FYS3150/FYS4150
Project 1

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1 Motivation

Solutions of the Poissons equation are a vital component to many aspects physics.

2 a)

Formulation

Given the following ODE with boundary conditions,

$$-u''(x) = f(x), \quad x \in (0,1), \quad u(0) = u(1) = 0. \tag{1}$$

and the dicretized form following a symmetric Taylor expansion

$$-\frac{v_{i+1} + v_{i-1} - 2v_i}{h^2} = f_i \quad \text{for} \quad i = 1, \dots, n, \quad v_0 = v_n = 0 \tag{2}$$

we are going to show it can be written as system of linear equations of the form:

$$\mathbf{Av} = \tilde{\mathbf{b}}, \tag{3}$$

Solution

Multipling the discretized equation (2) by h^2 we get:

$$-v_{i-1} + 2v_i - v_{i+1} = h^2 f_i \quad \text{for} \quad i = 1, \dots, n$$

Filling in for i and choosing $\tilde{b_i} = h^2 f_i$ we obtain the following set of equations:

$$\begin{aligned}
2v_1 - v_2 &= \tilde{b_1} \\
-v_1 + 2v_2 - v_3 &= \tilde{b_2} \\
&\vdots \\
-v_{i-1} + 2v_i - v_{i+1} &= \tilde{b_i} \\
&\vdots \\
-v_{n-1} + 2v_n &= \tilde{b_n}
\end{aligned}$$

Now one can easily see that this system of linear equations can written on the form of (3), where

$$\mathbf{A} = \begin{pmatrix} 2 & -1 & 0 & \dots & \dots & 0 \\ -1 & 2 & -1 & 0 & \dots & \dots \\ 0 & -1 & 2 & -1 & 0 & \dots \\ & \dots & \dots & \dots & \dots & \dots \\ 0 & \dots & & -1 & 2 & -1 \\ 0 & \dots & & 0 & -1 & 2 \end{pmatrix}, \quad \mathbf{v} = \begin{pmatrix} v_1 \\ v_2 \\ \dots \\ \dots \\ \dots \\ v_n \end{pmatrix}, \quad \tilde{\mathbf{b}} = \begin{pmatrix} \tilde{b}_1 \\ \tilde{b}_2 \\ \dots \\ \dots \\ \dots \\ \tilde{b}_n \end{pmatrix}.$$