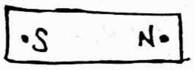
Magnetic effect of current

Some points related to magnetism-

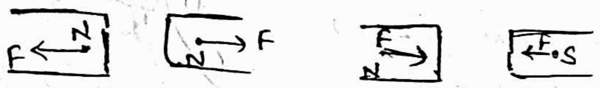
(i) Magnet has two poles-

(a) North pole (N pole)

(b) South pole (S pole)



(ii) Like poles repel each other and unlike poles attract each other.



(iii) Magnetic field- It is that space surrounding a magnet with in which when other magnets and magnetic materials are brought in then they experience attractive or repulsive force.

Note-Magnetic materials are Fe, Ni, Co etc.

(iv) Magnetic field intensity- It is defined as the magnetic force experienced by a unit north pole placed at any point in magnetic field.

If a test north pole of strength m_o experiences magnetic force F at any point in magnetic field then the magnetic field intensity at that point is given by

$$B = \frac{F}{m_o}$$

$m_o \rightarrow F$

$1 \rightarrow F/m_o$

In vector form

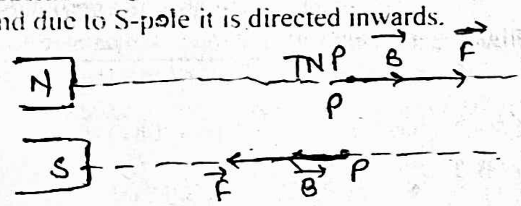
$$\vec{B} = \frac{\vec{F}}{m_o}$$

$P \longrightarrow \vec{F}$

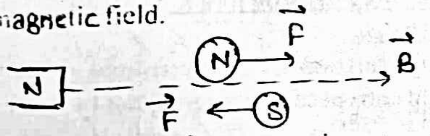
It is a vector quantity and its direction is same as the magnetic force experienced by test north pole. Its S.I. unit is tesla and C.G.S. emu is gauss.

1 tesla (T) = 10^4 gauss (G)

Note(a)- Magnetic field due to N-pole is directed outwards and due to S-pole it is directed inwards.



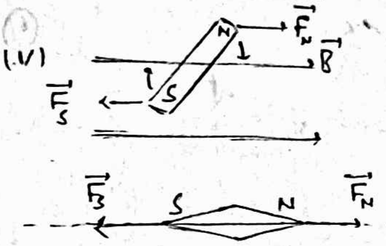
(b) When a N-pole is placed in external magnetic field then it experiences force in the direction of magnetic field and when a S-pole is placed in external magnetic field then it experiences force in the direction opposite to the magnetic field.

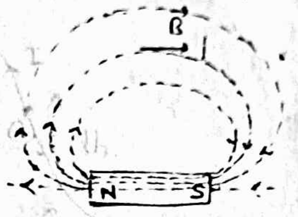


(v) Behaviour of a magnet in external magnetic field - When a magnet is placed in external magnetic field then its N-pole experiences force in the direction of magnetic field where as its S-pole experiences force in the direction opposite to magnetic field. The two forces acting on the magnet combine to form a couple whose tendency is to rotate magnet in the direction of magnetic field.

That is why when a magnetic needle (which is a small magnet) is placed in magnetic field then it orient itself in the direction of magnetic field.

(vi) Magnetic lines of force -Those imaginary smooth curves along which a unit N-pole would move if it is free to do so are called magnetic lines of force.





These lines represent magnetic field. The tangent drawn at any point of these lines gives us the direction of magnetic field intensity at that point.

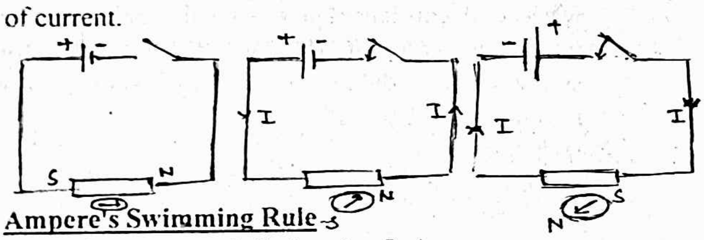
Oersted's experiment

Ques.-Describe Oersted's experiment.

Ans.- In 1820, Oersted found that when the flow of electric current takes place through a conductor then a magnetic needle placed below the conductor gets deflected. On reversing the direction of electric current the direction of deflection of magnetic needle also gets reversed.

As magnetic needle field can only be deflected by magnetic field, therefore from his experiment Oersted concluded that a current carrying conductor generates magnetic field.

The phenomenon of generation of magnetic field by a current carrying conductor is called magnetic effect of current.

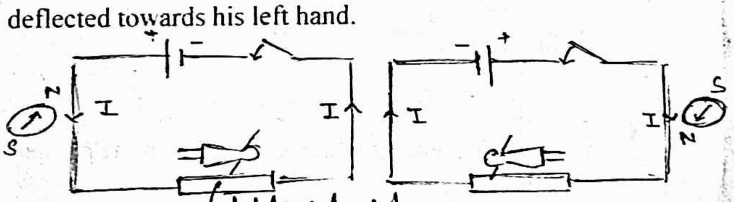


Ampere's Swimming Rule

Ques.-State Ampere's Swimming Rule.

Ans.- In order to find out the direction of deflection of magentic neddle in Oersted's experiment, Ampere gave a rule which is known as Ampere's swimming rule.

It states that when a person swims parallel to a current carrying conductor in the direction of electric current by keeping his face towards the magnetic needle such that the electric current enters through his feet and leaves through his head then the N-pole of magnetic needle gets deflected towards his left hand.



Biot-savart law (laplace law)

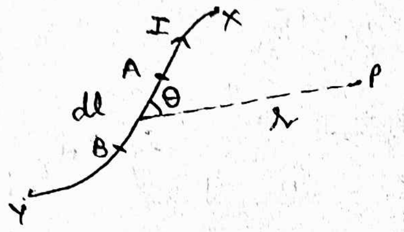
Ques.-State and explain Biot-savart law for the magnetic produced by a current element.

Ans.- In 1820, Biot-savart on the basis of their experiment established an expression for the magnetic field intensity generated by a current carrying conductor which is called Biot-savart law.

Consider a small element AB of length dl of a conductor which is carrying currentI as shown in fig.

On the basis of their experiment Biot-savart found that magnetic field intensity, due to the current carrying element AB at point P which is at a distance r from it is-

(i) directly proportional to the current flowing through



the element.

i.e. $dB \propto I$

(ii) directly proportional to the length of the element

i.e $\quad dB \propto d\ell$

(iii) directly proportional to the sin of the angle between the line joining the observation point to the element and the direction of electric current.

i.e $\quad dB \propto \sin\theta$

and (iv) inversely proportional to the square of the distance of observation point from the element.

i.e $\quad dB \propto \dfrac{1}{r^2}$

Combining all the above relations, we get

$$dB = \frac{I\,d\ell \sin\theta}{r^2}$$

or $\quad dB = k\dfrac{I\,d\ell \sin\theta}{r^2}$ -----(i)

where k is a constant of proportionality and is called electromagnetic constant. The value of k depends upon the system of unit used and the nature of medium in which the conductor is placed.

(I) S.I. system

(a) In vacuum-

$$k = \frac{\mu_0}{4\pi}$$

where μ_0 is another constant which is called absolute permeability of free space (vaccum).

$$\mu_0 = 4\pi \times 10^{-7}\ \mathrm{TmA^{-1}}$$

$\therefore \quad k = \dfrac{\mu_0}{4\pi} = \dfrac{4\pi \times 10^{-7}\mathrm{TmA^{-1}}}{4\pi}$

or $\quad k = 10^{-7}\,\mathrm{TmA^{-1}}$

From relation (i)

or $\quad \boxed{dB = \dfrac{\mu_0}{4\pi}\dfrac{I\,d\ell \sin\theta}{r^2}}$

(b) In medium-

$$k = \frac{\mu}{4\pi}$$

where μ is constant which is called absolute permeability of medium.

$\because \quad \mu = \mu_r\mu_0$

where μ_r is the relative permeability of medium

$\therefore \quad k = \dfrac{\mu}{4\pi} = \dfrac{\mu_r\mu_0}{4\pi}$

From relation (i)

or $\quad \boxed{dB = \dfrac{\mu_r\mu_0}{4\pi}\dfrac{I\,d\ell \sin\theta}{r^2}}$

(II) In C.G.S. (electromagnetic) system-

(a) In vaccum-

$$k = \frac{\mu_0}{4\pi}$$

Here, $\mu_0 = 4\pi$ Gcmabamp^{-1}

$\therefore \quad k = \dfrac{\mu_0}{4\pi} = \dfrac{4\pi}{4\pi} = 1$ Gcmabamp^{-1}

From relation (i)

⊗ shows direction (Inwards)

⊙ Outwards direction

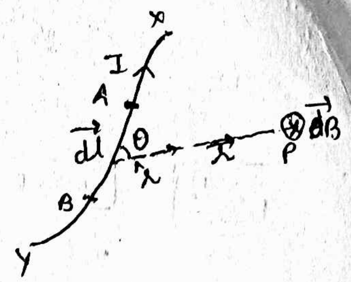
$$\boxed{dB = \frac{I\,d\ell \sin\theta}{r^2}}$$

(b) In medium-

$$k = \frac{\mu}{4\pi}$$

$\because \quad \mu = \mu_r\mu_0$

$\therefore \quad k = \dfrac{\mu}{4\pi} = \dfrac{\mu_r\mu_0}{4\pi}$



From relation (i)

$$\boxed{dB = \mu_r \frac{I\,d\ell \sin\theta}{r^2}}$$

Note- (i) Vector form of Biot-savart law- In vector form the Biot savart law can be written as-

$$\boxed{\vec{dB} = \frac{\mu_0}{4\pi}\frac{I(\vec{d\ell} \times \hat{r})}{r^2}}$$

where $\hat{r}$ is the unit vector along the position vector of observation point w.r.t. the element of current carrying conductor.

Multiplying both num. and deno. by r we get

$$\vec{dB} = \frac{\mu_0}{4\pi}\frac{I(\vec{d\ell} \times \hat{r})r}{r^2 . r}$$

or $\quad \vec{dB} = \dfrac{\mu_0}{4\pi}\dfrac{I(\vec{d\ell} \times \vec{r})}{r^3}$

Thus, the direction of magnetic field intensity $\vec{dB}$ at any point in the magnetic field due to current carrying element is same as that of $\vec{d\ell} \times \hat{r}$.

(ii) The direction of $\vec{d\ell}$ is always taken same as that of electric current. It is called element vector and $I\vec{d\ell}$ is called current element vector.

(iii) $\dfrac{\text{Relative permeability}}{\text{of a medium}} = \dfrac{\text{Absolute permeability of that medium}}{\text{Absolute permeability of vaccum}}$

i.e. $\quad \mu_r = \dfrac{\mu}{\mu_0}$

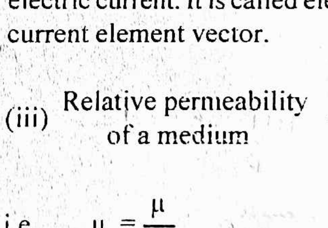
or $\quad \mu = \mu_r\mu_0$

As relative permeability of a medium is equal to the ratio of two similar physical quantities therefore it is a unitless and dimensionless physical quantity.

Magnetic field intensity due to a long straight current carrying conductor

Ques.-Derive the relation for magnetic field intensity due to a long straight current carrying conductor by using Biot-Savart law.

Ans.-Consider a long straight conductor which carries current I in the upward direction. We have to find out MFI due to the current carrying conductor at a point P which is at a ⊥ distance a from it.



From point P draw a ⊥ PM (equal to a) on the conductor. In order to find out MFI at point P, consider an element AB of the conductor whose length is $d\ell$ which is

... distance ℓ from point M. If the distance of the observation point P from the element AB be r and the angle between the $\vec{d\ell}$ and $\vec{r}$ be θ then the MFI due to the element at observation point P is given by-

or $\quad dB = \frac{\mu_0}{4\pi} \frac{I\,d\ell \sin\theta}{r^2}$ ---(i)

$\because \quad \cancel{\theta + \beta = 180^\circ}$

$\therefore \quad \cancel{\beta = 180^\circ - \theta}$

$\therefore \quad \cancel{\sin\beta = \frac{MP}{PB}}$

or $\quad \cancel{\sin(180^\circ - \theta) = \frac{a}{r}}$

or $\quad \cancel{\sin\theta = \frac{a}{r}}$

~~From relation (i)~~

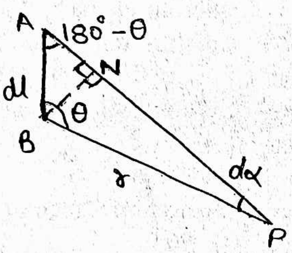
$\cancel{dB = \frac{\mu_0}{4\pi} \frac{Id\ell}{r^2} \frac{a}{r}}$

or $\quad \cancel{dB = \frac{\mu_0}{4\pi} \frac{Iad\ell}{r^3}}$ ----(ii)

In fig. ∠MPB = α and ∠APB = dα

From point B draw BN ⊥ PA

In rt. Δ BNP



$\sin d\alpha = \frac{BN}{BP}$

$\because$ dα is very small

$\Rightarrow \sin d\alpha \simeq d\alpha$

$\therefore \quad d\alpha = \frac{BN}{r}$

or $\quad BN = r\,d\alpha$ ----(ii)

Also, in Δ ABP

$\angle BAN + \theta + d\alpha = 180^\circ$

or $\quad \angle BAN = 180^\circ - \theta - d\alpha$

or $\quad \angle BAN \simeq 180^\circ - \theta \quad [\because$ dα is very small]

Again in Δ BNA

or $\quad \sin(180^\circ - \theta) = \frac{BN}{AB}$

or $\quad \sin\theta = \frac{BN}{d\ell}$

or $\quad BN = d\ell \sin\theta$ -----(iii)

From relation (ii) and (iii)

$d\ell \sin\theta = r\,d\alpha$

or $\quad \cancel{d\ell = \frac{r\,d\alpha}{\sin\theta}}$

Substituting the above value of $d\ell\sin\theta$ in relation (i)

$\cancel{dB = \frac{\mu_0}{4\pi} \frac{Ir\,d\alpha}{r\sin\theta}}$ $\quad dB = \frac{\mu_0}{4\pi} \frac{Ir\,d\alpha}{r^2}$

or $\quad \cancel{dB = \frac{\mu_0}{4\pi} \frac{Ia\,d\alpha}{a \times r}}$

or $\quad dB = \frac{\mu_0}{4\pi} \frac{I}{r} d\alpha$ ---(v)

In rt. Δ BNP

$\cos\alpha = \frac{PN}{PB}$

or $\quad \cos\alpha = \frac{a}{r}$

or $\quad r = \frac{a}{\cos\alpha}$

From relation (v)

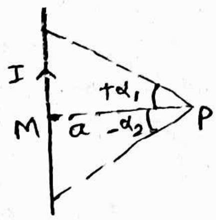
$dB = \frac{\mu_0}{4\pi} \frac{I}{a/\cos\alpha} d\alpha$

or $\quad dB = \frac{\mu_0}{4\pi} \frac{I}{a} \cos\alpha\, d\alpha$

If the angle substended by the top and bottom of the straight conductor at the observation point be α_1 and $-\alpha_2$ respectively.

Then, by integrating the above relation with in proper limits MFI due to the whole conductor at the observation point can be calculated as follows-

$\int_0^B dB = \int_{-\alpha_2}^{\alpha_1} \frac{\mu_0}{4\pi} \frac{I}{a} \cos\alpha\, d\alpha$



or $\quad [B]_0^B = \frac{\mu_0}{4\pi} \frac{I}{a} \int_{-\alpha_2}^{\alpha_1} \cos\alpha\, d\alpha$

or $\quad B - 0 = \frac{\mu_0}{4\pi} \frac{I}{a} \left[\sin\alpha\right]_{-\alpha_2}^{\alpha_1}$

or $\quad B = \frac{\mu_0}{4\pi} \frac{I}{a} [\sin\alpha_1 - \sin(-\alpha_2)]$

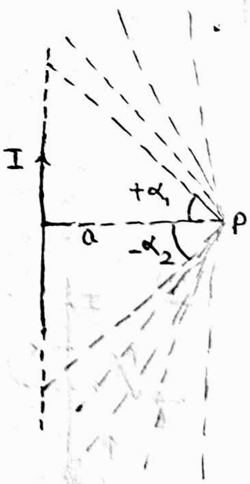
or $\quad \boxed{B = \frac{\mu_0}{4\pi} \frac{I}{a} [\sin\alpha_1 + \sin\alpha_2]}$

This is the required expression.

Particular cases- (i) When the straight conductor is of infinite length and the observation point lies between its two ends at a certain distance from it.

In this case-

$\alpha_1 \to \frac{\pi}{2}$ & $-\alpha_2 \to -\frac{\pi}{2}$



$\Rightarrow \quad \alpha_2 \to \frac{\pi}{2}$

$\therefore \quad B = \frac{\mu_0}{4\pi} \frac{I}{a} \left[\sin\frac{\pi}{2} + \sin\frac{\pi}{2}\right]$

or $\quad B = \frac{\mu_0 I}{4\pi a} [1 + 1]$

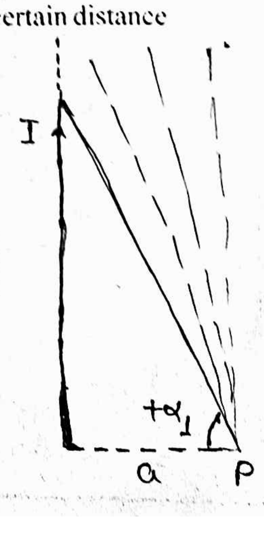
or $\quad B = \frac{\mu_0 I}{4\pi a} \cdot 2$

or $\quad \boxed{B = \frac{\mu_0}{4\pi} \frac{2I}{a}}$

(ii) When the conductor is of infinite length and the observation point lies at one of its ends at a certain distance from it.

In this case

$\alpha_1 \to \frac{\pi}{2}$ & $-\alpha_2 \to 0$



$\Rightarrow \quad \alpha_2 \to 0$

$\therefore \quad B = \frac{\mu_0}{4\pi} \frac{I}{a} \left[\sin\frac{\pi}{2} + \sin 0\right]$

or $\quad B = \frac{\mu_0 I}{4\pi a} [1 + 0]$

or $\quad \boxed{B = \frac{\mu_0 I}{4\pi a}}$

Note- (i) If nothing is given about the length of straight conductor then it should be assumed to be infinite length and the observation point should be assumed to be situated between the ends of conductor at a certain distance from it.

(ii) If the observation point lies in the direction of current carrying conductor then the MFI at the observation point is given by-

In this case

$$\alpha_1 = 0° \ \& \ -\alpha_2 = 0°$$

$$\therefore \quad B = \frac{\mu_0}{4\pi}\frac{I}{a}\left[\sin 0° + \sin 0°\right]$$

$$\text{or} \quad B = \frac{\mu_0 I}{4\pi a}\left[0+0\right]$$

$$\text{or} \quad B = 0$$

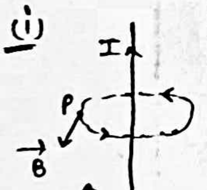
Direction of magnetic field due to straight current carrying conductor

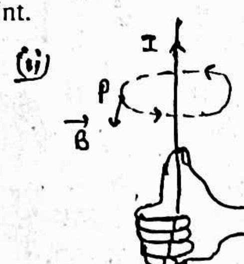
Ques.-State the rules for the determination of direction of magnetic field due to straight current carrying conductor.

Ans.- It can be find out by using three rules-

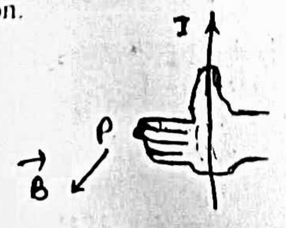
(i) Maxwell's (right handed) cork screw rule-It states that when a (right handed) cork screw which is placed parallel to a straight current carrying conductor is rotated in such a manner that it proceeds in the direction of electric current then the direction in which the thumb rotates gives us the direction of magnetic lines of force and the tangent drawn at any point of magnetic lines of force gives us the direction of MFI at that point.

(ii) Right hand thumb rule- It states that when a current carrying straight conductor in held in the right hand in such a manner that the thumb is in the direction of electric current then the direction in which the finger curl gives us the direction of magnetic lines of force. The tangent drawn at any point of magnetic lines of force gives us the direction of MFI at that point.





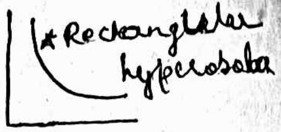
(iii) Right hand palm rule- It states that when the palm of right hand is stretched in such a manner that the thumb is directed in the direction of electric current and the fingures are directed towards the observation point then the direction of MFI is perpendicular to the open palm in the outward direction.



Note-(i) MFI due to a straight current carrying conductor of infinite length

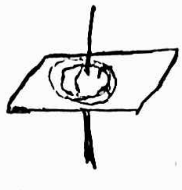
$$B = \frac{\mu_0}{4\pi}\frac{2I}{a}$$

$$\therefore \quad B \propto \frac{1}{a}$$



Graph between B and a is a rectangular hyperbola.

