

Types of substances

10

Capacitance

Ques.- What do you understand by conductors, insulators and semiconductors.(For reading)

Ans.- There are three different types of substances-

(1) Conductors- Those substances through which charge can easily flow are called conductors. In these substances there is excess of free electrons that is why when an electric field is applied across them the free electrons present inside them experience electric force in the direction opposite to the electric field and begin to move.

e.g.- Au, Cu, Al, Fe, etc.

(2) Insulators- Those substances through which the flow of charge does not take place are called insulators. In these substances there is deficiency of free electrons that is why when an electric field is applied across them electrons do not move and the flow of charge does not take place.

e.g. Rubber, paper, glass, wood etc.

(3) Semiconductors- Those substances whose conductivity lies between conductors and insulators are called semiconductors. In these substances there is neither excess nor deficiency of charge.

e.g.- Silicon, germanium etc.

Note- (i) Those insulators which exhibit electrical effects when placed in external electric field are called dielectric substances.

(ii) In conductors there are approx. 10^{23} free electrons in 1 cm^3 , in insulators there is approx. 1 free electrons in 1 cm^3 and in semiconductors there are approx. 10^{12} free electrons in 1 cm^3 .

Behaviour of conductor in external electric field

Ques.- Describe the behaviour of conductor in external electric field.(For reading)

Ans.- When a conductor is placed in external electric field E_0 then under the effect of applied electric field the free electrons present inside the conductor begin to drift in the direction opposite to the applied electric field because of the electric force acting on them in the direction opposite to the applied electric field.

Due to excess of electrons one surface of conductor becomes negatively charged and due to deficiency of electrons the other surface of conductor becomes positively charged. As a result of charge induced on the surface of conductor an additional electric field comes into action inside the conductor whose direction is opposite to the applied electric field which is called induced electric field E_i .

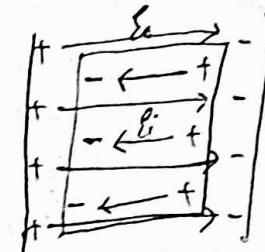
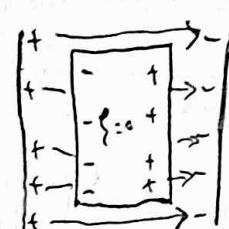
So long as the applied electric field remains dominant the drifting of electrons takes place in the direction opposite to the applied electric field and finally a stage is reached when the induced electric field becomes equal to the applied electric field.

Thus, in stable configuration the induced electric field E_i becomes equal and opposite to the applied electric field E_0 and the resultant electric field inside the conductor becomes zero.

$$\text{i.e. } \vec{E} = \vec{E}_0 + \vec{E}_i$$

$$\text{or, } \vec{E} = \vec{E}_0 + (-\vec{E}_0)$$

or $\vec{E} = 0$



Properties of a conductor situated in external electric field and properties of a charged conductor

Ques.- What are the properties of a conductor situated in external electric field and properties of a charged conductor (For conceptuals).

Ans.- (i) When a conductor is placed in external electric field then the net electric field inside the conductor is zero.

$$\text{Net electric field } E = E_0 - E_i$$

$$\text{In stable configuration } E_0 = E_i$$

$$\therefore E = 0$$

That is why electric lines of force do not pass through a conductor.

(ii) When charge is given to a conductor then due to mutual repulsion the charge gets distributed on the surface of the conductor. Therefore, the charge given to a conductor only resides on its surface and the charge inside a conductor is zero. Thus, in accordance with Gauss theorem the electric field intensity inside a charged conductor is also zero.

(iii) At any point on the surface of conductor the electric field is perpendicular to its surface.

(iv) As the electric field intensity inside a conductor is zero therefore electric potential at every point inside it is constant which is equal to the electric potential of the surface of conductor.

$$\text{In relation } E = -\frac{dV}{dr}, \text{ if } E = 0$$

$$\text{Then } 0 = -\frac{dV}{dr}$$

$$\therefore V = \text{constant}$$

(v) Surface charge density at any point of the surface of conductor varies inversely as the radius of curvature of surface at that point.

$$\text{i.e. Surface charge density } \propto \frac{1}{\text{Radius of curvature}}$$

$$\text{or } \sigma \propto \frac{1}{r}$$

$$\therefore r_A < r_B$$

$$\therefore \sigma_A > \sigma_B$$



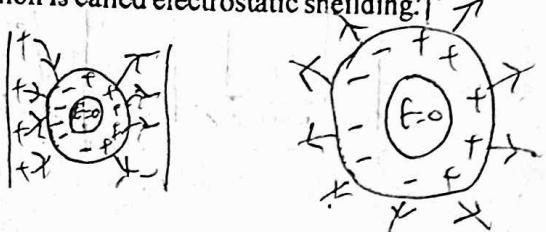
(vi) When charge is given to a conductor then the flow of charge takes place from the point of higher electric potential to the point of lower electric potential until the electric potential becomes equal at every point of charged conductor. Thus, the surface of a charged conductor behaves like equipotential surface.

Electrostatic Shielding

Ques.- What is electrostatic shielding ?

Ans.- When a conductor is placed in external electric field or when charge is given to it then in both the cases electric field intensity inside the conductor is zero. Thus,

when a sensitive instrument is placed inside a cavity present in the conductor then it can be shielded from electric field. In this manner a conductor having a cavity behaves like a shield for the sensitive instrument. This phenomenon is called electrostatic shielding.



Note- On the basis of this phenomenon it could be explained that why it is more safer to remain in car with windows closed as compared to standing outside the car under a tree, when there is probability of lightning in the near future.

Electrostatic induction

Ques.- What do understand by electrostatic induction ? (for conceptuals)

Ans.- When a charged conductor is brought close to an uncharged conductor then under the effect of electric force acting on free electrons in the direction opposite to the electric field they begin to drift (move) in the direction opposite to the electric field due to which at the surface of uncharged conductor which is nearer to charged conductor unlike charge is developed and at the other surface of uncharged conductor which is farther from the charged conductor like charge is developed. This phenomenon of induction of charge in uncharged conductor is called electrostatic induction and the charge developed on it is called induced charge.

Note- The charge developed on the nearer surface of uncharged conductor is bounded by the attractive force to the charged conductor therefore it is called bound charge and the charge present on the farther of uncharged conductor is free to move into another conductor under the effect of repulsive force therefore it is called free charge.

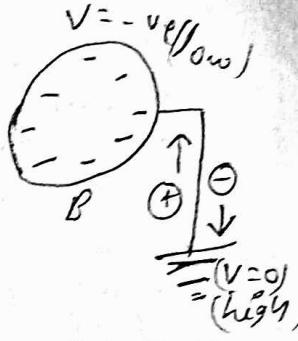
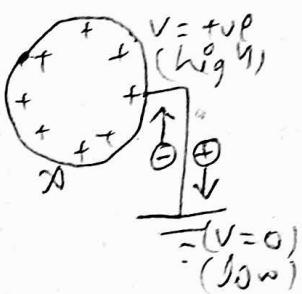
Electric potential of earth

Ques.- Why the electric potential of earth is taken to be zero ?

Ans.- Earth is a large conductor therefore when a certain amount of charge is given to it or taken from it then its electric potential remains unchanged. Moreover earth is a conductor therefore at every point of it its electric potential is same.

Because of the above two reasons the electric potential of earth has been chosen as standard for the measurement of electric potential and that is why its electric potential is taken to be zero.

