



Optimality of event-based policies for decentralized estimation over shared networks

Marcos M. Vasconcelos

`marcos@terpmail.umd.edu`

Dept. of Electrical and Computer Engineering
University of Maryland, College Park

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8-22-16

Applications

- Sensor networks - **one-time catastrophic events**



Oil & gas pipelines

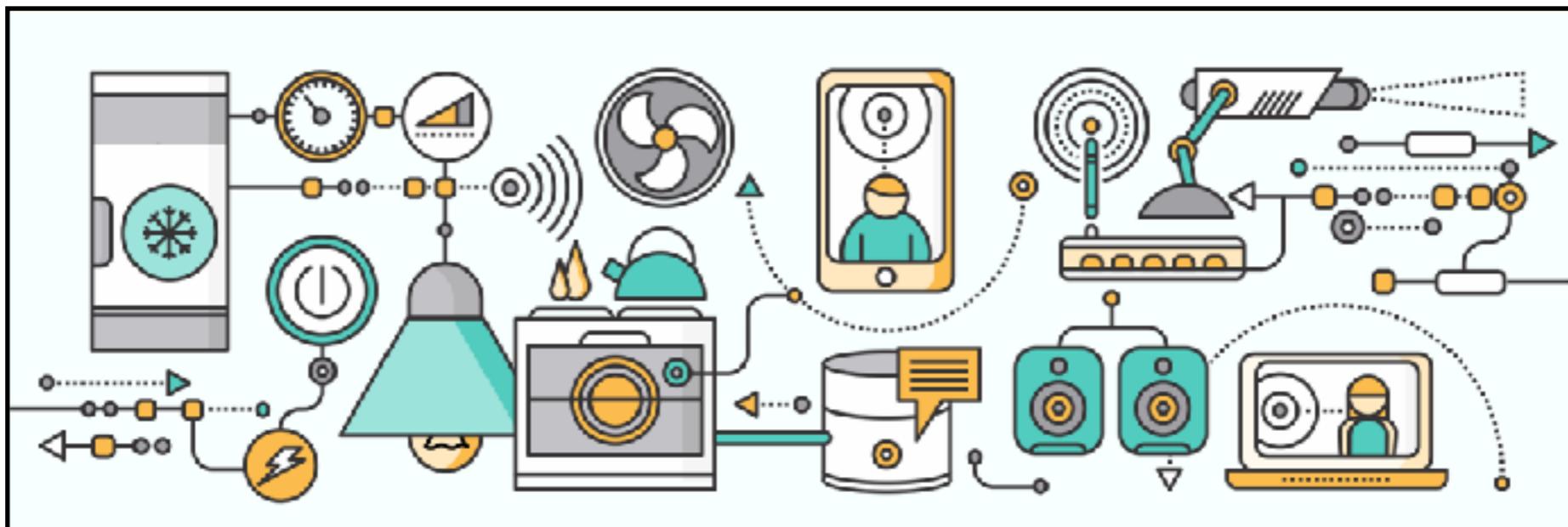


Powergrid

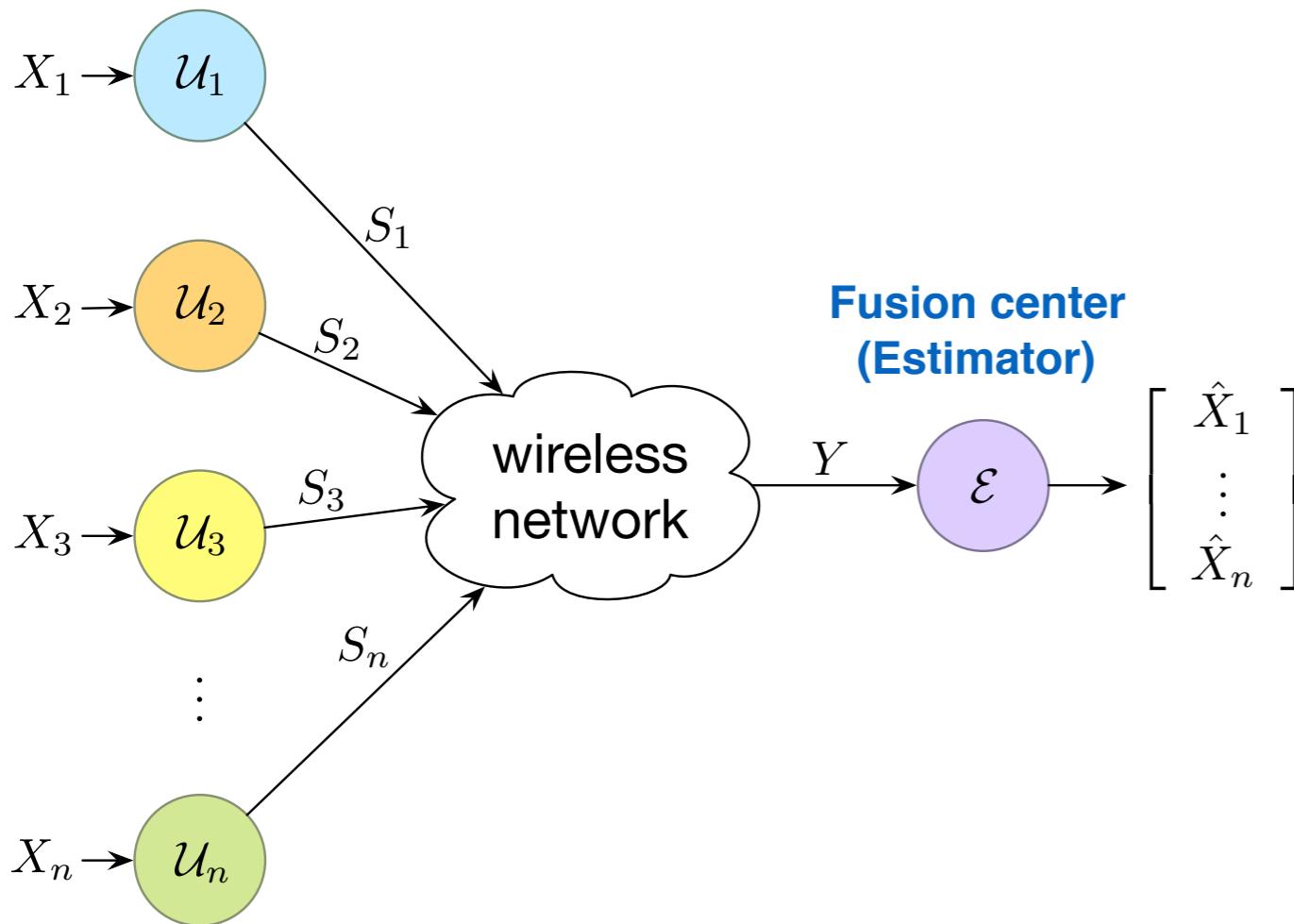


Bridges

- Internet-of-things - **real time wireless networking**



**Sensors
(Decision Makers)**



The setting

Channel models used in networked control

- Rate-limited channel¹
- Packet drop channel²
- **Collision channel**^{new}

- **One-shot estimation**
- **Decentralized = no coordination** between sensors

1. Nair et al, “Feedback control under data rate constraints: An overview,” *Proceedings IEEE* 2007.

2. Schenato et al, “Foundations of control and estimation over lossy networks,” *Proceedings IEEE* 2007.

Collision channel

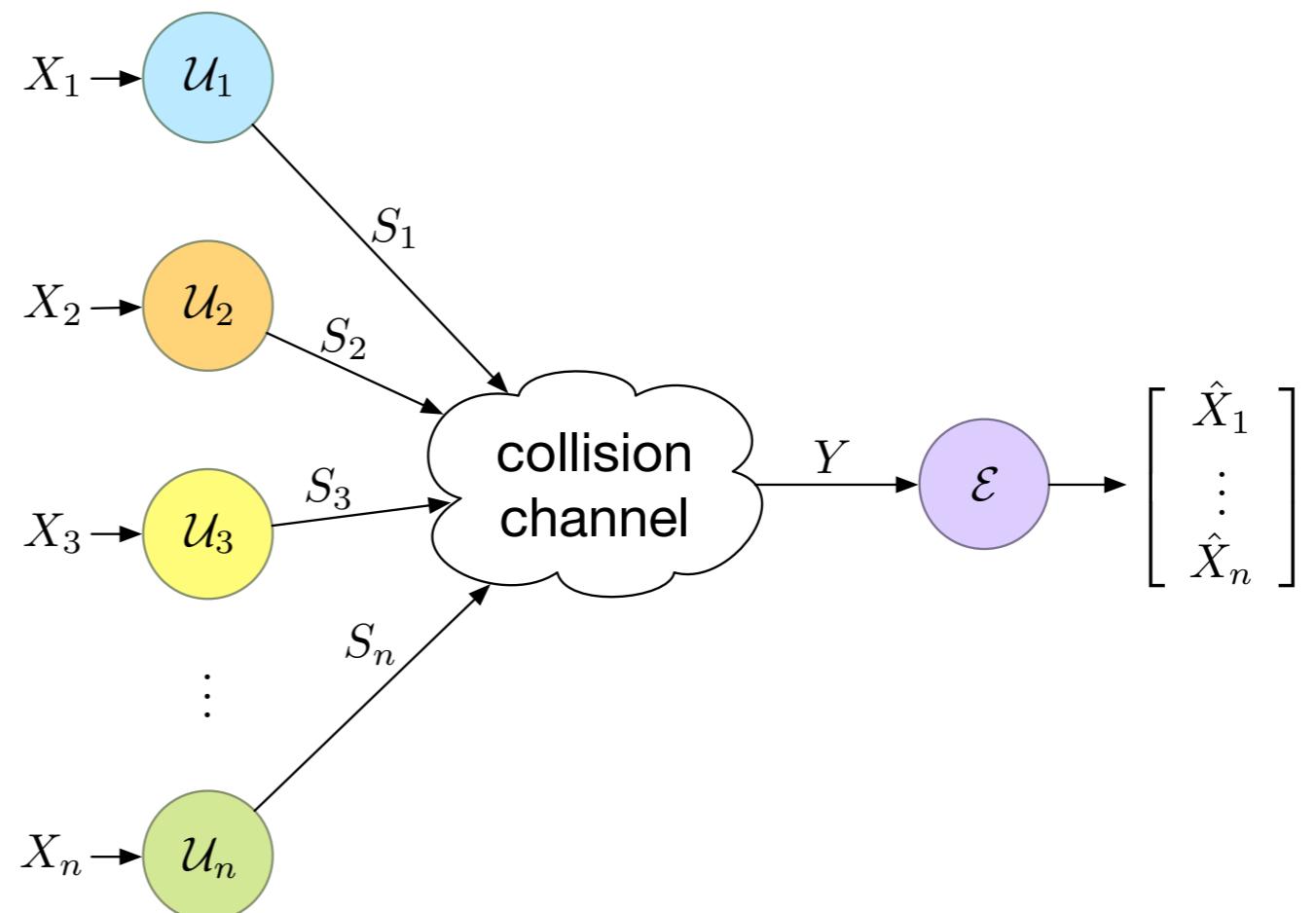
Model of **interference**:

- **Widely used in wireless communications**^{1,2}
- >1 transmission results in a **collision**
- Sensors decide whether to transmit or not

Decision variables: U_i

$$U_i = 1 \implies S_i = (i, X_i) \quad (\text{transmit})$$

$$U_i = 0 \implies S_i = \emptyset \quad (\text{stay silent})$$



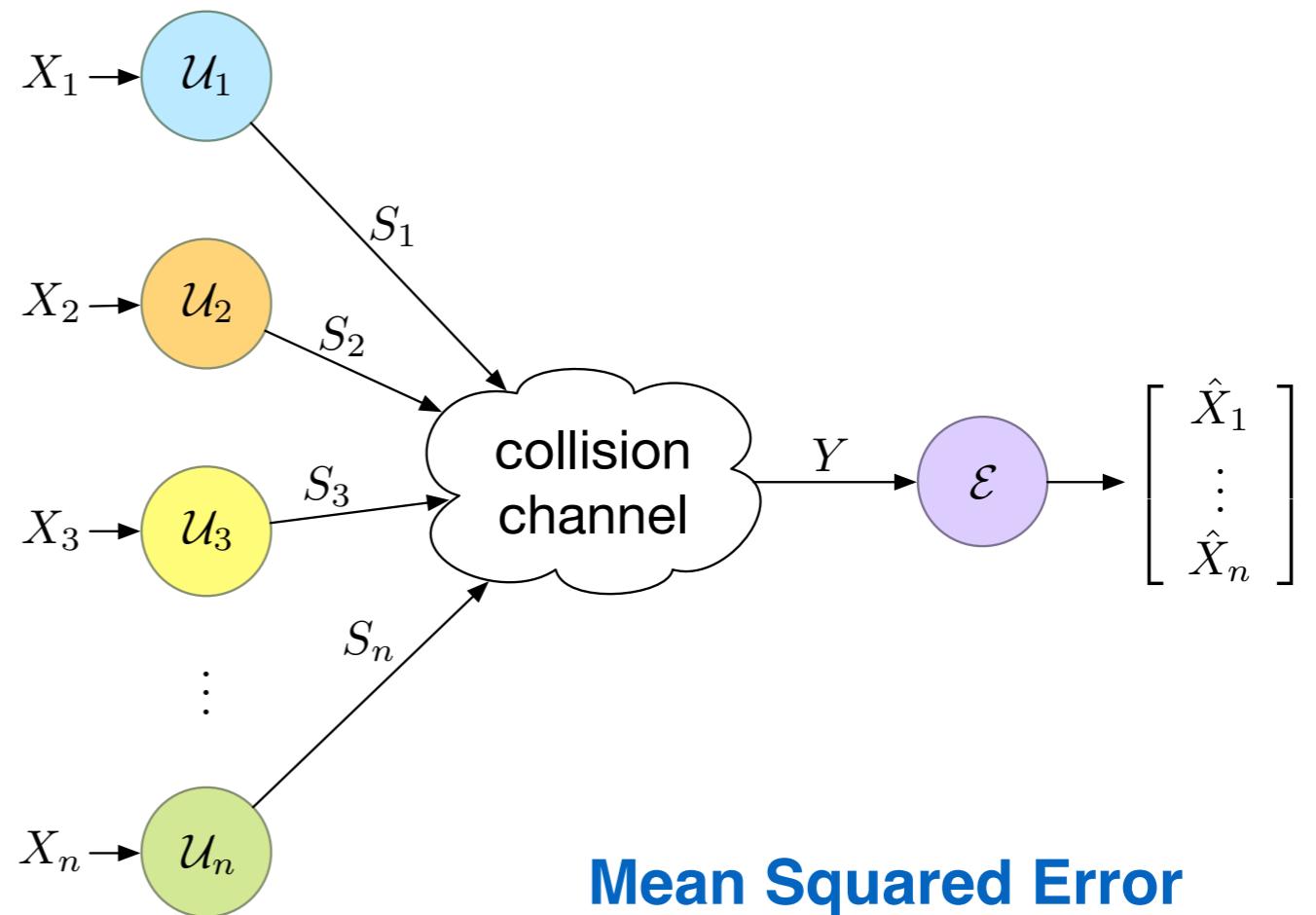
1. Goldsmith, *Wireless Communications*, 2005.
2. Bertsekas and Gallager, *Data Networks*, 1992.

