

MATLAB으로 배우는 선형대수학(2판)

연습문제 답 이용 안내

- 본 문제 풀이의 저작권은 이광연, 설한국과 한빛아카데미(주)에 있습니다.
- 이 자료를 무단으로 전제하거나 배포할 경우 저작권법 136조에 의거하여 최고 5년 이하의 징역 또는 5천만원 이하의 벌금에 처할 수 있고 이를 병과(併科)할 수도 있습니다.

Chapter 01. 연습문제 정답

1.1.

[정답] (a), (c), (f)

1.2.

[정답]

(a) $x = 4, y = -3$

(b) $x = 9, y = -1, z = -2$

(c) $x = 1, y = 0, z = -1$

(d) $x_1 = 7 - t - 2s, x_2 = -3 - t - s, x_3 = t, x_4 = s$

(e) $x = 7t/4, y = -t/2, z = -15t/4, w = t$

(f) $z = 1, y = 1, x = 2, w = 4, v = 8$

(g) $x = 1 - 2t - s, y = t, z = 2 - s, w = s$

(h) $x_1 = 1/2, x_2 = -1/2, x_3 = 0, x_4 = -1$

(i) $x_1 = -2, x_2 = -1, x_3 = 0, x_4 = 0, x_1 = 0, x_2 = 0, x_3 = 0, x_4 = 1$

(j) $v = 1 - t - 2s, w = 1 - t, x = t, y = s, z = 0$

1.3.

[정답]

(a)
$$\begin{bmatrix} 1 & -3 & 0 & -3 & 0 & -4 \\ 0 & 0 & 1 & 2 & 0 & 3 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(b)
$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -3 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

1.4.

[정답]

$$x = \frac{12}{7}, y = \frac{1}{7}$$

1.5.

[정답]

해가 없다.

1.6.

[정답] $a = \frac{16}{5}, b = -\frac{13}{10}, c = -\frac{31}{10}$

1.7.

[정답] $\alpha = \frac{3\pi}{2}, \beta = 0, \pi, \gamma = 0$

1.8.

[정답]

해가 없거나 무수히 많은 해를 가진다.

1.9.

[정답] $y = \frac{3}{2}x^2 - \frac{11}{2}x + 3$

1.10.

[정답]

$$A = -\frac{1}{4}, B = -\frac{5}{8}, C = \frac{3}{2}$$

1.11.

[정답]

$$x = 29, y = 16, z = 3$$

1.12.

[정답] $x = 1, y = 1, z = 1$
 $x = 1, y = 2, z = 2$
 $x = 1, y = 2, z = 3$

1.13.

[정답] $a = -3$

1.14.

[정답] 무수히 많은 해를 가진다.

1.15.

[정답]

(a) $a = 3$

(b) $a \neq 2$ 이고 $a \neq 3$

(c) $a = 2$

1.16.

[정답]

해가 없다.

1.17.

[증명 생략]

Chapter 02 연습문제 정답

2.1.

[정답]

(a) $a_{12} = -2, a_{22} = -3, a_{23} = 4$

(b) $b_{11} = 2, b_{31} = 5$

(c) $c_{13} = 2, c_{31} = 7, c_{33} = -1$

(d) $6, 3, -1$

2.2.

[정답] $x = 0, y = -3, z = -1, w = -1$

2.3.

[증명 생략]

2.4.

[정답]

(a) $\begin{bmatrix} 3 & 2 & 0 \\ 3 & -2 & 2 \\ 3 & 2 & 3 \end{bmatrix}$

(b) $\text{tr}4$

(c) $\begin{bmatrix} 4 & 1 & -4 \\ 8 & -1 & 3 \\ 5 & 2 & 6 \end{bmatrix}$

(d) 9

2.5.

[증명 생략]

2.6.

