

Marking Scheme: - a) 1.5 marks b) 3 marks c) 1 mark d) 1.5 marks.

Input: array A[0..n-1] of n numbers

Output: A is sorted in increasing order.

If $n = 2$ and $A[0] > A[1]$, then swap ($A[0]$, $A[1]$)

If $n > 2$ then {

Stooge-sort($A[0..\text{ceil}(2n/3)]$) // sort first two-thirds.

Stooge-sort($A[\text{floor}(n/3)..n]$) // sort last two-thirds.

Stooge-sort($A[0..\text{ceil}(2n/3)]$) // sort first two-thirds again.

}

- a) Each time the function stooge-sort is called 3 times on $2n/3$ elements and on each call the work done is constant.

$$T(n) = 3T\left(\frac{2}{3}n\right) + \Theta(1).$$

b)

$$\begin{aligned} T(n) &= 1 + 3T\left(\frac{2}{3}n\right) \\ &= 1 + 3 + 9T\left(\frac{4}{9}n\right) \\ &\quad \dots \\ &= 1 + 3 + 3^2 + \dots + 3^{\log_{3/2} n} \\ &= \frac{3^{\log_{3/2} n + 1} - 1}{3 - 1} \\ &= \Theta(3^{\log_{3/2} n}) \\ &= \Theta(3^{(\log_3 n)/(\log_3 3/2)}) \\ &= \Theta(n^{1/(\log_3 3/2)}) \\ &= \Theta(n^{2.71}). \end{aligned}$$

c) Yes.

- d) No. Insertion sort complexity is $O(n^2)$ which is equivalent to $O(n^{\log 9/4})$ (log base is 3/2) clearly $9/4 < 3$ hence performance of stooge-sort is worse than insertion sort.