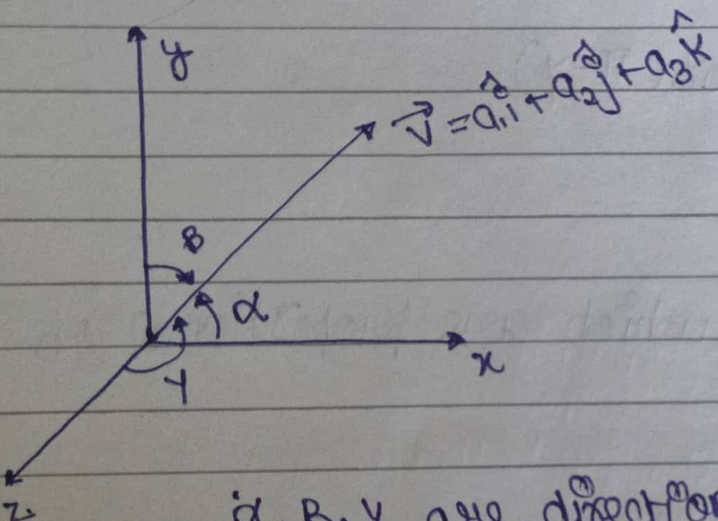
# 3-D geometry

## # Direction angle and direction cosine of a vector :




α, β, γ are direction angles of vector

cos α, cos β, cos γ are direction cosines of the vector

$\cos\alpha = \dfrac{\vec{V}\cdot\hat{i}}{|\vec{V}||\hat{i}|}$ $\cos\beta = \dfrac{\vec{V}\cdot\hat{j}}{|\vec{V}||\hat{j}|}$ $\cos\gamma = \dfrac{\vec{V}\cdot\hat{k}}{|\vec{V}||\hat{k}|}$

$\cos\alpha = \dfrac{a_1}{|\vec{V}|}$ $\cos\beta = \dfrac{a_2}{|\vec{V}|}$ $\cos\gamma = \dfrac{a_3}{|\vec{V}|}$

$$\boxed{\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1}$$ ★

$$\boxed{\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2}$$

## Direction Ratio and direction cosine of a line :

$\cos\alpha$ , $\cos\beta$ , $\cos\gamma$

$(l$ , $m$ , $n)$

$\cos(\pi-\alpha)$ , $\cos(\pi-\beta)$ , $\cos(\pi-\gamma)$

$(-l$ , $-m$ , $-n)$

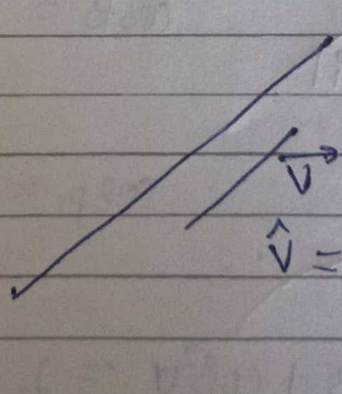
Three number $a, b, c$ which are proportional to $l, m, n$

$\left(\frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right)$

$$\left[\frac{a}{l} = \frac{b}{m} = \frac{c}{n} = \lambda\right]$$

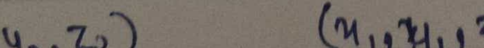
$$l^2 + m^2 + n^2 = 1$$

$$\Rightarrow \quad a^2 + b^2 + c^2 = \lambda^2$$



$\vec{v} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\hat{v} = l\hat{i} + m\hat{j} + n\hat{k}$




$$\vec{v} = (x_1 - x_2)\hat{i} + (y_1 - y_2)\hat{j} + (z_1 - z_2)\hat{k}$$

Q $L_1 : (l_1, m_1, n_1)$ , $(a_1, b_1, c_1)$

$L_2 : (l_2, m_2, n_2)$ , $(a_2, b_2, c_2)$

$$\cos\theta = l_1 l_2 + m_1 m_2 + n_1 n_2$$

$$\cos\theta = \frac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\left(\sqrt{a_1^2 + b_1^2 + c_1^2}\right)\left(\sqrt{a_2^2 + b_2^2 + c_2^2}\right)}$$

i) If $l_1 \perp l_2$

$$l_1 l_2 + m_1 m_2 + n_2 n_1 = 0$$

$$a_1 a_2 + b_1 b_2 + c_1 c_2 = 0$$

ii) If $l_1 \parallel l_2$

it can be $\pm 1$

$$\frac{l_1}{l_2} = \frac{m_1}{m_2} = \frac{n_1}{n_2}$$

Q1) Find the direction cosines of a line perpendicular to two lines whose dr's (direction ratio) are 1, 2, 3 & -2, 1, 4.

Solve) $L_1 = \hat{i} + 2\hat{j} + 3\hat{k}$

$L_2 = -2\hat{i} + \hat{j} + 4\hat{k}$

$$L_1 \times L_2 = \begin{vmatrix} i & j & k \\ 1 & 2 & 3 \\ -2 & 1 & 4 \end{vmatrix}$$

$\vec{L} = 5\hat{i} - 10\hat{j} + 5\hat{k}$

$\vec{L} = \hat{i} - 2\hat{j} + \hat{k}$

Direction cosine $\longrightarrow$

$\left(\frac{1}{\sqrt{6}}, \frac{-2}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right)$ Ans or $\left[\frac{-1}{\sqrt{6}}, \frac{2}{\sqrt{6}}, \frac{-1}{\sqrt{6}}\right]$

wala $(\pi - \alpha)$

Q.2) The direction cosine $l, m, n$ of two lines are connected by the relations $l + m + n = 0$ & $2lm + 2ln - mn = 0$, find them and also find the angle b/w them

solve) $m = (-n - l)$

$2l(-n-l) + 2ln - (-n-l)n = 0$

$-2l^2 - 2nl + 2ln + n^2 + ln = 0$

$-2l^2 + ln + n^2 = 0$

$l = n$, $-2l = n$

when $l = n$

$l : m : n$

$\frac{l}{n} : \frac{m}{n} : 1$ —— we divide by n eq (1)

$1 : -2 : 1$

$\left[\frac{1}{\sqrt{6}}, \frac{-2}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right)$

when $-2l = n$

$$\frac{l}{n} = -\frac{1}{2} \quad -1)$$

$$-\frac{1}{2} : \frac{m}{n} : 1$$

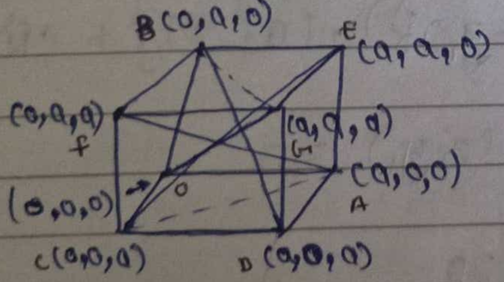
$$-\frac{1}{2} : -\frac{1}{2} : 1$$

$$\left[-\frac{1}{\sqrt{6}}, \frac{-1}{\sqrt{6}}, \frac{2}{\sqrt{6}}\right]$$

$$\cos\theta = \frac{-1+4}{6} = \frac{1}{2} \quad \theta = \frac{\pi}{3} \text{ or } \frac{2\pi}{3}.$$

Q.3) line makes $\alpha, \beta, \gamma, \delta$ with four diagonal of a cube prove that $\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta = \frac{4}{3}$

Solve)




$\vec{OG} = a(\hat{i} + \hat{j} + \hat{k})$ $\alpha$

$\vec{AF} = a(-\hat{i} + \hat{j} + \hat{k})$ $\beta$

$\vec{CE} = a(\hat{i} + \hat{j} - \hat{k})$ $\gamma$

$\vec{BD} = a(\hat{i} - \hat{j} + \hat{k})$ $\delta$

$$\vec{v} = l\hat{i} + m\hat{j} + n\hat{k}$$

$$\cos\alpha = \frac{a(l+m+n)}{a\sqrt{3}} = \frac{l+m+n}{\sqrt{3}}$$

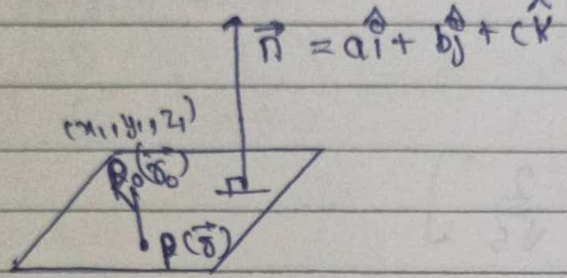
$$\cos\beta = \frac{a(-l+m+n)}{a\sqrt{3}} = \frac{-l+m+n}{\sqrt{3}}$$

$$3\left(\Sigma\cos^2\alpha\right) = 4$$

$$\cos\gamma = \frac{l+m-n}{\sqrt{3}}, \quad \cos\delta = \frac{l-m+n}{\sqrt{3}}$$

# PLANE

It is infinate surface such that the line segment joining any two points lieing on it completely lie in that surface.



