



Bacterial Quorum Sensing as a Networked Decision System

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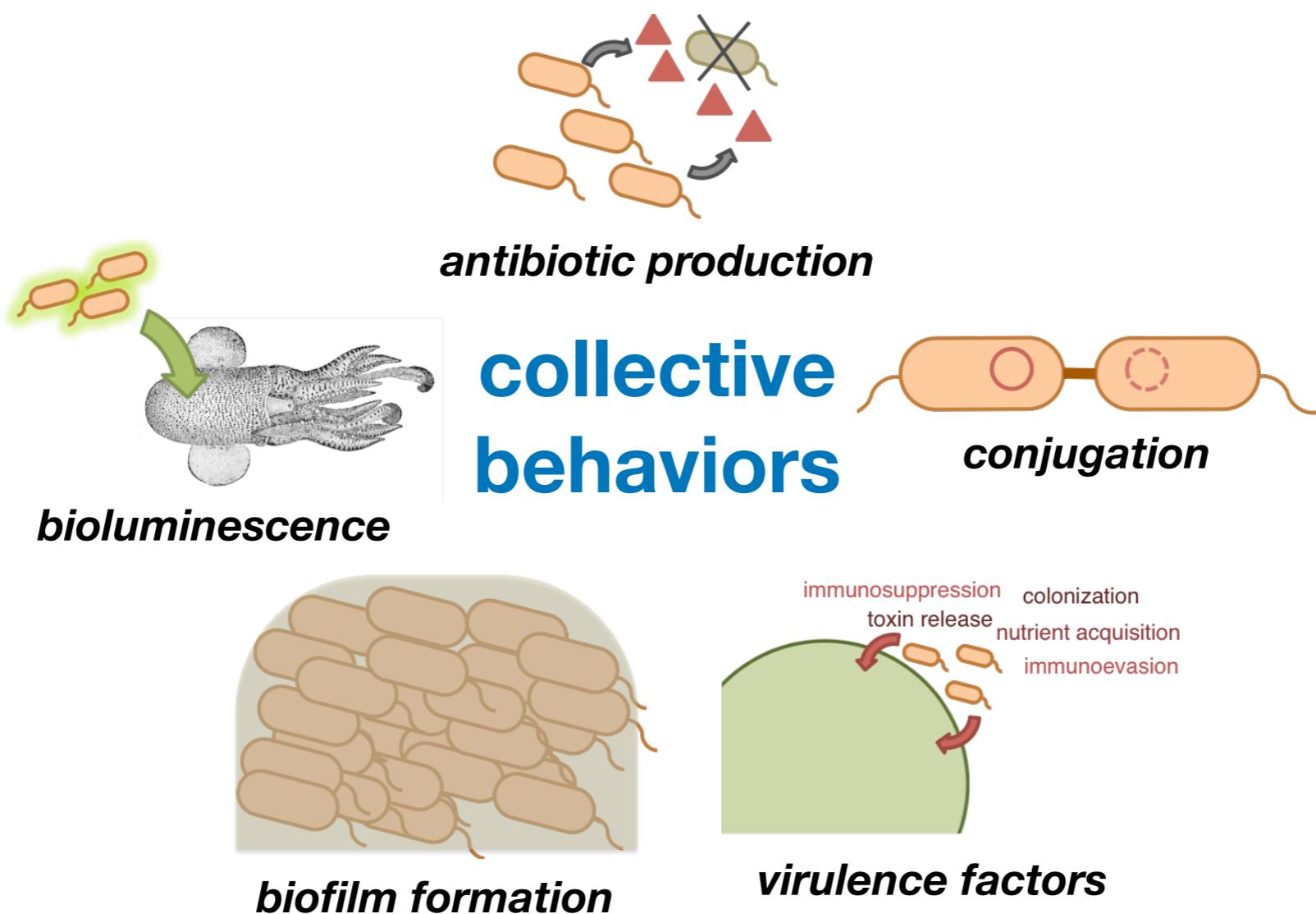
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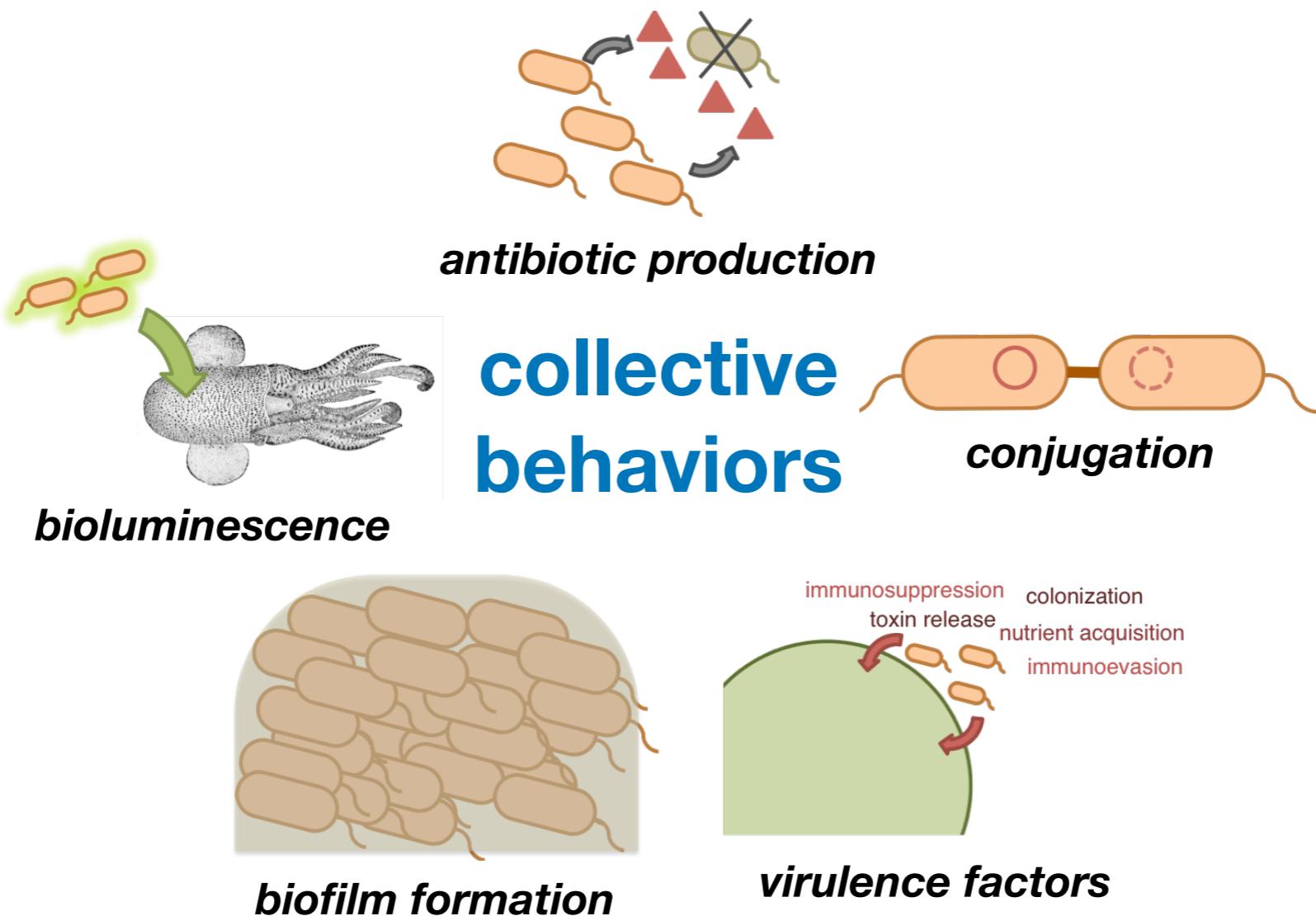
Quorum sensing

Mechanism used by bacteria to coordinate collective behavior

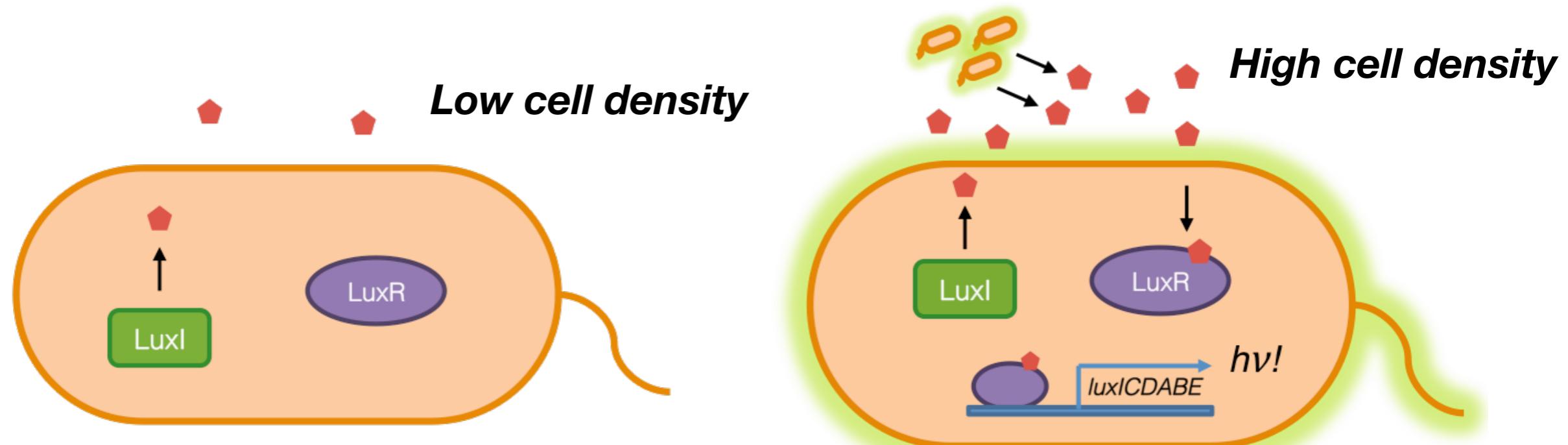
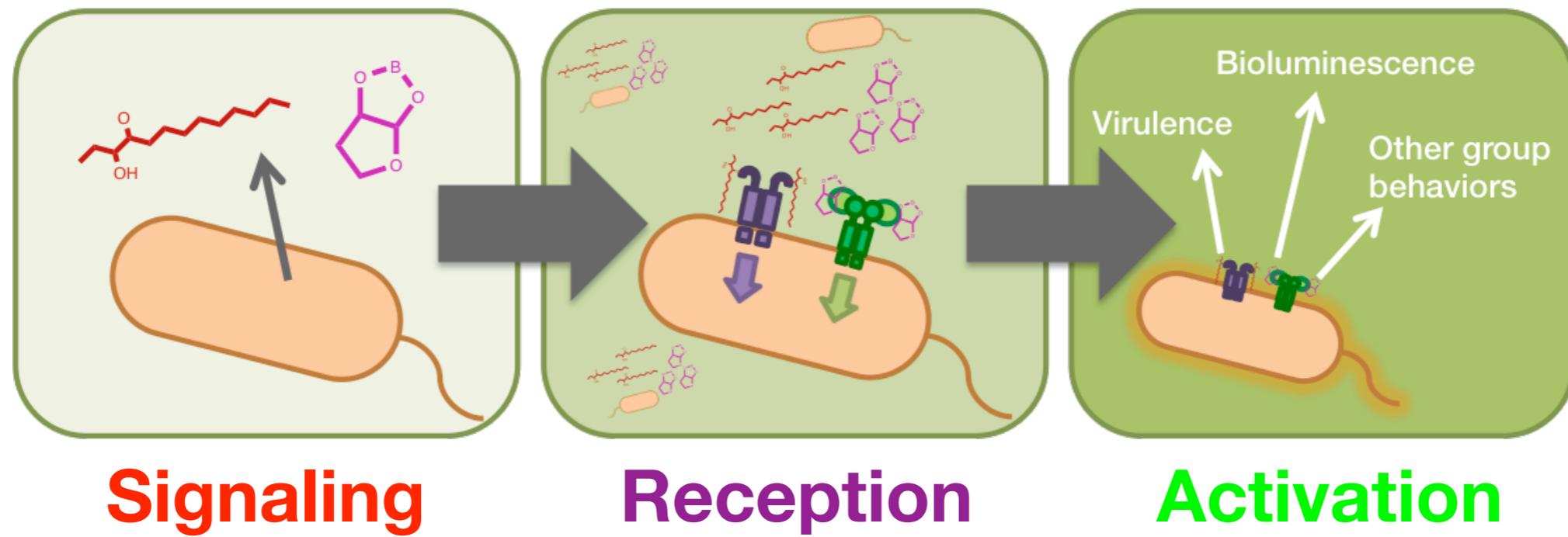


