Review of Stream Depletion Solutions, Behavior, and Calculations

Bruce Hunt[1]

Abstract: A review is given of some currently available solutions for the calculation of stream depletion from pumped aquifers. A brief history is given of the development of analytical and semianalytical models. A solution published in 2003, which is also capable of duplicating many previously published solutions, is described in detail. In this solution, the stream partially penetrates a Boulton delayed-yield aquifer and is shown herein to apply when different segments of a stream may be either gaining or losing water to the pumped aquifer. Excel computational tools freely available to practitioners are referenced and used in an illustration for the calculation of the 2003 solution, and the general behavior of the solution is discussed. Generalizations and additional solutions are discussed for streams and aquifers with finite width, for systems containing several aquifers and for abstraction wells that are either horizontal or slanted. The review concludes with a very brief discussion of the advantages and disadvantages of using numerical solutions for modeling stream-depletion behavior. **DOI: 10.1061/(ASCE)HE.1943-5584.0000768.** *© 2014 American Society of Civil Engineers.*

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Introduction

When a well is located in an aquifer near a stream, and when the stream is connected hydraulically to the pumped aquifer, then abstracting water from the well will also abstract water from the stream in a process that is usually referred to as stream depletion. Stream depletion is an important topic in hydrology since too much water abstraction from a stream can have severe effects on stream life and, in extreme cases, can even cause a streambed to become completely dry. The first analytical solution for stream depletion was obtained by Theis (1941), but a greatly accelerated rate of research has occurred in this area since 1990 as a result of environmental concerns with stream ecology. The purpose of this paper is to give a brief discussion of the historical development of stream-depletion solutions and to describe tools that are currently available for analyzing stream-depletion problems.

Development of Analytical and Semianalytical Models

The first unsteady analytical solution for stream depletion was obtained by Theis (1941) for the simplest hydrogeologic setting [Fig. 1(a)]: a straight river fully penetrating a homogeneous, isotropic aquifer of semi-infinite extent, which can be either confined or unconfined. Water is abstracted at a constant rate from a fully penetrating well. The solution assumes that changes in free surface elevations are small enough to allow linearization of the governing equations and that water is released instantaneously from aquifer storage.

At the time that Theis (1941) obtained the first stream depletion solution, the error and complementary error functions apparently had not yet been defined. Thus, the Theis solution was written in terms of a definite integral that had to be evaluated numerically. Later, however, Glover and Balmer (1954) obtained the same solution but expressed this solution in terms of the then known complementary error function, which can be evaluated by using well-known series approximations. However unfair this might be, most writers today refer to this solution as the Glover-Balmer solution rather than the Theis solution. The Theis (1941) or Glover-Balmer (1954) solution is given by

$$\frac{\Delta Q}{Q} = \operatorname{erfc}\left(\sqrt{\frac{\sigma L^2}{4Tt}}\right) \tag{1}$$

where ΔQ = stream depletion flow rate; Q = constant flow abstraction from the well; erfc = complementary error function; σ = specific yield for an unconfined aquifer or storativity for a confined aquifer; L = shortest distance between the well and stream edge; T = pumped aquifer transmissivity; and t = time. All variables in Eq. (1) occur in dimensionless combinations, so any consistent set of units may be used.

Jenkins (1968) developed dimensionless graphs for the volume and rate of stream depletion. Jenkins also took advantage of the fact that all of the governing equations in the Theis solution are linear with coefficients that do not depend upon time, and this allowed him to use superposition and time translation to calculate the result of pumping from an abstraction well for a finite period of time. An example of this is shown in Fig. 2. Since all of the solutions described in this section are the result of solving linear equations with coefficients that do not depend upon time, the methods of superposition and time translation used by Jenkins (1968) can also be used with any of the solutions described herein to model the result of pumping from a well for a finite period of time. The methods used by Jenkins (1968) are still used today by some water administrators to assess stream depletion (see for example Scott and Callander 1995; Sanders 1996; Environment Canterbury 2000).

Stream depletion resulting from cyclic pumping can be found in a similar way by adding further components (Jenkins 1968;

[1]Retired, 19 The Fairway, Whakatane, New Zealand 3120; formerly, Reader, Dept. of Civil Engineering, Univ. of Canterbury, Private Bag 4800, Christchurch, New Zealand. E-mail: bruce.hunt@kinect.co.nz

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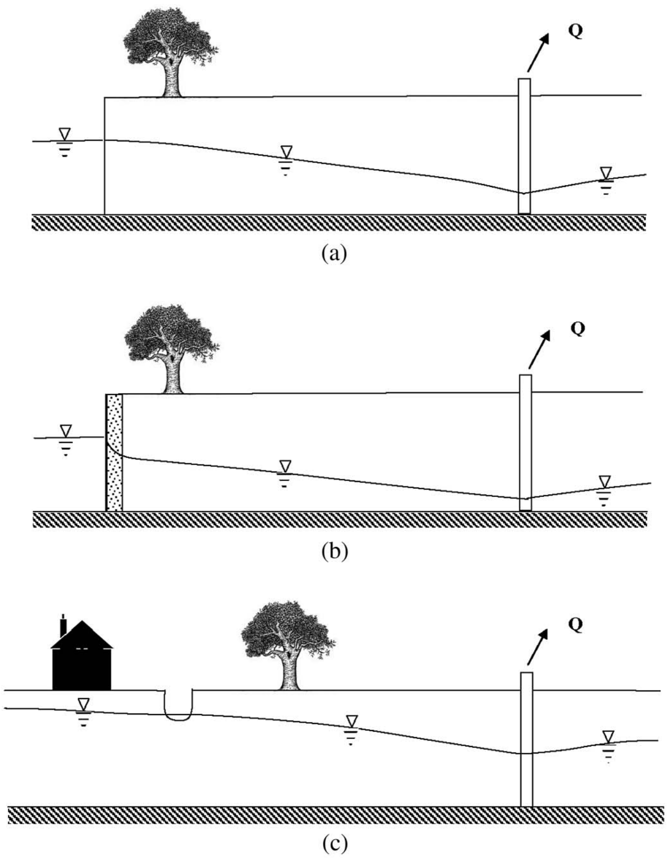

Fig. 1. Hydrogeology considered by (a) Theis (1941); (b) Hantush (1965); (c) Hunt (1999)

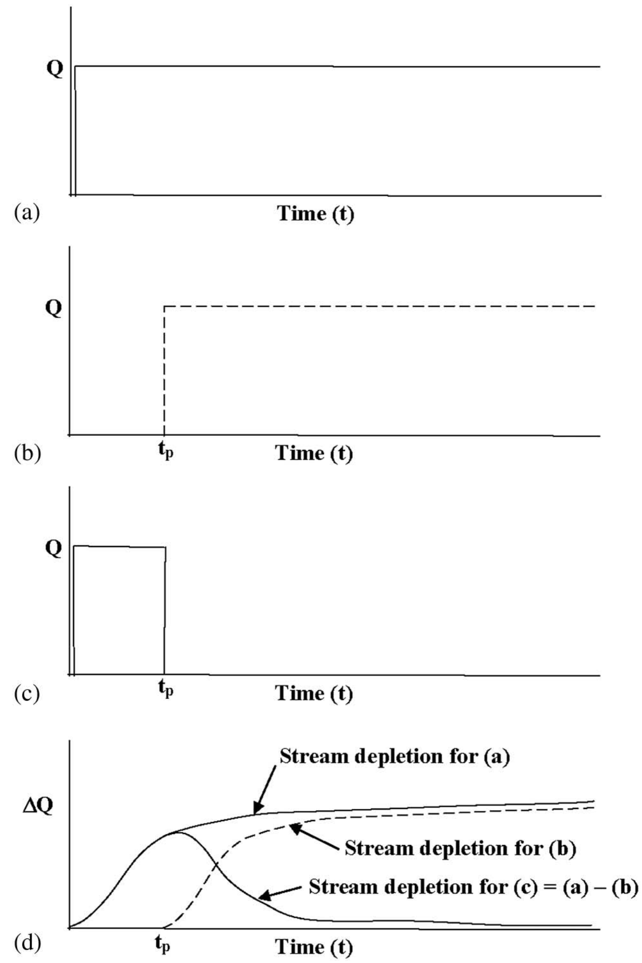

Fig. 2. Stream depletion (d) for a pulse (c) can be calculated as the stream depletion resulting from (a) pumping for an infinite time from $t = 0$ minus the stream depletion resulting from (b) pumping for an infinite time starting from $t = t_p$

Wallace et al. 1990). This entire procedure is identical with the use of superposition and time translation in applications of unit hydrograph theory.