$\vec{PR} \cdot \vec{n} = 0$

$(\vec{r} - \vec{r_0}) \cdot \vec{n} = 0$

$\vec{r} \cdot \vec{n} = \vec{r_0} \cdot \vec{n} = q$

$\boxed{\vec{r} \cdot \vec{n} = q}$ $\Rightarrow$ Scalar dot ~~form~~ (Product) form

$((x-x_1)\hat{i} + (y-y_1)\hat{j} + (z-z_1)\hat{k}) \cdot (a\hat{i} + b\hat{j} + c\hat{k}) = 0$

$a(x-x_1) + b(y-y_1) + c(z-z_1) = 0$

$ax + by + cz + d = 0$

$a^2 + b^2 + c^2 = 1$

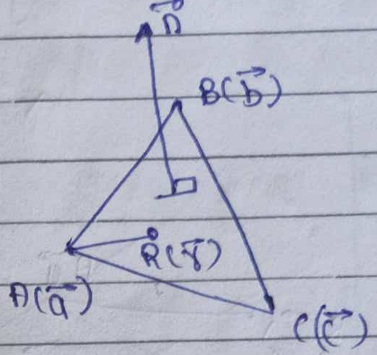
> $a\hat{i} + b\hat{j} + c\hat{k}$ is $\perp^{ar}$ line of plane

(II) Given 3 non-collinear point

$\vec{n} = \vec{AB} \times \vec{AC}$

$= (\vec{b}-\vec{a}) \times (\vec{c}-\vec{a})$

$\vec{n} = \vec{b}\times\vec{c} + \vec{b}\times\vec{a} + \vec{a}\times\vec{c}$




$\vec{AR} \cdot \vec{n} = 0$

$(\vec{r} - \vec{a}) \cdot \vec{n} = 0$

$$\boxed{\vec{r} \cdot \vec{n} = \vec{a} \cdot \vec{n}}$$

<u>Alternate way</u>

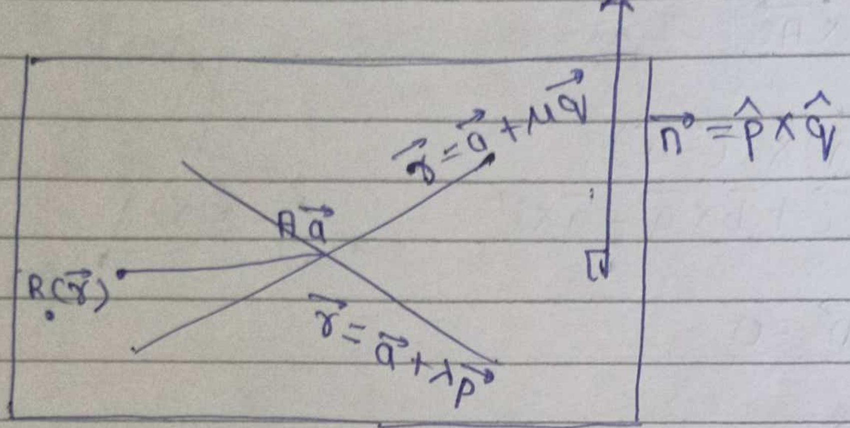
$\vec{AR}$, $\vec{AB}$ and $\vec{AC}$ are coplanar

$[\vec{AR}\ \vec{AB}\ \vec{AC}] = 0$

$$\begin{vmatrix} (x-a_1) & (y-a_2) & (z-a_3) \\ (b_1-a_1) & (b_2-a_2) & (b_3-a_3) \\ (c_1-a_1) & (c_2-a_2) & (c_3-a_3) \end{vmatrix} = 0$$

Now we get $eq^n$

III) plane containing 2 intersecting lines.




$\vec{AR} \cdot \vec{n} = 0$

$(\vec{r} - \vec{a}) \cdot \vec{n} = 0$

$\vec{r} \cdot \vec{n} = \vec{a} \cdot \vec{n}$

$$\boxed{[\vec{r}\ \vec{p}\ \vec{q}] = [\vec{a}\ \vec{p}\ \vec{q}]}$$

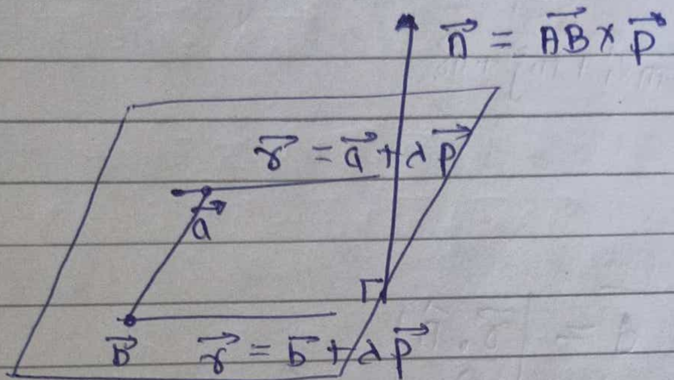
Alt: Parametric form

$\vec{AR}$, $\hat{p}$ and $\hat{q}$ are coplanar

$\vec{AR} = x\hat{p} + y\hat{q}$

$$\boxed{\vec{r} = \vec{a} + x\hat{p} + y\hat{q}}$$ where $x$ and $y$ are real parametre.

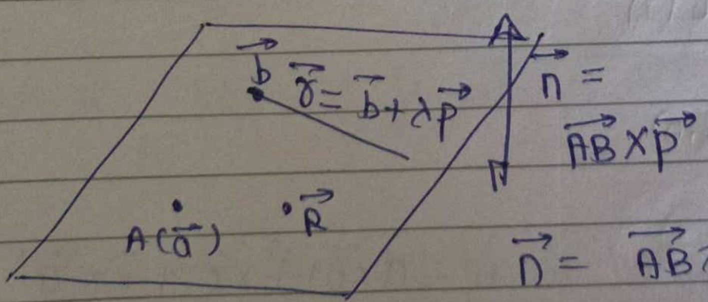
(IV) plane containing two parallel line.




