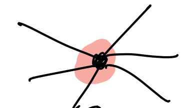
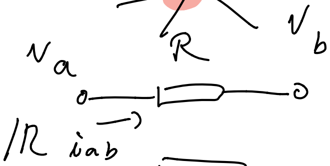
Kirchoff's Voltage Law

Kirchoff's Current Law

$$\begin{cases} \text{KVL} & \sum_{\text{lokuð lykkja}} v_i = 0 \\ \text{KCL} & \sum i_{in} = \sum i_{út} \end{cases}$$



Ohm $v_{ab} = i_{ab} \cdot R$

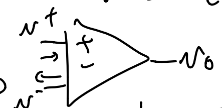


$i_{ab} = (v_a - v_b)/R$

Greining Rása

Jafngildisrásir

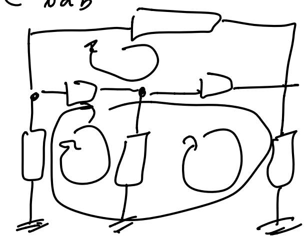
Op amp



$v^+ = v^-$

$i^+ = i^- = 0$

$v_0 \in \mathbb{R}$

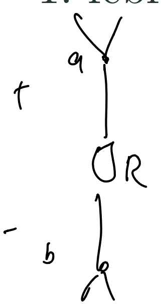


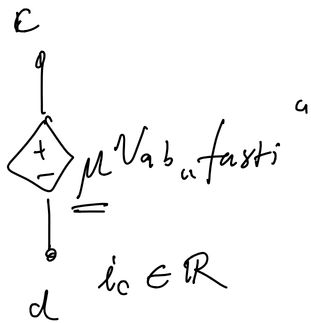
Ólafur Bjarki Bogason

1. febrúar 2021

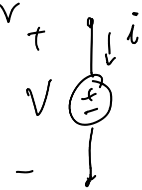
Viðnám $v_{ab} = i_{ab} \underline{\underline{R}}$

Óháðar lindir

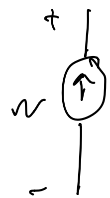




$i_c \in \mathbb{R}$



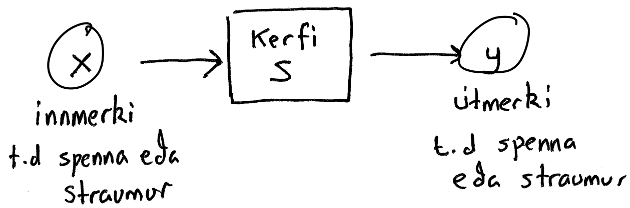
V fasti, $i \in \mathbb{R}$



I fasti en $v \in \mathbb{R}$

Línuleg kerfi/rásir

$y = S(x)$



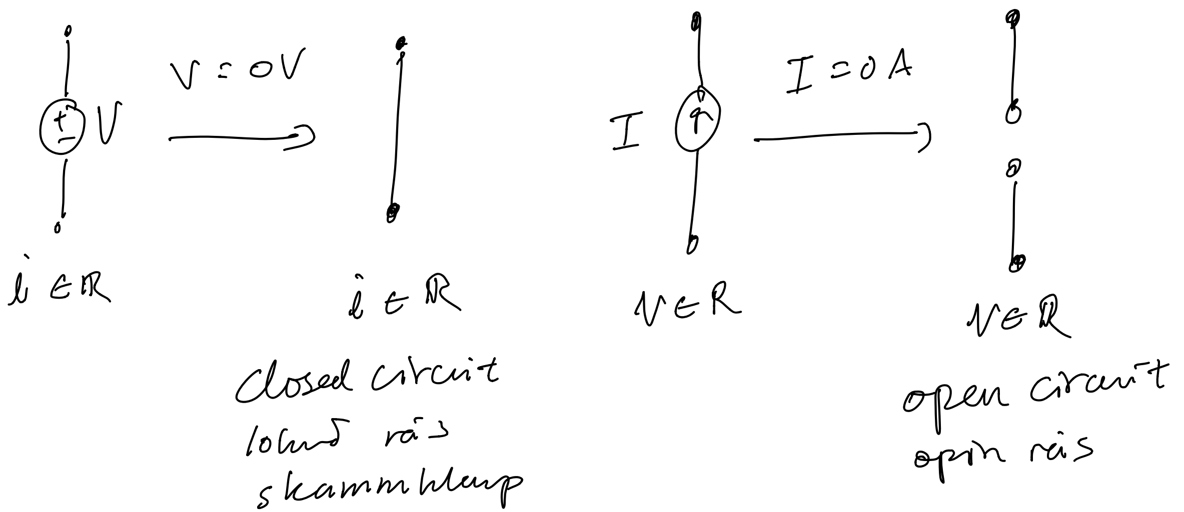
- Samband innmerkis og útmerkis er stundum skrifað $y = S(x)$
- Tvö innmerki x_1 og x_2; Tvö samsvarandi útmerki $\boxed{y_1 = S(x_1)}$ og $\boxed{y_2 = S(x_2)}$
- Fyrir **línuleg kerfi** gildir

$$\boxed{\alpha y_1 + \beta y_2 = S(\alpha x_1 + \beta x_2)} = \alpha S(x_1) + \beta S(x_2)$$

  fyrir alla stuðla α og β
- Dæmi um línuleg kerfi eru viðnám, þéttar og spólur

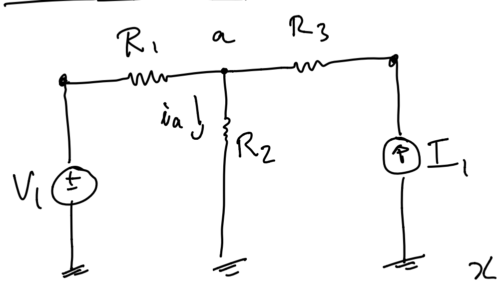
Samlagningareiginleiki

- Rás sem samanstendur af viðnámum, þéttum, spólum, stýrðum lindum er línuleg rás/kerfi
- Í línulegri rás með fleiri en einni óháðari lind, þá má reikna útmerki (spennu eða straum) í rás með því að leggja saman tillegg óháðu lindanna hverrar fyrir sig þegar hinar lindirnar eru núllstilltar.



Dæmi: Finna v_a

$V = 10V$, $R_1 = 4\Omega$ $R_2 = 2\Omega$ $R_3 = 2\Omega$ $I = 2A$



línulegt kerfi (vísir okkar)

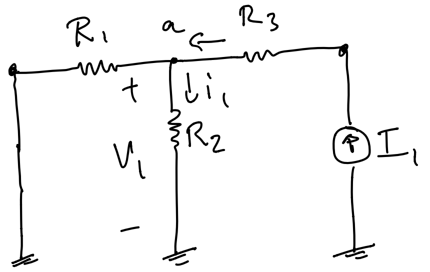
$$\alpha y_1 + \beta y_2 = S(\alpha x_1 + \beta x_2)$$

út/innmerki frá V_1 út/innmerki frá I_1

x, y eru (v, i) par

Lausn

① Núllstillum spennulind $V_1 = 0V$ & finnum v_1 & i_1, aka y_1



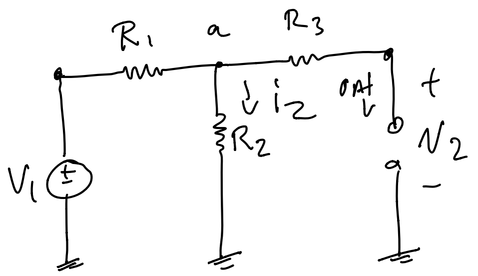
(straumdeiling)

$$i_1 = I_1 \frac{R_1}{R_1 + R_2} = \frac{4}{3} A$$

$$v_1 = R_2 i_1 = \frac{8}{3} V$$

$$y_1 = \Big(\underbrace{\tfrac{8}{3} V}_{v_1}, \underbrace{\tfrac{4}{3} A}_{i_1} \Big)$$

② Núllstillum straumlind $I_1 = 0A$ & finnum $y_2 = (v_2, i_2)$



(spennud.)

$$v_2 = V_1 \frac{R_2}{R_1 + R_2} = \frac{10}{3} V$$

$$i_2 = \frac{v_2}{R_2} = \frac{5}{3} A = \frac{V_1}{R_1 + R_2}$$

$$y_2 = \left(\frac{10}{3} V, \frac{5}{3} A \right)$$

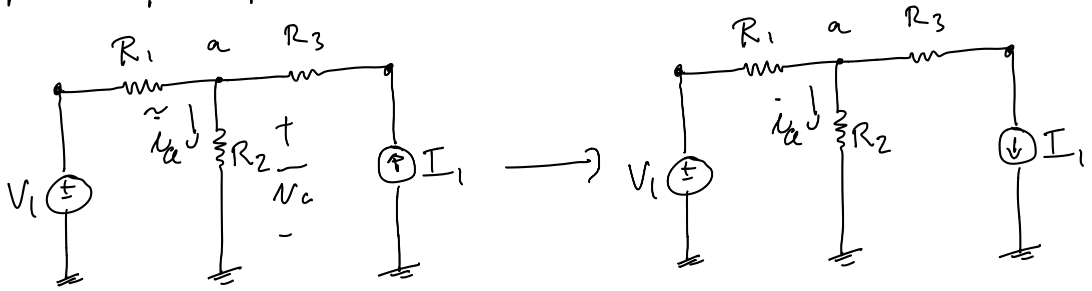
Þá er $(v_a, i_a) = y_1 + y_2$

$$= (v_1 + v_2, i_1 + i_2)$$

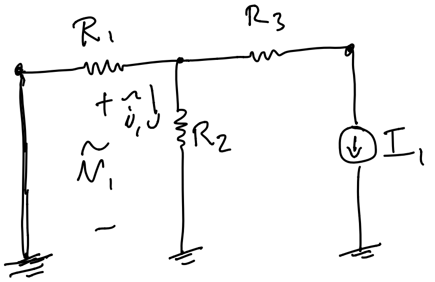
$$= \left(\frac{8}{3} + \frac{10}{3} V, \frac{4}{3} + \frac{5}{3} A \right) = \left(\frac{18}{3} V, \frac{9}{3} A \right)$$

$$= (6V, 3A)$$

Hvað ef stefna I_1 snýst við



Vitum að þetta hefur áhrif á (v_1, i_1)



