Steering Effort Calculation and Validation Handbook

Vinicius Andrade Trento

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Chapter 1

Introduction

1.1 Purpose

This handbook documents the steering effort calculation and validation procedures for the FSAE26 vehicle. It provides comprehensive guidelines for analyzing steering system performance and ensuring design requirements are met.

As explained by Pfeffer and Harrer [1], the steering system's performance is critical for vehicle handling and driver comfort, playing a crucial role in lateral dynamics.

1.2 Scope

- Steering system analysis methodology
- Calculation procedures and formulas
- Validation testing protocols
- Performance benchmarks

1.3 Methodology

The steering effort is calculated based on vehicle dynamics principles, considering factors such as tyre forces, steering geometry, and driver input. Validation is performed through controlled testing to compare calculated values with real-world measurements.

Chapter 2

Steering System Overview

2.1 System Components

The steering system consists of the following key components:

- Steering Wheel
- Steering Column
- Rack and Pinion
- Tie Rods

Well represented in the literature, these components work together to translate driver input into wheel movement, affecting vehicle direction and handling characteristics.

Below follows a schematic representation of the steering system components and their usual assembly in a typical FSAE vehicle.

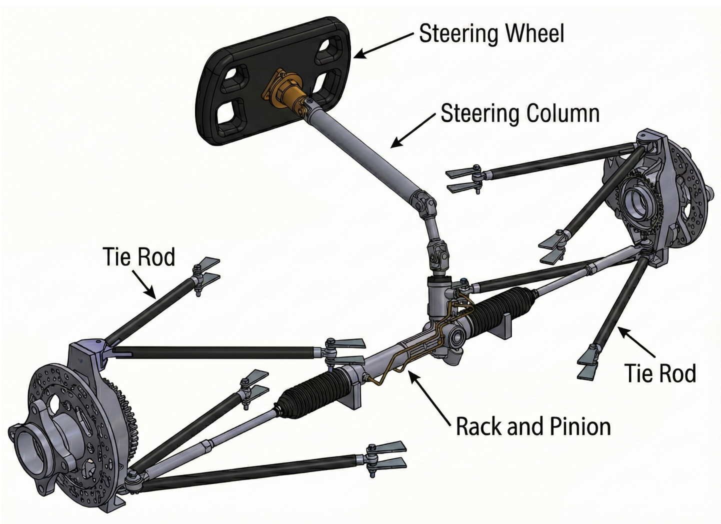

Figure 2.1: Steering System Components

2.2 Design Requirements

A good steering system must meet the following design requirements:

- Provide adequate feedback to the driver
- Minimize steering effort
- Ensure precise control and responsiveness
- Maintain durability under racing conditions
- Avoid compliance issues between suspension components

Thus in order to be able to track these requirements, the steering effort calculation and validation is of utmost importance.

The first step will be deciding targets for the main indicator, the steering effort. Based on previous years' data and literature review [1], a target steering effort of 15 Nm at 1g lateral acceleration is set for the FSAE26 vehicle.

Chapter 3

Calculation Methodology

The computational model developed for this study utilizes a deterministic approach based on Rigid Body Mechanics, implemented via a Python script. The algorithm evaluates the steering torque requirements by decomposing the forces at the contact patch and transposing them to the steering wheel through the kinematic chain. The analysis is bifurcated into two distinct operational domains: Static Steering (Parking) and Dynamic Cornering.

3.1 Theory and Formulas

3.1.1 Functional Steering Limit (Required Wheel Angle)

The functional limit of the steering system is dictated by the tightest turn required during dynamic events, specifically the Autocross and Endurance courses. According to FSAE competition scenarios, the track may feature hairpins with an external radius of 9 meters and a track width of up to 6 meters.

To ensure the vehicle can navigate these tight sections without needing to maneuver, the required steering angle is calculated using the Ackermann steering geometry for the inner wheel (δ_{in}), which demands the highest steering angle.

