Steady-State Error Design via Integral Control

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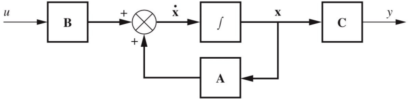
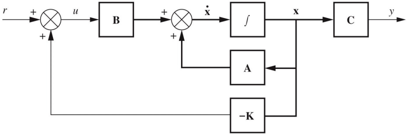
Controller Design Via State-Variable Feedback (Review)

First pass at state-space controller design employed state-variable feedback to place all system poles



Original system



With state-variable feedback

Placing system poles allows for transient-response manipulation

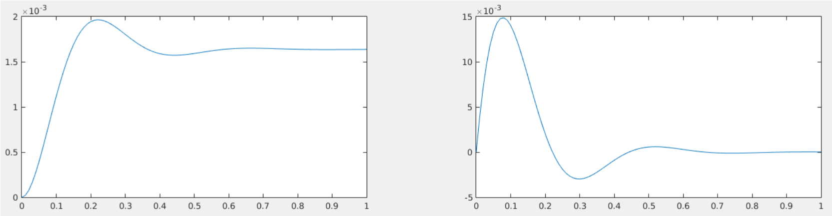
- Left with steady-state error

Motivation — Example With Steady-State Error

Given a DC motor with load having state-space representation

$$\dot{\bar{x}} = \begin{bmatrix} 0 & 1 \\ 0 & -5/3 \end{bmatrix} \bar{x} + \begin{bmatrix} 0 \\ 5/12 \end{bmatrix} u$$
$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \bar{x}$$

a state-variable controller with gain matrix $K = \begin{bmatrix} 610.62 & 30.909 \end{bmatrix}$ yields 0.55 second settling time and 20% overshoot as shown below.



Step response of θ_m and ω_m

Note that θ_m steady-state error is 0.9984

Adding Integral Control

Remove steady-state error by adding output feedback and an integrator as shown below

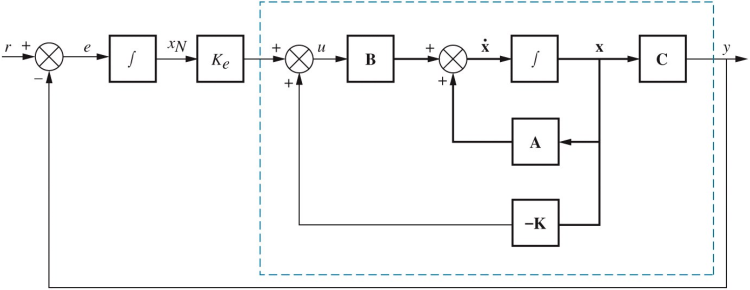


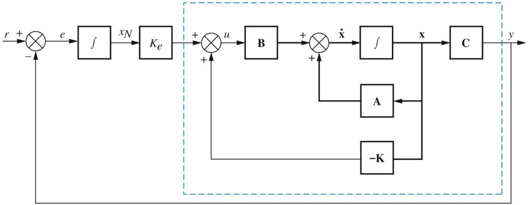
Figure 12.21

State-variable feedback system shown in dashed line

Introduces new state variable (increased system order):

$$x_N = \int (r - C\bar{x})$$

Integral Control State Equations



State and output equations

$$\dot{\bar{x}} = A\bar{x} + Bu$$
$$\dot{x}_N = -C\bar{x} + r$$
$$y = C\bar{x}$$

Matrix form, concatenate $\bar{x}$ and x_N

$$\begin{bmatrix}\dot{\bar{x}}\\ \dot{x}_N\end{bmatrix} = \begin{bmatrix}A & \bar{0}\\ -C & 0\end{bmatrix}\begin{bmatrix}\bar{x}\\ x_N\end{bmatrix} + \begin{bmatrix}B\\ 0\end{bmatrix}u + \begin{bmatrix}\bar{0}\\ 1\end{bmatrix}r$$
$$y = \begin{bmatrix}C & 0\end{bmatrix}\begin{bmatrix}\bar{x}\\ x_N\end{bmatrix}$$

Substitution

$$u = -K\bar{x} + K_e x_N = -\begin{bmatrix}K & -K_e\end{bmatrix}\begin{bmatrix}\bar{x}\\ x_N\end{bmatrix}$$

$$\begin{bmatrix}\dot{\bar{x}}\\ \dot{x}_N\end{bmatrix} = \begin{bmatrix}A & \bar{0}\\ -C & 0\end{bmatrix}\begin{bmatrix}\bar{x}\\ x_N\end{bmatrix} - \begin{bmatrix}BK & -BK_e\\ \bar{0}^T & 0\end{bmatrix}\begin{bmatrix}\bar{x}\\ x_N\end{bmatrix} + \begin{bmatrix}\bar{0}\\ 1\end{bmatrix}r$$
$$= \begin{bmatrix}A - BK & BK_e\\ -C & 0\end{bmatrix}\begin{bmatrix}\bar{x}\\ x_N\end{bmatrix} + \begin{bmatrix}\bar{0}\\ 1\end{bmatrix}r$$
$$y = \begin{bmatrix}C & 0\end{bmatrix}\begin{bmatrix}\bar{x}\\ x_N\end{bmatrix}$$

