

Bangladesh Army University of Science and Technology

Department of Computer Science and Engineering

Special Backlog Examination 2020

Course Code: CSE 1201

Time: 03 (Three) hours

Level-I Term-II

Course Title: Discrete Mathematics

Full Marks: 210

N.B. (i) Answer any three questions from each PART (ii) Use separate answer script for each PART
(iii) Marks allotted are indicated in the margin (iv) Special Instruction (if any)-----N/A-----

PART A

(Answer any three questions)

1. a) Write some applications of propositional logic. (6)
b) Prove that "The square of an even number is even" using indirect proof technique. (11)
c) Show that $(p \rightarrow q \equiv \neg q \rightarrow \neg p)$ is a logically equivalent using truth table. (10)
d) Express the following statement using predicate and quantifier. (8)
 - i. Everyone is perfect.
 - ii. Some electronics are in the correct place and are in excellent condition.
 - iii. There are two girls in your school those who are joyful.
 - iv. No student in your school has taken a course in Mathematics.

2. a) Define: i. Tautology ii. Contradiction iii. Contingency (6)
b) Use set builder notation and logical equivalence to establish the second De Morgan Law (10)
$$(A \cap B)' = A' \cup B'$$

c) Let f be the function from $\{a, b, c, d\}$ to $\{1, 2, 3, 4\}$ with $f(a)=4, f(b)=2, f(c)=1$ and $f(d)=3$. Is f bijection? (9)
d) For each of the relations, decide whether it is reflexive or symmetric, anti-symmetric, transitive, equivalence or partial order relation. (10)
A={1,2,3}
R1={}
R2={(1,1),(2,2),(3,3)}
R3={(1,1),(2,2),(3,3),(1,2),(2,1)}
R4={(1,1),(2,2),(3,3),(1,3),(2,3)}
R5={(1,1),(1,2),(2,3),(1,3)}
R6={(1,1),(1,3),(2,2),(2,3),(3,3)}
R7=AxA

3. a) Form a binary search tree for the words discreet, mathematics, structure, programming, python, java, php, ruby, algorithm (using alphabetical order). Then answer the below questions: (20)
 - i. What is the depth of the node 'algorithm' in the tree.
 - ii. Which are the ancestors of the node 'php'
 - iii. What is the siblings of the node 'programming'
 - iv. Which are the leaves of the binary tree.
 - v. Write the pre-order, in-order and post-order traversal of the tree.
b) What is the value of the following postfix expression, (10)
$$7 \ 8 * \ 3 \uparrow \ 2 \ 3 - \ 8 \ 2 / \ * \ -$$

c) Discuss the properties of tree. (5)

4. a) Define with figure: pseudo graph, cyclic graph, wheel graph, multigraph, directed graph. (15)
b) Briefly describe tree terminologies. (10)

- c) Draw a undirected graph represented by the given adjacency matrix.

(10)

$$\begin{bmatrix} 2 & 1 & 0 & 2 & 1 \\ 3 & 0 & 0 & 1 & 1 \\ 1 & 2 & 3 & 2 & 1 \\ 1 & 0 & 0 & 2 & 1 \end{bmatrix}$$

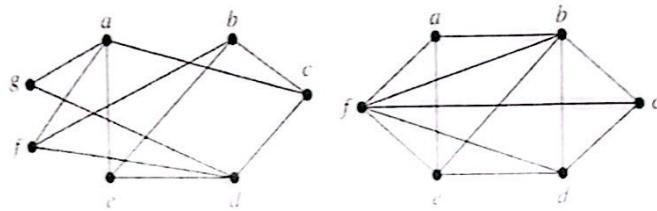
PART B

(Answer any three questions)

5. a) Each of the letters of the word DISCRETEMATH are written on separate pieces of paper that are then folded, put in a hat, and mixed thoroughly. One piece of paper is chosen (without looking) from the hat. What is the probability it is a 'E' or a 'T'? (7)
- b) Define arithmetic modulo m. Use the definition of addition and multiplication in Z_m to find $23 +_{11} 9$ and $23 \cdot_{11} 9$ (2+5=7)
- c) Write the algorithm to multiply two binary numbers and describe with example. (5+2=7)
- d) Prove that, "If n is a composite integer, then n has a prime divisor less than or equal to \sqrt{n} ". (7)
- e) How many bit strings of length four do not have three consecutive 1s? (7)
6. a) What is the probability that when two dice are rolled, the sum of the numbers on the two dice is prime number? (5)
- b) What is cryptography? Using the encryption function $E_n(x) = (x + n) \text{ mod } 26$ and decryption function $D_n(x) = (x - n) \text{ mod } 26$ encrypt the message "DISCRETE" and then decrypt also. (2+8=10)
- c) Prove that, "There must be infinitely many prime numbers". (10)
- d) Set A contains all the even numbers between 2 and 50 inclusive. Set B contains all the even numbers between 102 and 150 inclusive. What is the difference between the sum of elements of set B and that of set A? (5)
- e) If $a_2 = 5$ and $a_8 = 35$, what is the value of a_{30} ? (5)
7. a) Using mathematical induction proof that sum of the squares of first n positive integers is $\frac{n(n+1)(2n+1)}{6}$. (10)
- b) i. $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots + \frac{1}{1024} = ?$ (5+5=10)
ii. $\frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \dots + \infty = ?$
- c) What is the sum of the 40th and the 70th elements of the series defined as: (8)
 $s_1 = 281$
 $s_n = s_{n-1} - 5$
- d) Consider the following sequence of integers: 5, 11, 23, 47
What is the sum up to 6th element in this sequence? (7)
8. a) What is an algebraic structure? Write the main components of algebraic structure with example. (2+5=7)

a₉

- b) What is group? Is $\langle \mathbb{Z}, * \rangle$ a group? Prove that a Hexagon H with the set of its rotations, $H = \{R_0, R_1, R_2, R_3, R_4, R_5\}$ is a group when the structure is $\langle H, * \rangle$. [* operator denotes the addition of angles]. (2+9=11)
- c) Show that, a simple graph is bipartite if and only if it is a bi-chromatic graph. Determine whether graph G and H is bipartite or not. (6+6=12)



- d) Prove that, $\langle \mathbb{Z}_6, +, * \rangle$ is a commutative ring with unity. (5)

Bangladesh Army University of Science and Technology

Department of Computer Science and Engineering

Final (Online Live) Examination, Summer 2020

Course Code: CSE 1201

Time: 01 (One) hour

Level-1 Term-II

Course Title: Discrete Mathematics

Full Marks: 105

**N.B. (i) Answer any three questions from this PART (ii) Marks allotted are indicated in the margin
(iii) Symbols and abbreviations bear their standard meaning**

PART A

1. a) State the converse, contrapositive, and inverse of each of these conditional statements. 15

 - i) If it rained last night, then the sidewalk is wet.
 - ii) A quadrilateral has two pairs of parallel sides whenever it is a rectangle.
 - iii) Two angles have the same measure if they are congruent.

b) Is this compound proposition “[$(p \rightarrow q) \wedge (q \rightarrow r)] \wedge (p \wedge \neg r)$ ” , Tautology or Contradiction or Contingency? Show using truth table. 20

2. a) Define Rules of inference, Modus Ponens and Modus Tollens with examples 15

b) Show whether $(p \rightarrow q) \rightarrow (r \rightarrow s)$ and $(p \rightarrow r) \rightarrow (q \rightarrow s)$ are logically equivalent or not. Discuss whether \rightarrow operator is associative or not. 20

3. a) Given, $f(x) = 5x - 1$ and $g(x) = \frac{x}{5} + \frac{2}{5}$, show that $(fog)(x) = (gof)(x) = x$. 7

b) Define partial order and equivalence relations. Determine for the set $A=\{1, 2, 3\}$ from the list of relations below, whether each of the relations is partial order or equivalence or both or neither. 28

 - i) $R1=\{\}$
 - ii) $R2=\{(1,1),(2,2),(3,3)\}$
 - iii) $R3=\{(1,1),(2,2),(3,3),(1,2),(2,1)\}$
 - iv) $R4=\{(1,1),(2,2),(3,3),(1,3),(2,3)\}$
 - v) $R5=\{(1,1),(1,2),(2,3),(1,3)\}$
 - vi) $R6=\{(1,1),(1,3),(2,2),(2,3),(3,3)\}$
 - vii) $R7=A \times A$

4. a) Write the value of the following postfix expression: 7, 8, *, 3, ↑, 2, 3, -, 8, 2, /, *, - 15

b) Form a binary search tree for the words: the, quick, brown, fox, jumps, over, lazy, dog (using alphabetical order). Then answer the below questions: 20

 - i) What is the depth of the node ‘over’ in the tree?
 - ii) Which are the ancestors of the node ‘fox’?
 - iii) What are the siblings of the node ‘dog’?
 - iv) Which are the leaves of the binary tree?
 - v) Write the pre-order, in-order and post-order traversals of the tree.

