Bayesian Multi-Messenger Search Method for Common Sources of Gravitational Waves and High-Energy Neutrinos

Imre Bartos,[1,*] Doğa Veske,[2] Azadeh Keivani,[2] Zsuzsa Márka,[2] Stefan Countryman,[2] Erik Blaufuss,[3] Chad Finley,[4] and Szabolcs Márka[2]

[1]*Department of Physics, University of Florida, Gainesville, FL 32611*
[2]*Department of Physics, Columbia University, New York, NY 10027*
[3]*Department of Physics, University of Maryland, College Park, MD 20742, USA*
[4]*Oskar Klein Centre and Department of Physics, Stockholm University, SE-10691 Stockholm, Sweden*

Multi-messenger astrophysics is undergoing a transition towards low-latency searches based on signals that could not individually be established as discoveries. The rapid identification of signals is important in order to initiate timely follow-up observations of transient emission that is only detectable for short time periods. Joint searches for gravitational waves and high-energy neutrinos represent a prime motivation for this strategy. Both gravitational waves and high-energy neutrinos are typically emitted over a short time frame of seconds to minutes during the formation or evolution of compact objects. In addition, detectors searching for both messengers observe the whole sky continuously, making observational information on potential transient sources rapidly available to guide follow-up electromagnetic surveys. The direction of high-energy neutrinos can be reconstructed to sub-degree precision, making a joint detection much better localized than a typical gravitational wave signal. Here we present a search strategy for joint gravitational wave and high-energy neutrino events that allows the incorporation of astrophysical priors and detector characteristics following a Bayesian approach. We aim to determine whether a multi-messenger correlated signal is a real event, a chance coincidence of two background events or the chance coincidence of an astrophysical signal and a background event. We use an astrophysical prior that is model agnostic and takes into account mostly geometric factors. Our detector characterization in the search is mainly empirical, enabling detailed realistic accounting for the sensitivity of the detector that can depend on the source properties. By this means, we will calculate the false alarm rate for each multi-messenger event which is required for initiating electromagnetic follow-up campaigns.

I. INTRODUCTION

Multi-messenger astrophysics produced two foundational discoveries in 2017: the detection of a binary-neutron star merger through gravitational waves (GWs) and electromagnetic emission [1], and the observation of a blazar through high-energy neutrinos and electromagnetic emission [2]. The multimessenger science reach of the GW detectors had been enabled by decades of effort preceding the discovery [1, 3–34] .

The third leg of multi-messenger astrophysics will be the discovery of GWs and high-energy neutrinos from a common source [23, 26]. Such a detection could shed light to, e.g., how newly formed compact objects accelerate particles to extreme energies. In addition, some high-energy neutrinos are identified rapidly with localization accuracies much better than that available with GW detectors, which can guide observatories in their search of the electromagnetic counterparts of GW sources.

Several source candidates are considered to generate GWs and high-energy neutrinos, including core-collapse supernovae [21, 35], gamma-ray bursts (GRBs) (See e.g. [36, 37]), BNS mergers [38], neutron star-black hole mergers [39], soft gamma repeaters [40, 41], and microquasars [16]. Besides these candidate sources, searches might reveal unknown source populations or production mechanisms. Detecting even one joint source of GWs and high-energy neutrinos will significantly increase our understanding of the underlying mechanisms that create them [23, 26].

Searching for joint GW+high-energy neutrino (hereafter GW+neutrino) sources has only become viable in recent years with the advent of large-scale detectors, in particular the Advanced LIGO [42] and Advanced Virgo [43] observatories on the GW side, and the IceCube [44], ANTARES [45] and Pierre Auger [46] observatories on the neutrino side. Both sides will experience significant upgrades in the coming years. Advanced LIGO and Advanced Virgo are set to reach their design sensitivities within the next few years [47]. IceCube started an upgrade towards a second generation detector, IceCube-Gen2, with several times improved sensitivity [48]. Another neutrino detector, KM3NeT, is being constructed in the Mediterranean [49]. Due to these advances, our reach to GW and neutrino sources is set to rapidly increase in the near future and beyond.

While no joint GW+neutrino discovery has been confirmed to date, there has been significant effort to search for such events. Following the first observational constraints on common sources in 2011 [18], independent searches were carried out using Initial LIGO/Virgo and the partially completed ANTARES and IceCube detectors [24, 28]. With the completion of Advanced LIGO,

* Email: imrebartos@ufl.edu

several searches were carried out to find the neutrino counterpart of GW discoveries [30–32]. A separate search was carried out to find joint events for which neither the GW nor the neutrino signal could be independently confirmed to be astrophysical [REF O1].

Most of these searches were based on the analysis method developed by Baret et al. [20]. This method combines GW amplitude, neutrino reconstructed energy, temporal coincidence and directional coincidence to separate astrophysical events from chance coincidences. The method aims to be emission model agnostic and does not impose constraints on the source properties except by assuming that higher neutrino energy is more likely to indicate an astrophysical signal.

Following the success of the search method by Baret et al. [20] spanning over a decade, it is time to upgrade it to enhance its sensitivity and aid newly relevant real-time searches. Two particular motivations for the upgrade are to facilitate the incorporation of astrophysical information and detector characteristics in the search. Regarding astrophysical information, while it is beneficial to keep the search largely model independent, in many cases signal constraints can be specified that do not depend strongly on particular model. Regarding detector characteristics, a more complex detector model will improve sensitivity and accuracy, but requires the incorporation of prior information on these characteristics to the search.

In this paper, we present a new search algorithm for common sources of GWs and high-energy neutrinos based on Bayesian hypothesis testing. A Bayesian framework is a natural choice to incorporate prior astrophysical and detector information. Bayesian solutions are becoming more common in GW [50–53] and more recently multi-messenger data analysis [54–59].

The paper is organized as follows. The general idea for this analysis is described in Sec II, following by probabilities describing signal hypothesis in Sec III, null hypothesis in Sec IV and chance coincidence hypothesis in Sec V. We define the use of odds ratios in Section VI. We conclude in Section VII.

II. MULTI-MESSENGER SEARCH METHOD

To determine whether a multi-messenger coincident signal is a real event or a random coincidence, we formulate the problem in the context of Bayesian hypothesis testing. We further incorporate detector and background characteristics as well as astrophysical information of the messenger particle and its source.

