Viðnám $v = R i$ $i = R^{-1} v = G v$

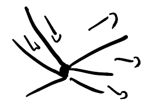
$R\,[\Omega]$ ohm

$G\,[\mho]$ mho

Þéttir $v = \frac{1}{C}\int_{-\infty}^{t} i \, d\tau = v(0^-) + \frac{1}{C}\int_{0^+}^{t} i \, d\tau$ $i = C \frac{dv}{dt} = C p v$

Spóla $v = L \frac{di}{dt}$ $i = \frac{1}{L}\int_{-\infty}^{t} v \, d\tau = i(0^-) + \frac{1}{L}\int_{0^+}^{t} v \, d\tau$

Greining Rása



Kerfisjöfnur

$p = \frac{d}{dt}$ $i_{inn} = i_{út}$ $v_{\text{í lykkju}} = 0$

$\frac{1}{p} = \int_{-\infty}^{t} \_\_ \, d\tau$ $\frac{d}{dt} i_{inn} = \frac{d}{dt} i_{út}$ $\frac{d}{dt} v_{\text{í lykkju}} = 0$

Ólafur Bjarki Bogason

4. mars 2021

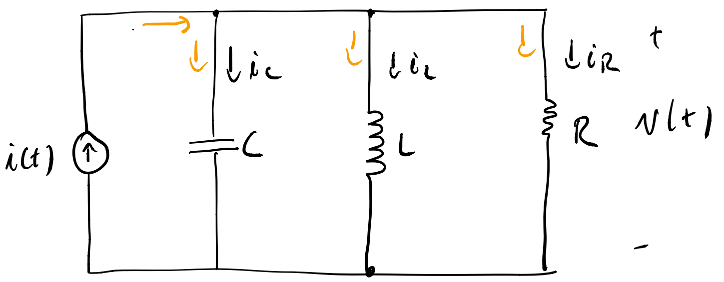
Inngangur

- Rafmagnsrás er mynduð úr samtengingu rásaeininga
- Þessar rásaeiningar hlíta lögmálum Kirchoffs:
  - KVL: Summa allra spennufalla eftir lokaðri leið í rás er núll
  - KCL: Summa allra strauma inn í hnútpunkt er núll
- Með því að beita lögmálum Kirchoffs á almennar rásir fáum við **kerfisjöfnur** sem eru almennt **tegur-diffurjöfnur** (e. integro differential equation)

Inngangur

- Hafi rásin fleiri en einn hnútpunkt eða möskva, þá fæst fleiri en ein tegur-diffurjafna; þessar jöfnur þarf að leysa saman
  - Viðnámsrásir eru bara sértilfelli af þessum almennu rásum
  - **Þegar orkugeymandi rásaeiningar (þéttar og spólur) bætast við breytast jöfnurnar úr venjulegum algebrískum jöfnum í tegur-diffurjöfnur**
- Almennt gildir að fyrir rás sem inniheldur n orkugeymandi rásaeiningar verður hæsti diffurkvótinn af gráðu n.
- Þá er talað um $n-$**tu gráðu rás** (kerfi)

Dæmi Finnum tegur-diffurjöfnu fyrir spennuna $v(t)$ ef $i(t)$ er gefinn.

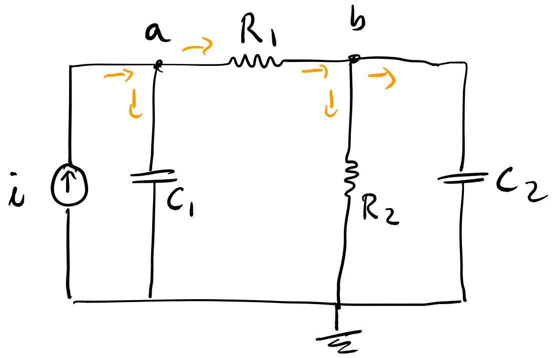


Með KCL fæst

$$i(t) = i_C(t) + i_L(t) + i_R(t)$$

Svo $$i(t) = C\frac{dv}{dt} + \frac{1}{L}\int_{-\infty}^{t} v(\tau)\,d\tau + \frac{1}{R}v$$

Dæmi Finnum tegur-diffurjöfnu fyrir $v_b(t)$ ef straumur i er gefinn



$$N_{jöfnur} = N_{hnútp.} + N_{VS} - 1 = 3 + 0 - 1 = \underline{\underline{2}}$$

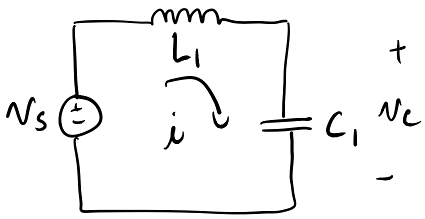
KCL í a: $i = C_1 \dfrac{dv_a}{dt} + G_1(v_a - v_b)$

KCL í b: $G_1(v_a - v_b) + G_2(v_b - 0) + C_2 \dfrac{dv_b}{dt}$

Þessar tvær jöfnur þarf að leysa saman, sem getur orðið ansi flókið...

$$R_1 C_1 C_2 \frac{d^2 v_b}{dt^2} + \frac{(R_1 + R_2)C_1 + R_2 C_2}{R_2} \frac{dv_b}{dt} + \frac{v_b}{R_2} = i(t)$$

