REGRESI LINIER

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Bahan Bacaan

- Pedregosa, F., Varoquaux, G., Gramfort, A., Michel, V., Thirion, B., Grisel, O., Blondel, M., Prettenhofer, P., Weiss, R., Dubourg, V. and Vanderplas, J., 2011. Scikit-learn: Machine learning in Python. the Journal of machine Learning research, 12, pp.2825-2830.
- Varoquaux, G., Buitinck, L., Louppe, G., Grisel, O., Pedregosa, F. and Mueller, A., 2015. Scikit-learn: Machine learning without learning the machinery. *GetMobile: Mobile Computing and Communications*, *19*(1), pp.29-33.
- https://scikit-learn.org/stable/index.html

Pengertian Regresi

x : variabel bebas y : variabel tak bebas

	ENGINESIZE	CYLINDERS	FUELCONSUMPTION_COMB	CO2EMISSIONS
0	2.0	4	8.5	196
1	2.4	4	9.6	221
2	1.5	4	5.9	136
3	3.5	6	11.1	255
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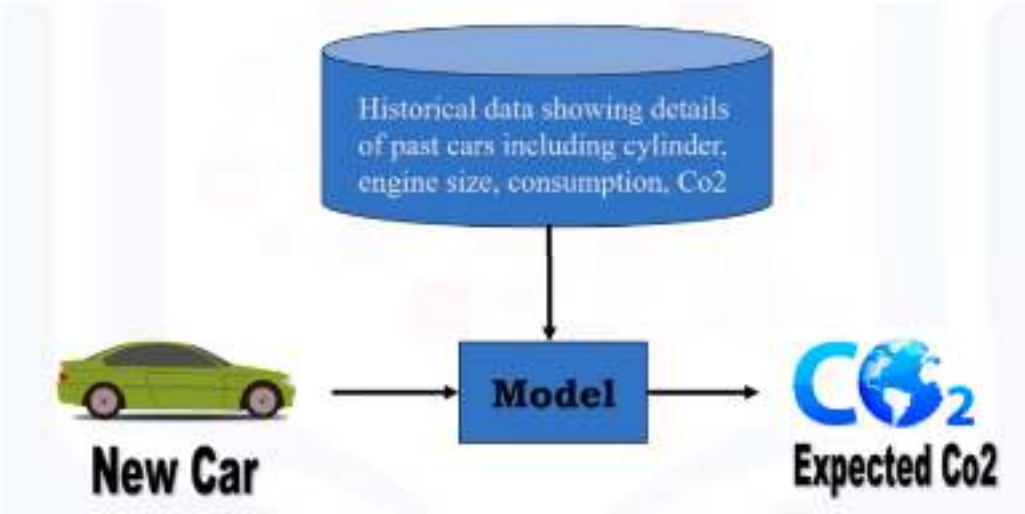
9	2.4	4	9.2	?

Regresi adalah proses
Memprediksi nilai kontinu

Model Regresi



Tipe Model Regresi

Regresi Sederhana:

- Regresi sederhana linier
- Regresi sederhana non-linier
- Contoh: memprediksi co2emission vs EngineSize dari semua mobil.

Regresi Variabel Jamak:

- Regresi variabel jamak linier
- Regreasi variabel jamak non-linier
- Contoh: meprediksi co2emission vs EngineSize dan Cylinders dari semua mobil.

Apa itu telaah data (*data understanding*)?

- Dilakukan setelah problem bisnis terdefinisikan sebagai hasil tahapan business understanding.
- Tujuan: mendapatkan gambaran utuh atas data.
- Dilanjutkan ke persiapan data (data preparation), jika pemahaman awal data cukup atau kembali ke business understanding jika definisi permasalahan bisnis harus direvisi.

Aplikasi Regresi

- Prakiraan penjualan produk
- Analisis kepuasan
- Estimasi harga
- Pendapatan pekerjaan
- dst.

