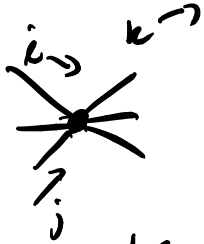
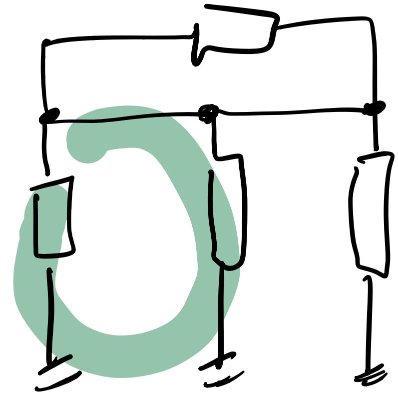


$\sum i_{in} = \sum i_{út}$

KCL



$\sum_{i=1}^{N} v_i = 0$

KVL

Greining Rása

Möskva- og hnútpunktajöfnur

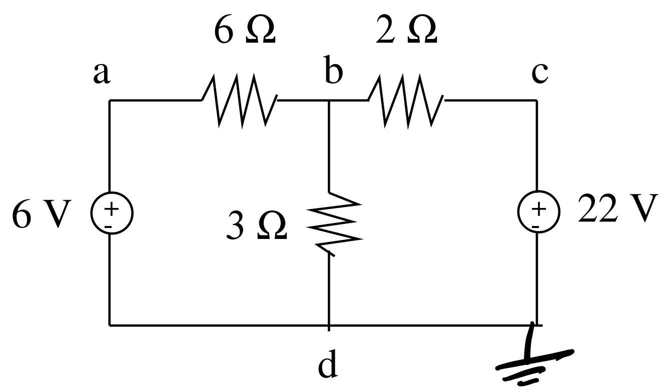
Ólafur Bjarki Bogason

18. Janúar 2021

Hnútpunktajöfnur

Hnútpunktur ekki hnútpunktur

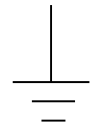
Markmiðið er að nota KCL og lögmál Ohms til að finna allar spennur og strauma í rásinni á eins skilvirkan hátt og mögulegt er



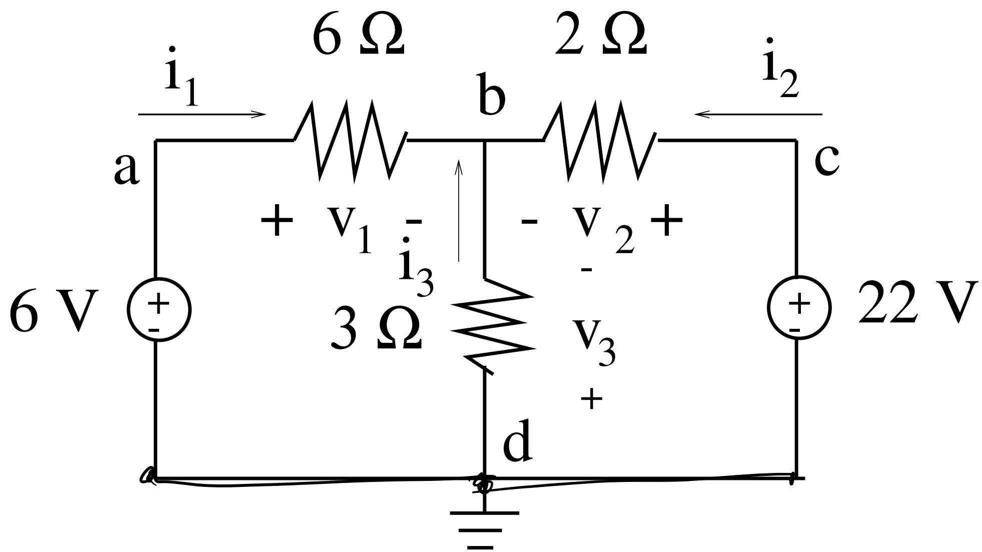
- Veljum einn hnútpunkt sem viðmiðunarpunkt. Spenna í sérhverjum öðrum hnútpunkti er þá skilgreind miðað við þennan viðmiðunarpunkt.

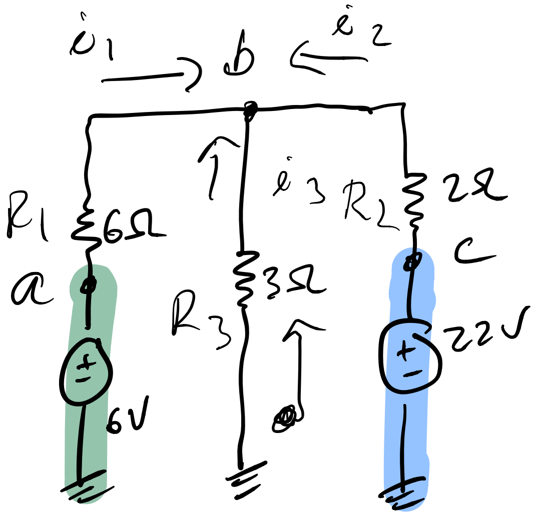
Hnútpunktajöfnur

- Spennan í viðmiðunarpunktinum er þá samkvæmt skilgreiningu núll (sbr. **jarðtenging**)



- Veljum t.d. hnútpunkt d sem jörð og skilgreinum strauma og spennur.





$V_a = 6V$

$V_c = 22V$

$V_b = ?$

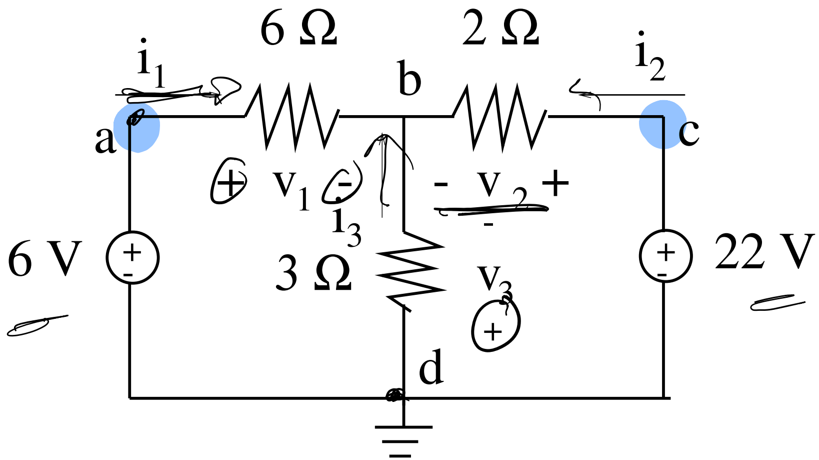
(KCL @ b)

