# 动态规划篇: 编辑距离问题

# 童咏昕

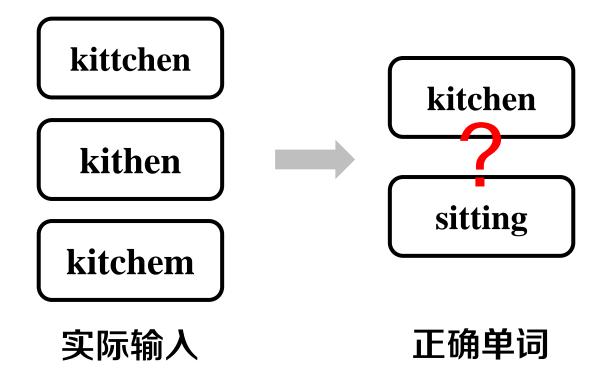
北京航空航天大学 计算机学院

中国大学MOOC北航《算法设计与分析》

# 问题背景



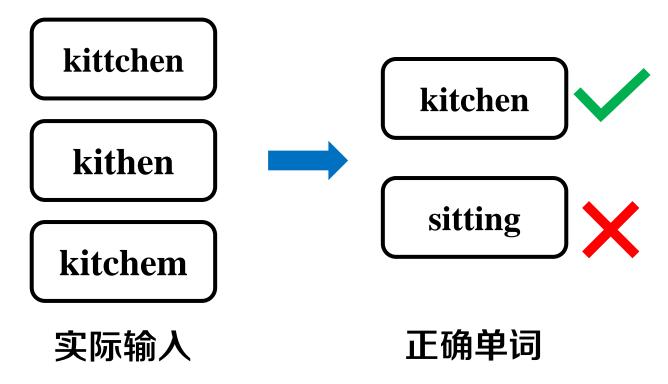
• 输入法自动更正



## 问题背景



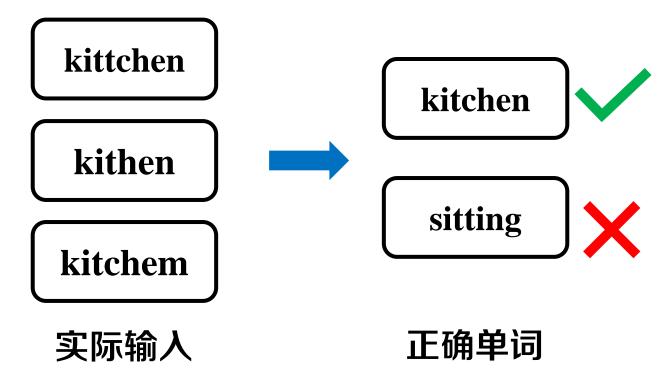
• 输入法自动更正



### 问题背景



• 输入法自动更正



问题: 如何衡量序列的相似程度?



• 基本思想

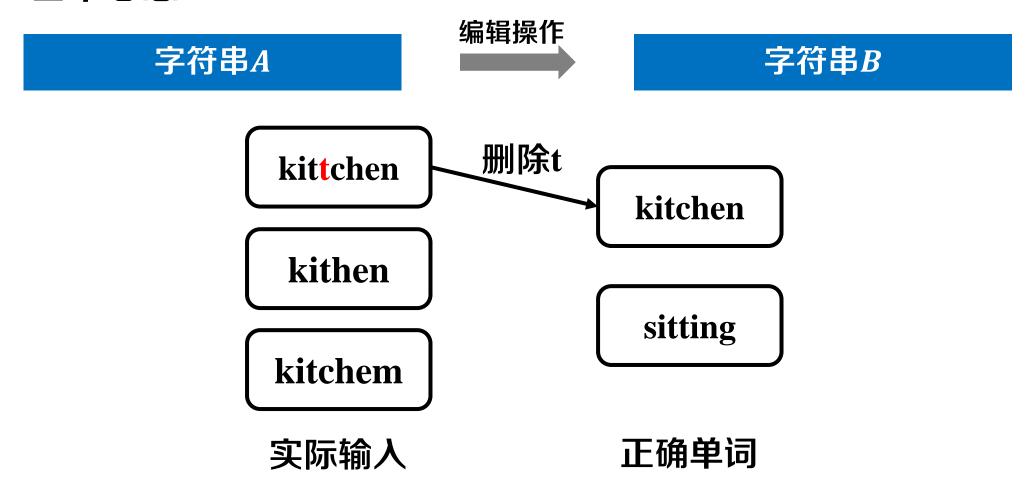
字符串A

编辑操作

字符串B

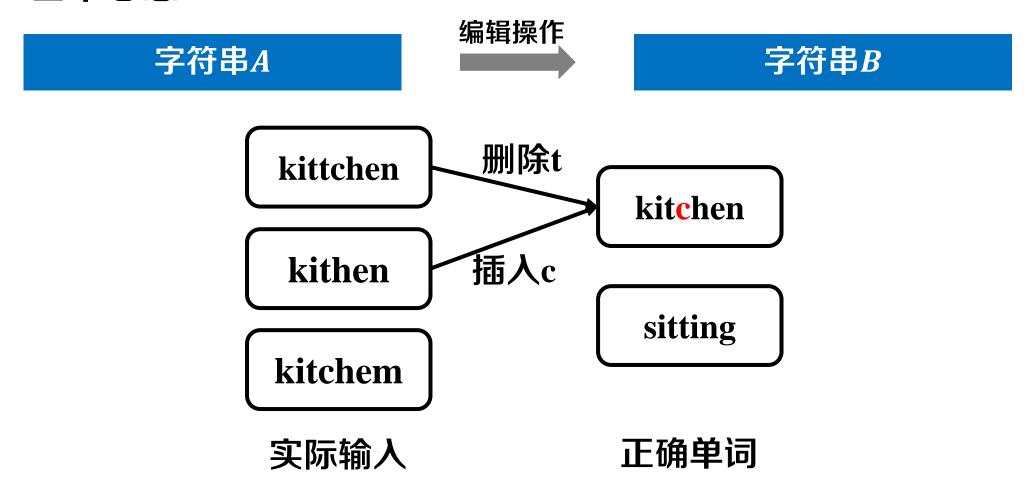


• 基本思想



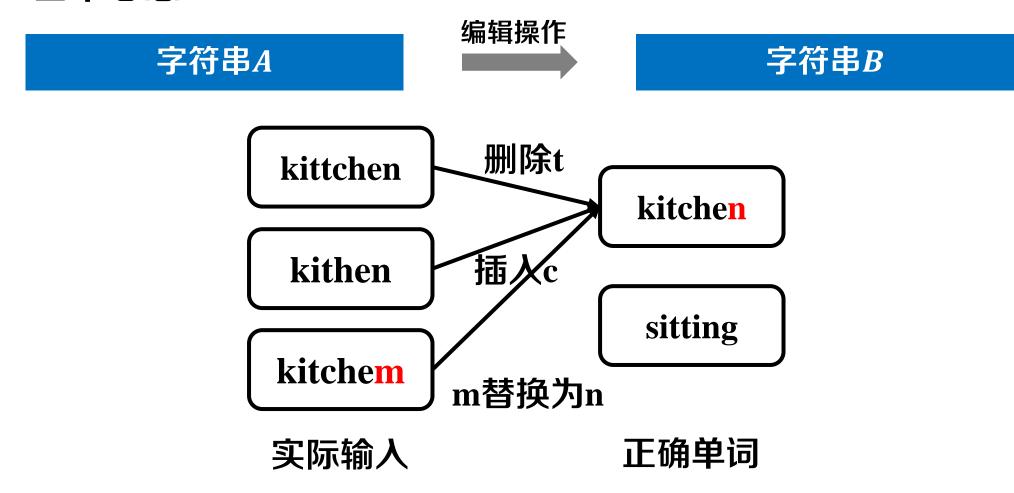


• 基本思想



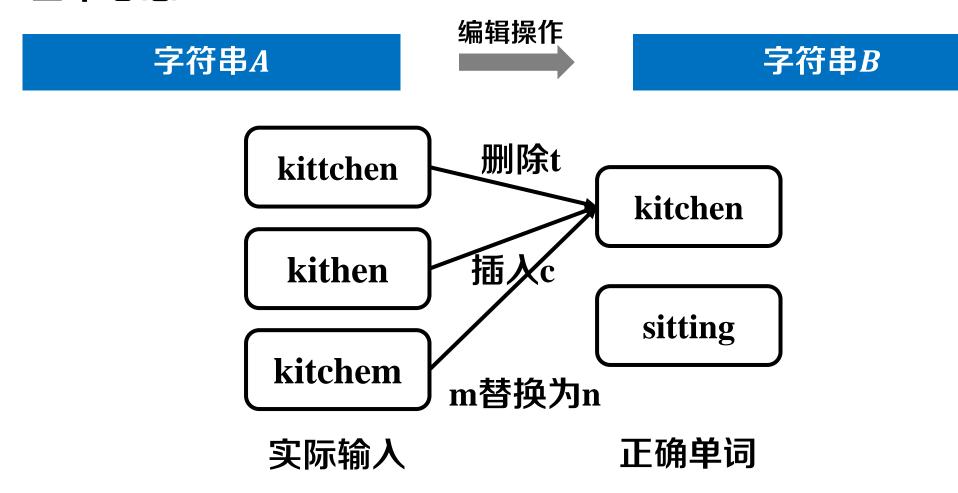


• 基本思想



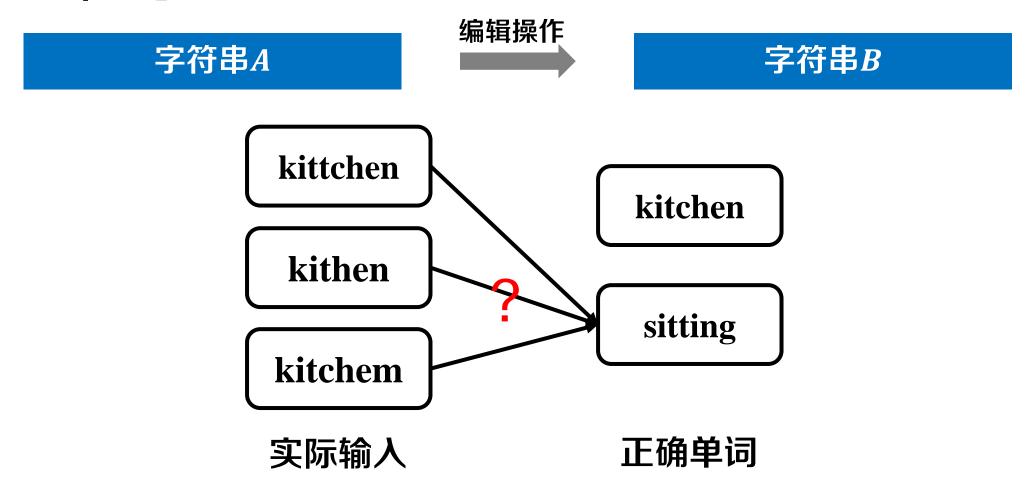


• 基本思想

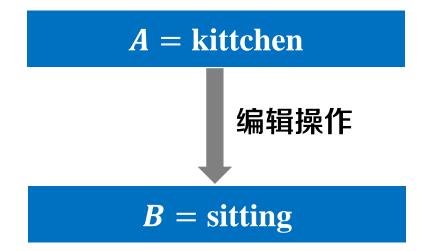




• 基本思想

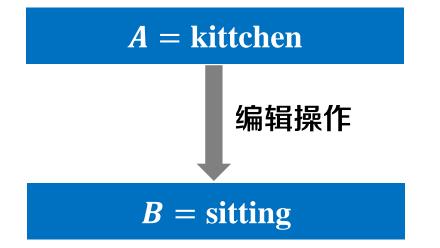






操作名称	操作示例
删除	kittchen → kitchen
插入	kithen → kit <mark>c</mark> hen
替换	kitchem → kitchen

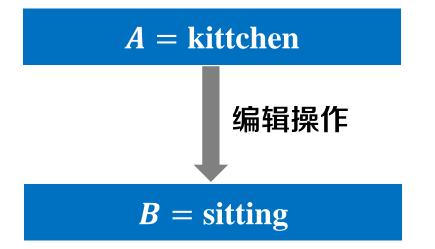




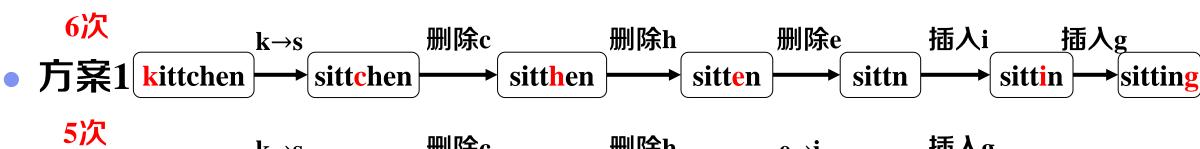
操作名称	操作示例
删除	kittchen → kitchen
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替换	kitchem → kitchen





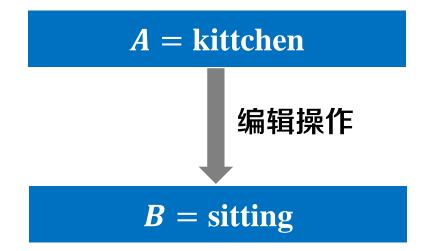


操作名称	操作示例
删除	kittchen → kitchen
插入	kithen → kit <mark>c</mark> hen
替换	kitchem → kitche <mark>n</mark>