Algoritma Regresi

- Linier Regression
- Polynomial Regression
- Support Vector Regression
- Decision Tree Regression
- Random Forest Regression
- LASSO Regression
- ANN Regression
- K-NN Regression
- dst.

Regresi Linier Sederhana

Regresi Linier Untuk Memprediksi Nilai Kontinu

x : variabel bebas

y : variabel tak bebas

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Nilai kontinyu / numerik

Topologi Regresi Linier

Regresi Linier Sederhana:

- Memprediksi co2emission vs EngineSize dari semua mobil
  - variabel bebas (x): EngineSize
  - variabel tak bebas (y): co2emission

Regresi Linier Variabel Jamak:

- Memprediksi co2emission vs EngineSize dan Cylinders dari semua mobil
  - variabel bebas (x): EngineSize, Cylinders, dst.
  - variabel tak bebas (y): co2emission

Cara Kerja Regresi Linier

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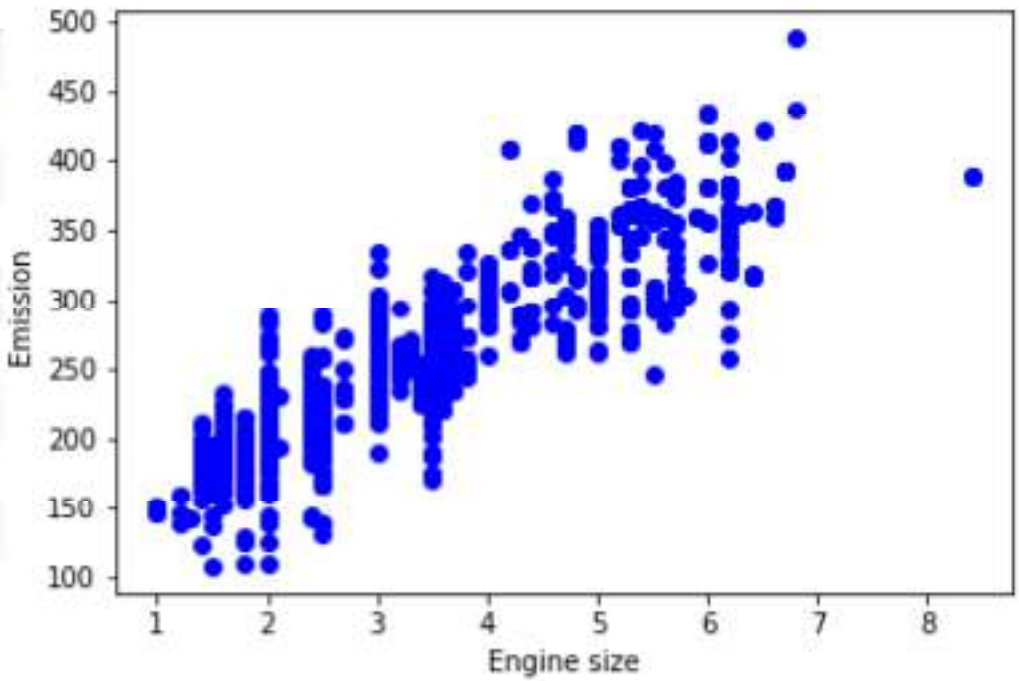

