Modelling the surface density profile of the primordial Solar Nebula by application of the MMSN model to the planets of our Solar System

Planetary Problem Set 1- Professor Gallagher

Alexandra Mulholland 03/10/2020

ABSTRACT

Accurate chemical data on elemental abundances within the cosmos is of paramount importance, particularly in the context of modelling from what the current Solar System may have initiated. The physical and chemical evolution of a stellar nebula are closely entwined and can be inferred strongly from chondritic meteorite, cometary and atmospheric data. This is because these samples preserve evidence of the environment in which they formed, unchanged by subsequent thermal activity within the Solar System as it evolved. The structure of the primitive nebula, however, is still a point of speculation. This report briefly investigates the Minimum Mass Solar Nebula model and its relevance in predicting the origins of the planetary system we see today. The surface density profile of the protoplanetary disk is modelled as a decay power-law of the form $\sigma(r) = \sigma_0 r^{-\alpha}$. The values obtained via the fitting approach adapted are in close agreement with those of other papers, which have all used varying fitting techniques. The values for σ_0 and α obtained in this investigation are ~2922.27gcm^{-2} and 1.92 respectively. Accepted values of other models range from ~1700 to 4300gcm^{-2} for σ_0 and from 1.5 to 2.0 for α. Polar integration across the disk, which is assumed to be of negligible thickness in this context, yields a Solar Nebula mass of ~2.33×10^{31}g.

INTRODUCTION AND SCIENTIFIC BACKGROUND

To adequately describe the formation of our Solar System from its temporal point of origin, it is important to be aware of the typical material condensation sequence which takes place in a protoplanetary disk. [1] Various gaseous elements condensate into solid form at specified temperature ranges and regions of the protoplanetary disks. The temperature distribution of the structure means that there are "snow lines" at which these condensates (i.e. ice grains) form, depending on their condensation temperature. These lines are imperative to the understanding of planetismal formation, and it should be noted that the positions of these lines vary as the disk evolves. For instance, [2] the Carbon Monoxide (CO) snow line has been hypothesised to drastically affect the bulk composition of planets at further radial distances from the protostar, and it has been established that the H_2O snow line is crucial in the context of giant planet formation in our Solar System.

[1] The typical condensation sequence of the cooling solar nebula is as follows: Firstly, refractory oxides and silicates condense at temperatures of around 1700K to 1400K. Iron and Nickel, which mainly comprise cores and mantles of the planets, deliquesce at temperatures ranging from 1350K to 1150K. Finally, volatile elements and molecules (those materials which have low melting and condensation temperatures) such as water, Sulphur, Carbon, CO and Nitrogen form between 600K to 100K. From a planet's density, one can infer its depletion in volatiles and hence the temperature at which condensation ceased for said planet. [3] As representation of the evolution of molecules within a disk, figure (5) of the appendix shows the radial distribution of various molecules along a typical protoplanetary disk's midplane.

The Minimum Mass Solar Nebula (MMSN) model is one by which many researchers base their analysis of exoplanetary disks. It is the framework of the Solar Nebula and its subsequent collapse into a circumstellar planetary system. [5] Our knowledge of cosmological abundances alongside condensation events means that the primordial mass of the protoplanetary Solar disk can be estimated accordingly. [6] The MMSN uses *ad hoc* measures to avail of the widely-accepted theory that each protoplanet originated essentially with cosmic composition.

The Jovian Planets: [7] Observation of the vibrational-rotational spectra of methane and ammonia in the IR spectrum suggest there are vast amounts of these molecules in the atmosphere of Jupiter. Wildt (1930) correctly predicted the presence of at least appreciable amounts of Hydrogen deep within the atmospheres of the Jovian planets, to restore the NH_3 and CH_4, which would otherwise be absent due to UV destruction. These planets are therefore considered to be primarily of solar composition (H and He). Uranus and Neptune, despite being less massive than Jupiter and Saturn, are considerably denser. [8] Models of the Solar System find that at around 20AU, beyond the snow lines of H_2O and NH_3, the molecules have condensed entirely. The density values for the Uranus and Neptune (~1.3gcm^{-3} and 1.7gcm^{-3}) suggest their atmospheres are gaseous, with an ice outer core and rocky inner core.

The terrestrial planets are comprised of materials which are substantially less abundant than those constituting the outer planets. The temperature distribution in the region where the inner planets formed suggests that the refractory materials will have entirely condensed prior to the planets' formation. [9] Silicates constitute the mantle layer and the most abundant metal, Iron, is mainly in their cores. One may note that Mercury, Venus and Earth all have similar densities of around 5.0-5.5gcm^{-3}, whereas Mars is less dense with a value of ~3.9gcm^{-3}. This can be explained as Mercury is less than half the mass of Mars, but with a heavier composition of metallic elements. Venus, Earth and Mars have similar compositions of mostly silicates.

METHOD

A familiarity with the material composition of the planets allows for a better understanding of the implementation of the MMSN model. [9] The model is built on the fundamental assumption that the planets today had accreted material from the primitive nebula. [5] Furthermore, an equivalent mass of solid solar nebula material is determined at the current planetary positions, and that [6] the protoplanets drew material only from a circumstellar annulus defined by these positions. The augmented planetary mass is assumed to originate from the annulus area of somewhat arbitrarily chosen boundaries, which are determined as follows, (using Venus as an example). The boundaries are defined as halfway to the planets on either side- for instance, for Venus, which lies between Mercury and Earth, and is at 0.72AU from the Sun. Mercury lies at 0.38AU and Earth of course at 1AU. The inner boundary of the annulus therefore lies at 0.72AU – (0.72-0.38)/2 AU = 0.55AU=0.83×10^{13} cm=r_{inner}. The outer boundary at 0.72AU+(1-0.72)/2 AU = 0.86AU = 1.29×10^{13}cm=r_{outer}. The ring-shaped zone from which Venus drew material is then $\pi\ r^2_{outer}$-πr^2_{inner}=3.06×10^{26}cm^2. As one can infer from the preceding process, this is extremely laborious if doing so for each planet. Therefore, a 'for' loop within the code is necessary (refer to the appendix). However, due to Mercury and Neptune being extremities of orbit, we treat their annuli separately. Here we treat their inner and outer radii to be at distances which are symmetric from their orbital positions. [10] The rings are nested and continuous and are assumed to account for the material abundances in their entirety.