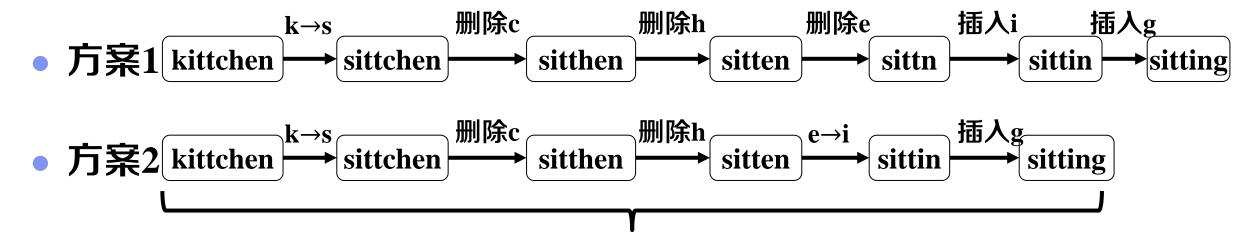


方案2 kittchen k→s sittchen 删除c sitthen 删除h sitten e→i sittin 插入g sitting



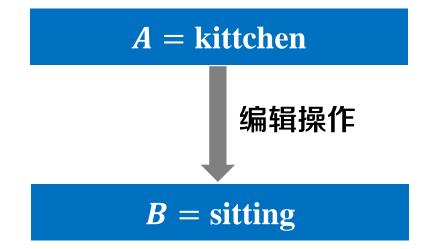


操作名称	操作示例
删除	kittchen → kitchen
插入	kithen → kit <b>c</b> hen
替换	kitchem → kitche <mark>n</mark>



编辑距离:编辑操作次数





操作名称	操作示例
删除	kittchen → kitchen
插入	kithen → kit <mark>c</mark> hen
替换	kitchem → kitchen





问题: 如何求出最少的编辑操作数(最小编辑距离)?



#### 编辑距离问题

#### **Minimum Edit Distance, MED**

#### 输入

• 长度为n的字符串s,长度为m的字符串t

#### 输出

• 求出一组编辑操作 $0 = \langle e_1, e_2, ... e_d \rangle$ , 令

优化目标

min |0|

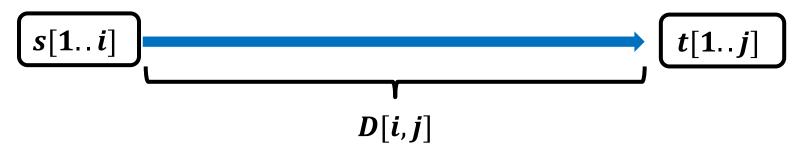
s.t. 字符串s经过o的操作后满足s=t

约束条件

#### 问题结构分析



- 给出问题表示
  - D[i,j]: 字符串S[1..i]变为t[1..j]的最小编辑距离



- 明确原始问题
  - D[n,m]: 字符串s[1..n]变为t[1..m]的最小编辑距离

问题结构分析



递推关系建立



自底向上计算



### 递推关系建立:回顾与启发

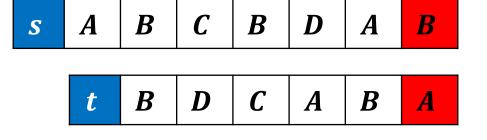


 $\boldsymbol{B}$ 

 $\boldsymbol{B}$ 

#### • 最长公共子序列

• 如果 $s_i \neq t_j$ 



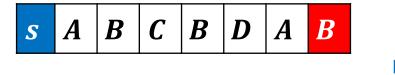
 $\boldsymbol{B}$  $\boldsymbol{B}$  $\boldsymbol{D}$  $\boldsymbol{B}$  $\boldsymbol{B}$  $\boldsymbol{A}$ D A B B A D S  $\boldsymbol{B}$  $\boldsymbol{B}$ A A

• 如果 $s_i = t_j$ 

### 递推关系建立



- 考察末尾元素
  - ●删除



 $t \mid B \mid D \mid C \mid A \mid B \mid A$ 

插入

s A B C B D A B ?

 $t \mid B \mid D \mid C \mid A \mid B \mid A$ 

● 替换

 $t \mid B \mid D \mid C \mid A \mid B \mid A$ 

问题结构分析

递推关系建立

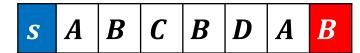


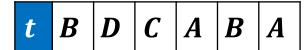
自底向上计算

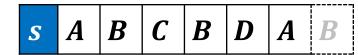




• 考察末尾元素: 删除







$$t \mid B \mid D \mid C \mid A \mid B \mid A$$

问题结构分析



递推关系建立

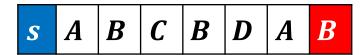


自底向上计算

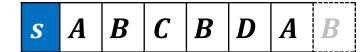




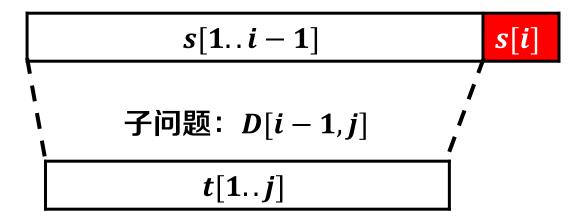
• 考察末尾元素: 删除



 $t \mid B \mid D \mid C \mid A \mid B \mid A$ 



 $t \mid B \mid D \mid C \mid A \mid B \mid A$ 



问题结构分析



递推关系建立

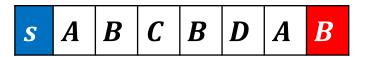


自底向上计算

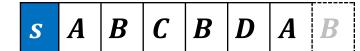




• 考察末尾元素: 删除



 $\boldsymbol{C}$ |B| $\boldsymbol{B}$ A  $\boldsymbol{D}$ 



 $\boldsymbol{B}$  $\boldsymbol{B}$ A A  $\boldsymbol{D}$ 

#### 问题结构分析



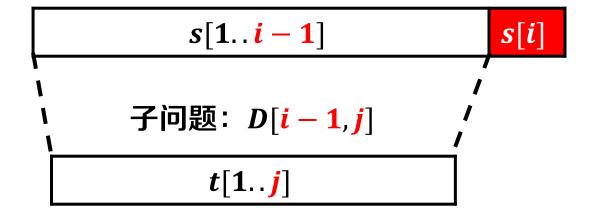
递推关系建立



自底向上计算



最优方案追踪



删除s[i] s[1..i-1]s[1..i]

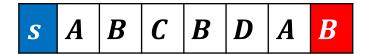
$$\begin{array}{c}
D[t-1,j] \\
t[1..j]
\end{array}$$

$$D[i,j]$$

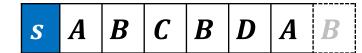
D[i-1,j]



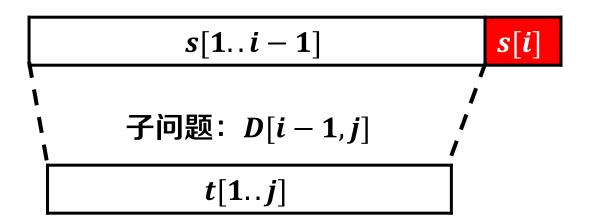
• 考察末尾元素: 删除



t B D C A B A



t B D C A B A



D[i,j] = D[i-1,j] + 1

问题结构分析



递推关系建立



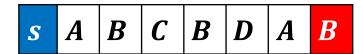
自底向上计算



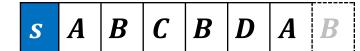


• 考察末尾元素: 删除

 $\bullet D[i,j] = D[i-1,j] + 1$ 



 $t \mid B \mid D \mid C \mid A \mid B \mid A$ 



 $t \mid B \mid D \mid C \mid A \mid B \mid A$ 





递推关系建立



自底向上计算



最优方案追踪



最优子结构



• 考察末尾元素: 插入

s A B C B D A B ?

 $t \mid B \mid D \mid C \mid A \mid B \mid A$ 

问题结构分析



递推关系建立

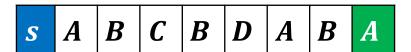


自底向上计算

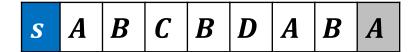




• 考察末尾元素: 插入



 $t \mid B \mid D \mid C \mid A \mid B \mid A$ 



 $t \mid B \mid D \mid C \mid A \mid B \mid A$ 

问题结构分析



递推关系建立

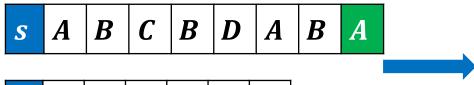


自底向上计算

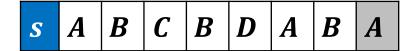




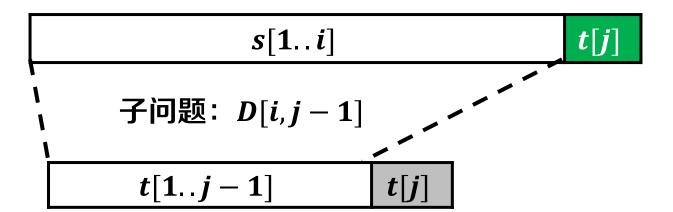
• 考察末尾元素: 插入



 $t \mid B \mid D \mid C \mid A \mid B \mid A$ 



 $t \mid B \mid D \mid C \mid A \mid B \mid A$ 



问题结构分析



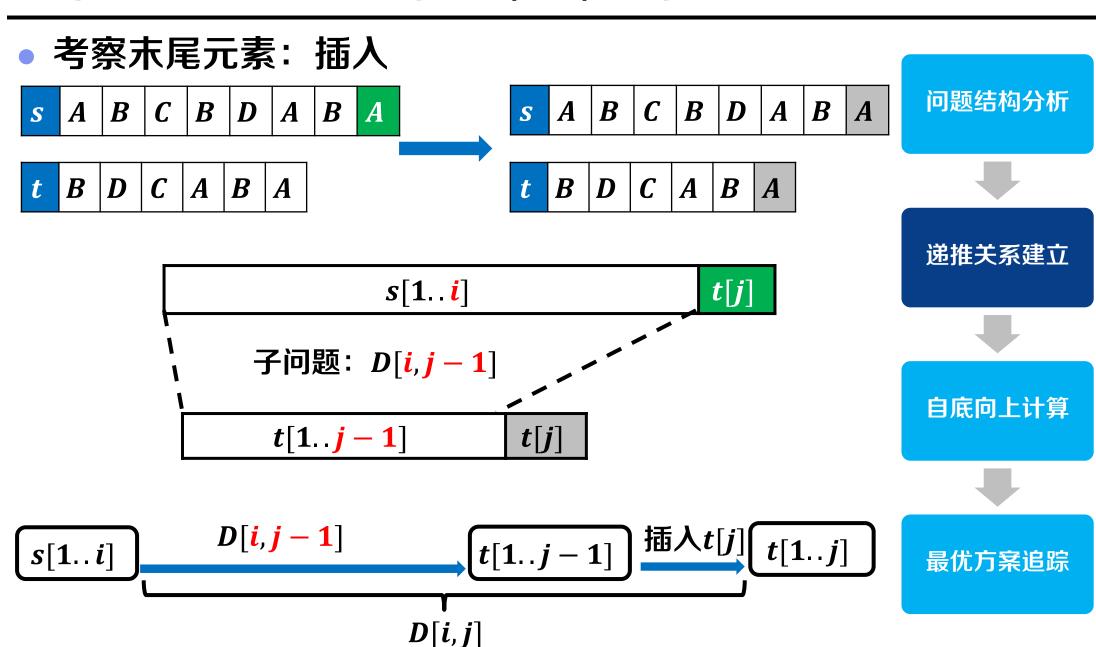
递推关系建立



自底向上计算







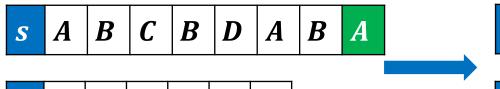


• 考察末尾元素: 插入

|A|B

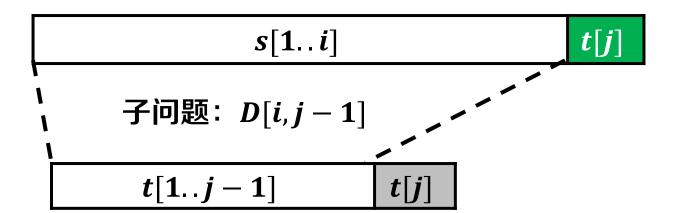
 $\boldsymbol{\mathcal{C}}$ 

 $D \mid$ 



s A B C B D A B A

t B D C A B A



• D[i,j] = D[i,j-1] + 1

问题结构分析



递推关系建立

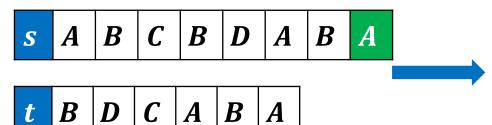


自底向上计算

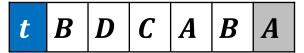




• 考察末尾元素: 插入



S A B C B D A B A



问题结构分析



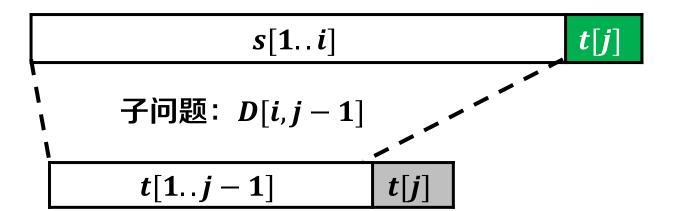
递推关系建立



自底向上计算



最优方案追踪

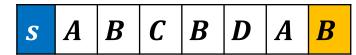


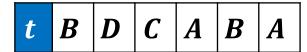
 $\bullet \left[D[i,j] = D[i,j-1] + 1\right]$ 

