Lab 3- Projectile motion under the influence of air resistance

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Aims

In this investigation, we coded a series of python scripts to simulate the motion produced for objects falling and projectiles travelling both with and without the effects of air resistance.

Introduction

The same assumption is made for both falling objects and projectiles, whereby air resistance is ignored. This means the object will follow a trajectory which is purely determined by the acceleration due to gravity. As this is always acting entirely vertically, the object will only have a constant acceleration in the vertical direction, while having a constant velocity in the horizontal direction throughout its motion.

However, when factoring in the effects of air resistance, the models described are not quite as simple. The calculation for trajectory is greatly complicated by air resistance and often causes the vertical and horizontal displacements to be significantly diminished. For relatively small objects which are moving at small velocities, this resistive force increases linearly with velocity. If we call F the air resistance, we say that F=bv. This equation is very suitable for bodies such as a drop of oil travelling very slowly. This was investigated in exercise 1 where the values for diameter (1.5x10^-6m) and velocity (5x10^-5m/s) of an oil drop were inputted into the code, and it was found that for such values of this magnitude, only the linear equation is relevant, and the quadratic term is neglected.

This quadratic term is relevant for larger objects travelling through air, whereby F=cv^2. Because of this relationship, it means that the air resistance will greatly increase with an increasing velocity. This was also shown in exercise 1 whereby the diameter and velocity of a baseball were inputted into the code. (These values were 0.07m and 5m/s respectively). In this case, the linear term was neglected as it is no longer representative of such an object's motion.

Now, with air resistance, the gravitational acceleration is not the only one responsible for the body's motion- the acceleration of a falling object will no longer be this constant value, g. Newton's second law is useful in comprehending this type of motion. An object will continue falling, with its speed increasing, until it reaches terminal velocity. This is where the resistive force of the air is equal to the magnitude of the object's weight and so there is no further acceleration. This terminal velocity, v_t, (for relatively small objects with small velocities) is given by:

$$v_t = \frac{mg}{b}$$

And the terminal velocity for an object which is greater in mass and has a greater velocity is given by:

$$v_t = (mg/c)^{1/2}$$

To expand of the definitions of b and c, we introduce the parameters B, C and D. Defining b=BD and c=CD^2, we can get a more expansive idea of what these coefficients represent. B and C are determined by the nature of the fluid medium which the object is falling through, and D is the diameter of this (spherical) object.

The diameter of the object will affect how fast it falls, just as with a heavier object will fall faster than one of the same size but is lighter, an object with a smaller diameter will fall faster than one with the same mass but a larger diameter.

We can therefore find the relationship $D \times v$ will scale with the function of air resistance, F(v). This is the case for objects undergoing projectile motion, and so the aim of exercise 1, was to plot graphs of the quadratic term and linear term of F(v) were plotted separately against $D \times v$, which were then compared.

Exercise 2 involved investigating vertical motion and how it is affected by air resistance. As stated earlier, Newton's second law is very useful to represent this motion. Let the acceleration of the falling object be a_y, (as the only component of the object's movement is in the y direction) which is equal to $\frac{dv_y}{dt}$. Therefore, Newton's second law can be written as $\frac{dv_y}{dt} = g - \frac{b}{m}v_y$ (after dividing through by mass, m, g=-9.81). Here, a code was plotted to obtain the behaviour of the vertical velocity variation with time. It is assumed that all the quantities on the right-hand side of Newton's equation, remain constant over small time intervals, Δt. This means that within the code, v_y is updated in steps of Δv_y. Upon integration of this form of Newton's second law we obtain:

$$v_y = v_t(1 - e^{-\left(\frac{b}{m}\right)t})$$

Where $v_t = \frac{mg}{b}$

A graph of mass versus the time taken for the object to reach the ground was then plotted to verify that an increased mass will lead to a decreased time to reach the ground.