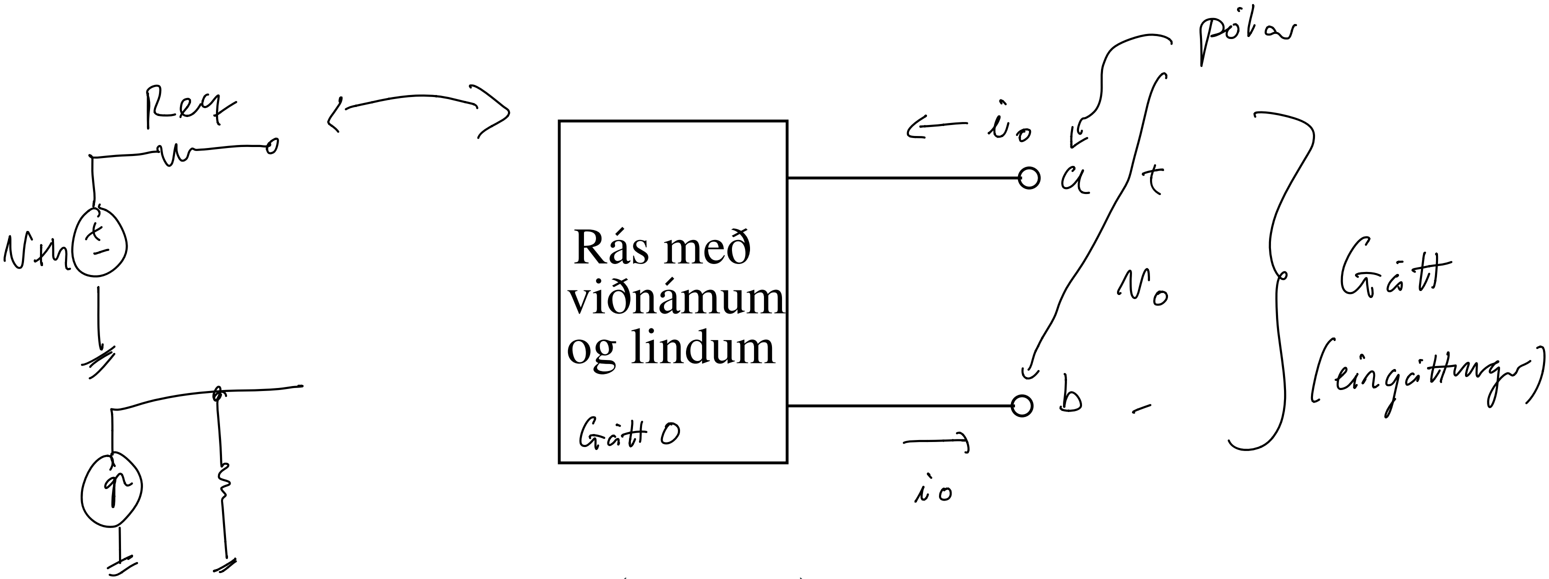
Hér er (straumd) $\tilde{i}_1 = -I_1 \dfrac{R_1}{R_2 + R_1} = -\dfrac{4}{3} A$

$$\tilde{v}_1 = \tilde{i}_1 R_1 = -\frac{8}{3} V$$

Þ $(\tilde{v}_a, \tilde{i}_a) = \left(\tilde{v}_1 + \underline{v_2},\ \tilde{i}_1 + \underline{i_2}\right) = \left(-\frac{8}{3} + \frac{10}{3} V,\ -\frac{4}{3} + \frac{5}{3} A\right)$

$$= \underline{\underline{\left(\frac{2}{3} V,\ \frac{1}{3} A\right)}}$$

Reglur Thevenin og Norton



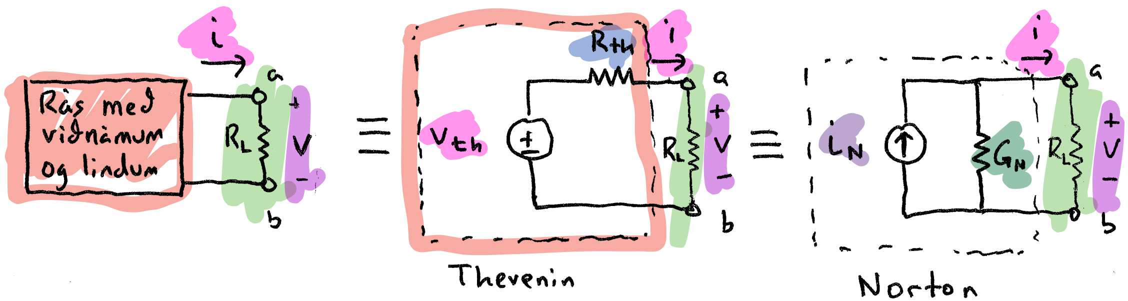
- Tveggja póla rásir (tvípólar) eru kallaðar jafngildar miðað við pólana (a-b) ef sami straumur streymir inn í báðar rásir þegar sama spenna er á milli pólanna; eða öfugt.
- Dæmi um slíkar jafngildisrásir eru jafngildisviðnám fyrir hliðtengingar og raðtengingar viðnáma.

Reglur Thevenin og Norton

Helmholtz (1853) Mayer (1926)

Thévenin (1883) Norton (1926)

- Thévenin og Norton sýndu fram á að rás sem inniheldur línuleg viðnám og lindir (spennu eða straum, stýrðar eða óháðar) hefur jafngildisrásir á forminu:



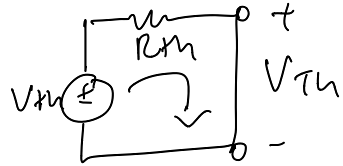
- Til að finna v_{Th}, R_{Th}, i_{N} og R_{N} höfum við rásirnar fyrst ótengdar (ytra viðnám $R_L = \infty$). Þá er augljóst að $i = 0$ og spennurnar V_{oc} eru þær sömu.
- Köllum spennuna V_{oc} **tómgangsspennuna**. Sjáum að

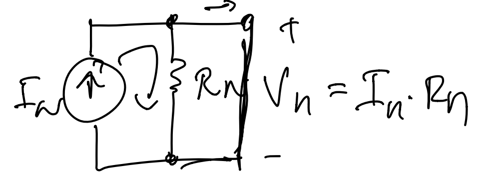
oc = open circuit

$$\boxed{V_{\mathrm{oc}} = V_{\mathrm{Th}} = I_{\mathrm{N}} R_{\mathrm{N}}}$$

$i = 0$

Reglur Thevenin og Norton





- Síðan skammhleypum við milli pólanna a og b (ytra viðnám $R_L = 0$). Þá er $v = 0$ og straumurinn I_{sc} rennur frá a til b
- Köllum strauminn I_{sc} **skammhlaupsstraum**
- Sjáum að

$$\boxed{I_{sc} = I_N = \frac{V_{Th}}{R_{Th}}}$$

- Berum saman ofangreindar jöfnur og sjáum að

$$\boxed{R_{Th} = R_N \equiv R_{eq}}$$

$$V_{oc} = V_{Th} = I_N R_N$$

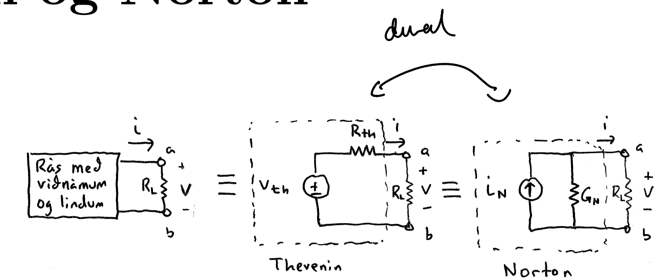
$$I_{sc} = I_N = \frac{V_{Th}}{R_{Th}} \qquad I_N = \frac{V_{Th}}{R_N}$$

Reglur Thevenin og Norton

V_{Th} R_{Th}

I_N R_N

R_{eq}

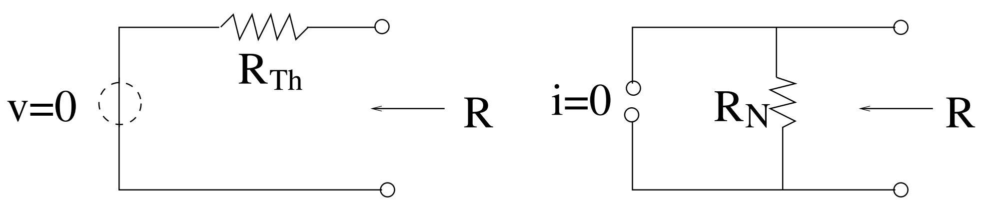


1. Spennulindin í Thévenin-rásinni er tómgangsspenna rásarinnar $V_{\mathrm{Th}} = V_{\mathrm{oc}}$
2. Straumlindin í Norton-rásinni er skammhlaupsstraumur rásarinnar $I_{\mathrm{N}} = I_{\mathrm{sc}}$
3. Raðtengda viðnámið í Thévenin-rásinni er jafnstórt og hliðtengda viðnámið í Norton-rásinni $R_{\mathrm{Th}} = R_{\mathrm{N}}$. Það er oft kallað **útgangsviðnám** R_o og stundum **jafngildisviðnám** R_{eq}
4. Lögmál Ohms tengir saman tómgangs- spennuna, skammhlaupsstrauminn og útgangsviðnámið

$$\boxed{V_{\mathrm{oc}} = I_{\mathrm{sc}} R_{\mathrm{Th}} = I_{\mathrm{sc}} R_{\mathrm{N}}}$$

Reglur Thevenin og Norton

- Einnig sjáum við að ef spennulindin í Thévenin-rásinni er núllstillt (skammhlaup) þá er R_{Th} það viðnám sem við sjáum á milli pólanna
- Sama gildir um Norton rásina; ef straumlindin er núllstillt (opin rás) þá er R_{N} það viðnám sem við sjáum á milli pólanna

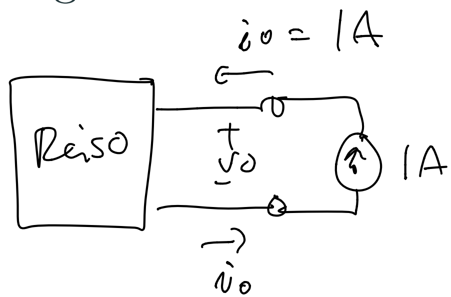


Reglur Thevenin og Norton

Fyrir hvaða rás sem er (viðnám og lindir) má finna jafngildis útgangsviðnám: $R_{eq} = R_{Th} = R_N$

1. Núllstilla allar <u>óháðar</u> lindir
   - setja skammhlaup fyrir spennulind ($v = 0$, i óþekkt)
   - opna rás fyrir straumlind ($i = 0$, v óþekkt)
2. Finna jafngildisviðnám milli pólanna.
   Algengasta leiðin er að setja 1 A prufustraum inn á rásina (milli pólanna) og finna hver spennan verður ~~á milli pólanna~~. Sú spenna er þá tölulega jafnstór og jafngildisviðnámið, þ.e

$$R_{eq} = R_N = R_{\mathrm{Th}} = \frac{V_{\mathrm{o}}\ [\mathrm{V}]}{1\ [\mathrm{A}]} = V_{\mathrm{o}}\ [\Omega]$$

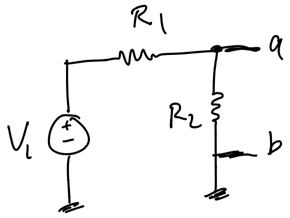


Reglur Thevenin og Norton

- Til að finna Thévenin- og Norton-jafngildisrásir fyrir tiltekna rás er nægilegt að finna tvær af stærðunum þremur R_{Th}, V_{oc} og I_{sc}. $V_{oc} = R_{eq} I_{sc}$
- Athuga ber að Thévenin- og Norton- jafngildisrásir eru aðeins jafngildar miðað við pólana (a og b); þær segja ekkert um hvað gerist inni í rásinni, t.d. afltöp.

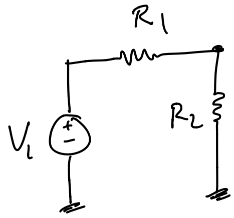
$$\boxed{V_{oc} = V_{Th} = R_{eq} I_{sc} = R_{eq} I_N}$$

Dæmi: Finna Thévenin & Norton jafngildisrásir yfir R_2 ef $R_1 = 2\Omega$, $R_2 = 3\Omega$, $V_1 = 10V$.



