Module 08:
§3.1–5 and §6.5
With Answers

§3.1–4

1. Write a state-space representation for the system in Figure 1 with the acceleration of the mass on the right as the output.

   Answer:

$$\dot{\bar{x}} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & -1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ v_1 \\ x_2 \\ v_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} f$$

$$y = \begin{bmatrix} 1 & 1 & -1 & -1 \end{bmatrix} \bar{x}$$

2. Write a state-space representation of the two-tank system shown in Figure 2 with the difference between the two tank heights as the output.

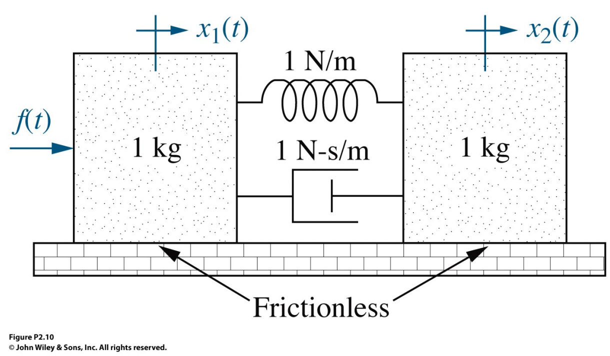
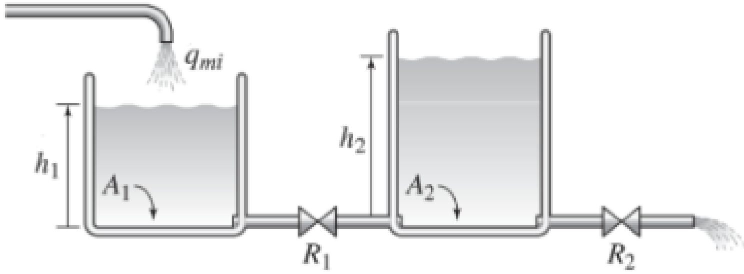


Figure 1: Two-block system



$$\rho A_1 \dot{h}_1 + \frac{\rho g}{R_1}(h_1 - h_2) = q_{mi}$$
$$\rho A_2 \dot{h}_2 - \frac{\rho g}{R_1}(h_1 - h_2) + \frac{\rho g}{R_2} h_2 = 0$$

Figure 2: Two-tank system with governing equations

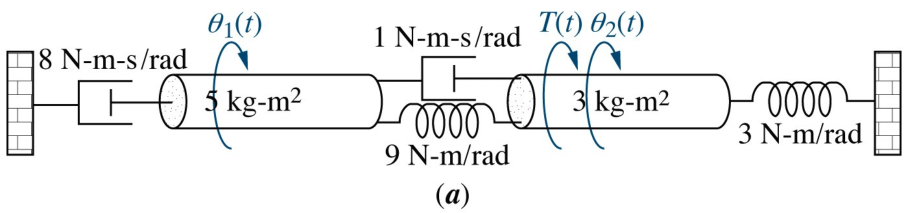


Figure P2.16a

Figure 3: Rotational system

Answer:

$$\begin{aligned}
\dot{\bar{x}} &= A\bar{x} + Bu \\
&= \begin{bmatrix} -\frac{g}{A_1 R_1} & \frac{g}{A_1 R_1} \\ \frac{g}{A_2 R_1} & -\frac{g}{(A_2)(R_1+R_2)} \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} + \begin{bmatrix} \frac{1}{\rho A_1} \\ 0 \end{bmatrix} q_{mi} \\
y &= C\bar{x} \\
&= \begin{bmatrix} 1 & -1 \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix}
\end{aligned}$$

3. Write a state-space representation for the system in Figure 3 with the torque on the wall from the damper as the output.

   Answer:

$$\dot{\bar{x}} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -9/5 & -9/5 & 9/5 & 1/5 \\ 0 & 0 & 0 & 1 \\ 3 & 1/3 & -4 & -1/3 \end{bmatrix} \begin{bmatrix} \theta_1 \\ \omega_1 \\ \theta_2 \\ \omega_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1/3 \end{bmatrix} \tau$$
$$y = \begin{bmatrix} 0 & 8 & 0 & 0 \end{bmatrix} \bar{x}$$

§3.5

1. Find the state-space representation in phase-variable form for

$$G(s) = \frac{100}{s^4 + 20s^3 + 10s^2 + 7s + 100}$$

Answer:

$$\dot{\bar{x}} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -100 & -7 & -10 & -20 \end{bmatrix} \bar{x} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 100 \end{bmatrix} r$$

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \bar{x}$$

2. Find the state-space representation in phase-variable form for

$$G(s) = \frac{30}{s^5 + 8s^4 + 9s^3 + 6s^2 + s + 30}$$

Answer:

$$\dot{\bar{x}} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ -30 & -1 & -6 & -9 & -8 \end{bmatrix} \bar{x} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 30 \end{bmatrix} r$$

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \end{bmatrix} \bar{x}$$

3. Starting with the transfer function $G(s) = \Theta_m/E_a$, write a state-space representation of a DC motor, where e_a is the input and the angular velocity, ω_m, is the output.

Answer:

$$\dot{\bar{x}} = \begin{bmatrix} 0 & 1 \\ 0 & -\left(\frac{1}{J_m}\right)\left(D_m + \frac{K_t K_b}{R_a}\right) \end{bmatrix} \begin{bmatrix} \theta_m \\ \omega_m \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{K_t}{R_a J_m} \end{bmatrix} e_a$$

$$y = \begin{bmatrix} 0 & 1 \end{bmatrix} \bar{x}$$

4. Find the state-space representation in phase-variable form for

$$G(s) = \frac{8s + 10}{s^4 + 5s^3 + s^2 + 5s + 13}$$

Answer:

$$\dot{\bar{x}} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -13 & -5 & -1 & -5 \end{bmatrix} \bar{x} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} r$$

$$y = \begin{bmatrix} 10 & 8 & 0 & 0 \end{bmatrix} \bar{x}$$

5. Find the state-space representation in phase-variable form for

$$G(s) = \frac{s^4 + 2s^3 + 12s^2 + 7s + 6}{s^5 + 9s^4 + 13s^3 + 8s^2}$$

Answer:

$$\dot{\bar{x}} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -8 & -13 & -9 \end{bmatrix} \bar{x} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} r$$

$$y = \begin{bmatrix} 6 & 7 & 12 & 2 & 1 \end{bmatrix} \bar{x}$$

§6.5

1. For the following system, determine how many eigenvalues are in the right-half plane, in the left-half plane, and on the $j\omega$-axis.

$$\dot{\bar{x}} = \begin{bmatrix} 0 & 1 & 3 \\ 2 & 2 & -4 \\ 1 & -4 & 3 \end{bmatrix} \bar{x} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix} \bar{x}$$

Answer: One eigenvalue is in the left-half plane, and the other two are in the right-half plane.

2. Use MATLAB to find the eigenvalues of the following system

$$\dot{\bar{x}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & -4 \\ -1 & 1 & 8 \end{bmatrix} \bar{x} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \bar{x}$$

Answer:

$$\lambda_{1,2,3} = 0.54,\ 1.0,\ 7.5$$

3. Determine whether or not the following system is stable.

$$\dot{\bar{x}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 3 \\ -3 & -4 & -5 \end{bmatrix} \bar{x} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & 1 & 1 \end{bmatrix} \bar{x}$$

Answer: $\lambda_{1,2,3} = -.68 \pm j1.7,\ -2.6 \ \rightarrow$ It's stable.