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#### FIT 3181 Deep Learning

Week 01: Mathematics and Machine Learning Revisit

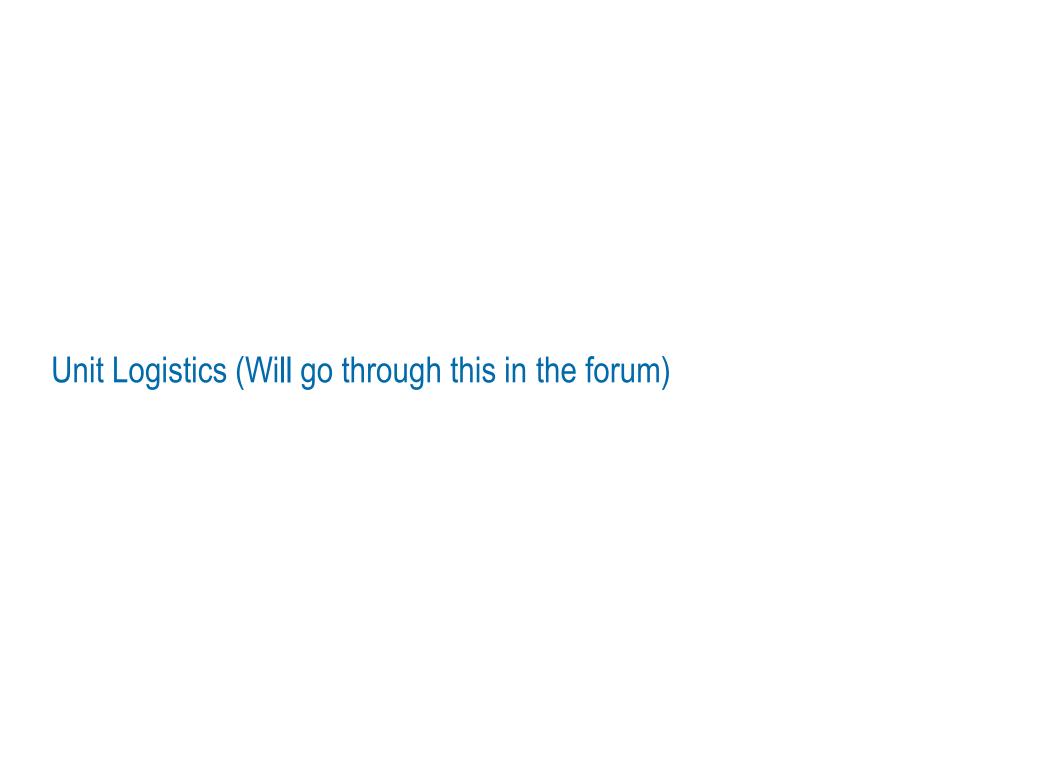
**Lecturer: Dr Lim Chern Hong** 

Email: lim.chernhong@monash.edu



## Outline

- Unit Logistics
- Mathematical Background Revisit
- Machine Learning Revisit
- Summary



FIT3181, 2022

#### FIT 3181 Team

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Trung

Lim



Thanh



Tony



Tuan



Md Mohaimenuzzaman



**James** 



Binh

## Lecture and consultation times

- <u>Lectures</u> for FIT 3181 will be pre-recorded
  - Please refer to the watch section in moodle
- Forum for FIT 3181 will be conducted via Zoom
  - Time: Tuesday 8:00 am 9:00 am
- Consultation will be conducted via Zoom (by appointment)
  - Please email to your tutor with heading [FIT3181]
- Important notes:
  - There will be 12 Forums in total, each is 1 hour long.
  - There will be 12 labs in total, each is 2 hours long.



# Tutorials (Lab)

- Make sure you have been allocated to <u>one</u> tutorial.
  - ☐ Three tutorial slots per week for FIT3181
- They will be on-campus
  - □ On-campus tutorial classes
    - ☐ Make sure you know the tutorial room
- If you miss a tutorial and would like to join the next (same) tutorial, you <u>must</u> email the tutor for permission and detail to attend.



#### Assessments

#### Tests, Assignments and Exam

- □ Two in-semester online tests: 10% each
  - During week 7 and week 12
  - Approximately 1 hour long, online test, questions will be randomised each time.
- Two assignments: 20% each
  - (Tentatively) due in week 7 and week 12 (Friday)
- One final exam (closed book): 40%
  - 10 minutes reading
  - Duration: 2 hours
- To pass this unit:
  - 45% or more in the final exam
  - 45% or more for assignments + in-semester tests.
  - An overall unit mark of 50% or more



## Assessments

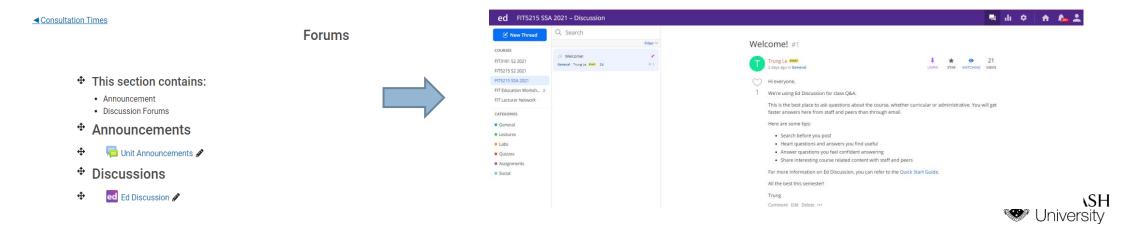
Tests, Assignments and Exam

- Every student is given a total quota of <u>4 days</u> for late submission
  - Use this quota wisely!
- □ Late submission
  - Subject to <u>marking penalty</u>
  - <u>5% deduction per</u> day once exceeding the quota
- Apply for extension:
  - Assignment extensions are only given when applied <u>before</u> the due date.
  - Supporting documents are <u>required</u>.
  - Maximum extension is 10 days.



## Communication

- Communication will be done mainly via emails and discussion (using Ed discussion via the link from Moodle site)
  - For FIT3181: email subject must start with "FIT3181: ..." and address to Dr Lim Chern Hong
    - If you don't follow this rule, your email might be <u>missed!</u>
  - I/we will normally answer your email within 2 working days
- Contact your lecturer, tutor for your concerns as soon as possible.
- Strongly recommend to use <u>Ed discussion</u> as much as possible!



# Learning objectives

- Fundamental elements of deep learning
- Basic and advanced concepts of machine learning, AI and deep learning
- Hand-on experience with practical deep learning models and framework

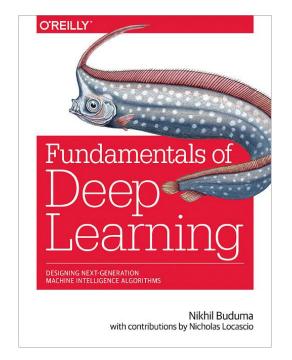


## Course structure

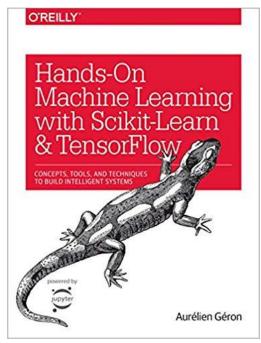
Wk	Lectures	Assessment
0	Revision and Preparation: Students are required to install Anaconda/Tensorflow before going to the tutorial 1	
1	Mathematics and Machine learning revisited	Assignment 1 released
2	Introduction to Deep Learning, from multilayer feedforward networks to modern deep neural networks (DNNs)	
3	Stochastic Gradient Descent and Optimization for Deep Learning	
4	Deep Learning for Vision (I): convolutional neural networks (CNNs)	
5	Training (very) deep networks:: optimizers and practical skills	
6	Deep Learning for Vision (II): network architectures, visualization, interpretability and robustness	
7	Deep learning for time-series and temporal data: RNNs and LSTMs	Online Quiz Test 1 (10%)  , Assignment 1 due (20%) Assignment 2 released
8	Representation and deep learning for language: word2vec, Slim and Glove	
	Semester Break	I
9	Seq2Seq and End-to-End Deep Models	
10	Autoencoder and Variational Autoencoder (VAE)	-
11	Deep generative models and GANs	Online Quiz Test 2 (10%), Assignment 2 due (20%)
12	Revision and Exam Information	



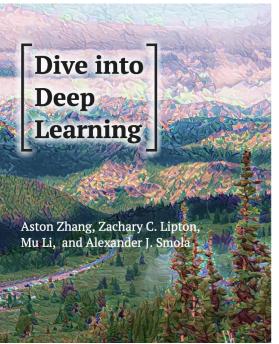
Textbook and References



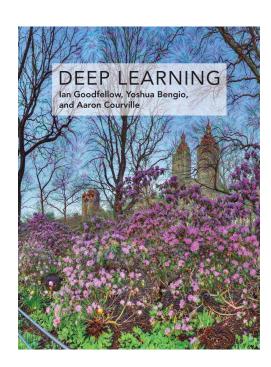
Introductory and hand-on







Intermediate and hand-on (but don't use tensorflow)



Advanced and theoretical

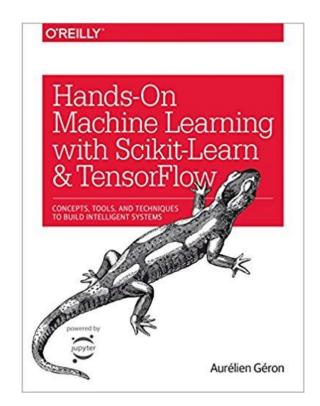
- No single textbook, some tutorial/lectures parts might use these books
- All of these books are <u>available</u> as online resources from Monash library.