Note- (i) Electric potential of a positively charged conductor is positive i.e. greater than earth and a negatively charged conductor is negative i.e. less than earth.
(ii) When a positively and negatively charged conductor is brought in contact with earth then the flow of the positive charge takes place from conductor A to earth and from earth to conductor B and the flow of negative charge (electrons) takes place from earth to conductor A and from conductor B to earth.



The flow of charge takes place until the electric potential of conductor becomes equal to that of earth i.e. zero.

Capacitance of a conductor

Ques.- What is meant by capacitance (capacity) of a conductor ? On what factors does it depends ?

Ans.- It is that physical quantity which measures the ability of a conductor to hold electric charges.

Consider a conductor which when given q amount of charge then the increase in its electric potential be V .

As the electric potential of the conductor is directly proportional to the amount of charge given to it i.e.

$$i.e. \quad q \propto V$$

$$\text{or} \quad q = CV$$

where C is the constant of proportionality which is called capacitance of the conductor.

\therefore Capacitance of a conductor

$$C = \frac{q}{V}$$

Thus, the capacitance of a conductor is equal to the ratio of the amount of charge given to the conductor and the increase in its electric potential.

Its value depends on

- (i) shape and size (dimensions) of the conductor.
- (ii) nature of medium in which the conductor is placed.
- (iii) presence of other conductors close to it.

It is a scalar quantity.

Unit- from relation $C = \frac{q}{V}$

S.I. unit of $C = \frac{C}{V} = \text{farad (F)}$

C.G.S. esu of $C = \frac{\text{statC}}{\text{statV}} = \text{statsfarad (statF)}$

C.G.S. emu of $C = \frac{\text{abC}}{\text{abV}} = \text{absfarad (abF)}$

Dimensions- From relation $C = \frac{q}{V}$

The dimensions Formula of $C = \frac{[AT]}{[ML^2T^3A^{-1}]}$

$$= [M^{-1}L^2T^4A^2]$$

Note- So long as the above factors remains unchanged

capacitance of a conductor is independent of-

- (i) charge given to it.
- (ii) increase in its electric potential
- (iii) nature of its material.

Ques.- Define S.I. and C.G.S. units of capacitance (capacity) of a conductor.

Ans.- In relation $C = \frac{q}{V}$

If $q = 1C$ and $V = 1V$

$$\text{Then } C = \frac{1C}{1V} = 1F$$

Thus, if on giving 1C of charge to a conductor the increase in its electric potential be 1V then its capacitance is said to be 1F.

Again in relation $C = \frac{q}{V}$

If $q = 1 \text{ statC}$ and $V = 1 \text{ statV}$

$$\text{Then } C = \frac{1 \text{ statC}}{1 \text{ statV}} = 1 \text{ statF}$$

Thus, if on giving 1statC of charge to a conductor increase in its electric potential be 1statV then its electric potential is said to be 1statF.

Capacitance of an isolated spherical conductor

Ques.-Derive the relation for the capacitance of an isolated spherical conductor.

Ans.- Consider an isolated spherical conductor of radius R.

If on giving q amount of charge to the conductor the increase in its electric potential be V then by definition the capacitance of spherical conductor $C = \frac{q}{V}$

Since, the electric potential of spherical conductor

$$V = \frac{q}{4\pi\epsilon_0 R}$$

$$\therefore C = \frac{q}{q/4\pi\epsilon_0 R}$$

$$\text{or } C = 4\pi\epsilon_0 R$$

If the conductor be placed in a medium of dielectric constant K than

$$V = 4\pi\epsilon_0 KR$$

This is the required expression.

In C.G.S. electrostatic system

$$\frac{1}{4\pi\epsilon_0} = 1$$

$$4\pi\epsilon_0 = 1$$

∴ In vacuum $C = R$

and in medium $C = \epsilon R$

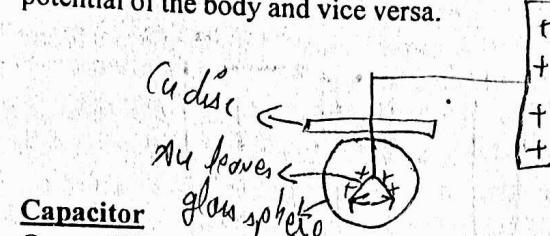
From the above relation it can be concluded that the capacitance of a spherical conductor is-

- (i) directly proportional to the dielectric constant of the medium in which the conductor is placed i.e. $C \propto K$.
- (ii) directly proportional to the radius of the spherical conductor i.e. $C \propto R$.

Note- 1. In C.G.S. electrostatic system the capacitance of an isolated spherical conductor in vacuum in numerically equal to its radius i.e. $C = R$

2. Electroscope- It is device which measures the electric potential of a charged body, the larger the divergence (separation) of gold leaves the greater will be the electric

potential of the body and vice versa.



Ques.- What is a capacitor (condenser) ?

Ans. - It is a device with the help of which capacitance of a conductor can be increased without changing its shape or size.

It is an arrangement of two conductors out of which one is isolated

and the other is earthed. The magnitude of charge present on any plate of

capacitor is called charge of capacitor. The two conductors of which a

capacitor is made of are called plates of capacitor.

Note- (i) Whatever be the shape or size of the two plates of capacitor the amount of charge present on them are always equal in magnitude and opposite in nature.

(ii) The total charge present on the plates of a capacitor is zero.

Principle of capacitor

Ques.- Describe the principle of capacitor (condenser) ?

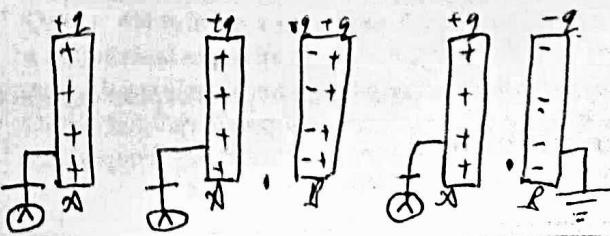
Ans. - Consider a positively charged conductor A as shown in fig. 1. When an uncharged conductor is brought close to it then the induction of unlike charge takes place on the nearer surface and the induction of like charge takes place on the farther surface of uncharged conductor as shown in fig. As the distance of unlike charge induced on the nearer surface is less than the distance of positive charge induced on the farther surface of uncharged conductor from conductor A therefore the -ve potential due to induced -ve charge will be more than the +ve potential due to induced +ve charge on conductor A. As a result of which the electric potential of conductor A decreases and in accordance with relation

$$C = \frac{q}{V} \text{ its capacitance increases.}$$

When the conductor B is earthed as shown in fig then the whole of the +ve charge induced on its farther surface will go into the earth and only -ve charge will remain on it due to which the electric potential of conductor A further decreases and in accordance with relation

$$C = \frac{q}{V} \text{ its capacitance further increases.}$$

From the above it can be concluded that whenever an uncharged earthed conductor is brought close to charged conductor then due to decrease in electric potential of charged conductor its capacitance increases. This is called the principle of capacitor.



Capacitance of a capacitor

Ques.- Define capacitance of a capacitor. What are its units and dimensions?

Ans.- The capacitance of a capacitor is defined as the ratio of magnitude of charge present on its isolated plate and the potential difference between its two plates.

