Electric potential

Ques.- Define electric potential. What are its units and dimensions ?

Ans.-Electric potential at any point in the electric field is defined as the work done in bringing a unit positive charge from infinity to that point

If the work done in bringing test q_0 from infinity to a point in the electric field be W then the electric potential at that point is given by

$$\boxed{V = \frac{W}{q_0}}$$

It is a scalar quantity. Its S.I. unit is JC^{-1} or volt (V), C.G.S. esu is $ergstatC^{-1}$ or statvolt (statV) and C.G.S. emu is $ergabC^{-1}$ or abvolt (abV). Its dimensional formula is $[ML^2T^{-3}A^{-1}]$.

Electric potential difference

Ques.- Define electric potential difference. What are its units and dimensions ?

Ans.- Electric potential difference between any two points in the electric field is defined as the work done in moving a unit positive charge from one point to the another point

If work done in moving it charge q_0 from one point to another point in the electric field be W then the electric potential difference between those points in given by.

$$\boxed{V_{AB} = \frac{W}{q_0}}$$

It is a scalar quantity. Its S.I. unit is JC^{-1} or volt (V), C.G.S. esu is $ergstatC^{-1}$ or statvolt (statV) and C.G.S. emu is erg abC^{-1} or abvolt (abV). Its dimensional formula is $[ML^2T^{-3}A^{-1}]$.

Note- (i) If electric potential at any point in the electric field be V then the work done in moving a charge q from infinity to that point is given by

$$W = qV$$

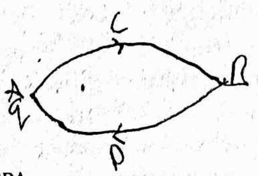
(ii) If the electric potential difference between two points in the electric field be V_{AB} then the work done in moving a charge q from one point to the other point is given by

$$W = qV_{AB}$$

(iii) Electric field is a conservative field i.e. work done by it does not depend upon the nature of path followed by the particle and it only depends upon the initial and final position of the particle.

$(iv) Work done by electric field in moving a charge in closed loop is always zero.

Explaination- Work done in moving a charge q from point A to B along C



$$W_{ACB} = qV_{BA}$$

and work done in moving charge q from point B to A along D

$$W_{BDA} = qV_{AB}$$

total work done $W = W_{ACB} + W_{BDA}$

$= qV_{BA} + qV_{AB}$

$= q(V_B - V_A) + q(V_A - V_B)$

$= qV_B - qV_A + qV_A - qV_B$

$= 0$

Definition of S.I. and C.G.S. esu of electric potential

Ques.- Define S.I. and C.G.S. units of electric potential and obtain relation between them.

Ans.- (a) S.I. unit- In relation

$$V = \frac{W}{q_0}$$

If $q_0 = 1C$ and $W = 1J$

then $V = \frac{1J}{1C} = 1V$

Thus, if the work done is bringing one coulomb of charge from infinity to any point in the electric field be one joule then the electric potential at that point is said to be one volt.

(b) C.G.S.esu- In relation

$$V = \frac{W}{q_0}$$

If $q_0 = 1statC$ and $W = 1erg$

then $V = \frac{1erg}{1statC} = 1statV$

Thus, if the work done is bringing one statcoulomb of charge from infinity to any point in the electric field be one erg then the electric potential at that point is said to be one statvolt.

Relation between S.I. and C.G.S. esu-

$$1V = \frac{1J}{1C}$$

or $1V = \frac{10^7 erg}{3\times10^9 \, statC}$

or $1V = \frac{1}{300}$ statV

Electric potential due to a point charge

Ques.- Obtain an expression for the electric potential due to a point charge at a point in space.

Ans.- Consider a point charge q at point O. We have to find out electric potential due to it at point P which is as a distance r from it.

In order to find out electric potential at point P, a ~~unit positive~~ test charge q_0 is taken at point A, which is at a distance x from point charge q.

