Optimal communication strategies in networked estimation

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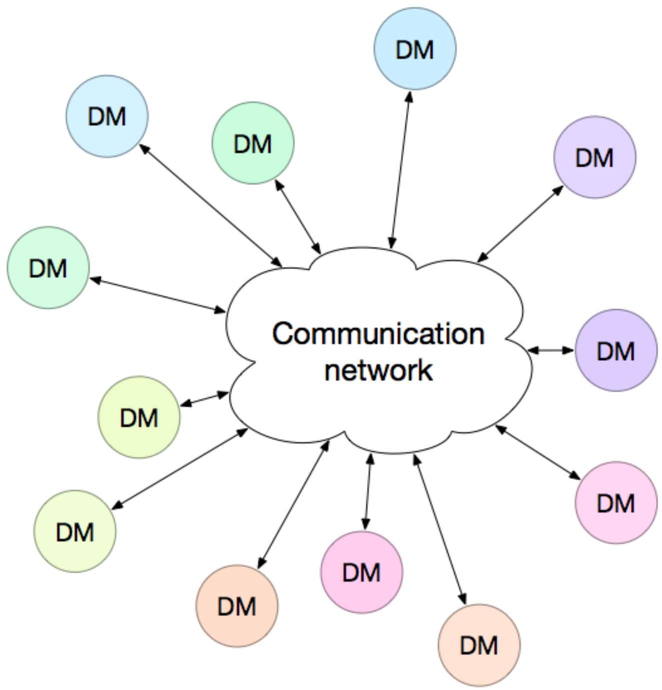
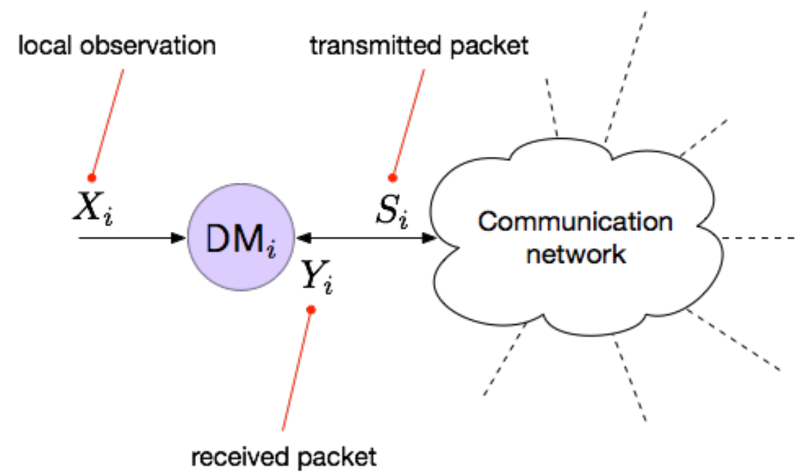
Networked multi-agent decision systems



Many applications



Many challenges

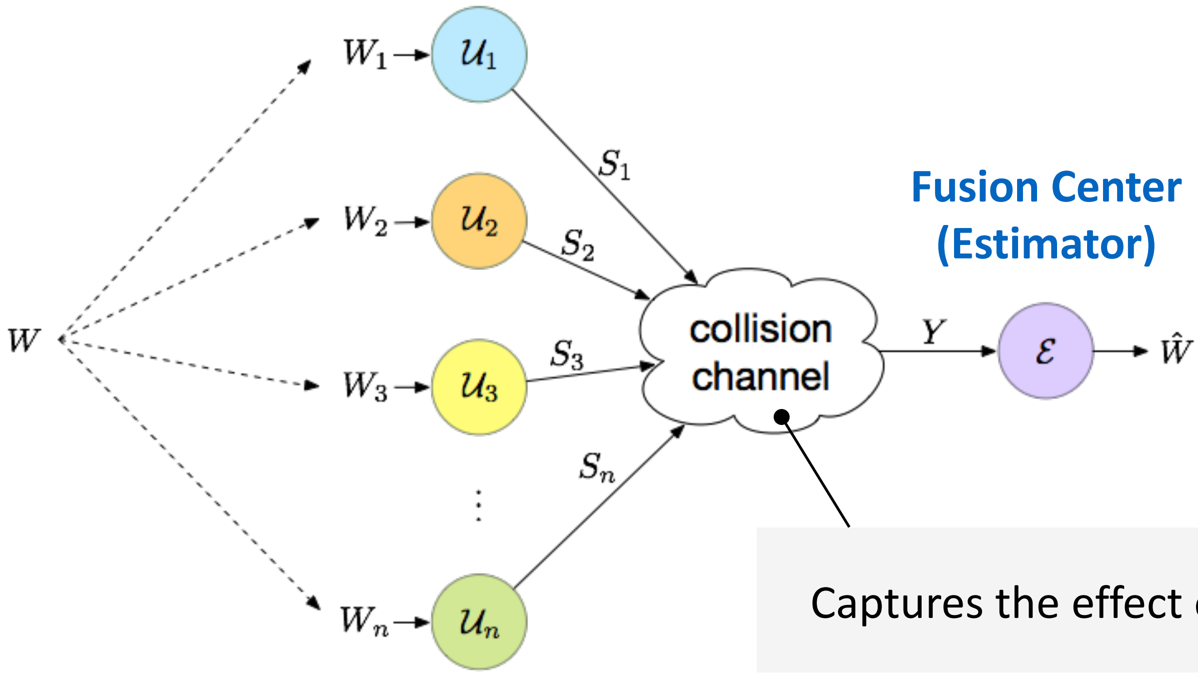
1. Networked control
2. Remote estimation
3. Sensor networks
4. Robotic networks

Communication is imperfect:
Delays, noise, quantization, congestion, packet drops, connectivity and interference