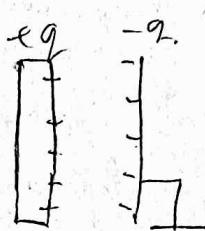
If on giving $+q$ amount of charge to the isolated plate of a capacitor the electric potential of its two plates be V_1 and V_2 then the capacitance of capacitor is given by

$$C = \frac{q}{V_1 - V_2}$$

If $V_1 = V$ and $V_2 = 0$

$$\text{Then, } C = \frac{q}{V - 0}$$

$$\text{or } C = \frac{q}{V}$$



It is a scalar quantity. Its S.I. unit is farad (F), C.G.S. esu is statfarad (statF) and dimensional formula is $[M^{-1}L^{-2}T^4A^2]$.

Note- Capacitor is represented by the symbol $\begin{array}{|c|}\hline | & | \\ \hline\end{array}$ and variable capacitor is represented by the symbol $\begin{array}{|c|}\hline | & | \\ \hline\end{array} \neq$

Parallel plate capacitor

Ques.- Explain the construction and working of parallel plate capacitor and derive expression of its capacitance.

Ans.- It consists of two identical parallel conducting plates P and Q which may be rectangular circular or of any other shape. The area of two plates be A and the separation between them be d. A medium of dielectric constant K is filled between the two plates in order to increase the capacitance of parallel plate capacitor.

When $+q$ charge is given to plate P then an equal amount of $-ve$ charge is induced on the nearer surface and an equal amount of $+ve$ charge is induced on the farther surface of plate Q. If the plate Q is brought in contact with earth then the whole of $+ve$ charge induced on its farther surface will go into the earth and only the negative charge remains on the plate Q.

Since the plates of parallel plate capacitor behaves like two parallel sheets of charge therefore electric field intensity at any point between the two plates is

$$E = \frac{\sigma}{K\epsilon_0}$$

where $\sigma = \frac{q}{A}$ is the surface charge density of two plates.

$$\therefore E = \frac{q}{K\epsilon_0 A}$$

Potential difference between the two plates of parallel plate capacitor.

$$V_p - V_q = Ed$$

$$\text{or } V = \frac{q}{K\epsilon_0 A} d$$

By definition the capacitance of parallel plate capacitor

$$C = \frac{q}{V}$$

$$\text{or } C = \frac{q}{\frac{q}{K\epsilon_0 A} d}$$

$$\text{or } C = \frac{K\epsilon_0 A}{d}$$

This is the required relation.

From the above relation it can be concluded that the capacitance of parallel plate capacitor is-

- (i) directly proportional to the dielectric constant of the medium present between the two plates i.e. $C \propto K$.
- (ii) directly proportional to the overlapping area of two plates i.e. $C \propto A$.
- (iii) inversely proportional to the distance between the two plates i.e. $C \propto \frac{1}{d}$

Note- Due to finite size of plates of parallel plate capacitor, slight fringing of electric field takes place at the end of two plates. This fringing of electric field can be reduced by placing the two plates close to each other.

Parallel plate capacitor partially filled with dielectric medium

Ques.- Derive expression for the capacitance of a parallel plate capacitor partially filled with dielectric medium.

Ans.- Consider a parallel plate capacitor which consists of two identical metallic plates of area A which are placed at a separation d. Out of the two plates one is isolated and the other is earthed. A dielectric slab of dielectric constant K and thickness t is placed in between the two plates such that it is parallel to the two plates. When $+q$ charge is given to the isolated plate P an equal amount of $-ve$ charge is induced on the nearer surface and an equal amount of $+ve$ charge is induced on the farther surface of the earthed plate. As the plate Q is earthed therefore the $+ve$ charge induced on its farther surface will go into the earth and only $-ve$ charge will remain on it.

Electric field intensity at any point in vacuum between the two plates $E_0 = \frac{\sigma}{\epsilon_0} = \frac{q}{\epsilon_0 A}$

and electric field intensity at any point in medium between the two plates $E = \frac{\sigma}{K\epsilon_0} = \frac{q}{K\epsilon_0 A}$

As the thickness of air column between the plates a parallel plate capacitor is $(d - t)$ and the thickness of dielectric slab is t therefore the potential difference between the plates of parallel plate capacitor

$$V = E_0(d - t) + Et$$

$$\text{or } V = \frac{q}{\epsilon_0 A}(d - t) + \frac{q}{K\epsilon_0 A} t$$

$$\text{or } V = \frac{q}{\epsilon_0 A} \left[(d - t) + \frac{t}{K} \right]$$

By definition the capacitance of partially filled parallel plate capacitor

$$C = \frac{q}{V}$$

$$\text{or } C = \frac{q}{\frac{q}{\epsilon_0 A} \left[(d - t) + \frac{t}{K} \right]}$$

$$C = \frac{\epsilon_0 A}{(d-t) + \frac{t}{K}}$$

$$\text{or } C = \frac{\epsilon_0 A}{d-t} \left(1 - \frac{1}{K}\right)$$

This is the required expression.

Particular cases- (i) When a metal plate of thickness t is placed between the two plates of parallel plate capacitor

$$C = \frac{\epsilon_0 A}{(d-t) + \frac{t}{\infty}}$$

$$\text{or } C = \frac{\epsilon_0 A}{d-t+0}$$

$$\text{or } C = \frac{\epsilon_0 A}{d-t}$$

(ii) When a metallic foil (thin metallic sheet) is placed parallel to the two plates of PPC

$$C = \frac{\epsilon_0 A}{d-0}$$

$$\text{or } C = \frac{\epsilon_0 A}{d}$$

(iii) When dielectric slabs of thickness t_1, t_2, t_3 and dielectric constants K_1, K_2, K_3 respectively between the plates of PPC

$$C = \frac{\epsilon_0 A}{(d-t_1-t_2-t_3) + \frac{t_1}{K_1} + \frac{t_2}{K_2} + \frac{t_3}{K_3}}$$

If $t_1+t_2+t_3 = d$

$$\text{Then } C = \frac{\epsilon_0 A}{\frac{t_1}{K_1} + \frac{t_2}{K_2} + \frac{t_3}{K_3}}$$

(iv) When the two plates of PPC are brought in electrical contact of a conducting medium is filled between the plates then

$$C = \frac{K \epsilon_0 A}{d}$$

$$\text{or } C = \frac{\infty \epsilon_0 A}{d}$$

$$\text{or } C = \infty$$

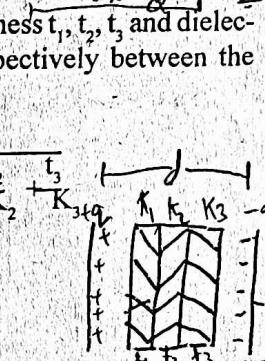
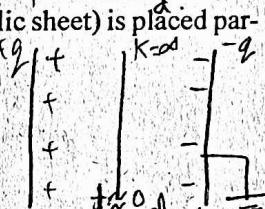
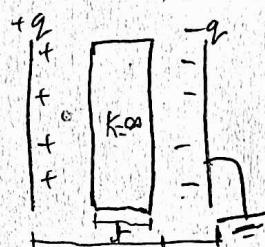
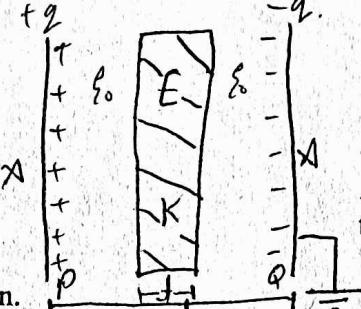
Such a capacitor is of no practical use because in such cases the plates of capacities do not hold electric charges (as the neutralization of electric charge takes place).

Capacitance of a spherical capacitor

Ques.- Obtain an expression for the capacitance of a spherical capacitor.

