Homework Assignment 4
Not-So-Simple Harmonic Oscillator

Practice Problems

These problems are graded on effort only.

Griffiths: P2.4, P2.7, P2.8

Additional Problems

Problem 1: Show that:

$$\frac{d}{dt}\langle \hat{x}\hat{p}\rangle = 2\langle \hat{T}\rangle - \langle x\frac{dV}{dx}\rangle$$

where

$$\hat{T} = \frac{\hat{p}^2}{2m}$$

is the kinetic energy. This is the quantum mechanical version of the virial theorem of classical mechanics. Hint: look at the technique we used to derive Ehrenfest's Theorem in the chapter one lecture notes.

Problem 2: For the nth stationary state of the harmonic oscillator, use the operator method of Griffith's Example 2.5 (p. 47) to:

(A) Show that $\langle x\rangle = \langle p\rangle = 0$.

(B) Calculate $\langle x^2\rangle$ and $\langle p^2\rangle$, and show that uncertainty principle is satisfied.

(C) Calculate $\langle x^7\rangle$. Hint: think before you calculate. What is the condition for a term to be non-zero?

Problem 3: Consider the Hermite Polynomials $H_n(u)$ that are part of the nth solution to the harmonic oscillator problem:

$$\psi_n(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} \frac{1}{\sqrt{2^n n!}} H_n(u) e^{-u^2/2}$$

(A) In lecture, we determined a recursion relation:

$$a_{m+2} = \frac{-2(n-m)}{(m+2)(m+1)}\, a_m$$

for both the even and odd power series solutions:

$$\begin{aligned} h_{\text{even}}(u) &= a_0 + a_2 u^2 + a_4 u^4 + \dots \\ h_{\text{odd}}(u) &= a_1 u + a_3 u^3 + a_5 u^5 + \dots \end{aligned}$$

Determine the terminating power series solutions for $n = 0, 1, 2, 3, 4$.

(B) Suppose at some n you have an even solution $h_{\text{even}}(u)$ which terminates after coefficient a_n. What will happen if you attempt to create an odd solution for n?

(C) The generating function:

$$\exp(-z^2 + 2zu) = \sum_{n=0}^{\infty} \frac{z^n}{n!} H_n(u)$$

allows us to read off the Hermite Polynomial from its Taylor Series expansion. Use this technique to determine $H_0(u)$,$H_1(u)$,$H_2(u)$,$H_3(u)$, and $H_4(u)$. Compare to your answers in part (A), and note that you are free to multiply your solutions by an arbitrary normalization factor.

(D) For some n, what is the term with the highest power of n in $H_n(u)$?

Problem 4: In lecture we defined the commutator of two operators $\hat{A}$ and $\hat{B}$ as:

$$[\hat{A}, \hat{B}] = \hat{A}\hat{B} - \hat{B}\hat{A}$$

.

(A) Show that:

$$[\hat{A}\hat{B}, \hat{C}] = \hat{A}[\hat{B}, \hat{C}] + [\hat{A}, \hat{C}]\hat{B}$$

(B) Prove the Jacobi Identity:

$$[\hat{A}, [\hat{B}, \hat{C}]] + [\hat{B}, [\hat{C}, \hat{A}]] + [\hat{C}, [\hat{A}, \hat{B}]] = 0$$

(This is the property of the algebra of commutators that replaces associativity)