Demo: Find diff eqn for $v_c(t)$ if $L_1 = 2H$ $C_1 = 3F$



KVL $v_s = v_L + v_c$

$$= L \frac{di}{dt} + v_c$$

$$i_L = i_C = i = C \frac{dv_c}{dt}$$

$$= LC \frac{d^2 v_c}{dt^2} + v_c$$

So $\underline{v_s(t) = 6 \frac{d^2 v_c}{dt^2} + v_c(t)}$

Virkjatáknun

- Til að geta höndlað tegur-diffurjöfnur eins og algebrískar jöfnur (þó ekki leyst þær) skilgreinum við virkjann p, diffurvirkjann

$$\boxed{p \equiv \frac{d}{dt}}$$

  Þá er

$$\boxed{pf = \frac{df}{dt}}$$

- Diffrum n sinnum

$$\boxed{p^n f = \frac{d^n f}{dt^n}}$$

Virkjatáknun

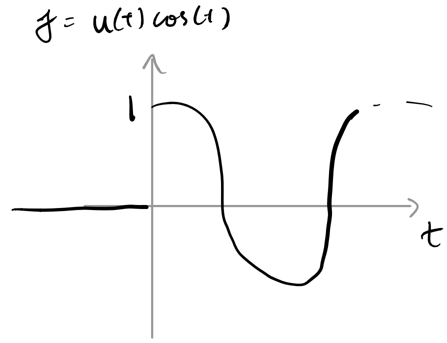
- Við skilgreinum $1/p$ sem tegrun

$$\boxed{\frac{1}{p}f \equiv \int_{-\infty}^{t} f(\tau)d\tau}$$

ef $f = 0$ fyrir $t < 0$ þá

$$\frac{1}{p}f \equiv \int_{0-}^{t} f(\tau)d\tau$$

<u>Dømi</u> $f(t) = \cos(t)\,u(t)$



a) Finnið pf

$$pf = \frac{d}{dt} f = -\sin(t)\,u(t) + \cos(t)\,\underline{\underline{\delta(t)}}$$

$$= -\sin(t)\,u(t) + \delta(t)$$

b) Finnið $\frac{1}{p}(pf)$

$$\frac{1}{p} pf = \int_{-\infty}^{t} \left(-\sin(\tau)\,u(\tau) + \delta(\tau) \right) d\tau$$

$$= \int_{0^-}^{t} (-\sin\tau)\, d\tau + \int_{0^-}^{t} \delta(\tau)\, d\tau \qquad \underline{t \geq 0}$$

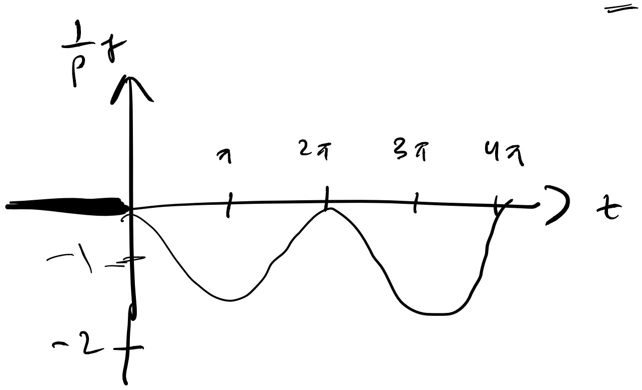
$$= \left[\cos\tau \right]_{0^-}^{t} + 1 = \cos(t) - 1 + 1 = \cos(t)$$

$$\frac{1}{p} pf = \cos(t)\,u(t) = f$$

Dæmi $f(t) = -\sin(t)u(t)$ Finnið & teiknið $\frac{1}{p}f$

$$\frac{1}{p}f = \int_{-\infty}^{t} \left(-\sin(\tau)u(\tau)\right)d\tau \overset{t\geq 0}{=} \int_{0^-}^{t} \left(-\sin(\tau)\right)d\tau = \left[\cos\tau\right]_{0^-}^{t} = (\cos t - 1)\,u(t)$$

0^-

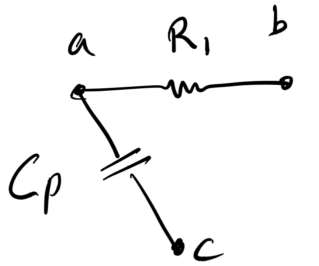


Dæmi Skrifið með virkjatákn $\underbrace{\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 3y + 3\cdot\int_{0^-}^{t} y\,d\tau}$ $= 2\cos t\cdot u(t)$

$p \equiv \frac{d}{dt} \qquad p^2 = \frac{d^2}{dt^2}$

$\frac{1}{p} = \int_{-\infty}^{t} \_\, d\tau$

$$p^2 y + 4py + 3y + 3\frac{1}{p}y = 2\cos t\, u(t)$$



$$\begin{bmatrix} G_1 + Cp & -G_1 & -Cp \\ -G_1 & G_1 & \\ -Cp & & Cp \end{bmatrix} \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = \begin{bmatrix} I_1 \\ 0 \\ 0 \end{bmatrix}$$

$Z \approx$ "G"

$i = C\cdot\frac{dv}{dt} = Cpv$

v_a

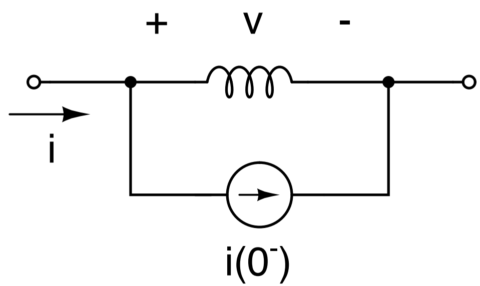
$\frac{dv_a}{dt}$

$\frac{d^2v_a}{dt^2}$

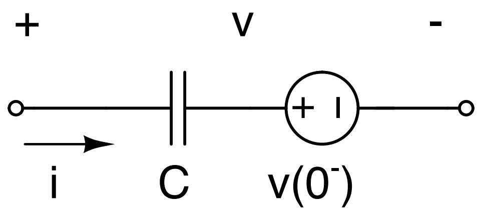
Virkjatáknun

- Eins og áður þá gerum við ráð að það kveikni á rásunum okkar við tímann $t = 0$ og notum lindir til að tákna byrjunargildi í þéttum og spólum