Ans.- It consists of two hollow concentric spheres A and B whose radii are a and b respectively. A medium of dielectric constant K is filled between the two spheres in order to increase its capacitance. When $+q$ charge is given to the inner sphere A through rod M then an equal amount of -ve charge is induced on the inner surface and an equal amount of +ve charge is induced on the outer surface of sphere B. When the outer sphere is earthed the whole of the +ve charge present on its outer surface will go into the earth and only -ve charge will remain on its inner surface.

Consider a point P between the two spheres at a distance r from its center O such that $a < r < b$ then the electric field intensity at point P is given by



$$E = \frac{q}{4\pi\epsilon_0 Kr^2}$$

Consider another point P' at a small distance dr from point P in the inward direction then the potential difference between points P and P' is given by

$$dV = -Edr \quad \therefore E = -\frac{dV}{dr}$$

$$\text{or } dV = -\frac{-q}{4\pi\epsilon_0 Kr^2} dr$$

By integrating above relation with in proper limits the potential difference between the two spheres can be calculated as follows-

$$\int_0^V dV = \int_b^a \frac{-q}{4\pi\epsilon_0 Kr^2} dr$$

$$\text{or } [V]_0^V = \frac{-q}{4\pi\epsilon_0 K} \int_b^a \frac{1}{r^2} dr$$

$$\text{or } V - 0 = \frac{-q}{4\pi\epsilon_0 K} \int_b^a r^2 dr$$

$$\text{or } V = \frac{-q}{4\pi\epsilon_0 K} \left[\frac{r^{2+1}}{2+1} \right]_b^a$$

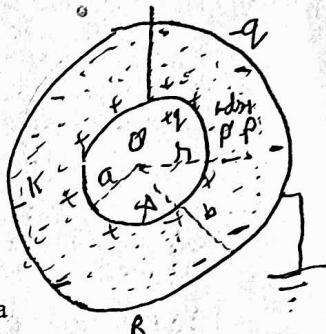
$$\text{or } V = \frac{-q}{4\pi\epsilon_0 K} \left[\frac{r^1}{-1} \right]_b^a$$

$$\text{or } V = \frac{q}{4\pi\epsilon_0 K} \left[\frac{r^1}{b} \right]_b^a$$

$$\text{or } V = \frac{q}{4\pi\epsilon_0 K} \left[\frac{1}{r} \right]_b^a$$

$$\text{or } V = \frac{q}{4\pi\epsilon_0 K} \left[\frac{1}{a} - \frac{1}{b} \right]$$

$$\text{or } V = \frac{q}{4\pi\epsilon_0 K} \left[\frac{b-a}{ab} \right]$$



By definition the capacitance of a spherical capacitor

$$C = \frac{q}{V}$$

$$\text{or } C = \frac{q}{\frac{q}{4\pi\epsilon_0 K} \left[\frac{b-a}{ab} \right]}$$

$$\text{or } C = \frac{4\pi\epsilon_0 Kab}{(b-a)}$$

This is the required expression.

Note- From the above relation it can be concluded that the capacitance of spherical capacitor

$$(i) C \propto K$$

$$(ii) C \propto ab$$

$$(iii) C \propto \frac{1}{b-a}$$

Capacitance of a cylindrical capacitor

Ques.- Obtain an expression for the capacitance of a cylindrical capacitor.

Ans.- It consists of two coaxial hollow cylinders A and B whose radii are a and b respectively and length is L . A medium of dielectric constant K is filled between the two

cylinders in order to increase its capacitance. When +q charge is given to the inner cylinder then an equal amount of -ve charge is induced on the inner surface and an equal amount of +ve charge is induced on the outer surface of cylinder B. When the outer cylinder B is earthed then the whole of the positive charge induced on its outer surface will go into the earth and only -ve charge remains on its inner surface.

Consider a point P between the two cylinders at a distance r from their axis such that $a < r < b$ then the electric field intensity at point P is given by

$$E = \frac{q}{2\pi\epsilon_0 K r l}$$

Consider another point P' at a small distance dr from point P in the inward direction then the potential difference between point P' and P is given by

$$dV = -Edr$$

$$\therefore E = -\frac{dV}{dr}$$

$$\text{or } dV = \frac{q}{2\pi\epsilon_0 K r l} dr$$

By integrating the above relation with in proper limits the potential difference between the two cylinders can be calculated as follows -

$$0 \int^V_b dV = \int^a_b \frac{q}{2\pi\epsilon_0 K r l} dr$$

$$\text{or } \left[V \right]_0^V = \frac{q}{2\pi\epsilon_0 K r l} \int_b^a \frac{1}{r} dr$$

$$\text{or } V - 0 = \frac{q}{2\pi\epsilon_0 K r l} \left[\log_e r \right]_b^a$$

$$\text{or } V = \frac{q}{2\pi\epsilon_0 K r l} \left[\log_e a - \log_e b \right]$$

$$\text{or } V = \frac{q}{2\pi\epsilon_0 K r l} \left[\log_e b - \log_e a \right]$$

$$\text{or } V = \frac{q}{2\pi\epsilon_0 K r l} \log_e \frac{b}{a}$$

By definition the capacitance of cylindrical capacitor

$$C = \frac{q}{V}$$

$$\text{or } C = \frac{q}{\frac{q}{2\pi\epsilon_0 K r l} \log_e \frac{b}{a}}$$

$$\text{or } C = \frac{2\pi\epsilon_0 K r l}{\log_e \frac{b}{a}}$$

$$\text{or } C = \frac{2\pi\epsilon_0 K r l}{2.303 \log_{10} \frac{b}{a}}$$

This is the required expression.

Note(i)- From the above relation it can be concluded that the capacitance of cylindrical capacitor.

$$(a) C \propto K \quad (b) C \propto l \quad (c) C \propto \frac{1}{\log_e \frac{b}{a}}$$

Note(ii)- Capacitance per unit length of cylindrical capacitor

$$\frac{C}{l} = \frac{2\pi\epsilon_0 K}{\log_e \frac{b}{a}}$$

$$\text{or } \frac{C}{l} = \frac{2\pi\epsilon_0 K}{2.303 \log_{10} \frac{b}{a}}$$

Series combination of capacitors

Ques.- What do you understand by series combination of capacitors ? Derive relation for its equivalent capacitance. What is its importance ?

Ans.- When two or more capacitors are connected in such a manner that the second plate of first capacitor is connected to the first plate of second capacitor, the second plate of second capacitor is connected to the first plate of third capacitor and in this manner the second plate of last capacitor is connected to the earth then such a combination of capacitors is called series combination of capacitor.

Consider three capacitors of capacitances C_1 , C_2 and C_3 , which are connected in series as shown in fig. When +q amount of charge is given to the first plate of first capacitor then an equal amount of negative charge will induce on the nearer surface and equal amount of positive charge will induce on the farther surface of second plate of first capacitor. Due to repulsion the +ve charge induced on the farther surface of second plate of first capacitor is transferred to the first plate of second capacitor. Again induction of charge takes place at the second plate of second capacitor and the positive charge induced on its farther surface will repel and go to the first plate of third capacitor. Again induction of charge takes place at the second plate of third capacitor and the positive charge induced on its farther surface will repeat and go into the earth. In this manner the amount of charge present on all the three capacitors is same i.e. q.

As the capacitances of capacitors are different therefore the potential difference across them will also be different.