Textbook and References

Machine Learning Landscape End-to-End Machine Learning Project Classification Training Models Part 1: Fundamentals of **Machine Learning** Support Vector Machines Decision Trees Ensemble Learning and Random Foreests Hand-on Machine Learning Dimensionality Reduction Scikit-Learn and TensorFlow Up and Running with TensorFlow (Geron, 2017) Introduction to Artificial Neural Networks Training Deep Neural Networks Distributing TensorFlow Across Devices and Servers Part 2: Neural Networks and Deep Learning Convolutional Neural Networks Recurrent Neural Networks Autoencoders Reinforcement Learning

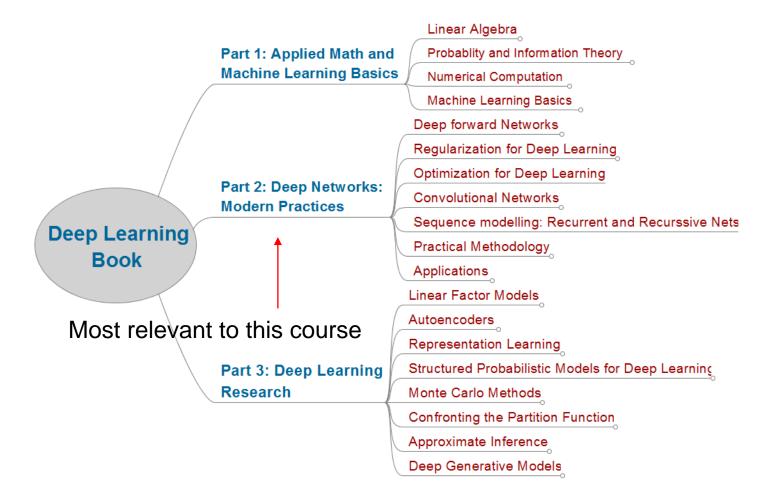
Deep learning part, using TF 2.x

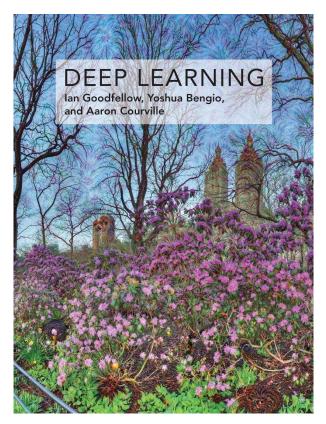


Intermediate and hand-on



#### Textbook and References





Advanced and theoretical



## Unit materials

#### Materials available in Moodle:

- Unit handbook
- Lecture slides and subsequent recorded lectures
- News and occasional required readings
- Tutorial and hand-on practical materials:
  - See Week 00 for preparation and revision: installation, anaconda, python, numpy, scikit-learn, etc.
  - Deep learning algorithms and models (DNN, CNNs, LSTM, Word2Vec, etc). Code is given for some of them.
  - Jupyter notebooks
  - ...
- Formal assessment instructions



#### How to succeed in this unit?

- Work on the hand-on tutorials and master them as quickly as you can
- Where applicable, attempt all exercises and questions during lectures and tutorials
- Revise key points after each lesson to prepare for the in-semester test
- Start on your assignments early!
- Build your knowledge systematically from lectures and recommended readings
  - If it helps, use some mindmap tool such as xmind

#### How to become extra ... ordinary?

- Read and learn beyond what is being taught
- Work on project/ideas that inspire you
- Be proactive, github and build DL profile

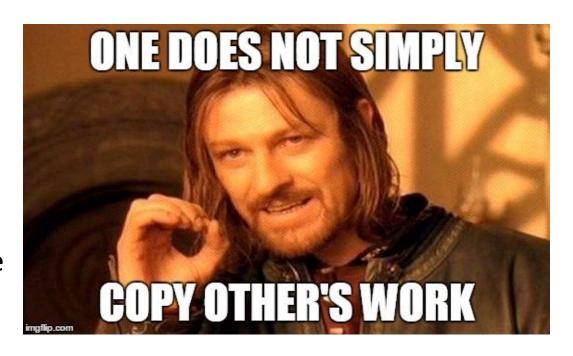


# Plagiarism Notice

Plagiarism:

"the practice of taking someone else's work or ideas and passing them off as one's own" (Oxford Dictionary)

 It is a <u>serious</u> academic offence and you can be <u>expelled</u> from the University!





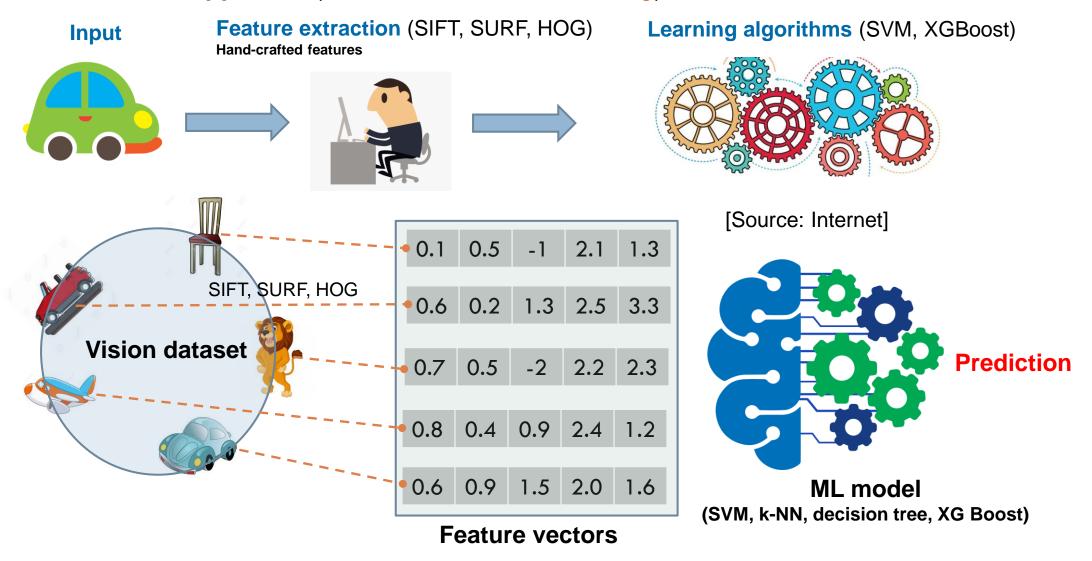


Linear algebra revision

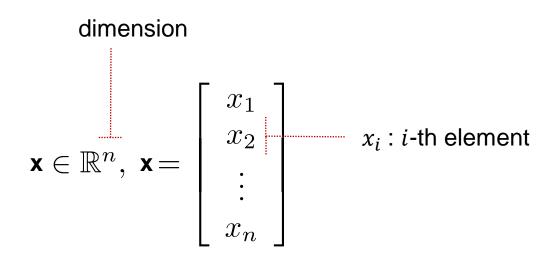
# Why do we need vectors? (Forum discussion)

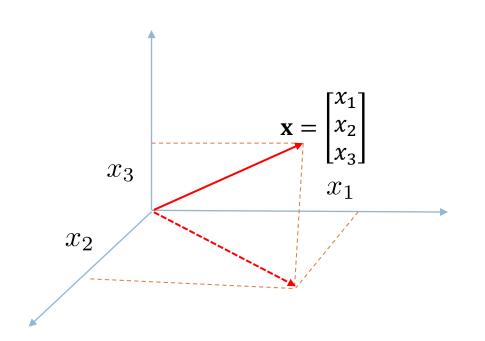
Hand-crafted feature

#### Traditional approach (hand-crafted feature learning)



#### n-dimensional vector





#### Operations on vector

transpose: column vector to row vector

$$\mathbf{x}^{\mathsf{T}} = [x_1 \ x_2 \ \dots \ x_n]$$

$$\mathbf{x} = \left[ egin{array}{c} x_1 \\ x_2 \\ \vdots \\ x_n \end{array} 
ight]$$

$$\mathbf{x} + \mathbf{y} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} = \begin{bmatrix} x_1 + y_1 \\ x_2 + y_2 \\ \vdots \\ x_n + y_n \end{bmatrix}$$
 addition

$$\mathbf{x}^{\mathsf{T}}\mathbf{y} = x_1y_1 + x_2y_2 + \ldots + x_ny_n$$

inner product

p-norm

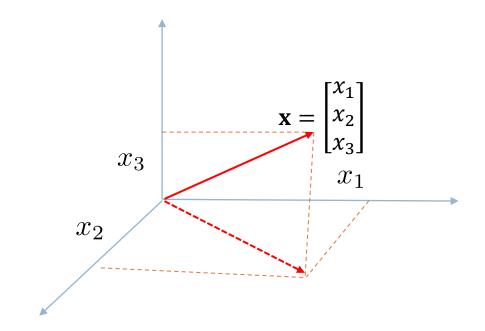
$$\|\mathbf{x}\|_p = (|x_1|^p + |x_2|^p + \dots + |x_n|^p)^{\frac{1}{p}}$$
 with  $p \ge 0$ 

- p = 0 : how many elements in x are non-zeros (sparsity)
- p = 1 : sum of absolute values of elements
- p = 2 : length of the vector
- $\|\mathbf{x}\|_{p} \geq 0$  and  $\|\mathbf{x}\|_{p} = 0$  if only if  $\mathbf{x} = \mathbf{0}$ .
- $\|\mathbf{k}\mathbf{x}\|_p = \|k\|\|\mathbf{x}\|_p$  for any scalar  $k \in \mathbb{R}$ .
- $||\mathbf{x} + \mathbf{y}||_p \le ||\mathbf{x}||_p + ||\mathbf{y}||_p$  (triangle inequality).