最优子结构



• 考察末尾元素: 替换





问题结构分析



递推关系建立

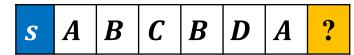


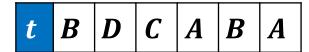
自底向上计算





• 考察末尾元素: 替换





问题结构分析



递推关系建立



自底向上计算



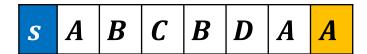
s[1..i-1]

子问题: D[i-1,j-1]

t[1..j-1]



• 考察末尾元素: 替换



 $t \mid B \mid D \mid C \mid A \mid B \mid A$ 



t B D C A B A

s[i]

t[j]





递推关系建立



自底向上计算



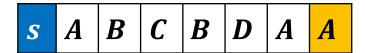
s[1..i-1]

子问题: D[i-1,j-1]

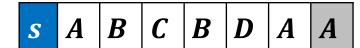
t[1..j-1]



• 考察末尾元素: 替换



 $t \mid B \mid D \mid C \mid A \mid B \mid A$ 



t B D C A B A

s[i]

#### 问题结构分析



递推关系建立



自底向上计算



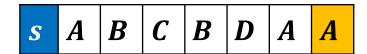
 $\begin{array}{c|c} s[1..i] \\ \hline \\ s[1..i-1] \\ \hline \end{array} \begin{array}{c} b[i-1,j-1] \\ \hline \\ t[1..j-1] \\ \hline \end{array} \begin{array}{c} t[1..j] \\ \hline \end{array}$ 

t[j]

D[i,j]

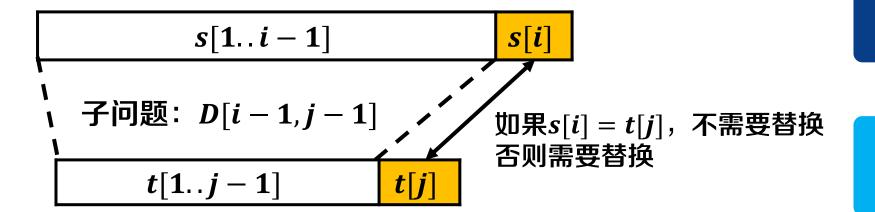


• 考察末尾元素: 替换



 $t \mid B \mid D \mid C \mid A \mid B \mid A$ 

t B D C A B A



•  $D[i,j] = D[i-1,j-1] + \begin{cases} 0, & \text{if } s[i] = t[j] \\ 1, & \text{if } s[i] \neq t[j] \end{cases}$ 

问题结构分析



递推关系建立



自底向上计算





• 考察末尾元素: 替换

 $t \mid B \mid D \mid C \mid A \mid B \mid A$ 

t B D C A B A

s[i]





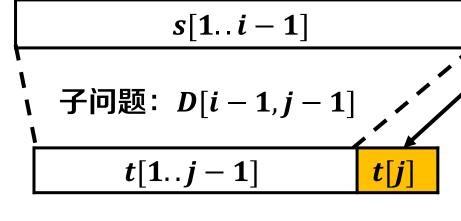
递推关系建立



自底向上计算



最优方案追踪



如果s[i] = t[j],不需要替换 否则需要替换

#### 递推关系建立: 构造递推公式



• 综合上面三种方式

• 
$$D[i,j] = \min egin{cases} D[i-1,j]+1 & ext{删除} \ D[i,j-1]+1 & ext{插入} \ D[i-1,j-1]+iggl\{0,\ if\ s[i]=t[j] \ 1,\ if\ s[i] 
otation iggl\}$$
 替换

问题结构分析



递推关系建立



自底向上计算



#### 递推关系建立: 构造递推公式



• 最小编辑距离 vs. 最长公共子序列

• 
$$D[i,j] = \min egin{cases} D[i-1,j]+1 & ext{删除} \ D[i,j-1]+1 & ext{插入} \ D[i-1,j-1]+iggl\{0,\ if\ s[i]=t[j] \ 1,\ if\ s[i] 
otation iggl\}$$
 替换

•  $C[i,j] = \begin{cases} \max\{C[i-1,j], C[i,j-1]\}, x_i \neq y_j \\ C[i-1,j-1] + 1 \end{cases}$ ,  $x_i = y_j$ 

问题结构分析



递推关系建立



自底向上计算

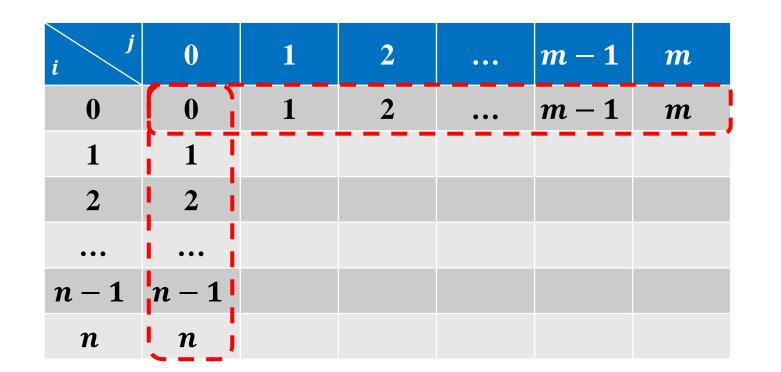


#### 自底向上计算:确定计算顺序



#### • 初始化

- D[i,0]=i
  - 把长度为i的串变为空串至少需要i次操作(删除)
- D[0,j]=j
  - · 把空串变为长度为j的串至少需要j次操作(插入)



问题结构分析



递推关系建立



自底向上计算



#### 自底向上计算:确定计算顺序



#### • 递推公式

• 
$$D[i,j] = \min egin{cases} D[i-1,j]+1 & ext{删除} \ D[i,j-1]+1 & ext{插入} \ D[i-1,j-1]+egin{cases} 0, \ if \ s[i]=t[j] \ 1, \ if \ s[i] 
eq t[j] \end{cases}$$
 替换

i j	0	1	2	•••	m-1	m
0	0	1	2	•••	m-1	m
1	1	0[i-1,i]	<b>– 1</b> ] + {	0	- 1, <i>j</i> ] +	
2	2	[ -, ]	-, (	$\begin{matrix} 1 & D[i] \end{matrix}$	-1, j] +	1
• • •	•••			D[i,j]		
n-1	n-1		D[i,j-1]	1] + 1		
$\boldsymbol{n}$	$\boldsymbol{n}$					

问题结构分析



递推关系建立



自底向上计算



#### 自底向上计算: 依次计算问题



#### • 递推公式

• 
$$D[i,j] = \min egin{cases} D[i-1,j]+1 & ext{删除} \ D[i,j-1]+1 & ext{插入} \ D[i-1,j-1]+iggl\{0,\ if\ s[i]=t[j]\ 1,\ if\ s[i] 
otag by$$

i j	0	1	2	•••	m-1	m
0	0	1	2	• • •	m-1	m
1	1					<del></del> >
2	2	4				<del>.=</del> >
•••	•••	<u> </u>				<del>.=</del> >
n-1	n-1	4				<del>.=&gt;</del>
n	n	<u> </u>			<b></b>	*

问题结构分析



递推关系建立



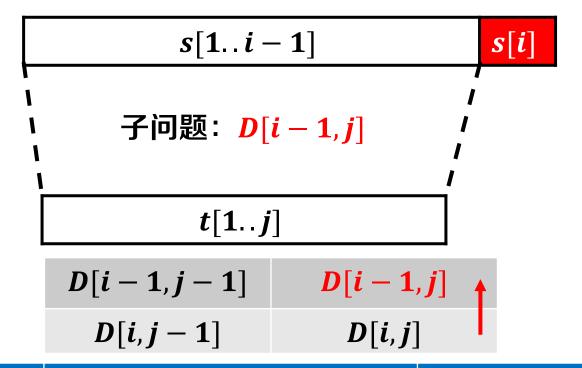
自底向上计算



#### 最优方案追踪:记录决策过程



• 追踪数组Rec,记录子问题来源



Rec[i,j]	子问题来源	操作
U	上侧,即 <i>D</i> [ <i>i</i> — 1, <i>j</i> ]	删除s[i]

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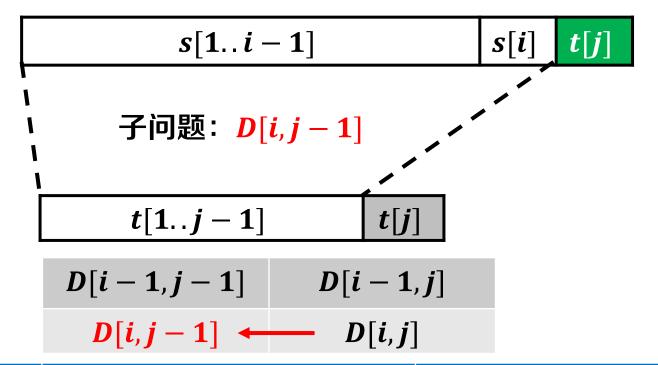
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U	上侧,即 <i>D</i> [ <i>i</i> — 1, <i>j</i> ]	删除s[i]
L	左侧,即 <i>D</i> [ <i>i,j</i> — 1]	插入 <i>t</i> [ <i>j</i> ]

问题结构分析



递推关系建立



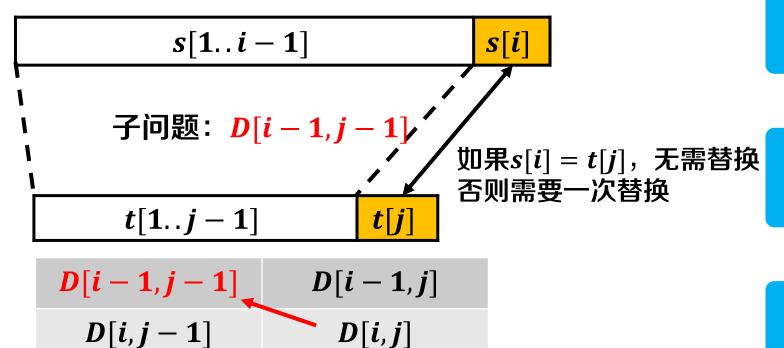
自底向上计算



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U	上侧,即 <i>D</i> [ <i>i</i> — 1, <i>j</i> ]	删除s[i]
L	左侧,即 <i>D</i> [ <i>i,j</i> — 1]	插入t[j]
LU	左上,即 <i>D</i> [ <i>i</i> – 1, <i>j</i> – 1]	用 $t[j]$ 替换 $s[i]$ /无操作

问题结构分析



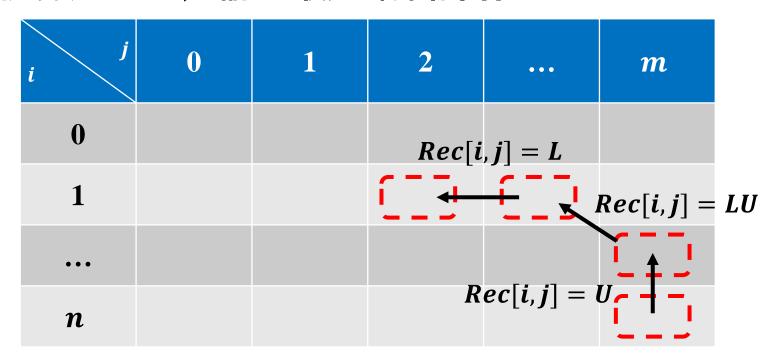
自底向上计算



#### 最优方案追踪:输出最优方案



#### • 根据数组Rec,输出最少编辑操作



Rec[i,j]	子问题来源	操作
U	上侧,即 <i>D</i> [ <i>i</i> — 1, <i>j</i> ]	删除s[i]
L	左侧,即 <i>D</i> [ <i>i,j</i> — 1]	插入t[j]
LU	左上,即 <i>D</i> [ <i>i</i> – 1, <i>j</i> – 1]	用 $t[j]$ 替换 $s[i]$ /无操作

问题结构分析



递推关系建立



自底向上计算





	1	2	3	4	5	6	7
S	A	В	C	В	D	A	В
t	В	D	C	A	В	A	

D								Rec							
i $j$	0	1	2	3	4	5	6	i	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6	0		L	L	L	L	L	L
1	1							1	U						
2	2		初始	· 化				2	U						
3	3							3	U						
4	4							4	U						
5	5							5	U						
6	6							6	U						
7	7							7	U						



