Ques.- Define charge. State its types.

Ans.- It is that property of a substance by virtue of which it exhibits electric and magnetic effects.

Types of charges- When an amber rod is rubbed with fur then the charge developed on amber rod is called negative charge and when a glass rod is rubbed with silk then the charge developed on glass rod is called positive charge.

Electricity

Ques.-Define electricity. State its types.

Ans.-It was defined by Thomas Brown as that form of energy by virtue of which substances acquire the property of attracting other light bodies.

Types of electricity-

(a) Static electricity- When the electricity developed on a body is not free to flow from one point to another point then it is called static electricity.

(b) Current electricity- When the electricity developed on a body is free to flow from one point to another point then it is called current electricity.

Frictional electricity

Ques.-What is frictional electricity? Give example.

Ans.-When two bodies are rubbed together then the electricity developed on bodies by virtue of friction is called frictional electricity.

e.g.- When an amber rod is rubbed with fur then due to frictional electricity it acquires the property of attracting other light bodies like bits of paper, feathers, dust particles etc.

Note-(i) In 600 B.C. It was discovered by Greek philosopher Thales of Miletus. He observed that when an amber rod is rubbed with fur then it acquires the property of attracting other light bodies like bits of paper, feathers, dust particles etc. (Amber is a yellow resinous substance found on the shores of Baltic sea.)

(ii) The word electricity is derived from Greek word "elektron" which means amber.

(iii) When a body acquires the property of attracting other light bodies then it is said to be electrified or charged.

(iv) In 1600 A.D. on the basis of his experiments William Gilbert divided substances into two categories-

(a) Those substances which behave like glass are called vitreous.

(b) Those substances which behave like amber are called resinous.

Resinous	Vitreous
Amber	Wool
Ebonite	Fur or catskin
Plastic	Flannel
Rubber	Nylon
Silk	Glass

Later vitreous substances were called positively charged and resinous substances were called negatively charged by Benjamin Franklin.

Electronic theory of charge

Ques.- What is electronic theory of charge and on the basis of it explain frictional electricity ?

Ans.- Every substance is made up of molecules and molecules are made up of atoms. At the centre of each atom there is a nucleus which contains electrically neutral neutrons and positively charged protons. The negatively charged electrons revolve around the nucleus in circular orbits. As the number of protons and electrons in an atom is same and the amount of charge present them is also equal and opposite therefore an atom is electrically neutral as a whole and hence the substances are also electrically neutral.

Explaination- When a substance is rubbed with another substance then the generation of heat takes place due to friction. If the heat generated is greater than the work function of one substance and less than that of other substance then the substance having low work function lose electrons and the substance having high work function gain electrons. Due to deficiency of electrons the substance having low work function becomes positively charged and due to excess of electrons the substance having high work function becomes negatively charged.

Note- (i) Like charges repel each other and unlike charges attract each other.

(ii) In frictional electricity the charges developed on two bodies are always equal in magnitude and opposite in nature.

(iii) In order to make a body positively charged some electrons have to be removed from it due to which its mass decreases and in order to make a body negatively charged some electrons have to be added to it due to which its mass increases.

(v) In physics, charge is taken as derived physical quantity.

Charge = electric current × time

i.e. $q = It$

It is scalar quantity and its sign tells us about its nature. Its S.I. unit is coulomb (C) its C.G.S. electrostatic unit is statcoulomb (statC) and its C.G.S. electromagnetic unit is abcoulomb (abC). Its dimentional formula is [AT].

$$1C = 3 \times 10^9 \text{ statcoulomb (statC)}$$

$$\text{and} \quad 1C = \frac{1}{10} \text{ abcoulomb (abC)}$$

$$\therefore 1abC = 3 \times 10^{10} \text{ statC}$$

$$\text{or} \quad 1abC = c \text{ statC}$$

where $c = 3 \times 10^{10}$ cms^{-1} is the speed of light is vacuum.

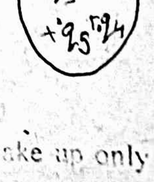
Additive nature of charge

Ques.-What is additive nature of charge ?

Ans.- When two or more charges are present on a body then the total charge present on the body is equal to the scalar sum of all the charges which are present on it. That is why charge is said to be additive in nature.

If five charges $-q_1$, $+q_2$, $+q_3$, $-q_4$ and $+q_5$ are present on a body then the total charge present on it is given by

$$q_{tot} = -q_1 + q_2 + q_3 - q_4 + q_5$$



Note- The sign of charge tells us about its nature and not its direction.

Quantisation of charge

Ques.-What is quantisation of charge?

Ans.- The charge present on a body can take up only certain specific values i.e the charge is not continuous but discrete in nature. This property of charge is called

Quantisation of charge. Mega electron volt (MeV):

In nature, the charge present on a body is an integral multiple of fundamental unit of charge called elementary charge which is denoted by 'e'. This elementary charge is equal to the magnitude of charge present on an electron or proton i.e. $e = 1.602 \times 10^{-19}$ C.

If the charge present on a body be q then in accordance with quantisation of charge

$$q = \pm ne$$

where n = 1, 2, 3, 4, 5......

Thus, the charges like +e, +2e, +3e..... and -e, -2e, -3e.... can only exist in nature and the fractional multiples of e cannot exist in nature.

Note- Quantization of charge was first experimentally verified by Millikan in 1913 through his famous oil drop experiment.

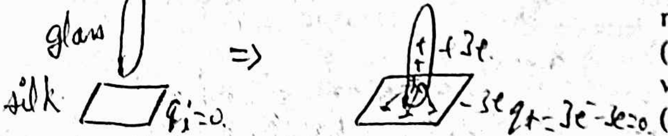
Conservation of charge

Ques.-What is conservation of charge?

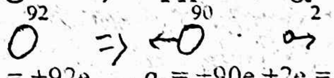
Ans.- The law of conservation of charge states that the total charge present on an isolated system always remains conserved. In other words in an isolated system the charge can neither be created nor it can be destroyed.

The law of conservation of charge is a universal law.

e.g.- (i) When a glass rod is rubbed with silk cloth then the total charge on the glass rod and the silk cloth before and after rubbing is zero .

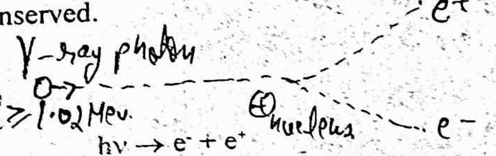


(ii) In nuclear fission, when a uranium-238 nucleus gets converted into thorium-234 nucules by emitting an α-particle then the total charge before and after fission remains conserved. $U^{238}_{92} \rightarrow Th^{234}_{90} + \alpha^{4}_{2}$



$q_i = +92e \qquad q_f = +90e + 2e = +92e$

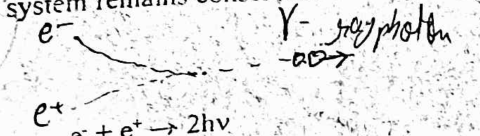
(ii) When a γ-ray photon having energy greater or equal to 1.02 MeV passes close by a nucleus then under the effect of nucleus its gets converted into an electron (e^-) and a positron (e^+). This phenomenon is called pair production and here also the total charge of the system remains conserved.



$h\nu \rightarrow e^- + e^+$

$q_i = 0 \quad q_f = -e + e = 0$

(iv)When an electron (e^-) and a positron (e^+) combine together they give rise to two γ-ray photons. This phenomenon is called annihilation of matter and here also the total charge of the system remains conserved.



$e^- + e^+ \rightarrow 2h\nu$

$q_i = -e + e = 0 \quad q_f = 0$

Note-(i) The law of conservation of charge is universal in nature i.e. charge remains coserved in all mechanical chemical, electrical, magnetic and nuclear processes.

Specific charge = $\frac{q}{m}$

(ii) The amount of charge present on a body is independent of its velocity i.e. whatever be the velocity of the body the amount of charge present on it remains constant (From Einstein's theory of relativity).

(iii) Charge can never exist without mass.

$(iv) In 1964, It was found that protons and neutrons are made up of quarks. One proton is made up of two up and one down qwark.

i.e 1proton = 2 up quark + 1 down quark (uud)

and one neutron is made up of one up and two down quarks.

i.e. 1 neotron = 1 up quark + 2 down quark (udd)

The charge present on up quark is $\frac{+2e}{3}$ and the charge present on down quark is $\frac{-e}{3}$.

Even after the discovery of quarks the quantization of charge remains uneffected. But the elementary charge is reduced to e/3.

Comparision between charge and mass

Ques.- Compare charge and mass.

Ans.-

Charge	Mass
(i) Charge is that property of a substance by virtue of which it exhibit electric and magnetic effects.	(i) Mass is the quantity of matter contained in a body.
(ii) Charge can not exist without mass.	(ii) Mass Can exist without charge.
(iii) Charge does not depend upon the velocity of a body.	(iii) Mass depends upon the velocity of body.
(iv) Two charges attract or repel one another.	(iv) Two masses always attract one another.
(v) Change is quantised.	(v) Mass is continuous i.e. not quantised.

Coulomb's Inverse Square law

Ques.- State Coulomb's Inverse Square law .Write mathematical formula.

Ans.- In the year 1785, French scientist Charles Augustine de Coulomb experimentally established a relation for the force acting between two stationary point charges with the help of torsion balance.

According to Coulomb's inverse square law the force of attraction or repulsion between two stationary point charges is directly proportional to the product of magnitude of two charges and inversely proportional to the square of distance between them. This force always act along the line joining the two charges.

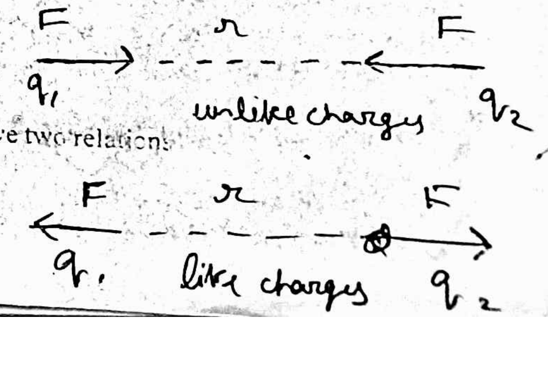
If two point charges q_1 and q_2 are placed at a separation r from each other then according to Coulomb's inverse square law

$F \propto q_1 q_2$

and $F \propto \frac{1}{r^2}$

Combining the above two relations

$F \propto \frac{q_1 q_2}{r^2}$



or $F = \frac{k\, q_1 q_2}{r^2}$ ---(i)

where k is a constant of proportionality which is called electrostatic constant.

The value of k depends upon the system of unit used and the nature of medium between the two charges.