By ClSL, the electrostatic force experienced by test charge q_0 ~~(unit positive charge)~~ at point A

$$F_e = \frac{1}{4\pi\varepsilon_0}\frac{qq_0}{x^2}$$

$$\cancel{F_e = \frac{1}{4\pi\varepsilon_0}\frac{q(1)}{x^2}}$$

$$\cancel{F_e = \frac{1}{4\pi\varepsilon_0}\frac{q}{x^2}}$$

In order to move charge q_0 through infinitely small distance dx towards point P a force has to be applied on it which should be equal in magnitude and opposite in direction to the electric force.i.e.

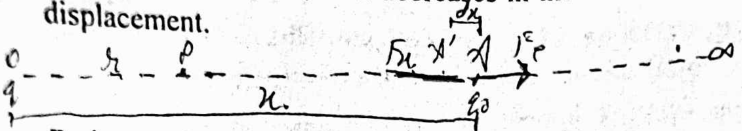
$$F_a = F_e = \frac{1}{4\pi\varepsilon_0}\frac{qq_0}{x^2} \text{ (in magnitude)}$$

Work done in moving ~~unit positive~~ test charge through infinitely small distance dx

$$dW = F_a(-dx)$$

or $dW = -\frac{1}{4\pi\varepsilon_0}\frac{qq_0}{x^2}dx$

Here (-) sign shows that x decreases in the direction of displacement.



By integrating the above relation within proper limits the total work done in moving charge q_0 from ∞ to point P can be calculated as follows-

$$\int_0^W dW = \int_\infty^r -\frac{1}{4\pi\varepsilon_0}\frac{qq_0}{x^2}\,dx$$

or $$\left[W\right]_0^W = -\frac{1}{4\pi\varepsilon_0}qq_0\int_\infty^r \frac{1}{x^2}\,dx$$

or $$W - 0 = -\frac{1}{4\pi\varepsilon_0}qq_0\int_\infty^r x^{-2}\,dx$$

or $$W = -\frac{1}{4\pi\varepsilon_0}qq_0\left[\frac{x^{-2+1}}{-2+1}\right]_\infty^r$$

or $$W = -\frac{1}{4\pi\varepsilon_0}qq_0\left[\frac{x^{-1}}{-1}\right]_\infty^r$$

or $$W = +\frac{1}{4\pi\varepsilon_0}qq_0\left[x^{-1}\right]_\infty^r$$

or $$W = \frac{1}{4\pi\varepsilon_0}qq_0\left[\frac{1}{x}\right]_\infty^r$$

or $$W = \frac{1}{4\pi\varepsilon_0}qq_0\left[\frac{1}{r}-\frac{1}{\infty}\right]$$

or $$W = \frac{1}{4\pi\varepsilon_0}qq_0\left[\frac{1}{r}-0\right]$$

or $$W = \frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r}$$

By definition, the electric potential at point P

$$V = W/q_0 \Rightarrow \frac{\frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r}}{q_0}$$

or $$\boxed{V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}}$$

This is the required expression of electric potential due to a point charge.

Particular cases- (i) If r = 0

then $$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{0} = \infty \text{ (not defined)}$$

Thus, the electric potential due to a point charge on itself is not defined.

(ii) If $r = \infty$

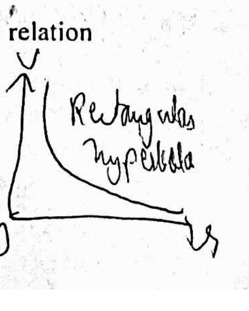
then $$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{\infty} = 0$$

Thus, the electric potential due to a point charge at ∞ is zero.