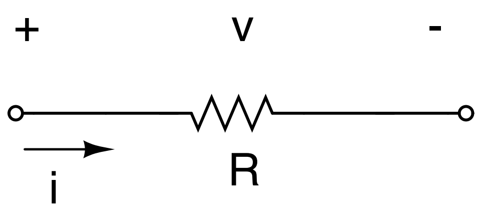
$$\int_{-\infty}^{t} \_\, d\tau = \int_{-\infty}^{0^-} \_\, d\tau + \int_{\underline{0^-}}^{t} \_\, d\tau$$



$$v = Lpi \quad \text{og} \quad i = \frac{1}{Lp}v + i(0^-)$$



$$v = \frac{1}{Cp}i + v(0^-) \quad \text{og} \quad i = Cpv$$



$$v = iR \quad \text{og} \quad i = \frac{1}{R}v$$

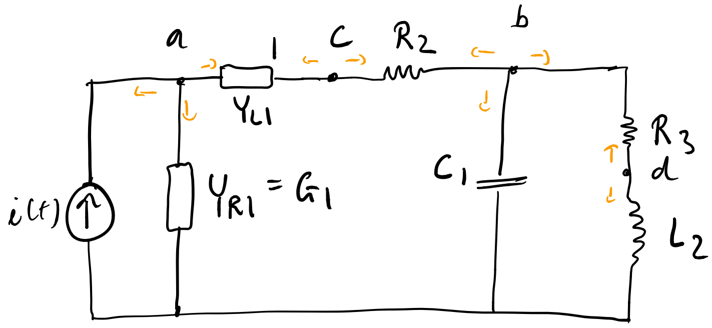
Samviðnám

- Lögmál Kirchoffs gilda alveg jafnt um rásir sem innihalda orkugeymandi rásaeiningar eins og um rásir sem innihalda eingöngu viðnám og lindir
- Til að setja upp hnútpunkta- eða möskvajöfnur fyrir tiltekna rás beitum við lögmálum Kirchoffs
- Með því að nota $p-$virkja táknunina fáum við jöfnurnar á snyrtilegu og samþjöppuðu formi

$\Longrightarrow$ Dæmi 9.7.

- Í ofangreindu dæmi má líta á $i(t)$ sem **innmerki** og v_{a}, v_{b}, sem **útmerki**

Dæmi Finnið p-virkja jöfn sem leysa rásina. Gerum ráð fyrir að $\forall\, i, v = 0 \quad t<0$



$R_1 = 2\Omega, \; R_2 = 4\Omega, \; R_3 = 8\Omega$

$L_1 = 2H, \; L_2 = 3H, \; C_1 = 2F$

$Z : i \to v$
$Z_c = \frac{1}{Cp}$
$Z_L = Lp$
$Z_R = R$

Munum: $i_R = \frac{1}{R} v_R \quad i_c = Cp\, v_c \quad i_L = \frac{1}{Lp} v_L$

$v_R = R\, i_R \quad v_c = \frac{1}{Cp} i_c \quad v_L = Lp\, i_L$

samleiðni

$i_c = C \frac{dv_c}{dt} \quad i_L = \frac{1}{L}\int_{-\infty}^{t} v$

$G_2(v_c - v_b)$

$Y : v \to i$

samviðnám

$Z : i \to v$

$Y_R = \frac{1}{R} = G \quad Y_c = Cp \quad Y_L = \frac{1}{Lp}$

$Z_R = R \quad Z_c = \frac{1}{Cp} \quad Z_L = Lp$

KCL: a: $-i(t) + Y_{R1}(v_a - 0) + Y_{L1}(v_a - v_c) = 0$

c: $Y_{L1}(v_c - v_a) + Y_{R2}(v_c - v_b) = 0$

b: $Y_{R2}(v_b - v_c) + Y_{C1}(v_b - 0) + Y_{R3}(v_b - v_d) = 0$

d: $Y_{R3}(v_d - v_b) + Y_{L2}(v_d - 0) = 0$

$$\begin{array}{c} \\ a \\ b \\ c \\ d \end{array}
\begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\
\begin{bmatrix} Y_{R1}+Y_{L1} & 0 & -Y_{L1} & 0 \\ 0 & Y_{R2}+Y_{C1}+Y_{R3} & -Y_{R2} & -Y_{R3} \\ -Y_{L1} & -Y_{R2} & Y_{L1}+Y_{R2} & 0 \\ 0 & -Y_{R3} & 0 & Y_{R3}+Y_{L2} \end{bmatrix} \end{array}
\begin{bmatrix} v_a \\ v_b \\ v_c \\ v_d \end{bmatrix} = \begin{bmatrix} i(t) \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

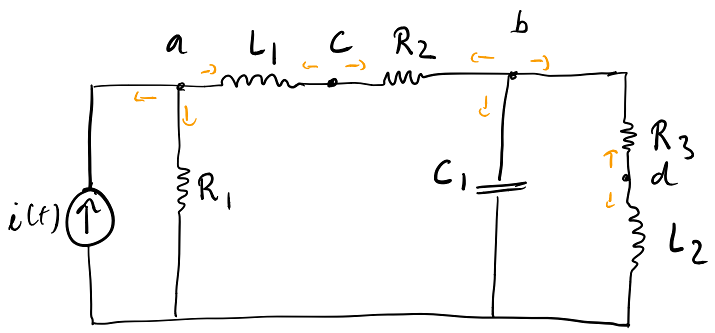
$Y_{R2} = \frac{1}{R_2} = \frac{1}{4} \mho$

$Y_{L1} = \frac{1}{L_1 p} = \frac{1}{2p}$

$Y_{C1} = C_1 p = 2p$

$$\begin{array}{c} \\ a \\ b \\ c \\ d \end{array}
\begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\
\begin{bmatrix} G_1 + \frac{1}{L_1 p} & & -\frac{1}{L_1 p} & \\ & G_2 + G_3 + Cp & -G_2 & -G_3 \\ -\frac{1}{L_1 p} & -G_2 & \frac{1}{L_1 p} + G_2 & \\ & -G_3 & & G_3 + \frac{1}{L_2 p} \end{bmatrix} \end{array}
\begin{bmatrix} v_a \\ v_b \\ v_c \\ v_d \end{bmatrix} = \begin{bmatrix} i \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Dæmi   Finnið p-virkja jöfnu sem leysa rásina.   Gerum ráð fyrir að $\forall i, v = 0 \quad t < 0$