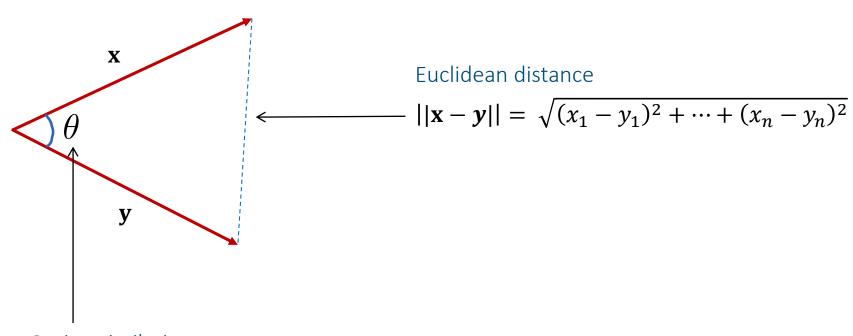
Length of a vector

length(
$$\mathbf{x}$$
) =  $\sqrt{x_1^2 + x_2^2 + \dots + x_n^2}$ 

$$\|\mathbf{x}\|_2 = \sqrt{x_1^2 + x_2^2 + \dots + x_n^2}$$



#### Distance between two vectors



Cosine similarity

$$\cos(\theta) = \cos(\mathbf{x}, \mathbf{y}) = \frac{\mathbf{x}^T \mathbf{y}}{||\mathbf{x}|| ||\mathbf{y}||} = \frac{\mathbf{x}^T \mathbf{y}}{\sqrt{\mathbf{x}^T \mathbf{x}} \sqrt{\mathbf{y}^T \mathbf{y}}} \rightarrow -1 \le \cos(\mathbf{x}, \mathbf{y}) \le 1$$

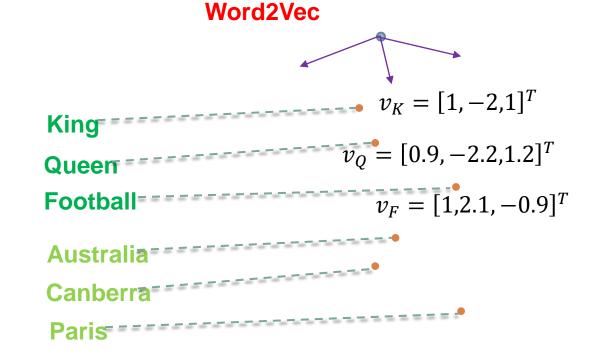
Cosine distance is defined as

$$d_{cosine}(\mathbf{x}, \mathbf{y}) = 1 - \cos(\mathbf{x}, \mathbf{y})$$

Cosine distance can be computed via Euclidean distance if vectors are made unit vectors! (why?)

# Example of cosine similarity (Forum discussion)

• Assume that a Word2Vec model transforms three words King, Queen, and Football to three vectors v<sub>K</sub>, v<sub>Q</sub>, and v<sub>F</sub> as shown in the figure. Between two words King and Football, which word is more similar to word Queen?



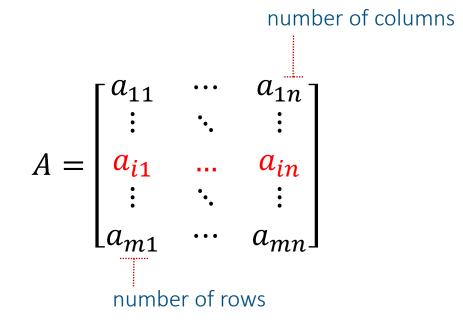
• 
$$cos\_sim(King, Queen) = \frac{v_K^T v_Q}{\|v_K\| \|v_Q\|} = \frac{1 \times 0.9 + (-2) \times (-2.2) + 1 \times 1.2}{\sqrt{1^2 + (-2)^2 + 1^2} \sqrt{0.9^2 + (-2.2)^2 + 1.2^2}} = 0.996$$

• 
$$cos\_sim(Football, Queen) = \frac{v_F^T v_Q}{\|v_F\| \|v_Q\|} = \frac{1 \times 0.9 + 2.1 \times (-2.2) + (-0.9) \times 1.2}{\sqrt{1^2 + 2.1^2 + (-0.9)^2} \sqrt{0.9^2 + (-2.2)^2 + 1.2^2}} = -0.72$$

King is more similar to Queen than Football.

## Matrix (2D tensor)

# number of columns $A \in \mathbb{R}^{m \times n}$ number of rows



$$A = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{i1} & \cdots & a_{in} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{bmatrix} \text{row vector}$$

$$A = \begin{bmatrix} a_{11} & \dots & a_{1j} & \dots & a_{1n} \\ \dots & \dots & \dots & \dots & \dots \\ a_{m1} & \dots & a_{nj} & \dots & a_{mn} \end{bmatrix}$$

## Matrix operation

$$A = egin{bmatrix} a_{11} & \cdots & a_{1n} \ dots & \ddots & dots \ a_{i1} & \cdots & a_{in} \ dots & \ddots & dots \ a_{m1} & \cdots & a_{mn} \end{bmatrix} \in \mathbb{R}^{m imes n}$$

$$C = A^T = egin{bmatrix} a_{11} & \cdots & a_{m1} \ dots & \ddots & dots \ a_{1j} & \cdots & a_{mj} \ dots & \ddots & dots \ a_{1n} & \cdots & a_{mn} \end{bmatrix} \in \mathbb{R}^{n imes m}$$

 $C = A^T$  if only if  $C_{ij} = A_{ji}$  for any  $1 \le i \le n, 1 \le j \le m$ 

$$B = egin{bmatrix} oldsymbol{b_{11}} & \cdots & oldsymbol{b_{1n}} \ drawtooldsymbol{drawtown} & drawtooldsymbol{drawtown} \ oldsymbol{b_{i1}} & \cdots & oldsymbol{b_{in}} \ oldsymbol{drawtown} & drawtooldsymbol{drawtown} \ oldsymbol{b_{m1}} & \cdots & oldsymbol{b_{mn}} \end{bmatrix} \in \mathbb{R}^{m imes n}$$

$$lpha A = egin{bmatrix} lpha a_{11} & \cdots & lpha a_{1n} \ drawtooldrow & \ddots & drawtooldrow \ lpha a_{i1} & \cdots & lpha a_{in} \ drawtooldrow & \ddots & drawtooldrow \ lpha a_{m1} & \cdots & lpha a_{mn} \end{bmatrix} \in \mathbb{R}^{m imes n}$$

$$C = A + B = egin{bmatrix} a_{11} + b_{11} & \cdots & a_{1n} + b_{1n} \ dots & \ddots & dots \ a_{i1} + b_{i1} & \cdots & a_{in} + b_{in} \ dots & \ddots & dots \ a_{m1} + b_{m1} & \cdots & a_{mn} + b_{mn} \end{bmatrix} \in \mathbb{R}^{m imes n}$$

# Matrix multiplication

$$A = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{i1} & \cdots & a_{in} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{bmatrix}$$

$$B = \begin{bmatrix} b_{11} & \cdots & b_{1j} & \cdots & b_{1p} \\ \cdots & \cdots & \cdots & \cdots \\ b_{n1} & \cdots & b_{np} \end{bmatrix}$$

$$c_{ij} = A[i,:]B[:,j] = \sum_{k=1}^{n} a_{ik}b_{kj}$$

- Given **two matr**ices  $A = [a_{ij}] \in \mathbb{R}^{m \times n}$  and  $B = [b_{ij}] \in \mathbb{R}^{n \times p}$ , we can compute the **multiplication** C = AB of A and B as
  - $C = [c_{ij}] \in \mathbb{R}^{m \times p}$
  - $c_{ij} = \sum_{k=1}^{n} a_{ik} b_{kj}$  for  $1 \le i \le m$  and  $1 \le j \le p$
  - $c_{ij} = A[i,:]B[:,j]$  which is the **dot product** of the **row** i in **A** and the **column** j in **B**.

