$$y'' + \alpha y' + \beta y = \underbrace{e^{-2t} \text{ eða } 4t+1}$$

$$f(t) = u(t) \cdot \sim$$

Greining Rása

Kerfi með sinuslaga innmerki

$$f(t) = \cos/\sin(\omega_0 t)$$

Ólafur Bjarki Bogason

8. apríl 2021

$$\cos(2t + 45^\circ)$$

$$\to A_0 \cos(2t + \_\_)$$

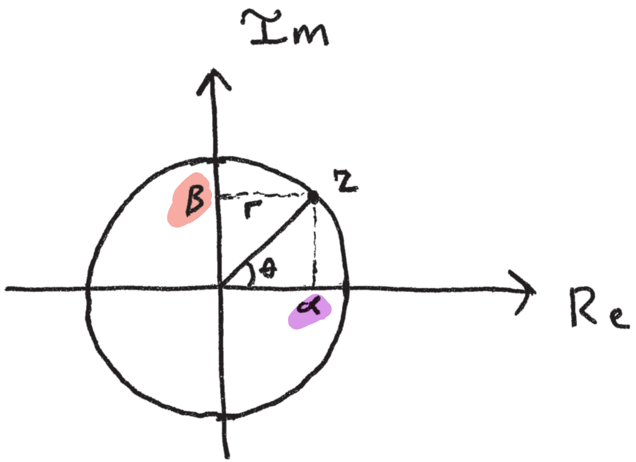
Inngangur

- Við viljum finna æstæða svörun línulegrar rásar við sínuslaga innmerki

$$v(t) = V_m cos(\omega t)$$

- Þessi svörun er einfaldlega sérlausnin á diffurjöfnu rásarinnar
- Við einblínum á sínuslaga innmerki vegna þess að
  1. Sínuslaga merki eru notuð í aflflutningsrásum (220 V (RMS) á 50Hz)
  2. Sínuslaga merki eru byggingareiningar lotubundinna merkja (Fourier greining)

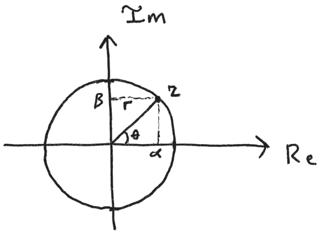
Tvinntölur



- Framsetning tvinntölu z

$$
\begin{aligned}
z &= \alpha + j\beta \quad \text{Rétthyrnt form} \\
&= r\angle\theta \quad \text{Pólform} \\
&= re^{j\theta} \quad \text{Veldisvísisform}
\end{aligned}
$$

Tvinntölur



- Sambandið á milli rétthyrnds forms og pólforms er eftirfarandi

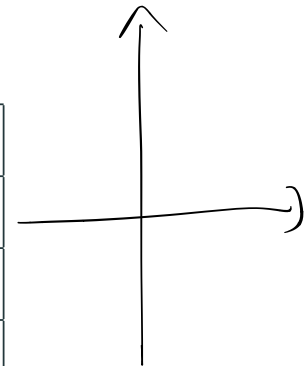
$$
\begin{aligned}
r &= \sqrt{\alpha^2 + \beta^2} \\
\theta &= \arctan\left(\frac{\beta}{\alpha}\right) \\
\alpha &= r\cos(\theta) \\
\beta &= r\sin(\theta)
\end{aligned}
$$

Tvinntölur á pólformi

- Þegar breyta á gildi úr rétthyrndu formi yfir í pólform þá þarf að hafa í huga:

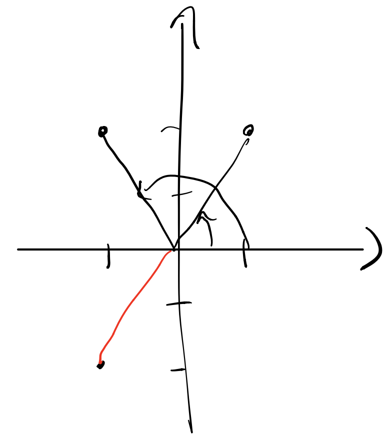
$$
\begin{aligned}
r &= \sqrt{\alpha^2 + \beta^2} \\
\theta &= \arctan(\frac{\beta}{\alpha}) \\
\alpha &= r\cos(\theta) \\
\beta &= r\sin(\theta)
\end{aligned}
$$

Fjórðungur	arctan gildi
I	gildið úr reiknivélinni
II	gildið úr reiknivélinni $+180°$
III	gildið úr reiknivélinni $+180°$
IV	gildið úr reiknivélinni



- Eingöngu þegar fært er yfir í pólform, það þarf ekkert að gera þegar breyta á gildi úr pólformi yfir í rétthyrnt form
- Það má alltaf leggja við eða draga frá $360°$

Tvinntölur á pólformi frh.