We will compare multiple hypotheses. Our signal hypothesis, H_s, is that all considered messengers originated from the same astrophysical source. Our null hypothesis, H_0, is that triggers in all messengers arose from the background. Additionally, we will consider a chance coincidence hypothesis, H_c, that one of the triggers is astrophysical, but the others are random coincidences from the background. We will neglect the possibility that different messengers from distinct astrophysical signals coincide as this is highly unlikely given our low signal rate.

For GWs we use the following observational information for the search: (i) detection time t_gw; (ii) reconstructed sky location probability density $\mathcal{P}_\mathrm{gw} = \mathcal{P}_\mathrm{gw}(\mathbf{\Omega})$, called the *skymap*, where $\mathbf{\Omega}$ is the source sky location; and (iii) GW statistic ρ_gw, which is a measure of the GW signal-to-noise ratio (SNR) and the event's consistency with background expectations [60]. We define a vector containing the measured properties of a GW trigger as

$$\mathbf{x}_\mathrm{gw} = \{t_\mathrm{gw}, \mathcal{P}_\mathrm{gw}, \rho_\mathrm{gw}\}. \tag{1}$$

For multiple source types, an additional variable could be the source-dependent gravitational waveform. We omit this as a factor in the following description.

For high-energy neutrinos, we assume that a single neutrino is observed. The used observational information for this neutrino includes (i) its detection time t_ν; (ii) its reconstructed sky location probability density $\mathcal{P}_\nu = \mathcal{P}_\nu(\mathbf{\Omega})$; and (iii) the reconstructed neutrino energy ϵ_ν. As high-energy neutrinos are not directly observed, the observed energy of the lepton produced in the neutrino interaction is taken as ϵ_ν. We assume that the reconstructed sky location can be described as a Gaussian distribution centered at reconstructed neutrino direction $\mathbf{\Omega}_\nu$, with reconstructed uncertainty σ_ν. We define a vector containing the measured properties of a neutrino trigger as

$$\mathbf{x}_\nu = \{t_\nu, \mathbf{\Omega}_\nu, \sigma_\nu, \epsilon_\nu\}. \tag{2}$$

We define a vector containing our model parameters for the signal hypothesis as

$$\boldsymbol{\theta} = \{t_\mathrm{s}, r, \mathbf{\Omega}, E_\mathrm{gw}, E_\nu\}, \tag{3}$$

where t_s is the reference time, r is the luminosity distance, $\mathbf{\Omega}$ is the sky location, E_gw is the isotropic-equivalent total GW energy, and E_ν is the isotropic-equivalent total high-energy neutrino energy emitted from the astrophysical event. The reference time can be thought of as the time of a relevant astrophysical event to which we compare the other times of arrival, delayed by the travel time of information to Earth at the speed of light. The neutrino energies considered here render the neutrino travel time practically the same as travel time at the speed of light.

III. SIGNAL HYPOTHESIS

We first introduce our signal hypothesis H_s. Given the observational data, the probability of the signal hypothesis being true can be written as $P(H_\mathrm{s}|\mathbf{x}_\mathrm{gw}, \mathbf{x}_\nu)$. We apply Bayes' rule to express this probability as

$$P(H_\mathrm{s}|\mathbf{x}_\mathrm{gw}, \mathbf{x}_\nu) = \frac{P(\mathbf{x}_\mathrm{gw}, \mathbf{x}_\nu|H_\mathrm{s})P(H_\mathrm{s})}{P(\mathbf{x}_\mathrm{gw}, \mathbf{x}_\nu)}, \tag{4}$$

Since we are interested in the ratio of such probabilities for different hypotheses, the denominator above will cancel out. We therefore omit its computation. To obtain the first term in the numerator, we write

$$P(\mathbf{x}_{\rm gw},\mathbf{x}_\nu|H_{\rm s}) = \int P(\mathbf{x}_{\rm gw},\mathbf{x}_\nu|\boldsymbol{\theta},H_{\rm s})P(\boldsymbol{\theta}|H_{\rm s})d\boldsymbol{\theta} \quad (5)$$

Since $\mathbf{x}_{\rm gw}$ and $\mathbf{x}_\nu$ are both dependent on $\boldsymbol{\theta}$ but are otherwise can be considered independent of each other, we can write

$$P(\mathbf{x}_{\rm gw},\mathbf{x}_\nu|\boldsymbol{\theta},H_{\rm s}) = P(\mathbf{x}_{\rm gw}|\boldsymbol{\theta},H_{\rm s})P(\mathbf{x}_\nu|\boldsymbol{\theta},H_{\rm s}) \quad (6)$$

We now specify the independent elements of Eqs. 4-6 in the context of our astrophysical and detection models.

A. Parameter priors ($H_{\rm s}$)

There are two prior probabilities that we need to compute in our signal hypothesis: $P(\boldsymbol{\theta}|H_{\rm s})$ and $P(H_{\rm s})$.

We first discuss the prior probability distribution of the parameters, $P(\boldsymbol{\theta}|H_{\rm s})$, which appears in Eq. 5 above and also appears on other occasions below. Here we review the role of each source parameter.

1. **Time ($t_{\rm s}$):** We assume that a signal is equally likely to occur at any time during an observation period. We further assume that no other parameter depends on the time of observation, therefore we can treat this probability independently. Taking the duration of the observation period to be $T_{\rm obs}$, the resulting prior probability distribution is

$$P(t_{\rm s}|H_{\rm s}) = \frac{1}{T_{\rm obs}}. \quad (7)$$

2. **Source distance (r):** We assume a uniform distribution of sources in the local universe, and that sources are not at cosmological distances due to the relevant distance range of current GW observatories. We further assume that a GW signal can be detected if its root-sum-squared GW strain $h_{\rm rss}$ is above a detection threshold $h_{\rm rss,0}$ [61]. The probability density that an observed GW+neutrino event occurred at distance r is independent of r since the volume in space in the distance range $[r, r+dr]$ is $\propto r^2dr$, but the probability of detecting a neutrino from the source falls as r^{-2}. This uniform distribution is valid up to the GW distance range $r_0 f_{\rm A}(\mathbf{\Omega}, t_{\rm GW})$ beyond which sources are not detected. Here, r_0 is the GW detection range for optimal source direction, and $f_{\rm A}(\mathbf{\Omega}, t_{\rm GW})$ is the antenna pattern of the GW detector network. The latter is the ratio of the projected GW amplitude in the detector and the actual amplitude at Earth. The range r_0 satisfies $r_0(E_{\rm gw}) \propto E_{\rm gw}^{1/2}$.