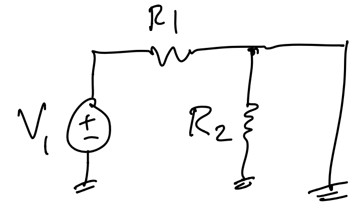
Lausn: Vantar að finna 2 af 3 stærðum V_{oc}, I_{sc}, R_{eq}

① Byrja að finna V_{oc} yfir R_2



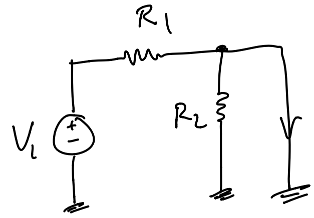
(spennu)

$$+\; V_{oc}\; - \qquad V_{oc} = V_1 \frac{R_2}{R_1+R_2} = 10\,\frac{3\Omega}{2\Omega+3\Omega} = 10\,\frac{3}{5} = \underline{6V}$$



$$\downarrow I_{sc} = \frac{V_1 - 0}{R_1}$$

② Finn næst I_{sc}

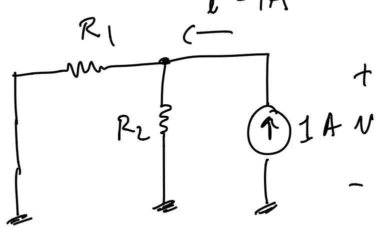


$$I_{sc} = \frac{V_1 - 0}{R_1} = \frac{10V}{2\Omega} = \underline{\underline{5A}}$$

③ Þá er $R_{eq} = \dfrac{V_{oc}}{i_{sc}} = \dfrac{6V}{5A} = \underline{\underline{1\frac{1}{5}\Omega}}$

③' Getum líka sett "prufustraum" 1A þegar $V_1 = 0V$

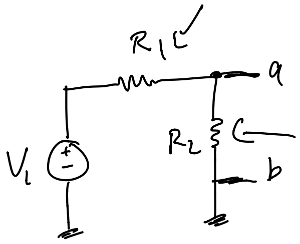
$$\boxed{R_{eq} = \frac{v}{1A} = v\;\Omega}$$



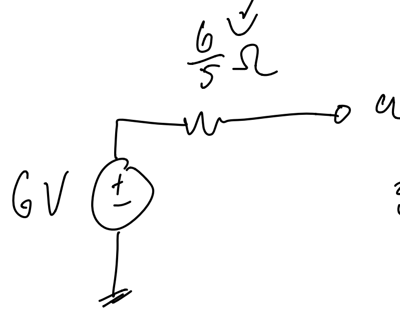
en $v = i\,\underline{\underline{R_{eq}}} = i\,(R_1 \parallel R_2)$

svo $R_{eq} = R_1 \parallel R_2 = \dfrac{2 \cdot 3}{2+3} = \underline{\underline{\dfrac{6}{5}\Omega}}$

$p = v \cdot i$

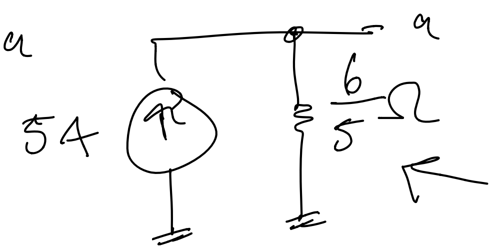


afltap > 0W



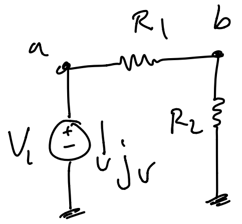
Thévenin

0W!



Norton

> 0W

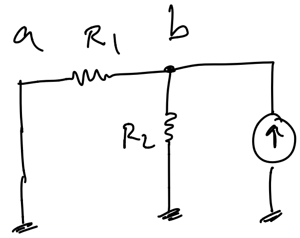


Auðvitað gætum við líka leyst þetta með MNA.

$$
\begin{array}{c|c}
 & \begin{matrix} a & \quad b & \quad V_1 \end{matrix} \\
\begin{matrix} a \\ b \\ V_1 \end{matrix} &
\left[\begin{array}{cc|c} G_1 & -G_1 & 1 \\ -G_1 & G_1+G_2 & 0 \\ \hline 1 & 0 & 0 \end{array}\right]
\end{array}
\begin{bmatrix} v_a \\ v_b \\ j_V \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ V_1 \end{bmatrix} \quad \text{eða} \quad \begin{bmatrix} v_a \\ v_b \\ j_V \end{bmatrix} = \begin{bmatrix} 10\text{ V} \\ 6\cancel{8}\text{ V} \\ -2\text{A} \end{bmatrix}
$$

① Byrja að finna v_{oc} yfir R_2, eða bara nema $v_{oc} = v_b$

② Set nú $V_1 = 0$ & 1A prufstraum inn



$v = \tilde{v}_b = 1\frac{1}{5}\text{ V}$ þá er $R_{eq} = \dfrac{\tilde{v}_b}{1\text{A}} = 1\frac{1}{5}\,\Omega$

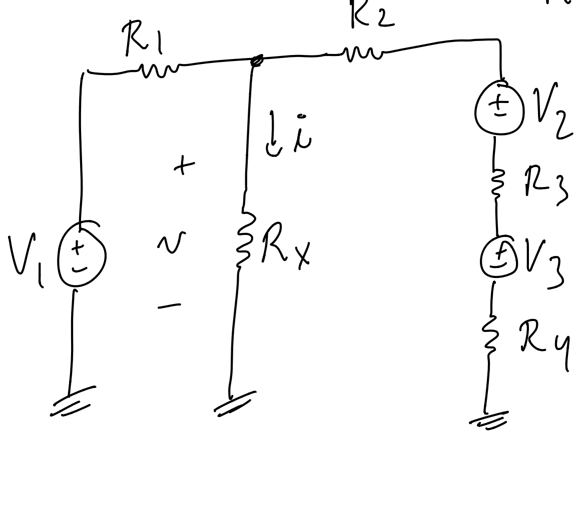
$V_1 = 0$

$$
\left[\begin{array}{cc|c} G_1 & -G_1 & 1 \\ -G_1 & G_1+G_2 & 0 \\ \hline 1 & 0 & 0 \end{array}\right]
\begin{bmatrix} \tilde{v}_a \\ \tilde{v}_b \\ \tilde{j}_V \end{bmatrix} = \begin{bmatrix} 0 \\ 1\text{A} \\ 0 \end{bmatrix} \quad \text{eða} \quad \begin{bmatrix} \tilde{v}_a \\ \tilde{v}_b \\ \tilde{j}_V \end{bmatrix} = \begin{bmatrix} 0\text{V} \\ 1\frac{1}{5}\text{ V} \\ 0.6\text{A} \end{bmatrix}
$$

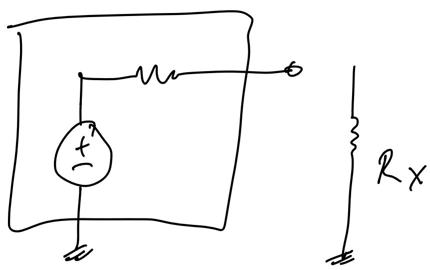
Dæmi fyrir ykkur $V_1 = 100\text{V}$, $V_2 = 50\text{V}$, $V_3 = 10\text{V}$, $R_4 = 2.3\,\Omega$

$R_1 = 4\,\Omega$, $R_2 = 2.2\,\Omega$, $R_3 = 1.5\,\Omega$,

$R_x = 5\,\Omega$



Finna i & v með Thevenin reglu.

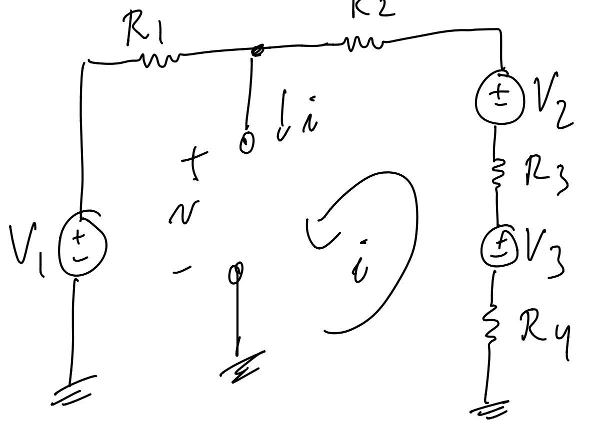


① $V_{oc} = ?$

② $I_{sc} = ?$

③ $V_{1,2,3} = 0$ & 1A prufstr.

$R_{eq} = \dfrac{V_{oc}}{I_{sc}}$

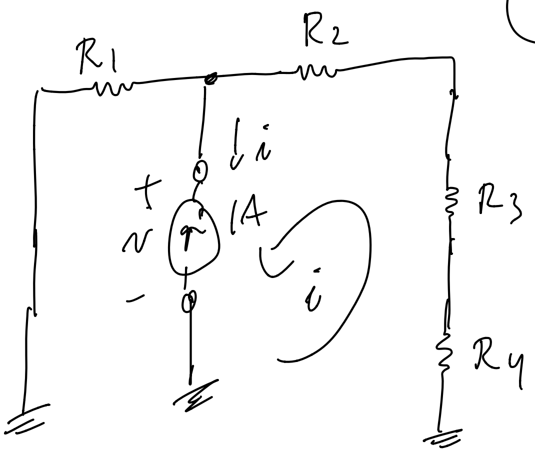


$$i = \frac{V_3 + V_2 - V_1}{R_1 + R_2 + R_3 + R_4} = \underline{-4A}$$

1

$$v_{oc} = V_1 + R_1 \cdot i = \underline{84V = V_{Th}}$$

2

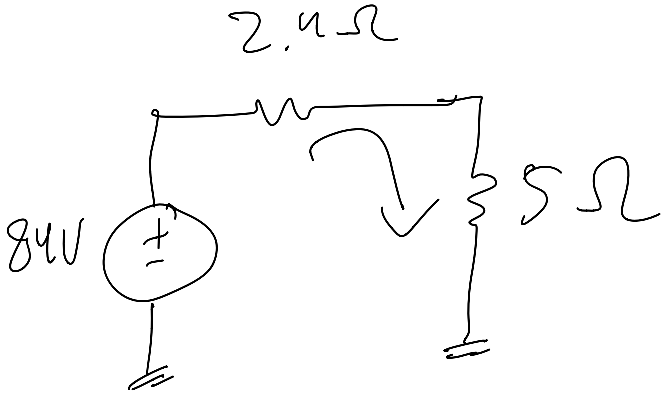


$$R_{eq} = \frac{v}{1A} = v$$

$$v = i \cdot R_{eq}$$

$$= \cancel{i}^{1A} \left(R_1 \parallel (R_2 + R_3 + R_4) \right)$$

$$= \underline{2.4\,\Omega}$$



$$i = \frac{V_{Th} - 0}{R_{eq} + R_X} = \underline{11.35A}$$

$$\underline{v = i \cdot R_X = 56.75V}$$

Reglur Thevenin og Norton

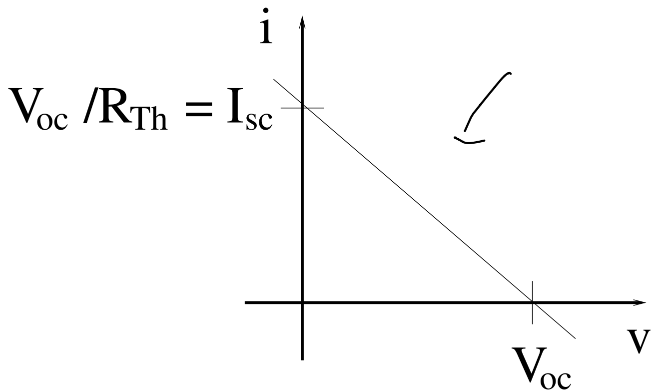
Skoðum nú $I - V$ kennilínur Thévenin- og Norton-rásanna.