$R_1 = 2\Omega, \; R_2 = 4\Omega, \; R_3 = 8\Omega$

$L_1 = 2H, \; L_2 = 3H, \; C_1 = 2F$

$V = Z i$

$Z_C = \frac{1}{Cp}$

$Z_L = Lp$

$Z_R = R$

Munum: $i_R = \frac{1}{R} v_R \quad i_C = Cp\, v_C \quad i_L = \frac{1}{Lp} v_L$

$v_R = R i_R \quad v_C = \frac{1}{Cp} i_C \quad v_L = Lp\, i_L$

Heft $N_{jöfnu} = N_{hnútp} + N_{VS} - 1 = 5 + 1 - 1 = \underline{\underline{5}}$

Skrifum KCL í a - d

$$a: \quad -i(t) + G_1(v_a - 0) + \frac{1}{L_1 p}(v_a - v_c) = 0$$

$$c: \quad \frac{1}{L_1 p}(v_c - v_a) + G_2(v_c - v_b) = 0$$

$$b: \quad G_2(v_b - v_c) + Cp\,(v_b - 0) + G_3(v_b - v_d) = 0$$

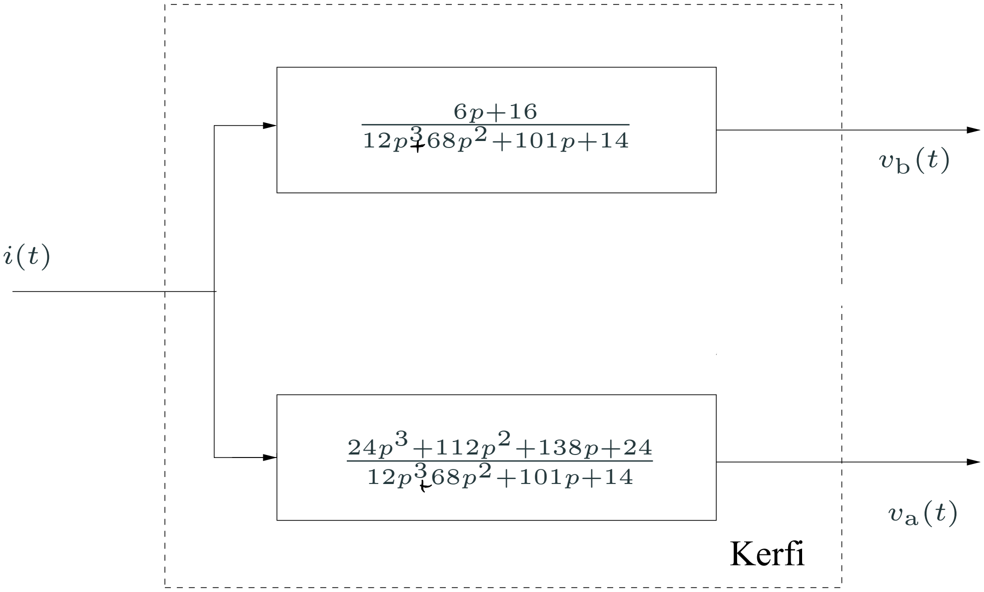
$$d: \quad G_3(v_d - v_b) + \frac{1}{L_2 p}(v_d - 0) = 0$$

$$\begin{array}{c} \\ a \\ b \\ c \\ d \end{array}
\begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\
\begin{bmatrix} G_1 + \frac{1}{L_1 p} & & -\frac{1}{L_1 p} & \\ & G_2 + G_3 + Cp & -G_2 & -G_3 \\ -L_1 p & -G_2 & \frac{1}{L_1 p} + G_2 & \\ & -G_3 & & G_3 + \frac{1}{L_2 p} \end{bmatrix} \end{array}
\begin{bmatrix} v_a \\ v_b \\ v_c \\ v_d \end{bmatrix} = \begin{bmatrix} i \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} v_a \\ v_b \\ v_c \\ v_d \end{bmatrix} = i \begin{bmatrix} \frac{48p^3 + 224p^2 + 276p + 48}{24p^3 + 136p^2 + 202p + 28} \\ \frac{12p + 32}{24p^3 + 136p^2 + 202p + 28} \\ \frac{96p^2 + 268p + 48}{24p^3 + 136p^2 + 202p + 28} \\ \frac{3p}{6p^3 + 34p^2 + 50.5p + 7} \end{bmatrix} = \frac{i}{24p^3 + 136p^2 + 202p + 28} \begin{bmatrix} 48p^3 + 224p^2 + 276p + 48 \\ 12p + 32 \\ 96p^2 + 268p + 48 \\ 12p \end{bmatrix}$$

Samviðnám

- Sýnum þetta með kassamynd



- Það sem stendur í hverjum kassa fyrir sig kallast **yfirfærslufall** og er táknað með $H(p)$

Samviðnám

- Yfirfærsluföll tengja innmerki og útmerki

$$\text{yfirfærslufall} \times \text{innmerki} = \text{útmerki}$$

- Ef útmerkið er spenna og innmerkið er straumur þá hefur rásafallið eininguna Ω og kallast **samviðnám** (e. impedance)
- Við notum táknið $Z(p)$ fyrir samviðnám
- Ef innmerkið er spenna og útmerkið er straumur þá hefur rásafallið eininguna $\mho$ og kallast samleiðni (e. admittance)
- Við notum táknið $Y(p)$ fyrir samleiðni

Samviðnám

samviðnám

e.g., $v = iR$

	viðnám	þéttir	spóla
$Z(p)$	R	$\frac{1}{Cp}$	Lp
$Y(p)$	$\frac{1}{R}$	Cp	$\frac{1}{Lp}$