3. **Energy ($E_{\rm gw}$ and E_ν):** We need to specify our dependency to energies. A naive choice can be independent log-uniform distributions over the energy ranges $[E^-_{\rm gw}, E^+_{\rm gw}], [E^-_\nu, E^+_\nu]$ with probability density

$$\begin{aligned} P(E_{\rm gw}, E_\nu|H_{\rm s}) &= P(E_{\rm gw}|H_{\rm s})P(E_\nu|H_{\rm s}) \\ &= (E_{\rm gw}E_\nu \log(\frac{E^+_{\rm gw}}{E^-_{\rm gw}})\log(\frac{E^+_\nu}{E^-_\nu}))^{-1} \end{aligned} \quad (8)$$

Throughout the paper instead of the expression for the specific log-uniform model, $P(E_{\rm gw}, E_\nu|H_{\rm s})$ will be used to express the universality of the method.

Overall, we find

$$P(\boldsymbol{\theta}|H_{\rm s}) = \begin{cases} \frac{P(E_{\rm gw},E_\nu|H_{\rm s})}{T_{\rm obs}N_1} & \text{if } r \le r_0(E_{\rm gw})f_{\rm A}(\mathbf{\Omega}, t_{\rm gw}) \\ 0 & \text{otherwise} \end{cases} \quad (9)$$

with suitable normalization constant N_1. Next, we consider the prior $P(H_{\rm s})$. This probability should depend on the expected detection rate of multi-messenger events, which is proportional to the volume within which a GW signal can be detected, the probability of detecting a neutrino from the event, and the neutrino beaming factor $f_{{\rm b},\nu}$. We assume here that within the neutrino beam, detection probability is constant at a fixed distance, and it is zero outside. The expected number of detected neutrinos can be written as

$$\langle n_\nu(E_\nu, r)\rangle = n_{\nu,51,100}\left(\frac{E_\nu}{10^{51}\,\text{erg}}\right)\left(\frac{r}{100\,\text{Mpc}}\right)^{-2} \quad (10)$$

Here, $n_{\nu,51,100}$ is a detector specific parameter. It is $n_{\nu,51,100} \approx 250$ for IceCube [62]. The rate of GW+neutrino detection will then be

$$R_{det} = \dot{n}_{{\rm gw}+\nu}f^{-1}_{b,\nu}\int_{E^-_\nu}^{E^+_\nu}dE_\nu\int_{E^-_{\rm gw}}^{E^+_{\rm gw}}dE_{\rm gw}P(E_{\rm gw},E_\nu|H_{\rm s})\int d\mathbf{\Omega}\int_0^{r_0(E_{\rm gw})f_{\rm A}(\mathbf{\Omega},t_{\rm gw})}dr\,r^2[1-\text{Poiss}(0,\langle n_\nu(E_\nu,r)\rangle)] \quad (11)$$

where the $d\mathbf{\Omega}$ integral is over the whole sky, $\dot{n}_{{\rm gw}+\nu}$ is a constant such that $\dot{n}_{{\rm gw}+\nu}P(E_{\rm gw},E_\nu|H_{\rm s})$ corresponds to the differential rate density of GW+neutrino events, and $\text{Poiss}(k,\lambda)$ is the Poisson probability density function with λ mean and k observed events. Although the rate seems to be dependent on detection time, it isn't due to the fact that time only rotates the antenna pattern and we integrate over the whole sky. The prior probability will be

$$P(H_\mathrm{s}) = \frac{1}{N_2} R_\mathrm{det} \tag{12}$$

with suitable normalization constant N_2. This constant will be the same for our null hypothesis and will be cancelled out, therefore its actual value is not relevant.

B. Gravitational waves (H_s)

We now consider the probability $P(\mathbf{x}_\mathrm{gw}|\boldsymbol{\theta}, H_\mathrm{s})$. We assume that the GW observables are only connected through the source parameters and are otherwise independent. This assumption is valid if we neglect the variation of the GW antenna pattern within the GW skymap. While this is only approximately true for most cases and its accuracy decreases for larger skymaps, this assumption simplifies the calculations below. For independent variables, we have:

$$P(\mathbf{x}_\mathrm{gw}|\boldsymbol{\theta}, H_\mathrm{s}) = P(t_\mathrm{gw}|\boldsymbol{\theta}, H_\mathrm{s}) P(\rho_\mathrm{gw}|\boldsymbol{\theta}, H_\mathrm{s}) P(\mathcal{P}_\mathrm{gw}|\boldsymbol{\theta}, H_\mathrm{s}) \tag{13}$$

The term $P(t_\mathrm{gw}|\boldsymbol{\theta}, H_\mathrm{s})$ should only depend on the difference $t_\mathrm{gw} - t_\mathrm{s}$. We adopt the model-agnostic assumption that the probability $P(t_\mathrm{gw}|t_\mathrm{s}, H_\mathrm{s})$ is uniform within a time window $t_\mathrm{gw} - t_\mathrm{s} \in [t^-_\mathrm{gw}, t^+_\mathrm{gw}]$ for suitable parameters t^-_gw and t^+_gw and is zero elsewhere:

$$P(t_\mathrm{gw}|t_\mathrm{s}, H_\mathrm{s}) = \begin{cases} (t^+_\mathrm{gw} - t^-_\mathrm{gw})^{-1} & \text{if } t_\mathrm{gw} - t_\mathrm{s} \in [t^-_\mathrm{gw}, t^+_\mathrm{gw}] \\ 0 & \text{otherwise} \end{cases} \tag{14}$$

For example, previous GW+neutrino searches used parameters $t^+_\mathrm{gw} = -t^-_\mathrm{gw} = 250\,\mathrm{s}$ [14, 20, 28, 30–32]. We assume that the other source parameters are independent of t_gw.

To understand the second term on the right hand side of Eq. 13, we make use of the fact that ρ_gw on average is proportional to the GW signal's amplitude at Earth, characterized by the root-sum-squared GW strain h_rss. Assuming here for simplicity that all gravitational waveforms are similar, the GW strain is fully determined by r, E_gw, $\boldsymbol{\Omega}$ and t_gw. The time dependence comes in due to Earth's rotation if we measure sky location in equatorial coordinates. Assuming that ρ_gw precisely describes h_rss, this term represents a constraint on the source parameters, which need to be such that they produce at Earth the h_rss value that corresponds to the measured ρ_gw value. This means that only the combination $E^{1/2}_\mathrm{gw} r^{-1} f_\mathrm{A}(\boldsymbol{\Omega}, t_\mathrm{gw})$ is constrained, where $f_\mathrm{A}(\boldsymbol{\Omega}, t_\mathrm{gw})$ is the direction-dependent antenna pattern of the GW detector network. This combination is proportional to the measured GW strain amplitude.