Spennan v í Thévenin-rásinni fæst samkvæmt KVL:

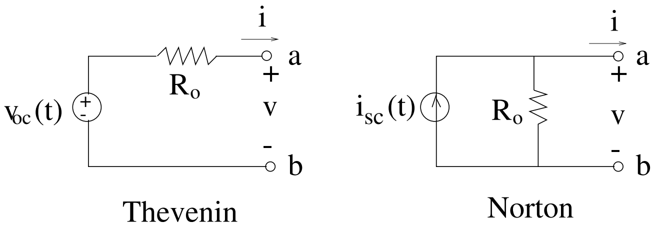
$$v = V_{\mathrm{oc}} - iR_{\cancel{\mathrm{o}}\mathrm{th}}$$

$$\boxed{i = -\frac{1}{R_{\mathrm{o}}}v + \frac{V_{\mathrm{oc}}}{R_{\cancel{\mathrm{o}}\mathrm{th}}} = -\frac{1}{R_{\cancel{\mathrm{o}}\mathrm{th}}}v + I_{\mathrm{sc}}}$$

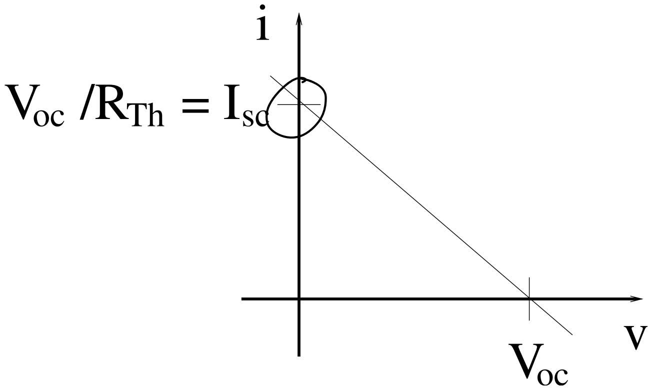
sem fæst einnig með KCL út frá Norton-rásinni.



Reglur Thevenin og Norton

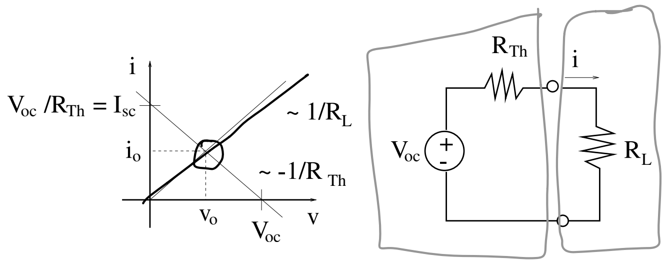


Kennilínan er því bein lína með hallatölu $-1/R_\mathrm{o}$ og skurðpunkt við $i-$ás í $i = I_\mathrm{sc}$.



Reglur Thevenin og Norton

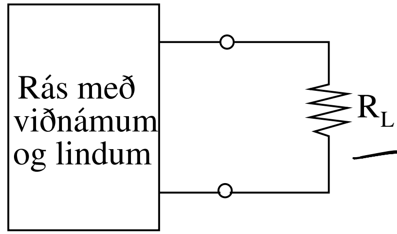
Tengjum viðnám milli pólanna á Thévenin- rásinni og teiknum $i - v$ kennilínu viðnámsins inn á sömu mynd og $i - v$ kennilínu Thévenin rásarinnar.



Skurðpunktur línanna segir til um þá spennu og þann straum sem uppfyllir skilyrði beggja rásahluta og er hann jafnframt eina lausnin $(v_\mathrm{o}, i_\mathrm{o})$.

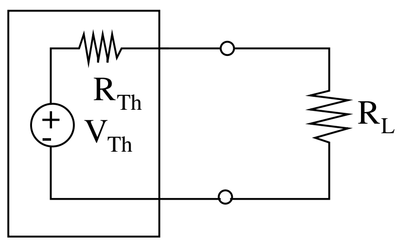
Hámarksaflflutningur

$\max(p = v \cdot i) = ?$



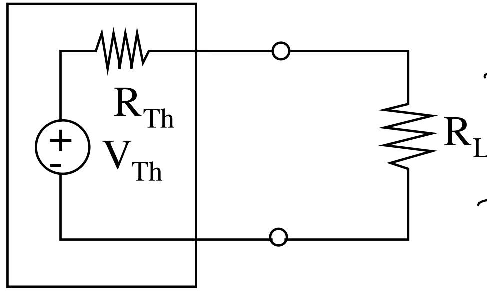
Hvernig á að velja R_L til að hámarka aflið í R_L ?

Finnum fyrst Thévenin-jafngildisrás fyrir rásina. Finnum síðan aflið í R_L, $P_L(t)$, sem fall af R_L.



Hámarksaflflutningur

$\boxed{p = v\,i}$



$+$ V_o $-$

$V = I \cdot R$

$p = v \cdot i = \dfrac{v^2}{R}$

Með spennudeilingu fæst að

$$v_{\rm o}(t) = V_{\rm Th}(t)\frac{R_L}{R_{\rm Th}+R_L}$$

og alfið í R_L er

$$P_L(t) = \frac{v_{\rm o}^2(t)}{R_L} = \frac{V_{\rm Th}^2(t)}{R_L}\frac{R_L^2}{(R_{\rm Th}+R_L)^2} = \frac{V_{\rm Th}^2(t)R_L}{(R_{\rm Th}+R_L)^2}$$

Diffrum með tilliti til R_L og setjum diffurkvótann núll

$$\frac{\partial P_L(t)}{\partial R_L} = 0$$

Hámarksaflflutningur

Hér er $f(R_L) = V_{Th}^2(t) R_L$

$g(R_L) = (R_{Th} + R_L)^2$

$$\frac{d}{dx} \frac{f}{g} = \frac{f'g - g'f}{g^2}$$

$$V_{\mathrm{Th}}^2 \frac{(R_L + R_{\mathrm{Th}})^2 - R_L 2(R_L + R_{\mathrm{Th}})}{(R_L + R_{\mathrm{Th}})^4} = 0$$

eða

$$R_L^2 + 2R_L R_{\mathrm{Th}} + R_{\mathrm{Th}}^2 - 2R_L^2 - 2R_L R_{\mathrm{Th}} = 0$$

eða

$$R_{\mathrm{Th}}^2 - R_L^2 = 0$$

og

$$R_{\mathrm{Th}} = R_L$$

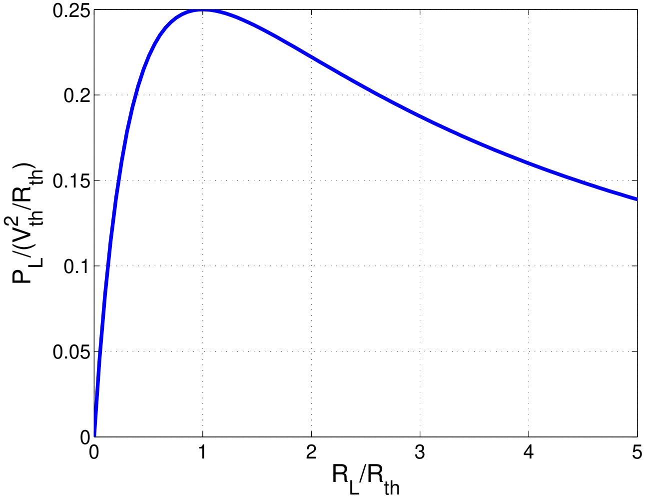
Hámarksaflflutningur

Hámarksaflið verður þá

$$(P_L(t))_{\max} = \frac{v_{\mathrm{o}}^2}{R_L} = \frac{V_{\mathrm{Th}}^2}{4R_{\mathrm{Th}}}$$

kallað mesta fánlegt afl og það er fáanlegt aðeins ef **álagsviðnámið** R_L er **aðhæft** að rásinni.

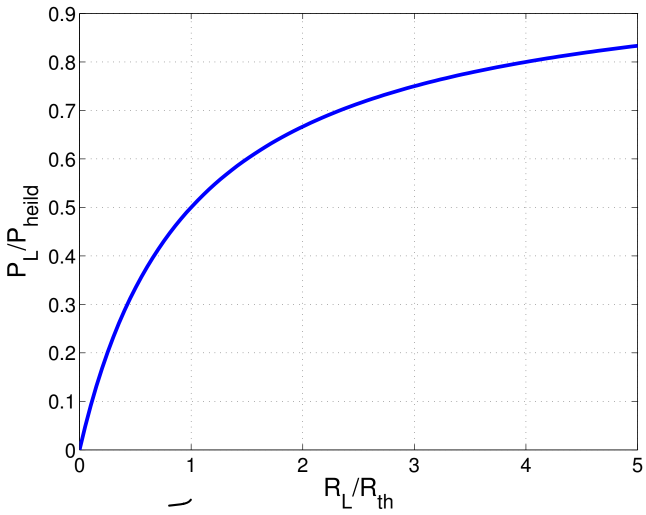
Hámarksaflflutningur



- Getum endurritað

$$\frac{P_L}{V_{\mathrm{Th}}^2/R_{\mathrm{Th}}} = \frac{R_{\mathrm{L}}/R_{\mathrm{Th}}}{(1 + R_{\mathrm{L}}/R_{\mathrm{Th}})^2}$$

Aflnýtni

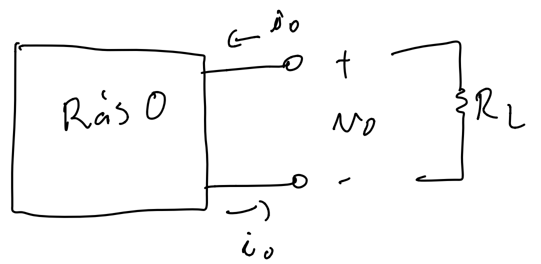


- Aflnýtni er afl sem eyðist í álagi deilt með heildarafli sem flutt er til rásar

$$\frac{P_L}{P_{heild}} = \frac{I^2 R_L}{I^2(R_L + R_{Th})}$$

$$= \frac{R_L}{R_L + R_{Th}}$$

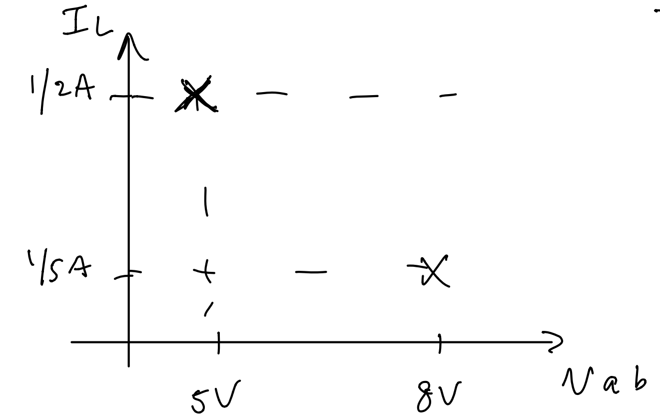
Dæmi Höfum „Rás 0" sem er óþekkt, en við viljum finna Thévenin / Norton jafngildisrás fyrir

