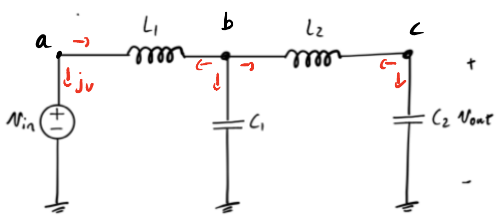
Dæmi 1 – Spólur og þéttar

Heimadæmi 9 Lausnir 21/03/21

Finnið $H(p) = v_{\text{out}}/v_{\text{in}}$ og diffurjöfnu sem tengir v_{out} og v_{in} saman.



Breyta	Gildi
C_1, C_2	1 F
L_1, L_2	2 H

Hér $Y_{Li} = \frac{1}{L_i p}$ $\quad Y_{Ci} = C_i p$ & $N_{\text{jöfnur}} = N_{\text{hnútpunktar}} + N_{vs} - 1$

$= 4 + 1 - 1 = \underline{\underline{4}}$

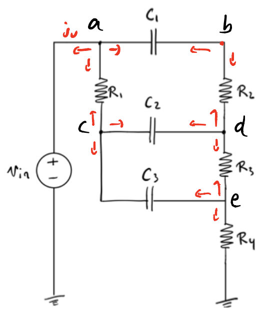
$$\left[\begin{array}{ccc|c} Y_{L1} & -Y_{L1} & 0 & 1 \\ -Y_{L1} & Y_{L1}+Y_{C1}+Y_{L2} & -Y_{L2} & 0 \\ 0 & -Y_{L2} & Y_{C2}+Y_{L2} & 0 \\ \hline 1 & 0 & 0 & 0 \end{array}\right] \left[\begin{array}{c} v_a \\ v_b \\ v_c \\ \hline j_u \end{array}\right] = \left[\begin{array}{c} 0 \\ 0 \\ 0 \\ \hline v_{in} \end{array}\right] \text{ svo } \left[\begin{array}{c} v_b \\ v_c \\ \hline j_u \end{array}\right] = \frac{v_{in}}{4p^4+6p^2+1} \left[\begin{array}{c} (2p^2+1) \\ 1 \\ -2p(p^2+1) \end{array}\right] \quad \& \quad v_a = v_{in}$$

Hér er $H(p) = \frac{v_{out}}{v_{in}} = \frac{v_c}{v_a} = \frac{1}{4p^4+6p^2+1}$

Diffurjafnan er því $\underline{\underline{\frac{d^4 v_{out}}{dt^4} + \frac{3}{2}\frac{d^2 v_{out}}{dt^2} + \frac{1}{4} v_{out} = \frac{1}{4} v_{in}}}$

Dæmi 2 – Fender Bassman Tone Stack

Rásin hér að neðan er rás úr gítarmagnara. Finnið $H(p) = v_{\text{out}}/v_{\text{in}}$, þar sem v_{out} er spennan í hnútpunktinum sem R_2, C_2 og R_3 tengjast í. Skrifið svo diffurjöfnu sem tengir v_{out} og v_{in} saman.



Breyta	Gildi
R_1, R_2, R_3, R_4	$1\,\Omega$
C_1, C_2, C_3	$1\,\text{F}$

Höfum $Y_{Ci} = C_i p = p$ Nú er $N_{\text{jöfnu}} = N_{\text{hnútapunkta}} + N_{\text{spennulind}} - 1 = 6 + 1 - 1 = \underline{\underline{6}}$

$Y_{Ri} = \frac{1}{R_i} = G_i = 1\,\mho$

$$\begin{array}{c|c}
 & \begin{matrix} a & b & c & d & e & V_{in} \end{matrix} \\
\begin{matrix} a \\ b \\ c \\ d \\ e \\ V_{in} \end{matrix} &
\left[\begin{array}{ccccc|c}
Y_{R_1}+Y_{C_1} & -Y_{C_1} & -Y_{R_1} & 0 & 0 & 1 \\
-Y_{C_1} & Y_{C_1}+Y_{R_2} & 0 & -Y_{R_2} & 0 & 0 \\
-Y_{R_1} & 0 & Y_{R_1}+Y_{C_2}+Y_{C_3} & -Y_{C_2} & -Y_{C_3} & 0 \\
0 & -Y_{R_2} & -Y_{C_2} & Y_{R_2}+Y_{C_2}+Y_{R_3} & -Y_{R_3} & 0 \\
0 & 0 & -Y_{C_3} & -Y_{R_3} & Y_{R_3}+Y_{C_3}+Y_{R_4} & 0 \\
\hline
1 & 0 & 0 & 0 & 0 & 0
\end{array}\right]
\end{array}
\begin{bmatrix} v_a \\ v_b \\ v_c \\ v_d \\ v_e \\ \hline j_v \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \hline v_{in} \end{bmatrix}$$

en $v_a = v_{in}$ &

$$\begin{bmatrix} v_b \\ v_c \\ v_d \\ v_e \\ \hline j_v \end{bmatrix} = \frac{v_{in}}{162p^3 + 648p^2 + 432p + 54} \begin{bmatrix} \frac{p}{6}\left(972p^2 + 3564p + 1944\right) \\ 108p^3 + 432p^2 + 324p + 54 \\ p\left(108p^2 + 324p + 162\right) \\ -p\left(108p^2 + 324p + 162\right) \end{bmatrix}$$

Nú er $\underline{\underline{H(p) = \frac{v_{out}}{v_{in}} = \frac{v_d}{v_a} = \frac{p\left(108p^2 + 324p + 162\right)}{162p^3 + 648p^2 + 432p + 54}}}$

eða $162\frac{d^3 v_{out}}{dt^3} + 648\frac{d^2 v_{out}}{dt^2} + 432\frac{d v_{out}}{dt} + 54 v_{out} = 108\frac{d^3 v_{in}}{dt^3} + 324\frac{d^2 v_{in}}{dt^2} + 162\frac{d v_{in}}{dt}$

eða $\underline{\underline{\frac{d^3 v_{out}}{dt^3} + 4\frac{d^2 v_{out}}{dt^2} + 2\tfrac{2}{3}\frac{d v_{out}}{dt} + \frac{1}{3} v_{out} = \frac{2}{3}\frac{d^3 v_{in}}{dt^3} + 2\frac{d^2 v_{in}}{dt^2} + \frac{d v_{in}}{dt}}}$