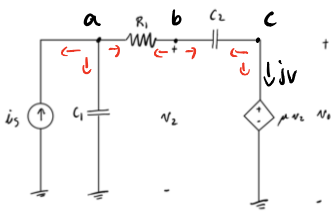
Lausnir leiðrétt 25/04/21

Dæmi 1 – Þrep- og impúlssvörun

$i_C = C \frac{dv_c}{dt}$

Finnið þrep- og impúlssvörun þegar i_s og v_0 eru inn- og útmerki. Athugið að v_2 er spennan í hnútapunktinum þar sem R_1 og C_2 tengjast saman.



Breyta	Gildi
R_1	$1\,\Omega$
C_1	$1\,\mathrm{F}$
C_2	$2\,\mathrm{F}$
μ	-2

Finnum fyrst þrep-ssvörun $\underline{i_s(t) = u(t)}$

Notum MNA $\quad N_{jöfn} = N_{hnútpunktar} + N_{VS} - 1 = 4 + 1 - 1 = \underline{4 \text{ jöfn}}$

KCL $\quad a: \; -i_s + Y_{C1}(v_a - 0) + Y_{R1}(v_a - v_b) = 0$

$b: \; Y_{R1}(v_b - v_a) + Y_{C2}(v_b - v_c) = 0$

$c: \; Y_{C2}(v_c - v_b) + j_v = 0$

KVL VCVS $\quad v_c - 0 = \mu v_b$ eða $-\mu v_b + v_c = 0$

Setjum í fylki

$$\begin{array}{c|c} & \begin{array}{cccc} a & b & c & \text{VCVS} \end{array} \\ \begin{array}{c} a \\ b \\ c \\ \text{VCVS} \end{array} & \left[\begin{array}{ccc|c} Y_{C1}+Y_{R1} & -Y_{R1} & 0 & 0 \\ -Y_{R1} & Y_{R1}+Y_{C2} & -Y_{C2} & 0 \\ 0 & -Y_{C2} & Y_{C2} & 1 \\ \hline 0 & -\mu & 1 & 0 \end{array}\right] \end{array} \begin{bmatrix} v_a \\ v_b \\ v_c \\ \hline j_v \end{bmatrix} = \begin{bmatrix} i_s \\ 0 \\ 0 \\ \hline 0 \end{bmatrix}$$

svo

$$\begin{bmatrix} v_a \\ v_b \\ v_c \\ \hline j_v \end{bmatrix} = \frac{i_s}{p(6p+7)} \begin{bmatrix} 6p+1 \\ 1 \\ -2 \\ 6p \end{bmatrix}$$

Hér er $\quad v_0 = v_c = \dfrac{-2 i_s}{p(6p+7)}$ $\quad$ eða $\quad 6p^2 v_0 + 7p v_0 = -2 i_s$

eða $\quad \underline{\dfrac{d^2 v_0}{dt^2} + \dfrac{7}{6}\dfrac{dv_0}{dt} = -\dfrac{2}{6} i_s = -\dfrac{1}{3}, \; t > 0}$

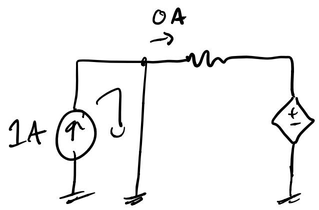
<u>Náttúrleg lausn</u> $\quad$ Gískum á lausn $\underline{v_{0n}(t) = A_0 e^{-\frac{7}{6}t} + A_1}$

<u>Sérlausn</u> $\quad$ Gískum á lausn $\left.\begin{array}{l} v_{0p}(t) = k_0 t + k_1 \\ v_{0p}'(t) = k_0 \end{array}\right\}$ $\frac{7}{6} k_0 = -\frac{2}{6}$ svo $\underline{k_0 = -\frac{2}{7}}$

<u>Heildarlausn</u> $\quad v_0(t) = v_{0n}(t) + v_{0p}(t) = A_0 e^{-\frac{7}{6}t} + A_1 - \frac{2}{7}t + k_1$, $\quad$ sameinum A_1 & k_1 í einn fasta $\tilde{A}_1$

$= A_0 e^{-7/6 t} - \frac{2}{7}t + \tilde{A}_1$

Skoðum rás við $\underline{t=0^+}$



Hér er $v_C(0^-) = v_C(0^+) = 0\,V$

$\frac{i_C(0^+)}{C} = v_C'(0^-) = v_C'(0^+) = 0\,\frac{V}{s}$ } á við um báða þætta!