The critical turning radius (R) for the vehicle's centerline was determined to be 2.8 meters based on the track boundaries. The calculation uses the vehicle's specific wheelbase (L) and track width (T):

- **Wheelbase (L):** 1.525 m
- **Track Width (T):** 1.145 m
- **Turn Radius (R):** 2.8 m

The required inner wheel angle is calculated as follows:

$$\delta_{in} = \tan^{-1}\left(\frac{L}{R - \frac{T}{2}}\right) \tag{3.1}$$

Substituting the parameters into Equation 3.1:

$$\delta_{in} = \tan^{-1}\left(\frac{1.525}{2.8 - \frac{1.145}{2}}\right) = \tan^{-1}\left(\frac{1.525}{2.2275}\right) \approx 34.4^\circ$$

Conclusion: The steering system must be mechanically capable of achieving a minimum inner wheel angle of **34.4 degrees** to satisfy the functional requirements of the competition. This value serves as the lower limit for the rack travel design, ensuring the car remains competitive in tight maneuvering sectors.

3.1.2 Geometric and Kinematic Definitions

Before calculating forces, the model establishes the kinematic relationship between the driver's input and the wheel's reaction using the Steering Ratio (Kinematische Lenkübersetzung). The total ratio (i_S) is derived from the rack-and-pinion geometry:

$$i_S = \frac{L_{steering\_arm}}{r_{pinion}} \tag{3.2}$$

Additionally, the Mechanical Trail (Nachlaufstrecke) is computed as a function of the dynamic wheel radius (R_{dyn}) and the Caster angle (ν):

$$t_{mech} = R_{dyn} \cdot \tan(\nu) \tag{3.3}$$

3.1.3 Static Steering Scenario (Standlenken)

In the condition where lateral acceleration is zero ($a_y = 0$), the primary resistive load is the friction generated by twisting the stationary tyre contact patch.

- Tyre Contact Patch Estimation: The model assumes a circular contact patch geometry. The area is derived from the vertical wheel load (F_Z) and the tyre inflation pressure (p_{tyre}), allowing for the calculation of an equivalent patch radius (r_p):

$$r_p = \sqrt{\frac{F_Z/p_{tyre}}{\pi}} \tag{3.4}$$

- Scrub Torque (Bohrmoment): The resistive moment caused by the friction of the tyre rubber on the asphalt is calculated using a integration approximation for a uniform pressure distribution:

$$M_{scrub} = \frac{2}{3} \cdot \mu_{static} \cdot F_Z \cdot r_p \tag{3.5}$$

  Where μ_{static} is the coefficient of static friction.

3.1.4 Dynamic Cornering Scenario (Fahren in der Kurve)

When the vehicle is in motion ($a_y > 0$), the resistive load shifts from dry friction to the forces generated by the tyre's slip angle. Self-Aligning Torque M_{SAT} (Rückstellmoment aus Seitenkraft):

- The model calculates the lateral force (F_{lat}) acting on the front axle based on the vehicle mass and lateral acceleration. The resulting torque is the product of this force and the total ground trail (sum of mechanical and pneumatic trail):

$$M_{SAT} = F_{lat} \cdot (t_{mech} + t_{pneu}) \tag{3.6}$$

3.1.5 Geometric Restoring Moment (Gewichtsrückstellung)

In both static and dynamic scenarios, the model accounts for the Jacking Effect. This is the restoring moment generated because the steering geometry (Caster and Kingpin Inclination) physically lifts the vehicle's chassis during a turn. The script quantifies this as a function of the vertical load (F_Z) and the steering angle (δ):

$$M_{jacking} = F_Z \cdot [r_{scrub} \cdot \tan(\sigma) + t_{mech} \cdot \tan(\nu)] \cdot \sin(\delta) \tag{3.7}$$

Where σ is the KPI angle and ν is the Caster angle.