[정답]

(a) $\begin{bmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{bmatrix}$

(b) $\begin{bmatrix} \frac{2}{7} & \frac{1}{7} \\ -\frac{1}{7} & \frac{2}{21} \end{bmatrix}$

(c) $\begin{bmatrix} \frac{1}{5} & \frac{1}{5} \\ -\frac{4}{15} & \frac{1}{15} \end{bmatrix}$

$$(d) \begin{bmatrix} \frac{19}{18} & -\frac{7}{6} & -\frac{11}{18} \\ -\frac{1}{9} & \frac{1}{3} & \frac{2}{9} \\ -\frac{7}{18} & \frac{1}{6} & \frac{5}{18} \end{bmatrix}$$

$$(e) \begin{bmatrix} -\frac{3}{10} & \frac{3}{20} & \frac{1}{5} \\ \frac{4}{15} & \frac{11}{30} & -\frac{1}{15} \\ \frac{1}{30} & -\frac{1}{60} & \frac{2}{15} \end{bmatrix}$$

(f) 비가역

$$(g) \begin{bmatrix} 1 & 0 & 0 & 0 \\ -\frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & -\frac{1}{3} & \frac{1}{3} & 0 \\ 0 & 0 & -\frac{1}{4} & \frac{1}{4} \end{bmatrix}$$

$$(h) \begin{bmatrix} \frac{7}{23} & -\frac{8}{23} & \frac{1}{23} & \frac{2}{23} \\ \frac{37}{46} & \frac{7}{46} & \frac{24}{23} & -\frac{21}{23} \\ \frac{17}{46} & -\frac{3}{46} & \frac{16}{23} & -\frac{14}{23} \\ \frac{9}{46} & -\frac{7}{46} & -\frac{1}{23} & \frac{2}{23} \end{bmatrix}$$

$$(i) \begin{bmatrix} \frac{1}{18} & \frac{5}{12} & -\frac{11}{36} & -\frac{1}{36} \\ -1 & \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ \frac{7}{18} & -\frac{7}{12} & -\frac{13}{36} & \frac{11}{36} \\ -\frac{5}{18} & \frac{5}{12} & \frac{1}{36} & -\frac{13}{36} \end{bmatrix}$$

$$(j) \begin{bmatrix} \frac{1}{k} & 0 & 0 & 0 \\ -\frac{1}{k^2} & \frac{1}{k} & 0 & 0 \\ \frac{1}{k^3} & -\frac{1}{k^2} & \frac{1}{k} & 0 \\ -\frac{1}{k^4} & \frac{1}{k^3} & -\frac{1}{k^2} & \frac{1}{k} \end{bmatrix}$$

2.7.

[정답] $k = -3$

2.8.

[정답]

(a) $x_1 = 0, x_2 = 0, x_3 = 0$ (b) $x_1 = 2, x_2 = 1, x_3 = -1$

2.9.

[정답]

$$A = \begin{bmatrix} -2 & \frac{7}{2} & -\frac{1}{2} & 0 \\ \frac{7}{2} & 1 & 1 & \frac{3}{2} \\ -\frac{1}{2} & 1 & 1 & 2 \\ 0 & \frac{3}{2} & 2 & 4 \end{bmatrix} + \begin{bmatrix} 0 & \frac{5}{2} & \frac{5}{2} & 0 \\ -\frac{5}{2} & 0 & -3 & \frac{3}{2} \\ -\frac{5}{2} & 3 & 0 & 0 \\ 0 & -\frac{3}{2} & 0 & 0 \end{bmatrix}$$

2.10.

[정답] $x \neq -2, 1, 4$

2.11.

[정답] $a = 0, b = 3$

2.12.

[정답] $a = -1, 3$

2.13.

[정답] $a + b - c \neq 0$

2.14.

[정답]

$$D^{-1} = \begin{bmatrix} \frac{1}{6} & 0 & 0 & 0 \\ 0 & -\frac{1}{8} & 0 & 0 \\ 0 & 0 & \frac{1}{7} & 0 \\ 0 & 0 & 0 & -\frac{1}{9} \end{bmatrix}$$

2.15.

[증명 생략]

2.16.