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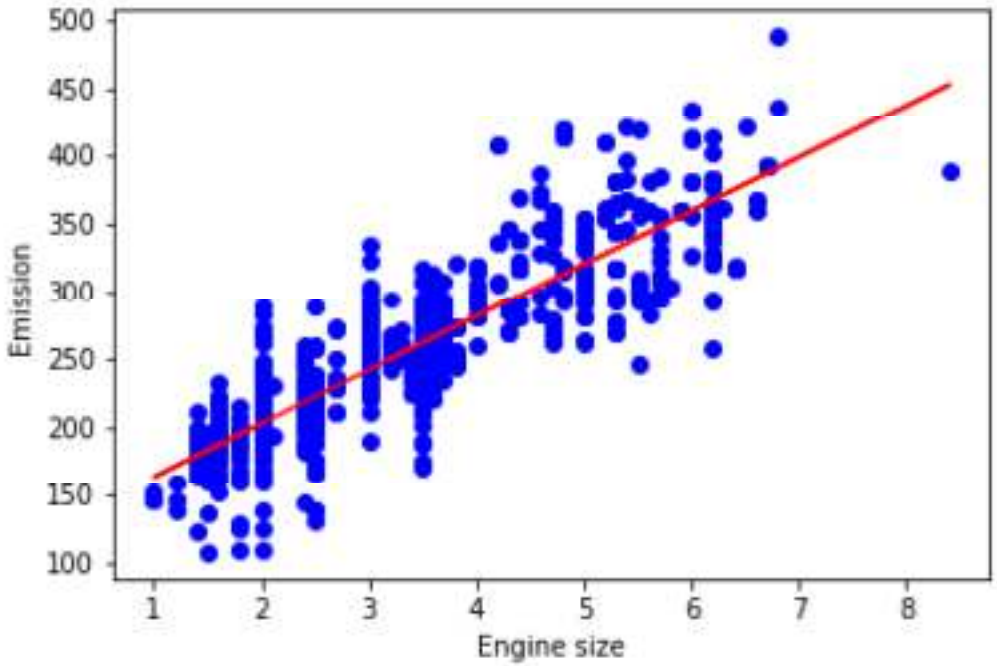

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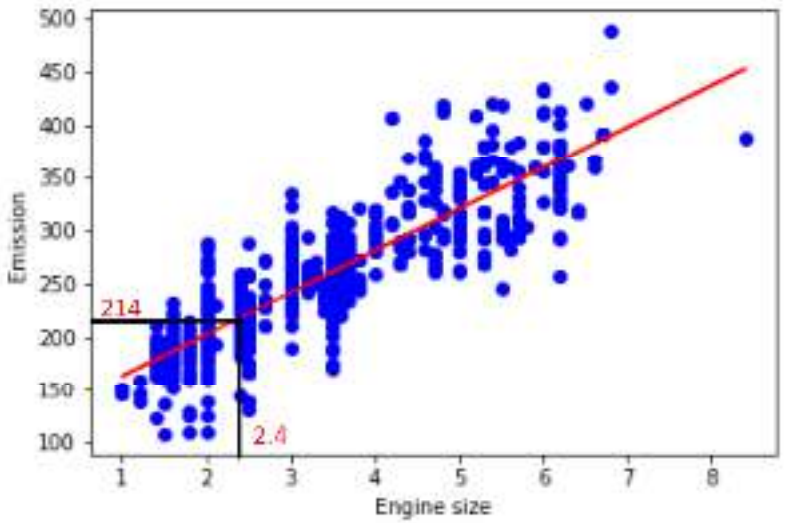