$(\vec{r} - \vec{a}) \cdot \vec{n} = 0$ where $\vec{n} = (\vec{b} - \vec{a}) \times \vec{p}$

$\vec{r} \cdot \vec{n} = \vec{a} \cdot \vec{n}$

(V) plane containing a line and a point which doesn't lie on the line




$\vec{n} = \vec{AB} \times \vec{p}$

$\vec{AR} \cdot \vec{n} = 0$

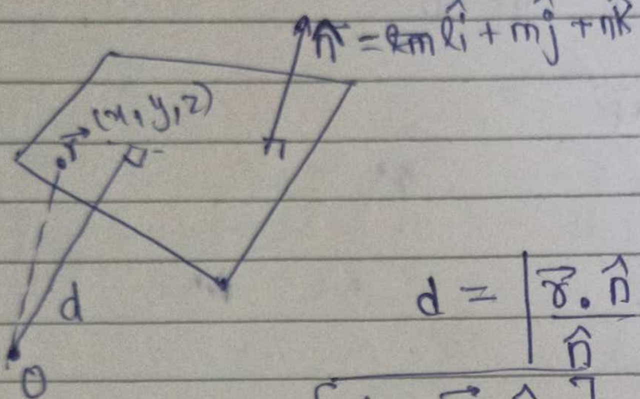
$(\vec{r} - \vec{a}) \cdot \vec{n} = 0$

$\vec{r} \cdot \vec{n} = \vec{a} \cdot \vec{n}$

## ~~(VI)~~ Normal form

Imp

> If d is the distance from the origin and l, m, n are D.C of normal then foot of $\perp^{ar}$ (ld, md, nd)




$d = \left| \frac{\vec{r} \cdot \hat{n}}{\hat{n}} \right|$

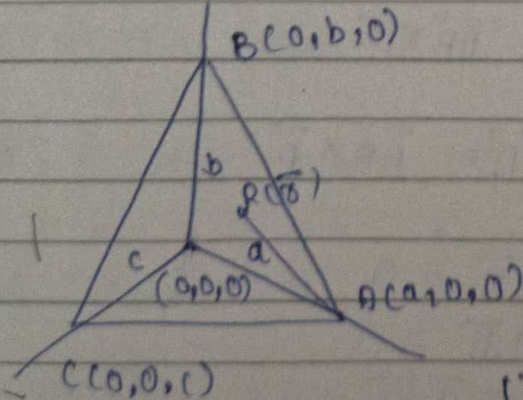
$\boxed{d = \vec{r} \cdot \hat{n}}$

$\hat{n}$ = unit vector along normal's direction.

$d = |\text{projection of OR on } \hat{n}|$

* $\boxed{lx + my + nz = d}$

## (VII) Intercept form




$\hat{n} = \vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}$

$\vec{AR} \cdot \vec{n} = 0$

$(\vec{r} - \vec{a}) \cdot \vec{n} = 0$

$\boxed{\vec{r} \cdot \vec{n} = \vec{a} \cdot \vec{n}}$

$$\vec{n} = (\vec{a}\hat{i} \times \vec{b}\hat{j}) + (\vec{b}\hat{j} \times \vec{c}\hat{k}) + (c\hat{k} \times a\hat{i})$$

$$\vec{n} = ab\hat{k} + bc\hat{i} + ca\hat{j} \quad \text{— 1)}$$

$$\cancel{\vec{x}}\ \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$(\vec{r} - \vec{a}) = (x-a)\hat{i} + y\hat{j} + z\hat{k} \quad \text{— (2)}$$

$$(\vec{r} - \vec{a})\cdot\vec{n} = bc(x-a) + cay + abz = 0$$

$$abc = cay + abz + bcx$$

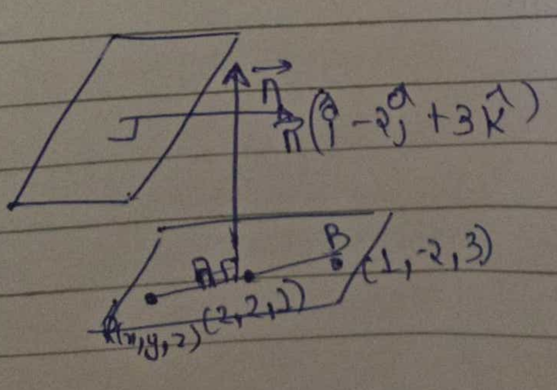
$$\boxed{1 = \frac{y}{b} + \frac{z}{c} + \frac{x}{a}} \quad \text{— Imp.}$$

Area of Plane segment ABC

$$ar\ ABC = \frac{1}{2}|\hat{n}| = \frac{1}{2}\sqrt{(bc)^2 + (ca)^2 + (ab)^2} \quad \text{— Imp.}$$

Q) Find the eqⁿ of plane passing through the points $(2,2,1)$ and $(1,-2,3)$ & perp. to plane $x - 2y + 3z + 4 = 0$

Solve)




$$AB = -\hat{i} - 4\hat{j} + 2\hat{k}$$

$$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 3 \\ -1 & -4 & +2 \end{vmatrix}$$

$$\vec{n} = (8)\hat{i} - (5)\hat{j} - 6\hat{k} \quad \text{— 1)}$$

~~AB~~ $\vec{Ar}\cdot\vec{n}=0$

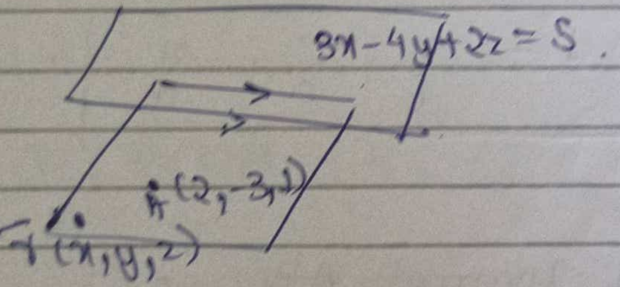
$(x-2)8-5(y-2)-6(z-1)=0$

$8x-16-5y+10-6z+6=0$

$8x-5y-6z=0$ Ans

Q2) Find the eqⁿ of the plane passing through the point $(2,-3,1)$ and parallel to the plane $3x-4y+2z=5$

Solve)




$\vec{n}=3\hat{i}-4\hat{j}+2\hat{k}$ is also ⊥ to that plane.

$\vec{Ar}\cdot\vec{n}=0$

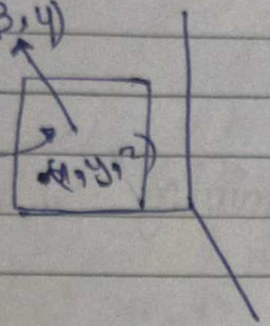
$3(x-2)-4(y+3)+2(z-1)=0$

$3x-6-4y-12+2z-2=0$

$3x-4y+2z=20$ Ans

Q3) find the $eq^n$ of the plane whose foot of normal from origin is $\alpha, \beta, \gamma$

Solve) $(\alpha, \beta, \gamma)$



$$\vec{n} \cdot \vec{r} = d$$

$$(\alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}) \cdot (\vec{r}) = \sqrt{\alpha^2 + \beta^2 + \gamma^2}$$

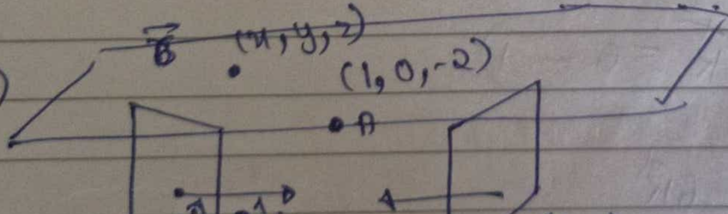
$\cancel{\alpha\hat{i} + \beta\hat{j} + \gamma}$

$\cancel{\alpha\hat{i} + \beta y}$ $\quad \alpha x\hat{i} + \beta y\hat{j} + \gamma z\hat{k} = \sqrt{\alpha^2 + \beta^2 + \gamma^2}$

$$(x-\alpha)\hat{i} + (y-\beta)\hat{j} + (z-\gamma)\hat{k} = 0$$

Q4) find the $eq^n$ of the plane passing through the point $(1, 0, -2)$ & perpendicular to plane $2x + y - z = 2$ & $x - y - z = 3$

Solve)



$$n_1 = 2\hat{i} + \hat{j} - \hat{k}$$

$$n_1 \times n_2 = n_{req}$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & -1 \\ 2 & 1 & -1 \end{vmatrix} = n_{req}$$

$$\hat{i}(2) - \hat{j}(1) + \hat{k}(3) = n_{req}$$

$$2\hat{i} - \hat{j} + 3\hat{k} = n_{req} \quad \text{--- 1)}$$

$\overrightarrow{AR} \cdot \overrightarrow{n_{req}} = 0$

$2(x-1) - y + 3(z+2) = 0$

Q. Find the eqn of the plane containing 2 intersecting lines

$\vec{r} = 2\hat{j} + \hat{k} + \lambda(\hat{i} - \hat{j} + \hat{k})$

$\vec{r} = 2\hat{i} + 3\hat{j} + 6\hat{k} + \mu(2\hat{i} + \hat{j} + 5\hat{k})$

Solve)