- $\alpha + j\beta$
  - Fjórðungur I: α og β bæði jákvæðar tölur
  - Fjórðungur II: α neikvæð og β jákvæð tala
  - Fjórðungur III: α og β bæði neikvæðar tölur
  - Fjórðungur IV: α jákvæð og β neikvæð tala
- Dæmi:

  $x_1 = 1 + j2 = \sqrt{5}\angle\arctan(\frac{2}{1}) = 2,236\angle 63,435^\circ$

  $x_2 = -1 + j2 = \sqrt{5}\angle\arctan(\frac{2}{-1}) + 180^\circ = 2,236\angle 116,565^\circ$

  $x_3 = -1 - j2 = \sqrt{5}\angle\arctan(\frac{-2}{-1}) + 180^\circ = 2,236\angle 243,435^\circ$

  $x_4 = 1 - j2 = \sqrt{5}\angle\arctan(\frac{-2}{1}) = 2,236\angle - 63,435^\circ$

  - Prófið að teikna vísamyndina, þá ætti að skýrast afhverju þetta er nauðsynlegt

$$\theta = \begin{cases} \arctan(\beta/\alpha) & \text{ef } \alpha > 0 \\ \arctan(\beta/\alpha) + \pi & \text{ef } \alpha < 0 \end{cases}$$

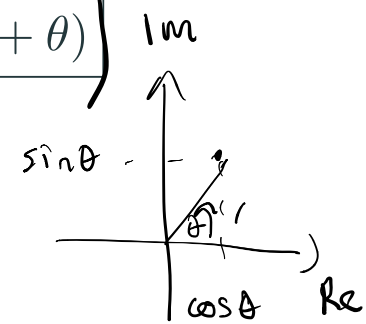
Vísar (e. phasors) $ii=-1$

$$e^{j\theta} = \cos\theta + j\sin\theta$$

$$e^{ix} = \cos x + i\sin x$$

- Hugsum okkur <u>tvinntöluspennu</u>

$$\boxed{v(t) = V_m e^{j(\omega t+\theta)} = V_m\big(\cos(\omega t+\theta) + j\sin(\omega t+\theta)\big)}$$



- Við getum skrifað

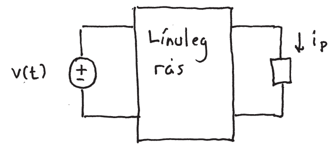
$$e^{x+y} = e^x e^y$$

$$\begin{aligned} v(t) &= \left(V_m e^{j\theta}\right) e^{j\omega t} \\ &= \left(V_m \angle\theta\right) e^{j\omega t} \end{aligned}$$

$$f = \frac{\omega}{2\pi}$$

- **Við köllum $V_m\angle\theta$ vísirinn fyrir $v(t)$**

Línuleg kerfi með sínuslaga innmerki



- G.r.f að línulegt kerfi hafi innspennu sem er sínuslaga

$$v(t) = V_m \cos(\omega t + \theta)$$

- Hún veldur sínuslaga straumsvörun

$$i_p(t) = I_m \cos(\omega t + \phi)$$

- **Svörunin er á sömu horntíðni og innspennan ef rásin er línuleg**

Dæmi Finna $v_1(t)+v_2(t)$ ef $v_1(t) = 10\cos(3t-76^\circ)$ & $v_2(t) = 5\cos(3t+71^\circ)$

Lausn

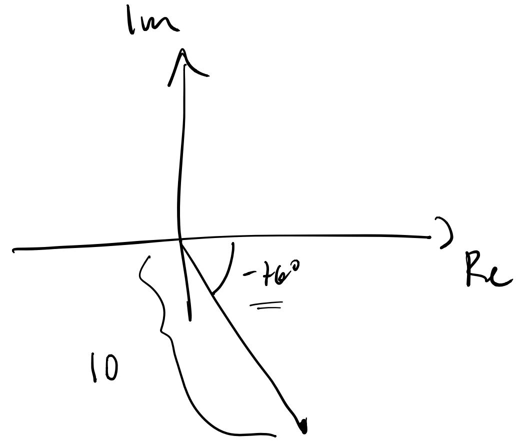
$$v_1(t) = V_1\cos(\omega t+\alpha), \quad V_1 = 10, \quad \omega = 3, \quad \alpha = -76^\circ$$

$$= \mathrm{Re}\left\{V_1 e^{j(\omega t+\alpha)}\right\}$$

$$v(t) = \left(V_m e^{j\theta}\right)e^{j\omega t} = \underbrace{(V_m\angle\theta)}_{\text{vísirinn}}e^{j\omega t}$$

$$\bar{V}_1 = V_1\angle\alpha = 10\angle-76^\circ \qquad v_1 = 2.42 - j9.7$$

$$v_1(t) = \mathrm{Re}\left\{\underbrace{V_1 e^{j\alpha}}_{V_1\angle\alpha}\cdot e^{j\omega t}\right\}$$



