Observability

KENNESAW STATE UNIVERSITY

Contents

Definition

When designing an observer, if any state variable has no effect upon output, then cannot estimate this state variable via output feedback

- If $\bar{x}(t_0)$ can be found from $u(t)$ and $y(t)$ measured over finite interval from t_0, the system is *observable*.
    - That is, deducing state variables from $u(t)$ and $y(t)$
- A state variable can can influence output either directly or through coupling of variables.

Determining Observability By Inspection

To consider observability, examine the following systems:

Example 1

$$\dot{\bar{x}} = \begin{bmatrix} -a_1 & 0 & 0 \\ 0 & -a_2 & 0 \\ 0 & 0 & -a_3 \end{bmatrix} \bar{x} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \bar{x} = x_1 + x_2 + x_3$$

This system is observable because all the state variables are coupled to the output.

Example 2

$$\dot{\bar{x}} = \begin{bmatrix} -a_1 & 0 & 0 \\ 0 & -a_2 & 0 \\ 0 & 0 & -a_3 \end{bmatrix} \bar{x} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & 1 & 1 \end{bmatrix} \bar{x} = x_2 + x_3$$

This system is unobservable because state variable, x_1, has no influence over the output.

Determining Observability in General

In general, state matrix is not diagonal

- Relationships between y and x_i not so clear

Use *observability matrix* for testing general systems:

The n^{th}-order system with state-space representation

$$\dot{\bar{x}} = A\bar{x} + Bu$$
$$y = C\bar{x}$$

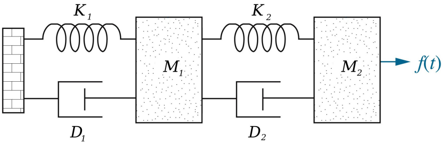
is observable if the *observability matrix*

$$O_M = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{bmatrix}$$

is of rank n.

Observability Example

Determine whether or not, if x_2 is the output, the system shown below is observable. Use the state vector, $\bar{x} = \begin{bmatrix} x_1 & v_1 & x_2 & v_2 \end{bmatrix}^{\mathsf{T}}$, and $K_1 = 2$, $D_1 = 3$, $M_1 = 1$, $K_2 = 9$, $D_2 = 4$, and $M_2 = 1$.



$$\dot{\bar{x}} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -11 & -7 & 9 & 4 \\ 0 & 0 & 0 & 1 \\ 9 & 4 & -9 & -4 \end{bmatrix} \bar{x} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} f$$

$$y = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix} \bar{x}$$

$$O_M = \begin{bmatrix} C \\ CA \\ CAA \\ CAAA \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 9 & 4 & -9 & -4 \\ -80 & -35 & 72 & 23 \end{bmatrix}$$

$\mathrm{rank}(O_M) = 4$

$\therefore$ The system is observable.

- It is possible to estimate a past state by measuring $u(t)$ and $y(t)$ over a finite time interval.