(I) S.I. system-

(a) In vacuum- $k = \frac{1}{4\pi\varepsilon_0}$

where ε_0 is another constant which is called absolute permittivity of free space.

$$\varepsilon_0 = 8.8542 \times 10^{-12} \; \frac{C^2}{Nm^2}$$

$$\therefore \quad k = \frac{1}{4\pi\varepsilon_0} = \frac{1}{4\pi \times 8.8542 \times 10^{-12}} = 9\times10^9 \; \frac{Nm^2}{C^2}$$

From relation (i)

$$\boxed{F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}}$$

(b) In medium- $k = \frac{1}{4\pi\varepsilon}$

where ε is called absolute permittivity of medium.

$\varepsilon = \varepsilon_r \varepsilon_0$ where ε_r is called relative permittivity of medium.

$$\therefore\; k = \frac{1}{4\pi\varepsilon} = \frac{1}{4\pi\varepsilon_0\varepsilon_r} = \frac{9\times10^9}{\varepsilon_r}\;\frac{Nm^2}{C^2}$$

From relation (i)

$$\boxed{F = \frac{1}{4\pi\varepsilon_0\varepsilon_r}\frac{q_1 q_2}{r^2}}$$

(II) C.G.S.(electrostatic) system-

(a) In vacuum- $k = \frac{1}{4\pi\varepsilon_0}$

$$\because\; \varepsilon_0 = \frac{1}{4\pi}\;\frac{statC^2}{dyn\ cm^2}$$

$$\therefore\quad k = \frac{1}{4\pi\varepsilon_0} = \frac{1}{4\pi\frac{1}{4\pi}} = 1\;\frac{dyn\ cm^2}{statC^2}$$

From relation (i)

$$\boxed{F = \frac{q_1 q_2}{r^2}}$$

(b) In medium- $k = \frac{1}{4\pi\varepsilon}$

$$\therefore\quad k = \frac{1}{4\pi\varepsilon} = \frac{1}{4\pi\varepsilon_0\varepsilon_r} = \frac{1}{\varepsilon_r}\frac{dyn\,cm^2}{statC^2}$$

From relation (i)

$$\boxed{F = \frac{1}{\varepsilon_r}\frac{q_1 q_2}{r^2}}$$

Note- (i) Electrostatic force (Coulombic force) obeys Newton's third law.

(ii) It is a central force i.e. it always act along the line joining the two charges.

(iii) It is a conservative force i.e. work done by it does not depend upon the path taken by the charges but only depends upon the initial and final position of the charged particle.

(iv) It is a strong force (as compared to gravitational force $F_e \simeq 10^{36} f_g$)

(v) Coulomb's inverse square law is valid for all distances ranging form interatomic distances to astronomical distances (but this law is not valid inside nucleus because inside nucleus the distance between protons is so small that they no longer behave like point charges).

(vi) Conditions for the validity of coulomb's inverse square law -

(a) The charges should be stationary.

(b) The charges should be point like.

(vii)(a) If $q_1q_2 < 0$ i.e. the product of two charges is -ve then the two charges are of opposite nature so they attract each other.

(b) If $q_1q_2 > 0$ i.e. the product of two charges is +ve then the two charges are of same nature so they repel each other.

Relative permittivity

Ques.- What do you understand by relative permittivity?

Ans.-The ratio of absolute permittivity of a medium and the absolute permittivity of free space is called relative permittivity or dielectric constant of that medium. It is denoted by ε_r or K.

i.e. Relative permittivity or dielectric constant

$$\varepsilon_r \text{ or } K = \frac{\varepsilon}{\varepsilon_0}$$

It is a unitless and dimensionless physical quantity.

Medium	Dielectric constant
Vacuum	1
Air	1.00029 ≃ 1.0003
Water	80.4
Metals	∞

Note- The word dielectric means insulator.

Definition of relative permittivity or dielectric constant (in terms of force)

Ques.- Define relative permittivity or dielectric constant in terms of force.

Ans.- Consider two point charges q_1 and q_2 placed at a certain distance r in vacuum then the force acting acting between them is given by

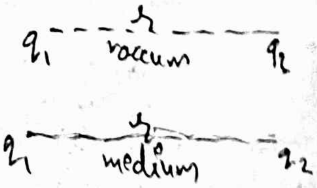
$$F_{vac} = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2} \quad \text{---(i)}$$

If the same two charges are placed at the same separation in a medium of dielectric constant ε_r then the force acting between them is given by

$$F_{med} = \frac{1}{4\pi\varepsilon_0\varepsilon_r}\frac{q_1q_2}{r^2} \quad \text{---(ii)}$$

Dividing relation (i) by (ii) we get

$$\frac{F_{vac}}{F_{med}} = \frac{\frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2}}{\frac{1}{4\pi\varepsilon_0\varepsilon_r}\frac{q_1q_2}{r^2}}$$



or $\frac{F_{vac}}{F_{med}} = \varepsilon_r$

Thus, the relative permittivity or dielectric constant of a medium is equal to the ratio of force acting between two point charges placed at a certain distance in vacuum and the force acting between the same two point charges placed at the same distance in that medium.

Definition of coulomb

Ques.- Define S.I. unit of charge i.e. coulomb.

Ans.- According to Coulomb's inverse square law the

Force acting between two point charges situated in vacuum is given by

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$$

If $q_1 = q_2 = q$, $r = 1m$ and $F = 9 \times 10^9 N$

$$9 \times 10^9 = 9 \times 10^9 \frac{qq}{(1)^2}$$

or $q^2 = 1$ i.e. $q = \pm 1$

$\therefore q = 1C$

Thus, if on placing two identical charges 1m apart in vacuum they repel one another by a force of 9×10^9 N then each of the charge is said to be 1C.

Note- (i) From relation

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2} \text{ we have } \frac{[AT]\times[AT]}{[MLT^{-2}]\times[L]^2}$$

$$\varepsilon_0 = \frac{1}{4\pi F}\frac{q_1 q_2}{r^2}$$

$\therefore$ S.I. unit of $\varepsilon_0 = \frac{C \times C}{N \times m^2} = \frac{C^2}{Nm^2} = [M^{-1}L^{-3}T^4A^2]$

(ii) From relation

$$F = \frac{kq_1q_2}{r^2} \text{ we have } \frac{[MLT^{-2}]\times[L^2]}{(AT)\times(AT)}$$

$$k = \frac{Fr^2}{q_1 q_2}$$

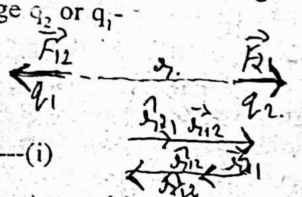
$\therefore$ S.I. unit of $k = \frac{Nm^2}{C \times C} = \frac{Nm^2}{C^2} = [ML^3T^{-4}A^{-2}]$

Coulomb's inverse square law in vector form

Ques.- State Coulomb's Inverse Square law in vector form.

Ans.- Consider two like point charges q_1 and q_2 placed at a separation r from one another in vacuum. Then the magnitude of force exerted by change q_2 or q_1-

$$F_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$$



In vector form

$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\hat{r}_{21} \quad \text{---(i)}$$

where $\hat{r}_{21}$ is the unit vecter along the position vector of charge q_1 w.r.t. charge q_2 i.e. $\vec{r}_{21}$.

The magnitude of force exerted by charge q_1 on charge q_2-

$$F_{21} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$$

In vector form

$$\vec{F}_{21} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\hat{r}_{12} \quad \text{---(ii)}$$

where $\hat{r}_{12}$ is the unit vecter along the position vector of charge q_1 w.r.t. charge q_2 i.e. $\vec{r}_{21}$.

$\because \hat{r}_{21} = -\hat{r}_{12}$

$\therefore$ From relation (i)

$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}(-\hat{r}_{12})$$

or

$$\vec{F}_{12} = \frac{-1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\hat{r}_{12}$$

or $\vec{F}_{12} = -\vec{F}_{21}$

From the above relation it can be conducted that

(a) coulombic force obeys Newton's third law.

(b) coulombic force is a central force.

Principle of superposition of forces

Ques.- What is principle of superposition of forces?

Ans.- It states that when two or more charges are present in space then the net force acting on a charge is equal to the vector sum of forces exerted by all other charges on it.

If five point charges q_1, q_2, q_3, q_4 and q_5 are present in space then the net force acting on charge q_5 is given by

$$\vec{F}_{5net} = \vec{F}_{51} + \vec{F}_{52} + \vec{F}_{53} + \vec{F}_{54}$$

Note- From the principle of superposition of forces, it can be concluded that the force acting between two charges is independent of presence of other charges in space.

Different types of charge distributions

Ques.- What do you understand by -

(a) Line charge distribution & line charge density

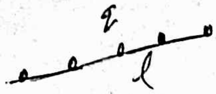
(b) Surface charge distribution & surface charge density

(c) Volume charge distribution & volume charge density

Ans-(a) **Line charge distribution**- When charge is distributed on a line then it is called line charge distribution. The amount of charge present on a unit length of line is called line charge density. It is denoted by λ.

If q be the amount of charge distributed uniformly on a line of length ℓ then the line charge density is given by

$$\lambda = \frac{q}{\ell}$$



It's S.I. unit is Cm^{-1}.

$Note- (Optional) Total force exerted by a line charge distribution on a point charge q_0 can be calculated as follows-

Line charge density

$$\lambda = \frac{dq}{d\ell}$$

or $dq = \lambda d\ell$

Force exerted by elementary charge dq on test charge q_0

$$d\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{dq\, q_0}{r^2}\hat{r}$$

Total force exerted by line charge on test charge q_0

$$\int_\ell d\vec{F} = \int_\ell \frac{1}{4\pi\varepsilon_0}\frac{dq\, q_0}{r^2}\hat{r}$$

or

$$\vec{F} = \frac{1\, q_0}{4\pi\varepsilon_0}\int_\ell \frac{dq}{r^2}\hat{r}$$

or

$$\vec{F} = \frac{1\, q_0}{4\pi\varepsilon_0}\int_\ell \frac{\lambda d\ell}{r^2}\hat{r}$$

(b) **Surface charge distribution**- When charge is distributed over a surface than it is called surface charge distribution. The amount of charge present on a unit area of surface is called surface charge density. It is denoted by σ

If q be the amount of charge distributed on a surface of area A then the surface charge density is given by

$$\sigma = \frac{q}{A}$$

It's S.I. unit is Cm^{-2}.