## Matrix multiplication with numpy

```
A = np.array([
        [10,20,30],
        [40,50,60]
])
A

array([[10, 20, 30],
        [40, 50, 60]])
```

#### Declare matrix A with the size $2 \times 3$

Declare matrix D with the size  $3 \times 4$ 

```
E = A.dot(D)
E
array([[ 930, 1160, 1320, 1560],
        [2010, 2510, 2910, 3450]])
```

 $E = A \times D$  has the size  $2 \times 4$ 

#### Tensor

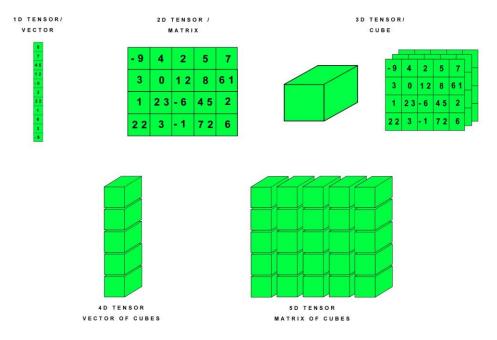
A tensor is simply a multidimensional array

0D tensor: a scalar

1D tensor: a vector

2D tensor: a matrix

3D tensor: a 3D hyperrectangle of numerals



Examples of tensors. [Source: Internet]



#### A small detour to calculus

- Calculus = mathematics of change (very important for deep learning)
- Properties of derivative:

$$f'(x) = \nabla f(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$(uv)' = u'v + uv'$$

$$(e^u)' = u'e^u$$

$$(\log u)' = \frac{u'}{u}$$

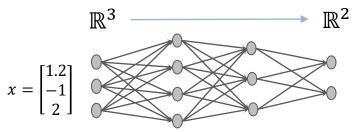
- Multi-variate function  $f: \mathbb{R}^n \to \mathbb{R}$  with  $y = f(x) = f(x_1, ..., x_n)$ .
  - Gradient/derivative:  $\frac{\partial f}{\partial x}(a) = \nabla_x f(a) = [\nabla_{x_1} f(a), \nabla_{x_2} f(a), \dots, \nabla_{x_n} f(a)].$
- Chain rule 
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$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial v} \times \frac{\partial v}{\partial x}$$

## Example

- Consider the function  $f(x, y, z) = \log(\exp(x) + \exp(y) + \exp(z))$ . What are  $f'_x = \nabla_x f$ ,  $f'_y = \nabla_y f$ , and  $f'_z = \nabla_z f$ ?
- $f(x, y, z) = \log(u) \text{ with } u = \exp(x) + \exp(y) + \exp(z).$

## Derivative for multi-variate functions



- Given a function  $f: \mathbb{R}^m \to \mathbb{R}^n$ 
  - $f(x) = (f_1(x), \dots, f_n(x))$  where  $f_1, \dots, f_n : \mathbb{R}^m \to \mathbb{R}^m$
  - **related** notion) is a matrix n by m (i.e., the Jacobian matrix).

iven a function 
$$f: \mathbb{R}^m \to \mathbb{R}^n$$

$$f(x) = (f_1(x), ..., f_n(x)) \text{ where } f_1, ..., f_n: \mathbb{R}^m \to \mathbb{R}$$

R and  $x = (x_1, ..., x_m)$ . Let denote  $y = f(x)$ .

The derivative of  $f$  at the point  $a \in \mathbb{R}^m$ , denoted by  $\nabla f(a)$  (function related notion) or  $\frac{\partial y}{\partial x}(a)$  (variable related notion) is a matrix  $n$  by  $m$  (i.e., the Jacobian  $\frac{\partial f}{\partial x_1}(a)$  ...  $\frac{\partial f}{\partial x_1}(a)$ 

#### **Jacobian matrix**

#### Chain rule 👄

- Given a function  $f: \mathbb{R}^m \to \mathbb{R}^n$ ,  $g: \mathbb{R}^n \to \mathbb{R}^p$ , denote  $h = g \circ f: \mathbb{R}^m \to \mathbb{R}^p$ , meaning that  $h(x) = g \circ f: \mathbb{R}^m \to \mathbb{R}^p$ g(f(x)). We further define y = f(x) and z = g(y) = g(f(x)) = h(x).
- For  $x \in \mathbb{R}^m$ ,  $\nabla h(x) = \nabla g(f(x)) \times \nabla f(x)$  or equivalently  $\frac{\partial z}{\partial x} = \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial x}$ .  $p \times m$   $p \times n$   $n \times m$  $\begin{array}{ccc}
  x & y = f(x) & z = g(y) = h(x) \\
  \mathbb{R}^m & \xrightarrow{f} & \mathbb{R}^n & \xrightarrow{g} & \mathbb{R}^p
  \end{array}$

### Example

$$y = f(x) = f(x_1, x_2, x_3) = (x_1^2 + x_2^2, x_2^2 + x_3^2 x_2)$$

$$f: \mathbb{R}^3 \to \mathbb{R}^2$$

$$f_1(x) = f_1(x_1, x_2, x_3) = x_1^2 + x_2^2$$

$$f_2(x) = f_2(x_1, x_2, x_3) = x_2^2 + x_3^2 x_2$$

$$\quad \frac{\partial y}{\partial x} = \nabla f \in \mathbb{R}^{2 \times 3}$$

$$\frac{\partial y}{\partial x} = \nabla_x f = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \frac{\partial f_1}{\partial x_3} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \frac{\partial f_2}{\partial x_3} \end{bmatrix} = \begin{bmatrix} 2x_1 & 2x_2 & 0 \\ 0 & 2x_2 + x_3^2 & 2x_2x_3 \end{bmatrix}$$

### Example

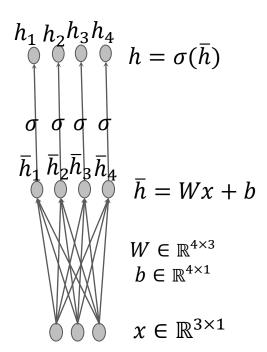
$$\bar{h} = Wx + b$$
 and  $h = \sigma(\bar{h})$ 

- $h = \sigma(Wx + b)$
- $\circ$   $\sigma$  is the activation function

$$\frac{\partial h}{\partial x} = \frac{\partial h}{\partial \overline{h}} \times \frac{\partial \overline{h}}{\partial x} = diag\left(\sigma'(\overline{h})\right)W$$

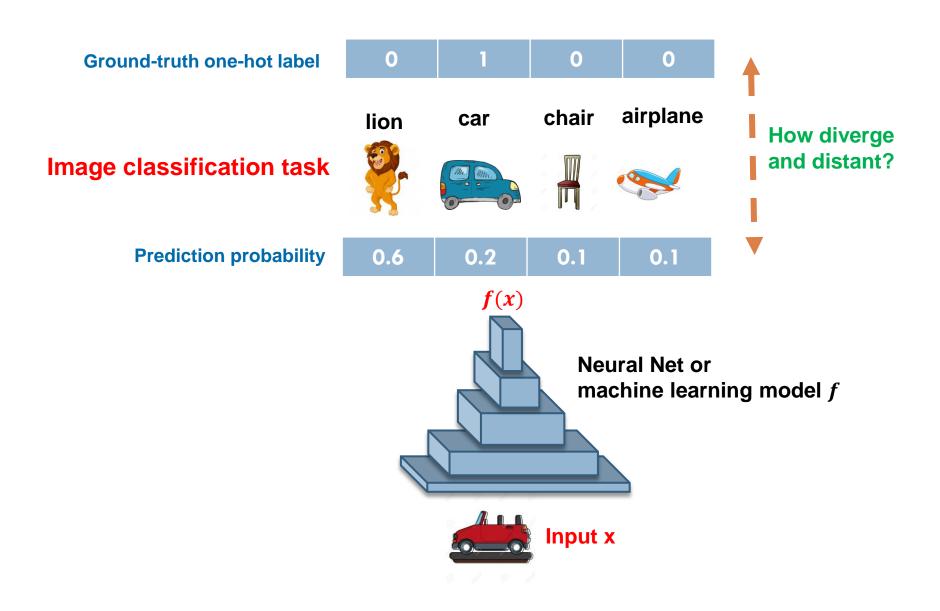
$$\frac{\partial h}{\partial \bar{h}} = \begin{bmatrix} \frac{\partial h_1}{\partial \bar{h}_1} & \frac{\partial h_1}{\partial \bar{h}_2} & \frac{\partial h_1}{\partial \bar{h}_3} & \frac{\partial h_1}{\partial \bar{h}_4} \\ \frac{\partial h_2}{\partial \bar{h}_1} & \frac{\partial h_2}{\partial \bar{h}_2} & \frac{\partial h_2}{\partial \bar{h}_3} & \frac{\partial h_2}{\partial \bar{h}_4} \\ \frac{\partial h_3}{\partial \bar{h}_1} & \frac{\partial h_3}{\partial \bar{h}_2} & \frac{\partial h_3}{\partial \bar{h}_3} & \frac{\partial h_3}{\partial \bar{h}_4} \\ \frac{\partial h_4}{\partial \bar{h}_1} & \frac{\partial h_4}{\partial \bar{h}_2} & \frac{\partial h_4}{\partial \bar{h}_3} & \frac{\partial h_4}{\partial \bar{h}_4} \end{bmatrix} = \begin{bmatrix} \sigma'(\bar{h}_1) & 0 & 0 & 0 \\ 0 & \sigma'(\bar{h}_2) & 0 & 0 \\ 0 & 0 & \sigma'(\bar{h}_3) & 0 \\ 0 & 0 & 0 & \sigma'(\bar{h}_4) \end{bmatrix} = diag(\sigma'(\bar{h}))$$

$$\Box \quad \frac{\partial \bar{h}}{\partial x} = W$$



Information theory revision

### Why need to work with discrete distributions?



### Revision of basic informatic quantities

Given two discrete distributions  $p = [p_i]_{i=1}^d$  ( $p_i \ge 0$  and  $\sum_{i=1}^d p_i = 1$ ) and  $q = [q_i]_{i=1}^d$  ( $q_i \ge 0$  and  $\sum_{i=1}^d q_i = 1$ ).

