CHAPTER 2. LINEAR FIXED EFFECTS MODELS

Basics

In this chapter, we consider fixed effects methods for data in which the dependent variable is measured on an interval scale and is linearly dependent on a set of predictor variables. We have a set of individuals ($i = 1, \ldots, n$), each of whom is measured at two or more points in time ($t = 1, \ldots, T$). Each time point will be referred to as a "period."

Here's the notation. We let y_{it} be the dependent variable. We have a set of predictor variables that vary over time, represented by the vector $\mathbf{x}_{it}$, and another set of predictor variables $\mathbf{z}_i$ that do not vary over time. (If you're not comfortable with vectors, you can interpret these as single variables). Our basic model for y is

$$y_{it} = \mu_t + \beta\mathbf{x}_{it} + \gamma\mathbf{z}_i + \alpha_i + \varepsilon_{it} \tag{2.1}$$

where μ_t is an intercept that may be different for each period, and β and γ are vectors of coefficients. Although Equation 2.1 seems to be strictly cross-sectional, there is nothing to prevent the $\mathbf{x}_{it}$ vector from including lagged versions of the x variables, except that one must have at least three periods to estimate a model with a lag of one period.

The two "error" terms, α_i and ε_{it}, behave somewhat differently from each other. There is a different ε_{it} for each individual at each point in time, but α_i only varies across individuals, not over time. We regard α_i as representing the combined effect on y of all unobserved variables that are constant over time. On the other hand, ε_{it} represents purely random variation at each point in time.

At this point, I'll make some rather strong assumptions about ε_{it}, namely, that each ε_{it} has a mean of zero, has a constant variance (for all i and t), and is statistically independent of everything else (except for y). The assumption of zero mean is not critical as it is only relevant for estimating the intercept. The constant variance assumption can sometimes be relaxed to allow for different variances for different t. Note that the ε_{it} at any one period is independent of $\mathbf{x}_{it}$ at any *other* period, which means that $\mathbf{x}_{it}$ is *strictly exogenous*. This assumption may be relaxed in some situations, but the issues involved are neither trivial nor purely technical. I'll discuss some of those issues in Chapter 6.

As for α_i, the traditional approach in fixed effects analysis is to assume that this term represents a set of n fixed parameters that can either be

directly estimated or removed in some way from the estimating equations. As noted in Chapter 1, we'll take a more modern approach in this chapter by assuming that α_i represents a set of random variables. Although we'll assume statistical independence of α_i and ε_{it}, we allow for *any* correlations between α_i and $\mathbf{x}_{it}$, the vector of time-varying predictors. And if we are not interested in γ, we can also allow for any correlations between α_i and $\mathbf{z}_i$. The inclusion of such correlations distinguishes the fixed effects approach from a random effects approach and allows us to say that the fixed effects method "controls" for time-invariant unobservables. At this point, we don't need to impose any restrictions on the mean and variance of α_i.

The Two-Period Case

Estimation of the model in Equation 2.1 is particularly easy when the variables are observed at only two periods ($T = 2$). The two equations are then

$$y_{i1} = \mu_1 + \beta\mathbf{x}_{i1} + \gamma\mathbf{z}_i + \alpha_i + \varepsilon_{i1}$$

$$y_{i2} = \mu_2 + \beta\mathbf{x}_{i2} + \gamma\mathbf{z}_i + \alpha_i + \varepsilon_{i2} \tag{2.2}$$

If we subtract the first equation from the second, we get the "first difference" equation:

$$y_{i2} - y_{i1} = (\mu_2 - \mu_1) + \beta(\mathbf{x}_{i2} - \mathbf{x}_{i1}) + (\varepsilon_{i2} - \varepsilon_{i1}) \tag{2.3}$$

which can be rewritten as

$$\Delta y_i = \Delta\mu + \beta\Delta\mathbf{x}_i + \Delta\varepsilon_i \tag{2.4}$$

where Δ indicates a difference score. Note that both α_i and $\gamma\mathbf{z}_i$ have been "differenced out" of the equation. Hence, we no longer have to be concerned about α_i and its possible correlation with $\Delta\mathbf{x}_i$. On the other hand, we also lose the possibility of estimating γ. Since $\mathbf{x}_{i1}$ and $\mathbf{x}_{i2}$ are each independent of ε_{i1} and ε_{i2}, it follows that $\Delta\mathbf{x}_i$ is independent of $\Delta\varepsilon_i$. This implies that one can get unbiased estimates of β by doing ordinary least squares (OLS) regression on the difference scores.

