12.5

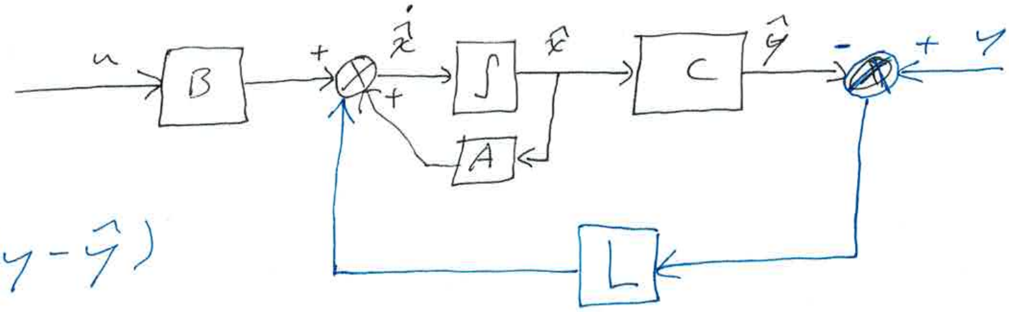
State feedback control based on each state variable by a gain

This not always feasible: excessive instrumentation required, variables unavailable for measurement, etc.

Estimate state variables with observer

- Full-order observer estimates all, reduced-order used when have measurements for some ~~available for measurement~~

Observer with feedback



$\dot{\hat{x}} = A\hat{x} + Bu$

$\hat{y} = C\hat{x}$

$\dot{\hat{x}} = A\hat{x} + Bu + L(y - \hat{y})$

$\hat{y} = C\hat{x}$

Want $\bar{x} - \hat{x}$ to go to zero quickly (relative to speed of plant dynamics)

$$\dot{\bar{x}} - \dot{\hat{x}} = A(\bar{x} - \hat{x}) - L(y - \hat{y}) = (A - LC)(\bar{x} - \hat{x})$$

$$\dot{\bar{e}}_x = (A - LC)\bar{e}_x$$

∴ Eigenvalues of $A-LC$ should be much more (10x) negative than those of $A-BK$

eg) Revisit module09:12.2.4 w/position as output. Design an observer.

$$\dot{\bar{x}} = \begin{bmatrix} 0 & 1 \\ -15 & -6 \end{bmatrix}\bar{x} + \begin{bmatrix} 0 \\ \frac{1}{2} \end{bmatrix} f, \quad y = \begin{bmatrix} 1 & 0 \end{bmatrix}\bar{x}, \quad K = \begin{bmatrix} 1650 & 68.0 \end{bmatrix}$$

Closed-loop system poles at $-20 \pm j20.98 \longrightarrow$ observer poles $-200 \pm j20.$

$$L = \begin{bmatrix} l_1 \\ l_2 \end{bmatrix}, \quad A - LC = \begin{bmatrix} 0 & 1 \\ -15 & -6 \end{bmatrix} - \begin{bmatrix} l_1 & 0 \\ l_2 & 0 \end{bmatrix} = \begin{bmatrix} -l_1 & 1 \\ -15 - l_2 & -6 \end{bmatrix}$$

$$|sI - (A - LC)| = \begin{vmatrix} s + l_1 & -1 \\ 15 + l_2 & s + 6 \end{vmatrix} = s^2 + (l_1 + 6)s + 6l_1 + 15 + l_2$$

Desired CP: $s^2 + 400s + 84016$

$$\therefore l_1 + 6 = 400, \quad 6l_1 + 15 + l_2 = 84016 \implies L = \begin{bmatrix} 394 \\ 37621 \end{bmatrix}$$

12.6

If the initial state, $\bar{x}(t_0)$, can be determined by observing the output, y for a finite amount of time, the system is observable.

- A requirement for full-order observer

eg1) $\dot{\bar{x}} = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 10 \end{bmatrix}\bar{x} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}u, \quad y = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}\bar{x}$ is not observable bc x_2 decoupled from x_1 and x_3 and the output has only x_2

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eg 2) $\dot{\bar{x}} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \bar{x} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u$, $y = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \bar{x}$ is observable

Can test with observability matrix: $\text{rank}(O_n) = n$

$$O_n = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{bmatrix}$$

eg) $O_{m1} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$, $\text{rank}(O_{m1}) = 1 \rightarrow$ unobservable

$O_{m2} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 2 \end{bmatrix}$, $\text{rank}(O_{m2}) = 3 \rightarrow$ observable

12.8

[state-variable feedback with integral action – block diagram]

[next slide: state and output equations, concatenated form]

Resulting system matrix: $\begin{bmatrix} A - BK & BK_e \\ -C & 0 \end{bmatrix}$, $\begin{bmatrix} \dot{\bar{x}} \\ \dot{x}_N \end{bmatrix} = \begin{bmatrix} A - BK & BK_e \\ -C & 0 \end{bmatrix} \begin{bmatrix} \bar{x} \\ x_N \end{bmatrix} + \begin{bmatrix} \bar{0} \\ 1 \end{bmatrix} r$

Eigenvalues are CL poles

$y = \begin{bmatrix} C & 0 \end{bmatrix} \begin{bmatrix} \bar{x} \\ x_N \end{bmatrix}$

eg) Revisit module 09: 12.2.4 & add integral control

$$A_{CI} = \begin{bmatrix} A - BK & Bk_e \\ -C & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -15 - k_1/2 & -6 - k_2/2 & k_e/2 \\ -1 & 0 & 0 \end{bmatrix}$$

$$\begin{vmatrix} s & -1 & 0 \\ 15 + \frac{k_1}{2} & s + 6 + \frac{k_2}{2} & -\frac{k_e}{2} \\ 1 & 0 & s \end{vmatrix} = s^3 + \left(6 + \frac{k_2}{2}\right)s^2 + \left(15 + \frac{k_1}{2}\right)s + \frac{k_e}{2}$$

$s_{1,2,3} = -100, -20 \pm j20.98$

$\Downarrow$

$s^3 + 140s^2 + 4839.9s + 83990$

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$\therefore K = \begin{bmatrix} 9650 & 268 \end{bmatrix}$

$k_e = 167980$

Note that if $T_s = 2s$ then $K = [66.8 \quad 16.0]$ and $k_e = 168$