Quorum sensing

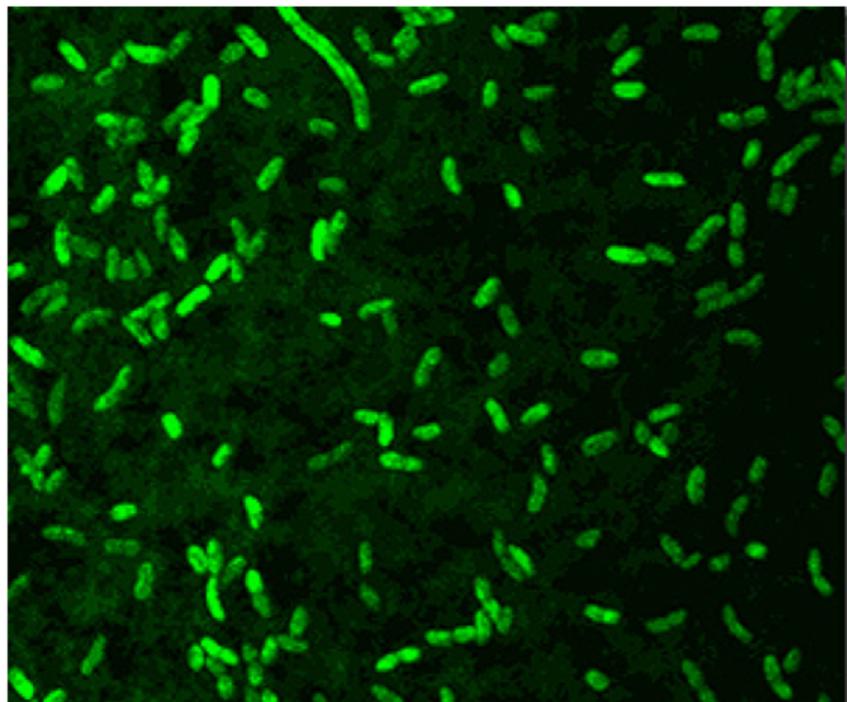
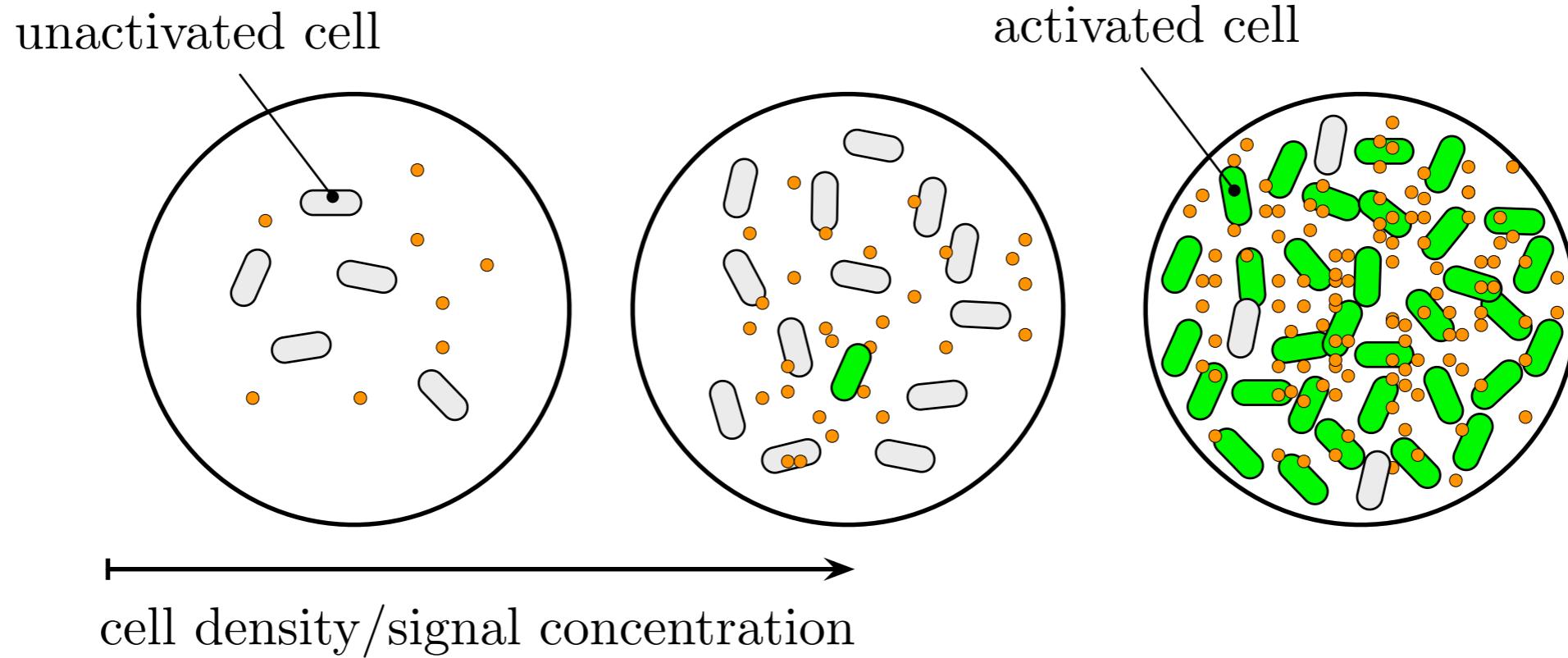
Enables bacteria to act as multicellular organisms!



How does it work?