[증명 생략]

2.17.

[증명 생략]

2.18.

$$[\text{정답}] \quad 3, \begin{bmatrix} 1 & r & r^2 + s \\ 0 & 1 & r \\ 0 & 0 & 1 \end{bmatrix}$$

2.19.

[증명 생략]

2.20.

[생략]

2.21.

[증명 생략]

2.22.

[증명 생략]

2.23.

[증명 생략]

2.24.

[증명 생략]

2.25.

[증명 생략]

2.26.

[정답]

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

2.27.

[증명 생략]

2.28.

[정답]

$$\begin{bmatrix} 2 & 4 \\ -1 & -2 \\ 3 & 3 \end{bmatrix}$$

2.29.

[증명 생략]

2.30.

[풀이]

$$(a) \ E_1 = \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{3} \end{bmatrix}, \ E_2 = \begin{bmatrix} 1 & 0 \\ \frac{2}{3} & 1 \end{bmatrix}$$

$$(b) \ \begin{bmatrix} 1 & 0 \\ \frac{2}{3} & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{3} \end{bmatrix}$$

2.31.

[풀이]

$$\begin{bmatrix} \frac{1}{2} \\ 0 \\ -\frac{1}{2} \end{bmatrix}$$

Chapter 03 연습문제 정답

3.1.

[정답]

- | | | |
|------------|--------|-------|
| (a) 15 | (b) 0 | (c) 4 |
| (d) $-abc$ | (e) 42 | (f) 4 |

3.2.

[정답]

 $\lambda = 3$ 또는 $\lambda = 2$

3.3.

[정답]

 $\lambda = 2$ 또는 $\lambda = 6$

3.4.

[정답]

 $r = 0, \frac{1}{3}, 4$

3.5.

[정답]

 $\frac{3}{4} \pm \frac{\sqrt{33}}{4}$

3.6.

[증명 생략]

3.7.

[정답]

 $a = 3, -2$

3.8.

[정답]

 $\lambda = 1, 2, 2$

3.9.

[정답]

80

3.10.

[정답]

64

3.11.

[정답]

$$(a) \begin{bmatrix} 18 & 10 & 6 \\ -17 & 1 & 10 \\ 6 & -28 & 2 \end{bmatrix}$$

$$(b) \begin{bmatrix} \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$$

3.12.

[정답]

$$(a) \frac{7}{6}$$

$$(b) \frac{9}{37}$$

3.13.

[정답]

$$\text{조건: } a^2 + 3a - 40 = 0$$

$$\text{해: } x = \frac{-2a-3}{13}t, \quad y = \frac{-a+18}{13}t, \quad z = t \quad (t: \text{실수})$$

3.14.

[증명 생략]

3.15.

[풀이]

(a) 증명 생략

$$(b) \cos\alpha = \frac{-a^2+b^2+c^2}{2bc}, \quad \cos\beta = \frac{a^2-b^2+c^2}{2ac}, \quad \cos\gamma = \frac{a^2+b^2-c^2}{2ab} \quad (\text{단, } abc \neq 0)$$

3.16.

[증명 생략]

3.17.

[증명 생략]

3.18.

[증명 생략]

3.19.

[증명 생략]

3.20.

[증명 생략]

3.21.

[증명 생략]

3.22.

[증명 생략]

3.23.

[정답]

$$\det(A) = 1 \text{ 또는 } \det(A) = 0$$

3.24.

[증명 생략]

3.25.

[정답]

$$p(x) = 1 - \frac{421}{36}x + \frac{251}{72}x^2 + \frac{215}{72}x^3 - \frac{7}{9}x^4$$

Chapter 04 연습문제 정답

4.1.

[정답]

- (a) 4개 (b) 4개 (c) 4개 (d) 3개 (e)
- $\sqrt{17}$

4.2.