(ii)Magnetic lines of force due to a straight current carrying conductor of infinite length are concentric circles whose centre lies at the conductor.



MFI due to a current carrying circular coil or loop

Ques.-Derive the expresssion of MFI due to a current carrying circular coil or loop.

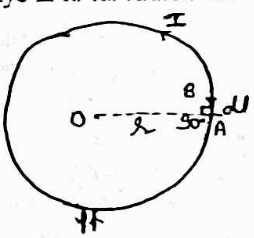
Ans.- Consider a circular coil of radius r and centre O in which electric current I in flowing in ACW direction.

In order to calculate MFI at the centre O of the coil. consider an element AB of the coil whose length is $d\ell$ and which is at a distance r from the centre O.

MFI due to the element AB at the centre O of the circular coil is given by

$$dB = \frac{\mu_0}{4\pi}\frac{I\,d\ell\sin\theta}{r^2}$$

As circumference of a circle is always ⊥ to its radius i.e. $\theta = 90°$



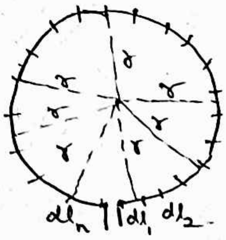
$$\therefore \quad dB = \frac{\mu_0}{4\pi}\frac{I\,d\ell\sin 90°}{r^2}$$

$$\text{or} \quad dB = \frac{\mu_0}{4\pi}\frac{I\,d\ell(1)}{r^2}$$

$$\text{or} \quad dB = \frac{\mu_0}{4\pi}\frac{I\,d\ell}{r^2} \quad \text{----(i)}$$

As the circular coil can be assumed to be made up of large number of elements. the MFI due to all of which is same (i.e. upwards) at the centre O of the coil. Therefore by intergrating both the sides of rel'n (i) within proper limits MFI at the centre of circular coil can be calculated as follows-

$$\int_\ell dB = \int_\ell \frac{\mu_0}{4\pi}\frac{I\,d\ell}{r^2}$$



$$\text{or} \quad B = \frac{\mu_0}{4\pi}\frac{I}{r^2}\int_\ell d\ell$$

$$\text{or} \quad B = \frac{\mu_0}{4\pi}\frac{I}{r^2}\ell$$

where $\ell = 2\pi r$ is the circuneference of circular coil.

$$\therefore \quad B = \frac{\mu_0}{4\pi}\frac{I}{r^2}2\pi r$$

$$\text{or} \quad \boxed{B = \frac{\mu_0}{4\pi}\frac{2\pi I}{r}}$$

If the number of turns in the coil be N then

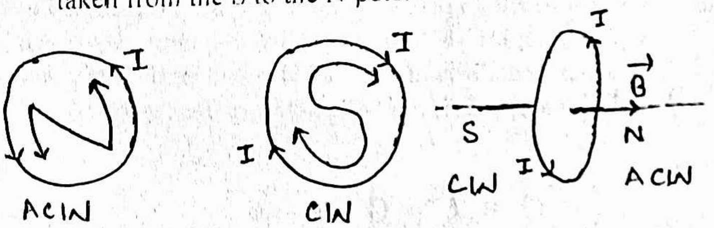
$$\boxed{B = \frac{\mu_0}{4\pi}\frac{2\pi NI}{r}}$$

This is the required expression.

Direction of Magnetic field- As the direcion of magnetic field due to all the length elements at the centre of the circular coil is same i.e. $\perp^r$ to the plane of the coil therefore when the direction of electric current in the coil is ACW then the direction of magnetic field at the centre is along the axis of the coil in the upward direction and when the direction of electric current in the coil is CW then the direction of magnetic field at the centre is along the axis of the coil in the downward direction.

Note- The direction of magnetic field due to a current carrying circular coil can also be find out by using end rule.

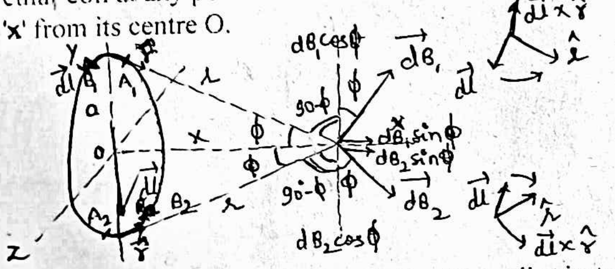
It states that the end of the coil from which the direction of electric current appears to be ACW behaves like the N-pole and the end from which the direction of electric current appears to be CW behaves like the S-pole of the magnet. The direction of magnetic field is always taken from the S to the N-pole.



MFI due to current carrying circular coil at any point on its axis

Ques.-Derive the expresssion of MFI due to current carrying circular coil at any point on its axis.

Ans.- Consider a circular coil of radius 'a' and centre O in which electric current 'I' in flowing in ACW direction. We have to find out MFI due to the current carrying circular coil at any point P on its axis which is at a distance 'x' from its centre O.



Consider a length element A_1B_1 of the coil at its top whose length is $d\ell$. If the distance of observation point from the length element be 'r' then the MFI due to it at the observation point is given by-

$$dB = \frac{\mu_0}{4\pi}\frac{I\,d\ell\sin\theta}{r^2}$$

Here $\theta = 90°$

$$\therefore \quad dB_1 = \frac{\mu_0}{4\pi}\frac{I\,d\ell\sin 90°}{r^2}$$

$$\text{or} \quad dB_1 = \frac{\mu_0}{4\pi}\frac{I\,d\ell\,(1)}{r^2}$$

$$\text{or} \quad dB_1 = \frac{\mu_0}{4\pi}\frac{I\,d\ell}{r^2} \quad \text{---(i)}$$

As the direction of $d\vec{B}_1$ is $\perp^r$ to plane containing $d\vec{\ell}$ and $\vec{r}$ i.e. along PM therefore it can be resolved into two components- (i) $dB_1\cos\phi$ in the vertical direction.

(ii) $dB_1\sin\phi$ in the horizontal direction.

Now consider another element A_2B_2 of the coil which is just diametrically opposite to the element A_1B_1.

MFI due to the element A_2B_2 at the observation point is given by-

$$\therefore \quad dB_2 = \frac{\mu_0}{4\pi}\frac{I\,d\ell\sin 90°}{r^2}$$

$$\text{or} \quad dB_2 = \frac{\mu_0}{4\pi}\frac{I\,d\ell\,(1)}{r^2}$$

$$\text{or} \quad dB_2 = \frac{\mu_0}{4\pi}\frac{I\,d\ell}{r^2}$$

As the direction of $d\vec{B}_2$ in also $\perp^r$ to the plane comtaining $d\vec{\ell}$ and $\vec{r}$ i.e. along PN therefore it can also be resolved into two components-

(i) $dB_2\cos\phi$ in the vertical direction.

(ii) $dB_2\sin\phi$ in the horizontal direction.

As the magnitude of dB_1 and dB_2 are same therefore their vertical components will cancel out each other and horizontal components will add up together.

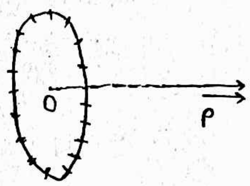
In this manner the whole coil can be resolved into infinite elements such that the vertical components of MFI due to each pair of diametrically opposite length elements will cancel out each other and horigontal components will add up together.

Thus the effective value of MFI at the observation point due to a length element is given by-

$$dB = dB_1\sin\phi$$

By integrating both the sides of the above relation with in proper limits the MFI due to the whole coil at point P can be calculated as follows-

$$\int_\ell dB = \int_\ell \frac{\mu_0}{4\pi}\frac{I\,d\ell\sin\phi}{r^2}$$



$$\text{or} \quad B = \frac{\mu_0}{4\pi}\frac{I\sin\phi}{r^2}\int_\ell d\ell$$

$$\text{or} \quad B = \frac{\mu_0}{4\pi}\frac{I\sin\phi}{r^2}\ell$$

where $\ell = 2\pi a$ is the circuneference of the coil.

$$\therefore \quad B = \frac{\mu_0}{4\pi}\frac{I\sin\phi}{r^2}2\pi a$$

$\because$ in rt. triangle ΔCOP $\sin\phi = \frac{a}{r}$

$$\therefore \quad B = \frac{\mu_0}{4\pi}\frac{Ia\,2\pi a}{r^2 . r}$$

$$\text{or} \quad B = \frac{\mu_0}{4\pi}\frac{2\pi I a^2}{r^3} \quad \text{---(i)}$$

Again in rectangle ΔCOP by pythagoras theorem

$$r^2 = a^2 + x^2$$

$$\text{or} \quad r = (a^2 + x^2)^{1/2}$$

From relation (i)

$$B = \frac{\mu_0}{4\pi}\frac{2\pi I a^2}{(a^2 + x^2)^{3/2}} = \frac{2\pi I a^2}{(a^2+x^2)^{3/2}}\frac{\mu_0}{4\pi}$$

If the circular coil has N turns then.

$$B = \frac{\mu_0}{4\pi}\,\frac{2\pi N I a^2}{(a^2+x^2)^{3/2}}$$

This is the required expression.

Particular cases- (i) When the observation point lies at the centre of the circular coil.

i.e. $x = 0$

then $$B = \frac{\mu_0}{4\pi}\,\frac{2\pi N I a^2}{(a^2+0^2)^{3/2}}$$

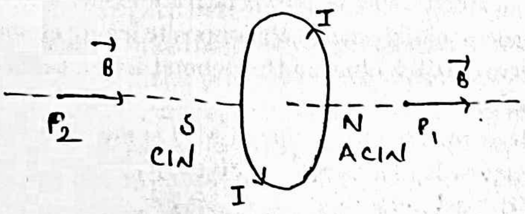
or $$B = \frac{\mu_0}{4\pi}\,\frac{2\pi N I}{a}$$

(ii) When the observation point lies at a very-very large distance from the centre 0 i.e. $x >> a$ then a^2 can be neglected as compared to x^2.

$$\therefore \quad B = \frac{\mu_0}{4\pi}\,\frac{2\pi N I a^2}{(x^2)^{3/2}}$$

or $$B = \frac{\mu_0}{4\pi}\,\frac{2\pi N I a^2}{x^3}$$

Direction of magnetic field- At any point on the axis of current carrying circular coil the direction of magnetic field can be find out by using end rule.



#Note- MFI at any point on the axis of a current carrying coil

$$B_{axis} = \frac{\mu_0}{4\pi}\,\frac{2\pi N I a^2}{(a^2+x^2)^{3/2}}$$

or $$B_{axis} = \frac{\mu_0}{4\pi}\,\frac{2\pi N I a^2}{a^3\left(1+\frac{x^2}{a^2}\right)^{3/2}}$$

or $$B_{axis} = \frac{\mu_0}{4\pi}\,\frac{2\pi N I}{a\left(1+\frac{x^2}{a^2}\right)^{3/2}}$$

or $$B_{axis} = \frac{B_{centre}}{\left(1+\frac{x^2}{a^2}\right)^{3/2}}$$

Ampere's Circuital Law

Ques.-State and prove Ampere's circuital law.

Ans.-It states that the line integral of magnetic field along a closed loop is equal to μ_0 times the net electric current threading through the area enclosed by that loop.

i.e $$\oint_l \vec{B}.\vec{dl} = \mu_0 I$$

where $\vec{B}$ is the MFI at any point of the loop $\vec{dl}$ is small element of the loop and I_{net} is the net electric current flowing through that loop.

The loop on which ampere's circuital law is applied is called ampere's circuit or ampere's loop.

It is always imaginary.

Proof- Consider a straight current carrying conductor of infinite length through which electric current I is flowing in upward direction. The MFI due to the current carrying straight coductor at a point P which is at a perpendicular distance a from it is given by

$$B = \frac{\mu_0}{4\pi}\,\frac{2I}{a}$$

or $$B = \frac{\mu_0 I}{2\pi a}$$

By taking conductor as centre and 'a' as radius draw a circle which passes through point P. This circle will behave like amperian loop. Now choose a small element of length dl of the ampere's circuit about point P. As MFI at point P is directed in the inward direction along the tangent to the ampere's circuit. Therefore, the angle between $\vec{B}$ and $\vec{dl}$ will be $0°$ and from symmetry we can conclude that the magnitude of MFI will be same at every point of ampere's circuit. Consider

$$\oint_l \vec{B}.\vec{dl} = \oint_l B\,dl\cos\theta$$

$$= \oint_l B\,dl\cos 0$$

$$= \oint_l B\,dl\,(1)$$

$$= \oint_l B\,dl$$

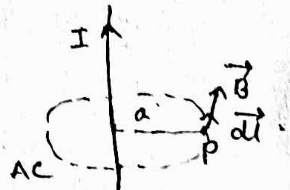
$$= B\oint_l dl$$

$$= Bl$$

$$= 2B\pi a$$

$$= \frac{\mu_0 I}{2\pi a}\,2\pi a$$

$$= \mu_0 I$$

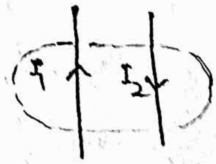


$$\therefore \quad \oint_l \vec{B}.\vec{dl} = \mu_0 I$$

This is the reqd. expression of ampere's circuital law.

Note- When the flow of electric current takes place through the ampere's circuit in opposite directions then the net electric current is calculated by subtracting the given electric currents.

$$\oint_l \vec{B}.\vec{dl} = \mu_0 I_{net}$$
$$= \mu_0(I_1 - I_2)$$



application (1) MFI due large no. wounde devi

In ideal solenoid, length is infinite.

Applications of ampere's circuital law

(1) MFI due to a current carrying solenoid -When a large no. of turns of insulated copper wire is closely wounded over an asbestos or china clay cylinder then the device obtained is called solenoid.

When the diameter of a solenoid is negligible as compared to its length then it is said to be ideal solenoid.

When an electric current is allowed to flow through a solenoid then the MF inside the sloenoid is uniform which is parallel to the axis of solenoid and the MF outside the solenoid is zero.

Consider a solenoid whose radius r is negligible as compared to its length and in which no.of turns per unit length be n. When I amount of electric current is allowed to flow through the solenoid then we have to find out the MFI B at any point inside the solenoid.

Let us choose a rectangular shaped ampere's circuit ABCD of length length 'ℓ' and breadth 'b' be placed partially inside and partially outside the solenoid.

Here,

$$\oint_{\ell} \vec{B}.\vec{d\ell} = \int_A^B \vec{B}.\vec{d\ell} + \int_B^C \vec{B}.\vec{d\ell} + \int_C^D \vec{B}.\vec{d\ell} + \int_D^A \vec{B}.\vec{d\ell}$$

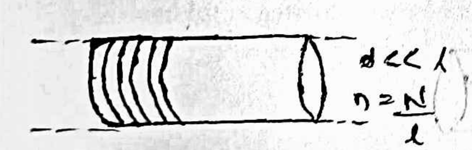
$$= \int_A^B Bd\ell\cos\theta + \int_B^C Bd\ell\cos\theta + \int_C^D Bd\ell\cos\theta + \int_D^A Bd\ell\cos\theta$$

$$= \int_A^B Bd\ell\cos0^\circ + \int_B^C Bd\ell\cos90^\circ + \int_C^D 0d\ell\cos0^\circ + \int_D^A Bd\ell\cos90^\circ$$

$$= \int_A^B Bd\ell + \int_B^C Bd\ell(0) + \int_C^D 0 + \int_D^A Bd\ell(0)$$

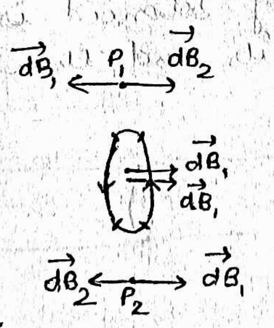
$$= B\int_A^B d\ell + 0 + 0 + 0$$

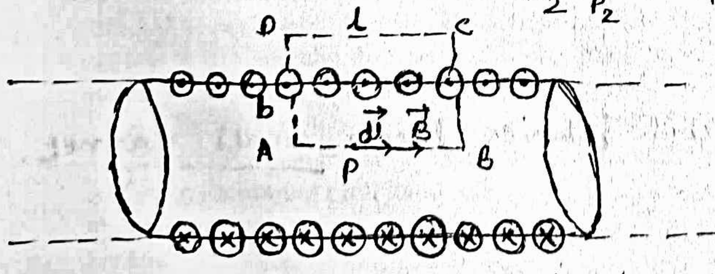
$$= B\ell$$



$d << \ell$

$n = \frac{N}{\ell}$





No. of turns of solenoid which passes through ampere's circuit

$N = n\ell$

$\therefore$ Net electric current which passes through ampere's circuit

$I_{net} = NI = n\ell I$

By ACL

$$\oint_{\ell} \vec{B}.\vec{d\ell} = \mu_0 I_{net}$$

or $\quad B\ell = \mu_0 n\ell I$

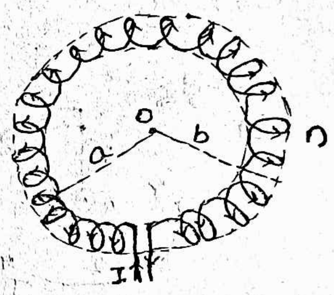
or $\quad \boxed{B = \mu_0 nI}$

This is the required expression.

Note- Due to an ~~ideal~~ solenoid MF is same as that of magnet i.e. inside the solenoid MLF are directed from its S to its N pole and outside MLF are directed from its N to S pole. (length of solenoid is finite)

(2) MFI due to a current carrying toroid- When an insulated copper wire is closely wounded over circular ring made of asbestos or china clay then the device obtained is called toroid.

Consider a toroid whose length be ℓ and no.of turns in a unit length of it be n .When I amount of electric current is allowed to flow through the toroid then from symmetry we can say that the magnitude of MFI is same at every point which lie at equidistant from the centre of toroid.

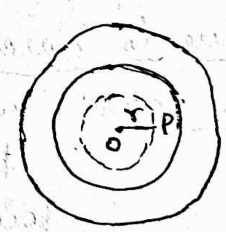


(i) When the observation point lies inside the toroid- By taking O as centre and r as radius draw a circle which behaves like ampere's circuit. By ACL

$$\oint_{\ell} \vec{B}.\vec{d\ell} = \mu_0 I_{net}$$

or $\quad \oint_{\ell} \vec{B}.\vec{d\ell} = \mu_0 0$

or $\quad \oint_{\ell} \vec{B}.\vec{d\ell} = 0$



$\therefore \quad B = 0$

(ii) When the observation point P lies inside the core of toroid- By taking O as centre and OP = r as radius draw a circle which will behave like ampere's circuit

No. of turns of toroid passing through the ampere's circuit $N = n\ell$

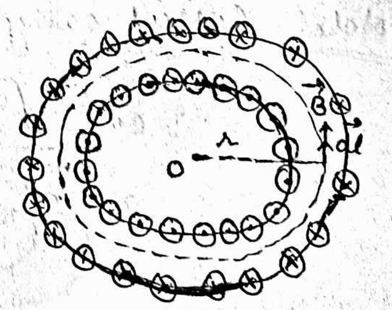
$\therefore$ Net electric current passing through the ampere's circuit, $I_{net} = NI = n\ell I$

By ACL

$$\oint_{\ell} \vec{B}.\vec{d\ell} = \mu_0 I_{net}$$

or $\quad \oint_{\ell} \vec{B}.\vec{d\ell} = \mu_0 n\ell I$

or $\quad \oint_{\ell} Bd\ell\cos 0^\circ = \mu_0 n\ell I$



[$\because$ B & $d\ell$ are in the same direction.]

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Note: when solenoid is converted into toroid, then direction of MFI is same as that of along the direction of its axis. (parallel to the axis)

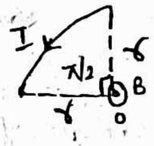
★ MFI due to current carrying circular arc at its centre.

$$B = \frac{\mu_0}{4\pi}\frac{I\alpha}{r}$$

$$\boxed{\begin{aligned} \because\ 2\pi &\longrightarrow \frac{\mu_0 I \times 2\pi}{4\pi r} \\ 1 \text{ rad} &\longrightarrow \frac{\mu_0 I}{4\pi r} \\ \alpha \text{ rad} &\longrightarrow \frac{\mu_0 I \alpha}{4\pi r} \end{aligned}}$$

Particular cases :-

1)

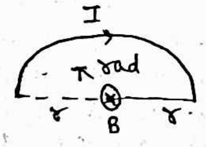


If $\alpha = \pi/2$

then $B = \frac{\mu_0}{4\pi}\cdot\frac{I}{r}\cdot\frac{\pi}{2}$

$= \frac{\mu_0 I}{8r}$ ⊙

2)

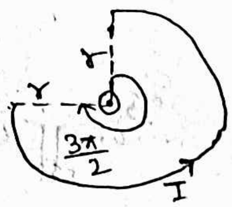


If $\alpha = \pi$

then $B = \frac{\mu_0}{4\pi}\cdot\frac{I}{r}\cdot\pi$

$= \frac{\mu_0 I}{4r}$ ⊗

3)



If $\alpha = \frac{3\pi}{2}$

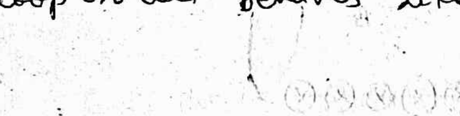
then $B = \frac{\mu_0}{4\pi}\cdot\frac{I}{r}\cdot\frac{3\pi}{2}$

$= \frac{3\mu_0 I}{8r}$ ⊙

★ MFI due to current carrying solenoid :-

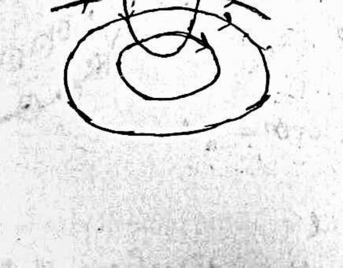
Particular cases: when the length of solenoid is infinite & observation pt. lie at one of its end, then MFI at the observation point,

$$B_{end} = \frac{B}{2}$$



$$\boxed{B_{end} = \frac{\mu_0 N I}{2}}$$

Note: Current carrying circular loop or coil behaves like a small magnet.



