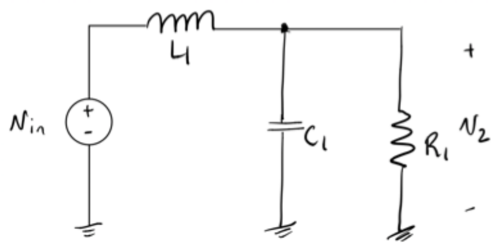
# Heimadæmi 8 - Lausnir

~~16/03/21~~ 24/03/21

## Dæmi 1 – RLC rás

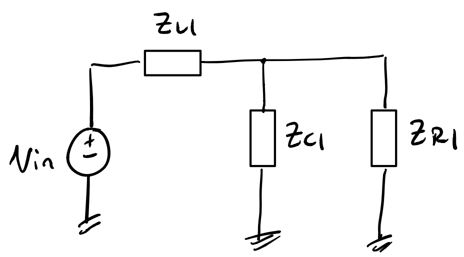
Finnið $H(p) = v_2/v_{in}$ og diffurjöfnu sem tengir $v_2$ og $v_{in}$ saman.




| Breyta | Gildi |
|---|---|
| $R_1$ | $8\,\Omega$ |
| $C_1$ | $2\,\mathrm{F}$ |
| $L_1$ | $1\,\mathrm{H}$ |

Munum að $Z_{R1} = R$, $Z_{L1} = L_1 p$ & $Z_{C1} = \frac{1}{C_1 p}$

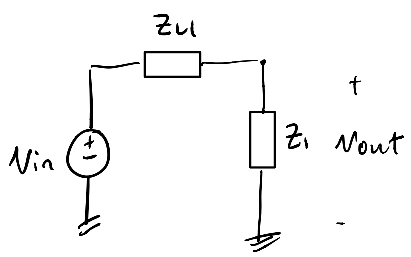
Endurritum rás með samviðnámum.




Tökum eftir að $Z_{C1}$ & $Z_{R1}$ eru hliðtengd.

Set $Z_1 = Z_{C1} \parallel Z_{R1} = \frac{Z_{C1} \cdot Z_{R1}}{Z_{C1} + Z_{R1}} = \frac{\frac{1}{C_1 p} \cdot R_1}{\frac{1}{C_1 p} + R_1}$

$$= \frac{R_1}{C_1 R_1 p + 1}$$

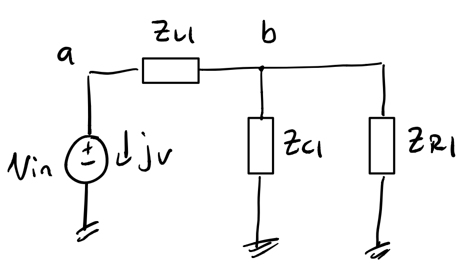



Með spennudeilingu fæst þá

$$\frac{V_{out}}{V_{in}} = \frac{Z_1}{Z_1 + Z_{L1}} = \frac{\frac{R_1}{C_1 R_1 p + 1}}{\frac{R_1}{C_1 R_1 p + 1} + L_1 p} = \frac{R_1}{R_1 L_1 C_1 p^2 + L_1 p + R_1}$$

$$= \frac{8}{16 p^2 + p + 8}$$

Getum sömuleiðis leyst með MNA $(Y = Z^{-1})$




$$\begin{bmatrix} Y_{L1} & -Y_{L1} & 1 \\ -Y_{L1} & Y_{L1} + Y_{C1} + Y_{R1} & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ j_V \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ V_{in} \end{bmatrix}$$

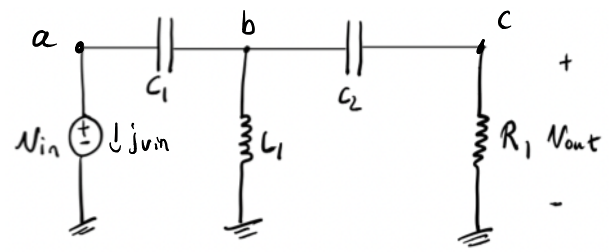
eða

$$\begin{bmatrix} V_a \\ V_{out} \\ j_V \end{bmatrix} = \begin{bmatrix} V_{in} \\ \frac{R_1}{R_1 L_1 C_1 p^2 + L_1 p + R_1} \\ -\frac{R_1 C_1 p + 1}{R_1 L_1 C_1 p^2 + L_1 p + R_1} \end{bmatrix}$$

