



# **COL215 DIGITAL LOGIC AND SYSTEM DESIGN**


Designing synchronous  
sequential circuits

23 August 2017

# Design problem

Design a synchronous circuit with the following specifications

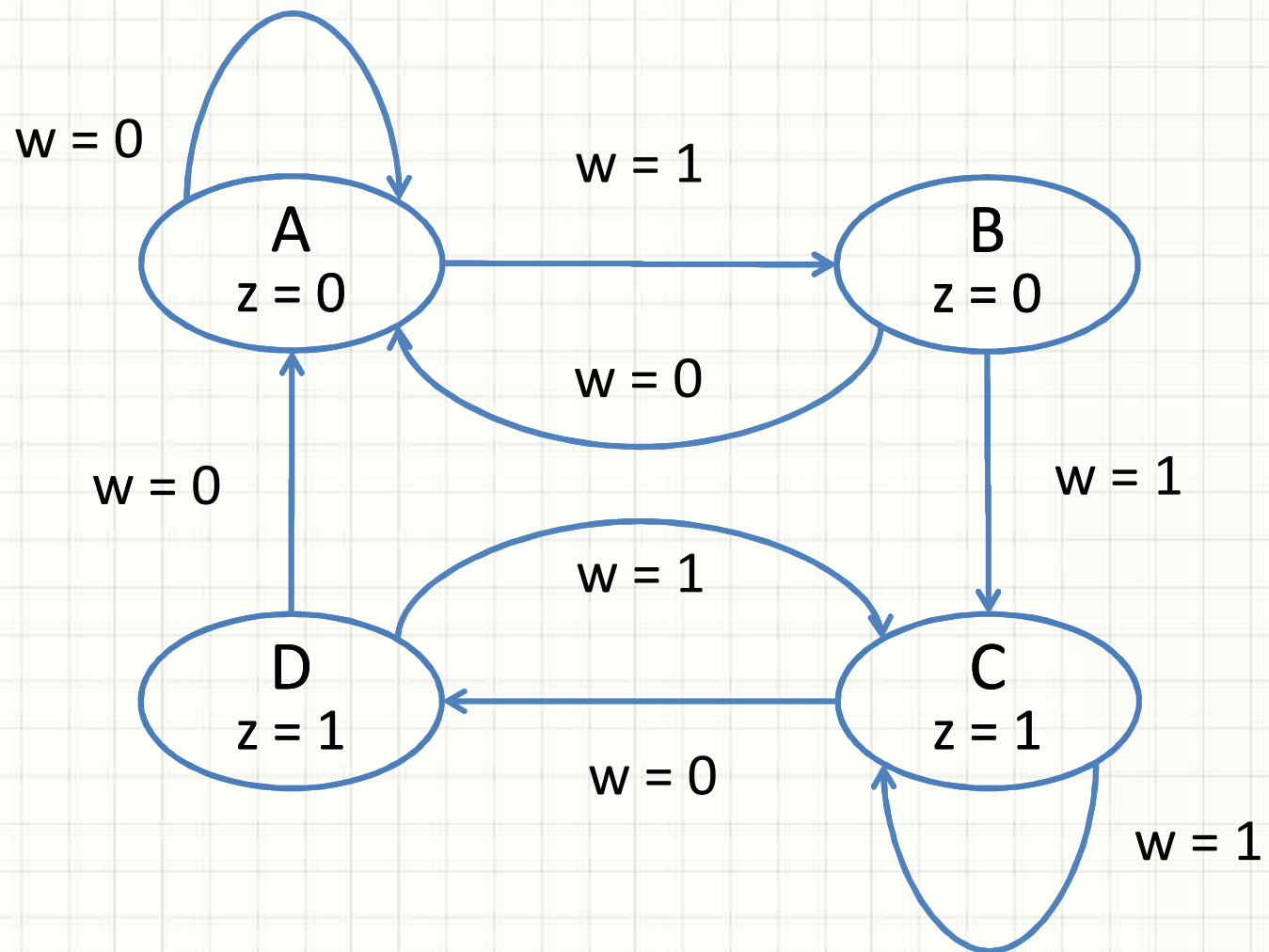
- It has one input  $w$  and one output  $z$
- Output  $z$  becomes 1, if during two immediately preceding clock cycles input  $w$  is 1
- Output  $z$  becomes 0, if during two immediately preceding clock cycles input  $w$  is 0
- Otherwise, output  $z$  remains unchanged
- Initially, output  $z$  is 0



