Plan For EAS Tagger

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November 20, 2025

1 Preliminaries: Differential Intensity of Extensive Air Showers

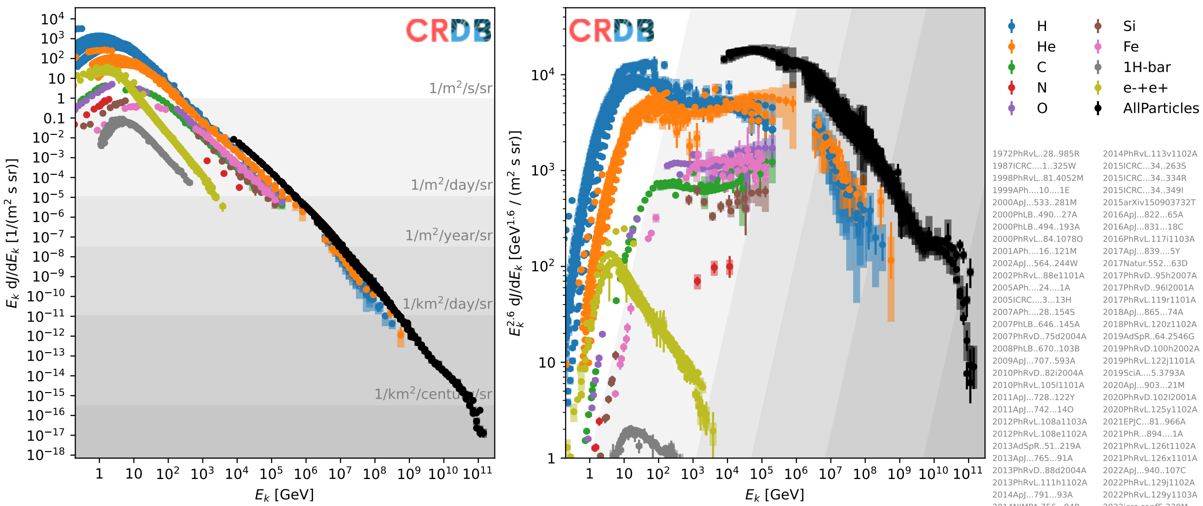

Figure 1: Incident differential intensity, from the CRDB.

The differential intensity J of cosmic rays incident at the top of the atmosphere (TOA) is defined as

$$J = \frac{dN}{dE\,dA\,dt\,d\Omega}$$

where E is the energy of the incident primary and t is time. The quantities $d\Omega$ and dA are the infinitesimal solid angle, relative to normal, and the infinitesimal area, both **at the TOA**. The intensity has units:

$$[J] = \left[\frac{1}{\epsilon \text{ m}^2 \text{ sr s}}\right]$$

where ϵ is some energy scale, e.g. 1 GeV, and sr is a dimensionless reminder to integrate over solid angle. The incident differential intensity at the top of the atmosphere is independent of direction and follows a power law:

$$J(E) = \alpha \left(\frac{E}{\epsilon}\right)^{-\gamma} \tag{1}$$

with $\gamma \sim 3$. The power law is only approximate. Our results below include an estimate for a minimum energy, and we expect only a narrow energy region above this minimum will contribute significantly. The parameters for the power law should therefore be taken for that energy range.

The differential intensity as tabulated by the Cosmic Ray Database (CRDB) is shown in Fig. 1. When plotting a power law of form:

$$\frac{dy}{dx} = \alpha x^{-\gamma}$$

is often preferable to use a log-log scale, so that the power law becomes the linear relationship:

$$\log\left(\frac{dy}{dx}\right) = -\gamma \log(x) + \log(\alpha)$$

It also useful to set the y-axis to:

$$x\frac{dy}{dx} = \frac{dy}{d(\log(x))}$$

which removes any dependence[1] on the units of x, and is appropriate for binning the x parameters on a log scale. Furthermore, for a decreasing power lower, a calculation shows:

$$\int_a^\infty \frac{dy}{dx}dx = \frac{1}{\gamma - 1}\, x\frac{dy}{dx}\bigg|_{x=a}$$

so the y-axis is proportional to the integrated rate. Returning to Fig. 1, note that indeed:

$$10^{-5} \cdot 24 \cdot 60 \cdot 60 = 0.86$$

is indeed nearly 1 muon per day.