We now apply this method to some real data. Our sample comes from the National Longitudinal Survey of Youth (NLSY; Center for Human Resource Research, 2002).[1] Drawing a subset of a much larger sample, we have 581 children who were interviewed in 1990, 1992, and 1994. Initially, we will work with just three variables that were measured in each of the three interviews:

ANTI	antisocial behavior (scale ranges from 0 to 6)
SELF	self-esteem (scale ranges from 6 to 24)
POV	coded 1 if family is in poverty, otherwise 0

At this point, we're going to ignore the observations in the middle year, 1992, and use only the data in 1990 and 1994. Our objective is to estimate a linear equation with ANTI as the dependent variable[2] and SELF and POV as independent variables:

$$\text{ANTI}_t = \mu_t + \beta_1\text{SELF}_t + \beta_2\text{POV}_t + \alpha + \varepsilon_t, \quad t = 1, 2 \tag{2.5}$$

By expressing the model in this way, we are assuming a particular direction of causation, specifically, that SELF and POV affect ANTI and not the reverse. We also assume that the effects are contemporaneous (no lagged effects of SELF and POV). Both these assumptions will be relaxed in Chapter 6. Last, we assume that β_1 and β_2 are the same at both periods, an assumption that we will relax very shortly. On the other hand, we let the intercept μ_t be different at each period, allowing for changes in the average level of antisocial behavior that are not a consequence of any changes in SELF or POV.

Let us begin by estimating Equation 2.5 separately for each period, using ordinary least squares regression. The results are shown in the first two columns of Table 2.1. Not surprisingly, in both years, poverty is associated with higher levels of antisocial behavior, whereas self-esteem is associated with lower levels. The coefficients are quite similar across the two years.

In neither of these two regressions are there any controls for time-invariant predictors, such as sex and race. Rather than putting such variables in the equation, however, we can control for *all* time-invariant predictors by doing the regression with difference scores. For each child and each variable, we subtract the 1990 value from the 1994 value and then regress the ANTI difference on the SELF difference and the POV difference. Since POV is a dummy variable, it might seem inappropriate to subtract one value from the other. But, in fact, dummy variables can be treated just like any other variables in this regard.

The results are in the last column of Table 2.1. Although the equation was estimated in the form of difference scores, the coefficients can be interpreted as if we had estimated Equation 2.5 directly. That is, they represent the effects of each variable in a given year on the value of the dependent variable in the

Table 2.1 OLS Regression of Antisocial Behavior on Self-Esteem and Poverty

	1990		*1994*		*Difference Score*	
	Coefficient	*Standard Error*	*Coefficient*	*Standard Error*	*Coefficient*	*Standard Error*
Intercept	2.375**	0.384	2.888**	0.447	0.209**	0.063
Self-esteem	−0.050**	0.019	−0.064**	0.021	−0.056**	0.015
Poverty	0.595**	0.126	0.547**	0.148	−0.036	0.128
R^2	0.05		0.04		0.02	

** $p < .01$.

same year. For self-esteem, the estimated coefficient in the difference score model is in between the coefficients for the two separate years and still highly significant. For poverty, on the other hand, the coefficient is dramatically smaller and no longer statistically significant.

It's fairly common for fixed estimates to vary markedly from those produced by other methods. In this case, one possible explanation is that the estimates for the poverty effect in the regressions for the separate years were spurious, reflecting the correlation between poverty and some time-invariant variables that affected antisocial behavior.

One should not be too hasty about that conclusion, however. Whenever conventional regression produces a significant coefficient but fixed effects regression does not, there are two possible explanations: (a) The fixed effects coefficient is substantially smaller in magnitude and/or (b) the fixed effects standard error is substantially larger. As already mentioned, standard errors for fixed effects coefficients are often larger than those for other methods, especially when the predictor variable has little variation over time. In fact, most of the variation in poverty is between girls, with only about 24% of the girls moving into or out of poverty between 1990 and 1994.

Nevertheless, the standard error of the poverty coefficient in the difference score model is about the same as the standard error in 1990 and smaller than the standard error in 1994. The conclusion, then, is that insufficient variation is not a problem here. There seems to be a real and substantial decline in the magnitude of the poverty effect when time-invariant variables are controlled. The general lesson is this: Whenever p values for fixed effects methods are noticeably different from those for other methods, always check both the coefficients and their standard errors.