Note- (Optional) Total force exerted by a surface charge distribution on a point charge q_o can be calculated as follows-

Surface charge density

$$\sigma = \frac{dq}{dA}$$

or $\quad dq = \sigma dA$

Force exerted by elementary charge dq on test charge q_o

$$\vec{dF} = \frac{1}{4\pi\varepsilon_0}\,\frac{dq\,q_o}{r^2}\,\hat{r}$$

Total force exerted by surface charge on test charge q_o

$$\int_s \vec{dF} = \int_s \frac{1}{4\pi\varepsilon_0}\,\frac{dq\,q_o}{r^2}\,\hat{r}$$

or $$\vec{F} = \frac{1\,q_o}{4\pi\varepsilon_0}\int_s \frac{dq}{r^2}\,\hat{r}$$

or $$\vec{F} = \frac{1\,q_o}{4\pi\varepsilon_0}\int_s \frac{\sigma dA}{r^2}\,\hat{r}$$

(c) **Volume charge distribution**- When charge is distributed in volume of a body then it is called volume charge distribution. The amount of charge present in a unit volume of a body is called volume charge density. It is denoted by ρ.

If q amount of charge be present in volume V of a body then its volume charge density is given by

$$\rho = \frac{q}{V}$$

It's S.I. unit is Cm^{-3}.

Note- (Optional) Total force exerted by a volume charge distribution on a point charge q_o can be calculated as follows-

Volume charge density

$$\rho = \frac{dq}{dV}$$

or $\quad dq = \rho dV$

Force exerted by elementary charge dq on test charge q_o

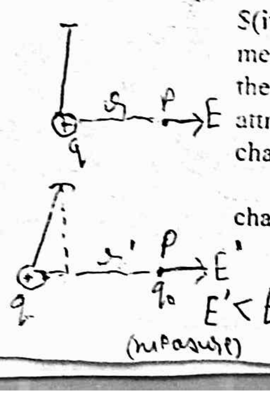
$$\vec{dF} = \frac{1}{4\pi\varepsilon_0}\,\frac{dq\,q_o}{r^2}\,\hat{r}$$

Total force exerted by volume charge on test charge q_o

$$\int_v \vec{dF} = \int_v \frac{1}{4\pi\varepsilon_0}\,\frac{dq\,q_o}{r^2}\,\hat{r}$$

or $$\vec{F} = \frac{1\,q_o}{4\pi\varepsilon_0}\int_v \frac{dq}{r^2}\,\hat{r}$$

or $$\vec{F} = \frac{1\,q_o}{4\pi\varepsilon_0}\int_v \frac{\rho dV}{r^2}\,\hat{r}$$



Electric field

Ques.- Define electric field.

Ans.- The concept of electric field was introduced by Michael Faraday in order to explain action at a disance.

The electric field is that space surrounding a charge with in which when other charges are brought in then they experience attractive or repulsive force.

Note(i)- Source charge- The charge which generates electric field is called source charge. It may be positive or negative.

(ii) Test charge- The charge which is used for the detection and the measurement of the effect of electric field is called test charge. It is always taken positive and is denoted by q_o.

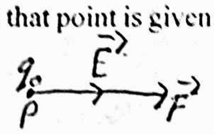
Electric field intensity

Ques.- Define electric field intensity.

Ans.- Electric field intensity at any point in the electric field is defined as the electrostatic force experienced by a unit positive charge placed at that point.

If the electrostatic force experienced by a test charge q_o, when it is placed at any point in the electric field be F then the electric field intensity at that point is given by

$$E = \frac{F}{q_o}$$



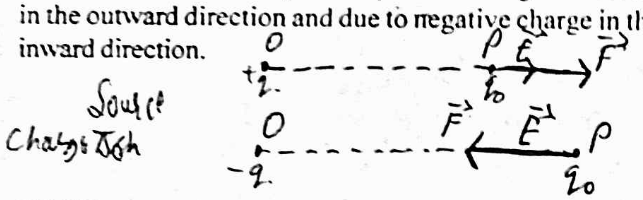
In vector form $\vec{E} = \frac{\vec{F}}{q_o}$

It is a vector quantity its direction is same as the direction of electrostatic force experienced by the test charge q_o.

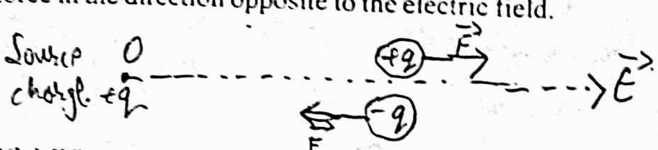
Its S.I. unit is NC^{-1}, C.G.S. esu is $dynstatC^{-1}$ and C.G.S. emu is $dynabC^{-1}$. Its dimentional formula is $[MLT^{-3}A^{-1}]$

Note(i)- Electric field intensity is that physical quantity which measures the effect of electric field at any point of it.

(ii) Electric field intensity due to positive charge is directed in the outward direction and due to negative charge in the inward direction.



(iii) When a positive charge is placed in external electric field then it experiences electrostatic force in the direction of electric field and when a negative charge is placed in external electric field then it experiences electrostatic force in the direction opposite to the electric field.



(iv) When test charge q_o is placed at a point in order to measure electric field intensity then due to its presence the electric field at that point gets modified (either due to attraction or repulsive between the source charge and test charge), when the source charge is free to move.

Therefore, in order to eliminate the effect of test charge, the electric field intensity is defined as

$$E = \lim_{q_o \to 0} \frac{F}{q_o}$$

~~Mathematically the value of~~

~~$$E = \frac{F}{q_0} \quad \text{and} \quad E = \lim_{q_0 \to 0} \frac{F}{q_0} \text{ are same.}$$~~

~~For mathematical simplicity, we prefer~~

~~$$E = \frac{F}{q_0}$$~~

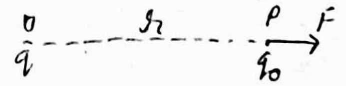
Electric field intensity due to a point charge

Ques.- Derive the expression for electric feild intensity due to a point charge.

Ans.- Consider point charge q at point O. We have to find out EFI due to point charge at point P which is at a distance r from it. In order to measure EFI at point P, consider a test charge q_0 at point P.

By CISL, the electrostatic force acting on test charge q_0 is

$$F = \frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r^2}$$



By definition, EFI at point P due to point charge q is given by

$$E = \frac{F}{q_0}$$

or
$$E = \frac{\frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r^2}}{q_0}$$

or
$$\boxed{E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}}$$

This is the reqd. expression.

If the charge be situated in a dielectric medium of dielectric constant ε_r then

$$E = \frac{1}{4\pi\varepsilon_0\varepsilon_r}\frac{q}{r^2}$$

In C.G.S. (electrostatic) system

(a) In vacuum $\quad E = \frac{q}{r^2}$

(a) In medium $\quad E = \frac{1}{\varepsilon_r}\frac{q}{r^2}$

Particular cases- (i) If r = 0

Then, $E = \frac{1}{4\pi\varepsilon_0}\frac{q}{0^2} = \infty$ (not defined)

Thus, the EFI due to a point charge on itself is not defined (therefore it is never taken into considertion).

(i) If $r = \infty$

Then, $E = \frac{1}{4\pi\varepsilon_0}\frac{q}{\infty^2} = 0$

Thus, the EFI due to a point charge at infinite distance is zero.

Graph between E and r-

From relation $E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$, we have

$$E \propto \frac{1}{r^2}$$

Note- From the above it can be concluded that the EFI due to a point charge extends upto infinity.

SEFI due to a point charge in terms of position vectors

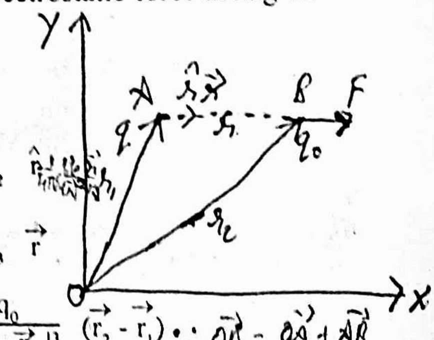
Ques.- Derive the expression for electric feild intensity due to a point charge in terms of position vectors.

Ans.- Consider a point charge q at point A. We have to find out the EFI due to the point charge at point B whose distance from point A be r. If the position vector of point charge be $\vec{r}_1$ and that of observation point be $\vec{r}_2$. Then the position vector of the observation point B w.r.t. point charge q is given by

$$\vec{AB} = \vec{OB} - \vec{OA}$$

or $\quad \vec{r} = \vec{r}_2 - \vec{r}_1$

In order to measure EFI at point B, consider a test charge q_0 at point B. By CISL the electrostatic force acting on test charge q_0 is given by



~~Then~~, $\vec{F}$ ~~E~~ $= \frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r^2}$

In vector form, $\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{qq_0}{|\vec{r}|^2}\hat{r}$

$\vec{F} = F\hat{r}$

or
$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{qq_0}{|\vec{r}|^3}\vec{r}$$

or
$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{qq_0}{|\vec{r}_2 - \vec{r}_1|^3}(\vec{r}_2 - \vec{r}_1) \quad \because \vec{OB} = \vec{OA} + \vec{AB}$$

By definition EFI at point B is given by

$$\vec{E} = \frac{\vec{F}}{q_0}$$

or
$$\vec{E} = \frac{\frac{1}{4\pi\varepsilon_0}\frac{qq_0}{|\vec{r}_2 - \vec{r}_1|^3}(\vec{r}_2 - \vec{r}_1)}{q_0}$$

or
$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{|\vec{r}_2 - \vec{r}_1|^3}(\vec{r}_2 - \vec{r}_1)$$

This is the required expression.

Electric lines of force or electric field lines

Ques.- What are electric lines of force? Write down their properties.

Ans.- The concept of electric lines of force was introduced by Michael Faraday. They give pictorial representation of electric field.

Electric lines of force are those imaginary smooth curves along which a unit positive charge would move, if it is free to do so.

Properties of electric lines of force- (i) They always emanate from positive charge and terminate at negative charge i.e. they never form closed loops.

(ii) The tangent drawn at any point of electric lines of force gives us the direction of EFI at that point.

(iii) Two electric lines of force can never intersect each other.

(iv) They always emanate normally and terminate normally at the surface of charged conductor.

(v) They possess longitudinal tension i.e. they have a tendency to contract lengthwise (on the basis of this property we can explain the force of attraction between unlike charges).

(vi) Electric lines of force possess lateral pressure i.e. they repel one another perpendicular to their length (on the basis of this property. we can explain the force of repulsion between like charges).

(vii) In the regions of strong electric field electric lines of force are closely spaced and in the regions of weak electric field electric field electric lines of force are widely spread.

(viii) EFI at any point in the electric field is equal to the number of electric lines of force which passes normally through a unit area chosen about that point.

(ix) Electric lines of force do not pass through a conductor.

Note-(i) Uniform electric field- The electric field at every point of which both the magnitude and direction of electric field intensity is same is called uniform electric field.

It is represented by equally spaced parallel electric lines of force.

(ii) Explaination of III property- If two electric lines of force interseet each other then at the point of intersection two tangents can be draqn which will give us two different direction of EFI at the came point. which is physically impossible. Therefore two electric lines of force can never interseet each other.

Electric dipole & electric dipole moment

Ques.-(a) What is electric dipole?