Diffurjafnan er þá $\frac{d^2 v_{out}}{dt^2} + \frac{1}{16}\frac{d v_{out}}{dt} + \frac{1}{2} v_{out} = \frac{1}{2} v_{in}$

# Dæmi 2 – Tveir þéttar, spóla og viðnám

Finnið $H(p) = v_{\text{out}}/v_{\text{in}}$ og diffurjöfnu sem tengir $v_{\text{out}}$ og $v_{\text{in}}$ saman.




| Breyta | Gildi |
|---|---|
| $R_1$ | $10\,\Omega$ |
| $C_1, C_2$ | $0.1\,\text{F}$ |
| $L_1$ | $2\,\text{H}$ |

Veit að $v_c = v_{out}$

Nota samviðnám, set upp í MNA með $N_{jöfn} = N_{hnútp.} + N_{vs} - 1 = 4 + 1 - 1 = \underline{4 \text{ jöfn}}$

$$
\begin{array}{c c}
 & \begin{array}{cccc} a & b & c & v_{in} \end{array} \\
\begin{array}{c} a \\ b \\ c \\ v_{in} \end{array} &
\begin{bmatrix} Y_{C_1} & -Y_{C_1} & 0 & 1 \\ -Y_{C_1} & Y_{C_1}+Y_{L_1}+Y_{C_2} & -Y_{C_2} & 0 \\ 0 & -Y_{C_2} & Y_{C_2}+Y_{R_1} & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}
\end{array}
\begin{bmatrix} v_a \\ v_b \\ v_c \\ j_{vin} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ v_{in} \end{bmatrix}
$$

Hér er $v_a = v_{in}$ svo þessi tvö eru jöfn $\longleftrightarrow$

$$
\begin{array}{c}
\begin{array}{ccc} b & c & v_{in} \end{array} \\
\begin{bmatrix} -Y_{C_1} & 0 & 1 \\ Y_{C_1}+Y_{L_1}+Y_{C_2} & -Y_{C_2} & 0 \\ -Y_{C_2} & Y_{C_2}+Y_{R_1} & 0 \end{bmatrix}
\end{array}
\begin{bmatrix} v_b \\ v_c \\ j_{vin} \end{bmatrix} = \begin{bmatrix} -Y_{C_1} v_{in} \\ +Y_{C_1} v_{in} \\ 0 \end{bmatrix}
$$

Þá er

$$
\begin{bmatrix} v_b \\ v_{out} \\ j_{vin} \end{bmatrix} = \frac{v_{in}}{R_1C_1C_2L_1p^3 + L_1(C_1+C_2)p^2 + R_1C_2p + 1} \begin{bmatrix} C_1L_1p^2(R_1C_2p+1) \\ C_1C_2L_1R_1p^3 \\ -C_1p(C_2L_1p^2 + C_2R_1p + 1) \end{bmatrix}
$$

Þá er

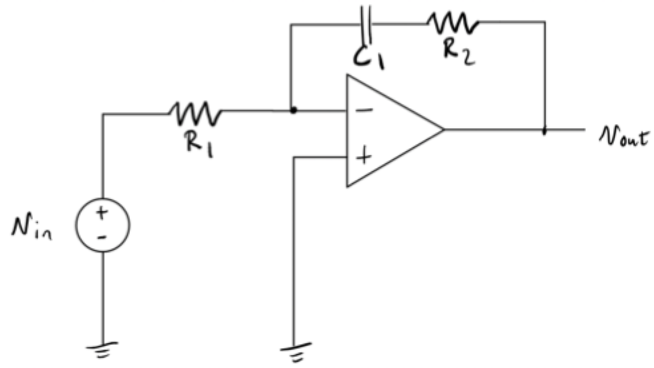
$$
\frac{v_{out}}{v_{in}} = \frac{C_1C_2L_1R_1p^3}{R_1C_1C_2L_1p^3 + L_1(C_1+C_2)p^2 + R_1C_2p + 1} = \frac{0.02p^3}{0.02p^3 + 0.04p^2 + 0.1p + 0.1} = \underline{\underline{\frac{p^3}{p^3 + 2p^2 + 5p + 5}}}
$$

sem jafngildir $p^3 v_{out} + \frac{1}{2}p^2 v_{out} + 5p\,v_{out} + 5v_{out} = p^3 v_{in}$

eða $\underline{\underline{\dfrac{d^3 v_{out}}{dt^3} + 2\dfrac{d^2 v_{out}}{dt^2} + 5\dfrac{dv_{out}}{dt} + 5v_{out} = \dfrac{d^3 v_{in}}{dt^3}}}$