Note, finally, that the intercept of 0.209 is highly significant. This coefficient represents the estimated change in antisocial behavior from Time 1 to Time 2 for a person who did *not* change in self-esteem or poverty.

Extending the Difference Score Method for the Two-Period Case

The basic fixed effects model of Equation 2.1 can be extended to allow the effects of **x** and **z** to vary over time. In the two-period case, we can write the equations with distinct coefficients at each period:

$$y_{i1} = \mu_1 + \beta_1\mathbf{x}_{i1} + \gamma_1\mathbf{z}_i + \alpha_i + \varepsilon_{i1}$$

$$y_{i2} = \mu_2 + \beta_2\mathbf{x}_{i2} + \gamma_2\mathbf{z}_i + \alpha_i + \varepsilon_{i2} \qquad (2.6)$$

Taking first differences and rearranging terms produces

$$y_{i2} - y_{i1} = (\mu_2 - \mu_1) + \beta_2(\mathbf{x}_{i2} - \mathbf{x}_{i1}) + (\beta_2 - \beta_1)\mathbf{x}_{i1} + (\gamma_2 - \gamma_1)\mathbf{z}_i + (\varepsilon_{i2} - \varepsilon_{i1}) \qquad (2.7)$$

which could also be written as

$$\Delta y_i = \Delta\mu + \beta_2\Delta\mathbf{x}_i + \Delta\beta\mathbf{x}_1 + \Delta\gamma\mathbf{z}_i + \Delta\varepsilon_i$$

There are three things to note about this equation. First, as before, α_i has dropped out, so we don't have to be concerned about its potential confounding effects. Second, **z** has *not* dropped out, and its coefficient vector is the difference in the coefficient vectors for the two time points. From this we learn that time-invariant variables whose coefficients vary over time *must* be explicitly included in the regression equation. Fixed effects only controls for time-invariant variables with time-invariant effects. Third, the equation now includes $\mathbf{x}_1$ as a predictor, and its coefficient vector is the difference in the coefficient vectors for the two time periods. Thus, for **z** and $\mathbf{x}_1$, tests for whether their coefficients are 0 are equivalent to testing whether $\beta_1 = \beta_2$ or $\gamma_1 = \gamma_2$.

Let's try this for the NLSY data. The data set includes several time-invariant variables that we'll examine as possible predictors:

BLACK	1 if child is black, otherwise 0
HISPANIC	1 if child is Hispanic, otherwise 0
CHILDAGE	child's age in 1990
MARRIED	1 if mother was currently married in 1990, otherwise 0
GENDER	1 if female, 0 if male
MOMAGE	mother's age at birth of child
MOMWORK	1 if mother was employed in 1990, otherwise 0

The first two variables, BLACK and HISPANIC, represent two categories of a three-category variable—the reference category being white, non-Hispanic. These seven variables will now be included in the difference score regression for antisocial behavior, along with the difference scores for self-esteem and poverty. The 1990 measures of self-esteem and poverty are also included.

Results shown in Table 2.2 are consistent with the findings in Table 2.1. The coefficient for self-esteem (the difference score) is about −.06 and is highly significant, while the coefficient for poverty (the difference score) is

Table 2.2 OLS Estimates of Extended Difference Score Model

	Coefficient	*Standard Error*	*p Value*
Intercept	−0.550	1.360	.6859
Self-esteem difference	−0.060	0.020	.0024
Poverty difference	0.031	0.156	.8446
Self-esteem 1990	−0.018	0.025	.4826
Poverty 1990	0.121	0.178	.4991
Black	−0.100	0.155	.5185
Hispanic	0.084	0.164	.6109
ChildAge	0.220	0.107	.0409
Married	−0.206	0.154	.1808
Gender	0.101	0.126	.4262
MomAge	−0.040	0.030	.1842
MomWork	−0.153	0.140	.2735

far from significant. Neither the 1990 self-esteem coefficient nor the 1990 poverty coefficient approaches statistical significance, indicating that the effects of self-esteem and poverty were stable over time. Of the seven time-invariant variables, only one—child's age in 1990—is statistically significant (just barely). That doesn't mean that the other six variables are not affecting antisocial behavior. Rather, it means that their effects in 1990 and 1994 are essentially the same.

