Homework Assignment 8
“What a long strange trips it’s been”

Practice Problems

These problems are graded on effort only.

Problem 1: Reproduce the derivation of generalized Ehrenfest theorem (Eq. 3.73).

Griffiths: P3.18, P3.19, P3.25, P3.26

Additional Problems

Problem 2: The Parseval-Plancherel identity:

$$\int_{-\infty}^{+\infty} |\Psi(x)|^2 \, dx \;=\; \int_{-\infty}^{+\infty} |\widetilde{\Psi}(p)|^2 \, dp$$

shows that if a position-space wave function $\Psi(x)$ is normalized, then so is it’s momentum-space wave function $\widetilde{\Psi}(p)$.

(A) Derive the Parseval-Plancherel identity by explicit integration. **Hint:** start with the LHS and write out $\Psi(x)$ and $\Psi^*(x)$ in terms of their Fourier transforms. Make sure you use two different dummy variables for the integral over momentum (e.g. dq and dp) . At this point, you should have **three** integrals: dx,dp, and dq. Look carefully and note that only two factors involve x. Move the integral dx to include just those factors, and use the orthogonality of the complex exponentials:

$$\frac{1}{2\pi\hbar} \int_{-\infty}^{+\infty} \exp((q-p)x/\hbar) \, dx = \delta(q-p)$$

This kills the integral over dx, then use the resulting δ-function to kill the integral of dq.

(B) Derive Parseval-Plancherel identity using the Dirac notation. Start with:

$$\langle \Psi | \Psi \rangle$$

then insert the identity:

$$\int dx \, |x\rangle \langle x| = 1$$

and identify:

$$\Psi(x) = \langle x | \Psi \rangle$$

Next, do the same thing but using the identity:

$$\int dp \, |p\rangle \langle p| = 1$$

and identify:

$$\widetilde{\Psi}(p) = \langle p|\Psi\rangle$$