WAVE OPTICS

Ques.- What is light ?

Ans.- It is that form of energy which is responsible for vision when light after being reflected from different objects enters into our eyes then the objects are visible to us.

Optics

Ques.- What is optics ?

Ans.- It is that branch of physics in which we study about light energy and the phenomena related to it. It is divided into two parts-

(i) Wave optics- It is that branch of optics in which we study about the various phenomena related to light by considering that light propagates in the form of wave. It is also called physical optics.

(ii) Ray optics- It is that branch of optics in which we study about the various phenomena related to light by considering that light propagates in the form of rays. It is also called geometrical optics.

Newton's corpuscular theory of light

Ques.- State Newton's corpuscular theory of light. Why did this theory fail ?

Ans.- In the year 1676 Newton proposed his corpuscular Theory of light whose main postutales are as follows-

(i) Every light source emit infinite particles of negligible mass which are called corpuscles and the propogation of light takes place through corpuscles.

(ii) These corpuscles travel in straight line with constant speed in a homogeneous medium and on changing the medium the velocity of corpuscles also changes.

(iii) The sensation of different colours of light due to the different sizes of corpuscles .

(iv) When corpuscles strike the the retina of eye then due to transmission of energy to the retina object are seen.

Causes of failure of Newton corpuscular theory-

(i) According to this theory the mass of sources should decrease with time but it does not happen in actual practice.

(ii) According to this theory the velocity of light should depend upon the temperature of source. Again it does not happen in actual practice.

(iii) Newton assumed that repulsive force act on corpuscles during the phenomena of reflection and refraction from denser to rarer medium and attractive force act on coupuscles during refraction from rarer to denser medium these assumptions are contrary and have no theoretical foundation .

(iv) According to Newton velocity of light is denser medium is greater than that in rarer medium which was late proved to be wrong.

(v) On the basis of this theory the phenomenon of interference, deffraction and polarisation could not be explained.

Because of all the above reason NCT of light was rejected.

Huygen's wave theory of light

Ques.- State Huygen's wave theory of light. What are its drawbacks ?

Ans.- In 1678 Huygen's proposed his wave theory of light the main postulates of this theory are-

(i) Light propagates in the form of wave.

(ii) Light wave travels in all directions with very high velocity. (3×10^8 m/s)

(iii) Whole universe it pervaded with a weight less, colour less and odourless medium called luminiferous ether whose elasticity is very high and density is very low.

(iv) The different colours of light is due to different wavelength of light.

(v) The light waves are longitudinal in nature (which later proved to be transverse in nature by Maxwell).

On the basis of Huygen theory the phenomena of transmission of energy through light, reflection, refraction interference and diffraction could be successfully explained. But this theory could not explain the phenomena of polarization, photoelectric effect, compton effect etc .

Drawbacks of Huygen wave theory of light -

(i) Huygen assumed that light waves are machanical in nature which is later proved to be wrong because Maxwell proved that light waves are electromagnetic waves.

(ii) Huygen assumed that light are longitudnal in nature which is again proved to be wrong because Maxwell established light waves are transverse in nature.

(iii) Huygen assumed that the whole universe is pervaded with luminiferous ether medium whose presence could not be detected by any experiment.

(iv) On the basis of Huyen's wave theory phenomena like polarisation, photoelectric effect and compton effect could not be explained

Wavefront

Ques.- Define wavefront. What are its types ?

Ans.- It is that imaginary surface at every point of which all the particles of medium vibrate in the same phase. The imaginary lines which are drawn perpendicular to wavefront are called rays. These rays gives us the direction of propagation of waves.

Wavefront are of four types-

(1) Spherical wavefront- The wavefront which are obtained at finite distance from a point source of light are called spherical wavefront. In the case of point source of light wavefront are concentric spheres whose centre lies at point source of light.

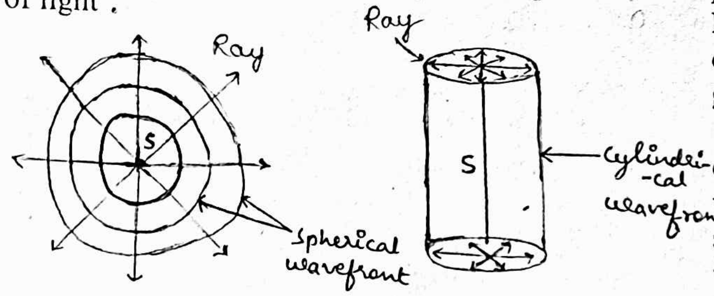
(2) Cylindrical wavefront- The wavefronts which are obtained at a finite distance from a line source of light

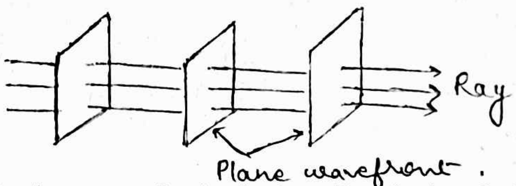
are called cylindrical wavefronts. In the case of line source of light wavefronts are coaxial cylinders whose axis lies at the line source of light.

(3) Plane wavefront- The wavefronts which are obtained at infinite distance from a point or line source of light are called plane wavefront.

At infinite distance from point or line source of light wavefront obtained are parallel planes and the rays perpendicular to them are parallel lines.

(4) Circular wavefront- The wavefronts which are obtained at finite distance from point source of light in a plane are called circular wavefront. In the case of point source of light wavefronts obtained in a plane are concentric circles whose centre lies at the point source of light .





Note- In the case of spherical, cylindrical and circular wavefronts rays are either diverging or converging where as in the case of plane wavefronts rays are palallel lines.

Huygen's theory of secondary wavelets

Ques.- State Huygen's theory of secondary wavelets.

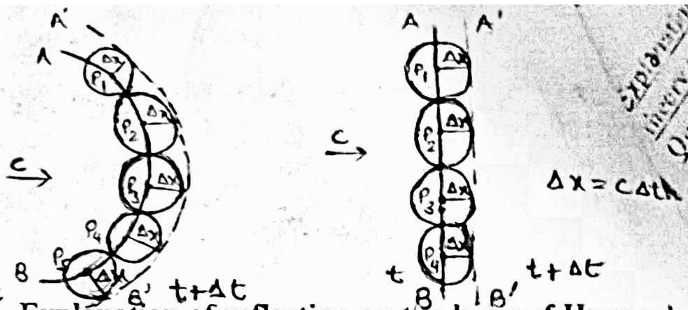
Ans.- It is geometrical method with the help of which the position of wavefront can be located at any instant. Its main postulates are-

(i) Every point of wavefront behaves like a source of light waves which are called secondary wavelets.

(ii) These secondary wavelets propagate in all directions with the velocity of original wave.

(iii) At any instant the forward envelope of secondary wavelets gives us the location of wavefront at that instant.

Let AB be the wavefront generated from a source of light at any instant t. We have to find out the new location of wavefront AB after time interval Δt. In time interval Δt the distance covered by the secondary wavelets generated from various points P_1, P_2, P_3......... of wavefront AB will be $c\Delta t$. By taking points P_1, P_2, P_3 as centres and $c\Delta t$ as radius, if spheres are drawn then the forward envelope A'B' of the sphere drawn will represent the new location of waveformt at time $t + \Delta t$.



Explanation of reflection on the basis of Huygen's theory of secondary wavelets

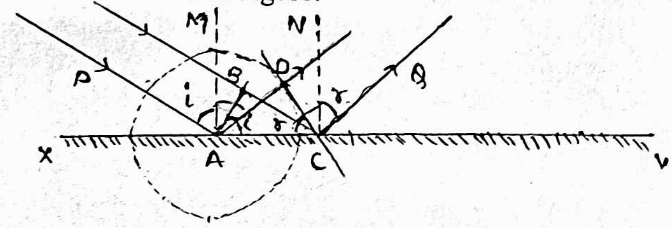
Ques.- Explain the phenomenon of reflection on the basis of Huygen's theory of secondary wavelets.

Ans.- Let XY be a reflecting surface and a parallel beam of light is allowed to incidence on it.If PA be the incident ray and AB be the incident wavefront then its point A will strike the reflecting surface first and point B will strike the reflecting surface after time t then the distance BC=ct . In this duration t the secondary wavelet generated from point A will cover a distance AD = ct.

By taking A as centre and ct as radius if a sphere is drawn then it will represent the location of secondary wavelet generated from point A after time t. A tangential surface CD drawn from point C of the secondary wavelet will represent the location of all the points which lie on incident wavefront AB, t time before. Since wavefront CD represents the reflected wavefront therefore the ray CQ which is drawn perpendicular to it represent the reflected ray.

From figure it is clear that the reflected ray, the normal and the incident ray all lie in the same plane. This is required the first law of reflection .

If the angle of incidence be i and the angle of reflection be r then in figure.



$\angle PAM = i \quad \& \angle NCQ = r$

$\angle MAB = \angle PAB - \angle PAM = 90^0 - i$

$\therefore \quad \angle BAC = \angle MAC - \angle MAB$

$= 90^0 - (90^0 - i) = i$

Again, $\angle DCN = \angle DCQ - \angle NCQ = 90^0 - r$

$\therefore \quad \angle DCA = \angle NCA - \angle DCN$

$= 90^0 - (90^0 - r) = r$

In ΔABC and ΔADC

$\angle ABC = \angle ADC = 90^0$

$BC = AD = ct$

and $AC = AC$

By RHS conqruency rule

$\Delta ABC \cong \Delta ADC$

$\therefore \quad \angle BAC = \angle DCA$

or $\quad i = r$

This is the required second law of reflection.

Explanation of refraction on the basis of Huygen's theory of secondary wavelets

Ques.- Explain the phenomenon of refraction on the basis of Huygen's theory of secondary wavelets.