Force experienced by charged particle in a uniform magnetic field

Ques.-Derive the expression of force experienced by charged particle in a uniform magnetic field.

Ans.- When a charged particle is projected in uniform magnetic field then it experiences a magnetic force in the direction which is $\perp$ to both the direction of velocity of charged particle and the magnetic field. This force is called Lorentz magnetic force.

Consider a charged particle of charge 'q' projected with velocity $\vec{v}$ at an angle 'θ' with the direction of uniform magnetic field $\vec{B}$.

Then the lorentz magnetic force experiences by the charged particle is-

(i) directly proportional to charge q of charged particle i.e. $F \propto q$

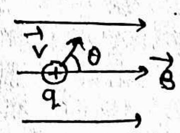
(ii) directly proportional to velocity v of charged particle i.e. $F \propto v$

(iii) directly proportional to the MFI B in which the charged particle moves.

ie. $F \propto B$

(iv) directly proportional to the sine of angle θ between the direction of the velocity of charged particle and the magnetic field.

i.e. $F \propto \sin\theta$



Combining all the above relations

$$F \propto qvB\sin\theta$$

or $F = kqvB\sin\theta$

where k is a constant of proportionality and its value depends upon the system of unit used.

In S.I. and C.G.S. electromagnetic system

$$k = 1$$

therefore $\boxed{F = qvB\sin\theta}$

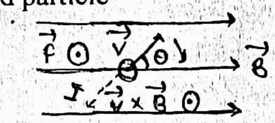
This is the required relation of lorentz magnetic force.

In vector form for positively charged particle

$$\vec{F} = q(\vec{v} \times \vec{B})$$

and for negatively charged particle

$$\vec{F} = -q(\vec{v} \times \vec{B})$$



From the above relation it is clear that the direction of LMF experienced by positively charged particle is same as $(\vec{v} \times \vec{B})$ and the direction of LMF experienced by negatively charged particle is opposite to that of $(\vec{v} \times \vec{B})$.

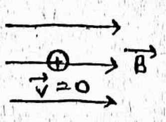
The direction of LMF acting on a charged particle which is moving in magnetic field can also be determined by using Flemming's left hand rule.

Particular cases-

(i) If v = 0

then $F = q0B\sin 0^\circ$

or $F = 0$

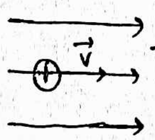


Thus when a charged particle is at rest in magnetic field then the LMF experienced by it is zero.

(ii) If θ = 0° or 180°

then $F = qvB\sin 0^\circ$

or $F = 0$



and $F = qvB\sin 180^\circ$

or $F = qvB0$

or $F = 0$

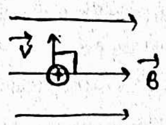
Thus when a charged particle moves parallel or antiparallel to the magnetic field the LMF experienced by it is 0.

(iii) If θ = 90°

then $F = qvB\sin 90^\circ$

or $F = qvB\,(1)$

or $F = qvB$

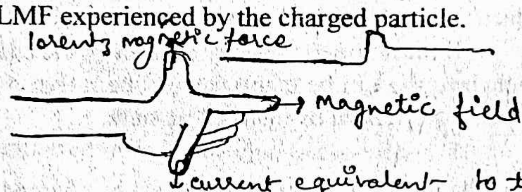


Thus when a charged particle moves in the $\perp$ direction of magnetic field then the LMF experienced by it is max.

Flemming's left hand rule

Ques.-State Flemming's left hand rule.

Ans.-It states that if the central finger, the fore finger and the thumb of the left hand are streched mutually $\perp$ to one another in such a manner that the central finger is directed towards the direction of the electric current equivalent to the moving charged particle and the fore finger is directed towards magnetic field then the thumb gives the direction of LMF experienced by the charged particle.



Note- S.I. and C.G.S. emu of MFI :-

From relation $F = qvB\sin\theta$ we have

$$B = \frac{F}{qv\sin\theta}$$

S.I. unit of $B = \frac{N}{C\frac{m}{s}}$

$$= \frac{N}{Am}$$

$= \text{tesla (T)}$ → In the honour of nicola Tesla.

C.G.S. emu of $B = \frac{dyn}{abC\frac{cm}{s}}$

$$= \frac{dyn}{abA\,cm}$$

$= \text{gauss (G)}$

Relation between S.I. and C.G.S. emu-

$$1T = \frac{1N}{Am}$$

or $1T = \dfrac{10^5\,dyn}{\frac{1}{10}\,abamp\;100cm}$

or $1T = \dfrac{10^4\,dyn}{abampcm}$

$\therefore$ $1T = 10^4 G$

Definitions of S.I. and C.G.S. emu of MFI

Ques.-Define S.I. and C.G.S. emu of MFI.

Ans.-From relation $F = qvB\sin\theta$ we have

$$B = \frac{F}{qv\sin\theta}$$

If q = 1C, v = 1m/s, θ = 90° and F = 1N

then $B = \dfrac{1}{1 \times 1\sin 90^\circ}$

or $B = \frac{1}{1\times1\times1}$

or $B = 1T$

Thus, if on projecting 1C of charge with the velocity of 1m/s in the $\perp$ direction of UMF the force experienced by the charged particle be 1N then the intensity of magnetic field is said to be 1T.

Again if q = 1 abC, v = 1 cm/s, $\theta = 90°$ and F = 1dyn

then $B = \frac{1}{1\times1\sin90°}$

or $B = \frac{1}{1\times1\times1}$

or $B = 1G$

Thus, if on projecting a charged particle of 1 abC with a velocity of 1cm/s in the $\perp$ direction of UMF the force experienced by the charged particle be 1dyne then the intensity of magnetic field is said to be 1G.

Note- (i) When a charged particle is projected in electric field as well as magnetic field then it will experience both electric force and the magnetic force.

If the amount of charge present on the charged particle be q the EFI be E and the MFI be B then the electric force experienced by charged particle

$\vec{F}_e = q\vec{E}$

and the magnetic force experienced by charged particle

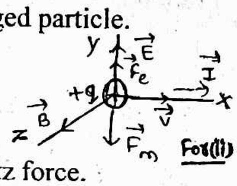
$\vec{F}_m = q(\vec{v}\times\vec{B})$

Net force experienced by charged particle.

$\vec{F} = \vec{F}_e + \vec{F}_m$

or $\vec{F} = q\vec{E} + q(\vec{v}\times\vec{B})$

or $\vec{F} = q[\vec{E} + (\vec{v}\times\vec{B})]$



This experssion is called lorentz force.

(ii) When a charged particle moves through electric field and magnetic field with constant velocity then the magnetic force acting on it must be balancing the electric force acting on it.

In equilibrium

$F_m = F_e$ ⇒ V = constt. a = zero

or $qvB\sin\theta = qE$ $F_{net} = zero$

or $vB(1) = E$ $F_m + F_e = 0$

or $\boxed{v = \frac{E}{B}}$

(iii) When a charged particle of charge q is projected with velocity v in a uniform magnetic field B then the LMF experienced by the charged particle.

$\vec{F} = q(\vec{v}\times\vec{B})$

From the above relation it can be concluded that the LMF always act $\perp$ to the velocity of charged particle.

$\therefore$ The instantaneous power imposed on the charged particle will be 0.

$P = \vec{F}.\vec{v}$

or $P = Fv\cos90°$

or $P = Fv(0)$

or $P = 0$

This implies that the magnetic force acting on charged particle does no work.

$W = Pt = 0t = 0$

From W.E. theorem

$W = \Delta K.E$

or $0 = \Delta K.E$

or $K.E = constant$

or $\frac{1}{2}mv^2 = constant$

or $v^2 = \frac{2\ constant}{m}$

or $v = \sqrt{\frac{2constant}{m}}$

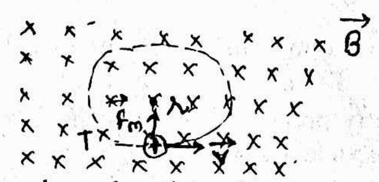
or $v = constant$

Thus when a charged particle moves in uniform magnetic field then its K.E. and hence its speed remains constant.

Motion of a charged particle in uniform magnetic field

Ques.-Describe the motion of a charged particle in uniform magnetic field when the initial velocity of charged particle is $\perp$ to the magnetic field.

Ans.-



Consider a charged particle of mass m and charge q which is projected with velocity v in the $\perp$ direcion of UMF and then the LMF experienced by the charged particle is given by

$\vec{F}_m = q(\vec{v}\times\vec{B})$

In magnitude $F_m = qvB\sin\theta$

Here $\theta = 90°$

$\therefore$ $F_m = qvB\sin90°$

or $F_m = qvB(1)$

or $F_m = qvB$

$\because$ The LMF always act $\perp$ to the velocity of charged particle therefore under the effect of LMF the charged particle well follow a circular path with constant speed in a plane $\perp$ to the magnetic field. The centripetal force required for circular motion of charged particle will be provided by the LMF.

i.e Centripetal force = LMF

or $\frac{mv^2}{r} = qvB$

where r is the radius of circular path followed by the charged particle.

or $\frac{mv^2}{qvB} = r$

or $\boxed{r = \frac{mv}{qB}}$

This is the required expression.

The time period of revolution of charged particle

$T = \frac{2\pi r}{v}$

or $T = \frac{2\pi}{v}\frac{mv}{qB}$

or $\boxed{T = \frac{2\pi m}{qB}}$

The frequency of revolution of charged particle

$\nu = \frac{1}{T}$

or $\nu = \frac{1}{\frac{2\pi m}{qB}}$

or $\boxed{\nu = \frac{qB}{2\pi m}}$

and the angular velocity of revolution of charged particle

$\omega = 2\pi\nu$

or $\omega = 2\pi \frac{qB}{2\pi m}$

or $\boxed{\omega = \frac{qB}{m}}$

From the above analysis it can be concluded that the time period, frequency and angular velocity of charged particle is independent of its speed.

Note- (i) Since the linear momentum of charged particle is p = mv therefore the radius of circular path followed by the charged particle.

$r = \frac{p}{qB}$

$\because p = \sqrt{2mE_k}$

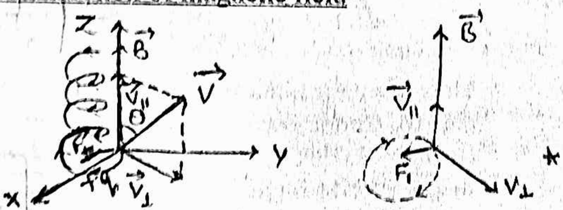
$\therefore \quad r = \frac{\sqrt{2mE_k}}{qB}$

If the charged particle is accelerated through a potential difference V before entering into the magnetic field then

$E_k = qV$

$\therefore \quad r = \frac{1}{B}\sqrt{\frac{2mV}{q}}$

(ii) Motion of a charged particle in uniform magnetic field when a charged particle is projected at an angle other than 90° with the direction of magnetic field-



Consider a charged particle of mass m and charge q which is projected with a velocity v at an angle θ with the direction of uniform magnetic field. Here velocity of the charged particle can be resolved into two components-

(i) $v_{\parallel} = v\cos\theta$ in the direction of magnetic field.

(ii) $v_{\perp} = v\sin\theta$ in the direction ⊥ to manetic field.

Because of $v_{\parallel}$ the LMF acting on the charged particle

$F_{\parallel} = qv_{\parallel}B\sin 0^\circ$

or $F_{\parallel} = qv_{\parallel}B0$

or $F_{\parallel} = 0$

As $F_{\parallel}$ is 0 therefore the charged particle will move with constant velocity $v_{\parallel}$ in the direction of magnetic field.

Because of $v_{\perp}$ the LMF acting on the charged particle.

$F_{\perp} = qv_{\perp}B\sin 90^\circ$

or $F_{\perp} = qv_{\perp}B(1)$

or $F_{\perp} = qv_{\perp}B$

As $F_{\perp}$ acts in the ⊥ direction of velocity of charged particle therefore the charged particle will move along a circular path in the plane ⊥ to magnetic field.

Due to superposition of both the types of motion the charged particle will move along a helical path.

As the centripetal force required for the circular motion of charged particle is provided by LMF

$\therefore$ Centripetal force = LMF

or $\frac{mv_{\perp}^2}{r} = qv_{\perp}B$

* Helical path → Path like spring

where r is the radius of helix.

$\frac{mv_{\perp}^2}{qv_{\perp}B} = r$

or $r = \frac{mv_{\perp}}{qB}$

or $\boxed{r = \frac{mv\sin\theta}{qB}}$

This is the required expression.

Time period of revolution of charged particle

$T = \frac{2\pi r}{v_{\perp}}$

or $T = \frac{2\pi}{v_{\perp}}\frac{mv_{\perp}}{qB}$

or $\boxed{T = \frac{2\pi m}{qB}}$

Frequency of revolution of the charged particle

$\nu = \frac{1}{T}$

or $\nu = \frac{1}{\frac{2\pi m}{qB}}$

or $\boxed{\nu = \frac{qB}{2\pi m}}$

Angular velocity of the charged particle

$\omega = 2\pi\nu$

or $\omega = 2\pi\frac{qB}{2\pi m}$

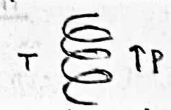
or $\boxed{\omega = \frac{qB}{m}}$

In this case also the time period, frequency and the angular velocity of charged particle is independent if its speed.

* Pitch of a helix- The linear distance covered by a charged particle in one time period is called pitch of a helix.

$p = v_{\parallel}T$

$= v\cos\theta \frac{2\pi m}{qB}$



Magnetic force experienced by a current carrying conductor placed in uniform magnetic field

Ques.-Obtain the expression for the magnetic force experienced by a current carrying conductor placed in uniform magnetic field.

Ans.- Consider a conductor of length ℓ and area of cross-section A in which the no. density of free electrons be n and the electric current flowing in it be I. The conductor is placed in a uniform magnetic field B in such a manner that the angle between the direction of electric current and the magnetic field be θ.

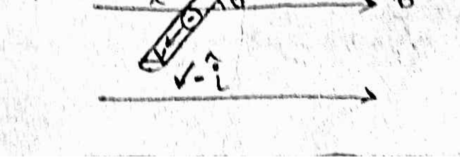
At any instant the total charge of free electrons present inside the conductor

$q = Ne$

$= nVe \; (\because N = nV)$

$= nA\ell e$

Due to the charges in motion the LMF experienced by the current carrying conductor



$\vec{F} = -q(\vec{v} \times \vec{B})$

If the unit vector in the direction of electric current I be $\hat{i}$ then the unit vector in the direction of $\vec{v}$ will be $-\hat{i}$.

∴ $\vec{F} = -nAle[v(-\hat{i}) \times \vec{B}]$

or $\vec{F} = +nAve[\ell\hat{i} \times \vec{B}]$

∵ nAve = I, electric current

and $\ell\hat{i} = \vec{\ell}$ length vector whose direction as same as electric current.

∴ $\vec{F} = I\vec{\ell} \times \vec{B}$

or $\vec{F} = I(\vec{\ell} \times \vec{B})$ In mag. $\boxed{F = I\ell B\sin\theta}$

This is the required expression.

From the above relation it can be concluded that the direction of LMF $\vec{F}$ experienced by the current carrying conductor is same as $\vec{\ell} \times \vec{B}$.

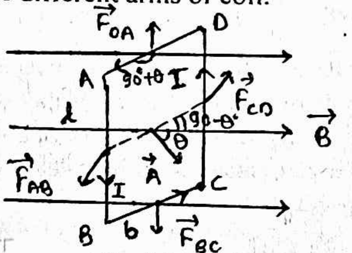
The direction of LMF can also be find out by using flemming's left hand rule.

Torque experienced by a current carrying coil placed in uniform magnetic field

Ques.-Derive the relation for the torque experienced by a current carrying coil placed in uniform magnetic field.

Ans.- Consider a rectangular coil ABCD whose length be ℓ and breadth be b which is placed in a uniform manetic field B in such a manner that the angle between the area vector and the direction of magnetic field be θ.

When I amount of electric current is allowed to flow through the coil in anticlockwise direction. Then the LMF will act on the different arms of coil.



LMF acting on arm AB of the coil

$F_{AB} = I\ell B \sin 90°$
$= I\ell B (1)$
$= I\ell B$

According to FLHR this force will act will act in the outward direction ⊥ to the plane of paper.

LMF acting on arm BC of the coil

$F_{BC} = IbB \sin(90° - \theta)$
$= IbB \cos\theta$

According to FLHR this force will act in the downward direction in the the plane of the paper.

LMF acting on arm CD

$F_{CD} = I\ell B \sin 90°$
$= I\ell B (1)$
$= I\ell B$

According to FLHR this force will act in the inward direction ⊥ to the plane of the paper.

and LMF acting on arm DA

$F_{DA} = IbB \sin(90° + \theta)$
$= IbB \cos\theta$

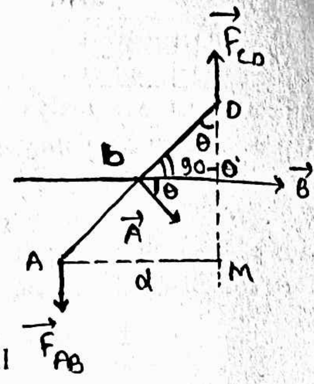
According to FLHR this force will act in the upward direction in the plane of the paper.

As the line of action of forces $\vec{F}_{BC}$ and $\vec{F}_{DA}$ coincide with one another therefore torque due to them will be zero. But, the line of action of forces F_{AB} and F_{CD} coincide, therefore torque will act on the coil due to... Now, the torque acting on current carrying conductor = magnitude of either force × ⊥ distance between the line of action of two forces

i.e. $\tau = F_{AB} d$ ---(i)

Here $F_{AB} = I\ell B$

and in rt. angle Δ AMD



$\sin\theta = \frac{AM}{AD}$

or $\sin\theta = \frac{d}{b}$

or $d = b\sin\theta$

From relation (i)

$\tau = I\ell B \sin\theta$

∵ $\ell b = A$ area of the coil

∴ $\tau = IAB \sin 0°$

If the no. of turns in the coil be N then

$\boxed{\tau = NIAB\sin\theta}$

This is the required expression.

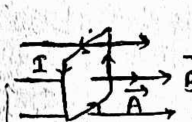
In vector form

$\vec{\tau} = N(\vec{A} \times \vec{B}) I = NI(\vec{A} \times \vec{B})$

From the above relation it can be concluded that the direction of torque $\vec{\tau}$ acting on the current carrying coil placed in uniform manetic field is same as $\vec{A} \times \vec{B}$.

Particular cases- (i) If θ = 0°

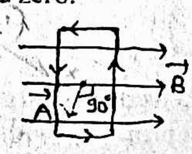
then $\tau = NIAB \sin 0°$
$= NIAB (0)$
$= 0$



Thus when the plane of the coil is ⊥ to the magnetic field then the torque experienced by it is zero.

(i) If θ = 90°

then $\tau = NIAB \sin 90°$
$= NIAB (1)$
$= NIAB$



Thus when the plane of the coil is parallel to the magnetic field then the torque experienced by it maximum.

Note-(i) If the angle between the plane of the coil and the manetic field is given then

$\theta + \alpha = 90°$

or $\theta = 90 - \alpha$

∴ $\tau = NIAB \sin\theta$

or $\tau = NIAB \sin(90° - \alpha)$

or $\tau = NIAB \cos\alpha$

(ii) Whatever be the shape of the loop the above results are valid-

(a) $F_{net} = 0$

(b) $\tau_{net} = NIAB \sin\theta$

~~= NIAB cosα~~

Galvanometer

Ques.-What is galvanometer ? State its types.

Ans.- It is a device which measures electric current flowing in an electric circuit.

They are of two types-

(i) Moving coil galvanometer- In these galvanometer the deflection of coil takes place between the poles of a magnet which is as rest. e.g. suspension type galvanometer, pivoted type galvanometer.

(ii) Moving magnet galvanometer- In these galvanometer the deflection of magnet takes place at the centre of a coil which is at rest. e.g. tangent galvanometer.

Force acting between two straight parallel current carrying conductors (wires)

Ques.-(a) Derive the expression of Force acting between two straight parallel current carrying conductors (wires). When this force is attractive or repulsive?

(b) On the basis of relation obtained define ampere.

Ans.-(a) Consider two straight parallel wires which are placed r distance apart in vaccum. When electric current is allowed to flow through them then they will exert LMF on each other because each current carrying wire is situated in the magnetic field generated by the other.