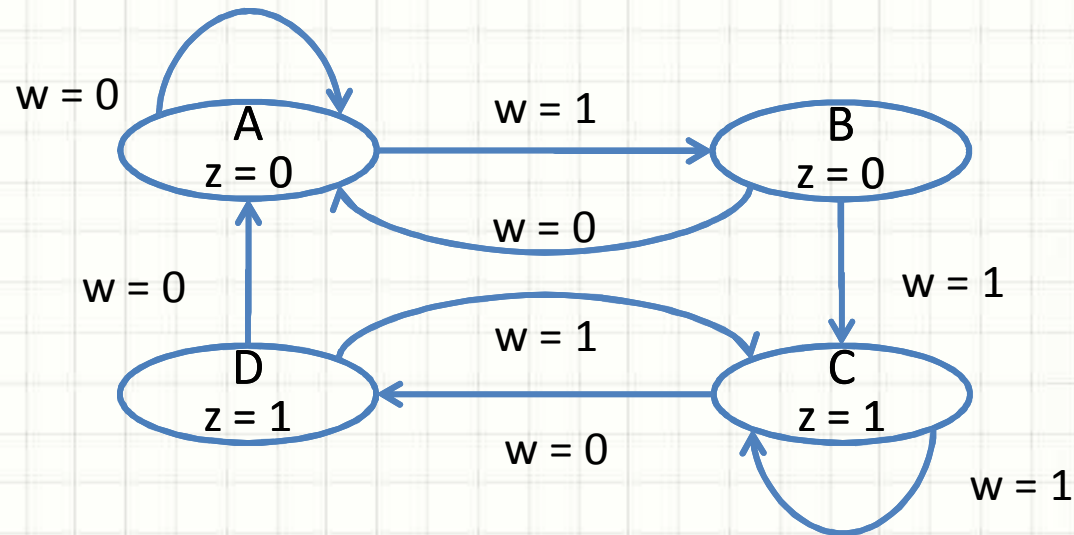
# How to design?

- Identify states
- A state indicates how the past input history influences the output
- When another input gets added to the history, state may change
- 'States' and 'state transitions' form 'state transition diagram'