Ans.- Consider a refracting surface XY separatng a rarer and denser medium in which the velocity light be c_1 and c_2 respectively. a parallel beam of light is allowed to incidence on it.If PA be the incident ray and AB be the incident wavefront then its point A will strike the refracting surface first and point B will strike the refracting surface after time t then the distance BC is equal to c_1t. In this duration t the secondary wavelets generated from point A will cover a distance AD equal to c_2t in the denser medium. By taking A as centre and c_2t as radius if a sphere in drawn then the sphere thus obtained will represent the location of secondary wavelet generated from point A at time t .The tangential plane CD drawn from point C on the secondary wavelet generated from point A will give us the location of all those points which lie on wavefront AB, t time before therefore the plane CD will represent refracted wavefront and the ray CQ which is drawn perpendicular to refracted wavefront represent refracted ray. From figure it is clear that the incident ray, refracted ray and the normal all lie in the same plane. This is the required first law of refraction.

$\because$ If the angle of incidence be i and the angle of refraction be r then,

$$\angle PAM = i \qquad \& \ \angle NCQ = r$$

$$\angle CAB = \angle MAC - \angle BAM$$
$$= 90^0 - (90^0 - i) = i$$

Again, $\angle DCN = \angle DCQ - \angle NCQ = 90^0 - r$

$\therefore \angle DCA = \angle NCA - \angle DCN$
$$= 90^0 - (90^0 - r) = r$$

In rt. ΔABC

$$\sin i = \frac{BC}{AC} \quad \text{---(i)}$$

and in rt. ΔADC

$$\sin r = \frac{AD}{AC} \quad \text{---(ii)}$$

Dividing rel'n (i) by (ii), we get

$$\frac{\sin i}{\sin r} = \frac{BC/AC}{AD/AC}$$

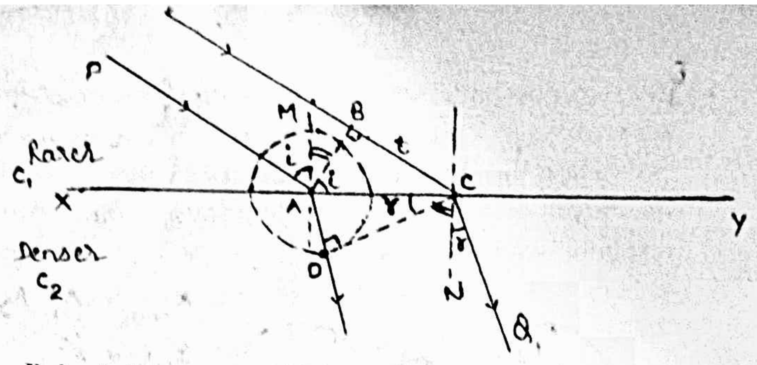
or $$\frac{\sin i}{\sin r} = \frac{BC}{AD}$$

$\because$ $BC = c_1 t$ and $AD = c_2 t$

$$\therefore \frac{\sin i}{\sin r} = \frac{c_1 t}{c_2 t}$$

or $$\frac{\sin i}{\sin r} = \frac{c_1}{c_2}$$

This is the Snell's law which is the reqd. second law of refraction



Principle of superposition of waves

Ques.- State principle of superposition of waves.

Ans.-- When two or more waves reach at a particle of medium then the resultant displacement of the particle of medium is equal to the vector sum of its displacements under the effect of individual waves. This is the principle of superposition of waves.

If under the effect n waves the individual displacement of a particle of medium be y_1, y_2, y_3 ....y_n. Then in accordance with principle of superposition of waves the resultant displacement of particle of medium under the effect of all wave will be given by

$$\vec{y} = \vec{y_1} + \vec{y_2} + \vec{y_3} \ldots\ldots\ldots\ldots + \vec{y_n}$$

The principle of superposition of waves is valid for all types of waves (mechanical as well as electromagnetic waves) except those waves whose amplitude of oscillation is very large namely laser waves and shock waves generated from supersonic jets and earthquakes.

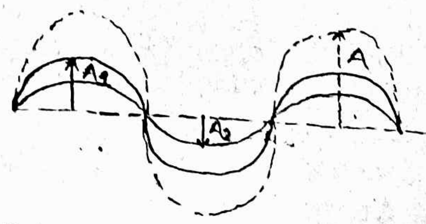
Interference of light

Ques- What do you mean by interference of light ? State its types.

Ans.-The phenomena of redistribution of energy in a medium due to superposition between the two waves having the same frequency and constant initial phase difference is called interference of light. There are two types of interference-

(1) Constructive interference- When two waves superpose with one another in such a manner that the crest of one wave lies over the crest of another wave and the trough of one wave lises over the trough of another wave that is the two waves are in the same phase .Then. the interference between them is said to be constructive.

In this case the two waves support each other as a result of which the amplitude and the intensity of resultant wave is maximum.



$A_{max} = A_1 + A_2$

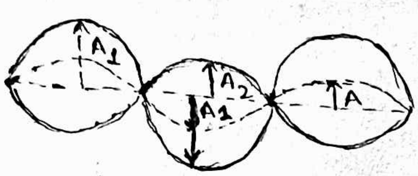
$I_{max} \propto A_{max}^2$

(2)Destructive interference- When two waves superpose one another in such a manner that the crest of one wave lies over the trough of another wave and the trough of one wave lies over the crest of another wave that is the two waves are out of phase. Then the

Interference between the two waves is said to be destructive.

In this case the two waves cancel out each other's effect. As a result of which the amplitude and the intensity of resultant wave is minimum.

$A_{min} = A_1 - A_2$



$I_{min} \propto A_{min}^2$

Conditions for permanent interference

Ques- What are the conditions for permanent interference ?

Ans.-(i) The frequency of two source should be same- If it is not so then at any point in the medium the phase and hence the intensity of resultant wave will change continuously with time and as a result of which there will be no interference.

(ii) The initial phase difference between the two waves should remain constant- If it is not so then at any point of medium the phase and hence the intensity of resultant wave will change continuously with time. As a result of which there will be no interference.

Thus for interference the two sources should be coherent.

Coherent sources

Ques- What are coherent sources ?

Ans.- Those sources whose frequencies are same and initial phase difference between them is also constant are called coherent sources. For interference the two source should be coherent.

Note- From a light source emission of light takes place due to transition of electrons from higher energy level to lower energy level. As there are infinite number of atoms in a light source whose behaviour is arbitrary which cannot be controlled by any mode therefore there will always be intial phase difference between two independent light sources which varies with time.Hence two independent light source con not be coherent.

Note-Exception two independent laser sources can be coherent.

Conditions for constructive and destructive interference

Ques- Derive an expression for the intensity at any point in the phenomenon of interference. Hence obtain the conditions for constructive and destructive interference.

Ans.- Consider two waves of same frequency but diff amplitudes A_1 and A_2 which are propagating along positive direction of x-axis. If the ~~initial~~ phase difference between the two waves be ϕ then under the effect of two waves the displacement of the particle of medium which lies at the origin is given by.

$$y_1 = A_1 \sin 2\pi\nu t = A_1 \sin \omega t \quad \text{----(i)}$$

$$y_2 = A_2 \sin (2\pi\nu t + \phi) = A_2 (\sin \omega t + \phi)$$

The above relations represent the equations of two waves. If the superposition takes place between these two waves then according to the pricinple superposition of wavesdisplacement of the particle lying at the origin will be $y = y_1 + y_2$

or $y = A_1 \sin \omega t + A_2 \sin (\omega t + \phi)$

or $y = A_1 \sin \omega t + A_2 (\sin \omega t \cos\phi + \cos\omega t \sin\phi)$

or $y = A_1 \sin \omega t + A_2 \sin \omega t \cos\phi + A_2 \cos\omega t \sin\phi$

or $y = (A_1 + A_2\cos\phi) \sin \omega t + A_2 \cos\omega t \sin\phi$

Let $A_1 + A_2\cos\phi = A \cos\theta$ ------(iii)

and $A_2 \sin\phi = A \sin\theta$ ------(iv)

where A and θ are any two constants.

Then $y = A\cos\theta \sin\omega t + A \sin\theta \cos\omega t$

or $y = A (\sin\omega t \cos\theta + \cos\omega t \sin\theta)$

or $y = A\sin (\omega t + \theta)$ ------(v)

This is the reqd. equation of resultant wave whose amplitude is A and phase difference with the first wave is θ.

Squaring and adding both the sides of eqn (iii) and (iv). we get

$$(A_1 + A_2\cos\phi)^2 + (A^2\sin\phi)^2 = (A\cos\theta)^2 + (A\sin\theta)^2$$

or $A_1^2 + A_2^2\cos^2\phi + 2A_1A_2\cos\phi + A_2^2\sin^2\phi = A^2\cos^2\theta + A^2\sin^2\theta$

or $A_1^2 + A_2^2 + 2A_1A_2\cos\phi = A^2$

or $A^2 = A_1^2 + A_2^2 + 2A_1A_2\cos\phi$

Taking square root

$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi} \quad \text{------(vi)}$$

This is the reqd. exp of amplitude of resultant wave.