The potential difference across the plates of I capacitor

$$V_1 = \frac{q}{C_1}$$

The potential difference across the plates of II capacitor

$$V_2 = \frac{q}{C_2}$$

and the potential difference across the plates of III capacitor

$$V_3 = \frac{q}{C_3}$$

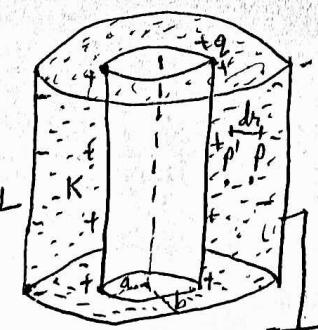
Total potential difference across the series combination of capacitors

$$V = V_1 + V_2 + V_3$$

$$\text{or } V = \frac{q}{C_1} + \frac{q}{C_2} + \frac{q}{C_3}$$

$$\text{or } V = q \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \right) \quad \text{---(i)}$$

If the equivalent capacitance of series combination of capacitors be C then



$$V = \frac{q}{C} \quad \text{---(ii)}$$

From relation (i) and (ii)

$$\frac{q}{C} = q \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \right)$$

$$\text{or } \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

Thus, the reciprocal of equivalent capacitance of series combination of capacitors is equal to the sum of reciprocals of capacitances of capacitors involved in series grouping.

Importance- When the desired capacitance is less than the capacitance of given capacitors then in order to obtain desired capacitance the given capacitors should be connected in series.

Note-(i) Generalization- When n capacitors of capacitances $C_1, C_2, C_3, \dots, C_n$ are connected in series then the equivalent capacitance is given by

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots + \frac{1}{C_n}$$

Particular case- If $C_1 = C_2 = C_3 = \dots = C_n = C$

$$\text{Then } \frac{1}{C_s} = \frac{1}{C} + \frac{1}{C} + \frac{1}{C} + \dots + \frac{1}{C}$$

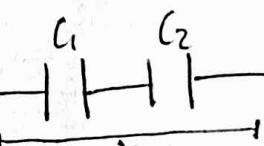
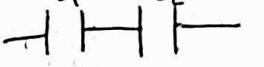
$$\text{or } \frac{1}{C_s} = \frac{n}{C} \quad \text{or } C_s = \frac{C}{n}$$

(ii) If two capacitors of capacitance C_1 and C_2 are connected in series

$$\text{Then } \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$\text{or } \frac{1}{C} = \frac{C_1 + C_2}{C_1 C_2}$$

$$\text{or } C = \frac{C_1 C_2}{C_1 + C_2}$$



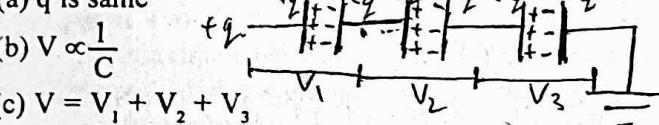
(iii) Potential division formula for two capacitors connected in series

$$\therefore V \propto \frac{1}{C}$$

$$\therefore V_1 = \frac{C_2 V}{C_1 + C_2} \quad \text{and} \quad V_2 = \frac{C_1 V}{C_1 + C_2}$$

(iv) In series combination of capacitors

(a) q is same



$$(b) V \propto \frac{1}{C}$$

$$(c) V = V_1 + V_2 + V_3$$

$$(d) C < C_1, C < C_2, C < C_3$$

Parallel combination of capacitors

Ques.- What do you understand by parallel combination of capacitors? Derive relation for its equivalent capacitance. What is its importance?

Ans.- When two or more capacitors are connected in such a manner that the first plate of all the capacitors are connected to one point and the second plate of all the capacitors are connected to the other point which is further in contact with earth then such a combination of capacitors is called parallel combination of capacitor.

Consider three capacitors of capacitances C_1, C_2 and C_3 which are connected in parallel as shown in fig. When $+q$ amount of charge is given to point A then the first plates of three capacitors will take large according to

their capacitances. An equal amount of negative charge will induce on the nearer surface and an equal amount of positive charge will induce on the farther surface of their second plates. As the second plates of all the capacitors are in contact with earth therefore the positive charge induced on their farther surface will repel and go into the earth.

Since all the three capacitors are connected between the same two points therefore the potential difference across each capacitor is same as the potential difference across the parallel combination of capacitors i.e. V .

As the capacitances of capacitors are different therefore the amount of charge present on them will also be different.

The amount of charge present on the plates of I capacitor $q_1 = C_1 V$

the amount of charge present on the plates of II capacitor $q_2 = C_2 V$

and the amount of charge present on the plates of III capacitor $q_3 = C_3 V$

Total charge present on the parallel combination of capacitors $q = q_1 + q_2 + q_3$

$$\text{or } q = C_1 V + C_2 V + C_3 V \quad \text{---(i)}$$

If the equivalent capacitance of parallel combination of capacitors be C then

$$q = CV \quad \text{---(ii)}$$

Comparing relation (i) and (ii)

$$CV = (C_1 + C_2 + C_3)V$$

$$\text{or } C = C_1 + C_2 + C_3$$

Thus, when two or more capacitors are connected in parallel then the equivalent capacitance of parallel combination is equal to the sum of capacitances of capacitors involved in parallel grouping.

Importance- When the desired capacitance is greater than the capacitances of given capacitors then the given capacitors should be connected in parallel.

Note-(i) Generalization- If n capacitors of capacitances $C_1, C_2, C_3, \dots, C_n$ are connected in parallel then the equivalent capacitance is given by

$$C_p = C_1 + C_2 + C_3 + \dots + C_n$$

$$\text{If } C_1 = C_2 = C_3 = \dots = C_n = C$$

$$\text{Then } C_p = nC$$

(ii) Charge division formula for two capacitors connected in parallel-

$$\therefore q \propto C$$

$$\therefore q_1 = \frac{C_1}{C_1 + C_2} q \quad \text{and} \quad q_2 = \frac{C_2}{C_1 + C_2} q$$



(iii) In parallel combination of capacitors

(a) V is same

(b) $q \propto C$

$$(c) q = q_1 + q_2 + q_3$$

$$(d) C = C_1 + C_2 + C_3$$

$$(e) C > C_1, C > C_2, C > C_3$$

Energy of a charged conductor

Ques.- Derive relation for energy of a charged conductor.

Ans.- The electric potential energy of a charged conductor is equal to work done in bringing the charge from infinity to the conductor.

ity to the conductor.

Consider a conductor of capacitance C which when given q amount of charge then its electric potential increases by V .

In order to calculate the work done in charging the conductor consider an intermediate position when the charge present on the conductor is q' and its electric potential is V' .

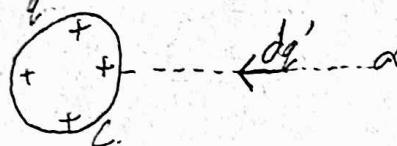
In this position, when a small change dq' is brought from infinity to the charged conductor then the work done is given by

$$dW = dq' V'$$

$$\therefore C = \frac{q'}{V'} \text{ or } V' = \frac{q'}{C}$$

$$\therefore dW = dq' \frac{q'}{C}$$

$$\text{or } dW = \frac{q'}{C} dq'$$



By integrating the above relation with in proper limits the total work done in charging the conductor can be calculated as follows-

$$0 \int^W_0 dW = \int^q_0 \frac{q'}{C} dq'$$

$$\text{or } \left[W \right]_0^W = \frac{1}{C} \int^q_0 q' dq'$$

$$\text{or } W - 0 = \frac{1}{C} \left[\frac{q'^2}{2} \right]_0^q$$

$$\text{or } W = \frac{1}{C} \left[\frac{q^2}{2} - \frac{0^2}{2} \right]$$

$$\text{or } W = \frac{1}{2} \frac{q^2}{C}$$

By definition, the electric potential energy of charged conductor

$$U = W$$

$$\text{or } U = \frac{1}{2} \frac{q^2}{C} \quad \text{---(i)}$$

$$\therefore q = CV$$

$$\therefore U = \frac{1}{2} \frac{(CV)^2}{C}$$

$$\text{or } U = \frac{1}{2} CV^2 \quad \text{---(ii)}$$

$$\therefore C = \frac{q}{V}$$

$$\therefore U = \frac{1}{2} \frac{q}{V} V^2$$

$$\text{or } U = \frac{1}{2} qV \quad \text{---(iii)}$$

The relation (i), (ii) and (iii) are the required expression of electric potential energy of a charged conductor.