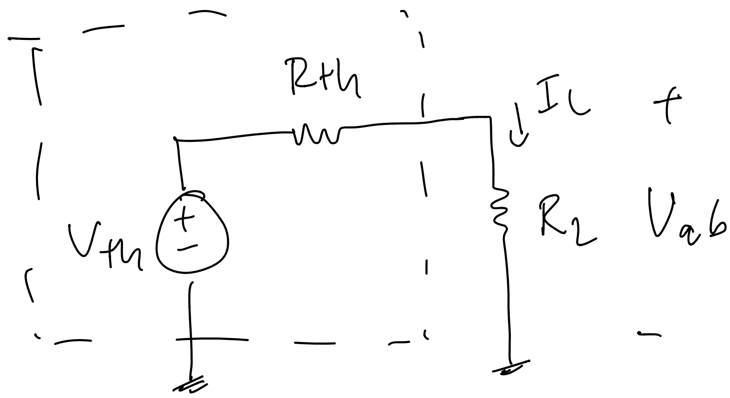


Setjum inn ólík R_L

V_{ab}	I_L
5V	$\frac{1}{2}$ A
8V	$\frac{1}{5}$ A

Lausn





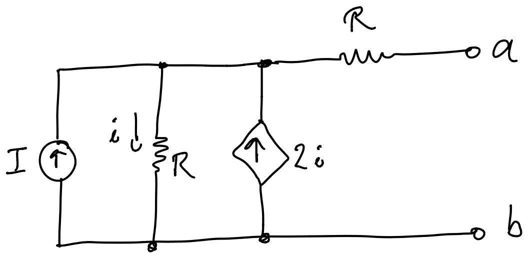
Þá $V_{ab} = \underline{V_{th}} - I_L \underline{R_{th}}$

óþekkt

Höfum
$$5 = V_{th} - \frac{1}{2} R_{th}$$
$$8 = V_{th} - \frac{1}{5} R_{th}$$

$$\begin{bmatrix} 1 & -\frac{1}{2} \\ 1 & -\frac{1}{5} \end{bmatrix} \begin{bmatrix} V_{th} \\ R_{th} \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \end{bmatrix} \text{ svo } \begin{bmatrix} V_{th} \\ R_{th} \end{bmatrix} = \begin{bmatrix} 10\,V \\ 10\,\Omega \end{bmatrix}$$

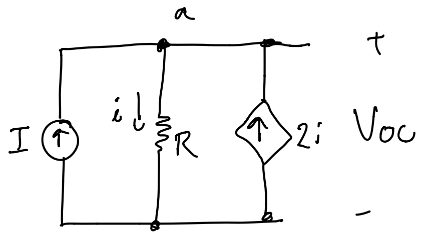
Dæmi: Finna Thévenin & Norton jafngildisrásir milli póla a & b, ef $I = 5A$



CCCS

Lausn: þurfum að finna V_{oc} (1) & R_{th} (2).

(1) V_{oc}, tómagangsspennan



KCL í a gefur: $I + 2i = (v_a - 0)\,G$

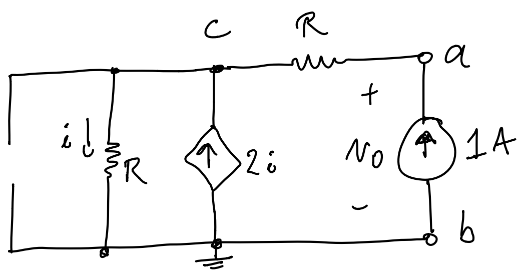
& Lögmál Ohm $i = (v_a - 0)\cdot G$

svo $I + 2 v_a G = v_a G$

eða $v_a = -\dfrac{I}{G} = -I\cdot R = -5R$

svo $V_{oc} = -5R$ V

(2)



CCCS

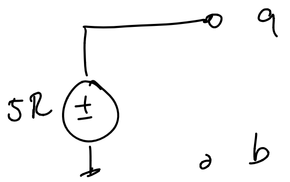
KCL í c gefur

$1 + 2i = i \Rightarrow i = -1A$

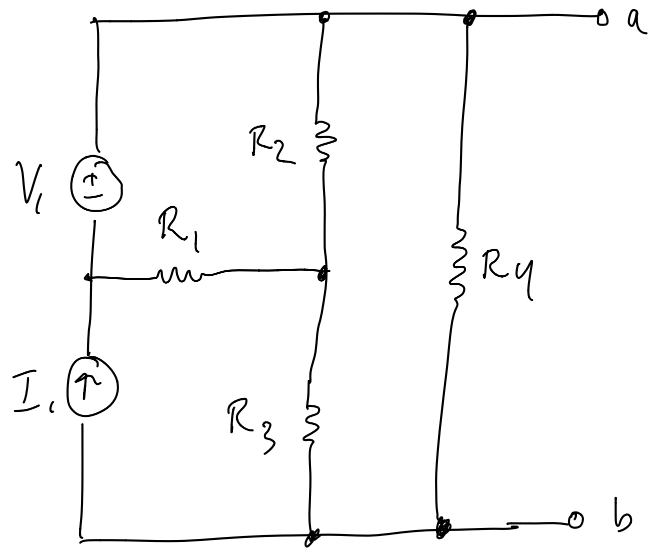
$v_o = 1\cdot R + i\cdot R = 0$

svo $R_{th} = 0\,\Omega$

Þá er Thévenin jafngildisrásin en þar sem $R_{th} = 0\,\Omega$ er Norton-jafngildisrás óskilgreind milli póla a & b.



Dæmi Finnum Norton & Thévenin jafngildisrásir milli póla a & b.



$V_1 = 6V$, $I_1 = 2mA$, $R_1 = 2k\Omega$, $R_2 = 4k\Omega$

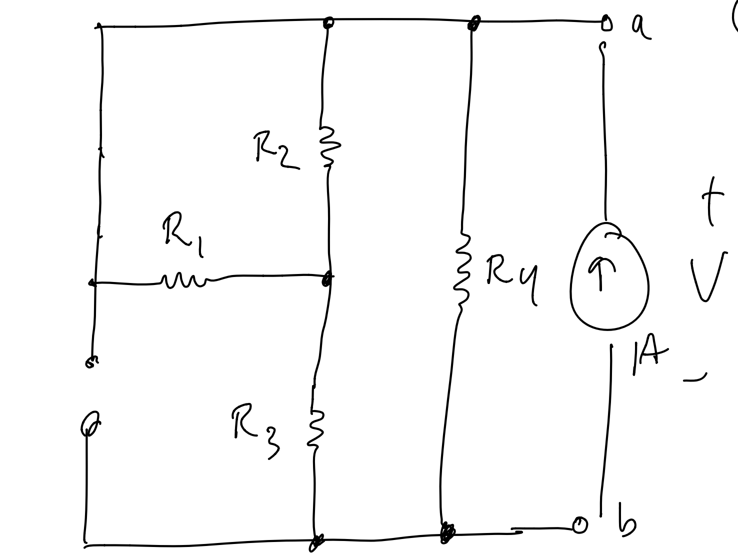
$R_3 = 2k\Omega$, $R_4 = 6k\Omega$

Lausn Finnum:

① R_{eq} jafngildisviðnám

② $V_{th} = V_{oc}$ tómagangsspenna

③ $I_{sc} = I_N$ skammhlaupsstrauminn

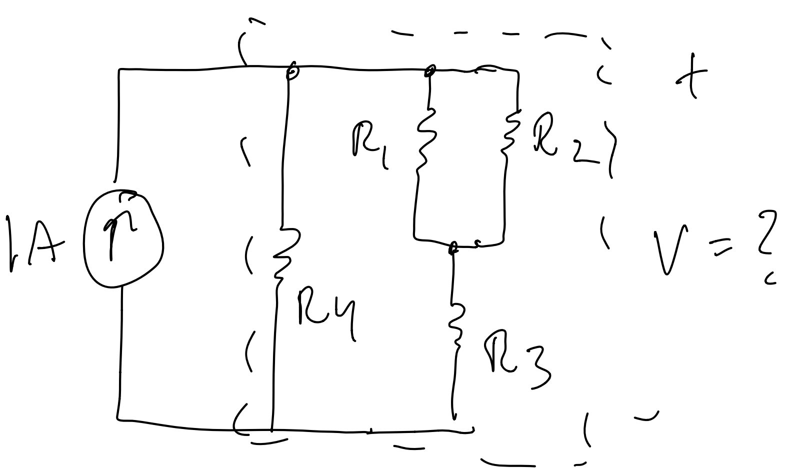


① Núllstillum óháðar lindir, Setjum 1A prufustraum & mælum spennu yfir straumlindina

$v = (1A) \cdot R_{eq}$

svo $\underline{\underline{R_{eq} = v\,[\Omega]}}$

Endurteiknum rásina...

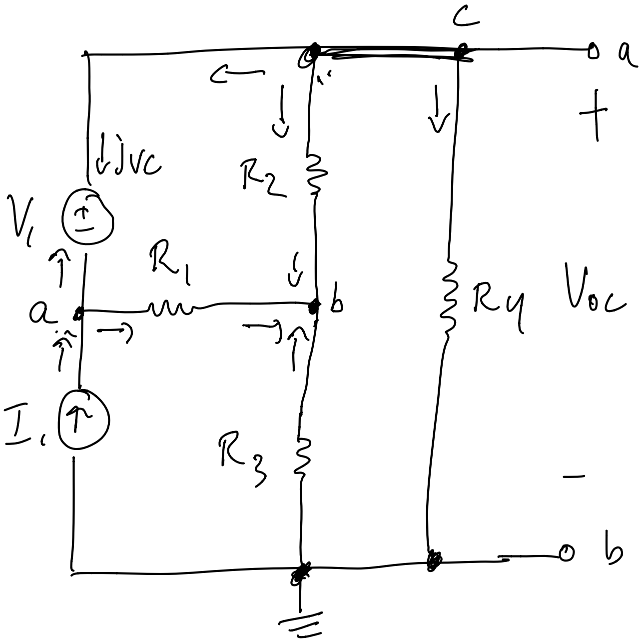


$R_{eq} = R_4 \parallel \left(R_1 \parallel R_2 + R_3\right)$

$= 6k\Omega \parallel \left(2k\Omega \parallel 4k\Omega + 2k\Omega\right)$

$\vdots$

$= \underline{\underline{\frac{15}{7} k\Omega}}$



② Finnum nú V_{oc} tómagangsspennu með hefðbundinni rásagreiningu

$$N_{jöfnu} = N_{hnútpunkta} + N_{VS} - 1$$
$$= 4 + 1 - 1 = \underline{4 \text{ jöf!}}$$

$$G_i = \frac{1}{R_i}$$

(a) $G_1(V_a - V_b) - j_{vc} = I_1$

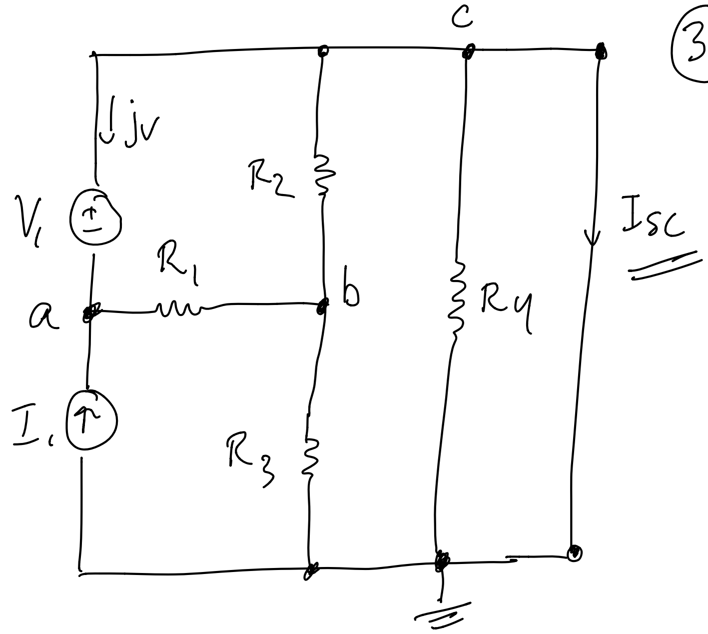
(c) $G_2(V_c - V_b) + G_4(V_c - 0) + j_{vc} = 0$