Quorum sensing



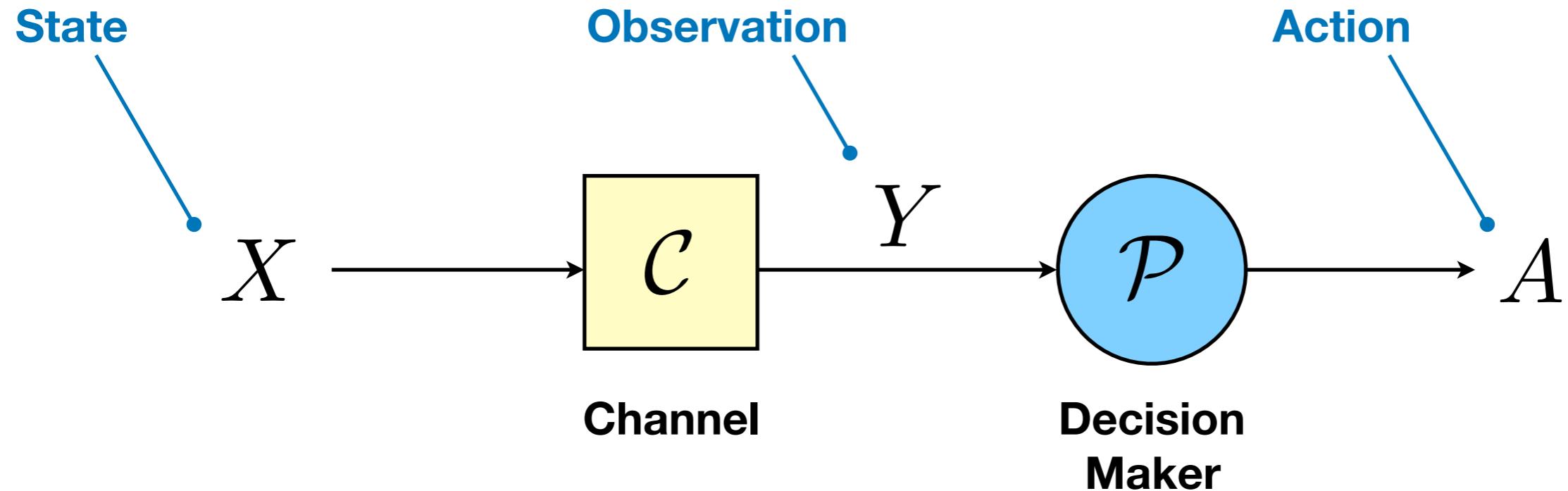
Goal

Cells are decision makers

Propose a new optimization-based model for QS

Calibrate our model from experimental data

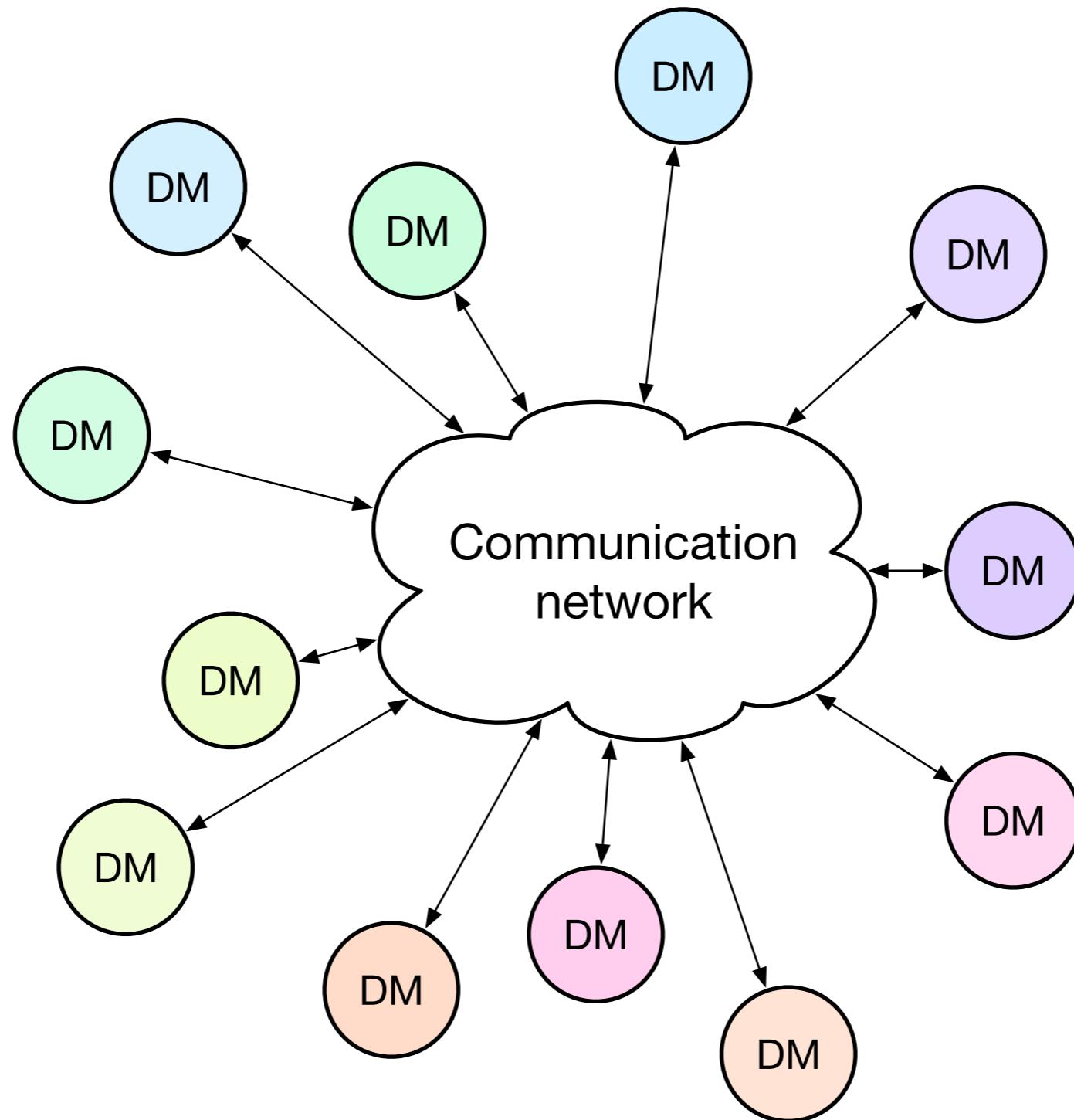
Bayesian decision system



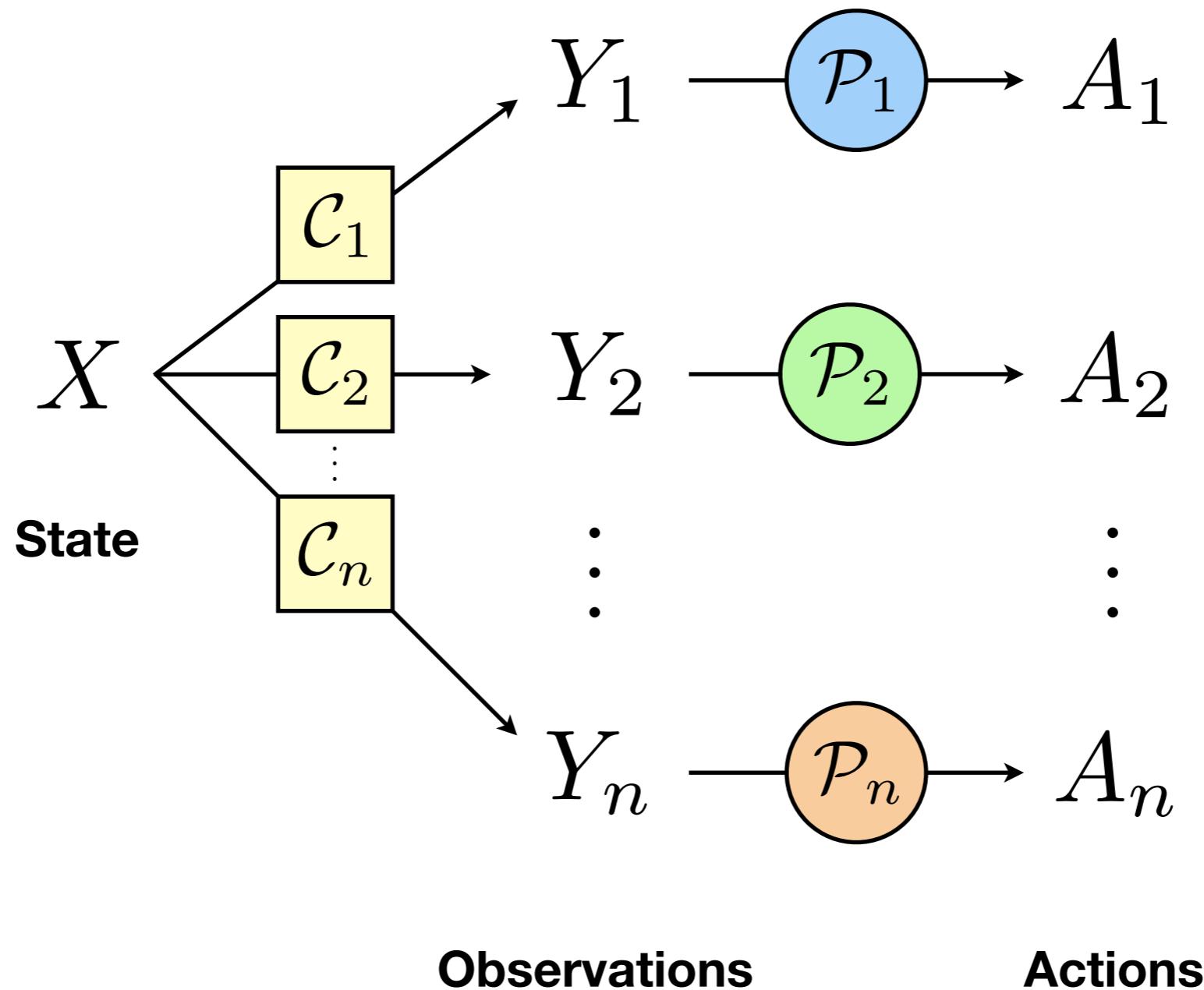
$$X \sim f_X$$

$$\underset{\mathcal{P}}{\text{maximize}} \quad \mathbf{E}[\mathcal{U}(A, X)]$$

Network decision system



Local observations, Global actions

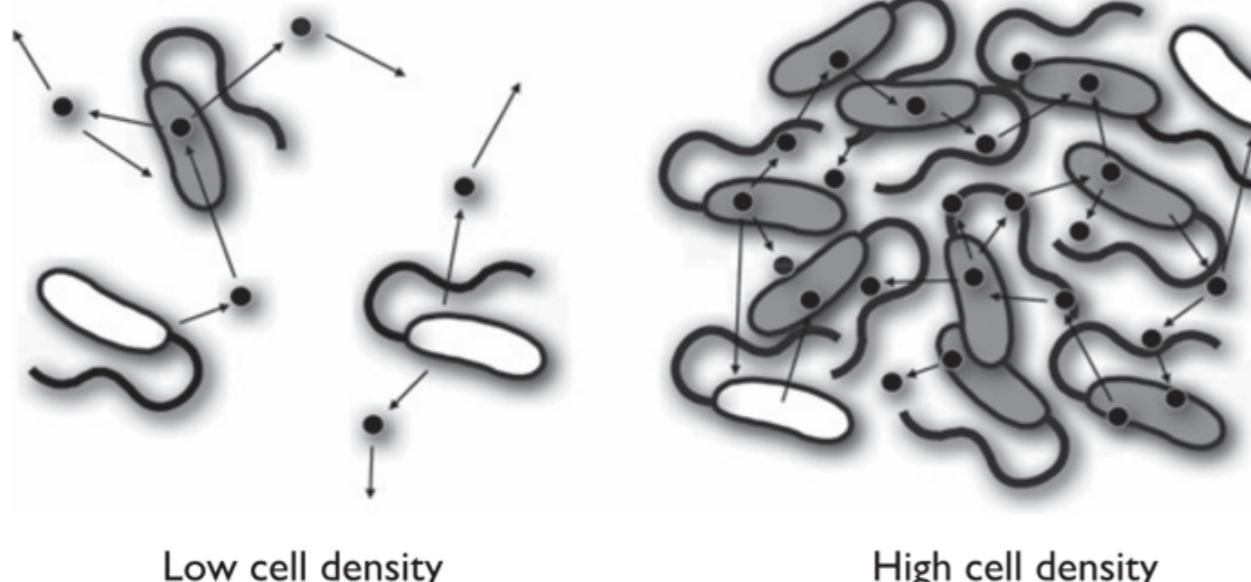


$$\underset{\mathcal{P}_1, \dots, \mathcal{P}_n}{\text{maximize}} \quad \mathbf{E}[\mathcal{U}(A_1, \dots, A_n, X)]$$