Basic framework

Team of sensors
(Decision Makers)

Fusion Center
(Estimator)

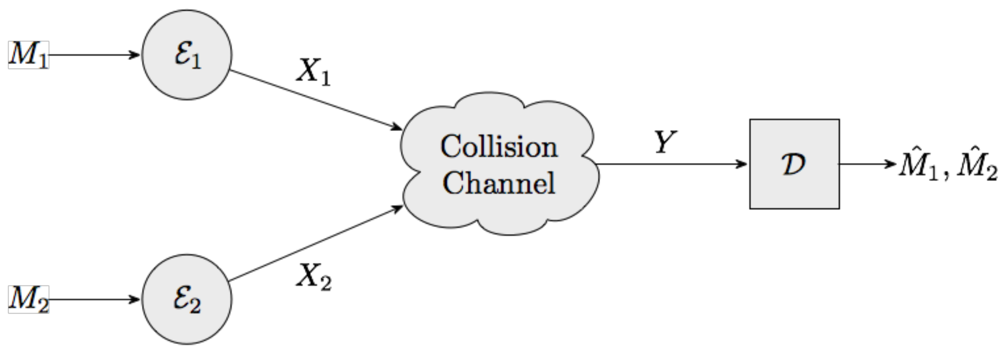


Captures the effect of **interference**

Design jointly optimal communication and estimation strategies

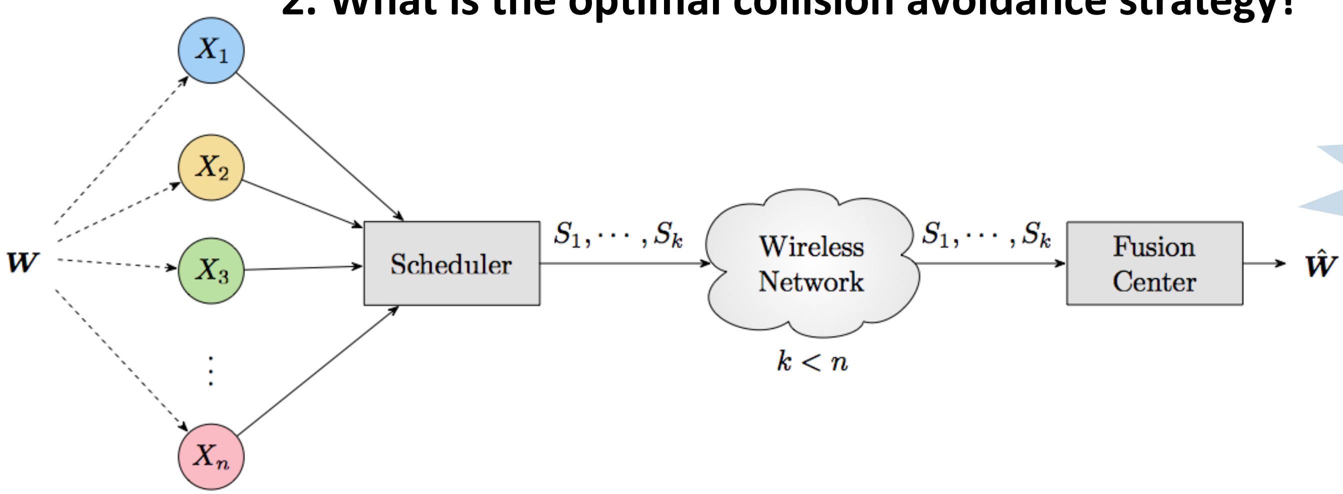
Many open questions

1. What is the capacity of the collision channel?



Information theory

2. What is the optimal collision avoidance strategy?

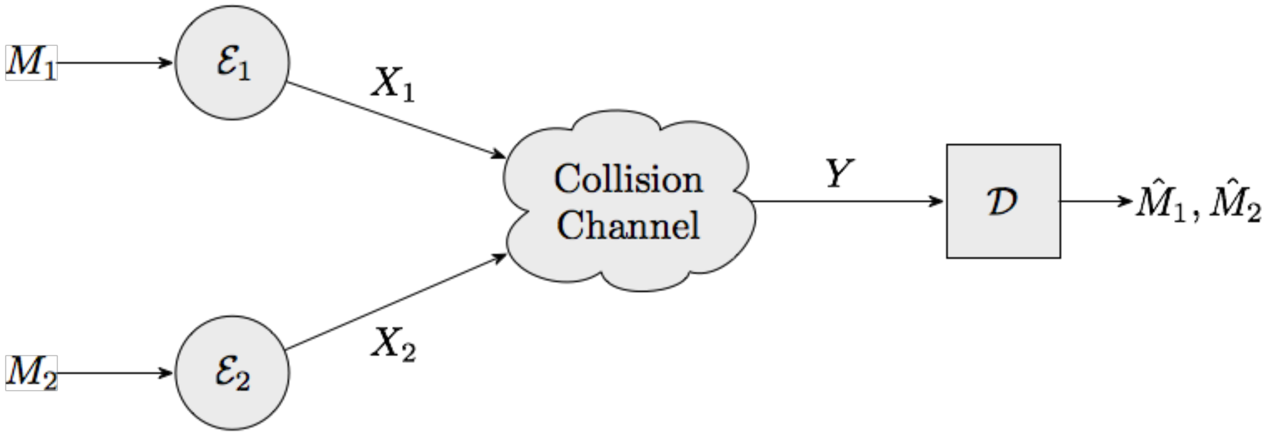


Statistical signal processing

The Collision MAC

Model of **interference**

- **Widely used in wireless communications**
- >1 transmission results in a **collision**
- Sensors decide whether to transmit or not