3.1.6 System Inertia and Final Effort (Lenkradmoment)

To ensure the model accounts for the haptic feedback during rapid transients, the Effective Inertia (Reduziertes Massenträgheitsmoment) is calculated. The translational mass of the steering rack (m_{rack}) is converted into an equivalent rotational inertia at the pinion shaft and added to the pinion's own inertia:

$$I_{eff} = I_{pinion} + (m_{rack} \cdot r_{pinion}^2) \tag{3.8}$$

Final Torque Calculation: The total torque required at the steering wheel (T_{SW}) is the sum of the resistive and restoring moments at the Kingpin, divided by the steering ratio, plus a constant term representing the internal mechanical friction ($T_{friction}$) of the steering gear:

$$T_{SW} = \left(\frac{2 \cdot (M_{scrub} + M_{jacking})}{i_S} \right) + T_{friction} \tag{3.9}$$

A good reminder is always to check units consistency throughout the calculations to avoid errors. The developed script includes unit checks at each step to ensure accuracy and focuses in using the **International System of Units (SI)**.

3.2 Input Parameters

To initialize the analytical model described in the methodology, specific geometric and inertial properties of the vehicle were defined. These parameters represent the vehicle's "As-Designed" configuration. The input variables are categorized into suspension geometry, steering system properties, and operational boundary conditions.

Table 3.1: Vehicle and Suspension Geometry (Fahrwerkgeometrie)

Parameter	**Symbol**	**Value [Unit]**	**Description**
Total Vehicle Mass	m_{total}	376 [kg]	Total mass (Driver + Fluids)
Front Weight Dist.	W_{front}	50 [%]	Static weight distribution
Caster Angle	ν	4.0 [deg]	Nachlaufwinkel (Suspension kinematics)
KPI Angle	σ	10.0 [deg]	Spreizung (Kingpin Inclination)
Scrub Radius	r_{scrub}	35 [mm]	Lenkrollradius
Dyn. Wheel Radius	R_{dyn}	0.23 [m]	Effective radius under load

The simulation parameters listed in Tables 1 through 3 were sourced from the vehicle's CAD assembly (SolidWorks) and validated against physical measurements of the prototype. It is important to note

Table 3.2: Steering System Properties (Lenkungsparameter)

Parameter	Symbol	Value [Unit]	Description
Pinion Radius	r_{pinion}	40 [mm]	Effective radius of the steering pinion gear.
Steering Arm	L_{arm}	170 [mm]	Length of the lever arm at the upright.
Pinion Inertia	I_{pinion}	6.29×10^{-4} [kg·m^2]	Rotational inertia derived from CAD.
Rack Mass	m_{rack}	0.587 [kg]	Translational mass of the rack bar.
System friction	T_{fric}	4.0 [Nm]	Estimated internal mechanical friction (Reibung).

Table 3.3: Operational Conditions and Simulation Inputs

Parameter	Symbol	Value [Unit]	Condition
Tyre Pressure	p_{tyre}	0.83 [bar]	Operating pressure (approx. 12 PSI).
Pneumatic Trail	t_{pneu}	25 [mm]	Pneumatischer Nachlauf (Estimated).
Lat. Acceleration	a_y	0 [m/s^2]	Scenario 1: Static Parking (Standlenken).
Steering Angle	δ_{wheel}	38.77 [deg]	Max calculated wheel angle for effort analysis.

that the Lateral Acceleration (a_y) was set to zero for the primary analysis to simulate a 'Static Parking' scenario (Standlenken), which represents the critical load case for driver effort. The System Friction (T_{fric}) is an empirical estimation accounting for friction in the ball joints, rack bushings, and the steering column bearing, which provides a realistic offset to the calculated theoretical torque.