Bangladesh Army University of Science and Technology

Department of Computer Science and Engineering

Final (Online Live) Examination, Summer 2020

Course Code: CSE 1201

Time: 01 (One) Hour

Level-1 Term-II

Course Title: Discrete Mathematics

Full Marks: 105

N.B. (i) Answer any three questions from this PART

(iii) Symbols and abbreviations bear their standard meaning

(ii) Marks allotted are indicated in the margin

PART B

5. a) Prove that, "If n is a composite integer, then n has a prime divisor less than or equal to \sqrt{n} ". 15
 b) Using the principle of mathematical induction, prove that 20

$$1 + 2 + 3 + 3 \cdot 4 + \dots + n(n+1) = 1/3\{n(n+1)(n+2)\}.$$

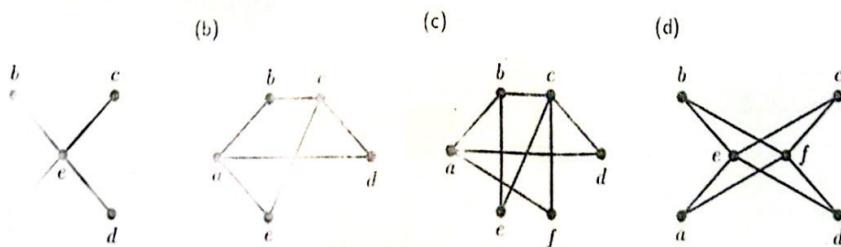
6. a) Each of the letters of the word MATHEMATICS are written on separate pieces of paper that are folded, put in a hat, and mixed thoroughly. One piece of paper is chosen (without looking) from the hat. What is the probability it is an 'M' or a 'T'? 7
 b) Using binomial theorem, Expand $(x+y)^{25}$ and then find the coefficient of $x^{10}y^{15}$ from the expansion. 13
 c) Use Euclidean division algorithm. What are the quotient and remainder when -17 is divided by 3? 15

7. a) Draw the graphs and represent them using adjacency matrices. 9
 i) S5 ii) C8 iii) W5

- b) Draw the undirected graph represented by the given adjacency matrix, 10

	a	b	c	d
a	1	3	2	0
b	3	0	4	2
c	2	4	0	1
d	0	2	1	0

- c) Show that, a simple graph is bipartite if and only if it is a bi-chromatic graph. Determine whether graph (a), (b), (c) and (d) is bipartite or not. 16



8. a) Prove that there must be infinitely many prime numbers. 15

- b) Define algebraic structure. Write the main components of algebraic structure with example. 10

- c) Find the sum of below series : 10

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots + \frac{1}{1024}$$

Bangladesh Army University of Science and Technology

Department of Computer Science and Engineering

Final Examination, Summer 2021

Course Code: CSE 1201

Time: 03 (Three) hours

Level-1 Term- II

Full Marks: 180

Course Title: Discrete Mathematics

- N.B. • The questions are of equal value.
 • Figures in the margin indicate full marks allotted to each question.
 • Symbols and abbreviations bear their standard meaning.
 • Use a separate answer script for each PART.
 • The corresponding course outcomes (COs) are given in the rightmost column.

PART- A (Marks:90)

(Answer any three questions including Q. No. 1)

- | | Marks | COs |
|---|-------|-----|
| 1. a) Translate the sentence and prove the proposition "If it is a holiday, then we will play football and we will play football when it is not a holiday" is a tautology. | 10 | CO2 |
| b) Translate the following nested quantifications in an English statement and determine the truth value of this statement where the domain of each variable consists of all real numbers. | 10 | CO2 |
| c) Prove that $(p \rightarrow r) \vee (q \rightarrow r)$ and $(p \wedge q) \rightarrow r$ are logically equivalent. | 10 | CO2 |
| 2. a) Find fog and gof , where $f(x) = x^2 + 1$ and $g(x) = x + 2$, are functions from \mathbb{R} to \mathbb{R} . | 8 | |
| b) Let f be the function from $\mathbb{Z}^+ \rightarrow \mathbb{Z}^+$ with $f(x) = x^2 + 1$. Is f invertible, and if it is, what is its inverse? | 8 | |
| c) Write down the properties of an algorithm. | 7 | |
| d) Calculate the Big-O for the following function: | 7 | |
| $f(x) = 3x^2 + 8x \log x$ | | |
| 3. a) Find the sum of the following series: | 7 | |
| $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots + \frac{1}{256}$ | | |
| b) Prove that the following statement is true by using mathematical induction: | 7 | |
| $1 + 3 + 5 + \dots + (2n - 1) = n^2$ | | |
| c) Prove that, "If n is a composite integer, then n has a prime divisor less than or equal to \sqrt{n} ". | 8 | |
| d) Using the encryption function $E(x) = (x + 6) \bmod 26$ and decryption function $D(x) = (x - 6) \bmod 26$, encrypt the message "DISCRETE MATH" and then decrypt it. | 8 | |
| 4. a) State the Pigeonhole Principle. Find the minimum number of students in a class such that five of them are born in the same month. | 8 | |
| b) Find the coefficient of $x^{12}y^6$ in the expansion of $(2x - 3y)^{18}$. | 8 | |
| c) One hundred students were surveyed about their preference between dogs and cats. The following two-way table displays data for the sample of students who responded to the survey. | 14 | |

Preference	Male	Female	Total
Prefers dogs	36	20	56
Prefers cats	10	26	36
No Preference	2	6	8
TOTAL	48	52	100

- i) Find the probability that a randomly selected student prefers dogs or

prefers cats.

- ii) Find the probability that a randomly selected student prefers dogs or is female.

PART- B (Marks:90)

(Answer any three questions including Q. No. 5)

Marks COs

5. a) Suppose, A is the father of B, C, and D. E and F are the grandchildren of A. E and G are brothers and their father is B. H and I are cousins and grandchildren of D. F and J are sisters and D is their father.

- Draw a tree for the above family line.
- Find the ancestor of node F.
- Calculate the height of the tree.
- Find out the leaves of the tree.
- Determine the orders in which a preorder and a postorder traversal visit the nodes of the tree.

6. a) Define partial order and equivalence relations. Determine for the set $A = \{1, 2, 3\}$ from the list of relations below, whether each of the relations is partial order or equivalence or both or neither. 15

- $R_1 = \{(1,1), (2,2), (3,3)\}$
- $R_2 = \{(1,1), (2,2), (3,3), (1,2), (2,1)\}$
- $R_3 = \{(1,1), (2,2), (3,3), (1,3), (2,3)\}$
- $R_4 = \{(1,1), (1,2), (2,3), (1,3)\}$
- $R_5 = \{(1,1), (1,3), (2,2), (2,3), (3,3)\}$

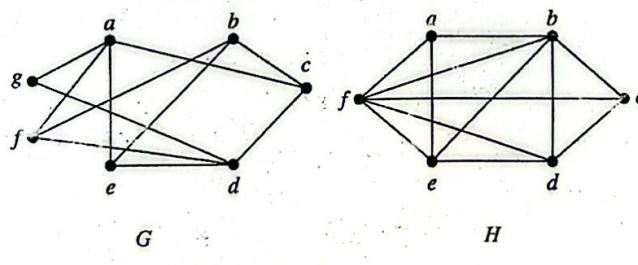
- b) Find the first five terms of the sequence defined by the following recurrence relation and initial condition. 8

$$a_n = a_{n-1} + 3a_{n-2}; \quad a_0 = 1, a_1 = 2$$

- c) Form a recurrence relation if the first eight terms are 1, 3, 4, 7, 11, 18, 29, 47. 7

7. a) Define with figure: pseudo graph, cyclic graph, wheel graph, multigraph. 10

- b) Show that a simple graph is bipartite if and only if it is a bi-chromatic graph. 12
Determine whether graph G and H is bipartite or not.