Diving equation (iv) by (iii) we get

$$\frac{A_2\sin\phi}{A_1 + A_2\cos\phi} = \frac{A\sin\theta}{A\cos\theta}$$

or $\dfrac{A_2\sin\phi}{A_1 + A_2\cos\phi} = \tan\theta$

or $\theta = \tan^{-1}\left[\dfrac{A_2\sin\phi}{A_1 + A_2\cos\phi}\right]$ ------(vii)

This is the reqd. expression of phase difference between the rusultant wave and the first wave. If the intensities of interfering waves be I_1 and I_2

then $I_1 \propto A_1^2 \Rightarrow I_1 = kA_1^2$

and $I_2 \propto A_2^2 \Rightarrow I_2 = kA_2^2$

Then, the intensity of the resultant wave

$I \propto A^2$

or $I = kA^2$

or $I = k(A_1^2 + A_2^2 + 2A_1A_2\cos\phi)$

or $I = kA_1^2 + kA_2^2 + 2A_1A_2 k\cos\phi$

or $I = kA_1^2 + kA_2^2 + 2\sqrt{k^2A_1^2A_2^2}\cos\phi$

or $I = kA_1^2 + kA_2^2 + 2\sqrt{k^2A_1^2A_2^2}\cos\phi$

or $I = I_1 + I_2 + 2\sqrt{I_1I_2}\cos\phi$ -----(viii)

Conditions for constructive interference- At the point of constructive interference intensity of resultant wave is maximum. This can be possible when the value of $\cos\phi$ is maximum.

$\because (\cos\phi)_{max} = 1$

$\cos\phi = \cos 0°$

$\phi = 2n\pi \pm 0$

or $\phi = 2n\pi$

where $n = 0,1,2,3,..............$

$\therefore \phi = 0, 2\pi, 4\pi, 6\pi, ...$

$\because$ Path difference $= \frac{\lambda}{2\pi}$ Phase difference

or $\Delta x = \frac{\lambda}{2\pi}\phi$

$\therefore \Delta x = \frac{\lambda}{2\pi} 2n\pi$

or $\Delta x = 2n\frac{\lambda}{2}$

$\therefore \Delta x = 0, \frac{2\lambda}{2}, \frac{4\lambda}{2}, \frac{6\lambda}{2},$

Thus, when the phase difference between the interfering waves 0 or even multiple of π and the path difference between them is 0 or even multiple of $\lambda/2$ then the interference between the waves is said to be constructive.

In this situation

$$A_{max} = \sqrt{A_1^2 + A_2^2 + 2A_1A_2(1)}$$
$$= \sqrt{A_1^2 + A_2^2 + 2A_1A_2}$$
$$= \sqrt{(A_1 + A_2)^2}$$
$$= A_1 + A_2$$

and
$$I_{max} = I_1 + I_2 + 2\sqrt{I_1I_2(1)}$$
$$= \sqrt{I_1}^2 + \sqrt{I_2}^2 + 2\sqrt{I_1}\sqrt{I_2}$$
$$= (\sqrt{I_1} + \sqrt{I_2})^2$$

Here $I_{max} > I_1 + I_2$

<u>Conditions for destructive interfernce</u> -At the points of destructive interference the intensity of resultant wave is minimum. This can be possible when the value of $\cos\phi$ is minimum

$\because (\cos\phi)_{max} = -1$

$\therefore \cos\phi = \cos\pi$

$\therefore \phi = 2n\pi \pm \pi$

or $\phi = (2n \pm 1)\pi$

or $\phi = (2n-1)\pi$

where $n = 1,2,3$

$\therefore \phi = \pi, 3\pi, 5\pi, ...$

$\because$ Path difference $= \frac{\lambda}{2\pi}$ Phase difference

or $\Delta x = \frac{\lambda}{2\pi}(2n-1)\pi$

or $\Delta x = (2n-1)\frac{\lambda}{2}$

$\therefore \Delta x = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}, ...$

Thus, when the phase difference between the interfering waves is an odd multiple of π and the path difference between them is an odd multiple of $\lambda/2$ then the interference between the waves is said to be destructive.

In this situation

$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2(-1)}$$
$$= \sqrt{(A_1 - A_2)^2}$$
$$= A_1 - A_2$$

and
$$I_{min} = I_1 + I_2 + 2\sqrt{I_1I_2}(-1)$$
$$= \sqrt{I_1}^2 + \sqrt{I_2}^2 - 2\sqrt{I_1I_2}$$
$$= (\sqrt{I_1} - \sqrt{I_2})^2$$

Here $I_{min} < I_1 + I_2$

<u>Note</u>- $$\frac{A_{max}}{A_{min}} = \frac{A_1 + A_2}{A_1 - A_2} = \frac{\frac{A_1}{A_2} + 1}{\frac{A_1}{A_2} - 1}$$

and $$\frac{I_{max}}{I_{min}} = \frac{(\sqrt{I_1} + \sqrt{I_2})^2}{(\sqrt{I_1} - \sqrt{I_2})^2} = \frac{\left(\sqrt{\frac{I_1}{I_2}} + 1\right)^2}{\left(\sqrt{\frac{I_1}{I_2}} - 1\right)^2}$$

<u>Young's double slit experiment</u>

<u>Ques.</u>- Describe Young's double slit experiment showing the phenomenon of interference.

<u>Ans.</u>- The experimental demonstration of interference was first performed by Thomas Young in the year 1801. Through this experiment it was established that light has wave nature.

<u>Experimental arrangement</u>- In the experiment performed by Thomas Young there is a narrow rectangular slit which is illuminated by a monochromatic source of light. At equal distance from slit S there are two narrow rectangular slits S_1 and S_2. Infront of the two slits there is a screen XY.

<u>Explanation</u>- When the slit S is illuminated by the monochromatic source of light then the propagation of wave takes place from the slit S towards the two slits S_1 and S_2 in the form of wavefronts (in fig. the wavefronts corresponding to crests are shown by bold curves and the wavefronts corresponding to trough are shown by dotted curves) when these wavefronts reaches the two slits then the slit S_1 and S_2 behave like the source of secondary wavelets which propogates again in the form of crests and troughs. When these wavefronts superpose in the medium then at those points where the crest of one wave falls on the crest of another wave trough of one wave falls on the trough of another wave the intensity of resultnat wave is maximum. At these points constructive interference takes place and are represented by (•) in the fig. Those points of the medium where the crest of one wave falls in trough of another wave and the trough of one wave falls on the crest of another wave the intensity of resultant wave is minimum. At these points destructive interference takes place and are shown by (×) in the fig.

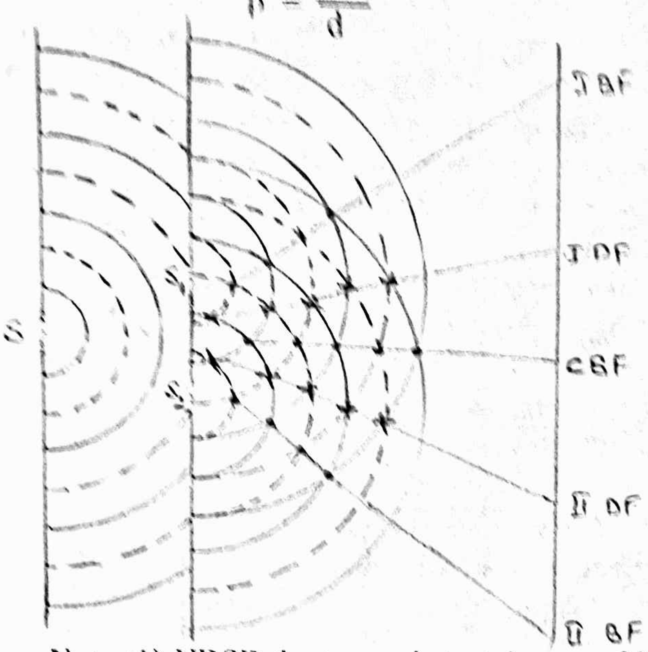
<u>Observations</u>-

(i) On the screen XY alternate bright and dark spots are obtained which are called bright and dark fringes and are represented by B and D.

(ii) The distance between two successive bright and dark fringes always remains the same which is called fringe width.

(iii) If the distance between two slits be d the wavelength of light be λ and the distance of the screeen from the two slits be D then fringe width

$$\beta = \frac{\lambda D}{d}$$



Note-(1) YDSE the two cohetent source of light are real.

(2) In the actual experiment performed by Young instead of rectangular slits he used pin holes and instead of monochromatic source of light he used sunlight.

Expression of fringe width

Ques.- Deduce an expression for fringe width in Young's double slit experiment. How can the wavelength of monochromatic light be found by this experiment?

Ans.- Consider a narrow rectangular slit which is illuminited with a monochromatic light of wavelength λ. At equal distances from it there are two narrow rectangelur slits S_1 and S_2 such that the distance between the two slits be d. Infront of the two slits there is a screen XY which is at a distance D from the two slits. If the perpendicular bisector of the line joining the two slits S_1 and S_2 is drawn then it intersects the screen XY at point O. The point O behave like the centre of the screen.

Consider point P on the screen which is at a distance y from its centre. The waves coming from the two slits will reach the point P after covering distances S_1P and S_2P. Draw perpendiculars S_1M and S_2N from the two slits on the screen.