For exercise 3, the aim was to investigate projectile motion under the influence of air resistance. For this motion, there is now a horizontal velocity component and a vertical one. Here, Newton's second law is once again applied to determine the acceleration in both directions, as now there is a horizontal deceleration due to effects of air resistance. The equations for horizontal and vertical acceleration are as follows:

$$\frac{dv_x}{dt} = -\frac{b}{m}v_x$$

$$\frac{dv_y}{dt} = g - \frac{b}{m}v_y$$

The minus sign for the horizontal component verifies that it is in the negative direction and so will impede the motion. This, as stated earlier, will then diminish the horizontal distance travelled, which is also the case for the height reached in the trajectory. Exercise 3 also involved finding the optimum angel for this motion, which is generally 45˚, and seeing if this is affected by air resistance.

Exercise 4 involved the same investigation but utilised different concepts- whereby the horizontal and vertical components are no longer treated separately. Air resistance now has a quadratic dependence on the velocity of the object, and so this relationship, shown below, was investigated where parameters of D and v_o were set and varied.

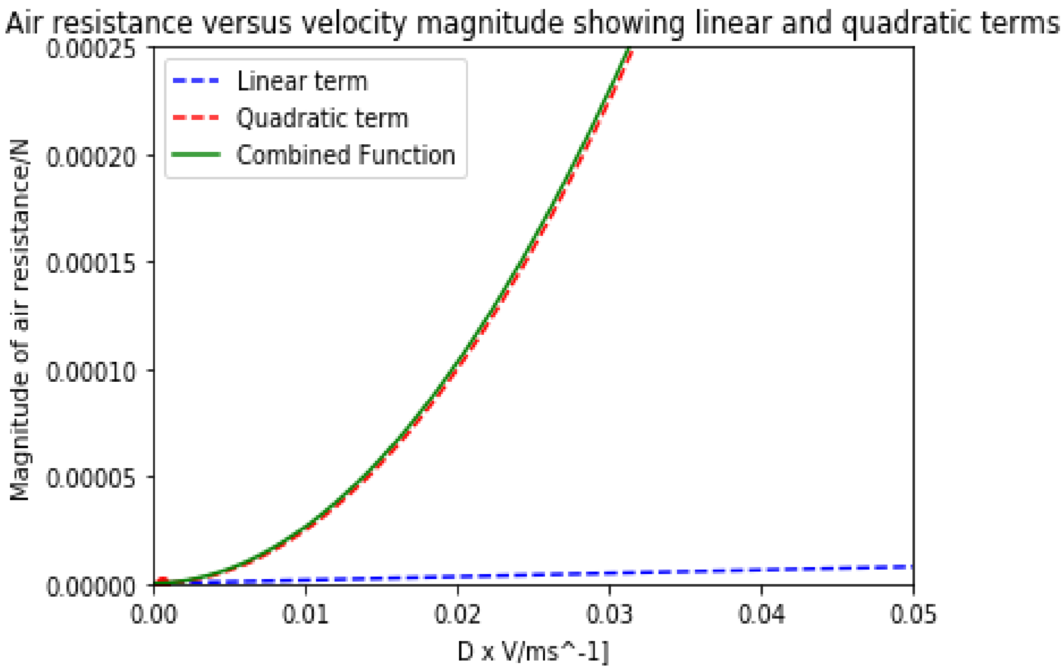
$$\frac{dv_x}{dt} = -\frac{c}{m}({v_x}^2 + {v_y}^2)^{1/2}v_x$$

$$\frac{dv_y}{dt} = g - \frac{c}{m}({v_x}^2 + {v_y}^2)^{1/2}v_y$$

Results

Exercise 1

Figure 1.1



The figure above displays the expected, whereby for small masses, with small velocities, the linear term (shown in blue) dominates and so the air resistance is proportional to the velocity of the object. For larger masses, with larger velocities, the quadratic term (red) dominates, whereby air resistance is proportional to the square of the velocity. The combined term will have this shape (shown in green) as overall, the equation will be $F(v) = bv + cv^2$ for the air resistance.