- c) Calculate the value of the following postfix expression: 8

$$6 \ 4 * 2 / 3 \ 5 + 4 \ 2 / * -$$

8. a) Prove that, "There must be infinitely many prime numbers". 8

- b) Use K-maps to minimize $F(x, y, z) = xy\bar{z} + x\bar{y}\bar{z} + \bar{x}yz + \bar{x}\bar{y}z$ and draw the minimized circuit using logic gates. 12

- c) Write the division algorithm. Calculate the quotient and remainder when -11 is divided by 3. 10

PART- A (Marks: 90)

(Answer any three questions including Q. No. 1)

- | | | Marks | COs |
|----|---|-------|-----|
| 1. | a) Translate the sentences and proof the propositions “You are a cricketer only if you can play football or if you can play cricket, then you can play football” and proposition “You are a cricketer and you can play cricket only if you can play football” are logically equivalent. Show both truth table and logical proof. | 10 | CO2 |
| b) | Define with example: Tautology, Contradiction and Contingency. Is this proposition “You can sing a song and you are a singer only if you can sing a song or you are a singer”, Tautology or Contradiction or Contingency? Show both truth table and logical proof of your answer. | 10 | CO2 |
| c) | Proof by contrapositive, “The square of an even number is even”. | 10 | CO2 |
| 2. | a) Write the contrapositive, converse, and inverse of the conditional statement “The home team wins whenever it is raining?” | 15 | |
| b) | For each of these collections of premises, what relevant conclusion or conclusions can be drawn? Explain the rules of inference used to obtain each conclusion from the premises. <ul style="list-style-type: none"> i) “If I take the day off, it either rains or snows.” “I took Tuesday off or I took Thursday off.” “It was sunny on Tuesday.” “It did not snow on Thursday.” ii) “Every computer science major has a personal computer.” “Ralph does not have a personal computer.” “Ann has a personal computer.” | 15 | |
| 3. | a) A bag contains 10 red marbles, 10 white marbles, and 10 blue marbles. What is the minimum no. of marbles you have to choose randomly from the bag to ensure that we get 4 marbles of same color? | 10 | |
| b) | A family of five is having portraits taken. <ul style="list-style-type: none"> i) How many ways can the family line up for the portrait? ii) How many ways can the photographer line up 3 family members? iii) How many ways can the family line up for the portrait if the parents are required to stand on each end? | 10 | |
| c) | Proof by contradiction that $\sqrt{2}$ is irrational. | 10 | |
| 4. | a) Let $f(x) = 5x + 4$ and $g(x) = 4x + 5$. Find the value of $f \circ g(2)$ and $g \circ f(2)$. | 10 | |
| b) | Let, $A = \{x: x \in \mathbb{Z}^+, x \text{ is prime number less than } 10\}$,
$B = \{x: x \in \mathbb{Z}^+, x \text{ is odd number less than } 10\}$ and
$C = \{x: x \in \mathbb{Z}^+, x \text{ is divisible by } 3 \text{ and less than } 10\}$
$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ | 12 | |
- Now find the following expressions.
- i) $A \cap B \cap C$
 - ii) $(A \cup B) \cap C$
 - iii) $(A - B) \cup (A - C)$
 - iv) $A' \cap B' \cap C'$



- c) Given, $f(x) = \frac{x+4}{2x-5}$ find, $f^{-1}(x)$. Verify your inverse by proving that, $(f \circ f^{-1}) = (f^{-1} \circ f) = x$.

8

PART- B (Marks: 90)

(Answer any three questions including Q. No. 5)

5. a) Consider the below code segments A and B:

Marks 15
COs CO3

A	B
$k := 0$	$k := 0$
$\text{for } i_1 := 1 \text{ to } n_1$	$\text{for } i_1 := 1 \text{ to } n_1$
$k := k + 1$	$\text{for } i_2 := 1 \text{ to } n_2$
$\text{for } i_2 := 1 \text{ to } n_2$.
$k := k + 1$.
.	.
.	.
$\text{for } i_m := 1 \text{ to } n_m$	$\text{for } i_m := 1 \text{ to } n_m$
$k := k + 1$	$k := k + 1$

- i) Identify which segment related to sum rule and which one is related to product rule.
 ii) Find the final value of k in both cases considering $n_1 = n_2 = \dots = n_m = 5$ and $m = 5$.

- b) Show that a simple graph is bipartite if and only if it is a bi-chromatic graph. Determine whether graph (a), (b) and (c) in fig. 5(b) is bipartite or not.

15 CO3

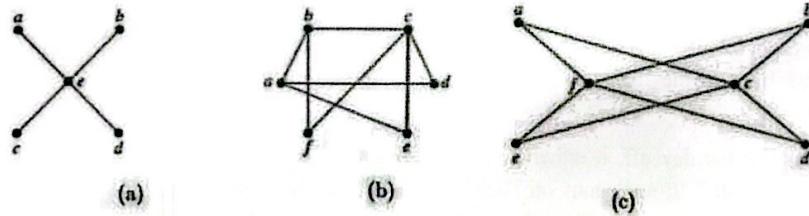


Figure 5(b) : Graphs

6. a) Consider a list of first names and last names of you, your parents and your grandparents then make a binary search tree from the list and find the depth and height of the tree.
 b) Prove by mathematical induction that the formula, $S_n = \frac{a(r^n - 1)}{r - 1}$ for the sum of the first n terms of the geometric sequence, holds.
 c) Determine the number of bit strings of length seven either start with a 0 bit or end with the two bits 11.

10

10

10

7. a) Calculate the value of the sum, $\sum_{i=1}^{\infty} \frac{i}{7^i}$.
 b) Determine the 7th term in the expansion of $(x-2y)^{12}$.
 c) Proof that, "There must be infinitely many prime numbers".

10

10

10

8. a) Justify that, $\langle Z_6, + \rangle$ is isomorphic to $\langle H, * \rangle$.
 b) Show that $(p \rightarrow q) \rightarrow (r \rightarrow s)$ and $(p \rightarrow r) \rightarrow (q \rightarrow s)$ are not logically equivalent. Discuss whether \rightarrow operator is associative or not.
 c) Translate the following nested quantifications into an English statement that expresses a mathematical fact assuming all real numbers to be the domain.

10

10

10

$$\forall x \forall y (((x \geq 0) \wedge (y < 0)) \rightarrow (x - y > 0))$$

Bangladesh Army University of Science and Technology

Department of Computer Science and Engineering

Referred/Improvement/Backlog Examination, Winter 2022

Course Code: CSE 1201

Time: 03 (Three) hours

Level-1 Term- II

Full Marks: 180

Course Title: Discrete Mathematics

- N.B. • The questions are of equal value.
• Figures in the margin indicate full marks allotted to each question.
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• Use a separate answer script for each PART.
• The corresponding course outcomes (COs) are given in the rightmost column.

PART- A (Marks: 90)

(Answer any three questions including Q. No. 1)

	Marks	COs
1. a) Translate the sentence and prove the proposition “If it is a holiday, then we will play football or we will play football when it is not a holiday” is a tautology.	15	CO2
b) Prove that $(p \rightarrow r) \vee (q \rightarrow r)$ and $(p \wedge q) \rightarrow r$ are logically equivalent.	15	CO2
2. a) Given, $f(x) = 3x - 2$ and $g(x) = \frac{x}{3} + \frac{2}{3}$, show that $(fog)(x) = (gof)(x) = x$.	12	
b) Calculate the Big-O for the following function:	10	
$f(x) = 3x^2 + 8x \log x$		
c) Write down the properties of an algorithm.	08	
3. a) Prove that the following statement is true by using mathematical induction:	14	
$1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$		
b) A coin is flipped four times where each flip comes up either heads or tails. Calculate the number of possible outcomes	16	
a) are there in total? b) contain exactly three heads? c) contains at least three heads? d) contain the same number of heads and tails?		
4. a) State the Pigeonhole Principle. Find the minimum number of students in a class such that five of them are born in the same month.	10	
b) Prove that $\overline{A \cup B} = \overline{A} \cap \overline{B}$, where universal set, $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$, $A = \{1, 2, 3, 4\}$ and $B = \{2, 4, 6, 8\}$	10	
c) What is the probability that when two dice are rolled, the sum of the numbers on the two dice is 7?	10	

PART- B (Marks:90)

(Answer any three questions including Q. No. 5)

	Marks	COs
5. a) Suppose, A is the father of B, C, and D. E and F are the grandchildren of A. E and G are brothers and their father is B. H and I are cousins and grandchildren of D. F and J are sisters and D is their father.	30	CO3
i) Draw a tree for the above family line. ii) Find the ancestor of node F. iii) Calculate the height of the tree. iv) Find out the leaves of the tree. v) Determine the orders in which a preorder and a postorder traversal visit the nodes of the tree.		