The Theis (1941), Glover and Balmer (1954), and Jenkins (1968) solutions, although still used extensively for making water right decisions, suffer from the fact that many of the simplifying assumptions on which the solutions are based are almost never fulfilled in practice. Spalding and Khaleel (1991) and Sophocleous et al. (1995) used numerical modeling to evaluate the errors associated with some of the various assumptions. They found that significant errors were related to three phenomena not included in Theis's idealized setting [Fig. 1(a)]: the existence of a semipervious layer (also called a clogging layer) between the stream and aquifer, partial penetration of the aquifer by the stream, and aquifer heterogeneity.

The importance of a clogging layer on the stream bed was recognized in an earlier solution obtained by Hantush (1965). He considered a hydrogeologic setting similar to that of Theis but included a semipervious clogging layer along the stream edge [Fig. 1(b)]. The Hantush (1965) solution is given by

$$\frac{\Delta Q}{Q} = \mathrm{erfc}\left(\sqrt{\frac{\sigma L^2}{4Tt}}\right) - \exp\left(\frac{Tt}{\sigma R^2} + \frac{L}{R}\right)\mathrm{erfc}\left(\sqrt{\frac{Tt}{\sigma R^2}} + \sqrt{\frac{\sigma L^2}{4Tt}}\right) \tag{2}$$

where R = retardation coefficient defined by

$$R = \frac{K}{K'} b' \tag{3}$$

in which K = aquifer hydraulic conductivity; K' = clogging layer hydraulic conductivity; and b' = clogging layer thickness. When the thickness of the clogging layer, b', approaches zero, the Hantush solution reduces to the Theis solution.

Hunt (1999) obtained the first solution for flow to an abstraction well beside a stream that partially penetrates an aquifer extending to infinity in all horizontal directions, as shown in Fig. 1(c). This was accomplished by obtaining solutions for the separate regions $0 < x < \infty$ and $-\infty < x < 0$. Then these two solutions were joined along the common boundary at $x = 0$, which coincides with the stream, by requiring that (1) drawdowns be continuous, and (2) flows be discontinuous by the amount of flow depleted from the stream. Flows depleted from the stream were assumed to be directly proportional to drawdown along the stream at $x = 0$, and the stream was modeled by using a zero width and a length in the horizontal direction from $y = -\infty$ to $y = \infty$. Boundary conditions along the stream at $x = 0$ were incorporated in the problem by appending to the governing partial differential equation a term linearly proportional to drawdown multiplied by a Dirac delta function. The Hunt (1999) solution is given by

$$\frac{\Delta Q}{Q} = \mathrm{erfc}\left(\sqrt{\frac{\sigma L^2}{4Tt}}\right) - \exp\left(\frac{\lambda^2 t}{4\sigma T} + \frac{\lambda L}{2T}\right)\mathrm{erfc}\left(\sqrt{\frac{\lambda^2 t}{4\sigma T}} + \sqrt{\frac{\sigma L^2}{4Tt}}\right) \tag{4}$$

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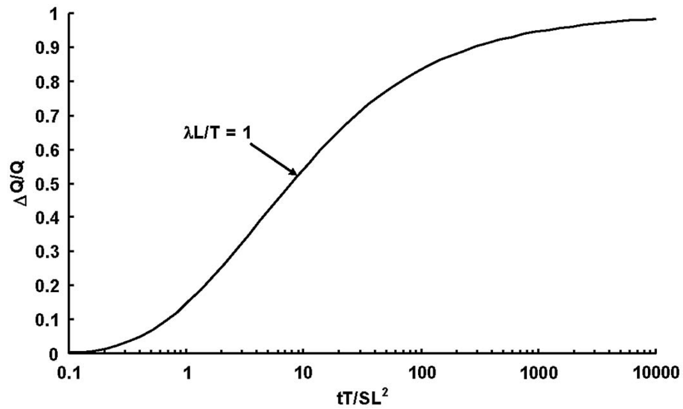

Fig. 3. A dimensionless plot of the Hunt (1999) solution for stream depletion

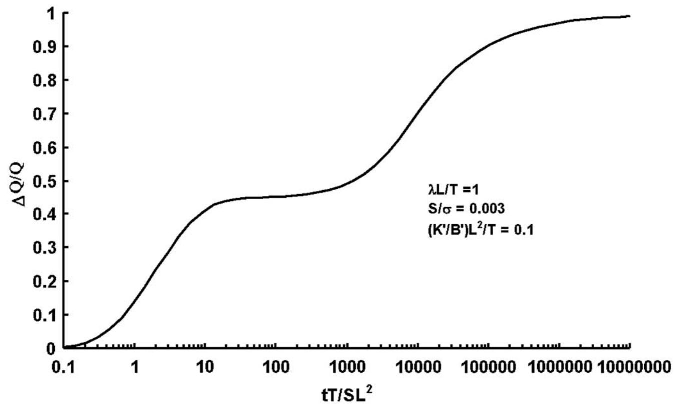

Fig. 4. Stream depletion in a delayed-yield aquifer showing the characteristic three changes in curvature

where λ = streambed conductive coefficient defined by

$$\lambda = \frac{b}{B''} K' \tag{5}$$

in which b = streambed width; K' = stream-bed hydraulic conductivity; and B'' = streambed thickness.

A typical dimensionless plot of total stream depletion versus time calculated from the Hunt (1999) solution is shown in Fig. 3, in which ΔQ = total flow depleted from the stream; Q = flow abstracted from the pumped well; t = time; T = pumped aquifer transmissivity; σ = specific yield (storage coefficient) of the pumped aquifer; and L = shortest distance between the pumped well and nearest stream edge. Since stream depletion is assumed to be directly proportional to the first power of drawdown along the stream, plots of drawdown at any point in the pumped aquifer versus time have exactly the same shape, although numerical values and units for the ordinates differ from those shown in Fig. 3 for stream depletion. Ward and Callander (2010) later examined the distribution of total stream depletion as a function of time and the spatial distribution of stream depletion as a function of distance along the stream.

Weir (1999) carried out stream depletion field experiments and obtained estimates for groundwater parameters by fitting the Hunt (1999) solution to measured values of observation well drawdowns and stream depletions. These results were later published in Hunt et al. (2001). However, measured drawdown and stream depletion curves were seen to vary slightly from the Hunt (1999) solution by having three changes in curvature, as shown in Fig. 4, rather than the single change in curvature shown in Fig. 3. This same behavior, which is sometimes even more pronounced in other studies, has been noticed in field measurements taken in numerous other investigations. Examples of this behavior appear in Nyholm et al. (2002), Kollet and Zlotnik (2003), Fox (2004), and Lough and Hunt (2006) and are reminiscent of behavior observed in delayed-yield flow to a well that was first analyzed by Boulton (1963). The most likely physical explanation for this type of behavior is that the pumped aquifer is overlain by an aquitard with a considerably smaller hydraulic conductivity. Before pumping begins, a free surface exists within the aquitard. Thus, when drawdowns in the underlying aquifer are increased by pumping from a well, a piezometric head differential is created across the overlying aquitard, and this causes water to move vertically downward through the aquitard to recharge the lower pumped aquifer. During this period, the pumped aquifer has a behavior similar to the leaky aquifer first devised by Hantush and Jacob (1955). However, as free surface elevations in the aquitard are lowered further, both the head differential across the aquitard and the resulting vertical recharge from the aquitard into the pumped aquifer are reduced. Eventually, the free surface level in the aquitard approaches the piezometric level along the common boundary between the aquitard and aquifer. Then vertical recharge from the aquitard ceases entirely because the piezometric head differential across the aquitard is now zero, and the pumped aquifer behaves as an unconfined aquifer with no vertical leakage. Hunt (2003) obtained a solution for stream depletion created by a well when abstracting water from a delayed-yield aquifer beneath a stream.

Description of the Hunt Solution

A sketch of the hydrogeology considered by Hunt (2003) is shown in Fig. 5. An aquitard containing a free surface overlies a relatively permeable pumped aquifer. The aquitard has a saturated thickness B', a hydraulic conductivity K', a specific yield σ, and a piezometric head distribution h'. The pumped aquifer has a transmissivity T, a storativity S, and a piezometric head distribution h. A stream of zero width at $x = 0$ and extending from $y = -\infty$ to $y = \infty$ partially penetrates the aquifer and has an aquitard thickness B'' beneath it. The pumped well has the horizontal coordinates $(x, y) = (L, 0)$ and is pumped at a constant rate Q from $t = 0$ to $t = \infty$. A free surface is present in the overlying aquitard before pumping begins at $t = 0$. Boulton (1973), Cooley and Case (1973), and Hunt (2003) showed that this aquifer hydrogeology was an accurate description of the hydrogeology described by the earlier apparently empirical solution of Boulton (1963) for his delayed-yield aquifer.