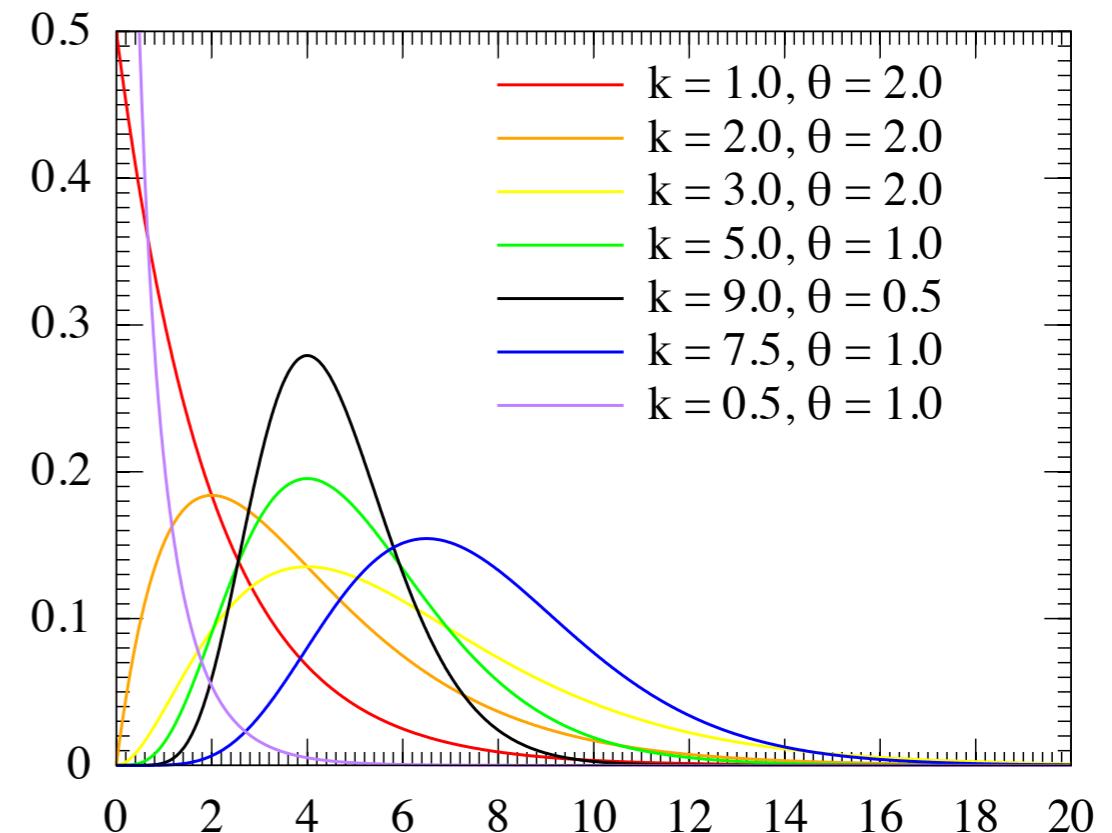
The model

State

Density of the colony



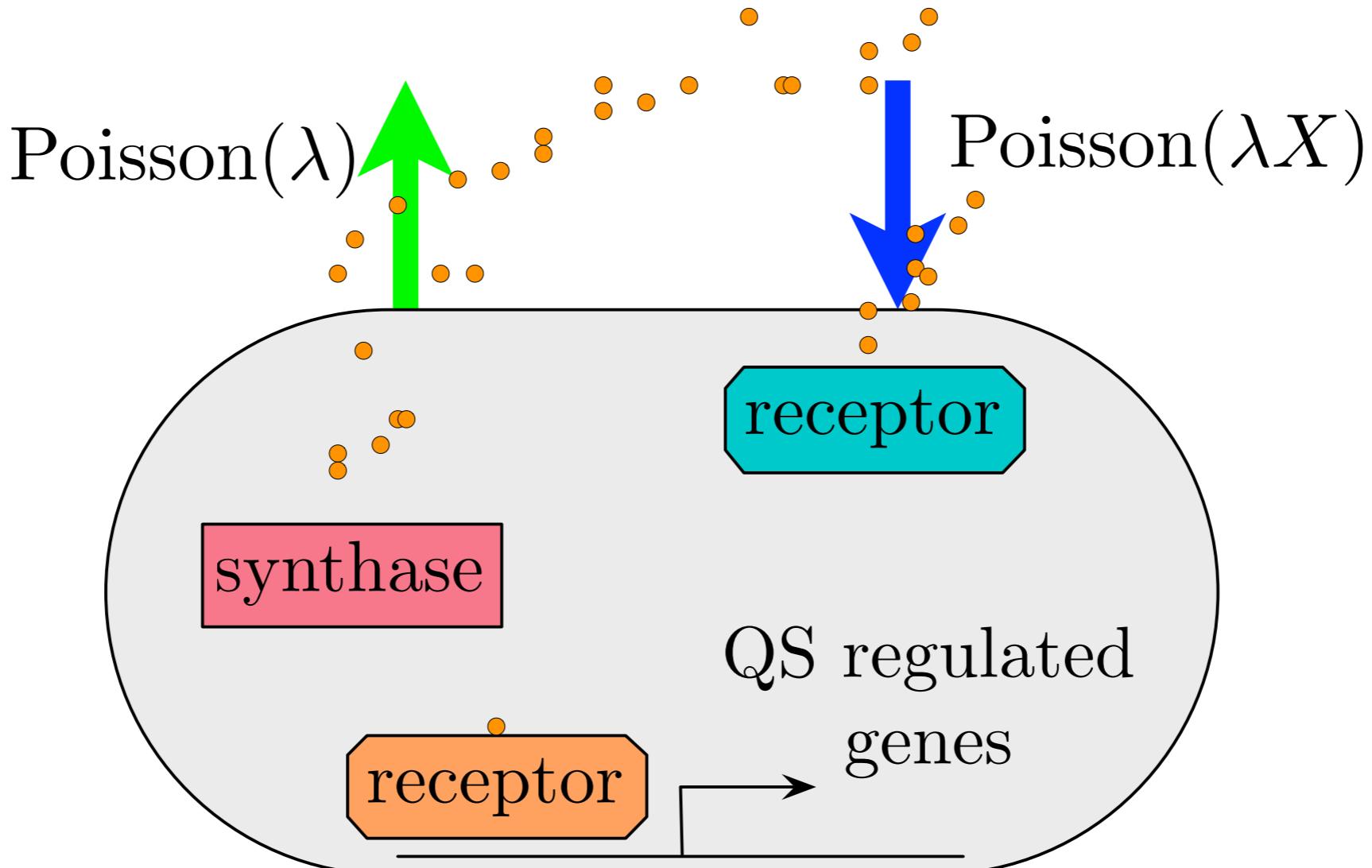
$X \sim \text{Gamma}(\kappa, \theta)$



[4] Darch et al (2012) - "Density-dependent fitness benefits in quorum-sensing bacterial populations"

[5] Clough et al (2005) - "Quantifying uncertainty associated with microbial count data: A Bayesian approach"

Sensing mechanism



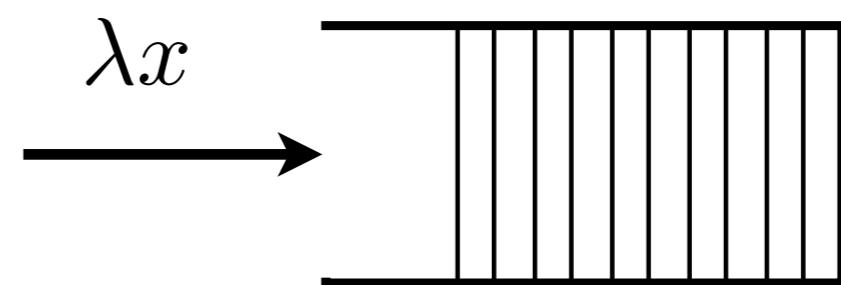
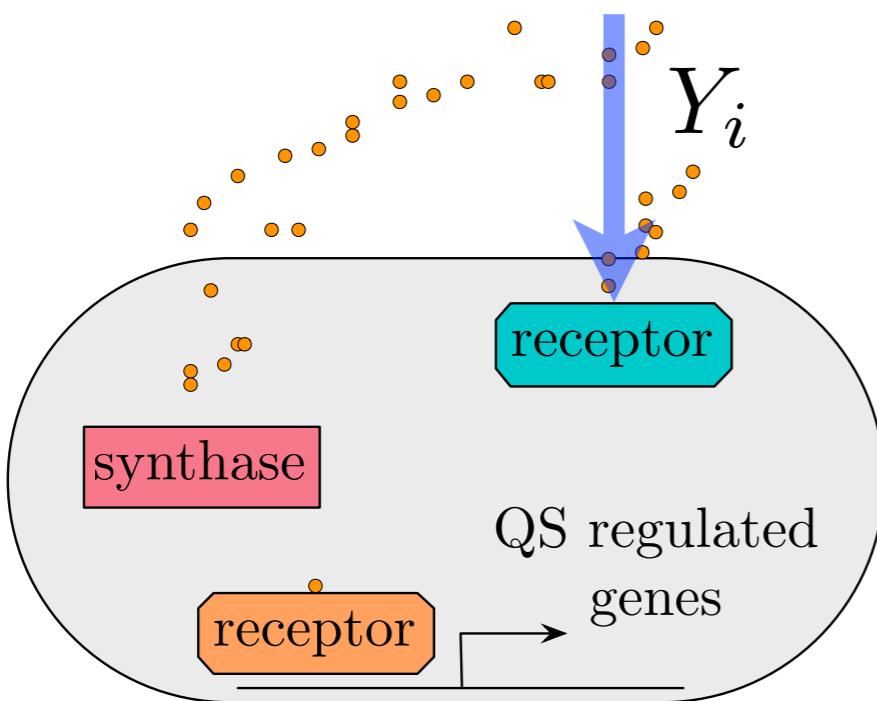
λ depends on the type of bacteria

Observations

Assumptions

Given X : $Y_i \perp\!\!\!\perp Y_j, i \neq j$

Given $X = x$: $Y_i \sim \text{Poisson}(\lambda x)$



Queueing analogy^[6]

Quorum size

quorum

noun [C] • **US** /'kwɔrəm, 'kwoʊr-/

★ the number of members who must be present at a meeting in order for decisions to be officially made

- Cambridge Dictionary

$$A_i = \begin{cases} 0 & i\text{-th cell is inactive} \\ 1 & i\text{-th cell is active} \end{cases}$$

Colony of N cells

Quorum of size τN

$$\sum_{k=1}^N A_k \geq \tau N$$

Public goods interpretation

Exofactor production is a **costly investment**

The **public benefit** is enjoyed by the **entire colony**

The benefit only kicks in **if a quorum is reached (risk)**

Pay-off

$$\mathcal{U}_i(A_i, A_{-i}, X) = \tau X \cdot \mathbf{1} \left(\sum_{k=1}^X A_k \geq \tau X \right) - c \cdot A_i$$

public benefit

activation cost

The diagram shows the pay-off function $\mathcal{U}_i(A_i, A_{-i}, X)$ enclosed in a yellow box. Inside the box, there is a term $\tau X \cdot \mathbf{1}$ followed by a large black bracket that covers the summation part of the equation. Below this bracket, a black dot is connected by a line to the text "public benefit". To the right of the equation, there is another yellow box containing the term $c \cdot A_i$. A black dot is connected by a line to the text "activation cost".