If the electric current flowing in the first wire be I_1 and the electric current flowing in same direction in second wire be I_2 then the MFI due to the first wire at the location of second wire will be $B = \frac{\mu_0}{4\pi}\frac{2I_1}{r}$ $\perp$ to the plane of paper in downward direction.

The LMF experienced by the ℓ length of second wire due to the magnetic field generated by first wire is given by

$$F_{21} = I_2 \ell B_1 \sin 90°$$

or $$F_{21} = I_2 \ell \frac{\mu_0}{4\pi}\frac{2I_1}{r} \quad (1)$$

or $$F_{21} = \frac{\mu_0}{4\pi}\frac{2I_1I_2\ell}{r} \quad \text{------(i)}$$

By FLHR the direction of LMF experienced by second wire will be towards the first wire.

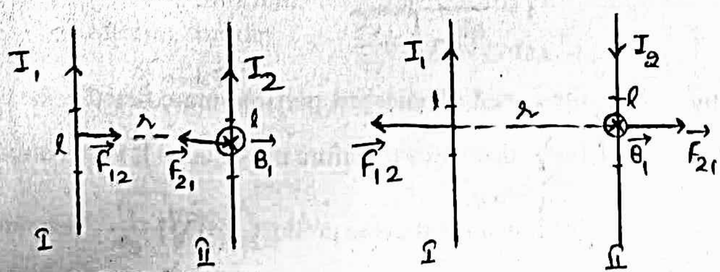
According to NTL the ℓ length of first wire will experience LMF which will be equal in magnitude and opposite in direction to the LMF experienced by ℓ length of second wire.

$$\boxed{F_{12} = \frac{\mu_0}{4\pi}\frac{2I_1I_2\ell}{r}}$$

and the force acting on unit length of each wire

$$\boxed{\frac{F}{\ell} = \frac{\mu_0}{4\pi}\frac{2I_1I_2}{r}}$$

When the direction of flow of current in the two wires be opposite then the two wires will repel each other but the magnitude of force acting on unit length of each wire will be the same.



(b)**Definition of S.I. unit of electric current** -

In relation

$$\frac{F}{\ell} = \frac{\mu_0}{4\pi}\frac{2I_1I_2}{r}$$

If $I_1 = I_2 = I$, $r = 1\text{m}$

and $\frac{F}{\ell} = 2\times10^{-7}\frac{N}{m}$

then $2\times10^{-7} = \frac{1\times10^{-7}\times2\times I\times I}{1}$

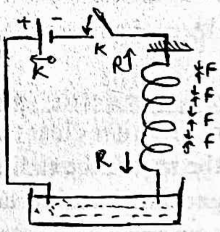
or $2\times10^{-7} = 2\times10^{-7} I^2$

or $I^2 = 1$

or $I = 1A$

Thus, 1 ampere is that amount of electric current which when allowed to flow through two straight parallel conductors of infinite length and negligible area of cross-section which are placed 1m apart in vaccum they attract each other by a force of 2×10^{-7} N/m of length.

Note- Experimental demonstration- Consider a spring whose one end its attached to a rigid support and other end is dipped in mercury contained in a vessel. A source of emf is connected between the rigid support and mercury through key K. When the key is closed then the flow of electric current takes place in the spring. As the direction of electric current in the adjacent turns of spring are same therefore attractive force comes into action between them as a result of which the contraction of spring takes place. As soon as the lower end of spring comes out of mecury the circuit becomes incomplete and the electric current flowing in the spring becomes zero. In this situation under the effect of restoring force the extension of spring takes place and its lower end again gets immersed into mercury due to which the circuit again becomes complete and the flow of electric current takes place in the spring. Due to the repeation of this process again and again and the oscillation spring takes place.



Cyclotron

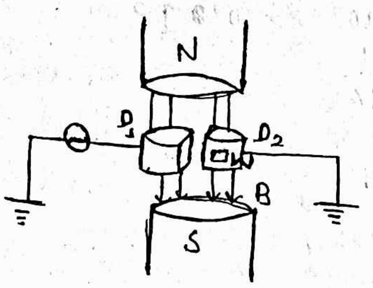
Ques.-Discuss the principle, construction and working of a cyclotron. What is the maximum kinetic energy acquired bvy the accelerated charged particles? give the limitations and uses of a cyclotron.

Ans.- It is a device which is used to accelerate positive ions like proton, deutrons etc. It was invented by Lawrence and Livingstone in 1934 to investigate nuclear structure.

Principle- When a positive charged particle is accelerated through oscillating electric feild by making it to cross the same electric feild again and again by making use of strong magnetic field then it acquire large kinetic energy.

Its operation is based on the fact that the time period revolution of a charged particle in a uniform magnetic field is independent of its speed (or velocity).

Construction- Fig shows the schematic view of the cylotron. The motion of charged particle takes place in two semi-circular disc like metal chambers D_1 and D_2 called dees. The straight section of dees are open so that the charge particle can move freely from D_1 to D_2 and vice-versa. The whole assembly is evacuted to minimize the collisions between charged particle and the air molecules. The two dees as are placed between the poles of a strong elec tromagnet and a high frequency alternating voltage is applied across the two dees.



Working- When positively charged particle is released at the centre of two dees then it moves in a semicircular path inside one of the dees under the effect of LMF and arrive in the gap between the dees in time T/2 where T is the time period of revolution which given by

$$T = \frac{2\pi m}{qB} = \frac{1}{f_c} \quad \text{---(i)}$$

$$\therefore \quad t = \frac{\pi m}{qB}$$

The frequency f_c of applied voltage is adjusted so that the polarities of the dees is reversed in the same time that it takes the charged particle to complete 1/2 of the revolution. This is called resonance condition. In this situation when the positively charged particle arrives at the edge of dee D_1, the dee D_2 is at lower potential and the ions are accelerated across the gap through potential difference $V(V_{D1}-V_{D2})$ due to which their K.E. increases by qV.

The radius of circular path followed by charged particle is given by

$$r = \frac{mv}{qB} \quad \text{---(ii)}$$

From the above relation it is clear that with the increase in K.E. and hence the speed, the radius of circular path followed by charged particle increases. The positively charged particles are repeatedly accelerated across the dees until they acquire the required energy and the radius of the circular path becomes equal to the radius of the dees. At this stage the charged particle leaves the system through an exit slit(window). At this point the speed of positively charged particle is given by

$$v = \frac{qB\ell}{m}$$

Negative plate is used to remove the +ve ion.

$$\text{or} \quad v_{max} = \frac{qBR}{m}$$

where R is the radius of dees

and the K.E. of positively charged particle is given by

$$K.E. = \frac{1}{2} mv^2$$

$$\text{or} \quad \boxed{K.E._{max} = \frac{q^2B^2R^2}{2m}}$$

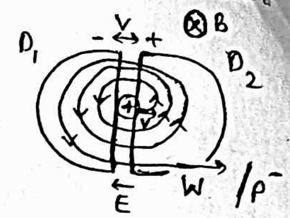
Uses- (i) It is used to bombard nuclei with highly energetic particle and to study the resulting nuclear reaction.
(ii) It is used to implant positive ions into solids and to modify their properties.
(iii) It is used in hospitals to produce radiations used in the treatment and diagonises of cancer.

Limitation- When the speed of positively charged particle increases its relativistic mass also increases in accordance with relation

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$$

∴ The time spent by positively charged particle inside a dee will become

$$t = \frac{T}{2} = \frac{2\pi m}{2qB}$$



$$\text{or} \quad t = \frac{\pi m}{qB}$$

$$\text{or} \quad t = \frac{\pi}{qB} \frac{m_0}{\sqrt{1 - v^2/c^2}}$$

From the above relation it is clear that with the increase in speed, time also increases this implies that the charged particle will take longer time to complete the semi-circular path then the time for half cycle of oscillating electric field. As a result of which the charged particle will not arrive in the gap between the two dees when the polarity of dees is reversed therefore it will not accelerate further. This problem can be overcome by adjusting the frequency of oscillating electric field in accordance with the frequency of +ve ion. Such cyclotron is called synchro-cyclotron or frequency modulated cyclotron.

Note- If the no. of completed revolution by the positively charged particle be N then the charged particle will accelerate through potential difference V, 2N times.

Total K.E. acquired by charged particle

$$K.E. = qV \times 2N$$

$$\text{But} \quad K.E. = \frac{q^2B^2R^2}{2m}$$

$$\therefore \quad \frac{q^2B^2R^2}{2m} = qV \times 2N$$

$$\text{or} \quad N = \frac{qB^2R^2}{4mV}$$

(ii) Cyclotron is suitable for accelerating heavy particles like proton, deutron, α-particle etc but not electrons. The reason is that due to small mass electrons gain speed very quickly as a result of the relativistic variation in mass of electrons increases quickly and it goes out of step with oscillating electric field.

(iii) Another method of overcoming limitation-

$$t = \frac{\pi m_0}{qB\sqrt{1 - \frac{v^2}{c^2}}}$$

As the speed of charged particle increases the factor $\sqrt{1-\frac{v^2}{c^2}}$ decreases therefore the value of B is increased in such a manner that the product $B\sqrt{1-\frac{v^2}{c^2}}$ remains constant and hence T also remains constant such a cyclotron is called synchrotron.

Suspension type moving coil galvanometer or D'Arsonaval type galvanometer

Ques.-Describe the principle, construction and working of a suspension type moving coil galvanometer or D'Arsonaval type galvanometer. Also give its merits & demerits.

[ϕ instead]

Ans. Construction- In suspension type moving coil galvanometer a coil made of thin copper wire is bounded on a rectangular $A\ell$ frame A. The core C of the coil is made of soft iron. Thus coil is suspended with the help of phosphorus bronze strip S from torsion head between the poles N and S a strong horse shoe magnet. the other end of coil is connected to the soft phosphorus bronze spring P. The poles of magnet are made cylindrically concave so that the magnetic field between them be radial. A small mirror M is attached to the suspension trip which rotates along with the coil. By falling light on plane mirror with the help of lamp-scale arrangement the deflection of coil is determined. The torsion head is connected to the adjusting screw T_1 and the spring is connected to the adjusting screw T_2. The whole apparatus is enclosed in a non magnetic box to protect it from air currents and dust.

Along the radius

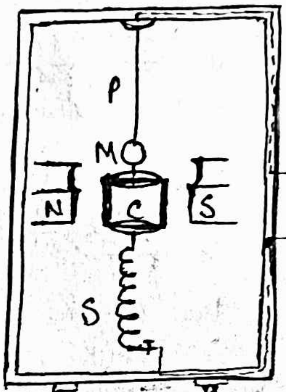
Principle- When a current carrying coil is suspended in UMF in such a manner that the plane of the coil coincides with the direction of magnetic field then a deflecting torque (couple) acts on the coil whose tendency is to bring the plane of the coil $\perp$ to the magnetic field.

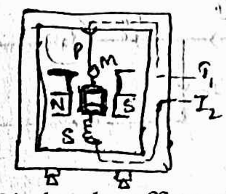
If the area of the coil be A ,the no. of turns in it be N, the MFI between the poles of the magnet be B and the current flowing in the coil be I then the deflecting torque acting on the coil is given by

$$\tau_{deflect} = NIAB\sin\theta$$

Here $\theta = 90°$

$$\therefore \quad \tau_{deflect} = NIAB \sin 90° = NIAB\,(1) = NIAB$$





Under the effect of deflecting torque as the coil rotates the suspension trip gets twisted and a restoing torque comes into action whose tendency is to bring the coil back to its original position. Thus the restoring torque opposes the rotation of the coil and is directly proportional to the deflection of the coil. When the restoring torque becomes equal to the deflecting torque then the coil acquires the state of equilibrium.

Restoring torque [ϕ is angular deflection]

$$\tau_{restore} = C\phi$$

where C is couple unit twist or torsional constant

In equilibrium per

$$\tau_{deflect} = \tau_{restore}$$

or $\quad I = \dfrac{C}{NAB}\phi$

or $\quad I = k\phi$

where $k = \dfrac{C}{NAB}$ is called galvanometer constant.

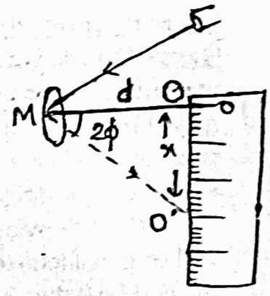
Thus electric current flowing in the coil of galvanometer is directly proportional to the deflection of coil. $I \propto \phi$

Working- First of all the base of SCG is made horizontal with the help of laveling screws so that the coil can rotate freely rotate between the poles of magnet. Then the light is allowed to fall on the plane mirror with the help of lamp-scale arrangement in such a manner that the reflected light gets focused on the zero mark of the scale. After this the electric current is allowed to flow through the coil of galvanometer by connecting it to the external circuit through adjusting screws T_1 and T_2 due to which the coil gets deflected along with the mirror and light focused on the scale also gets displaced.

If the $\perp$ distance of the scale from lamp be D and the angle of deflection of mirror be ϕ then the reflected light focused on scale gets displaced through angle 2ϕ.

In rt.angle Δ OMP

$$\tan 2\phi = \frac{OO'}{OM}$$



$\because 2\phi$ is small

$$\tan 2\phi \simeq 2\phi$$

$$\therefore \quad 2\phi = \frac{x}{d} \quad \text{or } \phi = \frac{x}{2d}$$

$$\therefore \quad \phi \propto x$$

But $\quad I \propto \phi$

$$\therefore \quad I \propto x$$

Thus the electric current flowing in the coil of galvanometer is directly proportional to the displacement of light on the scale.

Merits- (i) It is very sensitive, electric currents which are of the order 10^{-9}A can be measured which its help.

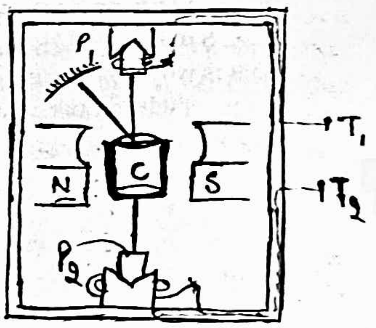
Demerits- (i) It is quite large therefore it can not be easily taken from one place to another.

(ii) Before taking reading it has to be adjusted which takes a lot of time.

Pivoted type galvanometer or Weston type galvanomter (Micro ammeter)

Ques.-Describe the principle, construction and working of a Pivoted type galvanometer or Weston type galvanomter. Also give its merits & demerits.

Ans.- Construction-



In pivoted type galvanometer there is a coil made of a thin copper wire which is bounded on an $A\ell$ frame. The core of the coil is made of soft iron. This coil is pivoted between two rigid supports with the help of knife edges P_1 and P_2. The coil is placed between two poles N and S of a strong horse shoe magnet which are cylindrically concave. Two oppositely bounded springs S_1 and S_2 are attached to the knife edges which provide the restoring torque. The two springs are connected to the adjusting screws T_1 and T_2. A pointer P is connected to the coil which rotates on graded scale S. The whole apparatus is enclosed in a non magnetic box in order to save it from air currents and dust particles.

Value of C depends on - i) Nature of material & shape of coil

Principle- Same as suspension type galvanometer.

Working- First of all pivoted type galvanometer is connected to external circuit in which electric current has to be measured through adjusting screw T_1 and T_2 due to which the coil gets deflected and the pointer attached to it also gets deflected. With the help of scale the electric current flowing in the circuit can be noted.

Merits-(i) It is small in size therefore it can easily be taken from one place to another.

(ii) It has not be adjusted before taking reading.

Demerits- Due to the friction between the kinfe edges and the pivots its sensitivity gets decreased therefore it can measure electric current which are of the order 10^{-6}A.

Current sensitivity of a galvanometer

Ques.-What do you mean by current sensitivitity of a galvanometer ? Write expression for it.

Ans.-The angular deflection produced in the coil of a galvanometer when a unit amount of electric current is allowed to flow through it is called its current sensitivity.

If on flowing I amount of electric current in the coil of galvanometer the angular deflection produced in the coil be θ then its current sensitivity is given by

$$S = \frac{\theta}{I}$$

figure of merit, $G \doteq \frac{1}{S}$

But $$I = \frac{C\theta}{NAB}$$

$$\therefore \quad S = \frac{\theta}{\frac{C\theta}{NAB}}$$

$$\boxed{S = \frac{NAB}{C}}$$

Thus the current sensitivity of a galvanometer-

(i) is directly proportional to the no. of turns in the coil

i.e. $S \propto N$

(ii) is directly proportional to area of coil

i.e. $S \propto A$

(iii) is directly proportional to MFI in the core of the coil

i.e. $S \propto B$

(That is why soft iron core is placed at the centre of coil.)

(iv) inversely proportional to the couple per unit of the suspension strip.

i.e. $S \propto \frac{1}{C}$

(That is why the suspension strip is made up of ~~quartz of~~ phosphor bronze as its couple per unit twist is ~~more~~ less.)

Note- Voltage sensitivity

$$S_v = \frac{\theta}{V}$$

or $$S_v = \frac{\theta}{IR} \doteq \frac{S}{R}$$

where R is the resistance of coil

or $$S_v = \frac{NAB}{CR}$$

Theory of shunt

Ques.-What is shunt ? Explain its principle. What are the advantages and disadvantages of using shunt with a galvanometer ?

Ans.- When a large amount of electric current is allowed to flow through a galvanometer then due to the large torque experienced by the coil the coil gets deflected sharply due to which its pointer may get broken by colliding against the stopper pin. In addition to this a large amount of heat is also generated in the coil due to which the coil may get burnt because of both the above factors the galvanometer may get damaged permanently.

In order to save galvanometer from being damaged a wire of small resistance called shunt is connected in parallel with the coil of galvanometer. As the resistance of shunt is very small as compared to the resistance of the coil of galvanometer therefore most of the main current flow through the shunt and only a part of it flows through the galvanometer. In this manner due to the presence of shunt the electric current flowing through the galvanometer remains with in safety limit and it is saved from being damaged. This is the theory of suhnt.

If the resistance of galvanometer be G and the resistance of shunt connected in parallel be S then I_g part of main current I flows thorugh the galvanometer and I_s part of it flows through the shunt. As the galvanometer and the shunt are connected in parallel.

Potential difference across galvanometer

= Potential difference across shunt

or $V_g = V_s$

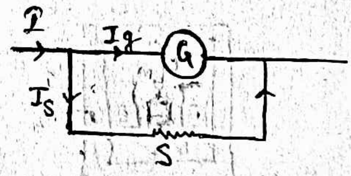
or $I_g G = I_s S$

or $$\frac{G}{S} = \frac{I_s}{I_g} \quad \text{—(i)}$$

Adding 1 to both the sides of above relation

or $$\frac{G}{S} + 1 = \frac{I_s}{I_g} + 1 \quad \text{—(i)}$$

or $$\frac{G+S}{S} = \frac{I_s + I_g}{I_s}$$



But $I_g + I_s = I$

$$\therefore \frac{G+S}{S} = \frac{I}{I_g}$$

$S << G$

or $$\boxed{I_g = \frac{IS}{G+S}}$$

$I \propto \frac{1}{R}$

$I_s >> I_G$

If $$I_g = \frac{I}{n}$$

then $$\frac{I}{n} = \frac{IS}{G+S}$$

or $G + S = nS$

or $G = nS - S$

or $G = (n - 1) S$

or $$\boxed{S = \frac{G}{n-1}}$$

Thus, when a shunt whose resistance is (n - 1)[th] part of the resistance of galvanometer is connected in parallel to it then the n[th] part of total current (main current) flows through the galvanometer.

Merits of shunt- (i) By using it the gavanometer can be saved from being permanently damaged.

(ii) By using it the range of galvanometer can be increased.

Demerits of shunt- By using it the sensitivity of galvanometer decreases.

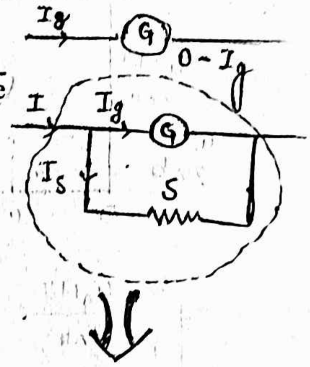
Ammeter

Ques.- What is ammeter? Explain how a galvanometer can be converted into ammeter?

Ans.- It is a current measuring device which is used for measuring large electric currents (in ampere).It is based on the theory of shunt.

Consider a galvanometer which can measure a maximum of I_g amount of current we have to convert it into ammeter which can measure a maximum of I amount of electric current. This can be done by connectring a small resistance ($\leq 10^{-2}\Omega$)called shunt in parallel to the galvanometer.On connecting the shunt only a small portion I_g of the total electric current I will pass through the galvanometer and the remaining electric current will pass through the shunt.

As the galvanometer and the shunt are connected in parallel.



$\therefore$ Current $\propto \dfrac{1}{\text{Resistance}}$

Here $\dfrac{I_s}{I_g} = \dfrac{G}{S}$

Adding 1 to both the sides

Here $\dfrac{I_s}{I_g} + 1 = \dfrac{G}{S} + 1$

or $\dfrac{I_s + I_g}{I_g} = \dfrac{G+S}{S}$

But $I_s + I_g = I$

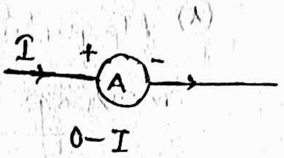
$\therefore \dfrac{I}{I_g} = \dfrac{G+S}{S}$

or $IS = I_g G + I_g S$

or $IS - I_g S = I_g G$

or $S(I - I_g) = I_g G$

or $\boxed{S = \dfrac{I_g G}{I - I_g}}$



Thus by connecting the shunt of above value in parallel to the galvanometer its range can be increased from $0 - I_g$ to $0 - I$.