Figure 1.2

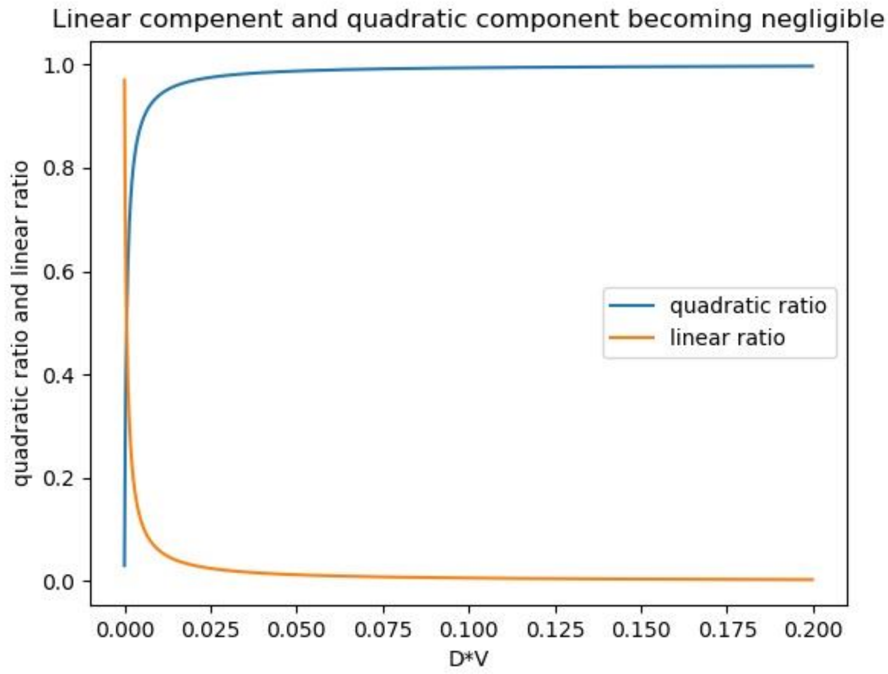


Figure 1.2 shows the expected trends, whereby for greater values of $D \times v$ lead to the linear term dying out and asymptotically tending to zero. The opposite happens for the quadratic term, which increases and asymptotically tends to a maximum value of 1. Both figures 1 and 2 are set for D=0.02 and B=1.6x10^-6.