In right ΔS_1MP by pythagorus theorem

$$S_1P^2 = S_1M^2 + MP^2$$
$$= S_1M^2 + (OP - OM)^2$$
$$= D^2 + \left(y - \frac{d}{2}\right)^2 \quad \text{---------(i)}$$

and in right ΔS_2NP by pythagorus theorem

$$S_2P^2 = S_2N^2 + NP^2$$
$$= D^2 + (OP + ON)^2$$
$$= D^2 + \left(y + \frac{d}{2}\right)^2 \quad \text{---------(ii)}$$

Subtracting equation (i) from (ii)

$$S_2P^2 - S_1P^2 = \left[D^2 + \left(y + \frac{d}{2}\right)^2\right] - \left[D^2 + \left(y - \frac{d}{2}\right)^2\right]$$
$$= \left[D^2 + y^2 + \frac{d^2}{4} + 2y\frac{d}{2}\right] - \left[D^2 + y^2 + \frac{d^2}{4} - 2y\frac{d}{2}\right]$$
$$= 2yd$$

or $\quad (S_2P - S_1P)(S_2P + S_1P) = 2yd$

or, $\quad S_2P - S_1P = \dfrac{2yd}{S_2P + S_1P}$

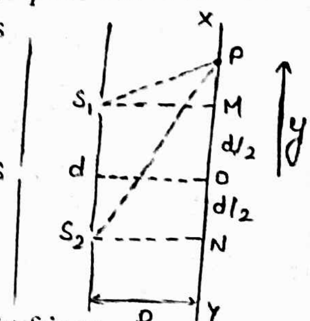
Here $S_2P \simeq S_1P \simeq D$ (having error but only 0.5

$\therefore$ Path difference $S_2P - S_1P = \dfrac{2yd}{D + D}$

or $\quad \Delta x = \dfrac{2yd}{2D}$

or $\quad \Delta x = \dfrac{yd}{D}$

For constructive interference the path difference between the two interfering waves



$$\Delta x = 2n\frac{\lambda}{2}$$

or $\quad \dfrac{yd}{D} = n\lambda$

or $\quad y = \dfrac{n\lambda D}{d}$

where n = 0, 1, 2, 3..........

When n = 0, $y_0 = 0$ central bright fringe

$n = 1, y_1 = \dfrac{\lambda D}{d}$ first bright fringe

$n = 2, y_2 = \dfrac{2\lambda D}{d}$ second bright fringe

— — — — — — — — — — —

and $n = n, y_n = \dfrac{n\lambda D}{d}$ n^{th} bright fringe

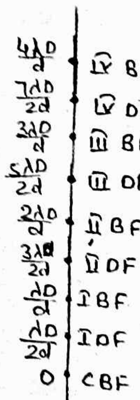
For destructive interference the path difference between the two interfering waves

$$\Delta x = (2n - 1)\frac{\lambda}{2}$$

or $\quad \dfrac{yd}{D} = (2n - 1)\dfrac{\lambda}{2}$

or $\quad y = (2n - 1)\dfrac{\lambda D}{2d}$

where n = 1, 2, 3.............



When $n = 1, y_1' = \dfrac{\lambda D}{2d}$ first dark fringe

$n = 2, y_2' = \dfrac{3\lambda D}{2d}$ second dark fringe

$n = 2, y_3' = \dfrac{5\lambda D}{2d}$ third dark fringe

— — — — — — — — — — —

and $n = n, y_n' = (2n-1)\dfrac{\lambda D}{2d}$ n^{th} dark fringe

In YDSE, Intensity of light coming from slit & width of the slit.

Width of bright fringe- The distance between two successive dark fringes is equal to the width of bright fringe.
ie. Width of bright fringe = Distance of (n + 1)th dark fringe - Distance of nth dark fringe

or $\beta_1 = y'_{n+1} - y'_n$

$$= \left[2(n+1) - 1\right]\frac{\lambda D}{2d} - (2n-1)\frac{\lambda D}{2d}$$

$$= \left[2n + 2 - 1 - 2n + 1\right]\frac{\lambda D}{2d} = \frac{\lambda D}{d}$$

Since the width of bright fringe is independent of the value of n therefore the width of all the bright fringes will be the same.

Width of Dark fringe- The distance between two succesive bright fringes is equal to the width of dark fringe.

i.e. Width of dark fringe = Distance of (n+1)th bright fringe - Distance of nth bright fringe

or $\beta_2 = y_{n+1} - y_n$

$$= (n+1)\frac{\lambda D}{d} - \frac{n\lambda D}{d}$$

$$= \frac{n\lambda D}{d} + \frac{\lambda D}{d} - \frac{n\lambda D}{d} = \frac{\lambda D}{d}$$

Since the width of dark fringe is independent of the value of n therefore the width of all the dark fringes will be the same.

From the above analysis it can be concluded that in interference pattern the widths of all bright and dark fringes are the same which is equal to $\frac{\lambda D}{d}$

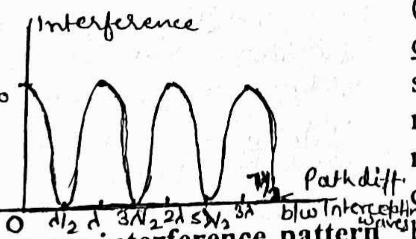
i.e. fringe width $\boxed{\beta = \frac{\lambda D}{d}}$

If the experimental values of β, D and d are known then by using the above relation the wavelength of light can be calculated.

Note- Intensity distribution curve for interference:-

$I_{max} = (\sqrt{I_1} + \sqrt{I_2})^2$
$= (\sqrt{I_0} + \sqrt{I_0})^2$
$= 4I_0$

$I_{min} = (\sqrt{I_0} - \sqrt{I_0})^2$
$= 0$



★★ Effect of various factors on interference pattern

Ques.- What is the effect of variopus factors on interference pattern when-

(i) white light is used in place of monochromatic light.
(ii) whole of the arrangement of YDSE is dipped in water.
(iii) one of the slit is partially closed.
(iv) a cellophane paper is placed infront of one of the two slits.
(v) one of the slit is covered with blue and the other slit is covered with red cellophane paper.
(vi) the distance between the two slits is made less than the wavelength of light.
\$(vii) the source slit is moved closer to the double slit plane.
\$(viii) the width of the source slit is increased.

Ans.-(i) When white light is used in place of monochromatic light - In this situation the central bright fringe will be white and all other fringes will be coloured (the fringe closest on either side ot the central white fringe is red and the farthest will appear blue) and due to overlapping of different colours of higher order fringes interference pattern will come unclear.

(ii) When whole of the arrangement of YDSE is dipped in water- In this situation the fringe width will decrease.

In air fringe width $\beta_a = \frac{\lambda_a D}{d}$ -----(i) $\lambda = \frac{c}{\nu}$

in water fringe width $\beta_w = \frac{\lambda_w D}{d}$ -----(ii)

Dividing relationn (ii) by (i)

$$\frac{\beta_w}{\beta_a} = \frac{\lambda_w}{\lambda_a}$$

$\because \lambda_w < \lambda_a$

$\therefore \beta_w < \beta_a$

(iii) When one of the slit is partially closed - In this situation the intensity of light coming from the slit which is partially closed decreases as a result of which the resultant intensity of light at the point constructive interference decreases and at the point of destructive interference increases. Due to which bright fringe becomes less bright and dark fringe becomes less dark.

(iv) When a cellophane paper is placed infront of one of the two slits- In this situation due to the absorption of light by the cellophane paper the intensity of light coming from one of the slit decreases. Due to which bright fringe becomes less bright and dark fringe becomes less dark.

(v) When one of the slit is covered with blue and the other slit is covered with red cellophane paper- In this situation if the source of light be white then the light of red colour will be obtained from the slit infront of which red cellophane paper is placed and the light of blue colour will be obtained from the slit infront of which blue cellophane paper is placed. Since the colours of light coming from the two slits are different therefore their frequency will also be different and interference pattern will not be obtained.

(vi) When the distance between the two slits is made less than the wavelength of light (ie. d < λ)- In this situation $\beta = \frac{\lambda D}{d}$

or $\frac{\beta}{D} = \frac{\lambda}{d}$

$\therefore 1 < \frac{\lambda}{d}$

$$\therefore \quad 1 < \frac{\beta}{D}$$

or $\quad \beta > D$

Therefore in this situation the central bright fringe will cover the whole screen and the interference pattern will not be obtained.

$(vii) <u>When the source slit is moved closer to the double slit plane</u> - If the size of the source be s and S its distance from the plane of two slits then for interference fringes to be seen, the condition $s/S < \lambda/d$ should be satisfied otherwise interference patterns produced by different parts of source overlap and no fringes are seen. As S decreases the interference pattern gets less and less sharp and when the source is brought so close that the condition $s/S < \lambda/d$ is not valid then the fringes will disappear.

$(viii) <u>When the width of the source slit is increased</u> - If the size of the source be s and S its distance from the plane of two slits then for interference fringes to be seen, the condition $s/S < \lambda/d$ should be satisfied otherwise interference patterns produced by different parts of source overlap and no fringes are seen. As s increases the interference pattern gets less and less sharp and when the source slit becomes so wide that the condition $s/S < \lambda/d$ is not valid then the fringes will disappear.

Interference and the law of conservation of energy

Ques.- Show that the phenomenon of interference is in accordance with the law of conservation of energy.

Ans.- Suppose the intensity of light coming from the two coherent sources be I_1 and I_2. If the two waves do not interfere then at any point of the medium the resultant intensity of light will be $I_1 + I_2$. But if the two wave interfere then due to constructive and destructive interference at one point the intensity of light will become maximum and at other point the intensity of light wil become minimum.

$$I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2}$$

and $$I_{min} = I_1 + I_2 - 2\sqrt{I_1 I_2}$$

Intensity of light at any point of medium

$$I_{av} = (I_1 + I_2 + 2\sqrt{I_1 I_2} + I_1 + I_2 - 2\sqrt{I_1 I_2})/2$$

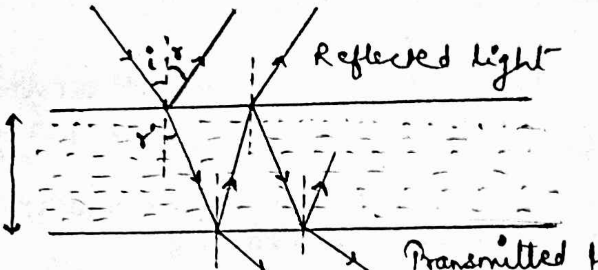
$$= I_1 + I_2$$

Thus in the phenomenon of interference neither creation nor destruction of energy takes place only the transfer of energy takes place from the points of destructive interference to the point of constructive interference i.e.only the redistribution of energy takes place in the phenomena of interference.Thus, the phenomena of interference is in accordance with law of conservation of energy.