KCL @ b

$$i_1 + i_2 + i_3 = 0$$

$$\frac{V_a - V_b}{R_1} + \frac{V_c - V_b}{R_2} + \frac{0 - V_b}{R_3} = 0$$

Hnútpunktajöfnur



- Sjáum strax að $v_\mathrm{a} = 6\ \mathrm{V}$ og $v_\mathrm{c} = 22\ \mathrm{V}$.
- Skrifum síðan KCL - jöfnu fyrir hnútpunktinn með óþekktu spennunni v_b

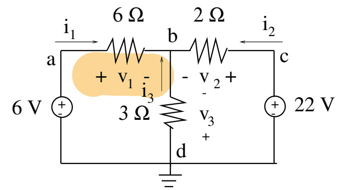
$$\boxed{i_1 + i_2 + i_3 = 0}$$

(3) + (ohm)

eða

$$\frac{v_1}{6\Omega} + \frac{v_2}{2\Omega} + \frac{v_3}{3\Omega} = 0$$

Hnútpunktajöfnur



$v_1 = v_a - v_b$

$v_2 = v_c - v_b$

$v_3 = 0 - v_b$

- Með KVL fæst síðan

$$v_1 = v_\mathrm{a} - v_\mathrm{b} = 6\mathrm{V} - v_\mathrm{b}$$

$$v_2 = v_\mathrm{c} - v_\mathrm{b} = 22\mathrm{V} - v_\mathrm{b}$$

$$v_3 = -v_\mathrm{b}$$

sem gefur

$$\frac{6 - v_\mathrm{b}}{6} + \frac{22 - v_\mathrm{b}}{2} + \frac{(-v_\mathrm{b})}{3} = 0$$

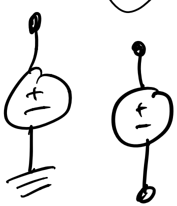
sem er leyst fyrir v_b og $v_\mathrm{b} = 12$ V og síðan er

$$i_1 = \frac{6 - v_\mathrm{b}}{6} = -1\mathrm{A}, \quad i_2 = \frac{22 - v_\mathrm{b}}{2} = 5\mathrm{A} \quad i_3 = \frac{-v_\mathrm{b}}{3} = -4\mathrm{A}$$

Hnútpunktajöfnur

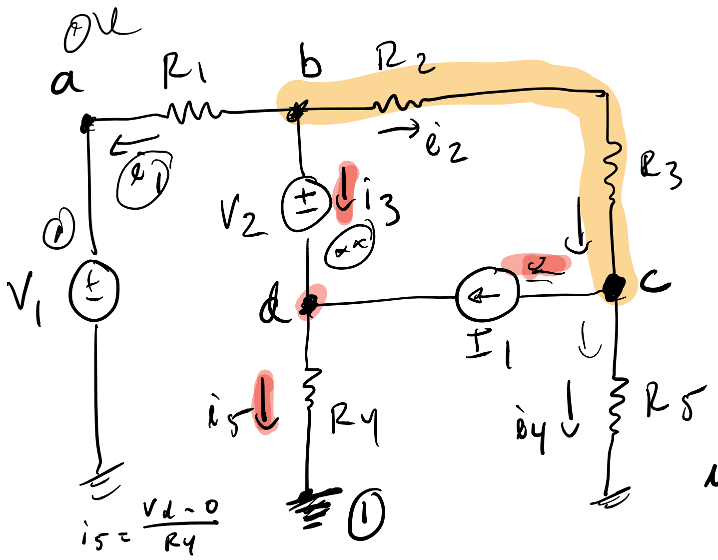
Kerfisbundin aðferð til að setja upp KCL - jöfnur fyrir alla þá hnútpunkta sem hafa óþekkta spennu:

1. Velja viðmiðunarpunkt og jarðtengja hann. Að öllu jöfnu er sá hnútpunktur valinn sem flestar spennulindir tengjast.
2. Skrifa eina jöfnu fyrir hverja spennulind: $V_a = V_{in}$



   - Ef lindin er jarðtengd er hnútpunktspennan lindarspennan
   - Einföld jafna sem gefur samband milli hnútpunktspennu beggja póla lindar
3. Skrifa KCL- jöfnu fyrir alla hnútpunkta sem eftir eru; nota lögmál Ohms til að breyta þeim yfir í hnútpunktaspennur
4. Leysa jöfnuhneppið sem fæst úr liðum 2. og 3.

$$\sum i \xrightarrow{(3)} \sum \frac{V_a}{R_a} \quad 2,3 \qquad \begin{bmatrix} \; \end{bmatrix} \begin{bmatrix} V_a \\ \vdots \end{bmatrix} = \begin{bmatrix} \; \end{bmatrix}$$



Finna spennur í öllum punktum & strauma i_{1-5} ef

$V_1 = 12V$, $V_2 = 22V$, $I_1 = 1A$, $R_5 = 8\Omega$

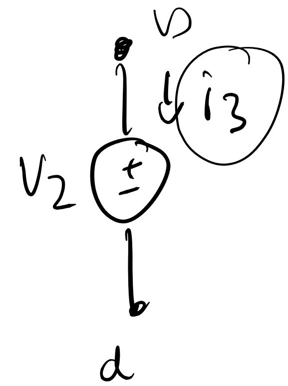
$R_1 = 2\Omega$, $R_2 = 4\Omega$, $R_3 = 1\Omega$, $R_4 = 1\Omega$

$i_2 = \frac{V_b - V_c}{R_2 + R_3}$

$i_4 = \frac{V_c - 0}{R_5}$

$i_5 = \frac{V_d - 0}{R_4}$

i_3 er straumur í V_2



(2) $V_a - 0 = V_1$ svo $V_a = V_1$ ok

$V_b - V_d = V_2$ (*)

(3) (KCL í b)

$i_1 + i_2 + i_3 = 0$

(Ohm)

eða $\frac{V_b - V_a}{R_1} + \frac{V_b - V_c}{R_2 + R_3} + i_3 = 0$

algebra (algebra)

$0.7 V_b - 0.2 V_c + i_3 = 6$ (**)

(KCL í c)

$i_2 = I_1 + i_4$

þá $\frac{V_b - V_c}{R_2 + R_3} = I_1 + \frac{V_c - 0}{R_5}$

e eða $0.2 V_b - 0.325 V_c = 1$ (***)

$i_3 + I_1 = i_5$ ohm

$i_3 + I_1 = \frac{V_d - 0}{R_4}$

eða $-V_d + i_3 = -1$ (****)