[정답]

- (a) $\mathbf{a} = \frac{1}{\sqrt{3}}(\mathbf{i} + \mathbf{j} + \mathbf{k})$ (b) $\mathbf{a} = \frac{1}{\sqrt{21}}(4, -1, 2)$
- (c) $\mathbf{a} = \frac{1}{\sqrt{10}}(3\mathbf{i} + \mathbf{j})$ (d) $\mathbf{a} = \frac{1}{\sqrt{30}}(-2, 1, -5)$

4.3.

[정답]

- (a) 내적 : 39
 외적 : $(-14, -5, -36)$
- (b) 내적 : 29
 외적 : $(-15, 9, -1)$

4.4.

[정답]

- (a) $(-22, -24, -116)$ (b) $(-4, 36, -26)$
- (c) $(0, 198, -132)$ (d) -198

4.5.

[정답]

- (a) $\frac{4}{5}$ (b) 0
- (c) $\frac{667}{940}$ (d) $\frac{649}{701}$

4.6.

[정답] 49

4.7.

[정답]

- (a) $\frac{|-7|}{\sqrt{26}} = \frac{7}{\sqrt{26}}$ (b) $\frac{17}{\sqrt{21}}$

4.8.

[정답] $24x + 12y + 8z - 24 = 0$

4.9.

[정답]

$$\begin{cases} x = -14t - 13 \\ y = -t - 17 \\ z = t \end{cases}$$

4.10.

[정답]

동일한 직선 위에 있지 않다.

4.11.

[정답]

$$2(x-2) - 3(y-3) + z + 1 = 0$$

4.12.

[정답] $-6x + 2y - 3z = 0$

4.13.

[풀이]

$$\left(-\frac{6}{7}, -\frac{2}{7}, \frac{11}{7} \right)$$

4.14.

[정답] $\frac{5817}{379}$

4.15.

[정답]

$$\sqrt{\frac{77}{6}}$$

4.16.

[정답]

$$\frac{51}{\sqrt{54}}$$

4.17.

[증명 생략]

4.18.

[증명 생략]

4.19.

[증명 생략]

4.20.

[증명 생략]

4.21.

[증명 생략]

4.22.

[증명 생략]

4.23.

[증명 생략]

4.24.

[증명 생략]

4.25.

[증명 생략]

4.26.

[증명 생략]

Chapter 05. 연습문제 정답

5.1.

[정답]

- | | |
|-------------|-------------|
| (a) 부분공간 | (b) 부분공간 아님 |
| (c) 부분공간 아님 | (d) 부분공간 아님 |

5.2.

[증명 생략]

5.3.

[정답]

$$\begin{bmatrix} 2 & 1 \\ 1 & -1 \\ 3 & -2 \end{bmatrix}$$

5.4.

[정답]

- | | |
|-------------|-------------|
| (a) 부분공간 아님 | (b) 부분공간 |
| (c) 부분공간 | (d) 부분공간 아님 |

5.5.

[정답]

- | | |
|----------|----------|
| (a) 일차독립 | (b) 일차독립 |
| (c) 일차독립 | (d) 일차독립 |

5.6.

[정답]

(c),(e)

5.7.

[증명 생략]

5.8.

[증명 생략]

5.9.

[증명 생략]

5.10.

[정답]

(a) 일차독립

(b) 일차독립

(c) 일차종속

5.11.

[정답]

 $\{(1,0,1,0), (0,1,-1,0), (0,0,1,0), (0,0,0,1)\}$

5.12.

[정답] 3차원

5.13.

[정답]

$$\left\{ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right\}$$

5.14.

[정답]

$$\begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ -3 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 4 \end{bmatrix}$$

5.15.

[증명 생략]

5.16.

[정답]

 $\{(10,11,7)\}$, 차원=1

5.17.

[정답]

$$\begin{bmatrix} 7 \\ 2 \end{bmatrix}$$

5.18.