Exercise 2

Figure 2.1

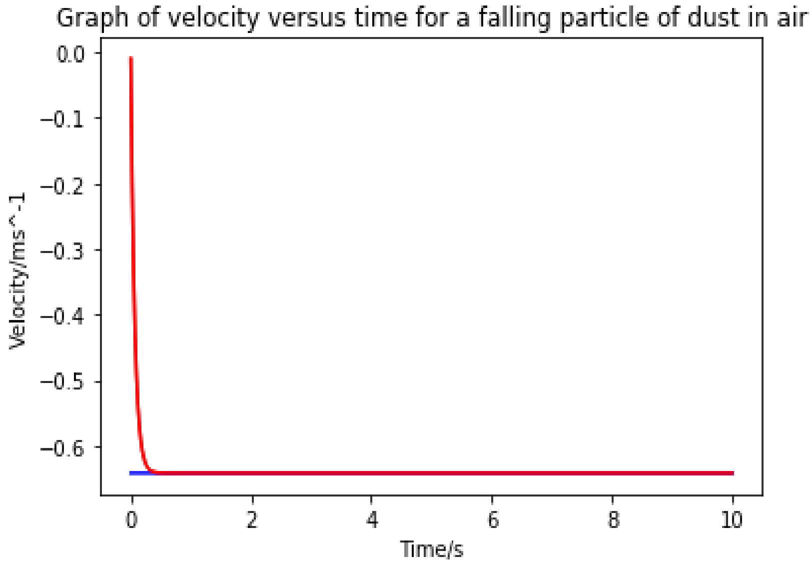
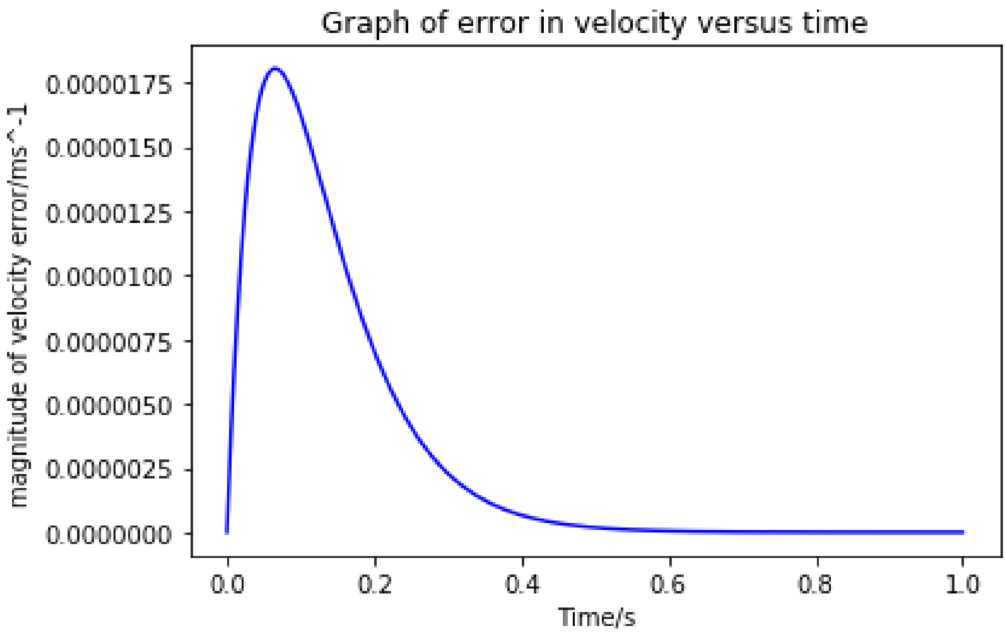


Figure 2.1, above, shows the velocity versus time graph for a particle with very low mass and diameter. The dust particle has a negative velocity as the sign convention has been adopted whereby the downward direction is negative. The magnitude of this velocity is increasing but is very low, as expected. Here, the initial velocity is set to 0.0m/s and the mass was calculated to be 1.047x10^-9kg for the dust particle. The terminal velocity for this was found to have a magnitude of roughly 0.6419m/s in the negative direction.

Figure 2.2



Graph 2.2 shows the error magnitude peaks at roughly 0.000018m/s at around 0.1 seconds. It allows for comparison of the results obtained from the code and those obtained from the analytical solution mentioned previously. The results from the code could be improved by perhaps reducing the time step intervals and using a more accurate value for gravity.