Part 1: MMSE estimation over the collision channel

$$W = [X_1, \dots, X_n]$$

$$X_i, \quad i \in \{1, \dots, n\}$$

- mutually independent
- **continuous** rvs
- supported on the real line
- arbitrarily distributed



Problem

$$\text{minimize} \quad \mathcal{J}(\mathcal{U}_1, \dots, \mathcal{U}_n) = \mathbf{E} \left[\sum_{i=1}^n (X_i - \hat{X}_i)^2 \right]$$

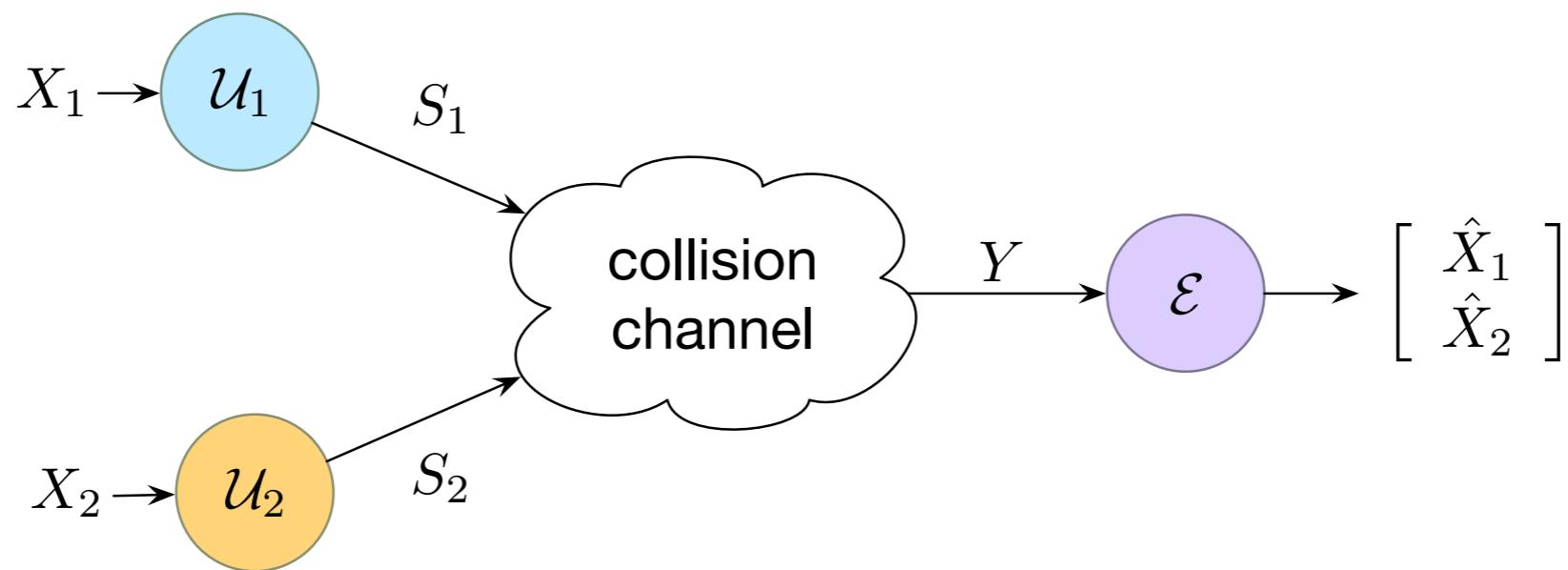
Stochastic policies

$$\text{prob}(U_i = 1 | X_i = x_i) = \mathcal{U}_i(x_i)$$

MMSE estimator

$$\mathcal{E}(y) = \mathbf{E}[W | Y = y]$$

Simplest case: two sensors



$$\mathbf{prob}(U_i = 1 | X_i = x_i) = \mathcal{U}_i(x_i)$$

Problem 1

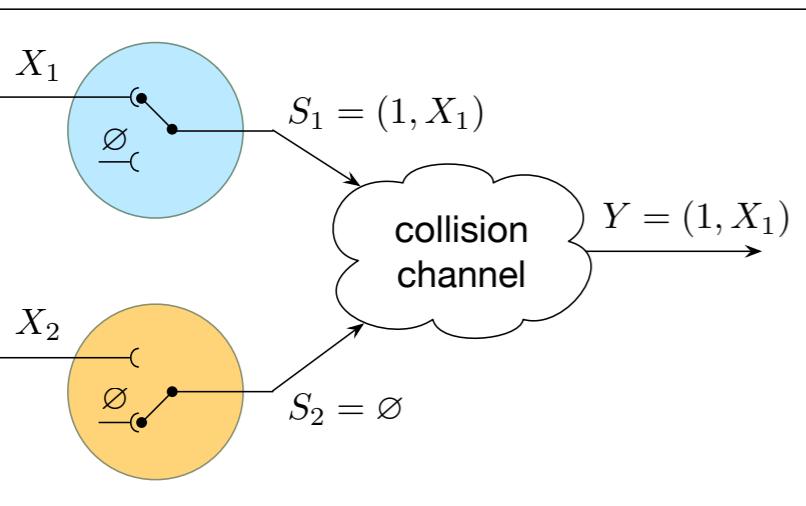
$$\text{minimize } \mathcal{J}(\mathcal{U}_1, \mathcal{U}_2) = \mathbf{E} \left[(X_1 - \hat{X}_1)^2 + (X_2 - \hat{X}_2)^2 \right]$$

$$\mathbb{U}_i = \{\mathcal{U} \mid \mathcal{U} : \mathbb{R} \rightarrow [0, 1]\}, \quad i \in \{1, 2\}$$

Collision channel

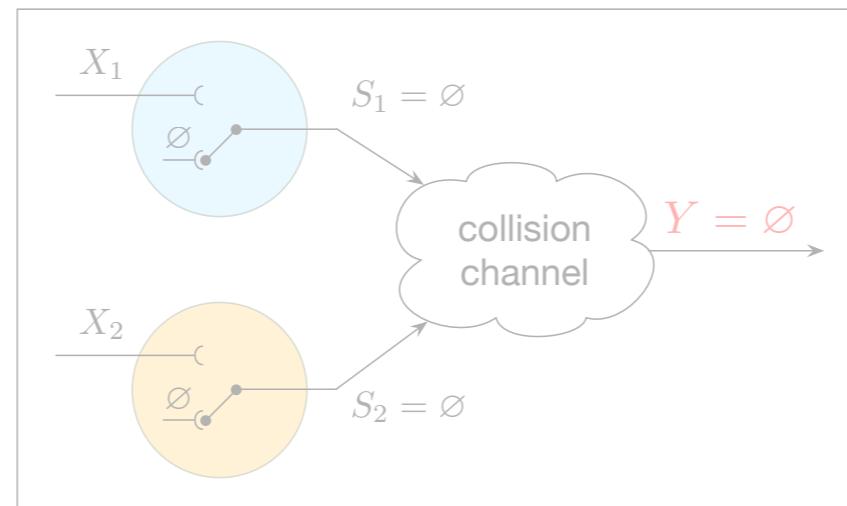
single transmission

$$U_1 = 1, U_2 = 0$$



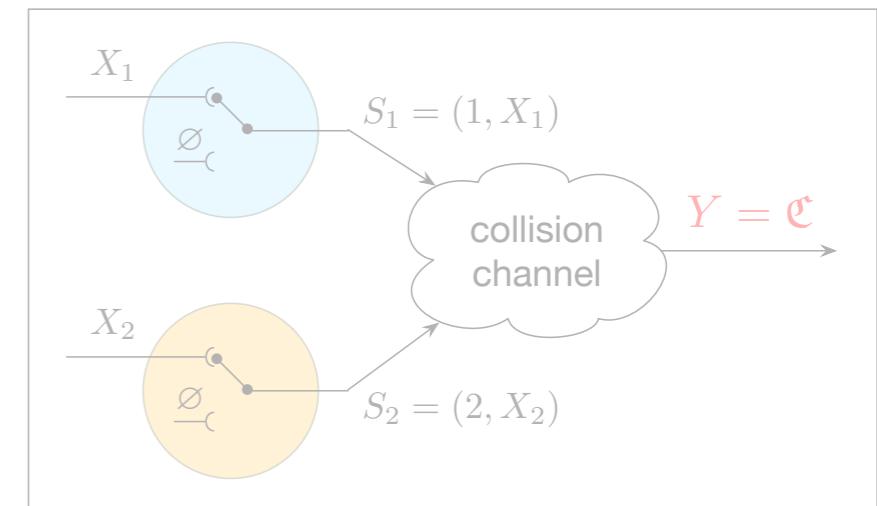
no transmissions

$$U_1 = 0, U_2 = 0$$



>1 transmissions

$$U_1 = 1, U_2 = 1$$



success!

no transmission \emptyset

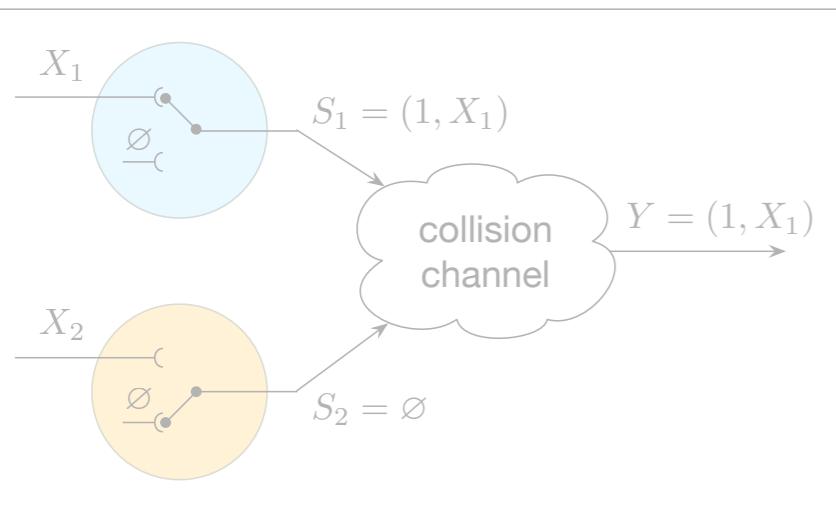
collision \mathfrak{C}

From the channel output we can always recover U_1 and U_2 .

Collision channel

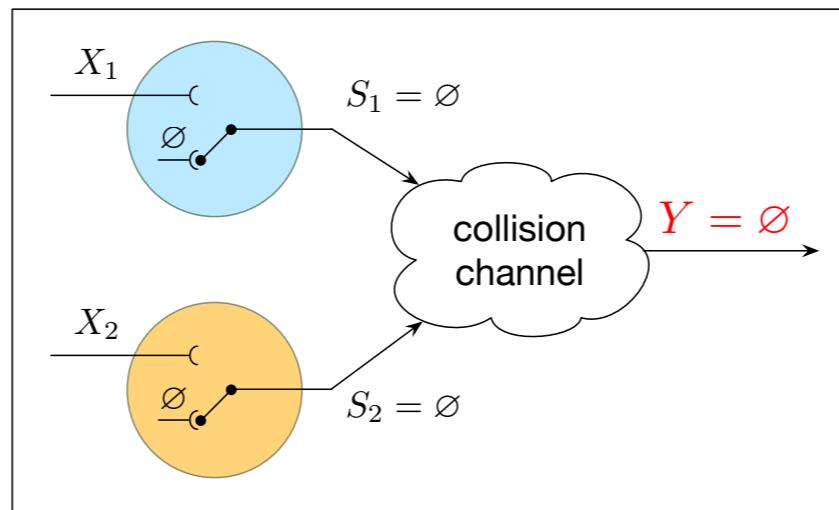
single transmission

$$U_1 = 1, U_2 = 0$$



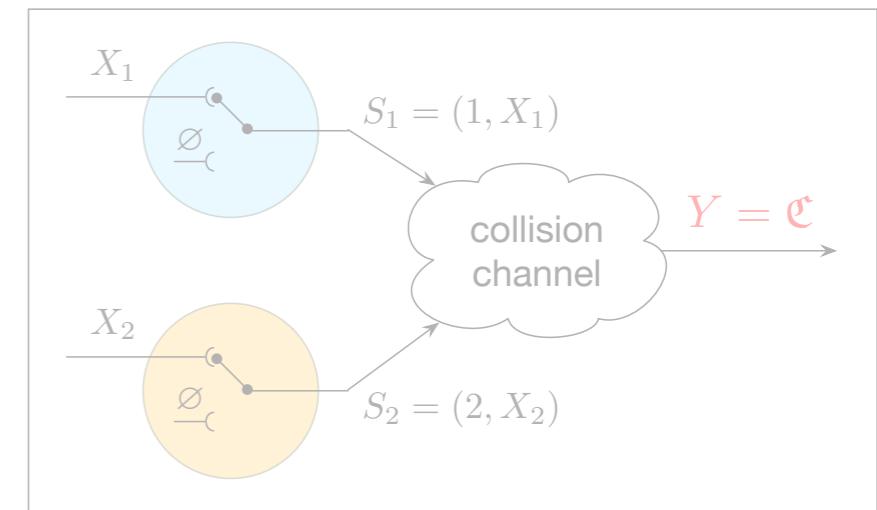
no transmissions

$$U_1 = 0, U_2 = 0$$



>1 transmissions

$$U_1 = 1, U_2 = 1$$



success!

no transmission \emptyset

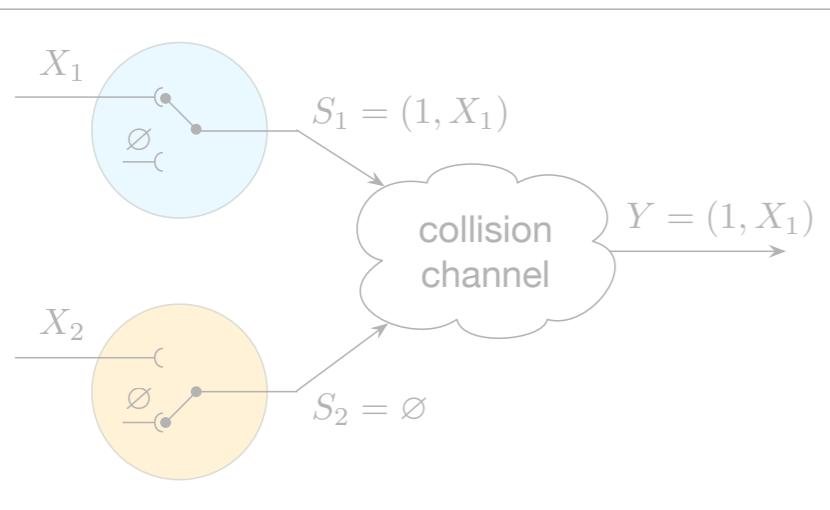
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From the channel output we can always recover U_1 and U_2 .

Collision channel

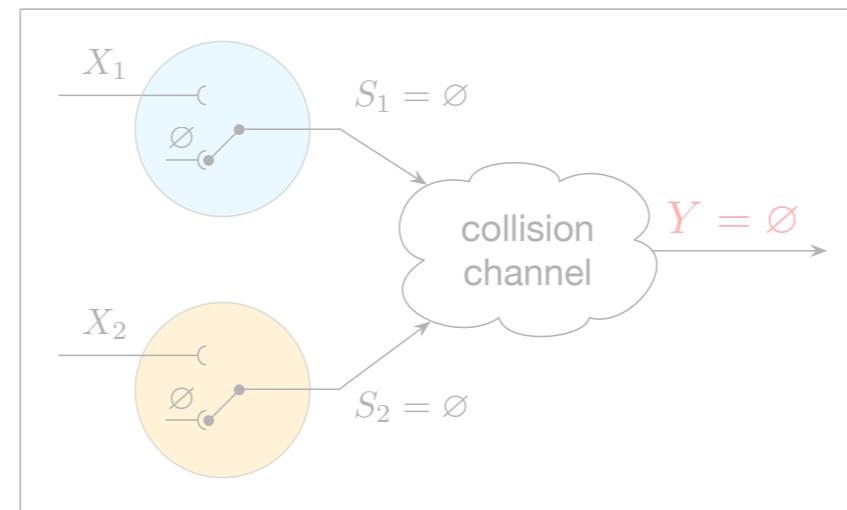
single transmission

$$U_1 = 1, U_2 = 0$$



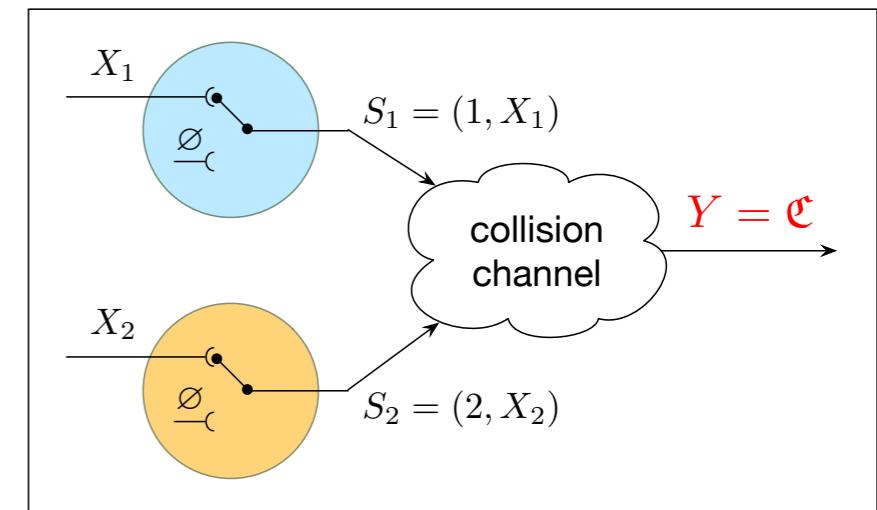
no transmissions

$$U_1 = 0, U_2 = 0$$



>1 transmissions

$$U_1 = 1, U_2 = 1$$



success!

no transmission \emptyset

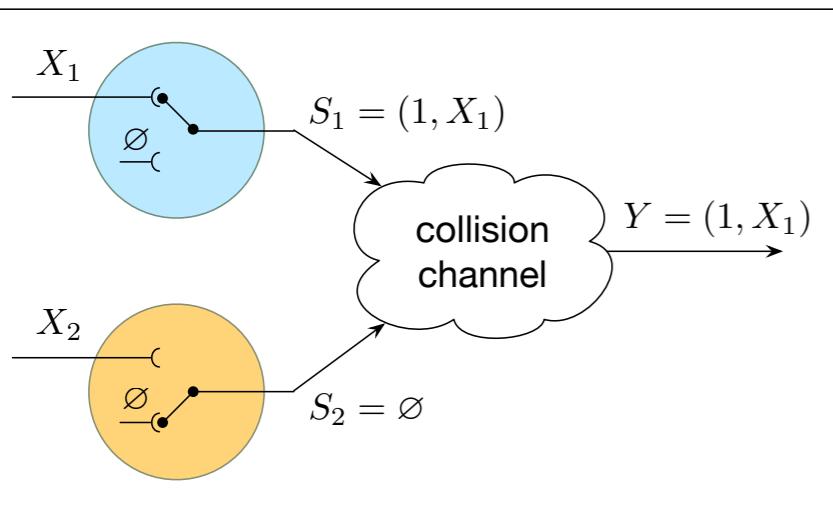
collision \mathfrak{C}

From the channel output we can always recover U_1 and U_2 .