# State transition diagram

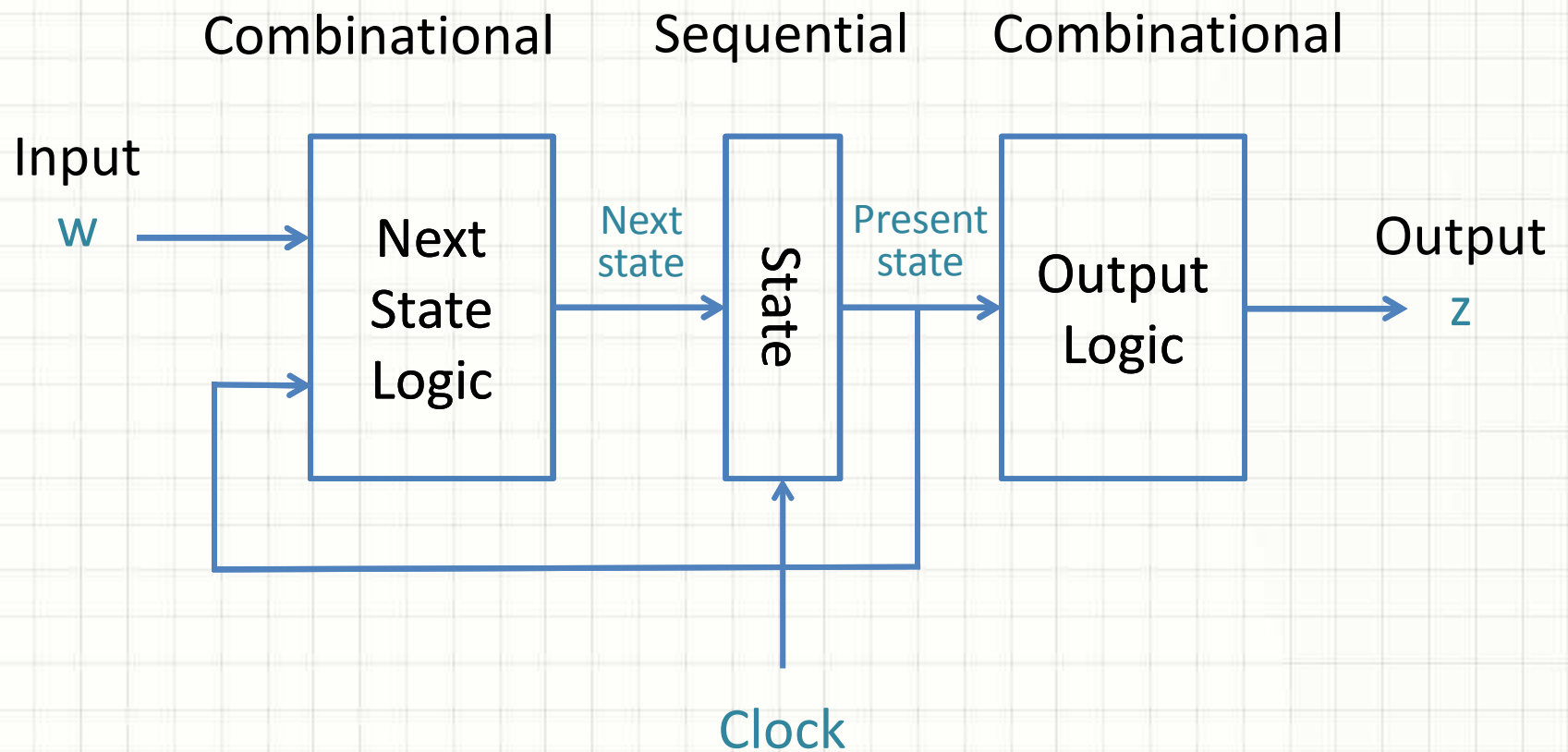


# State transition table



Present state	Next state		Output z
	w = 0	w = 1	
A	A	B	0
B	A	C	0
C	D	C	1
D	A	C	1

# Circuit structure



# Representing state

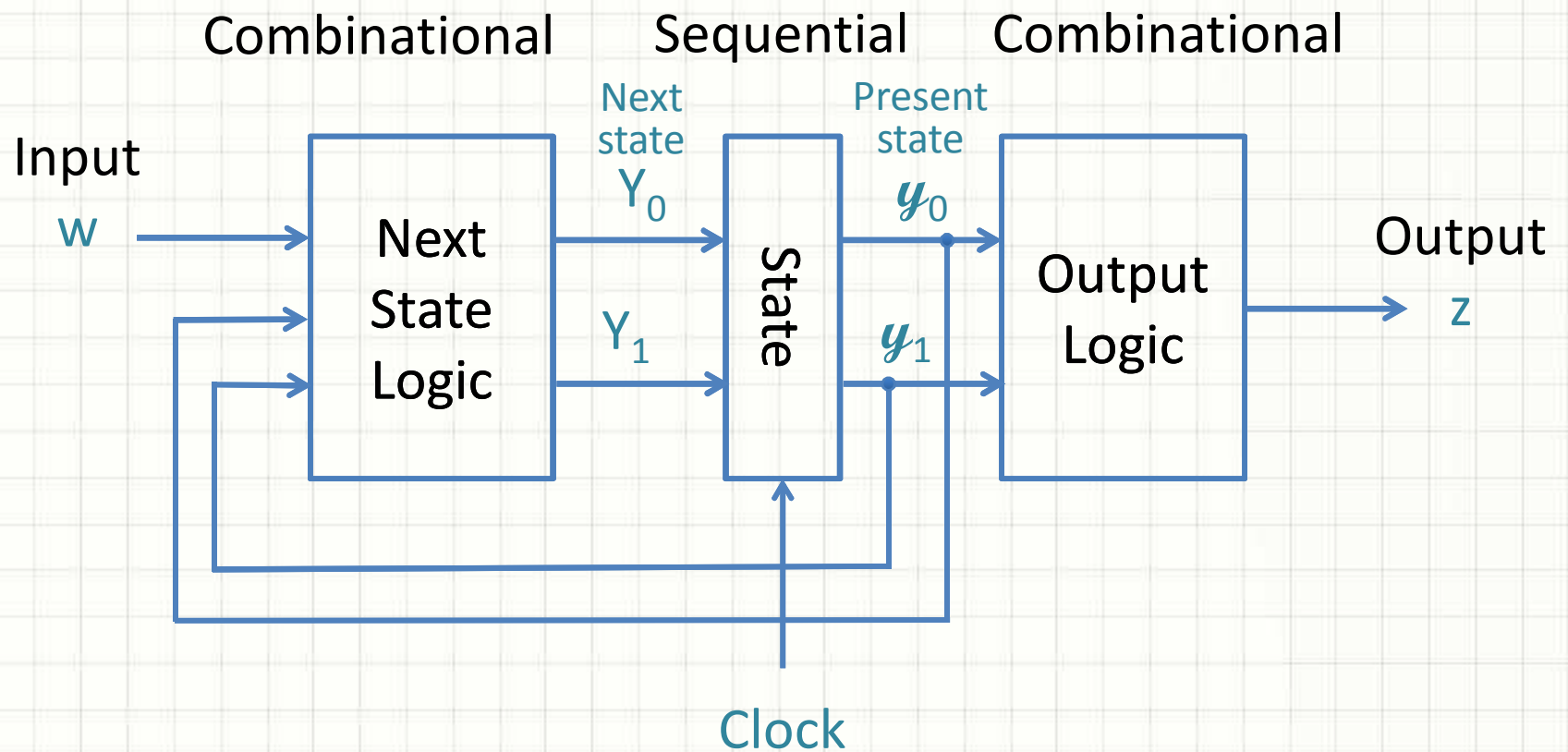
	$y_1$	$y_0$
A	0	0
B	0	1
C	1	0
D	1	1

Present state:  $y_1 y_0$

Next state:  $Y_1 Y_0$



# Circuit structure





# State transition table

$y_1 y_0$	$Y_1 Y_0$		Output $z$
	$w = 0$	$w = 1$	
0 0	0 0	0 1	0
0 1	0 0	1 0	0
1 0	1 1	1 0	1
1 1	0 0	1 0	1