(b) Define electric dipole moment. Write its units & dimensions.

Ans.-(a) When two equal and opposite charges are placed close to each other then this arrangement is called electric dipole.

In fig. the electric dipole is conetituting of two equal and opposite charges +q and -q which are placed at small separation 2ℓ or a.

$-q$ — 2ℓ or a — $+q$

e.g.- All polar molecules e.g. HF, HCl, H_2O etc. behave like electric dipoles.

(b) Electric dipole moment is defined as the product of magnitude of either charge of electric dipole and the distance between them. It is denoted by p.

If an electric dipole is made up of two charges +q and -q which are placed at a distance 2ℓ or a then its electric dipole moment is given by.

$$p = q\,2\ell \text{ or } p = qa$$

If is a vector quantity and its direction is always taken from -ve to +ve charge.

$-q \xrightarrow[\vec{p}]{2\ell \text{ or } a} +q$

In vector form $\vec{p} = q\vec{2\ell} = q\vec{a}$

where $\vec{2\ell}$ or $\vec{a}$ is the position vector of +ve charge w.r.t. -ve charge.

Its S.I. unit is C m, C.G.S. unit is statC cm and C.G.S. esu is abC cm. Its dimensional formula is [ATL].

Note- (i) The line joining the charges of electric dipole is called dipole axis and the electric field generated by electric dipole is called dipole field.

(ii) When $q \to \infty$ and $a \to 0$ then electric dipole is said to be ideal.

EFI due to electric dipole at any point on its axial line(end on position)

Ques.-Derive the relation for electic field intensity due an electric dipole at any point on its axial line(end on position of electric dipole).

Ans.-Consider an electric dipole AB which is made up of two charges +q and -q placed at a separation of 2ℓ from each another. We have to find out EFI at point P on the axial line of electric dipole which is at a distance r from its centre O.

EFI due to +q charge at point P

$$E_A = \frac{1}{4\pi\varepsilon_0}\frac{qq_0}{AP^2} \quad (\text{along } \vec{AP})$$

$\because$ $AP = OP - OA$

$= r - \ell$

$$\therefore \quad E_A = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r-\ell)^2} \quad \text{----(i)}$$

and EFI due to -q charge at point P

$$E_B = \frac{1}{4\pi\varepsilon_0}\frac{q}{BP^2} \quad (\text{along } \vec{PB})$$

$\because$ $BP = OP + OB$

$= r + \ell$

$$\therefore \quad E_B = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r+\ell)^2} \quad \text{----(ii)}$$

As $\vec{E}_A$ and $\vec{E}_B$ are oppositily directed and $|\vec{E}_A| > |\vec{E}_B|$ therefore the resultant EFI at point P is given by

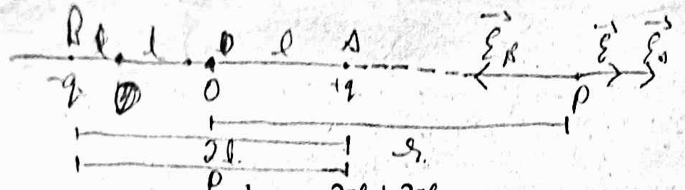
$$E = E_A - E_B \quad (\text{aong } \vec{AP})$$

or $$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r-\ell)^2} - \frac{1}{4\pi\varepsilon_0}\frac{q}{(r+\ell)^2}$$

or $$E = \frac{1}{4\pi\varepsilon_0}\, q\left[\frac{1}{(r-\ell)^2} - \frac{1}{(r+\ell)^2}\right]$$

or $$E = \frac{1}{4\pi\varepsilon_0}\, q\left[\frac{(r+\ell)^2 - (r-\ell)^2}{(r-\ell)^2\,.\,(r+\ell)^2}\right]$$

or $$E = \frac{1}{4\pi\varepsilon_0}\, q\left[\frac{r^2 + \ell^2 + 2r\ell - r^2 - \ell^2 + 2r\ell}{(r^2 - \ell^2)^2}\right]$$



or $E = \frac{1}{4\pi\varepsilon_0} q \frac{2r\ell + 2r\ell}{(r^2 - \ell^2)^2}$

or $E = \frac{1}{4\pi\varepsilon_0} q \frac{4r\ell}{(r^2 - \ell^2)^2}$

or $E = \frac{1}{4\pi\varepsilon_0} \frac{2(q2\ell)r}{(r^2 - \ell^2)^2}$

$\because$ $q2\ell = p$ electric dipole moment

$\therefore$ $\boxed{E = \frac{1}{4\pi\varepsilon_0} \frac{2pr}{(r^2 - \ell^2)^2}}$

This is the required expession of EFI due to electric dipole at any point on its axial line.

Particular case- If electric dipole be small i.e $\ell << r$ then ℓ^2 can be ignored as compared to r^2.

$\therefore$ $E = \frac{1}{4\pi\varepsilon_0} \frac{2pr}{(r^2)^2}$

or $\boxed{E = \frac{1}{4\pi\varepsilon_0} \frac{2p}{r^3}}$

Direction of electric field intensity- At any point on the axial line of electric dipole the direction of EFI is same as that of electric dipole moment i.e from -ve to +ve charge.

Note- The line which passes through both the charges of electric dipole is called its axial line.

EFI due to electric dipole at any point on its equitorial line (broad on position)

Ques.-Derive the relation for electric field intensity due an electric dipole at any point on its equitorial line(broad on position of electric dipole).

Ans.- Consider an electric dipole AB which is made up two charges +q and -q placed at a separation 2ℓ from each other. We have to find out EFI due to electric dipole at any point P on its equitorial line which is at a distance r from its centre O.

EFI due to +q charge at point P

$E_A = \frac{1}{4\pi\varepsilon_0} \frac{q}{AP^2}$ (along $\overrightarrow{AP}$)

$\because$ In rt Δ AOP by P.T.

$AP^2 = OP^2 + OA^2$

$= r^2 + \ell^2$

$\therefore$ $E_A = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2 + \ell^2}$ ---(i)

and EFI due to -q charge at point P

$E_B = \frac{1}{4\pi\varepsilon_0} \frac{q}{BP^2}$ (along $\overrightarrow{PB}$)

$\because$ In rt Δ BOP by P.T.

$BP^2 = OP^2 + OB^2$

$= r^2 + \ell^2$

$\therefore$ $E_B = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2 + \ell^2}$ --(ii)

Draw PQ||AB

If $\angle PAO = \angle PBO = \theta$

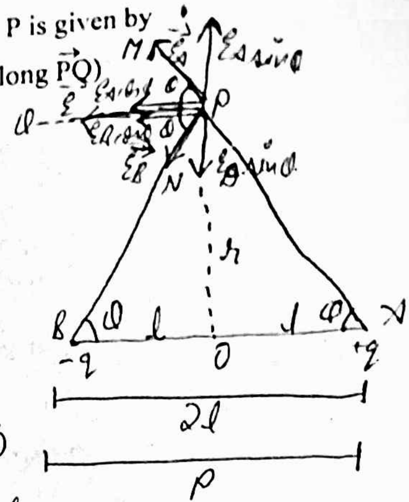
Then $\angle MPQ = \angle PAO = \theta$ (corres. $\angle$'s)

or $\angle NPQ = \angle PBO = \theta$ (alt. $\angle$'s)

Here $|\vec{E}_A| = |\vec{E}_B|$ and the angle between their direction is

$\therefore$ The resultant EFI at point P is given by

$E = 2E_A \cos\frac{2\theta}{2}$ (along $\overrightarrow{PQ}$)



or $E = 2E_A \cos\theta$

$\because$ $E_A = \frac{1}{4\pi\varepsilon_0} \frac{q}{(r^2 + \ell^2)}$

and in rt. Δ AOP

$\cos\theta = \frac{OP}{AP} = \frac{\ell}{(r^2 + \ell^2)^{1/2}}$

$\therefore$ $E = 2 \frac{1}{4\pi\varepsilon_0} \frac{q}{(r^2 + \ell^2)} \frac{\ell}{(r^2 + \ell^2)^{1/2}}$

or $E = \frac{1}{4\pi\varepsilon_0} \frac{2q\ell}{(r^2 + \ell^2)^{3/2}}$

$\because$ $2q\ell = p$ electric dipole moment

$\therefore$ $\boxed{E = \frac{1}{4\pi\varepsilon_0} \frac{p}{(r^2 + \ell^2)^{3/2}}}$

This is the required expression of EFI due to electric dipole at any point on its equitorial line.

Particular case- If electric dipole be small i.e. $\ell << r$ then ℓ^2 can be ignored as compared to r^2.

$\therefore$ $E = \frac{1}{4\pi\varepsilon_0} \frac{p}{(r^2)^{3/2}}$

or $E = \frac{1}{4\pi\varepsilon_0} \frac{p}{r^3}$

Direction of EFI- At any point on the equitorial line of electric dipole the direction of EFI is opposite to that of electric dipole moment i.e. from positive to negative charge.

Note- The line which passes through the center of the electric dipole and is perpendicular to its axis is called equitorial line of electric dipole.

$ EFI due to small electric dipole at any point

Ques.-Derive the relation for electric field intensity due to a small electric dipole at any point

Ans.- Consider a small electric dipole of electric dipole moment p. We have to find out EFI due to it at any point p which is at a distance r from its centre O such that the line joining the point P with centre O of electric dipole makes an angle θ with the dipole axis.

Here the electric dipole moment p of electric dipole can be resolved into two components-

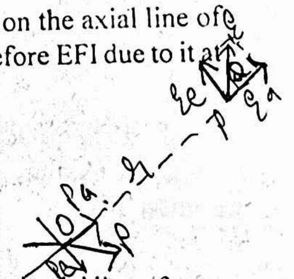
(a) axial component $p_a = p\cos\theta$

(b) equitorial component $p_e = p\sin\theta$

As the observation point lies on the axial line of component p_a of electric dipole. Therefore EFI due to it at point P is given by

$E_a = \frac{1}{4\pi\varepsilon_0} \frac{2P_a}{r^3}$

or $E_a = \frac{1}{4\pi\varepsilon_0} \frac{2p\cos\theta}{r^3}$ ---(i)



and the observation point lies on the equitorial line of component p_e of electric dipole therefore EFI due to its at p-oint P is given by

$$E_e = \frac{1}{4\pi\varepsilon_0}\frac{p_e}{r^3}$$

or $$E_e = \frac{1}{4\pi\varepsilon_0}\frac{p\sin\theta}{r^3} \quad \text{---(ii)}$$

Since. $\vec{E}_a$ is perpendicular to $\vec{E}_e$ therefore the magnitude of EFI at point P is given by

$$E = \sqrt{E_a^2 + E_e^2}$$

or $$E = \sqrt{\left(\frac{1}{4\pi\varepsilon_0}\frac{2p\cos\theta}{r^3}\right)^2 + \left(\frac{1}{4\pi\varepsilon_0}\frac{p\sin\theta}{r^3}\right)^2}$$

or $$E = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3}\sqrt{(2\cos\theta)^2 + (\sin\theta)^2}$$

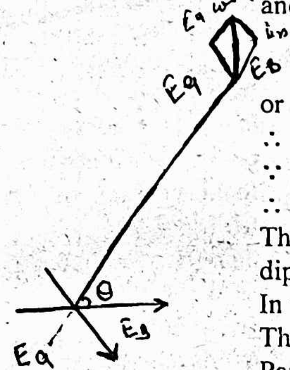
or $$E = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3}\sqrt{4\cos^2\theta + \sin^2\theta}$$

or $$E = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3}\sqrt{4\cos^2\theta + 1 - \cos^2\theta}$$

or $$E = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3}\sqrt{3\cos^2\theta + 1}$$

This is the required expression of EFI due to small electric dipole at any point.