Collision channel

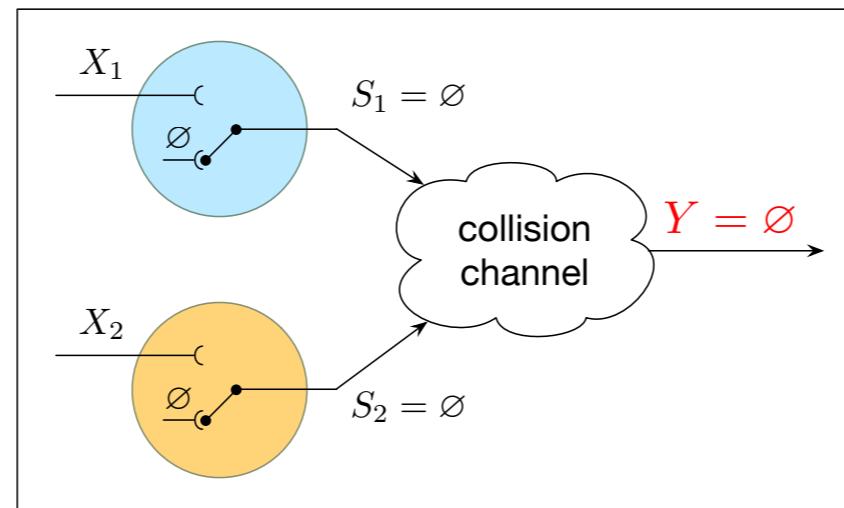
single transmission

$$U_1 = 1, U_2 = 0$$



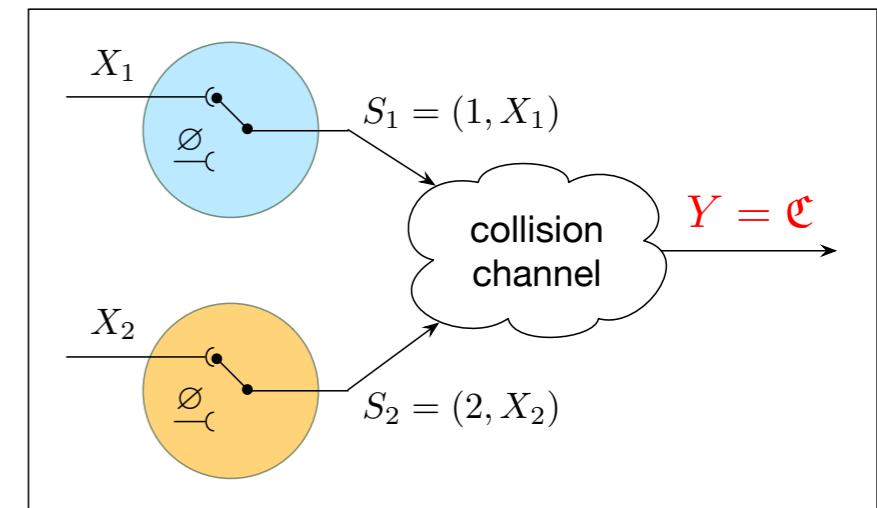
no transmissions

$$U_1 = 0, U_2 = 0$$



>1 transmissions

$$U_1 = 1, U_2 = 1$$



success!

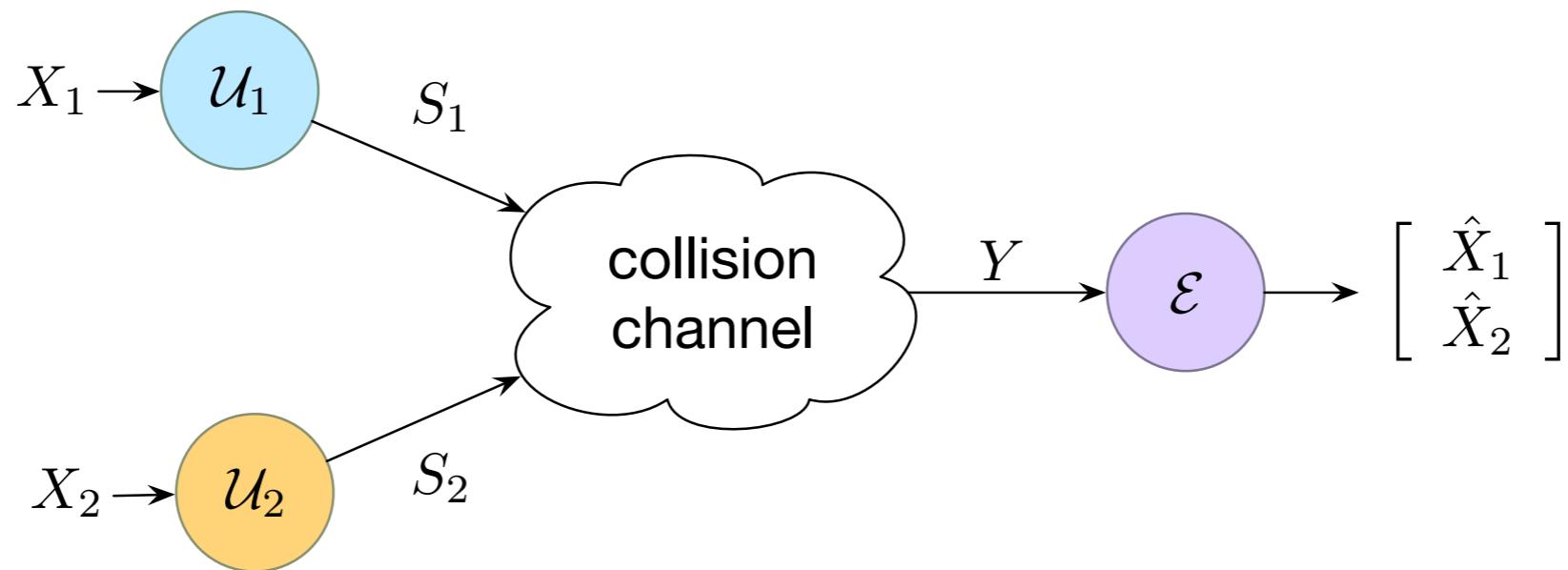
no transmission \emptyset

collision \mathfrak{C}

**The collision channel is fundamentally different
from the packet drop channel^{1,2}**

1. Sinopoli et al, “Kalman filtering with intermittent observations,” *IEEE TAC* 2004.
2. Gupta et al, “Optimal LQG control across packet-dropping links,” *Systems and Control Letters* 2007.

Why is this problem interesting?



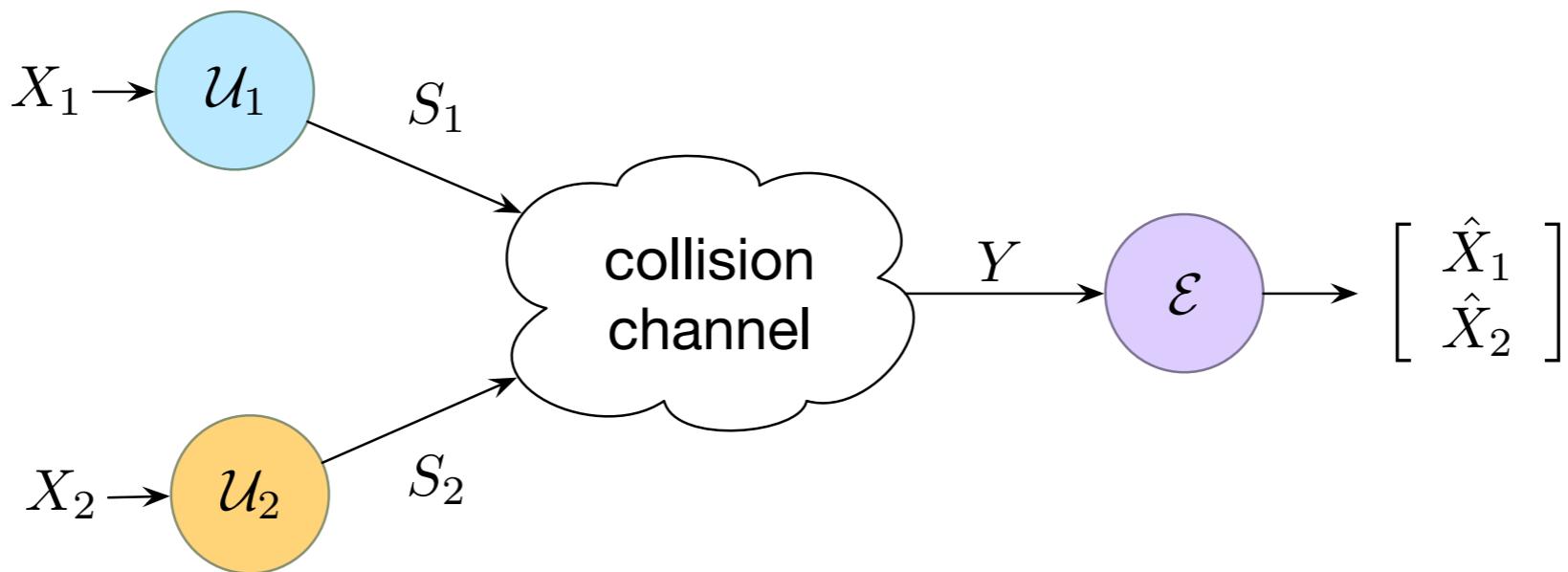
Problem 1

$$\text{minimize } \mathcal{J}(\mathcal{U}_1, \mathcal{U}_2) = \mathbf{E} \left[(X_1 - \hat{X}_1)^2 + (X_2 - \hat{X}_2)^2 \right]$$

Static **team-decision problem**
with **non-classical information** structure \implies **Non-convex**
(in most cases) **intractable**^{1,2}

1. Witsenhausen, "A counterexample in optimal stochastic control," *SIAM J. Control* 1968.
2. Tsitsiklis & Athans, "On the complexity of decentralized decision making and detection problems," *IEEE TAC* 1985.

Why is this problem interesting?



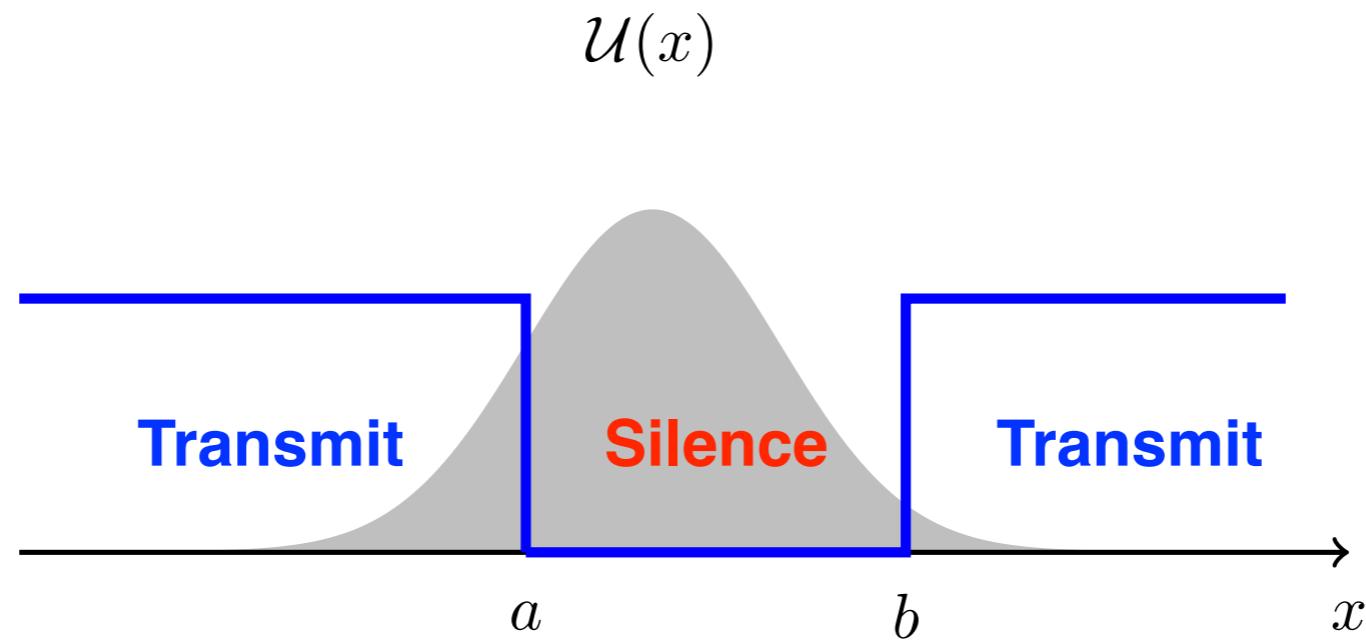
Problem 1

$$\text{minimize } \mathcal{J}(\mathcal{U}_1, \mathcal{U}_2) = \mathbf{E} \left[(X_1 - \hat{X}_1)^2 + (X_2 - \hat{X}_2)^2 \right]$$

Look for a class **parametrizable policies that contains an optimal strategy**

1. Witsenhausen, “A counterexample in optimal stochastic control,” *SIAM J. Control* 1968.
2. Tsitsiklis & Athans, “On the complexity of decentralized decision making and detection problems,” *IEEE TAC* 1985.

