

The statement in the question was false and to disprove it, you had to show that 5 elements can be sorted with at most 7 comparisons. The correct way to show this is to construct a comparison based algorithm that sorts the 5 elements with 7 comparisons in worst case. Following is one such algorithm:

Compare A to B and C to D. Without loss of generality (WLOG), suppose  $A > B$  and  $C > D$ . Compare A to C. WLOG, suppose  $A > C$ . Sort E into A-C-D. This can be done with two comparisons. Sort B into {E, C, D}. This can be done with two comparisons, for a total of seven. We are done at this point as we already know that  $A > B$ .

Marking Scheme:

1 mark - Recognising that the statement is false.

2 marks - Realizing that if an algorithm exists, then it must use exactly 7 moves.

3 marks - Trying to construct a comparison based algorithm and at least getting the first two comparisons ( $\langle A, B \rangle$  and  $\langle C, D \rangle$ ) right.

6 marks - Full solution correct.

A common incorrect solution was to use the decision tree argument to show that the lower bound on the number of comparisons required to sort 5 elements is  $\log(5!) \approx 6.9$  and then argue that as  $7 > 6.9$ , therefore the statement is false. This is incorrect as the existence of a lower bound does not necessarily mean that an algorithm exists that will achieve the lower bound. We have given such answers 2 marks.