Redistribution of charge between two charged conductors

Ques.- The capacities of two conductors are C_1 and C_2 , and their potentials are V_1 and V_2 , respectively. The conductors are joined by a wire. Calculate common potential, energy loss on combination and

charge on them ?

Ans.- Consider two charged conductors A and B of capacitances C_1 and C_2 , which when given q_1 and q_2 amount of charges then their electric potential becomes V_1 and V_2 , respectively.

Charge present on conductor A

$$q_1 = C_1 V_1$$

and charge present on conductor B

$$q_2 = C_2 V_2$$

Total Charge present on the two conductors

$$q = q_1 + q_2$$

$$\text{or } q = C_1 V_1 + C_2 V_2$$

When the two conductors are brought in electrical contact with the help of a conducting wire then the flow of charge takes place between them until their electric potential becomes equal.

Let the common potential of two conductor be V and in stable configuration the amount of charge on the two conductor be q'_1 and q'_2 , then by the law of conservation of charge,

Total final charge = Total initial charge

$$\text{or } q'_1 + q'_2 = q_1 + q_2$$

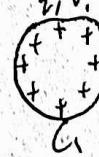
The common potential of two conductors after redistribution of charge

$$V = \frac{q_{\text{tot}}}{C_{\text{tot}}}$$

$$\text{or } V = \frac{q'_1 + q'_2}{C_1 + C_2}$$

$$\text{or } V = \frac{q_1 + q_2}{C_1 + C_2}$$

$$\text{or } V = \frac{C_1 V_1 + C_2 V_2}{C_1 + C_2}$$



Loss of energy during redistribution of charge-

Total initial energy of two conductors

$$U_i = \frac{1}{2} C_1 V_1^2 + \frac{1}{2} C_2 V_2^2$$

$$\text{or } U_i = \frac{1}{2} (C_1 V_1^2 + C_2 V_2^2)$$

and the total final energy of two conductors

$$U_f = \frac{1}{2} C_1 V^2 + \frac{1}{2} C_2 V^2$$

$$\text{or } U_f = \frac{1}{2} (C_1 + C_2) V^2$$

$$\text{or } U_f = \frac{1}{2} (C_1 + C_2) \left(\frac{C_1 V_1 + C_2 V_2}{C_1 + C_2} \right)^2$$

$$\text{or } U_f = \frac{1}{2} (C_1 + C_2) \frac{(C_1 V_1 + C_2 V_2)^2}{(C_1 + C_2)^2}$$

$$\text{or } U_f = \frac{1}{2} \frac{(C_1 V_1 + C_2 V_2)^2}{(C_1 + C_2)}$$

Change in energy $\Delta U = U_i - U_f$

$$= \frac{1}{2} (C_1 V_1^2 + C_2 V_2^2) - \frac{1}{2} \frac{(C_1 V_1 + C_2 V_2)^2}{(C_1 + C_2)}$$

$$= \frac{1}{2} \left[(C_1 V_1^2 + C_2 V_2^2) - \frac{(C_1 V_1 + C_2 V_2)^2}{(C_1 + C_2)} \right]$$

$$= \frac{1}{2} \left[C_1^2 V_1^2 + C_2^2 V_2^2 + C_1 C_2 V_1^2 + C_2 C_1 V_2^2 - \frac{C_1^2 V_1^2 + C_2^2 V_2^2 + 2 C_1 C_2 V_1 V_2}{(C_1 + C_2)} \right]$$

$$= \frac{1}{2} \left[\frac{C_1 C_2 V_1^2 + C_2 C_1 V_2^2 - 2 C_1 V_1 C_2 V_2}{(C_1 + C_2)} \right]$$

$$\begin{aligned} &= \frac{1}{2} \frac{C_1 C_2}{C_1 + C_2} (V_2^2 + V_1^2 - 2V_1 V_2) \\ &= \frac{1}{2} \frac{C_1 C_2}{C_1 + C_2} (V_2 - V_1)^2 \\ \text{i.e. } &\Delta U = \frac{1}{2} \frac{C_1 C_2}{C_1 + C_2} (V_2 - V_1)^2 \\ \therefore \Delta U &= +ve \quad \therefore U_i > U_f \end{aligned}$$

Thus, the loss of energy takes place during redistribution of charge between two charged conductors which appear in the form of heat & sound energy in the connecting wire during the flow of charge through it.

Particular case- If the electric potential of two conductors be same i.e. $V_1 = V_2 = V$

$$\begin{aligned} \text{i.e. } \Delta U &= \frac{1}{2} \frac{C_1 C_2}{C_1 + C_2} (V - V)^2 \\ \text{or } \Delta U &= \frac{1}{2} \frac{C_1 C_2}{C_1 + C_2} (0)^2 \\ \text{or } \Delta U &= 0 \end{aligned}$$

Thus, when two charged conductors having the same electric potential are brought in electrical contact then the redistribution of charge does not take place and hence the loss of energy does not take place.

Note- Again the redistribution of charge, the final charge of conductor A

$$q'_1 = C_1 V = C_1 \left[\frac{C_1 V_1 + C_2 V_2}{C_1 + C_2} \right] \quad \text{---(i)}$$

and the final charge on conductor B

$$q'_2 = C_2 V = C_2 \left[\frac{C_1 V_1 + C_2 V_2}{C_1 + C_2} \right] \quad \text{---(ii)}$$

Dividing relation (i) and (ii)

$$\begin{aligned} \frac{q'_1}{q'_2} &= \frac{C_1 V}{C_2 V} \\ \text{or } \frac{q'_1}{q'_2} &= \frac{C_1}{C_2} \\ \text{or } q' &\propto C \end{aligned}$$

Thus, when the two charged conductors are brought in electrical contact then the final charge of them is directly proportional to their electrical capacitance.

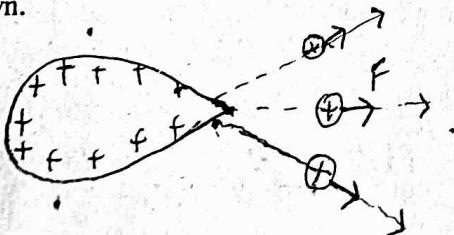
Dielectric strength

Ques.- What do you understand by dielectric strength of a medium?

Ans.- The maximum electric field intensity or potential gradient upto which the electrons do not come out from the molecules of a medium is called dielectric strength of that medium.

Dielectric strength of air is $3 \times 10^6 \text{ Vm}^{-1}$.