$Z : i \to v$

$Y : v \to i$

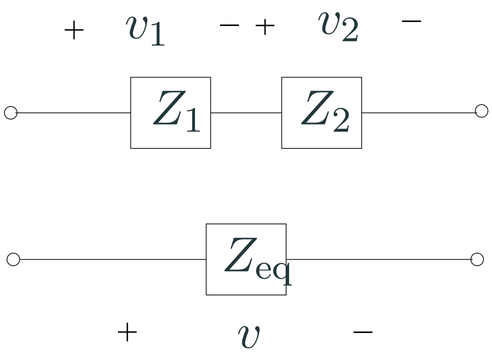
samleiðni

- Sjáum að

$$Z(p) = \frac{1}{Y(p)}$$

- Sýna má fram á að samviðnám $Z(p)$ hlíta sömu reglum og viðnám R varðandi raðtengingu, hliðtengingu o.þ.h.

Samviðnám



- Hér er

$$v = v_1 + v_2$$

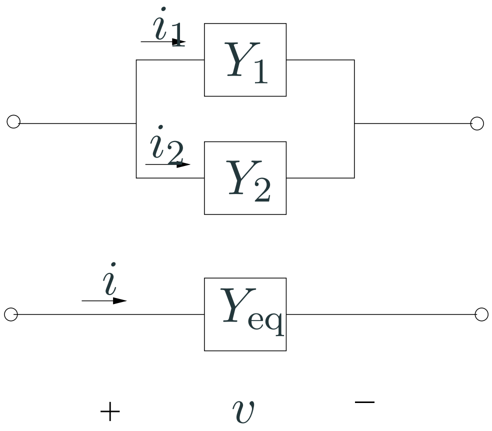
og

$$v = (Z_1(p) + Z_2(p))i = Z_{\text{eq}}(p)i$$

eða

$$Z_{\text{eq}} = (Z_1(p) + Z_2(p))$$

~~Samviðnám~~ Samleiðni



- Hér er

$$i = i_1 + i_2$$

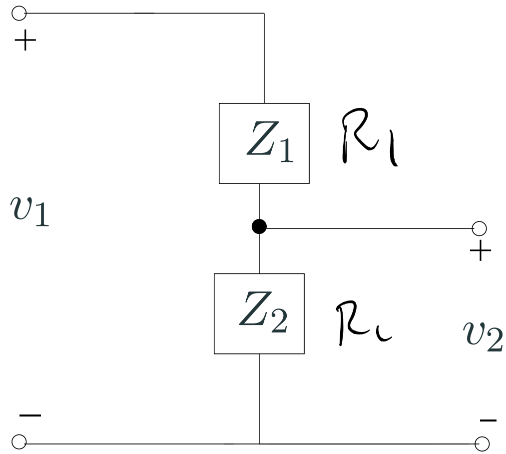
og

$$i = (Y_1(p) + Y_2(p))v = Y_{\mathrm{eq}}(p)v$$

eða

$$Y_{\mathrm{eq}} = (Y_1(p) + Y_2(p))$$

Samviðnám

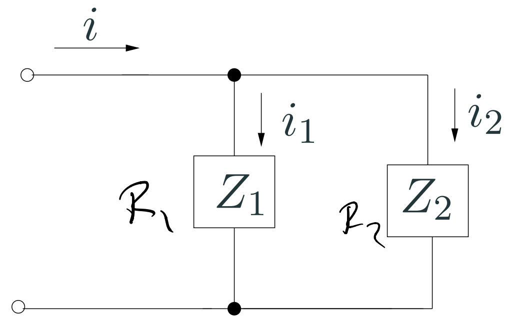


$$v_2 = \frac{R_2}{R_1 + R_2} v_1$$

- Eins gildir spennudeilingarformúlan

$$v_2 = \frac{Z_2(p)}{Z_1(p) + Z_2(p)} v_1$$

Samviðnám



- Eins gildir straumdeilingarformúlan

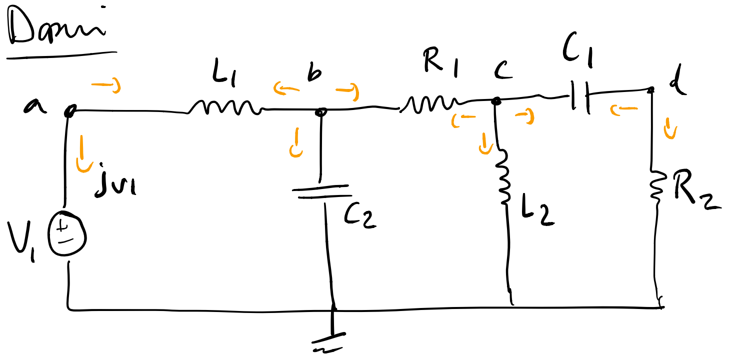
$$i_2 = \frac{Z_1(p)}{Z_1(p) + Z_2(p)} i$$

$$i_2 = i \frac{R_1}{R_1 + R_2}$$

Kerfisjöfnur

- Kerfisjafna er diffurjafna sem tengir háðu (óþekktu) breyturnar við lindirnar í rásinni
- Skrifum jöfnurnar venjulega þannig að hæsti diffurkvóti sé fyrstur og hafi stuðulinn 1

Dæmi



Finna MNA ef V_1 er þekkt & er innmerkið okkar

Notum $Z_{R_i} = R_i$, $Z_C = \frac{1}{Cp}$ & $Z_L = Lp$ $\quad i \to v$

$Y_{R_i} = G_i \quad Y_{C_i} = C_i p \quad Y_{L_i} = \frac{1}{L_i p} \quad v \to i$

$$N_{\text{jöfnur}} = N_{\text{hnútpunktar}} + N_{VS} - 1 = 5 + 1 - 1 = \underline{5 \text{ jöfnur}} \,/\, \text{hnútar}$$

