Experimental lab 1: Compton Scattering

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Aim and background

The aim of this experiment was to investigate the Compton Scattering Effect, whereby photons are directed at a charged particle, which is usually an electron, and scattered at various angles with a different final wavelength. The primary objective in this experiment was to then use the data collected and investigate the energy of the scattered photons, and how this depends on the incident angle at which the photons are fired.

The experiment was first demonstrated by Arthur Compton, who found the resulting energy of the incident photon to be decreased after the collision- a collision involving X-rays and a carbon target consisting of atoms with loosely bound electrons. He effectively proved that light cannot be solely treated as wave-like, but rather as a collection of particles also. The scattering which occurs is inelastic whereas the collision itself is elastic, whereby the kinetic energy is conserved. As the incident photon's energy significantly exceeds the bond energy of the outer electron to the atom to which it belongs, this bond energy is ignored, and the electron is viewed as "free".

The wavelength will become greater with an increased scattering angle, as we will show in our investigation, and so the value for Compton shift is increased. This change in wavelength and frequency is usually interpreted for electromagnetic waves but should not occur as a result of a change in direction- and so the incident x-rays had to be particles of energy, scattered from the collisions with the electrons. Compton showed that the frequency and wavelength shift both depends on the angle of scattering and can be calculated through fundamental kinematic approaches. In order to apply these and carry out a derivation of the equations involved, we must treat what is usually considered to be a massless photon, as a relativistic particle, and so can be treated as having a momentum.

The Klein-Nishina function indicates the cross-sectional pattern of photon scattering for a certain angle. For photons with a much higher rest energy than the electron it collides with, the scattering will be predominantly defined between the angles 0˚ and 60˚. This is seen in our results whereby the count rate at 30˚ for example was $175s^{-1}$ and the count rate for higher angles such as 90˚ was $25s^{-1}$.

The equations, depicting the scattered photon's final energy (1) and wavelength (2) after scattering are shown separately below.

1) Before derivation, we will define some of the terms involved.

E_{pi} is the photon's initial energy
E_{pf} is the photon's final energy
E_{ei} is the electron's initial energy
E_{ef} is the electron's final energy
And P is the term for momentum, where the subscripts have the same meanings.

From the law of the conservation of energy, we know that

$$E_{pi} + E_{ei} = E_{pf} + E_{ef}$$

Therefore

$$E_{pi} + m_e c^2 = E_{pf} + \sqrt{{P_e}^2 c^2 + {m_e}^2 c^4}$$

Bringing E_{pf} to the left-hand side and squaring both sides, we get:

$$(E_{pi} + m_e c^2 - E_{pf})^2 = {P_e}^2 c^2 + {m_e}^2 c^4$$

And from the conservation of momentum, we know:

$$\boldsymbol{P}_{pi} - \boldsymbol{P}_{pf} = \boldsymbol{P}_{ef}$$

We know this as we have assumed that the electron is initially stationary and so does not have an initial momentum. We are also treating the momentum as a vector quantity.

Squaring both sides, we get a $\cos\theta$ term, where θ is defined as the angle between the initial and final photon direction.

$${P_{pi}}^2 - 2P_{pi}P_{pf}cos\theta + {P_{pf}}^2 = {P_{ef}}^2$$

We know that for a photon, E=Pc, and so multiplying the above equation through by c^2, we have

$${E_{pi}}^2 - 2E_{pi}E_{pf}cos\theta + {E_{pf}}^2 = {P_{ef}}^2 c^2$$

Using our previous expression for the conservation of energy, and substituting in the expression above, we get

$${E_{pi}}^2 - 2E_{pi}E_{pf}cos\theta + {E_{pf}}^2 + {m_e}^2 c^4 = (E_{pi} - E_{pf} + m_e c^2)^2$$

Expanding the right-hand side and cancelling terms:

$$E_{pi} m_e c^2 - E_{pf} m_e c^2 - E_{pi}E_{pf} = -E_{pi}E_{pf}\cos\theta$$

Rearranging to get E_{pf} on its own, we have derived an expression for the final energy of the photon:

$$E_{pf} = \frac{E_{pi}}{1 + \left(\frac{E_{pi}}{mc^2}\right)}(1 - \cos\theta)$$

2) From this, we can use $E = \frac{hc}{\lambda}$ to find the final wavelength of the photon:

$$\frac{hc}{\lambda_f} = \frac{hc/\lambda_i}{(1 + hc/(\lambda_i m_e c^2))(1 - cos\theta)}$$

Cancelling terms and inverting the equation, we get:

$$\lambda_f = \lambda_i(1 + \frac{hc}{m_e c^2 \lambda_i}(1 - cos\theta))$$

Expanding the equation and rearranging, we have:

$$\lambda_f - \lambda_i = \frac{h}{m_e c}(1 - cos\theta)$$

Where $\frac{h}{m_e c}$ is known as the Compton wavelength. As it is a function of mass, the lower the masses involved, the greater the necessity to use quantum mechanics to explain the behaviour- whereas for larger masses the behaviour can be dealt with using classical methods.