Note 1.- When the value of electric field applied on a medium exceeds the dielectric strength of that medium then under the effect of electric force electrons begin to come out from the molecules of that substance. This phenomenon of breaking of dielectric molecules under the effect of applied electric field is called dielectric breakdown.



2.-Action of points- When charge is given to a conductor of irregular shape then the surface charge density at any point of conductor is inversely proportional to its radius of curvature at that point. Since the radius of curvature of a charged conductor is very small at sharp points therefore the surface charge density is very large at sharp points and hence electric field intensity is also very large at sharp points ($E = \sigma/\epsilon_0$). If the value of electric field intensity exceeds the dielectric strength of air at the sharp points, then the dielectric breakdown of air molecules takes place at the sharp points.

When this happens the electrons ejected from air molecules reach the conductor and neutralise the positive charge present on the conductor and the positive ion which remain in air begin to move away from the conductor under the effect of repulsive force. It seems as if electric discharge takes place at sharp point of conductor due to which stream of positive charge come out of the conductor from sharp point. This phenomena is called corona discharge and the stream of positive charge is called electric wind.

Van de Graff generator

Ques.- Describe Van de Graff generator under the following points-

- (i) Labelled diagram
- (ii) construction
- (iii) working
- (iv) merits
- (v) demerits.

Ans.- It was constructed by van-de-graph in 1931.

Principle- It works on the principles of action of points.

Construction- In Van de Graff generator a spherical metallic dome S is kept on two insulating pillars P₁ and P₂. An insulating belt which is made up of silk or rubber is allowed to rotate on two rollers R₁ and R₂ with the help of electric motor M. There is a metallic comb C₁ near the roller R₁ which is called spraying comb which is connected to the +ve terminal of a high voltage generator and there is another metallic comb C₂ near the roller R₂ called collecting comb which is connected to the spherical dome. There is a discharge tube whose one end lies inside the spherical dome and the other end is connected to earth. The whole arrangement is enclosed in a steel chamber in which highly pressurised air or inert gas is filled in order to avoid dielectric breakdown.

Working- First of all comb C₁ is charged to a potential of 10000 V by high voltage generator. As comb C₁ has sharp points the dielectric breakdown of air takes place at this sharp points due to which a stream of +ve charge begins to flow towards the belt. This +ve charge is carried by the belt along with it in the upward direction and when this +ve reaches in front of comb C₂ then the induction of -ve charge takes place on the point sharp points of comb C₂ and the induction of an equal amount of +ve charge takes place on the outer surface of dome. As the comb C₂ has sharp points dielectric breakdown of air takes place at its tips and a stream of -ve charge begins to flow towards the belt. When this -ve charge reaches the belt then the +ve charge present on the belt gets neutralised. In this manner the belt receives +ve charge from comb C₁ and transmits it to the dome through comb C₂. This process is repeated

again and again until the electric potential of dome becomes equal to or greater than 10^6 volt.

After this positively charged particle like proton, deuteron etc which is to be accelerated is generated in the discharge tube. Due to high potential difference between top and bottom of discharge tube acceleration of charged particle takes place from top to bottom. As a result of which it acquires very high kinetic energy which can be used as bombarding particle in nuclear research.

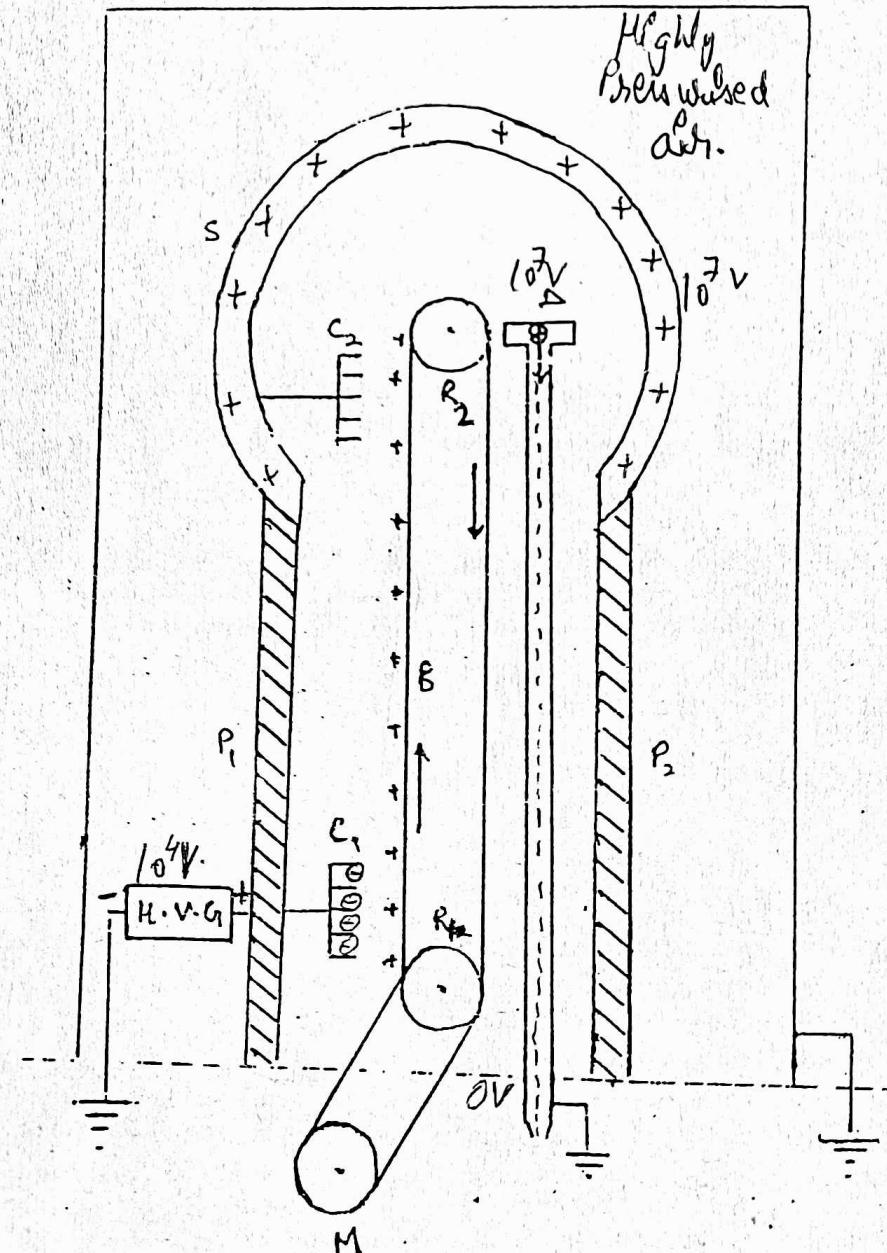
Merits- (i) It is used in nuclear research for artificial radioactive transmutation, nuclear fission etc.

(ii) It is used for generating high voltage of order of 10^6 volt.

(iii) It is used for accelerating charged particles.

Demerits- (i) It is very large in size.

(ii) Due to high potential difference it is quite dangerous.



Dielectrics

Ques.- What are dielectrics ? State their types.

Ans.- Those substances through which the flow of charge does not take place but they show electrical effect when placed in external electric field.