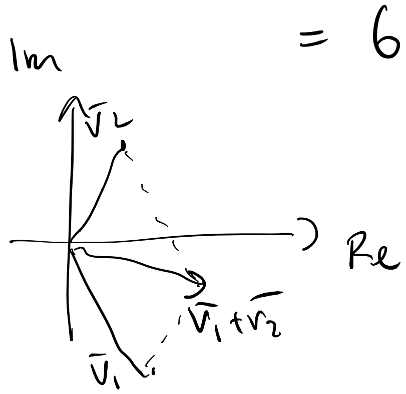
$$v_2(t) = V_2\cos(\omega t+\beta), \quad V_2 = 5 \quad \beta = 71^\circ$$

$$\bar{V}_2 = V_2\angle\beta = 5\angle71^\circ = 1.63 + j4.73$$

$$\bar{V}_1 + \bar{V}_2 = (2.42 - j9.7) + (1.63 + j4.73)$$

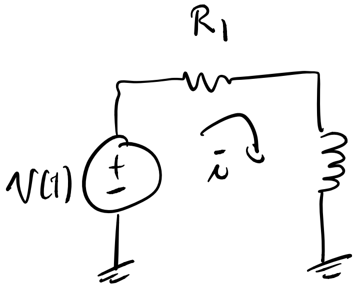
$$= 4.05 - j4.97$$

$$= 6.41\angle-50.8^\circ \qquad v_1 + v_2 = 6.41\cos(3t-50.8^\circ)$$



Dæmi Finna sérlausn $i_p(t)$ ef $v(t) = 4\cos(3t)$

$$R_1 = 1\Omega \quad L_1 = \tfrac{1}{2}H$$



Lausn $\quad i = \dfrac{v - 0}{R_1 + L_1 p}$

$$R_1 i + L_1 p i = v$$

eða $\quad pi + \dfrac{R_1}{L_1} i = \dfrac{1}{L_1} v \qquad\qquad f(g(x))' = f'(g(x))\,g'(x)$

$$i' + 2i = 8\cos(3t) \quad (*)$$

Gískum á $i_p(t) = k_1\cos(3t) + k_2\sin(3t) \qquad i_p'(t) = -3k_1\sin(3t) + 3k_2\cos(3t)$

Set inn í (*)

$$\left(-3k_1\sin(3t) + 3k_2\cos(3t)\right) + 2\left(k_1\cos(3t) + k_2\sin(3t)\right) = 8\cos(3t)$$

$$\left.\begin{aligned} 3k_2 + 2k_1 &= 8 \\ -3k_1 + 2k_2 &= 0 \end{aligned}\right\} \quad \begin{bmatrix} k_1 \\ k_2 \end{bmatrix} = \begin{bmatrix} \frac{16}{13} \\ \frac{24}{13} \end{bmatrix}$$

Þá er $\quad i_p(t) = \dfrac{16}{13}\cos(3t) + \dfrac{24}{13}\sin(3t)$ ← ekki fullnægjandi lausn

$$= 2.22\cos(3t - 56.3^\circ)$$

Línulegt kerfi með sínuslaga innmerki

- Ef innmerkið er tvinntölufall

$$v(t) = V_m e^{j(\omega t+\theta)}$$

- Þá er útmerkið einnig tvinntölufall

$$i_p(t) = I_m e^{j(\omega t+\phi)}$$

Línulegt kerfi með sínuslaga innmerki

- Ef finna á æstæða svörun við sínuslaga innmerki þá reynist eftirfarandi aðferð auðveldust!
    1. Breytum sínuslaga innmerki í tilsvarandi tvinntöluveldisvísisfall
    2. Greinum rás á hefðbundinn hátt
    3. Finnið útmerkið á tvinntöluformi
    4. Takið raunhluta útmerkisins til að fá lokaniðurstöðu

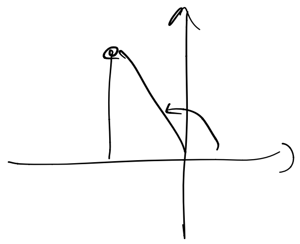
$$v(t) = V_m \cos(\omega t + \alpha) = \mathrm{Re}\left\{ V_m e^{j(\omega t + \alpha)} \right\}$$

Dapuni $\quad y'' + y' + \frac{4}{3}y = \underbrace{6\cos(2t+60^\circ)}_{f(t)}$ Finnid $y_p(t)$

Hiè ≈ $\quad f(t) = 6\cos(2t+60^\circ) = \mathrm{Re}\{\underbrace{6e^{j(2t+60^\circ)}}_{g(t)}\} =$

$$g(t) = 6e^{j2t}e^{j60^\circ}$$

$$= \bar{V}\underline{e^{j2t}} \qquad \bar{V} = 6e^{j60^\circ}$$



$$g'(t) = (2j)\bar{V}e^{j2t}$$

$$g''(t) = -4\bar{V}e^{j2t}$$

$$\tilde{y}_p = g(t) = \bar{V}e^{j2t}$$

$$\underbrace{\left(-4+2j+\frac{4}{3}\right)}_{-\frac{8}{3}+j2 \,=\, 3.333\angle 143^\circ}\bar{V}\cancel{e^{j2t}} = 6\angle 60^\circ\cancel{e^{j2t}}$$

