

Homework Assignment 1

Review

Practice Problems

(No Practice Problems)

Additional Problems

Problem 1: You have already covered¹ linear algebra in PHY 104A. Consider the real symmetric matrix:

$$A = \begin{pmatrix} 1 & 3 \\ 3 & 1 \end{pmatrix}$$

(a) Find the eigenvalues of the matrix by finding the roots of the characteristic polynomial:

$$\det(A - \lambda I) = 0$$

(b) Find the two eigenvectors \mathbf{u} and \mathbf{v} , and normalize them so that:

$$\mathbf{u} \cdot \mathbf{u} = \mathbf{v} \cdot \mathbf{v} = 1$$

(c) Note that $A = A^T$. Show that the two eigenvectors are orthogonal, i.e.:

$$\mathbf{u} \cdot \mathbf{v} = 0$$

(d) For the vector:

$$x = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

Find the values a and b such that:

$$x = a\mathbf{u} + b\mathbf{v}$$

Next problem on next page...

¹See Chapter 2, Section 11 of **Mathematical Methods in the Physical Sciences (Third Edition)** by M.L. Boas if you need a review.

Problem 2: Show that:

$$\begin{aligned}\frac{2}{L} \int_{-\frac{L}{2}}^{\frac{L}{2}} \sin\left(\frac{2\pi n}{L} x\right) \sin\left(\frac{2\pi m}{L} x\right) dx &= \delta_{nm} \\ \frac{2}{L} \int_{-\frac{L}{2}}^{\frac{L}{2}} \cos\left(\frac{2\pi n}{L} x\right) \cos\left(\frac{2\pi m}{L} x\right) dx &= \delta_{nm} \\ \frac{2}{L} \int_{-\frac{L}{2}}^{\frac{L}{2}} \sin\left(\frac{2\pi n}{L} x\right) \cos\left(\frac{2\pi m}{L} x\right) dx &= 0\end{aligned}$$

for integers n and m where:

$$\delta_{nm} = \begin{cases} 1 & \text{if } n = m \\ 0 & \text{otherwise} \end{cases}$$

You may use the trigonometric identities:

$$\begin{aligned}\sin \alpha \sin \beta &= \frac{1}{2} \{ \cos(\alpha - \beta) - \cos(\alpha + \beta) \} \\ \cos \alpha \cos \beta &= \frac{1}{2} \{ \cos(\alpha - \beta) + \cos(\alpha + \beta) \} \\ \cos \alpha \sin \beta &= \frac{1}{2} \{ \sin(\alpha + \beta) - \sin(\alpha - \beta) \}.\end{aligned}$$