Dielectric are of two types-

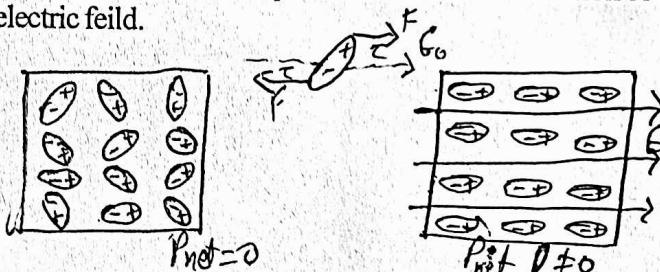
(i) Polar dielectrics- Those substances whose molecules behave like electric dipoles i.e. positive and negative centre of charge of whose molecules do not coincide e.g. HF, HCl, H₂O etc.

(ii) Non-polar dielectrics- Those substances whose molecules do not behave like electric dipoles i.e. +ve and -ve centre of charge of whose molecules coincide with one another e.g. O₂, CO₂, CH₄ etc.

Behaviour of polar dielectric in external electric field

Ques.- Describe the behaviour of polar dielectric in external electric field.

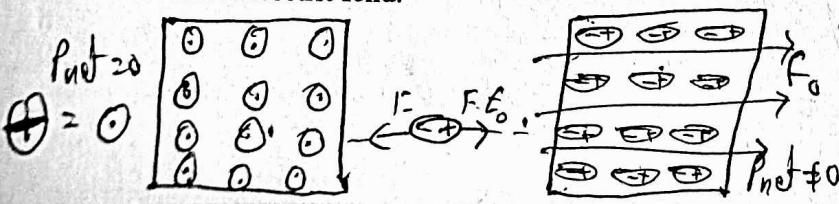
Ans.- When a polar dielectric is placed in external electric field then positive centre of charge of its molecules experience electric force in the direction of electric field and negative centre of charge of its molecules experience electric force in the direction opposite to electric field. As a result of which the molecules of polar dielectric experience a torque and reorient themselves in the direction of electric field. In this manner polar dielectrics acquire a net electric dipole moment in the direction of electric field.



Behaviour of non-polar dielectric in external electric field

Ques.- Describe the behaviour of non-polar dielectric in external electric field.

Ans.- When a non-polar dielectric is placed in external electric field then positive centre of charge of its molecules experience electric force in the direction of electric field and negative centre of charge of its molecules experience electric force in the direction opposite to electric field. As a result of which the positive and negative centre of charge gets slightly displaced and begin to behave like electric dipoles. In this manner non-polar dielectrics acquire a net electric dipole moment in the direction of electric field.



Polarization

Ques.- What is polarization ? Explain why the polarization of dielectric reduces electric field inside the dielectric.

Ans.- The phenomenon of alignment of polar molecules in the direction of electric field or the relative displacement of positive and negative centre of charge of non-polar molecules under the effect of external electric field is called polarization.

Due to phenomenon of polarization a dielectric substance acquires a net electric dipole moment in the direction of external electric field.

As a result of polarization an induced electric field (E_i) comes into action inside the dielectric substance because of induced charges whose direction is opposite to the applied electric field (E_o). Therefore the resultant electric field inside the dielectric is given by

$$E = E_o - E_i \quad E_o = \frac{q}{\epsilon_0} = \frac{q}{\epsilon_0 A}$$

Mathematically, it can be proved that

$$E = \frac{E_o}{K} \quad E = \frac{q}{\epsilon_0 K} = \frac{q}{\epsilon_0 A} = \frac{E_o}{K}$$

$$\therefore K > 1 \quad \therefore E < E_o$$

Polarization density

Ques.- What do you mean by polarization density ?

Ans.- The induced electric dipole moment developed per unit volume in a dielectric substance under the effect of external electric field is called polarization density.

If p be the induced electric dipole moment acquired by a molecule and N be the number of molecules present in a unit volume then polarization density is given by

$$P = Np$$

It is found that induced electric dipole moment is directly proportional to the resultant electric field intensity inside the dielectric.

$$\text{i.e. } P \propto E$$

$$\text{or } P = \chi_e E$$

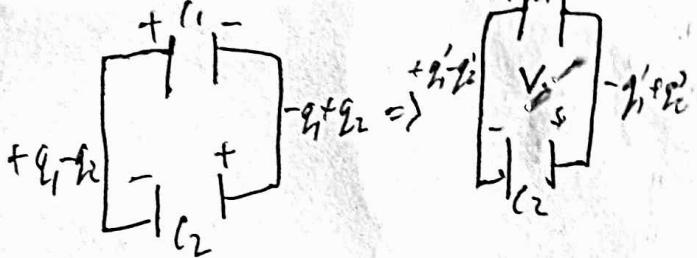
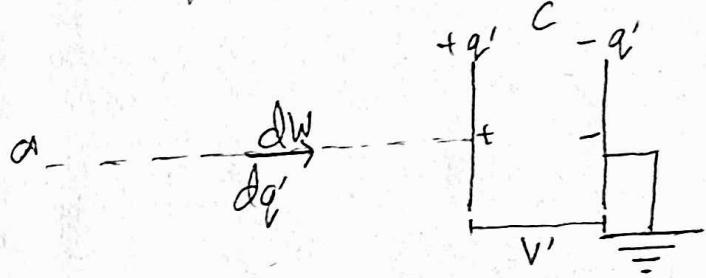
where χ_e is a constant of proportionality which is called electric susceptibility of the dielectric medium.

Note- Relation between electric susceptibility and dielectric constant of the dielectric medium-

$$\chi_e = \epsilon_0 (K-1)$$

~~* Energy of charged capacitor -~~
While deriving the expression of energy of charged capacitor work has been done in order to charge isolated plate of capacitor.

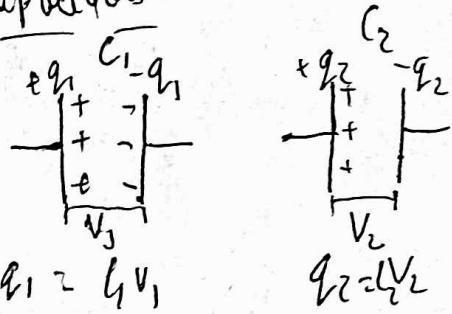
~~* Analysis is same as that of energy of charged conductors~~



$$V = \frac{q_1' + q_2'}{C_1 + C_2} = \frac{q_1 - q_2}{C_1 + C_2}$$

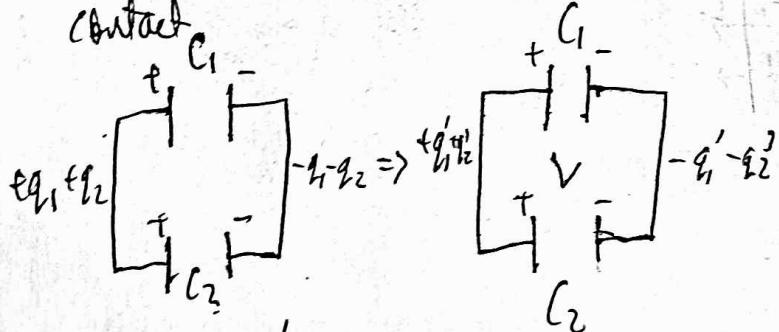
$$V = \frac{C_1 + C_2 - C_2 V_2}{C_1 + C_2}$$

* Redistribution of charge D/w discharged capacitors



$$q_1 = C_1 V_1, \quad q_2 = C_2 V_2$$

(i) When plates having like charge are brought into contact



$$V = \frac{q_1' + q_2'}{C_1 + C_2} = \frac{q_1 + q_2}{C_1 + C_2}$$

$$V = \frac{C_1 V_1 + C_2 V_2}{C_1 + C_2}$$

(ii) When plates having unlike charge are brought into contact