Greining Rása

Aflreikningur fyrir æstæða svörun

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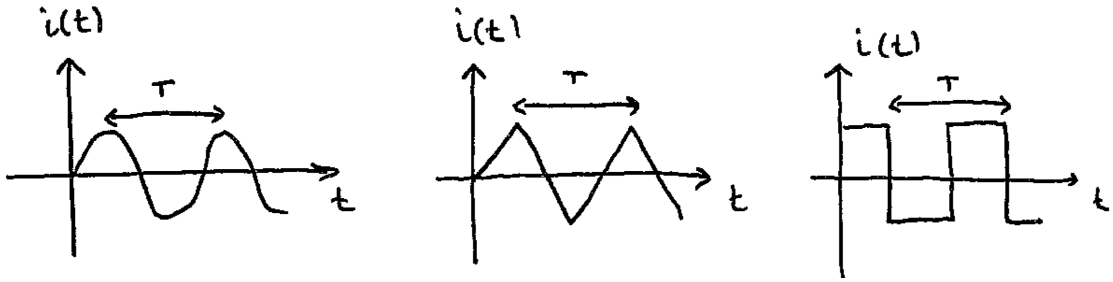
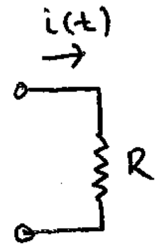
Lotubundin merki



- Lotubundin merki endurtaka sig á T sekúndu fresti

Virkt gildi (e. effective value (root mean square))



$p(t) = v(t)\, i(t)$

$p = iv = i^2 R$

- Virkt gildi (rms) lotubundins straums $i(t)$ er stærð þess DC straums I_{rms} sem veldur jafnmiklu afltapi í viðnáminu eins og lotubundni straumurinn

$$RI_{\text{rms}}^2 = \frac{R}{T} \int_{t_0}^{t_0+T} i^2(t)dt$$

Virkt gildi

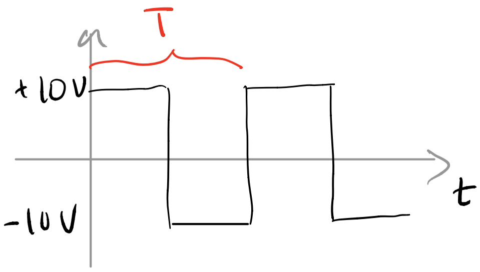
- Þar með er virkt gildi straumsins

$$\boxed{I_{\mathrm{rms}} = \left(\frac{1}{T}\int_{t_1}^{t_2} i^2(t)dt\right)^{1/2}}$$

- Á sama hátt er virkt gildi spennunnar skilgreint

$$\boxed{V_{\mathrm{rms}} \equiv \left(\frac{1}{T}\int_{t_1}^{t_2} v^2(t)dt\right)^{1/2}}$$

Dæmi Finnið RMS gildi á kassabylgju með ±10V útslag



$$V_{RMS} = \left(\frac{1}{T} \int_{t_1}^{t_1+T} 10^2 \, dt \right)^{1/2}$$

$$= \left(\frac{100}{T} (t_1 + T - t_1) \right)^{1/2}$$

$$= (100)^{1/2} = \underline{\underline{10 \text{ V}}}$$

Dæmi Finnið v_{RMS} fyrir $v(t) = \cos(2\pi t)$

$$\omega = 2\pi \quad f = \frac{\omega}{2\pi} = 1 \frac{1}{s} = 1 \text{ Hz} \quad \& \quad \underline{\underline{T = \frac{1}{f} = 1 \text{ s}}}$$

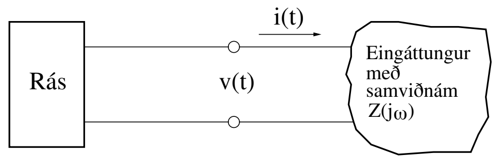
$$V_{RMS} = \left[\frac{1}{1} \int_0^1 \cos^2(2\pi t) \, dt \right]^{1/2}$$

$$= \left[\int_0^1 \left(\frac{1}{2} + \frac{1}{2} \cos(4\pi t) \right) dt \right]^{1/2}$$

$$= \left[\int_0^1 \frac{1}{2} dt + \underbrace{\int_0^1 \frac{1}{2} \cos(4\pi t) \, dt}_{0} \right]^{1/2}$$

$$= \left[\left[\frac{1}{2} t \right]_0^1 \right]^{1/2} = \underline{\underline{\frac{1}{\sqrt{2}} \text{ V}}}$$