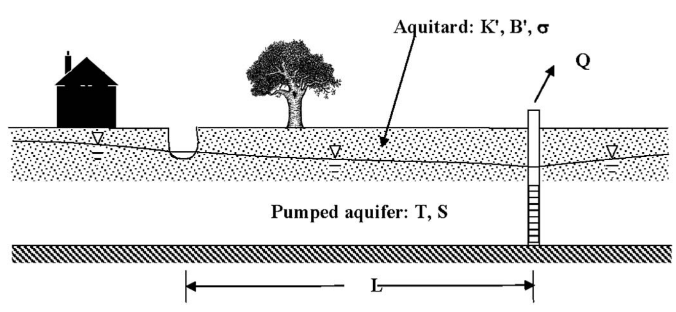

Fig. 5. Hydrogeology for a delayed-yield aquifer

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Therefore, this type of hydrogeology will be referred to herein as a delayed-yield aquifer.

The piezometric head distributions h_1 and h_1' in the aquifer and aquitard, respectively, which would exist if the new well were not inserted, can be described by the solution of the following equations:

$$T\left(\frac{\partial^2 h_1}{\partial x^2}+\frac{\partial^2 h_1}{\partial y^2}\right)=S\frac{\partial h_1}{\partial t}+\left(\frac{K'}{B'}\right)(h_1-h_1')+f(x,y,t)$$
$$-\delta(x)\lambda[h_1-h_0(x,t)] \quad (6)$$

$$\sigma\frac{\partial h_1'}{\partial t}+\left(\frac{K'}{B'}\right)(h_1'-h_1)=0 \quad (7)$$

$$\lim_{r\to\infty} h_1=g(x,y,t) \qquad \left(r\equiv\sqrt{x^2+y^2}\right) \quad (8)$$

$$h_1(x,y,0)=u(x,y) \quad (9)$$

$$h_1'(x,y,0)=v(x,y) \quad (10)$$

in which $h_0(x,t)$ = elevation of the water level in the stream; t = time; (x,y) = horizontal coordinates; $\delta(x)$ = Dirac's delta function; λ = streambed conductance coefficient; and $f(x,y,t)$ = prescribed function that can account for features such as existing wells. The function $g(x,y,t)$ = existing piezometric head distribution in the pumped aquifer at large distances from the new well before pumping begins, and $u(x,y)$ and $v(x,y)$ = piezometric head distributions in the pumped aquifer and aquitard, respectively, before pumping begins. Thus, piezometric head distributions in the aquitard and pumped aquifer before pumping begins will not, in general, be horizontal, and different sections of the stream may be either gaining or losing water, depending upon whether $[h_1-h_0(x,t)]$ is positive or negative.

If the new well is inserted, then piezometric heads h_2 and h_2' in the aquifer and aquitard, respectively, are described by the following equations:

$$T\left(\frac{\partial^2 h_2}{\partial x^2}+\frac{\partial^2 h_2}{\partial y^2}\right)=S\frac{\partial h_2}{\partial t}+\left(\frac{K'}{B'}\right)(h_2-h_1')+f(x,y,t)$$
$$+Q\delta(x-L)\delta(y)-\delta(x)\lambda[h_2-h_0(x,t)] \quad (11)$$

$$\sigma\frac{\partial h_2'}{\partial t}+\left(\frac{K'}{B'}\right)(h_2'-h_2)=0 \quad (12)$$

$$\lim_{r\to\infty} h_2=g(x,y,t) \qquad \left(r\equiv\sqrt{x^2+y^2}\right) \quad (13)$$

$$h_2(x,y,0)=u(x,y) \quad (14)$$

$$h_2'(x,y,0)=v(x,y) \quad (15)$$

in which $h_0(x,t)$ and the specified functions $f(x,y,t)$, $g(x,y,t)$, $u(x,y)$, and $v(x,y)$ are functions that are identical in Eqs. (6)–(10) and Eqs. (11)–(15).

Subtraction of Eqs. (11)–(15) from Eqs. (6)–(10) gives the following set of equations for the changes in piezometric heads created by inserting the pumped well

$$T\left(\frac{\partial^2 s}{\partial x^2}+\frac{\partial^2 s}{\partial y^2}\right)=S\frac{\partial s}{\partial t}+\left(\frac{K'}{B'}\right)(s-s')$$
$$-Q\delta(x-L)\delta(y)+\delta(x)\lambda s \quad (16)$$

$$\sigma\frac{\partial s'}{\partial t}+\left(\frac{K'}{B'}\right)(s'-s)=0 \quad (17)$$

$$\lim_{r\to\infty} s=0 \qquad \left(r\equiv\sqrt{x^2+y^2}\right) \quad (18)$$

$$s(x,y,0)=0 \quad (19)$$

$$s'(x,y,0)=0 \quad (20)$$

in which $s\equiv h_1-h_2$ and $s'\equiv h_1'-h_2'$ are the drawdowns (decreases in piezometric heads) in the pumped aquifer and the aquitard, respectively, that have been created by inserting the pumped well into the system. Final piezometric head distributions in the aquifer and aquitard can be obtained by subtracting the calculated drawdowns, s and s'', from piezometric head distributions, h_1 and h_1' that existed before pumping began. However, it is usually the drawdowns, s and s', caused by the pumping from the well that are of most interest in applications.

Eqs. (16)–(20) are the equations that were solved by Hunt (2003) to obtain the stream-depletion solution for a delayed-yield aquifer. Then the total flow depleted from the stream, $\Delta Q(t)$, was obtained by inserting the solution for $s(x,y,t)$ in the following equation:

$$\Delta Q(t)=\lambda\int_{-\infty}^{\infty}s(0,y,t)dy \quad (21)$$

in which λ = streambed conductance coefficient that is defined by Eq. (5). The streambed width, b, is assumed to be zero in the mathematical model but to retain a finite value in the physical model. In practice, λ is very unlikely to be estimated accurately from Eq. (5) but instead should be found by comparing calculated solutions of Eqs. (16)–(21) with solutions observed in field experiments.

It is sometimes claimed by people who are unfamiliar with the principle of superposition that the Hunt (2003) solution of Eqs. (16)–(21) is inapplicable to field problems in which free surface elevations and piezometric head distributions are not initially horizontal and different reaches of the stream are either gaining or losing water from seepage. The development shown with Eqs. (6)–(21) shows that this is not the case. In other words, the superposition principle allows the Hunt (2003) solution to be applied to field problems in which free surface elevations and piezometric head distributions are not initially horizontal, and different sections of the stream may be either gaining or losing water from seepage before pumping begins.

The solution of Eqs. (16)–(21) can be used to duplicate the Hunt (1999) solution by setting $K'=0$ and $S=\sigma$. Then Eqs. (17) and (20) show that $s'=0$ for all values of t, and Eqs. (16)–(21) reduce to the same equations solved by Hunt (1999). In other words, the Hunt (2003) solution can also be used for problems in which no aquitard overlies the pumped aquifer.

A less obvious result is that the solution of Eqs. (16)–(21) can be used to duplicate the Theis (1941) solution by setting $K'=0$, $S=\sigma$, and λ equal to a very large number, say 10,000. This can be shown in several ways. For example, taking the limit $\lambda\to\infty$ reduces Eq. (4) to Eq. (1), which is equivalent to letting the stream width, b, in Eq. (5) become infinite. Alternatively, integrating

Eq. (16) with respect to x about a small region close to $x = 0$ then letting $\lambda \to \infty$ shows that $s = 0$ at $x = 0$ for all values of time. A third way is to simply compare numerical values computed from Eq. (1) and the solution of Eqs. (16)–(21).

Inspection of Eqs. (2)–(4) shows that the Hunt (1999) solution will duplicate the Hantush (1965) solution if $\lambda = 2T/R$. Thus, the Hunt (2003) solution can also be used to duplicate the Hantush (1965) solution by setting $\lambda = 2T/R$, $K' = 0$, and $S = \sigma$. Thus, it is possible to use the Hunt (2003) solution to duplicate all previously published stream-depletion solutions.

The Hunt (2003) solution is often referred to as a semianalytical solution. This is because a closed-form analytical solution is obtained for the Laplace transform of the solution, but the Laplace transform is inverted numerically with an approximation known as the Stehfest algorithm.

