**Parameters**

* Link Length L1=L2=1702L\_1 = L\_2 = 1702L1​=L2​=1702 mm
* Mass m=15m = 15m=15 kg (mass is given but irrelevant for tension in space due to zero gravity)
* Distance between fixed end and free end in stowed condition ds=100d\_s = 100ds​=100 mm (0.1 m)
* Distance between fixed end and free end in deployed condition dd=3500d\_d = 3500dd​=3500 mm (3.5 m)

**Tension Calculation**

**Force Analysis in Stowed Condition**

* Hinges are loaded with torsion springs:
  + Hinge 1: 90 degrees
  + Hinge 2: 180 degrees
  + Hinge 3: 90 degrees

1. **Net Torsional Force**:
   * Spring constants: Let the spring constant be kkk (Nm/rad).
   * Torque due to each hinge spring:
     + T1=k⋅π2T\_1 = k \cdot \frac{\pi}{2}T1​=k⋅2π​
     + T2=k⋅πT\_2 = k \cdot \piT2​=k⋅π
     + T3=k⋅π2T\_3 = k \cdot \frac{\pi}{2}T3​=k⋅2π​
   * Net torque exerted by the springs on one side:
     + Tnet=k⋅π2+k⋅π+k⋅π2=2kπT\_{\text{net}} = k \cdot \frac{\pi}{2} + k \cdot \pi + k \cdot \frac{\pi}{2} = 2k\piTnet​=k⋅2π​+k⋅π+k⋅2π​=2kπ
   * Since there are hinges on both sides, the total torque is:
     + Ttotal=2×Tnet=4kπT\_{\text{total}} = 2 \times T\_{\text{net}} = 4k\piTtotal​=2×Tnet​=4kπ
2. **String Tension Calculation in Stowed Condition**:
   * The string tension must balance the total torque.
   * The string is anchored at the fixed end and attached to the free end.
   * Fs×0.1=4kπF\_s \times 0.1 = 4k\piFs​×0.1=4kπ
   * Fs=4kπ0.1=40kπF\_s = \frac{4k\pi}{0.1} = 40k\piFs​=0.14kπ​=40kπ

**Tension in the String During Deployed Condition**

* In the deployed condition, the torsion springs are in their free position and do not exert force on the links.
* The links are latched, and the string only needs to maintain the position.
* The residual tension required to keep the string taut is minimal, FdF\_dFd​.

**Summary of Tensions**

* **Stowed Condition**:
  + Fs=40kπF\_s = 40k\piFs​=40kπ (tension required to counteract the spring forces)
* **Deployed Condition**:
  + Minimal tension, FdF\_dFd​, just to keep the string taut.

**Numerical Example**

Let's assume a hypothetical spring constant kkk.

Assume k=1k = 1k=1 Nm/rad (for simplicity):

* **Stowed Condition**:
  + Fs=40×1×π≈125.6F\_s = 40 \times 1 \times \pi \approx 125.6Fs​=40×1×π≈125.6 N
* **Deployed Condition**:
  + Fd≈0F\_d \approx 0Fd​≈0 N (just enough to keep the string taut)

**Conclusion**

The tension in the string during the stowed condition is significantly higher due to the need to counteract the forces exerted by the torsion springs in the hinges. This tension reduces to a minimal value once the mechanism is deployed and latched in place.

To calculate the string tension while the mechanism is deploying at a constant velocity vvv, we need to consider the dynamic forces involved. Since the mechanism deploys gradually, the string tension will be affected by the rate of change of the angular positions of the links and the resulting forces on the hinges.

**Parameters and Assumptions:**

* Let vvv be the constant velocity at which the free end is moving away from the fixed end.
* kkk is the spring constant of the torsion springs.
* The hinges are initially loaded as described, and the deployment gradually releases the torsion springs.
* The deployment distance changes from ds=100d\_s = 100ds​=100 mm (0.1 m) to dd=3500d\_d = 3500dd​=3500 mm (3.5 m).

**Key Points:**

* The torsion springs exert forces that create torques at the hinges.
* The string controls the deployment speed, balancing the forces from the torsion springs.
* The string tension during deployment needs to counteract the torques from the springs dynamically.

