Optimization-based modeling of dynamics in bat prey capture

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Nature Inspired Flight Technologies and Ideas

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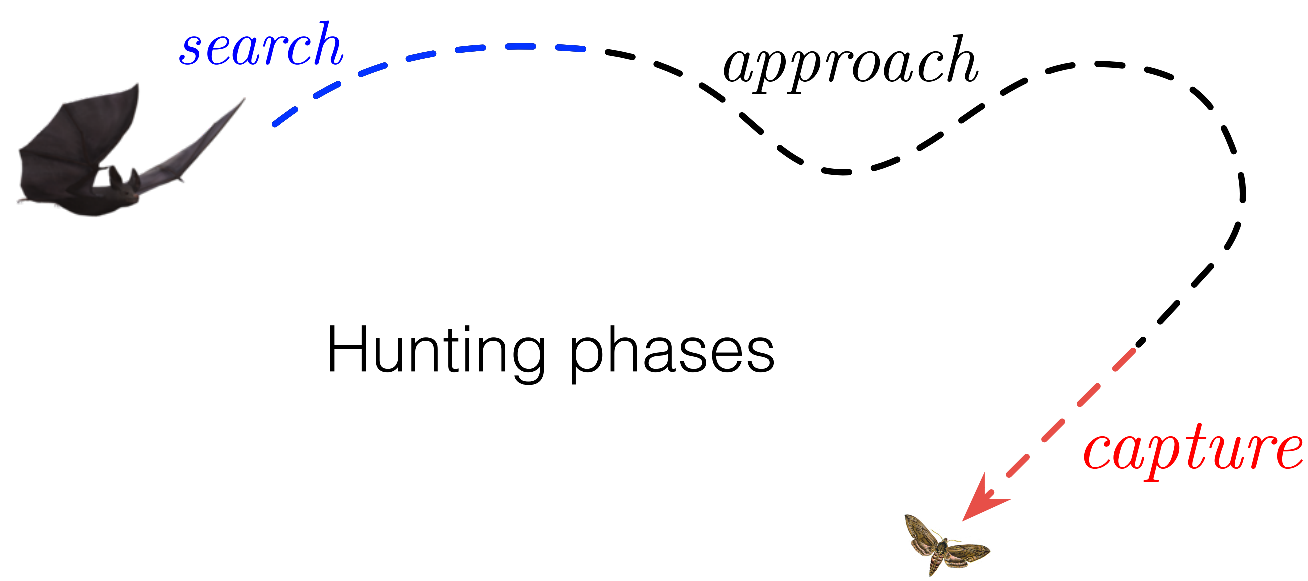
Outline

- **Motivation** and **basic setting**
- **Mathematical model**
  - dynamics
  - optimization
  - model predictive control
- **Preliminary numerical results**
- **Conclusion** and **future work**

Motivation

During prey pursuit bats behave in 3 modes:

1. **Search** (target detection)
2. **Approach** (tracking)
3. **Capture** - terminal buzz



Our hypothesis: There is a trade-off **Energy vs. Capture Time**

The bat would like to capture its prey as quickly as possible

The bat needs to resolve the location and velocity of the target

Abrupt steering and high acceleration are energetically costly

Goals:

1. Understand **the transition** from the approach to capture mode
2. Obtain efficient methods to **jointly optimize trajectory** and **capture time**