public benefit is proportional to the **size of the colony**

utility = growth rate of the colony

Optimization problem

Threshold policy

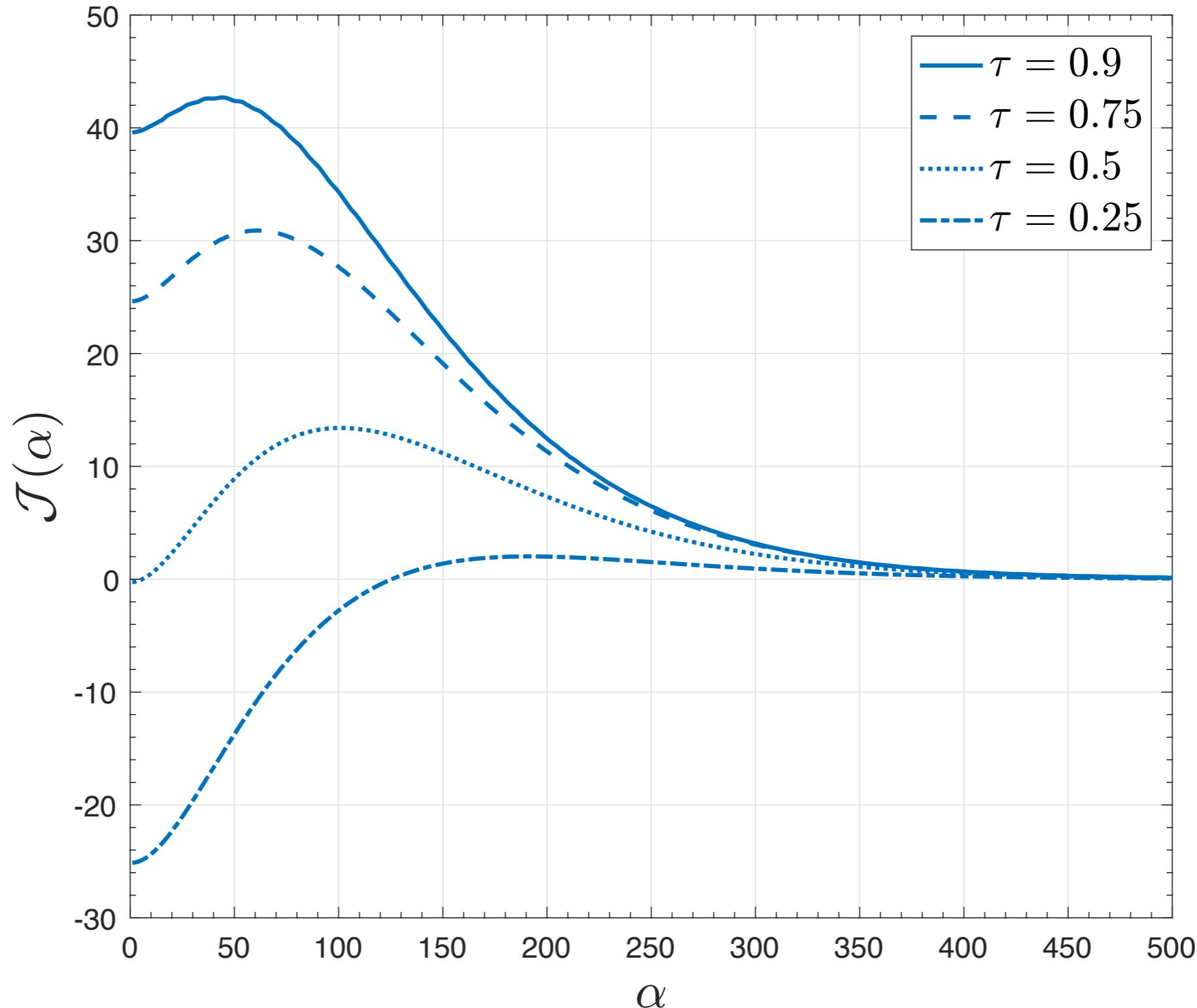
$$A_i = 1(Y_i \geq \alpha)$$

Find α that maximizes the function \mathcal{J}_i

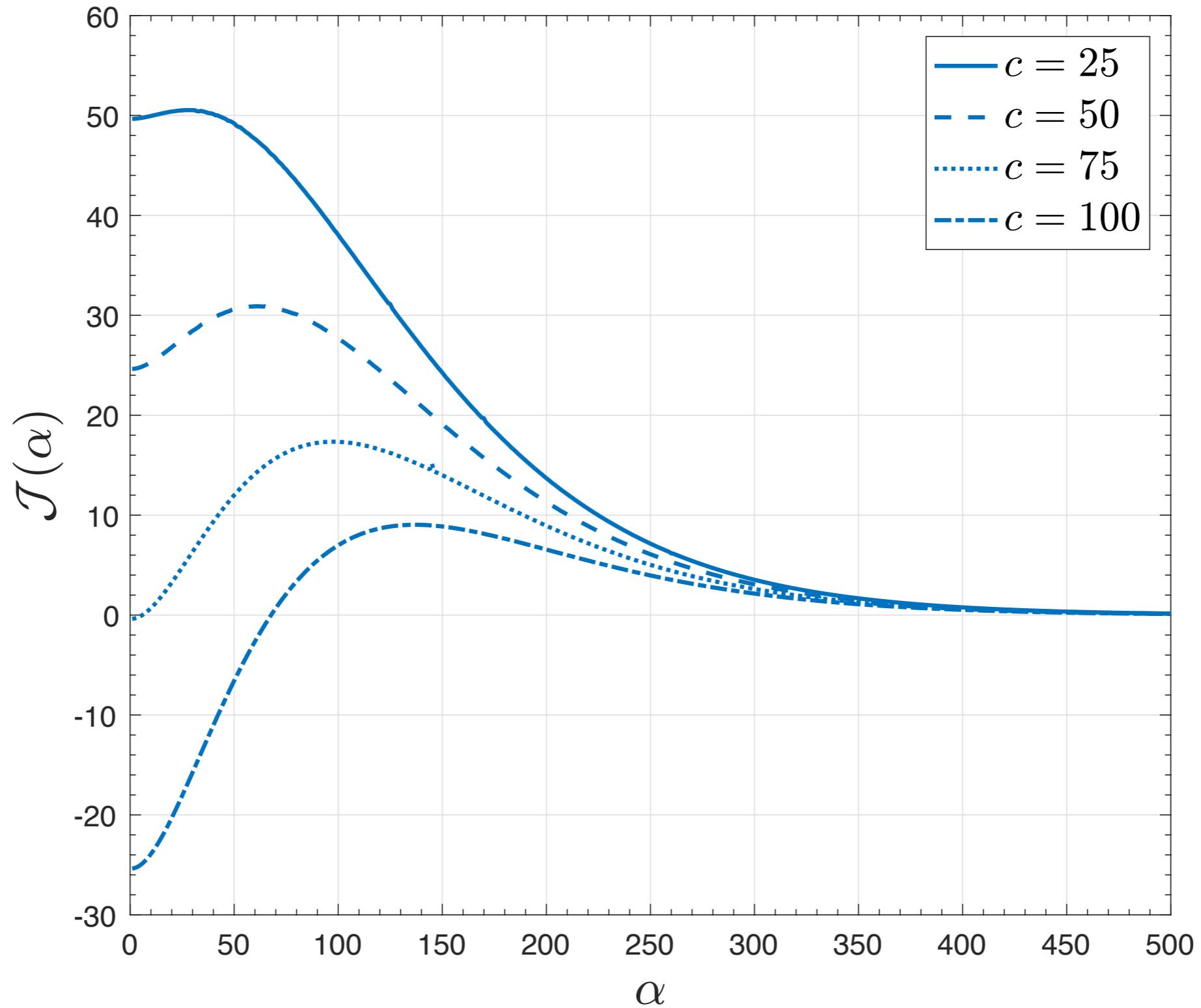
$$\mathcal{J}_i(\alpha) \triangleq \mathbf{E}[\mathcal{U}_i(A_i, A_{-i}, X)]$$

Threshold optimization

Quorum sensing

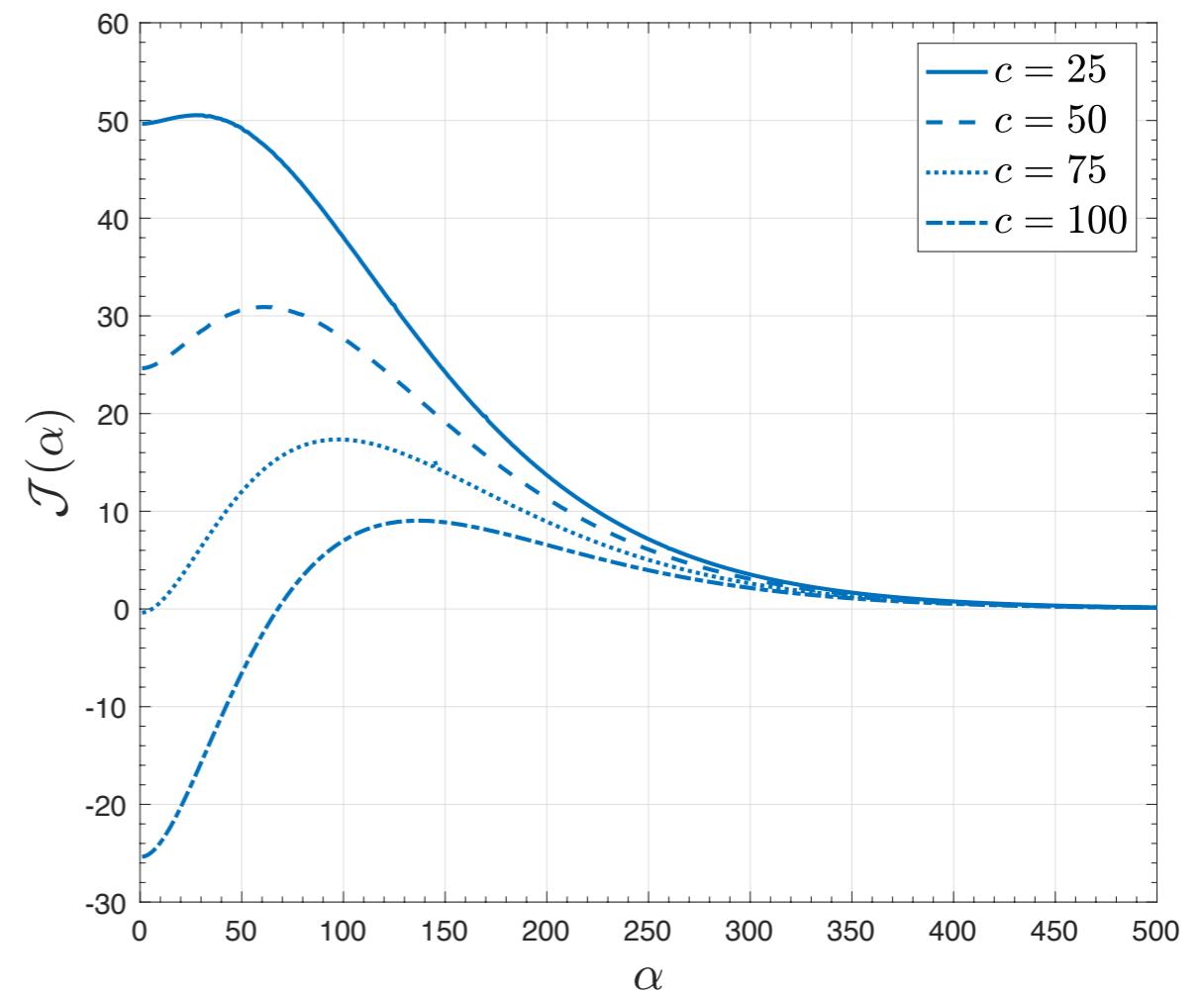
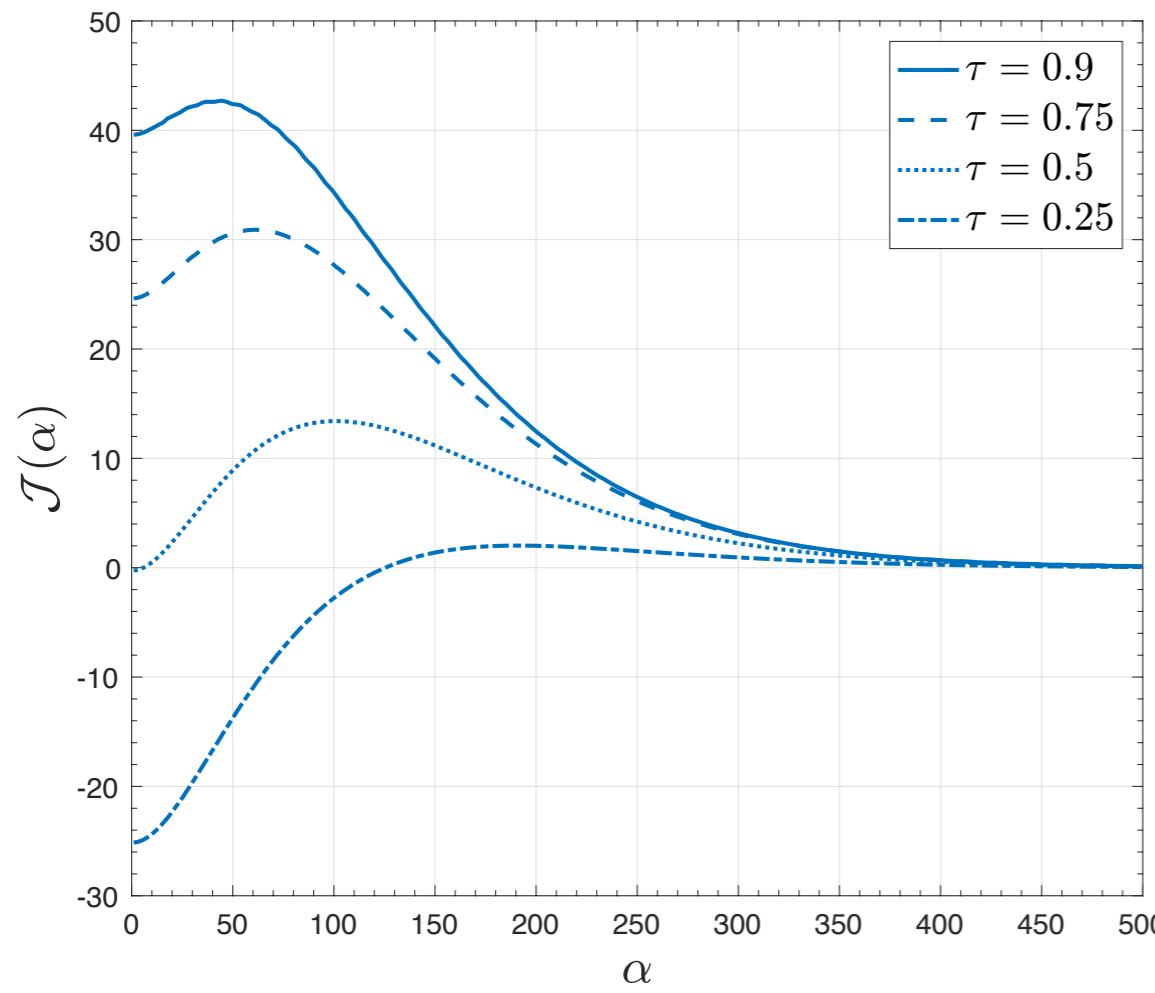