Using this formulation, and referring to our cosmic abundance knowledge, a unit-less quantity known as the 'F factor' can be determined. Knowing each planet's distance from the Sun and their angular extent in the sky allows us to determine their radius, and from this their volume. From the values of volume and mass (which we ascertain from Newton's form of Kepler's 3rd law), we can determine their densities.

Knowing these properties, and knowledge of cosmic abundances allows us to estimate of what the planet consists and how much of this material is present in the disk from which the planet could form. For instance: the planet Mercury has a density of around 5.43gcm^{-3}. From this density and knowing the temperature of the nebula in Mercury's development region, one can assume that this planet consists of mostly the metal Iron and perhaps some rock. As a solar nebula contracts and cools, metallic elements will be the first to condense to solids from their gaseous state i.e. they condense at higher temperatures. We can therefore assume that the iron has condensed entirely and makes up 100% of the mass of Mercury. Our knowledge of cosmic abundances tells us that Fe makes up around 0.285% of the mass of the solar nebula in Mercury's annulus of formation. Equating these two values such that 0.285% of the solar nebula mass makes up 100% of the mass of Mercury, we determine the F factor is 100/0.285 which is 350. Meaning the minimum mass required to make Mercury is 350 times its actual mass. These values were previously calculated for each planet and are provided in the appendix.

Multiplying each planet's F factor by their present mass is the mass required from the primitive Solar Nebula to form them. Dividing each quantity by their respective ring area produces a surface density value for the solar nebula prior to formation of the planetary system.

[9] Subsequently, a minimum-value estimation of the mass of the Solar Nebula can be inferred. For the purposes of this investigation, surface densities are used solely so as not to assume any characteristics of the vertical structure. From a radial distance-dependent logarithmic plot of the surface densities, a general fitting in the form of a power law is applied to the data. For reasons, which will become clear in the data, anomalous values of surface densities have been ignored. The modelling power law (in non-logarithmic form) is shown as equation (1a) below, and the latter equation (1b) is the logarithmic function which was fitted to the data.

$$\sigma(r) = \sigma_0 r^{-\alpha} \qquad \text{eq. (1a)}$$

$$\log_{10}\sigma(r) = \log_{10}\sigma_0 - \alpha\log_{10} r \qquad \text{eq. (1b)}$$

Where r is the radial distance from the protosun [AU], $\boldsymbol{\sigma}(\mathbf{r})$ is the surface density of the solar nebula [gcm^{-2}] and α is the r index of the power law which will be inferred from the model. From equation (1b) the gradient value is -α indicates the decay rate of the annular surface density of the nebula as a function of distance. Likewise, the intercept value of the fitting is the logarithmic function of the coefficient $\boldsymbol{\sigma_0}$ [gcm^{-2}].

By using a LOBF for logarithmic trend shown in figure (2), a value for the gradient and y-axis intercept is produced. From my chosen fitting approach, and subsequent primitive surface density profile of the protoplanetary disk, the minimum mass of solar nebula material required to construct the planets is deduced. Equation (2) describes the accretion of planetary mass in the annular model.

$$\iint_{R_S}^{R_F} \sigma_0 r^{-\alpha}\ \mathrm{rdr\,d\theta} \qquad \text{eq. (2)}$$

θ is the azimuthal angle over the disk and ranges from 0 to 2π. The surface density profile has no angular dependence and hence the integral can simply be multiplied by 2π in this polar integration.

The disk itself is one large annulus comprised of the modelled annuli of each planet and the asteroids. The inner radius of said annulus, R_S, I have taken to be the inner radius of Mercury's orbit (0.219AU). The outer radius of the disk, R_F, is taken to be the average distance to Pluto (39.5AU). The code performs numerical integration Simpson's rule. It should be noted that the integral can be evaluated analytically. I have demonstrated this by hand and inserted my solution into the Appendix. This serves to confirm the validity of the computations.

RESULTS AND DISCUSSION

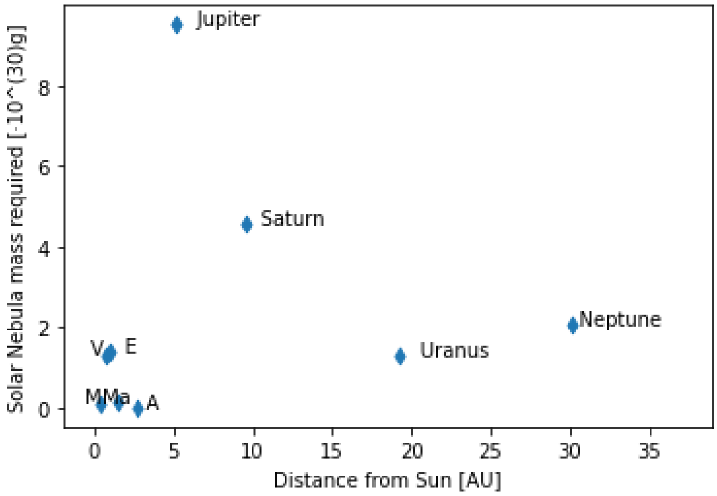

Figure 1: Plot of the primitive Solar Nebula mass of material which was required to form each of the planets of the Solar System. The mass is quoted in grams and is a function of distance in AU.