$V_b - V_d = V_2 \quad (*)$

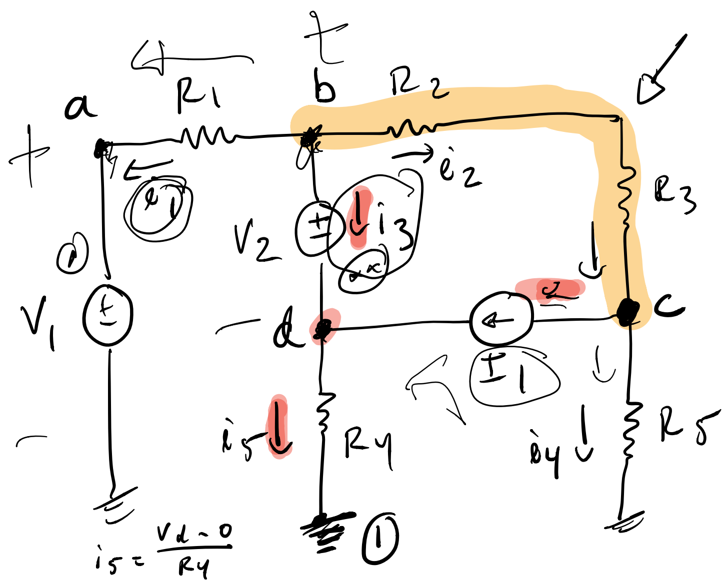
$0.2V_b - 0.325V_c = 1 \quad (***)$

$0.7V_b - 0.2V_c + i_3 = 6 \quad (**)$

$-V_d + i_3 = -1 \quad (****)$

$$\begin{array}{c} \\ b \\ c \\ d \\ i_3 \end{array}
\begin{array}{c} \begin{array}{cccc} b & c & d & i_3 \end{array} \\ \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0.7 & -0.2 & 0 & 1 \\ 0.2 & -0.325 & 0 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix} \end{array}
\begin{bmatrix} V_b \\ V_c \\ V_d \\ i_3 \end{bmatrix} = \begin{bmatrix} 22 \\ 6 \\ 1 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} V_b \\ V_c \\ V_d \\ i_3 \end{bmatrix} = \begin{bmatrix} 18V \\ 8V \\ -4V \\ -5A \end{bmatrix}$$



$i_1 = \dfrac{V_b - V_a}{R_1} = \dfrac{18-12}{2} = \dfrac{6}{2} = 3A$ ok

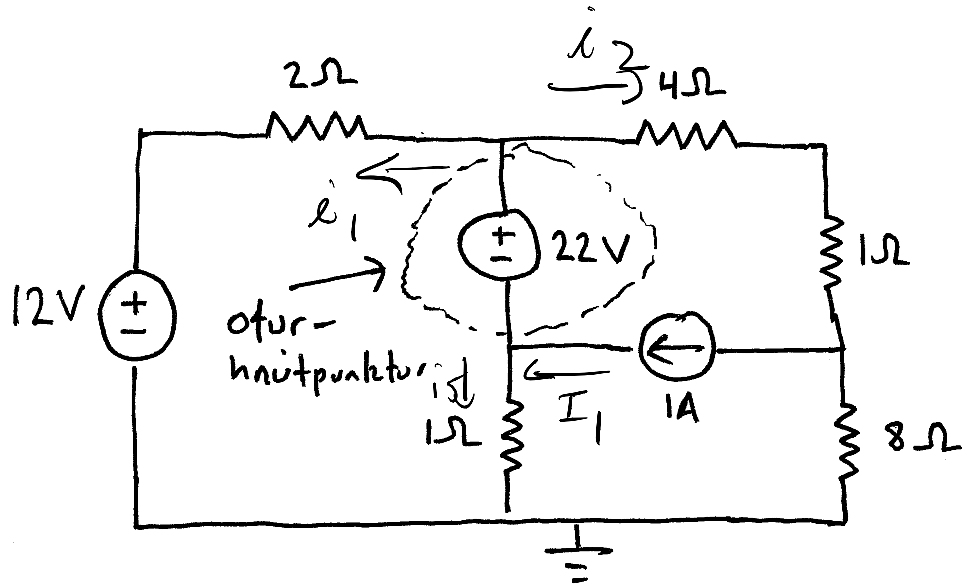
$i_2 = \dfrac{V_b - V_c}{R_2 + R_3} = \dfrac{18-8}{4+1} = \dfrac{10}{5} = 2A$ ok

$i_3 = -5A$

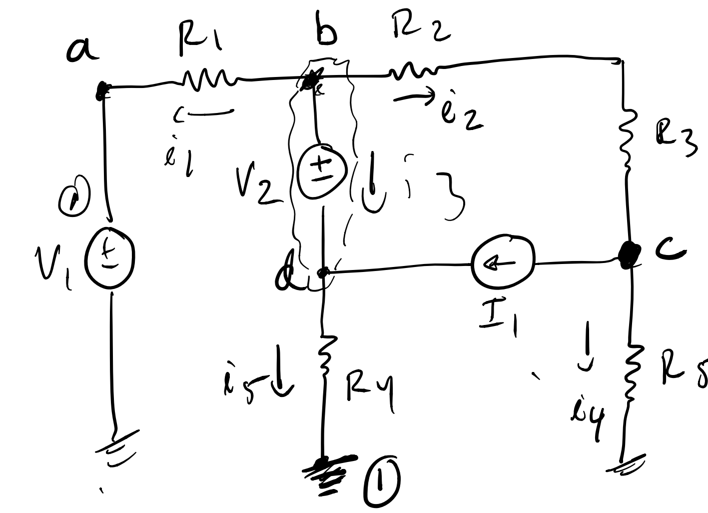
$i_4 = \dfrac{V_c - 0}{R_5} = \dfrac{8}{8} = 1A$

$i_5 = \dfrac{V_d - 0}{R_4} = \dfrac{-4V}{1} = -4A$

Hnútpunktajöfnur: Ofurhnútpunktar



- Við köllum spennulind (kjör/stýrð) sem tengist tveimur hnútpunktum sem hvorugur er jörð ofurhnútpunkt



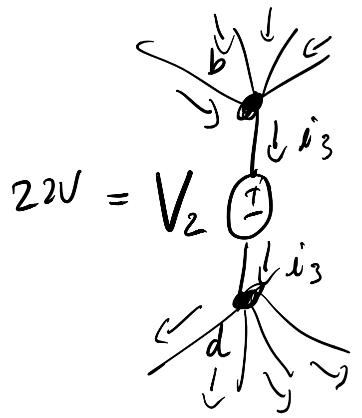
(KCL í ofn)

$$i_1 + i_2 + i_5 - I_1 = 0$$

(Ohm)

$$\frac{V_b - V_a}{R_1} + \frac{V_b - V_c}{R_1 + R_2} + \frac{V_d - 0}{R_4} - I_1 = 0$$

eða $0.7 V_b - 0.2 V_c + V_d = 7 \quad (*)$



$i_3 = \Sigma i_{in} = \Sigma i_{+pól}$

$\|$

$i_3 = \Sigma i_{út} = \Sigma i_{-pól}$

(KCL í c)

$$i_2 = I_1 + i_4$$

(Ohm)