Figure 2.3

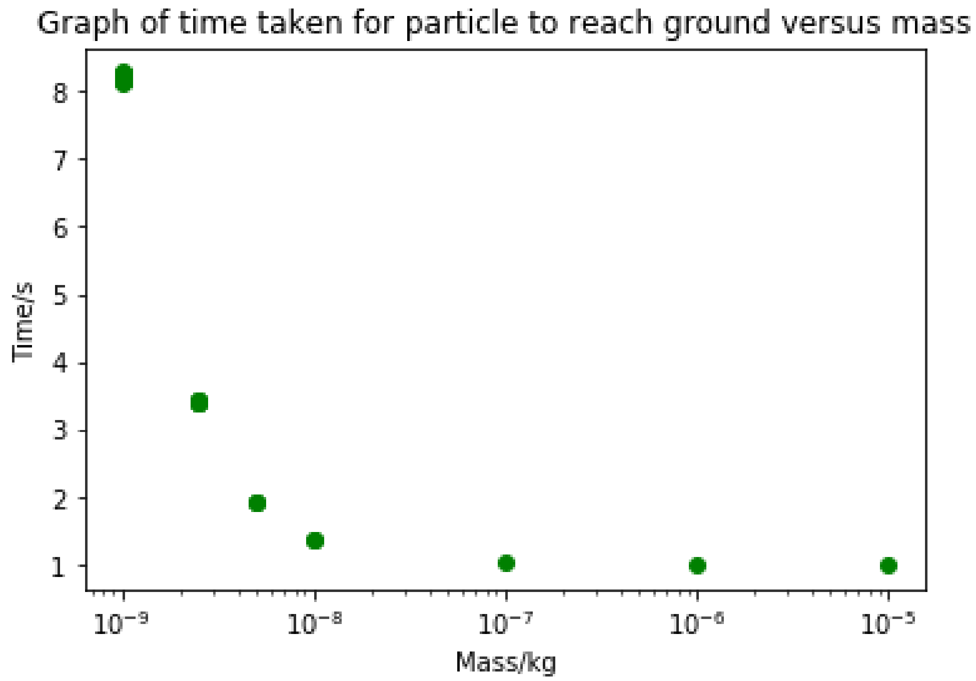


Figure 2,3 also shows the expected trend, whereby for a lighter mass, the particle will take a greater time to reach the ground. For instance, for a mass of 1×10^{-9}kg the particle takes the longest time. The graph then drops exponentially with mass and levels into linear relationship, e.g. for a mass of 1×10^{-7}kg, the particle takes just 1 second to reach the ground.

<u>Exercise 3</u>

Figure 3.1

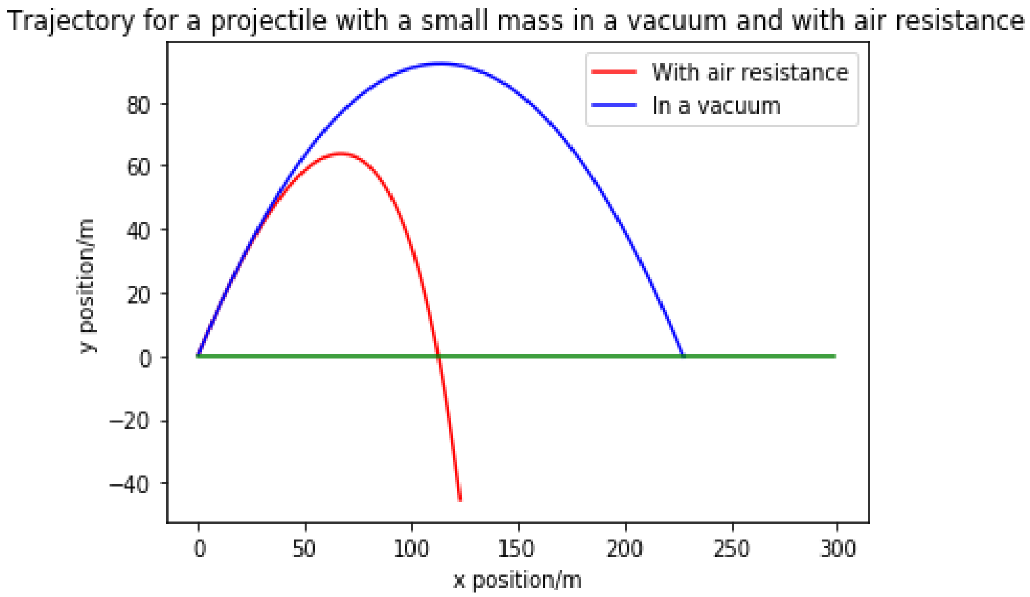


Figure 3.1 is slightly misleading in that it implies that the projectile travelling with the influence of air resistance travels beyond ground level, by over 40m. But overall, it shows the expected trend, whereby air resistance will restrict how fair the particle will movie in both the horizontal and vertical direction. Here, without the effects of air resistance, the particle will reach a vertical displacement of 100m, whereas when travelling in a vacuum, it will only reach a height of around 60m. The same applies to the range, whereby it is approximately halved when the particle undergoes air resistance. This verifies the expected behaviour for particles in such systems.