Figure 1, above, highlights the result of multiplying a planet's 'F factor' by its mass. The same applies to the asteroid belt between Mars and Jupiter. One can calculate the minimum mass required to form all the terrestrial planets is around $30{,}000\times10^{26}$g total. Whereas for the Jovian Planets, this value is excess of $180{,}000\times10^{26}$g- six times that of the inner planets (including the asteroid belt). The composition of the primitive solar nebular mass is ~98.4% hydrogen and helium (the main constituents of the gas giants). This would explain the sheer mass of Jupiter and Saturn. [6] Saturn, however, is considerably less in mass, Ramsey (1951) predicts that this was due to larger evaporation losses in Hydrogen than occurred in Jupiter. [12] The percentage of ices in the nebula is around 1.2%, explaining Uranus' and Neptune's relatively smaller value of solar nebula mass required to form them. Finally, the material constituents of refractories such as metals and silicates which constitute the inner terrestrial planets make up just 0.3% of the solar nebula mass.

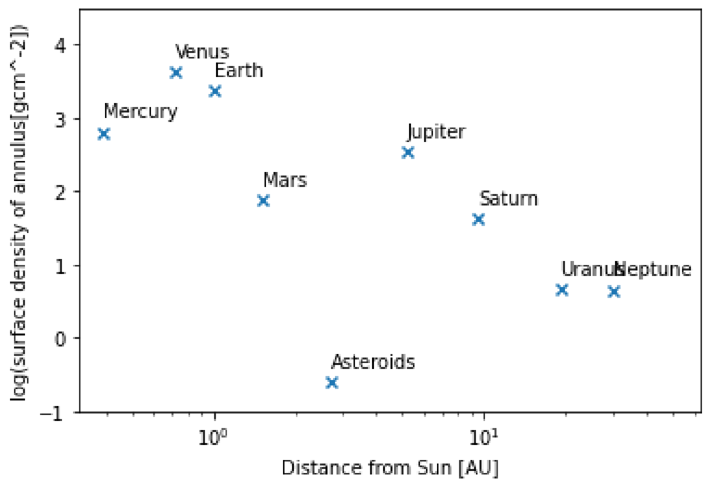

Figure 2: Plot of the logarithmic function of the annular surface density of each of the planets. This is given as a function of distance in AU, plotted on a logarithmic scale.

Figure 2 shows the log. values of the ring-shaped surface densities for each planet and the asteroids. One can vaguely see Venus, Earth and the Jovian planets follow a linear trend of negative gradient. This is emphasised in figure (4). Mercury, Mars and the asteroids are anomalously low in surface density value. In figure 3, where no logarithmic values are taken, the exponentially decaying power law is evident. (see equation 1a). Again, Mercury, Mars and the asteroids deviate from the expected trend. The reason for this digression are hypothesised briefly below.

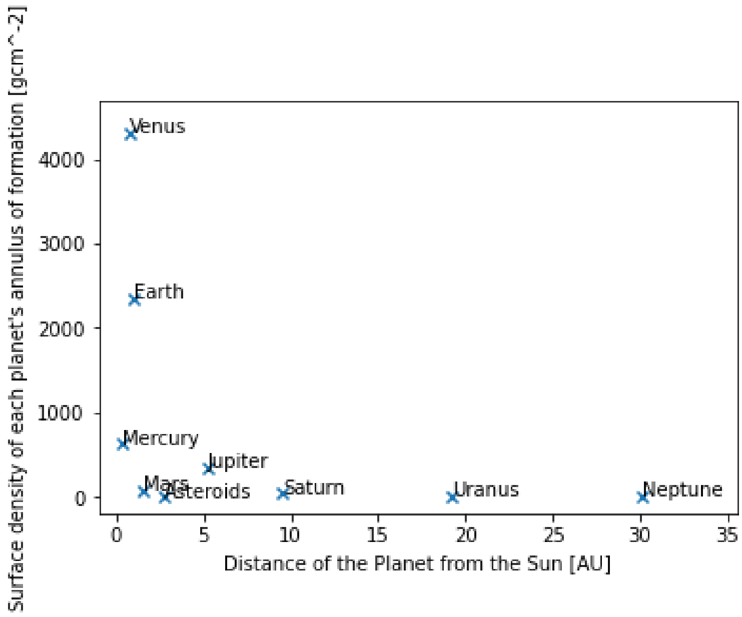

Figure 3: Plot of the annular surface density [gcm^{-2}] versus circumstellar distance [AU] of each of the planets.

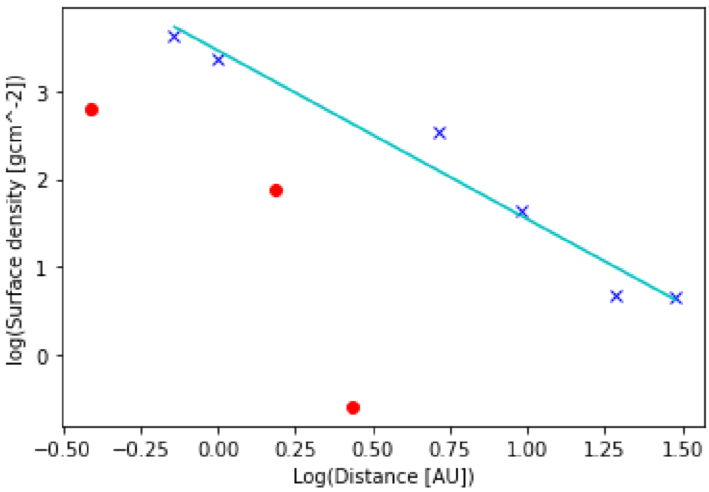

Figure 4: Plot of the logarithmic function of the Surface density [gcm-2] as a function of logarithmic distance [AU]. The Linear fit has been executed for the non-anomalous data.