eða $\dfrac{V_b - V_c}{R_2 + R_3} = I_1 + \dfrac{V_c - 0}{R_5}$

eða með algebru $0.2 V_b - 0.325 V_c = 1 \quad (**)$

Höfum einnig að $V_2 = V_b - V_d$

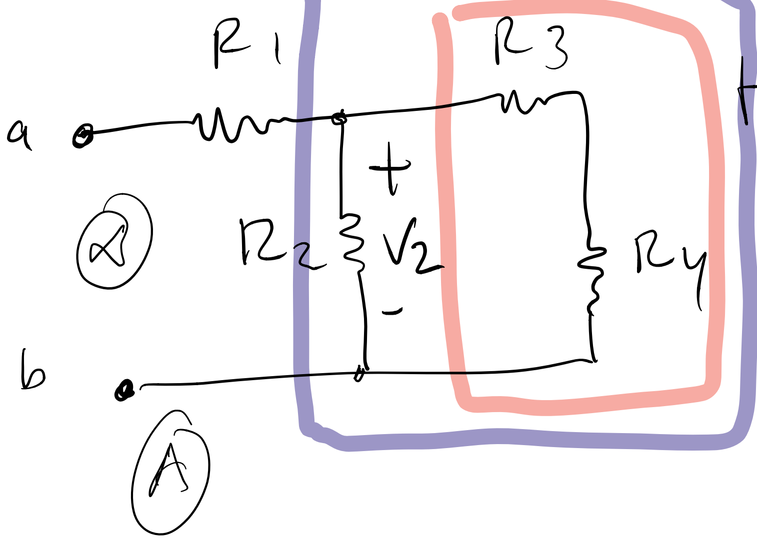
$$V_b - V_d = 22 \quad (***)$$

$$\begin{array}{c} \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{ccc} b & c & d \end{array} \\ \begin{bmatrix} 0.7 & -0.2 & 1 \\ 0.2 & -0.325 & 0 \\ 1 & 0 & -1 \end{bmatrix} \end{array} \begin{bmatrix} V_b \\ V_c \\ V_d \end{bmatrix} = \begin{bmatrix} 7 \\ 1 \\ 22 \end{bmatrix}$$

$$\begin{bmatrix} V_b \\ V_c \\ V_d \end{bmatrix} = \begin{bmatrix} 18V \\ 8V \\ -4V \end{bmatrix}$$

$$i_3 = -i_1 - i_2$$

$$= -\left(\frac{V_b - V_a}{R_1} + \frac{V_b - V_c}{R_1 + R_2} \right.$$



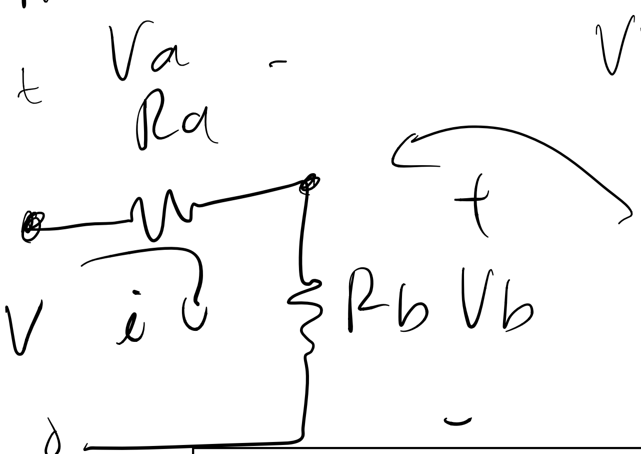
Hvert er jafngildisviðnám mill a & b, $R_{ab} = ?$

$$R_{ab} = R_1 + R_2 \parallel (R_3 + R_4)$$

$$\boxed{V = iR}$$

Hvað ef $R_1 = R_2 = R_3 = R_4 = 1\Omega$ & $V_{ab} = 10V$

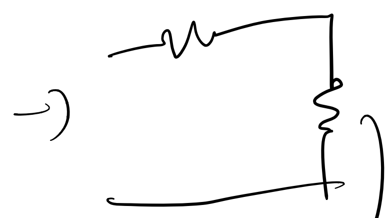
Hvað er V_2



$$V_b = V - V_a = V \frac{R_b}{R_b + R_a}$$

$$R_{eq} = \frac{R_2 (R_3 + R_4)}{R_2 + (R_3 + R_4)} = \frac{2}{3} R_1$$

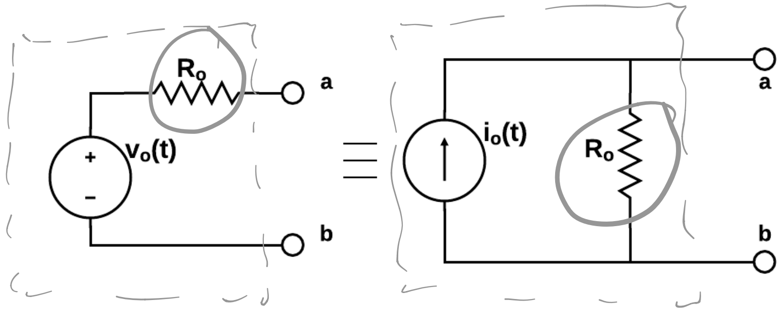
$$\boxed{R_a \parallel R_b = \frac{R_a R_b}{R_a + R_b}}$$



$$R_2 \parallel (R_3 + R_4)$$

$$V_2 = V_{ab} \cdot \frac{R_{eq}}{R_1 + R_{eq}} = 10 \frac{2/3}{1 + 2/3} = 4V$$

Umbreyting linda

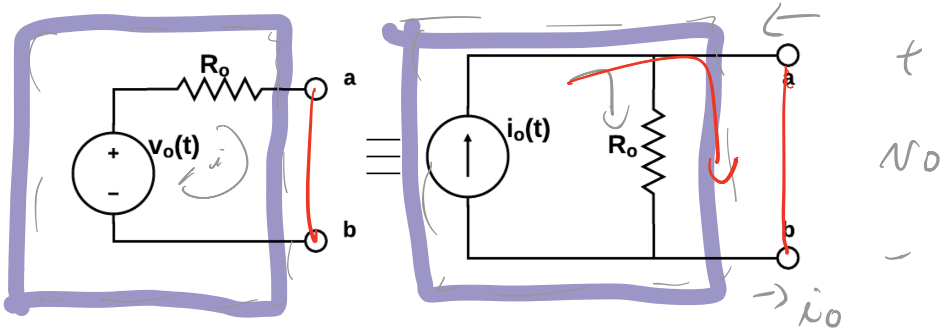


- Spennulind v_o með raðtengdu viðnámi R_o má alltaf umbreyta í straumlind i_o með hliðtengdu viðnámi R_o

$$i_o = \frac{v_o}{R_o}$$

Umbreyting linda

$p = v \cdot \cancel{i}^{\,0} = 0\ W$ $\qquad p = v_o i_o = i_o^2 R_o$ $\qquad v_o = R_o \cdot i_o$