Augnabliksafl



- Höfum áður skilgreint **augnabliksafl**

$$\boxed{p(t) = v(t)i(t)}$$

- Þegar sínuslaga innmerki með horntíðni ω er fætt inn á línulega rás verða allir straumar og spennur í rásinni sínuslaga með sömu horntíðni
- Gerum ráð fyrir að

$$i(t) = I_\mathrm{m}\cos(\omega t + \theta_\mathrm{i})$$

og

$$v(t) = V_\mathrm{m}\cos(\omega t + \theta_\mathrm{v})$$

Augnabliksafl

- Þá verður augnabliksaflið

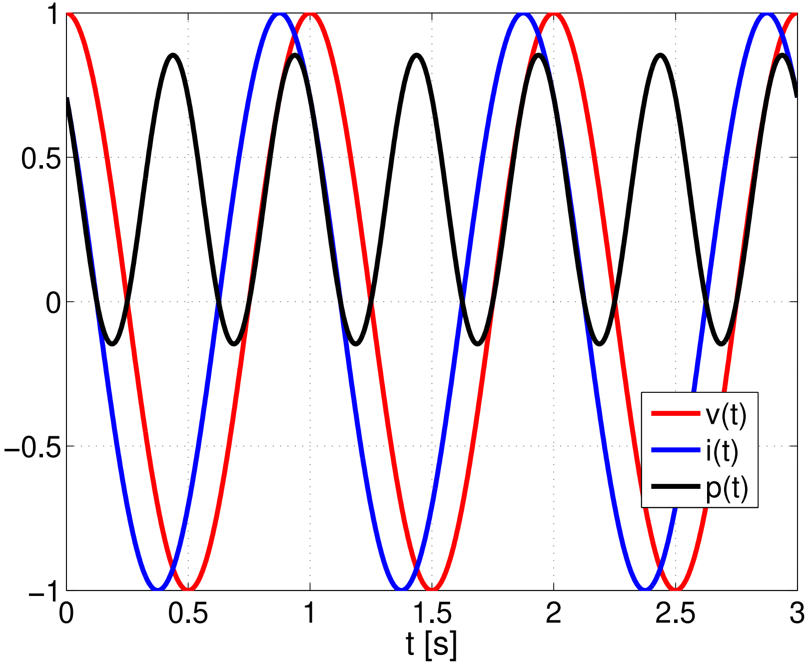
$$p(t) = v(t)i(t) = V_\mathrm{m}\cos(\omega t + \theta_\mathrm{v}) I_\mathrm{m}\cos(\omega t + \theta_\mathrm{i})$$

eða

$$p(t) = \underbrace{\frac{V_\mathrm{m} I_\mathrm{m}}{2}\cos(\theta_\mathrm{v} - \theta_\mathrm{i})}_{\text{fasti}}$$

$$+ \underbrace{\frac{V_\mathrm{m} I_\mathrm{m}}{2}\cos(2\omega t + \theta_\mathrm{v} + \theta_\mathrm{i})}_{\text{tvöföld horntídni}}$$

Augnabliksafl



Meðalafl

- Önnur mikilvæg stærð er meðalafl P eða P_{ave} sem er skilgreint sem meðalgildi augnabliksafls $p(t)$ yfir lotuna T
- Skilgreinum meðalafl

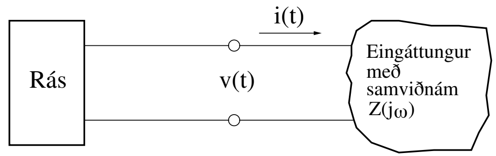
$$\boxed{P_{\text{ave}} = \frac{1}{T}\int_{t_0}^{t_0+T} p(t)dt}$$

eða

$$P_{\text{ave}} = \frac{V_{\text{m}} I_{\text{m}}}{2}\cos(\theta_{\text{v}} - \theta_{\text{i}}) = V_{\text{rms}} I_{\text{rms}}\cos(\theta_{\text{Z}})$$

- **Aflstuðull**: Stærðin $\cos(\theta_{\text{Z}})$ er kölluð aflstuðull (e. power factor, oft stytt í PF).

Tvinntöluafl



$$e^{j(\theta_v - \theta_i)} = \cos(\theta_v - \theta_i) + j\sin(\theta_v - \theta_i)$$

- Við höfum

$$P_{\mathrm{av}} = V_{\mathrm{rms}} I_{\mathrm{rms}} \cos(\theta_v - \theta_I)$$

- Með því að nota Euler jöfnuna þá er hægt að skrifa

$$P_{\mathrm{av}} = \mathrm{Re}\{V_{\mathrm{rms}} e^{j\theta_v} I_{\mathrm{rms}} e^{-j\theta_I}\}$$