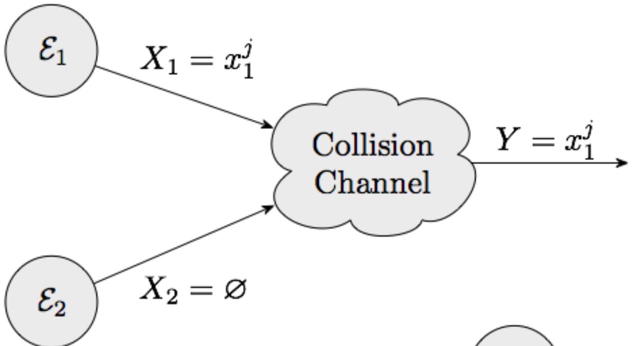
Input alphabet

$$\mathcal{X}_i = \left\{\varnothing, x_i^1, x_i^2, \cdots, x_i^{\Omega_i}\right\}, \quad i \in \{1, 2\}$$

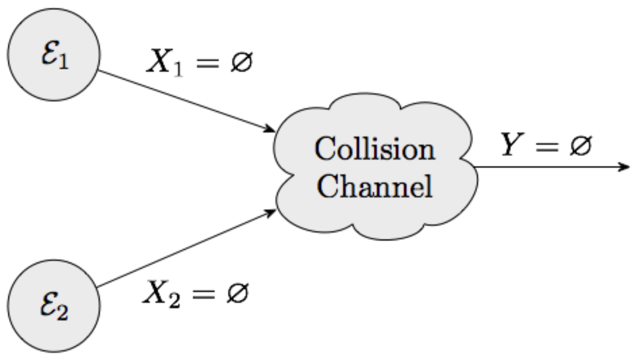
Output alphabet

$$\mathcal{Y} = \mathcal{X}_1 \cup \mathcal{X}_2 \cup \{\mathfrak{C}\}$$

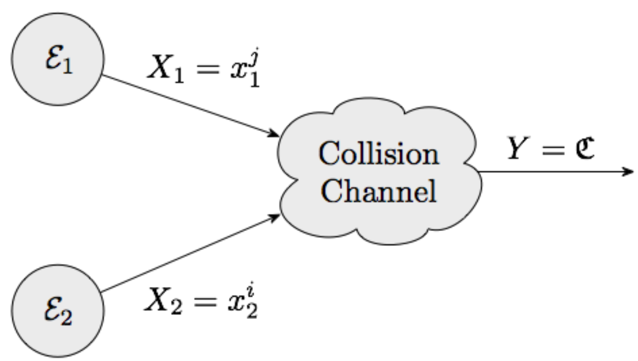
Collision channel



Success

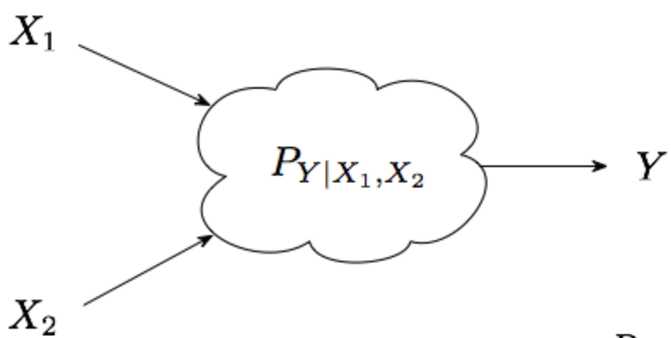


Erasure



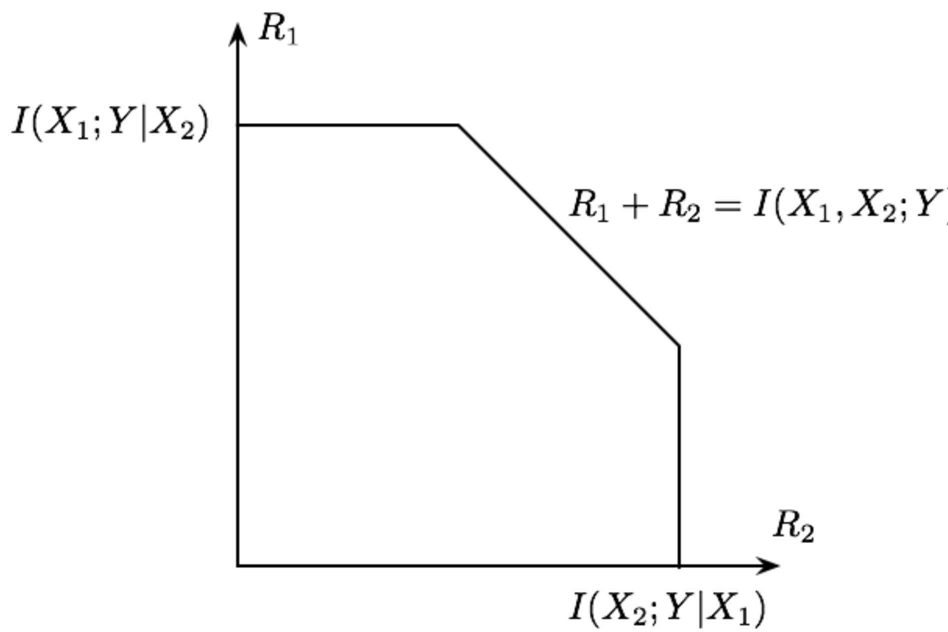
Collision!

