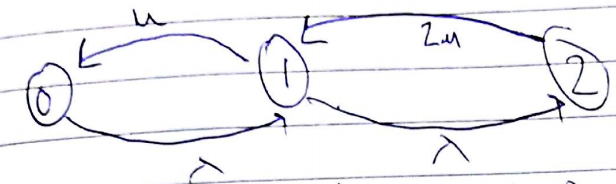
Tutorial 9

1. *A)* 

*P0*(*t*)=*P*(*t*)*Q K.F.E* with *P*(*t*)=(*P*0(*t*)*, P*1(*t*)*, P*2(*t*)) and  
*P0*(*t*)=*QP*(*t*) *K.B.E* with *P*(*t*)=(*P*0(*t*)*, P*1(*t*)*, P*2(*t*))*T*  
where *Pi*(*t*) = *P*[*X*(*t*) = *i*]

B) *π*0 = 1/(1+*ρ*+ *ρ* 2 /2) *, π*1 = *ρπ*0*, π*2 = (*ρ*2/2 )*π*0 , where *ρ = λ/µ*

Proportion of time = *π*0

1. Proportion of time a machine is in use = 1- Proportion of time a machine is not in use ,

where Pn is probability that n machines will not be in use.

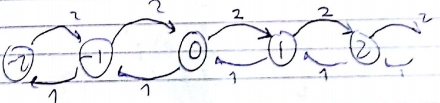
1. The generator matrix can be written as

So, using standard formuals,

π0 = 1/5

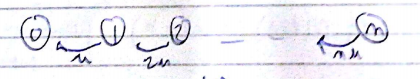
Π1 = 3/10

Π2 = 1/2



P’(t) = P(t)Q is the KFE

Also, *ρ = λ/µ* =2/1 > 1 , therefore equilibrium probability distribution does not exist.



Using KE, we get

Πn ‘(t) = -nµ Πn(t)

Πn(0) = 1

So, we get Πn(t) = e-nµt

Also, Πn-1’(t) = nµ Πn(t) – (n-1) µ Πn-1(t)

Integrating factor = e(n-1)µt

Using this we get

P = e-µt

Πn-1(t) = (n,n-1)\*(1-p)pn-1

Use PMI to prove that π ~ B(n,p)

1. A) *P*(*N*(*t* + *s*) − *N*(*t*) = *k* | *N*(*u*); 0 ≤ *u* ≤ *t*) = *P*(*N*(*t* + *s*) − *N*(*t*) = *k* ) (non overlapping intervals)

= *P*(*N*(*ts*) − *N*(*0*) = *k*) = P(N(s) = k) (homogeneity)

B) *P*(*N*(2*.*5) = 15*,N*(3*.*7) = 21*,N*(4*.*3) = 21)

= *P*(*N*(4.3) – P(3.7) = 0 *| N*(3*.*7) = 21*,N*(2.5) = 15) \* P( N(3.7)=21, N(2.5) =15)

= P(N(0.6)=0)\*P(N(3.7)-N(2.5)=6|N(2.5)=15)\* P(N(2.5)=15)

= P(N(0.6)=0)\* P(N(1.2)=6)\* P(N(2.5)=15)

= (e-12.5(12.5)15 / 15!)\* (e-6(6)6 / 6!)\* (e-3(3)0 / 0!)

1. Π’(t) = Π(t)Q
2. T1 ~ Exp(λ)

N(T1) | T1 ~ Pois(λT1)

E(N(T1)\*T1) = E(E(N(T1)T1 | T1))

= E(T1E(pois (λT1)))

= λE(T12)

= λ[1/ λ2 + 1/ λ2 ]

=2/ λ

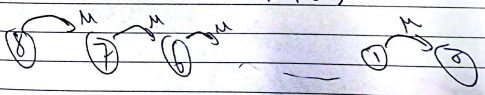
Var(N(T1)\*T1) = E(Var(N(T1)\*T1 | T1)) + Var(E(N(T1)T1|T1))

= E(λT1) + Var(λT1)

= λE(T1) + λ2 Var(T1)

= 1 + 1 = 2

1. Service time ~ Exp(µ)



1. Time spent = X1 + X2 + X3 + X4­ + … X8

~ Gamma(8, µ)

1. E(time spent in bank) = 8/ µ
2. Working ~ exp(λi)

Phone ~ exp(µi)

Xi = 1 if working

= 0 If on phone

X ( t) = ( X1 ( t ), X2 ( t ), X3 ( t ), X4 ( t))

Let t0 < t1 < t2 < t3 < t4 < ….. < tn < tn+1

We prove

P(X(tn+1) = j | X(t0) = i0 , ….. , X(tn­) = in) = P(X(tn+1) = j | X(tn) = in)

Each Xi(t) is a markov chain as

( X X 0 X) 🡪µ3 ( X X 1 X)

And ( X X 0 X) 🡸λ3 ( X X 1 X)

Similarly for the other Xi(t).

Solve πQ = 0

And then find value of π1111