We can verify that for an electron (which has a mass of 9.11×10^{-31} kg) and with the speed of light being roughly 3×10^{8} m/s and Planck's constant being 6.626×10^{-34} m^2 kg/s, that the Compton wavelength for a free electron is 2.42×10^{-12} meters using the equation for the Compton wavelength. Furthermore, using $E = \frac{hc}{\lambda}$, we can show that this corresponds to an energy value of 8.206×10^{-14} joules, dividing by the magnitude of the charge of an electron, this gives us 512901.9 eV or roughly 513 keV.

Experimental method

The radiation source used in this experiment was Caesium-137, which emits gamma rays with a main energy of 661. 657keV. The apparatus was set up with the radiation source kept in a lead container while the background radiation was measured. This was over a 15-minute period with 1 second intervals. This was the period and interval of time used for each of the angles at which the radiation source was directed. Throughout the experiment, gloves were worn as a safety precaution when handling the equipment.

The software used to measure the count rate along with the channels was CASSY Lab 2 and the high-power voltage supply was adjusted to a set point of 600V. The values set on the software were 512 channels of resolution, a gain value of -10.05 and the time settings as mentioned previously.

Following the background scan, a calibration run was carried out using americium-241 (which has a relatively lower activity of around 50kBq) and this will then be used to plot a graph of energy versus the channel number. From the gradient of this graph we can find the peak values of the subsequent readings taken at varying angles.

The count rate was measured for this and for angles of 30˚, 60˚, 90˚ and 120˚ in the clockwise direction. The radiation source was moved to these angle values and directed at the cylindrical scattering target, made of aluminium. The detector then measured the number of counts over 15 minutes for each angle. The spectra graph for each angle was plotted and saved, where they were compared on ORIGIN, and the results analysed.

Results and analysis

Figure 1

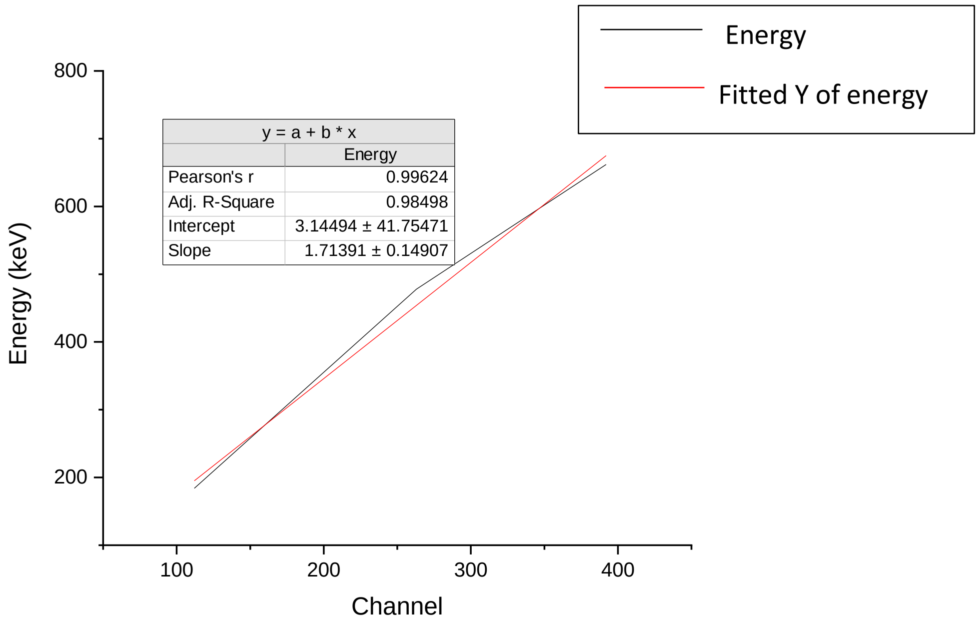

Figure 1 above shows a graph of energy versus channel number for the calibration run carried out at the beginning of the investigation. The gradient of this graph was calculated, and from this, the same graphs could be plotted for the angles at which the source was directed. This was done by matching the peaks of the calibration graph which displayed the number of events versus the channel number. We were given data at the beginning of the experiment, which displayed the energy value corresponding to the peaks which we had plotted.

This gradient was found to be roughly 1.71 keV/channel, with a clearly linear relationship, as expected. And so, for angles of 0˚ onwards, we could use this gradient value to calculate the corresponding energy values for each peak, as shown below. From these calculations also, the wavelength shift can also be found and hence the Compton wavelength.

