Parameters

- Link Length L1=L2=1702L_1 = L_2 = 1702L1=L2=1702 mm
- Mass m=15m = 15m=15 kg (mass is given but irrelevant for tension in space due to zero gravity)
- Distance between fixed end and free end in stowed condition ds=100d_s = 100ds=100 mm (0.1 m)
- Distance between fixed end and free end in deployed condition dd=3500d_d = 3500dd=3500 mm (3.5 m)

Tension Calculation

Force Analysis in Stowed Condition

- Hinges are loaded with torsion springs:
  - Hinge 1: 90 degrees
  - Hinge 2: 180 degrees
  - Hinge 3: 90 degrees

1. **Net Torsional Force**:
   - Spring constants: Let the spring constant be kkk (Nm/rad).
   - Torque due to each hinge spring:
     - T1=k·π2T_1 = k \cdot \frac{\pi}{2}T1=k·2π
     - T2=k·πT_2 = k \cdot \piT2=k·π
     - T3=k·π2T_3 = k \cdot \frac{\pi}{2}T3=k·2π
   - Net torque exerted by the springs on one side:
     - Tnet=k·π2+k·π+k·π2=2kπT_{\text{net}} = k \cdot \frac{\pi}{2} + k \cdot \pi + k \cdot \frac{\pi}{2} = 2k\piTnet=k·2π+k·π+k·2π=2kπ
   - Since there are hinges on both sides, the total torque is:
     - Ttotal=2×Tnet=4kπT_{\text{total}} = 2 \times T_{\text{net}} = 4k\piTtotal =2×Tnet=4kπ
2. **String Tension Calculation in Stowed Condition**:
   - The string tension must balance the total torque.
   - The string is anchored at the fixed end and attached to the free end.
   - Fs×0.1=4kπF_s \times 0.1 = 4k\piFs×0.1=4kπ
   - Fs=4kπ0.1=40kπF_s = \frac{4k\pi}{0.1} = 40k\piFs=0.14kπ=40kπ

Tension in the String During Deployed Condition

- In the deployed condition, the torsion springs are in their free position and do not exert force on the links.
- The links are latched, and the string only needs to maintain the position.
- The residual tension required to keep the string taut is minimal, FdF_dFd.

Summary of Tensions

- **Stowed Condition**:
    - Fs=40kπF_s = 40k\piFs=40kπ (tension required to counteract the spring forces)
- **Deployed Condition**:
    - Minimal tension, FdF_dFd, just to keep the string taut.

Numerical Example

Let's assume a hypothetical spring constant kkk.

Assume k=1k = 1k=1 Nm/rad (for simplicity):

- **Stowed Condition**:
    - Fs=40×1×π≈125.6F_s = 40 \times 1 \times \pi \approx 125.6Fs=40×1×π≈125.6 N
- **Deployed Condition**:
    - Fd≈0F_d \approx 0Fd≈0 N (just enough to keep the string taut)

Conclusion

The tension in the string during the stowed condition is significantly higher due to the need to counteract the forces exerted by the torsion springs in the hinges. This tension reduces to a minimal value once the mechanism is deployed and latched in place.

To calculate the string tension while the mechanism is deploying at a constant velocity vvv, we need to consider the dynamic forces involved. Since the mechanism deploys gradually, the string tension will be affected by the rate of change of the angular positions of the links and the resulting forces on the hinges.

Parameters and Assumptions:

- Let vvv be the constant velocity at which the free end is moving away from the fixed end.
- kkk is the spring constant of the torsion springs.
- The hinges are initially loaded as described, and the deployment gradually releases the torsion springs.
- The deployment distance changes from ds=100d_s = 100ds=100 mm (0.1 m) to dd=3500d_d = 3500dd=3500 mm (3.5 m).

Key Points:

- The torsion springs exert forces that create torques at the hinges.

- The string controls the deployment speed, balancing the forces from the torsion springs.
- The string tension during deployment needs to counteract the torques from the springs dynamically.

Steps to Determine String Tension During Deployment:

1. **Determine Angular Velocities:**
   - As the mechanism deploys, the links rotate about the hinges.
   - The angular velocity ω of each link can be related to the linear velocity v of the free end.
2. **Relation Between Linear and Angular Velocity:**
   - For each link of length L, the angular velocity ω is related to the linear velocity v by $\omega = \frac{v}{L}$.
   - Here, $L = 1702$ mm = 1.702 m (length of each link).
3. **Calculate the Dynamic Forces:**
   - The force due to the torsion spring in each hinge changes as the mechanism deploys.
   - The torque T exerted by each spring is a function of the angle θ.
   - The tension in the string F_t must balance the resultant torques dynamically.
4. **Dynamic Equilibrium:**
   - During deployment, the net torque due to the springs and the force exerted by the string must be in dynamic equilibrium.
   - The string tension F_t at any instant can be determined by the torques from the springs and the rate of deployment.