Discrete Multiple Access Channel



The capacity region of a DMAC is the convex-hull of (R_1, R_2) satisfying:

$$R_1 \leq I(X_1; Y|X_2)$$
$$R_2 \leq I(X_2; Y|X_1)$$
$$R_1 + R_2 \leq I(X_1, X_2; Y)$$



Capacity of the Collision MAC

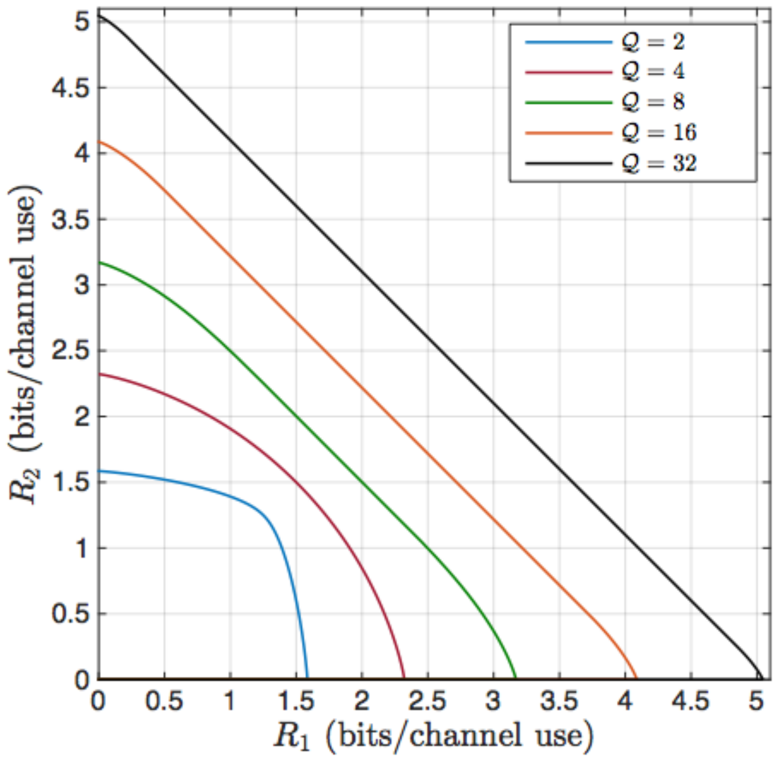
Theorem

The capacity region of the Collision MAC is the convex hull of (R_1, R_2) satisfying

$$R_1 \leq h(\theta_1) + \log Q_1 \cdot (1 - \theta_1) \cdot \theta_2$$
$$R_2 \leq h(\theta_2) + \log Q_2 \cdot (1 - \theta_2) \cdot \theta_1$$

θ_i is the prob. of node i being in **silence**
Q_i is the alphabet cardinality of node i

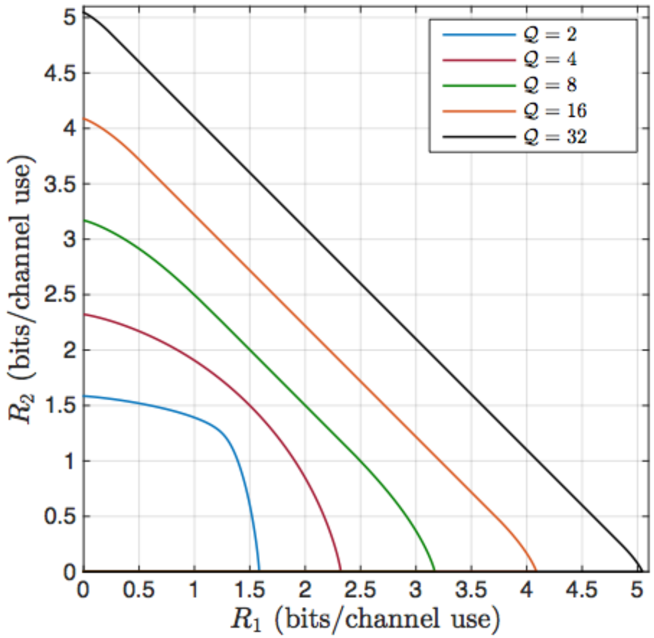
$$\theta_i = P_{X_i}(\varnothing)$$



New insights

For small alphabets, capacity achieving
codes require non-trivial use of no-transmission symbol

For large alphabets, the channel behaves
like a multiplier channel, i.e. TDMA approaches the capacity



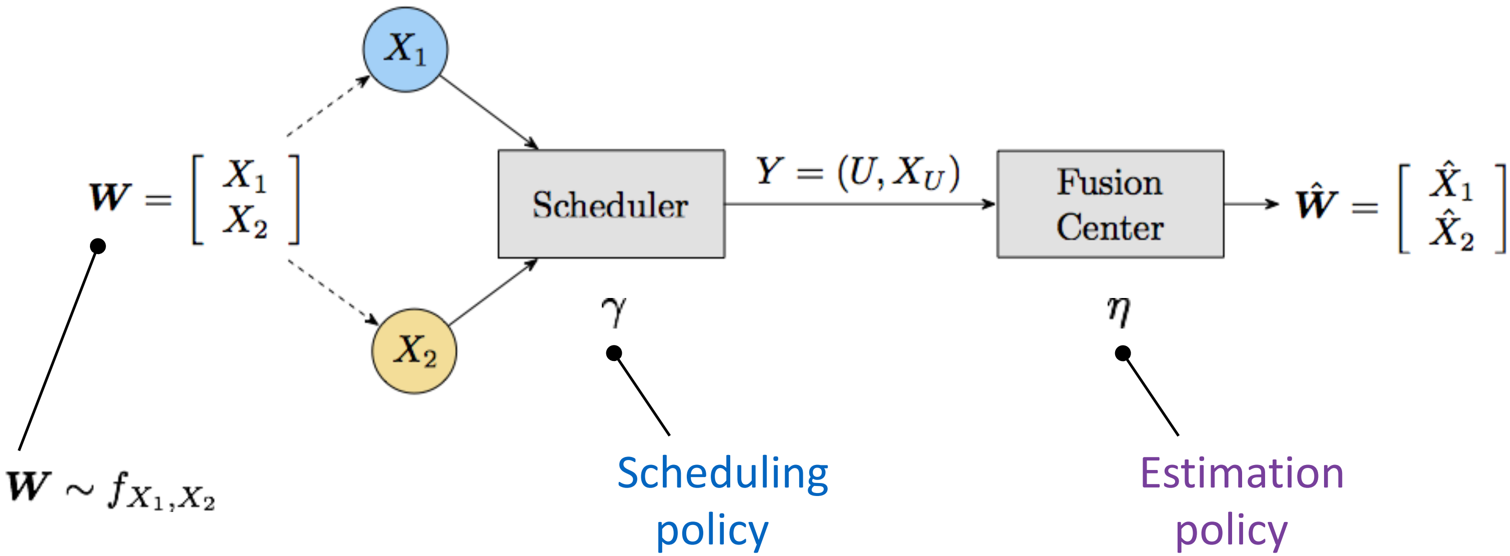
Achieving capacity requires coordination