(b) $G_1(V_a - V_b) + G_2(V_c - V_b) + G_3(0 - V_b) = 0$

$$\begin{array}{c c} & \begin{array}{cccc} a & b & c & V_1 \end{array} \\ \begin{array}{c} a \\ b \\ c \\ V_1 \end{array} & \begin{bmatrix} G_1 & -G_1 & 0 & -1 \\ G_1 & -G_1-G_2-G_3 & G_2 & 0 \\ 0 & -G_2 & G_2+G_4 & 1 \\ -1 & 0 & 1 & 0 \end{bmatrix} \end{array} \begin{bmatrix} V_a \\ V_b \\ V_c \\ j_{vc} \end{bmatrix} = \begin{bmatrix} I_1 \\ 0 \\ 0 \\ V_1 \end{bmatrix}$$

$$\begin{bmatrix} V_a \\ V_b \\ V_c \\ j_{vc} \end{bmatrix} = \begin{bmatrix} 6/7 \text{ V} \\ 12/7 \text{ V} \\ 48/7 \text{ V} \\ -0.0024 \text{ A} \end{bmatrix}$$

$$\underline{V_{th} = V_{oc} = V_c = 48/7 \text{ V}}$$

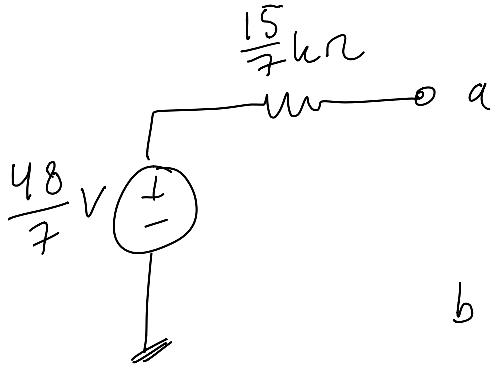


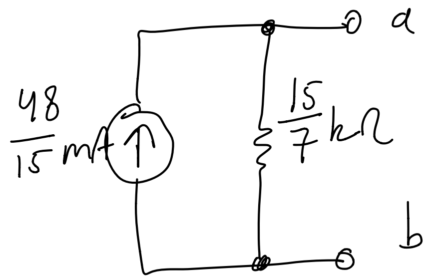
(3) Finnum nú skammhlaupsstrauminn með lögmáli Ohm.

$$I_{sc} = I_N = \frac{V_{oc}}{R_{eq}} = \frac{V_{th}}{R_{th}}$$

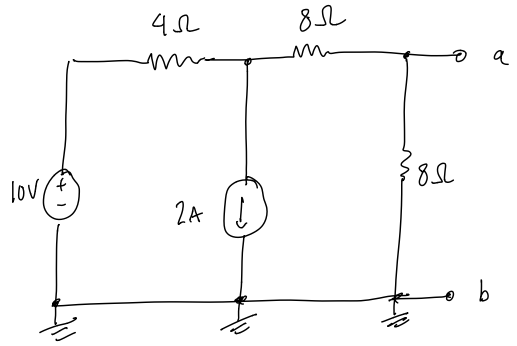
$$= \frac{48/7\ V}{15/7\ k\Omega} = \frac{48}{15}\ mA$$

Thévenin & Norton jafngildisrásirnar yfir póla a & b eru þá...

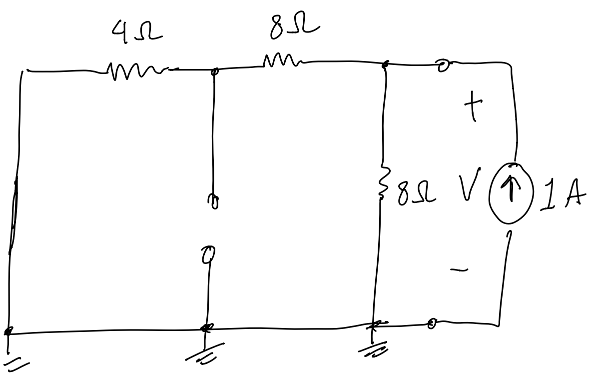




Dæmi: Hvert er Thévenin viðnámið á milli póla a & b.



Lausn: Núllstillum óháðar lindir, setjum 1A prufustraum & finnum spennu.



$V = (1A) \cdot R_{eq}$ svo $R_{eq} = V\ [\Omega]$

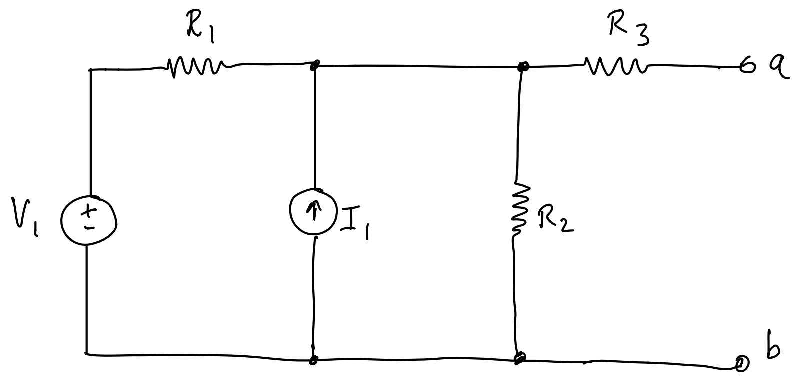
Hér er $R_{eq} = 8 \,\|\, (4+8) = 8 \,\|\, 12$

$$= \frac{8 \cdot 12}{8+12} = \frac{8 \cdot 12}{20}$$

$$= \frac{2 \cdot 12}{5} = \frac{24}{5}\,\Omega$$

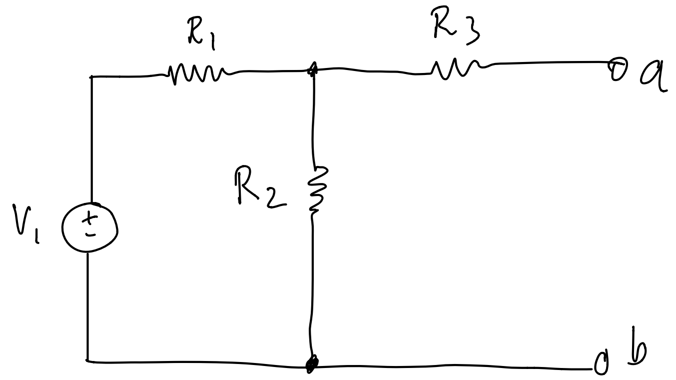
Nokkur dæmi. Lausnir fylgja á næstu síðum. Æfið ykkur fyrst sjálf!

Dæmi 1 Finnið Thévenin jafngildisrásina á milli póla a & b



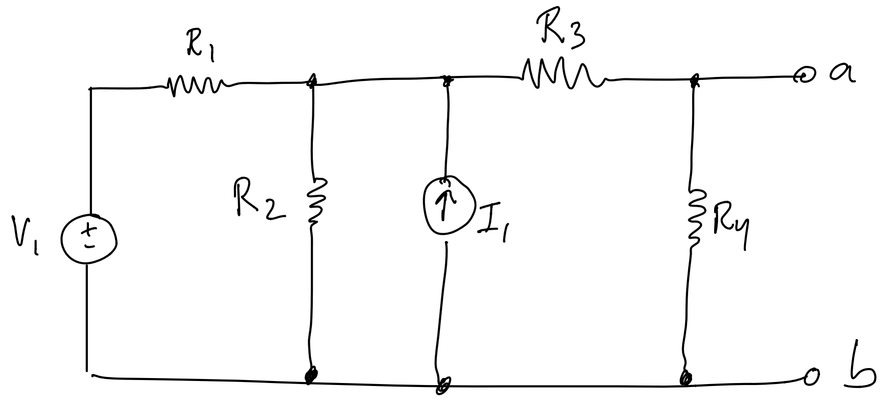
$V_1 = 10V$, $R_1 = 5\Omega$, $R_2 = 20\Omega$
$R_3 = 6\Omega$ $I_1 = 2A$

Dæmi 2 Finnið Thévenin jafngildisrásina á milli póla a & b



$V_1 = 24V$ $R_1 = 100\Omega$ $R_2 = 200\Omega$
$R_3 = 150\Omega$

Dæmi 3 Finnið Thévenin jafngildisrásina á milli póla a & b

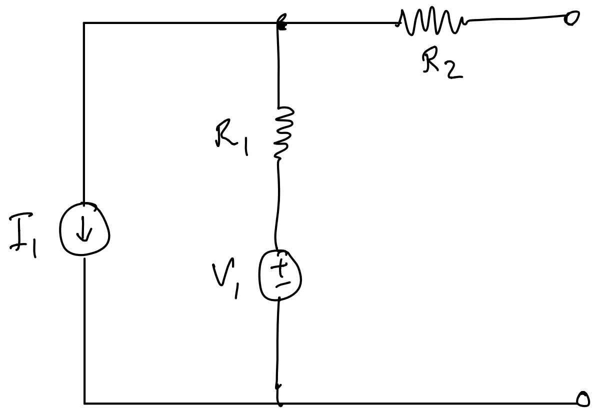


$V_1 = 20V$ $R_1 = 1k\Omega$ $R_2 = 4k\Omega$
$I_1 = 24A$ $R_3 = 1k\Omega$ $R_4 = 4k\Omega$

Dæmi 4 (Heimadæmi 2020) (Gott að skoða lausn betur...)