Figure 4, above, shows (in red) the data points of Mercury, Mars and the asteroids (from left to right) and (in blue) the data points which obey the expected power law. (See equation 1b). Python's 'polyfit' technique produced values of the gradient and intercept to be ~ -1.92 and ~3.47 respectively. From equation (1b), these correspond to an α value of 1.92 and a $\boldsymbol{\sigma_0}$ value of approximately 2922.27gcm^{-2}. Thus, equation (1b) becomes:

$$\log_{10} \sigma(r) = \log_{10}(2922.27) - 1.92 \log_{10} r$$

And hence, the power law (which models the annular surface density of the solar nebula as a function of distance) becomes:

$$\sigma(r) = 2922.27 r^{-1.92}$$

Papers regarding this model have various fitting approaches by which they obtain the power law, and the corresponding values can vary significantly. [11] Hayashi (1981) obtained a value of 1700gcm^{-2} for $\boldsymbol{\sigma_0}$ and an r index value of -1.5. [9] Weidenschilling's (1977) fitting approach resulted in values of 4200gcm^{-2} and -1.5 also. The discrepancy in my result could be due to the chosen Iron abundances of the planets, and the assumed mass of the asteroid belt: Weidenschilling assumed a value of ~0.03×10^{26}g, whereas I assumed this mass was ~0.1×10^{26}g. [10] Kuchner adapted the model to extrasolar nebula and finds a $\boldsymbol{\sigma_0}$ and an α value of 4230gcm^{-2} and 1.8 respectively. [7]

Lewis (2004) adopts the approach whereby the masses of Mars, the asteroids and Jupiter are conflated, and spread this combined mass over their total annulus area. He did the same for Neptune and Pluto. From this, the value of -α was found to be -1.8 and $\boldsymbol{\sigma_0}$, 3300gcm^{-2}. Lewis accepts a generally accepted range of values for α to be from 1.5 to 2.0. The values obtained via my fitting approach agree with this accepted range, and $\boldsymbol{\sigma_0}$ is also admissible.

Calculating the minimum mass of primitive solar nebula material required to form the planets and asteroids via eq.(2) yielded the following estimation: 2.33×10^{31}g. This result was produced by the numerical integration technique, Simpson's rule. However, as stated earlier, the surface density profile can be integrated analytically (shown in the appendix). This method yielded an approximate result of 2.375×10^{31}g. The generally accepted result is usually stated as a ratio to the current solar mass: $\frac{\mathbf{M_{nebula}}}{\mathbf{M_{sun}}}$ and is, on average ~0.01. However, [6] Kuiper (1956) predicts a value of around 0.06 for this ratio. The numerical integration of the surface density profile yields a ratio of ~0.0117. Overall, the results obtained via my chosen power-law fitting (where Mercury's, Mars' and the asteroids data values are neglected) are within an expected range.

Figure (7) in the appendix shows Kuiper's findings for the heliocentric mass distribution of the primitive nebula material, which is also demonstrates the anomalous nature of the data for Mercury, Mars and the asteroid belt. [7] Mercury's surface density value may be a result of the influence of the Sun's gravity, where significant amounts of material accreted onto the Sun. The plausible causes of the deficit in nebular mass values for Mars and the asteroid belt are explored by Weidenschilling.

The otherwise monotonic surface density suggests that there was previously significantly more mass of solid matter in the regions of the asteroid belt and Mars' ring of formation. Hypotheses as to what removed said matter involve planetismals orbiting in a more eccentric manner, entering the regions of anomaly and displacing the material which would have otherwise formed Mars.

CONCLUSION

[6] This investigation explored whether protoplanetary annular surface densities of the primitive Solar System could be derived from our knowledge of planetary compositions. Using the fundamental assumptions set by the MMSN model, an estimation of the total mass of Solar Nebula material necessary to form the planets was calculated. I obtained a general power law describing the variation of surface density of the disk, and this was in reasonable agreement with those determined using other fitting techniques. The power law arrived at here was of the general form $\sigma(r) = 2922.27r^{-1.92}$. The $\boldsymbol{\sigma_0}$ value here (2922.27gcm^{-2}) seems appropriate when compared to those obtained by Weidenschilling, Lewis and Hayashi, which were 4320, 3300 and 1700gcm^{-2} respectively. Furthermore, accepted values for the r decay index (-α) ranges from -1.5 to -2.0 in general. My value, although slightly higher in magnitude than those obtained by Weidenschilling and Hayashi, is as expected when ignoring the influence which Mars, the asteroids *and* Mercury have on the fitting. Consistent with the values obtained by researchers in this field, the minimum mass of the primitive material required to form the planets, was ~2.33×10^{-31}g ($\frac{1}{100}th$ of the mass of the Sun currently). This model is one which is very applicable to extrasolar systems also (the MMEN model), but makes several drastic assumptions in order to obtain the minimum mass value. A non-steady state interpretation of a general primitive planetary disk may significantly alter the results obtained by the MMSN. [6] This is because the inferred minimum mass may not have been present in the constructed annuli at the time of formation of each planet. The model in this context neglects the vertical structure of the disk as well, and assumes it is negligible when compared to the diameter. Furthermore, planetary migration is certainly a contributing factor as the disk evolves. Nevertheless, this model provides a strong foundation by which the mass distribution of the Solar Nebula and extrasolar nebulae can be constructed.

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APPENDIX

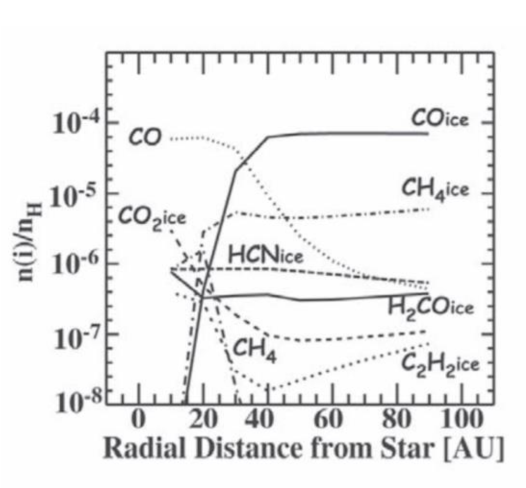

Figure 5: The radial distribution of molecules of an extrasolar protoplanetary disk. [3]

Table 1: Data set used in this investigation. F values are multiplied by the planets' respective masses to calculate the minimum mass of solar nebula material to form them.