We therefore write the probability as a constraint

$$P(\rho_\mathrm{gw}|\boldsymbol{\theta}, H_\mathrm{s}) = \delta\left[\rho_\mathrm{gw} - \kappa_0 E^{1/2}_\mathrm{gw} r^{-1} f_\mathrm{A}(\boldsymbol{\Omega}, t_\mathrm{gw})\right] \tag{15}$$

Where δ is the Dirac-delta and κ_0 is an appropriate constant that depends on the GW search algorithm and needs to be empirically determined.

Next, we look at the term $P(\mathcal{P}_\mathrm{gw}|\boldsymbol{\theta}, H_\mathrm{s})$. Using Bayes' theorem, we write

$$P(\mathcal{P}_\mathrm{gw}|\boldsymbol{\theta}, H_\mathrm{s}) = \frac{P(\boldsymbol{\theta}|\mathcal{P}_\mathrm{gw}, H_\mathrm{s}) P(\mathcal{P}_\mathrm{gw}|H_\mathrm{s})}{P(\boldsymbol{\theta}|H_\mathrm{s})} \tag{16}$$

Here, assuming that our reconstructed GW skymap is accurate, we have

$$P(\boldsymbol{\theta}|\mathcal{P}_\mathrm{gw}, H_\mathrm{s}) = \frac{\mathcal{P}_\mathrm{gw}(\boldsymbol{\Omega})}{T_{obs} N_1} \begin{cases} P(E_\mathrm{gw}, E_\nu|H_\mathrm{s}) & \text{if } r \le r_0(E_\mathrm{gw}) f_\mathrm{A}(\boldsymbol{\Omega}, t_\mathrm{gw}) \\ 0 & \text{otherwise} \end{cases}, \tag{17}$$

since the skymap determines the probability of the signal coming from a given sky location $\boldsymbol{\Omega}$. We assumed that no parameter other than $\boldsymbol{\Omega}$ is constrained by the GW skymap. The distance term here arises similarly to as it did for $P(\boldsymbol{\theta}|H_\mathrm{s})$ in Eq. 9.

Regarding $P(\mathcal{P}_\mathrm{gw}|H_\mathrm{s})$, we assume that the distribution of $\mathcal{P}_\mathrm{gw}$ is independent of the underlying hypothesis, i.e. $P(\mathcal{P}_\mathrm{gw}|H_\mathrm{s}) = P(\mathcal{P}_\mathrm{gw})$. This term appears for the alternative hypothesis as well, it cancels out and therefore we can ignore it here.

Putting everything together, we have for the GW term

$$P(\mathbf{x}_{\rm gw}|\boldsymbol{\theta}, H_{\rm s}) = \delta\left[\rho_{\rm gw} - \kappa_0 E_{\rm gw}^{1/2} r^{-1} f_{\rm A}(\boldsymbol{\Omega}, t_{\rm gw})\right] \mathcal{P}_{\rm gw}(\boldsymbol{\Omega}) \cdot \begin{cases} (t_{\rm gw}^{+} - t_{\rm gw}^{-})^{-1} & \text{if } t_{\rm gw} - t_{\rm s} \in [t_{\rm gw}^{-}, t_{\rm gw}^{+}] \\ 0 & \text{otherwise} \end{cases} \tag{18}$$

C. High-energy neutrinos ($H_{\rm s}$)

We now turn our attention to the high-energy neutrino term $P(\mathbf{x}_\nu|\boldsymbol{\theta}, H_{\rm s})$ in Eq. 6. We treat the temporal term similarly to the GW case. We assume that the time difference $t_\nu - t_{\rm s}$ is the only relevant temporal value. We further use a uniform probability density within the time interval $[t_\nu^-, t_\nu^+]$, and 0 outside:

$$P(t_\nu|t_{\rm s}, H_{\rm s}) = \begin{cases} (t_\nu^{+} - t_\nu^{-})^{-1} & \text{if } t_\nu - t_{\rm s} \in [t_\nu^{-}, t_\nu^{+}] \\ 0 & \text{otherwise} \end{cases} . \tag{19}$$

The remaining neutrino observables, $\boldsymbol{\Omega}_\nu$, σ_ν and ϵ_ν, are not independent. The sensitivity of neutrino detectors varies with both energy and sky location, and localization accuracy depends on source direction and energy.

Let us take the remaining neutrino term $P(\boldsymbol{\Omega}_\nu, \sigma_\nu, \epsilon_\nu|r, \boldsymbol{\Omega}, E_\nu, H_{\rm s})$. We assume that the signal distribution of ϵ_ν follows a power law, therefore the neutrino spectrum is independent of the source distance. Such a power-law distribution is typical in neutrino emission models [63]. Consequently, parameters r and E_ν do not affect the probability here.

The directional uncertainty variable σ_ν depends on ϵ_ν as well as $\boldsymbol{\Omega}_\nu$. As most information on the signal is included in the reconstructed ϵ_ν as well as $\boldsymbol{\Omega}_\nu$ values, we ignore ϵ_ν here.

For the remaining observables and parameters, we use the chain rule to write

$$P(\boldsymbol{\Omega}_\nu, \epsilon_\nu|\boldsymbol{\Omega}, H_{\rm s}) = P(\epsilon_\nu|\boldsymbol{\Omega}_\nu, \boldsymbol{\Omega}, H_{\rm s}) P(\boldsymbol{\Omega}_\nu|\boldsymbol{\Omega}, H_{\rm s}) \tag{20}$$

The neutrino energy reconstruction does not depend on the reconstructed direction since the angular distance between $\boldsymbol{\Omega}_\nu$ and $\boldsymbol{\Omega}$ is expected to be small compared to the scale on which energy dependence is relevant. Therefore, the first term on the right in Eq. 20 does not depend on $\boldsymbol{\Omega}_\nu$.