Finnið Thévenin jafngildisrásina á milli póla a & b

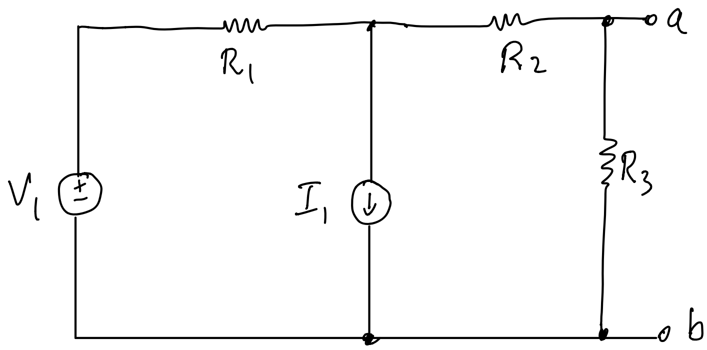
$V_1 = 10V \quad I_1 = 20mA \quad R_1 = 5k\Omega \quad R_2 = 10k\Omega$



Dæmi 5 (Heimadæmi 2020)

Finnið Thévenin jafngildisrásina á milli póla a & b

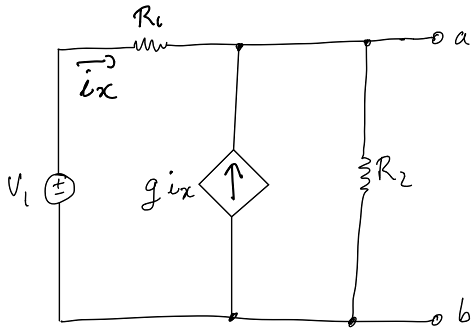
$V_1 = 10V, I_1 = 2A, R_1 = 4\Omega, R_2 = 8\Omega, R_3 = 8\Omega$



Dæmi 6

Finnið Thévenin jafngildisrásina á milli póla a & b

$V_1 = 10V, \; g = 9, \; R_1 = 100\Omega, \; R_2 = 10\Omega$

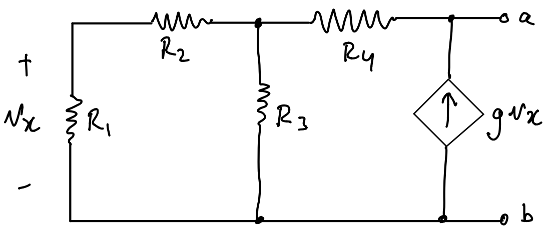


Dæmi 7

Finnið Thévenin jafngildisrásina milli póla a & b

$R_1 = 50\Omega \quad R_2 = 200\Omega \quad R_3 = 150\Omega \quad R_4 = 100\Omega$

$g = \frac{1}{100}$



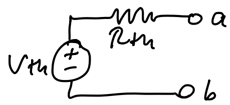
Lausnir við Dæmum

Skrefin eru alltaf þau sömu:

① Finna tómagangsspennu $v_{oc} = v_{th}$

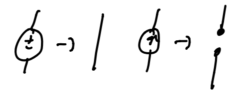
② Núllstilla óháðar lindir, setja 1A „prufustraum" á milli a & b, þá er $R_{eq} = v$ [Ω] spennan yfir straumlindina.

③ Teiknum Thévenin jafngildisrás



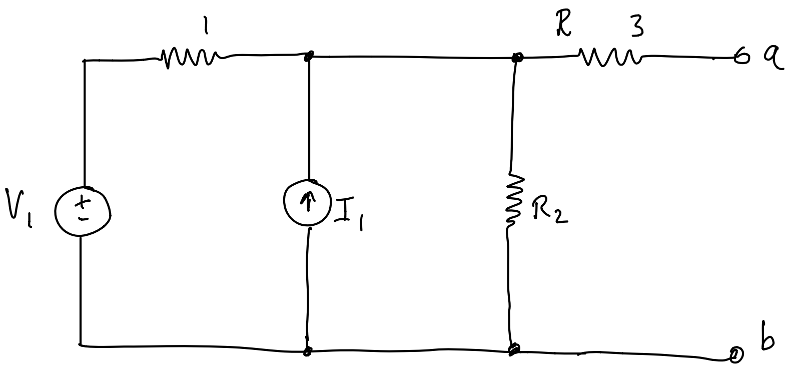
Gott að muna:

1. $G = 1/R$
2. núllstillum lindir



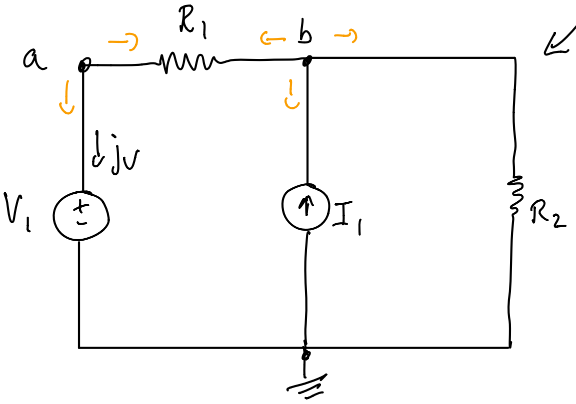
Lausn 1

$V_1 = 10V$, $R_1 = 5\Omega$, $R_2 = 20\Omega$, $R_3 = 6\Omega$, $I_1 = 2A$



R

① R_3 er ótengt svo rásin lítur svona út. Finnum núna spennuna yfir R_2 með hnútpunktagreiningu



$N_{jöfnur} = N_{hnútpunktar} + N_{spennulindir} - 1 = 3 + 1 - 1 = \underline{3}$

KCL í a: $G_1(v_a - v_b) + j_V = 0$

KCL í b: $G_1(v_b - v_a) + G_2(v_b - 0) - I_1 = 0$

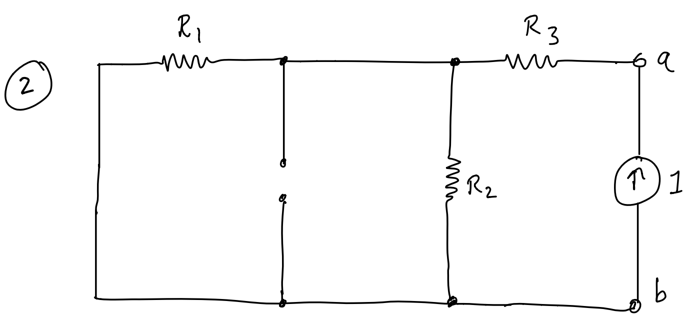
KVL yfir V_1: $(v_a - 0) = V_1$

Setjum í jöfnukerfi & leysum. Þá fæst $\underline{V_{oc} = v_b = 16V}$

MNA

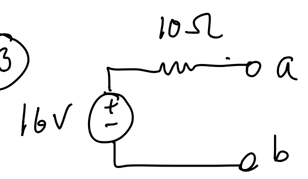
$$\begin{array}{c} \\ a \\ b \\ V_1 \end{array}
\begin{array}{c} \begin{array}{ccc} a & b & V_1 \end{array} \\ \begin{bmatrix} G_1 & -G_1 & 1 \\ -G_1 & G_1+G_2 & 0 \\ 1 & 0 & 0 \end{bmatrix} \end{array}
\begin{bmatrix} v_a \\ v_b \\ j_V \end{bmatrix} = \begin{bmatrix} 0 \\ I_1 \\ V_1 \end{bmatrix}
\text{ svo } \begin{bmatrix} v_a \\ v_b \\ j_V \end{bmatrix} = \begin{bmatrix} 10V \\ 16V \\ \frac{6}{5}A \end{bmatrix}$$

②

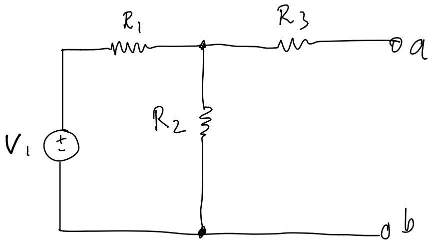


Hér er $\underline{R_{eq} = R_3 + (R_1 \| R_2) = 10\,\Omega}$

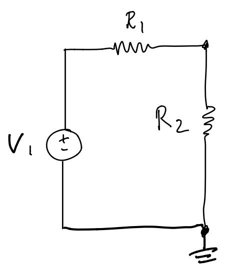
③



Lausn 2 $V_1 = 24V$ $R_1 = 100\Omega$ $R_2 = 200\Omega$ $R_3 = 150\Omega$



(1) R_3 er ótengt svo það tekst ekki með. Finnum V_{oc} sem er spenna yfir R_2



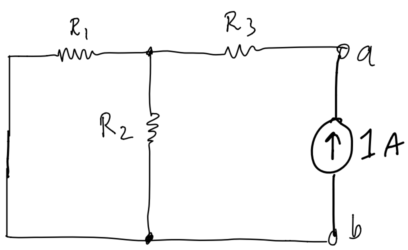
Hér (spennudeiling) $V_{R_2} = V_1 \dfrac{R_2}{R_1 + R_2} = 24 \dfrac{200}{200 + 100} = 24 \cdot \dfrac{2}{3} = 16V$

svo $\underline{V_{OC} = V_{th} = 16V}$

(2)

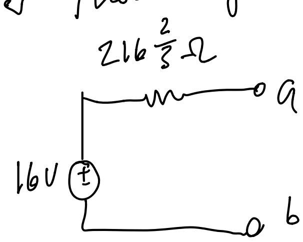
Hér er $R_{eq} = R_3 + R_1 \| R_2$

$= 150 + \dfrac{100 \cdot 200}{100 + 200} = 150 + 66\frac{2}{3} = \underline{\underline{216\frac{2}{3}\Omega}}$



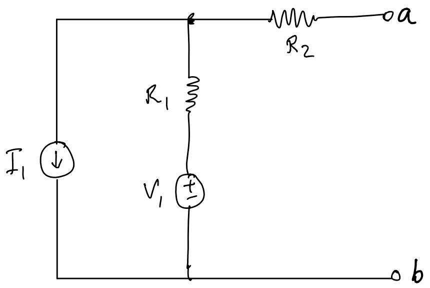
(3)

þá er Thévenin jafngildisrásin

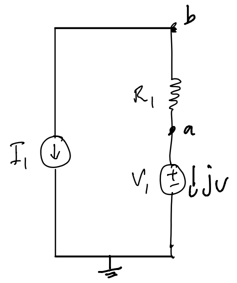


Lausn 4

$V_1 = 10V$ $I_1 = 20mA$ $R_1 = 5k\Omega$ $R_2 = 10k\Omega$



(1) R_2 er ótengt, svo rásin lítur út svona ...



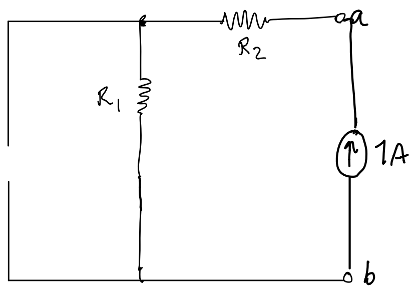
Höfum $N_{jöfn} = N_{hnútp.} + N_{vs} - 1 = 3 + 1 - 1 = \underline{3 \text{ jöfn}}$

Tökum eftir að $V_{oc} = v_b$. Setjum nú upp í MNA fylki

$$
\begin{array}{c|ccc}
 & a & b & V_1 \\
a & G_1 & -G_1 & 1 \\
b & -G_1 & G_1 & 0 \\
V_1 & 1 & 0 & 0
\end{array}
\begin{bmatrix} v_a \\ v_b \\ j_V \end{bmatrix} = \begin{bmatrix} 0 \\ -I_1 \\ V_1 \end{bmatrix} \quad \text{svo} \quad \begin{bmatrix} v_a \\ v_b \\ j_V \end{bmatrix} = \begin{bmatrix} 10V \\ -90V \\ -20mA \end{bmatrix}
$$

Þá er $\underline{\underline{V_{oc} = V_b = -90V}}$

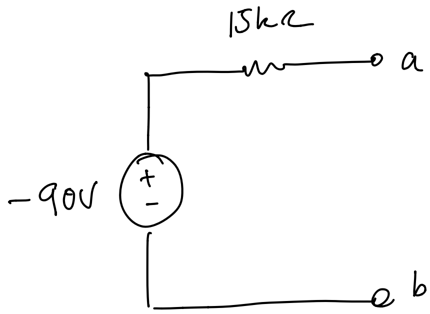
(2)



Hér er $\underline{\underline{R_{eq} = R_1 + R_2 = 15k\Omega}}$

(3)

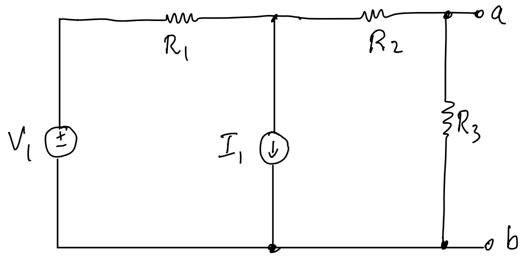
Þá er Thévenin jafngildisrás yfir póla a & b ...