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 2 & 1 & 5 \end{vmatrix} = 0$$

$\hat{i}(-6) - \hat{j}(3) + \hat{k}(+3) = 0$

$-6\hat{i} - 3\hat{j} + 3\hat{k} = 0$

$2\hat{i} + \hat{j} - \hat{k} = 0 \quad -1)$

Point

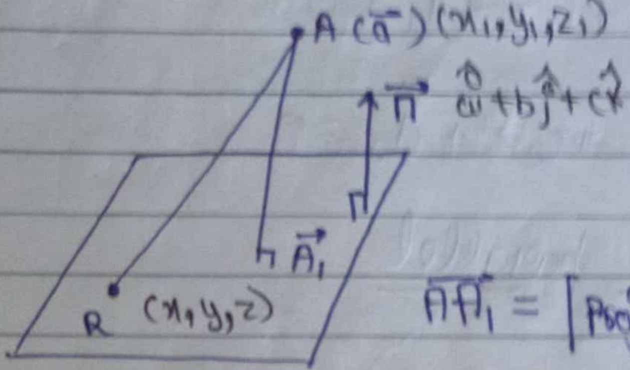
$\vec{A} = 2\hat{i} + 3\hat{j} + 6\hat{k}$

$\overrightarrow{AR} \cdot \vec{n} = 0$

$\Rightarrow (x-2)2 + 1(y-3) - 1(z-6) = 0$

$\Rightarrow 2x + y - z + 1 = 0$

# Perpendicular Distance of a point from plane




$\vec{AA_1} = [\text{Projection of } \vec{AR} \text{ along } \vec{n}]$

$$= \left|\frac{\vec{AR}\cdot\vec{n}}{|\vec{n}|}\right|$$

$\vec{r}\cdot\vec{n} = q$

$$\vec{AA_1} = \left|\frac{(\vec{r}-\vec{a})\cdot\vec{n}}{|\vec{n}|}\right|$$

$$= \frac{\vec{r}\cdot\vec{n} - \vec{a}\cdot\vec{n}}{\vec{n}}$$

$$\boxed{\vec{AA_1} = \left|\frac{q-\vec{a}\cdot\vec{n}}{\vec{n}}\right|}$$

$$= \left|\frac{q-(ax_1+by_1+cz_1)}{\sqrt{a^2+b^2+c^2}}\right|$$

$$= \left|\frac{ax_1+by_1+cz_1+d}{\sqrt{a^2+b^2+c^2}}\right|$$

$ax_0+by_0+cz_0+d=0$

$(x_1,y_1,z_1)$

## Condition for perpendicular or parallel plane.

$$\Pi_1 = a_1x + b_1y + c_1z + d_1 = 0$$

$$\Pi_2 = a_2x + b_2y + c_2z + d_2 = 0$$

(I) $\Pi_1$ & $\Pi_2$ are parallel

then $\hat{n}_1 = \lambda \hat{n}_2$

$$a_1\hat{i} + b_1\hat{j} + c_1\hat{k} = \lambda(a_2\hat{i} + b_2\hat{j} + c_2\hat{k})$$

$$a_1 = \lambda a_2$$

$$\frac{a_1}{a_2} = \lambda = \frac{b_1}{b_2} = \frac{c_1}{c_2} \neq \frac{d_1}{d_2}$$

(II) $\Pi_1$ & $\Pi_2$ are identical

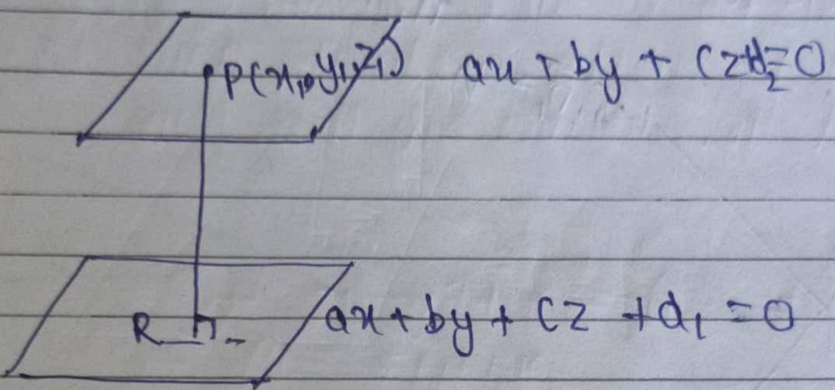
$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = \frac{d_1}{d_2}$$

(III) $\Pi_1$ & $\Pi_2$ are perpendicular

$$\hat{n}_1 \cdot \hat{n}_2 = 0$$

$$a_1a_2 + b_1b_2 + c_1c_2 = 0$$

# Distance b/w two parallel planes.




$$PR = \left| \frac{ax_1 + by_1 + cz_1 + d_1}{\sqrt{a^2 + b^2 + c^2}} \right|$$

$$\boxed{PR = \left| \frac{d_1 - d_2}{\sqrt{a^2 + b^2 + c^2}} \right|}$$

when written vectorially

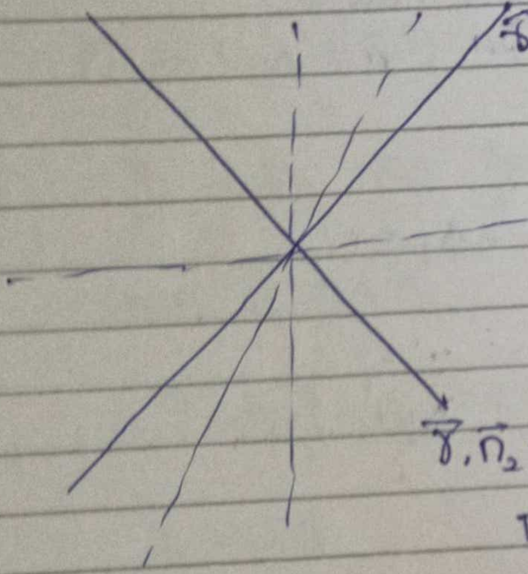
$$\vec{r} \cdot \vec{n} = q_1 \quad , \quad \vec{r} \cdot \vec{n} = q_2$$

$$\text{distance} = \left| \frac{q_1 - q_2}{|\vec{n}|} \right|$$

# family of plane

$\pi_1 : a_1x+b_1y+c_1z+d_1=0$

$\vec{r}.\vec{n_1}=q_1$



$\vec{r}.\vec{n_2}=q_2$

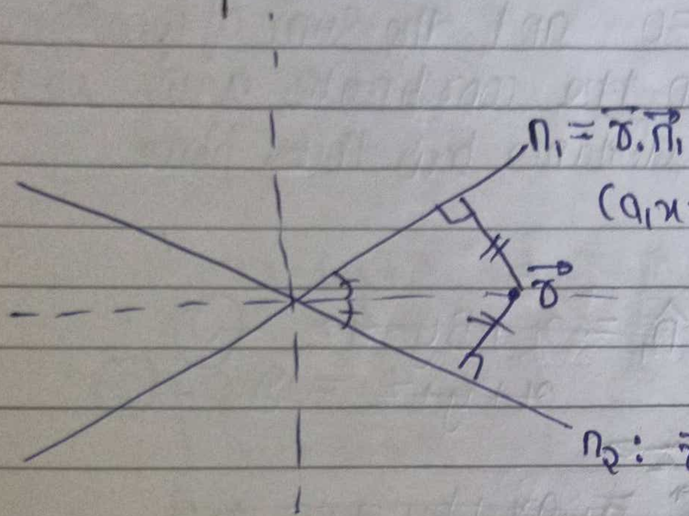
$\pi_2 : a_2x+b_2y+c_2z+d_2=0$

$\pi_1+\lambda\pi_2=0$

$(\vec{r}.\vec{n_1}-q_1)+\lambda(\vec{r}.\vec{n_2}-q_2)=0$

$$\boxed{\begin{gathered}\vec{r}.(\vec{n_1}+\lambda\vec{n_2})=q_1+q_2\\ \text{or}\\ a_1x+b_1y+c_1z+d_1+\lambda(a_2x+b_2y+c_2z+d_2)=0\end{gathered}}$$