Deterministic threshold policies



Threshold policy

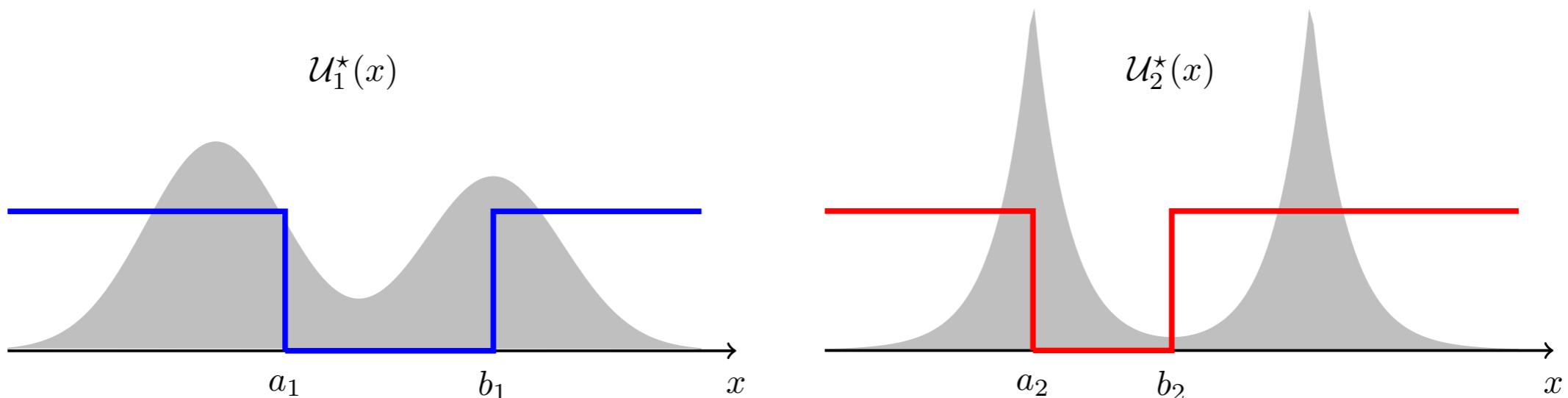
$$\mathcal{U}(x) = \begin{cases} 0 & a \leq x \leq b \\ 1 & \text{otherwise} \end{cases}$$

1. Imer & Basar, "Optimal estimation with limited measurements", *Int. Journal of Syst., Cont. and Comm.* 2010.
2. Lipsa & Martins, "Remote state estimation with communication costs for first-order LTI systems". *IEEE TAC* 2011.

Characterization of team-optimal policies

Theorem

There exists an team-optimal pair of **threshold policies** for Problem 1.



Sketch of Proof:

- Step 1: Equivalent single DM problem
- Step 2: Lagrange duality for infinite dimensional LPs

Main idea

Team-optimality

$$\mathcal{J}(\mathcal{U}_1^*, \mathcal{U}_2^*) \leq \mathcal{J}(\mathcal{U}_1, \mathcal{U}_2), \quad (\mathcal{U}_1, \mathcal{U}_2) \in \mathbb{U}_1 \times \mathbb{U}_2$$

$$\begin{array}{c} \Rightarrow \\ \Leftarrow \end{array}$$

Person-by-person optimality

$$\mathcal{J}(\mathcal{U}_1^*, \mathcal{U}_2^*) \leq \mathcal{J}(\mathcal{U}_1, \mathcal{U}_2^*), \quad \mathcal{U}_1 \in \mathbb{U}_1$$

$$\mathcal{J}(\mathcal{U}_1^*, \mathcal{U}_2^*) \leq \mathcal{J}(\mathcal{U}_1^*, \mathcal{U}_2), \quad \mathcal{U}_2 \in \mathbb{U}_2$$

$$(\mathcal{U}_1^*, \mathcal{U}_2^*) \in \mathbb{U}_1 \times \mathbb{U}_2 \longrightarrow (\check{\mathcal{U}}_1^*, \check{\mathcal{U}}_2^*) \in \mathbb{U}_1 \times \mathbb{U}_2$$

$$\mathcal{J}(\mathcal{U}_1^*, \mathcal{U}_2^*) \geq \mathcal{J}(\check{\mathcal{U}}_1^*, \check{\mathcal{U}}_2^*)$$

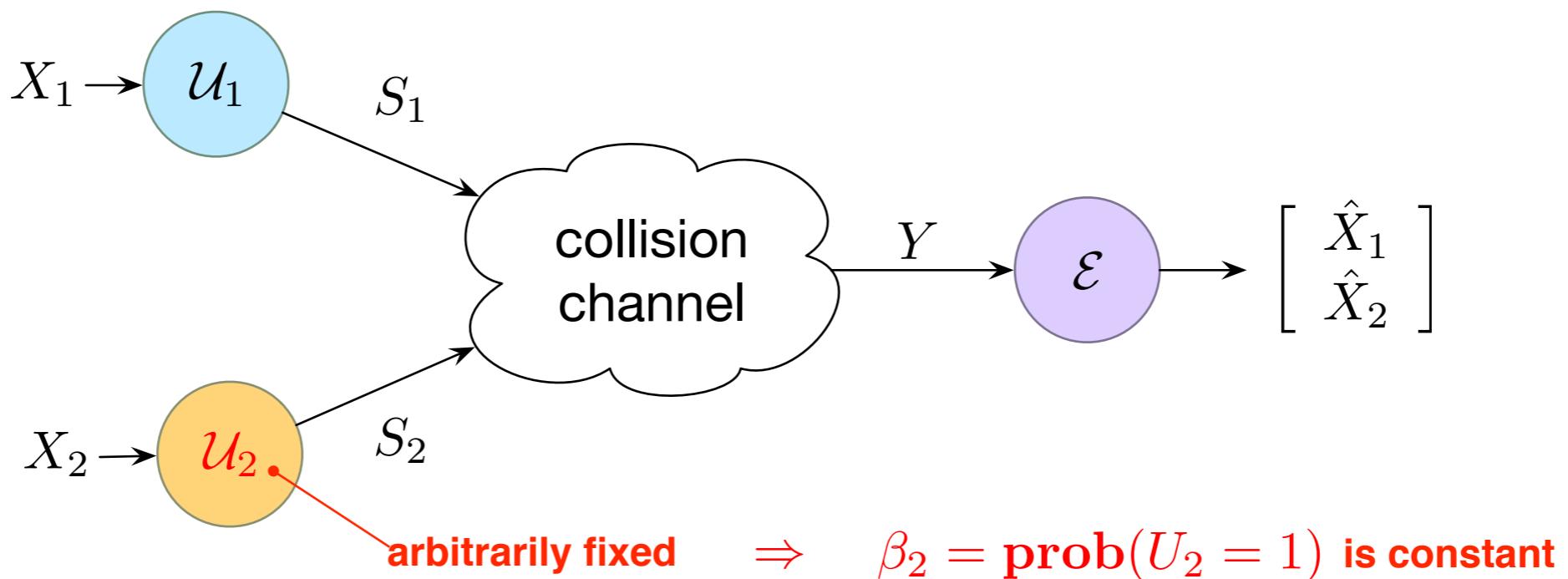
threshold
policies

Given any pair of person-by-person optimal policies

construct a new pair with **equal or better cost**,
where each policy is **threshold**

1. Yuksel & Basar, *Stochastic networked control systems*, Birkhauser 2013.
2. Mahajan et al, "Information structures in optimal decentralized control," CDC 2012.

Remote estimation with communication costs



Original cost:

$$\mathcal{J}(\mathcal{U}_1, \mathcal{U}_2) = \mathbf{E} \left[(X_1 - \hat{X}_1)^2 + (X_2 - \hat{X}_2)^2 \right]$$

Cost from the perspective of DM₁:

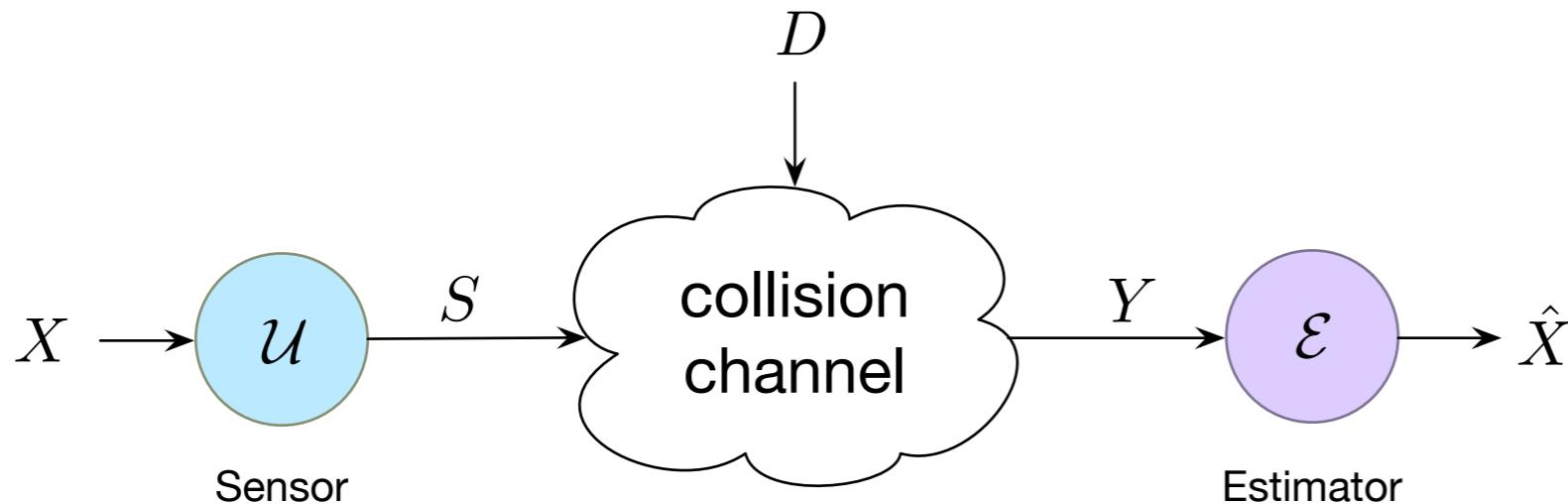
$$\mathcal{J}_1(\mathcal{U}_1) = \mathbf{E} \left[(X_1 - \hat{X}_1)^2 \right] + \rho_2 \cdot \text{prob}(U_1 = 1) + \theta_2$$

do not depend on \mathcal{U}_1

Communication cost:

$$\rho_2 = \mathbf{E} \left[(X_2 - \hat{X}_2)^2 | U_1 = 1 \right] - \mathbf{E} \left[(X_2 - \hat{X}_2)^2 | U_1 = 0 \right] \geq 0$$

Single DM subproblem



$$D \sim \mathcal{B}(\beta)$$

Determines if the channel
is occupied or not

$$X \perp\!\!\!\perp D$$

Problem 2

$$\text{minimize } \mathcal{J}(\mathcal{U}) = \mathbf{E}[(X - \hat{X})^2] + \rho \cdot \mathbf{prob}(U = 1)$$

$$\mathbf{prob}(U = 1 | X = x) = \mathcal{U}(x) \quad \mathbb{U} = \{\mathcal{U} \mid \mathcal{U} : \mathbb{R} \rightarrow [0, 1]\}$$

Lemma

There exists an optimal **threshold policy** for Problem 2.

Sketch of Proof

1. Express the cost as

$$\mathcal{J}(\mathcal{U}) = \mathbf{E}\left[\beta(X - \hat{x}_{\mathfrak{C}})^2 + \rho \mid U = 1\right] \cdot \text{prob}(U = 1) + \mathbf{E}\left[(X - \hat{x}_{\emptyset})^2 \mid U = 0\right] \cdot \text{prob}(U = 0)$$

$\hat{x}_{\mathfrak{C}} = \mathbf{E}[X \mid U = 1]$ $\hat{x}_{\emptyset} = \mathbf{E}[X \mid U = 0]$

2. After **introducing two linear constraints** and a **change of variables**, we have:

$$\begin{aligned} \text{prob}(U = 1) &= \alpha \\ \mathbf{E}[X \mid U = 0] &= \gamma \end{aligned}$$

$$\mathcal{G}(x) = \frac{1 - \mathcal{U}(x)}{1 - \alpha}$$

$$\begin{aligned} \underset{\mathcal{G} \in L^2_\mu(\mathbb{R})}{\text{minimize}} \quad & \mathbf{E}[X^2 \mathcal{G}(X)] \\ \text{subject to} \quad & \mathbf{E}[X \mathcal{G}(X)] = \gamma \\ & \mathbf{E}[\mathcal{G}(X)] = 1 \\ & 0 \leq \mathcal{G}(x) \leq \frac{1}{1 - \alpha} \end{aligned}$$

**moment optimization problem
with variable bounds
(convex)**

1. Akhiezer, *The Classical Moment Problem*, 1965.
2. Byrnes & Lindquist, "A convex optimization approach to generalized moment problems," Springer 2003.

Sketch of Proof

3. The Lagrange dual function is

$$\mathcal{C}^*(\nu) = -\nu_1 - \nu_0\gamma - \frac{1}{1-\alpha} \mathbf{E} [(X^2 + \nu_0 X + \nu_1)^-]$$

strong duality holds^{1,2}

4. The solution to the primal problem is

$$\mathcal{G}_{\nu^*}(x) = \begin{cases} \frac{1}{1-\alpha} & \text{if } x^2 + \nu_0^* x + \nu_1^* \leq 0 \\ 0 & \text{otherwise} \end{cases}$$

5. In the original optimization variable:

$$\mathcal{U}_{\nu^*}(x) = \begin{cases} 0 & \text{if } x^2 + \nu_0^* x + \nu_1^* \leq 0 \\ 1 & \text{otherwise} \end{cases} \implies$$

$$\mathcal{U}^*(x) = \begin{cases} 0 & \text{if } a \leq x \leq b \\ 1 & \text{otherwise} \end{cases}$$

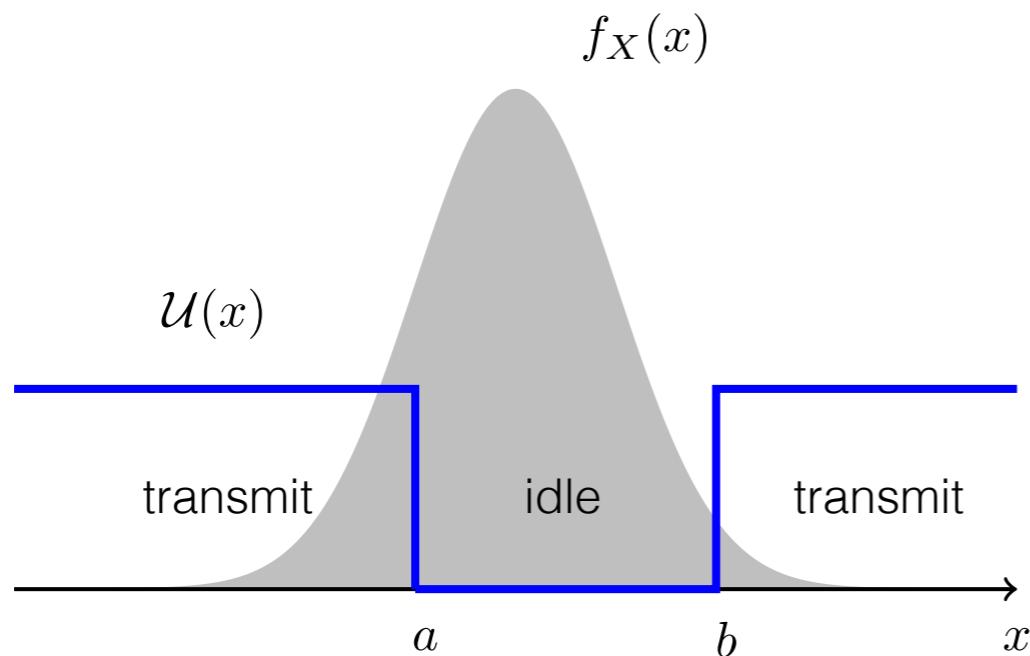


1. Borwein & Lewis, "Partially finite convex programming, Part I: Quasi relative interiors and duality theory," Math. Prog. 1992.
2. Limber & Goodrich, "Quasi interiors, Lagrange multipliers, and Lp spectral estimation with lattice bounds," JOTA 1993.