Quorum sensing



Quorum sensing

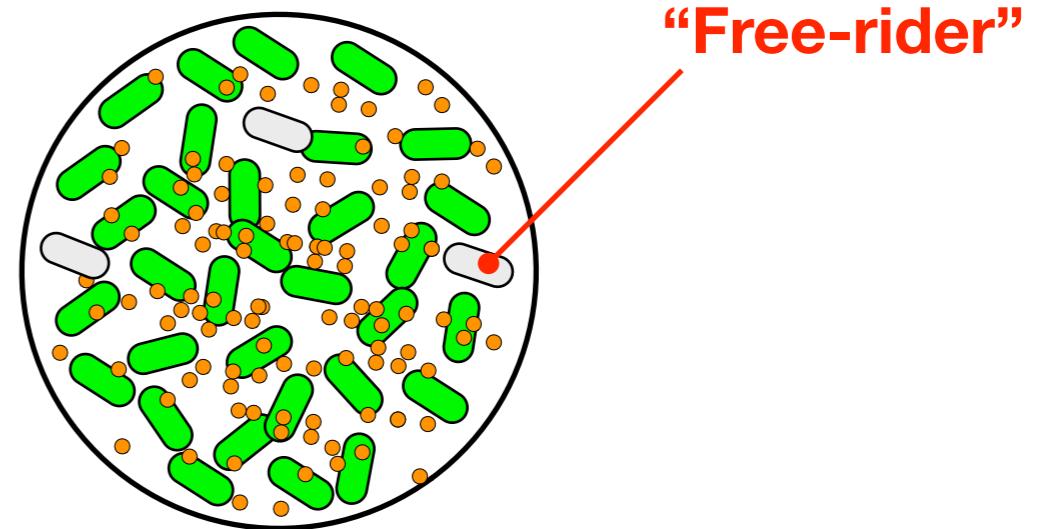
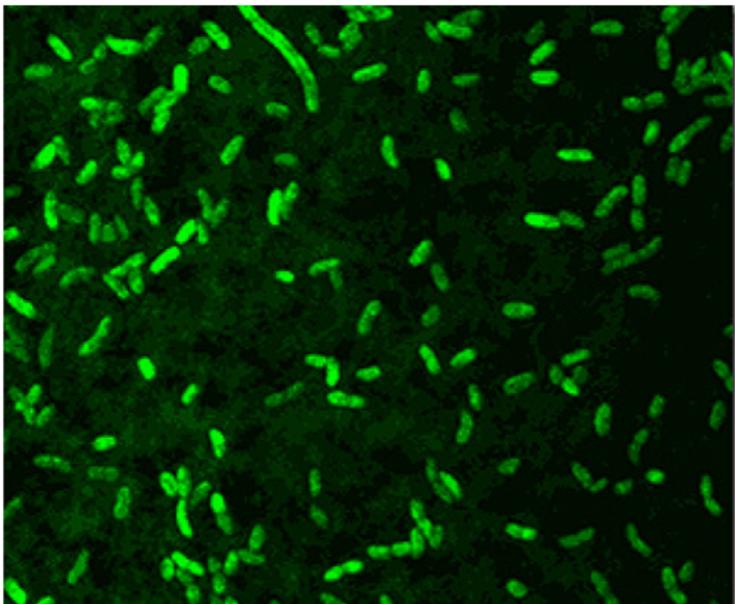
\mathcal{J} is unimodal in α



Free-riding bacteria

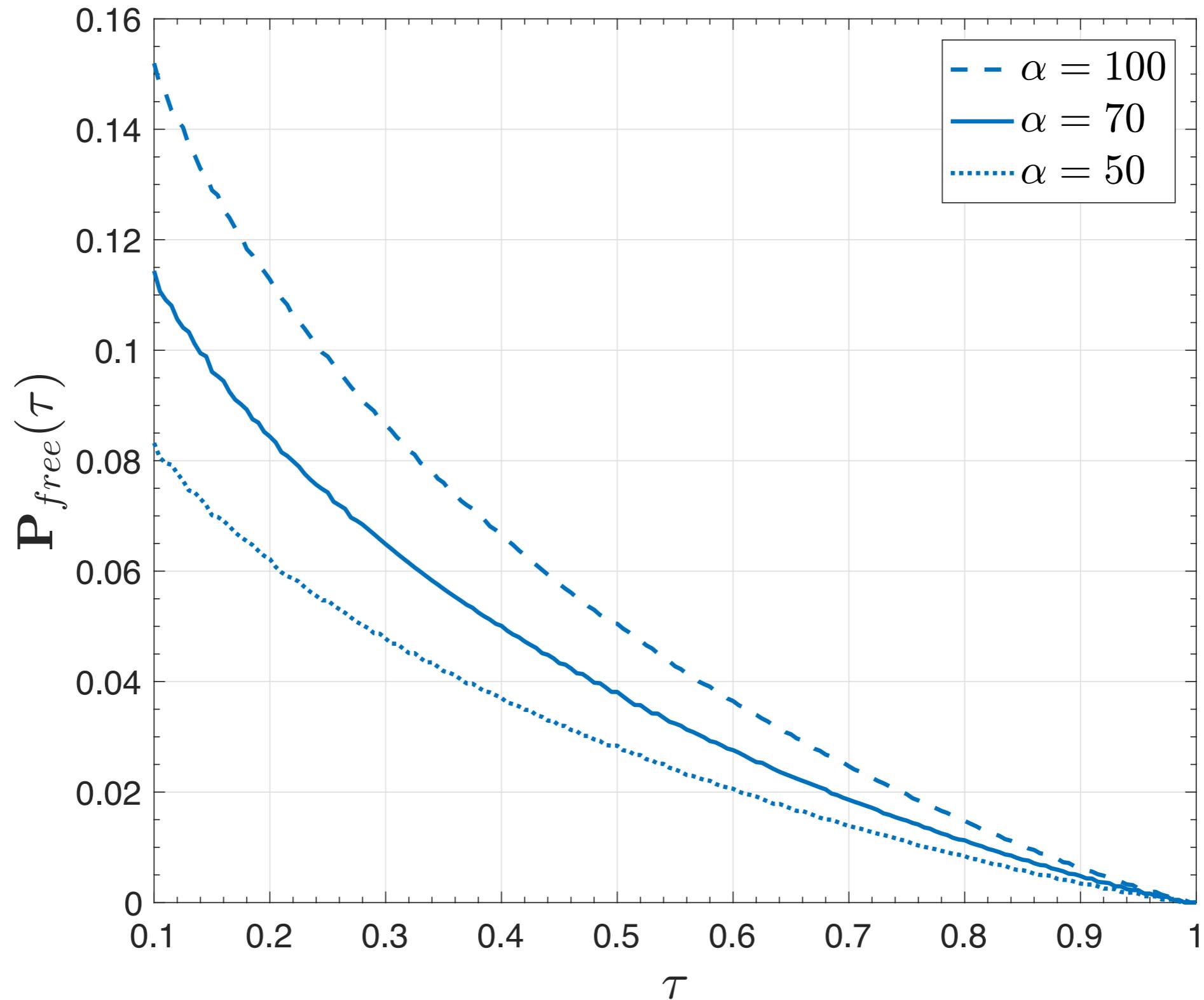
Free riding (cheating)

Agent that **benefits** from
public goods but is **inactive**



$$P_{free}(\tau) = P\left(A_i = 0 \mid \sum_{k \neq i} A_k \geq \tau X\right)$$

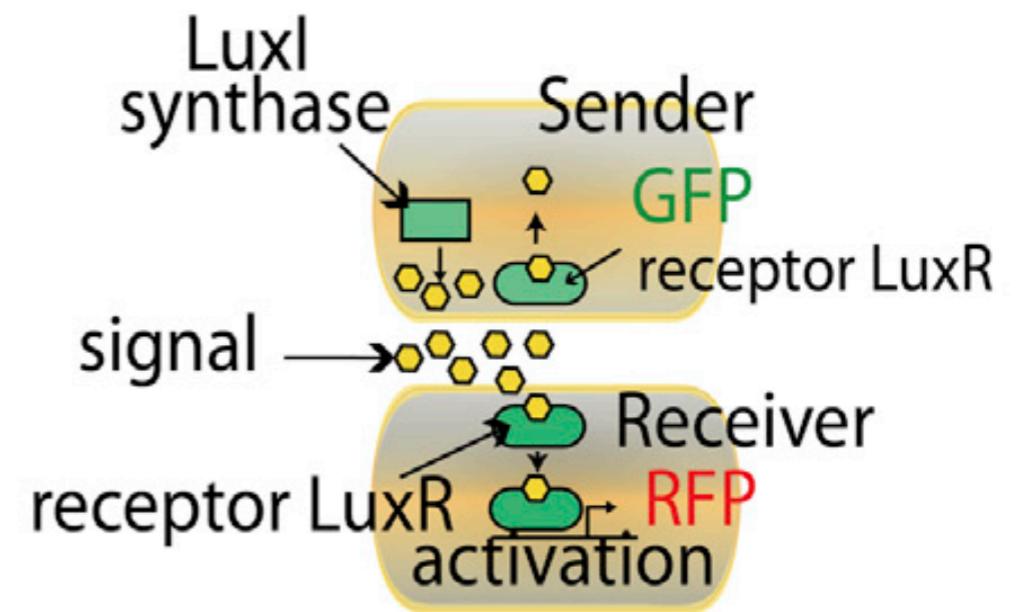
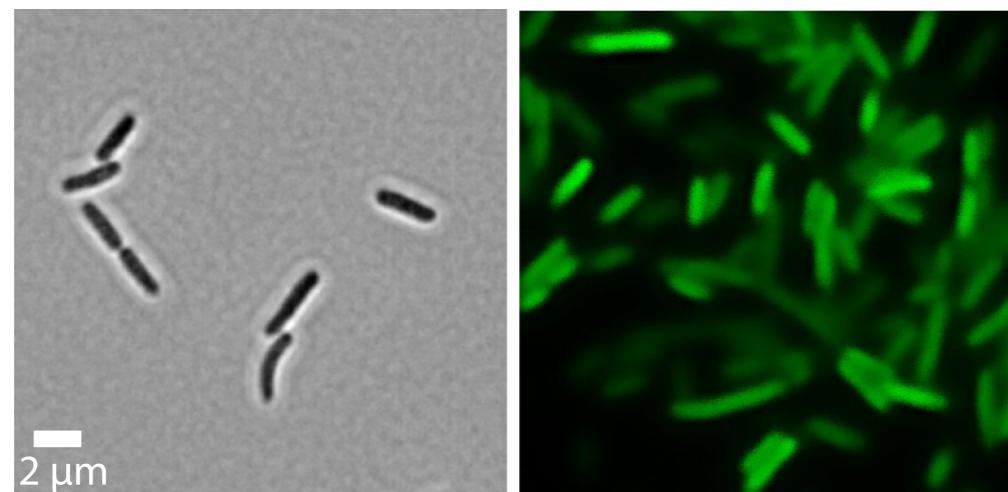
Free riding