# Bisector plane




$n_1 = \vec{r}\cdot\vec{n_1} = q_1$

$(a_1x + b_1y + c_1z + d_1) = 0$

$n_2 : \vec{r}\cdot\vec{n_2} = q_2$

$(a_2x + b_2y + c_2z + d_2) = 0$

$$\left|\frac{\vec{r}\cdot n_1 - q_1}{\hat{n}_1}\right| = \left|\frac{\vec{r}\cdot n_2 - q_2}{\hat{n}_2}\right|$$

$$\left(\frac{\vec{r}n_1 - q_1}{|\vec{n}_1|}\right) = \pm\left(\frac{\vec{r}\cdot n_2 - q_2}{|\vec{n}_2|}\right)$$

$$\left|\frac{a_1x + b_1y + c_2z + d_1}{\sqrt{a_1^2 + b_1^2 + c_1^2}}\right| = \pm\left[\frac{a_2x + b_2y + c_2z + d_2}{\sqrt{a_2^2 + b_2^2 + c_2^2}}\right]$$

Q1: find the eqⁿ of the plane which is parallel to the plane

$x+5y-4z+5=0$ and the sum of whose intercept on the coordinate axis is 19 units. Also, find the distance b/w these plane

Solve)

$\cancel{\hat{n}_1 = \hat{x}+}$ $\cancel{\hat{n}_1 = x+5y-4z}$ — ①

$\cancel{x+y+z=19}$ — ②

$\cancel{x+5y-4z}$

$\cancel{\vec{P_2} = ax+by+cz+d}$

$P_1 = x+5y-4z+5=0$

normal same

$\cancel{0 = x+5y-4z+d_2}$

$\cancel{19 = x+y+z}$

$\frac{x}{d} + \frac{y}{d/5} + \frac{z}{-d/4} = 0$

$d + \frac{d}{5} - \frac{d}{4} = 19$

$d = 20$

req plane $= x+5y-4z-20=0$

$\perp$ distance $= \frac{25}{\sqrt{1+25+16}} \Rightarrow \frac{25}{\sqrt{42}}$

Q2) find the eqⁿ of plane parallel to $2x-6y+3z=0$ & at a distance of 2units from the point $(1,2,-3)$

Solve) let point

$$2x-6y+3z+d_1=0$$

$$\left|\frac{2-12-9+d_1}{\sqrt{4+36+9}}\right|=2$$

$$\frac{-19+d_1}{\sqrt{49}}=\pm 2$$

$-19+d_1=14$ or $-19+d_1=-14$

$d_1=33$     $d_1=5$

$$2x-6y+3z+33=0$$

$$2x-6y+3z+5=0$$

Imp

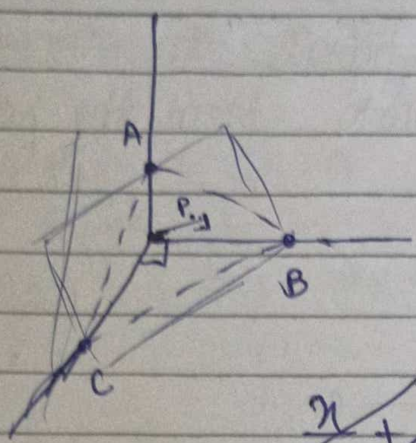
Q3) A plane which always remain at a constant ⊥ distance 'p' from the origin cuts the coordinate axis at points A, B, C find the locus of

(a) centroid of the plane face ABC

(b) ~~ct~~ centre of the tetrahedron OABC (O is origin)

(c)

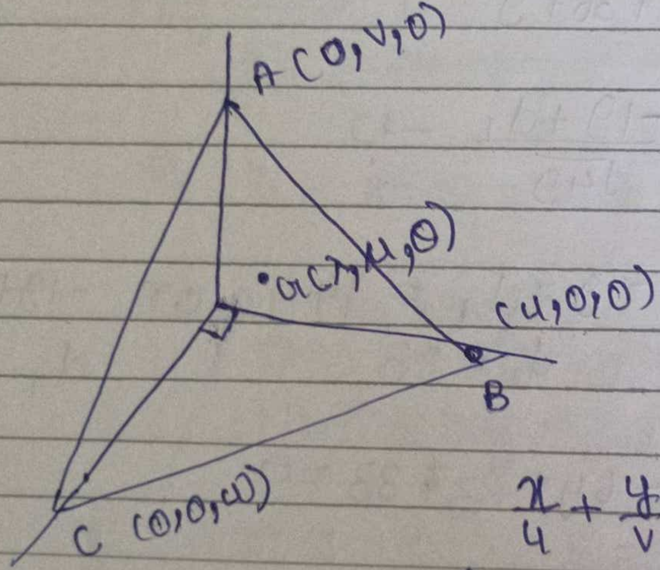
Solve)




$$\frac{x}{A} + \frac{y}{B} + \frac{z}{C} = 1 \quad \text{(i)}$$

$$\cancel{\frac{x}{A} + B}$$

Solve)




$$\frac{x}{u} + \frac{y}{v} + \frac{z}{w} = 1$$

$$\left| \frac{1}{\sqrt{\frac{1}{u^2} + \frac{1}{v^2} + \frac{1}{w^2}}} \right| = P$$

$\cancel{\lambda = 3}$. $3\lambda = u$

$3\mu = v$

$3\theta = w$

$$\frac{1}{P^2} = \frac{1}{u^2} + \frac{1}{v^2} + \frac{1}{w^2}$$

$$\frac{1}{P^2} = \frac{1}{9\lambda^2} + \frac{1}{9\mu^2} + \frac{1}{9\theta^2}$$

$$\frac{9}{P^2} = \frac{1}{\lambda^2} + \frac{1}{\mu^2} + \frac{1}{\theta^2}$$

DPP = 8

Ex-1st 1, 12

centre of tetrahydron

$$\frac{16}{p^2} = \frac{1}{\lambda^2} + \frac{1}{\mu^2} + \frac{1}{\theta^2} \quad \underline{\text{Ans}}$$

Imp Q. find the eqⁿ of the plane containing the line of intersection of the plane $\vec{r}\cdot\vec{n_1} = q_1$, $\vec{r}\cdot\vec{n_2} = q_2$, ~~$\vec{r}\cdot\vec{n_3}$~~ and is parallel to the line of intersection of the plane $\vec{r}\cdot\vec{n_3} = q_3$, $\vec{r}\cdot\vec{n_4} = q_4$