Colours of thin films

Ques.- Why colours are seen in thin films? Explain on the basis of phenomenon of interference.

Ans.- When sunlight falls on soap bubble or on thin layer of oil formed on the surface of water, whose thickness is of the order of wavelength of light (1000 or 10^{-6}m). Then due to interference between the light waves reflected from the upper and lower surface of thin film different colours are observed in it. At any point of the film only that colour is seen for which the condition of constructive interference is fulfilled. The colour which is seen at any point depends upon the nature of material of film, the thickness of film and the point of observation.

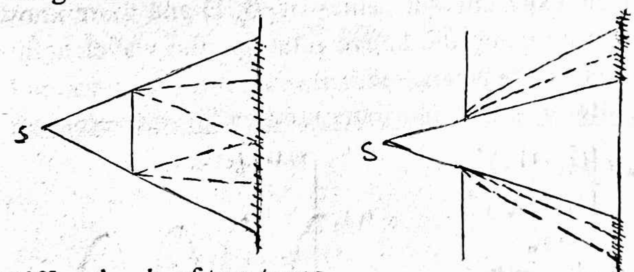


Note- The different colours are observed in film only when its thickness is of the order of wavelength of light. If its thickness is less than the wavelength of light then it appears dark and if its thickness is greater than the wavelength of light then it appears white.

Diffraction

Ques.- What is diffraction of light ? Distinguish between Fresnel's diffraction and Fraunhoffer's diffraction.

Ans. - The phenomena of bending of light across the edge of a narrow slit or an obstacle whose size is of the order of wavelenght of light and the encroachment of the geometrical shadow is called diffraction.



<u>Diffraction is of two types</u>-

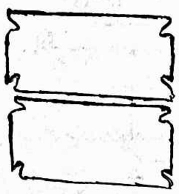
(i)<u>Fresnel diffraction</u>- In Fresnel diffraction the obstacle is at finite distance from a point or line source of light ,as a result of which the wavefront incident on the obstacle are either spherical or cylindrical .

In Fresnel diffraction the different points of a wavefront reach the obstacle at different times. Therefore there is initial phase difference between the secondary wavelets generated from the different points of the obstacle.Diffraction at a straight edge is an example of Fresnel diffraction.

(ii)<u>Fraunhoffer diffraction</u> -In Fraunhoffer diffraction the obstacle is at infinite distance from a point or line source of light, as a result of which wavefront incident on the obstacle is plane.

In Fraunhoffer diffraction the diffent points of a wavefront reach the obstacle simultaneously therefore there is no initial phase difference between the secondary wavelets generated from the different points of the obstacle. Diffraction at a single slit is an example of Fraunhoffer diffraction.

Note- Seeing the single slit diffraction pattern - When two razer bladed are placed very closed to one another to form a narrow slit then on illuminating the slit with a source of light alternate dark and bright bands of diffraction pattern are observed.



Experimental demonstration of diffraction at a straight edge-

Ques.-Describe the experiment of diffraction at a straight edge under the following headings- (i) Arrangement (ii) Observations (iii) Intensity distribution curve

Ans.-Arrangement- In figure S in a slit which is illuminated by a monochromatic source of light. Infront of the slit there is a sharp edge PQ behind which a screen XY is placed.

Observation - According to the linear propagation of light the part OX of the screen should be iluminated and the part OY should be completely dark but on the screen it is found that-

(i) The part OY is not completely dark but some part of it namely OA is illuminated. The intensity of light decreases from O to A and become zero at A, beyond A the screen is completely dark.

(ii) The part OX is not uniformly illuminated but alternate dark and bright fringes are obtained in this part.

(iii) When we move from O to X intensity of bright fringes and their width both decreases and the intensity of dark fringes increases.

(iv) Beyond point X uniform illumination is obtained on the screen.

* Polarisation :- The phenomena of restricting the vibrations of electric field vector of light in one direction ⊥r to the direction of propagation of light is called Polarisation.

* Unpolarised Light :- The light in which vibrations of EF vectors take place in all possible direction, in plane ⊥ to direction of propagation of light is called unpolarised light.

Conclusion- From the above experiment it can be concluded that light does not travel in straight line and the bending of light takes place accors the edge of obstacle and whose size is of the order of wavelength of light.

Note- Seeing the diffraction at a straight edge - When a small circular obstacle is placed in the path of light from a distant source due to diffraction of light from its edges a bright spot is observed at the centre of shadow.

Diffraction between interference and diffraction

Ques.-write some difference between interference and diffraction.

Ans.-

Interference	Diffraction
(i) Interference takes place due to the superposition between the secondary waves which come from the two coherent sources.	(i) Diffraction takes place due to the superposition between the secondary waves generated from the same wavefront of a wave.
(ii) In interference pattern width of all fringes is same.	(ii) In diffraction pattern width of central bright fringe is twice the width of any secondary maxima.
(iii) In interference pattern the intensity of all bright fringes is same	(iii)In diffraction pattern intensity of bright fringes decreases as we move away from central bright fringe.
(iv) In interference pattern the dark fringes are compelety dark as a result of which contrast between bright and dark fringes is good.	(iv) In diffcration pattern dark fringes are not completely dark as a result of which contrast between bright and dark fringes is poor .

Polarisation, unpolarised light & polarised light

Ques.-(a) What is polarisation ?

(b) What do you mean by unpolarised light & polarised light? Give their pictorial representations.

Ans.-(a) The phenomena of lack of symmetry of light wave about the direction of propagation is called polarisation.

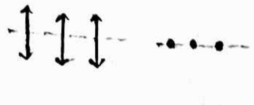
(b) The light in which the electric field vectors vibrate in all possible directions which are perpendicular to direction of propagation of light is called unpolarised light

The light in which the electric field vectors vibrate in a plane perpendicular to direction of propagation of light is called polarised light.

Representation of unpolarised and polarised light-

In unpolarised light the vibration of electric field vectors taken place in all possible directions where as in unpolarised light the vibrations of electric vectors takes place in a plane. They are represented as

(i) Polarised :-



(ii) unpolarised :-

(iii) Partially polarised :-

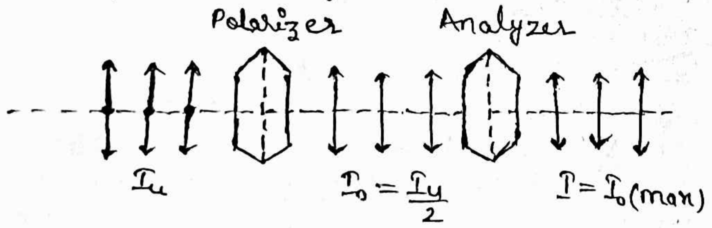
In the figure arrow represent those electric field vectors which vibrate in the plane of the paper and dots represent those electric field vectors which vibrate in the plane which is perpendicular to the plane of the paper.

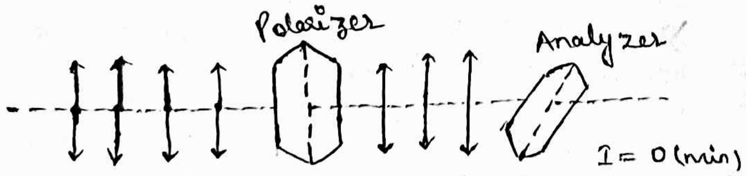
Experimental demonstration of polarisation of light

Ques.-Describe an experiment demonstrating the polarisation of light.

Ans.- In unpolarised light vibrations of electric field vectors take place symmetrically in all possible directions which are perpendicular to the direction of propagation of light. When this light is allowed to fall on tourmaline crystal then only those electric field vectors pass through the crystal whose vibrations are parallel to its axis and the remaining electric field vectors whose vibrations are perpendicular to its axis are absorbed by the crystal. As the vibrations of electric field vectors in transmitted light takes place only in one direction in a plane parallel to the direction of propagation of light therefore it is called plane polarised light. When the plane polarised light emerging from the first tourmaline crystal is allowed as fall on a second crystal whose axis is parallel to the first crystal then all vibrations emerging from the first crystal will also pass through the second crystal and the intensity of transmitted light will be maximum. But if the second crystal is rotated through 90° then its axis will become perpendicular to the first crystal in this situation vibrations coming from first crystal will be absorbed by the second crystal and the intensity of transmitted light becomes minimum, in this position the two crystal are said to be crossed. The first crystal which is responsible for the polarisation of light is called polariser and the second crystal with the help of which the light emerging from the first crystal is analysed is called analyser.

Applicable for transverse waves





Note-1. Transverse waves are those under whose effect particles of medium vibrate in the perpendicular direction of the direction of propagation of wave and longitudinal waves are those in which the particles of the medium vibrate in the direction of propagation of wave.

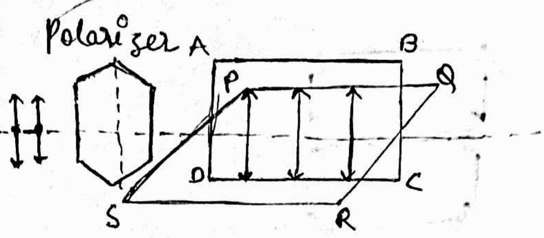
Note-2. Longitudinal waves connot be polarised where as transverse waves can be polarised therefore polarisation is the phenomena on the basis of which we can differentiate between longitudinal and tranverse waves

Planes of vibration & polarisation

Ques.-With the help of diagram explain the plane of vibration and the plane of polarisation.

