Module 09:
§12.1–3
With Answers

§12.2

1. For the system in Figure 1, find a state-space representation of the closed-loop system with state-variable feedback if the position of mass 2 is the output. Also find the matrix, $\lambda I - (A - BK)$.

   Answer:

$$\dot{\bar{x}} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{K_1+K_2}{M_1} & -\frac{D_1+D_2}{M_1} & \frac{K_2}{M_1} & \frac{D_2}{M_1} \\ 0 & 0 & 0 & 1 \\ \frac{K_2}{M_2} & \frac{D_2}{M_2} & -\frac{K_2}{M_2} & -\frac{D_2}{M_2} \end{bmatrix} \bar{x} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{M_2} \end{bmatrix} f$$

$$y = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix} \bar{x}$$

$$\lambda I - (A - BK) = \begin{bmatrix} \lambda & -1 & 0 & 0 \\ \frac{K_1+K_2}{M_1} & \lambda + \frac{D_1+D_2}{M_1} & -\frac{K_2}{M_1} & -\frac{D_2}{M_1} \\ 0 & 0 & \lambda & -1 \\ \frac{-K_2+k_1}{M_2} & \frac{-D_2+k_2}{M_2} & \frac{K_2+k3}{M_2} & \lambda + \frac{D_2+k_4}{M_2} \end{bmatrix}$$

2. For the plant

$$G(s) = \frac{100(s+10)}{s(s+3)(s+12)}$$

   design the phase-variable feedback gains to yield 5% overshoot and a peak time of 0.3 second.
   Answer:

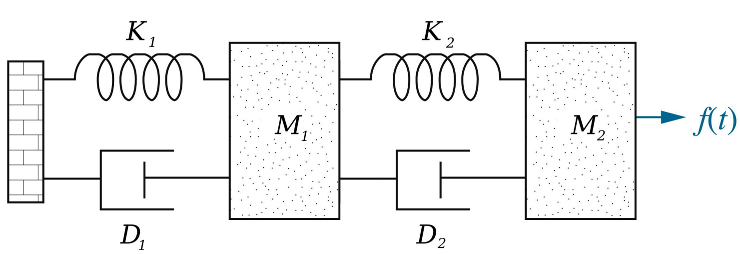

Figure 1: Two DoF SMD system

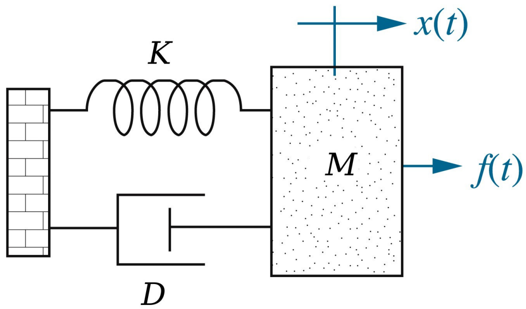

Figure 2: One DoF SMD system

$K = \begin{bmatrix} 2090 & 373 & 15.0 \end{bmatrix}$

3. Given the following open-loop plant,

$$G(s) = \frac{20}{(s+2)(s+4)(s+8)}$$

design a controller to yield a 15% overshoot and a 0.75 second settling time. Place the third pole 10 times as far from the imaginary axis as the dominant pole pair.

Answer:

$K = \begin{bmatrix} 5610 & 619 & 50 \end{bmatrix}$

4. Find the phase-variable gains that will yield 5% overshoot and 0.2 second settling time for the system in Figure 2, using the following impedance values: $K = 30$, $D = 12$, and $M = 2$.

Answer:

$K = \begin{bmatrix} 1650 & 68.0 \end{bmatrix}$

§12.3

1. Determine the controllability of a system with the following state equation.

$$\dot{\bar{x}} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \bar{x} + \begin{bmatrix} 1 \\ -1 \end{bmatrix} u$$

Answer:

$$C_M = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$\text{rank}(C_M) = 1$$

$$\therefore \text{uncontrollable}$$

2. Find the controllability matrix, C_M, and its rank for the system pictured below.

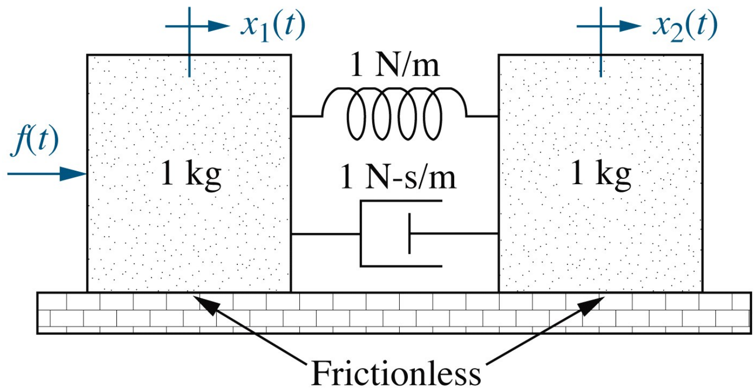

Figure P2.10

Answer:

$$C_M = \begin{bmatrix} 0 & 1 & -1 & 1 \\ 1 & -1 & 1 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 1 & -1 & 0 \end{bmatrix}$$

$\text{rank}(C_M) = 4$

3. Find the controllability matrix, C_M, and its rank for the system pictured below.

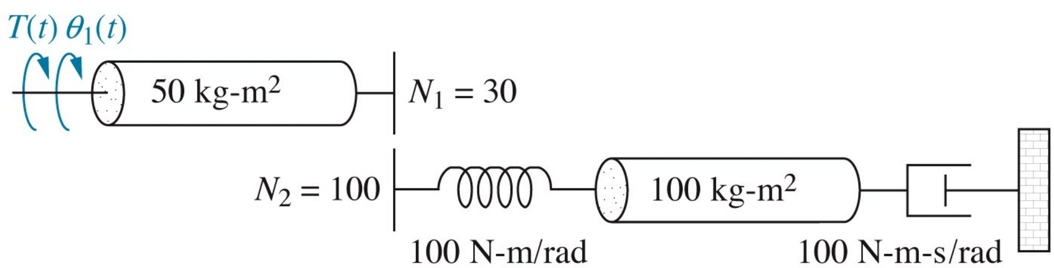

Figure P3.6

Answer:

$$C_M = \frac{1}{1000}\begin{bmatrix} 0 & 6 & 0 & -1.1 \\ 6 & 0 & -1.1 & 0 \\ 0 & 0 & 0 & 6 \\ 0 & 0 & 6 & -6 \end{bmatrix}$$

$\text{rank}(C_M) = 4$