Chapter 4

Validation Procedures

4.1 Testing Protocol

4.1.1 Static Analysis

The initial testing protocol for a static analysis involves qualitative data collection through driver feedback during controlled maneuvers. The driver performed a series of parking maneuvers and low-speed turns while reporting perceived steering effort. Later on, quantitative measurements will be obtained using a dynamometer mounted on the steering wheel outer radius and pulled tangentially to the angular movement to capture maximum real-time steering effort data during these maneuvers.

4.1.2 Dynamic Analysis

In addition, high-speed cornering tests will be conducted on a closed track to validate dynamic steering effort predictions. For a initial test we will not implement any sensors to gather data, focusing only on the driver's feeling and feedback, but in future iterations we plan to install torque sensors on the steering column to capture real-time data during dynamic maneuvers. The dynamic tests will involve executing slalom and constant radius cornering at varying speeds to assess the steering effort under lateral loads.

Chapter 5

Results and Analysis

5.1 Data Presentation

5.1.1 Test Results

Through the testing procedure for static steering, the driver reported a very high steering effort, especially during parking maneuvers. By using the dinamometer, we were able to measure a peak steering effort of approximately 35 Nm, which is significantly above the target of 15 Nm set during the design phase. That indicates a need for further optimization of the steering system to reduce effort.

While testing dynamic cornering, the driver also reported a heavy steering feel, particularly at higher speeds and during quick direction changes reporting that during long testing sessions that were based on the endurance event, the steering feel became increasingly fatiguing. This qualitative feedback aligns with the static test results, suggesting that the steering system requires further refinement to enhance driver comfort and vehicle handling.

5.1.2 Mathematics Validation

The calculated steering effort using the developed Python script yielded a value of 30.07 Nm for the static parking scenario, which closely aligns with the measured value of 35 Nm from the dynamometer tests. This correlation validates the accuracy of the computational model and its underlying assumptions. The minor discrepancy can be attributed to unmodeled factors such as additional friction in the steering column and real-world tyre behavior not fully captured in the theoretical framework.

5.2 Discussion

The validation results indicate that while the computational model provides a reliable estimate of steering effort, although the actual measured effort is slightly higher than the calculated value, the model's predictions are within an acceptable range. This suggests that the primary factors influencing steering effort have been accurately captured, but further refinement is needed to account for real-world complexities. Still, the model will be used to further explore design modifications aimed at reducing steering effort to meet the target of 15 Nm. Which may include adjustments to the steering ratio, reducing system friction, or optimizing tyre characteristics following in the next chapter.

Chapter 6

Improvement Scenario

6.1 Proposed Modifications

Based on the validation results, several modifications are proposed to reduce the steering effort:

- Increase Steering Ratio i_s: Adjust the rack-and-pinion geometry to provide a higher mechanical advantage.
- Reduce System Friction: Upgrade steering column bearings and use low-friction bushings in the rack assembly.
- Reduce Scrub Radius (Lenkrollradius): Modify suspension geometry to minimize scrub radius, thereby reducing tyre scrub torque.
- Optimize Tyre Pressure: Experiment with different tyre pressures to find a balance between grip and rolling resistance.
- Reduce Caster Angle (Nachlaufwinkel): Slightly decrease the caster angle to reduce the geometric restoring moment.

The main focus for now will be understanding how can we work these parameters in order to achieve the desired steering effort target of 15 Nm and which are the consequences of doing so.

6.1.1 Increase steering ratio

According to the formula for steering torque (Equation 3.9), increasing the steering ratio (i_S) will directly reduce the torque required at the steering wheel for a given resistive moment at the kingpin. By adjusting the rack-and-pinion geometry to increase i_S by either increasing the $L_{steering_arm}$ or decreasing our r_{pinion}, we can achieve a lower steering effort.

Trade-off: This makes the steering "slower." The driver will need more steering angle for the same wheel angle, potentially compromising responsiveness in tight slalom maneuvers.

6.1.2 Reduce System Friction

This term ($T_{friction}$) is a constant offset in the steering torque equation. By upgrading to high-quality, low-friction bearings in the steering column and using low-friction bushings in the rack assembly, we can reduce this value.