Remarks

1. Valid for **any continuous probability distribution**
2. **Vector observations** and **any number of sensors**

Assumption:
Finite 1st and 2nd
moments
(req. for strong duality)

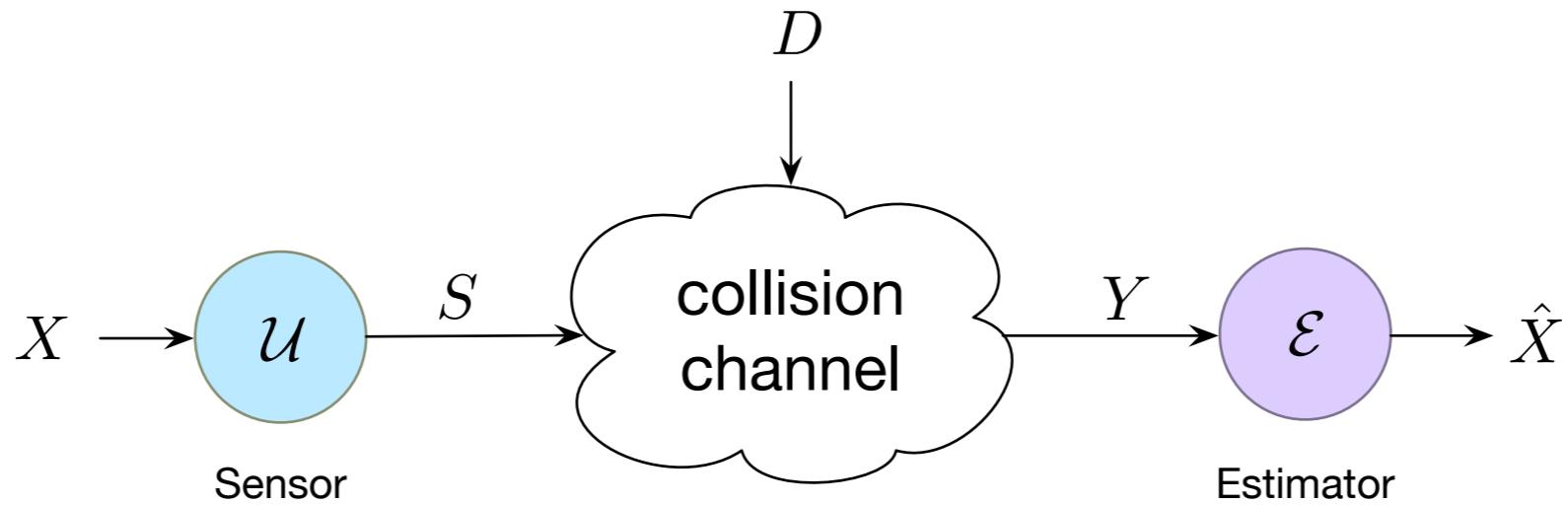


Additional assumption:

The fusion center can decode the indices of all sensors involved in a collision

How do we compute the optimal thresholds?

Part 2. Computing optimal thresholds



$$\mathcal{U}(x) = \begin{cases} 0 & a \leq x \leq b \\ 1 & \text{otherwise} \end{cases}$$

$$\mathcal{E}(y) = \begin{cases} x & y = x \\ \hat{x}_\emptyset & y = \emptyset \\ \hat{x}_{\mathfrak{C}} & y = \mathfrak{C} \end{cases}$$

$$\mathcal{J}(a, b, \hat{x}_\emptyset, \hat{x}_{\mathfrak{C}}) = \int_{[a, b]} (x - \hat{x}_\emptyset)^2 f_X(x) dx + \int_{\bar{\mathbb{R}} \setminus [a, b]} [\beta(x - \hat{x}_{\mathfrak{C}})^2 + \rho] f_X(x) dx$$

binary quantization with **asymmetric** distortion

1. Lloyd, "Least squares quantization in PCM", *IEEE IT* 1982.
2. Fleischer, "Sufficient conditions for achieving minimum distortion in a quantizer", *IEEE Int. Conv. Rec.*, 1964.

Binary quantization with asymmetric distortion

minimize $\mathcal{J}(a, b, \hat{x}_\emptyset, \hat{x}_{\mathfrak{C}})$
subject to $a \leq b$

$$x \in [a^*, b^*] \Leftrightarrow (x - \hat{x}_\emptyset)^2 \leq \beta(x - \hat{x}_{\mathfrak{C}})^2 + \rho$$

necessary optimality condition

Let $\hat{x} = (\hat{x}_\emptyset, \hat{x}_{\mathfrak{C}})$ is the pair representation points for no-transmission and collision symbols

$$a(\hat{x}), b(\hat{x}) = \frac{1}{1-\beta} \left[(\hat{x}_\emptyset - \beta \hat{x}_{\mathfrak{C}}) \pm \sqrt{\beta(\hat{x}_\emptyset - \hat{x}_{\mathfrak{C}})^2 + (1-\beta)\rho} \right]$$

Define a new cost: $\mathcal{J}_q(\hat{x}) = \mathcal{J}(a(\hat{x}), b(\hat{x}), \hat{x}_\emptyset, \hat{x}_{\mathfrak{C}})$

minimize $\mathcal{J}_q(\hat{x})$

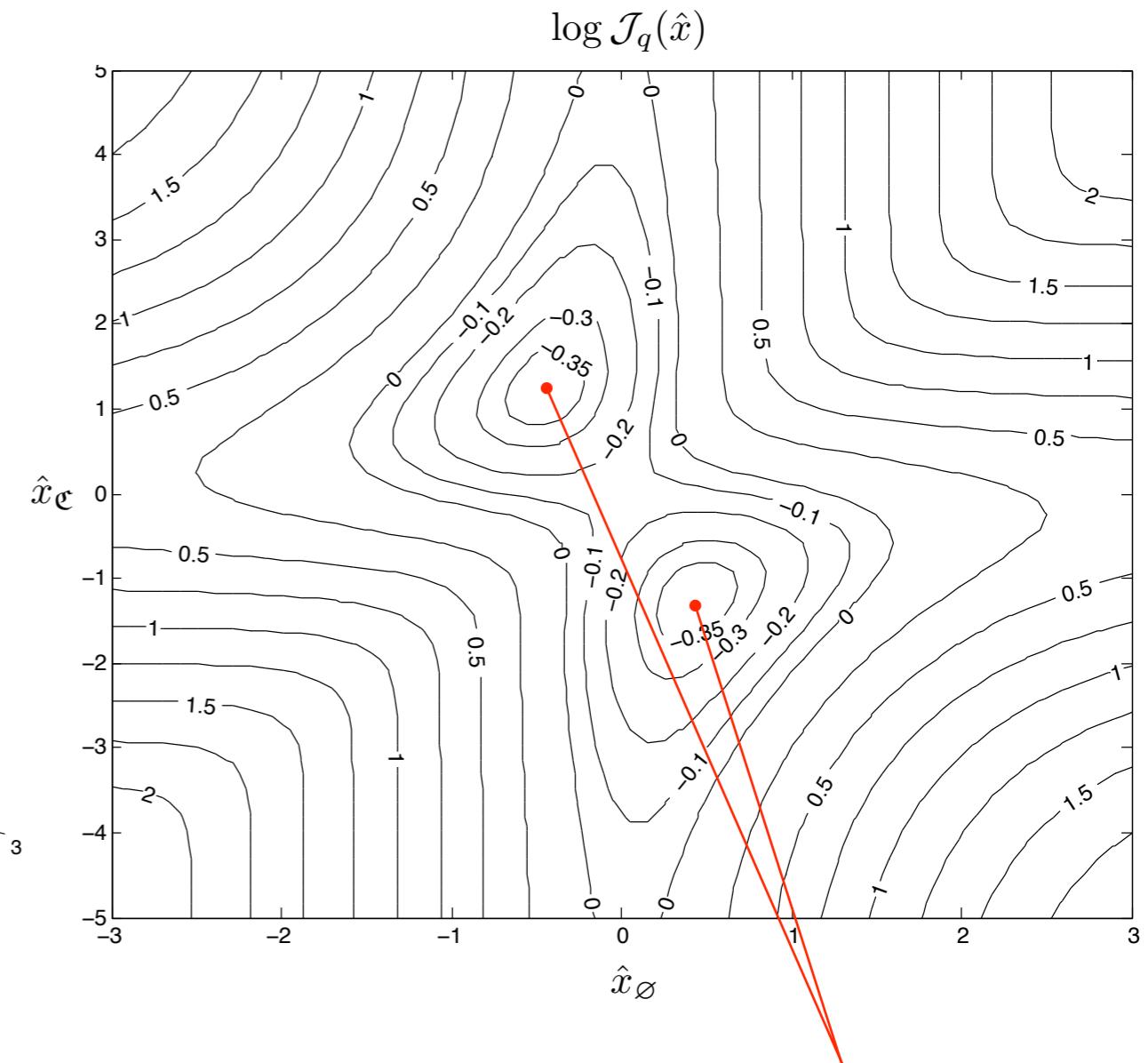
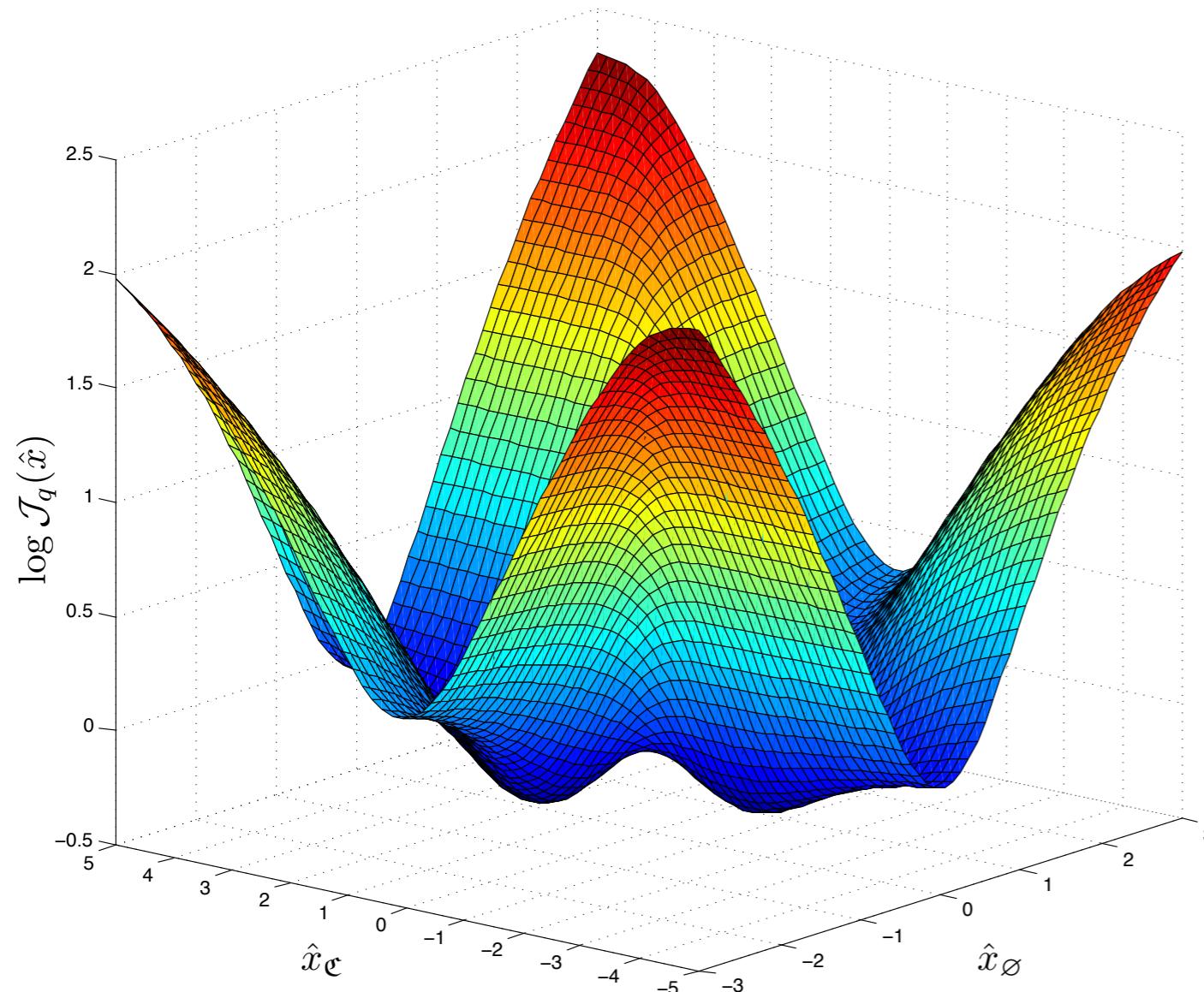
unconstrained problem

Quantizer distortion function

$$X \sim \mathcal{N}(0, 1)$$

$$\beta = 0.5$$

$$\rho = 1$$



non-unique minima

Neither convex nor quasi-convex

Continuously differentiable

Modified Lloyd-Max Algorithm

$$\underset{\hat{x} \in \mathbb{R}^2}{\text{minimize}} \quad \mathcal{J}_q(\hat{x})$$

$$\nabla \mathcal{J}_q(\hat{x}) = 0 \quad \longleftrightarrow \quad \hat{x} = \mathcal{F}(\hat{x})$$

Lloyd's Map

$$\mathcal{F}(\hat{x}) = \begin{bmatrix} \mathbf{E}\left[X | X \in [a(\hat{x}), b(\hat{x})]\right] \\ \mathbf{E}\left[X | X \notin [a(\hat{x}), b(\hat{x})]\right] \end{bmatrix}$$

Modified Lloyd-Max

$$\hat{x}^{(0)} \neq (0, 0)$$

$$\hat{x}^{(k+1)} = \mathcal{F}(\hat{x}^{(k)}), \quad k = 0, 1, \dots$$

Step 1 From $\hat{x}^{(k)}$ update the thresholds $a(\hat{x}^{(k)})$ and $b(\hat{x}^{(k)})$

Step 2 Compute the centroids of the new quantization regions

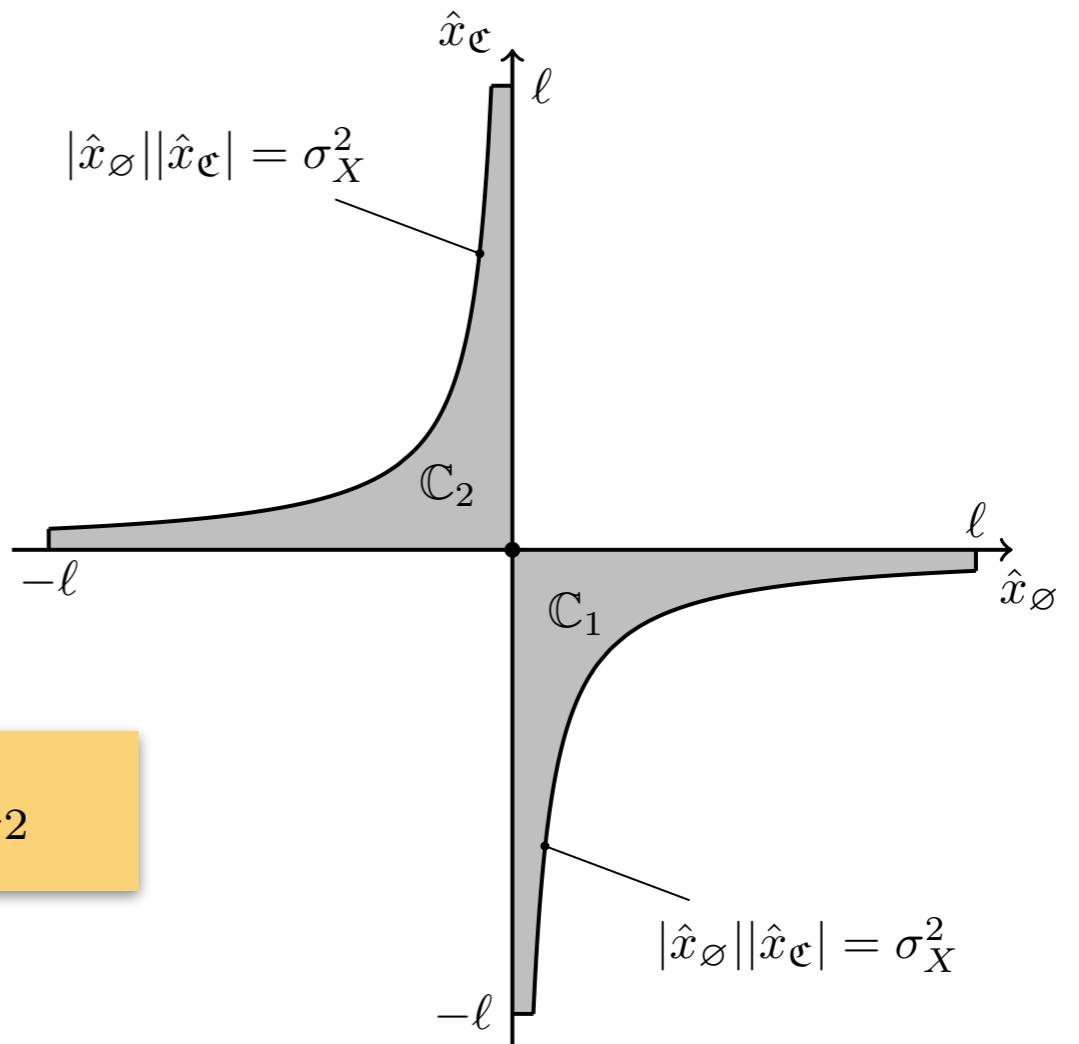
Convergence

Theorem:

For $X \sim \mathcal{N}(0, \sigma_X^2)$ the Modified Lloyd-Max algorithm is **globally convergent** to a local minimum of $\mathcal{J}_q(\hat{x})$.