	1	2	3	4	5	6	7		D[i-1,j]+1
S	A	В	C	В	D	A	В	$D[i,j] = \min \langle$	D[i,j-1]+1 $[0,if s[i]=t[i]]$
t	В	D	C	A	В	A			$D[i-1,j-1] + \begin{cases} 0, if \ s[i] = t[j] \\ 1, if \ s[i] \neq t[j] \end{cases}$

i $j$	0	1	2	3	4	5	6	i	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6	0		L	L	L	L	L	L
1	1							1	U						
2	2							2	U						
3	3							3	U						
4	4							4	U						
5	5							5	U						
6	6							6	U						
7	7							7	U						



	1	2	3	4	5	6	7		D[i-1,j]+1 删除A
S	A	В	C	В	D	A	В	$D[i,j] = \min \left\{$	D[i,j-1]+1 $(0,if s[i] = t[i]$
t	B	D	C	A	В	A			$D[i-1,j-1] + \begin{cases} 0, if \ s[i] = t[j] \\ 1, if \ s[i] \neq t[j] \end{cases}$

D[i-1,j]Rec D L U U U U U U U



	1	2	3	4	5	6	7	
S	_A_	В	C	В	D	A	В	$D[i,j] = \min \begin{cases} D[i,j-1] + 1 & 插入B \\ (0,if,s[i] = t[i]) \end{cases}$
t	<b>B</b>	D	C	A	В	A		$D[i,j] = \min \left\{ D[i-1,j-1] + \begin{cases} 0, if \ s[i] = t[j] \\ 1, if \ s[i] \neq t[j] \end{cases} \right\}$

D			D[i]	-1,j				Rec							
i $j$	0	1	2	3	4	5	6	i $j$	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6	0		L	L	L	L	L	L
1	1							1	$\mathbf{U}$						
2	2							2	$\mathbf{U}$						
3	3		D[i,]	[j-1]				3	$\mathbf{U}$						
4	4							4	$\mathbf{U}$						
5	5							5	$\mathbf{U}$						
6	6							6	$\mathbf{U}$						
7	7							7	U						



	1	2	2	3	4	5	6	7					[i-1,j]		删除A
S	A	] <u>I</u>	3	C	В	D	A	В	D[i,j]	= min	{		i,j-1		插入B = t[i]
t	В	] I	)	$\mathbf{C}^{\mathbf{S}}$	$i] \neq t[$	j]	A				D[i -	- <b>1</b> , <b>j</b> –	$1] + \left\{ 1 \right\}$	, if $s[i]$	$\neq t[j]$
D			D[i]	i-1,	<i>j</i> ]			Rec					A칱	替换为I	3
i	0	1	2	3	4	5	6	i $j$	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6	0		L	L	L	L	L	L
1	1			D	$\overline{[i-1]}$	(j-1]		1	$\mathbf{U}$						
2	2							2	$\mathbf{U}$						
3	3		D[i,	j-1				3	$\mathbf{U}$						
4	4							4	$\mathbf{U}$						
5	5							5	$\mathbf{U}$						
6	6							6	$\mathbf{U}$						
7	7							7	$\mathbf{U}$						



	1	2	3	4	5	6	7		D[i-1,j]+1
S	A	В	C	В	D	A	В	$D[i,j] = \min \langle$	D[i,j-1]+1 $(0,if s[i] = t[i]$
t	В	D	C $S$	$[i] \neq t$	j]	A			$D[i-1,j-1] + \begin{cases} 0, if \ s[i] = t[j] \\ 1, if \ s[i] \neq t[j] \end{cases}$
			_						

$\boldsymbol{D}$			D[i]	-1,j				Rec							
i	0	1	2	3	4	5	6	i $j$	0	1	2	3	4	5	6
0	0_	1	2	3	4	5	6	0		L	L	L	L	L	L
1	1	1		D[	i-1,	$\overline{j-1}$		1	U	LU					
2	2							2	U						
3	3		D[i,j]	[-1]				3	U						
4	4							4	U						
5	5							5	U						
6	6							6	U						
7	7							7	U						



	1		2	3	4	5	6	7				_	[i-1,j]		
S	A		B _	C	В	D	A	В	D[i,j]	= min	. }	D[	i, j — 1] (0	+1 angle	=t[i]
t	В	(_j	)	C	A S	$[t] \neq t[$	<i>j</i> ]				D[i -	1, j	$1] + \begin{cases} 1 \\ 1 \end{cases}$	, if s[i] , if s[i]	$\neq t[j]$
D					D[i-1]	L, <b>j</b> ]		Rec							
i $j$	0	1	2	3	4	5	6	i $j$	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6	0		L	L	L	L	L	L
1	1	1	2			O[i-1]	., <b>j</b> — <b>1</b> ]	1	$\mathbf{U}$	LU	LU				
2	2							2	U						
3	3			<b>D</b> [	i, j-1			3	$\mathbf{U}$						
4	4							4	U						
5	5							5	U						
6	6							6	U						
7	7							7	IJ						



	1	2	3	4	5	6	7		D[i-1,j]+1
S	A	В	C	В	D	A	В	$D[i,j] = \min \langle$	D[i,j-1]+1 $(0.if s[i] = t[i]$
t	В	D	$\left\{ \mathbf{C}\right\}$	A	В	A			$D[i-1,j-1] + \begin{cases} 0, if \ s[i] = t[j] \\ 1, if \ s[i] \neq t[j] \end{cases}$

j i	0	1	2	3	4	5	6	i	0	1	2	3	4	
0	0	1	2	3	4	5	6	0		L	L	L	L	
1	1	1	2	3				1	U	LU	LU	LU		
2	2							2	U					
3	3							3	U					
4	4							4	U					
5	5							5	U					
6	6							6	U					
7	7							7	U					



	1	2	2	3	4	5	6	7					[i-1,j]		
S	A	, F	3	C	В	D	A	B	D[i,j]	= min	}	_	[i,j-1]		= t[j]
t	В	I	)	C	A	B	s[i] = t				D[i -	-1,j	$egin{aligned} egin{aligned} egin{aligned\\ egin{aligned} egi$	if s[i]	$\neq t[j]$
D						D	[i-1, j]	j] c					无	需替换	
i $j$	0	1	2	3	4	5	6	i	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6	0		L	L	L	L	L	L
1	1	1	2	3	3		D[	$\overline{i-1}$ , $j$	<b>-1</b> ]	LU	LU	LU	LU		
2	2							2	U						
3	3					D[i,	j-1	3	$\mathbf{U}$						
4	4							4	$\mathbf{U}$						
5	5							5	$\mathbf{U}$						
6	6							6	$\mathbf{U}$						
7	7							7	$\mathbf{U}$						



	1	2	3	4	5	6	7		D[i-1,j]+1
S	A	В	C	В	D	A	В	$D[i,j] = \min \langle$	D[i,j-1]+1 $(0.if s[i] = t[i]$
t	В	D	C	A	В	A			$D[i-1,j-1] + \begin{cases} 0, if \ s[i] = t[j] \\ 1, if \ s[i] \neq t[j] \end{cases}$

i $j$	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1	1	1	2	3	3	4	
2	2						
3	3						
4	4						
5	5						
6	6						
7	7						

i $j$	0	1	2	3	4	5	6
0		L	L	L	L	L	L
1	$\mathbf{U}$	LU	LU	LU	LU	L	
2	$\mathbf{U}$						
3	$\mathbf{U}$						
4	U						
5	$\mathbf{U}$						
6	$\mathbf{U}$						
7	$\mathbf{U}$						



	1	2	3	4	5	6	7		D[i-1,j]+1
S	A	В	C	В	D	A	В	$D[i,j] = \min \langle$	D[i,j-1]+1 $(0.if s[i] = t[i]$
t	В	D	C	A	В	A			$D[i-1,j-1] + \begin{cases} 0, if \ s[i] = t[j] \\ 1, if \ s[i] \neq t[j] \end{cases}$

i	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6_
1	1	1	2	3	3	4	5
2	2						
3	3						
4	4						
5	5						
6	6						
7	7						

i	0	1	2	3	4	5	6
0		L	L	L	L	L	L
1	$\mathbf{U}$	LU	LU	LU	LU	L	LU
2	$\mathbf{U}$						
3	$\mathbf{U}$						
4	U						
5	$\mathbf{U}$						
6	U						
7	$\mathbf{U}$						



	1 2	2,	3	4	5	6	7		D[i-1,j]+1
S	AB	3_,	C	В	D	A	В	$D[i,j] = \min \langle$	D[i,j-1]+1 $[0,if s[i]=t[i]]$
t	B	)	C	A	В	A			$D[i-1,j-1] + \begin{cases} 0, if \ s[i] = t[j] \\ 1, if \ s[i] \neq t[j] \end{cases}$

i	0	1	2	3	4	5	6	i	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6	0		L	L	L	L	L	L
1	1	1	2	3	3	4	5	1	U	LU	LU	LU	LU	L	LU
2	2	1						2	U	LU					
3	3							3	U						
4	4							4	U						
5	5							5	U						
6	6							6	U						
7	7							7	U						



	1	2	3	4	5	6	7		D[i-1,j]+1
S	A	B	C	В	D	A	В	$D[i,j] = \min \langle$	D[i,j-1]+1 $-(0.if s[i] = t[i]$
t	В	D	C	A	В	A			$D[i-1,j-1] + \begin{cases} 0, if \ s[i] = t[j] \\ 1, if \ s[i] \neq t[j] \end{cases}$

i j	0	1	2	3	4	5	6	i
0	0	1	2	3	4	5	6	
1	1	1	2	3	3	4	5	
2	2	[1	2	)				
3	3							
4	4							
5	5							
6	6							
7	7							

i $j$	0	1	2	3	4	5	6
0		L	L	L	L	L	L
1	$\mathbf{U}$	LU	LU	LU	LU	L	LU
2	U	LU	LU				
3	$\mathbf{U}$						
4	U						
5	$\mathbf{U}$						
6	U						
7	$\mathbf{U}$						



	1	2	3	4	5	6	7		D[i-1,j]+1
S	A	B			D	A	В	$D[i,j] = \min \langle$	D[i,j-1]+1 $D[i,j-1]+1$
t	В	D	$\mathbb{C}$	A	В	A			$D[i-1,j-1] + \begin{cases} 0, if \ s[i] = t[j] \\ 1, if \ s[i] \neq t[j] \end{cases}$

i $j$	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1	1	1	2	3	3	4	5
2	2	1	2	3			
3	3						
4	4						
5	5						
6	6						
7	7						

i $j$	0	1	2	3	4	5	6
0		L	L	L	L	L	L
1	$\mathbf{U}$	LU	LU	LU	LU	${f L}$	LU
2	$\mathbf{U}$	LU	LU	LU			
3	$\mathbf{U}$						
4	U						
5	$\mathbf{U}$						
6	U						
7	$\mathbf{U}$						



	1	2	3	4	5	6	7		D[i-1,j]+1
S	A	B			D	A	В	$D[i,j] = \min \langle$	D[i,j-1]+1 $D[i,j-1]+1$
t	В	D	C	$\mathbf{A}$	В	A			$D[i-1,j-1] + \begin{cases} 0, if \ s[i] = t[j] \\ 1, if \ s[i] \neq t[j] \end{cases}$

i $j$	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1	1	1		3	3	4	5
2	2	1	2	3	4		
3	3						
4	4						
5	5						
6	6						
7	7						

j	0	1	2	3	4	5	6
0		L	L	L	L	L	L
1	$\mathbf{U}$	LU	LU	LU	LU	${f L}$	LU
2	U	LU	LU	LU	LU		
3	$\mathbf{U}$						
4	U						
5	$\mathbf{U}$						
6	U						
7	$\mathbf{U}$						



	1	2	3	4	5	6	7		D[i-1,j]+1
S	A	B	C			A	В	$D[i,j] = \min \langle$	D[i,j-1]+1 $(0.if s[i] = t[i]$
t	В	D	C	A	В	A			$D[i-1,j-1] + \begin{cases} 0, if \ s[i] = t[j] \\ 1, if \ s[i] \neq t[j] \end{cases}$

i	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1	1	1	2	3	3	4	5
2	2	1	2	3	4	3	
3	3						
4	4						
5	5						
6	6						
7	7						

i j	0	1	2	3	4	5	6
0		L	L	L	L	L	L
1	$\mathbf{U}$	LU	LU	LU	LU	${f L}$	LU
2	$\mathbf{U}$	LU	LU	LU	LU	LU	
3	$\mathbf{U}$						
4	$\mathbf{U}$						
5	$\mathbf{U}$						
6	U						
7	$\mathbf{U}$						



	1	2	3	4	5	6	7		D[i-1,j]+1
S	A	B	C	В	D	A	В	$D[i,j] = \min \langle$	D[i,j-1]+1 $(0.if s[i] = t[i]$
t	В	D	C	A	В	A			$D[i-1,j-1] + \begin{cases} 0, if \ s[i] = t[j] \\ 1, if \ s[i] \neq t[j] \end{cases}$

i $j$	0	1	2	3	4	5	6	i $j$	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6	0		L	L	L	L	L	L
1	1	1	2	3	3	4	5	1	U	LU	LU	LU	LU	L	LU
2	2	1	2	3	4	3	4	2	U	LU	LU	LU	LU	LU	L
3	3							3	U						
4	4							4	U						
5	5							5	U						
6	6							6	U						
7	7							7	U						