Figure 3.2

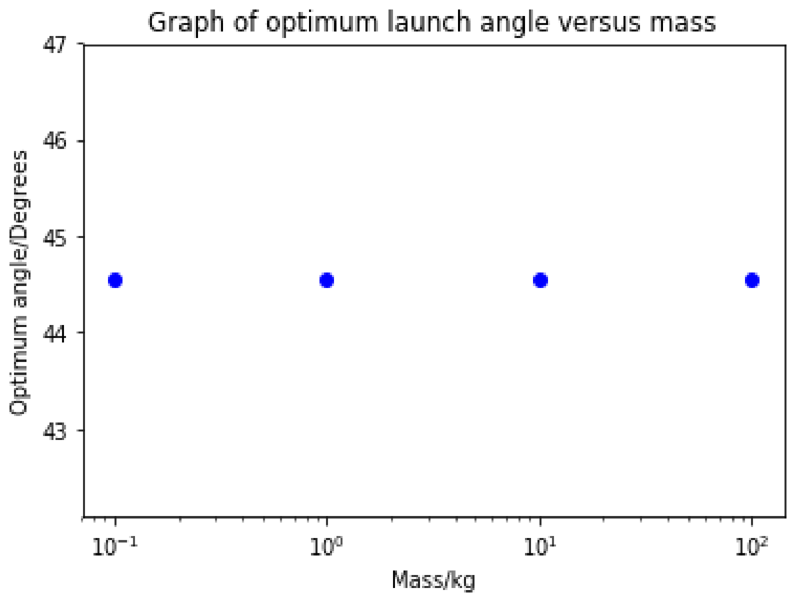


Figure 3.2 is a clearly constant relationship between mass and maximum launch angle. It is expected that for all particles, no matter of size or shape, that the optimum launch angle will be 45˚. The graph shows an optimum angle slightly below this value, but very close to that expected, being approximately 44.5454˚