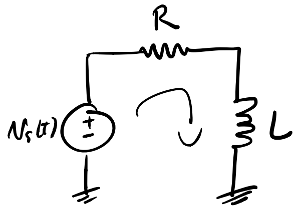
$$\bar{V} = \frac{6\angle 60^\circ}{3.333\angle 143^\circ} = \frac{6e^{j60^\circ}}{3.33e^{j143^\circ}} = \frac{6}{3.333}e^{j(60-143)}$$

$$= 1.8\angle -83^\circ$$

$$y_p(t) = \mathrm{Re}\{\bar{V}e^{j2t}\} = \{1.8e^{j(2t-83)^\circ}\}$$

$$= \underline{1.8\cos(2t-83^\circ)\ \mathrm{V}}$$

Dæmi 12.4



$$v_s = \begin{cases} 10\cos(2t) & , t<0 \\ 30\cos(2t) & , t>0 \end{cases}$$

$L = 8\,H \quad R = 24\,\Omega$

Finna $i(t)$, $t>0$

Lausn $\quad Lpi + Ri = v_s \quad$ eða $\quad pi + \frac{R}{L} i = \frac{1}{L} v_s$

$$pi + 3i \overset{\circledast}{=} \frac{1}{8} v_s = \frac{1}{8} \begin{cases} 10\cos 2t & t<0 \\ 30\cos 2t & t>0 \end{cases}$$

Náttúrleg lausn

$$\left.\begin{aligned} i_n(t) &= Ae^{st} \\ i_n'(t) &= sAe^{st} \end{aligned}\right\} \qquad \begin{aligned} i' + 3i &= 0 \\ s\cancel{Ae^{st}} + 3\cancel{Ae^{st}} &= 0 \\ s &= -3 \end{aligned}$$

$$\underline{i_n(t) = Ae^{-3t}}$$

Sérlausn $\quad$ Við $t>0$ fæst $v_s(t) = 30\cos(2t)$ ✓

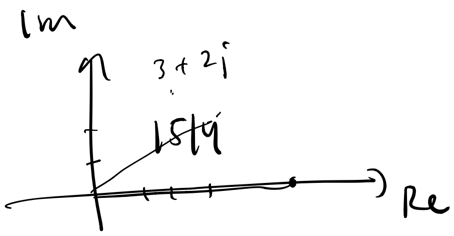
$$\bar{V}_s = 30\angle 0^\circ$$

$$i_p(t) = \bar{I} e^{j2t}$$

$$i_p'(t) = (2j)\bar{I} e^{j2t} \quad \text{setjum inn í } \circledast$$

$$2j\bar{I}\cancel{e^{j2t}} + 3\bar{I}\cancel{e^{j2t}} = \frac{1}{8} \cdot \bar{V}_s \cancel{e^{j2t}}$$

$$\bar{I} = \frac{15/4}{3+2j} = \frac{15}{4\sqrt{13}} \tan^{-1}\left(-\frac{2}{3}\right) = \underline{1.04\angle -33.7^\circ}$$



$$\frac{15/4}{3+2j} = \frac{15/4}{\sqrt{3^2+2^2}\, e^{j\theta}}$$

$$\begin{aligned} r &= \sqrt{\alpha^2+\beta^2} \\ \theta &= \arctan(\frac{\beta}{\alpha}) \\ \alpha &= r\cos(\theta) \\ \beta &= r\sin(\theta) \end{aligned}$$

$$\theta = \operatorname{atan}\left(\frac{2}{3}\right) \approx 33.7$$

$$\frac{15/4}{\sqrt{13}} e^{-j\theta} = \frac{15}{4\sqrt{13}} e^{-j33.7}$$

$$i_p(t) = Re\left\{ \bar{I} e^{j2t} \right\} = Re\left\{ 1.04\, e^{j(2t-33.7°)} \right\}$$

$$= 1.04 \cos(2t - 33.7°) \text{ A}$$

$$i(t) = i_n(t) + i_p(t) = 1.04 \cos(2t - 33.7°) + Ae^{-3t}$$

Før $t<0$ er $v_s(t) = 10\cos(2t)$ så $\bar{V}_s = 10 \angle 0°$

$$2j\bar{I}\cancel{e^{j2t}} + 3\bar{I}\cancel{e^{j2t}} = \frac{1}{8} \cdot \bar{V}_s \cancel{e^{j2t}}$$

$$\bar{I} = \frac{5/4}{3+j2} = \frac{5}{4\sqrt{13}} \angle \tan^{-1}\left(-\frac{2}{3}\right)$$

$$= 0.347 \angle -33.7°$$

$$i(t) = 0.347 \cos(2t - 33.7°)$$

ingen impulser

↓

Ved $t=0^-$ $i(0^-) = 0.347\cos(-33.7°) = \underline{\underline{0.289 = i(0^+)}}$