KCL

a : $j_{V1} + Y_{L1}(v_a - v_b) = 0$

b : $Y_{L1}(v_b - v_a) + Y_{C2}(v_b - 0) + Y_{R1}(v_b - v_c) = 0$

c : $Y_{R1}(v_c - v_b) + Y_{L2}(v_c - 0) + Y_{C1}(v_c - v_d) = 0$

d : $Y_{C1}(v_d - v_c) + Y_{R2}(v_d - 0) = 0$

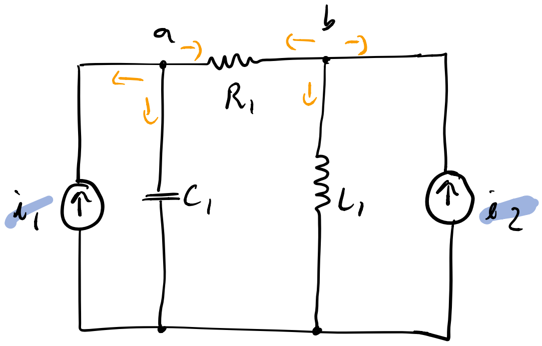
KVL yfir V_1 : $(v_a - 0) = V_1$

$$N_{\text{jöfnur}} = N_{hp} + N_{VS} - 1 = 5 + 1 - 1 = \underline{\underline{5}}$$

$$\begin{array}{c} \\ a \\ b \\ c \\ d \\ V_1 \end{array}
\begin{array}{c} \begin{array}{ccccc} a & b & c & d & V_1 \end{array} \\
\left[\begin{array}{cccc|c}
Y_{L1} & -Y_{L1} & 0 & 0 & 1 \\
-Y_{L1} & Y_{L1}+Y_{C2}+Y_{R1} & -Y_{R1} & 0 & 0 \\
0 & -Y_{R1} & Y_{R1}+Y_{L2}+Y_{C1} & -Y_{C1} & 0 \\
0 & 0 & -Y_{C1} & Y_{C1}+Y_{R2} & 0 \\
\hline
1 & 0 & 0 & 0 & 0
\end{array}\right] \end{array}
\left[\begin{array}{c} v_a \\ v_b \\ v_c \\ v_d \\ \hline j_V \end{array}\right]
=
\left[\begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ \hline V_1 \end{array}\right]$$

útmerki $\quad$ innmerki

Demo: Finna diffekv. som tar spänningen i knutpunkterna vid insignaler i_1 & i_2. $R_1 = 1\Omega$ $L_1 = 1H$ $C_1 = 1F$.



$Z_R \neq Z_C$ $\quad Z_R = R \quad Z_L = Lp \quad Z_C = \frac{1}{Cp}$

$Z_{R1} = 1 \quad Z_{L1} = p \quad Z_{C1} = \frac{1}{p} = p^{-1} \neq 1$

Z_{R1}, Z_{L1}, Z_{R1} ern ehli jetfrid!

$$N_{jöfr} = N_{knutp.} + N_{vs} - 1 = 2 \text{ jöfr}$$

KCL i a: $-i_1 + Y_{C1}(v_a - 0) + Y_{R1}(v_a - v_b) = 0$

b: $Y_{R1}(v_b - v_a) + Y_{L1}(v_b - 0) - i_2 = 0$

$$\begin{array}{c|cc} & a & b \\ \hline a & Y_{C1}+Y_{R1} & -Y_{R1} \\ b & -Y_{R1} & Y_{R1}+Y_{L1} \end{array} \begin{bmatrix} v_a \\ v_b \end{bmatrix} = \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

$Y_{R1} = 1 \quad Y_{L1} = p \quad Y_{C1} = p^{-1}$

eller

$$\begin{array}{c|cc} & a & b \\ \hline a & p^{-1}+1 & -1 \\ b & -1 & p+1 \end{array} \begin{bmatrix} v_a \\ v_b \end{bmatrix} = \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

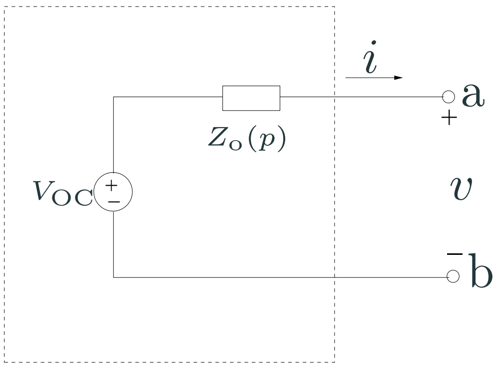
Het $N_{jöfr} = N_{np} + N_{vs} - 1 = 3 + 0 - 1 = 2$ nota $Y_{Ci} = C_i p$ $\quad Y_{Ri} = \frac{1}{R_i}$ $\quad Y_{Li} = \frac{1}{L_i p}$

$$\begin{bmatrix} Y_{R1}+Y_{C1} & -Y_{R1} \\ -Y_{R1} & Y_{L1}-Y_{R1} \end{bmatrix} \begin{bmatrix} v_a \\ v_b \end{bmatrix} = \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} \quad \text{svo} \quad \begin{bmatrix} v_a \\ v_b \end{bmatrix} = \frac{1}{p^2+p+1} \begin{bmatrix} i_1(p+1) + i_2 p \\ i_1 + (p+1) i_2 \end{bmatrix}$$

Som diffekv.