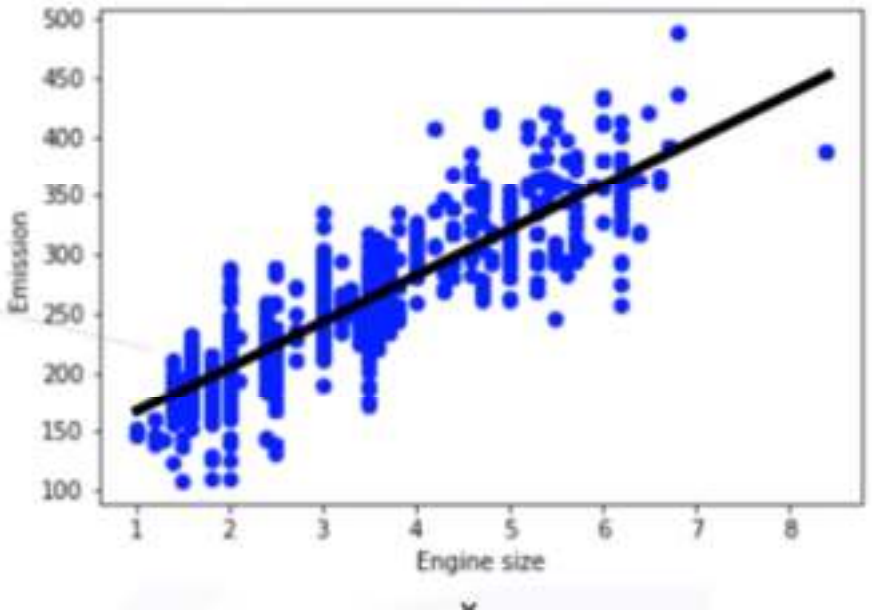
Cara Kerja Regresi Linier

$$\hat{y} = \theta_0 + \theta_1 x_1$$

response variable

a single predictor

y



Cara Mencari Parameter Model Terbaik

$x_1 = 5.4$ independent variable

$y = 250$ actual Co2 emission of x1

$\hat{y} = \theta_0 + \theta_1 x_1$

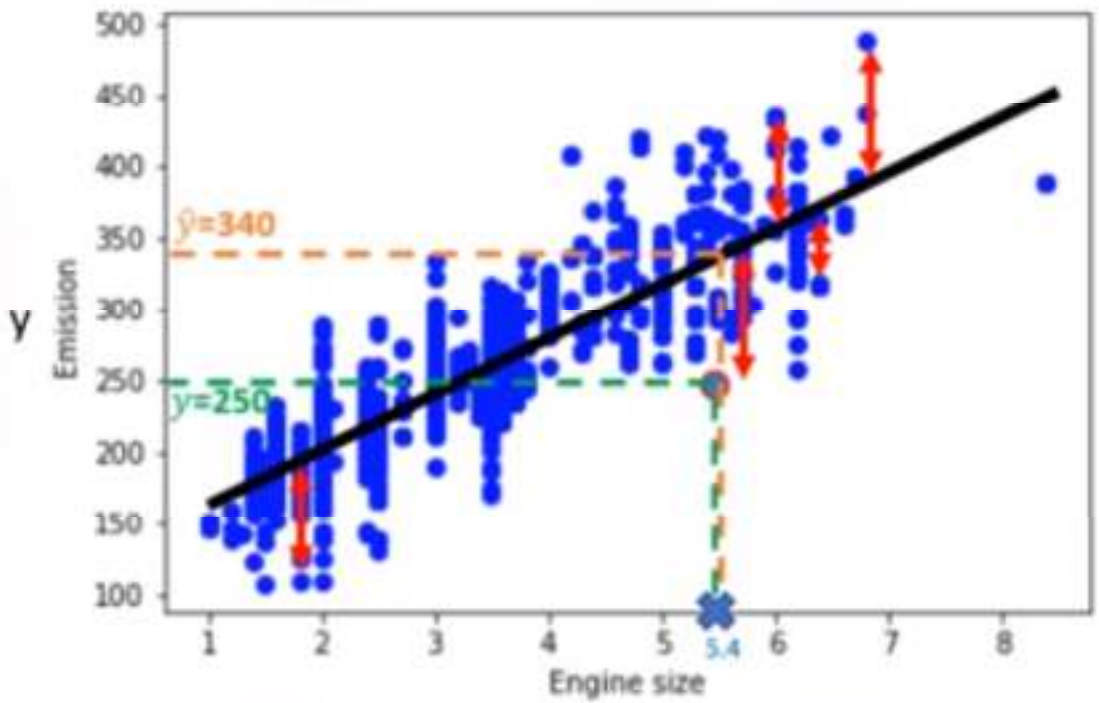
$\hat{y} = 340$ the predicted emission of x1

