Module 10:
§12.5–6 and §12.8
With Answers

§12.5

1. Design an observer with poles at $-100 \pm 50j$ for the system

$$\dot{\bar{x}} = \begin{bmatrix} -10 & 1 \\ -21 & 0 \end{bmatrix} \bar{x} + \begin{bmatrix} 0 \\ 25 \end{bmatrix} u$$
$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \bar{x}$$

Answer:

$$L = \begin{bmatrix} 190 \\ 12\,479 \end{bmatrix}$$

2. A controller with $K = \begin{bmatrix} 12 & 2 \end{bmatrix}$ was designed for the following system. Design an observer for it.

$$\dot{\bar{x}} = \begin{bmatrix} -8 & 1 \\ -210 & 0 \end{bmatrix} \bar{x} + \begin{bmatrix} 0 \\ 20 \end{bmatrix} u$$
$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \bar{x}$$

Answer:

$$L = \begin{bmatrix} 472 \\ 76\,790 \end{bmatrix}$$

3. Given the system below, find K to yield 0.5 s settling time and 10% overshoot. Then design an observer for the system.

$$\dot{\bar{x}} = \begin{bmatrix} 0 & 1 \\ -21 & -10 \end{bmatrix} \bar{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$
$$y = \begin{bmatrix} 25 & 0 \end{bmatrix} \bar{x}$$

Answer:

$$K = \begin{bmatrix} 162 & 6.00 \end{bmatrix}$$

$$L = \begin{bmatrix} 6.00 \\ 672 \end{bmatrix}$$

4. Design gains for an observer with poles $s_{1,2,3} = -50,\ -25 \pm 43.301j$ for the system below using the phase-variable state-space representation.

$$G(s) = \frac{23}{(s+1)(s+2)(s+5)}$$

Answer:

$$L = \begin{bmatrix} 84.0 \\ 3590 \\ 61\,800 \end{bmatrix}$$

§12.6

1. Find the observability matrix and its rank for all the systems in the previous section. *Answer:*

$$O_{M1} = \begin{bmatrix} 1 & 0 \\ -10 & 1 \end{bmatrix}$$

$$\text{rank}(O_{M1}) = 2$$

$$O_{M2} = \begin{bmatrix} 1 & 0 \\ -8 & 1 \end{bmatrix}$$

$$\text{rank}(O_{M2}) = 2$$

$$O_{M3} = \begin{bmatrix} 25 & 0 \\ 0 & 25 \end{bmatrix}$$

$$\text{rank}(O_{M3}) = 2$$

$$O_{M4} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\text{rank}(O_{M4}) = 3$$

2. Determine by inspection whether or not the following system is observable and state why or why not, then verify your answer using MATLAB.

$$\dot{\bar{x}} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 7 & 3 \\ 0 & 10 & 1 \end{bmatrix} \bar{x} + \begin{bmatrix} 0 \\ 5 \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \bar{x}$$

Answer: The system is not observable because x_1 is decoupled from x_2 and x_3 and since the output does not include x_1 directly, there's no information available about it. The rank of O_M is two.

§12.8

1. For a system with the following open-loop transfer function, use phase variables and design a state-space controller to yield 10% overshoot, 0.5 second settling time, and $e_{\text{step}}(\infty) = 0$.

$$G(s) = \frac{s+2}{s^2 - s - 2}$$

Answer:

Either

$$K = \begin{bmatrix} 34 & 19.0 \end{bmatrix}$$
$$k_e = 183$$

if third pole cancels zero, or

$$K = \begin{bmatrix} -2840 & 57.0 \end{bmatrix}$$
$$k_e = 3660$$

if third pole 5× farther from imaginary axis than dominant pair.

2. Repeat the previous problem for a system with the following open-loop transfer function.

$$G(s) = \frac{s+3}{s^2 + 7s + 10}$$

Answer:

Either

$$K = \begin{bmatrix} 38.0 & 12.0 \end{bmatrix}$$
$$k_e = 183$$

if third pole cancels zero, or

$$K = \begin{bmatrix} 813 & 49.0 \end{bmatrix}$$
$$k_e = 2440$$

if third pole 5× farther from imaginary axis than dominant pair.