Note-Graph between V and r- From relation

$$V = \frac{q}{4\pi\varepsilon_0 r}$$ we have

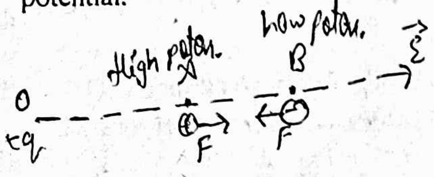
$$V \propto \frac{1}{r}$$



Note- (i) As the work done by external agency is positive and field is negative therefore the electric potential due to positive charge is positive and due to negative is negative. That is why the sign of charge should be taken in the expression of electric potential.

(ii) Electric potential is that physical quantity which decides the direction of flow of charge.

The positive charge always move from the point of higher electric potential to the point of lower electric potential and the negative charge always move from the point of lower electric potential to the point of higher electric potential.



(iii) Electric potential decreases in the direction of electric field and vice-versa.

Relation between electric field intensity and electric potential

Ques.- Derive relation between electric field intensity and electric potential.

Ans.- Consider a point charge q at point O and a unit positive test charge q_0 at point P which is at a distance r from it.

In order to displace unit positive charge through infinitely small distance dr from point P to P' towards charge q, a force has to be applied on it which should be equal in magnitude and opposite in direction to electric force acting on it.

i.e. $F_a = F_e$ (in magnitude)

Work done in moving unit positive charge by infinitely small distance dr.

$$dW = -F_a dr$$

or $$dW = -F_e dr$$

Since $$E = \frac{F_e}{q_0} = \frac{F_e}{1} = F_e$$

$\therefore$ $$dW = -E dr$$

By definition, the potential difference between points P & P' = dW

$\therefore$ $$dV = -E dr$$

or $$E = -\frac{dV}{dr}$$

Here (-) sign shows that electric potential decreases is the direction of electric field.

Thus, the EFI at any point in the electric field is equal to negative of the rate of charge of electric potential w.r.t. distance at that point.

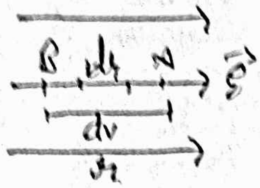
Note- When the electric field is uniform then the relation between EFI and electric potential can be obtained as follows-

or $$E = -\frac{dV}{dr}$$

or $$dV = -E dr$$

Integrating the above relation within proper limits

$$\int dV = \int -E\,dr$$



or $$\int dV = -E\int dr$$

or $\quad dV = -Er$

Numerically $\quad V = Er$

(Here r should be in the direction of electric field)

Potential gradient

Ques.- Define potential gradient. What are its units and dimensions.

Ans.- The rate of change of electric potential with distance at any point in the electric field is called potential gradient at that point. It is represented by ∇ (atled)

Thus, potential gradient $\boxed{\nabla = \frac{dV}{dr}}$

Also $\quad \frac{dV}{dr} = -E \qquad \left(\because E = -\frac{dV}{dr}\right)$

$\therefore \quad \nabla = -E \quad$ or $E = -\nabla$

In vector form $\vec{\nabla} = -\vec{E} \quad$ or $\vec{E} = -\vec{\nabla}$

Here (-) sign shows that the direction of potential gradient is opposite to that of EFI i.e. from low electric potential to high electric potential.

Thus, electric field intensity at any point in the electric field is equal to the negative of potential gradient at that point.

Its S.I. unit is Vm^{-1}, C.G.S. esu is $statVcm^{-1}$ and dimensional formula is $[MLT^{-3}A^{-1}]$

Note- Since $E = -\nabla$ therefore the dimensions and units of E are same as that of ∇.

Equipotential surface

Ques.- What do you understand by equipotential surface? Write down its properties.

Ans.- It is that imaginary surface at every point of which the value of electric potential is same.

Properties of equipotential surface- (i) The work done in moving a charge on an equipotential surface is always zero.

(ii) Equipotential surfaces are always perpendicular to electric lines of force.