Sketch of Proof^{1,2}:

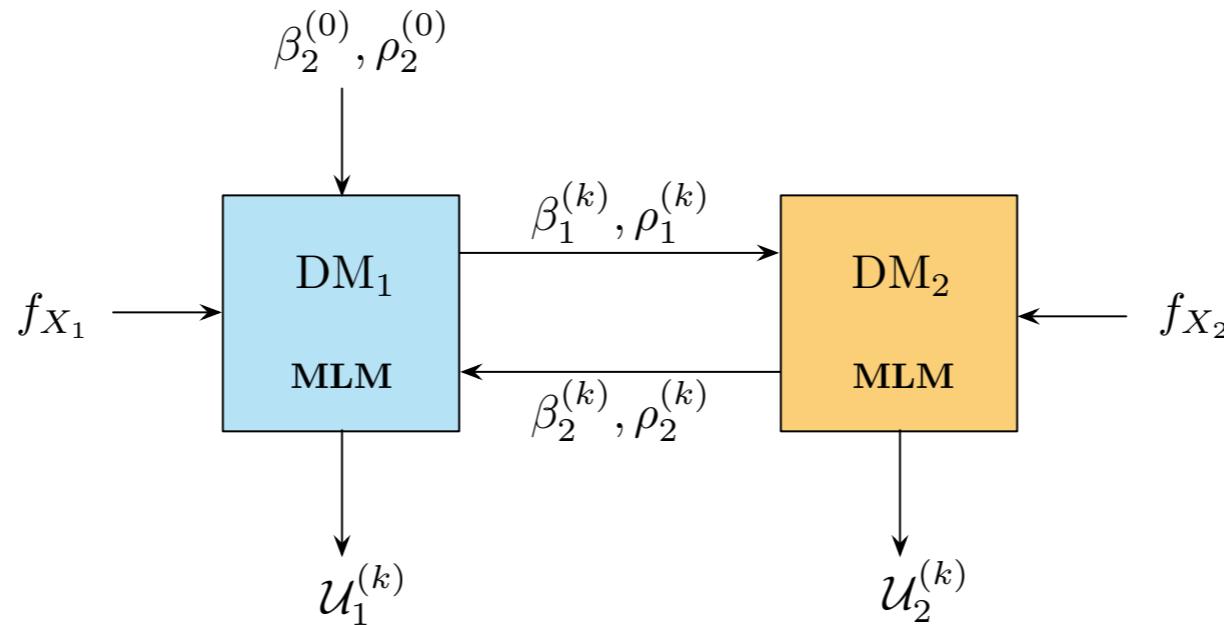
- Find a compact set \mathbb{C} that contains all the critical points of $\mathcal{J}_q(\hat{x})$ such that $\mathcal{F}(\mathbb{C}) \subset \mathbb{C}$



$$\mathbb{C} = \mathbb{C}_1 \cup \mathbb{C}_2$$

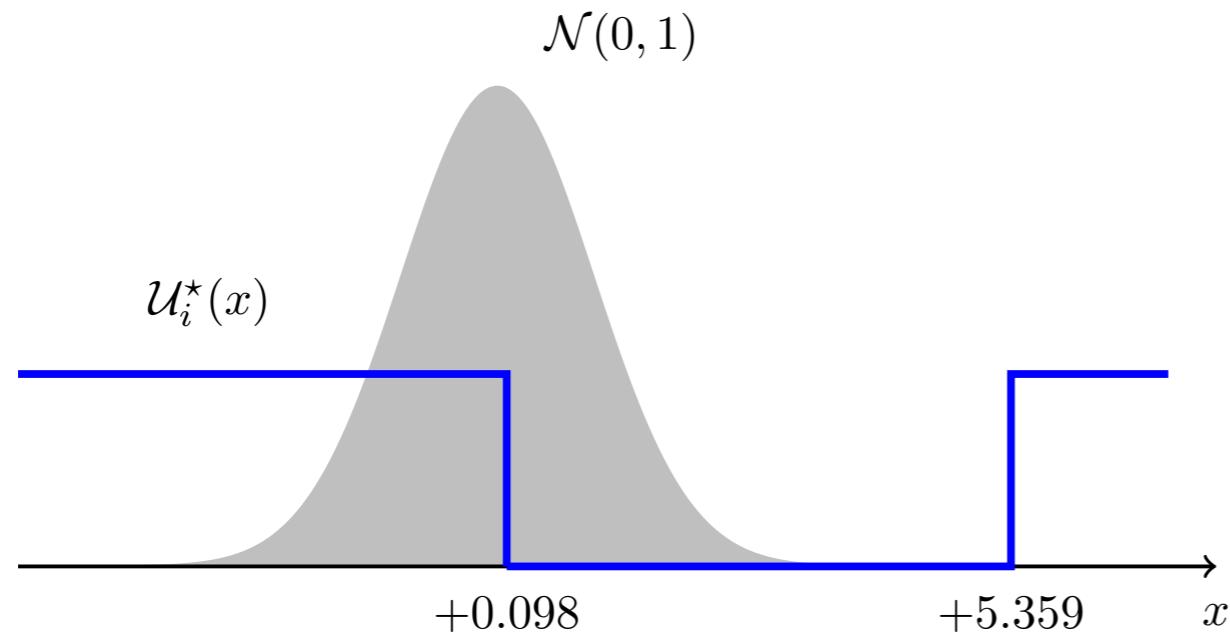
1. Du et. al., “Conv. of the Lloyd algorithm for computing Voronoi tessellations,” *SIAM Num. Analysis* 2006.
2. Vasconcelos & Martins, “Optimal thresholds for remote estimation over the collision channel,” *CDC* 2015.

Numerical procedure



Repeat until the cost cannot be further reduced

Example $X_1, X_2 \sim \mathcal{N}(0, 1)$



i.i.d. observations, symmetric pdf

asymmetric thresholds

$$\mathcal{J}(U_1^*, U_2^*) = 0.54$$

Gain of 46% over
scheduling policies

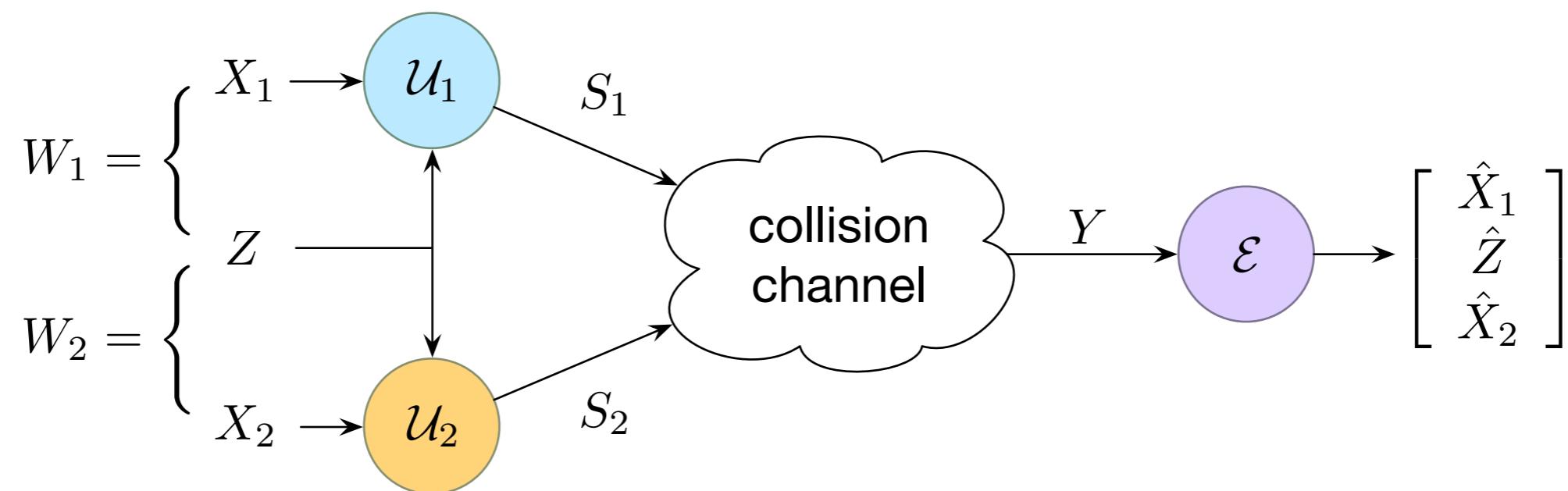
Collision channel with common and private observations

$$W = \begin{bmatrix} X_1 \\ Z \\ X_2 \end{bmatrix} \quad W_i = \begin{bmatrix} X_i \\ Z \end{bmatrix}$$

private observation
common observation

$$f_W = f_Z \cdot f_{X_1|Z} \cdot f_{X_2|Z}$$

$$X_1 \leftrightarrow Z \leftrightarrow X_2$$



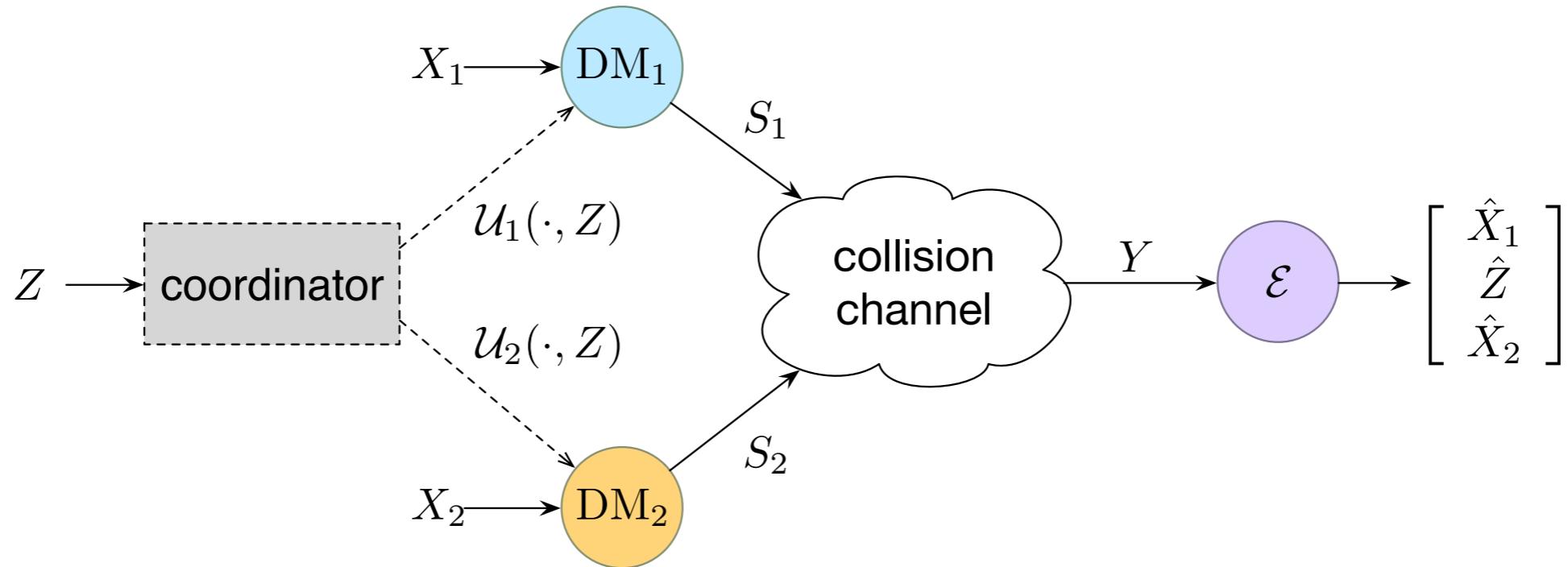
Problem 3

minimize

$$\mathcal{J}(\mathcal{U}_1, \mathcal{U}_2) = \mathbf{E} \left[(X_1 - \hat{X}_1)^2 + (Z - \hat{Z})^2 + (X_2 - \hat{X}_2)^2 \right]$$

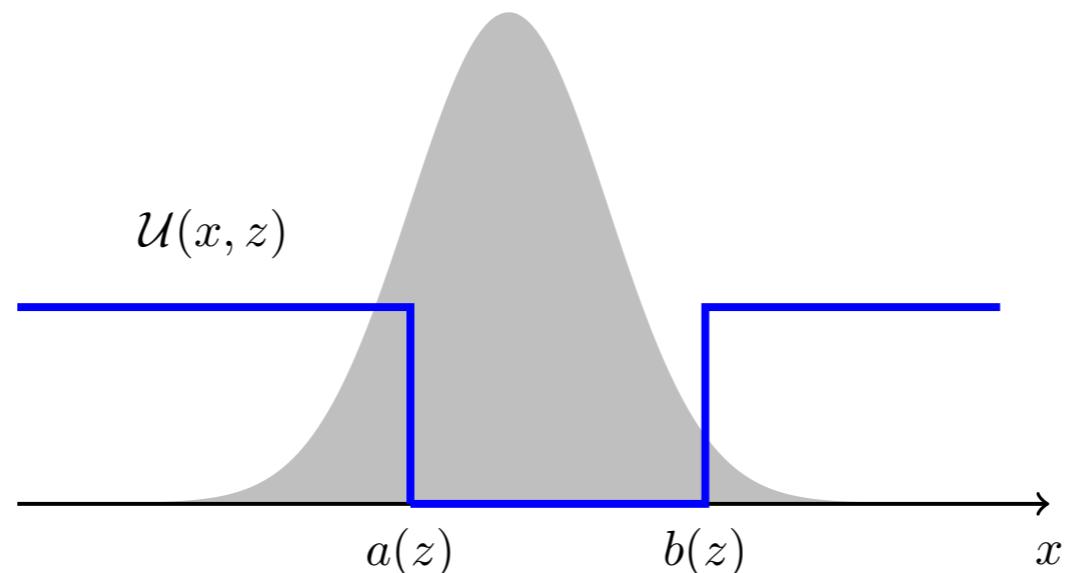
Common information approach

Common information¹ can be used to **simplify** and **characterize** optimal solutions of team problems.



$$\text{minimize} \quad \mathcal{J}^z(\mathcal{U}_1, \mathcal{U}_2) = \mathbf{E} \left[(W - \hat{W})^T (W - \hat{W}) \mid Z = z \right]$$

Threshold policy on private information



Threshold policy on private information

$$U(x, z) = \begin{cases} 0 & a(z) \leq x \leq b(z) \\ 1 & \text{otherwise} \end{cases}$$

Theorem:

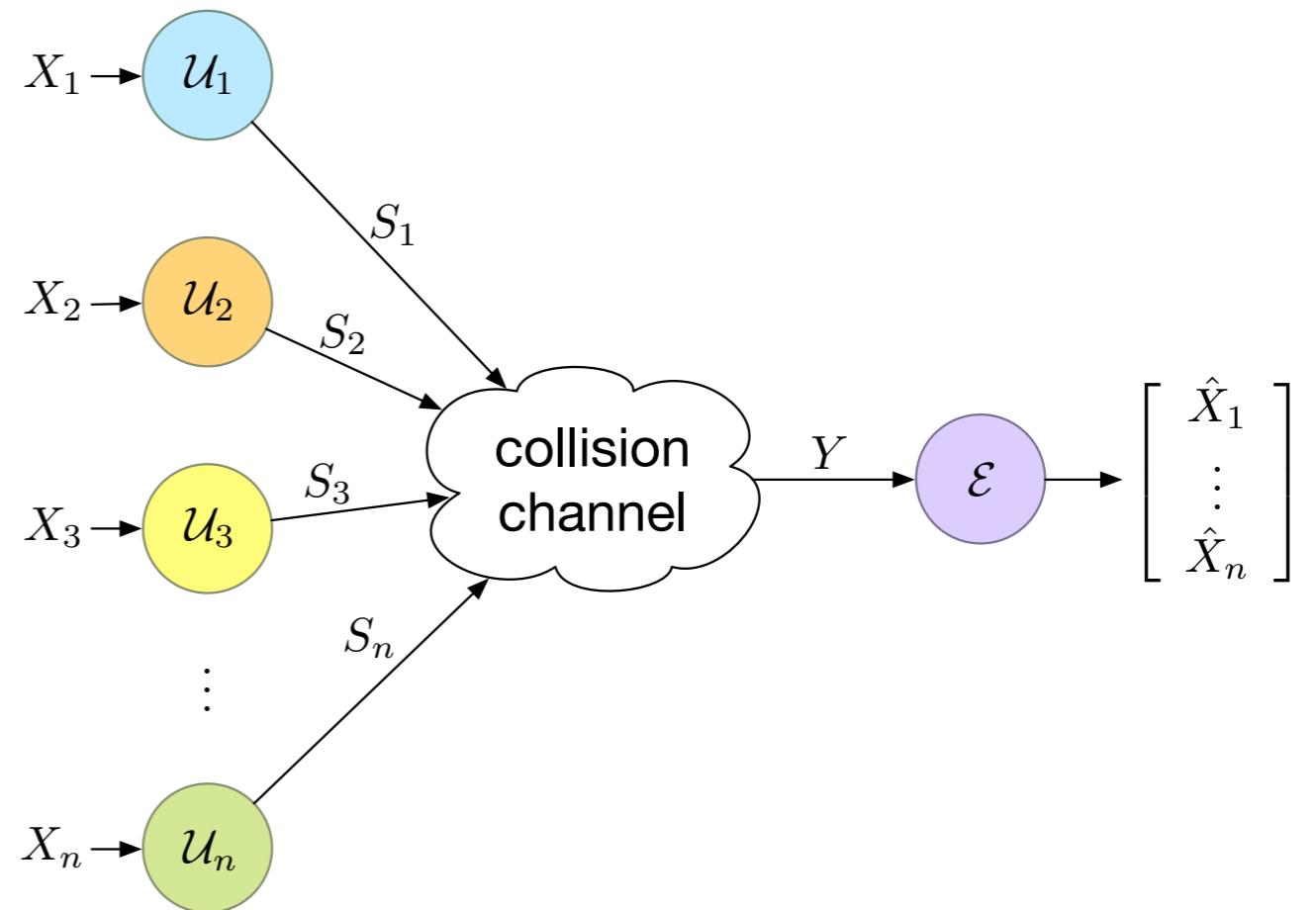
There exists a team-optimal pair of **threshold policies on private information** for Problem 3.