Resistance of ammeter

$$R_a = \frac{GS}{G+S}$$

$\therefore R_a < S << G$

As the resistance of ammeter is very-very small therefore when it is connected in series to an electrical circuit the change in the resistance of electrical circuit is negligible due to which the change in the value of electric current flowing in the circuit is also negligible Thus, when an ammeter is connected in series to an electrical circuit the error in the reading of ammeter is negligible that why it is connected in series to a circuit.

When the resistance of ammeter is zero then there will be no error in its reading but such an ammeter can not be realized into actual practice therefore the resistance of an ideal ammeter is zero.

Note- If by chance ammeter be connected in parallel to an electrical circuit then due to its small resistance most of the current will flow through it which will damage ammeter.

Voltmeter

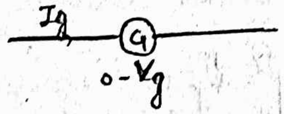
Ques.-What is voltmeter? Explain how a galvanometer can be converted into voltmeter?

Ans.-It is a device which is used for measuring potential difference between two points of an electrical circuit

Consider a galvanometer of resistance G which can measure a maximum of I_g amount of electric current and hence a maximum V_g amount of potential difference ($V_g = I_g G$).

In order to convert galvanometer into voltmeter which can measure a maximum of V amount of potential difference a large resistance R ($\geq 2000\Omega$)has to be connected in series to the galvanometer. In this situation the maximum potential difference V which can be measured is given by

$V = I_g (G + R)$

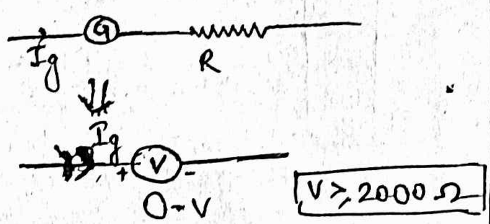


or $\dfrac{V}{I_g} = G + R$

or $\boxed{R = \dfrac{V}{I_g} - G}$

$\because V_g = I_g G$

or $I_g = \dfrac{V_g}{G}$



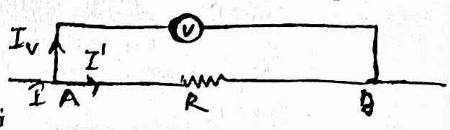
$\therefore R = \dfrac{V}{V_g/G} - G$

or $R = \dfrac{VG}{V_g} - G$

or $\boxed{R = \left(\dfrac{V}{V_g} - 1\right) G}$

When the resistance of above value of connected in series to the galvanometer then its range increases from $0 - V_g$ to $0 - V$.

Resistance of voltmeter-



$R_v = G + R$

$\therefore R_v > R >> G$

As the resistance of voltmeter is very large therefore when it is connected in parallel between any two points of an electrical circuit the current drawn by it is negligible due to which change in current flowing between those two points is negligible and the error in the reading of voltmeter is also negligible.

If the resistance of voltmeter be infinity then there will be no error in its reading but such galvanometer can not be realized in to actual practice. Thus the resistance of an ideal voltmeter is infinity(∞).

Note- If by chance the voltmeter be connected in series in an electrical circuit then due to increase in (equivalent) resistance the electric current flowing through the circuit will decrease appreciably and the potential difference which has tc be measured will also decrease considerably.

$$B \oint_l dl(1) = \mu_0 n I$$

or $Bl = \mu_0 n l I$

or $\boxed{B = \mu_0 n I}$

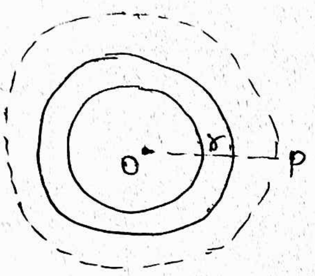
$\because$ $n = \frac{N}{l}$

$\therefore$ $B = \mu_0 \frac{N}{l} I$

$\because$ $l = 2\pi r$

$\therefore$ $\boxed{B = \frac{\mu N I}{2\pi r}}$

(iii) When the observation point P lies outside the toroid-
By taking O as centre and $OP = r$ as radius draw a circle which will behave like ampere's circuit.
By ACL



$$\oint_l \vec{B}.\vec{dl} = \mu_0 I_{net}$$

or $\oint_l \vec{B}.\vec{dl} = \mu_0 0$

or $\oint_l \vec{B}.\vec{dl} = 0$

$\therefore$ $B = 0$

Note- (i) If in numericals inner and outer radius of toroid are given say 'a' and 'b' respectively. Then the value of r is taken as

$$r = \frac{a+b}{2}$$

(ii) If in the core of solenoid and toroid a medium of relative magnetic permeability μ_r is present then the value of MFI is taken as

$$B = \mu_r \mu_0 n I$$

Note- MFI due to a current carrying circular arc at its centre-

$$\boxed{B = \frac{\mu_0}{4\pi} \frac{I\alpha}{r}}$$

Particular cases-

(a) When $\alpha = 90° = \frac{\pi}{2}$

$$B = \frac{\mu_0}{4\pi} \frac{I}{r} \frac{\pi}{2} = \frac{\mu_0 I}{8r}$$

(b) When $\alpha = 180° = \pi$

$$B = \frac{\mu_0}{4\pi} \frac{I}{r} \pi = \frac{\mu_0 I}{4r}$$

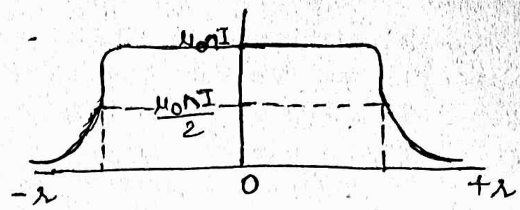
(c) When $\alpha = 270° = \frac{3\pi}{2}$

$$B = \frac{\mu_0}{4\pi} \frac{I}{r} \frac{3\pi}{2} = \frac{3\mu_0 I}{8r}$$

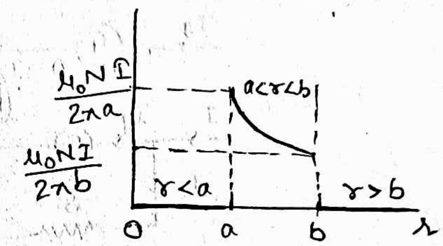
★ Graph :- (a) for solenoid:

$B_{inside} = \mu_0 n I$

$B_{end} = \frac{\mu_0 n I}{2}$



(b) for toroid:



$B = \frac{\mu_0 N I}{2\pi r} \Rightarrow B \propto \frac{1}{r}$

⊛ Magnetic force acting b/w two moving charged particles

$$F_m = \frac{\mu_0}{4\pi} \frac{q_1 v_1 q_2 v_2}{r^2}$$

Magnetism

Magnet- Those substances which attract magnetic materials are called magnets. The properly by virtue of which magnet attract magnetic materials is called magnetism.

Magnets are of two types-

1. Natural magnets- Naturally made magnets are called natural magnets. They are irregular is shape and have low attracting power.e.g.-magnets made of magnetite(Fe_3O_4)

2. Artificial magnets- Man made magnets are called artificial magnets and have high attracting power. e.g. magnets made of steel, iron, ticonal, alnico etc. Artificial magnets are again of two types-

(a) Temporary magnets- Those magnets whose magnetic property is temporary are called temporary magnets. They are made of soft iron.

(b) Permanent magnet- Those magnets whose magnet property is permanent are called permanent magnets. They are made of steel.

Magnetic poles- When iron filings are sprinkeled around a magnet then it is found that the concentration of iron filings is maximum at the two points near the ends and minimum at the centre of the magnet.

The points near ends of a magnets where its attractive power is maximum are called magnetic poles and the points where attractive power of magnet is minimum is called neutral point.

Every magnet has two poles wihch are equal in strength and opposite in nature-

(i) North pole (N-pole)

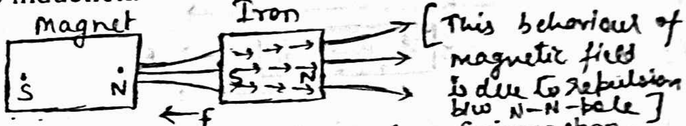
(ii) South pole (S-pole)

When a magnet is freely suspended then it orient itself in N-S direction. The pole pointing towards north is called N-pole and pole pointing towards south is called S-pole.

Properties of a magnet-

1. When a magnet is freely suspended then it orient itself in N-S direction.
2. A magnet attracts other magnetic materials towards it.
3. Like poles of magnets repel and unlike poles of magnet attract each other.
4. Two poles of a magent can not be separated from each another.
5. A magnet induces magnetism in other magnetic substances by magnetic induction.

Note- 1. Magnetic induction- When a magnet is brought close to a magnetic substance then under the effect of its magnetic feild the atomic magnets present inside the magnetic substance orient themselves in the direction of magnetic feild. As a result of which opposite pole induces on that surface of magnetic substance which lies close to magnet and identical pole induces on its farther surface. This phenomenon is called magnetic induction.



2. If a magnet is broken into a number of pieces then each piece behave like a magnet with two equal and opposite poles. This implies that poles of a magnet can not be seperated and magnetic monopoles do not exist.

3. When a pole of a magnet is brought close to the unlike pole of another magnet then it gets attracted towards it. Also when a pole of magnet is brought closer to a magnetic material then it gets attracted towards it due to magnetic induction but when a pole of a magnet is brought closer to the like pole of another magnet then they repel each other. Thus repulsion is the surest test of magnetism (i.e if two rods repel each other then only we can say that the two rods are magnet).

Definitions of few terms

(1) Magnetic axis- The line joining the poles of a magnet is called its magnetic axis.

The distance between the poles of a magnets is called its magnetic length. The magnetic length of a bar magnet is approx. 5/6 th part of its geometrical length.

$l_m = \frac{5}{6} l_g$

(2) Magnetic meridian- The vertical plane which passes through the magnetic axis of a freely suspended magnet on the surface of earth is called magnetic meridian.

(3) Molecular theory of magnetism- It was proposed by Weber and Ewing. According to this theory-

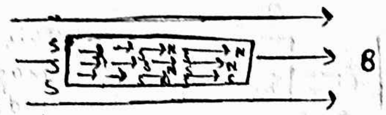
1. Every molecule of a magnetic substance behaves like a magnet having a N and S pole.
2. In an unmagnetised substance these molecular magnets are randomly oriented and form closed chains.

In this manner the molecular magnets cancel out each others effect and the resultant magnetism remains zero.

(3) During the process of magnetisation the reorientation of these molecular magnets takes place and they get aligned themselves in the direction of magnetic feild due to which one end of magnetic sub

4) stray magnetic field is also the reason. * Small magnet behaves like dipole.

stance behaves like N-pole and the other end behaves like S-pole.



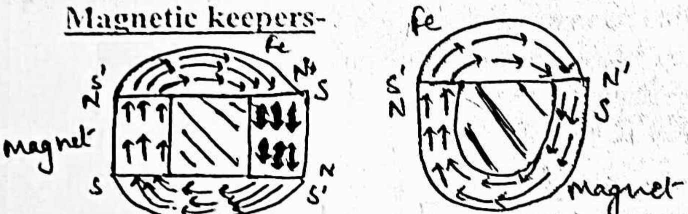
Note- The alignment of molecular magnets is permanent in the case of steel therefore steel is used for making permanent magnets.

As soon as the magnetic feild is removed the alignment of molecular magnets is disturbed in soft iron therefore it is used for making electromagnets.

Causes of demagnetisation-

1. When a magnet is heated then the thermal energy of its molecular magnets increases due to which their amplitude of vibration also increases . As a result of which the unlike poles of molecular magnets which lie in adjacent rows come close to one another and they form closed chains due to which the magnet gets demagnetised.
2. For the same reason when a magnet is hammered it gets demagnetised.
3. When a magnet is kept open for long time then due to repulsion between like poles of molecular magnets present at the ends of magnet the molecular magnets gets randomly oriented and form closed chains due to which the magnet gets demagnetised.

Magnetic keepers-



They are soft iron pieces by using which the magnetic properties of a magnet can be retained (saved) for ling time.

In the case of two bar magnets two magnetic keepers and in the case of a horse shoe magnet only one magnetic keeper is reqd. as shown in fig.

Pole strength

Ques.-Define pole strength.State its S.I. unit and dimensions.

Ans.-It is that physical quantity which measures the ability of poles of a magnet to attract magnet materials towards them. It is denoted by 'm'

It is a scalar quantity. Its SI unit is Am and dimensional formula is [AL].

Its value depends upon-

1. Nature of material of magnet.
2. Area of cross-section of magnet ($m \propto A$)

Note- The strength of two poles of a magent is always same

Magnetic dipole and Magnetic dipole moment

Ques.-(a)Define magnetic dipole.

(b)Define magnetic dipole moment.State its S.I. unit and dimensions.

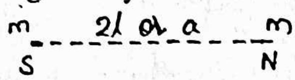
Ans.- An arrangement of two equal and opposite magnetic poles which are placed close to one another is called magnetic dipole.

Small magnets behave like magnetic dipole.

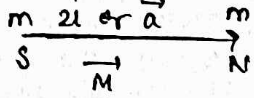
Magnetic dipole moment of a magnet is defined as the product of the strength of either pole and the distance between the two poles.

The pole strength of a magnet be m and the distance between the two poles be 2ℓ or a then its magnetic dipole moment is given by

$$M = m\,2\ell$$
$$= ma$$



It is a vector quantity. Its direction is always taken from S to N- pole.



In vector form

$$\vec{M} = m\,2\vec{\ell} = M\vec{a}$$

where $2\vec{\ell}$ or $\vec{a}$ is the position vector of N-pole w.r.t S-pole.Its S.I.unit is Am^2 and dimensional formula is $[AL^2]$.

Note (I)-Magnetic dipole moment is that physical quantity which measures the ability of magnetic dipole to rotate in external magnetic feild.

Note-(II) a. When a magnet is divided into two identical parts along its magnetic axis.

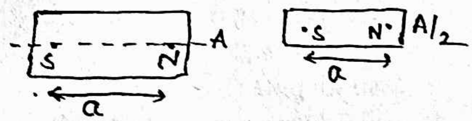
$$M = ma$$

For each part

$$A' = \frac{A}{2} \quad \text{and} \quad a' = a$$

$$\because \quad m \propto A$$

$$\therefore \quad m' = \frac{m}{2}$$



$$\text{and} \quad M' = m'a'$$
$$= \frac{m}{2} a$$
$$= \frac{M}{2}$$

Generalization- If a magnet be divided into R identical parts parallel to its axis

$$\text{then } m' = \frac{m}{R}$$

$$\text{and } M' = \frac{M}{R}$$

(b) When a magnet is divided into two identical parts $\perp$ to its magnetic axis.

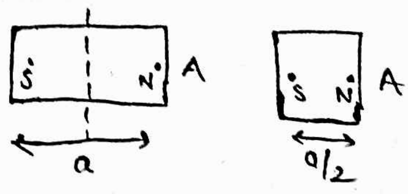
$$M = ma$$

For each part

$$A' = A, \quad \text{and} \quad a' = \frac{a}{2}$$

$$\therefore \quad m' = m$$

$$\text{and} \quad M' = m'a'$$
$$= m\frac{a}{2}$$
$$= \frac{M}{2}$$



* current carrying loop behaves like dipole.

Generalization- When the magnet is divided into C parts ⊥ to its axis.

$$m' = m \quad \text{and} \quad M' = \frac{M}{C}$$

(c) When a magnet is divided into four identical parts (two along the axis and two ⊥ to the axis)

For each part

$$A' = \frac{A}{2} \quad \text{and} \quad a' = \frac{a}{2}$$

$$\therefore \quad m' = \frac{m}{2} \ \& \ M' = m'a'$$

$$= \frac{M}{2}\frac{a}{2}$$

$$= \frac{M}{4}$$

Generalization- When a magnet is divided into R parts along its axis and C parts ⊥ to its magnetic axis simultaneously.

$$\text{then } m' = \frac{M}{R} \quad \& \quad M' = \frac{M}{RC}$$

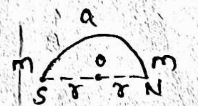
2. Equivalent MDM of a magnetic wire.

(a) When the wire is bent into a semi circle.

$$\frac{2\pi r}{2} = a$$

$$\text{or} \quad \pi r = a$$

$$\text{or} \quad r = \frac{a}{\pi}$$

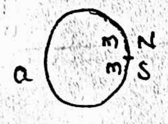


$$a' = 2r = \frac{2a}{\pi}$$

$$\therefore \quad M = m'a'$$

$$= m \times \frac{2a}{\pi}$$

$$= \frac{2M}{\pi}$$



(b) When the wire is bent into circle.

$$a' = 0$$

$$M' = ma'$$

$$= m\,(0)$$

$$= 0$$

Magnetic field and magnetic field intensity

Ques.-(a)Define magnetic field.

(b)Define magnetic field intensity.State its units and dimensions.

Ans.-(a) The space surrounding a magnet with in which when other mgnetic substances are brought in then they experience attractive force is called magnetic feild.

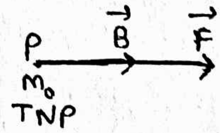
The magnetic force experienced by unit N-pole placed at any point in the magnetic feild is called magnetic feild intensity at that point it is denoted of by B or I.

If the magnetic force experienced by test N-pole of pole strength m_0 at any point in the magnetic feild be F then the MFI at that point is given by

$$B = \frac{F}{m_0}$$

In vector form

$$\vec{B} = \frac{\vec{F}}{m_0}$$



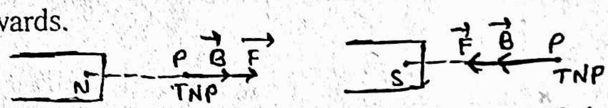
It is a vector quantity and its direction is same as the direction of magnetic force experienced by test N-pole.

Its S.I. unit is $NA^{-1}m^{-1}$ or tesla (T), C.G.S. unit is dynab$A^{-1}cm^{-1}$ or gauss (G) and dimensional formula is $[MT^{-2}A^{-1}]$

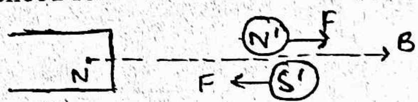
Note- (i) (a) Source pole- The pole which generates MF.It can be N or S- pole.

(b)Test pole-The pole which is used for measuring and testing MF. It is always N-pole.

(ii) MF due to N-pole of a magnet is directed outwards and the MF due to S-pole of a magnet is directed inwards.



(iii) When a N-pole is placed in external MF then it experiences magnetic force in the direction of MF and when S-pole is placed in external MF then it experienced force in the direction opposite to the MF.



Coulomb's inverse square law of magnetism

Ques.-State Coulomb's inverse square law of magnetism. Write its expression in S.I. and C.G.S. system.

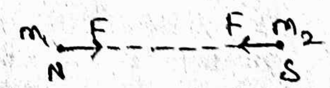
Ans.- This law was established by coulomb in the year 1785 on the basis of experiments performed by him with the help of torsional balance.

It states that the force of attraction or repulsion between two magnetic poles is directly proportional to the product of their pole strength and inversely proportional to the square of the distance between them.This force always act along the line joining the two poles.

If two magnetic poles of pole strength m_1 and m_2 are placed at seperation r then accordance with CISL the magnetic force acting between them is

$$F \propto m_1 m_2$$

$$\text{and} \quad F \propto \frac{1}{r^2}$$



Combining the above two factors

$$F \propto \frac{m_1 m_2}{r^2}$$

$$\text{or} \quad F \propto k\frac{m_1 m_2}{r^2} \quad \text{----(i)}$$

where k is a constt of proportionality and is called eleletro magnetic constt.

The value k depends on-

1 The nature of medium between the two poles

Poles are of different magnet.

2. The system of unit used.

(I) <u>In S.I. system</u>

(a) In vacuum

$$k = \frac{\mu_0}{4\pi}$$

where $\mu_0 = 4\pi \times 10^{-7}$ TmA^{-1} is called absolute permeability of free space (vacuum).

$$k = \frac{\mu_0}{4\pi} = \frac{4\pi \times 10^{-7}}{4\pi}$$

$= 10^{-7}$ TmA^{-1}

From rel'n (i)

$$\boxed{F = \frac{\mu_0}{4\pi}\frac{m_1 m_2}{r^2}}$$

(b) In medium

$$k = \frac{\mu}{4\pi}$$

where μ absolute permeability of medium .

$$k = \frac{\mu}{4\pi} = \frac{\mu_r \mu_0}{4\pi}$$

$= \mu_r \times 10^{-7}$ TmA^{-1}

From rel'n (i)

$$\boxed{F = \frac{\mu_0 \mu_r}{4\pi}\frac{m_1 m_2}{r^2}}$$

(ii) <u>In C.G.S. (electromagnetic) system</u>

(a) In vacuum

$$k = \frac{\mu_0}{4\pi}$$

where $\mu_0 = 4\pi$ GcmabA^{-1}

$$\therefore k = \frac{\mu_0}{4\pi} = \frac{4\pi}{4\pi}$$

$= 1$GcmabA^{-1}

From rel'n (i)

$$\boxed{F = \frac{m_1 m_2}{r^2}}$$

(b) In medium

$$k = \frac{\mu}{4\pi}$$

where $\mu = \mu_0 \mu_r$

$$\therefore \ k = \frac{\mu}{4\pi} = \frac{\mu_r \mu_0}{4\pi}$$

$= \mu_r \times 1$

$= \mu_r$ GcmabA^{-1}

From rel'n (i)

$$\boxed{F = \frac{\mu_r m_1 m_2}{r^2}}$$

MFI due to an isolated pole

Ques.-Derive relation for MFI due to an isolated pole.