String Tension During Deployment:

For simplicity, let's assume a linear deployment rate and calculate the tension at an arbitrary position d between 0.1 m and 3.5 m.

1. **Torque from Hinges During Deployment:**
   - The angular displacement changes linearly with deployment.
   - Let $\theta(t)$ be the angular displacement at time t.
   - The torque at any hinge is $T(t) = k \cdot \theta(t)$.
2. **Net Torque at Intermediate Position:**
   - At an intermediate deployment position d, the angular displacements for each hinge can be interpolated between the initial and final states.
   - If $d(t)$ is the deployment distance at time t, $d(t) = v \cdot t$.
3. **Calculate Angular Displacements:**

- For Hinge 1 (initially 90 degrees): θ1(t)=π2⋅(1−d(t)−0.13.4)\theta_1(t) = \frac{\pi}{2} \cdot \left(1 - \frac{d(t) - 0.1}{3.4}\right)θ1(t)=2π⋅(1−3.4d(t)−0.1).
- For Hinge 2 (initially 180 degrees): θ2(t)=π⋅(1−d(t)−0.13.4)\theta_2(t) = \pi \cdot \left(1 - \frac{d(t) - 0.1}{3.4}\right)θ2(t)=π⋅(1−3.4d(t)−0.1).
- For Hinge 3 (initially 90 degrees): θ3(t)=π2⋅(1−d(t)−0.13.4)\theta_3(t) = \frac{\pi}{2} \cdot \left(1 - \frac{d(t) - 0.1}{3.4}\right)θ3(t)=2π⋅(1−3.4d(t)−0.1).

4. **Net Torque from Springs:**
    - T1(t)=k⋅θ1(t)T_1(t) = k \cdot \theta_1(t)T1(t)=k⋅θ1(t)
    - T2(t)=k⋅θ2(t)T_2(t) = k \cdot \theta_2(t)T2(t)=k⋅θ2(t)
    - T3(t)=k⋅θ3(t)T_3(t) = k \cdot \theta_3(t)T3(t)=k⋅θ3(t)
    - Total torque on one side: Tnet(t)=T1(t)+T2(t)+T3(t)T_{\text{net}}(t) = T_1(t) + T_2(t) + T_3(t)Tnet(t)=T1(t)+T2(t)+T3(t)
    - Total torque considering both sides: Ttotal(t)=2×Tnet(t)T_{\text{total}}(t) = 2 \times T_{\text{net}}(t)Ttotal(t)=2×Tnet(t)
5. **String Tension During Deployment:**
    - The string tension FtF_tFt at any distance ddd can be calculated using the net torque:
        - Ft(t)×d(t)=Ttotal(t)F_t(t) \times d(t) = T_{\text{total}}(t)Ft(t)×d(t)=Ttotal(t)
        - Ft(t)=Ttotal(t)d(t)F_t(t) = \frac{T_{\text{total}}(t)}{d(t)}Ft(t)=d(t)Ttotal(t)

Numerical Example:

Assume a hypothetical spring constant k=1k = 1k=1 Nm/rad and v=0.1v = 0.1v=0.1 m/s (as an example):

1. At t=0t = 0t=0 (initially):
    - d(t)=0.1d(t) = 0.1d(t)=0.1 m
    - θ1(0)=π2\theta_1(0) = \frac{\pi}{2}θ1(0)=2π, θ2(0)=π\theta_2(0) = \piθ2(0)=π, θ3(0)=π2\theta_3(0) = \frac{\pi}{2}θ3(0)=2π
    - Tnet(0)=2πT_{\text{net}}(0) = 2\piTnet(0)=2π
    - Ttotal(0)=4πT_{\text{total}}(0) = 4\piTtotal(0)=4π
    - Ft(0)=4π0.1≈125.6F_t(0) = \frac{4\pi}{0.1} \approx 125.6Ft(0)=0.14π≈125.6 N
2. At t=35t = 35t=35 seconds (final):
    - d(t)=3.5d(t) = 3.5d(t)=3.5 m
    - θ1(35)≈0\theta_1(35) \approx 0θ1(35)≈0, θ2(35)≈0\theta_2(35) \approx 0θ2(35)≈0, θ3(35)≈0\theta_3(35) \approx 0θ3(35)≈0
    - Tnet(35)≈0T_{\text{net}}(35) \approx 0Tnet(35)≈0

- Ttotal(35)≈0T_{\text{total}}(35) \approx 0Ttotal(35)≈0
- Ft(35)≈0F_t(35) \approx 0Ft(35)≈0