[정답]

$$(a) [\mathbf{x}]_T = \begin{bmatrix} \frac{4}{3} \\ \frac{1}{3} \\ \frac{8}{3} \end{bmatrix}, [\mathbf{y}]_T = \begin{bmatrix} 1 \\ 1 \\ -3 \end{bmatrix}$$

$$(c) [\mathbf{x}]_S = \begin{bmatrix} \frac{19}{8} \\ -\frac{3}{8} \\ \frac{9}{8} \end{bmatrix}, [\mathbf{y}]_S = \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}$$

$$(b) \begin{bmatrix} \frac{1}{8} & \frac{5}{8} & -\frac{3}{4} \\ \frac{7}{8} & \frac{11}{8} & \frac{3}{4} \\ \frac{3}{8} & -\frac{1}{8} & -\frac{1}{4} \end{bmatrix}$$

$$(d) \begin{bmatrix} \frac{1}{8} & \frac{5}{8} & -\frac{3}{4} \\ \frac{7}{8} & \frac{11}{8} & \frac{3}{4} \\ \frac{3}{8} & -\frac{1}{8} & -\frac{1}{4} \end{bmatrix}^{-1}$$

5.19.

[정답]

(a) 직교

(b) 직교 아님

5.20.

[풀이]

-1

5.21.

[풀이]

$$\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$$

5.22.

[정답]

$$(a) T = \left\{ \mathbf{y}_1 = \begin{bmatrix} 0.26726 \\ 0.53452 \\ 0.80178 \end{bmatrix}, \mathbf{y}_2 = \begin{bmatrix} -0.87287 \\ -0.21822 \\ 0.43644 \end{bmatrix}, \mathbf{y}_3 = \begin{bmatrix} -0.40825 \\ 0.81650 \\ -0.40825 \end{bmatrix} \right\}$$

$$(b) T = \left\{ \mathbf{y}_1 = \begin{bmatrix} \frac{379}{1257} \\ \frac{379}{419} \\ -\frac{379}{1257} \end{bmatrix}, \mathbf{y}_2 = \begin{bmatrix} \frac{1531}{1636} \\ \frac{434}{1971} \\ \frac{657}{2387} \end{bmatrix}, \mathbf{y}_3 = \begin{bmatrix} -\frac{461}{2525} \\ \frac{505}{1383} \\ \frac{461}{505} \end{bmatrix} \right\}$$

5.23.

[정답]

$$R = \begin{bmatrix} \sqrt{5} & \sqrt{5} \\ 0 & \sqrt{5} \end{bmatrix}$$

5.24.

[정답]

$$(a) Q = \begin{bmatrix} -\frac{747}{4174} & -\frac{889}{1136} & -\frac{963}{1615} \\ \frac{498}{2087} & -\frac{1400}{2249} & \frac{963}{1292} \\ -\frac{671}{703} & -\frac{20}{2249} & \frac{963}{3230} \end{bmatrix}, R = \begin{bmatrix} -\frac{4174}{249} & \frac{3638}{231} & \frac{3638}{693} \\ 0 & -\frac{1347}{374} & -\frac{449}{374} \\ 0 & 0 & * \end{bmatrix}$$

$$(b) Q = \begin{bmatrix} 0.26726 & -0.87287 & -0.40825 \\ 0.53452 & -0.21822 & 0.81650 \\ 0.80178 & 0.43644 & -0.40825 \end{bmatrix}, R = \begin{bmatrix} 3.74166 & 5.85975 & 7.21605 \\ 0 & 0.65465 & 0.87287 \\ 0 & 0 & 0.40825 \end{bmatrix}$$

5.25.

[정답]

$$T = \left\{ y_1 = \begin{bmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{bmatrix}, y_2 = \begin{bmatrix} \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{6}} \\ -\sqrt{\frac{2}{3}} \end{bmatrix}, y_3 = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \\ 0 \end{bmatrix} \right\}$$

$$v = (v \cdot y_1)y_1 + (v \cdot y_2)y_2 + (v \cdot y_3)y_3$$

5.26.