Planet	Mass (x10^26 g)	*F*	Distance (AU)
Mercury	3.3	350	0.387
Venus	48.7	270	0.723
Earth	59.8	235	1
Mars	6.4	235	1.524
Asteroids	0.1	200	2.7
Jupiter	19,040	5	5.203
Saturn	5,695	8	9.523
Uranus	870	15	19.208
Neptune	1,032	20	30.087

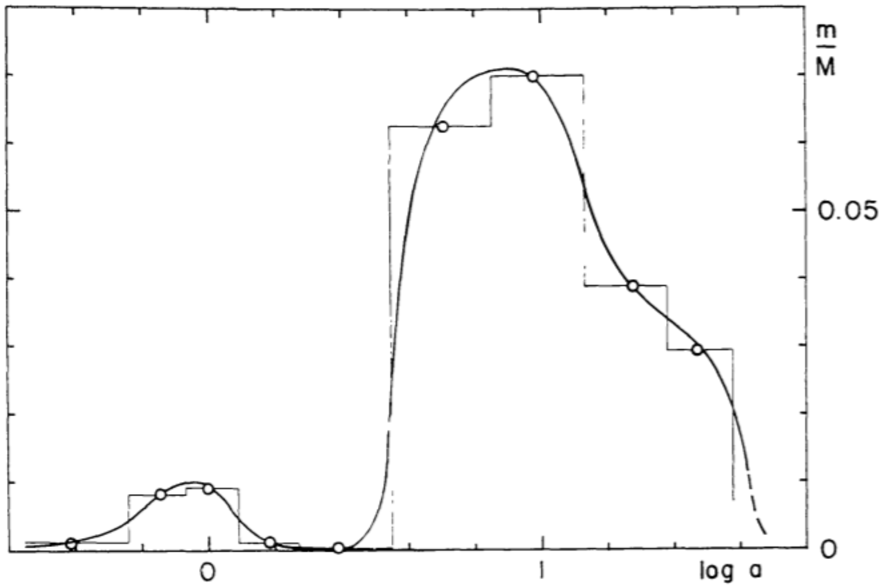

Figure 6: [6] Kuiper's model of the heliocentric distribution of mass in the primitive Solar Nebula.

ANALYTICAL INTEGRATION OF SURFACE DENSITY PROFILE:

Here, I have included a slightly different method of calculating the minimum mass of solar nebula material required to form the planets and asteroids of our Solar System. The integration was initially carried out by numerical means, but here I have done so analytically.

$\sigma(r) = \sigma_0 r^{-\alpha}$

By my fitting: $-\alpha \approx -1.9225... \approx -1.92$

$\sigma_0 \approx 2922.27... \, gcm^{-2} \approx 2922.3$

Mass of Solar nebula:

$$M = \int_0^{2\pi}\int_{R_{start}}^{R_{final}} \sigma(r)\, r\, dr\, d\theta = \iint_{\theta\ r} \sigma(r)\, dA$$

I set R_{start} to be Mercury's annulus' inner radius: $3.2762... \times 10^{12}$ cm = 0.218.. AU ≈ 0.2AU

and R_{Final} to be the average distance to Pluto ≈ 39.5AU

$$\therefore M = \int_0^{2\pi}\int_{0.2}^{39.5} r^{-1.92}\, r\, dr\, d\theta \quad \times \sigma_0$$

2

$$M = 2\pi \int_{0.2}^{39.5} r^{-1.92}\, r\, dr \quad \times \sigma_0$$

$$M = 2\pi \int_{0.2}^{39.5} r^{-0.92}\, dr \quad \times \sigma_0$$

$$= 2\pi \left[\frac{r^{0.08}}{0.08} \right]_{0.2}^{39.5} \quad \times \sigma_0$$

$$= \left[\left(\frac{39.5^{0.08}}{0.08} \right) - \left(\frac{0.2^{0.08}}{0.08} \right) \right] 2\pi \times 2922.3$$

$$= \left[16.77\ldots - 10.989\ldots \right] 2\pi \times 2922.3$$

$$= 36.3\ldots \times 2922.3$$

3

$$= 106131.0702$$

$$1\,AU = 1.496 \times 10^{13}\,cm$$

$$1\,cm = 6.684 \times 10^{-14}\,AU$$

$$\therefore \frac{106131.0702}{(6.684 \times 10^{-14})^2} = 2.375 \times 10^{31}\,g$$

= mass of solar nebula material required to form the planets

c.f. the numerical integration in code $\sim 2.33 \times 10^{31}\,g$

One can see within the hand-written attachment that I have compared the analytical result closely agrees with that result via numerical means. The analytical result for the solar nebula mass is approximately $0.0119M_{sun}$.

<u>ADDITIONAL INFORMATION:</u>

[7] Methods by which particles may accrete to subsequently form planetismals:

Some inter-grain forces which favour accretion include magnetic attraction, electrostatic interactions and the 'sticking' force because of the partial fusion of grains upon impact. Magnetic attraction occurs between ferromagnetic materials which below their Curie temperature (beyond which their magnetic properties are lost). The dipole moments of their atoms are aligned with any external magnetic field, resulting in attraction between grains, such as magnetite (a large constituent of meteors) at short-range. Electromagnetic attraction is evident between grains due to their positive charge and the electron distribution within silicate grains (which are dielectric).