Given the source direction as a parameter, the probability of reconstructing ϵ_ν for a detected neutrino depends on the energy- and direction-dependent effective area $A_{\rm eff}(\epsilon_\nu, \boldsymbol{\Omega})$ of the neutrino detector, as well as the source power spectral density. Here we ignore the difference between true and reconstructed energy when calculating the effective area as this should not significantly change its value. We take the neutrino spectral density to be $dN_\nu/d\epsilon_\nu \propto \epsilon_\nu^{-2}$, which is the standard spectrum expected from Fermi processes [63]. With these dependences, we write

$$P(\epsilon_\nu|\boldsymbol{\Omega}_\nu, \boldsymbol{\Omega}, H_{\rm s}) = \frac{1}{N_\epsilon} A_{\rm eff}(\epsilon_\nu, \boldsymbol{\Omega}) \epsilon_\nu^{-2}, \tag{21}$$

where

$$N_\epsilon = \int d\boldsymbol{\Omega} \int_{\epsilon_{\rm min}}^{\infty} \epsilon_\nu^{-2} A_{\rm eff}(\epsilon_\nu, \boldsymbol{\Omega}) d\epsilon_\nu \tag{22}$$

where the $d\boldsymbol{\Omega}$ integral is over all sky and $\epsilon_{\rm min}$ is the minimum reconstructable energy.

For the second term on the right-hand side in Eq. 20, we adopt the Normal distribution [64]

$$P(\boldsymbol{\Omega}_\nu|\boldsymbol{\Omega}, H_{\rm s}) = \frac{1}{2\pi\sigma_\nu^2} {\rm e}^{-\frac{|\boldsymbol{\Omega}_\nu - \boldsymbol{\Omega}|^2}{2\sigma_\nu^2}} \tag{23}$$

Putting everything together, we have for the neutrino term

$$P(\mathbf{x}_\nu|\boldsymbol{\theta}, H_{\rm s}) = \frac{1}{N_\epsilon} A_{\rm eff}(\epsilon_\nu, \boldsymbol{\Omega}) \epsilon_\nu^{-2} \frac{1}{2\pi\sigma_\nu^2} {\rm e}^{-\frac{|\boldsymbol{\Omega}_\nu - \boldsymbol{\Omega}|^2}{2\sigma_\nu^2}} \cdot \begin{cases} (t_\nu^{+} - t_\nu^{-})^{-1} & \text{if } t_\nu - t_{\rm s} \in [t_\nu^{-}, t_\nu^{+}] \\ 0 & \text{otherwise} \end{cases} \tag{24}$$

D. Combination of probabilities ($H_{\rm s}$)

We can combine the above results to obtain the probability of the joint event being a signal by taking Eqs. 18 and 24 and substituting them into Eq. 6. We then substitute Eq. 6 into Eq. 5.

To solve Eq. 5, we further substitute $P(\boldsymbol{\theta}|H_{\rm s})$ from Eq. 9. Finally, we can substitute Eq. 5 and Eq. 12 into Eq. 4, obtaining $P(H_{\rm s}|\mathbf{x}_{\rm gw}, \mathbf{x}_\nu)$, other than the factor $P(\mathbf{x}_{\rm gw}, \mathbf{x}_\nu)$ which will cancel out in comparison to the alternative hypothesis.

IV. NULL HYPOTHESIS

We now move to our null hypothesis H_0. Given the observational data, the probability of the null hypothesis being true can be written as $P(H_0|\mathbf{x}_{\rm gw}, \mathbf{x}_\nu)$. We apply

Bayes' rule to express this probability as

$$P(H_0|\mathbf{x}_{\rm gw},\mathbf{x}_\nu) = \frac{P(\mathbf{x}_{\rm gw},\mathbf{x}_\nu|H_0)P(H_0)}{P(\mathbf{x}_{\rm gw},\mathbf{x}_\nu)}, \tag{25}$$

The denominator here will cancel out with the same denominator in the signal hypothesis, therefore we do not need to further consider it. To compute the first term in the numerator on the right-hand side, we write

$$P(\mathbf{x}_{\rm gw},\mathbf{x}_\nu|H_0) = \int P(\mathbf{x}_{\rm gw},\mathbf{x}_\nu|\boldsymbol{\theta}_0,H_0)P(\boldsymbol{\theta}_0|H_0)d\boldsymbol{\theta}_0, \tag{26}$$

where the parameter vector for our null hypothesis only has one parameter, the time of the background event:

$$\boldsymbol{\theta}_0 = \{t_0\}. \tag{27}$$

Since the background events for GW and neutrino observations are independent, we can write

$$P(\mathbf{x}_{\rm gw},\mathbf{x}_\nu|\boldsymbol{\theta}_0,H_0) = P(\mathbf{x}_{\rm gw}|\boldsymbol{\theta}_0,H_0)P(\mathbf{x}_\nu|\boldsymbol{\theta}_0,H_0) \tag{28}$$

We will now specify the independent elements of Eqs. 25, 26 and 28 in the context of our background model.

A. Parameter priors (H_0)

There are two prior probabilities that we need to compute in our null hypothesis: $P(\boldsymbol{\theta}_0|H_0)$ and $P(H_0)$.

We first discuss the prior probability distribution of the parameters, $P(\boldsymbol{\theta}_0|H_0)$, which appears in Eq. 26 above. We assume that a background event is equally likely to occur at any time during the observation period. This gives us

$$P(t_0|H_0) = \frac{1}{T_{\rm obs}}. \tag{29}$$

which is the same as our Eq. 7 for the corresponding prior in the signal hypothesis.

Next, we consider $P(H_0)$. This probability depends on the expected detection rate of background events, which in turn is a combination of the rate of GW and neutrino background events separately. Considering background rates for the GW and neutrino channel to be $R_{\rm gw,bg}$ and $R_{\nu,\rm bg}$ we have the proportionality

$$P(H_0) = \frac{1}{N_2}R_{\rm bg,det} = \frac{1}{N_2}R_{\rm gw,bg}R_{\nu,\rm bg}(t_{\rm gw}^+ + t_\nu^+ - t_{\rm gw}^- - t_\nu^-), \tag{30}$$

where $R_{\rm bg,det}$ is the expected false multi-messenger detection rate from background events. The normalization factor N_2 will cancel out with the same factor in the signal hypothesis, see Eq. 12.