3. At intermediate t=17.5t = 17.5t=17.5 seconds (halfway):
   - d(t)=1.75d(t) = 1.75d(t)=1.75 m
   - θ1(17.5)=π2⋅0.5≈0.785\theta_1(17.5) = \frac{\pi}{2} \cdot 0.5 \approx 0.785θ1(17.5)=2π⋅0.5≈0.785 rad
   - θ2(17.5)=π⋅0.5≈1.57\theta_2(17.5) = \pi \cdot 0.5 \approx 1.57θ2(17.5)=π⋅0.5≈1.57 rad
   - θ3(17.5)=π2⋅0.5≈0.785\theta_3(17.5) = \frac{\pi}{2} \cdot 0.5 \approx 0.785θ3(17.5)=2π⋅0.5≈0.785 rad
   - Tnet(17.5)=k⋅(0.785+1.57+0.785)≈3.14T_{\text{net}}(17.5) = k \cdot (0.785 + 1.57 + 0.785) \approx 3.14Tnet(17.5)=k⋅(0.785+1.57+0.785)≈3.14
   - Ttotal(17.5)=2×3.14≈6.28T_{\text{total}}(17.5) = 2 \times 3.14 \approx 6.28Ttotal(17.5)=2×3.14≈6.28
   - Ft(17.5)=6.281.75≈3.59F_t(17.5) = \frac{6.28}{1.75} \approx 3.59Ft(17.5)=1.756.28≈3.59 N

These values illustrate how the tension changes dynamically with deployment.

To calculate the holding torque needed for the motor to keep the string taut during the stowed condition, we need to ensure that the motor can counteract the total torque exerted by the torsion springs in the hinges.

Parameters and Assumptions

- Spring stiffness k=0.0398k = 0.0398k=0.0398 Nm/rad
- Initial angular displacements:
  - Hinge 1: θ1,stowed=90∘=π2\theta_{1, \text{stowed}} = 90^\circ = \frac{\pi}{2}θ1,stowed=90∘=2π radians
  - Hinge 2: θ2,stowed=180∘=π\theta_{2, \text{stowed}} = 180^\circ = \piθ2,stowed=180∘=π radians
  - Hinge 3: θ3,stowed=90∘=π2\theta_{3, \text{stowed}} = 90^\circ = \frac{\pi}{2}θ3,stowed=90∘=2π radians

Calculate Torque from Each Hinge

The torque exerted by each hinge spring in the stowed condition is given by:

- Torque at Hinge 1:

T1,stowed=k⋅θ1,stowed=0.0398⋅π2T_{1, \text{stowed}} = k \cdot \theta_{1, \text{stowed}} = 0.0398 \cdot \frac{\pi}{2}T1,stowed=k⋅θ1,stowed=0.0398⋅2π

- Torque at Hinge 2:

T2,stowed=k·θ2,stowed=0.0398·πT_{2, \text{stowed}} = k \cdot \theta_{2, \text{stowed}} = 0.0398 \cdot \piT2,stowed=k·θ2,stowed=0.0398·π

- Torque at Hinge 3:

T3,stowed=k·θ3,stowed=0.0398·π2T_{3, \text{stowed}} = k \cdot \theta_{3, \text{stowed}} = 0.0398 \cdot \frac{\pi}{2}T3,stowed=k·θ3,stowed=0.0398·2π

Total Torque on One Side

Adding the torques for one side:

Tnet, stowed=T1,stowed+T2,stowed+T3,stowedT_{\text{net, stowed}} = T_{1, \text{stowed}} + T_{2, \text{stowed}} + T_{3, \text{stowed}}Tnet, stowed=T1,stowed+T2,stowed+T3,stowed

Substituting the values:

Tnet, stowed=0.0398·π2+0.0398·π+0.0398·π2=0.0398·2·π2+0.0398·π=0.0398·2·π2+0.0398·πT_{\text{net, stowed}} = 0.0398 \cdot \frac{\pi}{2} + 0.0398 \cdot \pi + 0.0398 \cdot \frac{\pi}{2} = 0.0398 \cdot 2 \cdot \frac{\pi}{2} + 0.0398 \cdot \pi = 0.0398 \cdot 2 \cdot \frac{\pi}{2} + 0.0398 \cdot \piTnet, stowed=0.0398·2π+0.0398·π+0.0398·2π=0.0398·2·2π+0.0398·π=0.0398·2·2π+0.0398·π

Simplifying further:

Tnet, stowed=0.0398·π+0.0398·π=0.0398·2π=0.0796π NmT_{\text{net, stowed}} = 0.0398 \cdot \pi + 0.0398 \cdot \pi = 0.0398 \cdot 2\pi = 0.0796\pi \, \text{Nm}Tnet, stowed =0.0398·π+0.0398·π=0.0398·2π=0.0796πNm

Total Torque Considering Both Sides

Since there are hinges on both sides of the mechanism:

Ttotal, stowed=2×Tnet, stowed=2×0.0796π=0.1592π NmT_{\text{total, stowed}} = 2 \times T_{\text{net, stowed}} = 2 \times 0.0796\pi = 0.1592\pi \, \text{Nm}Ttotal, stowed=2×Tnet, stowed =2×0.0796π=0.1592πNm

Motor Holding Torque

Therefore, the motor needs to provide a holding torque that matches the total torque exerted by the springs to keep the mechanism stowed:

Tmotor=Ttotal, stowed=0.1592π NmT_{\text{motor}} = T_{\text{total, stowed}} = 0.1592\pi \, \text{Nm}Tmotor=Ttotal, stowed=0.1592πNm

Numerical Value

Tmotor≈0.1592×3.14159≈0.5 NmT_{\text{motor}} \approx 0.1592 \times 3.14159 \approx 0.5 \, \text{Nm}Tmotor≈0.1592×3.14159≈0.5Nm

Conclusion

The motor needs to provide a holding torque of approximately **0.5 Nm** to hold the string taut and keep the mechanism in the stowed condition, counteracting the combined torque exerted by the torsion springs in the hinges.

To generalize the calculation of the motor holding torque based on the spring constant kkk, we will derive an equation that takes into account the contributions from all the torsion springs in the hinges during the stowed condition.

Parameters and Assumptions

- Spring stiffness kkk (Nm/rad)
- Initial angular displacements in the stowed condition:
  - Hinge 1: θ1,stowed=90∘=π2\theta_{1, \text{stowed}} = 90^\circ = \frac{\pi}{2}θ1,stowed=90∘=2π radians
  - Hinge 2: θ2,stowed=180∘=π\theta_{2, \text{stowed}} = 180^\circ = \piθ2,stowed =180∘=π radians
  - Hinge 3: θ3,stowed=90∘=π2\theta_{3, \text{stowed}} = 90^\circ = \frac{\pi}{2}θ3,stowed=90∘=2π radians

Torque Calculation for Each Hinge

1. **Torque at Hinge 1:**

T1,stowed=k⋅θ1,stowed=k⋅π2T_{1, \text{stowed}} = k \cdot \theta_{1, \text{stowed}} = k \cdot \frac{\pi}{2}T1,stowed=k⋅θ1,stowed=k⋅2π

2. **Torque at Hinge 2:**

T2,stowed=k⋅θ2,stowed=k⋅πT_{2, \text{stowed}} = k \cdot \theta_{2, \text{stowed}} = k \cdot \piT2,stowed=k⋅θ2,stowed=k⋅π

3. **Torque at Hinge 3:**

T3,stowed=k⋅θ3,stowed=k⋅π2T_{3, \text{stowed}} = k \cdot \theta_{3, \text{stowed}} = k \cdot \frac{\pi}{2}T3,stowed=k⋅θ3,stowed=k⋅2π

Total Torque on One Side

Adding the torques for one side:

Tnet, stowed=T1,stowed+T2,stowed+T3,stowed=k⋅π2+k⋅π+k⋅π2T_{\text{net, stowed}} = T_{1, \text{stowed}} + T_{2, \text{stowed}} + T_{3, \text{stowed}} = k \cdot \frac{\pi}{2} + k \cdot \pi + k \cdot \frac{\pi}{2}Tnet, stowed=T1,stowed+T2,stowed+T3,stowed=k⋅2π+k⋅π+k⋅2π

Simplifying the expression:

Tnet, stowed=k(π2+π+π2)=k(π+π)=k⋅2πT_{\text{net, stowed}} = k \left(\frac{\pi}{2} + \pi + \frac{\pi}{2} \right) = k \left(\pi + \pi \right) = k \cdot 2\piTnet, stowed=k(2π+π+2π)=k(π+π)=k⋅2π

Total Torque Considering Both Sides

Since there are hinges on both sides of the mechanism:

Ttotal, stowed=2×Tnet, stowed=2×k⋅2π=4kπT_{\text{total, stowed}} = 2 \times T_{\text{net, stowed}} = 2 \times k \cdot 2\pi = 4k\piTtotal, stowed=2×Tnet, stowed=2×k⋅2π=4kπ

Motor Holding Torque

Therefore, the motor holding torque required to keep the mechanism in the stowed condition is:

Tmotor=Ttotal, stowed=4kπ NmT_{\text{motor}} = T_{\text{total, stowed}} = 4k\pi \, \text{Nm}Tmotor=Ttotal, stowed=4kπNm

Conclusion

The motor holding torque TmotorT_{\text{motor}}Tmotor needed to keep the string taut and maintain the mechanism in the stowed condition, given the spring constant kkk, is:

Tmotor=4kπ NmT_{\text{motor}} = 4k\pi \, \text{Nm}Tmotor=4kπNm