Så nå gjenstår å finne A

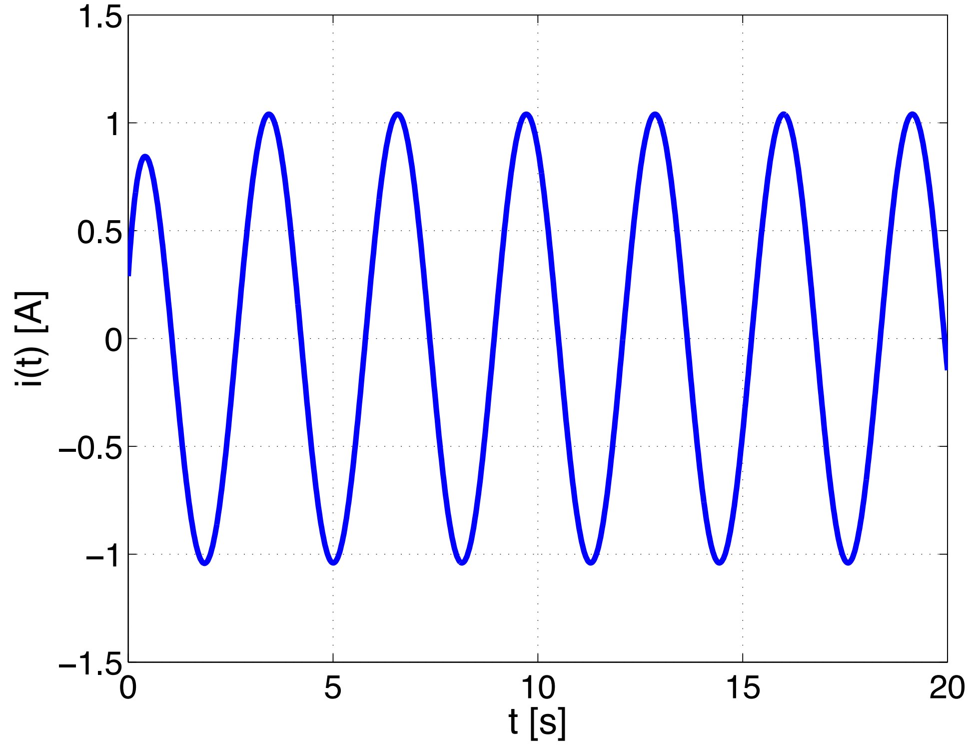
$i(0^+) = 0.289 = 1.04 \cos(\cancel{2t}^{\to 0} - 33.7^\circ) + A$

eða $A = -0.576$

þá er heildarlausnin okkar

$$i(t) = 1.04 \cos(2t - 33.7^\circ) - 0.576 e^{-3t} \quad t > 0$$

Dæmi 12.4



Samviðnám og samleiðni fyrir sínuslaga innmerki

- Hugsum okkur að tvinntölustraumur

$$\hat{i}(t) = \boldsymbol{I} e^{j\omega t}$$

$v = L i'$

fari um spólu og spennan yfir spóluna sé

$i = C v'$

$$\hat{v}(t) = \boldsymbol{V} e^{j\omega t}$$

- Sambandið milli straums og spennu spólunnar er $\hat{v}(t) = L\frac{d\hat{i}}{dt}$ eða

$$\boldsymbol{V} e^{j\omega t} = j\omega L \boldsymbol{I} e^{j\omega t}$$

- Þannig að hlutfall spennuvísisins $\boldsymbol{V}$ og straumvísisins $\boldsymbol{I}$ sem við köllum **samviðnám** $Z(j\omega)$ er

$$\boxed{Z_L(j\omega) = \frac{\boldsymbol{V}}{\boldsymbol{I}} = j\omega L}$$

$v = Z i$

Samviðnám og samleiðni fyrir sínuslaga innmerki

$i = Cv' = Cj\omega v$

- Með sömu röksemdafærslu má leiða út að samviðnám fyrir þétti er

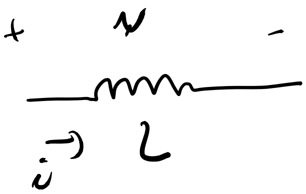
$$\boxed{Z_C(j\omega) = \frac{1}{j\omega C}}$$

- **Almennt gildir að samviðnám (e. impedance) fyrir sínuslaga innmerki $Z(j\omega)$ er skilgreint sem spennuvísir deilt með straumvísi**

$$\boxed{Z(j\omega) = \frac{\boldsymbol{V}}{\boldsymbol{I}}}$$

Dæmi 12.5

Finnið $i_p(t)$ ef $L = \frac{1}{2} H$ & $v(t) = 10\cos(5t + 15^\circ)$, $\bar{V} = 10\angle 15^\circ$



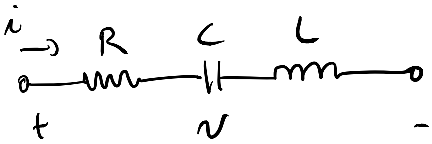
$$Z(j\omega) = j\omega L = j\frac{5}{2}$$

$$\bar{V} = \bar{I} \cdot Z \quad \& \quad \bar{I} = \frac{\bar{V}}{Z} = \frac{10\angle 15^\circ}{\frac{5}{2}\angle 90^\circ} = \frac{10e^{j15}}{\frac{5}{2}e^{j90}}$$

$$= 4\angle -75^\circ$$

$$i_p(t) = Re\left\{\bar{I}e^{j5t}\right\} = \underline{\underline{4\cos(5t - 75^\circ)}}$$

Dæmi 10.6 $\quad R = 3\Omega \quad L = 2H \quad C = \frac{1}{30} F \quad i(t) = 15\cos(3t + 30^\circ)$ Finnum $v(t)$