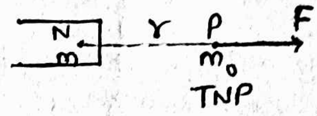
Ans.-Consider an isolated N- pole whose pole strength be m. We have to calculate MFI due to the isolated N-pole at a point P which is at a distance r from it.

In order to calculate MFI at point P, consider a test N-pole of pole strength m_0 at point P then in accordance with CISL the magnet force experienced by the test N-pole is given by

$$F = \frac{\mu_0}{4\pi}\frac{m m_0}{r^2}$$

By definition, MFI at point P

$$B = \frac{F}{m_0}$$



$$B = \frac{\frac{\mu_0}{4\pi}\frac{m m_0}{r^2}}{m_0}$$

$$\boxed{B = \frac{\mu_0}{4\pi}\frac{m}{r^2}}$$

If the pole and the observation point both lie in a medium of relative permeability μ_r

then $\boxed{B = \frac{\mu_r \mu_0}{4\pi}\frac{m}{r^2}}$

This is the reqd expression.

<u>Particular cases</u>- (i) If r = 0

then $B = \frac{\mu_0}{4\pi}\frac{m}{0^2}$

or $B = \infty$ (not defined)

(ii) If $r = \infty$

then $B = \frac{\mu_0}{4\pi}\frac{m}{\infty^2}$

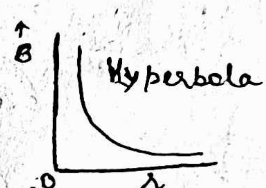
or $B = 0$

<u>Graph between B and r</u>-From rel'n

$$B = \frac{\mu_0}{4\pi}\frac{m}{r^2}$$

we have

$$B \propto \frac{1}{r^2}$$



~~**MFI due to an isolated pole**~~

MFI at any point on the axial line of magnet

Ques.-Derive expression for MFI due to a magnet at any point on its axis.

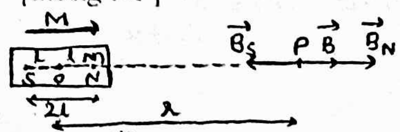
Ans.- The line which passes through both the poles of a magnet is called its axial line. Consider a magnet whose pole strength be m and length be 2ℓ We have to find out MFI at any point P on the axial line of magnet which is at a distance r from its centre O.

MFI due to N-pole of magnet at the observation point P

$$B_N = \frac{\mu_0}{4\pi}\frac{m}{NP^2} \quad [\text{along } \overrightarrow{NP}]$$

But NP = OP - ON

$= r - \ell$



$$\therefore \quad B_N = \frac{\mu_0}{4\pi}\frac{m}{(r-\ell)^2} \quad \text{--(i)}$$

and MFI due to S-pole of magnet at point P

$$B_S = \frac{\mu_0}{4\pi}\frac{m}{SP^2} \quad [\text{along } \overrightarrow{PS}]$$

But SP = OP + OS

$= r + \ell$

$$\therefore \quad B_N = \frac{\mu_0}{4\pi}\frac{m}{(r+\ell)^2} \quad \text{--(ii)}$$

$\because \quad |\vec{B_N}| > |\vec{B_S}|$

and they are oppositely directed

Resolving $\vec{B_N}$ & $\vec{B_s}$ at point P into horizontal & vertical components, the vertical components will cancel out each other & their horizontal components will add up together.

∴ Resultant MFI at point P

$$B = B_N - B_S \,[\text{along } \vec{NP}]$$

or $$B = \frac{\mu_0}{4\pi}\frac{m}{(r-\ell)^2} - \frac{\mu_0}{4\pi}\frac{m}{(r+\ell)^2}$$

or $$B = \frac{\mu_0 m}{4\pi}\left[\frac{1}{(r-\ell)^2} - \frac{1}{(r+\ell)^2}\right]$$

or $$B = \frac{\mu_0 m}{4\pi}\left[\frac{(r+\ell)^2 - (r-\ell)^2}{(r-\ell)^2\,(r+\ell)^2}\right]$$

or $$B = \frac{\mu_0 m}{4\pi}\left[\frac{(r^2+\ell^2+2r\ell)-(r^2+\ell^2-2r\ell)}{\{(r-\ell)(r+\ell)\}^2}\right]$$

or $$B = \frac{\mu_0 m}{4\pi}\left[\frac{r^2+\ell^2+2r\ell - r^2 - \ell^2 + 2r\ell}{(r^2-\ell^2)^2}\right]$$

or $$B = \frac{\mu_0 m}{4\pi}\left[\frac{4r\ell}{(r^2-\ell^2)^2}\right]$$

or $$B = \frac{\mu_0}{4\pi}\frac{2\times 2m\ell\times r}{(r^2-\ell^2)^2}$$

∵ $2m\ell = M$ magnetic dipole moment of magnet

∴ $$\boxed{B = \frac{\mu_0}{4\pi}\frac{2Mr}{(r^2-\ell^2)^2}}$$

This is the reqd expression.

Particular case- If magnet be small

i.e. $\ell \ll r$

then ℓ^2 can be ignored as compared to r^2

∴ $$B = \frac{\mu_0}{4\pi}\frac{2Mr}{(r^2)^2}$$

or $$\boxed{B = \frac{\mu_0}{4\pi}\frac{2M}{r^3}}$$

This is the reqd expression of MFI due to a small magnet at any point on its axial line.

Direction of MFI- The direction of MFI at any point of the axial line of a magnet is from S to N pole of the magnet or same as magnetic dipole moment.

MFI on the equitorial line of a magnet

Ques.-Derive expression for MFI on the equitorial line of a magnet.

Ans.- The line which passes through the centre of a magnet and is perpendicular to its axis is called its equitorial ~~axial~~ line. Consider a magnet NS whose pole strength be m and length be 2ℓ. We have to find out MFI due to the magnet at any point P on its equitorial line which is at distance r from its distance r from its center O.

MFI due to N-pole at point P.

$$B_N = \frac{\mu_0}{4\pi}\frac{m}{NP^2}\ [\text{along } \vec{NP}]$$

But in rt. Δ NOP by pythagorus theorem

$$NP^2 = OP^2 + ON^2$$
$$= r^2 + \ell^2$$

∴ $$B_N = \frac{\mu_0}{4\pi}\frac{m}{(r^2+\ell^2)} \quad \text{------(i)}$$

and MFI due to S-pole of magnet at point P

$$B_S = \frac{\mu_0}{4\pi}\frac{m}{SP^2}\ [\text{along } \vec{PS}]$$

But in rt. Δ SOP by PT

$$SP^2 = OP^2 + OS^2 = r^2 + \ell^2$$

∴ $$B_N = \frac{\mu_0}{4\pi}\frac{m}{(r^2+\ell^2)} \quad \text{--(ii)}$$

Let ∠PSO = ∠PNO = θ

Draw PQ parallel to NS

∴ ∠TPQ = ∠PNO = θ (corresponding ∠'s)

and ∠UPQ = ∠PSO = θ (alternate ∠'s)

∵ $|\vec{B_N}| = |\vec{B_S}|$

and the angle between the direction of $\vec{B_N}$ and $\vec{B_S}$ is 2θ

∴ Resultant MFI at point P

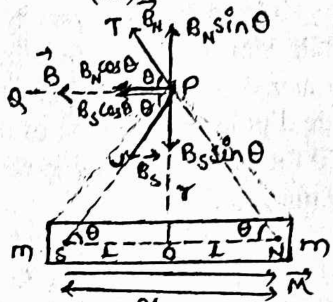
$$B = B_N\cos\theta\ ~~[\text{along } PQ]~~ + B_S\cos\theta\ (\text{along } \vec{PQ})$$

or $$B = 2B_N\cos\theta \quad \text{---(iii)}$$

∵ $$B_N = \frac{\mu_0}{4\pi}\frac{m}{(r^2+\ell^2)}$$

and in ractangle Δ PNO

$$\cos\theta = \frac{ON}{NP}$$
$$= \frac{\ell}{(r^2+\ell^2)^{1/2}}$$



From relation (iii)

$$B = \frac{2\mu_0}{4\pi}\frac{m}{(r^2+\ell^2)}\frac{\ell}{(r^2+\ell^2)^{1/2}}$$

or $$B = \frac{\mu_0}{4\pi}\frac{2m\ell}{(r^2+\ell^2)^{3/2}}$$

∵ $2m\ell = M$ magnetic dipole moment of magnet

$$\boxed{B = \frac{\mu_0}{4\pi}\frac{M}{(r^2+\ell^2)^{3/2}}}$$

This is the reqd expression.

Particular case- If the magnet be small

i.e. $i \ll r$.

then ℓ^2 can be ignored as compared to r^2

∴ $$B = \frac{\mu_0}{4\pi}\frac{M}{(r^2)^{3/2}}$$

or $$\boxed{B = \frac{\mu_0}{4\pi}\frac{M}{r^3}}$$

This is the reqd expression of magnetic field intensity due to a small magnet at any point on equitorial line.

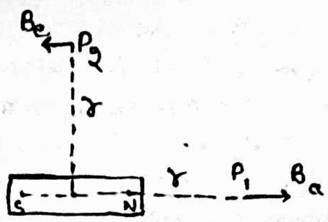
Direction of MFI- The direction of MFI at any point on equitorial line of a magnet is from N to S or opposite to that of magnetic dipole moment.

Note- Comparision of MFI due to magnet at any point on its axial and equitorial line- For the same distance r of abs. point from the centre of magent of magnetic dipole moment M. MFI of any point on its axial line.

or $$B_a = \frac{\mu_0}{4\pi}\frac{2M}{r^3} \quad \text{--(i)}$$

and MFI at any point on its equitorial line

or $$B_e = \frac{\mu_0}{4\pi}\frac{M}{r^3} \quad \text{---(ii)}$$



Dividing rel'n (i) by (ii)

or $$\frac{B_a}{B_e} = \frac{\frac{\mu_0}{4\pi}\frac{2M}{r^3}}{\frac{\mu_0}{4\pi}\frac{M}{r^3}}$$

or $\frac{B_a}{B_e} = 2$

or $B_a = 2B_e$

Thus MFI at any point on the axial line of a magnet is twice the MFI at any point on the equitorial line at the same distance.

Note- If nothing be given about the length of magnet then the magnet should be assumed to be small.

Torque acting on a magnet placed in uniform magnetic field

Ques.-Derive expression for torque acting on a magnet placed in uniform magnetic field.

Ans.- Consider a magnet NS of length 2ℓ and pole strength m width is placed in uniform MF B in such a manner that the angle between the direction of magnetic dipole moment $\vec{M}$ of the magnet and the MF $\vec{B}$ be θ then the force acting on N-pole of the magnet in the direction of MF

$F_N = mB$

and the force acting on S-pole of magnet in the direction of magnetic force

$F_S = mB$

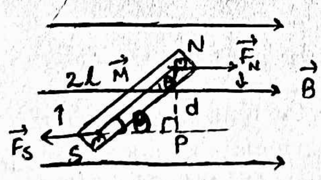
As the two forces are equal in magnetic and opposite in direction, therefore they will constitute to a form a couple and a torque will act on the magnet whose tendency is to bring it in the direction of magnetic field. It is also called restoring torque.

Restoring torque acting on the magnet = Magnitude of either force × ⊥ distance between the line of action of two forces.

∴ $\tau = F_n d$ --(i)

But $F_N = mB$

in ractangle Δ NPS



$\sin\theta = \frac{NP}{NS}$

$= \frac{d}{2\ell}$

or $d = 2\ell\sin\theta$

From rel'n (i)

$\tau = mB2\ell\sin\theta$

or $\tau = m2\ell B\sin\theta$

But $2m\ell = M$ magnetic dipole moment of magnet

∴ $\boxed{\tau = MB\sin\theta}$

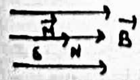
This is the reqd expression. In vector form $\boxed{\vec{\tau} = \vec{M}\times\vec{B}}$

Particular cases-

(i) When θ = 0° or 180°

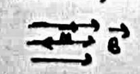
$\tau = MB \sin 0° = MB (0)$

$= 0$ (minimum)



and $\tau = MB \sin 180° = MB (0)$

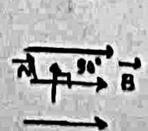
$= 0$ (minimum)



(ii) When θ = 90°

then $\tau = MB \sin 90° = MB (1)$

$= MB$ (maximum)



Note- Definition of MDM in terms of torque

In rel'n $\tau = MB\sin\theta$

If B= 1 and θ = 90°

then $\tau = M.1.\sin 90°$

$\tau = M.1.1$

$\tau = M$

Thus MDM of a magnet is numerically equal to the torque experienced by it when it is placed ⊥ to a unit magnetic field.

WD in rotating a magnet placed in uniform magnetic field

Ques.-Derive expression for WD in rotating a magnet placed in uniform magnetic field.

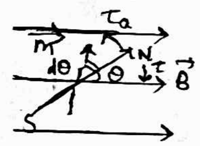
Ans.- Consider a magnet NS of magnetic dipole moment M which is placed in uniform MF B such that the angle between $\vec{M}$ and $\vec{B}$ be θ.

In this position the restoring torque acting on the magnet is given by

$\tau = MB\sin\theta$

In order to rotate the magnet by an infinitely small angle dθ in the direction opposite to the magnetic field a torque has to be applied on the magnet which is equal in magnitude and opposite in the direction of the restoring torque.

Applied torque



$\tau_a = \tau$

$= MB\sin\theta$

WD in rotating mgnets by small angle dθ is given by

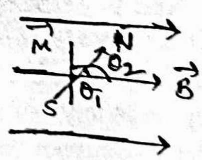
$dW = \tau d\theta$

$dW = MB\sin\theta\, d\theta$

By integrating the above rel'n with in proper limits the WD in rotating the magnet from angle θ_1 to θ_2 can be calculated as follows-

$$\int_0^W dW = \int_{\theta_1}^{\theta_2} MB\sin\theta\, d\theta$$

$$[W]_0^W = MB\int_{\theta_1}^{\theta_2}\sin\theta\, d\theta$$



$$dw - 0 = MB\,[\cos\theta]_{\theta_1}^{\theta_2}$$

or $W = MB\,[(-\cos\theta_2) - (-\cos\theta_1)]$

or $W = MB\,[\cos\theta_1 - \cos\theta_2]$

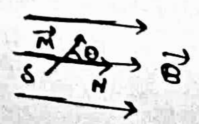
or $\boxed{W = MB\,(\cos\theta_1 - \cos\theta_2)}$

This is the reqd expression.

If the magnet be rotated from stable equibrium position (from the direction of magnetic field) by an angle θ then

$W = MB\,(\cos 0° - \cos\theta)$

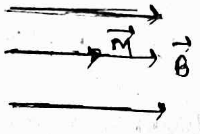
or $\boxed{W = MB\,(1 - \cos\theta)}$



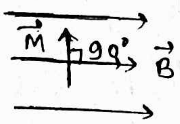
Particular cases- (i) If θ = 0°

then $W = MB\,(1 - \cos\theta)$

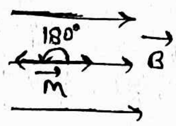
$= MB (1 - \cos 0°)$
$= MB (1 - 1)$
$= 0$ (minimum)



(ii) If $\theta = 90°$
then $W = MB (1 - \cos 90°)$
$= MB (1 - 0)$
$= MB$



(iii) If $\theta = 180°$
then $W = MB (1 - \cos 180°)$
$= MB (1 + 1)$
$= 2 MB$ (maximum)



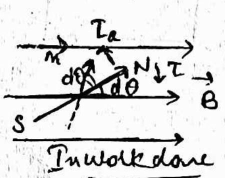
P.E. of a magnet in uniform MF

Ques.-Derive expression for P.E. of a a magnet placed in uniform magnetic field.

Ans.-The P.E. of a magnet placed on uniform magnetic field is equal to the WD in rotating the magnet from its standard position to the given position.

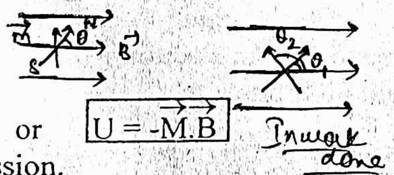
Consider a magnet NS of MDM M which is placed in a uniform magnetic field B such that the angle between $\vec{M}$ and $\vec{B}$ be θ then the work alone in rotaing in magnet from standard position ($\theta_1 = 90°$) to given position ($\theta_2 = \theta$) is

$W = MB (\cos\theta_1 - \cos\theta_2)$
or $W = MB (\cos 90° - \cos\theta)$
or $W = MB (0 - \cos\theta)$
or $W = - MB \cos\theta$



By definition this work is stored in the magnet in the form of P.E.

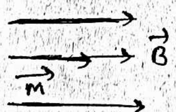
P.E of magnet
$U = W$



$\boxed{U = -MB \cos\theta}$ or $\boxed{U = -\vec{M}.\vec{B}}$

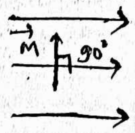
This is the reqd expression.

Particular cases- (i) If $\theta = 0°$
then $U = - MB \cos 0°$
$= - MB (1)$
$= - MB$



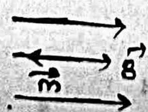
In this position the magnet will be stable equibrium position.

(ii) If $\theta = 90°$
then $U = - MB \cos 90°$
$= - MB (0)$
$= 0$



In this position the magnet will be in standard position.

(i) If $\theta = 180°$
then $U = - MB \cos 180°$
$= - MB (-1)$
$= + MB$



In this position the magnet will be in unstable equibrium position.

Magnetic field lines

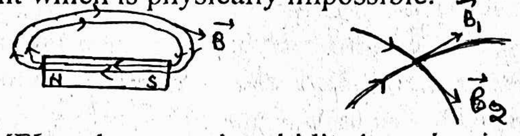
Ques.-What are magnetic feild lines? Give their properties.

Ans.-These are those imaginary smooth curves along which a unit N-pole would move if it free to do so.

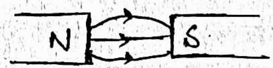
The concept of magnetic field lines was first given by Michael Faraday and they give us the pictorial representation of magnetic field.

Properties of magnetic field lines-

1. MFL always form closed loops. Outside a magnet they run from N to S and inside a magnet they run from S to N.
2. The tangent drawn at any point of MFL gives us the direction of magnetic field intensity at that point.
3. Two MFL do not intersect each other because when the two MFL will intersect then at the point of intersection two tangents can be drawn which will give us two different directions of magnetic field at the same point which is physically impossible.



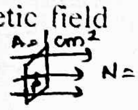
4. MFL ~~and~~ possess longitidinal tension i.e their tendency is to shrink lengthwise on the basis of this property, we can explain the force of attraction between two unlike poles.



5. MFL ~~and~~ possess lateral pressure i.e. they repel one another ⊥ to their length. On the basis of this property, we can explain the force of repulsion between two like poles.

9. NFL can pass through conductor

6. In the region of strong magnetic field MFL are closely spaced and in the region of weak magnetic field MFL are far apart.
7. MFL emerge at different angles from N-pole and also enter at different angles at the S-pole of magnet.
8. The number of MFL which passes normally through the unit area chosen about a point in magnetic field gives us the magnitude of MFI at that point.



Note- 1. Difference between MFL and ELF-
(i) MFL always form close loops where as ELF emanate from positive charge and terminate at negative charge.
(ii) MFL emerge at different angles from the N-pole and enter at different angles into S-pole of a magnet where as ELF always emerge and terminate normally at the surface of charged conductor.
(iii) MLF can pass through a conductor like Fe, Ni, Co etc where as ELF do not pass through a conductor.

Uniform magnetic field

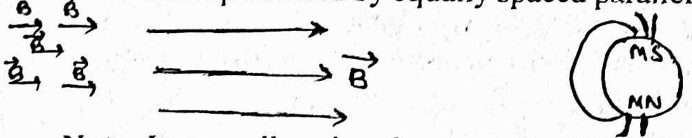
Ques.-What is uniform magnetic field ? How is it rep

Magnetic field lines are different from Magnetic lines of force.

resented?

Ans.-It is that magnetic field at every point of which the magnitude and direction of MFI is same.

It is represented by equally spaced parallel MFL.



Note- In a small region the magnetic field of earth is uniform therefore it is represented by equally spaced parallel MFL.

Magnetic Dipole moment of a current carrying loop or solenoid

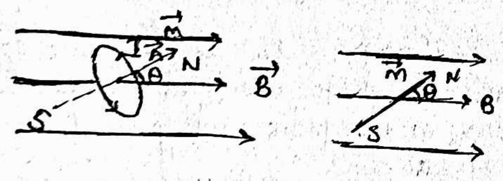
Ques.- Show that a current carrying loop behaves like a magnetic dipole. Derive an expression for its magnetic dipole moment.