#### Kullback-Leibler (KL) divergence between p, q

- $\square KL(p,q) = \sum_{i=1}^{d} p_i \log \frac{p_i}{q_i}$
- $\square$   $KL(p,q) \ge 0$  and KL(p,q) = 0 if only if p = q

#### Entropy of the distribution p

$$\Box H(p) = \sum_{i=1}^{d} p_i \log \frac{1}{p_i} = -\sum_{i=1}^{d} p_i \log p_i$$
 where  $0 \times \log 0 = 0$ 

#### $lue{}$ Cross-entropy (CE) divergence between $m{p}$ , $m{q}$

- $\Box CE(p,q) = -\sum_{i=1}^{d} p_i \log q_i.$
- $\Box CE(p,q) = KL(p,q) + H(p).$

### Example

- Consider p = [0.1,0.4,0.5] and q = [0.2,0.5,0.3]. Compute KL(p,q), CE(p,q), and H(p).
  - $KL(p,q) = 0.1 \times \log \frac{0.1}{0.2} + 0.4 \times \log \frac{0.4}{0.5} + 0.5 \times \log \frac{0.5}{0.3} \approx 0.042$
  - $_{\circ}$   $CE(p,q) = -0.1 \times \log 0.2 0.4 \times \log 0.5 0.5 \times \log 0.3 \approx 0.452$
  - $_{\circ}$   $H(p) = 0.1 \times \log \frac{1}{0.1} + 0.4 \times \log \frac{1}{0.4} + 0.5 \times \log \frac{1}{0.5} \approx 0.41$

- Consider p = [0,1,0] and q = [0.2,0.3,0.5]. Compute CE(p,q).
  - $CE(p,q) = -0 \times \log 0.2 1 \times \log 0.3 0 \times \log 0.5 \approx -\log 0.3 = 0.523$

**Question:** when the entropy is maximized or minimized?

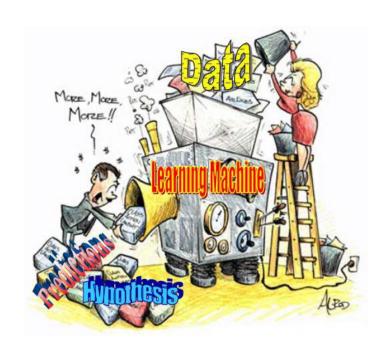




### Machine learning

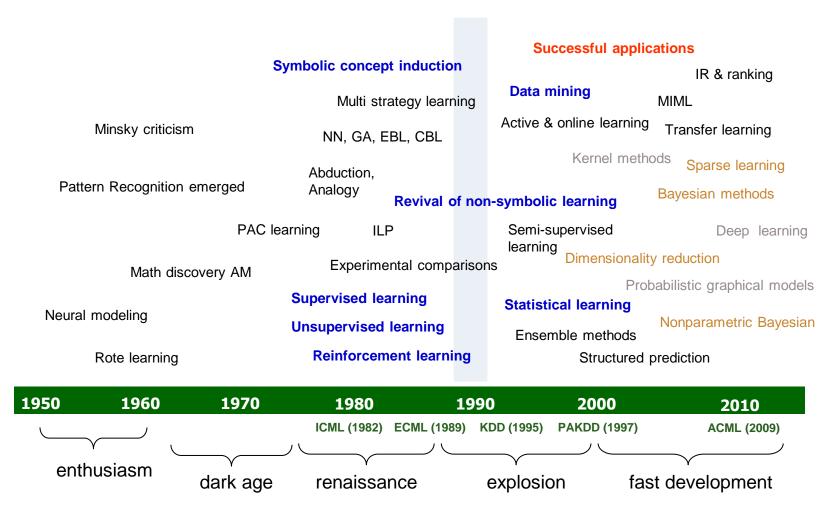
- "Field of study that gives computers the ability to learn without being explicitly programmed" (Arthur Samuel, 1959).
- "Machine learning sits at the crossroads of computer science, statistics and a variety of other disciplines concerned with automatic improvement over time, and inference and decision-making under uncertainty." (Jordan & Michell, 2015).
- "A computer program is said to learn from experience E with respect to some class of tasks T and performance measure P, if its performance at tasks in T, as measured by P, improves with experience E", (Tom Mitchell, 1997)

M.I. Jordan and T. Mitchell, "Machine Learning: Trends, perspectives, and prospects", *Science*, 17 July 2015.



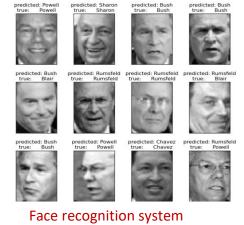
(from Eric Xing lecture notes)

#### A brief history of ML



[credit: Prof Bao Ho, JAIST]

### Machine learning





**Detecting Spam Emails** 



Detect credit card fraud

#### Experience E Task T Performance P

dataset of thousands of given a new photo, known faces recognise the name of the face

Label set: names of all

people.

how **accurate** the recognition is

Measure: Accuracy



Experience E	Task T	Performance P
Examples of spam emails and not-spam emails	To assign a label "spam" or "not-spam" to an email Label set: <b>spam</b> and <b>not spam</b>	how <b>accurate</b> spam email can be detected Measure: <b>accuracy</b>

Task T

fraud

Experience L
Data collected for
credit-card transactions
deemed as fraud and
not-fraud

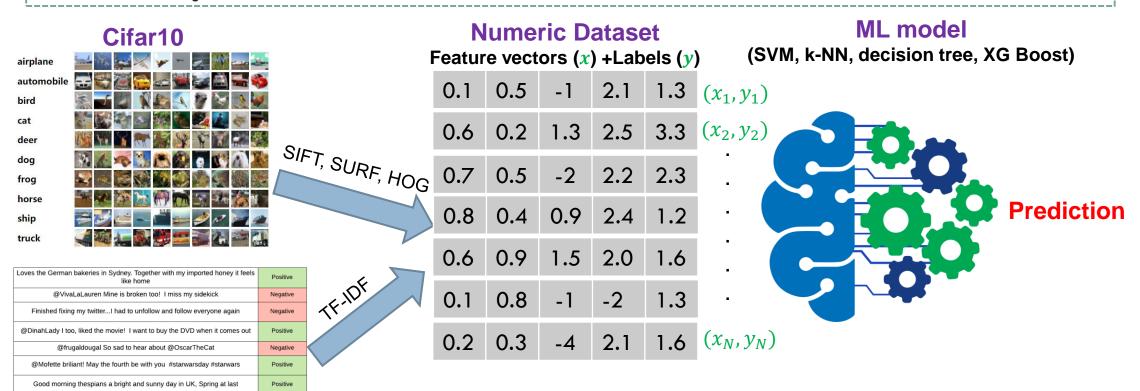
Experience F

To assign a label 'fraud' or "not fraud" to a given credit-card transaction Label set: **fraud** and **not**  how accurate a creditcard fraud transaction can be detected. Measure: accuracy

Performance P

### Feature extraction (Forum discussion)

- Most machine learning algorithms (e.g, Support Vector Machine or XG Boosts) only work
   with numeric data
  - o Given **structural data** such as images or texts, we **extract numeric features** that preserve characteristics of raw data
  - Feature extraction methods
    - Images: SIFT, SURF, and HOG.
    - Texts: bag of words and TF-IDF.



@DowneyisDOWNEY Me neither! My laptop's new, has dvd burning/ripping software but I just can't copy the files somehow!