Part 3. MAP estimation over the collision channel

$$W = [X_1, \dots, X_n]$$

$$X_i, \quad i \in \{1, \dots, n\}$$

- mutually independent
- **discrete** rvs
- supported on \mathbb{X}_i
- arbitrarily distributed



Probability of error

$$\text{minimize } \mathcal{J}_B(\mathcal{U}_1, \dots, \mathcal{U}_n) = \text{prob}(W \neq \hat{W})$$

MAP estimator

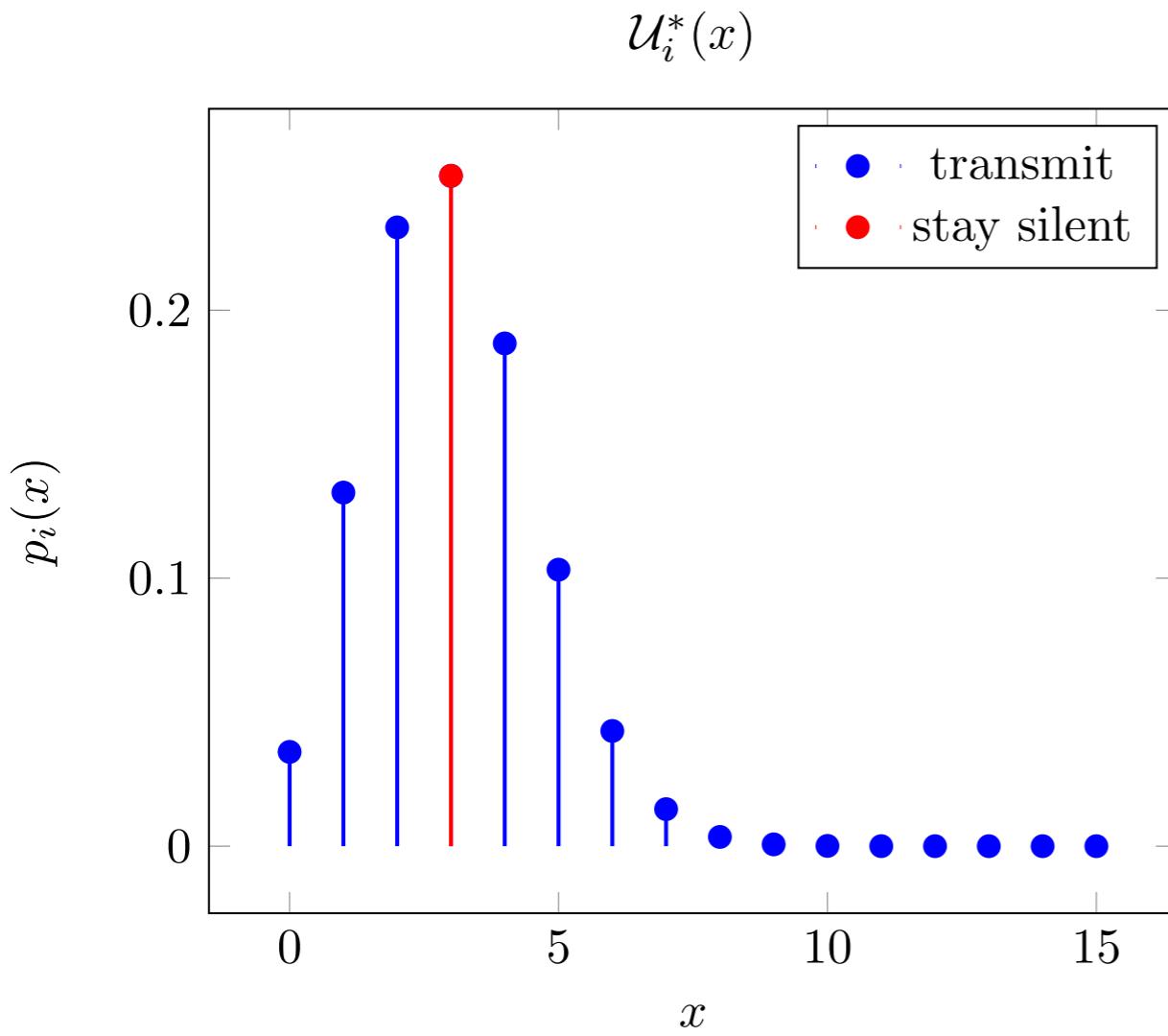
$$\mathcal{E}(y) = \arg \max_{w \in \mathbb{W}} \text{prob}(W = w | Y = y)$$

$$\mathbb{W} = \mathbb{X}_1 \times \cdots \times \mathbb{X}_n$$

Structural result

Theorem

There exists a team-optimal strategy where each sensor transmits **all but the most likely** observation.



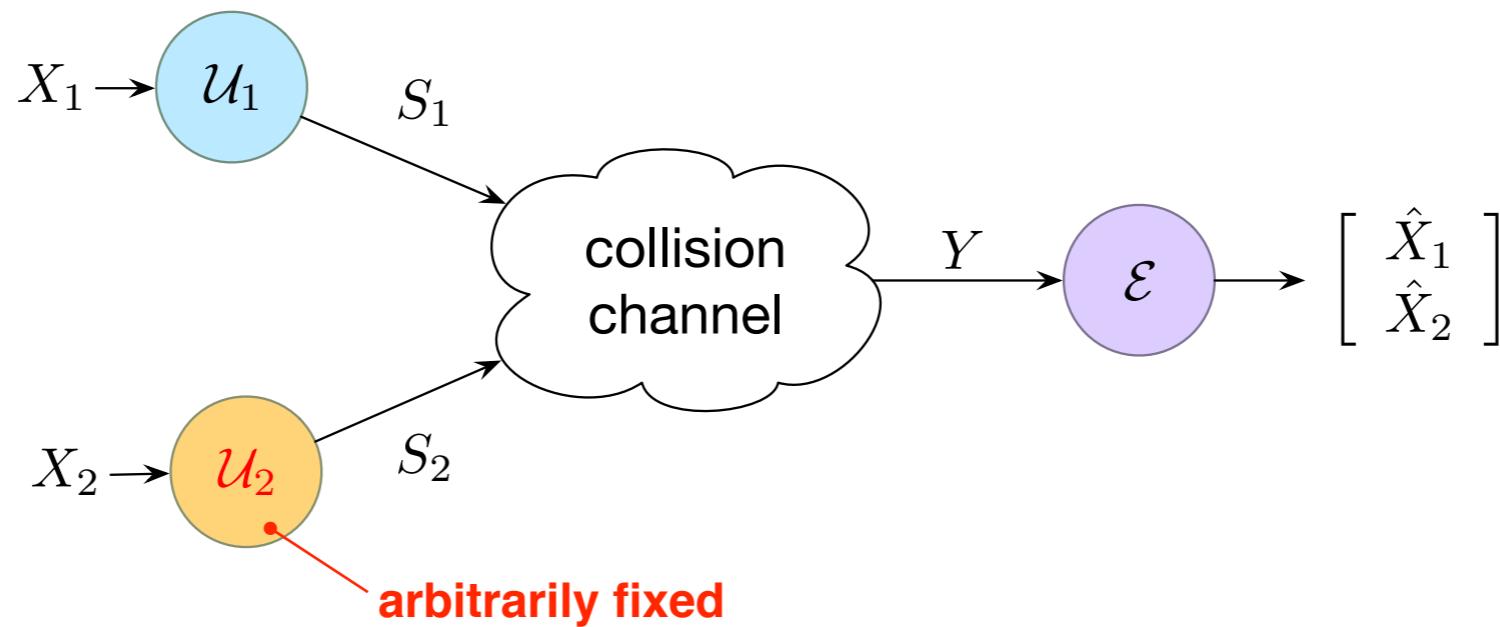
$$\mathcal{U}^* = (\mathcal{U}_1^*, \dots, \mathcal{U}_n^*)$$

$$\mathcal{U}_i^*(x) = \begin{cases} 0 & \text{if } x = x_{i,[1]} \\ 1 & \text{otherwise.} \end{cases}$$

$$[\mathbb{X}_i] = \{x_{i,[1]}, x_{i,[2]}, x_{i,[3]}, \dots\}$$

$$p_i(x_{i,[1]}) \geq p_i(x_{i,[2]}) \geq p_i(x_{i,[3]}) \geq \dots$$

Sketch of proof



Original cost: $\mathcal{J}_B(\mathcal{U}_1, \mathcal{U}_2) = \text{prob}(\{X_1 \neq \hat{X}_1\} \cup \{X_2 \neq \hat{X}_2\})$

$$\mathcal{J}_B(\mathcal{U}_1, \mathcal{U}_2) = 1 - \tau_2 \max_{x \in \mathbb{X}_1} \mathcal{U}_1(x)p_1(x) - \varrho_2 \sum_{x \in \mathbb{X}_1} \mathcal{U}_1(x)p_1(x) - (\beta_2 + \varrho_2) \max_{x \in \mathbb{X}_1} (1 - \mathcal{U}_1(x)) p_1(x)$$

$$\tau_2 = \max_{x \in \mathbb{X}_2} \mathcal{U}_2(x)p_2(x)$$

where: $\varrho_2 = \max_{x \in \mathbb{X}_2} (1 - \mathcal{U}_2(x)) p_2(x)$

$$\beta_2 = \sum_{x \in \mathbb{X}_2} \mathcal{U}_2(x)p_2(x)$$

$$\beta_2, \varrho_2, \tau_2 \geq 0$$

$$\beta_2 \geq \tau_2$$

important!

Single DM subproblem

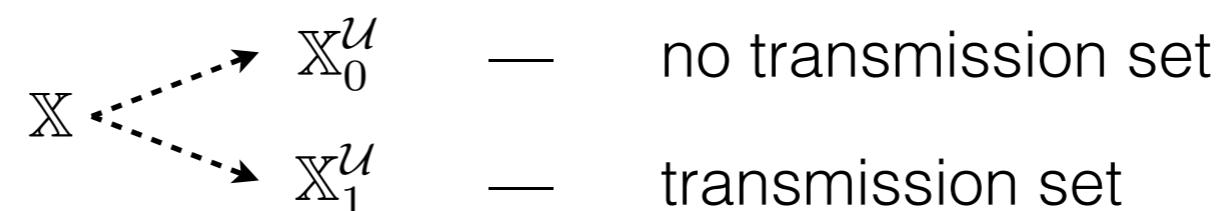
For $\beta, \varrho, \tau \in [0, 1]$ such that $\beta \geq \tau$:

$$\begin{aligned} & \text{minimize} && \tilde{\mathcal{J}}_B(\mathcal{U}) \\ & \text{subject to} && 0 \leq \mathcal{U}(x) \leq 1, \quad x \in \mathbb{X} \end{aligned}$$

$$\tilde{\mathcal{J}}_B(\mathcal{U}) \stackrel{\text{def}}{=} 1 - \tau \max_{x \in \mathbb{X}} \mathcal{U}(x)p(x) - \varrho \sum_{x \in \mathbb{X}} \mathcal{U}(x)p(x) - (\varrho + \beta) \max_{x \in \mathbb{X}} (1 - \mathcal{U}(x)) p(x)$$

concave function

1. Constrain to **deterministic policies**



Sketch of Proof

The cost becomes:

$$\tilde{\mathcal{J}}_B(\mathcal{U}) = 1 - \tau \max_{x \in \mathbb{X}_1^{\mathcal{U}}} p(x) - \varrho \sum_{x \in \mathbb{X}_1^{\mathcal{U}}} p(x) - (\varrho + \beta) \max_{x \in \mathbb{X}_0^{\mathcal{U}}} p(x)$$

1. “Converse” part:

$$\sum_{x \in \mathbb{X}_1^{\mathcal{U}}} p(x) \leq 1 - \max_{x \in \mathbb{X}_0^{\mathcal{U}}} p(x) \implies \tilde{\mathcal{J}}_B(\mathcal{U}) \geq 1 - \varrho - \tau \max_{x \in \mathbb{X}_1^{\mathcal{U}}} p(x) - \beta \max_{x \in \mathbb{X}_0^{\mathcal{U}}} p(x)$$

$$\beta \geq \tau$$

$$\tilde{\mathcal{J}}_B(\mathcal{U}) \geq 1 - \varrho - \tau p_{[2]} - \beta p_{[1]}$$

2. “Achievability” part:

$$\mathcal{U}^*(x) = \begin{cases} 0 & \text{if } x = x_{[1]} \\ 1 & \text{otherwise} \end{cases}$$

$$\tilde{\mathcal{J}}_B(\mathcal{U}^*) = 1 - \varrho - \tau p_{[2]} - \beta p_{[1]}$$