Ans.- Plane of vibration - The plane which contains the vibrations of electirc field vectors and the direction of propagation of plane polarised light is called plane of vibration. In figure ABCD is the plane of vibration.

Plane of polarisation - The plane which is perpendicular to the plane of vibration and which contains the direction of propagation of plane polarised light is called plane of polarisation. In figure PQRS is the plane of polarisation



Polarisation by reflection (Brewster's law)

Ques.-Explain polarisation by reflection. State and prove Brewster's law of polarisation.

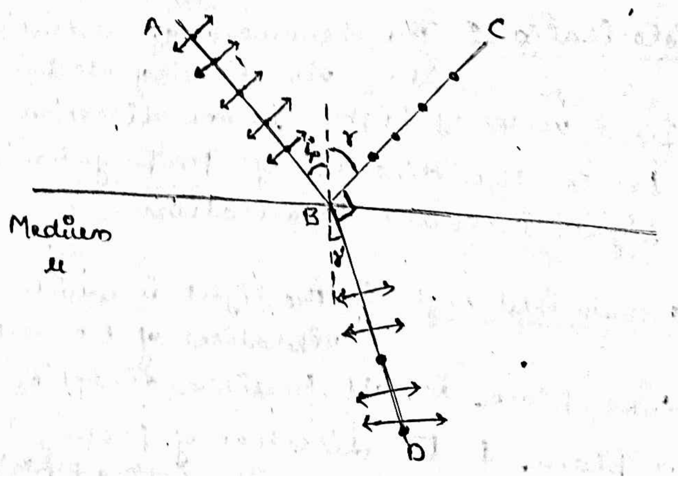
Ans.- In the year 1808, Malus experimentally found that the light reflected from the surface is partially polarised. Later in the year 1811 Brewster experimentally found that the amount of polarisation of reflected light depends upon the angle of incidence and for a particular angle of incidence the reflected light is completely polarised which contains only those electric field vector whose vibration is the perpendicular plane to incidence. This angle of incidence is called angle of polarisation and is denoted by i_p.

Brewster's law states that the tangent of angle of polarisation is equal to the refractive index of the reflecting medium

i.e. $\tan i_p = \mu$

where μ is the refractive index of the reflecting mediem.

Suppose a beam AB of unpolarised light falls on reflecting surface at angle of polarisation then it is found that the reflected light BC is completely polarised and the refracted light BD is partially polarised and they are mutually perpendicular.



$\angle CBD = 90^0$

If the angle of reflection be r and the angle of refraction r'

then $r + \angle CBD + r' = 180^0$

or $r' + r = 90^0$

or $r' = 90^0 - r$

From Snell's law

$$\frac{\sin i_p}{\sin r'} = \mu$$

or $$\frac{\sin i_p}{\sin (90^0 - r)} = \mu$$

or $$\frac{\sin i_p}{\cos r} = \mu$$

or $$\frac{\sin i_p}{\cos i_p} = \mu \quad (\because r = i_p)$$

or $$\boxed{\tan i_p = \mu}$$

This relation is known as Brewster's law.

Polarisation by scattering

Ques.-Explain polarisation by scattering.

Ans.- When clear sky is seen through tourmaline crystal then on rotating the crystal it is observed that the intensity of light twice becomes maximum and twice becomes minimum i.e. zero. It means that the light received on the earth is plane polarised. Actually the sunlight is scattered by air molecules and becomes plane polarised.

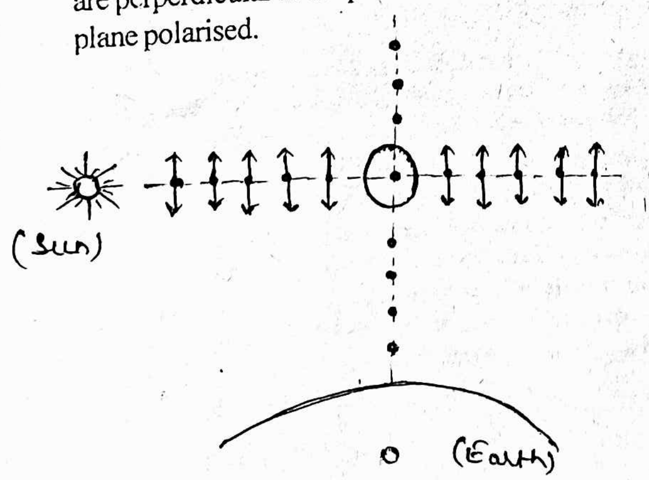
Explanation - When unpolarised sunlight falls on air molecules its elelctirc field vectors vibrate in all directions. Which can be resolved into perpendicular components-

(i) Parallel to the plane of the paper represented by

(ii) Perpendicular to the plane of the paper represented by

When unpolarised light falls on air molecules then the formation of dipole takes place in the molecules which vibrate in both the directions.

The vibrations which are parallel to the plane of paper are in the direction of observer therefore they cannot produce electromagnetic wave in the direction of observer. That is the why, the electromagnetic waves which reach the eyes of observer contains only those vibrations which are perperdicular to the plane of the paper and hence it is plane polarised.



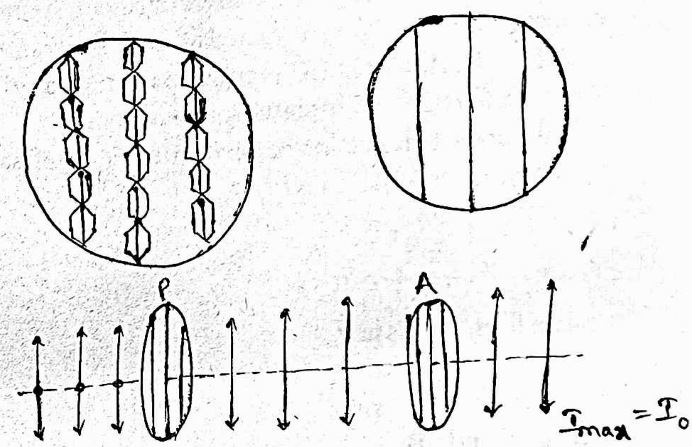
Polaroids

Ques.-What are polaroids? How they polarise an unpolarise beam of light? Explain some uses of polaroids.

Ans.- In 1932 E.H land manufactured a cheap device with the help of which light can be polarised in large cross-section which is called polaroid. It works on the principle of selective absorbtion of light.

Construction - Polaroids are made by arranging small crystals of herapathite on a thin nitrocellulose film in such a manner that their crystalline axes be parallel. In order to impart stability to the film it is mounted between two thin glass plates.

Working - When unpolarised light falls on a polaroid then the vibrations which are parallel to its tranamission axis gets transmitted. If a polaroid P_1 is placed in front of a light source then the light emerging form it is plane polarised. When another polarised P_2 is placed in front of first polaroid so that their transmission axis be parallel then the light polarised by P_1 will pass through P_2 also. If the polarised P_2 is rotated through 90^0 then the transmission axis of two polarised become perpendicular to one another and no light emerges from it. In this situation the two polaroids are said to be crossed.



Uses of polaroids -(i) Polaroids are used in sun glasses to cut the glare of light reflected from horizontal surface.

(ii) Polaroids are used in head lights and windscreen of vehicles to cut the glare of light.

(iii) Polaroids are used in shooting and observing three dimensional movies.

(iv) Polaroids are used in cameras to cut the glare of light while taking photographs of clouds.

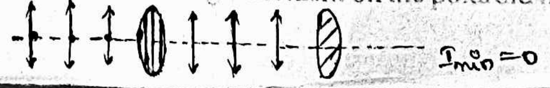
(v) Polaroids are used to control the amount of light which enter in aeroplanes and trains.

(vi) Polaroids are used for the detection of nature of light-

a-If on rotating a polaroid once the intensity of transmitted light does not change then the light incident on the polaroid must be unpolarised.

b-If on rotating a polaroid once the intensity of transmitted light twice becomes maximum and twice becomes minimum but not zero then the light incident on the polaroid must be partially polarised.

c-If on rotating a polaroid once the intensity of transmitted light twice becomes maximum and twice becomes zero then the light incident on the polaroid must be polarised.



Polarisation by reflection (Brewster's law)

Ques.-What is Brewster's law? Prove that for the light incident on a transparent surface at Brewster's angle, the reflected and refacted rays are normal to each other.

Ans.- In the year 1808, Malus experimentally found that the light reflected from the surface is partially polarised. Later in the year 1811 Brewster experimentally found that the amount of polarisation of reflected light depends upon the angle of incidence and for a particular angle of incidence the reflected light is completely polarised which contains only those electric field vector whose vibration is the perpendicular plane to incidence. This angle of incidence is called angle of polarisation and is denoted by i_p.

Brewster's law states that the tangent of angle of polarisation is equal to the refractive index of the reflecting medium

i.e. $\tan i_p = \mu$

where μ is the refractive index of the reflecting mediem.

Suppose a beam AB of unpolarised light falls on reflecting surface at angle of polarisation then the beam is reflected along BC and refracted along BD. It is found that the reflected light BC is completely polarised and the refracted light BD is partially polarised.

If the angle of reflection be r and the angle of refraction r' then from Brewster's law

$$\tan i_p = \mu$$

or $$\frac{\sin i_p}{\cos i_p} = \mu \quad \text{-----(i)}$$

Again form Snell's law

$$\mu = \frac{\sin i_p}{\sin r'} \quad \text{----(ii)}$$

From (i) and (ii)

$$\frac{\sin i_p}{\cos i_p} = \frac{\sin i_p}{\sin r'}$$

or $$\frac{1}{\cos i_p} = \frac{1}{\sin r'}$$

or $\sin r' = \cos i_p$

$\because$ $i_p = r$

$\therefore$ $\sin r' = \cos r$

or $\sin r' = \sin(90^0 - r)$

or $r' = 90^0 - r$

or $r' + r = 90^0$

Here $r + \angle CBD + r' = 180^0$

or $\angle CBD + 90^0 = 180^0$

or $\angle CBD = 90^0$

Thus, at the angle of polarisation the reflected ray is perpendicular to the refracted ray.