[정답]

$$\begin{bmatrix} \frac{\sqrt{3}}{3} & \frac{\sqrt{3}}{3} & \frac{\sqrt{3}}{3} \\ 0 & \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{6}}{3} & -\frac{\sqrt{6}}{6} & -\frac{\sqrt{6}}{6} \end{bmatrix}$$

5.27.

[정답]

$$a=0, b=-\sqrt{\frac{2}{3}}, c=\sqrt{\frac{1}{3}}$$

5.28.

[증명 생략]

5.29.

[증명 생략]

Chapter 06 연습문제 정답

6.1.

[증명 생략]

6.2.

[증명 생략]

6.3.

[증명 생략]

6.4.

[증명 생략]

6.5.

[정답]

- (a) $(1, -15)$ (b) $(-9, 11)$
 (c) $\{t(-2, 3, 4) | t \in \mathbb{R}\}$ (d) \mathbb{R}^2

6.6.

[정답]

- (a) $-26x^2 + 7x - 9$ (b) $-\frac{1572}{199}x^2 - 3x + \frac{4552}{1189}$
 (c) $\{0\}$ (d) P_2

6.7.

[정답]

- (a) $x_1 = 3w_1 - 2w_2$
 $x_2 = -w_1 + w_2$
 (b) $x_1 = w_1 - 2w_2 + 4w_3$
 $x_2 = -w_1 + 2w_2 - 3w_3$
 $x_3 = -w_1 + 3w_2 - 5w_3$

6.8.

[정답]

$$\begin{bmatrix} \frac{\sqrt{3}+3}{4} \\ \frac{\sqrt{3}+1}{4} \end{bmatrix}$$

6.9.

[정답]

(a) 3

(b) 1

6.10.

[정답]

$$\begin{bmatrix} 1 & -4 & -3 \\ -3 & 3 & -1 \\ -4 & 3 & -3 \end{bmatrix}$$

6.11.

[정답]

(1) R^2 의 임의의 기저(2) \emptyset (3) $\text{rank}(T) = 2, \text{nullity}(T) = 0$

6.12.

[정답]

(a) $\begin{bmatrix} -7 & 0 \\ 0 & 11 \end{bmatrix}$

(b) $\begin{bmatrix} 7 & -1 \\ 1 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 0000 \\ 0000 \\ 0000 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 2 & 1 & 0 \\ 1 & 5 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix}$

6.13.

[풀이]

1

6.14.

[증명 생략]

6.15.

[정답]

표준행렬 : $\begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$

$L(\mathbf{x}) = \begin{bmatrix} k \\ 2k \end{bmatrix}$

6.16.

[증명 생략]

6.17.

[증명 생략]

6.18.

[정답]

(a) 증명 생략

(b) $\{A \in M_n \mid A = A^T\}$ (c) $\{A = [a_{ij}] \in M_n \mid a_{ii} = 0, i = 1, 2, \dots, n\}$

6.19.

(※ 2쇄에 다음 내용을 추가할 예정입니다.)

단, $[L_1]_{B', B} = \begin{bmatrix} 1 & 0 \\ 1 & 2 \\ -1 & 0 \end{bmatrix}$, $[L_2]_{B, B'} = \begin{bmatrix} 1 & 2 & 1 \\ -2 & 2 & 1 \end{bmatrix}$ 이다.

[정답]

$$\begin{bmatrix} 2 & 4 \\ -1 & 4 \end{bmatrix}$$

6.20.

[증명 생략]

Chapter 07 연습문제 정답

7.1.