Ans.- Consider a current carrying loop in which the number of turns be N, its area be A and the current flowing in it be I. The loop is placed in uniform magnetic feild B in such a manner that the angle between $\vec{A}$ and $\vec{B}$ be θ then the torque experienced by current carrying loop is given by

$$\vec{\tau} = NI\ (\vec{A} \times \vec{B})$$

or $\vec{\tau} = NI\vec{A} \times \vec{B}$ ---(i)

If the equivalent MDM of the current carrying loop be $\vec{M}$ then the angle between $\vec{M}$ and $\vec{B}$ will also be θ.



∴ The torque experienced by equivalent MDM of current carrying loop when placed in UMF is given by

$\vec{\tau} = \vec{M} \times \vec{B}$ --(ii)

Comparing rel'n (i) and (ii)

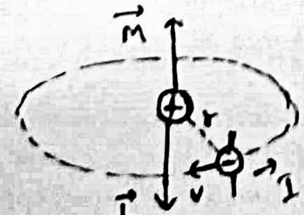
$$\vec{M} = NI\vec{A}$$

This is the reqd. expression of MDM of current carrying loop.

Magnetic dipole moment of a revolving elelctron (MDM of an atom)

Ques.- Derive an expression for the magnetic dipole moment of a revolving elelctron revolving around a nucleus. Define Bohr magneton and find its value?

Ans.-



Consider an electron of mass m_e and charge e which is revolving in a circular orbit of radius r around a +ve nucleus in clockwise direction. Then the angular momentum of electron due to its orbital motion is given by

$L = m_e vr$ ---(i)

If the time period of revolution of electron be T then the current equivalent to the motion of electron

$$I = \frac{q}{t} = \frac{e}{T}$$

But $T = \frac{2\pi r}{v}$

or $I = \frac{ev}{2\pi r}$

MDM of atom, M = IA

or $M = \frac{ev}{2\pi r} \times \pi r^2$

or $\boxed{M = \frac{evr}{2}}$ ---(ii)

From rel'n (i)

or $r = \frac{L}{m_e v}$

Substituting the value of r in rel'n (ii)

$$M = \frac{ev}{2} \cdot \frac{L}{m_e v}$$

or $\boxed{M = \frac{eL}{2m_e}}$ --(iii)

As the $\vec{M}$ and $\vec{L}$ are oppositely directed therefore in vector form

or $\vec{M} = \frac{-e\vec{L}}{2m_e}$

By according to Bohr's theory, the angular momentum of an electron in a stationary orbit is an integral multiple of $\frac{h}{2\pi}$

i.e. $L = \frac{nh}{2\pi}$ where n = 1, 2, 3,.....

Substituting the value of L in rel'n (iii)

$$M = \frac{e}{2m_e} \cdot \frac{nh}{2\pi}$$ [n is principle Quantum Number]

$\boxed{M = \frac{nhe}{4\pi m_e}}$ ---(iv)

The above rel'n gives magnetic moment of an electron due to its orbital motion.

Bohr magneton- The MDM of a hydrogen atom due to orbital motion of an electron in its first orbit is called bohr magneton. It is denoted by μ_B.

In rel'n (iv) if n = 1

then $M = \mu_B$,

∴ $\mu_B = \frac{1.he}{4\pi m_e}$

∵ $h = 6.63 \times 10^{-34}$ Js, $e = 1.6 \times 10^{-19}$ C,

and $m_e = 9.1 \times 10^{-31}$ kg

∴ $\mu_B = \frac{6.63 \times 10^{-34} \times 1.6 \times 10^{-19}}{4\pi \times 9.1 \times 10^{-31}}$

or $\mu_B = 9.27 \times 10^{-24}$ Am^2

Neutral points

Ques.-Define neutral points.

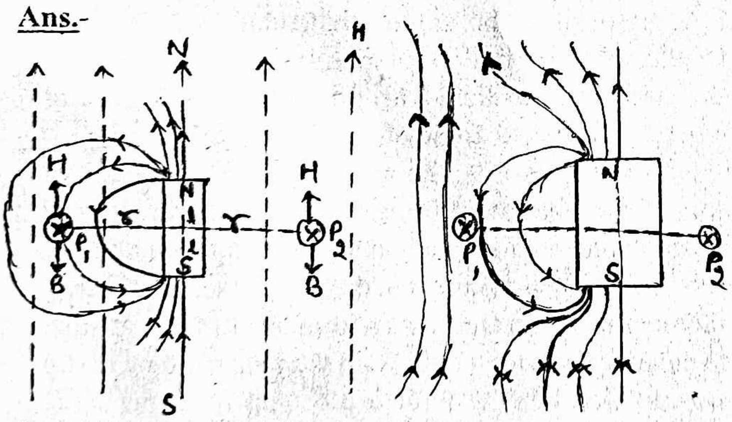
Ans.- While drawing MFL of a magnet in the horizontal plane (with the help of magnetic needle) certain points are obtained in the horizontal plane at which the magnetic field intensity B of a magnet is equal and opposite to the horizontal component of earth's magnetic field H. At these points resultant magnetic field intensity becomes zero. When a magnetic needle is placed at such points then it can orient in any direction. These points at which the resultant MFI is zero are called neutral points.

Note- As in the horizontal plane only the horizontal component of earth's mangnetic field is effective. Therefore while doing experiments in horizontal plane only the horizontal component of earth's magnetic field should be taken into account which remains constant in a small region, i.e. why it is represented by equally spaced parallel MFL which run from South to North.

MFL when N-pole of magnet is directed towards north

Ques.- Sketch MFL when when the magnet is placed with its north pole towards north pole of earth.How will you find out the magnetic moment of a bar magnet by locating its neutral points?

Ans.-



Consider a magnet NS whose MDM be M and length be 2ℓ which is placed in horizontal plane in such a manner that its N pole is directed towards north.

In this situation when MLF are drawn with the help of a magnetic needle then two neutral points are obtained on the equitorial line of the magnet at which the magnetic needle does not acquire any preferred direction. These points are called neutral points and at these points the magnetic field of magnet is equal and opposite to the horizontal component of earth's magnetic field.

If the distance of neutral points from the centre of the magnet on the equitorial line be r then at neutral points

$$B = H$$

But the MFI due to magnet at neutral points

$$B = \frac{\mu_0}{4\pi}\frac{M}{(r^2+\ell^2)^{3/2}}$$

$$\therefore \quad \frac{\mu_0}{4\pi}\frac{M}{(r^2+\ell^2)^{3/2}} = H$$

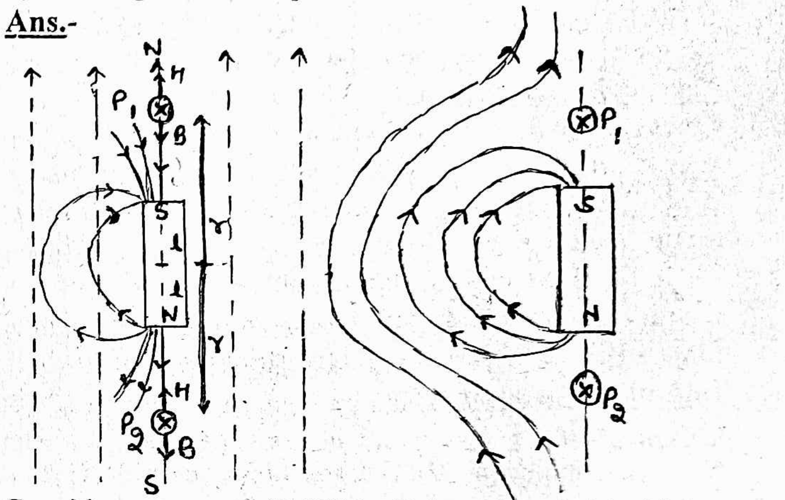
or $$M = \frac{4\pi H\ (r^2+\ell^2)^{3/2}}{\mu_0}$$

Now by substituting the value of r, ℓ and H in the above rel'n the MDM of the magnet can be find out experimentally.

MFL when N-pole of magnet is directed towards south

Ques.- Sketch MFL when when the magnet is placed with its north pole towards south pole of earth.How will you find out the magnetic moment of a bar magnet by locating its neutral points?

Ans.-



Consider a magnet of MDM of M and length 2ℓ which is placed in horizontal plane in such a manner that its N-pole is directed towards south.

In this situation when MFL are drawn with the help of a magnetic needle then two neutral points are obtained on the axial line of the magnet at which the compass needle does not acquire any preferred direction. These points are called neutral points and at these points the magnetic field of magnet will be equal and opposite to the horizontal component of earth's magnetic field.

If the distance of the neutral points from the centre of magnet be r then at neutral points

$$B = H$$

But the MFI due to magnet at neutral points

$$B = \frac{\mu_0}{4\pi}\frac{2Mr}{(r^2-\ell^2)^2}$$

$$\therefore \quad \frac{\mu_0}{4\pi}\frac{2Md}{(r^2-\ell^2)^2} = H$$

or $$M = \frac{4\pi H\ (r^2-\ell^2)^2}{\mu_0 2r}$$

By substituting the values of r, ℓ and H in the above relation the MDM of a magnet can be find out experimentally.

Note- Between the magnet and the neutral points the magnetic field of magnet is dominant. Therefore the direction of magnetic field is same as that of magnet but beyond the neutral points the earth's magnetic field is dominant therefore the direction of magnetic field is same as that of earth.

Gauss law in magnetism

Ques.- State Gauss law in magnetism. What are its consequences?

Ans.- It states that the surface integeral of magnetic field over any closed surface is always equal to zero.

i.e. $\oint_S \vec{B}.\vec{ds} = 0$

Consequences- 1. When a certain number of MFL leave a surface then an equal number of MFL also enter into the closed surface.

2. Magnetic poles always exists in unlike pairs of equal strength i.e. magnetic monopoles do not exist.

Terrestrial magnetism

Ques.- What do mean by terrestrial magnetism?

Ans.- Earth behaves like a magnet whose poles lie slightly inside its surface. Magnetic N-pole lies in Antarctica and magnetic S-pole lies in Canada.

Thus the earth behaves like a magnet whose length is slightly less than the diameter of earth. The magnetism of earth is called terrestrial magnetism.

Definitions of some terms

Ques.- Define-

(1). Geographic axis (2). Geographic equator (3). Geographic meridian (4). Magnetic axis (5). Magnetic equator (6). Magnetic meridian

Ans.- 1. Geographic axis-The axis about which the earth rotates is called its geographic axis. It joins geographic N and geographic S-pole of earth.

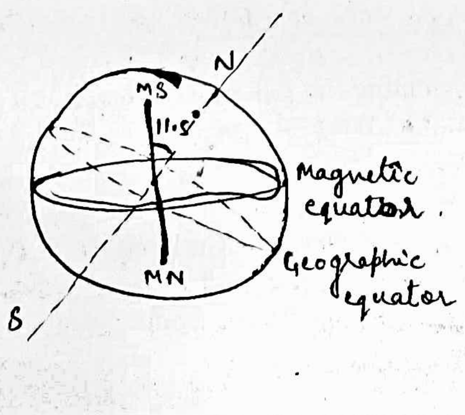
2. Geographic equator- The largest circle on the surface of earth whose plane is perpendicular to geographic axis is called geographic equator.

3. Geographic meridian- The vertical plane at any point on the surface of earth which passes through the geographic axis is called geographic meridian at that point.

4. Magnetic axis- The line joining the magnetic poles of earth is called magnetic axis.

5. Magnetic equator- The largest circle on the surface of earth whose plane is perpendicular to magnetic axis is called magnetic equator.

6. Magnetic meridian- The vertical plane at any point on the surface of earth which passes through the magnetic axis is called magnetic meridian at that point.



Note- 1. Magnetic N is located at 70.5°S, 84°E, (latitude and longitude) and magnetic S is located at 70.5°N and 96°W.

2. The angle between geographic axis and magnetic axis is approx. 11.5°.

Elements of earth's magnetic field

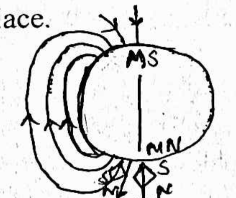
Ques.- What are elements of earth's magnetic field?

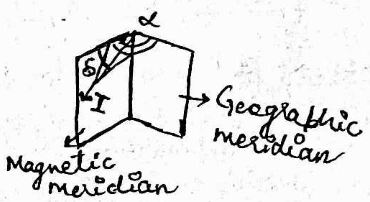
OR

What do mean by (1) Angle of declination
(2). Angle of dip
(3). Earth's horizontal magnetic field intensity

Ans.- 1. Angle of declination- The angle between the magnetic meridian and the geographic meridian at any place on the surface of earth is called angle of declination at that place. It is denoted by α.

Its value varies from place to place from 0 to 11.5° on the surface of earth. With the help of angle of declination we can locate magnetic meridian at any place if the position of geographic meridian is known at that place.





2. Angle of dip- The angle substended by a magnetic needle, which is free to rotate about horizontal axis in the magnetic meridian, with the horizontal direction at any place is called angle of dipole of that place.

In other words, the angle substended by the resultant magnetic field of earth with the horizontal direction at any place is called angle of dip at that place.

Its value is different at different places on the surface of earth. Its value varies from 90° to 0° from pole to equator and vice-versa. With the help of angle of dip we can find out the direction of resultant magnetic field of earth at any place. It is denoted by δ.

Note-(a) In southern hemisphere the S- pole of magnetic needle is directed towards earth and in northern hemisphere the N-pole of magnetic needle is directed towards earth.

(b) The needle which is free to rotate about horizontal axis in magnetic meridian (vertical N-S plane) is also called dip needle.

(c) The needle which is free to rotate about a vertical axis in the horizontal plane is called compass needle.

3. Earth's horizontal magnetic field intensity-The resultant magnetic field intensity of earth at any place on its surface can be resolved into two components-

Horizontal component which is called earth's horizontal magnetic field intensity.

2. Vertical component which is called earth's vertical magnetic field intensity.

If the value of earth's horizontal magnetic field intensity is known at any place on the surface of earth then with its help the magnitude of earth's resultant magnetic field intensity can be calculated at that place.

Note- (a). The value of earth's resultant magnetic field intensity at the poles is approx 0.66G and at the equator is 0.33 G.

Thus the average value of earth's resultant magnetic field intensity is approx is 0.495 G {0.5G}

(b). By knowing the value of angle of diclination, angle of dip and the earth's horizontal magnetic field intensity at any place on the surface of earth, the magnitude and the direction of earth's resultant magnetic field can be find out at that place. These are called the elements of earth's magnetic field.

Relation between H, V, I and δ

Ques.- Establish relation between H, V, I and δ?

Ans.-Let the plane OABC represents the magnetic meridian

In rectangle Δ OAB

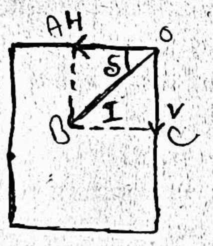
[H-horizontal component of Earth magnetic field]

$$\cos\delta = \frac{OA}{OB}$$

or $$\cos\delta = \frac{H}{I}$$

or $$\boxed{H = I\cos\delta} \quad \text{---(i)}$$

and $$\sin\delta = \frac{AB}{OB}$$



or $$\sin\delta = \frac{OC}{OB}$$

or $$\sin\delta = \frac{V}{I}$$

or $$V = I\sin\delta \quad \text{---(ii)}$$

Squaring both the sides of rel'n (i) and (ii) and then adding them we get

$$H^2 + V^2 = (I\sin\delta)^2 + (I\cos\delta)^2$$

or $$= I^2\sin^2\delta + I^2\cos^2\delta$$

or $$= I^2(\sin^2\delta + \cos^2\delta)$$

or $$= I^2(1)$$

or $$= I^2$$

or $$\boxed{I = \sqrt{H^2 + V^2}} \quad \text{---(iii)}$$

Dividing rel'n (ii) by (i)

$$\frac{V}{H} = \frac{I\sin\delta}{I\cos\delta}$$

or $$\frac{V}{H} = \tan\delta$$

or $$\boxed{\tan\delta = \frac{V}{H}} \quad \text{--(iv)}$$

Particular cases-

1. At magnetic equator δ = 0°

$$\therefore \quad H = I\cos 0° = I$$

and $$V = I\sin 0° = 0$$

Thus at magnetic equator the earth's resultant magnetic field is horizontal.

2. At magnetic poles δ = 90°

$$\therefore \quad H = I\cos 90° = 0$$

and $$V = I\sin 90° = I$$

Thus, at magnetic poles the earth's resultant magnetic field is vertical.

Causes of earth's magnetism

Ques.- What are the causes of earth's magnetism ?

Ans.-1. The outer core of earth is in molten state due to high temperature. Due to the radiations emitted by radioactive substances present in earth the molten mass gets ionized. Due to the rotation of earth about its axis relative motion is developed between the molten mass and the earth as a result of which the ions present in molten mass constitutes electric currents which are responsible for earth's magnetic field.

2. Due to the solar wind and ultraviolet rays released by sun and cosmic rays coming from outer space the outer most layer of earth's atmosphere gets ionized which is called ionosphere. Due to the rotation of earth about its axis a relative motion is developed between the ionosphere and the earth. As a result of which the charged particles present in ionosphere constitutes electric currents which are responsible for earth's magnetism.

3. As the sun rays fall normally on equator and obliquely at the poles. The air near equator becomes hot and light which rises upwards and move towards poles. Its position is occupied by cold and heavy air which move from poles to equator. As the charged particles are present in earth's atmosphere due to cosmic rays and ultraviolet rays these charged particles move alongwith the air and constitutes electric currents which are responsible for earth's magnetism.

O-Green
N-Pink

Note-The solar wind consists of charged particles that emerge continuously from sun. The charged particles of solar wind get trapped near the magnetic poles of earth. They ionize the atmosphere above these poles which causes specticular display of light in the shape giant curtains in the atmosphere In the arctic region it is called Aurora borialis or Northren lights and in the Antarctica region it is called Aurora australis or Southern lights.

As India is located far away from poles therefore these lights are not visible in India.

Relation between real and apparent angle fo dip

Ques.- What is real and apparent angle of dip? Derive relation between them.

Ans.-Real angle of dip δ- When a magnetic needle is suspended in magnetic meridian then the angle substended by it with the horigontal direction is called real angle of dip.

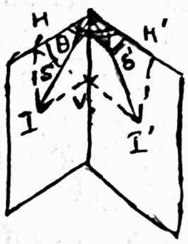
Apparant angle of dip δ'- When a magnetic needle is suspended in any plane other than the magnetic meridian then the angle substended by it with the horizontal direction is called apparent angle of dip.

For the real angle of dip

$$\tan\delta = \frac{V}{H}$$

and for apparent angle of dip

$$\tan\delta' = \frac{V}{H'}$$



or $\tan\delta' = \dfrac{V}{H\cos\theta}$

or $\boxed{\tan\delta' = \dfrac{\tan\delta}{\cos\theta}}$

Here $\cos\theta < 1$

$\therefore$ $\tan\delta' > \tan\delta$

or $\delta' > \delta$

Particular cases- When dipole needle is suspended in a plane which is $\perp$ to magnetic meridian i.e. or $\theta = 90°$ then

or $\tan\delta' = \dfrac{\tan\delta}{\cos 90°}$

or $\tan\delta' = \dfrac{\tan\delta}{0}$

or $\tan\delta' = \tan 90°$

$\therefore$ $\delta' = 90°$

Thus when a dipole needle is suspended in a plane which is $\perp$ to magnetic meridian the dipole needle becomes vertical.

Classification of magnetic materials-

Relation between real and apparent angle fo dip

Ques.- Define (i) Magnetic intensity (ii) Intensity of magnetisation (iii) Magnetic induction or magnetic flux denity (iv)Absolute magnetic permeability (v)Relative magnetic permeability (vi)Magnetic susceptibility

Ans.(i) **Magnetic intensity**- Consider a toroid having n turns per unit length carrying current I.

Then magnetic induction in the core of toroid

$$B_0 = \mu_0 nI$$

The product nI is called magnetic intensity (H)

$\therefore$ $B_0 = \mu_0 H$

or $H = \dfrac{B_0}{\mu_0} = nI$

It is defined as the number of ampere turns flowing per unit length of a toroid which produces a magnetic induction B_0 in its core.

Its S.I unit is Am^{-1} and dimensional formula is $[AL^{-1}]$

Note- The older C.G.S unit of magnetic intensity is oersted.

In rel'n $H = \dfrac{B_0}{\mu_0}$

If $H = 1$ oers and $B_0 = 1G$

then 1 oers $= \dfrac{1G}{\mu_0}$

$$= \frac{10^{-4}T}{4\pi\times 10^{-7}TmA^{-1}}$$

$$\simeq 80\ Am^{-1}$$

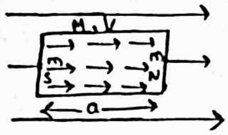
(ii) **Intensity of magnetisation (I)**- When a substances is placed in external magentic field then the MDM acquired by a unit volume of it is called intensity of magnetisation.