Skilgreinum tvær stærðir

$$\mathbf{V}_{\mathrm{rms}} = V_{\mathrm{rms}} \angle \theta_v \quad \mathbf{I}_{\mathrm{rms}} = I_{\mathrm{rms}} \angle \theta_I$$

- Þá má skrifa

$$P_{\mathrm{av}} = \mathrm{Re}\{\mathbf{V}_{\mathrm{rms}} \mathbf{I}^*_{\mathrm{rms}}\}$$

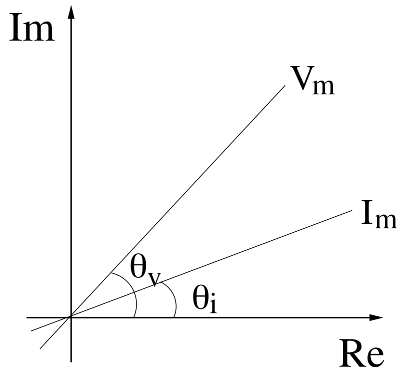
Tvinntöluafl

- Skoðum tvinntöluaflið

$$
\begin{aligned}
S &= \mathbf{V}_{\mathrm{rms}} \mathbf{I}^*_{\mathrm{rms}} \\
&= P_{\mathrm{av}} + jQ \\
&= |S| \angle (\theta_v - \theta_i) \\
&= |S| \cos(\theta_v - \theta_i) + j|S| \sin(\theta_v - \theta_i) \\
S &= \frac{1}{2} \mathbf{V} \mathbf{I}^*
\end{aligned}
$$

Stærðin $|S|$ kallast **sýndarafl** og hefur eininguna VA (volt-amps), og stærðin Q kallast **launafl** og hefur eininguna VAr (volt-amps reactive).

Augnabliksafl og meðalafl



- Það að þekkja aflstuðulinn segir ekki allt um hornið þar eð

$$\cos(\theta_\mathrm{v} - \theta_\mathrm{i}) = \cos(\theta_\mathrm{i} - \theta_\mathrm{v})$$

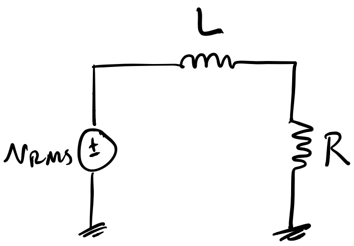
- Til að lýsa þessu horni er talað um seinkaðan aflstuðul ef straumur er á eftir spennu eða álag sé span (e: lagging power factor)
- og flýttan aflstuðul ef straumur er á undan spennu og álag rýmd (e. leading power factor).

Dæmi 13.3

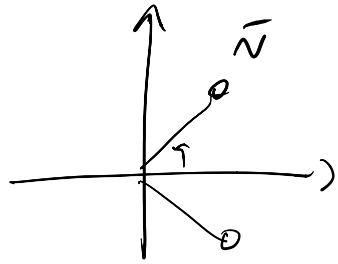
$V_{RMS} = 120\cos(\omega_0 t + 30^\circ)$ en $\omega_0 = 377$ rad/s

$L = 100\,mH$

$R = 15$



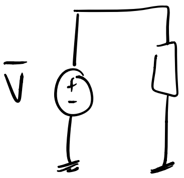
$\bar{V}_{RMS} = 120\angle 30^\circ$



Finnið sýndarafl, meðalafl, launafl & aflstuðul.

$x x^* = |x|^2$

Lausn



$\bar{Z} = R + j\omega L = 15 + j\,377 \cdot 100 \cdot 10^{-3} = 15 + j\,37.7 = 40.6\angle 68.3^\circ$

$\bar{I}_{RMS} = \dfrac{\bar{V}_{RMS}}{\bar{Z}} = \dfrac{120\angle 30^\circ}{40.6\angle 68.3^\circ} = \underline{2.96\angle -38.3^\circ}$

Tvinntöluaflið er $S = \bar{V}_{RMS}\,\bar{I}^*_{RMS} = (120\angle 30^\circ)(2.96\angle 38.3^\circ)$

$= 355\angle 68.3^\circ = 131 + j\,300$

$= P_{av} + j\,Q$

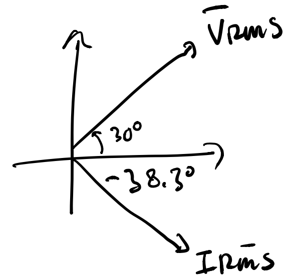
Sýndarafl $|S| = 355$ VA

Meðalafl $P_{av} = 131$ W

Launafl $Q = 300$ VAr

Aflstuðull er $\cos(\theta_v - \theta_i) = \cos(30^\circ + 38.3^\circ)$

$\approx \underline{0.369}$



Straumur er á eftir spennu => "lagging".