Eqs. (16)–(21) can be used to describe flow in a Hantush-Jacob (1955) leaky aquifer by setting $\sigma = \infty$. [The term Hantush-Jacob leaky aquifer is used herein to describe the hydrogeology that was assumed by Hantush and Jacob (1955) to derive their well-known solution for radial flow to a pumped well in an infinite aquifer.] Then Eqs. (17) and (20) show that $s' = 0$ for all values of time, and the second term on the right side of Eq. (16) reduces to the Hantush leaky-aquifer term. The Hantush-Jacob solution for flow to a well in a leaky aquifer that extends to infinity in all horizontal directions calculates steady-flow drawdowns that decrease exponentially with distance from the pumped well. Since streamflow depletion is directly proportional to drawdown along the stream, placing a well beside a stream in a Hantush-Jacob leaky aquifer creates a steady-flow stream depletion that decreases exponentially with the distance L between the pumped well and the stream. However, this is a completely artificial situation that never exists in practice since the specific yield, σ, of any aquitard or aquifer never exceeds unity. In fact, this observation calls into question the entire practice of using a Hantush-Jacob leaky aquifer to model long-term flow in any hydrological setting unless the aquitard above the pumped aquifer is bounded on top by a body of water with a constant water level. In that case, and only in that case, will $s' = 0$ for all time.

The Hunt (2003) solution can be used to approximate flow to a well beside a stream in a Hantush-Jacob leaky aquifer by setting σ equal to a very large number, say 10^8 or 10^{10}. Then the initial part of the solution is virtually identical with the exact solution for a Hantush-Jacob leaky aquifer. At extremely large times, however, stream depletion and drawdowns will start to increase again as water stored in the aquitard begins to run out. This is because σ, although very large, is still a finite number. Modeling flow depletion with a Hantush-Jacob leaky aquifer solution certainly minimizes estimates for streamflow depletion, but it also models a completely artificial hydrological setting that never exists in practice.

Calculation of the Hunt Solution

Evaluation of the Hunt (2003) solution is relatively complicated. However, this solution can be evaluated easily with groundwater analysis tools that can be downloaded without charge from either of the web pages http://www.civil.canterbury.ac.nz/staff/bhunt.shtml or https://sites.google.com/site/brucehuntsgroundwaterwebsite.

The first of these tools is entitled Function.xls and is downloaded from the Canterbury website under the heading "Excel spreadsheet for analysis." Function.xls contains user-defined functions in Excel that can be used to calculate both the Hunt (2003) solution and numerous other functions that are useful in both groundwater flow and applied mathematics. User-defined functions are functions computed from computer programs stored within the visual basic editor. These computer programs must be invoked by inserting specific call names and arguments when users construct spreadsheet programs for particular applications. In other words, these are not fill-in-the-blanks spreadsheet programs that automatically compute answers when users insert a few numbers in certain cells. Rather, user-defined-programs require users to exercise their own ingenuity when constructing spreadsheet programs for particular applications.

Function.xls will work as downloaded in versions of Excel up through Excel 2003. However, new security provisions in Excel 2007 and Excel 2010 require that some changes be made in these newer versions of Excel before Function can be used. The following procedure allows Function.xls to be used in Excel 2007 and Excel 2010:

1. Download Function.xls from this web site, choosing the option to open Function in Excel. Then use "Save As" to save Function as an "Excel Macro-Enabled Workbook."
2. Click on the "Microsoft Office Button" on the top left corner for Excel 2007 or "File" in Excel 2010.
3. Click on "Excel Options" in Excel 2007 or "Options" in Excel 2010.
4. Click on "Trust Center" in the left margin of the Excel Options drop-down menu.
5. Click on "Trust Center Settings" on the right side of the Trust Center menu.
6. Click on the "Disable all macros with notification" button under "Macro Settings."
7. Click "OK" twice to exit.

A security warning will appear above the Function spreadsheet whenever Function is opened. Click on "Options," then on the "Enable this content" button and finally on "OK" in Excel 2007 or on the "Enable Content" box beside the security warning "Macros have been disabled" in Excel 2010. The programs in Function.xlsm can now be used.

The second of the tools on the Canterbury website is a 66-page manual entitled "Analysis using Function.xls manual." This pdf file contains references, explanations, numerous examples and descriptions of solutions programmed in Function.xls for 20 different groundwater problems. It is essential reading for people who want to know what functions are programmed in Function.xls and how to use these programs for spreadsheet calculations. Hunt (2005) gave an earlier description of a few of these programs, but this description was incomplete at the time. Furthermore, programs for numerous solutions have been added since 2005 and are documented in Sheet 1 of Function.xls.

Fig. 5 shows a definition sketch for flow to a well beside a stream in a delayed-yield aquifer. The following variables are defined in this sketch: Q = well abstraction rate; T = pumped aquifer transmissivity; S = pumped aquifer storativity; L = shortest distance between the well and stream; x = coordinate normal to the stream; y = coordinate parallel to the stream; K' = aquitard hydraulic conductivity; σ = aquitard specific yield; B' = aquitard saturated thickness; b = stream width; and B'' = aquitard thickness beneath the stream. Function.xls computes stream depletion with the following equation:

$$\frac{\Delta Q}{Q} = Q\_4\left[\frac{tT}{SL^2}, \frac{(K'/B')L^2}{T}, \frac{S}{\sigma}, \frac{\lambda L}{T}\right] \tag{22}$$

where ΔQ = stream depletion; t = time; λ = streambed resistance coefficient = $K'b/B''$; and Q_4 = the call name for the user-defined program.

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Values for λ are found most accurately by comparing measured and calculated drawdown values in an observation well. An example of this procedure is given in Lough and Hunt (2006). Drawdowns in the pumped aquifer are calculated in Function.xls with the following equation:

$$\frac{sT}{Q} = W\_4\left[\frac{x}{L},\frac{y}{L},\frac{tT}{SL^2},\frac{\lambda L}{T},\frac{(K'/B')L^2}{T},\frac{S}{\sigma}\right] \tag{23}$$

where s = drawdown at the point (x, y); and W_4 = the call name for the user-defined program. All variable groupings in Eqs. (22) and (23) are dimensionless, so any consistent system of units may be used for calculations.

Since user-defined functions are accessible only in Excel spreadsheets that have the required programs attached to a visual basic editor, computations must be carried out either in Function.xls or in a copy of Function.xls that has been renamed. Therefore, open either Function.xls or a copy of Function.xls and enter in Sheet 2 the data shown in rows 1 through 9 in Fig. 6. Select cells A1:C2 and click on "Insert, Name and Create" to create the names a, b, and n for the contents of cells A2, B2, and C2, respectively, in versions of Excel up to and including Excel 2003. In later versions of Excel, a similar process is carried out by selecting cells A1:C2, clicking on the "Formulas" tab at the top of the page, clicking on "Create from Selection" above "Defined Names" at the top of the page and then clicking on "Create names in the top row" in the "Create Names from Selection" drop-down menu. Values for a, b, and n allow n equally spaced points to be calculated on a $\log_{10}$ scale from 10^a to 10^b by using the formula

$$t_k = 10^{M_k} \qquad (k = 1, 2, \ldots, n) \tag{24}$$

where

$$M_k = a + (k-1)(b-a)/(n-1) \qquad (k = 1, 2, \ldots, n) \tag{25}$$

Apply this formula by typing a 1 in cell A10, entering the equation

$$= A10 + 1 \tag{26}$$

in cell A11 and dragging the contents of cell A11 down until the value of n (in this case 50) is reached. (Dragging a formula downward is accomplished by left-clicking on the bottom right corner of the cell containing the formula and dragging the cursor downward while keeping the left click depressed.) Then calculate the corresponding n values for t by entering in cell B10 the formula

$$= 10\hat{}[a + (\mathrm{A10} - 1) * (b - a)/(n - 1)] \tag{27}$$

Duplicate this formula either by dragging it downward or by double-clicking on the bottom right corner of cell B10. In this particular case, 50 points are calculated that are equally spaced on a $\log_{10}$ scale from $t = 10^{-3}$ to $t = 10^3$ days.

Name the cell values in cells B6:H6 by selecting cells B5:H6 and proceeding as explained in the previous paragraph. Then calculate the stream depletion, ΔQ, in cell C10 by typing in cell C10 the following formula, which is obtained from Eq. (22)

$$= Q * Q\_4[\mathrm{B10} * T/(S * L^2), KB * L^2/T, S/\mathrm{sigma}, \mathrm{lambda} * L/T] \tag{28}$$

where KB represents (K'/B') in Eq. (22). Since the variables Q, T, S, L, lambda, sigma, and KB have been named, these variable names may be typed directly into the equation in cell C10. An easier way, however, is to left-click on the cell containing the value for a variable, and the name of that variable (rather than its value) will appear in the formula. Calculate all of the remaining values for ΔQ either by double-clicking on the bottom, right corner of cell C10 or by dragging the formula in cell C10 down to cell C59.