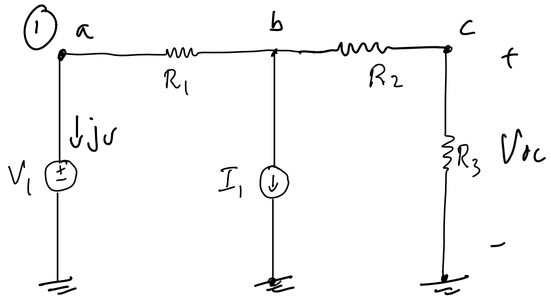
Athugið að ef I_1 flæddi inn í a

þá væri $\underline{v_b = +110V}$

Lausn 5 $V_1 = 10V,\ I_1 = 2A,\ R_1 = 4\Omega,\ R_2 = 8\Omega,\ R_3 = 8\Omega$



(1)



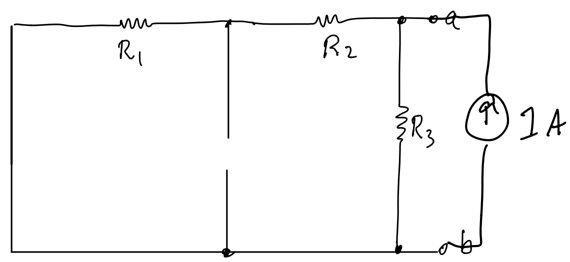
Hér er $N_{jöfn} = N_{hnútpunkta} + N_{VS} - 1 = 4 + 1 - 1 = \underline{4}$

Tökum eftir að $V_{oc} = V_c$. Stillum þannig upp MNA fylkinu.

$$
\begin{array}{c|cccc}
 & a & b & c & V_1 \\
\hline
a & & & & \\
b & & & & \\
c & & & & \\
V_1 & & & &
\end{array}
\begin{bmatrix} G_1 & -G_1 & 0 & 1 \\ -G_1 & G_1+G_2 & -G_2 & 0 \\ 0 & -G_2 & G_2+G_3 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}
\begin{bmatrix} V_a \\ V_b \\ V_c \\ j_V \end{bmatrix}
=
\begin{bmatrix} 0 \\ -I_1 \\ 0 \\ V_1 \end{bmatrix}
\quad \text{svo} \quad
\begin{bmatrix} V_a \\ V_b \\ V_c \\ j_V \end{bmatrix}
=
\begin{bmatrix} 10V \\ 8/5\,V \\ 4/5\,V \\ -2.1A \end{bmatrix}
$$

eða $\underline{V_{oc} = V_{th} = V_c = \frac{4}{5}V}$

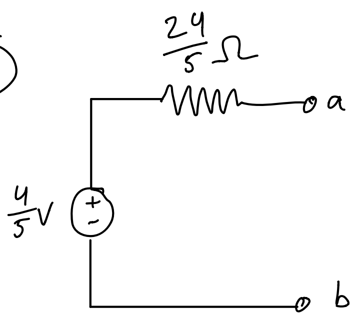
(2)



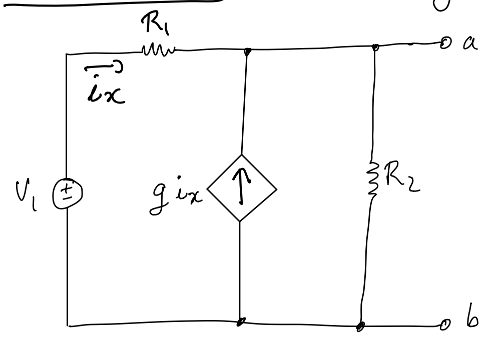
Því $R_{eq} = R_3 \,\|\, (R_1 + R_2)$

$= \underline{4.8\,\Omega}$

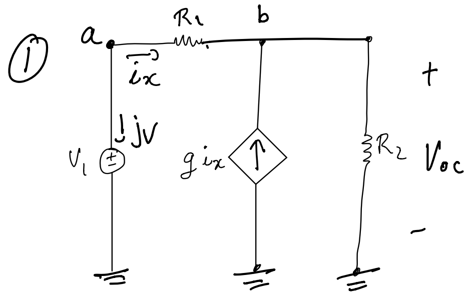
(3)



Lausn 6 $V_1 = 10V$, $g = 9$, $R_1 = 100\Omega$, $R_2 = 10\Omega$



①



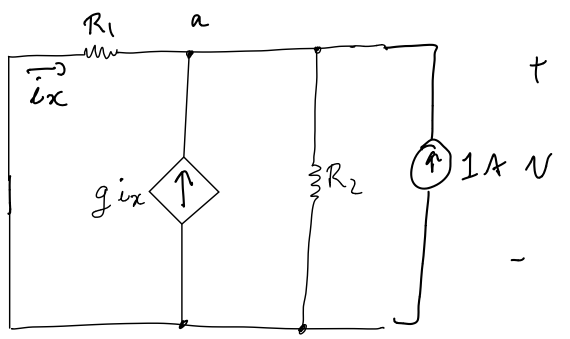
Höfum $N_{jöf} = N_{nútpunkta} + N_{VS} - 1 = 2 + 1 - 1 = \underline{2}$

Tökum eftir að $v_{oc} = v_b$ & $\boxed{i_x = -j_V}$ stillum svo upp MNA fylkin

$$\begin{bmatrix} G_1 & -G_1 & 1 \\ -G_1 & G_1 + G_2 + g & \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} v_a \\ v_b \\ j_V \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ V_1 \end{bmatrix} \quad \text{svo} \quad \begin{bmatrix} v_a \\ v_b \\ j_V \end{bmatrix} = \begin{bmatrix} 10V \\ 5V \\ -\frac{1}{20}A \end{bmatrix}$$

Nú er $\underline{\underline{V_{oc} = V_{th} = v_b = 5V}}$

② Núllstillum óháðar lindir & setjum „prufustraum" & notum MNA. $v = v_a$



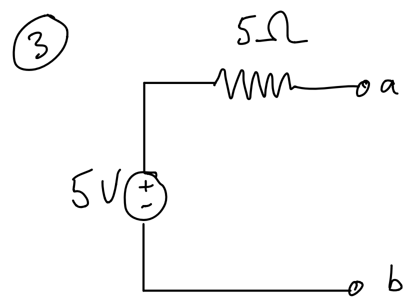
Höfum $i_x = G_1(0 - v_a)$

& KCL í a $\quad \underline{\underline{i_x}} + g\,\underline{\underline{i_x}} + 1 + G_2(0 - v_a) = 0$

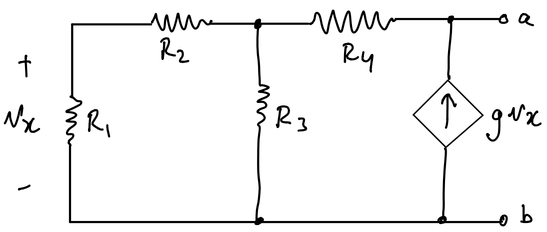
$$\text{svo} \quad \begin{bmatrix} -G_2 & g+1 \\ G_1 & 1 \end{bmatrix} \begin{bmatrix} v_a \\ i_x \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \end{bmatrix} \quad \text{eða} \quad \begin{bmatrix} v_a \\ i_x \end{bmatrix} = \begin{bmatrix} 5V \\ -\frac{1}{20}A \end{bmatrix}$$

Nú er $\underline{\underline{R_{eq} = R_{th} = v = v_a = 5\ \Omega}}$

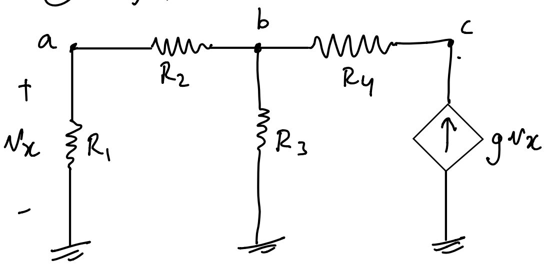
③



Dæmi 7 $R_1 = 50\Omega,\ R_2 = 200\Omega,\ R_3 = 150\Omega,\ R_4 = 100\Omega,\ g = \frac{1}{100}$



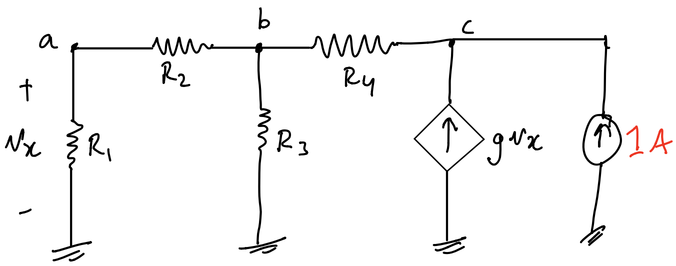
① $N_{jöfnur} = N_{hnútpunkta} + N_{VS} - 1 = 4 + 0 - 1 = \underline{3}$ ⇒ tek eftir að $v_x = V_a$. Set upp MNA fylki:



$$\begin{array}{c|c} & \begin{array}{ccc} a & b & c \end{array} \\ \begin{array}{c} a \\ b \\ c \end{array} & \begin{bmatrix} G_1+G_2 & -G_2 & \\ -G_2 & G_2+G_3+G_4 & -G_4 \\ -g & -G_4 & G_4 \end{bmatrix} \end{array} \begin{bmatrix} V_a \\ v_b \\ v_c \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \text{svo} \quad \begin{bmatrix} V_a \\ v_b \\ v_c \end{bmatrix} = \begin{bmatrix} 0V \\ 0V \\ 0V \end{bmatrix}$$

Nú er $\underline{v_{oc} = v_c = 0V}$ <=> Engar óháðar lindir

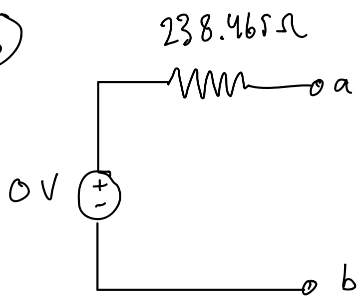
② Engar óháðar lindir svo MNA fylki er eins. Dálkurinn það eina sem breytist



$$\begin{array}{c|c} & \begin{array}{ccc} a & b & c \end{array} \\ \begin{array}{c} a \\ b \\ c \end{array} & \begin{bmatrix} G_1+G_2 & -G_2 & \\ -G_2 & G_2+G_3+G_4 & -G_4 \\ -g & -G_4 & G_4 \end{bmatrix} \end{array} \begin{bmatrix} V_a \\ v_b \\ v_c \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1A \end{bmatrix} \quad \text{svo} \quad \begin{bmatrix} V_a \\ v_b \\ v_c \end{bmatrix} \approx \begin{bmatrix} 23.077\,V \\ 115.385\,V \\ 238.465\,V \end{bmatrix}$$

Hér er $\underline{R_{eq} = v_c = 238.465\Omega}$

③



eða