	1	2	3	4	5	6	7		D[i-1,j]+1
S	A	В	C	В	D	A	В	$D[i,j] = \min \langle$	D[i,j-1]+1 $(0.if s[i] = t[i]$
t	В	D	C	A	В	A			$D[i-1,j-1] + \begin{cases} 0, if \ s[i] = t[j] \\ 1, if \ s[i] \neq t[j] \end{cases}$

i j	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1	1	1	2	3	3	4	5
2	2	$\begin{bmatrix} 1 \end{bmatrix}$	2	3	4	3	4
3	3	2					
4	4						
5	5						
6	6						
7	7						

i j	0	1	2	3	4	5	6
0		L	L	L	L	L	L
1	U	LU	LU	LU	LU	L	LU
2	U	LU	LU	LU	LU	LU	L
3	U	U					
4	U						
5	U						
6	U						
7	U						



	1	2	3	4	5	6	7		D[i-1,j]+1
S	A		C		D	A	В	$D[i,j] = \min \langle$	D[i,j-1]+1 $D[i,j-1]+1$
t	В	D	C	A	В	A			$D[i-1,j-1] + \begin{cases} 0, if \ s[i] = t[j] \\ 1, if \ s[i] \neq t[j] \end{cases}$

i $j$	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1	1	1	2	3	3	4	5
2	2	1	2	3	4	3	4
3	3	2	2				
4	4						
5	5						
6	6						
7	7						

i $j$	0	1	2	3	4	5	6
0		L	L	L	${f L}$	L	L
1	U	LU	LU	LU	LU	L	LU
2	U	LU	LU	LU	LU	LU	L
3	$\mathbf{U}$	U	LU				
4	U						
5	U						
6	U						
7	U						



	1	2	3	4	5	6	7		D[i-1,j]+1
S	A		C		D	A	В	$D[i,j] = \min \langle$	D[i,j-1]+1 $D[i,j-1]+1$
t	В	D	C	A	В	A			$D[i-1,j-1] + \begin{cases} 0, if \ s[i] = t[j] \\ 1, if \ s[i] \neq t[j] \end{cases}$

i	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1	1	1	2	3	3	4	5
2	2	1	2	3	4	3	4
3	3	2	2	2			
4	4						
5	5						
6	6						
7	7						

i j	0	1	2	3	4	5	6
0		L	L	L	L	L	L
1	$\mathbf{U}$	LU	LU	LU	LU	L	LU
2	$\mathbf{U}$	LU	LU	LU	LU	LU	L
3	$\mathbf{U}$	$\mathbf{U}$	LU	LU			
4	$\mathbf{U}$						
5	$\mathbf{U}$						
6	U						
7	$\mathbf{U}$						



	1	2	3	4	5	6	7		D[i-1,j]+1
S	A	В	C	В	D	A	В	$D[i,j] = \min \langle$	D[i,j-1]+1 $(0.if s[i] = t[i]$
t	В	D	C	A	В	A			$D[i-1,j-1] + \begin{cases} 0, if \ s[i] = t[j] \\ 1, if \ s[i] \neq t[j] \end{cases}$

D

i $j$	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1	1	1	2	3	3	4	5
2	2	1	2	3	4	3	4
3	3	2	2	2	3		
4	4						
5	5						
6	6						
7	7						

i $j$	0	1	2	3	4	5	6
0		L	L	L	L	L	L
1	$\mathbf{U}$	LU	LU	LU	LU	L	LU
2	$\mathbf{U}$	LU	LU	LU	LU	LU	L
3	$\mathbf{U}$	$\mathbf{U}$	LU	LU	L		
4	$\mathbf{U}$						
5	$\mathbf{U}$						
6	$\mathbf{U}$						
7	$\mathbf{U}$						



	1	2	3	4	5	6	7		D[i-1,j]+1
S	A	В	C	В	D	A	В	$D[i,j] = \min \langle$	D[i,j-1]+1 $(0.if s[i] = t[i]$
t	В	D	C	A	В	A			$D[i-1,j-1] + \begin{cases} 0, if \ s[i] = t[j] \\ 1, if \ s[i] \neq t[j] \end{cases}$

D

i $j$	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1	1	1	2	3	3	4	5
2	2	1	2	3	4	3	4
3	3	2	2	2	3	4	
4	4						
5	5						
6	6						
7	7						

i	0	1	2	3	4	5	6
0		L	L	L	L	L	L
1	U	LU	LU	LU	LU	L	LU
2	U	LU	LU	LU	LU	LU	L
3	U	U	LU	LU	L	L	
4	U						
5	U						
6	U						
7	$\mathbf{U}$						



	1	2	3	4	5	6	7		D[i-1,j]+1
S	A	В	C	В	D	A	В	$D[i,j] = \min \langle$	D[i,j-1]+1 $(0.if s[i] = t[i]$
t	В	D	C	A	В	A			$D[i-1,j-1] + \begin{cases} 0, if \ s[i] = t[j] \\ 1, if \ s[i] \neq t[j] \end{cases}$

D

i $j$	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1	1	1	2	3	3	4	5
2	2	1	2	3	4	3	4
3	3	2	2	2	3	4	4
4	4						
5	5						
6	6						
7	7						

i $j$	0	1	2	3	4	5	6
0		L	L	L	L	L	L
1	$\mathbf{U}$	LU	LU	LU	LU	L	LU
2	$\mathbf{U}$	LU	LU	LU	LU	LU	L
3	$\mathbf{U}$	$\mathbf{U}$	LU	LU	L	L	LU
4	$\mathbf{U}$						
5	$\mathbf{U}$						
6	U						
7	U						



	1	2	3	4	5	6	7		D[i-1,j]+1
S	A	В	C	B	D	A	В	$D[i,j] = \min \langle$	D[i,j-1]+1 $(0.if s[i] = t[i]$
t	В	D	C	A	В	A			$D[i-1,j-1] + \begin{cases} 0, if \ s[i] = t[j] \\ 1, if \ s[i] \neq t[j] \end{cases}$

i $j$	0	1	2	3	4	5	6	i $j$	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6	0		L	L	L	L	L	L
1	1	1	2	3	3	4	5	1	$\mathbf{U}$	LU	LU	LU	LU	L	LU
2	2	1	2	3	4	3	4	2	$\mathbf{U}$	LU	LU	LU	LU	LU	L
3	3_	2	2	2	3	4	4	3	$\mathbf{U}$	$\mathbf{U}$	LU	LU	L	L	LU
4	4	3						4	U	LU					
5	5							5	$\mathbf{U}$						
6	6							6	U						
7	7							7	U						



	1	2	3	4	5	6	7		D[i-1,j]+1
S	A	В	C	B	D	A	В	$D[i,j] = \min \langle$	D[i,j-1]+1 $(0.if s[i] = t[i]$
t	В	D	C	A	В	A			$D[i-1,j-1] + \begin{cases} 0, if \ s[i] = t[j] \\ 1, if \ s[i] \neq t[j] \end{cases}$

i $j$	0	1	2	3	4	5	6	i $j$	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6	0		L	L	L	L	L	L
1	1	1	2	3	3	4	5	1	$\mathbf{U}$	LU	LU	LU	LU	L	LU
2	2	1	2	3	4	3	4	2	$\mathbf{U}$	LU	LU	LU	LU	LU	L
3	3	2_	2	2	3	4	4	3	$\mathbf{U}$	U	LU	LU	L	L	LU
4	4	3	3					4	$\mathbf{U}$	LU	LU				
5	5							5	$\mathbf{U}$						
6	6							6	$\mathbf{U}$						
7	7							7	$\mathbf{U}$						



	1	2	3	4	5	6	7		D[i-1,j]+1
S	A	В	C	B	D	A	В	$D[i,j] = \min \langle$	D[i,j-1]+1 $(0.if s[i] = t[i]$
t	В	D	$\mathbb{C}$	A	В	A			$D[i-1,j-1] + \begin{cases} 0, if \ s[i] = t[j] \\ 1, if \ s[i] \neq t[j] \end{cases}$

i $j$	0	1	2	3	4	5	6	i $j$	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6	0		L	L	L	L	L	L
1	1	1	2	3	3	4	5	1	$\mathbf{U}$	LU	LU	LU	LU	L	LU
2	2	1	2	3	4	3	4	2	$\mathbf{U}$	LU	LU	LU	LU	LU	L
3	3	2	2	2	3	4	4	3	$\mathbf{U}$	U	LU	LU	L	L	LU
4	4	3	3	3				4	U	LU	LU	LU			
5	5							5	$\mathbf{U}$						
6	6							6	U						
7	7							7	U						



	1	2	3	4	5	6	7		D[i-1,j]+1
S	A	В	C	B	D	A	В	$D[i,j] = \min \langle$	D[i,j-1]+1 $-(0.if s[i] = t[i]$
t	В	D	C	A	В	A			$D[i-1,j-1] + \begin{cases} 0, if \ s[i] = t[j] \\ 1, if \ s[i] \neq t[j] \end{cases}$

D

i	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1	1	1	2	3	3	4	5
2	2	1	2	3	4	3	4
3	3	2	2	2	3	4	4
4	4	3	3	3	3		
5	5						
6	6						
7	7						

i	0	1	2	3	4	5	6
0		$\mathbf{L}$	${f L}$	L	L	L	L
1	$\mathbf{U}$	LU	LU	LU	LU	${f L}$	LU
2	$\mathbf{U}$	LU	LU	LU	LU	LU	L
3	$\mathbf{U}$	$\mathbf{U}$	LU	LU	L	L	LU
4	$\mathbf{U}$	LU	LU	LU	LU		
5	$\mathbf{U}$						
6	U						
7	U						



	1	2	3	4	5	6	7		D[i-1,j]+1
S	A	В	C	B	D	A	В	$D[i,j] = \min \langle$	D[i,j-1]+1 $(0.if s[i] = t[i]$
t	В	D	C	A	B	A			$D[i-1,j-1] + \begin{cases} 0, if \ s[i] = t[j] \\ 1, if \ s[i] \neq t[j] \end{cases}$

D

i $j$	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1	1	1	2	3	3	4	5
2	2	1	2	3	4	3	4
3	3	2	2	2	3	4_	4
4	4	3	3	3	3	3	
5	5						
6	6						
7	7						

i	0	1	2	3	4	5	6
0		L	L	L	L	L	L
1	$\mathbf{U}$	LU	LU	LU	LU	L	LU
2	$\mathbf{U}$	LU	LU	LU	LU	LU	L
3	$\mathbf{U}$	$\mathbf{U}$	LU	LU	L	L	LU
4	$\mathbf{U}$	LU	LU	LU	LU	LU	
5	$\mathbf{U}$						
6	U						
7	$\mathbf{U}$						



	1	2	3	4	5	6	7		D[i-1,j]+1
S	A	В	C	B	D	A	В	$D[i,j] = \min \langle$	D[i,j-1]+1 $(0.if s[i] = t[i]$
t	В	D	C	A	В	A			$D[i-1,j-1] + \begin{cases} 0, if \ s[i] = t[j] \\ 1, if \ s[i] \neq t[j] \end{cases}$

i	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1	1	1	2	3	3	4	5
2	2	1	2	3	4	3	4
3	3	2	2	2	3	4	4_
4	4	3	3	3	3	3	4
5	5						
6	6						
7	7						

i	0	1	2	3	4	5	6
0		L	L	L	L	L	L
1	$\mathbf{U}$	LU	LU	LU	LU	L	LU
2	U	LU	LU	LU	LU	LU	L
3	$\mathbf{U}$	$\mathbf{U}$	LU	LU	L	L	LU
4	$\mathbf{U}$	LU	LU	LU	LU	LU	L
5	$\mathbf{U}$						
6	U						
7	$\mathbf{U}$						



	1	2	3	4	5	6	7		D[i-1,j]+1
S	A	В	C	В	D	A	В	$D[i,j] = \min \langle$	D[i,j-1]+1 $(0.if s[i] = t[i]$
t	В	D	C	A	В	A			$D[i-1,j-1] + \begin{cases} 0, if \ s[i] = t[j] \\ 1, if \ s[i] \neq t[j] \end{cases}$

i $j$	0	1	2	3	4	5	6	i $j$	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6	0		L	L	L	L	L	L
1	1	1	2	3	3	4	5	1	U	LU	LU	LU	LU	L	LU
2	2	1	2	3	4	3	4	2	U	LU	LU	LU	LU	LU	L
3	3	2	2	2	3	4	4	3	U	U	LU	LU	L	L	LU
4	4	3	3	3	3	3	4	4	U	LU	LU	LU	LU	LU	L
5	5	4	3					5	U	U	LU				
6	6							6	U						
7	7							7	U						



	1	2	3	4	5	6	7		D[i-1,j]+1
S	A	В	C	В	D	A	В	$D[i,j] = \min \langle$	D[i,j-1]+1 $(0.if s[i] = t[i]$
t	В	D	C	A	В	A			$D[i-1,j-1] + \begin{cases} 0, if \ s[i] = t[j] \\ 1, if \ s[i] \neq t[j] \end{cases}$

i $j$	0	1	2	3	4	5	6	i $j$	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6	0		L	L	L	L	L	L
1	1	1	2	3	3	4	5	1	$\mathbf{U}$	LU	LU	LU	LU	L	LU
2	2	1	2	3	4	3	4	2	U	LU	LU	LU	LU	LU	L
3	3	2	2	2	3	4	4	3	U	U	LU	LU	L	L	LU
4	4	3	3	3	3	3	4	4	U	LU	LU	LU	LU	LU	L
5	5	4	3	4	j			5	U	U	LU	LU			
6	6							6	U						
7	7							7	U						