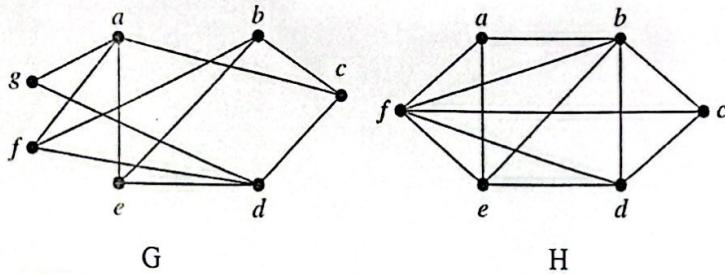
6. a) Define partial order and equivalence relations. Determine for the set $A = \{1, 2, 3\}$ from the list of relations below, whether each of the relations is partial order or equivalence or both or neither. 20

- i) $R1 = \{(1,1), (2,2), (3,3)\}$
- ii) $R2 = \{(1,1), (2,2), (3,3), (1,2), (2,1)\}$
- iii) $R3 = \{(1,1), (2,2), (3,3), (1,3), (2,3)\}$
- iv) $R4 = \{(1,1), (1,2), (2,3), (1,3)\}$
- v) $R5 = \{(1,1), (1,3), (2,2), (2,3), (3,3)\}$

- b) Prove that, "If n is a composite integer, then n has a prime divisor less than or equal to \sqrt{n} ". 10

7. a) Distinguish between graph and tree with example. 10

- b) Determine whether graph G and H is bipartite or not. 12



- c) Calculate the value of the following postfix expression: 08

$$6 \ 4 * \ 2 / \ 3 \ 5 + \ 4 \ 2 / * \ -$$

8. a) Write the division algorithm. Calculate the quotient and remainder when -11 is divided by 3 . 10

- b) Draw the graphs and represent them using adjacency matrices. 12

- i) K5
- ii) C8
- iii) W5

- c) Prove that there must be infinitely many prime numbers. 08

Bangladesh Army University of Science and Technology

Department of Computer Science and Engineering

Referred/Improvement/Backlog Examination, Winter/Summer 2022

Course Code: CSE 1201

Time: 03 (Three) hours

Level-1 Term-II

Full Marks: 180

Course Title: Discrete Mathematics

N.B. • The questions are of equal value.

- Figures in the margin indicate full marks allotted to each question.
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PART- A

(Answer any three questions including Q. No. 1)

	Marks	COs
1. a) A discrete mathematics class contains 1 mathematics major who is a freshman, 12 mathematics majors who are sophomores, 15 computer science majors who are sophomores, 2 mathematics majors who are juniors, 2 computer science majors who are juniors, and 1 computer science major who is a senior. Express each of these statements in terms of quantifiers and then determine its truth value.	15	CO2
i) There is a student in the class who is a junior. ii) Every student in the class is a computer science major. iii) There is a student in the class who is neither a mathematics major nor a junior. iv) Every student in the class is either a sophomore or a computer science major. v) There is a major such that there is a student in the class in every year of study with that major.		
b) Show that the premises "If you send me an e-mail message, then I will finish writing the program," "If you do not send me an e-mail message, then I will go to sleep early," and "If I go to sleep early, then I will wake up feeling refreshed" lead to the conclusion "If I do not finish writing the program, then I will wake up feeling refreshed."	15	CO2
2. a) Write the contrapositive, converse, and inverse of the following conditional statements: i) A positive integer is a prime only if it has no divisors other than 1 and itself. ii) If it snows tonight, then I will stay at home. iii) When I stay up late, it is necessary that I sleep until noon.	15	
b) For each of the following collections of premises, what relevant conclusion or conclusions can be drawn? Explain the rules of inference used to obtain each conclusion from the premises. i) "If I take the day off, it either rains or snows." "I took Tuesday off or I took Thursday off." "It was sunny on Tuesday." "It did not snow on Thursday." ii) "Every computer science major has a personal computer." "Ralph does not have a personal computer." "Ann has a personal computer."	15	
3. a) Assume that in a group of six people, each pair of individuals consists of two friends or two enemies. Show that there are either three mutual friends or three mutual enemies in the group.	10	
b) Suppose that a cookie shop has four different kinds of cookies. How many different ways can six cookies be chosen? Assume that only the type of cookie, and not the individual cookies or the order in which they are chosen, matters.	10	
c) Prove that if n is an integer and n^2 is odd, then n is odd.	10	
4. a) Determine whether the function $f(x) = x^2$ where $x \in Z^+$ from the set of integers to the set of integers is one-to-one.	10	

- b) The English alphabet contains 21 consonants and five vowels. How many strings of six lowercase letters of the English alphabet contain

- i) exactly one vowel?
- ii) exactly two vowels?
- iii) at least one vowel?
- iv) at least two vowels?

- c) Given, $f(x) = \frac{x+4}{2x-5}$ find, $f^{-1}(x)$. Verify your inverse by proving that, $(f \circ f^{-1}) = (f^{-1} \circ f) = x$.

8

PART- B
(Answer any three questions including Q. No. 5)

5. a) Translate in two ways each of these statements into logical expressions using predicates, quantifiers, and logical connectives. First, let the domain consists of the students in your class and second, let it consists of all people.
- i) Everyone in your class has a cellular phone.
 - ii) Somebody in your class has seen a foreign movie.
 - iii) There is a person in your class who cannot swim.
 - iv) All students in your class can solve quadratic equations.
 - v) Some student in your class does not want to be rich.

Marks COS
15 CO3

- b) Show that a simple graph is bipartite if and only if it is a bi-chromatic graph. Determine whether graph G and H in Figure 5(b) is bipartite or not.

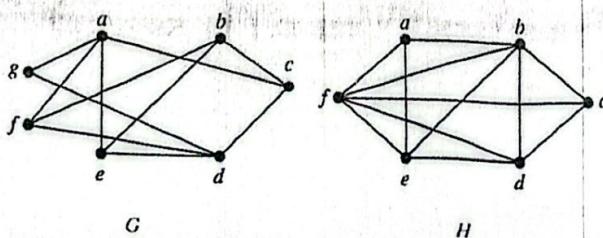


Figure 5(b) : Graphs

6. a) Prove $\forall k \in \mathbb{Z} \quad k \equiv 1 \pmod{3} \Rightarrow k^3 \equiv 1 \pmod{9}$ using direct proof technique. 15
- b) Suppose, $A = \{x : x \text{ is an even positive integer less than } 10\}$, $B = \{1, 3, 5, 7, 9\}$ and $C = \{x \in \mathbb{Z}^+ \mid x \text{ is divide by } 3 \text{ and } x < 10\}$ then derives the below sets:
 i) $(A \cup B)/C$
 ii) $(A \cap B)UC$
 iii) Prove that : $(BUC) = \overline{B} \cap \overline{C}$ (Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$)
- a) What is the probability that when two dice are rolled, the sum of the numbers on the two dice is 7? 10
- b) You randomly draw a marble out of a bag that contains 20 total marbles. 12 of the marbles in the bag are blue. What is $P(\text{draw a blue marble})$? 10
- c) Each of the letters of the word MISSISSIPPI are written on separate pieces of paper that are then folded, put in a hat, and mixed thoroughly. One piece of paper is chosen (without looking) from the hat. What is the probability it is an 'I'? 10

8. a) Define group. Is $\langle \mathbb{Z}, * \rangle$ a group? Prove that a Hexagon H with the set of its rotation set $H = \{R_0, R_1, R_2, R_3, R_4, R_5\}$ is a group when the structure is $\langle H, * \rangle$. [* operator denotes the addition of angle]. 15
- b) Consider the following tree in Figure 8(b) and answer the below questions: 15
- Find the depth of node 5 in the tree.
 - Calculate the height of the tree.
 - Write the pre-order, in-order, post-order traversal sequences of the tree.

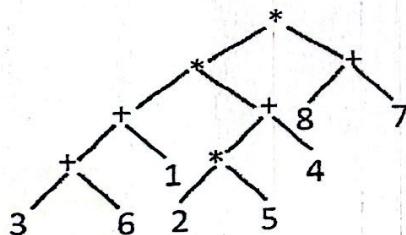


Figure 8(b): Tree

Course Title: Discrete Mathematics

- N.B. • The questions are of equal value.
 • Figures in the margin indicate full marks allotted to each question.
 • Symbols and abbreviations bear their standard meaning.
 • Use separate answer script for each PART.
 • The corresponding course learning outcomes (CLOs) are given in the right most column.