A First-Difference Method for Three or More Periods per Individual

When each individual is observed at three or more points in time ($T > 2$), it's not so obvious how to extend the methods we just considered. For the NLSY data, we actually have three years of data—1990, 1992, and 1994. One possible approach is to construct and estimate two first-difference equations. Starting with Equation 2.2, we have

$$y_{i2} - y_{i1} = (\mu_2 - \mu_1) + \beta(\mathbf{x}_{i2} - \mathbf{x}_{i1}) + (\varepsilon_{i2} - \varepsilon_{i1})$$
$$y_{i3} - y_{i2} = (\mu_3 - \mu_2) + \beta(\mathbf{x}_{i3} - \mathbf{x}_{i2}) + (\varepsilon_{i3} - \varepsilon_{i2}) \qquad (2.8)$$

These equations can be estimated separately by OLS, and each will give unbiased estimates of β. The first two columns of Table 2.3 show the results for the NLSY data. The negative coefficients for self-esteem are highly significant in both difference equations and are roughly of the same magnitude. Poverty is far from significant in both equations. The intercepts represent the change from one period to the next, after adjusting for the two predictor variables. Although there was an increase in antisocial behavior for each 2-year interval, only the change from 1992 to 1994 was statistically significant.

Under the assumption that β doesn't vary over time, the two equations should be estimated simultaneously for optimal efficiency. This can be accomplished by creating a single data set with two records for each person, one with the difference scores for the first equation and the other with the difference scores for the second equation. There should also be a dummy variable distinguishing the first record from the second record. And also, there should be a variable with a common ID number for the two records from each person.

The third column of Table 2.3 shows the results of applying OLS to this combined data set of 1,162 records. Not surprisingly, the coefficients for self-esteem and poverty are in between their respective coefficients in the first two columns. The standard errors are appreciably lower, however,

Table 2.3 First-Difference Regressions of Antisocial Behavior on Self-Esteem and Poverty

	1992–1994 OLS		*1990–1992 OLS*		*Combined OLS*		*Combined GLS*	
	Coefficient	*Standard Error*	*Coefficient*	*Standard Error*	*Coefficient*	*Standard Error*	*Coefficient*	*Standard Error*
Intercept	0.71**	0.059	0.040	0.053	0.045	0.056	0.05	0.056
Self-esteem	−0.072**	0.016	−0.039**	0.014	−0.055**	0.010	−0.055**	0.010
Poverty	0.216	0.136	0.197	0.133	0.213	0.095	0.139	0.094
Equation dummy					0.122	0.080	0.122	0.094

**$p < .01$.

because more information is being used. The intercept can be interpreted as an estimate of $\mu_2 - \mu_1$ while the coefficient for the equation dummy is an estimate of $(\mu_3 - \mu_2) - (\mu_2 - \mu_1)$. Both are positive, indicating an increase in antisocial behavior from Time 1 to Time 2, and a more rapid increase from Time 2 to Time 3. But neither is statistically significant.

Although the combined OLS estimates should be unbiased, they ignore the fact that $\varepsilon_2 - \varepsilon_1$ is likely to be negatively correlated with $\varepsilon_3 - \varepsilon_2$ because they share a common component, ε_2, with opposite signs. This implies that the coefficient estimates may not be fully efficient and the standard error estimates may be biased. We can solve this problem by estimating the correlation between the error terms and then using generalized least squares (GLS) to take account of that correlation.

Most comprehensive statistical packages have routines to do GLS. Such programs typically require the specification of an ID variable so that observations from the same individual can be identified. I used the **xtreg** command in Stata with the **pa** option, which estimates the linear model using GLS.[3] The GLS results, shown in the last column of Table 2.3, are very similar to the OLS coefficient estimates and standard errors in the preceding column.

The first-difference method can be easily extended to more than three periods per individual. For T periods per individual, $T - 1$ records are created, each with difference scores between adjacent time points for all variables. Additionally, there should be a variable containing a common ID number for all observations from the same individual, and a variable or a set of dummy variables to distinguish the different records. The regression is then estimated on the entire set of records, using GLS to adjust for correlations among the error terms. Unless T is large, for example, greater than 10, it's probably best to allow the error correlation matrix to be unstructured. That is, the matrix would allow for a different correlation between each pair of error terms. With larger T, it may be preferable to impose a simplified structure to reduce the number of distinct correlations that need to be estimated. For more details, see Greene (2000).