Echolocation

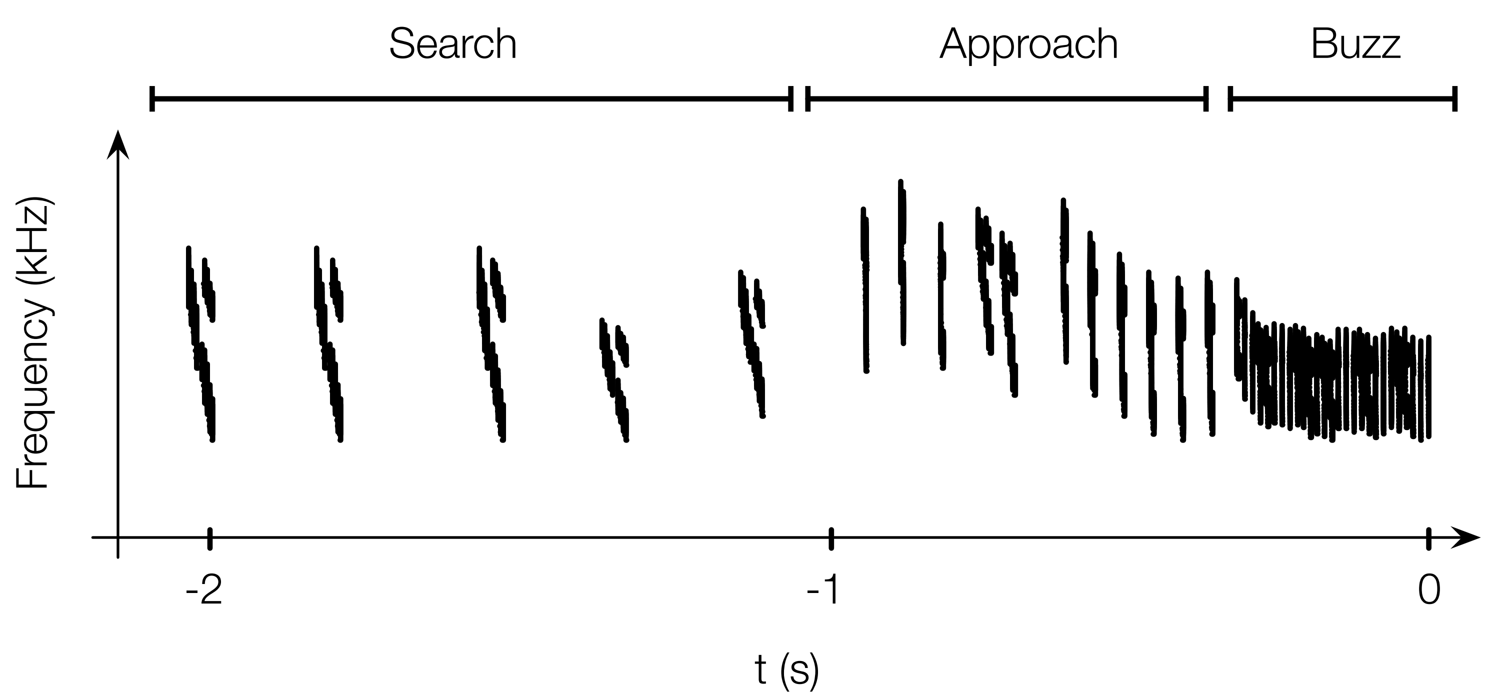
1. Active sensing system

The bat produces the signals used to sense its environment

2. Provides information on:

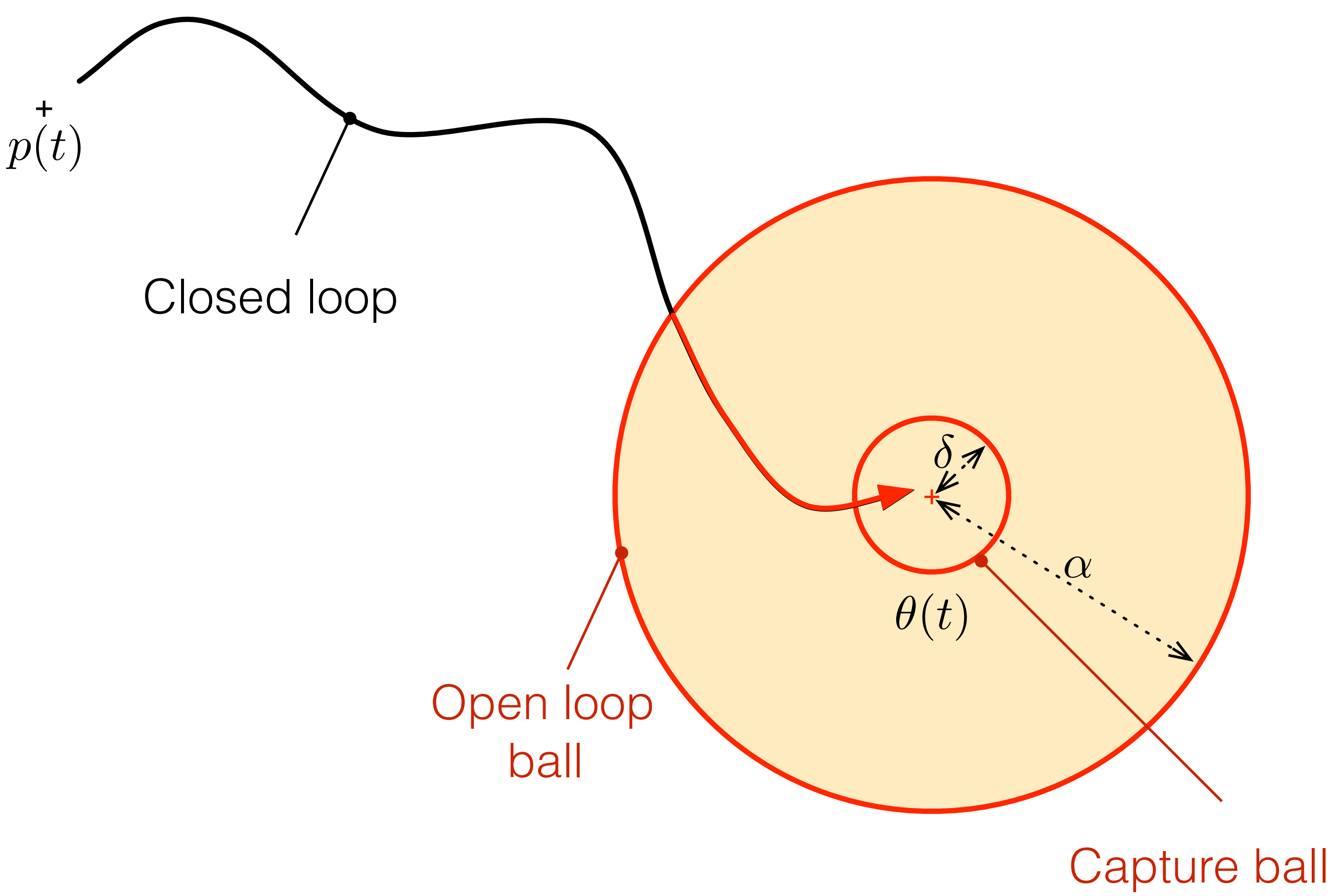
- **distance** between target and bat
- **relative velocity** of the target
- size of the target
- azimuth and elevation
- target texture

"As a bat approaches and attacks an insect, the bandwidth of individual calls increases and **the rate at which calls are produced also increases**, allowing the animal to more accurately and rapidly sample the position and features of a sonar target." - **Ulanovsky and Moss '08**



Mathematical model

Use a point-mass particle to model the dynamics of the bat



1. During the approach phase, the bat uses **echolocation** to move toward the target

2. Outside the ball echolocation provides the bat with its **relative distance** and **velocity** to the target

3. Inside the ball the bat **cannot make measurements** and enters the capture phase relying on information accumulated until then

When should the bat initiate capture?