$\bar{I} = 15\angle 30^\circ \qquad \hat{i}(t) = 15\, e^{j(3t+30)}$

$i(t) = Re\{\hat{i}\}$

$\omega = 3$ rad/s

$$\bar{V} = Z\bar{I} \qquad Z(j\omega) = Z_R + Z_C + Z_L$$

$$= R + \frac{1}{j\omega C} + j\omega L$$

$$\boxed{\frac{1}{i} = \frac{i}{ii} = -i}$$

$$= 3 + \frac{1}{j\frac{1}{10}} + j6 = 3 - 10j + j6 = \underline{3 - j4}$$

$$= \underline{5\angle -53.1^\circ}$$

$$\bar{V} = \bar{I}Z = (15\angle 30^\circ)(5\angle -53.1^\circ) = 75\angle -23.1^\circ$$

$$v(t) = Re\{\bar{V}e^{j3t}\} = \underline{\underline{75\cos(3t - 23.1^\circ)}}$$

Samviðnám

- Samviðnám er almennt tvinntala

$$Z(j\omega) = R(\omega) + jX(\omega)$$

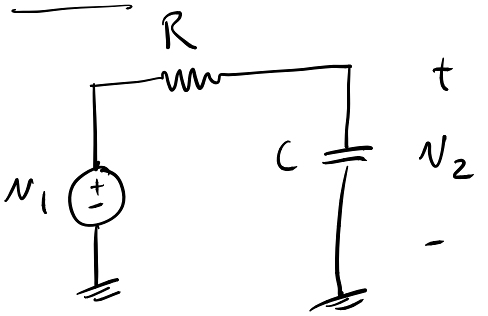
  þar sem raunhlutinn kallast raunviðnám (e. resistance) og þverhlutinn kallast launviðnám (e. reactance)

- Samleiðnin $Y = 1/Z$ er einnig tvinntala sem við getum ritað

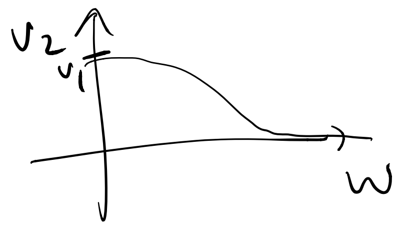
$$Y(j\omega) = G(\omega) + jB(\omega)$$

  þar sem raunhlutinn er kallaður leiðni (e. conductance) og þverhlutinn launleiðni (e. susceptance)

Dæmi



$$\bar{Z}_R = R$$
$$\bar{Z}_C = \frac{1}{j\omega C}$$



a) v_1 er sínuslaga, lýsið hegðun rásarinnar við ólíka tíðni

$$v_2 = \frac{Z_C}{Z_R + Z_C} v_1 = \frac{1}{1 + \frac{Z_R}{Z_C}} v_1 = \frac{1}{1 + j\omega RC} v_1$$

$$\boxed{\begin{aligned} &\frac{j}{j + j \cdot j\omega RC} v_1 \\ &= \frac{j}{j - \omega RC} v_1 \\ &= \frac{-j}{\omega RC - j} v_1 \end{aligned}}$$

ω lágt → $v_2 \approx v_1$ ekkert spennufall yfir R

ω hátt → $v_2 \approx 0$ öll spenna yfir R, C virkar eins og skammhlaup

b) $v_1(t) = 10\cos(\omega t)$ $\underline{R = 1\,M\Omega}$ $C = \frac{1}{10}\mu F$ $\bar{V}_1 = 10\angle 0^\circ$

Finnum $\underline{v_2(t)}$ fyrir i) $\omega = 1$ rad/s ii) $\omega = 10$ rad/s iii) $\omega = 50$ rad/s

$$\bar{V}_2 = \frac{-j}{\omega CR - j} \bar{V}_1$$

i) $$\bar{V}_2 = \frac{-j}{1 \cdot \frac{1}{10} \cdot 10^{-6} \cdot 1 \cdot 10^{6} - j} \cdot 10\angle 0^\circ = \frac{-j}{0.1 - j} \cdot 10\angle 0^\circ = \frac{10\angle -90^\circ}{1.005\angle -84.3^\circ}$$

$$= 9.95\angle -5.7^\circ$$

ii) $$\bar{V}_2 = \frac{-j}{10 \cdot 0.1 - j} \cdot 10\angle 0^\circ = \frac{10\angle -90^\circ}{\sqrt{2}\angle -45^\circ} = 7.07\angle -45^\circ$$

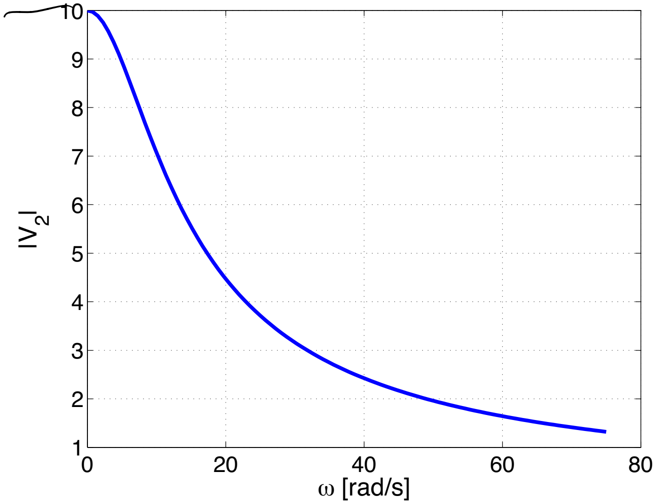
iii) $$\bar{V}_2 = \frac{-j}{50 \cdot 0.1 - j} 10\angle 0^\circ = \frac{10\angle -90^\circ}{5.099\angle -11.3^\circ} = 1.96\angle -78.7^\circ$$