* Human eye

The limit of resolution of human eye is equal to that min. angular displacement (separation) b/w two objects at which they can be resolved by human eye.

limit of resolution, $d\theta = 1'$

$$= \frac{1^\circ}{60}$$

$$= \frac{1}{60} \times \frac{\pi}{180} \text{ rad}$$

$$= 0.0003 \text{ rad}$$

Resolving power $= \frac{1}{d\theta} = \frac{1}{0.0003} = 3440$

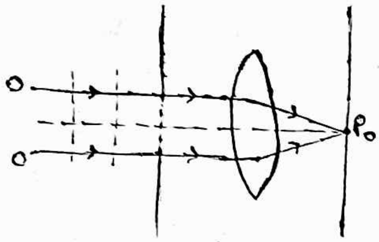
(*) We can check, that light is polarised or not by using toumaline crystal.

* If by rotating a toumaline crystal 360°, if we get twice max. and twice min. light.

: In diffraction at single slit, Intensity of central maxima ∝ (Width of slit)^2 $\Rightarrow I \propto a^2$

Diffraction at a Single Slit : When a plane wavefront falls on a single slit then the generation of secondary wavelets takes place from different points of incident wavefront. Due to interference between these wavelets the diffraction pattern is obtained on the screen.

Central maxima : The formation of central maxima takes place from those wavelets the path difference between whose extremes is zero. Therefore al the wavelets interfere constructively at point P the intensity of central maxima is maximum.



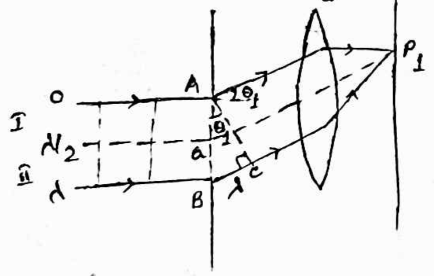
First Secondary Minima ; The formation of first secondary minima takes place from those wavelets whose deviation from the initial direction is θ_1 and path difference between whose extremes is λ. The incident wavefront can be divided into two parts so that the path difference between the secondary wavelets coming from the corresponding points of two parts is $\lambda/2$. Therefore all the wavelets which reach at point P_1 interfere destructively to which the intensity of first secondary minima is zero.

In rt ΔACB $\quad \sin\theta_1 = \dfrac{BC}{AB}$

or $\quad \sin\theta_1 = \dfrac{\lambda}{a}$

If θ_1 be small then $\sin\theta_1 \approx \theta_1$

$\therefore \quad \theta_1 = \dfrac{\lambda}{a}$



Second Secondary Minima : The formation of secondary minima takes place from those wavelets whose deviation from the initial direction is θ_2 and the path difference between whose extremes is 2λ. The incident wavefront can be divided into four parts so that the path difference between the secondary wavelets coming from the corresponding points of 1st and 2nd part is $\lambda/2$. Similar is the situation for 3rd and 4th part. Therefore all the wavelets which reach at point P_2 interfere destructively due to which the intensity of second secondary minima is zero.

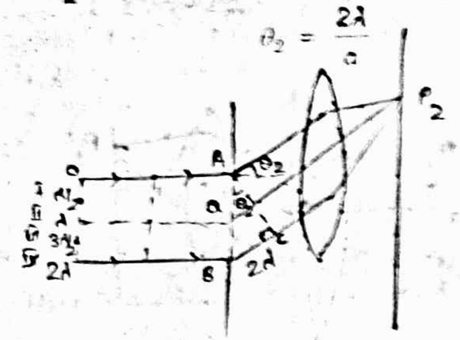
In rt ΔACB $\quad \sin\theta_2 = \dfrac{BC}{AB}$

$\sin\theta_2 = \dfrac{2\lambda}{a}$

or

If θ_2 be small then $\sin\theta_2 = \theta_2$

$\theta_2 = \dfrac{2\lambda}{a}$



n^{th} Secondary Minima : The formation of n^{th} secondary minima takes place from those wavelets whose deviation from the initial direction is θ_n and the path difference between whose extremes is $n\lambda$. The incident wavefront can be divided into $2n$ parts so that the path difference between the secondary wavelets coming from the corresponding points of 1st and 2nd part is $\lambda/2$. Similar is the situation for 3rd and 4th part, 5th and 6th part ...... $(2n-1)^{th}$ and $2n^{th}$ part. Therefore all the wavelets which reach at point P_n interfere destructively due to which the intensity of n^{th} secondary minima is zero.

In rt ΔACB $\quad \sin\theta_n = \dfrac{BC}{AB}$

or $\quad \sin\theta_n = \dfrac{n\lambda}{a}$

If θ_n be small then $\sin\theta_n \approx \theta_n$

$\therefore \quad \theta_n = \dfrac{n\lambda}{a}$

First Secondary Maxima : The formation of first secondary maxima takes place from those wavelets whose deviation from the initial direction is θ_1' and the path difference between whose extremes is $3\lambda/2$. The indent wavefront can be divided into three parts so that the path difference between the secondary wavelets coming from the corresponding points of 1st and 2nd part is $\lambda/2$. Therefore they interfere destructively and cancel out each others effect. But the secondary wavelets coming from the different points of third part interfere constructively at point P_1', therefore the intensity of first secondary maxima is 1/3rd the central maxima.

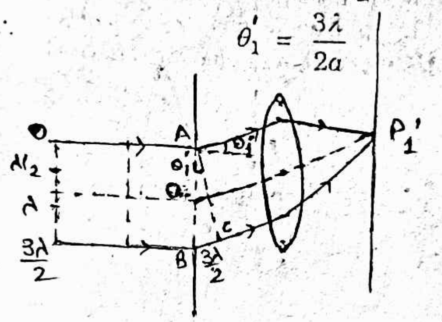
In ΔACB $\quad \sin\theta_1' = \dfrac{BC}{AB}$

or $\quad \sin\theta_1' = \dfrac{3\lambda/2}{a}$

or $\quad \sin\theta_1' = \dfrac{3\lambda}{2a}$

If θ_1' be small then $\sin\theta_1' \approx \theta_1'$

$$\therefore \quad \theta_1' = \frac{3\lambda}{2a}$$



Second Secondary Maxima : The formation of second secondary maxima takes place from those wavelets whose deviation from the initial direction is θ_2' and the path difference between whose extremes is $5\lambda/2$. The incident wavefront can be divided into five parts so that the path difference between the secondary wavelets coming from the corresponding points of 1st and 2nd part is $\lambda/2$. Similar is the situation for 3rd and 4th part. Therefore the wavelets coming from the four parts interfere destructively and cancel each others effect. But the secondary wavelets coming from the different points of fifth part interfere constructively at point P_2'. Therefore the intensity of second secondary maxima is 1/5th the central maxima.

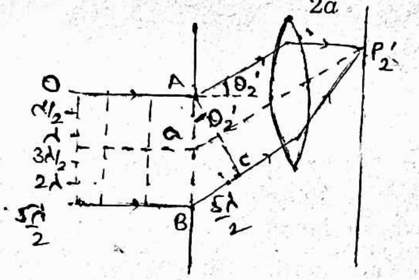
In rt ΔACB $\quad \sin\theta_2' = \dfrac{BC}{AB}$

or $\quad \sin\theta_2' = \dfrac{5\lambda/2}{a}$

or $\quad \sin\theta_2' = \dfrac{5\lambda}{2a}$

If θ_2' be small then $\sin\theta_2' \approx \theta_2'$

$$\therefore \quad \theta_2' = \frac{5\lambda}{2a}$$



n^{th} Secondary Maxima : The formation of n^{th} secondary maxima from those wavelets whose deviation from the initial direction is θ_n' and the path difference between whose extremes is $(2n+1)\lambda/2$. The incident wavefront can be divided into $(2n+1)$ parts so that the path difference between the secondary wavelets coming from the corresponding points of 1st and 2nd part is $\lambda/2$. Similarly is the situation for 3rd and 4th, 5th and 6th ...... $(2n-1)^{th}$ and $2n^{th}$ part. Therefore the wavelets coming from these $2n$ parts interfere destructively and cancel each others effect. But the secondary wavelets coming from the different points of $(2n+1)^{th}$ part interfere constructively at point P_n', therefore the intensity of n^{th} secondary maxima is $1/(2n+1)^{th}$ of the central maxima.