<u>Direction of EFI</u>- If the EFI at pont P makes an angle α with OP then



$$\tan\alpha = \frac{E_e}{E_a}$$

or $$\tan\alpha = \frac{\frac{1}{4\pi\varepsilon_0}\frac{p\sin\theta}{r^3}}{\frac{1}{4\pi\varepsilon_0}\frac{2p\cos\theta}{r^3}}$$

or $$\tan\alpha = \frac{1}{2}\tan\theta$$

or $$\boxed{\alpha = \tan^{-1}\left(\frac{1}{2}\tan\theta\right)}$$

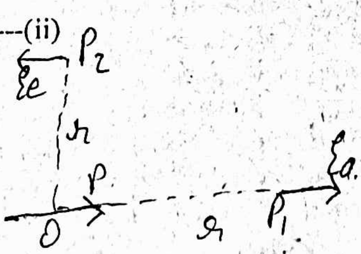
<u>Note</u>- EFI due to small electric dipole at any point on its axial line

or $$E_a = \frac{1}{4\pi\varepsilon_0}\frac{2p}{r^3} \quad \text{-----(i)}$$

EFI due to electric dipole at any point which is at the same distance on the equitorial line

or $$E_e = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3} \quad \text{-----(ii)}$$

Dividing relation (i) by (ii)



$$\frac{E_a}{E_e} = \frac{\frac{1}{4\pi\varepsilon_0}\frac{2p}{r^3}}{\frac{1}{4\pi\varepsilon_0}\frac{p}{r^3}}$$

or $$\frac{E_a}{E_e} = 2$$

or $$E_a = 2E_e$$

Thus for the same distance of observation point the EFI on axial line is twice the EFI on equitorial line.

<u>Torque acting on an electric dipole placed in uniform electric field</u>

<u>Ques.</u>-Derive the expression of torque acting on an electric dipole placed in uniform electric field.

<u>Ans.</u>- Consider an electric dipole which is made of two charges +q and -q placed at a separation 2ℓ from each other. This electric dipole is placed in uniform electric field such that the angle between the electric dipole moment $\vec{p}$ and the electric field $\vec{E}$ be θ.

The force experienced by the charge +q in the direction of electric field.

$$\vec{F}_1 = +q\vec{E}$$

and the force experienced by the charge -q in the direction opposite to the electric field

$$\vec{F}_2 = -q\vec{E}$$

As the two forces acting on electric dipole are equal in magnitude and opposite in direction therefore they will constitute to form a couple. Due to this couple, a torque will act on electric dipole whose tendency is to bring electric dipole in the direction of electric field. Therefore, it is called restoring torque.

The restoring torque acting an electric dipole is equal to product of magnitide of either force and the perpendicular distance between the line of action of two forces.

i.e $\quad \tau = F_1 d$

$\because \quad F_1 = qE$

and in rt. Δ AMB

$$\sin\theta = \frac{AM}{AB} = \frac{d}{2\ell}$$

or $\quad d = 2\ell\sin\theta$

$\therefore \quad \tau = qE\, 2\ell\sin\theta$

$\because \quad q2\ell = p$ electric dipole moment

$\therefore \quad \boxed{\tau = pE\sin\theta}$

This is the required expression of torque acting on electric dipole placed in uniform electric field.

In vector form $\vec{\tau} = \vec{p} \times \vec{E}$

The direction of $\vec{\tau}$ is same as that of $\vec{p} \times \vec{E}$

<u>Particular cases</u>- (i) If $\theta = 0°$

then $\quad \tau = pE\sin 0°$

or $\quad \tau = pE\,(0)$

or $\quad \tau = 0$ (min)

(ii) If $\theta = 90°$

then $\quad \tau = pE\sin 90°$

or $\quad \tau = pE\,(1)$

or $\quad \tau = pE$ (max)

(i) If $\theta = 180°$

then $\quad \tau = pE\sin 180°$

or $\quad \tau = pE\,(0)$

or $\quad \tau = 0$ (min)

<u>Note</u>- (i) When an electric dipole is placed in uniform electric field then the net force acting on it is zero

$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2$$
$$= +q\vec{E} - q\vec{E}$$
$$= 0$$

(ii) In relation $\tau = pE\sin\theta$

If E = 1 and $\theta = 90°$

then $\quad \tau = p.1.\sin 90°$

or $\quad \tau = p.1.1$

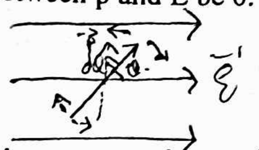
or $\quad \tau = p$

Thus, when an electric dipole is placed normal to unit electric field then its electric dipole moment is numerically equal to the torque acting on it.

Work done in rotating electric dipole in uniform electric field

Ques.-Derive the relation for work done in rotating an electric dipole placed in uniform electric field.

Ans.- Consider an electric dipole of electric dipole moment $\vec{p}$ which is placed in uniform electric field $\vec{E}$ such that the angle between $\vec{p}$ and $\vec{E}$ be θ.



Then, the restoring torque acting on electric dipole

$$\tau = pE\sin\theta$$

In order to rotate electric dipole by an infinetesimally small angle dθ in the direction opposite to the electric field, a torque has to be applied by exter nal agency on electric dipole which should be equal in magnitude and opposite in direction to the restoring torque.

$\therefore$ Applied torque $\tau' = \tau$

or $\quad \tau' = pE\sin\theta$

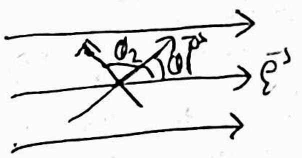
Work done in rotating electric dipole through small angle dθ

$$dW = \tau' d\theta$$

or $\quad dW = pE\sin\theta\, d\theta$

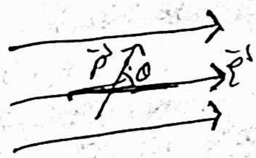
By integrating the above relation with in proper limits the work done in rotating electric dipole from angle θ_1 to θ_2 can be calculated as follows-

$$\int_0^W dW = \int_{\theta_1}^{\theta_2} pE\sin\theta\, d\theta$$



or $\quad \left[W\right]_0^W = pE\int_{\theta_1}^{\theta_2}\sin\theta\, d\theta$

or $\quad W - 0 = pE\left[-\cos\theta\right]_{\theta_1}^{\theta_2}$



or $\quad W = pE\left[(-\cos\theta_2) - (-\cos\theta_1)\right]$

or $\quad W = pE\left[-\cos\theta_2 + \cos\theta_1\right]$

or $\quad W = pE(\cos\theta_1 - \cos\theta_2)$

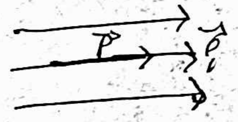
This is the required expression.

If the electric dipole be in the direction of electric field initially then thw work done in rotating electric dipole by an angle θ is given by.

$$W = pE(\cos 0° - \cos\theta)$$

or $\quad W = pE(1 - \cos\theta)$

Particular cases- (i) If $\theta = 0°$



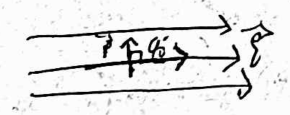
then $\quad W = pE(1 - \cos 0°)$

or $\quad W = pE(1 - 1)$

or $\quad W = pE(0)$

or $\quad W = 0$ (min)

(ii) If $\theta = 90°$

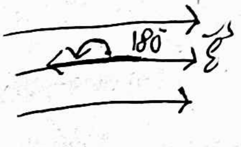


then $\quad W = pE(1 - \cos 90°)$

or $\quad W = pE(1 - 0)$

or $\quad W = pE$ (max)

(iii) If $\theta = 180°$



then $\quad W = pE(1 - \cos 180°)$

or $\quad W = pE(1 + 1)$

or $\quad W = 2pE$ (max)

Potential energy of an electric dipole placed in uniform electric field

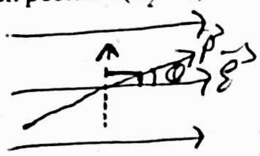
Ques.-Derive the relation for the potential energy of an electric dipole placed in uniform electric field.

Ans.- It is defined as the work done in bringing the two charges of an electric dipole from infinity to their respective positions in electric field.

Potential energy of an electric dipole in uniform electric field is equal to the work done in rotating electric dipole from standard position to the given position.

Consider an electric dipole of electric dipole moment $\vec{p}$ placed in uniform electric field $\vec{E}$ such that the angle between the $\vec{p}$ and $\vec{E}$ be θ.

Then, the work done in rotating electric dipole from standard position (θ = 90°) to the given position ($\theta_2 = 0$) is given by



$$W = pE(\cos\theta_1 - \cos\theta_2)$$

or $\quad W = pE(\cos 90° - \cos\theta)$

or $\quad W = pE(0 - \cos\theta)$

or $\quad W = -pE\cos\theta$

By definition the potential energy of electric dipole is

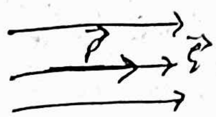
$$U = W$$

or $\quad \boxed{U = -pE\cos\theta}$

or $\quad U = -\vec{p}.\vec{E}$

This is the required expression.

Particular cases- (i) If θ = 0°



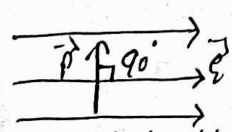
then $\quad U = -pE\cos 0°$

or $\quad U = -pE(1)$

or $\quad U = -pE$ (min)

In this position, the electric dipole is in stable equilibrium.

(ii) If θ = 90°



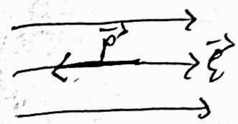
then $\quad U = -pE\cos 90°$

or $\quad U = -pE(0)$

or $\quad U = 0$

In this position, the electric dipole is in standard position.