In versions of Excel up to and including Excel 2003, plot the results just computed by selecting cells B10:C59 and clicking on "Insert", "Chart" and "XY(Scatter)." Then follow the four steps in the Chart Wizard by clicking on "Next" after completing each step. In Step 4, the writer almost always prefers to embed the chart as an object in the sheet, as shown in Fig. 7. The chart may be modified after it has been made by right-clicking on any part of the plot that needs to be changed and left-clicking on any of the options that appear on the screen in a drop-down

	A	B	C	D	E	F	G	H	I
1	**a**	**b**	**n**						
2	-3	3	50						
3									
4	**Units**	**m^3/day**	**m^2/day**	**Dimless**	**m**	**m/day**	**Dimless**	**days**	
5	**Symbol**	**Q**	**T**	**S**	**L**	**lambda**	**sigma**	**KB**	
6	**Value**	3000	1000	0.0001	100	1	0.1	0.001	
7									
8		**Calculated points for plot**							
9	**k**	**t (days)**	**ΔQ (L/s)**						
10	1	0.001	57.54227						
11	2	0.001326	76.74802						
12	3	0.001758	99.69554						
13	4	0.00233	126.6653						
14	5	0.003089	157.9226						
15	6	0.004095	193.7029						
16	7	0.005429	234.1872						
17	8	0.007197	279.4666						
18	9	0.009541	329.4942						
19	10	0.012649	384.0231						
20	11	0.016768	442.5333						
21	12	0.02223	504.151						
22	13	0.029471	567.5742						
23	14	0.039069	631.0267						
24	15	0.051795	692.2799						

Fig. 6. Spreadsheet calculations for stream depletion

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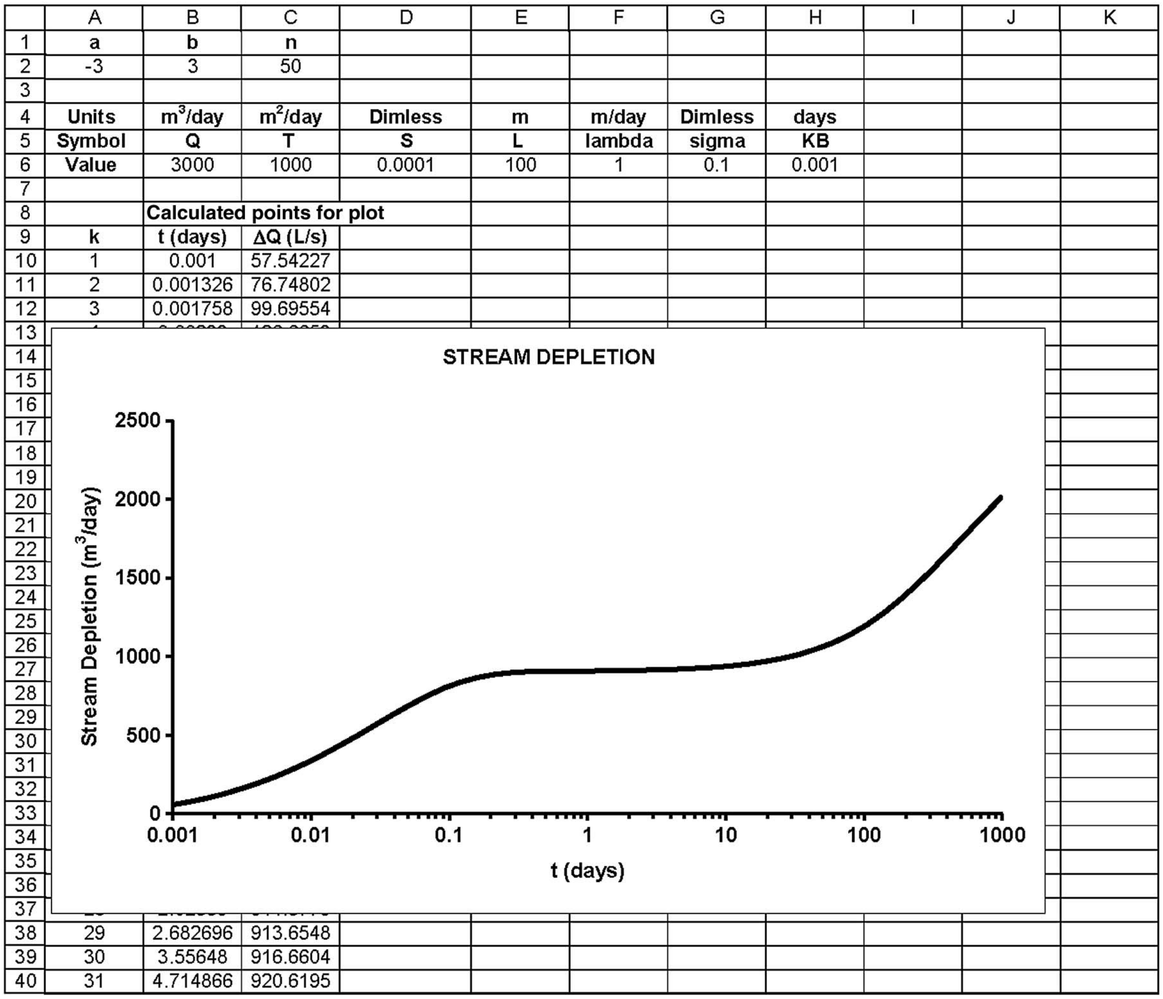

Fig. 7. A calculated stream-depletion plot embedded in its worksheet

menu. For example, right-clicking on the horizontal axis and choosing "Format Axis" allows the linear scale to be changed to a logarithmic scale.

In later versions of Excel, select cells B10:C59, click on the "Insert" tab at the top of the page, click on "Scatter" above "Charts" at the top of the page and choose the first option in the "Scatter" drop-down menu. Finally, choose the desired chart layout at the top of the page. Options for deleting, inserting, or changing various parts of the chart can be obtained by right-clicking on various parts of the chart.

Computations for drawdown in an observation well are carried out in a similar way and are shown in Fig. 8 for a well at the point $(x, y) = (50{,}100)$ m. This spreadsheet differs from the one shown in Fig. 7 only by the inclusion of the observation well coordinates in cells I6:J6 and by inserting in cell C10 the following formula:

$$= (Q/T) * W\_4[x/L, y/L, \mathrm{B10} * T/(S * L^2), \mathrm{lambda} * L/T, KB * L^2/T, S/\sigma] \tag{29}$$

Eq. (29) follows from Eq. (23).

Field observations of drawdown can also be shown in the plot for Fig. 8. This is done in versions of Excel up to and including Excel 2003 by typing the field data in columns, selecting both columns of data and clicking on "Edit" and "Copy." Then click on the embedded plot (shown in Fig. 8) and click on "Edit" and "Paste Special." This causes a drop-down menu to appear in which the user must ensure that "Add cells as New series," "Values (Y) in Columns," and "Categories (X Values) in First Column" are all ticked. Marker points used for the experimental points can be edited by right-clicking on the plotted points and then left-clicking on "Format Data Series." Fig. 9 shows the end result for the example considered herein.

In later versions of Excel, select both columns of field data and click on "Copy" above Clipboard at the top of the page. Then left click on the box surrounding the plot, click on "Paste" above "Clipboard" at the top of the page and choose "Paste Special" in the "Paste Options" drop-down menu. Finally, choose "Categories (X Values) in First Column" in the "Paste Special" drop-down menu.