Lack of coordination leads to a
non-cooperative game

Existence of a unique Nash-equilibrium

4. Vasconcelos & Mitra, "The collision multiple access channel without feedback," Allerton Conference 2017

How to avoid collisions?

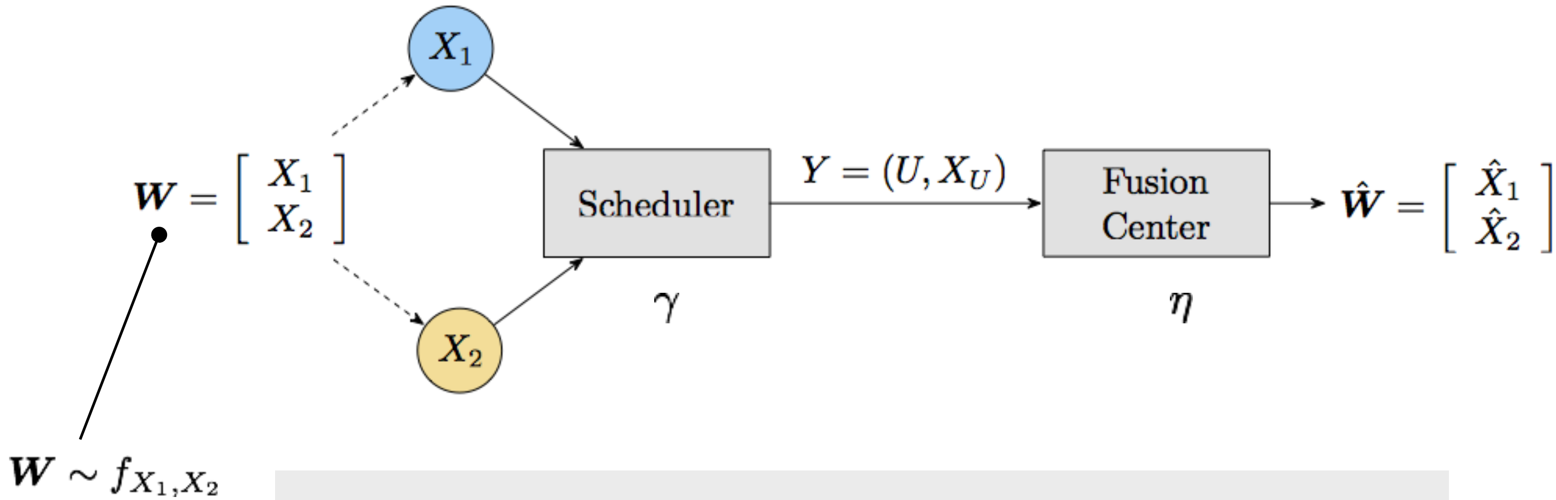


Team decision problem

Find **scheduling** and **estimation** policies that jointly minimize the following cost

$$\underset{(\gamma,\eta)\in\Gamma\times\mathrm{H}}{\text{minimize}} \quad \mathcal{J}(\gamma,\eta) = \mathbf{E}\Big[(X_1 - \hat{X}_1)^2 + (X_2 - \hat{X}_2)^2\Big]$$

Observation-driven scheduling



$$\underset{(\gamma,\eta)\in\Gamma\times\mathrm{H}}{\text{minimize}} \quad \mathcal{J}(\gamma,\eta) = \mathbf{E}\Big[(X_1 - \hat{X}_1)^2 + (X_2 - \hat{X}_2)^2\Big]$$

Open-loop scheduling: let the sensor with the **largest variance** transmit

Observation-driven scheduling[5]: let the sensor with the **"largest measurement"** transmit

5. Vasconcelos & Mitra, "Observation-driven sensor scheduling," ICC 2017

Notions of optimality

Team-optimality

$$\mathcal{J}(\gamma^*,\eta^*) \leq \mathcal{J}(\gamma,\eta), \quad (\gamma,\eta) \in \Gamma \times \mathrm{H}$$

$$\Longrightarrow$$

$$\not\Longleftarrow$$

Person-by-person optimality

$$\mathcal{J}(\gamma^*,\eta^*) \leq \mathcal{J}(\gamma,\eta^*), \quad \gamma \in \Gamma$$
$$\mathcal{J}(\gamma^*,\eta^*) \leq \mathcal{J}(\gamma^*,\eta), \quad \eta \in \mathrm{H}$$

$$\underset{(\gamma,\eta)\in\Gamma\times\mathrm{H}}{\text{minimize}} \quad \mathcal{J}(\gamma,\eta) = \mathbf{E}\left[(X_1 - \hat{X}_1)^2 + (X_2 - \hat{X}_2)^2\right]$$