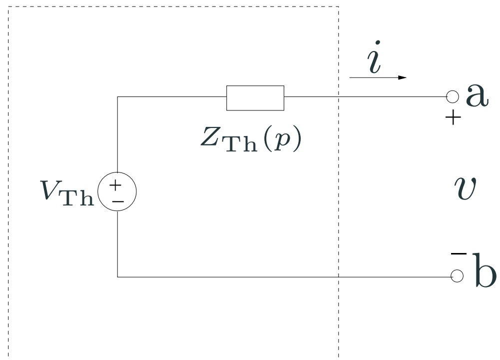
$$\begin{cases} \frac{d^2 v_a}{dt^2} + \frac{dv_a}{dt} + v_a = \frac{di_1}{dt} + i_1 + \frac{di_2}{dt} \\ \frac{d^2 v_b}{dt^2} + \frac{dv_b}{dt} + v_b = i_1 + \frac{di_2}{dt} + i_2 \end{cases}$$

Thévenin- og Norton rásir með $p-$virkjanum



- Við getum fundið Thévenin- og Norton jafngildisrásir fyrir rásir sem innihalda þétta og spólur, eins og fyrir venjulegar viðnámsrásir
  rétt
- Munurinn er sá að nota verður <u>$p-$virkja samviðnám</u> $Z(p)$
- Tómgangsspenna, skammhlaupsstraumur og útgangsviðnám verða þá föll af $p-$virkjanum

Thévenin- og Norton rásir með $p-$virkjanum



$v = V_{Th} - Z_{th}\, i$

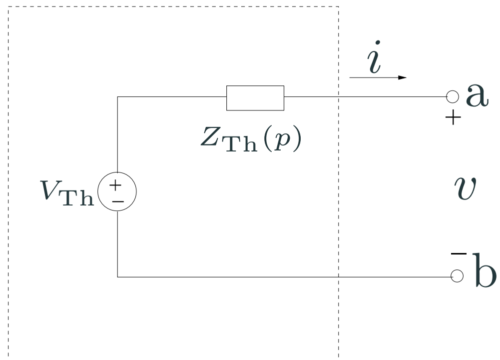
- Spennu-straumkennilínan (fyrir pólana) fyrir einhverja rás er á forminu

$$\boxed{i = \frac{V_{\mathrm{Th}}}{Z_{\mathrm{Th}}} - \frac{1}{Z_{\mathrm{Th}}} v}$$

eða

$$\boxed{v = V_{\mathrm{Th}} - Z_{\mathrm{Th}} i}$$

Thévenin- og Norton rásir með $p-$virkjanum



$$\boxed{v = V_{\mathrm{Th}} - Z_{\mathrm{Th}} i}$$

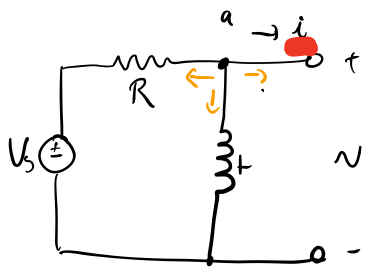
- Með lögmálum Kirchhoffs getum við alltaf fundið hnútpunktajöfnu fyrir eftri pólinn (a) á forminu

$$\boxed{v = \sum_{j=1}^{n} ((\text{virki}j)(\text{innri lind}j)) - (\text{virki}\phi) i(t)}$$

þar sem n er fjöldi innri linda

Dæmi   Finnum jöfnu á þessu formi fyrir gef. v s

$$v = \sum_{j=1}^{n}((\text{virki}\,j)(\text{innri lind}\,j)) - (\text{virki}\,\phi)\,i(t)$$



$$v = \boxed{}\, v_s - \boxed{}\, i$$

KCL í a : $Y_R(v_a - v_s) + Y_L(v_a - 0) + i = 0$

$$Y_R = \frac{1}{R} \qquad Y_L = \frac{1}{Lp}$$

svo $\dfrac{v_a - v_s}{R} + \dfrac{v_a}{Lp} + i = 0$

$v = v_a$ $\quad \dfrac{v - v_s}{R} + \dfrac{v}{Lp} + i = 0$

Zeq eða virk ϕ

þáttum og leysum fyrir v

$$v = \boxed{\frac{Lp}{R + Lp}}\, v_s - \boxed{\frac{R\,Lp}{R + Lp}}\, i$$

virki í v_s eða innri lind

V_{th}

Thévenin- og Norton rásir með $p-$virkjanum

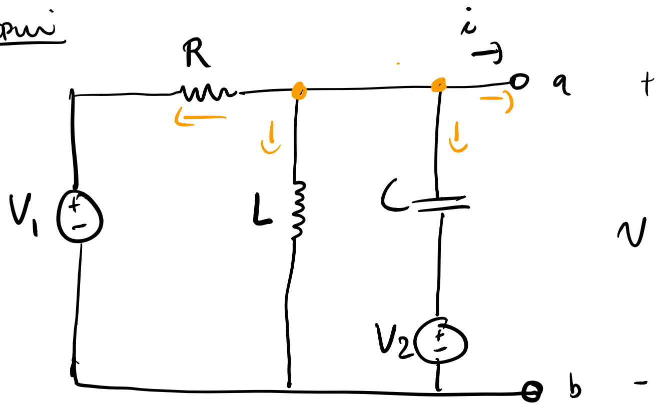
Samanburður gefur að

$$\boxed{V_{\mathrm{Th}} = \sum_{j=1}^{n}((\text{virki}j)(\text{innri lind}j))}$$

og

$$\boxed{Z_{\mathrm{Th}} = (\text{virki}\phi)}$$

Dæmi



Finnum v á forminu

$$v = \sum_{i=1}^{2} (\text{virki } i)(\text{inn lind } i) + (\text{virki})\, i(t)$$

Finna svo V_{th} & Z_{th}.