(iii) In the region of strong electric field equipotential surfaces are closely spaced and in the region of weak electric field equipotential surfaces are widely spaced.

$$E = -\frac{dV}{dr}$$

If $\quad dV = \text{constant}$

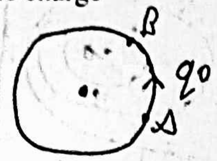
$$E \propto -\frac{1}{dr}$$

(iv) Two equipotential surfaces can never intersect each other.

If two equipotential surfaces intersect each other at the point of intersection there will be two different values of electric potential which is physically impossible.

Note-(i) Explaination of I property- When a test charge q_0 is moved from point A to B on an equipotential surface then the work done in moving the charge

$W = q_0 (V_B - V_A)$



$\because \quad V_B = V_A = A$

then $\quad W = q_0 (V - V)$

or $\quad W = q_0.0$

or $\quad W = 0$

$(Optional)(ii) Explaination of II property-When a test charge q_0 is moved by infinitely small distance ds along an equipotential surface then the work done is given by

$dW = \vec{F_e}.\vec{ds}$

$\because \quad dW = 0$

$\therefore \quad 0 = F_e\, ds \cos\theta$

or $\quad 0 = \cos\theta$

or $\quad \cos\theta = \cos 90°$

$\therefore \quad \theta = 90°$

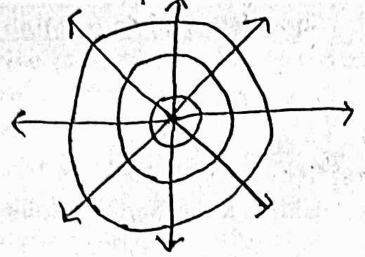
As electric force is along electric lines of force and displacement is along equipotential surface therefore electric lines of force and equipotential surface are perpendicular.

Equipotential surfaces due to various charge configuration

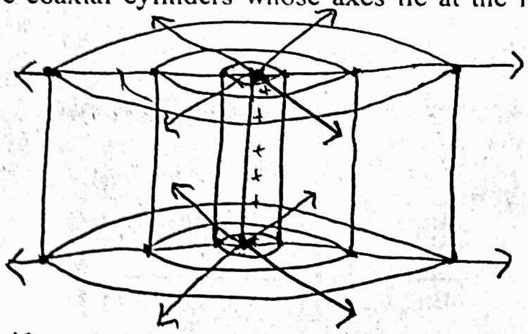
Ques.- Draw equipotential surfaces due to following charge configuration-

(i) Due to a point charge (ii) Due to a line charge (iii) In uniform electric field (iv) Due to an electric dipole (v) Due to two equal charges of same sign

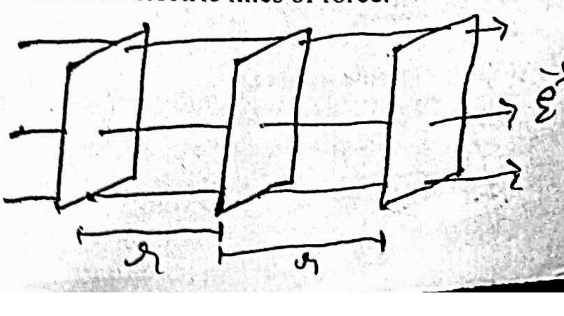
Ans.-(i) Due to a point charge-In this case equipotential surfaces are concentric spheres whose centres lie at the point charge.



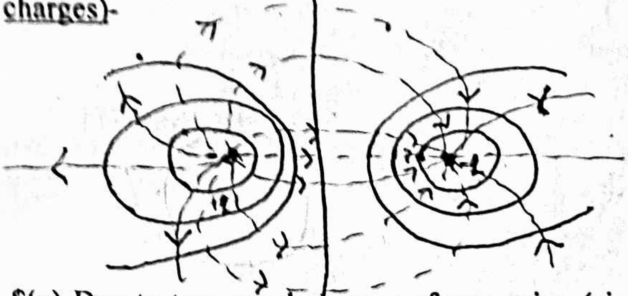
(ii) Due to a line charge- In this case equipotential surfaces are coaxial cylinders whose axes lie at the line charge.