Mathematical model

State representation based on two quantities that the bat observes through echolocation:

relative distance:

$$p_x(t) - \theta_x(t)$$
$$p_y(t) - \theta_y(t)$$

relative velocity:

$$v_x^{\mathrm{rel}}(t) = \frac{d}{dt}(p_x(t) - \theta_x(t))$$
$$v_y^{\mathrm{rel}}(t) = \frac{d}{dt}(p_x(t) - \theta_y(t))$$

Our preliminary model assumes a **stationary target**

$$\theta_x(t) = \theta_y(t) = 0, \quad t \geq 0$$

Define the **position and velocity** as state variables and the **acceleration** as control input

$$\begin{bmatrix} \dot{p}_x \\ \dot{p}_y \\ \dot{v}_x \\ \dot{v}_y \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} p_x \\ p_y \\ v_x \\ v_y \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} a_x \\ a_y \end{bmatrix}$$

Our framework can also support more realistic (nonlinear and 3D) dynamics

Mathematical model

Which can be discretized in intervals of duration Δ

$$\begin{bmatrix} p_x \\ p_y \\ v_x \\ v_y \end{bmatrix}_{k+1} = \underbrace{\begin{bmatrix} 1 & 0 & \Delta & 0 \\ 0 & 1 & 0 & \Delta \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{A} \begin{bmatrix} p_x \\ p_y \\ v_x \\ v_y \end{bmatrix}_{k} + \underbrace{\begin{bmatrix} \frac{1}{2}\Delta^2 & 0 \\ 0 & \frac{1}{2}\Delta^2 \\ \Delta & 0 \\ 0 & \Delta \end{bmatrix}}_{B} \begin{bmatrix} a_x \\ a_y \end{bmatrix}_{k}$$

$$x_k = \begin{bmatrix} p_x \\ p_y \\ v_x \\ v_y \end{bmatrix}_{k} \quad u_k = \begin{bmatrix} a_x \\ a_y \end{bmatrix}_{k}$$

motor system noise +
other external disturbances

$$x_{k+1} = Ax_k + Bu_k + w_k$$

$$w_k \sim \mathcal{N}(0, \Sigma_w)$$

Echolocation provides **noisy** measurements of the state variables

$$y_k = x_k + v_k$$

$$v_k \sim \mathcal{N}(0, \Sigma_v)$$

Kalman filter

$$\hat{x}_k = \bar{x}_k + L(y_k - \bar{x}_k)$$

$$\bar{x}_k = A\bar{x}_{k-1} + Bu_{k-1}$$

Optimization problem

The **Energy vs. Capture Time trade-off** is represented by the following cost

$$J(u,N) = \mathbf{E}\left[\underbrace{\sum_{k=0}^{N-1} c(x_k,u_k)}_{\text{Energy cost per stage}} + \underbrace{F(x_N)}_{\text{Terminal cost}}\right] + \underbrace{\gamma \cdot N}_{\text{Capture time}}$$

$$c(x_k,u_k) = x_k' Q x_k + u_k' R u_k$$

$$F(x_N) = p_N' Q_f p_N$$

or

$$F(x_N) = \begin{cases} +\infty & \text{if } \ p_N \notin \mathcal{B}_\delta \\ 0 & \text{if } \ p_N \in \mathcal{B}_\delta \end{cases}$$

Optimization problem

$$
\begin{aligned}
\underset{u,N}{\text{minimize}} \quad & J(u, N) \\
\text{subject to} \quad & x_{k+1} = A x_k + B u_k + w_k, \quad k = 0, \ldots, N-1 \\
& u_k \in \mathcal{U}, \quad k = 0, \ldots, N-1
\end{aligned}
$$

$$p_k \in \mathcal{B}_\alpha \rightarrow \text{enter capture phase}$$

$$p_k \notin \mathcal{B}_\alpha \rightarrow \text{continue to approach}$$

Difficulties:

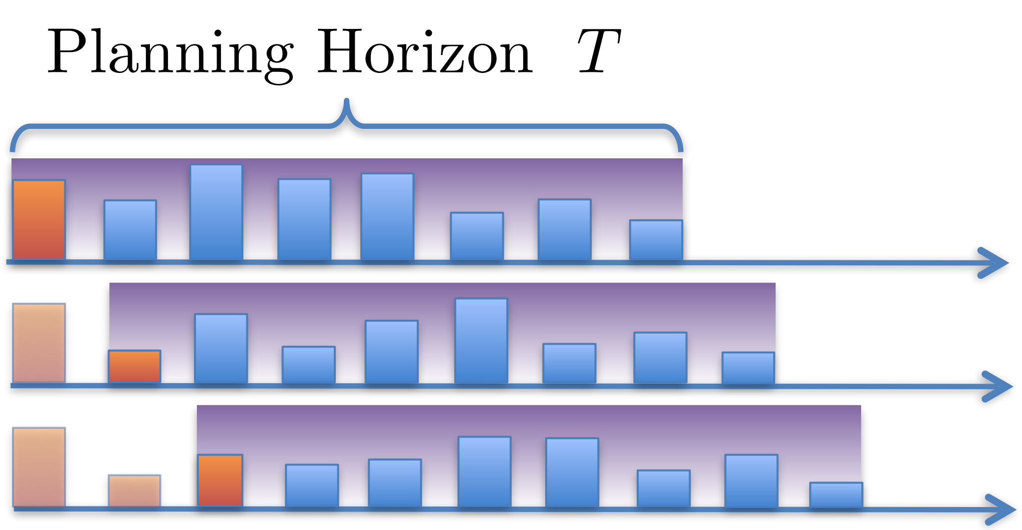
1. The analytical solution is an **open research problem**
2. It is more likely that the bat uses **near optimal** strategies than truly optimal ones
3. Optimal solutions would not account for a **limited cognitive capacity** of the bat

