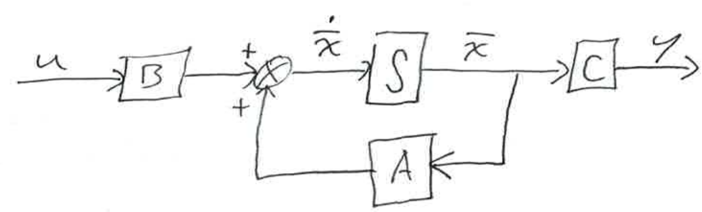
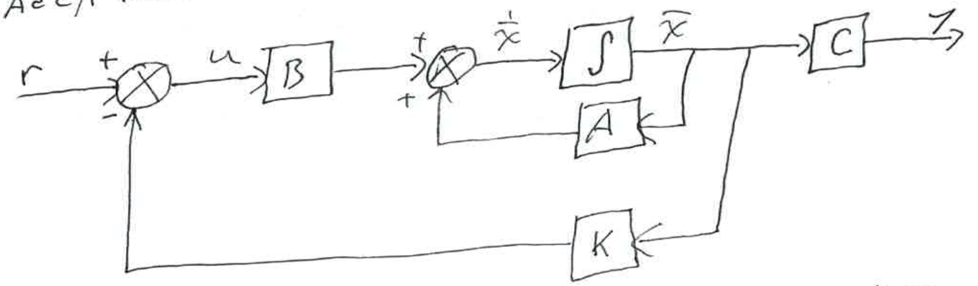
12.2

Time-domain block diagram of SS rep

$$\dot{\bar{x}} = A\bar{x} + Bu$$
$$y = C\bar{x}$$



Add~~ing~~ feedback of scaled state variables ~~to~~ alters system dynamics.



$$\dot{\bar{x}} = A\bar{x} + (B)(r - K\bar{x}) = \underline{\underline{(A - BK)\bar{x} + Br}}$$

$y = C\bar{x}$

Single input systems $\rightarrow$ ~~to~~ K is row vector (of n elements)

~~Ei~~ New system matrix, but eigenvalues of system matrix are still poles

$$\det(\lambda I - (A - BK)) = 0$$

Procedure for placing system poles:

1. Write characteristic ~~eqn~~ polynomial (CP) w/unknown gains
2. Determine desired system poles + write resulting ~~CE~~ CP
3. Solve for gains by equating coefficients from ① + ②

eg) Revisit system for which PD controller designed: m06_9.3.2

$G_p = \dfrac{1}{(s)(s+10)(s+20)} = \dfrac{1}{s^3 + 30s^2 + 200s}$, $G_c = (674)(s + 5.58)$

$\therefore s_{1,2,3} = -5.02,\ -12.5 \pm j24.3 \rightarrow$ ~~Simulation: T_s~~ $T_{s,calc} = 0.320$, $OS_{calc} = 20.0\%$

simulation: $T_s = 0.344$, $OS = 11.7\%$

① SS rep of uncompensated system

$$\dot{\bar{x}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -200 & -30 \end{bmatrix}\bar{x} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}u, \quad y = [1 \ 0 \ 0]\bar{x}$$

$$K = [k_1 \ k_2 \ k_3] \rightarrow BK = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ k_1 & k_2 & k_3 \end{bmatrix}$$

$$|\lambda I - (A - BK)| = \left| \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} - \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -k_1 & -200-k_2 & -30-k_3 \end{bmatrix} \right| = \begin{vmatrix} \lambda & -1 & 0 \\ 0 & \lambda & -1 \\ k_1 & 200+k_2 & \lambda+30+k_3 \end{vmatrix}$$

$$= \lambda^3 \cancel{+ 87.4\lambda^2}\ \lambda^2(30 + k_3) + \lambda(200 + k_2) + k_1$$

② $\lambda_{1,2,3\ desired} = (5)(-12.5),\ -12.5 \pm j24.3 \rightarrow \lambda^3 + \cancel{\lambda^2 (30}\ 87.4\lambda^2 + 2310\lambda + 46700$

③ $k_1 = 46700$, $200 + k_2 = 2310$, $30 + k_3 = 87.4$ $\therefore K = [46700 \quad 2110 \quad 57.4]$

simulation:

$T_s = 0.319$

$OS = 17.8\%$

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Two comments:

1. Can improve transient behavior relative to targets by moving third pole farther
2. Gains are already high

12.3

To get state vector to any value, ~~requires say~~ system must be controllable
- Required in order to be able to place poles

eg)
$$\dot{\bar{x}} = \begin{bmatrix} -5 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & -3 \end{bmatrix} \bar{x} + \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} u, \quad y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \bar{x}$$

$$BK = \begin{bmatrix} 0 & 0 & 0 \\ k_1 & k_2 & k_3 \\ k_1 & k_2 & k_3 \end{bmatrix}, \quad |\lambda I - (A - BK)| = \begin{vmatrix} \lambda+5 & 0 & 0 \\ k_1 & \lambda+4+k_2 & k_3 \\ k_1 & k_2 & \lambda+3+k_3 \end{vmatrix}$$

$$= (\lambda+5)((\lambda+4+k_2)(\lambda+3+k_3) - k_2 k_3)$$

$$= \lambda^3 + \lambda^2(12+k_2+k_3) + \lambda(47+8k_2+9k_3) + 60 + 15k_2 + 20k_3$$

System is overconstrained: 3 equations (coeffs of characteristic polynomial), but only two unknowns

$\therefore$ Cannot place all three poles.

There exists a simple test for controllability of time-invariant systems.

$$\text{rank}(C_M) = n \iff \text{system is controllable},$$

where n is system order and $C_M = [B \;\; AB \cdots A^{n-1}B]$

eg)
$$C_M = \left[\begin{matrix} 0 \\ 1 \\ 1 \end{matrix} \;\; \begin{bmatrix} -5 & & \\ & -4 & \\ & & -3 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \;\; \begin{bmatrix} -5 & & \\ & -4 & \\ & & -3 \end{bmatrix}\begin{bmatrix} -5 & & \\ & -4 & \\ & & -3 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right] = \begin{bmatrix} 0 & 0 & 0 \\ 1 & -4 & 16 \\ 1 & -3 & 9 \end{bmatrix}$$

$\text{rank}(C_M) < 3 \rightarrow$ system is uncontrollable