Values for aquifer parameters are obtained by adjusting values for T, S, lambda, sigma, and KB for the calculated plot (cells C6, D6, F6, G6, and H6, respectively, in Fig. 8) until the calculated plot provides a good fit to the experimental points. (The calculated results and corresponding plot change almost instantaneously as soon as a new value is entered.) Fig. 10 shows the end result of this fitting process, which is simplified considerably by realizing which parts of the plot are controlled by values for the different parameters. This process is described in more detail at the end of the next section and is also illustrated in Lough (2004) and Lough and Hunt (2006), where an example can be seen of fitting a calculated curve to actual field data rather than the artificially generated data shown in Figs. 9 and 10. Comparison of the fitted parameter values

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	A	B	C	D	E	F	G	H	I	J	K
1	a	b	n								
2	-3	3	50								
3											
4	Units	m³/day	m²/day	Dimless	m	m/day	Dimless	days	m	m	
5	Symbol	Q	T	S	L	lambda	sigma	KB	x	y	
6	Value	3000	1000	0.0001	100	1	0.1	0.001	50	100	
7											
8		Calculated points for plot									
9	k	t (days)	Drawdown (m)								
10	1	0.001	0.205676506								
11	2	0.001326	0.254761892								
12	3	0.001758	0.306444139								
13	4	0.00233	0.35977289								
14											
15											
16											
17											
18											
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29											
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31											
32											
33											
34											
35											
36											
37											
38											
39	[illegible]	[illegible]	[illegible]								
40	31	4.714866	0.919563968								
41	32	6.250552	0.921520782								
42	33	8.286428	0.924085301								
43	34	10.98541	0.927435274								

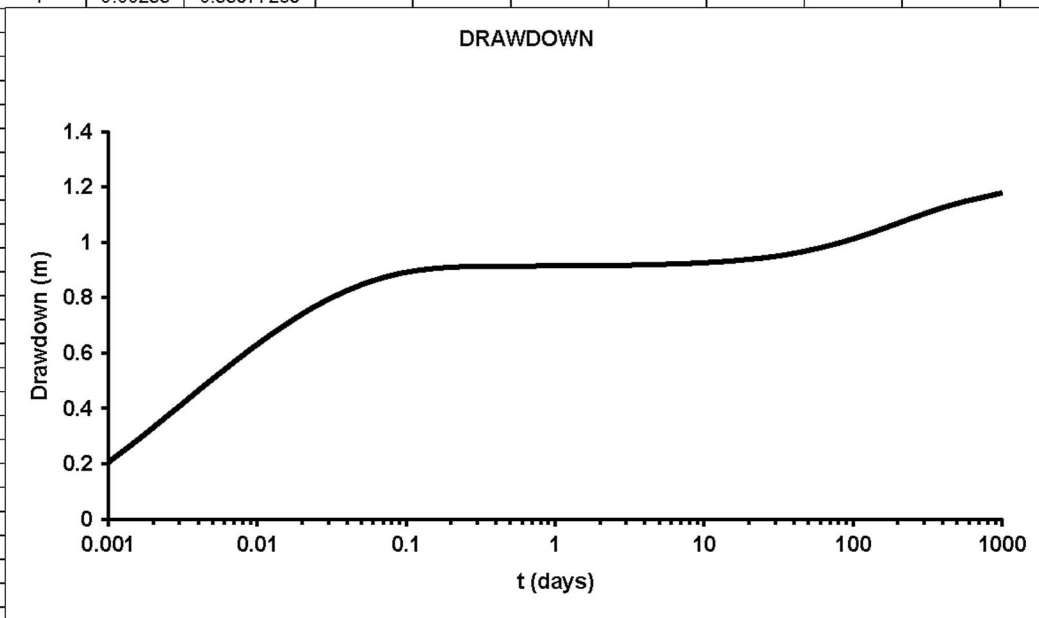

Fig. 8. Calculated drawdowns plotted for an observation well

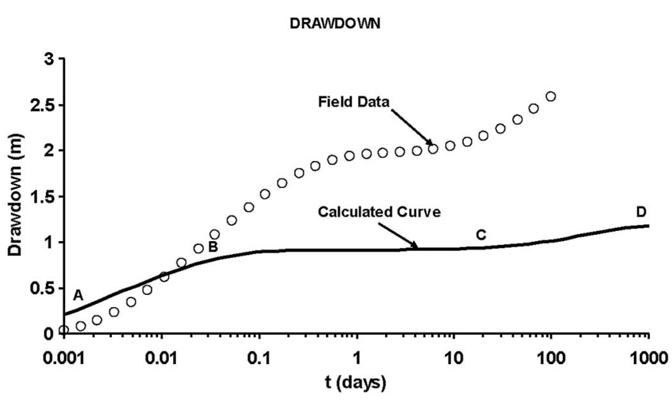

Fig. 9. Drawdown field data superimposed on the plot in Fig. 8

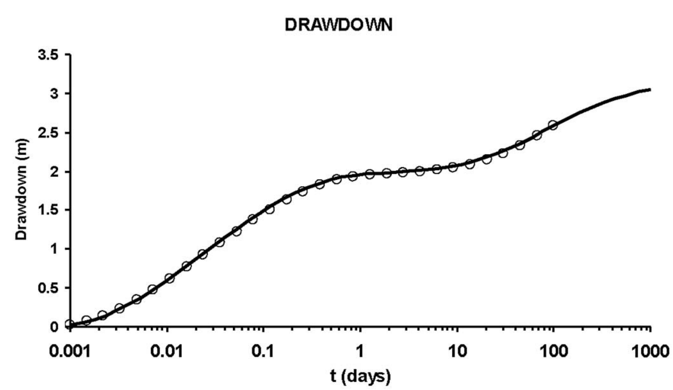

Fig. 10. Adjusted fit of field data to calculated curve in Fig. 9

(T, S, σ, λ, K'/B') = (500, 0.0003, 0.017, 0.2, 0.0006) (units omitted for printing brevity) with the parameters (T, S, σ, λ, K'/B') = (530, 0.0003, 0.02, 0.1, 0.0006) that were used to generate the artificial field data suggests that λ is the most difficult parameter to determine accurately. This is because the biggest influence of λ is to determine the steady-flow asymptote at extremely large values of time, and the experimental data does not contain drawdown values for values of time that are sufficiently large to show this steady-flow asymptote.

Readers who want to make use of a fill-in-the-blanks spreadsheet for stream-depletion problems can download a spreadsheet program for this purpose at the following page on the Environment

Canterbury website: http://ecan.govt.nz/services/online-services/tools-calculators/Pages/groundwater-tools.aspx#stream-depletion-xls. This spreadsheet, which is based on computational programs contained in Function.xls, calculates solutions using the Hunt (2003) stream-depletion solution. However, it will not allow users to calculate solutions for any of the other numerous programs contained in Function.xls. For this reason, readers are advised to learn to construct their own spreadsheet solutions using the techniques just described.

Solution Behavior

In some problems stream depletion and drawdown measurements in an observation well appear to approach a condition of steady flow in which flows and drawdowns appear to approach horizontal asymptotes within a few days after pumping starts. This condition is illustrated by segment abc for the drawdown curve shown in Fig. 11. In actual fact, this is a condition of pseudo steady flow since stream depletion and drawdowns will start to increase again as free surface levels in the aquitard draw down and begin to approach piezometric levels along the common boundary between the overlying aquitard and the underlying pumped aquifer. Then vertical recharge from the aquitard slows down drastically, and drawdown values in the pumped aquifer start to increase once again. Pumping tests are often not carried out for sufficiently long periods of time to get past this period of pseudo steady flow. Consequently, these aquifers may be analyzed incorrectly as Hantush-Jacob (1955) leaky aquifers.

When pumping continues for a sufficiently long period of time, stream depletion and drawdown values eventually approach a condition of true steady flow. At this point, stream depletion and flow abstraction from the well become equal. This is illustrated by segment de in Fig. 11. The time required to approach this condition may require a pumping period of the order of thousands of days. Although this period of steady flow can be modeled with an idealized mathematical model, conditions in a field situation are not likely to remain constant for a long enough period of time to reach this condition of steady flow. This illustrates why it is important to use an unsteady flow model with superposition and time translation to model realistic pumping schedules when applying for water consents.

It was shown previously how the superposition principle can be applied with the Hunt (2003) solution to model stream depletion along streams that have some stream segments that gain water from groundwater flow and other segments that lose water to groundwater flow. At first glance it may appear that abstracting water from a well near a region of gaining stream flow will intercept part or all of the flow that is already seeping through the ground and will not take flow directly from the stream itself. This is an illusion, however, since intercepted seepage that would ordinarily enter the stream has now been removed and will no longer be available to replenish the stream flow. Thus, the abstraction well will still cause a reduction in stream flow.