PART- A (Marks: 90)

(Answer any three questions including Q. No. 1)

		Marks	CLOs
1. a)	A discrete mathematics class contains 1 mathematics major who is a freshman, 12 mathematics majors who are sophomores, 15 computer science majors who are sophomores, 2 mathematics majors who are juniors, 2 computer science majors who are juniors, and 1 computer science major who is a senior. Express each of the following statements in terms of quantifiers and then determine its truth value.	15	CLO1
	i) There is a student in the class who is neither a mathematics major nor a junior. ii) Every student in the class is either a sophomore or a computer science major. iii) There is a major such that there is a student in the class in every year of study with that major.		
b)	State the converse, contrapositive, and inverse of each of the following conditional statements. i) If it snows today, I will ski tomorrow. ii) I come to class whenever there is going to be a quiz. iii) A positive integer is a prime only if it has no divisors other than 1 and itself.	15	CLO1
2. a)	Use set builder notation and logical equivalences to proof that $(A \cup B)' = A' \cap B'$ and show the outputs using Venn diagram.	10	
b)	Given $A - B = \{1,5,7,8\}$, $B - A = \{2,10\}$ and $A \cap B = \{3,6,9\}$. Determine the values of A and B .	10	
c)	Given, $f(x) = \frac{x+4}{2x-5}$ find, $f^{-1}(x)$. Verify your inverse by proving that, $(f \circ f^{-1}) = (f^{-1} \circ f) = x$.	10	
3. a)	For each of the following collections of premises, draw the relevant conclusion or conclusions. Explain the rules of inference used to obtain each conclusion from the premises. i) "If I take the day off, it either rains or snows." "I took Tuesday off or I took Thursday off." "It was sunny on Tuesday." "It did not snow on Thursday." ii) "Every computer science major has a personal computer." "Ralph does not have a personal computer." "Ann has a personal computer."	15	
b)	Define group. Is $\langle Z, * \rangle$ a group? Prove that a Hexagon H with the set of its rotation set $H = \{R_0, R_1, R_2, R_3, R_4, R_5\}$ is a group when the structure is $\langle H, * \rangle$. [$*$ operator denotes the modular addition of angle].	15	
4. a)	Assume that in a group of six people, each pair of individuals consists of two friends or two enemies. Show that there are either three mutual friends or three mutual enemies in the group.	10	
b)	Suppose that a cookie shop has four different kinds of cookies. How many different ways can six cookies be chosen? Assume that only the type of cookie, and not the individual cookies or the order in which they are chosen, matters.	10	

- c) Prove that, if n is an integer and n^2 is odd, then n is odd.

Marks CLOs
15 CLO3

5. a) Consider the following statements, of which the first three are premises and the fourth is a valid conclusion.

"All hummingbirds are richly colored."
 "No large birds live on honey."
 "Birds that do not live on honey are dull in color."
 "Hummingbirds are small."

Let $P(x)$, $Q(x)$, $R(x)$, and $S(x)$ be the statements " x is a hummingbird," " x is large," " x lives on honey," and " x is richly colored," respectively. Assuming that the domain consists of all birds, express the statements in the argument using quantifiers and $P(x)$, $Q(x)$, $R(x)$, and $S(x)$.

- b) Consider the following tree in figure 5(b) and answer the below questions: 15 CLO3
- Find the depth of node 5 in the tree.
 - Calculate the height of the tree.
 - Write the pre-order, in-order and post-order traversal sequences of the tree.

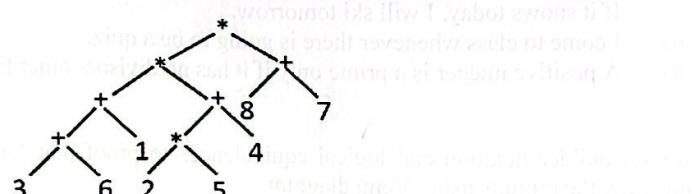


Figure 5(b): Tree.

6. a) Define the following terms with examples. Tautology, Contradiction and Contingency. Is this proposition "You can do programming and you are a programmer only if you can do program or you are a programmer", Tautology or Contradiction or Contingency? Show both truth table and logical proof of your answer. 15
- b) Write briefly about isomorphism. Determine whether or not S_3 (Equilateral Triangle) isomorphic to Z_6 or H (Regular Hexagon) by constructing necessary tables. 15
7. a) Show that a simple graph is bipartite if and only if it is a bi-chromatic graph. Determine whether graph G and H in figure 7(a) is bipartite or not. 15

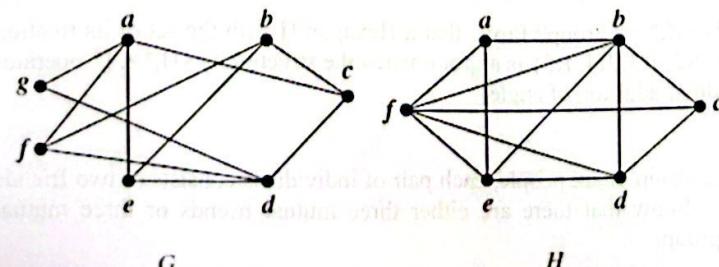


Figure 7(a): Graphs.

- b) Draw a decision tree that orders the elements of the list a, b, c . 15

- Proof that, there must be infinitely many prime numbers. 10
- b) Write the division algorithm with suitable example. 10
- c) Proof that, if n is a composite integer, then n has a prime divisor less than or equal to \sqrt{n} . 10

Course Title: Discrete Mathematics

- N.B. • The questions are of equal value.
 • Figures in the margin indicate full marks allotted to each question.
 • Symbols and abbreviations bear their standard meaning.
 • Use separate answer script for each PART.
 • The corresponding course learning outcomes (CLOs) are given in the right most column.

PART- A (Marks: 90)

(Answer any three questions including Q. No. 1)

- | | Marks | CLOs |
|--|-------|------|
| 1. a) Show that the premises "If you send me an e-mail message, then I will finish writing the program," "If you do not send me an e-mail message, then I will go to sleep early," and "If I go to sleep early, then I will wake up feeling refreshed" lead to the conclusion "If I do not finish writing the program, then I will wake up feeling refreshed." | 15 | CLO2 |
| b) Express the following statement using predicate and quantifier. | 15 | CLO2 |
| i) No one is perfect.
ii) All tools are in the correct place and are in excellent condition.
iii) There is a person in your school who is not happy. | | |
| 2. a) Define inverse function and injective function with examples. Determine whether the function $f(x) = x^2 + 2x - 5$ where $x \in \mathbb{Z}^+$ from the set of integers to the set of integers is one-to-one or not? | 10 | |
| b) Given, $f(x) = 3x - 2$ and $g(x) = \frac{x}{3} + \frac{2}{3}$, show that $(f \circ g)(x) = (g \circ f)(x) = x$. | 10 | |
| c) Given, $f(x) = \frac{x+4}{2x-5}$ find, $f^{-1}(x)$. | 10 | |
| 3. a) For each of the following collections of premises, draw the relevant conclusion or conclusions. Explain the rules of inference used to obtain each conclusion from the premises. | 15 | |
| i) "If I take the day off, it either rains or snows." "I took Tuesday off or I took Thursday off." "It was sunny on Tuesday." "It did not snow on Thursday."
ii) "Every computer science major has a personal computer." "Ralph does not have a personal computer." "Ann has a personal computer." | | |
| b) Define group. Is $\langle \mathbb{Z}, * \rangle$ a group? Prove that a Hexagon H with the set of its rotation set $H = \{R_0, R_1, R_2, R_3, R_4, R_5\}$ is a group when the structure is $\langle H, * \rangle$. [$*$ operator denotes the modular addition of angle]. | 15 | |
| 4. a) The English alphabet contains 21 consonants and five vowels. How many strings of six lowercase letters of the English alphabet contain, | 15 | |
| i) exactly one vowel?
ii) exactly two vowels?
iii) at least two vowels? | | |
| b) Prove $\forall k \in \mathbb{Z} \ k \equiv 1 \pmod{3} \rightarrow k^3 \equiv 1 \pmod{9}$ using direct proof technique. | 15 | |

PART- B (Marks: 90)

(Answer any three questions including Q. No. 5)

Marks 15 CLO3

5. a) Consider the following statements, of which the first three are premises and the fourth is a valid conclusion.

"All hummingbirds are richly colored."
"No large birds live on honey."
"Birds that do not live on honey are dull in color."
"Hummingbirds are small."

Let $P(x)$, $Q(x)$, $R(x)$, and $S(x)$ be the statements "x is a hummingbird," "x is large," "x lives on honey," and "x is richly colored," respectively. Assuming that the domain consists of all birds, express the statements in the argument using quantifiers and $P(x)$, $Q(x)$, $R(x)$, and $S(x)$.

- b) Form a binary search tree for the set $A = \{12, 20, 6, 7, 10, 13, 15, 34, 5, 8\}$. Then answer the below questions: 15 CLO3
- Find the depth of node 34 in the tree.
 - Calculate the height of the tree.
 - Write the pre-order, in-order, post-order traversal of the tree.