Dummy Variable Method for Two or More Periods per Individual

Although the multiple-difference-score method is a reasonable way to estimate a fixed effects model for the multiperiod case, the name "fixed effects" is usually reserved for a different method, one that can be implemented either by dummy variables or by constructing mean deviations. The results produced by the fixed effects method are not identical to those produced by the difference-score method, although they will usually be very similar. In the two-period case, the two methods give identical results.

The dummy variable method requires a data set with a rather different structure: one record for each period for each individual. For the NLSY data, for example, the required data set has three records for each of the 581 children, for a total of 1,743 records. The time-varying variables have the same variable names on each record but different values. For any time-invariant variables, their values are simply replicated across the multiple records for each individual. There should also be an ID variable with a common value for all the records for each individual. Last, there should be a variable distinguishing the different periods for each individual. For the NLSY data, for example, the variable TIME has values of 1, 2, and 3, corresponding to 1990, 1992, and 1994. Table 2.4 shows the first 15 records of this data set, corresponding to the first five persons.

Table 2.4 Data Set With Three Observations per Person (First Five Persons)

ID	*TIME*	*ANTI*	*SELF*	*POV*	*GENDER*
1	1	1	21	1	1
1	2	1	24	1	1
1	3	1	23	1	1
2	1	0	20	0	1
2	2	0	24	0	1
2	3	0	24	0	1
3	1	5	21	0	0
3	2	5	24	0	0
3	3	5	24	0	0
4	1	2	23	0	0
4	2	3	21	0	0
4	3	1	21	0	0
5	1	1	22	0	1
5	2	0	23	0	1
5	3	0	24	0	1

To implement the method, it's necessary to construct a set of dummy variables to distinguish the individuals in the data set. In our example, that means 580 dummy variables to represent the 581 children. Many statistical packages can do this automatically by specifying the ID variable as a categorical variable. If the TIME variable is also specified as categorical, two dummy variables will be created to distinguish the three different years. One can then use OLS to estimate the coefficients. The coefficients for the dummy variables created from the ID variable are actually the estimates of the α_i in Equation 2.1, under the constraint that one of them is equal to 0.

I did this in Stata using the `reg` command,[4] with results shown in the left-hand panel of Table 2.5. Only the coefficients for the first nine dummy variables are shown.

Table 2.5 Regression of Antisocial Behavior on Self-Esteem and Poverty, Dummy Variable Method

	Fixed Effects			*Conventional OLS*		
	Coefficient	*Standard Error*	*p*	*Coefficient*	*Standard Error*	*p*
SELF	−0.055	0.010	.00	−0.067	0.011	.00
POV	0.112	0.093	.23	0.518	0.079	.00
TIME_2	0.044	0.059	.45	0.051	0.090	.58
TIME_3	0.211	0.059	.00	0.223	0.091	.01
ID_2	−0.887	0.819	.28			
ID_3	4.131	0.811	.00			
ID_4	1.057	0.819	.20			
ID_5	−0.536	0.819	.51			
ID_6	0.040	0.820	.96			
ID_7	2.170	0.821	.01			
ID_8	0.910	0.820	.27			
ID_9	−0.276	0.819	.74			

Comparing the results in Table 2.5 with those in the last column of Table 2.3 (obtained with the first-difference method), we see that the coefficient and standard error for self-esteem are virtually identical. The coefficient for poverty is a little smaller using the dummy variable method, but it is far from significant with either method. The coefficients for TIME_2 and TIME_3 represent contrasts with the omitted category (TIME_1). We see that antisocial behavior increases, on average, over time, and that TIME_3 is significantly higher than TIME_1.

For comparison, the right-hand panel of Table 2.5 gives the OLS estimates of the coefficients without the inclusion of the 580 dummy variables. As we saw in the two-period case, the big difference in the results for the two methods is that the coefficient for POV is much larger for conventional OLS, and highly significant. Thus, the apparent effect of poverty on self-esteem disappears when we adjust for all between-person differences and focus only on within-person changes. It's also of some interest to compare the standard errors. The standard error for the POV coefficient is larger for the fixed effects estimate, a fairly typical result that stems from not using the between-person variation. On the other hand, for the coefficients for SELF and the two TIME dummies, the fixed effects standard errors are actually smaller than those for conventional OLS. Why the difference? It's all a matter of the relative magnitudes of within- and between-person variation. For POV, 70% of the variation is between persons, while for SELF the figure is only 53%.[5] For the TIME dummies, all the variation is within person and none is between. The best situation for a fixed effects analysis is when all the variation on a time-varying predictor is within persons, but there's still a lot of between-person variation on the response variable.