# Karnaugh maps

$y_1 y_0$	$Y_1 Y_0$	
	$w = 0$	$w = 1$
0 0	0 0	0 1
0 1	0 0	1 0
1 0	1 1	1 0
1 1	0 0	1 0

$y_1 y_0$	Output $z$
0 0	0
0 1	0
1 0	1
1 1	1

# Karnaugh maps

$y_1 y_0$	$Y_1 Y_0$	
	$w = 0$	$w = 1$
0 0	0 0	0 1
0 1	0 0	1 0
1 1	0 0	1 0
1 0	1 1	1 0

$y_1 y_0$	Output $z$
0 0	0
0 1	0
1 1	1
1 0	1

# Karnaugh maps

$Y_1$

$y_1 y_0$	$w = 0$	$w = 1$
0 0	0	0
0 1	0	1
1 1	0	1
1 0	1	1

$Y_0$

$y_1 y_0$	$w = 0$	$w = 1$
0 0	0	1
0 1	0	0
1 1	0	0
1 0	1	0

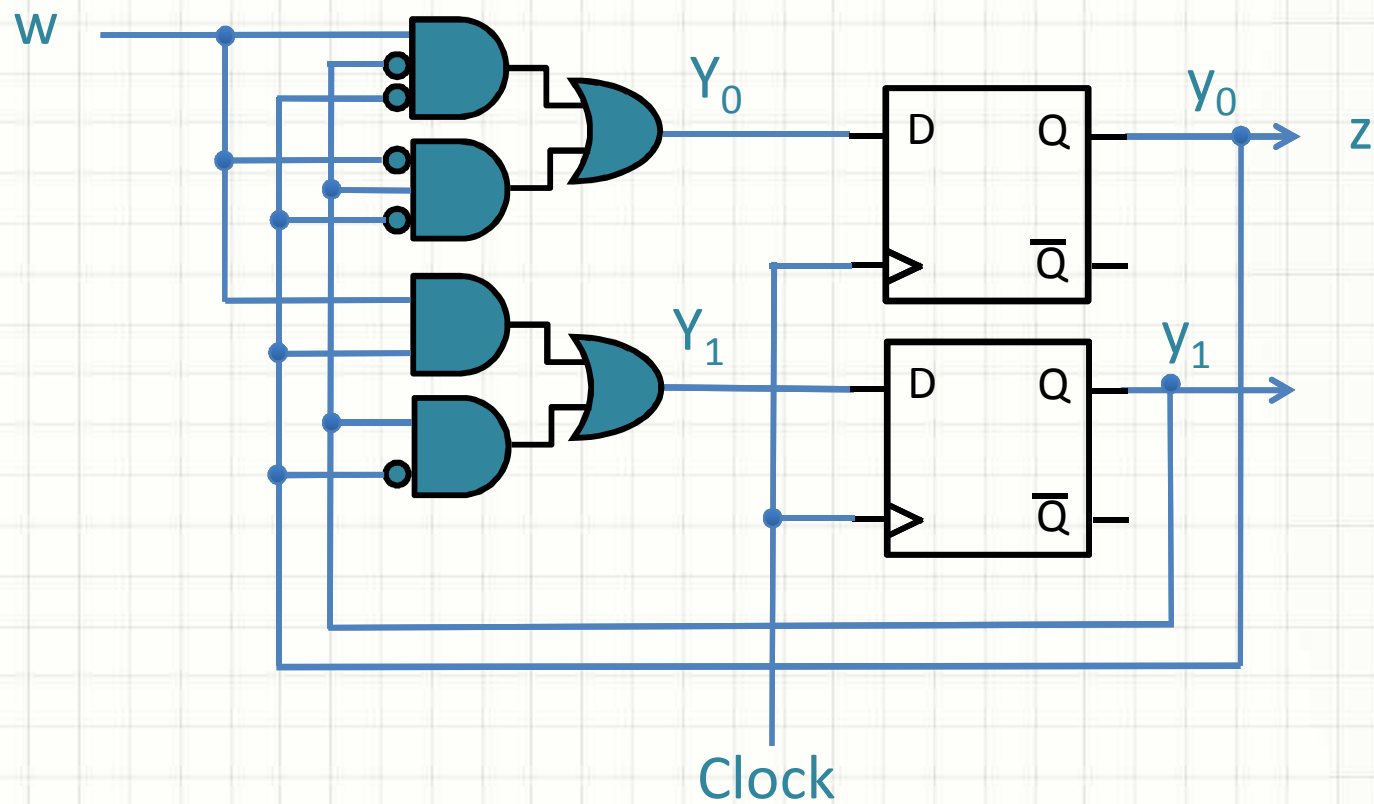
$y_1 y_0$	Output $z$
0 0	0
0 1	0
1 1	1
1 0	1