(iii) If θ = 180°



then $\quad U = -pE\cos 180°$

or $\quad U = -pE(-1)$

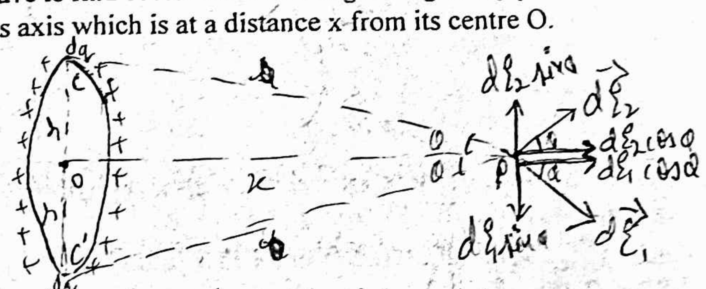
or $\quad U = +pE$ (max)

In this position, the electric dipole is in unstable equilibrium.

EFI due to a uniform charged circular loop or ring

Ques.-Derive the relation for electric field intensity due to a uniform charged circular loop or ring at any point on its axis.

Ans.- Consider a uniformly charged ring of radius r on which q amount of charge is distributed uniformly. We have to find out EFI due to charged ring at any point P on its axis which is at a distance x from its centre O.



Consider a charge element dq of charged ring at point C. EFI due to this charge element at point P or $dE_1 = \frac{1}{4\pi\varepsilon_0}\frac{dq}{CP^2}$ (along $\overrightarrow{CP}$)

In rt. Δ COP by P.T.

$$CP^2 = OC^2 + OP^2$$
$$= r^2 + x^2$$

$$\therefore \quad dE_1 = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2 + x^2} \quad \text{----(i)}$$

Again consider a charge element dq of charged ring at point C'. EFI due to this charge element at point P.

or $dE_2 = \frac{1}{4\pi\varepsilon_0}\frac{dq}{C'P^2}$ (along $\overrightarrow{C'P}$)

In rt. Δ C'OP by P.T.

$$C'P^2 = OC'^2 + OP^2$$
$$= r^2 + x^2$$

$$\therefore \quad dE_2 = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2 + x^2} \quad \text{----(ii)}$$

When the two EFI vectors at point P are resolved into vertical and horizontal components then the vertical components will cancel out each other and the horizontal components will add up together.

Similarly by resolving the whole charged ring into charge elements it can be proved that the vertical components of all charge elements which are present at diametrically opposite points will cancel out each other and only the horizontal components will add up together.

Thus the effective value of EFI due to a charge element at point P.

(ii) when the observer point lies at very very large distance from the ring i.e. $x >> r$ $= x^2 >>>> r^2$

$$dE = dE_1 \cos\theta$$

$$\because \quad dE_1 = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2 + x^2}$$

and in rt. Δ COP

then, $E = \frac{1}{4\pi\varepsilon_0}\frac{qx}{(x^2)^{3/2}}$ (ignoring r^2 as compared to x^2)

$$\cos\theta = \frac{OP}{CP}$$

$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{x^2}$

$$\text{or} \quad \cos\theta = \frac{x}{(r^2 + x^2)^{1/2}}$$

$$\therefore \quad dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{(r^2 + x^2)}\frac{x}{(r^2 + x^2)^{1/2}}$$

$$\text{or} \quad dE = \frac{1}{4\pi\varepsilon_0}\frac{dq\,x}{(r^2 + x^2)^{3/2}}$$

EFI due to whole charged ring at point P can be calculated by integrating both the sides of the above relation for the whole ring.

$$\int_l dE = \int_l \frac{1}{4\pi\varepsilon_0}\frac{dq\,x}{(r^2 + x^2)^{3/2}}$$

$$\text{or} \quad E = \frac{1}{4\pi\varepsilon_0}\frac{x}{(r^2 + x^2)^{3/2}}\int_l dq$$

$$\text{or} \quad E = \frac{1}{4\pi\varepsilon_0}\frac{x}{(r^2 + x^2)^{3/2}}\,q$$

$$\text{or} \quad \boxed{E = \frac{1}{4\pi\varepsilon_0}\frac{qx}{(r^2 + x^2)^{3/2}}}$$

This is the required expression of EFI at any point on its axis.

Particular case(i) When the observation point lies at the centre of charged ring $E = \frac{1}{4\pi\varepsilon_0}\frac{q0}{(r^2+0^2)^{3/2}}$ Ans

i.e. at x = 0

$= 0$

Electric flux

Ques.-(a) Define electric flux. What are its units and dimensions?

(b) What are the types of electric flux?

Ans.- The number of electric lines of force which pass normally through a surface which is placed in electric field is called electric flux linked with that surface. It is denoted by ϕ_E.

If a surface of area S be placed in uniform electric field E such that the angle between $\vec{S}$ and $\vec{E}$ be θ then the electric flux linked with that surface is given by

$$\phi_E = (E\cos\theta)S$$
$$\text{or} \quad \phi_E = ES\cos\theta$$
$$\text{or} \quad \phi_E = \vec{E}.\vec{S}$$

Particular cases- (i) When θ = 0°

then $\phi_E = ES\cos 0°$

or $\phi_E = ES(1)$

or $\phi_E = ES$ (max)

Thus, when a surface is placed perpendicular to the electric field then the electric flux linked with it is maximum.

(ii) When θ = 90°

then $\phi_E = ES\cos 90°$

or $\phi_E = ES(0)$

or $\phi_E = 0$

Thus, when a surface is placed parallel to the electric field then the electric flux linked with it is zero.

Unit- From relation $\phi_E = ES\cos\theta$

S.I. unit of $\phi_E = \frac{N}{C}.m^2$

$= Nm^2C^{-1}$

and C.G.S. esu of $\phi_E = \frac{dyn}{statC}cm^2$

$= dyn\,cm^2\,statC^{-1}$

Dimensions- From relation $\phi_E = ES\cos\theta$

Dimensional formula of $\phi_E = [MLT^{-3}A^{-1}][L^2] = [ML^3T^{-3}A^{-1}]$

(b) Types of electric flux- Electric flux is of three types-

(i) Positive electric flux- When electric lines of force come out of a surface then the electric flux linked with it is said to be positive.

(ii) Negative electric flux- When electric lines of force enter into a surface then the electric flux linked with it is said to be negative.

(iii) Zero electric flux- When the number of electric lines of force which come out of a surface is equal to the number of electric lines of force which enter into that surface then the electric flux linked with that surface is said to be zero.

In figure the electric flux linked with surface S_1 is positive with surface S_2 is negative and with surface S_3 is zero.

Note- (i) When the electric field is non-uniform then in order to calculate electric flux linked with the surface first of all it is divided into small area elements ($d\vec{S}$) then the electric flux linked with an area element is calculated and finally integration is done in order to calculate electric flux linked with the whole surface.

Electric flux linked with an area element.

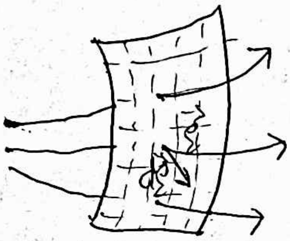
$$d\phi_E = \vec{E}.d\vec{S}$$

electric flux linked with the whole surface

$$\phi_E = \int_S \vec{E}.d\vec{S}$$

If the surface be closed, then

$$\phi_E = \oint_S \vec{E}.d\vec{S}$$



(ii) Area is a vector quantity and its direction is always taken perpendicular to the surface in the outward direction.

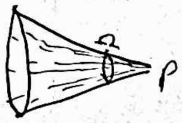
Solid angle

Ques.- Define solid angle . Prove that the solid angle substended by a sphere at its center is zero.

Ans.- The three dimensional angle which is obtained by joining all the points present on the boundary of a surface to a given point is called solid angle substended by that surface at the given point.

If the area chosen on the surface of a sphere be dS then the solid angle substended by the chosen area at the centre of sphere is given by

$$\text{Solid angle} = \frac{\text{Area of surface}}{(\text{Radius of sphere})^2}$$



or $\oint_S d\Omega = \oint_S \frac{dS}{R^2}$

or $\Omega = \frac{1}{R^2} \oint_S dS$

or $\Omega = \frac{1}{R^2} .S$

$\because$ $S = 4\pi R^2$ surface area of sphere

$\therefore$ $\Omega = \frac{1}{R^2} .4\pi R^2$

or $\Omega = 4\pi$ sr

Note- Similarly it can be proved that the solid angle substended by any closed surface at any point inside it is equal to 4π sr.

Gauss theorem

Ques.- State and prove Gauss theorem.

Ans.- C.F. Gauss established a relation between the total electric flux linked with a closed surface and the net charge enclosed by that surface which is called Gauss theorem.

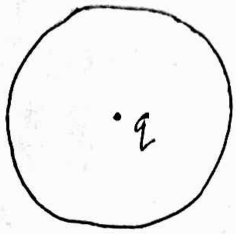
It states that the total electric flux linked with a closed surface is $\frac{1}{\varepsilon_0}$ times the net charge enclosed by that surface.

If the net charge enclosed by a closed surface be q then in accordance with Gauss theorem the total electric flux linked with that surface is given by

$$\phi_E = \frac{1}{\varepsilon_0} q$$

$\because$ $\phi_E = \oint_S \vec{E}.d\vec{S}$

$\therefore$ $\oint_S \vec{E}.d\vec{S} = \frac{1}{\varepsilon_0} q$



The surface on which Gauss theorem is applied is called Gaussian surface.

Proof- Consider a point charge q at point O. By taking O as centre and r as radius draw a sphere.

Electric field intensity due to point charge q at any point P on the surface of sphere is given by

$$E = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \quad (\text{along } \vec{OP})$$

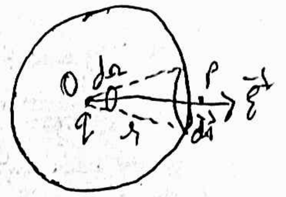
Choose a small area element dS about point P. Then the electric flux linked with the area element

$$d\phi_E = EdS\cos\theta = EdS$$

$$d\phi_E = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} dS$$

By integrating the above relation for the whole surface the total electric flux linked with the whole surface of sphere can be calculated as follows-

$$\oint_S d\phi_E = \oint_S \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} dS$$



or $\phi_E = \frac{1}{4\pi\varepsilon_0} q \oint_S \frac{dS}{r^2}$

$\because$ $\frac{dS}{r^2} = d\Omega$ solid angle substended by area element at the centre of sphere.

$\therefore$ $\phi_E = \frac{1}{4\pi\varepsilon_0} q \oint_S d\Omega$

or $\phi_E = \frac{1}{4\pi\varepsilon_0} q\Omega$

$\because$ $\Omega = 4\pi$ sr solid angle substended by the closed surface of sphere at its centre.

$\therefore$ $\phi_E = \frac{1}{4\pi\varepsilon_0} q\, 4\pi$

or $\boxed{\phi_E = \frac{1}{\varepsilon_0} q}$

This is the required mathemetical expression of Gauss theorem.

