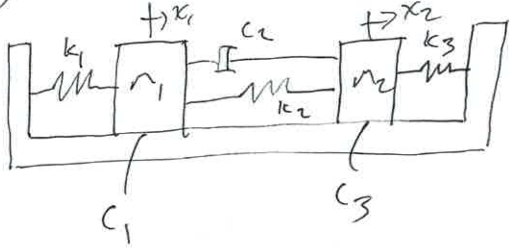
3.1 - 3.4

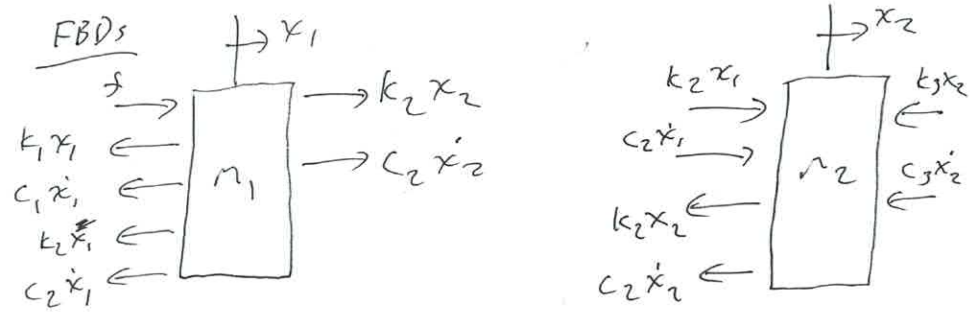
State-space representation includes state variables and input
- Write a first-order diff. eq. for each state variable in terms of state variables + input

Representation also includes output in terms of state variables and input
- Considering only SISO w/no feedforward

eg) Revisit early system + find SS rep w/ x_2 as output



FBDs



EoM

$$f-(k_1+k_2)x_1-(c_1+c_2)\dot{x}_1+k_2x_2+c_2\dot{x}_2=m_1\ddot{x}_1$$

$$k_2x_1+c_2\dot{x}_1-(k_2+k_3)x_2-(c_2+c_3)\dot{x}_2=m_2\ddot{x}_2$$

SS rep

$$\dot{\bar{x}}=A\bar{x}+Bu \quad , \quad \bar{x}=[x_1 \ \ v_1 \ \ x_2 \ \ v_2]^T , \quad u=f$$

$$y=C\bar{x} \quad , \quad y=x_2$$

$$\begin{bmatrix}\dot{x}_1\\ \dot{v}_1\\ \dot{x}_2\\ \dot{v}_2\end{bmatrix}=\begin{bmatrix}0 & 1 & 0 & 0\\ -\frac{k_1+k_2}{m_1} & -\frac{c_1+c_2}{m_1} & \frac{k_2}{m_1} & \frac{c_2}{m_1}\\ 0 & 0 & 0 & 1\\ \frac{k_2}{m_2} & \frac{c_2}{m_2} & -\frac{k_2+k_3}{m_2} & -\frac{c_2+c_3}{m_2}\end{bmatrix}\begin{bmatrix}x_1\\ v_1\\ x_2\\ v_2\end{bmatrix}+\begin{bmatrix}0\\ \frac{1}{m_1}\\ 0\\ 0\end{bmatrix}f$$

$$y=[0 \ \ 0 \ \ 1 \ \ 0]\bar{x}$$

~~TFs with zeros:~~

$$\cancel{R(s)\rightarrow\boxed{\frac{N_G(s)}{D_G(s)}}\xrightarrow{C(s)} \; = \; R(s)\rightarrow\boxed{\frac{1}{D_G(s)}}\xrightarrow{X_1(s)}\boxed{N_G(s)}\xrightarrow{C(s)}}$$

~~Use phase variables x_1 + its derivatives~~

~~eg)~~ $$\cancel{G(s)=\frac{b_2s^2+b_1s+b_0}{s^3+a_2s^2+a_1s+a_0}}$$

$$\cancel{\frac{X_1(s)}{R(s)}=\frac{1}{s^3+a_2s^2+a_1s+a_0}}$$

$$\cancel{\therefore \dot{x}_3=-a_0x_1-a_1x_2-a_2x_3+r \quad , \quad y=b_0x_1+b_1x_2+b_2x_3=[b_0 \ b_1 \ b_2]\bar{x}}$$