**Steps to Determine String Tension During Deployment:**

1. **Determine Angular Velocities:**
   * As the mechanism deploys, the links rotate about the hinges.
   * The angular velocity ω\omegaω of each link can be related to the linear velocity vvv of the free end.
2. **Relation Between Linear and Angular Velocity:**
   * For each link of length LLL, the angular velocity ω\omegaω is related to the linear velocity vvv by ω=vL\omega = \frac{v}{L}ω=Lv​.
   * Here, L=1702L = 1702L=1702 mm = 1.702 m (length of each link).
3. **Calculate the Dynamic Forces:**
   * The force due to the torsion spring in each hinge changes as the mechanism deploys.
   * The torque TTT exerted by each spring is a function of the angle θ\thetaθ.
   * The tension in the string FtF\_tFt​ must balance the resultant torques dynamically.
4. **Dynamic Equilibrium:**
   * During deployment, the net torque due to the springs and the force exerted by the string must be in dynamic equilibrium.
   * The string tension FtF\_tFt​ at any instant can be determined by the torques from the springs and the rate of deployment.

**String Tension During Deployment:**

For simplicity, let’s assume a linear deployment rate and calculate the tension at an arbitrary position ddd between 0.1 m and 3.5 m.

1. **Torque from Hinges During Deployment:**
   * The angular displacement changes linearly with deployment.
   * Let θ(t)\theta(t)θ(t) be the angular displacement at time ttt.
   * The torque at any hinge is T(t)=k⋅θ(t)T(t) = k \cdot \theta(t)T(t)=k⋅θ(t).
2. **Net Torque at Intermediate Position:**
   * At an intermediate deployment position ddd, the angular displacements for each hinge can be interpolated between the initial and final states.
   * If d(t)d(t)d(t) is the deployment distance at time ttt, d(t)=v⋅td(t) = v \cdot td(t)=v⋅t.
3. **Calculate Angular Displacements:**
   * For Hinge 1 (initially 90 degrees): θ1(t)=π2⋅(1−d(t)−0.13.4)\theta\_1(t) = \frac{\pi}{2} \cdot \left(1 - \frac{d(t) - 0.1}{3.4}\right)θ1​(t)=2π​⋅(1−3.4d(t)−0.1​).
   * For Hinge 2 (initially 180 degrees): θ2(t)=π⋅(1−d(t)−0.13.4)\theta\_2(t) = \pi \cdot \left(1 - \frac{d(t) - 0.1}{3.4}\right)θ2​(t)=π⋅(1−3.4d(t)−0.1​).
   * For Hinge 3 (initially 90 degrees): θ3(t)=π2⋅(1−d(t)−0.13.4)\theta\_3(t) = \frac{\pi}{2} \cdot \left(1 - \frac{d(t) - 0.1}{3.4}\right)θ3​(t)=2π​⋅(1−3.4d(t)−0.1​).
4. **Net Torque from Springs:**
   * T1(t)=k⋅θ1(t)T\_1(t) = k \cdot \theta\_1(t)T1​(t)=k⋅θ1​(t)
   * T2(t)=k⋅θ2(t)T\_2(t) = k \cdot \theta\_2(t)T2​(t)=k⋅θ2​(t)
   * T3(t)=k⋅θ3(t)T\_3(t) = k \cdot \theta\_3(t)T3​(t)=k⋅θ3​(t)
   * Total torque on one side: Tnet(t)=T1(t)+T2(t)+T3(t)T\_{\text{net}}(t) = T\_1(t) + T\_2(t) + T\_3(t)Tnet​(t)=T1​(t)+T2​(t)+T3​(t)
   * Total torque considering both sides: Ttotal(t)=2×Tnet(t)T\_{\text{total}}(t) = 2 \times T\_{\text{net}}(t)Ttotal​(t)=2×Tnet​(t)
5. **String Tension During Deployment:**
   * The string tension FtF\_tFt​ at any distance ddd can be calculated using the net torque:
     + Ft(t)×d(t)=Ttotal(t)F\_t(t) \times d(t) = T\_{\text{total}}(t)Ft​(t)×d(t)=Ttotal​(t)
     + Ft(t)=Ttotal(t)d(t)F\_t(t) = \frac{T\_{\text{total}}(t)}{d(t)}Ft​(t)=d(t)Ttotal​(t)​