$\underline{\underline{v_2 = v_0 = 0\,V}}$

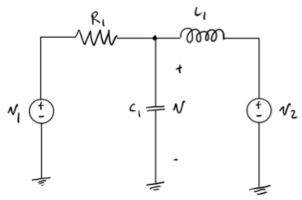
Vitum að $v_0(0^+) = 0 = A_0 + \tilde{A}_1$

$v'(0^+) = 0 = \left(-\frac{7}{6}\right)A_0 - \frac{2}{7}$ svo $\begin{bmatrix} 1 & 1 \\ -\frac{7}{6} & 0 \end{bmatrix}\begin{bmatrix} A_0 \\ \tilde{A}_1 \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{2}{7} \end{bmatrix}$ eða $\begin{bmatrix} A_0 \\ \tilde{A}_1 \end{bmatrix} = \begin{bmatrix} -\frac{12}{49} \\ +\frac{12}{49} \end{bmatrix}$

Þá er þrepsvörun $\underline{\underline{w_0(t) = -\frac{12}{49}e^{-\frac{7}{6}t} + \frac{12}{49} - \frac{2}{7}t}}$ & impulssvörun $h_0(t) = \frac{dw_0}{dt} = \frac{2}{7}e^{-\frac{7}{6}t} - \frac{2}{7} = \underline{\underline{\frac{2}{7}\left(e^{-\frac{7}{6}t} - 1\right)}}$

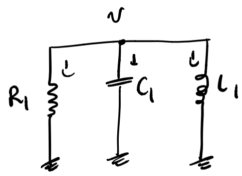
Dæmi 2

Gefið er að $v_1(t) = -50u(-t)$V og $v_2(t) = 125u(-t)$V. Hve stórt viðnám þarf að raðtengja við R_1 til að fá markdempun? Með þetta nýja viðnám í rásinni finnið $v(t)$ fyrir $t > 0$.



Breyta	Gildi
R_1	$25\,\Omega$
C_1	$16\,\mu$F
L_1	$1/16$ H

Skoðum rásina við $t>0$ KCL gefur $v Y_{R_1} + v Y_{C_1} + v Y_{L_1} = 0$



eða $\frac{v}{R_1} + v C_1 p + v \frac{1}{L_1 p} = 0$

eða $\frac{L_1 p}{R_1} v + v L_1 C_1 p^2 + v = 0$

eða $v p^2 + \frac{1}{R_1 C_1} p v + \frac{1}{L_1 C_1} v = 0$

svo kennijafnan v $s^2 + \frac{1}{R_1 C_1} s + \frac{1}{L_1 C_1} = 0$

með lausn $s^2 + 2\alpha s + \omega_0^2 = 0$ svo $2\alpha = \frac{1}{R_1 C_1}$ & $\omega_0^2 = \frac{1}{L_1 C_1}$

sem er markdempuð ef $\alpha = \omega_0$ ($s^2 + 2\alpha s + \alpha^2 = 0$)

eða $\frac{1}{2\tilde{R}_1 C_1} = \sqrt{\frac{1}{L_1 C_1}}$ eða $\tilde{R}_1 = \frac{\sqrt{L_1 C_1}}{2 C_1} = \frac{1}{2}\sqrt{\frac{L_1}{C_1}}$

svo $R_{add} = \frac{1}{2}\sqrt{\frac{L_1}{C_1}} - R_1 = \underline{6.25\,\Omega}$ þarf að bæta við

Notum nú $\tilde{R}_1$ í rás & leysum $s^2 + \frac{1}{\tilde{R}_1 C_1} s + \frac{1}{L_1 C_1} = 0$

eða $s^2 + 2000 s + 10^6 = 0 = (s + 1000)(s + 1000)$

svo $\underline{\alpha = 1000} = s_1 = s_2$

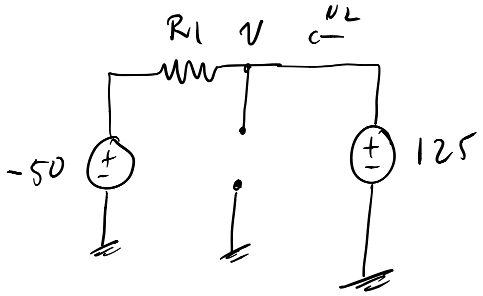
$\boxed{x(t) = e^{-\alpha t}(A_1 t + A_2)}$