6. a) Use mathematical induction to prove that the sum of the squares of first n natural numbers is $\frac{n(n+1)(2n+1)}{6}$. 15
- b) Write briefly about isomorphism. Is S_3 (Equilateral Triangle) isomorphic to Z_6 or H (Regular Hexagon)? 15

7. a) Show that a simple graph is bipartite if and only if it is a bi-chromatic graph. Determine whether graph F, G and H in figure 7(a) is bipartite or not. 15

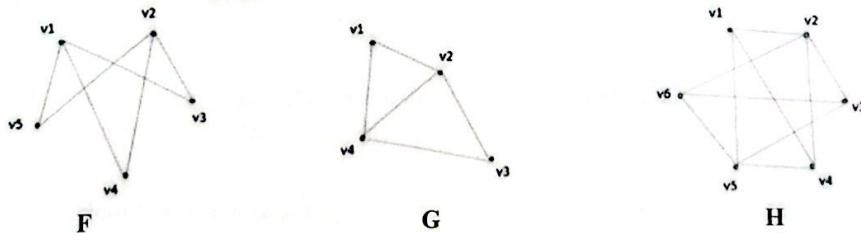


Figure 7(a): Graphs.

- b) Find the sum of following series: 15
- $\frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \dots + \frac{1}{59049}$
 - $1 + 4 + 9 + 16 + \dots + 1024$
 - $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots \infty$

8. a) Justify whether $[\neg p \wedge (p \vee q) \rightarrow q]$ is a tautology, contradiction or contingency using both tabular and logical proof. 10
- b) Let $Q(x)$ be the statement " $x < 2$." What is the truth value of the quantification $\forall x Q(x)$, where the domain consists of all real numbers? 10
- c) Prove that "The square of an even number is even", using Indirect Proof technique. 10

Bangladesh Army University of Science and Technology

Department of Computer Science and Engineering

Final (Online Live) Examination, Winter 2021

Course Code: CSE 1201

Time: 01 (One) hour

Level-I Term-II
Course Title: Discrete Mathematics
Full Marks: 60

- NB:
- Answer two questions including the compulsory question no.1 from PART A
 - Figures in the margin indicate full marks allotted to each question.
 - Symbols and abbreviations bear their standard meaning.
 - The corresponding course outcomes (COs) are given in the right most column.
 - Special instructions (if any)

PART-A

Marks COs

1. a) State the converse, contrapositive, and inverse of each of the following conditional statements.
- If it snows tonight, then I will stay at home.
 - I go to the beach whenever it is a sunny summer day.
 - When I stay up late, it is necessary that I sleep until noon.
- b) Show that $(p \rightarrow q) \rightarrow (r \rightarrow s)$ and $(p \rightarrow r) \rightarrow (q \rightarrow s)$ are not logically equivalent. Discuss whether \rightarrow operator is associative or not.
- c) Translate the following nested quantifications into an English statement that expresses a mathematical fact assuming all real numbers to be the domain.
 $\forall x \forall y ((x > 0) / (y < 0)) \rightarrow (x \cdot y > 0)$
2. a) Suppose, $A = \{x : x \text{ is an odd positive integer less than } 10\}$, $B = \{x : x \text{ is a prime number less than } 10\}$ and $C = \{x \in \mathbb{Z}^+ \mid x \text{ is divisible by } 3 \text{ and } x < 10\}$. Derive the following sets:
 - $(A \cup B) / C$
 - $(A \cap B) / C$
 - Prove that: $(\overline{B \cup C}) = \overline{B} \cap \overline{C}$ (Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$)
- b) Find $(f \circ g)$, $(g \circ f)$, $(f \circ f)$, $(g \circ g)$ for the following functions:
 $f(x) = \sqrt{2x + 3}$,
 $g(x) = x^2 + 1$
- c) Given, $f(x) = \frac{x+4}{2x-5}$, find $f^{-1}(x)$. Prove that, $(f \circ f^{-1}) = (f^{-1} \circ f) = x$.
3. a) Find the sum of the following series:
i) $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots + \frac{1}{2048}$
ii) $\frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \dots + \infty$
- b) Prove by mathematical induction that the formula, $S_n = \frac{a(r^n - 1)}{r - 1}$ for the sum of the first n terms of the geometric sequence, $a, ar, ar^2, \dots, ar^{n-1}$, holds.
- c) Set A contains all the even numbers between 2 and 50 inclusive. Set B contains all the even numbers between 102 and 150 inclusive. What is the difference between the sum of elements of set B and that of set A?

- NB:
- Answer two questions including the compulsory question no.4 from PART B
 - Figures in the margin indicate full marks allotted to each question.
 - Symbols and abbreviations bear their standard meaning.
 - The corresponding course outcomes (COs) are given in the right most column.
 - Special instructions (if any)

PART-B

Marks COs
18 CO2

4. a) Consider the following code segments A and B:

A	B
$k := 0$ <i>for</i> $i_1 := 1$ to n_1 $k := k + 1$ <i>for</i> $i_2 := 1$ to n_2 $\quad \quad \quad \vdots$ $k := k + 1$ $\quad \quad \quad \vdots$ <i>for</i> $i_m := 1$ to n_m $k := k + 1$	$k := 0$ <i>for</i> $i := 1$ to n <i>for</i> $i_2 := 1$ to n_2 $\quad \quad \quad \vdots$ <i>for</i> $i_m := 1$ to n_m $k := k + 1$

- i) Identify which segment is related to sum rule and which one is related to product rule.
ii) Find the final value of 'k' in both cases considering $n_1 = n_2 = \dots = n_m = 7$.
- b) Write the division algorithm. What are the quotient and remainder when -77 is divided by 5?
- c) Define arithmetic modulo m. Use the definition of addition and multiplication in Z_m to find $7 +_{11} 9$ and $7 \cdot_{11} 9$

5. a) Define partial order and equivalence relations. Determine for the set $A = \{1, 2, 3\}$ from the list of relations below, whether each of the relations is partial order or equivalence or both or neither.

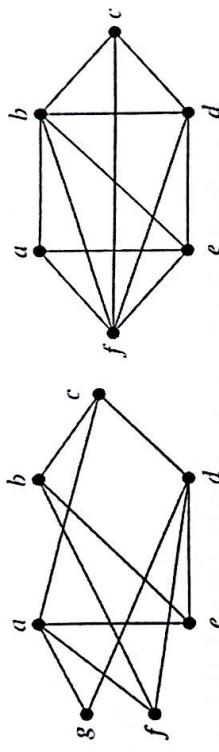
- i) $R1 = \{\}$
ii) $R2 = \{(1,1), (2,2), (3,3)\}$
iii) $R3 = \{(1,1), (2,2), (3,3), (1,2), (2,1)\}$
iv) $R4 = \{(1,1), (2,2), (3,3), (1,3), (2,3)\}$
v) $R5 = \{(1,1), (1,2), (2,3), (1,3)\}$
vi) $R6 = \{(1,1), (1,3), (2,2), (2,3), (3,3)\}$
vii) $R7 = A \times A$

- b) Draw a directed graph represented by the given adjacency matrix,

$$\begin{bmatrix} 1 & 0 & 2 & 2 & 1 \\ 1 & 2 & 1 & 1 & 1 \\ 2 & 2 & 1 & 2 & 1 \\ 1 & 0 & 3 & 1 & 1 \end{bmatrix}$$

05

6. a) Show that a simple graph is bipartite if and only if it is a bi-chromatic graph. 12
Determine whether graph G and H in fig. 6(a) is bipartite or not.



G

Fig. 6(a)

- b) Determine the 7th term in the expansion of $(x-2y)^{12}$ 06
c) How many bit strings of length seven either start with a 0 bit or end with the two bits 06
11?
d) What is the minimum number of students required in a discrete mathematics class to be sure that at least six will receive the same grade, if there are five possible grades, A, B, C, D, and F?

H

Fig. 6(a)

Bangladesh Army University of Science and Technology

Department of Computer Science and Engineering

Referred/Improvement/Backlog (Online Live) Examination, Winter 2021 Level-1 Term-II
Course Code: CSE 1201 Course Title: Discrete Mathematics
Time: 01 (One) hour Full Marks: 60

NB: • Answer two questions including the compulsory question no.1 from PART A

• Figures in the margin indicate full marks allotted to each question.

• Symbols and abbreviations bear their standard meaning.