Model predictive control

The case for MPC instead of optimal control and stochastic control

Model Predictive Control

1. Based on a **heuristics**
2. Requires **no analytical solutions**
3. **Performance is close to optimal** in most cases



Model predictive control

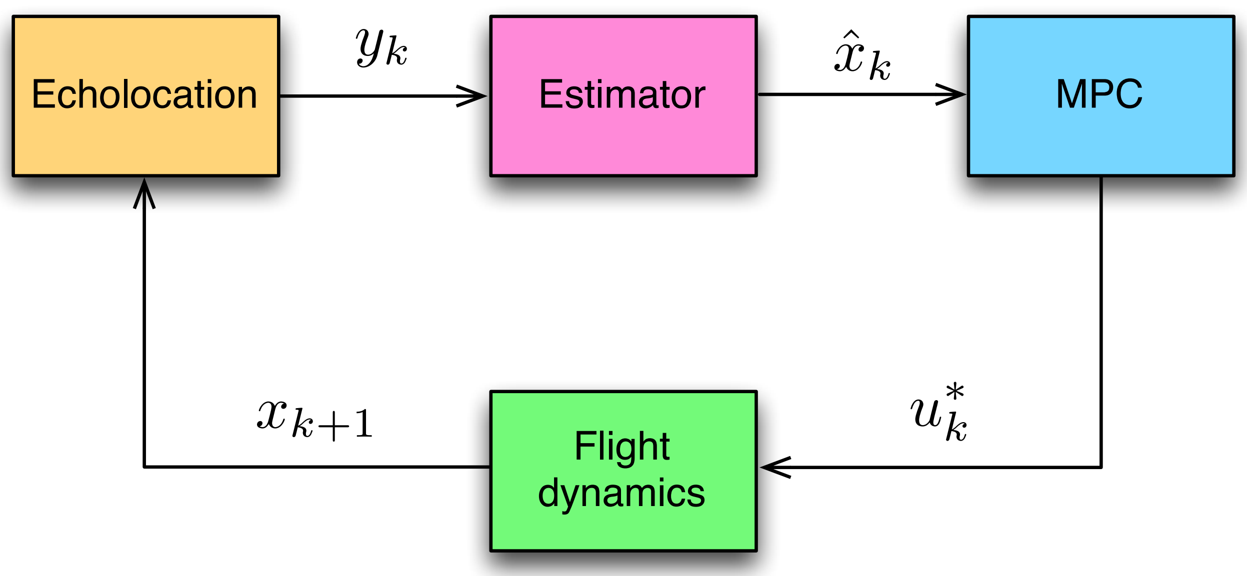
Using MPC is like playing a game of chess:

1. **predict** your opponent's moves over the next few steps
2. **select your strategy** based on your predictive model
3. **use only the first move** in the sequence

According to *nature.com* chess players typically use

$T = 8$

closed loop phase



MPC = Model Predictive Control

Step 1: Use the output of a **Kalman filter** to form a predictive model of the dynamics

Step 2: Solve a finite optimization problem

Step 3: Apply first element of the control sequence to the system

Model predictive control

$\hat{x}_k$

Kalman filter output

$$
\begin{array}{ll}
\text{minimize} & \displaystyle\sum_{t=k}^{T+k-1} (x_t' Q x_t + u_t' R u_t) + x_{T+k}' Q_f x_{T+k} \\
\text{subject to} & x_{t+1} = A x_t + B u_t, \quad t = k, \ldots, T+k-1 \\
& \|u_t\|_\infty \leq B, \quad t = k, \ldots, T+k-1 \\
& \|u_{t+1} - u_t\|_\infty \leq S, \quad t = k, \ldots, T+k-1 \\
& x_k = \hat{x}_k
\end{array}
$$

$u_k^\star$

Flight dynamics input

Optimize over the variables $x_k, \ldots x_{T+k}$

$u_k, \ldots, u_{T+k-1}$

Convex optimization problem that can be efficiently solved

The planning horizon T is a parameter that depends on the cognitive capacity of the bat

Preliminary numerical results

This framework allows us to perform **many** numerical experiments!