$$Y_1 = w \cdot y_0 + y_1 \cdot y_0'$$

$$Y_0 = w \cdot y_1' \cdot y_0' + w' \cdot y_1 \cdot y_0' \\ = (w \cdot y_1' + w' \cdot y_1) \cdot y_0'$$

$$z = y_1$$

# Complete circuit

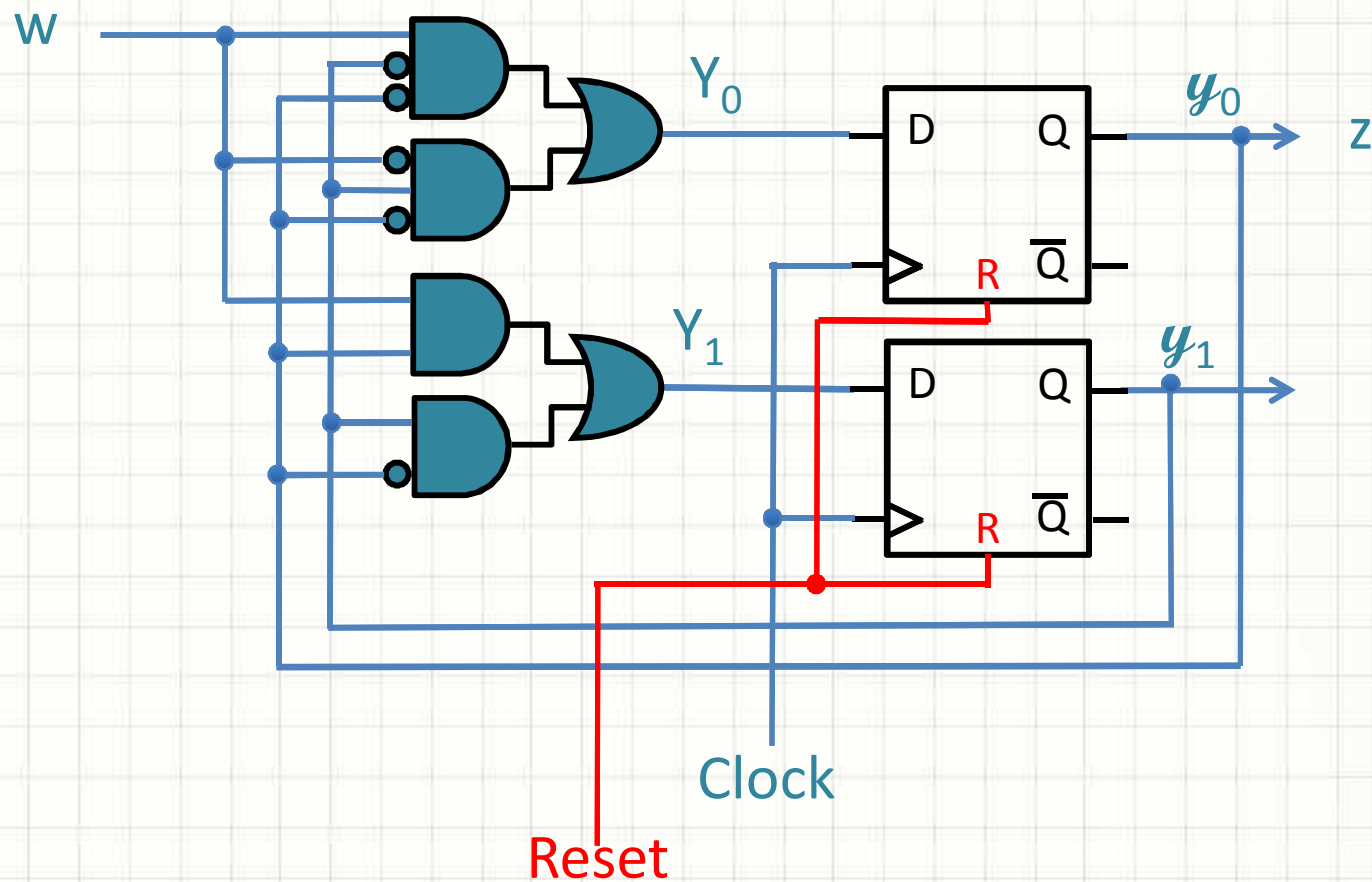


$$Y_1 = w.y_0 + y_1.y_0'$$

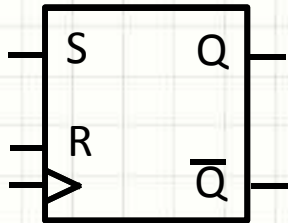
$$Y_0 = w.y_1'.y_0' + w'.y_1.y_0' \\ = (w.y_1' + w'.y_1).y_0'$$