<u>Markdempuð rás</u> hefur náttúrlega svörun $v(t) = e^{-\alpha t}(A_1 t + A_2)$

finnum með byrjunarskilyrðum $v(0^+)$ & $v'(0^+)$

$v'(t) = (-\alpha) e^{-\alpha t}(A_1 t + A_2) + e^{-\alpha t} A_1$

Rás við $t=0^-$ $\quad i_c = C v_c'$ & $v_L = L i_L'$

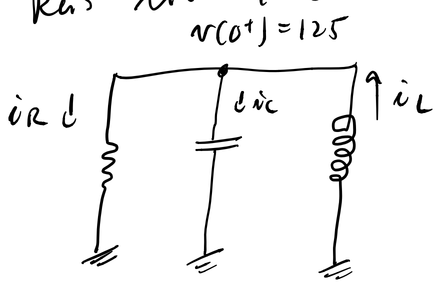


engir impulsar

Hér er $v(0^-) = 125 = v(0^+)$

$$i_L(0^-) = \frac{(125 - (-50))}{R_1} = 5.6A = i_L(0^+)$$

Rás við $t=0^+$



$$i_c(0^+) = i_L(0^+) - i_R(0^+)$$

$$= 5.6 - \frac{125}{R_1} \approx \underline{\underline{1.6 A}}$$

Nú er $v_c'(0^+) = \frac{i_c(0^+)}{C} \approx \underline{\underline{10^5 \frac{V}{s}}}$

Þá er $v(0^+) = 125 = A_2$

$v'(0^+) = 10^5 = -\alpha A_2 + A_1$

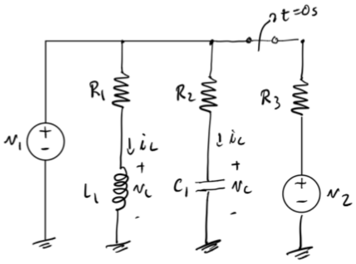
$$\begin{bmatrix} 0 & 1 \\ 1 & -\alpha \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} = \begin{bmatrix} 125 \\ 10^5 \end{bmatrix}$$

Þá er $\begin{bmatrix} A_1 \\ A_2 \end{bmatrix} = \begin{bmatrix} 2.25 \cdot 10^5 \\ 125 \end{bmatrix}$

& $\underline{\underline{v(t) = e^{-10^3 t}(125 + 2.25 \cdot 10^5 t) \quad t \geq 0}}$

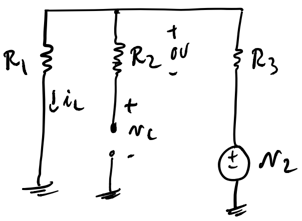
Dæmi 3

Ef gefið er að $v_1(t) = 10u(t)$V og $v_2(t) = 50$V, finnið v_c, i_L og v_L rétt fyrir og eftir núll. Finnið næst heildarsvörun $v_c(t)$.



Breyta	Gildi
R_1	$8\,\Omega$
R_2	$4\,\Omega$
R_3	$2\,\Omega$
C_1	$1/30\,$F
L_1	$6\,$H

Skoðum rás við $t=0^-$

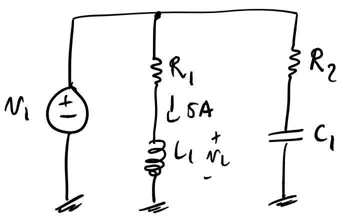


$$v_c(0^-) = v_2 \frac{R_1}{R_1 + R_3} = 50 \frac{8}{8+2} = 40V$$

$$i_L(0^-) = \frac{v_2 - 0}{R_1 + R_3} = \frac{50}{8+2} = 5A$$

Engar impúlsar svo $\underline{v_c(0^-) = v_c(0^+) = 40V}$ & $\underline{i_L(0^-) = i_L(0^+) = 5A}$

Skoðum rás við $t=0^+$



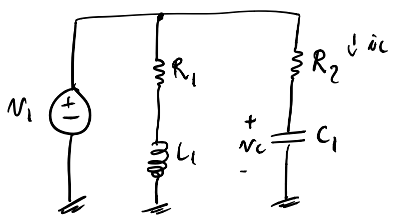
Veit að $i_L(0^+) = \dfrac{v_1(0^+) - v_L(0^+)}{R_1}$

eða $v_L(0^+) = -i_L(0^+)R_1 + v_1(0^+)$

$$= -5 \cdot 8 + 10 = \underline{-30V}$$

Engar impúlsar svo $\underline{v_L(0^+) = v_L(0^-)} = -30V$

Skoðum nú rás við $t>0$



$$v_c = v_1 \frac{Z_{C1}}{Z_{R2} + Z_{C1}} = v_1 \frac{\frac{1}{C_1 p}}{R_2 + \frac{1}{C_1 p}} = v_1 \frac{1}{C_1 R_2 p + 1}$$

svo $\dfrac{dv_c}{dt} + \dfrac{1}{R_2 C_1} v_c = v_1$ eða $\dfrac{dv_c}{dt} + 7.5 v_c = 10$