[정답]

$$(a) \left\{ \begin{bmatrix} -\sqrt{2} \\ 1 \end{bmatrix}, \begin{bmatrix} \sqrt{2} \\ 1 \end{bmatrix} \right\}, -3\sqrt{2}+5, 3\sqrt{2}+5$$

$$(b) \left\{ \begin{bmatrix} -\frac{2584}{2889} \\ \frac{1292}{2889} \end{bmatrix}, \begin{bmatrix} -\frac{985}{1393} \\ -\frac{985}{1393} \end{bmatrix} \right\}, -4, 2$$

$$(c) \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} \frac{1}{2} \\ 0 \\ 1 \end{bmatrix} \right\}, 1, 2, 3$$

$$(d) \left\{ \begin{bmatrix} \frac{209}{362} \\ -\frac{209}{362} \\ -\frac{877}{1519} \end{bmatrix}, \begin{bmatrix} -\frac{571}{989} \\ \frac{571}{989} \\ \frac{209}{362} \end{bmatrix}, \begin{bmatrix} -\frac{571}{989} \\ \frac{571}{989} \\ \frac{209}{362} \end{bmatrix} \right\}, \frac{115262}{115261}, \frac{230521}{230522}, \frac{230521}{230522}$$

$$(e) \left\{ \begin{bmatrix} -\frac{803}{839} \\ \frac{295}{1018} \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -\frac{295}{1018} \\ -\frac{803}{839} \\ 0 \end{bmatrix} \right\}, -\frac{1069}{120}, -1, \frac{2498}{1309}$$

$$(f) \left\{ \begin{bmatrix} 0 \\ \frac{985}{1393} \\ 0 \\ -\frac{985}{1939} \end{bmatrix}, \begin{bmatrix} -\frac{521}{991} \\ 0 \\ \frac{3275}{4379} \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ \frac{985}{1393} \\ 0 \\ \frac{985}{1393} \end{bmatrix}, \begin{bmatrix} -\frac{3725}{4379} \\ 0 \\ -\frac{521}{991} \\ 0 \end{bmatrix} \right\}, -1, -\frac{610}{987}, 1, \frac{1597}{987}$$

7.2.

[정답]

(a) 고윳값 $-3\sqrt{2}+5$ 에 대한 고유공간의 기저 $\begin{bmatrix} -\sqrt{2} \\ 1 \end{bmatrix}$,

고윳값 $3\sqrt{2}+5$ 에 대한 고유공간의 기저 $\begin{bmatrix} \sqrt{2} \\ 1 \end{bmatrix}$

(b) 고윳값 -1 에 대한 고유공간의 기저 $\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$,

(c) 고윳값 1 에 대한 고유공간의 기저 $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$,

고윳값 2 에 대한 고유공간의 기저 $\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$,

고윳값 3 에 대한 고유공간의 기저 $\begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix}$

(d) 고윳값 1 에 대한 고유공간의 기저 $\begin{bmatrix} 4 \\ 4 \\ 3 \\ 52 \end{bmatrix}$,

고윳값 2 에 대한 고유공간의 기저 $\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$,

고윳값 3 에 대한 고유공간의 기저 $\begin{bmatrix} 0 \\ -3 \\ 1 \\ 17 \end{bmatrix}$,

고윳값 9 에 대한 고유공간의 기저 $\begin{bmatrix} 0 \\ 0 \\ 7 \\ 8 \end{bmatrix}$,

7.3.

[정답]

(a) 증명 생략

$$(b) A^5 = -86A^2 - 102A + 198I, \quad A^{-1} = \frac{1}{9}(A^2 + 5A + 3I)$$

7.4.

[증명 생략]

7.5.

[증명 생략]

7.6.

[증명 생략]

7.7.

[정답]

$$\begin{bmatrix} \frac{985}{1393} & \frac{684}{721} \\ \frac{985}{1393} & \frac{228}{721} \end{bmatrix}$$

7.8.

[증명 생략]

7.9.

[정답]

$$(a) \text{고윳값 } 0, \frac{1}{4}, 1$$

$$\text{대응하는 고유벡터 } \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$(b) P = \begin{bmatrix} -1 & -2 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

$$\text{대각화 } \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{4} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

(c) 증명 생략

7.10.