$$z = y_1$$

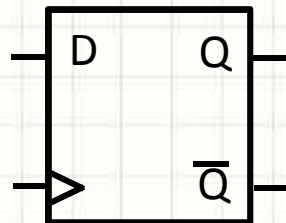
# Complete circuit



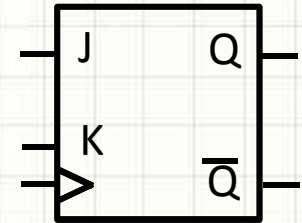
# Can we use J-K flip flops instead of D?



S	R	Q(t+1)
0	0	Q (t)
0	1	0
1	0	1
1	1	XXX



D	Q(t+1)
0	0
1	1



J	K	Q(t+1)
0	0	Q (t)
0	1	0
1	0	1
1	1	$\bar{Q} (t)$

$$D = J.\bar{Q} + \bar{K}.Q$$

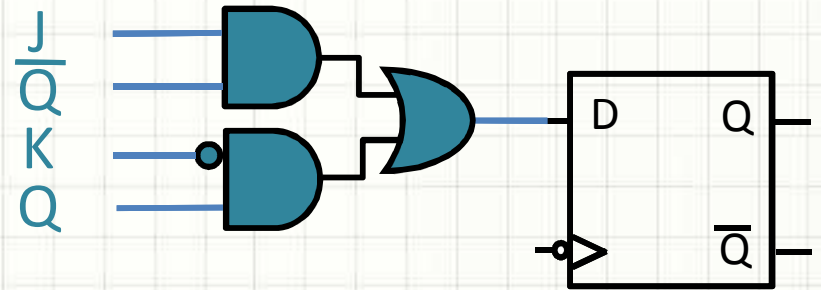
$$S = J.\bar{Q}, R = K.Q$$



# J-K flip flop made using D FF or SR FF

Implementing J-K FF  
using D FF

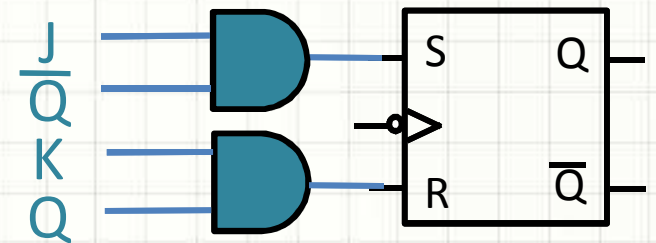
$$D = J \cdot \bar{Q} + \bar{K} \cdot Q$$