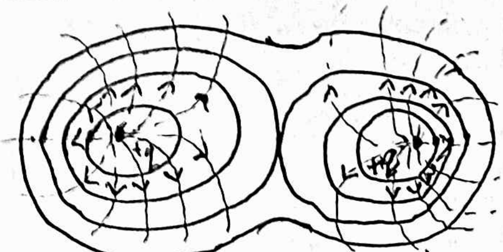
(iii) In uniform electric field- In this case equipotential surfaces are equally spaced parallel planes which are perpendicular to electric lines of force.



(iv) Due to an electric dipole (two equal and opposite charges)-



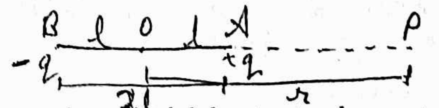
$(v) Due to two equal charges of same sign (similar charges)-



Electric potential due to electric dipole at any point on its axial line

Ques.- Derive the relation for electric potential due to electric dipole at any point on its axial line (end on position.)

Ans.- Consider an electric dipole AB which is made up of two charges +q and -q placed at a separation 2ℓ. We have to find out electric potential at point P which is at a distance r from its centre O on its axial line.



Elecetric potential due to +q charge at point P

$$V_A = \frac{1}{4\pi\varepsilon_0}\frac{q}{AP}$$

$\because$ $AP = OP - OA = r - \ell$

$$\therefore \quad V_A = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r-\ell)} \quad \text{---(i)}$$

and electric potential due to -q charge at point P

$$V_B = \frac{1}{4\pi\varepsilon_0}\frac{q}{BP}$$

$\because$ $BP = OP + OB = r + \ell$

$$\therefore \quad V_B = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r+\ell)} \quad \text{---(ii)}$$

Electric potential due to electric dipole at point P

$$V = V_A + V_B$$

or $$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r-\ell)} - \frac{1}{4\pi\varepsilon_0}\frac{q}{(r+\ell)}$$

or $$V = \frac{1}{4\pi\varepsilon_0} q \left[\frac{1}{(r-\ell)} - \frac{1}{(r+\ell)}\right]$$

or $$V = \frac{1}{4\pi\varepsilon_0} q \frac{r+\ell-r+\ell}{(r-\ell)(r+\ell)}$$

or $$V = \frac{1}{4\pi\varepsilon_0}\frac{q2\ell}{(r^2-\ell^2)}$$

$\because$ $q2\ell = p$ electric dipole moment

$$\therefore \quad \boxed{V = \frac{1}{4\pi\varepsilon_0}\frac{P}{(r^2-\ell^2)}}$$

This is the required expression.

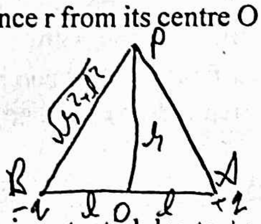
Particular case- If the electric dipole be small i.e. $\ell << r$ then neglecting ℓ^2 as compared to r^2, we get

$$\boxed{V = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^2}}$$

Electric potential due to electric dipole at any point on its equitorial line

Ques.- Derive the relation for electric potential due to electric dipole at any point on its equitorial line (broad on position.)

Ans.- Consider an electric dipole AB which is made up of two charges +q and -q placed at a separation 2ℓ. We have to find out electric potential at point P which is at a distance r from its centre O on its equitorial line.