	1	2	3	4	5	6	7		D[i-1,j]+1
S	A	В	C	В	D	A	В	$D[i,j] = \min \langle$	D[i,j-1]+1 $(0.if s[i] = t[i]$
t	В	D	C	A	В	A			$D[i-1,j-1] + \begin{cases} 0, if \ s[i] = t[j] \\ 1, if \ s[i] \neq t[j] \end{cases}$

i $j$	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1	1	1	2	3	3	4	5
2	2	1	2	3	4	3	4
3	3	2	2	2	3	4	4
4	4	3	3	3_	3	3	4
5	5	4	3		4		
6	6						
7	7						

i $j$	0	1	2	3	4	5	6
0		L	L	L	L	L	L
1	$\mathbf{U}$	LU	LU	LU	LU	${f L}$	LU
2	$\mathbf{U}$	LU	LU	LU	LU	LU	L
3	$\mathbf{U}$	$\mathbf{U}$	LU	LU	L	${f L}$	LU
4	U	LU	LU	LU	LU	LU	L
5	$\mathbf{U}$	$\mathbf{U}$	LU	LU	LU		
6	U						
7	$\mathbf{U}$						



	1	2	3	4	5	6	7		D[i-1,j]+1
S	A	В	C	В	D	A	В	$D[i,j] = \min \langle$	D[i,j-1]+1 $(0.if s[i] = t[i]$
t	В	D	C	A	В	A			$D[i-1,j-1] + \begin{cases} 0, if \ s[i] = t[j] \\ 1, if \ s[i] \neq t[j] \end{cases}$

D

i $j$	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1	1	1	2	3	3	4	5
2	2	1	2	3	4	3	4
3	3	2	2	2	3	4	4
4	4	3	3	3	3	3	4
5	5	4	3	4	4	4	
6	6						
7	7						

i	0	1	2	3	4	5	6
0		L	L	L	L	${f L}$	L
1	$\mathbf{U}$	LU	LU	LU	LU	${f L}$	LU
2	$\mathbf{U}$	LU	LU	LU	LU	LU	L
3	$\mathbf{U}$	$\mathbf{U}$	LU	LU	L	${f L}$	LU
4	$\mathbf{U}$	LU	LU	LU	LU	LU	${f L}$
5	$\mathbf{U}$	$\mathbf{U}$	LU	LU	LU	LU	
6	$\mathbf{U}$						
7	$\mathbf{U}$						



	1	2	3	4	5	6	7		D[i-1,j]+1
S	A	В	C	В	D	A	В	$D[i,j] = \min \langle$	D[i,j-1]+1 $(0.if s[i] = t[i]$
t	В	D	C	A	В	A			$D[i-1,j-1] + \begin{cases} 0, if \ s[i] = t[j] \\ 1, if \ s[i] \neq t[j] \end{cases}$

D

i $j$	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1	1	1	2	3	3	4	5
2	2	1	2	3	4	3	4
3	3	2	2	2	3	4	4
4	4	3	3	3	3	3	4
5	5	4	3	4	4	4	4
6	6						
7	7						

i	0	1	2	3	4	5	6
0		L	L	L	L	L	L
1	$\mathbf{U}$	LU	LU	LU	LU	L	LU
2	U	LU	LU	LU	LU	LU	L
3	$\mathbf{U}$	U	LU	LU	L	L	LU
4	$\mathbf{U}$	LU	LU	LU	LU	LU	L
5	$\mathbf{U}$	$\mathbf{U}$	LU	LU	LU	LU	LU
6	$\mathbf{U}$						
7	U						



LU

LU

LU

	1	2	3	4	5	6	7		D[i-1,j]+1
S	A	В	C	В	D	A	В	$D[i,j] = \min \langle$	D[i,j-1]+1 $(0.if s[i] = t[i]$
t	B	D	C	A	В	A			$D[i-1,j-1] + \begin{cases} 0, if \ s[i] = t[j] \\ 1, if \ s[i] \neq t[j] \end{cases}$

i	0	1	2	3	4	5	6	i $j$	0	1	2	3	4	5
0	0	1	2	3	4	5	6	0		L	L	L	L	L
1	1	1	2	3	3	4	5	1	$\mathbf{U}$	LU	LU	LU	LU	${f L}$
2	2	1	2	3	4	3	4	2	U	LU	LU	LU	LU	LU
3	3	2	2	2	3	4	4	3	$\mathbf{U}$	U	LU	LU	L	${f L}$
4	4	3	3	3	3	3	4	4	U	LU	LU	LU	LU	LU
5	5	4	3	4	4	4	4	5	$\mathbf{U}$	U	LU	LU	LU	LU
6	6	5						6	U	U				
7	7							7	$\mathbf{U}$					



	1	2	3	4	5	6	7		D[i-1,j]+1
S	A	В	C	В	D	A	В	$D[i,j] = \min \langle$	D[i,j-1]+1 $(0.if s[i] = t[i]$
t	В	D	C	A	В	A			$D[i-1,j-1] + \begin{cases} 0, if \ s[i] = t[j] \\ 1, if \ s[i] \neq t[j] \end{cases}$

i	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1	1	1	2	3	3	4	5
2	2	1	2	3	4	3	4
3	3	2	2	2	3	4	4
4	4	3	3	3	3	3	4
5	5	4	3	4	4	4	4
6	6	5	4				
7	7						

i	0	1	2	3	4	5	6
0		L	L	L	L	L	L
1	$\mathbf{U}$	LU	LU	LU	LU	L	LU
2	U	LU	LU	LU	LU	LU	L
3	U	U	LU	LU	L	L	LU
4	U	LU	LU	LU	LU	LU	L
5	U	U	LU	LU	LU	LU	LU
6	$\mathbf{U}$	U	U				
7	U						



	1	2	3	4	5	6	7		D[i-1,j]+1
S	A	В	C	В	D	A	В	$D[i,j] = \min \langle$	D[i,j-1]+1 $(0.if s[i] = t[i]$
t	В	D	C	A	В	A			$D[i-1,j-1] + \begin{cases} 0, if \ s[i] = t[j] \\ 1, if \ s[i] \neq t[j] \end{cases}$

i	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1	1	1	2	3	3	4	5
2	2	1	2	3	4	3	4
3	3	2	2	2	3	4	4
4	4	3	3	3	3	3	4
5	5	4	3	4_	4	4	4
6	6	5	4	4			
7	7						

i j	0	1	2	3	4	5	6
0		L	L	L	L	L	L
1	$\mathbf{U}$	LU	LU	LU	LU	${f L}$	LU
2	$\mathbf{U}$	LU	LU	LU	LU	LU	L
3	$\mathbf{U}$	$\mathbf{U}$	LU	LU	${f L}$	${f L}$	LU
4	$\mathbf{U}$	LU	LU	LU	LU	LU	L
5	$\mathbf{U}$	U	LU	LU	LU	LU	LU
6	$\mathbf{U}$	U	$\mathbf{U}$	LU			
7	$\mathbf{U}$						



	1	2	3	4	5	6	7		D[i-1,j]+1
S	A	В	C	В	D	A	В	$D[i,j] = \min \langle$	D[i,j-1]+1 $(0.if s[i] = t[i]$
t	В	D	C	A	В	A			$D[i-1,j-1] + \begin{cases} 0, if \ s[i] = t[j] \\ 1, if \ s[i] \neq t[j] \end{cases}$

i	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1	1	1	2	3	3	4	5
2	2	1	2	3	4	3	4
3	3	2	2	2	3	4	4
4	4	3	3	3	3	3	4
5	5	4	3	4	4_	4	4
6	6	5	4	4	4		
7	7						

i	0	1	2	3	4	5	6
0		L	L	L	L	L	L
1	$\mathbf{U}$	LU	LU	LU	LU	${f L}$	LU
2	U	LU	LU	LU	LU	LU	L
3	$\mathbf{U}$	U	LU	LU	${f L}$	L	LU
4	U	LU	LU	LU	LU	LU	L
5	$\mathbf{U}$	$\mathbf{U}$	LU	LU	LU	LU	LU
6	$\mathbf{U}$	$\mathbf{U}$	$\mathbf{U}$	LU	LU		
7	$\mathbf{U}$						



	1	2	3	4	5	6	7		D[i-1,j]+1
S	A	В	C	В	D	A	В	$D[i,j] = \min \langle$	D[i,j-1]+1 $(0.if s[i] = t[i]$
t	В	D	C	A	B	A			$D[i-1,j-1] + \begin{cases} 0, if \ s[i] = t[j] \\ 1, if \ s[i] \neq t[j] \end{cases}$

i	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1	1	1	2	3	3	4	5
2	2	1	2	3	4	3	4
3	3	2	2	2	3	4	4
4	4	3	3	3	3	3	4
5	5	4	3	4	4	4_	4
6	6	5	4	4	4	5	
7	7						

i	0	1	2	3	4	5	6
0		L	L	L	L	${f L}$	L
1	$\mathbf{U}$	LU	LU	LU	LU	${f L}$	LU
2	$\mathbf{U}$	LU	LU	LU	LU	LU	L
3	$\mathbf{U}$	$\mathbf{U}$	LU	LU	L	${f L}$	LU
4	$\mathbf{U}$	LU	LU	LU	LU	LU	${f L}$
5	$\mathbf{U}$	U	LU	LU	LU	LU	LU
6	U	U	U	LU	LU	LU	
7	$\mathbf{U}$						



	1	2	3	4	5	6	7		D[i-1,j]+1
S	A	В	C	В	D	A	В	$D[i,j] = \min \langle$	D[i,j-1]+1 $(0.if s[i] = t[i]$
t	В	D	C	A	В	A			$D[i-1,j-1] + \begin{cases} 0, if \ s[i] = t[j] \\ 1, if \ s[i] \neq t[j] \end{cases}$

i $j$	0	1	2	3	4	5	6	i $j$	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6	0		L	L	L	L	L	L
1	1	1	2	3	3	4	5	1	$\mathbf{U}$	LU	LU	LU	LU	L	LU
2	2	1	2	3	4	3	4	2	U	LU	LU	LU	LU	LU	L
3	3	2	2	2	3	4	4	3	U	U	LU	LU	L	L	LU
4	4	3	3	3	3	3	4	4	U	LU	LU	LU	LU	LU	L
5	5	4	3	4	4	4	4	5	$\mathbf{U}$	U	LU	LU	LU	LU	LU
6	6	5	4	4	4	5	4	6	U	U	U	LU	LU	LU	LU
7	7							7	U						



	1	2	3	4	5	6	7		D[i-1,j]+1
S	A	В	C	В	D	A	B	$D[i,j] = \min \langle$	D[i,j-1]+1 $[0,if s[i]=t[i]$
t	В	D	C	A	В	A			$D[i-1,j-1] + \begin{cases} 0, if \ s[i] = t[j] \\ 1, if \ s[i] \neq t[j] \end{cases}$

i	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1	1	1	2	3	3	4	5
2	2	1	2	3	4	3	4
3	3	2	2	2	3	4	4
4	4	3	3	3	3	3	4
5	5	4	3	4	4	4	4
6	6	5	4	4	4	5	4
7	7	6					

i	0	1	2	3	4	5	6
0		L	L	L	L	L	L
1	U	LU	LU	LU	LU	L	LU
2	U	LU	LU	LU	LU	LU	L
3	U	U	LU	LU	L	L	LU
4	U	LU	LU	LU	LU	LU	L
5	U	U	LU	LU	LU	LU	LU
6	U	U	U	LU	LU	LU	LU
7	U	LU					



LU

	1	2	3	4	5	6	7		D[i-1,j]+1
S	A	В	C	В	D	A	B	$D[i,j] = \min \left\{ \right.$	D[i,j-1]+1 $[0.if s[i] = t[i]$
t	В	D	C	A	В	A			$D[i-1,j-1] + \begin{cases} 0, if \ s[i] = t[j] \\ 1, if \ s[i] \neq t[j] \end{cases}$

i	0	1	2	3	4	5	6	i	0	1	2	3
0	0	1	2	3	4	5	6	0		L	L	L
1	1	1	2	3	3	4	5	1	$\mathbf{U}$	LU	LU	LU
2	2	1	2	3	4	3	4	2	$\mathbf{U}$	LU	LU	LU
3	3	2	2	2	3	4	4	3	$\mathbf{U}$	U	LU	LU
4	4	3	3	3	3	3	4	4	$\mathbf{U}$	LU	LU	LU
5	5	4	3	4	4	4	4	5	$\mathbf{U}$	U	LU	LU
6	6	5	4	4	4	5	4	6	$\mathbf{U}$	U	U	LU
7	7	6	5					7	$\mathbf{U}$	LU	U	