CODE

```
import numpy as np
import matplotlib.pyplot as plt
import math as math
from numpy.polynomial.polynomial import polyfit
from scipy.integrate import simps
import pandas as pd

#Planetary Problem set 1- Prof Gallagher
#Alexandra Mulholland


#My technique: Given a table of the planets and their masses in grams,
#the factor F for each and their distance in AU from the Sun.


#Knowing these properties, we can determine F as follows:
#a planet’s density [gcm^-3] and knowledge of cosmic abundances allows us to
#estimate from what the planet consists and how much of this material is
#present in the disk from which the planet could form. For instance:
#The planet Mercury has a density of around 5.43gcm-3.
#We can assume that iron has condensed entirely and makes up 100% of the
#mass of Mercury. Our knowledge of cosmic abundances tells us
#that Fe makes up around 0.285% of the mass of the solar nebula (in Mercury's
#annulus). Equating these two values such that we are saying that 0.285% of
#the solar nebula mass makes up 100% of the mass of Mercury, we determine the
#F factor is 100/0.285 which is 350.
#Meaning the minimum mass required to make Mercury is 350 times its mass.

#The ‘solar composition mass’ is the solar nebula mass required to form the
#individual planets. I will plot these mass values against their distance
#from the sun and label them by their planet names.
#Knowing the F values and mass values of each planet, we can simply multiply
#these 2 values for each planet to determine the primordial solar nebula mass
#required to form each planet.

#-----------------------------------------------------------------------------
#PART 1: Plotting the mass of solar nebula material required to form each
#	planet versus distance
#-----------------------------------------------------------------------------

#Listing the planets' masses in an array, in the order in which they formed
#from the Sun. Then listing the F factors in this same order in an array and
#multiplying the two arrays. The result I will then plot against distances
#from the Sun, (these values I will also put in an array).
#Order=Mercury, Venus, Earth, Mars, Asteroids, Jupiter, Saturn, Uranus, Neptune

planets = np.array([3.3,48.7,59.8,6.4,0.1,19040.0,5695.0,870.0,1032.0])
#These values are in grams and are all multiplied by 10^26 below:
planet_mass=planets*10**26
print("mass in grams of each planet =",planet_mass)

#Inputting the F factor for each planet and asteroids
factor=np.array([350,270,235,235,200,5,8,15,20])
print('F=',factor)

#Multiplying the planet mass array and F factor array together: each array
#element is therefore the minimuum mass of primordial solar nebula required to
#form each planet: 'mass_nebular'
mass_nebular=np.multiply(factor,planet_mass)
print('Solar Nebula mass=',mass_nebular)
#These are very high values (~10^30g) and for the sake of asthetic appeal
#of the graph, I am going to divide all the values by 10^30, call this 'solar'
#and label the graph accordingly
```

```
solar=mass_nebular*(1/10**30)
print('Solar nebula mass (/10^30g)=',solar)


#Below is an array of the distances at which the planets and asteroid belt
#formed. The order of distances in the array is obviously in the order of the
#planetary masses, and hence the min solar nebula masses required to form them.
dist_au=np.array([0.387,0.723,1.0,1.524,2.7,5.203,9.523,19.208,30.087])
print('Distance from sun[au]=',dist_au)
#I am calculating the distances in cm as for subsequent tasks, calculations
#will be conducted in cgs units
#1AU=1.496*10^13cm
dist_cm=dist_au*1.496e13
#Note that these distances, both in AU and cm, are in the form of an ARRAY
print('Distance from sun [cm]=',dist_cm)


#Now to plot the minimum solar nebular mass required for each planet to form,
#against their distance from the Sun at which the subsequently formed (in AU)
plt.scatter(dist_au,solar, marker='d')
plt.xlabel("Distance from Sun [AU]")
plt.ylabel("Solar Nebula mass required [·10^(30)g]")

#Labelling each point on the plot, Mercury, Venus, Earth and Mars are too close
#in solar nebula mass value and so I have labelled them by their initials
#(except for mars which is Ma)
name=np.array(['M', 'V', '     E', 'Ma', '    A', '      Jupiter',
              '     Saturn','      Uranus', '    Neptune'])
for i, txt in enumerate(name):
    plt.annotate(txt,(dist_au[i]-1.0,solar[i]))
#I will entitle the graphs in the writeup, as they are conventionally written
#underneath the graphs
#Setting a range of the x axis so as to fit the names of the planets within
#the boundaries: 'Neptune' overlaps the boundary otherwise
plt.xlim([-2,39])
plt.show()
#Can see Venus in Earth are very close in value min nebular mass value
#Comment on this- densities, masses, F factor


#-----------------------------------------------------------------------------
#PART 2: Plotting the surface densities of the solar nebula material required
#       to form each planet
#-----------------------------------------------------------------------------

#***Plotting the surface density of minimum solar material required to form
#each planet. This is very tedious to calculate by hand using the
#Minimum Mass Model. The process is as follows: eg for Venus
#Distribute the minimum mass requirement for each planet’s formation over
#an annulus centred on the planet (as in, the planet lies within the annulus,
#between the inner and outer boundaries. The boundaries are defined as
#halfway to the planets on either side- for instance, for Venus, which lies
#between Mercury and Earth, is at 0.72AU from the sun. Mercury lies at 0.38AU
#and Earth, of course, at 1AU. Let the distance to Mercury be D1, to Venus D2
#and to Earth, D3. The inner boundary of the annulus therefore lies at
# 0.72AU – (0.72-0.38)/2 AU = 0.55AU=0.83×10^13 cm=r(inner).
#i.e. D2-((D2-D1)/2) = r_inner
#The outer boundary at 0.72AU+(1-0.72)/2 AU = 0.86AU = 1.29×10^13cm=r(outer).
#i.e. D2+((D3-D2)/2) = r_outer
#The area of this annulus is πr^2(outer)-πr^2(inner)=3.06×10^26cm^2.