Exercise 4

Figure 4.1a

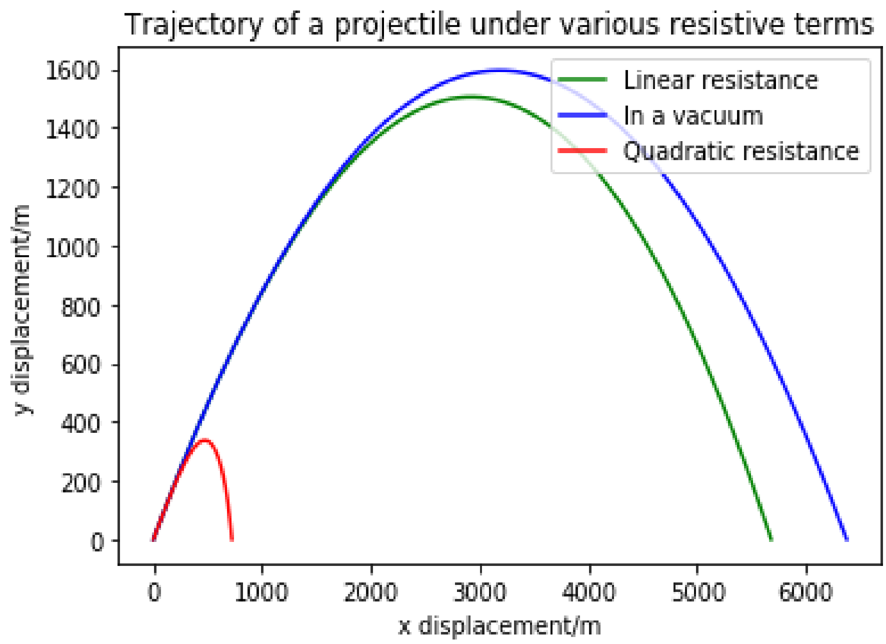


Figure 4.1a shows how the trajectory of a particle will be affected by differing air resistance functions. Here v was set to 250m/s, H was set to 0.0 and R was set to 0.0. The mass of this object was defined to be 1x10^-6kg, with b and c set to 0.5x10^-8 and 2.5x10^-9 respectively.

Figure 4.1b

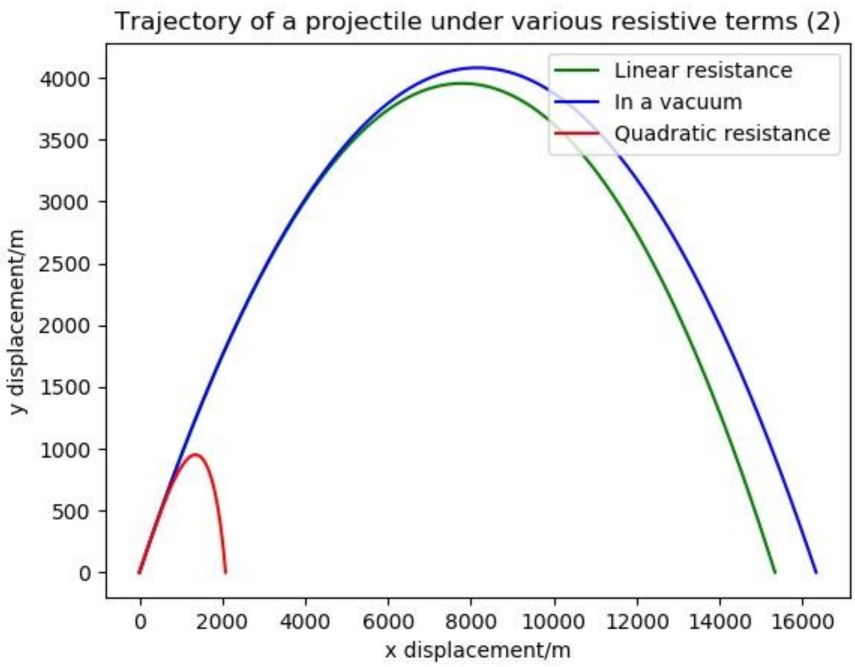


Figure 4.1b shows the same results but with a different initial value for v, which was varied to 400m/s and the mass increased to 3x10^-6kg. (This change was set about by changing the particle's diameter and hence volume). Here we can see that the maximum heights and ranges reached are much higher. For example, the range for the particle in a vacuum increased by about 10,000 metres. This is due to the increased velocity from 250m/s to 400m/s. However, the same trend is shown whereby the particle travelling under the influence of the quadratic air resistance term reaches much lower displacement values, as expected.

Conclusion

All the graphs plotted show the expected trends, although the code could be augmented in order to truly represent the particle's motion- for example in figure 3.1, the plot indicates the particle has travelled through the ground when acted upon by air resistance.

But overall, the trajectories of the projectiles were well represented for both free travel and those experiencing air resistance forces. Exercise 1 verifies that, for certain ranges of velocity and size of a particle, the linear term will dominate, and beyond these ranges, the quadratic term will dominate.

Exercise 2 clearly demonstrates the motion of a vertically falling object when acted upon by air resistance, and shows that upon comparison with the analytical solution, changes could be made to the code to increase accuracy.

Parts 3 and 4 investigated motion of a projectile under the influence of air resistance, but with the added task of determining the optimum angle, and if this was affected by air resistance- which it is not. It was then shown how trajectories can change due to air resistance when the type of particle and its initial velocities were varied, and again the results are as expected.