Experimental results

Bacteria strain

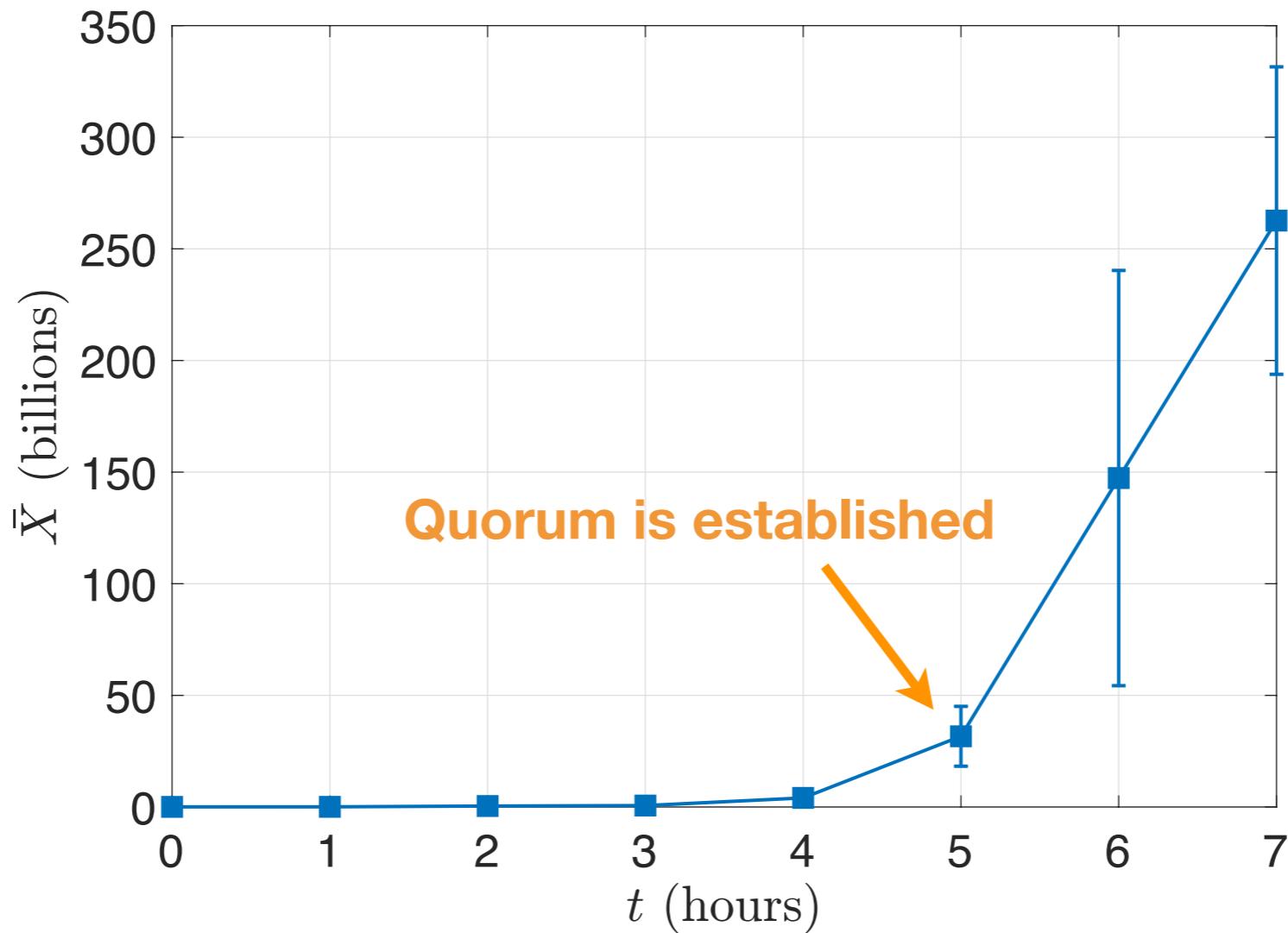
E. Coli ptd103 LuxI/R



Threshold $\alpha = 70\text{nM}$

Auto-inducer emission rate $\lambda = 2.3 \times 10^{-9} \text{ nmol}$

Experimental data



t (h)	\bar{X} (10^9 cells/L)	σ_X (10^9 cells/L)	$\lambda\bar{X}$ (nM)	p
0	0.0465	0.0140	0.1070	0
1	0.0653	0.0011	0.1503	0
2	0.4267	0.0833	0.9813	0
3	0.6400	0.1637	1.4720	0.0135
4	4.0667	0.5033	9.3533	0.1038
5	31.6667	13.4079	72.8333	0.8538
6	147.3333	93.0017	338.8667	0.9953
7	262.6667	68.8573	604.1333	1

Sharp transition happens here!