#Therefore, knowing from part 1 the mass of nebula material required to make
#Venus (1.3149x10^30g), the surface density of the region of the solar nebula
#where Venus formed is (1.3149x10^30)g / (3.06x10^26)cm^2 = 4281gcm^-2


#I have decided to redefine the distances of each planet from the sun in
```

```
#the form of a list
distlist_cm=[5.7895200e12, 1.0816080e13, 1.496e13, 2.279904e13,
            4.0392e13,7.783688e13, 1.4246408e14, 2.8735168e14, 4.5010152e14]

#Creating an empty list for the inner radius and an empty list for the
#outer radius of each annulus
r_inner=[]
r_outer=[]
#And an empty list for the area of the annuli, the surface density (sigma)
#of each annulus, and the log function of surface denities
area=[]
sigma=[]
log_sig=[]

#Here, I am looping through all the distances in cm so the inner and outer
#radii are calculated for each annulus, and the corresponding areas of each.
#Then the surface density of each is calculated by dividing the solar mass
#nebula by these areas. So distlist_cm is a list of all the planets' distances.

#len(list) returns the length of the list, shown in the line below
print('Mumber of planets etc =',len(distlist_cm))
#This prints 9: M,V,E,M,ASTEROIDS,J,S,U,N

print('2nd index value in list =',distlist_cm[1])
#Showing that index numbering in python lists starts from zero, as Venus'
#(the second planet) distance is printed
print('Last index value=',distlist_cm[8])
#Neptune's distance- the last index in list

#We must remember that Mercury does not have a planet adjacent to it which
#is closer to the Sun
#And so the 'for' loop used later cannot be used for Mercury- I will calculate
#this separately. Mercury's index in the distance list is 0.

#The same scenario applies to Neptune, as Pluto is not considered a planet,
#Neptune has no planet further from the Sun by which we can estimate the
#outer radius of its annulus of formation- so we must calculate Neptune's
#annulus separately also.

#MERCURY ANNULUS AREA CALCULATION:
r_inner.append(distlist_cm[0]-((distlist_cm[1]-distlist_cm[0])/2))
#Many research papers which also apply the MMSN model assume that Mercury and
#Neptune extend equal distances inward and outward from their orbits. i.e.
#for Mercury, the distance halfway to Venus is considered to be BOTH its inner
#annulus radius AND its outer annulus radius (when subtracted and added to its
#own distance from the Sun, respectively)
#distlist_cm[1] is VENUS
r_outer.append(distlist_cm[0]+((distlist_cm[1]-distlist_cm[0])/2))
area.append((math.pi*np.square(r_outer[0]))-(math.pi*np.square(r_inner[0])))
sigma.append(mass_nebular[0]/area[0])
log_sig.append(math.log10(sigma[0]))

print('Mercury annulus outer radius =',r_outer[0])
print('Mercury annulus inner radius =',r_inner[0])
print('Mercury annulus area =',area[0])
print('Mercury surface density =',sigma[0])


#The 'for' loop statement below is simply for indeces which range from the
#second index (Venus' distance) to 7th index (Uranus). This is because we are
#calculating Mercury's and Neptune's annulus of formation and subsequent
#surface desnities separately from the rest

#REMAINING PLANETS AND ASTEROID BELT ANNULUS AREA CALCULATION:
#Ignoring Mercury as has no planet on inner side of it, and Neptune as has no
#planet outside its orbit and calculate them separately.
```

```
length=len(distlist_cm)
print('length=',len(distlist_cm))

for i in range(1,8):
    r_inner.append(distlist_cm[i]-((distlist_cm[i]-distlist_cm[i-1])/2))
    #print('r_inner=',r_inner[i]) #checking to make sure goes
    #up to but NOT inlcuding Neptune
    #above, I am calculating the inner radius of each annulus, see Venus
    #example above for comparison
    r_outer.append(distlist_cm[i]+((distlist_cm[i+1]-distlist_cm[i])/2))
    #print('r_outer=',r_outer[i])
    #Again, see Venus example equation above for clarification***
    area.append((math.pi*(np.square(r_outer[i])))-math.pi*(np.square(r_inner[i])))
    sigma.append(mass_nebular[i]/area[i])
    log_sig.append(math.log10(sigma[i]))

#NEPTUNE'S OUTER RADIUS FOR ITS ANNULUS OF FORMATION
r_inner.append(distlist_cm[8]-((distlist_cm[8]-distlist_cm[7])/2))
r_outer.append(distlist_cm[8]+((distlist_cm[8]-distlist_cm[7])/2))
area.append((math.pi*np.square(r_outer[8]))-(math.pi*np.square(r_inner[8])))
sigma.append(mass_nebular[8]/area[8])
log_sig.append(math.log10(sigma[8]))
print('Neptune inner=',r_inner[8]) #Same as Uranus' outer radius
print('Neptune outer=',r_outer[8]) #Correct: I checked both by hand

#A planet's annulus' inner radius should be the same as the adjacent planet's
#(which is closer to the sun's) annulus' outer radius


#au_conversion=>>1.496e13cm
distlist_au= [dist/1.496e13 for dist in distlist_cm]
print('distances in AU from sun =',distlist_au)
#Converting my list of distances in cm to AU
#I am labelling my points again but this time with their full names
#as they will fit without overlap
names=np.array(['Mercury', 'Venus', 'Earth', 'Mars', 'Asteroids', 'Jupiter',
               'Saturn','Uranus', 'Neptune'])
plt.scatter(distlist_au,log_sig, marker='x')
for i, txt in enumerate(names):
    plt.annotate(txt,(distlist_au[i],log_sig[i]+0.2))
plt.xscale('log')#Using a logarithmic scale for the distance axis
plt.xlabel('Distance from Sun [AU]')
plt.ylabel('log(surface density of annulus[gcm^-2])')
plt.xlim([10**(-0.5),10**1.8])
plt.ylim([-1,4.5])
plt.show()
#setting limits on each axes so the names of the planets fit on graph
#Again, I'll put titles in the writeup

#Plotting the surface density as a function of distance, not on a logarithmic
#scale, so as to show the clear exponential of the data
plt.scatter(distlist_au,sigma,marker='x')
plt.xlabel('Distance of the Planet from the Sun [AU]')
plt.ylabel("Surface density of each planet's annulus of formation [gcm^-2]")
plt.xlim(-1,35.5)
plt.ylim(-200,4700)
for i, txt in enumerate(names):
    plt.annotate(txt,(distlist_au[i],sigma[i]))
plt.show()
#Plotting the values on a non-log scale shows clearly a trend which would fit
#a power law. However, anomalies in the data are obvious eg Mercury, Mars and