The problem with the dummy variable method is that the computational requirement of estimating coefficients for all the dummy variables can be quite burdensome, especially in large samples where it may be beyond the capacity of the software or the machine memory. Fortunately, there is an alternative algorithm—the mean deviation method—that produces exactly the same results. The one drawback is that it doesn't give estimates for the coefficients of the dummy variables representing different persons, but those are rarely of interest anyway.

The mean deviation algorithm works like this. For each person and for each time-varying variable (both response and predictor variables), we compute the means over time for that person:

$$\bar{y}_i = \frac{1}{n_i}\sum_t y_{it}$$

$$\bar{\mathbf{x}}_i = \frac{1}{n_i}\sum_t \mathbf{x}_{it}$$

where n_i is the number of measurements for person i. Then we subtract the person-specific means from the observed values of each variable:

$$y^*_{it} = y_{it} - \bar{y}_i$$

$$\mathbf{x}^*_{it} = \mathbf{x}_{it} - \bar{\mathbf{x}}_i$$

Finally, we regress y^* on $\mathbf{x}^*$, plus variables to represent the effect of time. This is sometimes called a "conditional" method because it conditions out the coefficients for the fixed effects dummy variables.

If you construct the deviation scores yourself and then use an ordinary regression program to estimate the coefficients, you will get the correct OLS estimates for all the coefficients. However, the standard errors and p values will not be correct. That's because the calculation of the degrees of freedom is based on the number of variables in the specified model, when it should actually include the number of dummy variables implicitly used to represent different persons in the sample (580 for the NLSY data). Formulas are available to correct the standard errors and p values (Judge, Hill, Griffiths, & Lee, 1985), but it's much easier to let the software do it for you. For example, the **xtreg** command[6] in Stata does the correct calculations for a fixed effects model; SAS does it with the ABSORB statement in PROC GLM.

Using the **xtreg** command, I specified a fixed effects model (FE option) with ID as the variable that identifies records from the same person. Results were identical to the first five lines of Table 2.5. **xtreg** also reports several additional statistics that are specific to fixed effects models:

1. An F test of the null hypothesis that all the coefficients for the fixed effects dummy variables are zero. In this case, the p value is less than .0001, so the null hypothesis can be confidently rejected. This is equivalent to saying that there is evidence for person-level unobserved heterogeneity. That is, there are stable differences in antisocial behavior between persons that are not fully accounted for by the measured predictor variables.

2. An estimate of the proportion of variance in the dependent variable that is attributable to the fixed effects (the α_is), labeled "rho (fraction of variance due to u_i)." In this case, the estimate is 0.64.

3. An estimate of the correlation between the fixed effects α_i and $\hat{\beta}\mathbf{x}_{it}$, the estimated linear combination of the time-varying predictors. In a random effects model, this correlation is assumed to be 0. For these data, the correlation is .068.

4. Three R^2s: within, between, and overall. The within R^2 is just the usual R^2 calculated for the regression using the mean deviation variables. Here it's .033. The between R^2 is the squared correlation between the person-specific mean of y and the *predicted* person-specific mean of y. In this case, it's .041. Finally, the overall R^2 (.036 in this case) is the squared correlation between y itself and the predicted value of y. All three of these R^2s are calculated using predicted values based on the estimated regression coefficients but *not* using the coefficients for the fixed effects dummy variables. If you include those (using the dummy variable method), the R^2 goes up to .73 for these data.

As noted before, one characteristic of this method is the inability to estimate coefficients for time-invariant predictors. This is evident from the fact that subtracting the person-specific mean of a time-invariant predictor from the individual values (which are the same at all periods) yields a value of 0 for all persons. Keep in mind, however, that we are still controlling all time-invariant predictors even though they drop out of the equation. In the next section, moreover, we'll see how to test whether the effects of those variables are themselves time invariant.

Interactions With Time in the Fixed Effects Method

In the two-period case, we saw how to extend the method to allow the coefficients for predictor variables to change over time. For a time-varying predictor, we simply added the variable measured at time 1 to the model. For a time-invariant predictor, we added the variable itself to the model. For the dummy variable method (or the equivalent mean deviation method), these extensions are accomplished by including interactions with time.