Proportion of activated cells

Calibrating the model

Estimate the quorum coefficient

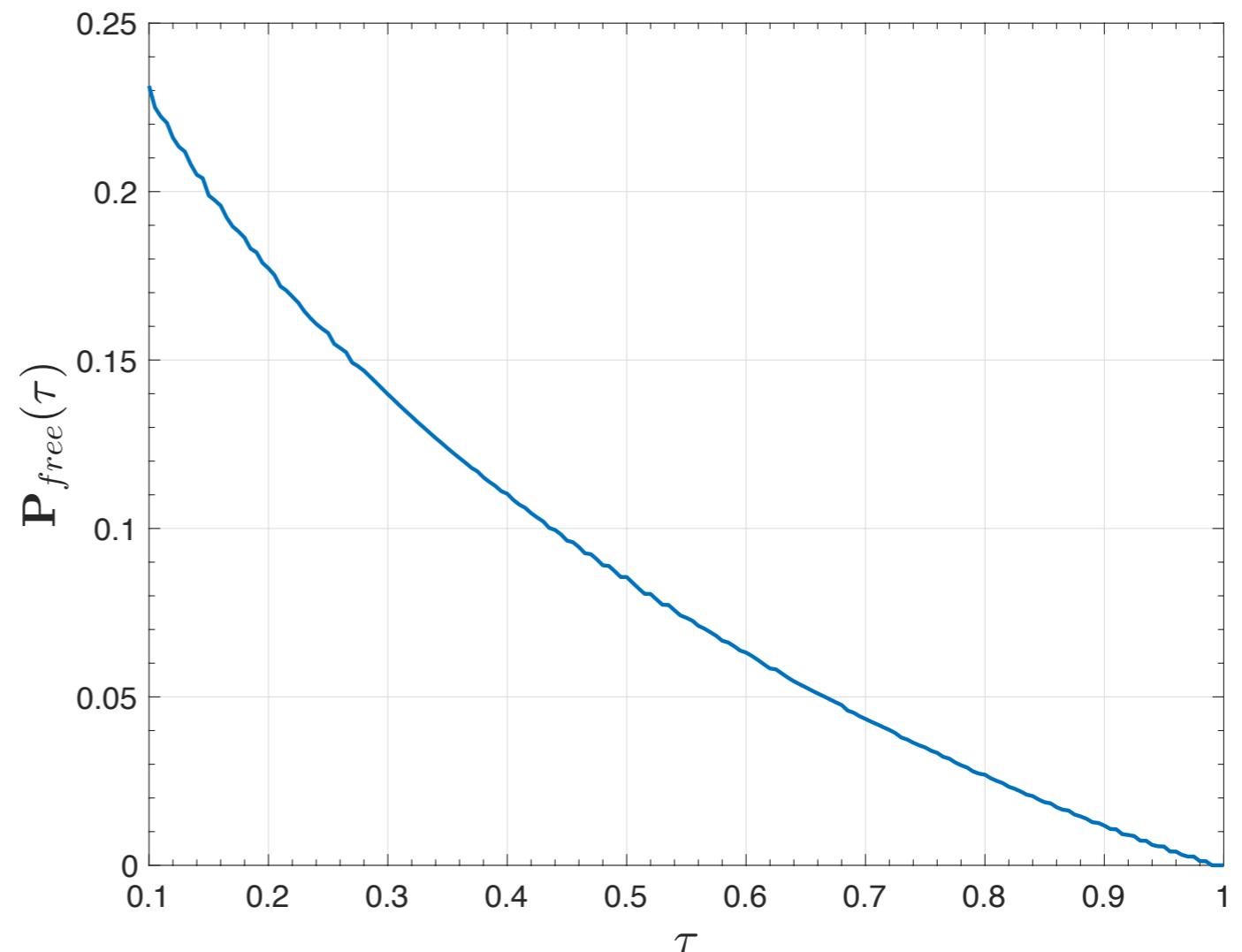
$$\bar{X} = 31.6667$$

$$\sigma = 13.4079$$

$$\kappa = 5.5780$$

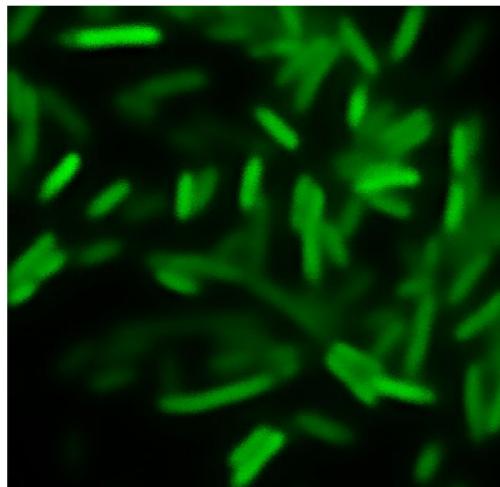
$$\theta = 5.6770$$

$$X \sim \text{Gamma}(\kappa, \theta)$$



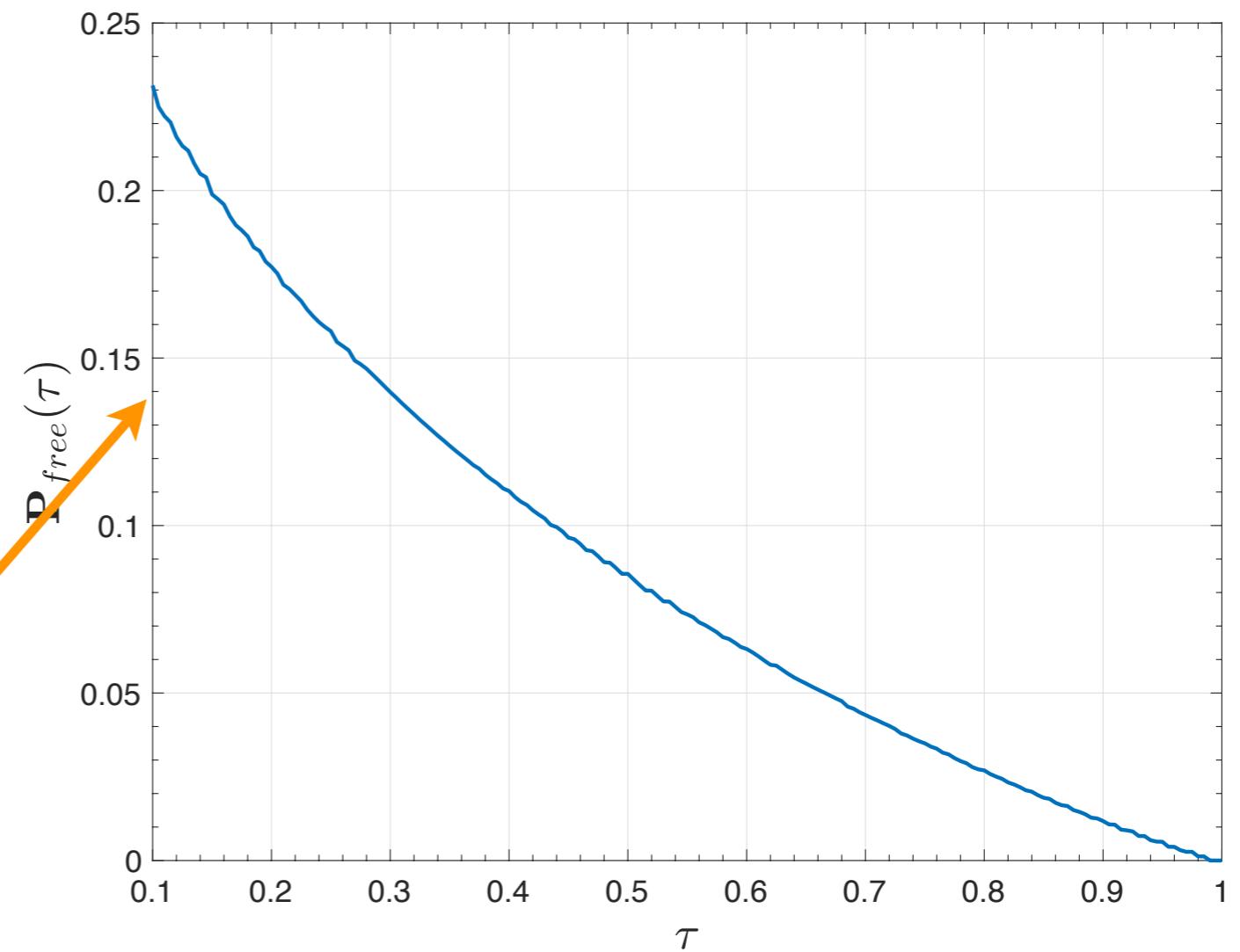
Calibrating the model

Estimate the quorum coefficient



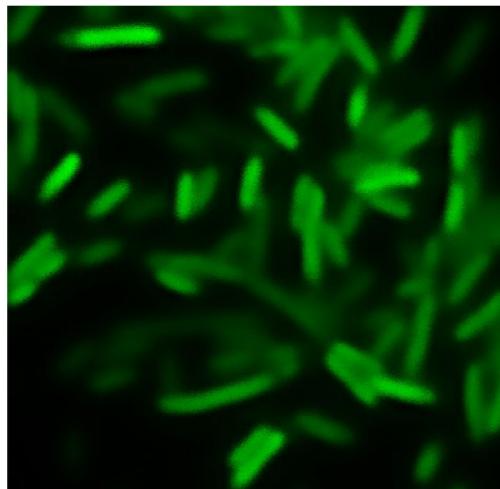
Frequency of free-riders

$$p \approx 0.14$$



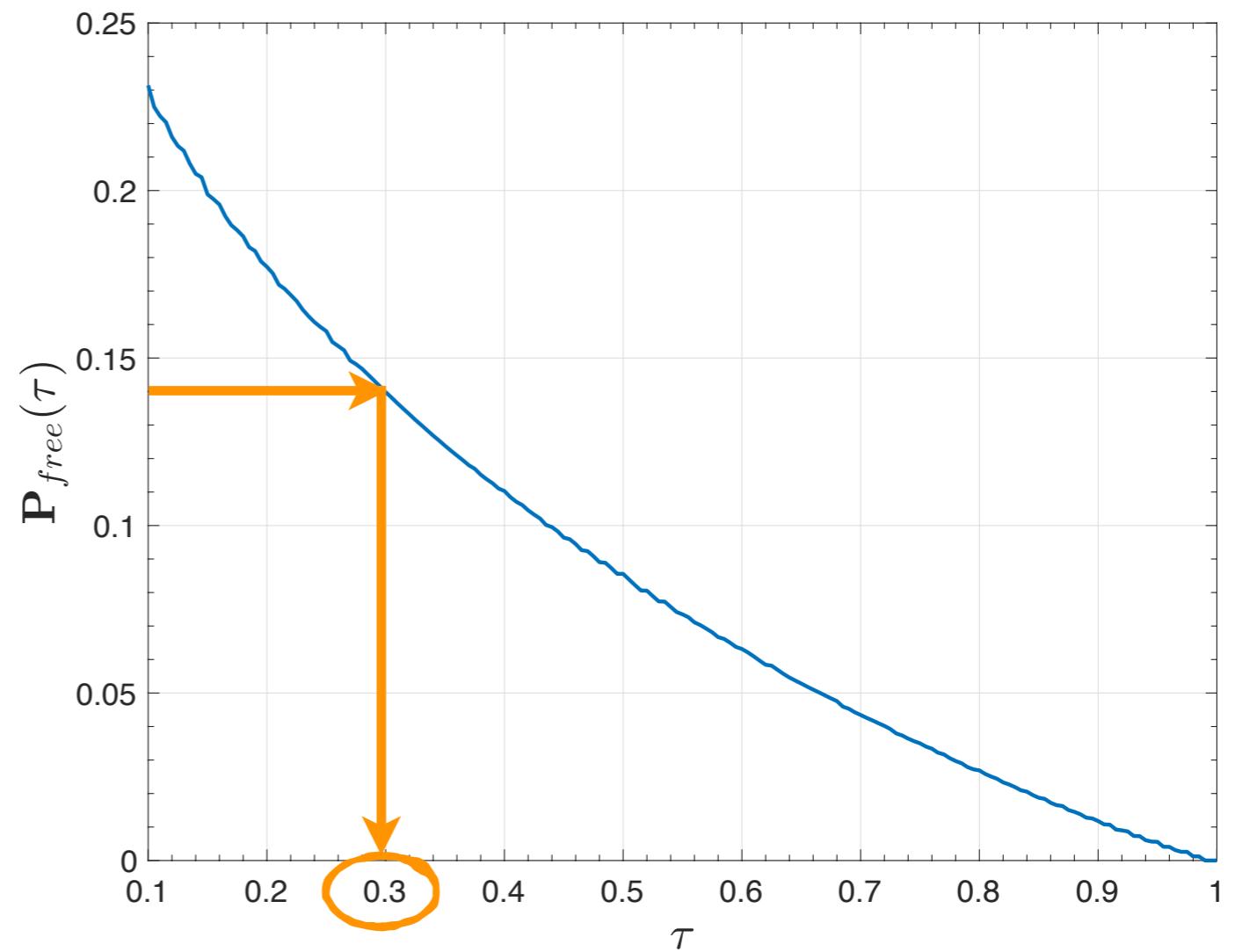
Calibrating the model

Estimate the quorum coefficient



Frequency of free-riders

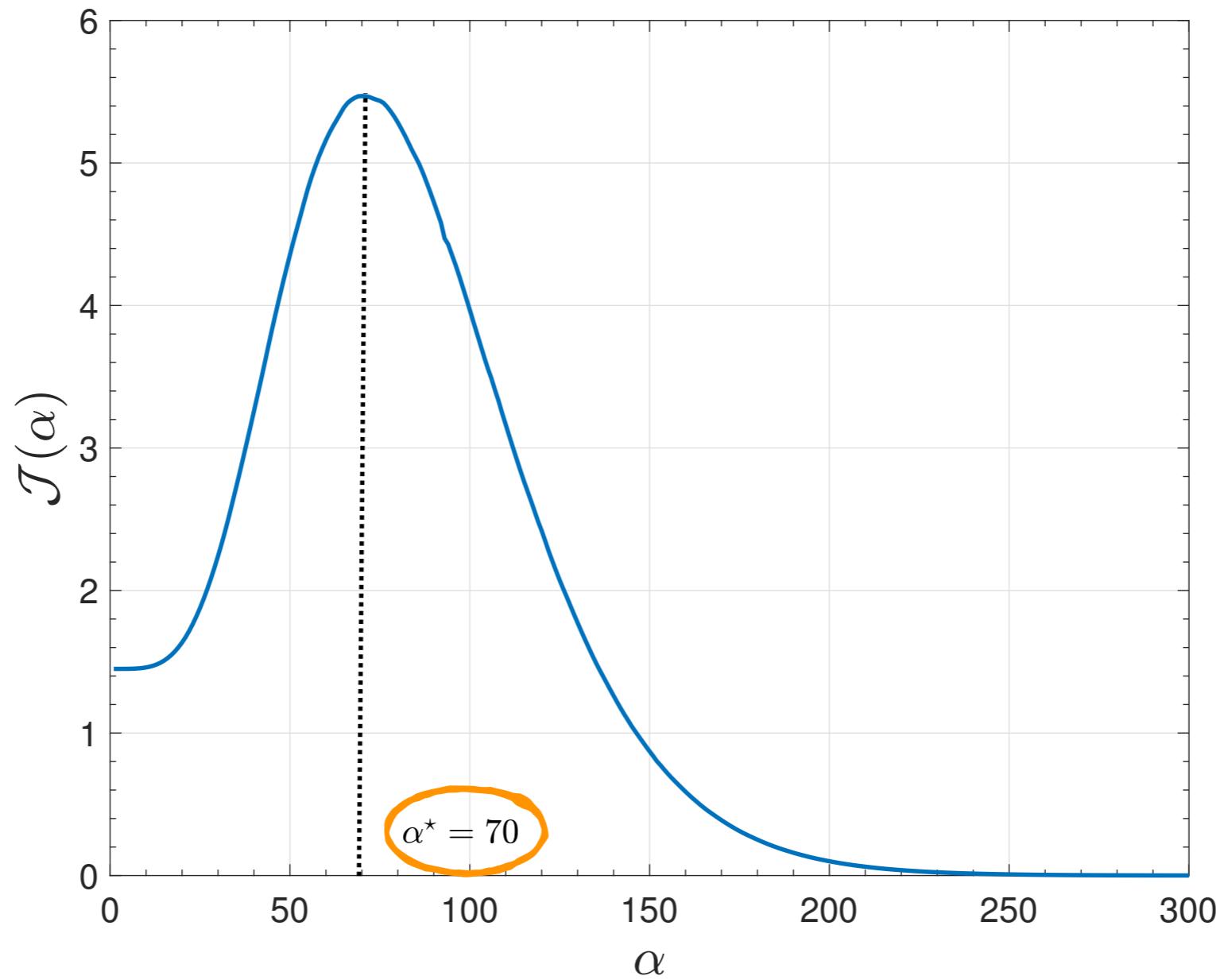
$$p \approx 0.14$$



$$\hat{\tau} = 0.3$$

Calibrating the model

Adjust the activation cost



$$\hat{c} \approx 20.25$$

Summary

New decision theoretic model of Quorum Sensing

Cells are decision makers

Nature is the “system designer”

Formulate new hypothesis on the trade-offs between:

- 1. growth rate vs. activation cost**
- 2. growth rate vs free-riding (cheating)**

Future work

Our model does not account for dynamics

Sequential model

1. Signal accumulation/degradation
2. Population dynamics

Open questions

How do cells aggregate information?

How do bacteria avoid the “Tragedy of the Commons”?