• The corresponding course outcomes (COs) are given in the right most column.
• Special instructions (if any)

PART-A

Marks

COs

- | | | |
|--|----|-----|
| 1. a) Proof that proposition "You can sing a song only if you are a singer or if you can dance, then you are a singer" and proposition "you can sing a song and you can dance only if you are a singer" are logically equivalent. Also show the truth table. | 12 | CO2 |
| b) Justify whether $[p \wedge (p \vee q)] \rightarrow q$ is a tautology, contradiction or contingency using both tabular and logical proof. | 10 | CO2 |
| c) Let $Q(x)$ be the statement " $x < 2$." What is the truth value of the quantification $\forall x Q(x)$, where the domain consists of all real numbers? | 08 | CO2 |
| 2. a) Define proposition. Proof that "The square of an even number is even", using Indirect Proof technique. | 12 | |
| b) Find $(f \circ g)$, $(g \circ f)$, $(f \circ f)$, $(g \circ g)$ for the following functions:
$f(x) = \sqrt{2x+3}, \quad g(x) = x^2 + 1$ | 09 | |
| c) Define inverse function and injective function with examples. Determine whether the function $f(x) = x^2 + 2x + 5$ where $x \in \mathbb{Z}^+$ from the set of integers to the set of integers is one-to-one or not? | 09 | |
| 3. a) Prove by mathematical induction that the formula, $S_n = \frac{a(r^n - 1)}{r - 1}$ for the sum of the first n terms of the geometric sequence, holds. | 12 | |
| b) Prove $\forall k \in \mathbb{Z} \ k \equiv 1 \pmod{3} \rightarrow k^3 \equiv 1 \pmod{9}$ using direct proof technique. | 10 | |
| c) Calculate the value of the sum $\sum_{i=1}^{\infty} \frac{2^{i-1}}{2^{i+1}} = ?$ | 08 | |

NB: • Answer two questions including the compulsory question no.4 from PART B

• Figures in the margin indicate full marks allotted to each question.

• Symbols and abbreviations bear their standard meaning.

• The corresponding course outcomes (COs) are given in the right most column.

• Special instructions (if any)

PART-B

4. a) Express the following statement using predicate and quantifier. Marks COs
 i) No one is perfect. 12 CO2
 ii) All tools are in the correct place and are in excellent condition.

- iii) There is a person in your school who is not happy.
 iv) No student in your class has taken a course in logic programming.

- b) Translate the sentences and propose the proposition "You are a cricketer only if you can play cricket or if you can play football, then you are a cricketer" and proposition "you can play cricket and you can play football only if you are a cricketer" are logically equivalent. Show both truth table and logical proof.

- c) Define arithmetic modulo m. Use the definition of addition and multiplication in Z_m to find $7 +_{11} 9$ and $7 \cdot_{11} 9$. 08 CO2

5. a) A bag contains 10 red marbles, 10 white marbles, and 10 blue marbles. What is the minimum no. of marbles you have to choose randomly from the bag to ensure that we get 4 marbles of same color? 06

- b) Define the following: (i) Full m-ary tree (ii) Balanced m-ary tree (iii) Complete m-ary tree. Give one example of each type of tree of height 4. 06

- c) Each of the letters of the word MISSISSIPPI are written on separate pieces of paper that are then folded, put in a hat, and mixed thoroughly. One piece of paper is chosen (without looking) from the hat. What is the probability it is an 'I'?

- d) A family of five is having portraits taken.
 i) How many ways can the family line up for the portrait?
 ii) How many ways can the photograph line up 3 family members?
 iii) How many ways can the family line up for the portrait if the parents are required to stand on each end?

- 12

6. a) Show that a simple graph is bipartite if and only if it is a bi-chromatic graph. Determine whether graph (a), (b) and (c) in fig. 6(a) is bipartite or not. 25

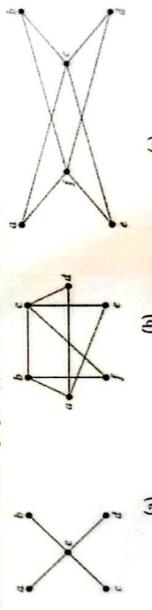


Fig. 6(a)

- b) Determine the value of golden ratio using line segment. 05

Bangladesh Army University of Science and Technology

Department of Computer Science and Engineering

Preferred/Improvement/Backlog Examination, Summer 2019

Course Code: CSE 1201

Time: 03 (Three) hours

Level-1 Term-II

Course Title: Discrete Mathematics

Full Marks: 210

N.B. (i) Answer any three questions from each PART
(iii) Marks allotted are indicated in the margin

(ii) Use separate answer script for each PART
(iv) Special Instruction (if any)-----N/A-----

PART A

(Answer any three questions)

1. a) What are the conjunction and disjunction of the propositions P and Q, where P is the proposition "Today is Friday" and Q is the proposition "It is raining". 06
b) Prove that "The square of an even number is even" using indirect proof technique. 11
c) Show that $(p \rightarrow q) \rightarrow r$ and $p \rightarrow (q \rightarrow r)$ are not logically equivalent using truth table. 10
d) Express the following statement using predicate and quantifier.
 - i. Everyone in your class is friendly.
 - ii. There is someone older than 21 years.
 - iii. Not everybody is your friend.
 - iv. At least one of your friend is perfect.08

2. a) Define cardinality, power set. What is the power set of the set {5, 8, 9, 0}. 06
b) Let $A=\{0,2,4,6,7,8\}$, $B=\{0,2,3,5,7,9\}$ and $C=\{1,2,3,4,5,6,8,9\}$, Find
 - i. $(A \cap B) \cup C$
 - ii. $(A \cup C) \cap B$
 - iii. $A \cup (B-C)$
 - iv. $A - (A \cap B \cap C)$10
c) Briefly discuss about one to one function and onto function with example 09
d) What rule of inference is used in each of these arguments?
 - i. Anik is a physics major. Therefore, Anik is either a physics major or a chemistry major.
 - ii. Moon is a science major and a commerce major. Therefore, Moon is a science major.
 - iii. If it snows today, the university will close. The university is not close today. Therefore, it did not snow today.10

3. a) Form a binary search tree for the words discreet, mathematics, structure, programming, python, java, php, ruby, algorithm (using alphabetical order). Then answer the following questions:
 - i. What is the depth of the node 'algorithm' in the tree.
 - ii. Which are the ancestors of the node 'php'
 - iii. What is the siblings of the node 'programming'
 - iv. Which are the leaves of the binary tree.

Write the pre-order, in-order and post-order traversal of the tree.

b) Find $f \circ g$ and $g \circ f$, where $f(x) = x^2 + 5$ and $g(x) = x + 7$, are functions from \mathbb{R} to \mathbb{R} . 10
c) Define: Symmetric and anti-symmetric. 0520

4. a) Define with figure: pseudo graph, cyclic graph, wheel graph, multigraph, directed graph. 15
b) Differentiate between graph and tree. 10

Draw a directed graph represented by the given adjacency matrix,

$$\begin{bmatrix} 2 & 0 & 1 & 2 & 1 \\ 1 & 3 & 1 & 0 & 1 \\ 0 & 2 & 1 & 2 & 1 \\ 1 & 0 & 2 & 3 & 1 \end{bmatrix}$$

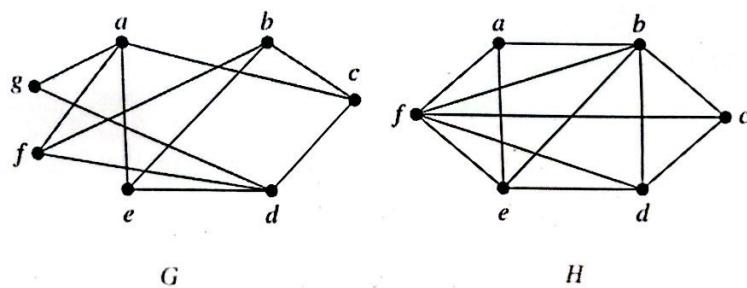
PART B

(Answer any three questions)