B. Gravitational waves (H_0)

We now consider the GW component $P(\mathbf{x}_{\rm gw}|\boldsymbol{\theta}_0,H_0)$. We assume that the measured GW parameters for the background are independent. We can then define the probabilities of measuring each parameter independently:

$$P(\mathbf{x}_{\rm gw}|\boldsymbol{\theta}_0,H_0) = P(t_{\rm gw}|\boldsymbol{\theta}_0,H_0)P(\mathcal{P}_{\rm gw}|\boldsymbol{\theta}_0,H_0)P(\rho_{\rm gw}|\boldsymbol{\theta}_0,H_0) \tag{31}$$

Starting with the first term on the right-hand side, we expect the probability distribution of detection time for a background event to be independent of time, therefore we adopt a uniform distribution within the allowed time window. The allowed time window is defined by what time difference between a GW and neutrino signal candidate we analyze for coincidence. We therefore have

$$P(t_{\rm gw}|t_0,H_0) = \begin{cases}(t_{\rm gw}^+ - t_{\rm gw}^-)^{-1} & \text{if } t_{\rm gw}-t_0 \in [t_{\rm gw}^-, t_{\rm gw}^+] \\ 0 & \text{otherwise}\end{cases} \tag{32}$$

Since we considered a signal model which also assumes uniform distribution within the allowed time window, we have $P(t_{\rm gw}|t_0,H_0) = P(t_{\rm gw}|t_{\rm s},H_{\rm s})$.

The distribution of $\rho_{\rm gw}$ depends on the detector properties as well as the properties of the reconstruction algorithm. We therefore estimate this distribution empirically, using observed $\rho_{\rm gw}$ from the background, obtained by time shifting data between multiple GW observatories and carrying out the full analysis algorithm over this time shifted data. We denote the empirically established distribution of $\rho_{\rm gw}$ with $P_{\rm emp}(\rho_{\rm gw}|H_0)$.

Considering the term $P(\mathcal{P}_{\rm gw}|\boldsymbol{\theta}_0,H_0)$, the GW skymap should not depend on the time of the event. In addition, we do not have any prior information on $P(\mathcal{P}_{\rm gw}|H_0)$, therefore we assume that it is independent of $\mathcal{P}_{\rm gw}$. Since there is a similar term in our signal hypothesis, these two cancel out. We therefore ignore this term in the following.

Putting everything together, we have for the background GW term

$$P(\mathbf{x}_{\rm gw}|\boldsymbol{\theta}_0,H_0) = P_{\rm emp}(\rho_{\rm gw}|H_0)\begin{cases}(t_{\rm gw}^+ - t_{\rm gw}^-)^{-1} & \text{if } t_{\rm gw}-t_0 \in [t_{\rm gw}^-, t_{\rm gw}^+] \\ 0 & \text{otherwise}\end{cases} \tag{33}$$

C. High-energy neutrinos (H_0)

Next, we examine $P(\mathbf{x}_\nu|\boldsymbol{\theta}_0, H_0)$ in Eq. 28. We first separate the temporal term which we assume to be independent of the other parameters. We assume that the time of arrival of a background neutrino signal is time-independent, and take a uniform probability distribution within the allowed time window. The allowed time window is defined by what time difference between a neutrino signal candidate we analyze for coincidence. We therefore have

$$P(t_\nu|t_0, H_0) = \begin{cases} (t_\nu^+ - t_\nu^-)^{-1} & \text{if } t_\nu - t_0 \in [t_\nu^-, t_\nu^+] \\ 0 & \text{otherwise} \end{cases} \tag{34}$$

Since our signal model also considers a uniform distribution within the allowed time window, we have $P(t_\nu|t_0, H_0) = P(t_\nu|t_\mathrm{s}, H_\mathrm{s})$.

The remaining measured parameters will not be independent of each other. In particular, the reconstructed neutrino direction and energy are interconnected. For simplicity, we ignore the directional uncertainty parameter σ_ν, as it is essentially determined by the reconstructed direction and the reconstructed neutrino energy.

We therefore need to examine the probability $P(\boldsymbol{\Omega}_\nu, \epsilon_\nu|\boldsymbol{\theta}_0, H_0)$. Given a sufficient number of observed background events, this probability can be estimated empirically using observed data. Let $\{\boldsymbol{\Omega}_{\nu,\mathrm{i}}, \epsilon_{\nu,\mathrm{i}}\}, i \in N_{\nu,\mathrm{obs}}$ be the reconstructed parameters observed set of $N_{\nu,\mathrm{obs}}$ neutrino candidates. We then have the empirical estimate

$$\begin{aligned} P_\mathrm{emp}(\boldsymbol{\Omega}_\nu, \epsilon_\nu|H_0) = \frac{1}{N_{\nu,\mathrm{obs}}} \frac{4}{\Delta_{\boldsymbol{\Omega}}^2} \frac{\epsilon_\nu^{(\max)} - \epsilon_\nu^{(\min)}}{2\Delta_\epsilon} \\ \sum_{i \in N_{\nu,\mathrm{obs}}} \left[|\boldsymbol{\Omega}_\nu - \boldsymbol{\Omega}_{\nu,\mathrm{i}}| < \Delta_{\boldsymbol{\Omega}} \;\; \& \;\; |\epsilon_\nu - \epsilon_{\nu,\mathrm{i}}| < \Delta_\epsilon \right], \end{aligned} \tag{35}$$

where we use here the bracket notation such that $[P]$ is 1 if P is true and 0 if P is false. We further introduced constants Δ_Ω and Δ_ϵ, which should be selected such that the uncertainty on the probability estimate is minimal. The values $\epsilon_\nu^{(\max)}$ and $\epsilon_\nu^{(\min)}$ correspond to the maximum and minimum values of the detectors sensitive energy band, respectively.

For detectors such as IceCube, one can effectively treat the reconstructed energy dependent only on the reconstructed declination, and not the right ascension. This simplifies the above empirical probability density.

Putting everything together, we have for the background neutrino term

$$P(\mathbf{x}_\nu|\boldsymbol{\theta}_0, H_0) = P_\mathrm{emp}(\boldsymbol{\Omega}_\nu, \epsilon_\nu|H_0) \begin{cases} (t_\nu^+ - t_\nu^-)^{-1} & \text{if } t_\nu - t_0 \in [t_\nu^-, t_\nu^+] \\ 0 & \text{otherwise.} \end{cases} \tag{36}$$

D. Combination of probabilities (H_0)

We can combine the above results to obtain the probability of the joint event being from the background by taking Eqs. 33 and 36 and substituting them into Eq. 28. Then Eq. 28 along with Eq. 29 should be substituted into Eq. 26. Finally, Eq. 26 and Eq. 30 should be substituted into Eq. 25. Eq. 25 will miss a normalization factor both from Eq. 30 and from the denominator on the right side, both of which cancel out upon calculating the Bayes factor.