Table 1

Angle/degrees	Channel value at which peak 1 occurred	Peak 2	Peak 3
0°	122	256	396
30°	120	255	394
60°	118	254	394
90°	117	253	392
120°	115	249	391
150°	112	248	389

Table 1 shows the expected trend, whereby for lower angles of scattering, the final energy of the scattered photon is higher than those that were scattered at higher angles. We can conclude this as the channel number corresponds to the energy values. For instance, at 0˚, the channel value at which the third peak occurred was 396, whereas the channel value for a 150˚ scatter was 389.

Figure 2

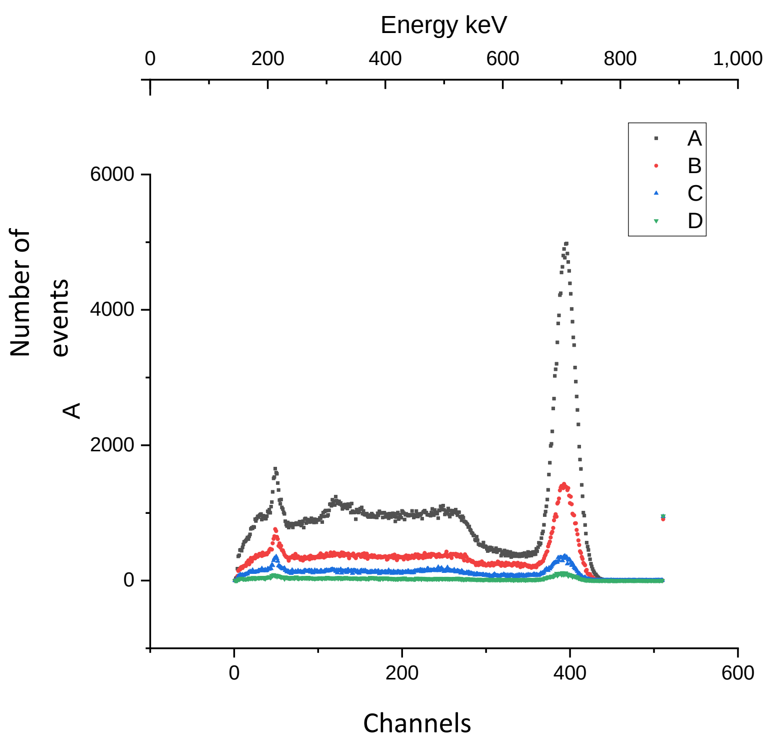

Figure 2 above, which has an inserted energy scale as the top axis, shows the number of events versus the channel number. Here, A=0˚, B=30˚, C=60˚ and D= 90˚. It is clearly following the expected trend whereby the higher the angle of scattering, the less impacts the detector receives and hence a lesser number of events recorded. It also shows that photons which haven't been deflected through large angles have a higher final energy. This can be verified from the Compton equation, whereby a larger angle means a greater wavelength shift. The energy values shown on the top axis verify that the photon will lose energy to the recoil electron, and so its final energy is diminished.

From figure 1, the gradient found can be multiplied by the channel numbers and so the energy for subsequent angles can be calculated, as carried out below.

Channel number x Gradient = Energy (keV)

E = hc/λ (where E is in joules)

$$\lambda_f - \lambda_i = \frac{h}{m_e c}(1 - cos\Theta)$$

Using these equations, we can find the Compton wavelength and verify that our measurements were valid.

122 x 1.7139 = 209.095 keV = 3.347 $\times 10^{-14}$ kJ -- First peak at 0°

120 x 1.7139 = 205.67 keV = 3.293 $\times 10^{-14}$ kJ -- First Peak at 30°

ΔE= 5.4$\times 10^{-16}$ kJ

2.424$\times 10^{-12}$ m x (1-cos30) =3.248$\times 10^{-13}$ m

6.626$\times 10^{-34}$ x2.997$\times 10^{8}$/3.248$\times 10^{13}$ m = 6.114 $\times 10^{-13}$ J

Conclusion

The results of this investigation verify that with an increased angle of scattering, the wavelength of the scattered photon is also increase, and so resulting greater Compton shift. This therefore corresponds to a decrease in energy of the photon. This is consistent for all angles- a greater angle means a greater Compton shift, as seen from a greater space between peaks. This is also verified from the calculations carried out above, resulting in the expected Compton wavelength for a photon. It is clearly seen that upon collision with the electron in the shell, the photon will lose some of its energy to this electron, hence the scattering angle is proportional to the energy lost by the photon.

Upon repetition of this experiment, I would like to investigate the amount of back-scatter in such an experiment, as it was not possible to do so with the set up involved. Furthermore, I would like to have a greater range energy values from the calibration peaks- this would help improve the accuracy of the experiment.

References

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