# Department of Mathematics

## Tutorial Sheet 5

## MTL 106

1. Let X and Y be independent random variables. The range of X is $\{1, 3, 4\}$ and the range of Y is $\{1, 2\}$.

   Partial information on the probability mass function is as follows:

   $$p_X(3) = 0.50; \quad p_Y(2) = 0.60; \quad p_{X,Y}(4, 2) = 0.18 .$$

   (a) Determine $p_X, p_Y$ and $p_{X,Y}$ completely.

   (b) Determine $P(|X - Y| \geq 2)$.

2. Evaluate all possible marginal and conditional distributions if (X, Y) has the following joint probability distribution:

   (a) $P(X = j, Y = k) = q^{k-j} p^j, j = 1, 2, \ldots$ and $k = j + 1, j + 2, \ldots \quad q = 1 - p$

   (b) $P(X = j, Y = k) = \frac{15!}{j!k!(15-j-k)!} \left(\frac{1}{2}\right)^j \left(\frac{1}{3}\right)^k \left(\frac{1}{6}\right)^{15-j-k}$

   for all admissible non negative integral values of j and k.

3. Show that

   $$F(x, y) = \begin{cases} 0, & x < 0, y < 0 \text{ or } x + y < 1 \\ 1, & otherwise \end{cases}$$

   is not a distribution function.

4. Find k, if the joint probability density of (X , Y) is

   $$f(x, y) = \begin{cases} ke^{-3x-4y}, & x > 0, y > 0 \\ 0, & otherwise \end{cases}$$

   Also find the probability that the value of X falls between 0 and 1 while Y falls between 0 and 2.

5. Consider a transmitter sends out either a 0 with probability p, or a 1 with probability $(1 - p)$, independently of earlier transmissions. Assume that the number of transmissions within a given time interval is Poisson distributed with parameter $\lambda$. Find the distribution of number of 1's transmitted in that same time interval?

6. Suppose that the two-dimensional random variable (X, Y) has joint pdf

   $$f(x, y) = \begin{cases} 1, & 0 < x, y < 1 \\ 0, & otherwise \end{cases}$$

   Find the pdf of $X + Y$.

7. Romeo and Juliet have a date at a given time, and each, independently, will be late by an amount of time, denoted by X and Y respectively, that is exponentially distributed with parameter $\lambda$. Find the pdf of, $X - Y$, the difference between their times of arrival?

8. (a) Let X, Y and Z be independent and identically distributed random variables each having a uniform distribution over the interval $[0, 1]$. Find the joint density function of $(V, W)$ where $V = XY$ and $W = Z^2$.

(b) The random variable X represents the amplitude of cosine wave; Y represents the amplitude of a sine wave. Both are independent and uniformly distributed over the interval (0,1). Let R represent the amplitude of their resultant, i.e., $R^2 = X^2 + Y^2$ and $\theta$ represent the phase angle of the resultant, i.e., $\theta = \tan^{-1}(Y/X)$. Find the joint and marginal pdfs of $\theta$ and R.

9. Let X and Y be continuous random variables having joint distribution which is uniform over the square which has corners at $(2, 2), (-2, 2), (-2, -2)$ and $(2, -2)$. Determine $P(|Y| > |X| + 1)$.

10. Suppose that the two-dimensional random variable (X, Y) has joint pdf

$$f(x, y) = \begin{cases} kx(x-y), & 0 < x < 2, -x < y < x \\ 0, & otherwise \end{cases}$$

Find k. Evaluate $P(X < 1 / Y = 0.5)$ and $P(Y < 1.5 / X = 1)$.

11. Let A, B and C be independent random variables each with uniform distributed on interval (0, 1). What is the probability that $Ax^2 + Bx + C = 0$ has real roots?