Solve) $\vec{r}\cdot\vec{n_1} + \lambda\vec{r}\cdot\vec{n_2} = q_1 + \lambda q_2$

$$\vec{r}\cdot(\vec{n_1} + \lambda\vec{n_2}) = q_1 + \lambda q_2$$

$$(\vec{n_1} + \lambda\vec{n_2})\cdot(\vec{n_3}\times\vec{n_4})$$

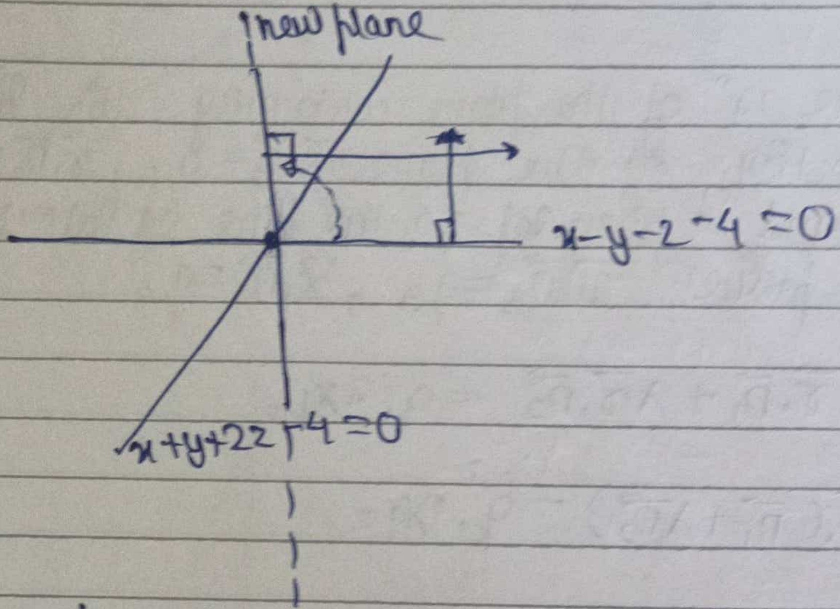
$$(\vec{n_3}\times\vec{n_4})\cdot\vec{n_1} + \lambda(\vec{n_3}\times\vec{n_4})\cdot\vec{n_2} = 0$$

$$[\vec{n_1}\ \vec{n_3}\ \vec{n_4}] + \lambda[\vec{n_2}\ \vec{n_3}\ \vec{n_4}] = 0$$

$$\lambda = \frac{-[\vec{n_1}\ \vec{n_3}\ \vec{n_4}]}{[\vec{n_2}\ \vec{n_3}\ \vec{n_4}]}$$

Q The plane $x-y-z=4$ is rotated through $90^\circ$ about its line of intersection with the plane $x+y+2z=4$ find its eqn in new position

Solve)




using family of plane

$$\Pi_1 + \lambda\Pi_2 = 0$$

$$x+y+2z-4+\lambda(x-y-z-4)=0$$

$$(\lambda+1)x+(1-\lambda)y+(2-\lambda)z-4-4\lambda=0$$

Since $\hat{n}_1 \cdot \hat{n}_2 = 0$

$$(\hat{i}-\hat{j}-\hat{k})\cdot\left[(\lambda+1)\hat{i}+(1-\lambda)\hat{j}+(2-\lambda)\hat{k}\right]=0.$$

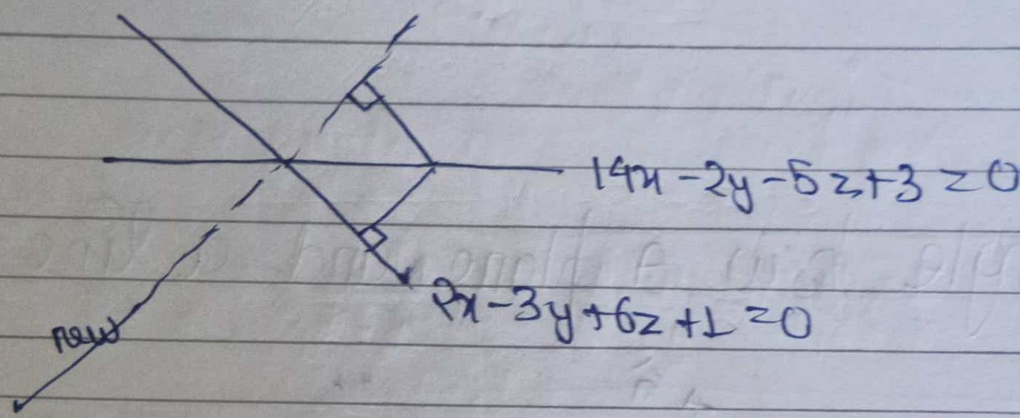
$$\lambda+1+\lambda-1-2+\lambda=0$$

$$\lambda=\frac{2}{3}$$

required plane $x+y+2z-4+\frac{2}{3}(x-y-z-4)=0$

Q Find the reflection of plane $2x-3y+6z+1=0$ in the plane $14x-2y-5z+3=0$.

Solu)




using family of plane.

$$\pi_1 + \lambda \pi_2 = 0$$

$$14x-2y-5z+3 + \lambda(2x-3y+6z+1) = 0$$
$$(14+2\lambda)x-(2+3\lambda)y-(5-6\lambda)z+(3+\lambda)=0$$

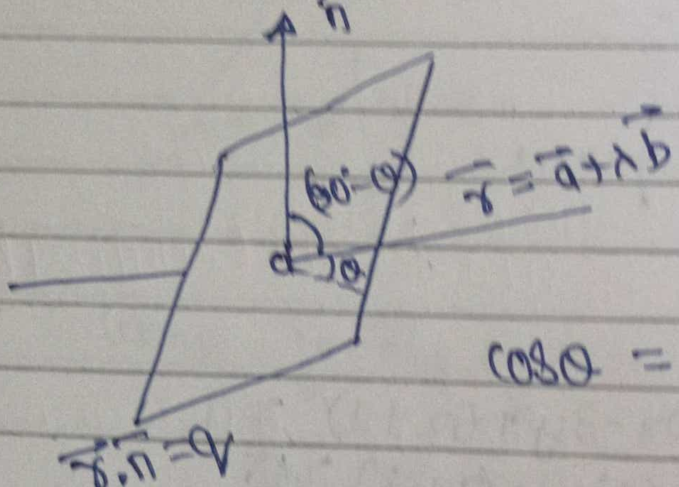
taking any point of mirror $\left(0,0,\frac{3}{5}\right)$

$$\left|\frac{\frac{18}{5}+1}{7}\right| = \left|\frac{(5-6\lambda)\frac{3}{5}}{\sqrt{(14+2\lambda)^2+(2+3\lambda)^2+(5-6\lambda)^2}}\right|$$

$$\left|\frac{23}{35}\right| = \left|\frac{\frac{3-18\lambda}{5}}{\sqrt{(14+2\lambda)^2+(2+3\lambda)^2+(5-6\lambda)^2}}\right|$$

$$\left|\frac{23}{35}\right| = \left|\frac{(15-18\lambda)}{5}\right|$$

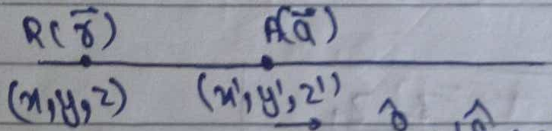
# Angle b/w A plane and a line




$$\cos\theta = \frac{\vec{n}\cdot\vec{b}}{|\vec{n}||\vec{b}|}$$

# Line

## Symmetric form




$$\vec{p} = a\hat{i} + b\hat{j} + c\hat{k}$$

$$\vec{r} = \vec{a} + \lambda\vec{p}$$

$$\boxed{\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c} = \lambda}$$

eqⁿ of x-axis

$$\frac{x-0}{1} = \frac{y-0}{0} = \frac{z-0}{0}$$

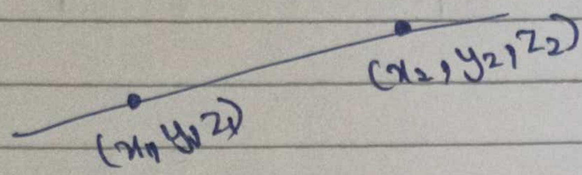
y-axis

$$\frac{x-0}{0} = \frac{y-0}{1} = \frac{z-0}{0}$$

z-axis

$$\frac{x-0}{0} = \frac{y-0}{0} = \frac{z-0}{1}$$

(ii) when two points are given




$$\boxed{\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}}$$

## ~~U~~ Asymmetric form of line

$$\left.\begin{aligned} a_1x + b_1y + c_1z + d_1 &= 0 \\ a_2x + b_2y + c_2z + d_2 &= 0 \end{aligned}\right\} \text{line of intersection}$$

$a_1x + b_1y + c_1z + d_1 = 0 = a_2x + b_2y + c_2z + d_2$

Put $z = 0$

$a_1x + b_1y + d_1 = 0$

$a_2x + b_2y + d_2 = 0$

we get points $(x_0, y_0, 0)$

Put $y = 0$

$a_1x + c_1z + d_1 = 0$

$a_2x + c_2z + d_2 = 0$

we get point $(x_1, 0, y_1)$

$$\boxed{\frac{x - x_0}{x_0 - x_1} = \frac{y - y_0}{y_0 - y_1} = \frac{z - z_0}{z_0 - z_1}}$$

Q1) Find the eqn of the line of intersection of planes
$x+y-2z=8$ & $3x-y+4z=12$ in the symmetric form.