5. a) Proof that, $\overline{A \cap B} = \overline{A} \cup \overline{B}$ by using set builder notation and logical equivalences. (5)
- b) Use mathematical induction to prove that the sum of the first n odd positive integers is n^2 . (6)
- c) Find the sum of the following series : (2x4=8)
- i) $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots + \frac{1}{1024}$
- ii) $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots \infty$
- d) Define recurrence relation. Write the Fibonacci sequence formula and then find the golden ratio from the sequence. (2+5=7)
- e) What is the sum of the 40th and the 70th elements of the series defined as: $s_1=281$ and $s_n=s_{n-1}-5$. (5)
- f) Find the 125th term in the arithmetic sequence 4, -1, -6, -11, ... (4)
6. a) Write the definitions of partial order and equivalence relations. Determine for the set $A=\{1, 2, 3\}$ from the list of relations below, whether each of the relations is partial order or equivalence or both or neither. (7x2=14)
- i) $R1=\{\}$
 ii) $R2=\{(1,1),(2,2),(3,3)\}$
 iii) $R3=\{(1,1),(2,2),(3,3),(1,2),(2,1)\}$
 iv) $R4=\{(1,1),(2,2),(3,3),(1,3),(2,3)\}$
 v) $R5=\{(1,1),(1,2),(2,3),(1,3)\}$
 vi) $R6=\{(1,1),(1,3),(2,2),(2,3),(3,3)\}$
 vii) $R7=A \times A$
- b) Define arithmetic modulo m . Use the definition of addition and multiplication in Z_m to find $7 +_{11} 9$ and $7 \cdot_{11} 9$ (2+5=7)
- c) Write the algorithm to add two binary numbers and describe with example. (5+2=7)
- d) Prove that, "If n is a composite integer, then n has a prime divisor less than or equal to \sqrt{n} ". (7)
7. a) Prove that, "There must be infinitely many prime numbers". (5)
- b) What is the probability that when two dice are rolled, the sum of the numbers on the two dice is 7? (5)
- c) What is cryptography? Using the encryption function $E_n(x) = (x + n) \text{ mod } 26$ and decryption function $D_n(x) = (x - n) \text{ mod } 26$ encrypt the message "BAUST CSE" and then decrypt also. (2+6=8)
- d) Each of the letters of the word MATHEMATICS are written on separate pieces of paper that are then folded, put in a hat, and mixed thoroughly. One piece of paper is chosen (without looking) from the hat. What is the probability it is a 'M' or a 'T'? (6)
- e) How many bit strings of length five do not have three consecutive 1s? (6)
- f) A special die is made in the shape of an octahedron: The die has 8 equal faces marked with the numbers 1 to 8. If the die is thrown once, what is the probability that the face that lands uppermost has a prime number? (5)

What is an algebraic structure. Write the main components of algebraic structure with example. (2+5=7)

- b) What is group? Is $\langle \mathbb{Z}, * \rangle$ a group? Prove that a Hexagon H with the set of its rotations $H = \{R_0, R_1, R_2, R_3, R_4, R_5\}$ is a group when the structure is $\langle H, * \rangle$. [* operator denotes the addition of angles]. (2+9=11)
- c) Show that, a simple graph is bipartite if and only if it is a bi-chromatic graph. Determine whether graph G and H is bipartite or not. (6+6=12)



- d) Prove that, $\langle \mathbb{Z}_6, +, * \rangle$ is a commutative ring with unity. (5)

Bangladesh Army University of Science and Technology

Department of Computer Science and Engineering

Final Examination, Summer 2019

Course Code: CSE 1201

Time: 03 (Three) hours

Level-1 Term-II

Course Title: Discrete Mathematics

Full Marks: 210

N.B. (i) Answer any three questions from each PART
(iii) Marks allotted are indicated in the margin

(ii) Use separate answer script for each PART
(iv) Special Instruction (if any)-----N/A-----

PART A

(Answer any three questions)

1. a) What are the contrapositive, the converse and the inverse of the conditional statement "The home team wins whenever it is raining"? 06
b) Prove that, "The square of an even number is even" using indirect proof technique. 11
c) Show using truth table that, $(p \rightarrow q) \wedge (q \rightarrow r) \rightarrow (p \rightarrow r)$ is a tautology. 10
d) Express the following statement using predicate and quantifier.
 - i. No one is perfect.
 - ii. All tools are in the correct place and are in excellent condition.
 - iii. There is a person in your school who is not happy.
 - iv. No student in your class has taken a course in logic programming.08
2. a) Define cardinality and power set. What is the power set of the set {5,8,9}? 06
b) Use set builder notation and logical equivalence to establish the second De Morgan Law $(A \cap B) = \bar{A} \cup \bar{B}$. 10
c) Briefly discuss one to one function and onto function with examples. 09
d) For each of the following relations, decide whether it is reflexive, symmetric, antisymmetric, transitive, equivalence or partial order relation. 10
 $A = \{1,2,3\}$
 $R_1 = \{\}$
 $R_2 = \{(1,1), (2,2), (3,3)\}$
 $R_3 = \{(1,1), (2,2), (3,3), (1,2), (2,1)\}$
 $R_4 = \{(1,1), (2,2), (3,3), (1,3), (2,3)\}$
 $R_5 = \{(1,1), (1,2), (2,3), (1,3)\}$
 $R_6 = \{(1,1), (1,3), (2,2), (2,3), (3,3)\}$
 $R_7 = A \times A$.
3. a) Form a binary search tree for the words discrete, mathematics, structure, programming, python, java, php, ruby, algorithm (using alphabetical order). Then answer the following questions:
 - i. What is the depth of the node 'algorithm' in the tree?
 - ii. Which are the ancestors of the node 'php'?
 - iii. What are the siblings of the node 'programming'?
 - iv. Which are the leaves of the binary tree?
 - v. Write the pre-order, in-order and post-order traversal of the tree.20
b) What is the value of the following postfix expression: $32 * 2 \uparrow 53 - 84 /* - ?$ 10
c) Draw a full 4-ary tree where the height of the tree is four. 05
4. a) Define with figure: pseudo graph, cyclic graph, wheel graph, multigraph and directed graph. 15
b) Differentiate between graph and tree. 10

- c) Draw a directed graph represented by the given adjacency matrix,

$$\begin{bmatrix} 1 & 0 & 2 & 2 & 1 \\ 1 & 2 & 1 & 1 & 1 \\ 2 & 2 & 1 & 2 & 1 \\ 1 & 0 & 0 & 3 & 1 \end{bmatrix}$$

PART B

(Answer any three questions)

5. a) Set A contains all the even numbers between 2 and 50 inclusive. Set B contains all the even numbers between 102 and 150 inclusive. What is the difference between the sum of elements of set B and set A? 05
- b) Use mathematical induction to prove that the sum of the squares of first n natural numbers is $\frac{n(n+1)(2n+1)}{6}$. 06
- c) Find the sum of the following series: 2x4=8
- i) $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots + \frac{1}{2048}$
- ii) $\frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \dots$ up to infinitely many terms.
- d) Define recurrence relation. Consider the following sequence of integers: 5, 11, 23, 47. What is the 6th element in this sequence? 2+5=7
- e) What is the sum of the 40th and the 70th elements of the series defined as: $S_1 = 281$ and $S_n = S_{n-1} - 5$? 05
- f) If $a_2 = 5$ and $a_8 = 35$, what is the value of a_{30} ? 04
6. a) Write the division algorithm. What are the quotient and remainder when -17 is divided by 3? 2+5=7
- b) Define arithmetic modulo m . Use the definition of addition and multiplication in Z_m to find $7 +_{11} 9$ and $7 \cdot_{11} 9$. 2+5=7
- c) Define Binomial theorem. Find the coefficient of $x^{10} y^{15}$ from the expansion of $(x + y)^{25}$. 2+5=7
- d) Write the algorithm to add two binary numbers. Describe with an example. 5+2=7
- e) Prove that, "If n is a composite integer, then n has a prime divisor less than or equal to \sqrt{n} ". 07
7. a) Prove that, "There must be infinitely many prime numbers". 05
- b) What is the probability that when two dice are rolled, the sum of the numbers on the two dice is 9? 05
- c) What is cryptography? Using the encryption function $E_n(x) = (x + n) \bmod 26$ and decryption function $D_n(x) = (x - n) \bmod 26$, encrypt the message "BAUST CSE" and then decrypt also. 2+6=8
- d) Each of the letters of the word "MISSISSIPPI" are written on separate pieces of paper that are then folded, put in a hat, and mixed thoroughly. One piece of paper is chosen (without looking) from the hat. What is the probability it is an "I"? 06
- e) Throw a die 2 times. What is the probability of getting two different sides? 06

- i) A special die is made in the shape of an octahedron: The die has 8 equal faces marked with the numbers 1 to 8. If the die is thrown once, what is the probability that the face that lands uppermost has a prime number? 05
8. a) Define algebraic structure. Write the main components of algebraic structure with examples. 2+5=7
- b) What is group? Is $\langle S_3, * \rangle$ a group where, $S_3 = \{R_0, R_1, R_2, F_0, F_1, F_2\}$? 2+9=11
- c) Prove that, $\langle Z_6, + \rangle$ is isomorphic to $\langle H, * \rangle$, where $Z_6 = \{0, 1, 2, 3, 4, 5\}$ and $H = \{R_0, R_1, R_2, R_3, R_4, R_5\}$. 12
- d) Prove that, $\langle Z_6, +, * \rangle$ is a commutative ring with unity. 05