**Numerical Example:**

Assume a hypothetical spring constant k=1k = 1k=1 Nm/rad and v=0.1v = 0.1v=0.1 m/s (as an example):

1. At t=0t = 0t=0 (initially):
   * d(t)=0.1d(t) = 0.1d(t)=0.1 m
   * θ1(0)=π2\theta\_1(0) = \frac{\pi}{2}θ1​(0)=2π​, θ2(0)=π\theta\_2(0) = \piθ2​(0)=π, θ3(0)=π2\theta\_3(0) = \frac{\pi}{2}θ3​(0)=2π​
   * Tnet(0)=2πT\_{\text{net}}(0) = 2\piTnet​(0)=2π
   * Ttotal(0)=4πT\_{\text{total}}(0) = 4\piTtotal​(0)=4π
   * Ft(0)=4π0.1≈125.6F\_t(0) = \frac{4\pi}{0.1} \approx 125.6Ft​(0)=0.14π​≈125.6 N
2. At t=35t = 35t=35 seconds (final):
   * d(t)=3.5d(t) = 3.5d(t)=3.5 m
   * θ1(35)≈0\theta\_1(35) \approx 0θ1​(35)≈0, θ2(35)≈0\theta\_2(35) \approx 0θ2​(35)≈0, θ3(35)≈0\theta\_3(35) \approx 0θ3​(35)≈0
   * Tnet(35)≈0T\_{\text{net}}(35) \approx 0Tnet​(35)≈0
   * Ttotal(35)≈0T\_{\text{total}}(35) \approx 0Ttotal​(35)≈0
   * Ft(35)≈0F\_t(35) \approx 0Ft​(35)≈0
3. At intermediate t=17.5t = 17.5t=17.5 seconds (halfway):
   * d(t)=1.75d(t) = 1.75d(t)=1.75 m
   * θ1(17.5)=π2⋅0.5≈0.785\theta\_1(17.5) = \frac{\pi}{2} \cdot 0.5 \approx 0.785θ1​(17.5)=2π​⋅0.5≈0.785 rad
   * θ2(17.5)=π⋅0.5≈1.57\theta\_2(17.5) = \pi \cdot 0.5 \approx 1.57θ2​(17.5)=π⋅0.5≈1.57 rad
   * θ3(17.5)=π2⋅0.5≈0.785\theta\_3(17.5) = \frac{\pi}{2} \cdot 0.5 \approx 0.785θ3​(17.5)=2π​⋅0.5≈0.785 rad
   * Tnet(17.5)=k⋅(0.785+1.57+0.785)≈3.14T\_{\text{net}}(17.5) = k \cdot (0.785 + 1.57 + 0.785) \approx 3.14Tnet​(17.5)=k⋅(0.785+1.57+0.785)≈3.14
   * Ttotal(17.5)=2×3.14≈6.28T\_{\text{total}}(17.5) = 2 \times 3.14 \approx 6.28Ttotal​(17.5)=2×3.14≈6.28
   * Ft(17.5)=6.281.75≈3.59F\_t(17.5) = \frac{6.28}{1.75} \approx 3.59Ft​(17.5)=1.756.28​≈3.59 N

These values illustrate how the tension changes dynamically with deployment.  
  
To calculate the holding torque needed for the motor to keep the string taut during the stowed condition, we need to ensure that the motor can counteract the total torque exerted by the torsion springs in the hinges.