Our initial results do not include the optimization of capture time

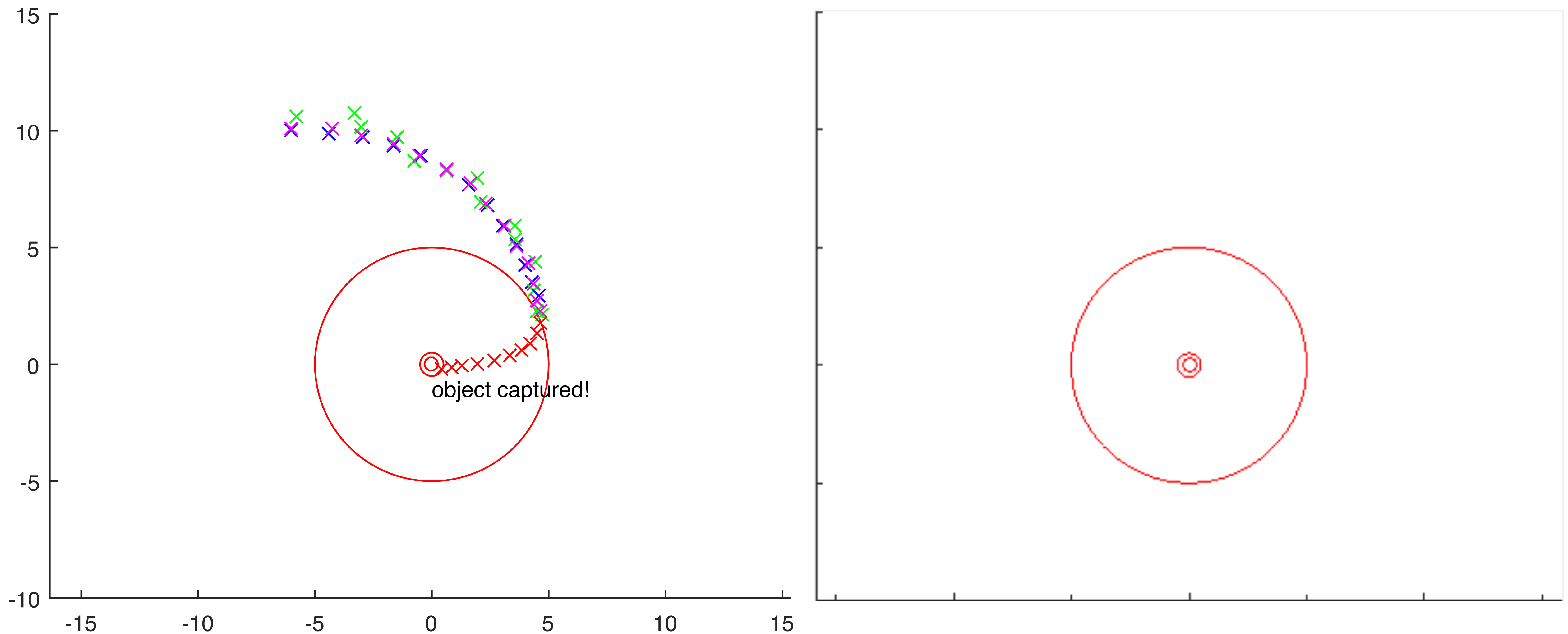
Fix $\Delta = 0.5 \quad T = 10 \quad \delta = 0.5$

- While the bat is **outside the larger ball**, perform MPC
- When the bat enters the **blind stage**, use the entire sequence of controls computed in the last round of MPC
- If the capture ball is reached, stop the simulation