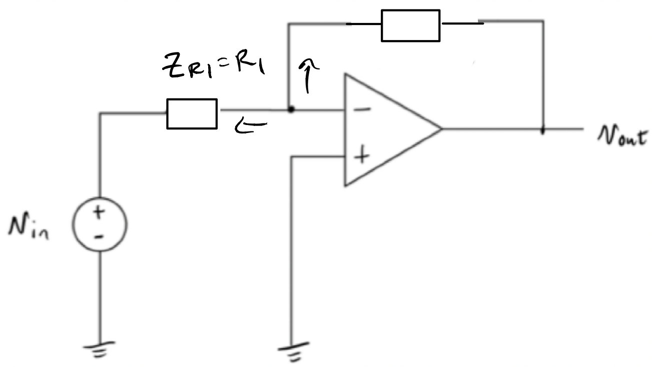
## Dæmi 3 – Aðgerðarmagnari, þéttir og viðnám

Gerið ráð fyrir fullkomnum aðgerðarmagnara og finnið $H(p) = v_{\text{out}}/v_{\text{in}}$.




Endurskrifum með sam viðnám

$$Z_1 = \frac{1}{C_1 p} + R_2 = \frac{R_2 C_1 p + 1}{C_1 p}$$




Veit að fyrir fullkominn aðgerðarm. gildir $v^- = v^+$ og $i^- = i^+ = 0A$

en $v^+ = 0V$ svo $v^- = 0V$

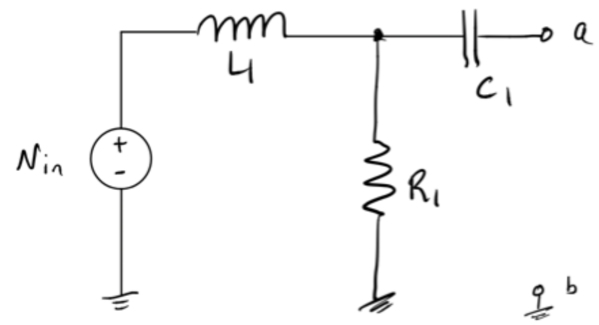
KCL við mínus pól aðgerðarmagnara gefur

$$Y_{R1}(\cancel{v^-}^{\,0} - v_{in}) + Y_1(\cancel{v^-}^{\,0} - v_{out}) = 0 \quad \text{svo} \quad Y_{R1} v_{in} = -Y_1 v_{out}$$

svo $$\frac{v_{out}}{v_{in}} = -\frac{Y_{R1}}{Y_1} = -\frac{Z_1}{Z_{R1}} = \frac{R_2 C_1 p + 1}{R_1 C_1 p}$$

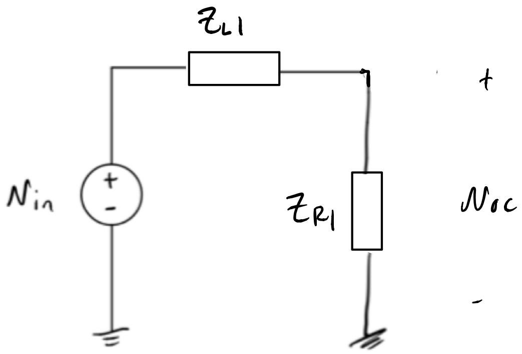
# Dæmi 4 – Thévenin og orkugeymandi rásaeiningar

Finnið Thévenin jafngildisrás milli póla $a$ og $b$




| Breyta | Gildi |
|---|---|
| $R_1$ | $10\,\Omega$ |
| $C_1$ | $0.1\,\mathrm{F}$ |
| $L_1$ | $2\,\mathrm{H}$ |

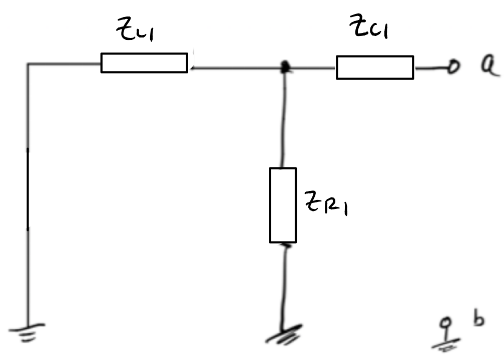
Finnum fyrst tómagangsspennu $V_{oc} = V_{th}$