**Parameters and Assumptions**

* Spring stiffness k=0.0398k = 0.0398k=0.0398 Nm/rad
* Initial angular displacements:
  + Hinge 1: θ1,stowed=90∘=π2\theta\_{1, \text{stowed}} = 90^\circ = \frac{\pi}{2}θ1,stowed​=90∘=2π​ radians
  + Hinge 2: θ2,stowed=180∘=π\theta\_{2, \text{stowed}} = 180^\circ = \piθ2,stowed​=180∘=π radians
  + Hinge 3: θ3,stowed=90∘=π2\theta\_{3, \text{stowed}} = 90^\circ = \frac{\pi}{2}θ3,stowed​=90∘=2π​ radians

**Calculate Torque from Each Hinge**

The torque exerted by each hinge spring in the stowed condition is given by:

* Torque at Hinge 1:

T1,stowed=k⋅θ1,stowed=0.0398⋅π2T\_{1, \text{stowed}} = k \cdot \theta\_{1, \text{stowed}} = 0.0398 \cdot \frac{\pi}{2}T1,stowed​=k⋅θ1,stowed​=0.0398⋅2π​

* Torque at Hinge 2:

T2,stowed=k⋅θ2,stowed=0.0398⋅πT\_{2, \text{stowed}} = k \cdot \theta\_{2, \text{stowed}} = 0.0398 \cdot \piT2,stowed​=k⋅θ2,stowed​=0.0398⋅π

* Torque at Hinge 3:

T3,stowed=k⋅θ3,stowed=0.0398⋅π2T\_{3, \text{stowed}} = k \cdot \theta\_{3, \text{stowed}} = 0.0398 \cdot \frac{\pi}{2}T3,stowed​=k⋅θ3,stowed​=0.0398⋅2π​

**Total Torque on One Side**

Adding the torques for one side:

Tnet, stowed=T1,stowed+T2,stowed+T3,stowedT\_{\text{net, stowed}} = T\_{1, \text{stowed}} + T\_{2, \text{stowed}} + T\_{3, \text{stowed}}Tnet, stowed​=T1,stowed​+T2,stowed​+T3,stowed​

Substituting the values:

Tnet, stowed=0.0398⋅π2+0.0398⋅π+0.0398⋅π2=0.0398⋅2⋅π2+0.0398⋅π=0.0398⋅2⋅π2+0.0398⋅πT\_{\text{net, stowed}} = 0.0398 \cdot \frac{\pi}{2} + 0.0398 \cdot \pi + 0.0398 \cdot \frac{\pi}{2} = 0.0398 \cdot 2 \cdot \frac{\pi}{2} + 0.0398 \cdot \pi = 0.0398 \cdot 2 \cdot \frac{\pi}{2} + 0.0398 \cdot \piTnet, stowed​=0.0398⋅2π​+0.0398⋅π+0.0398⋅2π​=0.0398⋅2⋅2π​+0.0398⋅π=0.0398⋅2⋅2π​+0.0398⋅π

Simplifying further:

Tnet, stowed=0.0398⋅π+0.0398⋅π=0.0398⋅2π=0.0796π NmT\_{\text{net, stowed}} = 0.0398 \cdot \pi + 0.0398 \cdot \pi = 0.0398 \cdot 2\pi = 0.0796\pi \, \text{Nm}Tnet, stowed​=0.0398⋅π+0.0398⋅π=0.0398⋅2π=0.0796πNm

**Total Torque Considering Both Sides**

Since there are hinges on both sides of the mechanism:

Ttotal, stowed=2×Tnet, stowed=2×0.0796π=0.1592π NmT\_{\text{total, stowed}} = 2 \times T\_{\text{net, stowed}} = 2 \times 0.0796\pi = 0.1592\pi \, \text{Nm}Ttotal, stowed​=2×Tnet, stowed​=2×0.0796π=0.1592πNm

**Motor Holding Torque**

Therefore, the motor needs to provide a holding torque that matches the total torque exerted by the springs to keep the mechanism stowed:

Tmotor=Ttotal, stowed=0.1592π NmT\_{\text{motor}} = T\_{\text{total, stowed}} = 0.1592\pi \, \text{Nm}Tmotor​=Ttotal, stowed​=0.1592πNm

**Numerical Value**

Tmotor≈0.1592×3.14159≈0.5 NmT\_{\text{motor}} \approx 0.1592 \times 3.14159 \approx 0.5 \, \text{Nm}Tmotor​≈0.1592×3.14159≈0.5Nm

**Conclusion**

The motor needs to provide a holding torque of approximately **0.5 Nm** to hold the string taut and keep the mechanism in the stowed condition, counteracting the combined torque exerted by the torsion springs in the hinges.