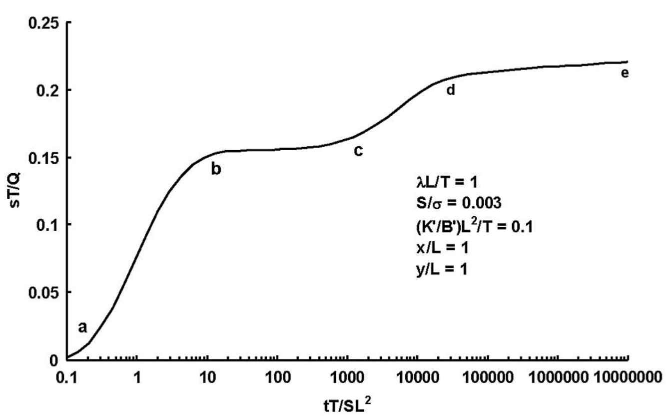

Fig. 11. Drawdown in an observation well for stream depletion in a delayed-yield aquifer

An accurate calculation of stream depletion requires an accurate estimate for the various hydrogeologic parameters that appear in the stream depletion solution. These parameters are listed below:

- T = pumped aquifer transmissivity,
- S = pumped aquifer storativity,
- σ = aquitard specific yield,
- K'/B' = ratio of hydraulic conductivity to saturated thickness for the aquitard (also referred to as aquitard conductance), and
- λ = streambed conductance coefficient.

These parameters cannot be measured accurately in a laboratory but instead must be found by comparing a calculated drawdown curve with an experimental drawdown curve measured in the field with a pumping test. As noted previously, an application of this procedure to actual field data was carried out by Lough (2004) and was later published by Lough and Hunt (2006).

The problem of determining parameters by using a comparison of calculated and measured drawdown curves in an observation well is often referred to as an inverse problem. Inverse problems are notoriously difficult to solve accurately, and one recommended procedure is to adjust parameter values in the calculated solution until a good fit is obtained between calculated and measured drawdown curves. (A pessimist would call this a trial-and-error procedure, while an optimist would refer to it as a method of successive approximation!) This means that it is essential to understand how the various parameters affect different parts of a drawdown curve. In particular, values for T control the slope of segments ab and cd in Fig. 11. [Segments ab and cd have the same asymptotic slope in the Hunt (2003) solution, although in some applications the length of these segments may not be sufficiently long to fully approach these asymptotes. An illustration of this is shown in Fig. 11, where segment cd is not sufficiently long to approach the same slope as segment ab.] The horizontal position of segment ab is controlled by values of S, and the horizontal position of segment cd is controlled by values of σ. Values of K'/B' have a major influence on the vertical elevation of segment bc, and values of λ have a major influence on the vertical elevation of segment de and a less important influence on the vertical elevation of segment bc.

Some Generalizations and Additional Solutions

Butler et al. (2001) obtained a solution for the case when the stream and pumped aquifer both have finite widths. The aquifer beneath the stream was modeled with a Hantush-Jacob leaky aquifer, and the two finite-width aquifers on both sides of the stream were modeled with the same type of aquifer considered by Theis (1941). Hunt (2008) obtained a solution when the finite-width aquifers on both sides of the stream were modeled with Boulton delayed-yield aquifers. The Hunt (2008) solution can be used to reproduce the Butler et al. (2001) solution by setting $K'/B' = 0$ and replacing S with σ. The Hunt (2008) solution showed that the zero stream width approximation used in the Hunt (2003) solution can be applied with confidence to calculate stream depletion values when the pumped well is more than one stream width from the nearest

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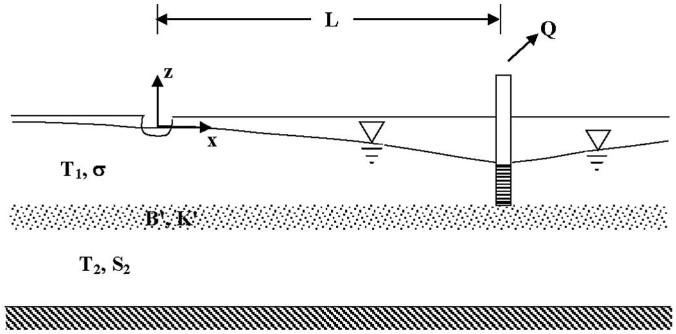

Fig. 12. Flow to a well beside a stream in a two-aquifer system

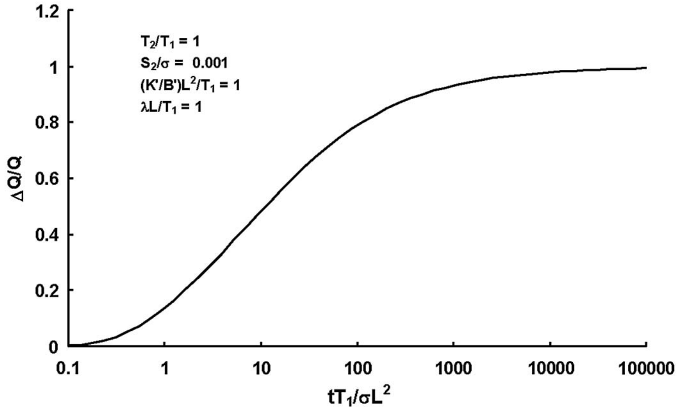

Fig. 13. Flow depletion using typical parameter values for the problem shown in Fig. 12

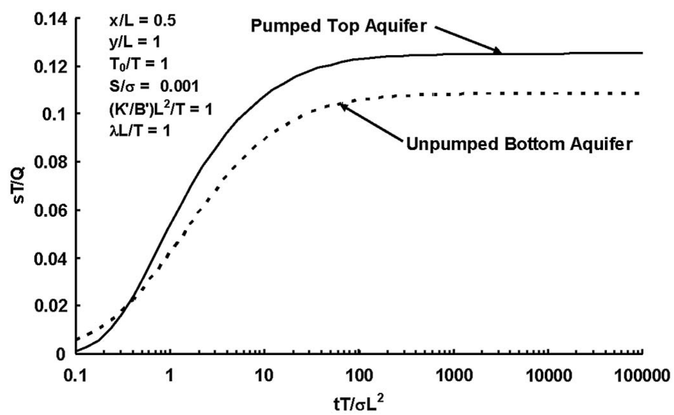

Fig. 14. Drawdowns using typical parameters for the problem shown in Fig. 12

edge of the stream. It was also found that drawdown values could be calculated with reasonable accuracy with the zero stream width solution when the pumped well is at least five stream widths from the nearest stream boundary. Additional calculations showed that stream depletion values and drawdown values are relatively sensitive to the spacing of the lateral impermeable aquifer boundaries at larger values of time. At smaller values of time, however, differences calculated for a finite-width aquifer and an infinitely wide aquifer are negligible. Programs for calculating stream depletion and drawdown values from the Hunt (2008) solution are available in Function.xls.

Hunt (2009) obtained a stream depletion solution when the pumped aquifer was underlain by an aquitard and a second unpumped aquifer, as shown in Fig. 12. The top pumped aquifer was identical with the aquifer shown in Fig. 1(c) (i.e., the free surface in the top aquifer was not contained within an aquitard, as shown in Fig. 5 for a delayed-yield aquifer). These calculations showed that the three changes in curvature that are characteristic of stream depletion and drawdown curves in a delayed-yield aquifer can only be obtained by giving the ratio of elastic storativity of the bottom unpumped aquifer to the specific yield of the top pumped aquifer a value that exceeded one. In practice, this ratio always has a value that is much less than one. (Typically, this ratio is about 0.001.) This result shows that the delayed-yield behavior often observed in stream depletion problems cannot be explained by the presence of an aquitard and unpumped aquifer beneath the top pumped aquifer. Calculations also showed that stream depletion values calculated for the two-aquifer model could be closely approximated with the Hunt (1999) solution by replacing the single-aquifer transmissivity in the Hunt (1999) solution with the sum of both aquifer transmissivities in the two-aquifer model. The implication is that this same approximation can be made when using the Hunt (2003) solution for a two-aquifer model when only the top aquifer is pumped. Programs for calculating stream depletion and drawdown values for the two-aquifer model are available in Function.xls.

Some example calculations for the stream-depletion problem shown in Fig. 12 are plotted in Figs. 13 and 14. The plot in Fig. 13 shows total flow depletion from the stream, and, as just noted, there is no hint of the presence of the three changes in curvature that are characteristic of stream-depletion plots in a delayed-yield aquifer. Drawdowns plotted in Fig. 14 show for larger values of time that drawdowns in the lower unpumped aquifer are less than drawdowns in the pumped aquifer in a region around the pumped well. Thus, even as steady flow is approached after an infinite period of time, an upward piezometric gradient still exists across the aquitard, and seepage moves upward from the unpumped lower aquifer into the pumped aquifer in a region near the well. Since the stream furnishes the only inexhaustible supply of aquifer recharge to the system, water in the top aquifer near the stream must seep downward across the aquitard into the bottom aquifer to supply water that is eventually abstracted from the bottom aquifer by the pumped well. The fact that drawdowns are shown in Fig. 14 to be greater in the bottom aquifer than in the top pumped aquifer at early values of time simply shows that the boundary of the region in which water from the top aquifer recharges the bottom aquifer moves with time.