### Elements of machine learning

## ① Data

Vector  $\mathbf{x} \in \mathbb{R}^d$ 

Image x

Sentence x

Speech x

**Ground-truth label**  $y \in Y$ 





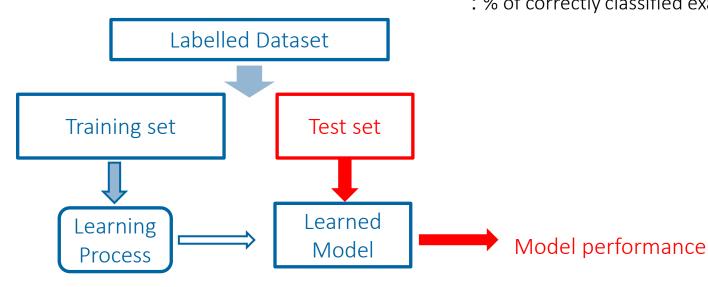
Why should I memorize something I can so easily write it on a tiny (paper and take to the exam

# 2 Model

- $f: X \to Y$
- X is data space, Y is label space
- Classification:  $Y = \{1, 2, ..., M\}$
- Regression:  $Y = \mathbb{R}$
- Prediction:  $f(x) = \hat{y} \in Y$

# 3 Assessment

- How good is the model doing its tasks?
- Performance metrics
  - Accuracy, Recall, Precision, F-score
  - $Accuracy = \frac{\#Correct\ Predictions}{\#All\ Predictions}$ : % of correctly classified examples



Machine learning pipeline.

### Classification and Regression

#### Two main tasks in machine learning

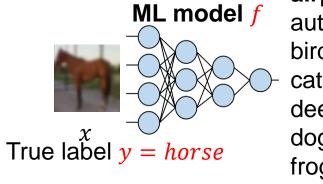
- 1. Classification and regression
- Other tasks in ML include density estimation, clustering, anomaly detection, recommendation, forecasting, and ranking

#### Classification

- The label set  $Y = \{1, 2, ..., M\}$  and the data space X (vectors, images).
- Training set  $\mathbf{D}=\{(x_1,y_1),...,(x_N,y_N)\}$  where  $x_i\in\mathbb{R}^d$  and  $y_i\in Y=\{1,2,...,M\}$
- 3. Learn a model  $f: X \to Y$
- Given data example x, render the prediction  $\hat{y} = f(x) \in \{1, 2, ..., M\}$

#### Regression

- The label set  $Y = \mathbb{R}$  and the data space X (vectors, images).
- Training set  $D = \{(x_1, y_1), ..., (x_N, y_N)\}$  where  $x_i \in \mathbb{R}^d$  and  $y_i \in Y = \mathbb{R}$
- Learn a model  $f: X \to Y$
- Given data example x, render the prediction  $\hat{y} = f(x) \in \mathbb{R}$



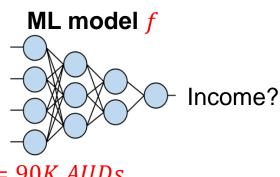
airplane?(0.4)
automobile(0.0)?
bird?(0.0)
cat?(0.05)
deer?(0.2)
dog?(0.05)
frog?(0.0)
horse?(0.3)
ship?(0.0)
truck?(0.0)

### Human info

Age, position, job, qualification, and etc.

 $\chi$ 

True label y = 90K AUDs



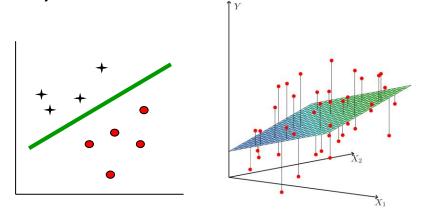
### Supervised learning (Forum discussion)

Problem: Learn a function from data to relate the inputs with outputs. Training data include output information (labels).

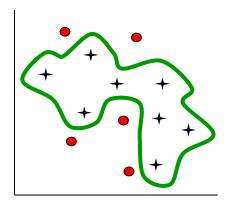
Data: 
$$D = \{(x_1, y_1), ..., (x_N, y_N)\}$$

Function:  $f: X \to Y$ 

```
x \in \mathbb{R}^d = feature y \in \{1,2,...,M\} = a discrete label (classification), y \in \mathbb{R} = a continuous value (regression)
```



e.g., <u>linear</u> functions

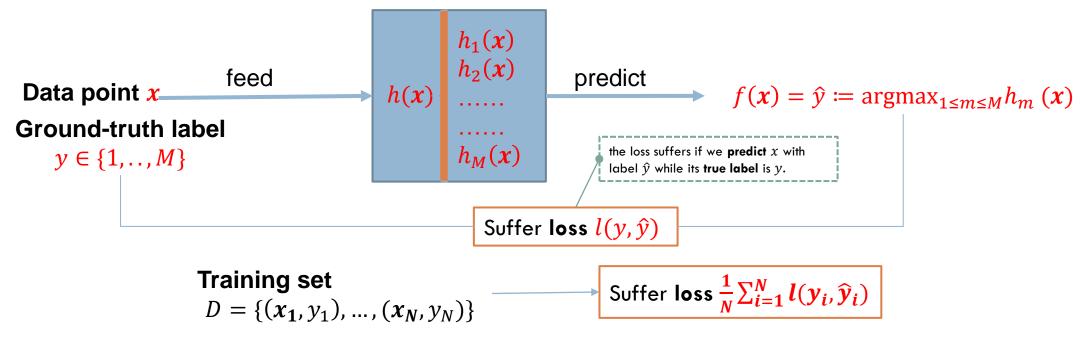


e.g., <u>nonlinear</u> functions

### Discriminative machine learning

#### Classification

#### Discriminative ML model h



- A data point x receives **discriminative values**  $h_1(x)$ , ...,  $h_M(x)$  from the model
  - $h_m(x)$  represents the **possibility** to classify x to class m for  $1 \le m \le M$
- $\square$  We use these discriminative values to predict the label  $\hat{y}$  as
  - $\hat{y} = \operatorname{argmax}_{1 \le m \le M} h_m(x)$ , meaning the class with highest discriminative value
- The prediction x with the predicted label  $\hat{y}$  suffers a loss
  - $l(\hat{y}, y)$  where l is a **loss function** (if  $\hat{y} = y$  then  $l(\hat{y}, y) = 0$ ).
- Given a training set  $D = \{(x_1, y_1), ..., (x_N, y_N)\}$ , the loss incurred is

#### Softmax activation function

#### Use to transform real-valued discriminative scores to discrete probabilities

$$h = [h_m]_{m=1}^M \to p = \operatorname{softmax}(h) \coloneqq \left[\frac{\exp\{h_m\}}{\sum_{i=1}^M \exp\{h_i\}}\right]_{m=1}^M$$

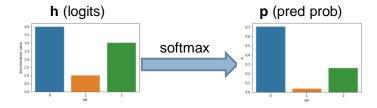
- $p = [p_m]_{m=1}^M$  becomes a distribution over classes  $\{1, ..., M\}$ 
  - $p_m \ge 0 \ (1 \le m \le M) \ \text{and} \ \sum_{m=1}^M p_m = 1$ .

#### Assume that we do sentiment classification (analysis)

- Data point x is a sentence (e.g.,  $x = "it is \ a \ beautiful \ day \ to \ day"$ ), we need to predict **sentiment** of this sentence
- Three sentiment labels: positive (happy, class 1), negative (sad, class 2), neural (none happy and sad, class 3)
- $_{\circ}$  The model gives three **discriminative values** for x
  - $h_1 = 2$ ,  $h_2 = -1$ ,  $h_3 = 1$  means that **highest possibility** to **classify** x to the **class 1** with the label **positive**.
- $_{\circ}$  We apply **softmax** function on the **discriminative scores** h
  - $p = \operatorname{softmax}(h)$

$$p_1 = \frac{\exp\{2\}}{\exp\{2\} + \exp\{-1\} + \exp\{1\}} \approx 0.705, p_2 = \frac{\exp\{-1\}}{\exp\{2\} + \exp\{-1\} + \exp\{1\}} \approx 0.035, p_3 = \frac{\exp\{1\}}{\exp\{2\} + \exp\{-1\} + \exp\{1\}} \approx 0.259$$

• p = [0.70538451, 0.03511903, 0.25949646] are the **probabilities** to **classify** x to the **classes 1, 2, 3** respectively.



**Softmax** transforms real-valued discriminative scores (ranged in  $(-\infty, +\infty)$ ) to probability values (ranged in [0,1]) but preserving the order.

#### Cross-entropy loss

- □ Given M labels 1,2, ..., M, a label  $y \in \{1, ..., M\}$  has two equivalent forms:
  - Numeric form:  $y \in \{1, ..., M\}$  is an index in  $\{1, ..., M\}$ .
  - o One-hot vector form:  $1_y = [0, ..., 0, 1_y, 0, ..., 0]$  the vector of zeros except an unique 1 in the y-th position.
- Label set {happy, sad, neural} with 1: happy, 2: sad, and 3: neural
  - $\circ$  [1,0,0]  $\rightarrow$  happy (numeric label 1), [0,1,0]  $\rightarrow$  sad (numeric label 2), [0,0,1]  $\rightarrow$  neural (numeric label 3)
- Given two discrete distributions over classes:  $p = [p_m]_{m=1}^M$  and  $q = [q_m]_{m=1}^M$ , the cross-entropy divergence between p and q
  - $_{\circ}$  CE $(p,q) \coloneqq -\sum_{m=1}^{M} p_m \log q_m$  which measures how far (divergent) it is between p and q.
  - $_{\circ}$  CE $(p,q) \geq \mathrm{H}(p) \coloneqq -\sum_{m=1}^{M} p_{m} \log p_{m}$  (H is the **Shannon entropy**).

