

①

We can use binary search algorithm to search the entire array and find the position. To do so, we need to find the mid element compare it with our element if it is equal we have the index else we would either search the left or right part of array.

① There will be a total n cases to be considered as our key can be present in one of the n positions.

Since there can be only $\log_2 n$ maximum comparisons.

At each i th step

that we have match

for each position i the probability¹ will be

no of comparisons $\leftarrow \begin{matrix} (i-1) \\ n \end{matrix} \rightarrow$ no of indices which will have i comparisons. total no of indices possible.

③

so

$$\sum_{i=1}^{\log_2 n} \frac{i \cdot 2^{i-1}}{2^m - 1}$$

$$S_n = \frac{1}{2^m - 1} + \frac{2 \cdot 2}{2^m - 1} + \frac{3 \cdot 2^2}{2^m - 1} + \dots$$

$$2S_n = \frac{2}{2^m - 1} + \frac{2 \cdot 2^2}{2^m - 1} + \dots$$

$$-S_n = \frac{1}{2^m - 1} + \frac{2}{2^m - 1} + \frac{2^2}{2^m - 1} + \dots + \frac{2^{\log_2 n}}{2^m - 1} = \frac{2 \cdot \log_2 n}{2^m - 1}$$

$$-S_n = \frac{1}{2^m - 1} \left(\frac{1(1-2)}{1-2} \right)$$

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$$S_n = - \left[\left(\frac{1}{2^m - 1} \right) \left(\frac{2^{\log_2 n + 1} - 1}{2 - 1} \right) - \frac{2^{\log_2 n}}{2^m - 1} \right]$$

$$S_n = \frac{2^{\log_2 n} \log_2 n}{2^m - 1} + \frac{(1 - 2^{\log_2 n + 1})}{2^m - 1}$$

$$= \frac{n \log_2 n}{n} + \frac{(1 - 2 \cdot 2^{\log_2 n})}{n}$$

$$= \log_2 n + \frac{1}{n} - \left(\frac{2 \cdot n}{n} \right)$$

$$= O(\log_2 n)$$

$$= O(m)$$