Ward and Lough (2011) obtained a stream-depletion solution for the two-layer leaky aquifer system shown in Fig. 12 when water is pumped from the bottom aquifer rather than from the top aquifer. Calculations carried out for the case when the top unconfined aquifer has a transmissivity with the same order of magnitude as the transmissivity of the bottom pumped aquifer showed that (1) drawdowns in the bottom pumped aquifer had a behavior typical of Boulton's delayed yield aquifer (i.e., the drawdown curve for the bottom aquifer had three changes in curvature), and (2) stream depletion values may develop more rapidly than for the Hunt (2003) solution, in which the free surface is contained entirely within an overlying aquitard. Eventually, however, the Ward and Lough (2011) and the Hunt (2003) solutions for stream depletion both approach the same steady-flow asymptote, where well abstraction and stream depletion become equal.

Some of the characteristic behaviors observed by Ward and Lough (2011) in their stream-depletion solution are shown in

Figs. 15 and 16. In common with the Boulton (1963) solution for flow to a well in a delayed-yield aquifer, the drawdown curve for the bottom pumped aquifer in Fig. 16 has three changes in curvature. This suggests that the delayed-yield behavior of a pumped aquifer requires either a free surface within an aquitard above the pumped aquifer or else an unconfined aquifer with at least one aquitard between the free surface and the pumped aquifer. Despite the three changes in curvature present in the drawdown curve for the bottom pumped aquifer, however, the drawdown curve for the top unpumped aquifer in Fig. 16 and the stream-depletion curve in Fig. 15 have only one change in curvature. Second, if a stream is present in the system, then the stream will furnish the only inexhaustible source of recharge. This means that steady flow will be achieved after an infinite period of pumping and that steady-flow stream depletion will equal well abstraction when steady flow is reached. Third, since all recharge furnished to the lower pumped aquifer must first pass from the stream into the top aquifer before ultimately seeping into the bottom pumped aquifer, drawdowns at all points in the top aquifer must always be less than drawdowns at underlying points in the bottom pumped aquifer. Programs for calculating the Ward and Lough (2011) solution have been added to Function.xls.

Hunt and Scott (2005, 2007) compared numerical solutions, using both a MODFLOW finite-difference model and a semianalytical model, for a well abstracting water from the bottom aquifer

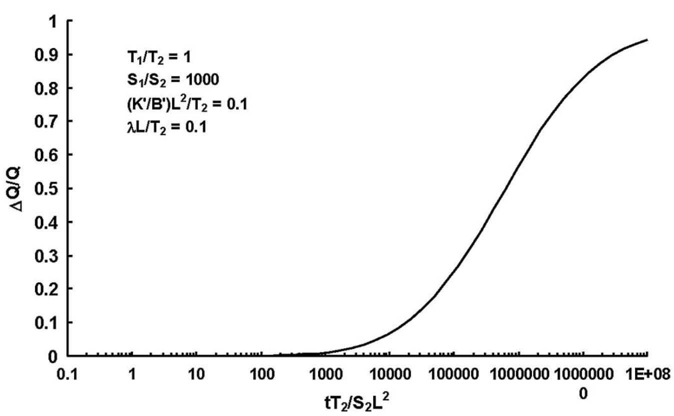

Fig. 15. Stream depletion calculated from the Ward and Lough (2011) solution for the hydrogeology shown in Fig. 12 when water is pumped from the bottom aquifer

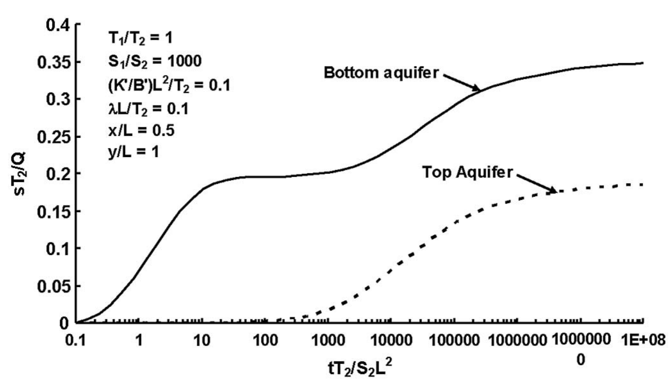

Fig. 16. Drawdowns calculated from the Ward and Lough (2011) solution for the hydrogeology shown in Fig. 12 when water is pumped from the bottom aquifer

in a two-aquifer system. There was no stream present in this problem, and, since no inexhaustible source of recharge was present, true steady flow was never achieved. Nevertheless, some conclusions drawn from these results are equally applicable to the stream-depletion problem considered herein. First, apart from the final steady-flow portion of the drawdown curves shown in Fig. 11, drawdown curves for the Hunt and Scott (2005, 2007) solutions and the Ward and Lough (2011) solution had similar behaviors. In particular, the drawdown curve for the bottom pumped aquifer had an initial pseudo steady-flow response that typifies a delayed-yield response to pumping, whereas the top unpumped, unconfined aquifer did not contain a similar pseudo steady-flow response. Second, drawdowns in the bottom pumped aquifer were everywhere greater than drawdowns at overlying points in the top aquifer, which is in agreement with the result shown in Fig. 16 for the solution obtained by Ward and Lough (2011). Third, the simpler Boulton (1963) solution was found to be applicable to the two-aquifer problem if the transmissivity of the top aquifer is less than or equal to 5% of the bottom pumped aquifer transmissivity. Thus, the Hunt (2003) solution can be expected to apply to stream-depletion problems when the overlying aquitard containing a free surface has a transmissivity that is less than or equal to 5% of the pumped aquifer transmissivity. Fourth, the MODFLOW finite-difference model accounted for compressibility effects in the aquitard sandwiched between the two aquifers, whereas the semianalytic model neglected aquitard compressibility. This allowed a comparison to be made to investigate the effects of aquitard compressibility, and it was found that aquitard compressibility could be neglected if the ratio of aquitard storativity to pumped aquifer storativity has an order of magnitude that does not exceed one. The implication of course is that similar results hold for stream-depletion problems.

Tsou et al. (2010) obtained a solution for stream depletion in a semi-infinite aquifer when the pumped well is either horizontal or slanted. This was accomplished by first obtaining the solution for flow to a point sink beside a stream. Then solutions for point sinks were distributed along the axis of either a horizontal or slanted well to obtain a solution flow to a line sink. The pumped aquifer was confined, occurred on only one side of the stream, and extended from the stream edge to infinity.

Numerical Models

The development of numerical models in the 1970s and 1980s overcame some of the limitations imposed by the simplifying assumptions on which analytical and semianalytical models are based. The flexibility of 3D numerical modeling allows the representation of general hydrologic settings that include heterogeneous aquifers, multiple aquifers, stream bends, more than one stream and partly penetrating rivers and wells. Models become increasingly complex, and the most difficult part of the modeling process is to obtain enough accurate data to calibrate the model. It therefore becomes time demanding and relatively expensive to set up, calibrate, and verify models. Numerical groundwater models have been used to assess effects on stream flow. For example, Spalding and Khaleel (1991), Sophocleous et al. (1995), and Clausen et al. (1993) evaluated analytical solutions and carried out sensitivity analysis with respect to abstraction regimes. Van Lanen and van de Weerd (1993) evaluated the effect of land-use change and climatic changes. Although numerical models are particularly valuable for sensitivity analysis, for large-scale catchment modeling or for streams and rivers of special interest, it is usually simpler and much more efficient to use either analytical or semianalytical models to assess stream depletion effects for a proposed groundwater abstraction scheme.

Conclusions

A brief history has been given of the development of analytical and semianalytical models for the solution of stream-depletion problems. A stream-depletion solution published in 2003, which is also capable of duplicating all previously published solutions, has been discussed in detail. In the 2003 solution the stream partially penetrates a Boulton delayed-yield aquifer and is modeled with an infinitely long straight line that has zero thickness. It has been explained why the 2003 solution applies for piezometric head levels in the pumped aquifer that are not initially horizontal when pumping begins, in which case different segments of the stream may be either gaining or losing water to the pumped aquifer. Excel computational tools that are available to practitioners without charge have been referenced and used in an illustration for the calculation of the 2003 solution, and the general behavior of the solution has been discussed. Generalizations and additional solutions have been described and referenced for streams and aquifers with finite width, for systems containing several aquifers and for abstraction wells that are either horizontal or slanted. Finally, the review has concluded with a very brief discussion of the advantages and disadvantages of using numerical solutions for modeling stream-depletion behavior.

Acknowledgments

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