Electric potential due to +q charge at point P

$$V_A = \frac{1}{4\pi\varepsilon_0}\frac{q}{AP}$$

$\because$ $AP = \sqrt{OP^2 + OA^2} = \sqrt{r^2 + \ell^2}$

$$\therefore \quad V_A = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r^2+\ell^2)^{1/2}} \quad \text{---(i)}$$

and electric potential due to -q charge at point P

$$V_B = \frac{1}{4\pi\varepsilon_0}\frac{-q}{BP}$$

$\because$ $BP = \sqrt{OP^2 + OB^2} = \sqrt{r^2 + \ell^2}$

$$\therefore \quad V_B = \frac{1}{4\pi\varepsilon_0}\frac{-q}{(r^2+\ell^2)^{1/2}} \quad \text{---(ii)}$$

Electric potential due to electric dipole at point P

$$V = V_A + V_B$$

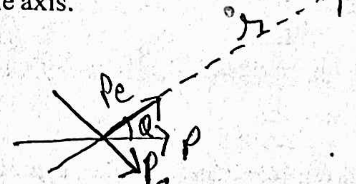
$$\therefore \quad V = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r^2+\ell^2)^{1/2}} \quad \frac{1}{4\pi\varepsilon_0}\frac{q}{(r^2+\ell^2)^{1/2}}$$

or $\boxed{V = 0}$

$Electric potential at any point due to small electric dipole

Ques.- Derive the relation for electric potential at any point due to small electric dipole.

Ans.- Consider a small electric dipole whose electric dipole moment be p. We have to find out electric potential due to electric dipole at point P which is at a distance r from its centre O such that the line joining OP makes an angle θ with the dipole axis.



Here the electric dipole moment of electric dipole can be resolved into two components-

(i) axial component $p_a = p\cos\theta$

(ii) equitorial component $p_e = p\sin\theta$

Electric potential due to axial component at point P

$$V_a = \frac{1}{4\pi\varepsilon_0}\frac{p_a}{r^2}$$

$$V_a = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2} \quad \text{----(i)}$$

and electric potential due to equitorial component at point P

$$V_e = 0 \quad \text{---(ii)}$$

$\therefore$ Electric potential due to electric dipole at point P.

$$V = V_a + V_e$$

or $$V = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2} + 0$$

or $$\boxed{V = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2}}$$

This is the required expression.

Electric potential energy of a system of charges

Ques.- What is electric potential energy of a system of charges ? Derive the relation for electric potential energy of a system of two & three point charges.

Ans.- Electric potential energy of a system of charges is equal to work done in bringing those charges from infinity to their respective positions.

(a) When the system is made up of two point charges-
Consider a system which is made up of two point charges q_1 and q_2 which are placed at a separation r from each other.

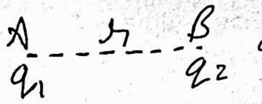
Let us assume that initially the two charges be at infinity. Now the work done in bringing charge q_1 form ∞ to point A

$$W_1 = = q_1 V_A = q_1(0) = 0$$

and the work done in bringing charge q_2 from ∞ to point B

$$W_2 = q_2 V_1$$

or $$W_2 = q_2 \frac{q_1}{4\pi\varepsilon_0 r}$$



or $$W_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}$$

Total work done bringing the two charges from ∞ to their respective positions.

$$W = W_1 + W_2$$

or $$W = 0 + \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}$$

or $$W = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}$$

By definition, this work done gets stored in the system in the form of potential energy.

$\therefore$ $U = W$

or $$\boxed{U = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}}$$

This is the required expression.

(b) When the system iis made up of three point charges-
Consider a system when is made up of three point charges q_1, q_2 and q_3 such that the separation between q_1 and q_2 be r_{12}, q_2 and q_3 be r_{23} and q_3 and q_1 be r_{31}.