Example: the i.i.d. case

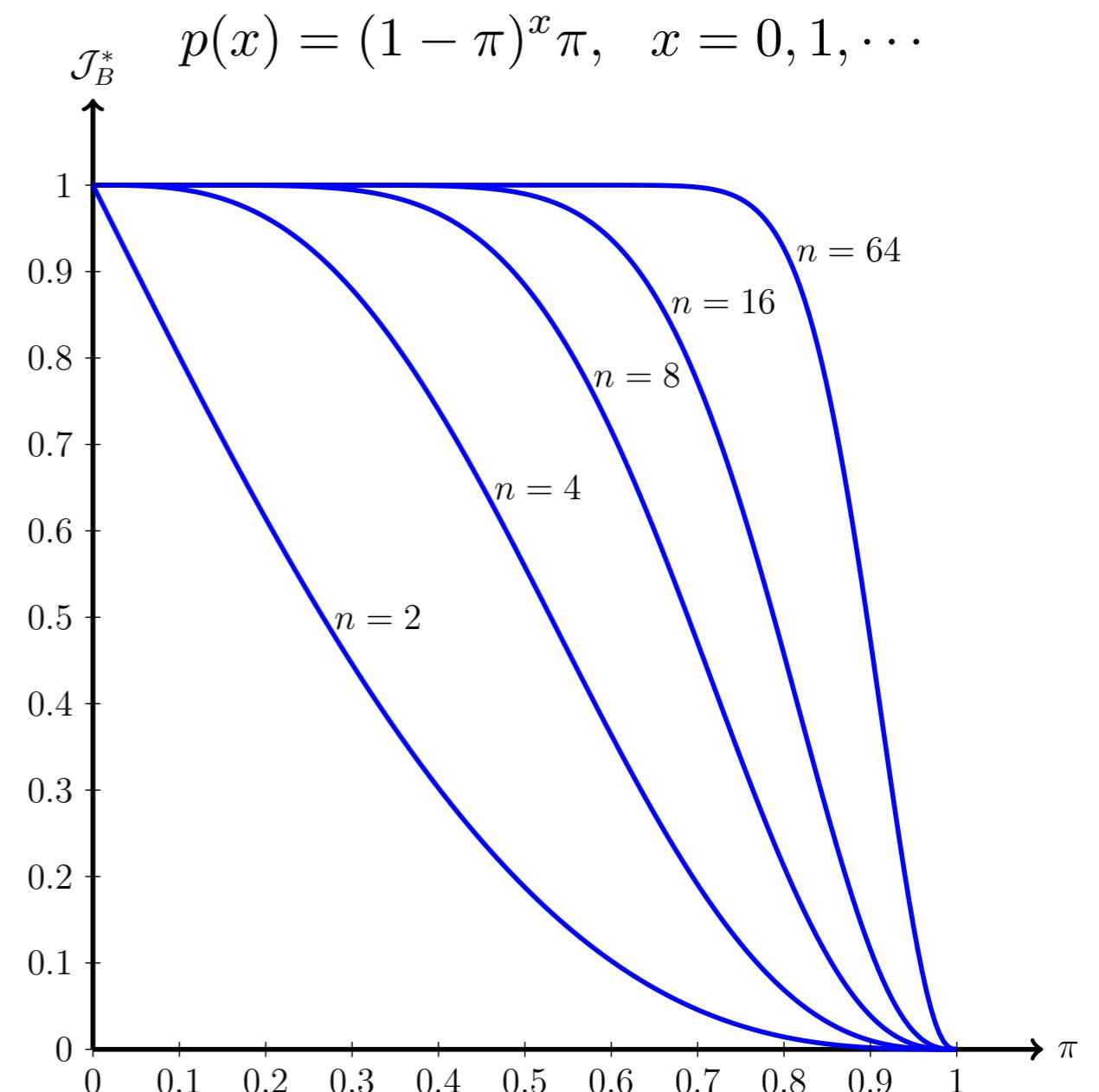
$$X_i \sim p(x), \quad x \in \mathbb{X}, \quad i \in \{1, \dots, n\}$$

$$\begin{aligned} \mathcal{J}_B(\mathcal{U}^*) &= 1 - np_{[1]}^{n-1}(1 - p_{[1]} - p_{[2]}) \\ &\quad - (p_{[1]} + p_{[2]})^n \end{aligned}$$

Two observations:

$$\mathcal{J}_B^{\text{bin}}(\mathcal{U}^*) = 0$$

$$\lim_{n \rightarrow \infty} \mathcal{J}_B(\mathcal{U}^*) = 1$$



Summary

1. **New class of problems** in remote estimation
2. Structural results for the MSE & Prob. of Error cases
3. **Optimality of event-based policies**
4. Algorithm to compute optimal thresholds & its convergence
5. Extensions: dependent observations & n sensors

Future work

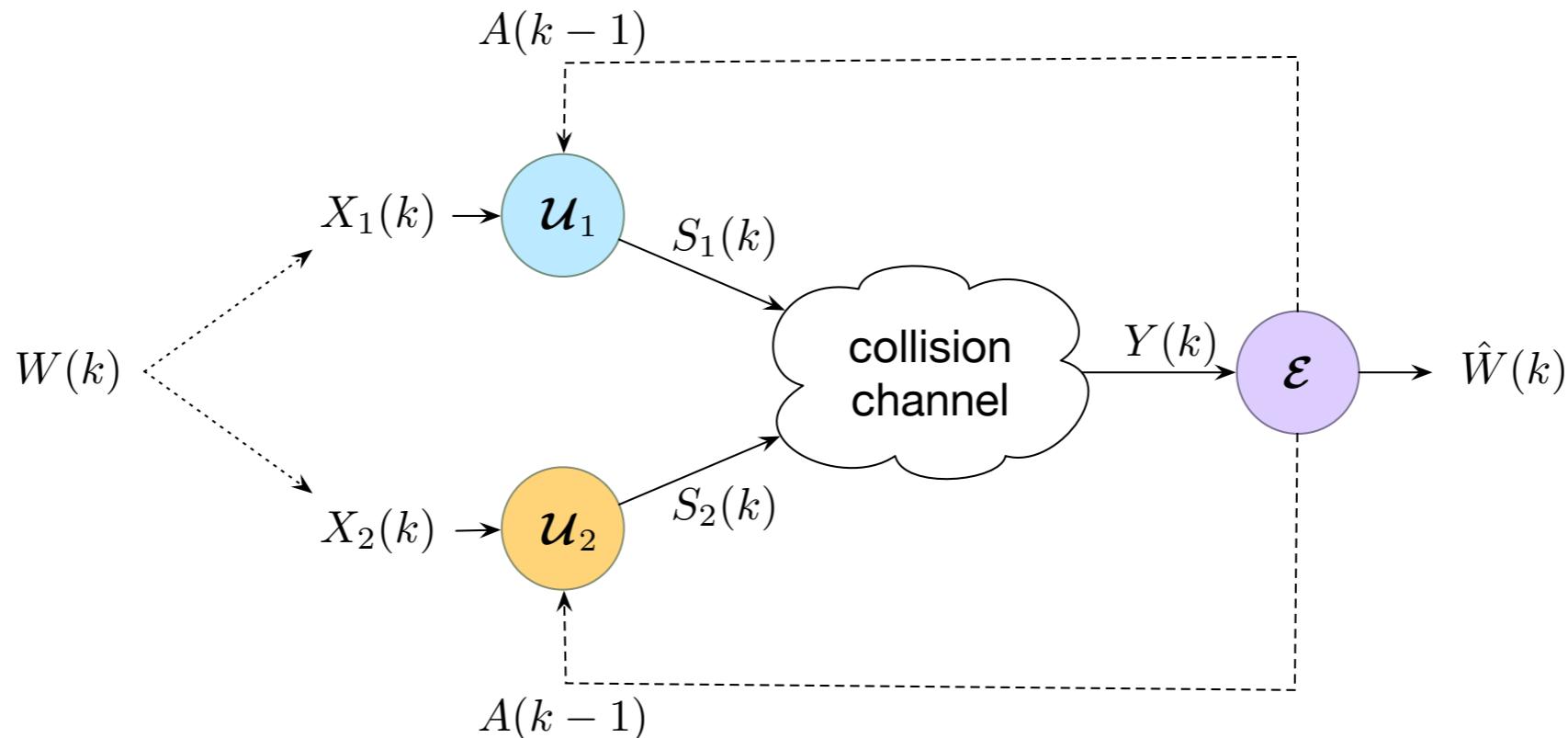
1. Extend to **arbitrarily correlated observations**
2. Different types of scalability results
3. **The sequential case** with acknowledgements
4. Lower and upper bounds on estimation error
5. Applications in biological communications

Publications

- [1] M. M. Vasconcelos and N. C. Martins. Estimation over the collision channel: Structural results. In *Proceedings of the 51st Annual Allerton Conference on Communication, Control and Computing*, pages 1114 – 1119, Monticello, IL, October 2013.
- [2] M. M. Vasconcelos and N. C. Martins. Remote estimation games over shared networks. In *Proceedings of the 52nd Annual Allerton Conference on Communication, Control and Computing*, pages 12 – 18, Monticello, IL, October 2014.
- [3] M. M. Vasconcelos and N. C. Martins. Optimal threshold strategies for remote estimation over the collision channel with communication costs. In *Proceedings of the 54th IEEE Conference on Decision and Control*, pages 1112 – 1119, Osaka, Japan, December 2015.
- [4] M. M. Vasconcelos and N. C. Martins. Estimation of discrete random variables over the collision channel with minimum probability of error. *To be submitted to IEEE Transactions on Automatic Control*, 2016.
- [5] M. M. Vasconcelos and N. C. Martins. *Principles of cyber-physical systems*, chapter A survey on remote estimation problems. Cambridge University Press, 2016.
- [6] M. M. Vasconcelos and N. C. Martins. The structure of optimal communication policies for remote estimation over the collision channel with private and common observations. In *Proceedings of the 55th IEEE Conference on Decision and Control (accepted)*, Las Vegas, NV, December 2016.
- [7] M. M. Vasconcelos and N. C. Martins. Optimal estimation over the collision channel. *Accepted to IEEE Transactions on Automatic Control*, (4), 2017.

Appendix

The sequential case

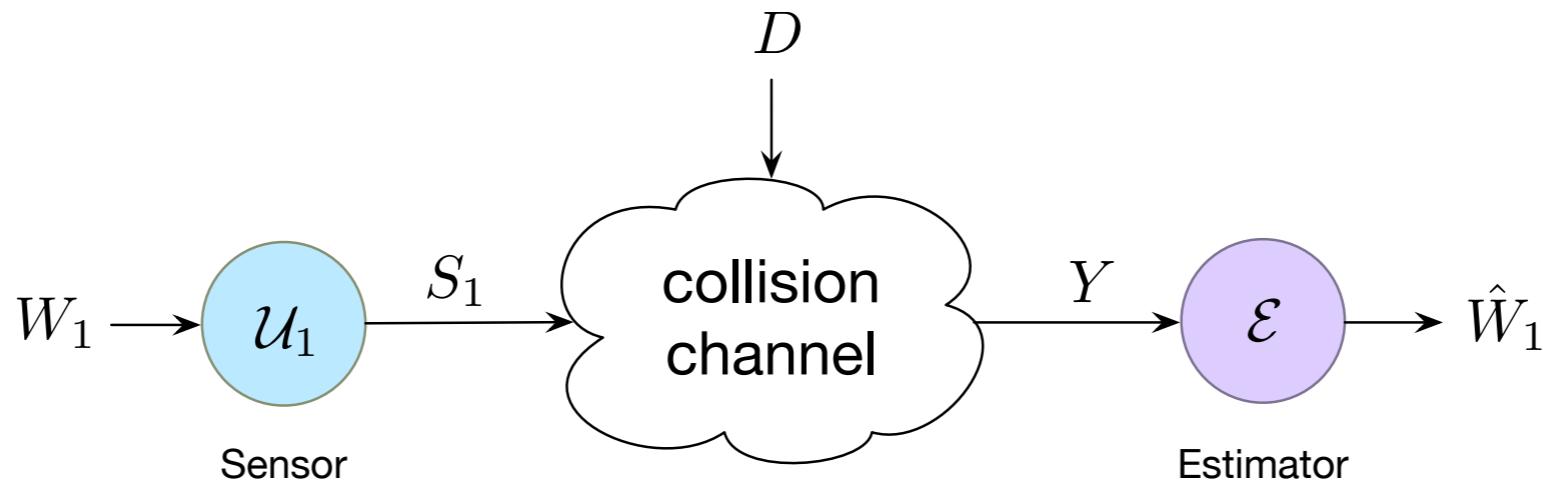


$$\mathcal{J}(\mathcal{U}_1, \mathcal{U}_2, \mathcal{E}) = \sum_{k=0}^T \mathbf{E} \left[d(W(k), \hat{W}(k)) \right]$$

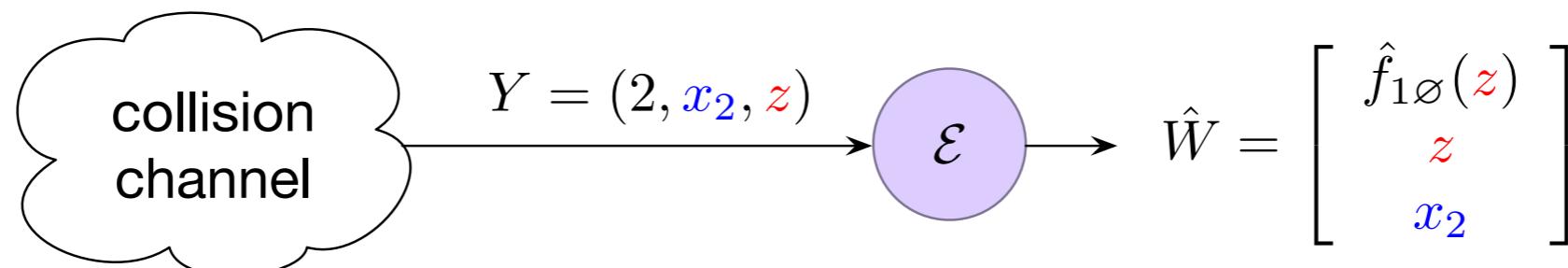
1. Mahajan, "Optimal decentralized control of coupled subsystems with control sharing", *IEEE TAC* 2013.
2. Bobrovsky & Zakai "A lower bound on the estimation error for Markov processes". *IEEE TAC* 1975.
3. Weiss & Weinstein. "A lower bound on the mean-square error in random parameter estimation". *IEEE IT* 1985.

Dependent observations: two difficulties

The event that there is a concurring transmission is
not independent of W_1 .



A successful transmission made by sensor 2
contains side information for the estimation of X_1 .

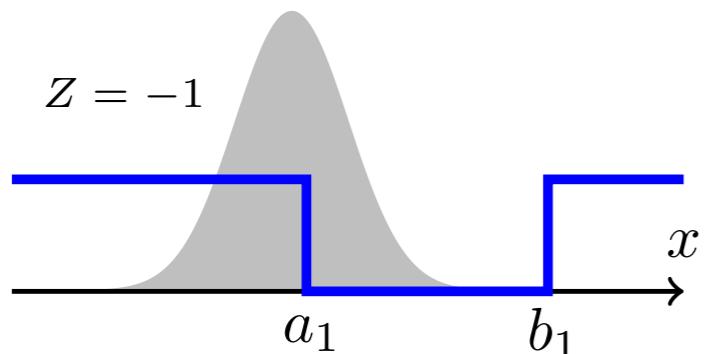


Scheduling vs. event-based policies

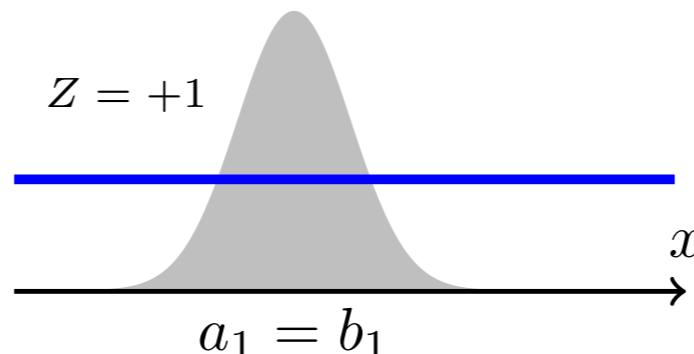
Example

$$X_1, X_2 \sim \mathcal{N}(0, 1)$$

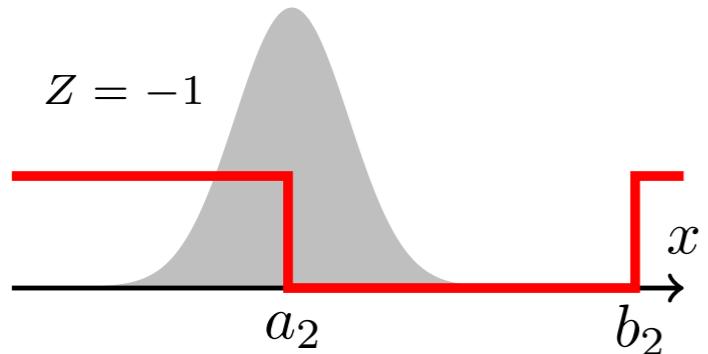
$$Z = \begin{cases} +1 & \text{w.p. } p \\ -1 & \text{w.p. } 1 - p \end{cases}$$



(a) Communication policy \mathcal{U}_1



p	\mathcal{J}^*
0	0.54
0.1	0.59
0.2	0.63
0.3	0.68
0.4	0.73
0.5	0.78



(b) Communication policy \mathcal{U}_2

$$p = 0.5 \implies \mathcal{J}(\mathcal{U}_1^*, \mathcal{U}_2^*) = 0.78$$

Combination of **scheduling** and **event-based** policies.

Gain of 22% over scheduling policies