Preliminary numerical results

Capture with nonzero initial and terminal velocities

Animation



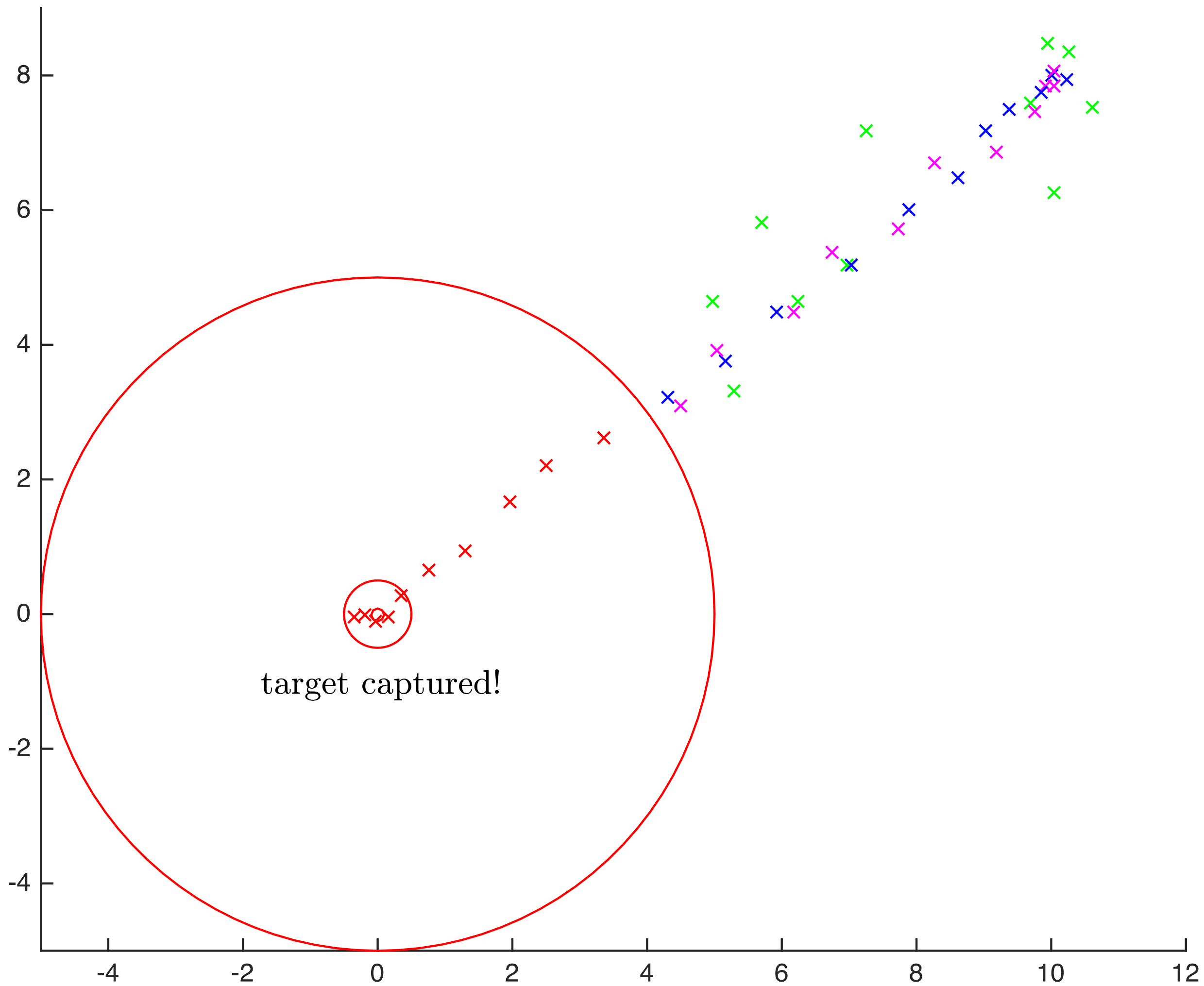
x - Position

x - Noisy measurement

x - Estimate

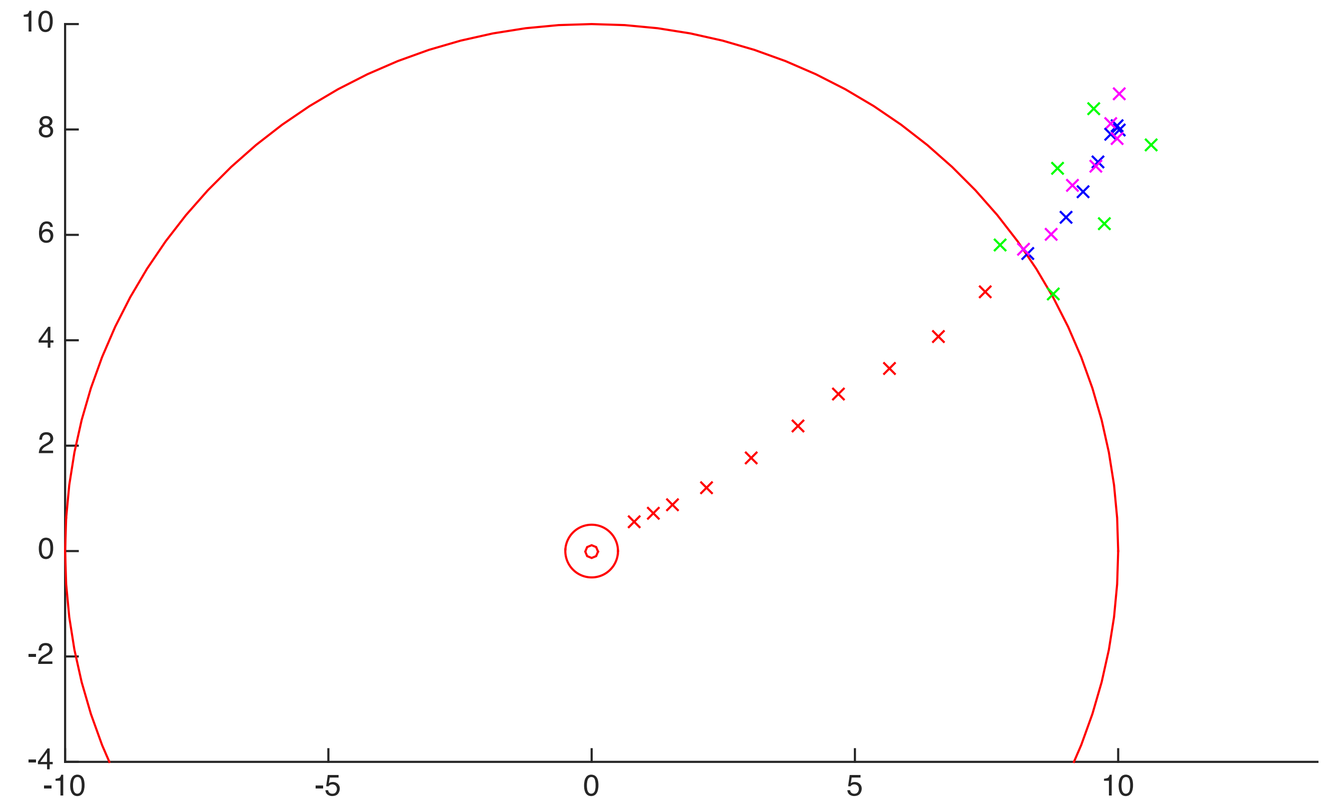
Preliminary numerical results

Homing with zero initial and terminal velocities