i) $\omega = 1 \frac{rad}{s}$ $v_2(t) = 9.95 \cos(t - 5.7°)$ V

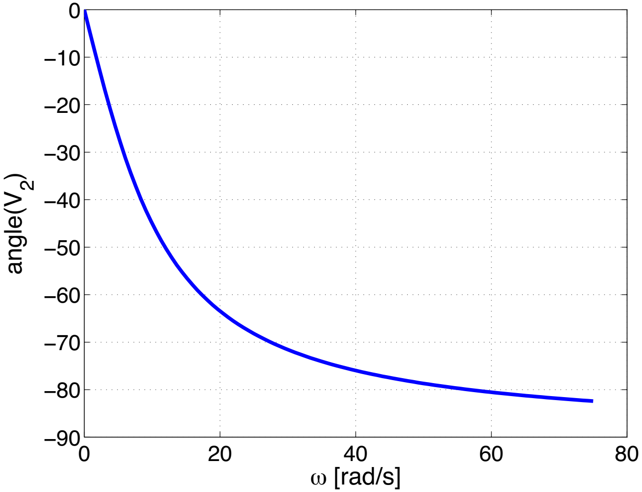
ii) $\omega = 10 \frac{rad}{s}$ $v_2(t) = 7.07 \cos(10t - 45°)$ V

iii) $\omega = 50 \frac{rad}{s}$ $v_2(t) = 1.96 \cos(50t - 78.7°)$ V

Dæmi 12.7

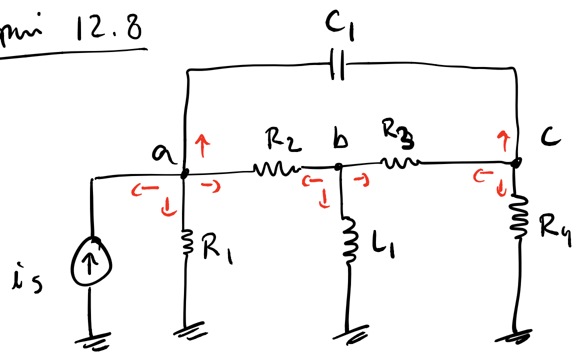


10
9
8
7
6
5
4
3
2
1
|V2|
0
20
40
60
80
ω [rad/s]



0
−10
−20
−30
−40
−50
−60
−70
−80
−90
angle(V2)
0
20
40
60
80
ω [rad/s]

Dæmi 12.8



$i_s(t) = 10\cos(2t+30°)$ $\quad \bar{I}_s = 10\angle 30°$

$R_1 = R_2 = R_3 = 1\Omega$ $\quad R_4 = 2\Omega$

$L_1 = 2H$ $\quad C_1 = 2F$ $\quad$ Hz ω

$$\boxed{\omega = 2\,\frac{rad}{s}}$$

Finna spenn yfir R_4.

$Z_R = R$ $\quad Z_L = j\omega L$ $\quad Z_C = \frac{1}{j\omega C}$

$Y_R = \frac{1}{R}$ $\quad Y_L = \frac{1}{Z_L}$ $\quad Y_C = \frac{1}{Z_C}$

$Y_{R1} = Y_{R2} = Y_{R3} = 1\,℧$ $\qquad Y_{L1} = \frac{1}{j2 2} = \frac{1}{j4} = -\frac{j}{4}$