V. CHANCE COINCIDENCE HYPOTHESIS

We finally calculate the probability for the chance coincidence hypothesis H_c. Given the observational data the probability of the chance coincidence hypothesis being true can be written as $P(H_\mathrm{c}|\mathbf{x}_\mathrm{gw}, \mathbf{x}_\nu)$. H_c can be separated into two parts, one of which considers a background neutrino event and a foreground gravitational wave event denoted by H_c^gw; and the other one considers a background gravitational wave event and a foreground neutrino event denoted by H_c^ν. Since these two cases are mutually exclusive and complementary to each other for the chance coincidence hypothesis we can write $P(H_\mathrm{c}|\mathbf{x}_\mathrm{gw}, \mathbf{x}_\nu) = P(H_\mathrm{c}^\mathrm{gw}|\mathbf{x}_\mathrm{gw}, \mathbf{x}_\nu) + P(H_\mathrm{c}^\nu|\mathbf{x}_\mathrm{gw}, \mathbf{x}_\nu)$. We again apply Bayes' rule

$$\begin{aligned} P(H_\mathrm{c}|\mathbf{x}_\mathrm{gw}, \mathbf{x}_\nu) &= P(H_\mathrm{c}^\mathrm{gw}|\mathbf{x}_\mathrm{gw}, \mathbf{x}_\nu) + P(H_\mathrm{c}^\nu|\mathbf{x}_\mathrm{gw}, \mathbf{x}_\nu) \\ &= \frac{P(\mathbf{x}_\mathrm{gw}, \mathbf{x}_\nu|H_\mathrm{c}^\mathrm{gw})P(H_\mathrm{c}^\mathrm{gw}) + P(\mathbf{x}_\mathrm{gw}, \mathbf{x}_\nu|H_\mathrm{c}^\nu)P(H_\mathrm{c}^\nu)}{P(\mathbf{x}_\mathrm{gw}, \mathbf{x}_\nu)} \end{aligned} \tag{37}$$

Here the denominator again cancels out with the same denominator in the signal and null hypotheses. Now consider the first term in the numerator

$$\begin{aligned} P(\mathbf{x}_\mathrm{gw}, \mathbf{x}_\nu|H_\mathrm{c}^\mathrm{gw}) = \\ \int P(\mathbf{x}_\mathrm{gw}, \mathbf{x}_\nu|\boldsymbol{\theta}_\mathrm{c}^\mathrm{gw}, H_\mathrm{c}^\mathrm{gw})P(\boldsymbol{\theta}_\mathrm{c}^\mathrm{gw}|H_\mathrm{c}^\mathrm{gw})d\boldsymbol{\theta}_\mathrm{c}^\mathrm{gw} \end{aligned} \tag{38}$$

where

$$\boldsymbol{\theta}_\mathrm{c}^\mathrm{gw} = \{t_\mathrm{c}^\mathrm{gw}, r, \boldsymbol{\Omega}, E_\mathrm{gw}\} \tag{39}$$

Here, t_c^gw is the time of the GW signal and the other parameters are the same as defined in Section III. Since a background neutrino event and a foreground gravitational wave event are independent, we can write

$$P(\mathbf{x}_{\rm gw},\mathbf{x}_\nu|\boldsymbol{\theta}_{\rm c}^{\rm gw},H_{\rm c}^{\rm gw}) = P(\mathbf{x}_{\rm gw}|\boldsymbol{\theta}_{\rm c}^{\rm gw},H_{\rm c}^{\rm gw})P(\mathbf{x}_\nu|\boldsymbol{\theta}_{\rm c}^{\rm gw},H_{\rm c}^{\rm gw}) \tag{40}$$

Similarly for the second term in Eq. 37

$$P(\mathbf{x}_{\rm gw},\mathbf{x}_\nu|H_{\rm c}^{\nu}) = \int P(\mathbf{x}_{\rm gw},\mathbf{x}_\nu|\boldsymbol{\theta}_{\rm c}^{\nu},H_{\rm c}^{\nu})P(\boldsymbol{\theta}_{\rm c}^{\nu}|H_{\rm c}^{\nu})d\boldsymbol{\theta}_{\rm c}^{\nu} \tag{41}$$

where $\boldsymbol{\theta}_{\rm c}^{\nu}$ vector has the components

$$\boldsymbol{\theta}_{\rm c}^{\nu} = \{t_{\rm c}^{\nu}, r, \boldsymbol{\Omega}, E_\nu\} \tag{42}$$

$t_{\rm c}^{\nu}$ is the time of arrival of the neutrino signal and the other parameters are the same as defined in Section III. Since a background gravitational wave event and a foreground neutrino event are independent, we can write

$$P(\mathbf{x}_{\rm gw},\mathbf{x}_\nu|\boldsymbol{\theta}_{\rm c}^{\nu},H_{\rm c}^{\nu}) = P(\mathbf{x}_{\rm gw}|\boldsymbol{\theta}_{\rm c}^{\nu},H_{\rm c}^{\nu})P(\mathbf{x}_\nu|\boldsymbol{\theta}_{\rm c}^{\nu},H_{\rm c}^{\nu}) \tag{43}$$

A. Parameter priors ($H_{\rm c}^{\rm gw}$)

Here our prior $P(\boldsymbol{\theta}|H_{\rm c}^{\rm gw})$ is the same $P(\boldsymbol{\theta}|H_{\rm s})$ in Section III A, since both only depend on the gravitational wave signal parameters and not the neutrino parameters:

$$P(\boldsymbol{\theta}^{\rm gw}|H_{\rm c}^{\rm gw}) = \begin{cases} \frac{r^2 P(E_{\rm gw})}{T_{\rm obs}N_1} & \text{if } r \le r_0(E_{\rm gw})f_{\rm A}(\boldsymbol{\Omega},t_{\rm gw}) \\ 0 & \text{otherwise} \end{cases} \tag{44}$$

Next we calculate the prior $P(H_{\rm c}^{\rm gw})$, which is proportional to the gravitational wave rate and background neutrino rate $R_{\nu,\rm bg}$:

$$P(H_{\rm c}^{\rm gw}) = \frac{1}{N_2}R_{\nu,\rm bg}(t_{\rm gw}^{+}+t_{\nu}^{+}-t_{\rm gw}^{-}-t_{\nu}^{-})\dot{n}_{\rm gw+\nu}\int_{E_{\rm gw}^{-}}^{E_{\rm gw}^{+}} dE_{\rm gw}P(E_{\rm gw}|H_{\rm s})\int d\boldsymbol{\Omega}\int_0^{r_0(E_{\rm gw})f_{\rm A}(\boldsymbol{\Omega},t_{\rm gw})} dr\, r^2. \tag{45}$$