Unfortunately, finding team-optimal solutions is very difficult[6,7]

6. Witsenhausen - “A counterexample in optimal stochastic control” *SIAM J. Control* 1968
7. Tsitsiklis & Athans - “On the complexity of decentralized decision making and detection problems” *IEEE TAC* 1985

MAX scheduling

Theorem 1

$$\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_2^2 \end{bmatrix}\right) \Longrightarrow \left(\gamma^{\max}, \eta^*_{\gamma^{\max}}\right) \text{ is person-by-person optimal}$$

Theorem 2

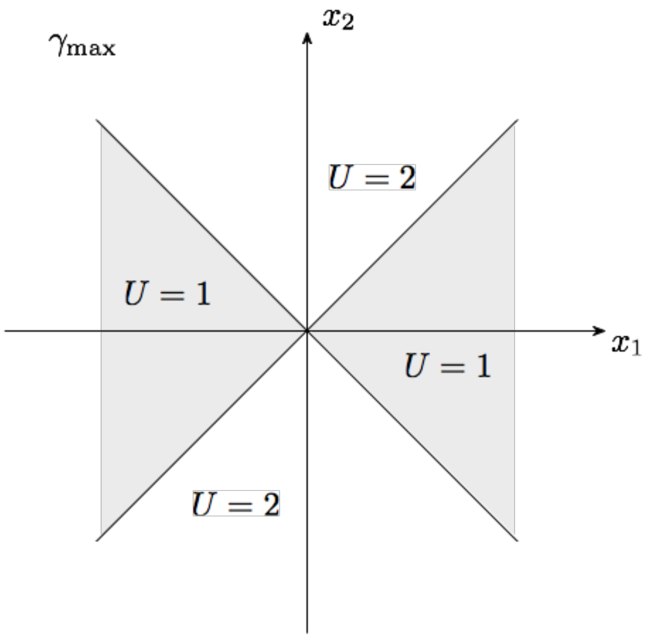
$$\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}\right) \Longrightarrow \left(\gamma^{\max}, \eta^*_{\gamma^{\max}}\right) \text{ is person-by-person optimal}$$

MAX scheduling policy

$$\gamma^{\max}(x_1, x_2) = \begin{cases} 1, & \text{if } |x_1| \geq |x_2| \\ 2, & \text{otherwise} \end{cases}$$

MMSE estimator

$$\eta^*_\gamma(y) = \mathbf{E}\big[\boldsymbol{W} \mid Y = y\big]$$



Performance of MAX scheduling

USC Viterbi

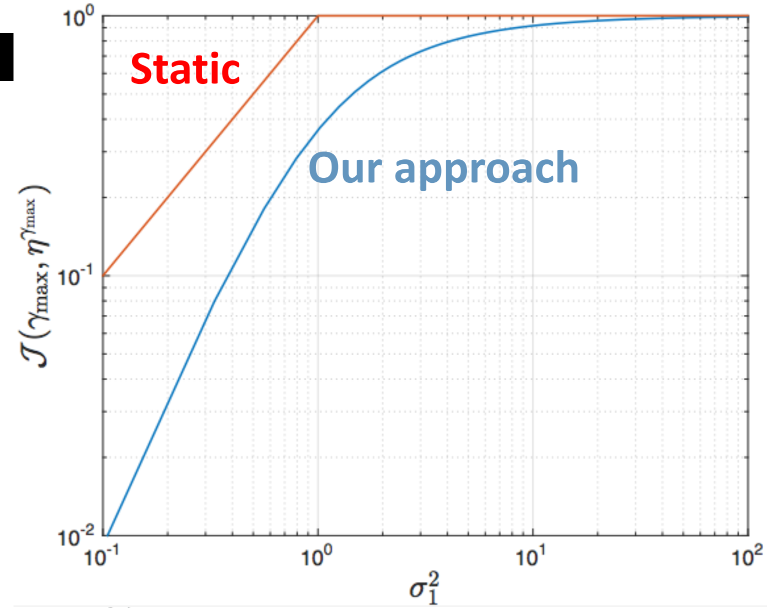
Independent Gaussian Sources

$$\mathcal{J}(\gamma_{\max}, \eta^{\gamma_{\max}}) = \mathbf{E}\Big[\min\left\{X_1^2, X_2^2\right\}\Big]$$

Observation-driven sensor scheduling

$$\bar{\mathcal{J}}(\sigma_1^2, \sigma_2^2) = \min\{\sigma_1^2, \sigma_2^2\}$$

"Open-loop" sensor scheduling



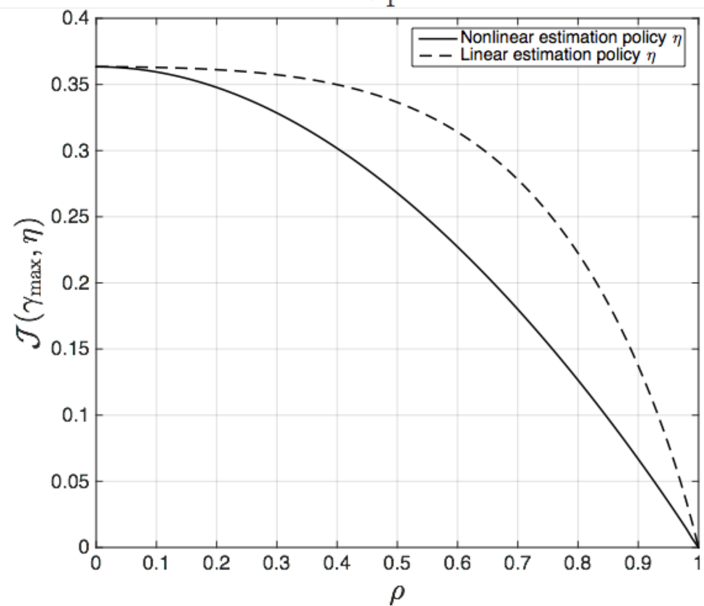
Symmetrically Correlated Gaussian Sources