$Y_{R4} = \frac{1}{2}\,℧$

$Y_{C1} = j\omega C = j\,2\cdot 2 = +j4$

a : $-i_s + Y_{R1}(v_a - 0) + Y_{C1}(v_a - v_c) + Y_{R2}(v_a - v_b) = 0$

b : $Y_{R2}(v_b - v_a) + Y_{L1}(v_b - 0) + Y_{R3}(v_b - v_c) = 0$

c: $Y_{R3}(v_c - v_b) + Y_{R4}(v_c - 0) + Y_{C1}(v_c - v_a) = 0$

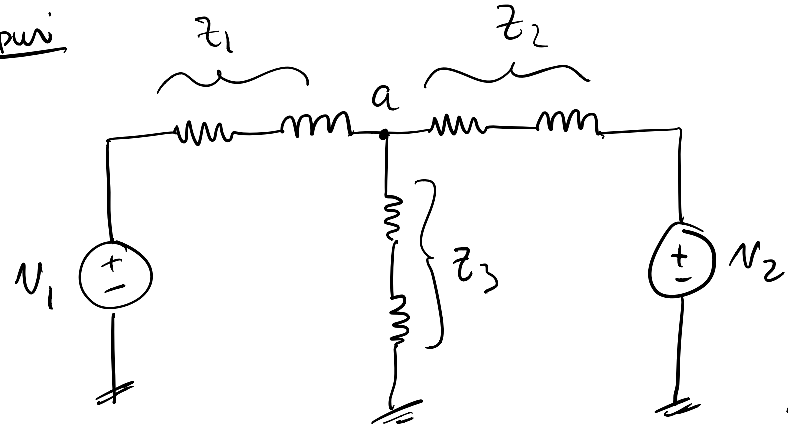
$$\begin{array}{c c} & \begin{array}{ccc} a & b & c \end{array} \\ \begin{array}{c} a \\ b \\ c \end{array} & \begin{bmatrix} Y_{R1}+Y_{R2}+Y_{C1} & -Y_{R2} & -Y_{C1} \\ -Y_{R2} & Y_{R2}+Y_{L1}+Y_{R3} & -Y_{R3} \\ -Y_{C1} & -Y_{R3} & Y_{R3}+Y_{R4}+Y_{C1} \end{bmatrix} \end{array} \begin{bmatrix} \bar{V}_a \\ \bar{V}_b \\ \bar{V}_c \end{bmatrix} = \begin{bmatrix} \bar{I}_s \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 2+j4 & -1 & -j4 \\ -1 & 2-\frac{j}{4} & -1 \\ -j4 & -1 & \frac{3}{2}+j4 \end{bmatrix} \begin{bmatrix} \bar{V}_a \\ \bar{V}_b \\ \bar{V}_c \end{bmatrix} = \begin{bmatrix} \bar{I}_s \\ 0 \\ 0 \end{bmatrix}$$

$\bar{V}_c = 6.47\angle 44° =$

$$\underline{v_c(t) = 6.47\cos(2t + 44°)}$$

Dæmi:



$Z_1 = 1.4\Omega + j1.6\ \Omega$

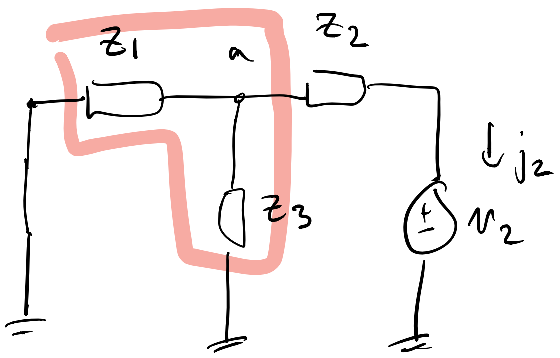
$Z_2 = 11.66\Omega + j8.75\Omega$

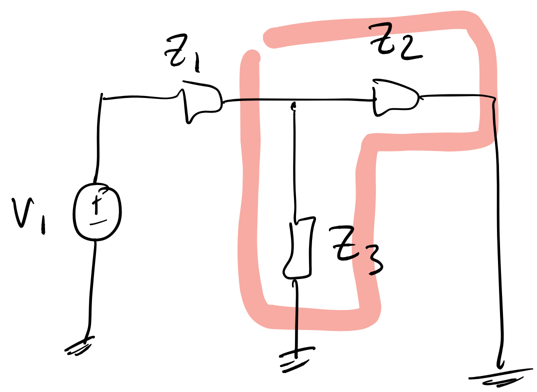
$Z_3 = 0.8\Omega + j1\Omega$

$v_1 = 460\cos(\omega_0 t) \quad v_2 = 451\cos(\omega_0 t)$

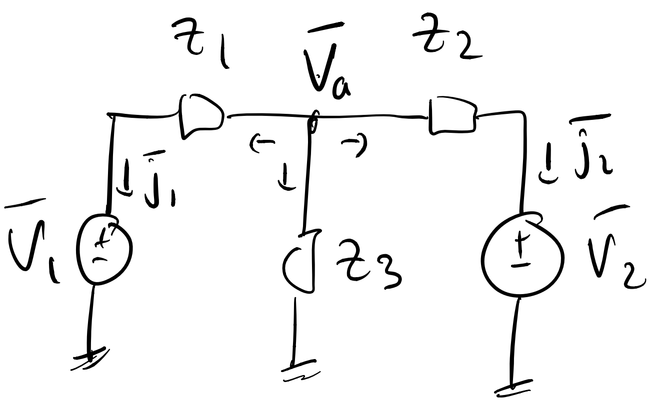
$\omega_0 = 377$ rad/s

Finnum v_a at $\omega_0 = 377$ rad/s $\quad \bar{V}_1 = 460\angle 0^\circ \quad \bar{V}_2 = 451\angle 0^\circ$





$$\bar{V}_a = \frac{Z_1 \| Z_3}{Z_2 + Z_1 \| Z_3}\bar{V}_2 + \frac{Z_2 \| Z_3}{Z_1 + Z_2 \| Z_3}\bar{V}_1$$



$$\bar{j}_1 + Y_3(\bar{V}_a - 0) + \bar{j}_2 = 0$$

$$\bar{j}_1 = Y_1(\bar{V}_a - V_1)$$

$$\bar{j}_2 = Y_2(\bar{V}_a - \bar{V}_2)$$

$$\underline{\bar{V}_a = 430.5\angle -0.7^\circ}$$