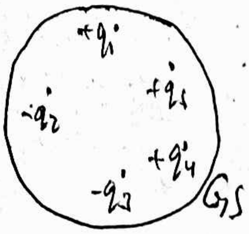
Note- (i) Gaussian surface ~~can be~~ is always real or imaginary.

(ii) In the expression of Gauss theorem, E is the resultant electric field intensity due to all the charges present in space where q is the net charge enclosed by the Gaussian surface.

(iii) If a number of charges are enclosed by a Gaussian surface then the total electric flux linked with it can be calculated as follows-

By Gauss theorem

$$\phi_E = \frac{1}{\varepsilon_0} q_{net}$$



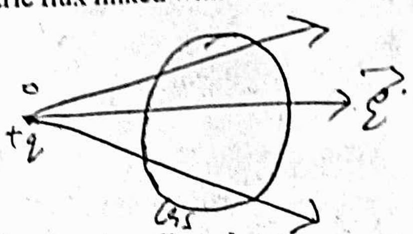
or $\phi_E = \frac{1}{\varepsilon_0} (+q_1 - q_2 - q_3 + q_4 + q_5)$

(iv) When a closed surface is placed in external electric field then the net electric flux linked with it is zero.

$$\phi_E = \frac{1}{\varepsilon_0} q$$

or $\phi_E = \frac{1}{\varepsilon_0} 0$

or $\phi_E = 0$



Electric field intensity due to a line charge

Ques.- Derive the relation for electric field intensity due to a line charge by using Gauss theorem.

Ans.- Consider a line of infinite length such that the amount of charge present on its unit length be λ. We have to find out electric field intensity due to line charge at point P which is at a distance r from the line charge.

By taking the line charge as axis when a cylinder of length ℓ and radius r is drawn then it behaves like Gaussian surface

Charge enclosed by the Gaussian surface $q = \lambda\ell$

$$\left[\because \lambda = \frac{q}{\ell}\right]$$

As the electric field due to the line charge is directed radially outwards and by symmetry the magnitude of electric field intensity is same at every point of curved surface of cylinder therefore electric field due to line charge will be parallel to the circular faces and $\perp$ to the curved part of Gaussian surface.

$\phi_E = \phi_{E1} + \phi_{E2} + \phi_{E3}$

or $\phi_E = 0 + 0 + ES_3$

or $\phi_E = ES_3$

$\because$ $S_3 = 2\pi r\ell$ area of curved part of Gaussian surface

$\therefore$ $\phi_E = E2\pi r\ell$ ----(i)

By Gauss theorem, the total electric flux linked by the Guassian surface

$$\phi_E = \frac{1}{\varepsilon_0} q \quad \text{---(ii)}$$

From relation (i) and (ii)

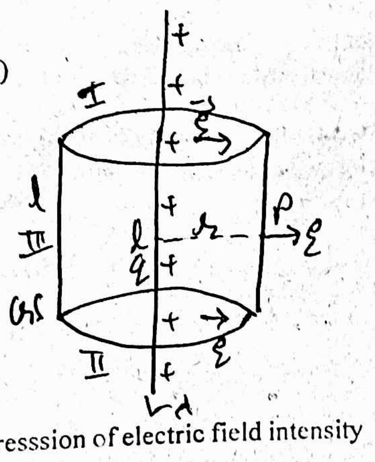
$$E2\pi r\ell = \frac{1}{\varepsilon_0} q$$

or $E = \frac{q}{2\pi\varepsilon_0 r\ell}$

$\because$ $q = \lambda\ell$

$\therefore$ $E = \frac{\lambda\ell}{2\pi\varepsilon_0 r\ell}$

or $\boxed{E = \frac{\lambda}{2\pi\varepsilon_0 r}}$



This is the required expresssion of electric field intensity due to a line charge.

Particular cases- (i) When r = 0

or $E = \frac{\lambda}{2\pi\varepsilon_0 0}$

or $E = \infty$

Thus, the electric field intensity due to a line charge on itself is not defined.

(ii) When $r = \infty$

or $E = \frac{\lambda}{2\pi\varepsilon_0 \infty}$

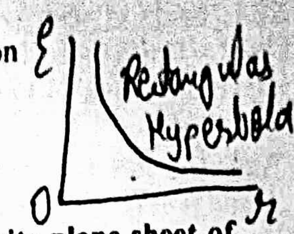
or $E = 0$

Thus, the electric field intensity due to a line charge at infinity is zero.

Graph between E and r- From relation

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

or $E \propto \frac{1}{r}$



Electric field intensity due to infinite plane sheet of charge

Ques.- Derive the relation for electric field intensity due to infinite plane sheet of charge by using Gauss theorem.

Ans.- Consider a plane sheet of infinite extension such that the amount of charge present on its unit area be σ. We have to find out electric field intensity due to plane sheet of charge at point P which is at a distance r from the sheet of charge.

Take another point P on the other side of plane sheet of charge at a $\perp$ distance r from it. By taking PP' as axis draw a cylinder area of whose circular faces be A. This cylinder will behave like Gaussian surface.

Charge enclosed by the Gaussian surface

$$q = \sigma A \quad \left[\because \sigma = \frac{q}{A}\right]$$

As the electric field due to plane sheet of charge is directed $\perp$ to it in the outward direction therefore electric field due to plane sheet of charge will be $\perp$ to circular faces and parallel to curved part of Gaussian surface and by symmetry the magnitude of electric field intensity is same at every point of circular faces of Guassian surface.

Total electric flux linked with the Guassian surface

$\phi_E = \phi_{E1} + \phi_{E2} + \phi_{E3}$

or $\phi_E = EA + EA + 0$

or $\phi_E = 2EA$ -----(i)

By Guass theorem, the total electric flux linked by with the Gaussian surface $\phi_E = \frac{1}{\varepsilon_0} q$ ----(ii)

From relation (i) and (ii)

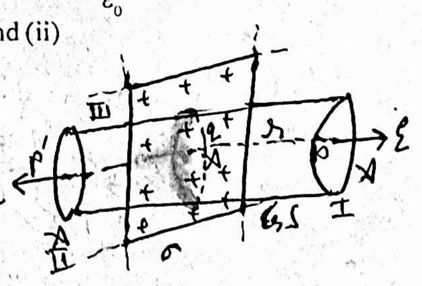
$$2EA = \frac{1}{\varepsilon_0} q$$

or $E = \frac{q}{2\varepsilon_0 A}$

$\because$ $q = \sigma A$

$\therefore$ $E = \frac{\sigma A}{2\varepsilon_0 A}$

or $\boxed{E = \frac{\sigma}{2\varepsilon_0}}$

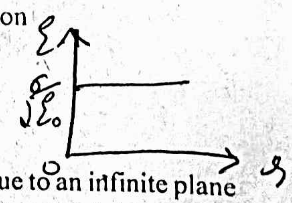


This is the required expression of electric field intensity due to infinite plane sheet of charge.

Graph between E and r- From relation

$$E = \frac{\sigma}{2\varepsilon_0} \quad \text{we have}$$

$E \propto r^0$



Note-As the electric field intensity due to an infinite plane sheet of charge is independent of distance of observation point therefore electric field due to it is uniform.

Electric field intensity due to two infinite plane parallel sheets of charge

Ques.- Derive the relation for electric field intensity due to due to two infinite plane parallel sheets of charge.

Ans.- Consider two infinite plane parallel sheets of charge A and B having like charges such that their surface charge densities be $+\sigma_A$ and $+\sigma_B$.

If the electric field in the positive x direction is taken positive and the electric field in the negative x-direction is taken negative then

Electric field intensity at point P

$$E_P = -E_A - E_B$$
$$= -\frac{\sigma_A}{2\varepsilon_0} - \frac{\sigma_B}{2\varepsilon_0}$$
$$= \frac{-1}{2\varepsilon_0}(\sigma_A + \sigma_B)$$

Electric field intensity at point Q

$$E_Q = E_A - E_B$$
$$= \frac{\sigma_A}{2\varepsilon_0} - \frac{\sigma_B}{2\varepsilon_0}$$
$$= \frac{1}{2\varepsilon_0}(\sigma_A - \sigma_B)$$

Electric field intensity at point R

$$E_R = E_A + E_B$$
$$= \frac{\sigma_A}{2\varepsilon_0} + \frac{\sigma_B}{2\varepsilon_0}$$
$$= \frac{+1}{2\varepsilon_0}(\sigma_A + \sigma_B)$$

Particular case- If $\sigma_A = \sigma_B = \sigma$

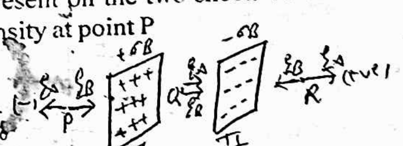
then $E_P = \frac{-1}{2\varepsilon_0}(\sigma + \sigma) = \frac{-1}{2\varepsilon_0}2\sigma = \frac{-\sigma}{\varepsilon_0}$

and $E_Q = \frac{1}{2\varepsilon_0}(\sigma - \sigma) = 0$

and $E_R = \frac{+1}{2\varepsilon_0}(\sigma + \sigma) = \frac{+1}{2\varepsilon_0}2\sigma = \frac{+\sigma}{\varepsilon_0}$

Thus, in the case of two infinite plane parallel sheets charge having like charges of equal surface charge densities the electric field is zero between the two sheets and electric field is uniform outside the two sheets.

Note- If the charge present on the two sheets be unlike then electric field intensity at point P



$$E_P = -E_A + E_B$$
$$= \frac{-\sigma_A}{2\varepsilon_0} + \frac{\sigma_B}{2\varepsilon_0}$$
$$= \frac{1}{2\varepsilon_0}(-\sigma_A + \sigma_B)$$

electric field intensity at point Q

$$E_Q = E_A + E_B$$
$$= \frac{\sigma_A}{2\varepsilon_0} + \frac{\sigma_B}{2\varepsilon_0}$$
$$= \frac{1}{2\varepsilon_0}(\sigma_A + \sigma_B)$$

electric field intensity at point R

$$E_R = E_A - E_B$$
$$= \frac{\sigma_A}{2\varepsilon_0} - \frac{\sigma_B}{2\varepsilon_0}$$
$$= \frac{1}{2\varepsilon_0}(\sigma_A - \sigma_B)$$

Particular case- If $\sigma_A = \sigma_B = \sigma$

then $E_P = \frac{1}{2\varepsilon_0}(-\sigma + \sigma) = \frac{1}{2\varepsilon_0}0 = 0$

$E_Q = \frac{1}{2\varepsilon_0}(\sigma + \sigma) = \frac{2\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0}$

and $E_R = \frac{+1}{2\varepsilon_0}(\sigma - \sigma) = \frac{1}{2\varepsilon_0}0 = 0$

Thus, in the case of two infinite plane parallel sheets charge having equal and opposite surface charge densities. The electric field is non-zero and uniform between the two sheets and electric field is zero outside the two sheets.