Example: Integral Control of DC Motor with Load

Given the DC motor from earlier, with state-space representation

$$\dot{\bar{x}} = \begin{bmatrix} 0 & 1 \\ 0 & -5/3 \end{bmatrix} \bar{x} + \begin{bmatrix} 0 \\ 5/12 \end{bmatrix} u$$
$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \bar{x}$$

Design the integral controller by using the state equation,

$$\begin{aligned}
\begin{bmatrix} \dot{\bar{x}} \\ \dot{x}_N \end{bmatrix} &= \begin{bmatrix} [A - BK] & BK_e \\ -C & 0 \end{bmatrix} \begin{bmatrix} \bar{x} \\ x_N \end{bmatrix} + \begin{bmatrix} \bar{0} \\ 1 \end{bmatrix} r \\
&= \begin{bmatrix} \left[\begin{bmatrix} 0 & 1 \\ 0 & -\frac{5}{3} \end{bmatrix} - \begin{bmatrix} 0 \\ \frac{5}{12} \end{bmatrix} \begin{bmatrix} k_1 & k_2 \end{bmatrix} \right] & \begin{bmatrix} 0 \\ \frac{5}{12} \end{bmatrix} K_e \\ -\begin{bmatrix} 1 & 0 \end{bmatrix} & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_N \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} r \\
&= \begin{bmatrix} 0 & 1 & 0 \\ -\frac{5}{12}k_1 & -(\frac{5}{3} + \frac{5}{12}k_2) & \frac{5}{12}K_e \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_N \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} r
\end{aligned}$$

to place poles via the characteristic equation,

$$\det \left(sI - \begin{bmatrix} [A - BK] & BK_e \\ -C & 0 \end{bmatrix} \right) = s^3 + \left(\frac{5}{3} + \frac{5}{12} k_2 \right) s^2 + \frac{5}{12} k_1 s + \frac{5}{12} K_e = 0$$

Example Continued

Desired poles come from second-order approximation.

Settling time and percent overshoot $\rightarrow \zeta = 0.456$ and $\omega_n = 16.0$

$$s_{1,2} = -7.27 \pm j14.2$$

Third pole placement to preserve 2nd-order approximation

$$s_3 = -40$$

These yield desired characteristic equation:

$$s^3 + 54.55s^2 + 836.2s + 10,180 = 0$$

Compare to system characteristic equation to find gains

$$s^3 + \left(\frac{5}{3} + \frac{5}{12}k_2\right)s^2 + \frac{5}{12}k_1 s + \frac{5}{12}K_e = 0$$

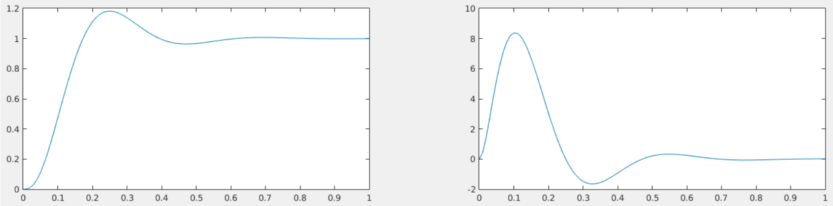
$$\therefore K = \begin{bmatrix} 2,007 & 126.9 \end{bmatrix}$$

$$K_e = 24,430$$

☆ Note that these gains are very high and thus difficult to implement.

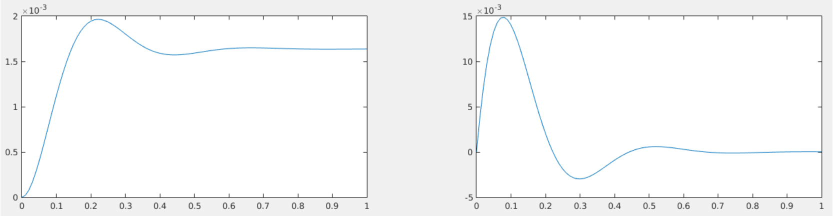
Example Output

Compare output of integral controller



Step response of θ_m and ω_m — integral control

with that of non-integral controller



Step response of θ_m and ω_m — no integrator