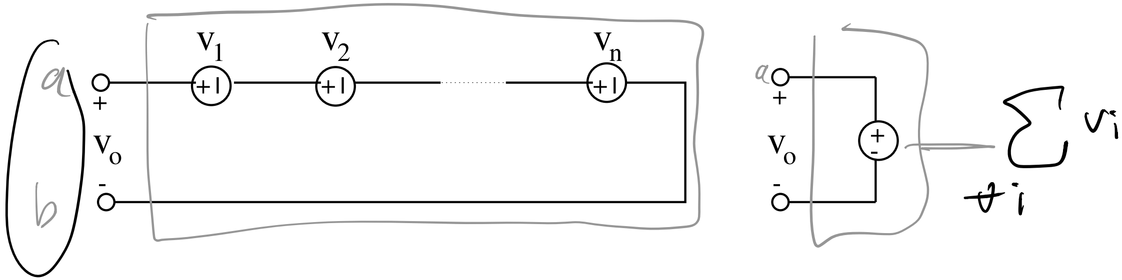


- Athuga ber að þetta eru aðeins jafngildar rásir miðað við pólana a og b.
- Séu rásirnar ótengdar eyðist ekkert afl í rásinni til vinstri á myndinni en í rásinni til hægri eyðist aflið
- Sé skammhleypt milli a og b eyðist ekkert afl í rásinni til hægri en í rásinni til vinstri eyðist aflið

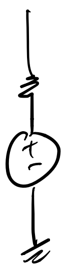
$$p_o = i_o^2 R_o$$

Raðtenging og hliðtengin linda

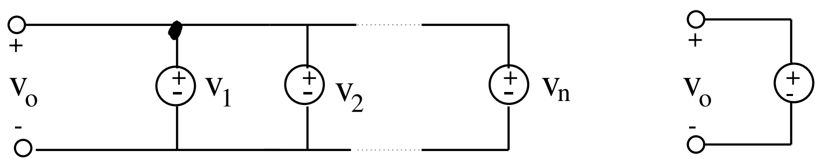
Raðtenging spennulinda



Með KVL fæst



$$\boxed{v_o = \sum_{i=1}^{n} v_i}$$

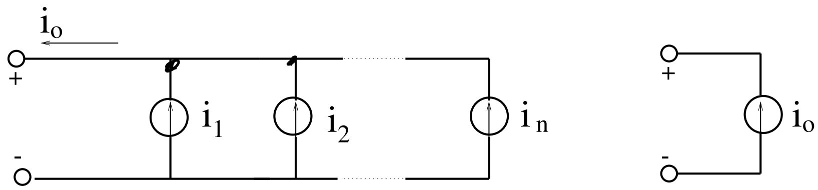


Hliðtenging spennulinda gengur ekki nema allar séu eins

Raðtenging og hliðtenging linda

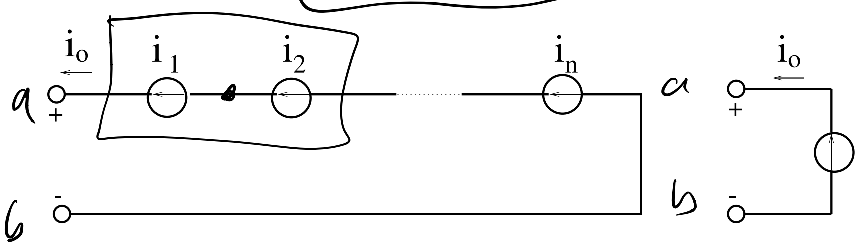
Hliðtenging straumlinda

$i_0 = i_1 + i_2 + \dots + i_n$



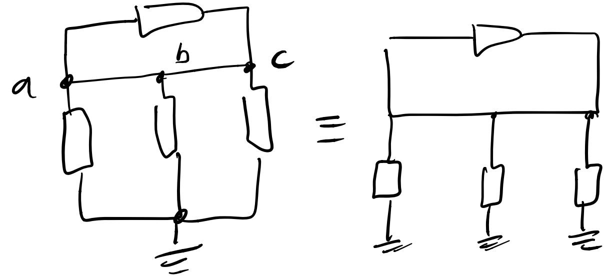
Með KCL fæst

$$\boxed{i_o = \sum_{i=1}^{n} i_i}$$



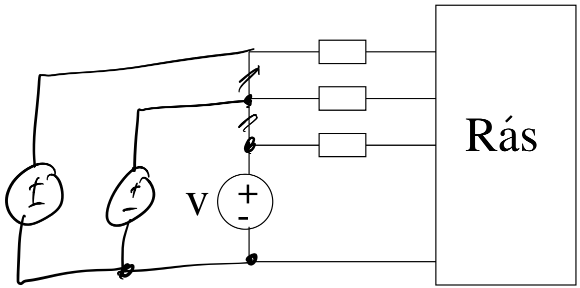
Raðtenging straumlinda gengur ekki nema allar séu eins

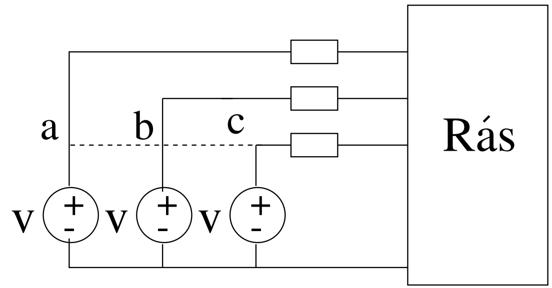
Raðtenging og hliðtenging linda



Spennulind

Oft getur verið sniðugt að skipta einni lind út fyrir margar lindir

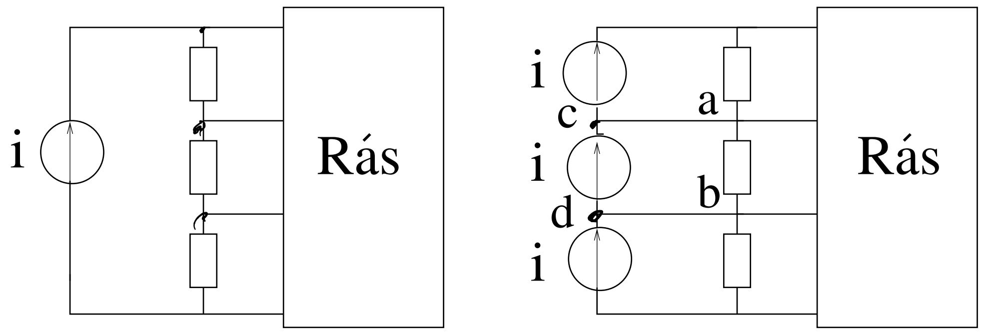




Spennumunur milli punkta a og b er núll (enginn straumur) svo að fjarlægja má tenginguna. Hið sama gildir fyrir b - c og c - a.