OVER

3.5

Numerous valid SS reps exist for a given system

Can convert TF to SS using phase variables

- $G(s)$ with no zeros: use phase variables of ~~y~~ output & its derivatives
  - For n^{th} order TF w/output y, have n state variables being $x_1 = y$ & its $n-1$ derivatives.

    $$x_2 = \frac{dy}{dt}, \quad x_3 = \frac{d^2y}{dt^2}, \ldots$$

  - Converting from TF to SS based on eqn

    $$\frac{d^ny}{dt^n} + a_{n-1}\frac{d^{n-1}y}{dt^{n-1}} + \ldots + a_1\frac{dy}{dt} + a_0 y = b_0 u$$

    which yields state eqns

    $$\dot{x}_1 = x_2$$
    $$\dot{x}_2 = x_3$$
    $$\vdots$$
    $$\dot{x}_n = -a_0x_1 - a_1x_2 - \ldots - a_{n-1}x_n + b_0 u$$

eg) Given $G(s) = \frac{C(s)}{R(s)} = \frac{27}{s^3+3s^2+s+13}$, Find SS rep using phase variables

Cross multiply then take inverse Laplace

$$s^3C + 3s^2C + sC + 13C = 27R \longrightarrow \dddot{c} + 3\ddot{c} + \dot{c} + 13c = 27r$$

$n = 3 \rightarrow 3$ state variables: $x_1 = c$, $x_2 = \dot{c}$, $x_3 = \ddot{c}$

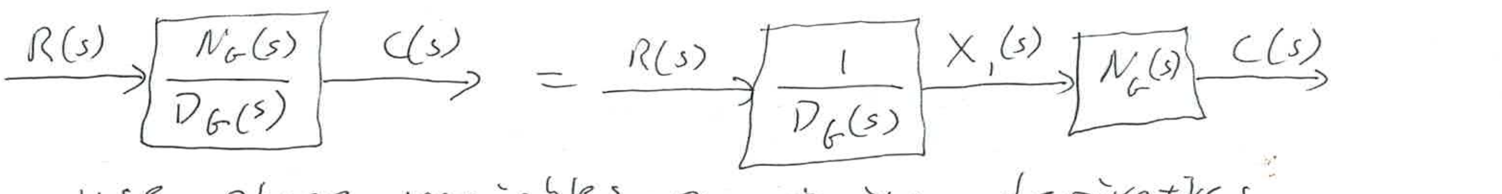
rearrange to get

$$\dddot{c} = -13c - \dot{c} - 3\ddot{c} + 27r$$
$$\dot{x}_3 = -13x_1 - x_2 - 3x_3 + 27r$$

$$\dot{\bar{x}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -13 & -1 & -3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 27 \end{bmatrix} r$$

$$c = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}\bar{x}$$

- $G(s)$ with zeros:



use phase variables x_1 & its derivatives

eg) $$G(s) = \frac{b_2s^2 + b_1s + b_0}{s^3 + a_2s^2 + a_1s + a_0}$$

OVER →

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$$\frac{X_1(s)}{R(s)} = \frac{1}{s^3 + a_2 s^2 + a_1 s + a_0} \quad \therefore \quad \dot{x}_3 = -a_0 x_1 - a_1 x_2 - a_3 x_3 + r$$

$$b_2 x_3 + b_1 x_2 + b_0 x_1 = c$$

$$\dot{\bar{x}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a_0 & -a_1 & -a_2 \end{bmatrix} \bar{x} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} r$$

$$c = \begin{bmatrix} b_0 & b_1 & b_2 \end{bmatrix} \bar{x}$$

6.5

~~Given a~~ In terms of SS representation, what's criterion for stability?

Solutions to $\det(\lambda I - A) = 0$ have negative real parts.

eg) Is the following system stable?

$$\dot{\bar{x}} = \begin{bmatrix} 0 & 1 \\ -75 & -3 \end{bmatrix} \bar{x} + \begin{bmatrix} 0 \\ 70 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \bar{x}$$

$$|\lambda I - A| = \begin{vmatrix} \lambda & -1 \\ 75 & \lambda + 3 \end{vmatrix} = \lambda^2 + 3\lambda + 75 = 0 \rightarrow \lambda_{1,2} = -\frac{3}{2} \pm \frac{1}{2}\sqrt{9 - 300}$$

$$= -1.5 \pm 8.53i$$

$\boxed{\text{stable}}$

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