Next we are interested in the flux of incident cosmic ray particles when projected (without any attenuation or scattering, this is just for accounting) to the surface of the earth. We first approximate the earth as an infinite plane (appropriate for our thin atmosphere, as we shall see) and calculate the angle-integrade differential flux as: simply:

$$F = \int J \cos\theta d\Omega = 2\pi J \int_0^{\pi/2} \cos\theta\, \sin\theta\, d\theta$$

where we integrate only over the upper hemisphere, have included a factor of $\cos\theta$ for a horizontal surface, and used the fact that J is independent of direction. Computing the integral we find that:

$$F = \pi J \tag{2}$$

For an atmosphere of height h and earths radius R and impact parameter b, at TOA we have incident angle α and:

$$\sin\alpha = \frac{b}{R+h}$$

and at ground we have incident nalge $beta$ and:

$$\sin\beta = \frac{b}{R}$$

so that:

$$(R+h)\sin\alpha = R\sin\beta \tag{3}$$

It is left as an exercise to compute the Jacobian:

$$\frac{d\Omega_\alpha}{d\Omega_\beta} = \left(\frac{R}{R+h}\right)^2 \frac{\cos\beta}{\sqrt{1 - \left(\frac{R}{R+h}\right)^2 \sin^2\beta}}$$

and show that the resulting incident flux per unit horizontal area (now accounting for earth's shape) is:

$$F = 0.92\,\pi\, J$$

for TOA at 100 km. Note that α and β will be recycled as parameters after this exercise!

[1] Notice that $d(log(x/X)) = d(log(x))$ for any constant X.

2 Local Muon Flux from Extensive Air Showers

A critical consideration for the feasibility of studying extensive air showers in the lab, using a local network of cell phones, is the rate at which the muon flux in the lab is detectable by the phone network. A typical phone has an area times efficiency of

$$A_{\text{eff}} = 2 \times 10^{-5}\ \text{m}^2$$

We define a detectable muon flux Φ as the density at which a network of N_{P} phones can be expected to register at least one hit. So for $N_{\text{P}} = 1000$, we have:

$$\Phi_{\text{D}} = \frac{1}{A_{\text{eff}}\ N_{\text{P}}} = 50\ \text{muons/m}^2 \tag{4}$$

We saw in the preceding section that the incident flux of cosmic rays follows a power law:

$$J(E) = \alpha \left(\frac{E}{\epsilon}\right)^{-\gamma}$$

and the angle-integrated differential flux:

$$F(E) = 0.92\ \pi\ J(E)$$

where we can set 0.92 to one if we want to approximate the earth as an infinite plane. The number of muons N produced by a shower of energy E also follows a power law:

$$N(E) = \beta \left(\frac{E}{\epsilon}\right)^{\eta} \tag{5}$$

Suppose that these N muons have lateral profile give by:

$$\Phi(r) = \Phi(0) \exp(-r/\lambda)$$

where λ is the length scale of the muon lateral profile. The condition:

$$\int_0^\infty \Phi(r)\, 2\pi r\, dr = N$$

implies that:

$$\Phi(0) = \frac{N(E)}{2\pi\lambda^2} \tag{6}$$

and

$$\Phi(r) = \frac{N(E)}{2\pi\lambda^2}\ \exp(-r/\lambda) \tag{7}$$

The radius at which the flux Φ drops to Φ_{D} is:

$$r = -\lambda \ln\left(\frac{2\pi\lambda^2\Phi_{\text{D}}}{N}\right)$$

which means the area A_{D} over which there is a detectable muon flux is give by:

$$A_{\text{D}}(E) = \pi\lambda^2\ \left(\ln \frac{2\pi\lambda^2\Phi_{\text{D}}}{N(E)}\right)^2$$

which is only valid ($A_{rmD} > 0$) when:

$$\Phi(0) = \frac{N(E)}{2\pi\lambda^2} > \Phi_D$$

which sets a minimum energy, as we shall see. Using the power law, note that:

$$\frac{N(E)}{\Phi_{\rm D}\, 2\pi\lambda^2} = \frac{\beta}{\Phi_{\rm D}\, 2\pi\lambda^2}\left(\frac{E}{\epsilon}\right)^{\eta} = \left(\frac{E}{\kappa\epsilon}\right)^{\eta}$$

where

$$\kappa = \left(\frac{\Phi_{\rm D} 2\pi\lambda^2}{\beta}\right)^{1/\eta} \tag{8}$$

and so:

$$A_{\rm D}(E) = \pi\lambda^2\eta^2 \left(\ln\frac{E}{\kappa\epsilon}\right)^2 \tag{9}$$

which is only valid for $E > \kappa\epsilon$ (which you can verify is equivalent to requiring that $\Phi(0) > \Phi_{\rm D}$).