Let us assume that initially the three charges be at unfinity. Then the work done in bringing charge q_1 from ∞ to pont A.

$$W_1 = = q_1 V_A = q_1(0) = 0$$

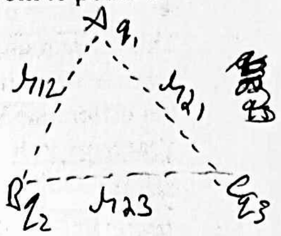
the work done in bringing charge q_2 from ∞ to point B

$$W_2 = q_2 V_B$$

or $$W_2 = q_2 \frac{q_1}{4\pi\varepsilon_0 r_{12}}$$

and the work done in bringing charge q_3 from to point C

$$W_3 = q_3 V_C$$



or $$W_3 = q_3(V_1 + V_2)$$

or $$W_3 = q_2\left(\frac{q_1}{4\pi\varepsilon_0 r_{31}} + \frac{q_2}{4\pi\varepsilon_0 r_{23}}\right)$$

or $$W_3 = \frac{1}{4\pi\varepsilon_0}\frac{q_3 q_1}{r_{31}} + \frac{1}{4\pi\varepsilon_0}\frac{q_3 q_2}{r_{23}}$$

Total work done bringing the three charges from ∞ to their respective positions

$$W = W_1 + W_2 + W_3$$

or $$W = 0 + \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}} + \frac{1}{4\pi\varepsilon_0}\frac{q_3 q_1}{r_{31}} + \frac{1}{4\pi\varepsilon_0}\frac{q_3 q_2}{r_{23}}$$

or $$W = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_2}{r_{12}} + \frac{q_2 q_3}{r_{23}} + \frac{q_3 q_1}{r_{31}}\right)$$

By definition, this work done is stored in the system in the form of electric potential energy.

$\therefore$ $U = W$

or $$\boxed{U = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_2}{r_{12}} + \frac{q_2 q_3}{r_{23}} + \frac{q_3 q_1}{r_{31}}\right)}$$

This is the required expression.

Note- When the system is made up of n point charges- (Optional)

$$U = \frac{1}{2}\left[\frac{1}{4\pi\varepsilon_0}\sum_{\substack{i,j=1 \\ i\neq j}}^{n}\frac{q_i q_j}{r_{ij}}\right]$$

Velocity and kinetic energy acquired by a charged particle in uniform electric field

Ques.- Derive the relation for velocity and kinetic energy acqulned by a charged particle in uniform electric field.

Ans.- Consider a charged particle of mass m and charge q in a uniform electric field E. Then the electric force acting a charged particle

$$F_e = qE$$

From NSL, the acceleration of the charged particles

$$a = \frac{F_e}{m} \quad \frac{qE}{m}$$

If the charged particle be intitially at rest then the velocity acquired by it in moving through a distance r can be calculated as follows-

$$v^2 - u^2 = 2ar$$

or $$v^2 - 0^2 = \frac{2qEr}{m}$$

or $$v^2 = \frac{2qEr}{m}$$

$\because$ Er = V potential difference between point A and B.

$\therefore$ $$v^2 = \frac{2qV}{m}$$

or $$\boxed{v = \sqrt{\frac{2qV}{m}}}$$

This is the required expression of velocity acquired by a charged particle when it is allowed to accelerated through a potential difference V.

$$K.E. = \frac{1}{2} mv^2$$

or $$K.E. = \frac{1}{2} m \frac{2qV}{m}$$

or $$\boxed{K.E. = qV}$$

This the required expression of K.E. acquired by a charged particle when it is allowed to accelerate through a potential difference V.

Electron volt

Ques.- Define electron volt.

Ans.- In relation $E_k = qV$

If $q = 1e$ and $v = 1V$

then $E_k = 1e \times 1V$

or $E_k = 1eV$

Thus, when a charge of one electron is allowed to accelerate through a potential difference of 1V then the K.E. acquired by it is said to be one electron volt.

$$\begin{aligned} 1eV &= 1e \times 1V \\ &= 1.6\times10^{-19}\ C\times1V \\ &= 1.6\times10^{-19}\ CV \\ &= 1.6\times10^{-19}\ J \end{aligned}$$

Note- 1Megaelectronvolt (MeV) $= 10^6$ eV

$$\begin{aligned} &= 10^6 \times 1.6\times10^{-19}\ J \\ &= 1.6\times10^{-13}\ J \end{aligned}$$