Error = $y - \hat{y}$

$= 250 - 340$

$= -90$

$$MSE = \frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2$$



Estimasi Parameter

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X_1 (ENGINESIZE) $\quad$ y (CO2EMISSIONS)

$$\hat{y} = \theta_0 + \theta_1 x_1$$

$$\theta_1 = \frac{\sum_{i=1}^{s}(x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{s}(x_i - \bar{x})^2}$$

$$\bar{x} = (2.0 + 2.4 + 1.5 + \dots)/9 = 3.34$$

$$\bar{y} = (196 + 221 + 136 + \dots)/9 = 256$$

$$\theta_1 = \frac{(2.0 - 3.34)(196 - 256) + (2.4 - 3.34)(221 - 256) + \dots}{(2.0 - 3.34)^2 + (2.4 - 3.34)^2 + \dots}$$

$$\theta_1 = 39$$

$$\theta_0 = \bar{y} - \theta_1 \bar{x}$$

$$\theta_0 = 256 - 39 * 3.34$$

$$\theta_0 = 125.74$$

$$\hat{y} = 125.74 + 39 x_1$$

Prediksi dengan Model Regresi Linier

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$$\hat{y} = \theta_0 + \theta_1 x_1$$

$$Co2Emission = \theta_0 + \theta_1\ EngineSize$$

$$Co2Emission = 125 + 39\ EngineSize$$

$$Co2Emission = 125 + 39 \times 2.4$$

$$Co2Emission = 218.6$$

Kelebihan Regresi Linier

- Ringan
- Tidak perlu tuning parameter
- Mudah dipahami dan diinterpretasikan

Lab

- Jalankan file Jupyter Notebook untuk Regresi Linier Sederhana

Regresi Linier Variabel Jamak

Contoh Regresi Linier Variabel Jamak

Efektivitas variabel-variabel bebas terhadap prediksi

- Apakah kegelisahan, kehadiran dosen, dan jenis kelamin mempunyai efek pada kinerja ujian mahasiswa?

Prediksi dampak perubahan

- Seberapa besar kenaikan/penurunan tekanan darah terhadap kenaikan/penurunan BMI dari pasien?

Prediksi Nilai Kontinu pada Regresi Linier Variabel lamak

X: Independent variable Y: Dependent variable

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$$Co2\ Em = \theta_0 + \theta_1 Engine\ size + \theta_2 Cylinders + \ ...$$

$$\hat{y} = \theta_0 + \theta_1 x_1 + \theta_2 x_2 + \ ... + \theta_n x_n$$

$$\hat{y} = \theta^T X$$

$$\theta^T = [\theta_0, \theta_1, \theta_2, ...] \qquad X = \begin{bmatrix} 1 \\ x_1 \\ x_2 \\ ... \end{bmatrix}$$

MSE Untuk Menunjukkan Error Pada Model

$\hat{y} = \theta^T X$

$\hat{y}_i = 140$ the predicted emission of x_i

$y_i = 196$ actual value of x_i

$y_i - \hat{y}_i = 196 - 140 = 56$ residual error

$$MSE = \frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2$$

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Estimasi Parameter Regresi Linier Variabel Jamak

Cara-cara mengestimasi parameter θ

Least Squares
- Operasi aljabar linier
- Perlu waktu yang lama untuk dataset yang besar (lebih dari 10000 baris)

Algoritma optimisasi
- Gradient Descent
- Metode yang sesuai apabila dataset sangat besar

Prediksi Menggunakan Regresi Linier Variabel Jamak

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$$\hat{y} = \theta^T X$$

$\theta^T = [125, 6.2, 14, \ldots]$

$$\hat{y} = 125 + 6.2x_1 + 14x_2 +$$

$Co2Em = 125 + 6.2EngSize + 14\ \text{Cylinders} + \ldots$

$Co2Em = 125 + 6.2 \times 2.4 + 14 \times 4 + \ldots$

$Co2Em = 214.1$

Lab

- Jalankan file Jupyter Notebook untuk Regresi Linier Variabel Jamak

Terima Kasih