To generalize the calculation of the motor holding torque based on the spring constant kkk, we will derive an equation that takes into account the contributions from all the torsion springs in the hinges during the stowed condition.

**Parameters and Assumptions**

* Spring stiffness kkk (Nm/rad)
* Initial angular displacements in the stowed condition:
  + Hinge 1: θ1,stowed=90∘=π2\theta\_{1, \text{stowed}} = 90^\circ = \frac{\pi}{2}θ1,stowed​=90∘=2π​ radians
  + Hinge 2: θ2,stowed=180∘=π\theta\_{2, \text{stowed}} = 180^\circ = \piθ2,stowed​=180∘=π radians
  + Hinge 3: θ3,stowed=90∘=π2\theta\_{3, \text{stowed}} = 90^\circ = \frac{\pi}{2}θ3,stowed​=90∘=2π​ radians

**Torque Calculation for Each Hinge**

1. **Torque at Hinge 1:**

T1,stowed=k⋅θ1,stowed=k⋅π2T\_{1, \text{stowed}} = k \cdot \theta\_{1, \text{stowed}} = k \cdot \frac{\pi}{2}T1,stowed​=k⋅θ1,stowed​=k⋅2π​

1. **Torque at Hinge 2:**

T2,stowed=k⋅θ2,stowed=k⋅πT\_{2, \text{stowed}} = k \cdot \theta\_{2, \text{stowed}} = k \cdot \piT2,stowed​=k⋅θ2,stowed​=k⋅π

1. **Torque at Hinge 3:**

T3,stowed=k⋅θ3,stowed=k⋅π2T\_{3, \text{stowed}} = k \cdot \theta\_{3, \text{stowed}} = k \cdot \frac{\pi}{2}T3,stowed​=k⋅θ3,stowed​=k⋅2π​

**Total Torque on One Side**

Adding the torques for one side:

Tnet, stowed=T1,stowed+T2,stowed+T3,stowed=k⋅π2+k⋅π+k⋅π2T\_{\text{net, stowed}} = T\_{1, \text{stowed}} + T\_{2, \text{stowed}} + T\_{3, \text{stowed}} = k \cdot \frac{\pi}{2} + k \cdot \pi + k \cdot \frac{\pi}{2}Tnet, stowed​=T1,stowed​+T2,stowed​+T3,stowed​=k⋅2π​+k⋅π+k⋅2π​

Simplifying the expression:

Tnet, stowed=k(π2+π+π2)=k(π+π)=k⋅2πT\_{\text{net, stowed}} = k \left( \frac{\pi}{2} + \pi + \frac{\pi}{2} \right) = k \left( \pi + \pi \right) = k \cdot 2\piTnet, stowed​=k(2π​+π+2π​)=k(π+π)=k⋅2π

**Total Torque Considering Both Sides**

Since there are hinges on both sides of the mechanism:

Ttotal, stowed=2×Tnet, stowed=2×k⋅2π=4kπT\_{\text{total, stowed}} = 2 \times T\_{\text{net, stowed}} = 2 \times k \cdot 2\pi = 4k\piTtotal, stowed​=2×Tnet, stowed​=2×k⋅2π=4kπ

**Motor Holding Torque**

Therefore, the motor holding torque required to keep the mechanism in the stowed condition is:

Tmotor=Ttotal, stowed=4kπ NmT\_{\text{motor}} = T\_{\text{total, stowed}} = 4k\pi \, \text{Nm}Tmotor​=Ttotal, stowed​=4kπNm

**Conclusion**

The motor holding torque TmotorT\_{\text{motor}}Tmotor​ needed to keep the string taut and maintain the mechanism in the stowed condition, given the spring constant kkk, is:

Tmotor=4kπ NmT\_{\text{motor}} = 4k\pi \, \text{Nm}Tmotor​=4kπNm