Implementing J-K FF  
using SR FF

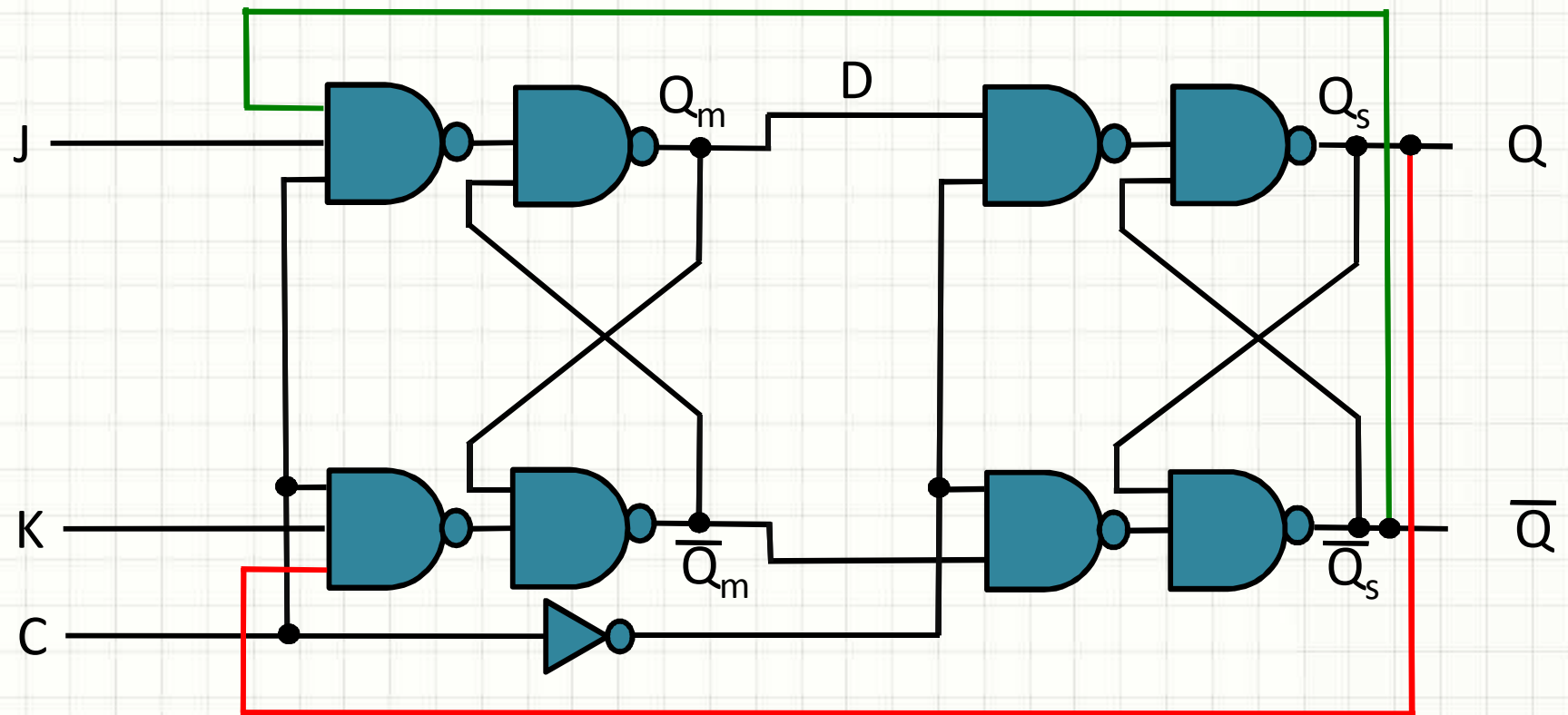
$$S = J \cdot \bar{Q}$$

$$R = K \cdot Q$$



# Direct implementation

## Master-Slave J-K Flip-Flop



# Defining J and K inputs

J	K	$Q(t + 1)$
0	0	$Q(t)$
0	1	0
1	0	1
1	1	$\overline{Q}(t)$

Transition	J K
$0 \Rightarrow 0$	0 -
$0 \Rightarrow 1$	1 -
$1 \Rightarrow 0$	- 1
$1 \Rightarrow 1$	- 0

Task: verify that these two tables are consistent with each other

# Transitions for the current example

$Y_1$

$y_1 y_0$	$w = 0$	$w = 1$
0 0	0 (0=>0)	0 (0=>0)
0 1	0 (0=>0)	1 (0=>1)
1 1	0 (1=>0)	1 (1=>1)
1 0	1 (1=>1)	1 (1=>1)

$Y_0$

$y_1 y_0$	$w = 0$	$w = 1$
0 0	0 (0=>0)	1 (0=>1)
0 1	0 (1=>0)	0 (1=>0)
1 1	0 (1=>0)	0 (1=>0)
1 0	1 (0=>1)	0 (0=>0)

These tables are same as those in slide 11  
with transitions shown in green

# J K inputs for the current example

J, K values for the required transitions are taken from the table on right and added to the tables below (shown in red)

Transition	J K
0 $\Rightarrow$ 0	0 -
0 $\Rightarrow$ 1	1 -
1 $\Rightarrow$ 0	- 1
1 $\Rightarrow$ 1	- 0

$J_1 K_1$

$y_1 y_0$	w = 0	w = 1
0 0	(0 $\Rightarrow$ 0) 0 -	(0 $\Rightarrow$ 0) 0 -
0 1	(0 $\Rightarrow$ 0) 0 -	(0 $\Rightarrow$ 1) 1 -
1 1	(1 $\Rightarrow$ 0) - 1	(1 $\Rightarrow$ 1) - 0
1 0	(1 $\Rightarrow$ 1) - 0	(1 $\Rightarrow$ 1) - 0

$J_0 K_0$

$y_1 y_0$	w = 0	w = 1
0 0	(0 $\Rightarrow$ 0) 0 -	(0 $\Rightarrow$ 1) 1 -
0 1	(1 $\Rightarrow$ 0) - 1	(1 $\Rightarrow$ 0) - 1
1 1	(1 $\Rightarrow$ 0) - 1	(1 $\Rightarrow$ 0) - 1
1 0	(0 $\Rightarrow$ 1) 1 -	(0 $\Rightarrow$ 0) 0 -