For the three-period NLSY data, Table 2.6 shows the results of including interactions between TIME (treated as categorical) and both time-varying and time-invariant predictors. Since TIME has three categories, there are two interactions with each predictor. Note that the time-invariant predictors do not have main effects included in the model. If we had tried to include them, the software would have dropped them from the model because they have no variation within persons.

For each of the interactions, the t statistic tests whether a coefficient at Time 2 or Time 3 is different from the coefficient at Time 1. Of the 18 interactions, only one (TIME_3*CHILDAGE) is statistically significant (p = .024). For that interaction, the coefficient of 0.227 indicates that the

Table 2.6 Interactions With Time

	Coefficient	*Standard Error*	*t*	*p*
TIME_2	0.291	1.245	0.23	.82
TIME_3	–0.444	1.258	–0.35	.72
SELF	–0.034	0.016	–2.08	.04
POV	0.097	0.130	0.75	.46
TIME_2 * SELF	–0.026	0.020	–1.28	.20
TIME_3 * SELF	–0.023	0.021	–1.09	.28
TIME_2 * POV	–0.112	0.152	–0.74	.46
TIME_3 * POV	0.099	0.155	0.64	.52
TIME_2 * BLACK	0.250	0.144	1.74	.08
TIME_3 * BLACK	–0.110	0.144	–0.77	.44
TIME_2 * HISPANIC	0.190	0.154	1.23	.22
TIME_3 * HISPANIC	0.075	0.153	0.49	.62
TIME_2 * CHILDAGE	0.076	0.100	0.76	.45
TIME_3 * CHILDAGE	0.227	0.100	2.26	.02
TIME_2 * MARRIED	–0.095	0.143	–0.67	.51
TIME_3 * MARRIED	–0.176	0.143	–1.23	.22
TIME_2 * GENDER	0.041	0.118	0.35	.73
TIME_3 * GENDER	0.107	0.118	0.91	.37
TIME_2 * MOMAGE	–0.027	0.028	–0.96	.34
TIME_3 * MOMAGE	–0.042	0.028	–1.52	.13
TIME_2 * MOMWORK	0.0137	0.131	1.05	.29
TIME_3 * SMOMWORK	–0.144	0.130	–1.11	.27

coefficient for CHILDAGE is 0.227 higher at Time 3 than at Time 1. Of course, with 18 tests it's a pretty good bet that at least one of them is statistically significant at the .05 level even if there's nothing really going on. A simultaneous test that all 18 interaction coefficients are equal to 0 yields a p value of .15.

Comparison With Random Effects Models

A popular alternative to the linear fixed effects model is the random effects or mixed model. This model is based on the same equation that we used for the fixed effects model:

$$y_{it} = \mu_t + \beta \mathbf{x}_{it} + \gamma \mathbf{z}_i + \alpha_i + \varepsilon_{it} \tag{2.9}$$

The crucial difference is that now, instead of treating α_i as a set of fixed numbers (which is equivalent to treating α_i as random but with all possible correlations with $\mathbf{x}_{it}$), we assume that α_i is a set of random variables with a specified probability distribution. For example, it is typical to assume that each α_i is normally distributed with a mean of 0, constant variance, and is independent of all the other variables on the right-hand side of the equation.

There's a lot of software available to estimate the random effects model. SAS can do it with the MIXED procedure. The `xtreg` command in Stata does GLS estimation of the random effects model by default. Table 2.7 presents the estimates produced by `xtreg`, both with and without time-invariant predictors.

The fact that the random effects method can include time-invariant predictors is the most apparent difference between the fixed and the random effects models. In this case, however, we see that the inclusion of those variables doesn't make much difference in the coefficients for the time-varying predictors, self-esteem, and poverty.

Like the conventional OLS estimates in Table 2.5 and unlike the fixed effects estimates, both variables have coefficients that are highly significant. The similarity of random effects estimates and OLS estimates is not surprising. If the random effects assumption that α is uncorrelated with all other variables is correct, both methods produce consistent (and therefore approximately unbiased estimates) of the coefficients in Equation 2.9. But when that assumption is incorrect, both methods will yield biased estimates.

Why is it that POV is highly significant in the random effects model but far from significant in the fixed effects model? As explained earlier, whenever a coefficient is significant in a random effects model but not in a