MMSE estimator

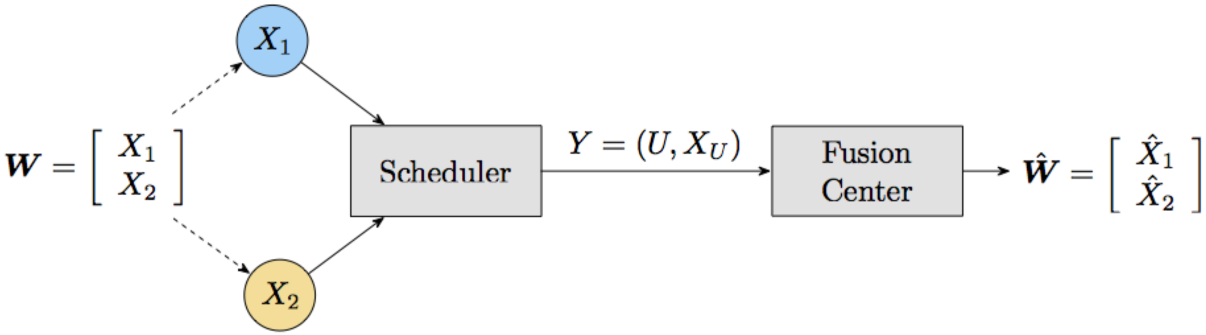
$$\eta^*_{\gamma_{\max},i}(x) = \frac{\int_{-|x|}^{|x|} \tau f_{X_i|X_j=x}(\tau)d\tau}{\int_{-|x|}^{|x|} f_{X_i|X_j=x}(\tau)d\tau}$$

LMMSE estimator

$$\eta_i(x) = \rho \cdot x$$



New insights

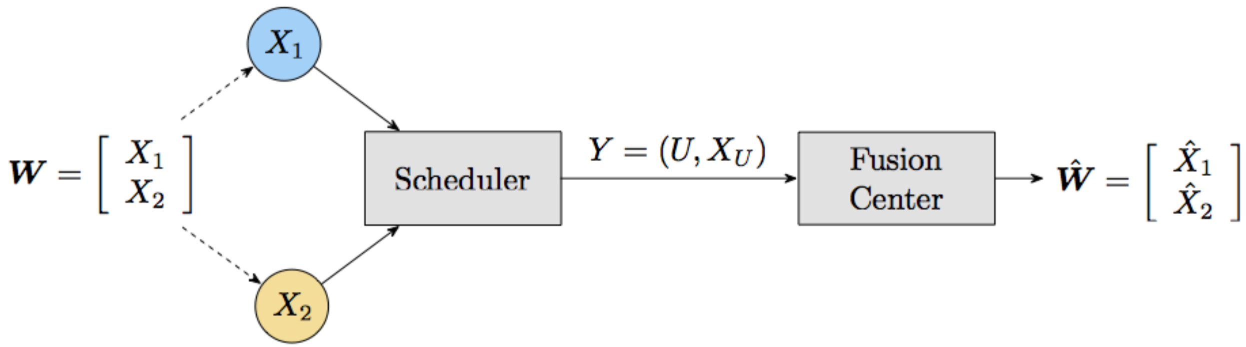


1. Introduced a **new approach to sensor scheduling**
2. Person-by-person optimality of **MAX scheduling**
3. A **nontrivial lower bound to estimation over the collision channel**

1. Extend these results to the **general multivariate Gaussian** case
2. Establish a **connection with compressive sensing** and **matrix completion**

Can we establish global optimality?

Scheduling of sources with arbitrary distributions



$\boldsymbol{W} \sim f_{X_1,X_2}$

Arbitrary!

Infinite dimensional optimization

$$\mathcal{J}(\gamma^*_\eta, \eta) = \mathbf{E}\Big[\min\Big\{\big(X_1 - \eta_1(X_2)\big)^2, \big(X_2 - \eta_2(X_1)\big)^2\Big\}\Big]$$