$\alpha = 5$

Successful capture

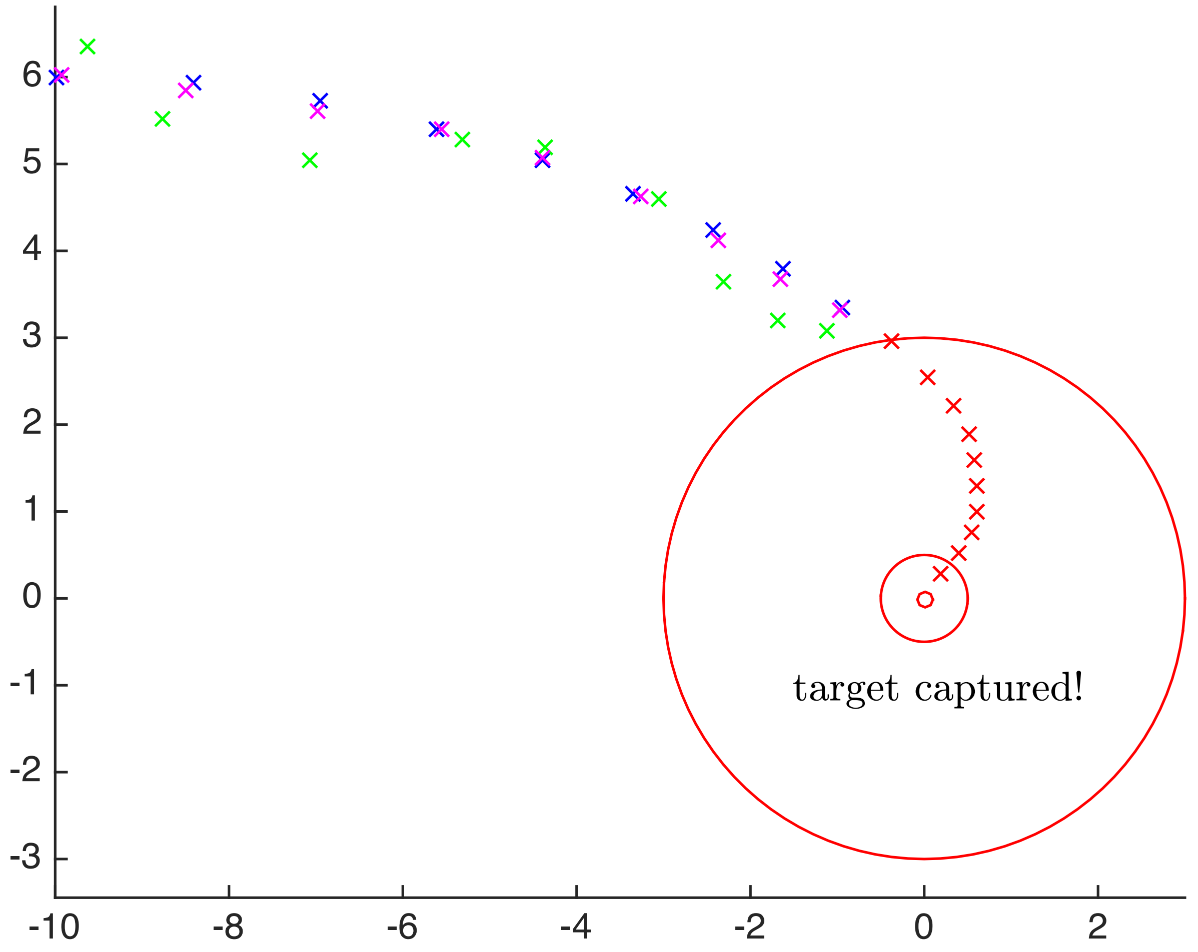


$\alpha = 10$

Failed attempt

Preliminary numerical results

Monte Carlo simulation: **rate of success**



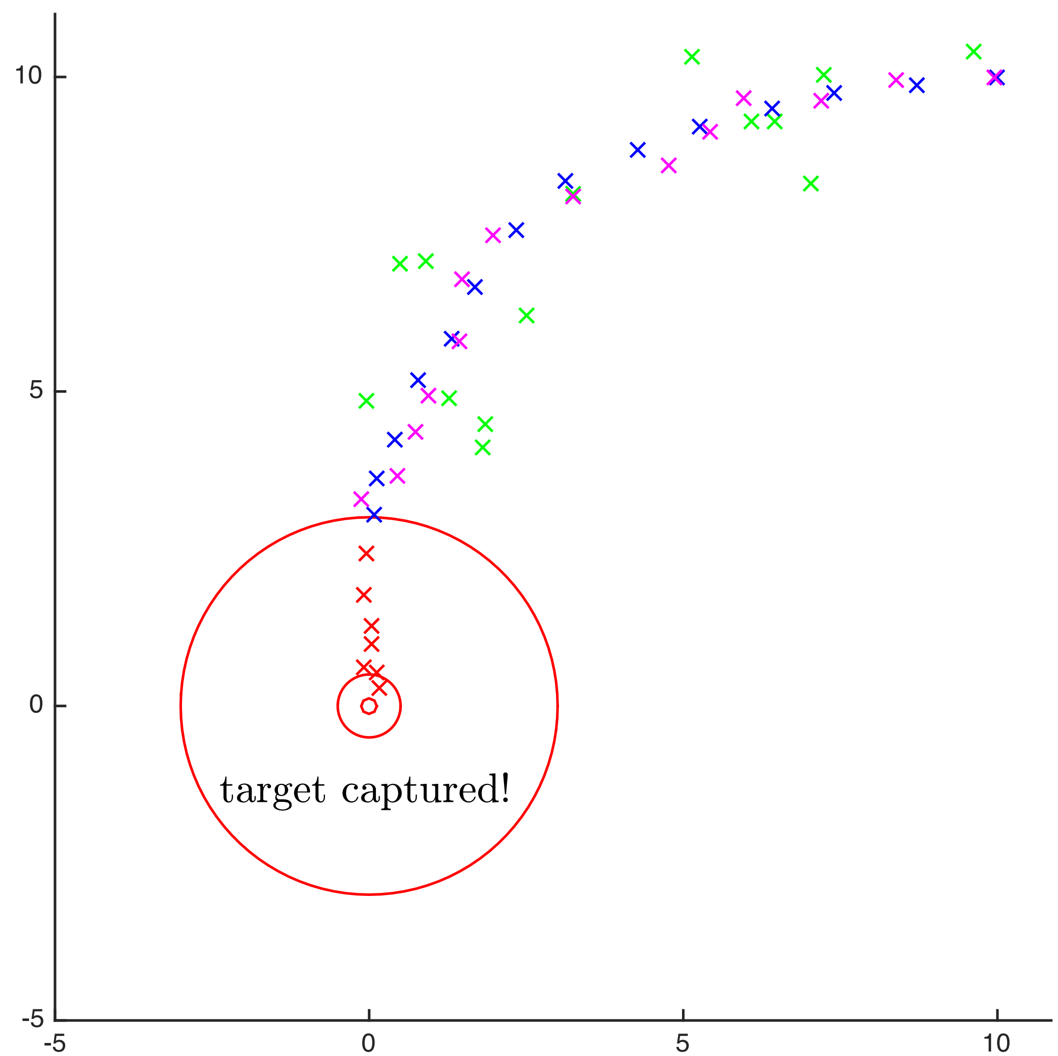
α	Rate of success
2	0.95
3	0.88
4	0.85
5	0.44