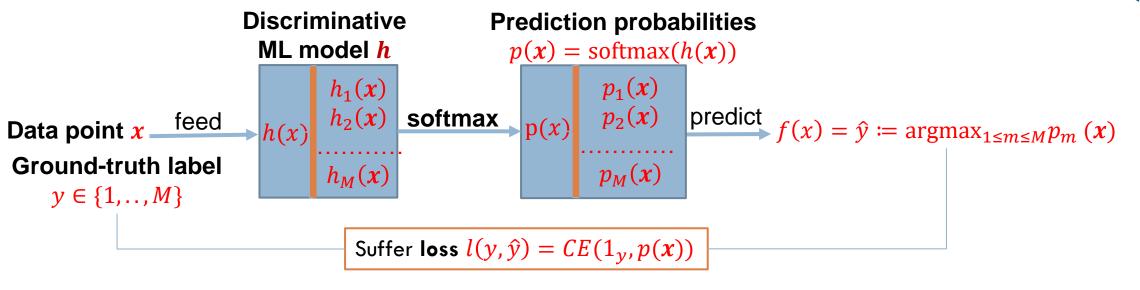
### Cross-entropy loss

- Cross-entropy loss
  - $l(y, \hat{y}) = CE(1_y, p) = -\log p_y$
  - $y \in \{1, ..., M\}$  is the ground-truth label of a given x and p is the prediction probabilities of x to classes
- Given a sentence x with the label positive/happy (the class 1), assume that our model predicts it with prediction probabilities p = [0.3,0.4,0.3], the crossentropy loss for this prediction
  - $l(y, \hat{y}) = CE([1,0,0], [0.3,0.4,0.3]) = -1.\log 0.3 0.\log 0.4 0.\log 0.3 = -\log 0.3 \approx 1.204$

Question: why is the cross-entropy loss always non-negative and when it is 0?

#### Discriminative machine learning

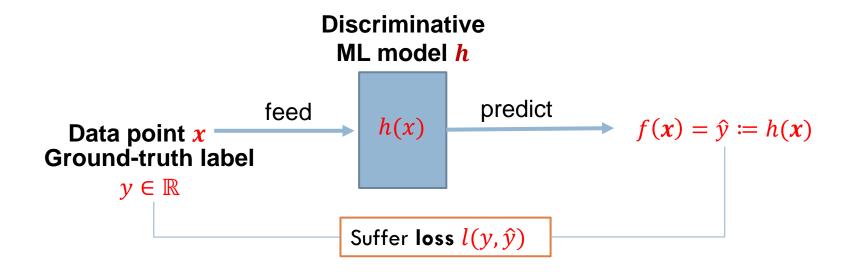
Classification with the cross-entropy loss



- We apply the softmax function:  $\mathbf{p}(x) = \mathbf{softmax}(\mathbf{h}(\mathbf{x}))$  to make p(x) prediction probabilities
  - $p_m(x) = p(y = m|x)$  is the **probability** to classify x to the class m for  $1 \le m \le M$
- The loss incurred for this prediction
  - $l(y, \hat{y}) = CE\left(1_y, p(x)\right) = -\log p_y(x) \text{ (negative log likelihood)}$
- □ The loss for the training set  $D = \{(x_1, y_1), ..., (x_N, y_N)\}$ 
  - $L(D,h) = \frac{1}{N} \sum_{i=1}^{N} CE(1_{y_i}, p(x_i)) = -\frac{1}{N} \sum_{i=1}^{N} \log p_{y_i}(x_i)$

#### Discriminative machine learning

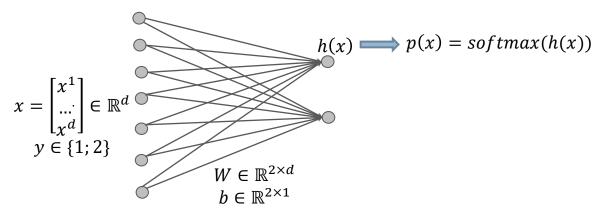
Regression



- Only one discriminative value  $h(x) \in \mathbb{R}$ , we use this to predict  $\hat{y} = f(x) \coloneqq h(x)$ .
- ☐ The **potential losses** could be
  - □ **L2 loss**:  $l(y, \hat{y}) = \frac{1}{2}(y \hat{y})^2$
  - $\Box \quad \mathsf{L1 \, loss} \colon l(y, \hat{y}) = |y \hat{y}|$
  - $\bullet$  -insensitive loss:  $l(y, \hat{y}) = \max\{0, |y \hat{y}| \epsilon\}$  where  $\epsilon \ge 0$ .

### Logistic regression (Forum discussion)

#### A simple feed-forward neural network



#### Computational process

- h(x) = Wx + b
  - $h(x) \in \mathbb{R}^{2 \times 1}$  contains **discriminative scores** with respect to classes 1 and 2.
- p(x) = softmax(h(x))
  - p(x) is the **prediction probabilities** with respect to classes 1 and 2.
- $\widehat{y} = \begin{cases} 1, & p_1(x) \ge p_2(x) \\ 2, & p_1(x) < p_2(x) \end{cases}$
- $l(y, \hat{y}) = CE\left(1_y, p(x)\right) = -\log p_y\left(x\right) \rightarrow \text{the cross entropy loss}.$

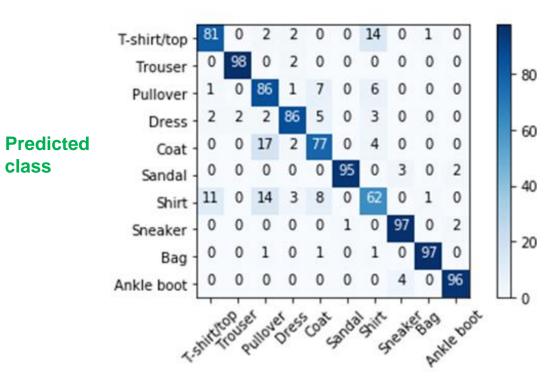
#### Training process

- Training set  $D = \{(x_1, y_1), ..., (x_N, y_N)\}$
- $L(D; W, b) = \frac{1}{N} \sum_{i=1}^{N} CE(1_{y_i}, p(x_i)) = -\frac{1}{N} \sum_{i=1}^{N} \log p_{y_i}(x_i)$
- $(W^*, b^*) = \operatorname{argmin}_{W,b} L(D; W, b)$



#### **Confusion matrix**

- How confusing a machine learning classifier is?
  - On-diagonal entries mean correct predictions.
  - Off-diagonal entries mean incorrect predictions.

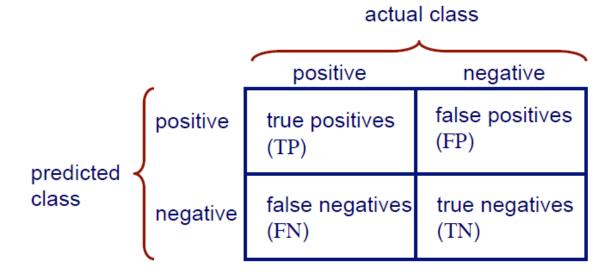


**Actual class** 

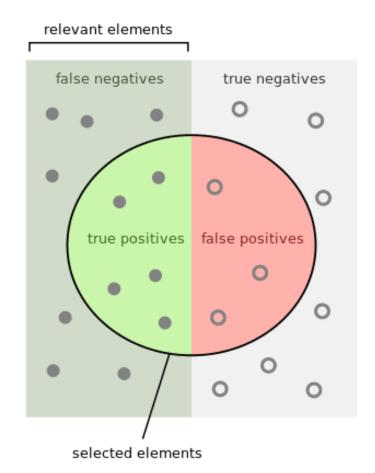
[Source: pythonhealthcare.org]

#### Other metrics for measuring performance

TP, FP, TN, FN and confusion matrix



- We may think of positive class as "class 1" and Negative class as "class 0".
- The second letter says what we predicted and the first letter says whether it was true or false.



# Other metrics for measuring performance

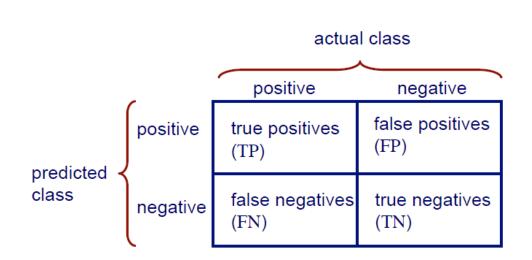
Precision, recall, and F-score

Recall = 
$$\frac{TP}{TP + FN}$$

Precision =  $\frac{TP}{TP + FP}$ 

F - score =  $2\frac{Precision \times Recall}{Precision + Recall}$ 

$$Accuracy = \frac{\#corrects}{\#examples} = \frac{TP + TN}{TP + TN + FP + TN}$$





### Example

- Example: Email Spam Detection
  - In test set: 10 spam, 20 non-spam
  - Positive = spam

#### **True Labels**

	SPAM (1)	NON-SPAM (0)
SPAM (1)	7	5
NON-SPAM (0)	3	15

- FP = 5, FN = 3
- Accuracy = 22/30
- Recall = 7/10
- Precision = 7/12
- F-score = ??