Náttúruleg lausn : $\underline{v_c(t) = Ae^{-7.5t}}$

Sérlausn Giskum á $v_c(t) = k_1 t + k_0$

$v_c'(t) = k_1$

$$\left.\begin{matrix} \\ \end{matrix}\right\} \quad k_1 + 10(k_1 t + k_0) = 10$$

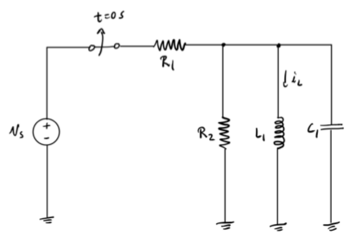
$k_1 = 0$

$k_0 = 10$

Heildarlausn $v_c(t) = Ae^{-7.5t} + 10$ $\quad v_c(0) = 40 = A + 10$ svo $\underline{v_c(t) = 30e^{-7.5t} + 10}$ $\quad t \geq 0$

Dæmi 4

Eftir að hafa verið lokaður í langan tíma opnast rofinn í rásinni hér að neðan við $t = 0$s. Finnið strauminn um spóluna $i_L(t)$ fyrir $t > 0$.

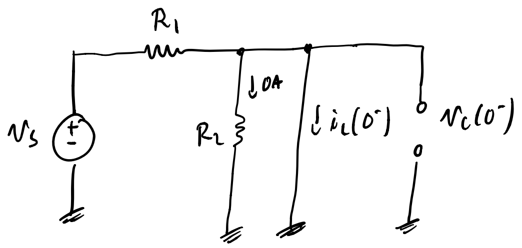


Breyta	Gildi
R_1	1 kΩ
R_2	500 Ω
L_1	2 H
C_1	2 µF
v_s	250 V

$i_c = C v_c'$

$v_L = L i_L'$

Við $t=0^-$ lítur rás svona út

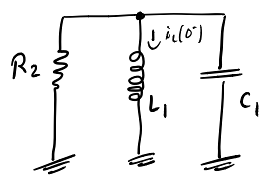


Hér er $i_L(0^-) = \frac{v_s - 0}{R_1} = \frac{250}{1k} = 0.25\ A$

Engar impúlsar eru í rásinni $i_L(0^-) = i_L(0^+) = \frac{1}{4} A$

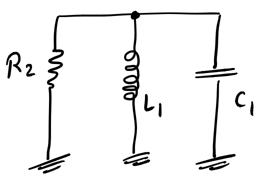
$v_c(0^-) = v_c(0^+) = 0\,V$

Við $t=0^+$ lítur rásin út svona



en: $i_L'(0^+) = \frac{v_L(0^+)}{L} = \frac{v_c(0^+)}{L} = 0\ \frac{A}{s}$

Við $t>0$ er rásin



$i_{R_1} + i_{L_1} + i_{C_1} = 0$

$R_1 v + L_1 p v + \frac{1}{C_1 p} v = 0$

eða $p^2 v + \frac{1}{R_1C_1} p v + \frac{1}{L_1C_1} v = 0$ (*)

Setjum nú $v = L p i_L$ inn í (*)

$p^2 (L p i_L) + \frac{1}{R_1C_1} p (L p i_L) + \frac{1}{L_1C_1}(L p i_L) = 0$

eða $p\left(\underbrace{p^2 i_L + \frac{1}{R_1C_1} p i_L + \frac{1}{L_1C_1} i_L}\right) = 0$

= fasti sem við veljum sem 0 (engin innmerki)

Diffurjafnan er þá $i_L'' + \frac{1}{R_1C_1} i_L' + \frac{1}{L_1C_1} i_L = 0$

Kennijafnan er $s^2 + \frac{1}{R_1C_1} s + \frac{1}{L_1C_1} = 0$ með rætur $s_1 = s_2 = -500 \Rightarrow$ markdeyfð!

Heildarlausn = Náttúrleg lausn

$$i(t) = e^{-s_1 t}(At + B)$$

$$i(0) = \frac{1}{4} = B$$

$$i'(t) = (-s_1)e^{-s_1 t}(At + B) + e^{-s_1 t}A$$

$$i'(0) = 0 = -s_1 B + A$$

svo $A = s_1 B = 125$

$$\underline{\underline{i(t) = e^{-500t}\left(125t + \frac{1}{4}\right) \quad t \geq 0}}$$