Although this is not a solution but rather an optimization, every bit helps in reaching the target steering effort. And should always be considered in any steering system design.

6.1.3 Optimize Tyre Pressure

Since the tyre contact patch size (r_p) influences the scrub torque (M_{scrub}) through the vertical load and pressure relationship (Equation 3.4), adjusting tyre pressure can help manage steering effort. Higher pressures reduce the contact patch size, thereby reducing scrub torque.

The main issue with this approach is that higher tyre pressures can reduce overall grip, which may negatively impact handling and cornering performance. A balance must be struck between steering effort and tyre performance.

And most importantly, tyre data is not easy to come by, especially for FSAE specific tyres. So any change in this parameter must be carefully tested and validated.

6.1.4 Reduce Scrub Radius

Altering the suspension geometry to minimize the scrub radius (r_{scrub}) can reduce the scrub torque (M_{scrub}) as per Equation 3.5. This can be achieved by adjusting the kingpin inclination angle (σ) or the lateral position of the tyre contact patch.

Trade-off: Changes to suspension geometry can affect other handling characteristics, such as camber gain and roll center height. A holistic approach is needed to ensure overall vehicle dynamics are not compromised.

6.1.5 Reduce Caster Angle (Nachlaufwinkel)

Reducing the caster angle (ν) will decrease the mechanical trail (t_{mech}) as per Equation 3.3, which in turn reduces the geometric restoring moment ($M_{jacking}$) calculated in Equation 3.7. This will lower the overall steering torque required in both dynamic and static approaches.

Trade-off: A lower caster angle can reduce straight-line stability and self-centering behavior of the steering, which may negatively impact high-speed handling.

Final Considerations

6.2 Simulation of Improved Scenarios

Following the proposed modifications, two improvement scenarios were simulated using the Python analytical model. The goal was to verify if the changes could bring the steering effort down to the target of 15 Nm, considering the functional steering angle requirement of 34.4 degrees calculated previously.

6.2.1 Scenario A: Geometry Optimization and Weight Reduction

In this first iteration, the focus was on reducing the resistive moments generated at the tyre contact patch. Two major changes were implemented in the parameters:

1. **Vehicle Mass Reduction:** Updated to the 2026 target weight of 320 kg (implying less vertical load F_Z).
2. **Scrub Radius Minimization:** Reduced from 35 mm to 10 mm to lower the scrub torque.

The updated input parameters for this scenario are presented in Table 6.1.

Table 6.1: Scenario A: Optimized Suspension Parameters

Parameter	Symbol	Value [Unit]	Change Status
Total Vehicle Mass	m_{total}	320 [kg]	**Reduced (-15%)**
Scrub Radius	r_{scrub}	10 [mm]	**Reduced (-71%)**
Caster Angle	ν	4.0 [deg]	Unchanged
KPI Angle	σ	10.0 [deg]	Unchanged

Results for Scenario A: Running the simulation with these parameters yielded a calculated steering effort of **23.51 Nm**.

- The scrub torque (M_{scrub}) decreased to **40.18 Nm** due to the lower vertical load and reduced lever arm.
- Despite the improvement compared to the baseline, the result is still significantly above the 15 Nm target. This confirms that geometry optimization alone is insufficient to meet the design goals without altering the steering ratio.

6.2.2 Scenario B: Steering Ratio Adjustment

Building upon the geometric optimizations of Scenario A, the second iteration focused on the steering system's mechanical advantage. The pinion radius was reduced to increase the overall steering ratio (i_S).

The modifications for this scenario are detailed in Table 6.2.