Computed from 100 trials

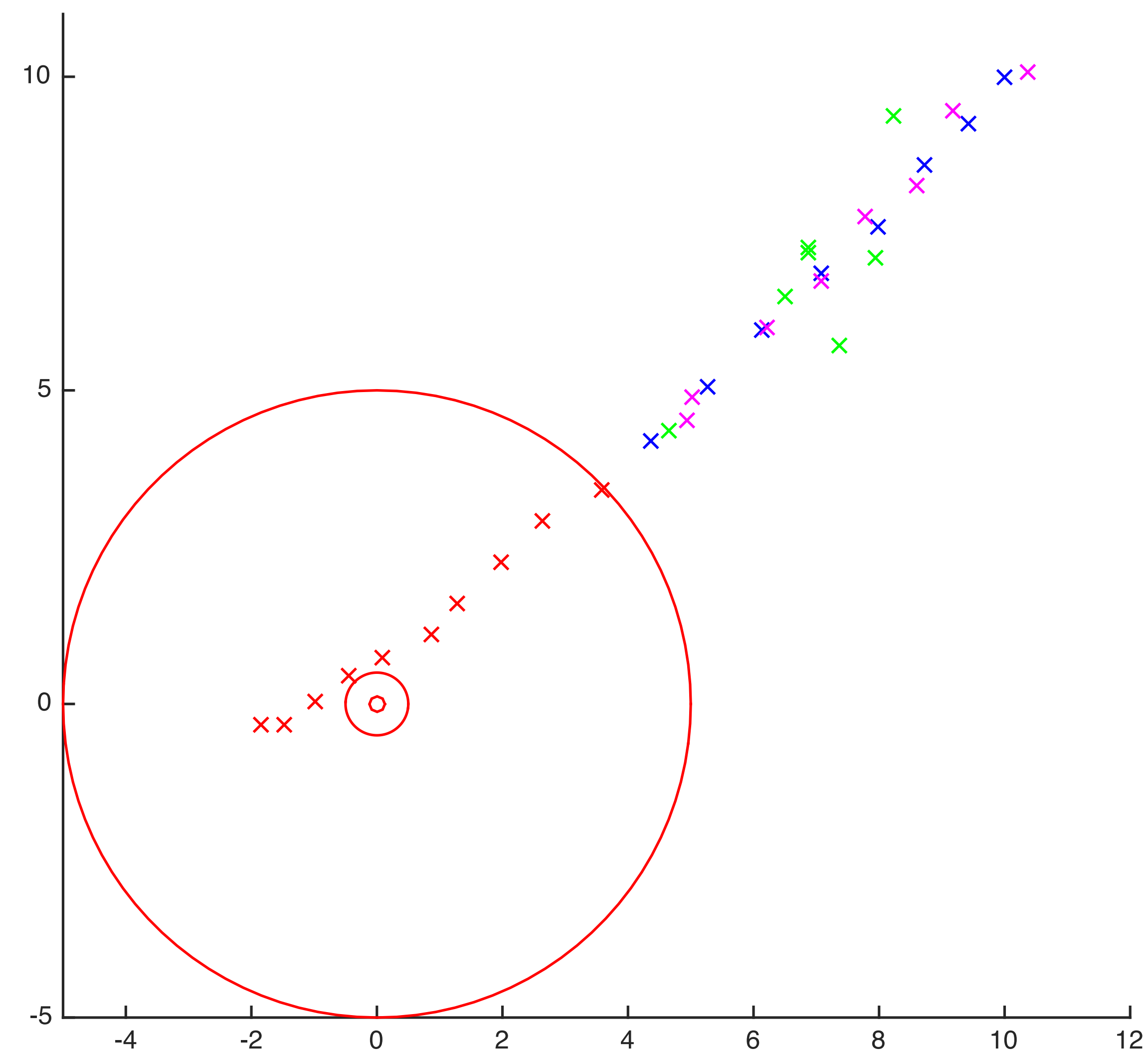
If the open loop ball **radius increases** there is a higher chance of **missing the target**

Preliminary numerical results

Homing with nonzero initial velocity



Successful capture



Failed attempt

The bat **may fail to capture** if it approaches the target too fast due to fewer collected measurements and noise in the open loop dynamics

Conclusions and future work

Conclusions:

1. A **simple but rich** model to study bats during prey capture
2. Optimization-based control is an **effective tool** to study bat's decision making
3. Observed a trade-off between how quickly the bat should **transition to capture** and the **rate of success**

Future work:

1. Incorporate the **risk of an evading target** in the cost
2. Optimize with respect to the time to transition to open loop - **optimal stopping**
3. Validate our hypothesis with **lab experiments**
4. Fine tune our optimization-based model using **real empirical data**