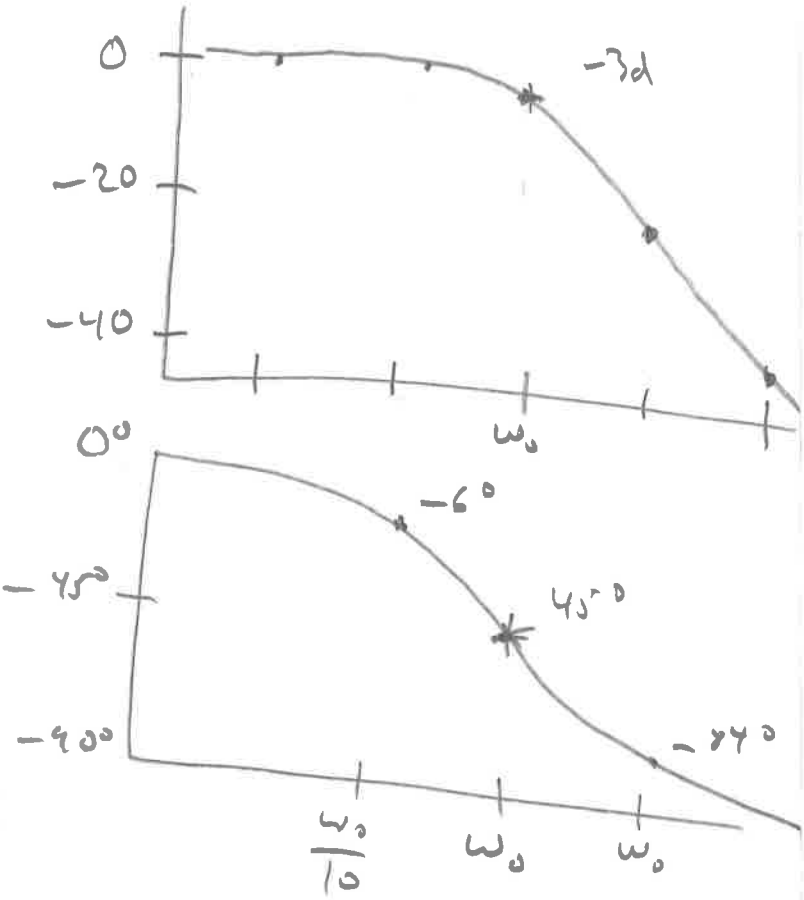


Summary

High Pass



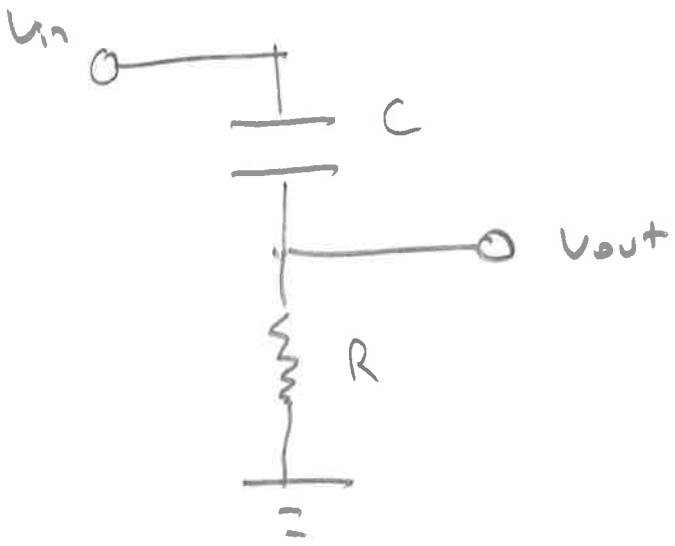
Low - Pass



(Bode Plots)

" Bah - dee "

High Pass

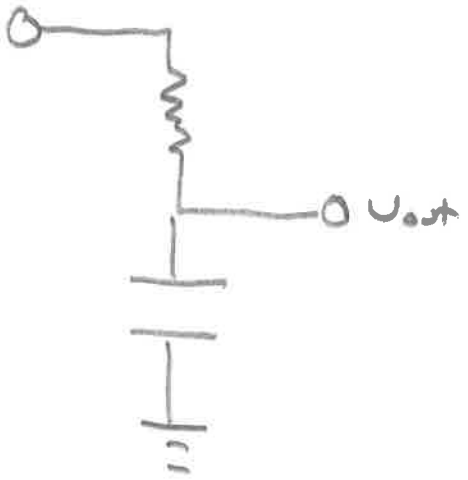


$$Z_1 = \frac{1}{j\omega C} \qquad Z_2 = R$$

$$V_{out} = \frac{R}{R + \frac{1}{j\omega C}} V_{in}$$

Changes Magnitude + Phase

Low - Pass Filter



$$Z_2 = \frac{1}{j\omega C} \qquad Z_1 = R$$

$$U_{out} = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} U_{in}$$

Magnitude of High-Pass

$$\frac{V_{out}}{V_{in}} = \frac{R}{R + \frac{1}{j\omega C}} = \frac{1}{1 - j\,\omega_0/\omega}$$

$$\omega_0 = \frac{1}{RC}$$

$$\left|\frac{V_{out}}{V_{in}}\right|^2 = \frac{1}{1 - j\,\omega_0/\omega} \quad \frac{1}{1 + j\,\omega_0/\omega}$$

$$= \frac{1}{1 + (\omega_0/\omega)^2}$$

$$dB \equiv 10 \log_{10}\left(\left|\frac{V_{out}}{V_{in}}\right|^2\right)$$

$$= -10 \log_{10}\left[1 + (\omega_0/\omega)^2\right]$$