B. Parameter priors ($H_{\rm c}^{\nu}$)

Similar to Eq. 7 we have uniform probability in time

$$P(t_{\rm c}^{\nu}|H_{\rm c}^{\nu}) = \frac{1}{T_{\rm obs}} \tag{46}$$

In the limiting case in which the expected number of detected neutrinos from a source is $\ll 1$, the probability of detecting a source is proportional to r^{-2}. Since the number of sources within a $[r, r+dr]$ shell is proportional to r^2, these two factors cancel out and $P(r|H_{\rm c}^{\nu}) = \text{const}$. We consider this uniform distribution up to a maximum value $r_{\nu,\rm max}$, which corresponds to the maximum distance at which sources could be detectable via GWs, therefore astrophysical neutrinos coming from beyond this distance are in effect background. While there is no actual GW source in the hypothesis $H_{\rm c}^{\nu}$, we are not concerned with neutrinos from sources beyond the GW horizon distance here as they are not relevant for the joint search. In the same limiting case, $P(\boldsymbol{\theta}_{\rm c}^{\nu}|H_{\rm c}^{\nu})$ does not depend on E_ν. Overall we have

$$P(\boldsymbol{\theta}_{\rm c}^{\nu}|H_{\rm c}^{\nu}) = \begin{cases} \frac{P(E_\nu)}{T_{obs}N_3} & \text{if } r \le r_{\nu,\rm max} \\ 0 & \text{otherwise} \end{cases} \tag{47}$$

with suitable N_3 normalization constant.

Next we consider $P(H_{\rm c}^{\nu})$, which depends on the rate of chance coincidence of nearby astrophysical neutrinos and false gravitational wave detection rate $R_{\rm gw,bg}$:

$$P(H_{\rm c}^{\nu}) = \frac{1}{N_2}R_{\rm gw,bg}\dot{n}_{\rm gw+\nu}f_b^{-1}(t_{\rm gw}^{+}+t_{\nu}^{+}-t_{\rm gw}^{-}-t_{\nu}^{-})\int_{E_\nu^{-}}^{E_\nu^{+}} dE_\nu P(E_\nu|H_{\rm s})\int_0^{r_{\nu,\rm max}} dr 4\pi\, r^2[1-\text{Poiss}(0,\langle N_\nu(E_\nu,r)\rangle)] \tag{48}$$

C. Gravitational waves ($H_{\rm c}^{\rm gw}$)

The term $P(\mathbf{x}_{\rm gw}|\boldsymbol{\theta}_{\rm c}^{\rm gw},H_{\rm c}^{\rm gw})$ is equal to the same term for our signal hypothesis, i.e. $P(\mathbf{x}_{\rm gw}|\boldsymbol{\theta},H_{\rm s})$ (see Eq. 18), since in both cases there is a detected astrophysical gravitational wave signal. The different scaling of some terms with respect to r, which is different due to the origin of the coincident neutrino in the two cases, cancels out.

D. High Energy neutrinos ($H_{\rm c}^{\rm gw}$)

The term $P(\mathbf{x}_\nu|\boldsymbol{\theta}_{\rm c}^{\rm gw},H_{\rm c}^{\rm gw})$ is equal to the same term for our null hypothesis, i.e. $P(\mathbf{x}_\nu|\boldsymbol{\theta}_0,H_0)$ (see Eq. 36), since in both cases there is a background neutrino event, and neither term depends on the GW signal.

E. Gravitational waves (H_c^ν)

The term $P(\mathbf{x}_{\rm gw}|\boldsymbol{\theta}_{\rm c}^\nu, H_{\rm c}^\nu)$ is equal to the same term for our background hypothesis, i.e. $P(\mathbf{x}_{\rm gw}|\boldsymbol{\theta}_0, H_0)$ (see Eq. 33), since in both cases there is a GW false detection from the background, and neither term depends on the neutrino signal.

F. High Energy neutrinos (H_c^ν)

The term $P(\mathbf{x}_\nu|\boldsymbol{\theta}_{\rm c}^\nu, H_{\rm c}^\nu)$ is equal to the same term for our signal hypothesis, i.e. $P(\mathbf{x}_\nu|\boldsymbol{\theta}, H_{\rm s})$ (see Eq. 24), since in both cases there is a detected astrophysical neutrino, and neither term depends on the GW signal.

G. Combination of probabilities (H_c)

This is done similarly to the signal and null hypothesis cases.

VI. ODDS RATIO

We test our signal hypothesis using odds ratios. We compare our signal hypothesis against both null and coincident hypotheses

$$\mathcal{O}_{\rm gw+\nu} = \frac{P(H_{\rm s}|\mathbf{x}_{\rm gw}, \mathbf{x}_\nu)}{P(H_0|\mathbf{x}_{\rm gw}, \mathbf{x}_\nu) + P(H_{\rm c}|\mathbf{x}_{\rm gw}, \mathbf{x}_\nu)}. \quad (49)$$

This comparison will be applicable both for (i) GW and neutrino candidates that are not independently established detections, and for detections that are already confirmed through one channel. For the prior case, the first term in the denominator will dominate, while in the latter case it will be the second term.

We empirically characterize the required threshold values based on background data and simulations, similarly to [O1 paper]. This background comparison also allows us to determine a false alarm

rate for the given event, which can be reported to initiate electromagnetic follow-up observations.

VII. CONCLUSIONS

We presented a search algorithm for common sources of GWs and high-energy neutrinos based on Bayesian hypothesis testing. This algorithm upgrades the method of Baret et al. [20] that was used in most prior joint searches. The main advantages of the new method are that (i) it incorporates astrophysical priors about the source that help differentiate between signal and background, while being largely independent of the specific astrophysical model in consideration; (ii) it incorporates a more realistic model of the detector background, for example by taking into account the direction dependent background rate and energy distribution. These detector properties are straightforward to establish empirically, and the method presents a straightforward way to incorporate them as priors.

In the presentation of the method, we made simplifications that make the algorithm easier to implement and can make the computation faster. As an example, we assumed that all GW and neutrino sources emit the same energy. It will be useful to study how these simplifications affect the sensitivity of the search, and how much model dependence they introduce. This will be carried out in a future work.

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