	1	2	3	4	5	6	7		D[i-1,j]+1
S	A	В	C	В	D	A	B	$D[i,j] = \min \left\langle \right.$	D[i,j-1]+1 $[0,if s[i]=t[i]$
t	В	D	C	A	В	A			$D[i-1,j-1] + \begin{cases} 0, if \ s[i] = t[j] \\ 1, if \ s[i] \neq t[j] \end{cases}$

i	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1	1	1	2	3	3	4	5
2	2	1	2	3	4	3	4
3	3	2	2	2	3	4	4
4	4	3	3	3	3	3	4
5	5	4	3	4	4	4	4
6	6	5	4	4	4	5	4
7	7	6	5	5			

i	0	1	2	3	4	5	6
0		L	L	L	L	L	L
1	$\mathbf{U}$	LU	LU	LU	LU	L	LU
2	$\mathbf{U}$	LU	LU	LU	LU	LU	L
3	$\mathbf{U}$	$\mathbf{U}$	LU	LU	${f L}$	L	LU
4	$\mathbf{U}$	LU	LU	LU	LU	LU	L
5	$\mathbf{U}$	$\mathbf{U}$	LU	LU	LU	LU	LU
6	$\mathbf{U}$	$\mathbf{U}$	$\mathbf{U}$	LU	LU	LU	LU
7	$\mathbf{U}$	LU	$\mathbf{U}$	LU			



	1	2	3	4	5	6	7		D[i-1,j]+1
S	A	В	C	В	D	A	B	$D[i,j] = \min \left\langle \right.$	D[i,j-1]+1 $[0,if s[i]=t[i]$
t	В	D	C	A	В	A			$D[i-1,j-1] + \begin{cases} 0, if \ s[i] = t[j] \\ 1, if \ s[i] \neq t[j] \end{cases}$

i	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1	1	1	2	3	3	4	5
2	2	1	2	3	4	3	4
3	3	2	2	2	3	4	4
4	4	3	3	3	3	3	4
5	5	4	3	4	4	4	4
6	6	5	4	4	4	5	4
7	7	6	5	5	5		

i $j$	0	1	2	3	4	5	6
0		L	L	L	L	L	L
1	$\mathbf{U}$	LU	LU	LU	LU	L	LU
2	$\mathbf{U}$	LU	LU	LU	LU	LU	L
3	$\mathbf{U}$	U	LU	LU	L	L	LU
4	$\mathbf{U}$	LU	LU	LU	LU	LU	L
5	$\mathbf{U}$	$\mathbf{U}$	LU	LU	LU	LU	LU
6	U	U	U	LU	LU	LU	LU
7	$\mathbf{U}$	LU	U	LU	LU		



	1	2	3	4	5	6	7		D[i-1,j]+1
S	A	В	C	В	D	A	B	$D[i,j] = \min \left\langle \right.$	D[i,j-1]+1 $[0,if s[i]=t[i]$
t	В	D	C	A	В	A			$D[i-1,j-1] + \begin{cases} 0, if \ s[i] = t[j] \\ 1, if \ s[i] \neq t[j] \end{cases}$

i	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1	1	1	2	3	3	4	5
2	2	1	2	3	4	3	4
3	3	2	2	2	3	4	4
4	4	3	3	3	3	3	4
5	5	4	3	4	4	4	4
6	6	5	4	4	4	5	4
7	7	6	5	5	5	4	

i $j$	0	1	2	3	4	5	6
0		L	L	L	L	L	L
1	U	LU	LU	LU	LU	L	LU
2	$\mathbf{U}$	LU	LU	LU	LU	LU	L
3	$\mathbf{U}$	U	LU	LU	L	L	LU
4	$\mathbf{U}$	LU	LU	LU	LU	LU	L
5	$\mathbf{U}$	U	LU	LU	LU	LU	LU
6	U	U	U	LU	LU	LU	LU
7	U	LU	U	LU	LU	LU	



	1	2	3	4	5	6	7		D[i-1,j]+1
S	A	В	C	В	D	A	B	$D[i,j] = \min \langle$	D[i,j-1]+1 $- (0.if s[i] = t[i]$
t	В	D	C	A	В	A			$D[i-1,j-1] + \begin{cases} 0, if \ s[i] = t[j] \\ 1, if \ s[i] \neq t[j] \end{cases}$

D

i $j$	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1	1	1	2	3	3	4	5
2	2	1	2	3	4	3	4
3	3	2	2	2	3	4	4
4	4	3	3	3	3	3	4
5	5	4	3	4	4	4	4
6	6	5	4	4	4	5	4
7	7	6	5	5	5	4	5

i	0	1	2	3	4	5	6
0		L	L	L	L	L	L
1	$\mathbf{U}$	LU	LU	LU	LU	L	LU
2	$\mathbf{U}$	LU	LU	LU	LU	LU	L
3	$\mathbf{U}$	U	LU	LU	L	L	LU
4	$\mathbf{U}$	LU	LU	LU	LU	LU	L
5	$\mathbf{U}$	U	LU	LU	LU	LU	LU
6	$\mathbf{U}$	U	$\mathbf{U}$	LU	LU	LU	LU
7	$\mathbf{U}$	LU	$\mathbf{U}$	LU	LU	LU	L



	1	2	3	4	5	6	7		D[i-1,j]+1
S	A	В	C	В	D	A	В	$D[i,j] = \min \langle$	D[i,j-1]+1 $(0.if s[i] = t[i]$
t	В	D	C	A	В	A			$D[i-1,j-1] + \begin{cases} 0, if \ s[i] = t[j] \\ 1, if \ s[i] \neq t[j] \end{cases}$

i $j$	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1	1	1	2	3	3	4	5
2	2	1	2	3	4	3	4
3	3	2	2	2	3	4	4
4	4	3	3	3	3	3	4
5	5	4	3		最优角	罕	4
6	6	5	4	4	4	3	4
7	7	6	5	5	5	4	5

i $j$	0	1	2	3	4	5	6
0		L	L	L	L	L	L
1	$\mathbf{U}$	LU	LU	LU	LU	L	LU
2	$\mathbf{U}$	LU	LU	LU	LU	LU	L
3	$\mathbf{U}$	$\mathbf{U}$	LU	LU	L	L	LU
4	$\mathbf{U}$	LU	LU	LU	LU	LU	L
5	$\mathbf{U}$	$\mathbf{U}$	LU	LU	LU	LU	LU
6	$\mathbf{U}$	U	U	LU	LU	LU	LU
7	$\mathbf{U}$	LU	$\mathbf{U}$	LU	LU	LU	L



	1	2	3	4	5	6	7		D[i-1,j]+1
S	A	В	C	В	D	A	В	$D[i,j] = \min \langle$	D[i,j-1]+1 $(0,if s[i] = t[i]$
t	В	D	C	A	В	A			$D[i-1,j-1] + \begin{cases} 0, if \ s[i] = t[j] \\ 1, if \ s[i] \neq t[j] \end{cases}$

操作:

插人A

Rec							
i $j$	0	1	2	3	4	5	6
0		L	L	L	L	L	L
1	$\mathbf{U}$	LU	LU	LU	LU	${f L}$	LU
2	U	LU	LU	LU	LU	LU	L
3	U	$\mathbf{U}$	LU	LU	${f L}$	L	LU
4	U	LU	LU	LU	LU	LU	L
5	U	$\mathbf{U}$	LU	LU	LU	LU	LU
6	U	$\mathbf{U}$	$\mathbf{U}$	LU	LU	LU	LU
7	U	LU	$\mathbf{U}$	LU	LU	LU	L



	1	2	3	4	5	6	7		D[i-1,j]+1
S	A	В	C	В	D	A	B	$D[i,j] = \min \langle$	D[i,j-1]+1 $(0.if s[i] = t[i]$
t	В	D	C	A	B	A			$D[i-1,j-1] + \begin{cases} 0, if \ s[i] = t[j] \\ 1, if \ s[i] \neq t[j] \end{cases}$

操作:

无需操作 插入A

i	0	1	2	3	4	5	6
0		L	L	L	L	L	L
1	$\mathbf{U}$	LU	LU	LU	LU	L	LU
2	$\mathbf{U}$	LU	LU	LU	LU	LU	L
3	$\mathbf{U}$	$\mathbf{U}$	LU	LU	L	L	LU
4	$\mathbf{U}$	LU	LU	LU	LU	LU	${f L}$
5	$\mathbf{U}$	$\mathbf{U}$	LU	LU	LU	LU	LU
6	$\mathbf{U}$	$\mathbf{U}$	$\mathbf{U}$	LU	LU	LU	LU
7	$\mathbf{U}$	LU	$\mathbf{U}$	LU	LU	LU	L



	1	2	3	4	5	6	7		D[i-1,j]+1
S	A	В	C	В	D	A	В	$D[i,j] = \min \langle$	D[i,j-1]+1 $(0.if s[i] = t[i]$
t	В	D	C	<b>A</b>	В	A			$D[i-1,j-1] + \begin{cases} 0, if \ s[i] = t[j] \\ 1, if \ s[i] \neq t[j] \end{cases}$

操作:

无需操作 无需操作 插人A

i $j$	0	1	2	3	4	5	6
0		L	L	L	L	L	L
1	$\mathbf{U}$	LU	LU	LU	LU	${f L}$	LU
2	U	LU	LU	LU	LU	LU	L
3	$\mathbf{U}$	$\mathbf{U}$	LU	LU	L	${f L}$	LU
4	U	LU	LU	LU	LU	LU	L
5	U	U	LU	LU	LU	LU	LU
6	U	U	U	LU	LU	LU	LU
7	$\mathbf{U}$	LU	$\mathbf{U}$	LU	LU	LU	${f L}$



	1	2	3	4	5	6	7		D[i-1,j]+1
S	A	В			D	A	В	$D[i,j] = \min \langle$	D[i,j-1]+1 $(0.if s[i] = t[i]$
t	В	D	$\mathbb{C}$	A	В	A			$D[i-1,j-1] + \begin{cases} 0, if \ s[i] = t[j] \\ 1, if \ s[i] \neq t[j] \end{cases}$

操作:

用C替换D 无需操作 无需操作 插人A

i $j$	0	1	2	3	4	5	6
0		L	L	L	L	L	L
1	$\mathbf{U}$	LU	LU	LU	LU	L	LU
2	$\mathbf{U}$	LU	LU	LU	LU	LU	L
3	$\mathbf{U}$	$\mathbf{U}$	LU	LU	L	L	LU
4	U	LU	LU	LU	LU	LU	${f L}$
5	$\mathbf{U}$	$\mathbf{U}$	LU	LU	LU	LU	LU
6	U	U	$\mathbf{U}$	LU	LU	LU	LU
7	$\mathbf{U}$	LU	$\mathbf{U}$	LU	LU	LU	L



	1	2	3	4	5	6	7		D[i-1,j]+1
S	A	В	C	B	D	A	В	$D[i,j] = \min \langle$	$D[i,j-1]+1 \ D[i,j-1]+1$
t	В	D	C	A	В	A			$D[i-1,j-1] + \begin{cases} 0, if \ s[i] = t[j] \\ 1, if \ s[i] \neq t[j] \end{cases}$

操作:

用D替换B 用C替换D 无需操作 无需操作 插人A

i	0	1	2	3	4	5	6
0		L	L	L	L	L	L
1	$\mathbf{U}$	LU	LU	LU	LU	${f L}$	LU
2	$\mathbf{U}$	LU	LU	LU	LU	LU	L
3	$\mathbf{U}$	$\mathbf{U}$	LU	LU	${f L}$	${f L}$	LU
4	$\mathbf{U}$	LU	LU	LU	LU	LU	L
5	$\mathbf{U}$	U	LU	LU	LU	LU	LU
6	U	U	$\mathbf{U}$	LU	LU	LU	LU
7	$\mathbf{U}$	LU	$\mathbf{U}$	LU	LU	LU	${f L}$



	1	2	3	4	5	6	7		D[i-1,j]+1
S	A	В	C	В	D	A	В	$D[i,j] = \min \langle$	D[i,j-1]+1 $(0.if s[i] = t[i]$
t	В	D	C	A	В	A			$D[i-1,j-1] + \begin{cases} 0, if \ s[i] = t[j] \\ 1, if \ s[i] \neq t[j] \end{cases}$

操作:

删除C 用D替换B 用C替换D 无需操作 无需操作 插人A

i	0	1	2	3	4	5	6
0		L	L	L	${f L}$	L	$\mathbf{L}$
1	$\mathbf{U}$	LU	LU	LU	LU	${f L}$	LU
2	U	LU	LU	LU	LU	LU	L
3	$\mathbf{U}$	U	LU	LU	${f L}$	${f L}$	LU
4	$\mathbf{U}$	LU	LU	LU	LU	LU	L
5	$\mathbf{U}$	$\mathbf{U}$	LU	LU	LU	LU	LU
6	U	$\mathbf{U}$	U	LU	LU	LU	LU
7	$\mathbf{U}$	LU	U	LU	LU	LU	${f L}$



	1	2	3	4	5	6	7		D[i-1,j]+1
S	A	B	C	В	D	A	В	$D[i,j] = \min \langle$	D[i,j-1]+1 $(0.if s[i] = t[i]$
t	B	D	C	A	В	A			$D[i-1,j-1] + \begin{cases} 0, if \ s[i] = t[j] \\ 1, if \ s[i] \neq t[j] \end{cases}$

操作:

无需操作 删除C 用D替换D 干需操作 无需操作 插入A

i $j$	0	1	2	3	4	5	6
0		L	L	L	L	L	L
1	$\mathbf{U}$	LU	LU	LU	LU	${f L}$	LU
2	$\mathbf{U}$	LU	LU	LU	LU	LU	L
3	$\mathbf{U}$	U	LU	LU	L	${f L}$	LU
4	$\mathbf{U}$	LU	LU	LU	LU	LU	L
5	$\mathbf{U}$	U	LU	LU	LU	LU	LU
6	$\mathbf{U}$	U	$\mathbf{U}$	LU	LU	LU	LU
7	$\mathbf{U}$	LU	$\mathbf{U}$	LU	LU	LU	L



	1	2	3	4	5	6	7		D[i-1,j]+1
S	A	В	C	В	D	A	В	$D[i,j] = \min \langle$	D[i,j-1]+1 $(0.if s[i] = t[i]$
t	В	D	C	A	В	A			$D[i-1,j-1] + \begin{cases} 0, if \ s[i] = t[j] \\ 1, if \ s[i] \neq t[j] \end{cases}$

操船无删用用无无插作除需除D替替操作需除D替操从系带的基础。

i	0	1	2	3	4	5	6
0		L	L	L	L	L	L
1	U	LU	LU	LU	LU	${f L}$	LU
2	$\mathbf{U}$	LU	LU	LU	LU	LU	L
3	$\mathbf{U}$	$\mathbf{U}$	LU	LU	L	${f L}$	LU
4	$\mathbf{U}$	LU	LU	LU	LU	LU	L
5	$\mathbf{U}$	$\mathbf{U}$	LU	LU	LU	LU	LU
6	U	U	U	LU	LU	LU	LU
7	$\mathbf{U}$	LU	$\mathbf{U}$	LU	LU	LU	L



```
输入: 字符串s和t
输出: s和t的最小编辑距离
n \leftarrow \text{length}(s)
m \leftarrow \text{length}(t)
新建D[0..n,0..m],Rec[0..n,0..m]两个二维数组
//初始化
for i \leftarrow 0 to n do
                                          初始化
    D[i,0] \leftarrow i
    Rec[i,0] \leftarrow "U"
end
for j \leftarrow 0 to m do
    D[0,j] \leftarrow j
    Rec[0,j] \leftarrow ``L"
```



```
for i \leftarrow 1 to n do
                                                 依次计算子问题
if s_i \neq t_i then
         c \leftarrow 1
         end
        replace \leftarrow D[i-1, j-1] + c
        delete \leftarrow D[i-1,j]+1
        insert \leftarrow D[i, j-1] + 1
        if replace = min\{delete, insert, replace\} then
            D[i,j] \leftarrow D[i-1,j-1] + c
           Rec[i,j] \leftarrow "LU"
         end
         else if insert = min\{delete, insert, replace\} then
            D[i,j] \leftarrow D[i,j-1] + 1
            Rec[i,j] \leftarrow "L"
         end
         else
            D[i,j] \leftarrow D[i-1,j] + 1
            Rec[i,j] \leftarrow "U"
         end
     end
 end
```



```
//动态规划
for i \leftarrow 1 to n do
    for j \leftarrow 1 to m do
                                                       替换/无需操作
        if s_i \neq t_j then
        \mathbf{end}
       replace \leftarrow D[i-1,j-1] + c
        delete \leftarrow D[i-1,j]+1
        insert \leftarrow D[i, j-1] + 1
        if replace = min\{delete, insert, replace\} then
           D[i,j] \leftarrow D[i-1,j-1] + c
           Rec[i,j] \leftarrow ``LU"
        end
        else if insert = min\{delete, insert, replace\} then
            D[i,j] \leftarrow D[i,j-1] + 1
           Rec[i,j] \leftarrow "L"
        end
        else
           D[i,j] \leftarrow D[i-1,j] + 1
           Rec[i,j] \leftarrow "U"
        end
    end
end
```



```
//动态规划
for i \leftarrow 1 to n do
   for j \leftarrow 1 to m do
        c \leftarrow 0
        if s_i \neq t_j then
         c \leftarrow 1
        end
        replace \leftarrow D[i-1, j-1] + c
        delete \leftarrow D[i-1,j]+1
       insert \leftarrow D[i, j-1] + 1
       if replace = min\{delete, insert, replace\} then
                                                                         采用替换操作
         + \mathcal{D}[i,j] \leftarrow \mathcal{D}[i-1,j-1] + c
           Rec[i,j] \leftarrow "LU"
        end
        else if insert = min\{delete, insert, replace\} then
            D[i,j] \leftarrow D[i,j-1] + 1
            Rec[i,j] \leftarrow "L"
        end
        else
            D[i,j] \leftarrow D[i-1,j] + 1
           Rec[i,j] \leftarrow "U"
        end
    end
end
```



```
//动态规划
for i \leftarrow 1 to n do
    for j \leftarrow 1 to m do
         c \leftarrow 0
         if s_i \neq t_j then
          c \leftarrow 1
         end
         replace \leftarrow D[i-1, j-1] + c
         delete \leftarrow D[i-1,j]+1
         insert \leftarrow D[i, j-1] + 1
        \begin{array}{l} \textbf{if } replace = min\{delete, insert, replace\} \textbf{ then} \\ | D[i,j] \leftarrow D[i-1,j-1] + c \end{array}
                                                                                      记录编辑距离和操作
            Rec[i,j] \leftarrow ``LU"
         else if insert = min\{delete, insert, replace\} then
              D[i,j] \leftarrow D[i,j-1] + 1
             Rec[i,j] \leftarrow "L"
         end
         else
              D[i,j] \leftarrow D[i-1,j] + 1
             Rec[i,j] \leftarrow "U"
         \mathbf{end}
    end
end
```



```
//动态规划
for i \leftarrow 1 to n do
   for j \leftarrow 1 to m do
        c \leftarrow 0
        if s_i \neq t_j then
         c \leftarrow 1
        end
        replace \leftarrow D[i-1, j-1] + c
        delete \leftarrow D[i-1,j]+1
        insert \leftarrow D[i, j-1] + 1
        if replace = min\{delete, insert, replace\} then
            D[i,j] \leftarrow D[i-1,j-1] + c
            Rec[i,j] \leftarrow "LU"
        \mathbf{end}
      \mathbf{f} else if insert = min\{delete, insert, replace\} then
                                                                                采取插入操作
            D[i,j] \leftarrow D[i,j-1] + 1
           Rec[i,j] \leftarrow ``L"
       end
        else
           D[i,j] \leftarrow D[i-1,j] + 1
           Rec[i,j] \leftarrow "U"
        end
    end
end
```



```
//动态规划
for i \leftarrow 1 to n do
    for j \leftarrow 1 to m do
        c \leftarrow 0
        if s_i \neq t_j then
         c \leftarrow 1
        \mathbf{end}
        replace \leftarrow D[i-1, j-1] + c
        delete \leftarrow D[i-1,j]+1
        insert \leftarrow D[i, j-1] + 1
        if replace = min\{delete, insert, replace\} then
            D[i,j] \leftarrow D[i-1,j-1] + c
           Rec[i,j] \leftarrow ``LU"
        end
        else if insert = min\{delete, insert, replace\} then
            D[i,j] \leftarrow D[i,j-1] + 1
            Rec[i,j] \leftarrow ``L"
        end
      /else
                                                                                  采取删除操作
          D[i,j] \leftarrow D[i-1,j] + 1
Rec[i,j] \leftarrow ``U"
        end
    end
end
```



## • Print-MED(*Rec*, s, t, i, j)

```
输入: 矩阵Rec, 字符串s,t, 位置索引i和j
输出: 操作序列
if i = 0 and j = 0 then
   return NULL
end
if Rec[i,j] = "LU" then
   Print-MED(Rec, s, t, i-1, j-1)
                                         采取替换操作
   if s_i = t_j then
      print "无需操作"
   \mathbf{end}
   else
      \operatorname{print} "用t_j替换s_i"
   end
end
else if Rec[i, j] = "U" then
   Print-MED(Rec, s, t, i - 1, j)
   print "删除s_i"
end
else
   Print-MED(Rec, s, t, i, j - 1)
   print "插入t_j"
end
```



## Print-MED(Rec, s, t, i, j)

```
输入: 矩阵Rec, 字符串s,t, 位置索引i和j
输出: 操作序列
if i = 0 and j = 0 then
   return NULL
end
if Rec[i, j] = "LU" then
  Print-MED(Rec, s, t, i - 1, j - 1)
                                          递归输出子问题方案
   if s_i = t_i then
      print "无需操作"
   \mathbf{end}
   else
      \operatorname{print} "用t_j替换s_i"
   end
end
else if Rec[i, j] = "U" then
   Print-MED(Rec, s, t, i - 1, j)
   print "删除s_i"
end
else
   Print-MED(Rec, s, t, i, j - 1)
   print "插入t_j"
end
```



## • Print-MED(*Rec*, s, t, i, j)

```
输入: 矩阵Rec, 字符串s,t, 位置索引i和j
输出: 操作序列
if i = 0 and j = 0 then
   return NULL
end
if Rec[i,j] = "LU" then
   Print-MED(Rec, s, t, i-1, j-1)
  if s_i = t_j then
                                 替换/无操作
      print "无需操作"
   end
  else
  ▮ | print "\mathbf{H}t_{j}替换s_{i}"
  end
end
else if Rec[i, j] = "U" then
   Print-MED(Rec, s, t, i - 1, j)
   print "删除s_i"
end
else
   Print-MED(Rec, s, t, i, j - 1)
   print "插入t_j"
end
```



## Print-MED(Rec, s, t, i, j)

```
输入: 矩阵Rec, 字符串s,t, 位置索引i和j
输出: 操作序列
if i = 0 and j = 0 then
   return NULL
end
if Rec[i,j] = "LU" then
   Print-MED(Rec, s, t, i - 1, j - 1)
   if s_i = t_i then
      print "无需操作"
   \mathbf{end}
   else
      \operatorname{print} "用t_j替换s_i"
   end
else if Rec[i,j] = "U" then
                                         采取删除操作
   Print-MED(Rec, s, t, i - 1, j)
   print "删除s_i"
end
else
   Print-MED(Rec, s, t, i, j - 1)
   print "插入t_j"
end
```



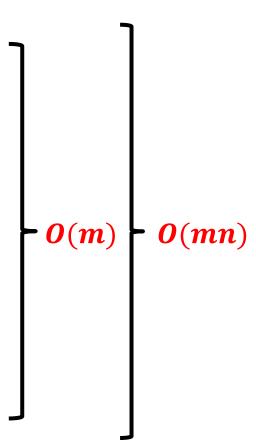
### • Print-MED(Rec, s, t, i, j)

```
输入: 矩阵Rec, 字符串s,t, 位置索引i和j
输出: 操作序列
if i = 0 and j = 0 then
   return NULL
end
if Rec[i,j] = "LU" then
    Print-MED(Rec, s, t, i - 1, j - 1)
    if s_i = t_i then
       print "无需操作"
   \mathbf{end}
    else
      \operatorname{print} "用t_j替换s_i"
    end
end
else if Rec[i, j] = "U" then
    Print-MED(Rec, s, t, i - 1, j)
   \operatorname{print} "删除s_i"
end
/else
                                           采取插入操作
    Print-MED(Rec, s, t, i, j - 1)
   print "插入t_i"
end
```

## 时间复杂度分析



```
//动态规划
for i \leftarrow 1 to n do
    for j \leftarrow 1 to m do
        c \leftarrow 0
        if s_i \neq t_j then
        \mathbf{e}\mathbf{n}\mathbf{d}
        replace \leftarrow D[i-1, j-1] + c
        delete \leftarrow D[i-1,j]+1
        insert \leftarrow D[i, j-1] + 1
        \mathbf{if}\ replace = min\{delete, insert, replace\}\ \mathbf{then}
            D[i,j] \leftarrow D[i-1,j-1] + c
            Rec[i,j] \leftarrow "LU"
        \mathbf{end}
        else if insert = min\{delete, insert, replace\} then
            D[i,j] \leftarrow D[i,j-1] + 1
            Rec[i,j] \leftarrow "L"
        end
        else
            D[i,j] \leftarrow D[i-1,j] + 1
            Rec[i,j] \leftarrow "U"
        end
    end
                                       时间复杂度: O(mn)
end
```





### 最长公共子序列

 $如果 s_i \neq t_i$ 

如果 $s_i = t_j$ 

$$C[i,j] = \begin{cases} \max\{C[i-1,j], C[i,j-1]\}, x_i \neq y_j \\ C[i-1,j-1] + 1, x_i = y_j \end{cases}$$

### 最小编辑距离

删除

插人

替换

$$t \mid B \mid D \mid C \mid A \mid B \mid A$$

$$oldsymbol{s} oldsymbol{A} oldsymbol{B} oldsymbol{B} oldsymbol{C} oldsymbol{B} oldsymbol{D} oldsymbol{A} oldsymbol{B} oldsymbol{2}$$

 $A \mid B \mid A$ 

s A

|B|

 $\boldsymbol{D}$ 

|B|

|D|A

B

?

 $t \mid B \mid D \mid C \mid A \mid B \mid A$ 

$$D[i,j] = \min egin{cases} D[i-1,j] + 1 \ D[i,j-1] + 1 \ D[i-1,j-1] + egin{cases} 0, if \ s[i] = t[j] \ 1, if \ s[i] 
otin terms \end{cases}$$