KCL í a : $i + Y_R(v_a - V_1) + Y_L(v_a - 0) + Y_C(v_a - V_2) = 0$

$v_a = v$

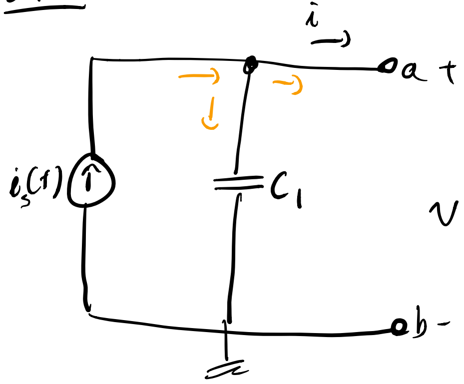
eða $$i + \frac{v - V_1}{R} + \frac{v}{Lp} + Cp(v - V_2) = 0$$

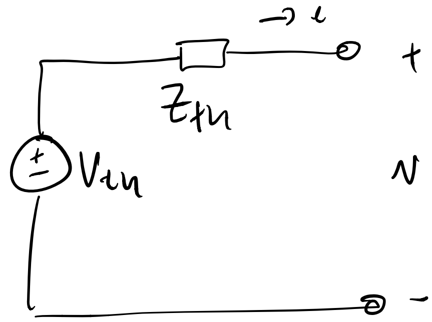
$$v\left(\underbrace{\frac{1}{R} + \frac{1}{Lp} + Cp}_{\frac{Lp + R + RCLp^2}{RLp}}\right) - v_1\left(\frac{1}{R}\right) - v_2\left(Cp\right) = -i$$

eða

$$v = \underbrace{\left(\underbrace{\frac{Lp}{RLCp^2 + Lp + R}}_{\text{virki 1}} v_1 + \underbrace{\frac{RLCp^2}{RLCp^2 + Lp + R}}_{\text{virki 2}} v_2\right)}_{V_{th} = V_{oc}(p)} - \underbrace{\underbrace{\frac{RLp}{\dots}}_{\text{virki } \emptyset}}_{Z_{th}(p)} i$$

Dæmi Finna Thévenin jafngildisrás ef $i_s(t) = 5u(t)$ & $C_1 = 2F$





$$v = V_{th} - Z_{th}\, i$$

KCL í a gefur $i = i_s(t) - i_c = 5u(t) - C_1 p v$

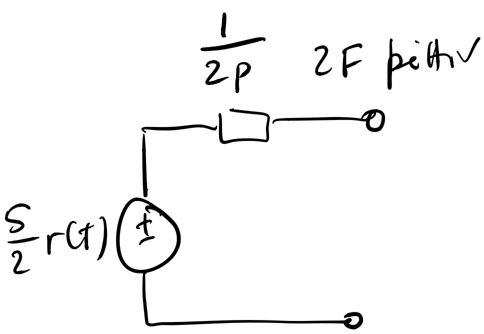
Leysum fyrir v

$$v = \underbrace{\frac{5}{Cp}u(t)}_{V_{th}} - \underbrace{\frac{1}{Cp}}_{Z_{th}}\, i$$

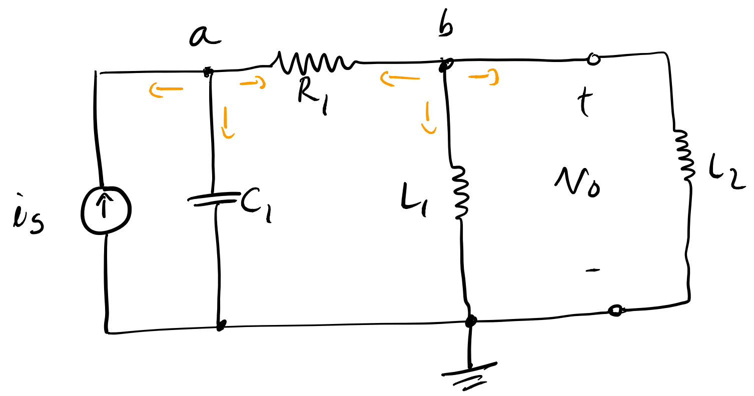
$$V_{th} = \frac{5}{2p}u(t) = \frac{5}{2}\int_{-\infty}^{t} u(\tau)\,d\tau = \left[\frac{5}{2}\int_{0^-}^{t} d\tau\right] u(t)$$

$$= \frac{5}{2}t\,u(t) = 5\,r(t)$$

$$Z_{th}(p) = \frac{1}{Cp} = \frac{1}{2p}$$



Dæmi Finnum diffurjöfnu fyrir $v_o(t)$ ef $Z_{R_1} = Z_{L_1} = Z_{C_1} = 1$ en $L_2 = 2H$



$$\boxed{Y_{R_i} = \frac{1}{R_i} \quad Y_{C_i} = C_i p \quad Y_{L_i} = \frac{1}{L_i p}}$$

$v = i \cdot R$ afnar $v \frac{1}{R} = i$

$Y: v \to i$

$Y_R = \frac{1}{R}$

Höfum $N_{jöfnu} = N_{hp} + N_{vs} - 1 = 3 + 0 - 1 = 2$ Höf $v_b = v_o$

$$\begin{cases} -i_s + Y_{C_1}(v_a - 0) + Y_{R_1}(v_a - v_b) = 0 \\ Y_{R_1}(v_b - v_a) + Y_{L_1}(v_b - 0) + Y_{L_2}(v_b - 0) = 0 \end{cases}$$

$$\begin{bmatrix} Y_{C_1} + Y_{R_1} & -Y_{R_1} \\ -Y_{R_1} & Y_{R_1} + Y_{L_1} + Y_{L_2} \end{bmatrix} \begin{bmatrix} v_a \\ v_b \end{bmatrix} = \begin{bmatrix} i_s \\ 0 \end{bmatrix} \text{ svo } \begin{bmatrix} v_a \\ v_b \end{bmatrix} = \frac{i_s}{2p^2 + 3p + 3} \begin{bmatrix} 2p + 3 \\ 2p \end{bmatrix}$$

svo
$$\begin{cases} 2\frac{d^2 v_a}{dt^2} + 3\frac{d v_a}{dt} + 3v_a = 2\frac{d i_s}{dt} + 3 i_s \\ 2\frac{d^2 v_o}{dt^2} + 3\frac{d v_o}{dt} + 3v_o = 2\frac{d i_s}{dt} \end{cases}$$