# Separating J and K values

$J_1$

$y_1 y_0$	$w'$	$w$
0 0	0	0
0 1	0	1
1 1	-	-
1 0	-	-

$K_1$

$y_1 y_0$	$w'$	$w$
0 0	-	-
0 1	-	-
1 1	1	0
1 0	0	0

$J_0$

$y_1 y_0$	$w'$	$w$
0 0	0	1
0 1	-	-
1 1	-	-
1 0	1	0

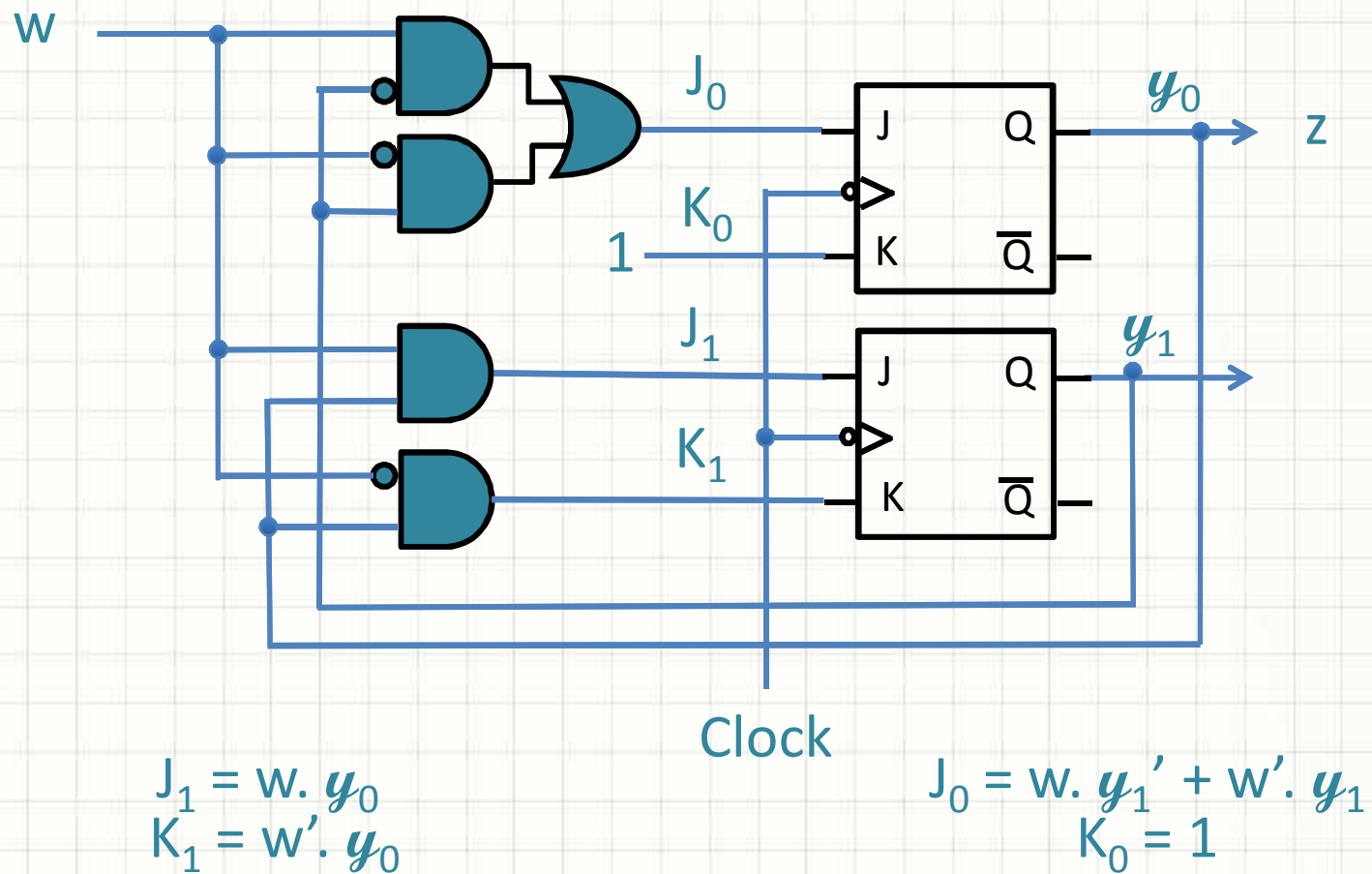
$K_0$

$y_1 y_0$	$w'$	$w$
0 0	-	-
0 1	1	1
1 1	1	1
1 0	-	-

$$\begin{aligned} J_1 &= w \cdot y_0 \\ K_1 &= w' \cdot y_0 \end{aligned}$$

$$\begin{aligned} J_0 &= w \cdot y_1' + w' \cdot y_1 \\ K_0 &= 1 \end{aligned}$$

# Final circuit







**THANKS**