Raðtenging og hliðtenging linda

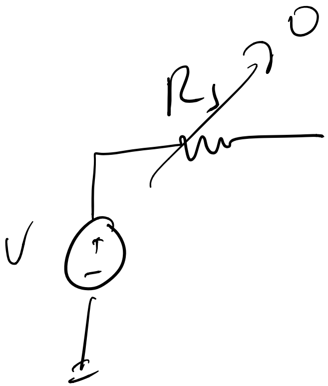
Straumlind

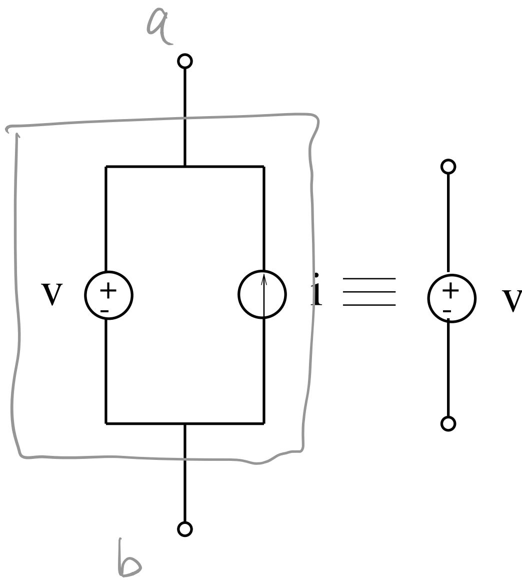


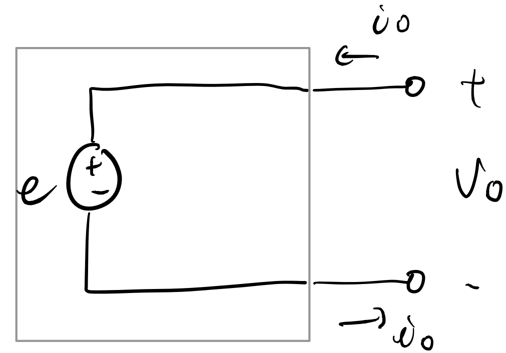
Óskilgreind spenna í punktum c og d, tengja má hvaða aðra hnútpunkta sem er við þá.

Enginn straumur er frá a til c og b til d, það hefur því ekki áhrif á verkun rásarinnar hvort hnútpunktarnir séu tengdir eður ei

Raðtenging og hliðtenging linda





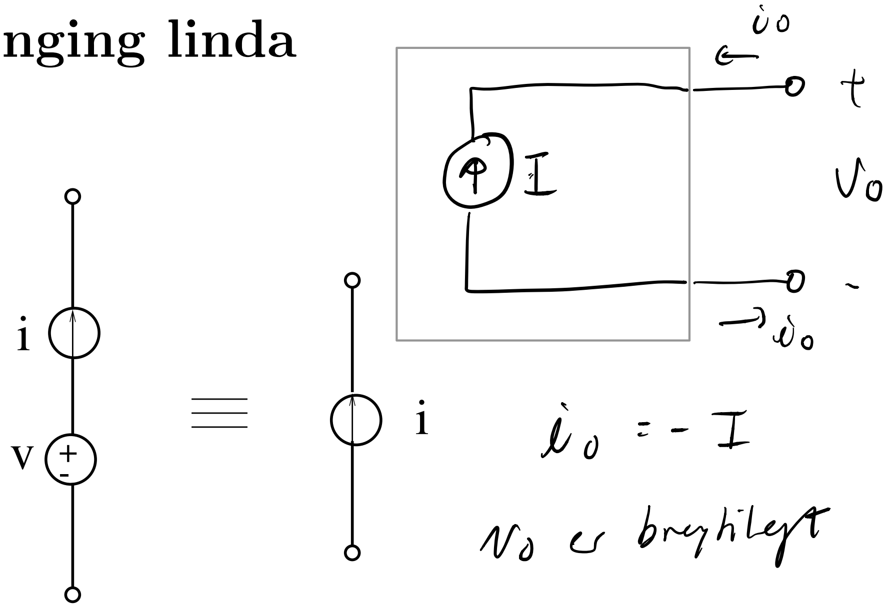


$v_0 = e$

i_0 er breytileg

- Spennulind með innra viðnám núll. Föst spenna en straumurinn í gegnum straumlind er óskilgreindur $\Longrightarrow$ jafngildir spennulind

Raðtenging og hliðtenging linda



$i_0 = -I$

V_0 er breytilegt

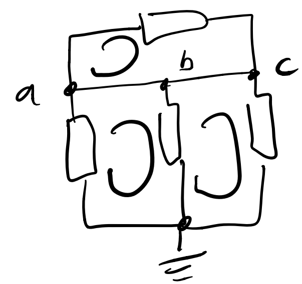
- Straumlind heldur ákveðnum straum óháð spennunni sem er óskilgreind
  $\Longrightarrow$ jafngildir straumlind

Lykkju- og möskvajöfnur

- Á sama hátt og hnútpunktajöfnur byggja á KCL þá eru til jöfnukerfi sem byggja á KVL, svokallaðar **lykkjujöfnur** eða **möskvajöfnur**
- Minnstu **lykkjur** í rás kallast **möskvar**.
- Möskvi er lykkja, en lykkja þarf ekki að vera möskvi. KVL umhverfis möskva gefur **möskvajöfnu**