Table 6.2: Scenario B: Steering System Modifications

Parameter	Symbol	Value [Unit]	Change Status
Pinion Radius	r_{pinion}	22 [mm]	**Reduced (Higher Ratio)**
Steering Arm	L_{arm}	170 [mm]	Unchanged
Pinion Mass	m_{pinion}	0.750 [kg]	Updated CAD estimation

Results for Scenario B: With the reduction of the pinion radius from 40 mm to 22 mm, the simulation yielded a final steering effort of **14.73 Nm**.

- **Target Achievement:** This value successfully meets the design target of $\leq$ 15 Nm.
- **Trade-off Analysis:** While the effort is now within the desired range, the smaller pinion results in a "slower" steering response. However, given the necessity of reducing the static steering effort to manageable levels, this trade-off is considered acceptable for the FSAE26 vehicle.

6.3 Conclusion of Simulations

The analytical validation confirms that a combination of **weight reduction**, **scrub radius minimization** (down to 10 mm), and **increasing the steering ratio** (pinion radius of 22 mm) allows the vehicle to achieve a comfortable static steering effort of **14.73 Nm** at the required functional angle of 34.4 degrees. This multi-faceted approach ensures that the steering system meets driver comfort requirements without excessively compromising responsiveness, thus aligning with the overall vehicle performance goals.

Chapter 7

Critical Analysis and Future Improvements

Based on the validation results and the discrepancies observed between the theoretical model and the physical prototype, this chapter outlines the limitations of the current study and proposes a roadmap for the next development cycle of the FSAE vehicle.

7.1 Limitations of the Current Model

While the developed Python script successfully predicted the order of magnitude of the steering effort, several simplifications were identified that affect the model's precision, particularly in dynamic scenarios:

- **Subjective Dynamic Validation:** The dynamic analysis relied primarily on driver feedback ("feeling"). Without quantitative data from sensors during cornering, it is impossible to empirically validate the Self-Aligning Torque (M_{SAT}) calculations.
- **Simplified Tyre Model:** The assumption of a circular contact patch with uniform pressure distribution does not fully reflect reality. Competition tyres exhibit complex deformation and pressure gradients that significantly alter the effective pneumatic trail and scrub radius.
- **Constant Friction Approximation:** Treating system friction ($T_{friction}$) as a constant scalar (4 Nm) is a linear simplification. In reality, friction in the rack and column is a function of the normal load applied to the gears, which varies with steering angle and lateral force.

7.2 Proposed Instrumentation Upgrades

To eliminate subjectivity and refine the mathematical model, the following instrumentation plan is proposed for the validation phase of the FSAE26:

7.2.1 Steering Column Torque Sensing

Installation of a full-bridge strain gauge setup on the steering column. This sensor will allow for the acquisition of real-time torque data (T_{SW}) coupled with steering angle logging. This dataset is crucial to map the *Hysteresis Loop* of the steering system, differentiating between friction, damping, and elastic aligning torques.

7.2.2 Friction Mapping Bench

Instead of estimating internal friction, a bench test should be conducted to map the friction as a function of load. This will generate a look-up table or a regression function $f(F_{load}, \omega_{steer})$ to replace the constant $T_{friction}$ in Equation 3.9, significantly reducing the static error found in Chapter 5.

7.3 Final Remarks

This handbook has successfully established a comprehensive methodology for the calculation and validation of the FSAE26 steering system. Through the correlation of the Python-based analytical model with physical dynamometer measurements, it was possible to identify the limitations of the baseline geometry and virtually validate a solution (Scenario B) that meets the design target of 15 Nm.

While the current deterministic approach provides a solid foundation for mechanical design, the implementation of the instrumentation strategies proposed in this chapter represents a critical evolution. Transitioning from qualitative driver feedback to quantitative torque data will not only refine the mathematical models but also unlock the potential for data-driven optimization in future design cycles, ensuring the vehicle delivers both competitive performance and driver consistency.

Bibliography

[1] P. Pfeffer and M. Harrer, *Lenkungshandbuch*, 2nd ed. Springer Vieweg Wiesbaden, 2013.