HOMEWORK 3 — TOM MULVEY
2/27/18

3.8) Prove if x is an odd int, then 9x+5 is even

Proof. Assume x is an odd integer, then..

$x = 2k + 1$ for some $k \in \mathbb{Z}$

It follows that $9x + 5$=$9(2k + 1) + 5$

=$18k + 14$

=$2(9k + 7)$ where 9k+7 is an integer

This is obvioulsy even. □

3.20)

Proof. *let*$x \in \mathbb{Z}$ Prove that $3x + 1$ is even $\iff$ $5x - 2$ is odd

Lemma 1. *Let $x \in \mathbb{Z}$. If $3x + 1$ is even then x is odd.*

Proof (lemma):We will prove this lemma by the contrapositive approach. If x is even, then $3x + 1$ is odd. If x is even, it follows that x = $2k$ for some $k \in \mathbb{Z}$

$3x + 1 = 3(2k) + 1 = 3*$*an even integer*$+1$

This is obviously odd.

Proof: Fot the forward direction, I will prove directly. *Let*$3x + 1$ be an even integer. by the lemma,the integer x is od. Since x is odd, $x = 2k + 1$ for some $k \in \mathbb{Z}$. Thus

$5x - 2 = 5(2k + 1) - 2 = 10k + 5 - 2 = 10k + 3$

$= 3(2k + 1) + 4k$

$= 3(2k + 1) + 2(2k)$

= 3∗an odd integer + an even integer

=an odd integer

For the reverse direction, I will prove by the contrapositive.

The proof is similar in the reverse direction as the forward. □

3.26) Prove that if $n \in \mathbb{Z}$ then $n^2 - 3n + 9$ is odd.

Proof. I will argue by cases that if n is either odd OR even, $n^2 - 3n + 9$ is odd.

CASE 1: n is even If n is even, it follows that n=$2k$ for some $k \in \mathbb{Z}$

thus $n^2 - 3n + 9 = (2k)^2 - 3(2k) + 9$

=$4k^2 - 6k + 9$

=$4k^2 - 3(2k + 3)$

=even integer - 3*(odd integer)

odd integer

CASE 2: n is odd

If n is odd, it follows that $n = 2k + 1$ for some $k \in \mathbb{Z}$

thus $n^2 - 3n + 9 = (2k + 1)^2 - 3(2k + 1) + 9$a

=$4k^2 + 4k + 1 - 6k - 3 + 9$

=$4k^2 - 2k + 7$

=$2(2k^2 - 1k) + 7$

=even integer + odd integer

=obviously odd

$n^2 - 3n + 9$ is odd when n is odd or evena □