Solve) $x+y-2z-8=0$ $\quad$ $3x-y+4z-12=0$

$x=0$

$y-2z=8$

$-y+4z=12$

$2z=20$

$z=10$ — (1)

$y=28$ — (2)

$(0,28,10)$

$\cancel{z}=\cancel{y}=z=0$

$x+y=8$

$3x-y=12$

$4x=20$

$x=5$

$y=3$

$(5,3,0)$

$$\frac{x-0}{5}=\frac{y-28}{-25}=\frac{z-10}{-10}$$

or

$$\frac{x}{1}=\frac{y-28}{-5}=\frac{z-10}{-2}$$

req eqn ↑

Q) find the points in which the line $x = 1+2t$, $y = -1-t$, $z = 3t$ meets the coordinate planes. that is $xy$, $yz$ and $zx$

Solve)

$xy$ plane $\Rightarrow$ $z=0$ at $xy$ plane

$t = 0$

$y = -1$, $z = 0$ $x = 1$

$(1, -1, 0)$

$zx$ plane $\Rightarrow$ $y = 0$ $t = -1$

$z = -3$ $x = -1$

$(-1, 0, -3)$

$yz$ plane $\Rightarrow$ $x = 0$ $t = -\frac{1}{2}$

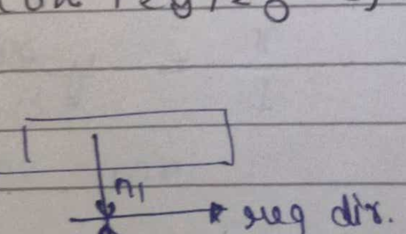
$y = -\frac{1}{2}$ $z = -\frac{3}{2}$ $(0, -\frac{1}{2}, -\frac{3}{2})$

Q) find eq$^n$ of line through $(1,4,-2)$ and parallel to the planes $(6x+2y+2z=3)$ and $x+2y-6z+4=0$

Soln) $\cancel{z=0 \quad y=0}$

$\cancel{6x+2z=3}$

$\cancel{x-6z=-4}$




$n_1 = 6\hat{i} + 2\hat{j} + 2\hat{k}$

$n_2 = \hat{i} + 2\hat{j} - 6\hat{k}$

$$n_1 \times n_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 6 & 2 & 2 \\ 1 & 2 & -6 \end{vmatrix}$$

$$= -16\hat{i} + 38\hat{j} + 10\hat{k}$$

$$\frac{x-1}{-16} = \frac{y-4}{38} = \frac{z+2}{10} \quad \text{Ans}$$

Q. Show that the lines
$L_1 : 3x + 2y + z - 5 = 0 = x + y - 2z - 3$
$L_2 : 8x - 4y - 4z = 0 = 7x + 10y - 8z$ are at right angle

Soln) $x = 0$

$2y + z = 5$
$2y - 4z = 6$
$-\quad +\quad -$

$5z = -1$

$z = -\frac{1}{5}$ —①

$2y + \frac{4}{5} = 6$

$2y = 6 - \frac{4}{5}$

$2y = \frac{26}{5}$

$y = \frac{13}{5}$ —②

$\left(0, \frac{13}{5}, -\frac{1}{5}\right)$

$y = 0$

$3x + z = 5$
$3x - 6z = 9$
$- \quad + \quad -$

$7z = -4$

$z = -\frac{4}{7}$ —①

$3x - \frac{4}{7} = 5$

$3x = 5 + \frac{4}{7}$

$3x = \frac{39}{7}$

$x = \frac{39}{21}$

$\left(\frac{39}{21}, 0, -\frac{4}{7}\right)$

$\frac{39}{7} + z = 5$

$z = 5 - \frac{39}{7}$

$z = -\frac{4}{7}$

$-\frac{4}{7}+\frac{1}{5}$

$\Rightarrow -\frac{20+7}{35} =$

$\frac{39}{21}\hat{i} - \frac{13}{5}\hat{j} - \frac{13}{35}\hat{k}$ — (2).

for case (ii)

$x = 0$

$(0, 0, 0)$

$-8y - 8z = 0$
$10y - 8z = 0$
$-\quad\ \ -\quad -$

$-18y = 0$
$y = 0$ — (2).

Alternative.

$$V_1 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & 1 \\ 1 & 1 & -2 \end{vmatrix}$$

$\hat{i}(-4-1) - \hat{j}(-6-1) + \hat{k}(3-2)$

$V_1 = -5\hat{i} + 7\hat{j} + \hat{k} = 0.$

$$V_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 8 & -4 & -4 \\ 7 & 10 & -8 \end{vmatrix}$$

$V_2 = 72\hat{i} + 36\hat{j} + 108\hat{k}$

$\cancel{\hat{i}(-8)} - \hat{j}(36)\hat{j}$

$V_1 \cdot V_2 = 0$

$-10 + 7 + 3 = 0.$

DPP-00,9

DPP 10 = Q9

Q) find the eqn of line passing through $(2,-1,-1)$ & parallel to the plane $4x+y+z+2=0$ & is perpendicular to the line of intersection of planes $x-y+z=0$ and $2x+y=0$

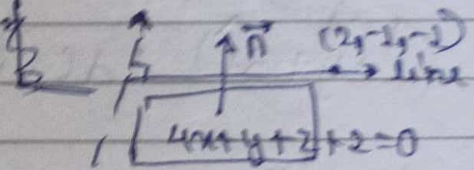
Soln) $\dfrac{x-2}{1} = \dfrac{y+1}{-13} = \dfrac{z+1}{9}$

$\vec{V} = \vec{n}_1 \times (\vec{n}_2 \times \vec{n}_3)$

$= (\vec{n}_1 \cdot \vec{n}_3)\vec{n}_2 - (\vec{n}_1 \cdot \vec{n}_2)\vec{n}_3$

$= 9(\hat{i}-\hat{j}+\hat{k}) - 4(2\hat{i}+\hat{j})$

$= \hat{i} - 13\hat{j} + 9\hat{k}$




Q) Find the distance of the point $A(1,0,-3)$ from the plane $x-y-z=9$ measured parallel to the line

$\dfrac{x-2}{2} = \dfrac{y+2}{3} = \dfrac{z-6}{-6}$

Solu) $\dfrac{x-1}{2} = \dfrac{y-0}{3} = \dfrac{z+3}{-6} = \lambda$

$x = (2\lambda+1) \quad y = 3\lambda \quad z = -3\lambda - 6$

$(2\lambda+1, 3\lambda, -3\lambda-6)$

$2\lambda+1-3\lambda+3\lambda+6 = 9$

$\lambda = 1$

$(3, 3, -9)$ $\quad \sqrt{4+9+36} = \sqrt{49} = 7$ Ans

Q. Find the shortest distance b/w the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$

& $\frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$

Solue)