If the volume of substances be V and the MDM acquired by it be M. Then, the intensity of magnetisation is given by

$$I = \frac{M}{V}$$

If the area of cross-section of the substance be A, its length be a and pole strength acquired by it be m

then $I = \dfrac{ma}{Aa}$



or $I = \dfrac{m}{A}$

Thus, the intensity of magnetisation may also be defined as the ratio of pole strength acquired by a substance when it is placed in external magnetic field and its area of cross-section.

Its S.I. unit is Am^{-1} and dimentional formula is $[AL^{-1}]$

(iii) **Magnetic induction or magnetic flux denity**- It is defined as the number of magnetic field lines which crosses normally a unit area of the substance when placed in external magnetic field.

Magnetic induction is propotional to the sum of magnetic intensity and intensity of magnenzation.

i.e. $B \propto (H + I)$

$\therefore$ $B = \mu_0 (H + I)$

where μ_0 is constant of propotionality and is called absolute permeability of free space.

Its S.I unit is $Nm^{-1}A^{-1}$ or T.

(iv) **Absolute magnetic permeability (μ)**- Magnetic permeability of a substance may be defined as the ratio of matnetic induction inside it when placed in external magnetic field and the magnetic intensity. If on placing a magnetic substance in external magnetic field whose magnetic intensity be H. the magnetic induction inside the substance be B then magnetic permeability

$$\mu = \frac{B}{H}$$

Its S.I unit is TmA^{-1} or NA^{-2} and its dimentional formula is

$[MLT^{-2}A^{-2}]$.

(v) **Relative magnetic permeability**- It is defined as the ratio of absolute magnetic permeability of a substance and the absolute magnetic permeability of free space (vacuum)

$$\mu_r = \frac{\mu}{\mu_0}$$

It is a unitless and dimensionless physical quantity.

Note- It measures the ability of a substance to permit the passage of magnetic field lines through it.

(vi) **Magnetic susceptibility (χ_m)**- It is defined as the ratio of intensity of magnetisation in a substances when placed in external magnetic field and the magnetic intensity.

If on placing a substance in external magnetic field of magnetic intensity H, the intensity of magnetisation in the substance be I then magnetic susceptibility

$$\chi_m = \frac{I}{H}$$

It is a unitless and dimensionless physical quantity.

Note- It measures the ability of a substances to acquire magnetism.

Relation between relative magnetic permeability and magnetic susceptibility

Ques.- Derive relation between relative magnetic permeability and magnetic susceptibility.

Ans.- ∵ Magnetic induction

$$B = \mu_0 (H + I)$$

Dividing both the sides of above rel'n by H

$$\frac{B}{H} = \frac{\mu_0 (H + I)}{H}$$

or $$\frac{B}{H} = \mu_0\left(1 + \frac{I}{H}\right)$$

or $$\mu = \mu_0 (1 + \chi_m)$$

or $$\frac{\mu}{\mu_0} = 1 + \chi_m$$

or $$\boxed{\mu_r = 1 + \chi_m}$$

This is the reqd. rel'n.

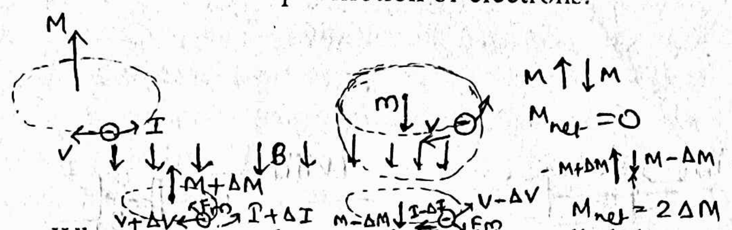
Diamagnetic substances

Ques.- What are diamagnetic substances? Explain the origin of diamagnetism on the basis of electron theory.

Ans.- Those substances which when placed in a magnetic field are feebly magnetised is the opposite direction of magnetic field are called diamagnetic substances. Generally diamagnetic substances are those in which valence electrons are paired.

e.g.-copper, zinc, bismuth, silver, gold, glass, water, sodium chlroride etc.

Diamagnetism on the basis of electrons theory (cause of diamagnetism)- In an atom electrons revolve in circular orbits and they behave like current carrying loops having magnetic dipole moments. In diamagnetic substance the valence electrons are paired, further the two electrons of a pair possess orbital motion in opposite direction. Thus, the two electrons of a pair possess magnetic moments equal in magnitude and opposite in direction and hence the magnetic moment of a pair of electrons due to their orbital motion is zero. For similar reasons, the diamagnetic atoms do not possess magnetic moment due to spin motion of electrons.



When an external magnetic field is applied then lorentz magnetic force begin to act on electrons. In case of an electron revolving in clockwise direction, the lorentz magnetic force acts radially inwards and its tends to increase the centripetal force on the electron due is which the velocity of the electron increses and hence the magnetic dipole moment of the electron increases to M + ΔM. In the case of an electron revolving in anticlockwise direction. The lorentz magnetic force acts radially outwards and its tends to decrease the centripetal force on the electron due to which the velocity of the electron and hence the magnetic dipole moment of the electron decreases to M - ΔM. Thus on applying magnetic field the pair of electrons possess a net magnetic dipole 2ΔM in a direction opposite to the direction of applied magnetic field. Thus, the diamagnetic substances gets slightly magnitued in the direction opposite to the applied magnetic field.

Properties of diamagnetic substances

Ques.- Describe some important properties of diamagnetic substances.

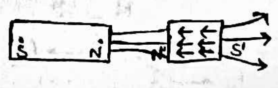
Ans.-Properties of diamagnetic substances-

(i) A diamagnetic substance is feebly repelled by a magnet.
Weakly

(ii) The tendency of diamagnetic substance is it move from the region of ~~weak~~ strong magnetic field to the region of weak ~~strong~~ magnetic field.

(iii) When a diamagnetic liquid contained in a watch glass is placed on two closely spaced poles of a magnet, it gets slightly depressed in the middle. However when the poles are moved apart the liquid gets slightly raised in the middle.

(ii)



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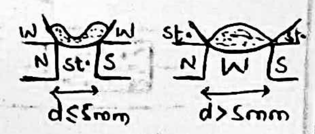

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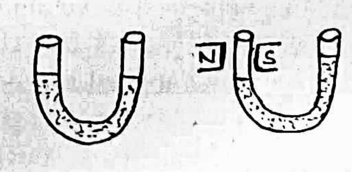
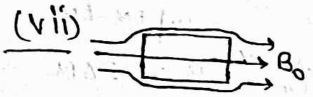


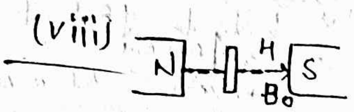
(iv) When one limb of a U-tube containing diamagnetic liquid is placed between the poles of a magnet then the diamagnetic liquid in that arm gets slightly depressed and in the other arm the diamagnetic liquid gets slightly raised.

(v) The relative magnetic permeability of a diamagnetic substances is less than 1 but positive.

(vi) The magnetic susceptibility of a diamagnetic substance is less than 1 but negative.

(vii) The tendency of magnetic field lines is to move away from diamagnetic substances.





(viii) When a rod of diamagnetic substance is suspended inside a magnetic field, it gets oriented is the perpendicular direction of magnetic field.

(ix) On the removal of applied magnetic field the diamagnetic property disappears.

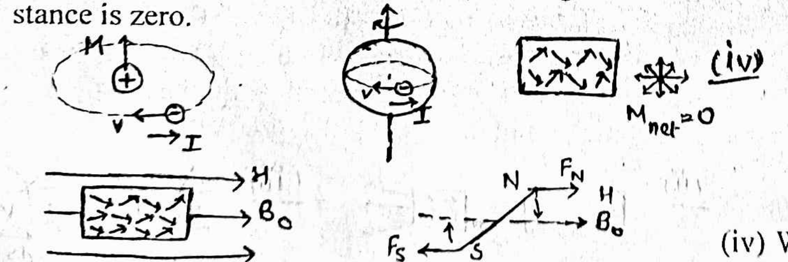
(x) The magnetic susceptibility of a diamagnetic substnace is independent of its temperature i.e. the diamagnetic substances do not obey Curie's law.

Paramagnetic substances

Ques.- What are paramagnetic substances? Explain the origin of paramagnetism on the basis of electron theory.

Ans.- Those substances which when placed in a magnetic field are feebly magnetised in the direction of magnetic field are called paramagnetic substances. Generally paramagnetic substances are those in which valence electrons are unpaired. e.g. alumunium, antimony, platinum, magnesium, sodium, copper chloride etc.

Paramagnetism on the basis of electron theory (cause of paramagnetism)- Every atom of a paramagnmetic substances possess magnetic dipole moment due to orbital motion and spin motion of valence unpaired electron. The interaction between the atomic magnets of a paramagnetic substance is weak therefore they are independent from one another. Due to thermal agitation these atomic magnets are randomly oriented so that the net magnetic dipole moment of the paramagnetic substance is zero.



When external magnetic field is applied the tendency of field is to align atomic magnets in its direction but the thermal agitation obstructs the alignment. But when the temperature is lowered and the applied magnetic field is increased, more and more atomic magnets align themselves in the direction of magnetic field. Thus the paramagnetic substances gets feebly magnetised in the direction of applied magnetic field.

Note- For paramagnetic substances, the intensity of magnetisation is

(i) directly proportional to the magnetic intensity i.e. I $\propto$ H

(ii) Inversely proportional to the absolute temperature of the substance i.e.

$$I \propto \frac{1}{T}$$

Combining the above two factors

$$I \propto \frac{H}{T}$$

or $$I = C\frac{H}{T}$$

where C is a constant of proportionality and is called Curie's constant.

$$\frac{I}{H} = \frac{C}{T}$$

or $$\chi_m = \frac{C}{T}$$

Thus, the magnetic susceptibility of a paramagnetic substance is inversely proportional to its absolute temperature. It is called Curie's law in magnetism.

Properties of paramagnetic substances

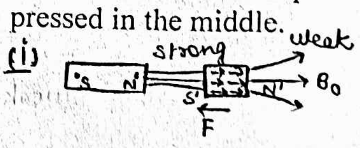
Ques.- Describe some important properties of paramagnetic substances.

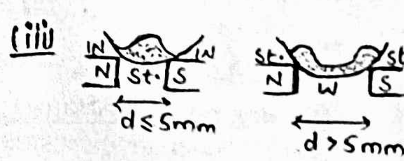
Ans.-Properties of paramagnetic substances-

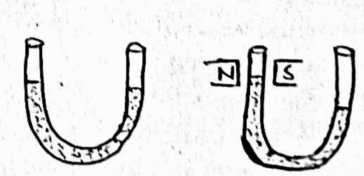
(i) A paramagnetic substances is feebly attracted by a magnet.

(ii) The tendency of paramagnetic substance is to move from the region of weak magnetic field to the region of strong magnetic field.

(iii) When a paramagnetic liquid contained in a watch glass is placed on two closely spaced poles of a magnet, it gets slightly raised in the middle. However when the poles are moved apart the liquid gets slightly depressed in the middle.







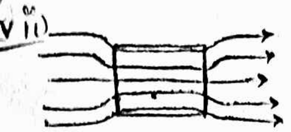
(iv) When one limb of a U tube containing paramagnetic liquid is placed between the poles of a magnet then the paramagnetic liquid in that arm gets slightly raised and in the other arm the paramagnetic liquid gets slightly depressed.

(v) The relative magnetic permeability of a paramagnetic substance is slightly greater than 1 but positive.

(vi) The magnetic susceptibility of a paramagnetic substance is small but positive.

(vii) The tendency of magnetic field lines is to pass through paramagnetic substances.

(vii)



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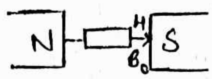


(viii) When a rod of paramagnetic substances is subspended inside a magnetic field it gets oriented parallel to the direction of magnetic field.

(ix) On the removal of applied magnetic field the paramagnetic property disappears.

(x) The magnetic susceptibility of a paramagnetic substances is inversely proportional to its temperature

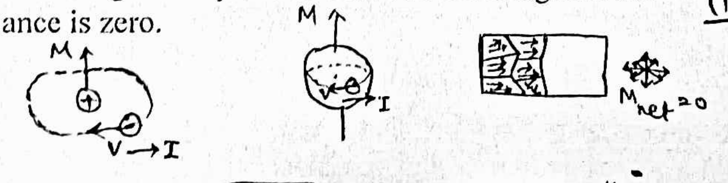
i.e. $\chi_m \propto \frac{1}{T}$

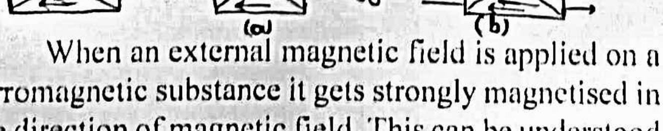
Ferromagnetic substances

Ques.- What are ferromagnetic substances? Explain the origin of ferromagnetism on the basis of electron theory.

Ans.-Those substances which when placed in a magnetic field are strongly magnetised in the direction of magnetic field are called ferromagnetic substance. Generally ferromagnetic substance are those in which valance electrons are unpaired and electrons of neighbouring atoms interact with one another e.g. iron, nickel cobalt etc.

Ferromagnetion on the basis of electron theory (cause of ferromagnetism)- Every atom of ferromagnetic substance possess magnetic dipole moment due to orbital motion and spin motion of valence unpaired electron. That is, every atom of ferromagnetic substance behaves like an atomic magnet. However in ferromagnetic substances atoms form a large number of small regions called domains. Each domain contains $\simeq 10^{11}$ atoms and $\simeq 10^{-18} m^3$ in volume. Due to quantum mechanical interaction between the unpaired electron of one atom with the unpaired electrons of neighbouring atoms, magnetic dipole moment of all the atoms align themselves in the same direction with in each domain. Thus, each domain behaves like a strong magnet. In the absence of magnetic field the magnetic moments of different domains are randomly oriented to that the resultant magnetic dipole moment of a ferromagnetic substance is zero.





When an external magnetic field is applied on a ferromagnetic substance it gets strongly magnetised in the direction of magnetic field. This can be understood in turns of-

(a) displacement of boundaries of domains i.e. domains which are oriented in the direction of magnetic field

increases in size and the domains which are oriented opposite to the magnetic field decreases in size. (fig a).

(b) rotation of domain i.e. the domains rotate till their magnetic dipole moments are aligned in the direction of applied magnetic field. This happens when the applied magnetic field is strong (fig. b)

When a ferromagnetic substance is heated then at a certain temperature called curie temperature the quantum mechanical interaction diappears and the ferromagnetic substance begin to behave like paramagnetic substance. Curie temperature for iron is 1043K, for nickel is 631K and for cobalt is 1394K.

Properties of ferromagnetic substances

Ques.- Describe some important properties of ferromagnetic substances.

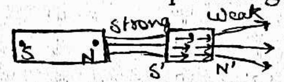
Ans.-Properties of ferromagnetic substances-

(i) A ferromagnetic substance is strongly attracted by a magnet.

(ii) The tendency of ferromagnetic substance is to move from the region of weak magnetic field to the region of strong magnetic field.

(iii) When a ferromagnetic substance in powdered form is taken in a watch glass is placed on two poles of a magnet it gets raised in the middle. However when the poles are moved aparts it gets depressed in the middle.

(ii)



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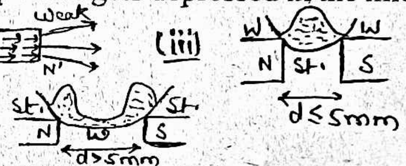


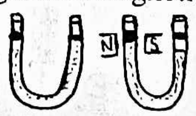
(iv) When one limb of a U tube containing ferromagnetic substance in the powdered form is placed between the poles of a magnet then the ferromagnetic substance in that arm gets raised and in the other arm it gets depressed.

(v) The relative magnetic permeability of a ferromagetic substance is much greater than 1 but positive.

(vi) The magnetic susceptibulity of a ferromagnetic substances is large but positive.

(vii) The tendency of magnetic field lines is to pass through ferromagnetic substance.

(iv)



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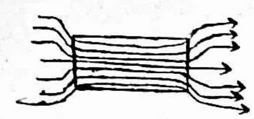

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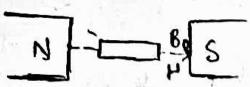


(viii) When a rod of ferromagnetic substance is suspended inside a magnetic field it gets oriented parallel to the direction of magnetic field.

(ix) On the removal of applied magnetic field the ferromagnetic property does not disappear.

(x) The magnetic susceptibility of a ferromagnetic substance decreases with the increase in temperature. But they do not obey Curie's law.

Note-(i) Ferromagnetic substances obey Curie's law above curie temperature.
(ii) Ferromagnetic substances obey Curie-Weiss law according to which

$$\chi_m \propto \frac{1}{T - T_c}$$

where T_c is curie's temperature.
The above law is valid only above curie's temperature.

Hysteresis

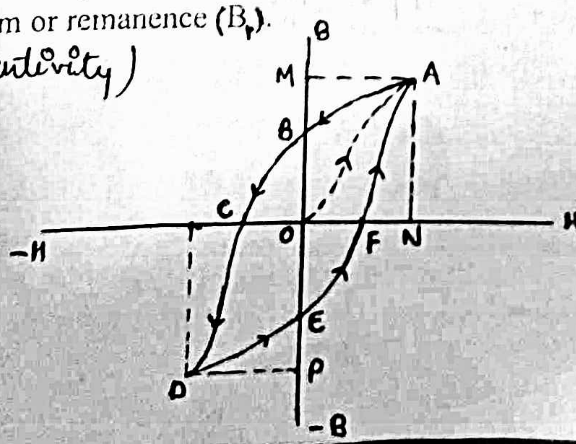
Ques.- Explain the phenomenon of hysteresis in magnetic materials. What is the significance of area of hysteresis loop?

Ans.- Consider a an unmagnetised steel ring on which a wire is wounded. Arrangemets are made to send the current in the wire which both varies in magnitude as well as direction.

When the current is increased from zero to maximum in one direction, the magneting field H also increases from zero to maximum in one direction and the corresponding values of magnetic induction B are determined. Now the current is decreased from maximum to zero, the magnetising field H also decreases from maximum to zero and the corresponding values of magnetic induction B are determined.

Next the current is increased from zero to maximum in the other direction the magnetising field H also increases from zero to maximum in the other direction and the corresponding values of magnetic induction B are determined. Again the current is decreased from maximum to zero. The magnetising field H also decreases from maximum to zero and the corresponding values of magnetic induction B are determined.

In this manner a cycle of magnetisation is completed. Using the values of H and B when a graph is plotted then the curve obtained is shown in fig. It is clear from the graph that when the magnetising field H is increased from zero to maximum ON, the induction B follows the curve OA. But when the magnitising field is brought to zero. The induction B does not follow back the curve OA but follows the curve AB. The same variation is observed throughout the curve. This lagging of magnetic induction B behind the magnetising field H is called hystersis. When the magnetising field H is reduced to zero some magnetic induction (OB or OE) is still present in the ring, it is called residual magnetism or remanence (B_r).
(retentivity)



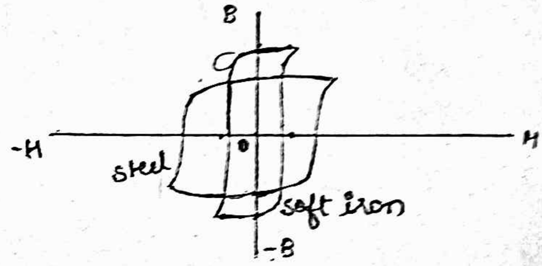
After the ring has been magnetised to saturation (OM or OP), a reverse magnetising field (OC or OF) is required to reduce the magnetic induction to zero. It is called coercivity or coercive force (H_c).

When a ferromagnetic substance is taken through a cycle of magnetisation then the heat produced in a unit volume of material is numerically equal to the area enclosed by the hystersis loop.

Comparision of hystersis loop for soft iron and steel-

Ques.- Compare hystersis loop for soft iron and steel.

Ans.-(i) The area enclosed by the hysteresis loop for iron is less than the area enclosed by the hyteresis loop for steel. Therefore the heat produced in a unit volume of iron will be less than the heat produced in a unit volume of steel over a cycle of magnetisation.



(ii) The magnetic saturation limit of soft iron is greater than that of steel which it acquires for the smaller value of magnetising field as compared to steel.
(iii) The retentivity of soft iron is greater than that of steel i.e on reducing the magnetising field to zero the residual magnetism in soft iron is greater that in steel.
(iv) The coercivity of steel is greater than that of soft iron i.e. the magnetising field required to destroy residual magnetism in steel is much greater that in soft iron.

Applications of ferromagnetic substances

Ques.- Write some applications of ferromagnetic substances.

Ans.-(i) Permanent magnets- The materials used for making permanent magnets should possess high permeability, high retentivity and coercivity. Steel possess high retentivity and high coercivity therefore the magnetism of steel can not be easily detroyed even if it is exposed to stray magnetic fields or handled roughly. Cobalt steel, carbon steel, alnico and ticonal are used for making parmanent magnets.
(ii) Electromagnets- The materials used for making electromagnets should possess high permeability, low retentivity and low coercivity. Since such a substance has to undergo magnetisation and demagnetsation again and again therefore the area of the hysteresis loop of the material used for making electromagnet should be small. All these requirements are fulfilled by soft iron therefore it is used for making electromagnets.

The electromagnets are used in electric bells, telephones, loudspeakers etc.Giant electromagnets are used in cranes to lift machinary and bulks of iron scrap.