Með spennudeilingu fyrst

$$V_{oc} = \frac{Z_{R1}}{Z_{R1} + Z_{L1}} v_{in} = \frac{R_1}{R_1 + L_1 p} v_{in} = \frac{10}{2p + 10} v_{in}$$

$$\underline{\underline{V_{oc} = \frac{5}{p + 5} v_{in}}}$$



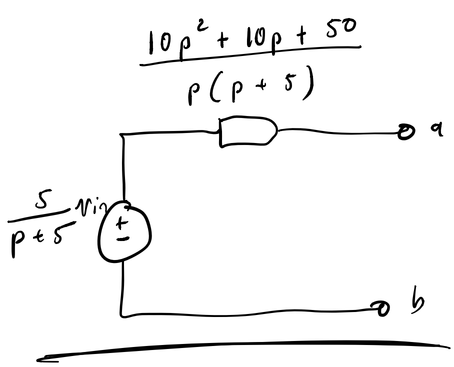

Næst tökum við eftir að rásin inniheldur engar háðar lindir. Þá er

$$Z_{th} = Z_{C1} + Z_{L1} \parallel Z_{R1} = \frac{1}{C_1 p} + \frac{L_1 p \cdot R_1}{L_1 p + R_1}$$

$$= \frac{(L_1 p + R_1) + L_1 p R_1 (C_1 p)}{C_1 p (L_1 p + R_1)} = \frac{R_1 L_1 C_1 p^2 + L_1 p + R_1}{L_1 C_1 p^2 + R_1 C_1 p}$$

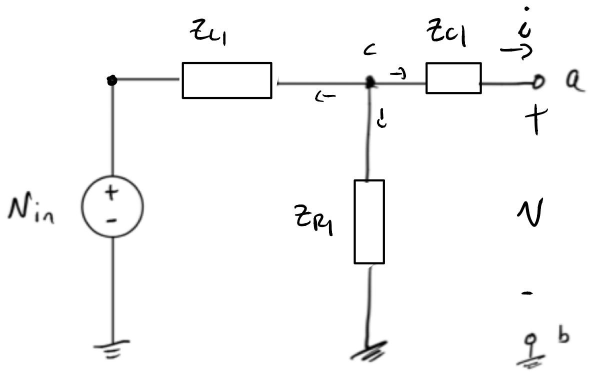
$$= \frac{2p^2 + 2p + 10}{0.2p^2 + p} = \frac{10p^2 + 10p + 50}{p(p+5)}$$

Þá fæst Thévenin jafngildisrásin



Við gett sömuleiðis loyst þetta dømi við því at finna $v_a = V_{th} - i Z_{th}$

þar sum $v$ & $i$ eru spenna yvir póla a & b og streymurin a yvir $C_1$




KCL í c: $Y_{L1}(v_c - v_{in}) + Y_{R1}(v_c - 0) + i = 0$

eða $v_c(Y_{L1} + Y_{R1}) = Y_{L1} v_{in} - i$

a: $Y_{C1}(v_a - v_c) + i = 0$

eða $v_a Y_{C1} - v_c Y_{C1} = -i$

Setjum upp í hneppi & loysum fyri $v_a$

$$\begin{array}{cc} & \begin{array}{cc} a & c \end{array} \\ \begin{array}{c} a \\ c \end{array} & \begin{bmatrix} 0 & Y_{L1} + Y_{R1} \\ Y_{C1} & -Y_{C1} \end{bmatrix} \end{array} \begin{bmatrix} v_a \\ v_c \end{bmatrix} = \begin{bmatrix} Y_{L1} v_{in} - i \\ -i \end{bmatrix}$$

Þá fæst $v_a = -\dfrac{10i(0.2p+1) + p(2ip - v_{in})}{p(0.2p+1)} = \dfrac{1}{0.2p+1} v_{in} - \dfrac{2p + 10 + 2p^2}{p(0.2p+1)} i$

$$= \underbrace{\frac{5}{p+5}}_{V_{th}} v_{in} - \underbrace{\frac{10p^2 + 10p + 50}{p(p+5)}}_{Z_{th}} i$$