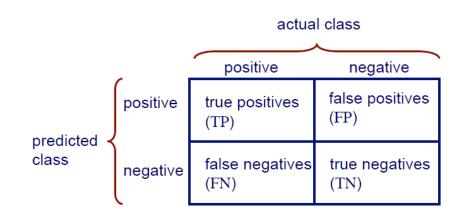
actual class

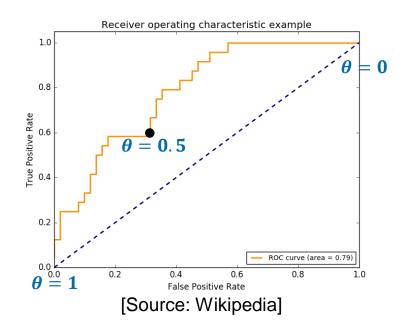
		positive	negative	
predicted <b>〈</b> class	positive	true positives (TP)	false positives (FP)	
	negative	false negatives (FN)	true negatives (TN)	

Predicted as

### ROC curve (Forum discussion)

- For binary classification
  - True positive rate (TPR) (Sensitivity)
    - TPR = TP/(TP + FN)
  - False positive rate (FRR)
    - FPR = FP / (FP + TN)
- When output also includes probability (e.g., logistic regression), we can compute ROC by varying the detection threshold.
  - ROC curve = true-positive rate (TPR) versus false-positive rate (FPR)
  - Consider a threshold  $\theta \in [0; 1]$ . Given a data point x, the ML model returns  $p(x) = \mathbb{P}(y = 1|x)$  (i.e., the **probability to predict** x as the label 1). We predict x as the **label 1** if  $p(x) = \mathbb{P}(y = 1|x) \ge \theta$  and the **label -1** otherwise.
  - We compute true positive rate (TPR) and false positive rate
     (FPR) for the entire training set and plot the point [TPR, FPR].
  - We vary  $\theta \in [0; 1]$  to gain the ROC curve.





#### Summary

#### Linear algebra :

Vectors, matrices and operations

#### Calculus

Derivative and chain rule

#### Information theory

Discrete distribution, KL divergence, entropy, and cross-entropy.

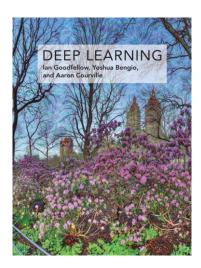
#### Machine learning revisit

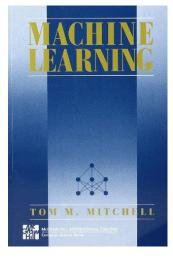
- Classification and regression
- Discriminative machine learning
- Confusion table and other metrics of interest (e.g., precision, recall, and F-score).

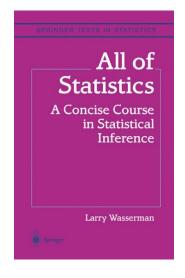


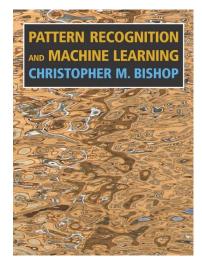
# Appendix

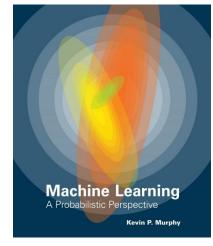
#### Other ML Books

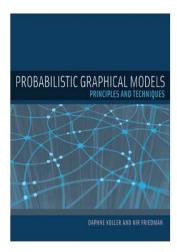


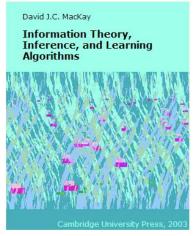


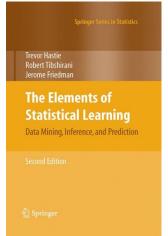


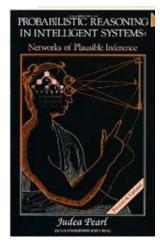


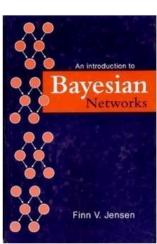














#### **The Symbolist School**

Tribes	Origins	Master Algorithm
Symbolists	Logic, philosophy	Inverse deduction



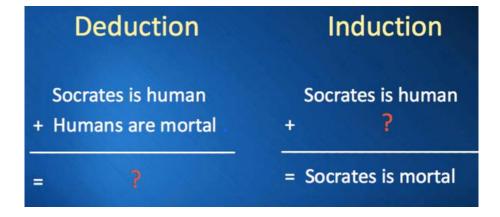




Tom Mitchell

Steve Muggleton

Ross Quinlan



Decision trees, C4.5, first-order propositional logic, rule-based reasoning systems, etc.

Tribes	Origins	Master Algorithm
Symbolists	Logic, philosophy	Inverse deduction
Evolutionary	Evolutionary biology	Genetic programming

#### The Evolutionary School



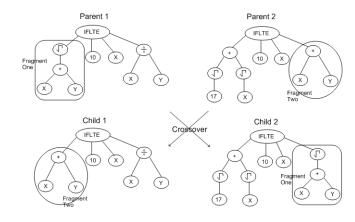




John Holland



**Hod Lipson** 



Genetic programming, genetic algorithms, fuzzy systems

The Connectionist

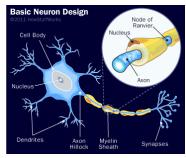
Tribes	Origins	Master Algorithm
Symbolists	Logic, philosophy	Inverse deduction
Evolutionary	Evolutionary biology	Genetic programming
Connectionists	Neuroscience	Backpropagation

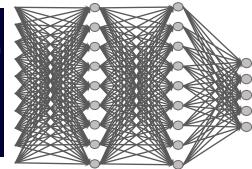






Geoff Hinton Yann LeCun Yoshua Bengio

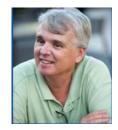




Deep Neural Networks, Convolutional Neural Networks, Deep learning, etc.

#### The Bayesians

Tribes	Origins	Master Algorithm
Symbolists	Logic, philosophy	Inverse deduction
Evolutionary	Evolutionary biology	Genetic programming
Connectionists	Neuroscience	Backpropagation
Bayesians	Statistics	Bayes' theorem

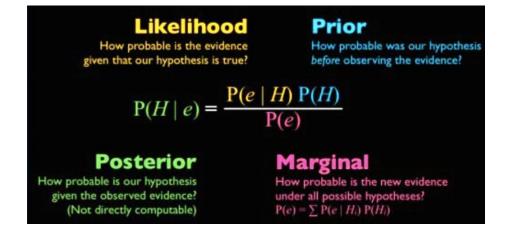






David Heckerman Judea Pearl

Michael Jordan



Bayesian networks classifiers, graphical models, Hidden Markov models, Conditional Random Fields, most of statistical machine learning approaches ...

#### The Analogizers

Tribes	Origins	Master Algorithm
Symbolists	Logic, philosophy	Inverse deduction
Evolutionary	Evolutionary biology	Genetic programming
Connectionists	Neuroscience	Backpropagation
Bayesian	Statistics	Bayes' theorem
Analogizers	Psychology	Support Vector Machines

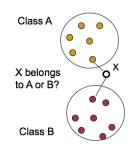


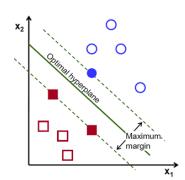


Vladimir Vapnik Peter Hart

**Douglas Hofstadter** 

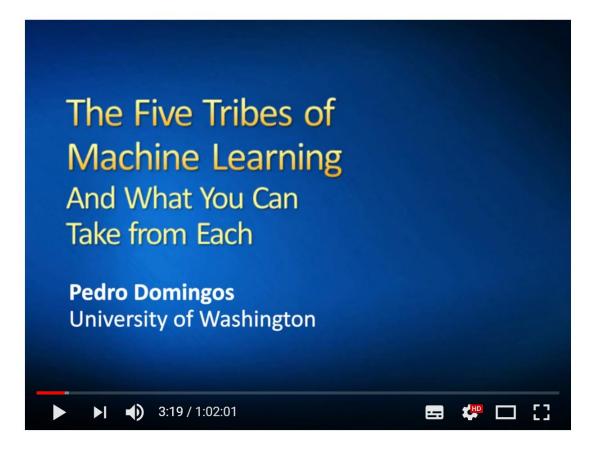
- Instance-based classification
  - Using most similar individual instances known in the past to classify a new instance
- Typical approaches
  - □ k-nearest neighbor approach
    - Instances represented as points in a Euclidean space





K-NN classifiers, Support vector machines, kernel machines, max-margin machines, instance-based classification, etc.

Approaches	Problem Solved	Solution Origin	Solution
Symbolists	Knowledge acquisition	Logic, philosophy	Inverse deduction
Evolutionary	Structure discovery (nature)	Evolutionary biology	Genetic programming
Connectionists	Credit assignment (nurture)	Neuroscience	Backpropagation and deep learning
Bayesian	Uncertainty	Statistics	Probabilistic Inference
Analogizers	Similarity	Psychology	Kernel machines



https://www.youtube.com/watch?v=E8rOVwKQ5-8