#the asteroid belt.

#-----------------------------------------------------------------------------
#PART 3: Fitting a surface density function of the form of a power law
#-----------------------------------------------------------------------------
```

```
print('Distance list in AU =',distlist_au)

#Making arrays of sigma (the surface density as a function of r, the
#distance from the sun) and the distances excluding mars and asteroids
#for best fit line

#We are attempting to fit a power law of the form sigma(r)=(sigma0)r^-(alpha)
#to our data of surface densities of solar material to form the planets

#My technique to complete this fitting will be as follows:
#I am going to ignore the anomalous nature of Mercury's, Mars' and the
#asteroids' surface densities. Numerous research papers on the subject have
#various ways of dealing with this data, some of which only account for the
#planets whose surface densities follow the power law described. See writeup
#for references to said papers.

#Hence, I am going to plot the data on a log scale, ignore Mercury, Mars and
#the asteroids, and plot a LOBF for the remaining planets. From this, I will
#determine the value of the gradient and the y-axis intercept. These values
#represent -alpha and log(sigma0) respectively.

#Here, I am printing all the data point values so can make sure that I have
#only included the non-anomalous data later.

#pd.DataFrame just prints the data in the form of a column so it is easier to
#read and compare values for my wee brain
print('Log(sigma), all data points = ',pd.DataFrame(log_sig))
print('r in AU, all data points = ', pd.DataFrame(distlist_au))

#Taking the log of all values of r in AU-->
log_distlist_au= [math.log10(i) for i in distlist_au]
print('log(r), all data points',pd.DataFrame(log_distlist_au))


#sigma(r)=sigma0 * r^-(alpha)
#log(sigma(r))=log(sigma0)-(alpha)log(r)

#Recall index notation: excluding index numbers 0,3 and 4, which are Mercury,
#Mars and the asteroids.
no_anomaly_logsig=np.array([log_sig[i] for i in [1,2,5,6,7,8]])
print('log(sigma) no Me, Ma or As',pd.DataFrame(no_anomaly_logsig))
#The above prints the expected values

no_anomaly_distances=np.array([log_distlist_au[j] for j in [1,2,5,6,7,8]])
print('log(r), no Me, Ma or As = ', pd.DataFrame(no_anomaly_distances))

#Now to plot the graph of log(sigma) versus log(distance in AU, r) with only
#the non-anomalous data points
x=no_anomaly_distances
y=no_anomaly_logsig
b, m = polyfit(x, y, 1)
#Ployfit = intrinsic to python, plotting in form of a straight line
plt.plot(x, y, 'bx')
plt.plot(x, b + m * x, 'c-')
plt.xlabel('Log(Distance [AU])')
plt.ylabel('log(Surface density [gcm^-2])')
#Defining and plotting the anomalous values of Mercury's, Mars' and the
#asteroids' surface densities and distances in AU-->
anomalous_dist=np.array([log_distlist_au[j] for j in [0,3,4]])
anomalous_logsig=np.array([log_sig[i] for i in [0,3,4]])
marker_colors = ['r']
plt.scatter(anomalous_dist,anomalous_logsig, c=marker_colors)
plt.show()
print('intercept = log10(sigma0) = ',b)
print('Sigma0 = ',10**b)
```

```
print('gradient = -alpha = ', m)

#-----------------------------------------------------------------------------
#PART 4: Total mass of solar nebula required to form the planets
#-----------------------------------------------------------------------------


#Adapting the fitting technique I have used to calculate the total minimum
#mass of solar nebula material required to form the planets and the asteroids

#M = integral of the surface density as fn of r integrated from the inner
#radius of the entire disk to the outer boundary of the disk
#Must also integrate over the entire azimuthal range. No polar integration
#is calculated in this model, as this would require assuming vertical density
#variation. We are essentially assuming the thickness of the disk is negligible

#So we have established that surface density varies as ~2922r^-1.92
#I will now integrate this power law over the area of the disk

#Must use a python Numerical integration technique. I will try scipy.integrate.
#I will also integrate from Mercury's annulus' inner radius to Pluto's distance

#To do this, I am going to add the distance to Pluto to the list of inner
#radii of each planet's annulus (but, this distance is obviously not an inner
#radius value!)
r_inner.append(5.9e14)
print('List of inner radii', pd.DataFrame(r_inner))

#Creating an array of inner annuli values so I can use this as a range over
#which I will complete the numerical integration of the surface density
r=np.array([r_inner[i] for i in range(len(r_inner))])

#Defining the function which I am going to integrate--> Here, I have used the
#printed values of alpha (the gradient value) and sigma0 (log10 of the
#intercept). This is the surface density relation as a function of r by my
#fitting technique. You will also see I have multiplied this by r as this is
#part of the polar integration (rdrd(theta)).
#Furthermore, the r values used are in cm and in order for the surface density
#relation to hold, the r's in the power law must be divided by 1AU in cm
def sigma_func(r):
    return (r*(2922.271172853227*(r/1.496e13)**(-1.922520218628957)))
#Using Simpson's rule - an intrinsic function of python
mass = simps(sigma_func(r),r)
print( 'Radial mass: ',mass)

#Must also mustiply by 2pi as we are integrating over the azimuthal area
#and the surface density function is independent of this angle (and so CAN
#simply multiply by 2pi)
mass_of_nebula=2*np.pi*mass
print('Solar Nebula mass [grams] = ',mass_of_nebula)
#This value is in grams and produces 2.3336528054865936e+31g
#Mass of sun=1.989 × 10^33 g and we predict the solar nebula should be about
#0.01*Msun

#I have completed the integral analytically by hand, so the previous code is
#merely an approximation of the surface density integral

#Calculating this ratio exactly, just for fun!
m_sun=1.989e33
ratio=mass_of_nebula/m_sun
print('Ratio of solar nebula mass to Solar mass = ',ratio)
#Success :)
```