Rás = Rásaeiningar + Tengingar

eðlisfræðileg módel
af raunveruleika



≡

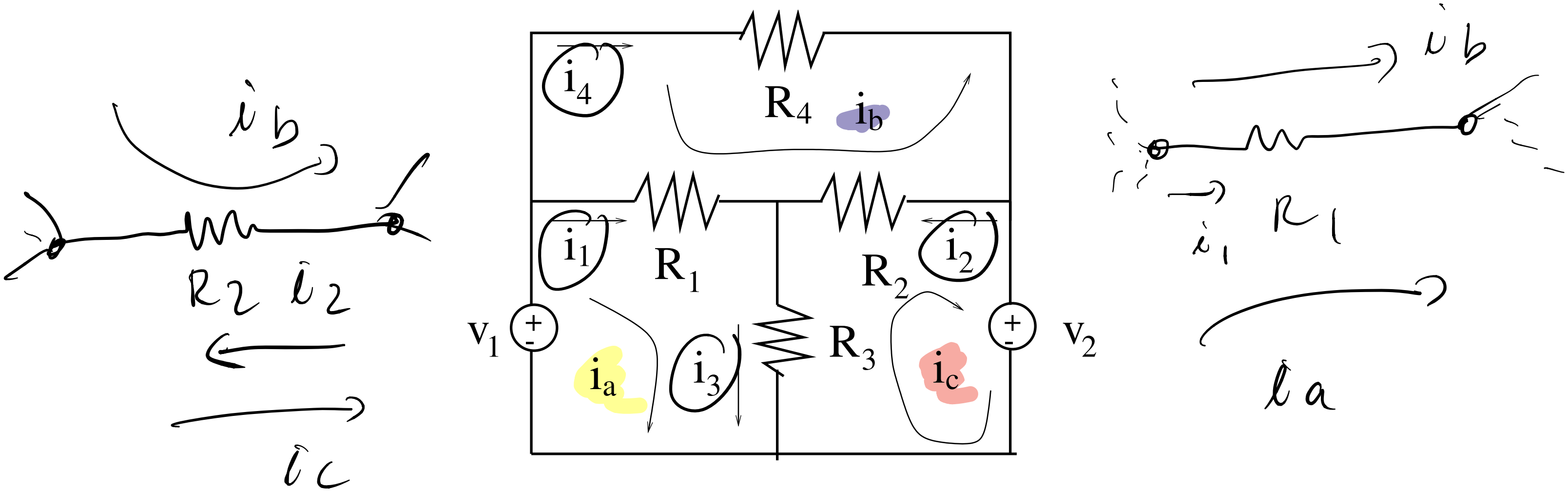
Va, Vb, Vc

Lykkju- og möskvajöfnur

KCL $I_1 = I_2 + 60$

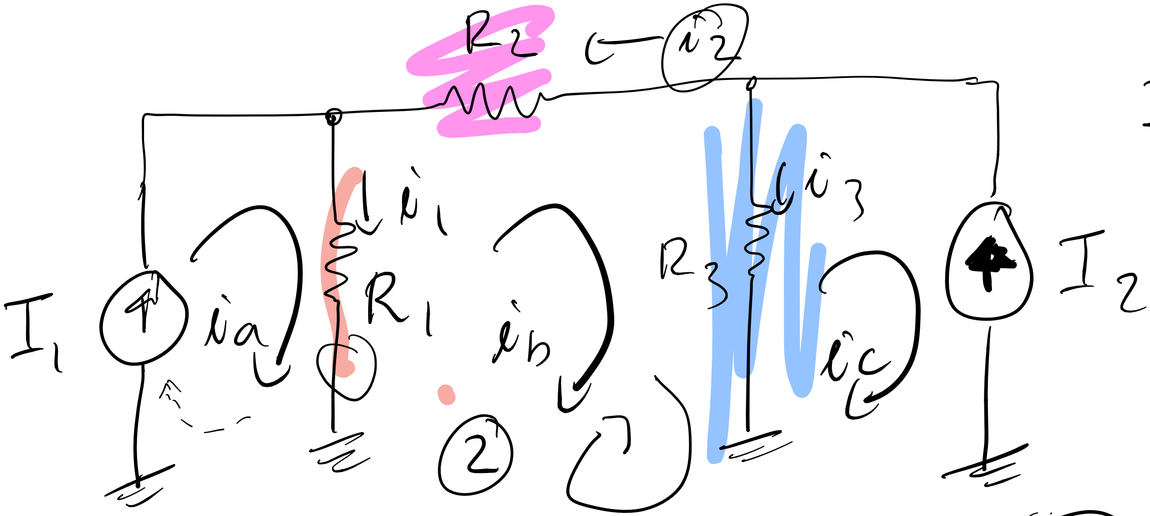
Ohm $\frac{V_A - V_B}{R_2} = - - -$

Hugsum okkur einn straum sem flýtur í hverjum möskva, svo kallað **möskvastrauma**



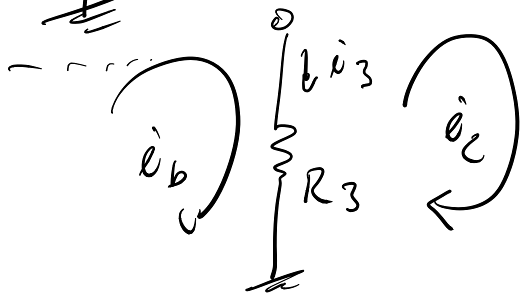
Hér er $\boxed{i_1 = i_a + i_b}$, $i_2 = -i_b - i_c$, $i_3 = i_a - i_c$ og $i_4 = -i_b$

þ.e. möskvastraumar eru straumþættir sem mynda straumana í hverri rásaeiningu fyrir sig



$I_1 = 2A$, $I_2 = 4A$

$R_1 = R_2 = R_3 = 1\Omega$



Finna i_a, i_b & i_c.

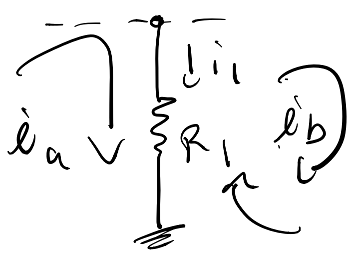
$\underline{\underline{i_a = I_1 = 2A}}$ $\quad i_3 = i_b - i_c$

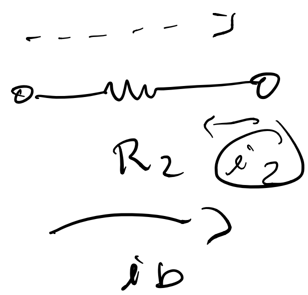
$\underline{\underline{i_c = -I_2 = -4A}}$

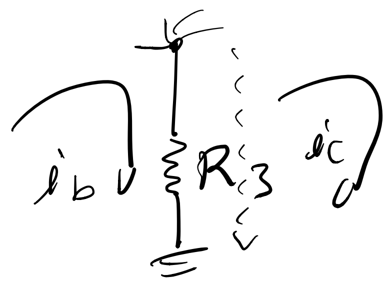
$V = iR$

(2) Skrifa KVL í lykkju 2.

$$R_1(i_b - i_a) + R_2 i_b + R_3(i_b - i_c) = 0$$







$$1(i_b - 2) + 1(i_b) + 1(i_b - (-4))$$

$\underline{\underline{i_b = -2/3 A}}$ $\quad i_2 = -i_b = -\left(-\frac{2}{3}\right) = \underline{\underline{\frac{2}{3}A}}$

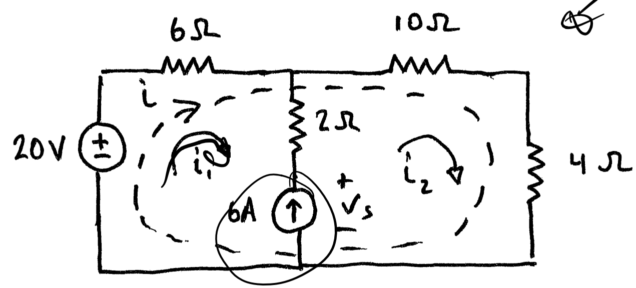
$i_1 = i_a - i_b = 2 - \left(-\frac{2}{3}\right) = \underline{\underline{\frac{8}{3}A}}$ $\quad i_3 = i_b - i_c = -\frac{2}{3} - (-4) = \underline{\underline{\frac{10}{3}A}}$