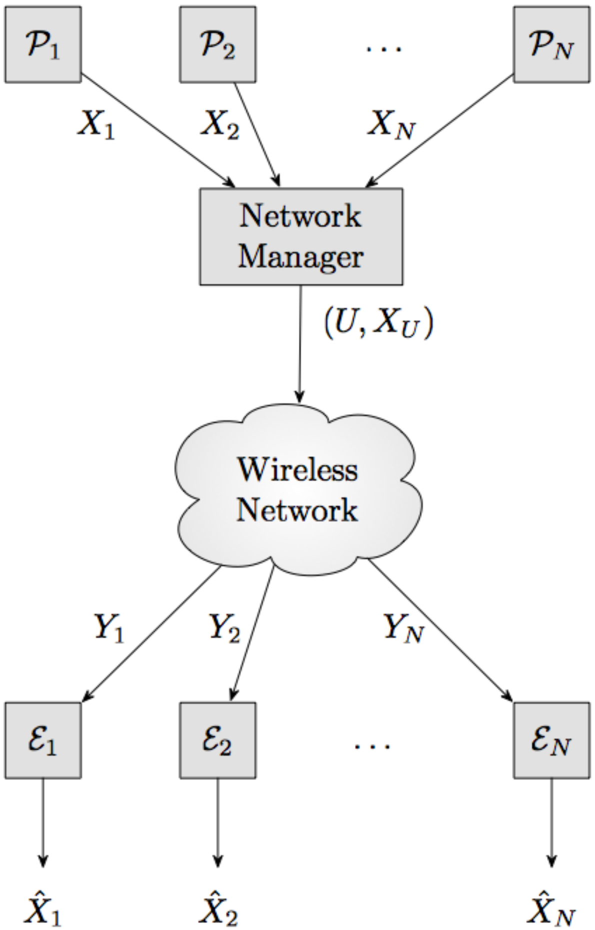
Constrain to affine estimators

$$\mathcal{J}(a,b,c,d) = \mathbf{E}\Big[\min\Big\{\big(X_1 - aX_2 - b\big)^2, \big(X_2 - cX_1 - d\big)^2\Big\}\Big]$$

- Nonconvex
- Difference-of-convex
- Convex-concave procedure
- Approximate subgradient

7. Vasconcelos & Mitra "On the joint design of schedulers for correlated random variables and their affine estimators," to be submitted to ACC 2018

Multiple sources & multiple receivers



Unicast network

A single link is active at a time

$$Y_i = \begin{cases} (U, X_U), & \text{if } \ U = i \\ \varnothing, & \text{otherwise} \end{cases}$$

Theorem

Unimodal densities

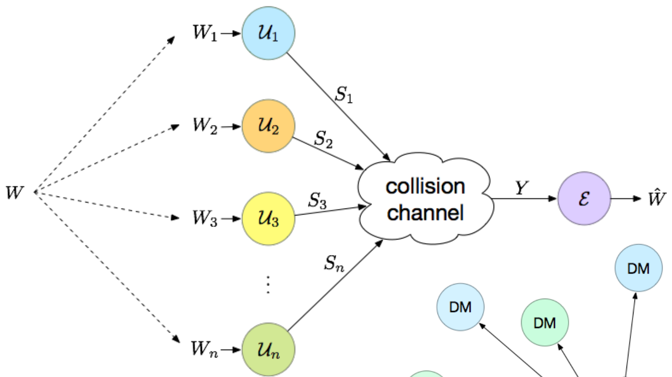
$\Downarrow$

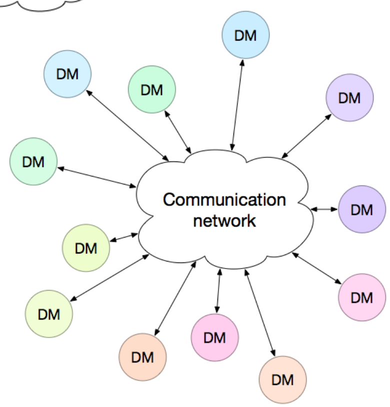
MAX scheduling is globally optimal!

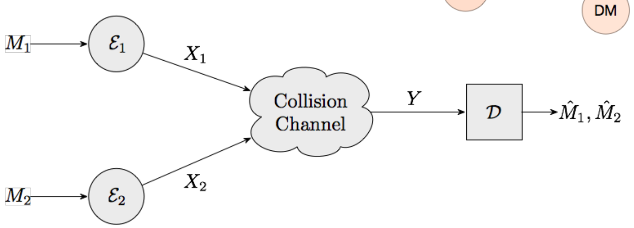
8. Vasconcelos, Nayyar & Mitra "Optimal sensor scheduling strategies in networked estimation," CDC 2017

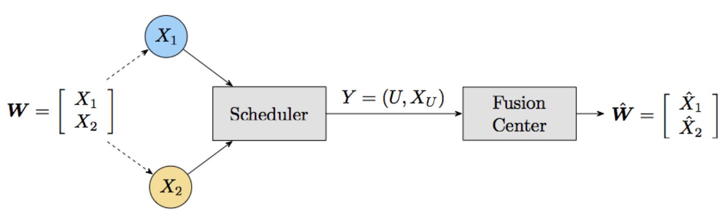
Summary

USC Viterbi









What are the limits of communication over the collision channel?

- Explicitly computed the capacity region
- How to operate at maximal communication rates

New opportunities

- Other collision channel models: feedback, $\mathfrak{C} = \varnothing$, …
- Find lower bounds on estimation performance

How to avoid collisions?

- Discovered the MAX scheduling strategies
- Proved its optimality in a number of scenarios

New opportunities

- Relationship with compressed sensing
- Apply these results on real data

Publications

Conference

1. "On the joint design of schedulers for correlated random variables and their affine estimators", M. M. Vasconcelos and U. Mitra, ***To be submitted*** American Control Conference, July 2018.
2. "The multiple-access collision channel without feedback: capacity region and a mutual information game", M. M. Vasconcelos and U. Mitra, *Allerton Conference on Communication, Control, and Computing*, October 2017 (***accepted***).
3. "Optimal sensor scheduling strategies in networked estimation", M. M. Vasconcelos, A. Nayyar and U. Mitra, *IEEE 56th IEEE Conference on Decision and Control*, December 2017 (***accepted***).
4. "Observation-driven sensor scheduling", M. M. Vasconcelos and U. Mitra, *IEEE International Conference on Communications*, May 2017.

Journal

1. "Observation-driven sensor scheduling and remote estimation of bivariate continuous sources, M. M. Vasconcelos and U. Mitra, ***In preparation***.
2. "Optimal sensor scheduling policies for sequential networked estimation", M. M. Vasconcelos, M. Gragani, A. Nayyar and U. Mitra, ***In preparation***.
3. "Fundamental limits of implicit communication over multiple access collision channels", M. M. Vasconcelos and U. Mitra, ***In preparation***.