We are interested in the quanity:

$$\frac{dN}{dEdt} = F(E)A_{\rm D}(E) = (0.92)\pi J(E)A_{\rm D}(E) \tag{10}$$

Plugging in the results below we obtain our main result:

$$\frac{dN}{dE\,dt} = \alpha\,(\pi\lambda\eta)^2 \left(\frac{E}{\epsilon}\right)^{\eta-\gamma}\left(\ln\frac{E}{\kappa\epsilon}\right)^2 \tag{11}$$

which we need only integrate to calculate the rate at which a point on the surface of the earth has a local muon flux exceeding Φ_D. Defining:

$$f(\kappa) = \int_\kappa^\infty x^{\eta-\gamma}\left(\ln\frac{x}{\kappa}\right)^2 dx \tag{12}$$

we can write the rate as:

$$\frac{dN}{dt} = \alpha\epsilon\,(\pi\lambda\eta)^2 f(\kappa) \tag{13}$$

3 Introduction

- Using rates from CRDB.
- Using Whitesons formula for the number of muons:

  $$N_\mu = 0.028(E/1\text{ GeV})^{0.93}$$

  Note that for $E = 10^{19}$ eV, this formular predicts 5×10^7 in good agreement with LHS of Fig. 2

- Muons have typical lateral extent of λ greater than Moliere radius. Assuming:

  $$\rho(r) = \frac{N}{2\pi\lambda^2}\ \exp(-x/\lambda)$$

  then $\lambda = 220$ puts $\rho(1\text{km}) = 2/\text{m}^2$ as in figure.

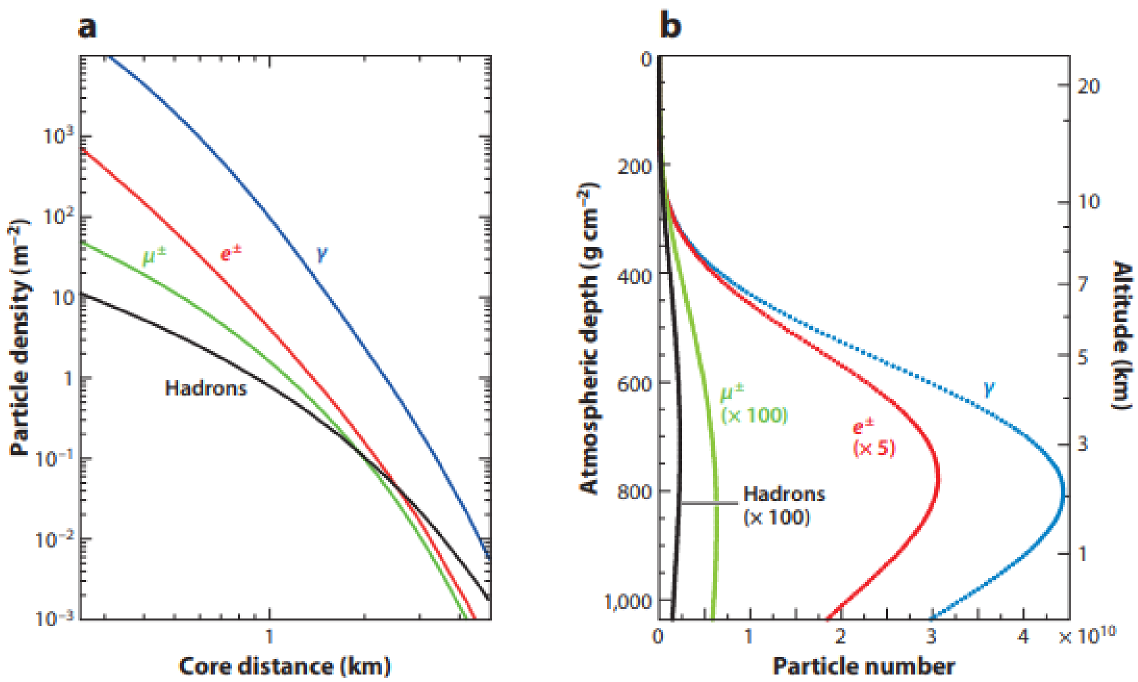

Figure 2

Average (*a*) lateral and (*b*) longitudinal shower profiles for vertical, proton-induced showers at 10^{19} eV. The lateral distribution of the particles at ground is calculated for 870 g cm^{-2}, the depth of the Pierre Auger Observatory. The energy thresholds of the simulation were 0.25 MeV for γ and $e^\pm$ and 0.1 GeV for muons and hadrons.

Figure 2: Extensive air showers, from Chapter 16 of "Gaissner Cosmic Rays and Particle Physics"

- Aim for $E \geq 10^{18}$ eV.
- Expected rate is:
$$R = \frac{1}{\mathrm{km}^2\,\mathrm{day}\,\mathrm{sr}}\,\pi(\pi\lambda^2) = 0.44\,/\,\mathrm{day}$$
- Total muons:
$$N_\mu = 6.6 \times 10^6$$
- And density at center:
$$\rho(0) = \frac{N_\mu}{2\pi\lambda^2} = 21/\ \mathrm{m}^2$$
- $A\epsilon = 2 \times 10^{-5}$, for 1000 phones: 0.02 m^2
- Muons hits in phone per shower: 0.42 per shower.
- Expect a phone hit coincident with tagger more than once per week.
- Taggers of size $0.1 m^2$ should be well suited.