Lykkju- og möskvajöfnur

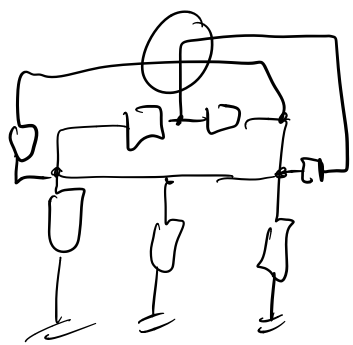
- Þegar greina á rás með möskvajöfnum er byrjað á að skilgreina möskvastraum í hverjum möskva fyrir sig, réttsælis eða rangsælis.
- Síðan er skrifuð KVL jafna fyrir hvern möskva með spennu yfir viðnám sem fall af möskvastraumnum í gegnum viðnámið.

Lykkju- og möskvajöfnur: Ofurmöskvar

$6A = i_2 - i_1$

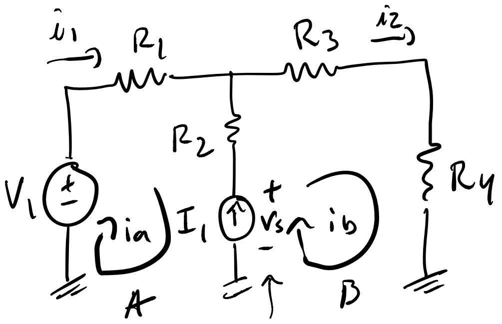


- **Sértilfelli**: Straumlind á milli tveggja möskva
  - Við getum sparað okkur eina KVL jöfnu með því að skilgreina **ofurmöskva**



non-planar rás

möskvagreining gengur ekki!



Finnum i_1 & i_2 ef $V_1 = 20V$, $I_1 = 6A$

$R_1 = 6\Omega$, $R_2 = 2\Omega$, $R_3 = 10\Omega$, $R_4 = 4\Omega$

i) Möskvagreining ii) ofurmöskvi iii) MNA

Lausn með möskvagr.

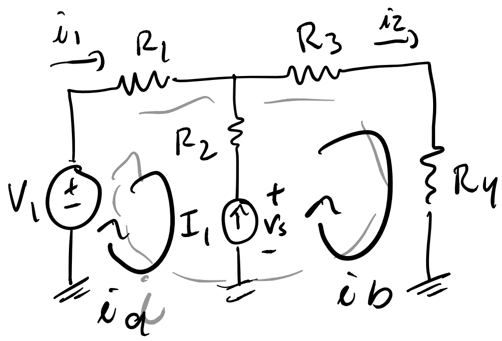
Möskvi A $-V_1 + R_1 i_a + 2(i_a - i_b) + V_s = 0$

Möskvi B $-V_s + R_2(i_b - i_a) + (R_3 + R_4) i_b = 0$

(*) $I_1 = i_b - i_a$

$$\begin{bmatrix} 8 & -2 & 1 \\ -2 & 16 & -1 \\ -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ V_s \end{bmatrix} = \begin{bmatrix} 20 \\ 0 \\ 6 \end{bmatrix}$$

notum algebru & fin

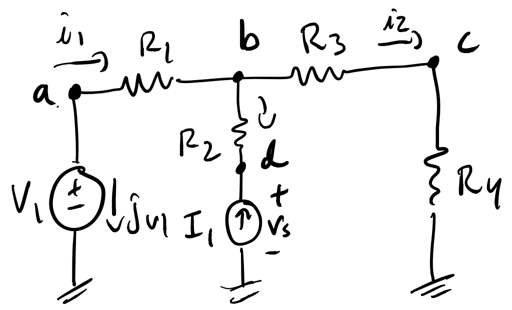


Lausn með ofurmöskva (notum i_1 & i_2)

$$-V_1 + R_1 i_a + (R_3 + R_4) i_b = 0$$

(*) $I_1 = i_b - i_a$

$$\begin{bmatrix} R_1 & R_3 + R_4 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} i_a \\ i_b \end{bmatrix} = \begin{bmatrix} V_1 \\ I_1 \end{bmatrix}$$



Leysum með MNA

Y : spenna í hnútpunkta
A : straum j í spennulindum
B : straum í straumlindum i_s
D : spenna e í spennulindum

$$\begin{bmatrix} Y & A \\ B & D \end{bmatrix} \begin{bmatrix} v_n \\ j \end{bmatrix} = \begin{bmatrix} i_s \\ e \end{bmatrix}$$

$$G_i = \frac{1}{R_i}$$

Höfum 4 hnútpunkta & 1 óháða spennulind svo við þurfum 5 jöfnur!

KCL

$$\begin{array}{c} \\ a \\ b \\ c \\ d \\ V_1 \end{array}
\begin{array}{c} \begin{array}{ccccc} a & b & c & d & V_1 \end{array} \\
\left[\begin{array}{cccc|c} G_1 & -G_1 & 0 & 0 & 1 \\ -G_1 & G_1+G_2+G_3 & -G_3 & -G_2 & 0 \\ 0 & -G_3 & G_3+G_4 & 0 & 0 \\ 0 & -G_2 & 0 & G_2 & 0 \\ \hline 1 & & & & \end{array}\right] \end{array}
\begin{bmatrix} v_a \\ v_b \\ v_c \\ v_d \\ j_{V1} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ I_1 \\ V_1 \end{bmatrix}
\begin{array}{c} (1) \\ (2) \\ (3) \\ (4) \\ (5) \end{array}$$

$$(1) \quad \overbrace{G_1(V_a - V_b)}^{i_1} + j_{V1} = 0$$

$$G_1(V_b - V_a) + G_2(V_b - V_d) + G_3(V_b - V_c) = 0$$

$$-G_1 V_a + V_b(G_1 + G_2 + G_3) + V_c(-G_3) + V_d(-G_2) = 0 \quad (2)$$

$$A\bar{x} = \bar{b} \quad \text{svo} \quad \bar{x} = A^{-1}\bar{b}$$

$$\begin{bmatrix} v_a \\ v_b \\ v_c \\ v_d \\ j_{V1} \end{bmatrix} = \begin{bmatrix} 20\,V \\ 39.2\,V \\ 51.2\,V \\ 51.2\,V \\ 3.2\,A \end{bmatrix}$$

og $i_1 = -j_{V1} = -3.2A$

$$i_2 = \frac{V_b - V_c}{R_3} = \frac{V_c - 0}{R_4}$$

$$= \underline{2.8A}$$