direction

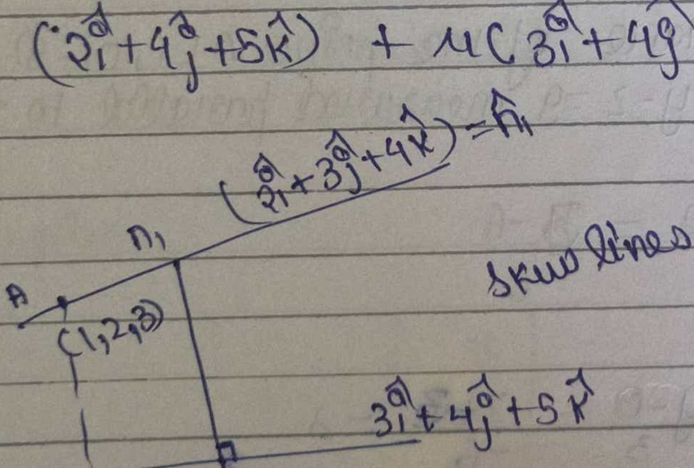
$2\hat{i}+3\hat{j}+4\hat{k}$

$(\hat{i}+2\hat{j}+3\hat{k}) + \lambda(2\hat{i}+3\hat{j}+4\hat{k})$

direction

$3\hat{i}+4\hat{j}+5\hat{k}$

$(2\hat{i}+4\hat{j}+5\hat{k}) + \mu(3\hat{i}+4\hat{j}+5\hat{k})$




$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix}$$

$\hat{i}(-1) - \hat{j}(10-12) + \hat{k}(8-9)$

$|\vec{n} \times \vec{n}_2| = -\hat{i}+2\hat{j}-\hat{k}$

$\vec{AB} = \hat{i}+2\hat{j}+2\hat{k}$

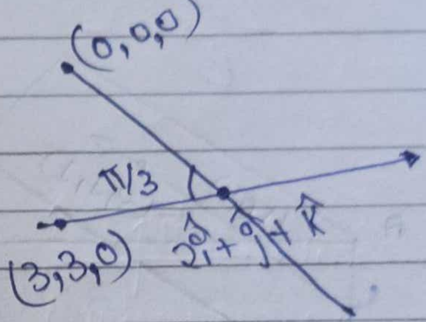
$$S.D = \frac{|\vec{AB}| \cdot |\vec{n_1 \times n_2}|}{|n_1 \times n_2|}$$

$$= \frac{-1+4-2}{\sqrt{6}}$$

$$\Rightarrow \frac{1}{\sqrt{6}}$$

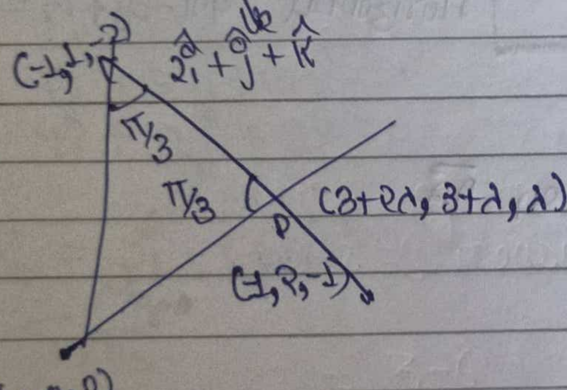
Q) Find the eq$^n$ of line passing through the origin which intersect the line $\frac{x-3}{2} = \frac{y-3}{1} = \frac{3}{1}$ at an angle of $\pi/3$

Solve) $2\hat{i} + \hat{j} + \hat{k}$ — ①




let line be $\vec{r}$

$\vec{p}$ // vector




$\vec{OP} =$

$$\vec{OP} = (3+2\lambda)\hat{i} + (3+\lambda)\hat{j} + \lambda\hat{k}$$

$$\cos\frac{\pi}{3} = \left|\frac{\vec{OP}\cdot\vec{P}}{|\vec{P}||\vec{OP}|}\right|$$

$$\frac{1}{2} = \left(\frac{6+4\lambda+3+\lambda+\lambda}{(\sqrt{6})(\sqrt{9+4\lambda^2+12\lambda+9+\lambda^2+6\lambda+\lambda^2}}\right)$$

$$\frac{1}{2} = \frac{6\lambda+9}{(\sqrt{6})(\sqrt{6\lambda^2+18\lambda+18}}$$

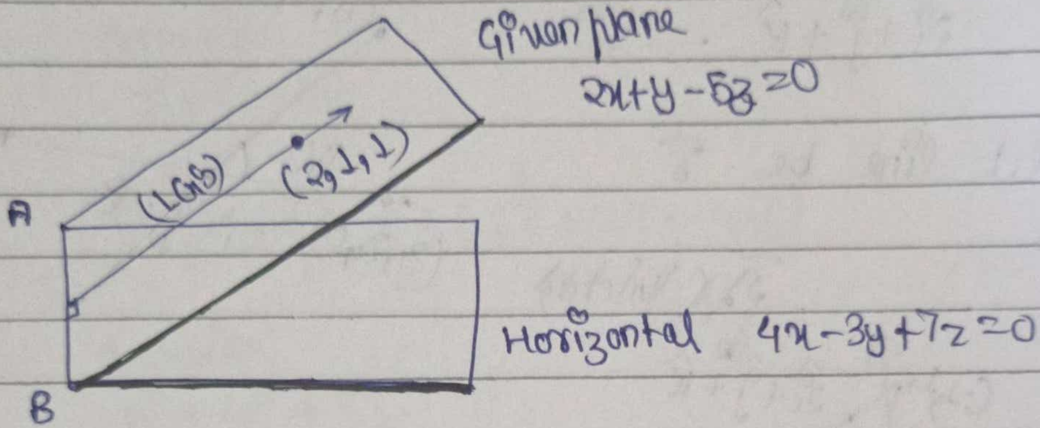
$$\frac{1}{4} = \frac{\cancel{2}\lambda+9}{\cancel{6}3\sqrt{\lambda^2+2\lambda+1}}$$

$$\sqrt{\lambda^2+2\lambda+1} = 2\lambda+3$$

$$\lambda^2+2\lambda+1 = 4\lambda^2+9+12\lambda$$

$$\lambda = -2, -1$$

# Line of greatest Slope




$\hat{n}_1 \times \hat{n}_2$ = direction $\vec{AB}$

$$\begin{vmatrix} i & j & k \\ 4 & -3 & 7 \\ 2 & 1 & -5 \end{vmatrix}$$

Again

$\hat{i}(15-7) - \hat{j}(-20-14) + \hat{k}(4+6) = 0$

$8\hat{i} + 34\hat{j} + 10\hat{k} = 0$

$2\hat{i} + 17\hat{j} + 8\hat{k} = 0.$ — (1).

$\hat{n}_2 = 2\hat{i} + \hat{j} - 5\hat{k} = 0$

$\hat{n}_2 \times (\hat{n}_1 \times \hat{n}_2)$ = direct of LOGS

$$\begin{vmatrix} i & j & k \\ 2 & 17 & 8 \\ 2 & 1 & -5 \end{vmatrix}$$

$\hat{i}(-65-5) - \hat{j}(-10-10) + \hat{k}(2-34) = 0$

$-70\hat{i} + 20\hat{j} - 32\hat{k} = 0$

$-35\hat{i} + 10\hat{j} - 16\hat{k} = 0$

Solve) $\hat{n}_1 \times \hat{n}_2$

$(4\hat{i} - 3\hat{j} + 7\hat{k}) \times (2\hat{i} + \hat{j} - 5\hat{k}) = 0$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -3 & 7 \\ 2 & 1 & -5 \end{vmatrix}$$

$\hat{i}(15-7) - \hat{j}(-20-14) + \hat{k}(4+6)$

$8\hat{i} + 34\hat{j} + 10\hat{k} = 0$

$4\hat{i} + 17\hat{j} + 5\hat{k} = 0 . - \vec{r}$

$\vec{n}_2 = 2\hat{i} + \hat{j} - 5\hat{k} = 0.$

$\vec{r} \times \vec{n}_2$

$$\begin{vmatrix} i & j & k \\ 4 & 17 & 5 \\ 2 & 1 & -5 \end{vmatrix}$$

$\hat{i}($

$\vec{n}_2 \times (\vec{n}_1 \times \vec{n}_2)$

$(\vec{n}_2 \cdot \vec{n}_2)\vec{n}_1 - (\vec{n}_2 \cdot \vec{n}_1)\vec{n}_2$

$\cancel{60\hat{i} + 30\hat{j} -}$