[정답]

(a) $(\lambda - 2)^2(\lambda - 8) = 0$

(b) $\lambda = 2 \leftrightarrow \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \left(-\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right), \left(-\frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}, -\frac{1}{\sqrt{6}}\right)$

(c) $\lambda = 8 \leftrightarrow \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \frac{1}{\sqrt{3}}(1, 1, 1)$

(d) $P = \begin{bmatrix} \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & 0 & \frac{2}{\sqrt{6}} \end{bmatrix}, D = \begin{bmatrix} 8 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

7.11.

[증명 생략]

7.12.

[정답]

(a) $\begin{bmatrix} 0 & 0 \\ 0 & 5 \end{bmatrix}$

(b) $\begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 9 \end{bmatrix}$

(c) $\begin{bmatrix} -\frac{408}{985} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{985}{408} \end{bmatrix}$

(d) $\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 16 \end{bmatrix}$

7.13.

[풀이]

$\begin{bmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 3 \end{bmatrix}$

7.14.

[정답]

$10^{10}, 9^{10}, (-8)^{10}, 7^{10}, (-1)^{10}$

7.15.

[증명 생략]

7.16.

[증명 생략]

7.17.

[증명 생략]

7.18.

[정답]

$$y_1(t) = 0, \quad y_2(t) = e^{3t}, \quad y_3(t) = -e^{3t}$$

Chapter 08 연습문제 정답

8.1.

[정답]

(a) $-1 + 4i$

(b) $5 - 5i$

(c) $-2 + 11i$

(d) $-\frac{2}{5} - \frac{1}{5}i$

(e) $30 + 15i$

(f) $-\frac{4}{25} - \frac{22}{25}i$

8.2.

[정답]

(a) 0

(b) $28 + 96i$

(c) 1

8.3.

[정답]

$x_1 = i, x_2 = 0, x_3 = -i$

8.4.

[정답]

(a) $2(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2})$

(b) $8(\cos \pi + i \sin \pi)$

(c) $3\sqrt{2}(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4})$

(d) $4\left(\cos\left(-\frac{\pi}{6}\right) + i \sin\left(-\frac{\pi}{6}\right)\right)$

8.5.

[정답]

(a) $\frac{721}{228}$

(b) $\frac{2251}{418}$

(c) $\frac{1941}{296}$

(d) $\frac{1171}{137}$

8.6.

[풀이]

195

8.7.

[정답]

(a) $2+5i$

(b) 0

8.8.

[정답]

(a) $7-5i$ (b) $-1-i$

8.9.

[정답]

에르미트 행렬 : (a), (b), (g), (h)

유니타리 행렬 : (d), (e), (g), (i)

8.10.

[정답]

$$(a) \quad U = \frac{1}{\sqrt{3}} \begin{bmatrix} 1-i & \frac{-1+i}{\sqrt{2}} \\ 1 & \frac{1}{\sqrt{2}} \end{bmatrix}, D = \begin{bmatrix} 3 & 0 \\ 0 & 0 \end{bmatrix}$$

$$(b) \quad U = \begin{bmatrix} 0 & 0 & 1 \\ \frac{881}{2158} + \frac{881}{2158}i & \frac{780}{1351} + \frac{780}{1351}i & 0 \\ -\frac{881}{1079} & \frac{780}{1351} & 0 \end{bmatrix}, D = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

8.11.

[증명 생략]

8.12.

[정답]

$$(a) \quad \sqrt{\frac{4}{3}}$$

$$(b) \quad \frac{1}{2} - \frac{1}{3}i$$

$$(c) \quad \frac{1}{2} + \frac{1}{3}i$$

8.13.

[정답]

$$T = \{\mathbf{v}_1 = (i, 0, 0), \mathbf{u}_2 = (0, 0, i), \mathbf{u}_3 = (0, -i, 0)\}$$

8.14.

[증명 생략]

8.15.

[증명 생략]

8.16.

[증명 생략]

8.17.

[증명 생략]

8.18.

[증명 생략]

8.19.

[증명 생략]

8.20.

[증명 생략]

8.21.

[증명 생략]

8.22.

[증명 생략]

8.23.

[증명 생략]

8.24.

[증명 생략]