In rt ΔACB $\quad \sin\theta_n' = \dfrac{BC}{AB}$

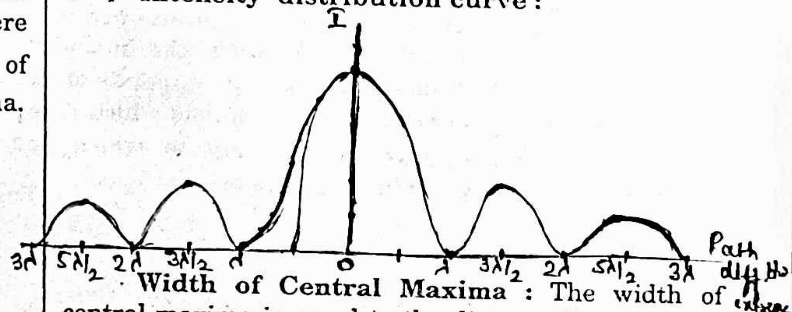
or $\quad \sin\theta_n' = \dfrac{(2n+1)\lambda/2}{a}$

or $\quad \sin\theta_n' = \dfrac{(2n+1)\lambda}{2a}$

If θ_n' be small then $\sin\theta_n' \approx \theta_n'$

$$\therefore \quad \theta_n' = \frac{(2n+1)\lambda}{2a}$$

→ **Intensity distribution curve :**



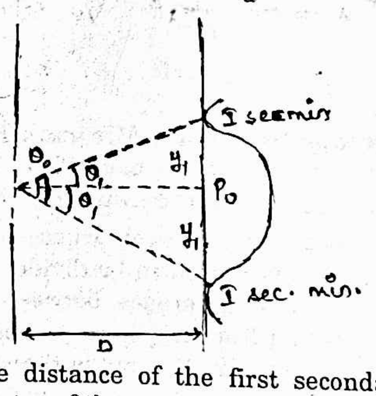
Width of Central Maxima : The width of central maxima is equal to the distance between first secondary minima on either side of P_0 (center of the screen).

For first secondary minima

$$\sin\theta_1 = \frac{\lambda}{a}$$

If θ_1 be small then $\sin\theta_1 \cong \theta_1$

$$\therefore \quad \theta_1 = \frac{\lambda}{a} \quad \text{...(i)}$$



If the distance of the first secondary minima from the centre of the screen be y and the distance of the screen from the slit be D then

$$\theta_1 = \frac{y_1}{D} \quad \text{...(ii)}$$

From equation (i) and (ii)

$$\frac{y_1}{D} = \frac{\lambda}{a}$$

or $y_1 = \dfrac{\lambda D}{a}$

Width of the central maxima $= 2y_1 = \dfrac{2\lambda D}{a}$

Note ; Angular width of the central maxima $= 2\theta_1 = 2y_1 / D$.

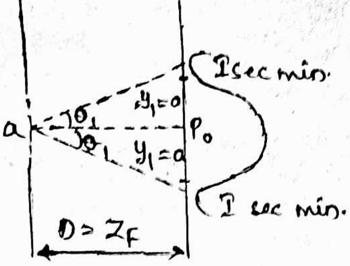
Fresnel Distance : The distance of the screen from the slit for which the spreading light due to diffraction from the centre of screen is equal to size of the slit is called Fresnel distance. It is denoted by Z_F.

For first secondary minima

$$\sin \theta_1 = \frac{\lambda}{a}$$

If θ_1 be small then $\sin \theta_1 \approx \theta_1$

$$\therefore \quad \theta_1 = \frac{\lambda}{a} \qquad ...(i)$$



If the distance of the first secondary minima from the centre of the screen by y and the distance of the screen from the slit be D then

$$\theta_1 = \frac{y}{D} \qquad ...(ii)$$

From equation (i) and (ii)

$$\frac{y}{D} = \frac{\lambda}{a}$$

From the definition of Fresnel distance

$$y = a \text{ and } D = Z_F$$

$$\therefore \quad \frac{a}{Z_F} = \frac{\lambda}{a}$$

$$\text{or} \quad Z_F = \frac{a^2}{\lambda}$$

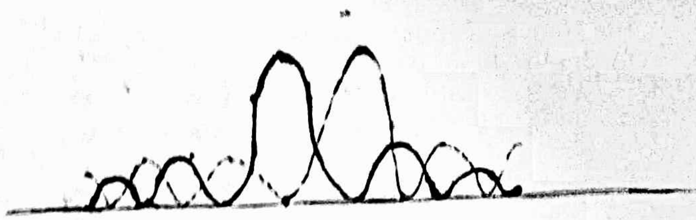
This is the required expression of Fresnel distance.

If $D < Z_F$ then there will be not be two much broadening by diffraction i.e. the light will approx travel along straight line and the concepts of ray optics will be valid. But if $D > Z_F$ then there will be two much broadening by diffraction i.e. the light will no longer travel along straight line and the concepts of wave optics will be valid. Thus ray optics is the limiting case of wave optics.

Rayleigh's Criterion of Limiting Resolution: According to Rayleigh's criterion of limiting resolution the two objects are said to be just resolved if the central maxima of (diffraction pattern of) first object lies at the first secondary minima of (diffraction pattern of) second object and the central maxima of (diffraction pattern of) second object lies at the first secondary minima of (diffraction pattern of) first object.

→ resolve :- To see different objects separately.

If the distance between the two objects is greater than this then they are said to be will resolved and if the distance between the two objects is less than this then they are said to be **unresolved.**



Limit of Resolution and Resolving Power of Optical Instruments : The smallest separation between the two point objects at which they appear just separated is called the limit of resolution of an optical instrument and the reciprocal of limit of resolution of an optical instrument is called its resolving power.

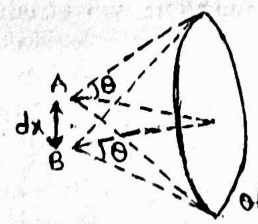
(a) Limit of Resolution of Microscope : It is defined as the least separation between two close objects, so that they appear just separated when seen through a microscope.

The least separation between the two objects is called limit of resolution which is given by

$$dx = \frac{1.22\lambda}{2\mu \sin \theta}$$

where λ is the wavelength of light used to illuminate the object. μ is the refractive index of medium between the objects and the objective of microscope and θ is the semivertical angle of cone formed by taking objects as apex and objective as base.

$$\text{Resolving power of microscope} = \frac{1}{dx} = \frac{2\mu \sin \theta}{1.22\lambda}$$



* $\mu \sin\theta$ – numerical aperture of microscope

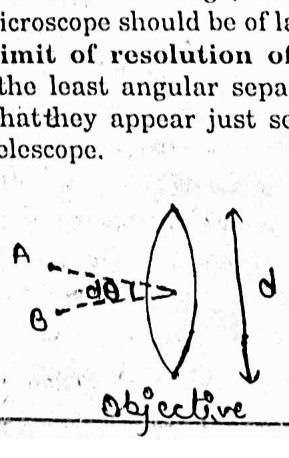
In order to increase resolving power :

(i) μ should be large i.e. the objects should be observed by placing them in a transparent liquid of high refractive index.

(ii) λ should be small i.e. the objects should be illuminated by light of small wavelength (like violet light).

(iii) θ should be large, for this objective of microscope should be of larger aperture.

(b) Limit of resolution of Telescope ; It is defined as the least angular separation between two objects so that they appear just separated when seen through a telescope.



The least angular separation between the two objects is called limit of resolution which is given by

$$d\theta = \frac{1.22\lambda}{d}$$

where λ is the wavelength of light coming from the objects and d is the diameter of aperture of objective of telescope.

$$\text{Resolving power of telescope} = \frac{1}{d\theta} = \frac{d}{1.22\lambda}$$

In order to increase resolving power :

(i) d should be large in the diameter of aperture of objective of telescope should be large.

(ii) λ should be small i.e. the wavelength of light coming from the objects should be small.

HUYGENS PRINCIPLE

(1) Behaviour of Prism : Consider a plane wavefront passing through a prism clearly the portion of the incident wavefront which travels through the greatest thickness of the prism has been delayed the most (since light travels slowly in glass as compared to air). This explain the tilt in the emerging wavefront.

(2) Behaviour of Convex Lens : Consider a plane wavefront passing through a convex lens. The central part of incident wavefront traverses the thickest portion of the convex lens and is delayed the most. Therefore the emerging wavefront has a depression at the centre. It is spherical and converges to a focus.

(3) Behaviour of Concave Mirror : Consider a plane wavefront which falls on a concave mirror. The central part of incident wavefront has to traverse greater distance before and after reflection as compared to edges. This again produces a spherical wavefront which converges to a focus.

Note :

POLARIZATION

Law of Malus : According to law of Malus, when a beam of completely plane polarized light is incident on an analyzer. The intensity of transmitted light I from the analyzer is directly proportional to the square of the cosine of angle θ between the planes of transmission of the analyzer and the polarizer.

i.e. $I \propto \cos^2\theta$

Suppose the plane of analyzer makes an θ with the plane of polarizer and plane polarized light of intensity I_0 and amplitude A is incident on the analyzer.

The amplitude A of the incident light can be resolved into two components.

(i) $A\cos\theta$ along the plane of transmission of analyzer.

(ii) $A\sin\theta$ perpendicular to the plane of transmission of analyzer.

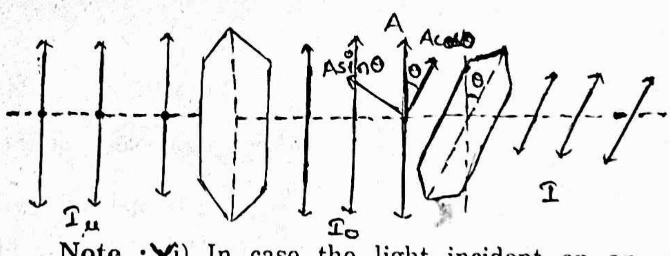
If there is no loss of light due to absorption while traversing the analyzer then the intensity of light transmitted from the analyzer is

$$I \propto (A\cos\theta)^2$$

or $$I = kA^2\cos^2\theta$$

or $$I = I_0\cos^2\theta$$

where $I_0 = kA^2$ is the intensity of incident plane polarized light.



Note : (i) In case the light incident on an polarized then $I = \frac{1}{2}I_0$. This is because in unpolarized light, the light vectors vibrate randomly in an plane perpendicular to the direction of propagation and therefore to determine the intensity of transmitted light, average value of $\cos^2\theta$ is to be used which is equal to $\frac{1}{2}$.

(ii) Graph between intensity of transmitted light and the angle between polarizer and analyzer.