Electric field intensity due to uniformly charged spherical shell

Ques.- Derive the relation for electric field intensity due to uniformly charged spherical shell by using Gauss theorem.

Ans.- Consider a spherical shell of radius R on which q amount of charge is distributed uniformly such that the amount of charge present on a unit surface area of shpere be σ.

We can find out the EFI due to uniformly charged spherical shell at three different positions-

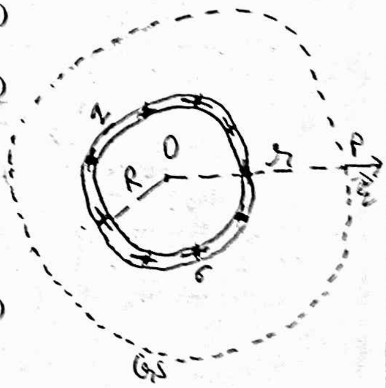
(i) When the observation point lies outside the spherical shell i.e. (r > R)- Consider a point P outside the spherical shell such that its distance from its centre O be r. By taking O as the centre and r as radius draw a sphere. This sphere will behave like Gaussian surface.

Total charge enclosed by the Gaussian surface

$$q = \sigma A \qquad \left[\because \sigma = \frac{q}{A}\right]$$
$$= \sigma 4\pi R^2$$

As the electric field due to positively charge spherical shell is directed radially outwards therefore the electric field will be perpendiuclar to the Gaussian surface and from symmetry the magnitude of EFI is same at every point of Gaussian surface.

Total electric flux linked with the Gaussian surface



$$\phi_E = ES$$
$$= E4\pi r^2 \quad \text{----(i)}$$

By Gauss theorem

$$\phi_E = \frac{1}{\varepsilon_0}q \quad \text{-----(ii)}$$

From relation (i) and (ii)

$$E4\pi r^2 = \frac{1}{\varepsilon_0}q$$

or $$E = \frac{q}{\varepsilon_0 4\pi r^2}$$

or $$\boxed{E = \frac{q}{4\pi\varepsilon_0 r^2}} \quad \text{----(iii)}$$

$\because$ $q = \sigma A = \sigma 4\pi R^2$

$\therefore$ $$E = \frac{\sigma 4\pi R^2}{4\pi\varepsilon_0 r^2}$$

or $$\boxed{E = \frac{\sigma R^2}{\varepsilon_0 r^2}} \quad \text{-----(iv)}$$

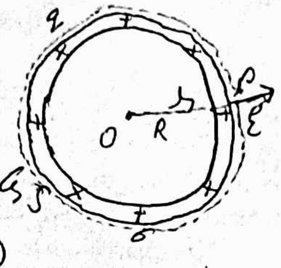
(ii) When the observation point lies on the surface of spherical shell (r = R)- Consider a point P on the surface of spherical shell.

Substituting r = R in equation (iii) and (iv), EFI at point P is given by

$$\boxed{E = \frac{q}{4\pi\varepsilon_0 R^2}} \quad \text{----(v)}$$

and $E = \frac{\sigma R^2}{\varepsilon_0 R^2}$

or $\boxed{E = \frac{\sigma}{\varepsilon_0}}$ -----(iv)



(iii) When the observation point lies inside the spherical shell (r < R)- Consider a point P inside the spherical shell such that its distance from centre O of the spherical shell be r.

By taking O as centre and r as radius draw sphere. This sphere will behave like Gaussian surface.

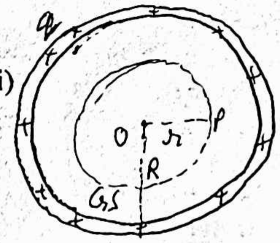
Total charge enclosed by Gaussian surface

$$q = 0$$

Substituting q = 0 in relation (iii) EFI at point P

$$E = \frac{0}{4\pi\varepsilon_0 r^2}$$

or $\boxed{E = 0}$ -----(vii)



Particular case- When $r = \infty$ then in this case $r > R$

or $E = \frac{q}{4\pi\varepsilon_0 \infty^2}$

or $E = 0$

Thus, due to a uniformly charged spherical shell EFI at infinity is zero.

Note- Graph between E and r- When $r > R$

$$E = \frac{q}{4\pi\varepsilon_0 r^2}$$

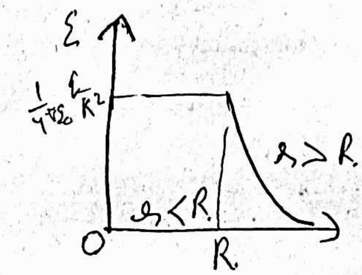
or $E \propto \frac{1}{r^2}$

When r = R

$$E = \frac{q}{4\pi\varepsilon_0 R^2}$$

and when r < R

$$E = 0$$



Electric field intensity due to uniformly charged solid sphere (non-conducting)

Ques.- Derive the relation for electric field intensity due to uniformly charged solid sphere (non-conducting) by using Gauss theorem.

Ans.- Consider a solid sphere of radius R in which q amount of charge is distributed uniformly such that the amount of charge present is a unit volume be ρ.

We can find out EFI due to the uniform by charged solid sphere at three different positions-

(i) When the observation point lies outside the sphere i.e (r > R)- Consider a point P outside the solid sphere such that its distance from its centre O be r.

By taking O as centre and r as radius draw a sphere. The sphere drawn will behave like Gaussian surface.

Total charge enclosed by Gaussian surface

$$q = \rho V \qquad \left[\because \rho = \frac{q}{V}\right]$$

or $q = \rho\frac{4}{3}\pi R^3$

As the electric field due to positively charged solid charged solid sphere is directed radially outwards therefore the electric field will be perpendiuclar to the Gaussian surface and from symmetry the magnitude of EFI is same at every point of Gaussian surface..

Total electric flux linked with the Gaussian surface

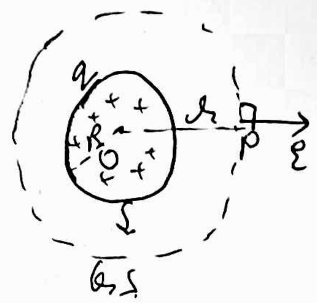
$$\phi_E = ES$$
$$= E4\pi r^2 \quad \text{---(i)}$$

By Gauss theorem

$$\phi_E = \frac{1}{\varepsilon_0}q \quad \text{----(ii)}$$

From relation (i) and (ii)

$$E4\pi r^2 = \frac{1}{\varepsilon_0}q$$



or $E = \frac{q}{\varepsilon_0 4\pi r^2}$

or $\boxed{E = \frac{q}{4\pi\varepsilon_0 r^2}}$ ---(iii)

$\because \quad q = \rho\frac{4}{3}\pi R^3$

$$\therefore \quad E = \frac{\rho\frac{4}{3}\pi R^3}{4\pi\varepsilon_0 r^2}$$

or $\boxed{E = \frac{\rho R^3}{3\varepsilon_0 r^2}}$ -----(iv)

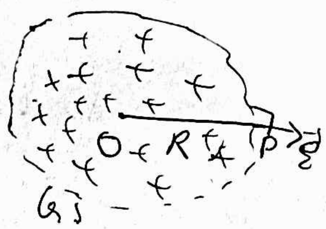
(ii) When the observation point lies at the surface of solid sphere (r = R)- Consider a point P on the surface of silid sphere.

Substituting r = R in relation (iii) and (iv) EFI at point P is

$$\boxed{E = \frac{q}{4\pi\varepsilon_0 R^2}} \quad \text{----(v)}$$

and $E = \frac{\rho R^3}{3\varepsilon_0 R^2}$

or $\boxed{E = \frac{\rho R}{3\varepsilon_0}}$ -----(iv)



(iii) When the observation point lies inside the solid sphere (r < R)- Consider a point P inside the charged sphere such that its distance from centre O of the sphere be r.

By taking O as the centre and r as radius draw a sphere, theis sphere will behave like Gaussian surface.

Total charge enclosed by the Gaussian surface

$$q' = \rho V' \qquad \left[\because \rho = \frac{q'}{V'}\right]$$

or $q' = \rho\frac{4}{3}\pi r^3$

As the electric field due to positively charged sphere is directed radially outwards and from symmetly the magnitude of EFI is same at every point of Gaussian surface.Therefore the electric field will be perpendiuclar to the Gaussian surface.

Total electric flux linked with the Gaussian surface

$$\phi_E = ES$$
$$= E4\pi r^2 \quad \text{-------(i)}$$

By Gauss theorem

$$\phi_E = \frac{1}{\varepsilon_0}q' \quad \text{-------(ii)}$$

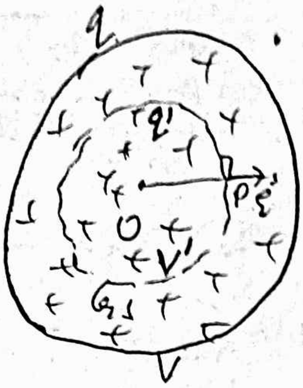
From relation (i) and (ii) we get

$$E4\pi r^2 = \frac{1}{\varepsilon_0}q'$$

the net charge ... with Gauss theorem the total or $E = \dfrac{q'}{\varepsilon_0 4\pi r^2}$

or $E = \dfrac{q'}{4\pi\varepsilon_0 r^2}$

$\because \quad q' = \rho \dfrac{4}{3}\pi r^3$



$\therefore \quad E = \dfrac{\rho \dfrac{4}{3}\pi r^3}{4\pi\varepsilon_0 r^2}$

or $\boxed{E = \dfrac{\rho r}{3\varepsilon_0}}$ ----(vii)

$\because \quad \rho = \dfrac{q}{\dfrac{4}{3}\pi R^3}$

$\therefore \quad E = \dfrac{\dfrac{q}{\dfrac{4}{3}\pi R^3} r}{3\varepsilon_0}$

or $\boxed{E = \dfrac{qr}{4\pi\varepsilon_0 R^3}}$ ----(viii) In terms of total charge present on it

Particular cases- (i) When r = 0 then in the case r < R

$\therefore \quad E = \dfrac{q.0}{4\pi\varepsilon_0 R^3}$ ----(viii)

or $E = 0$

Thus, due to uniformly charged solid sphere EFI at the centre is zero.

(ii) When $r = \infty$ then in the case r > R

$\therefore \quad E = \dfrac{q}{4\pi\varepsilon_0 \infty^2}$

or $E = 0$

Thus, due to uniformly charged solid sphere EFI at infinity is zero.

Note-(i)Graph between E and r- When r > R

then $E = \dfrac{q}{4\pi\varepsilon_0 r^2}$

$\therefore \quad E \propto \dfrac{1}{r^2}$

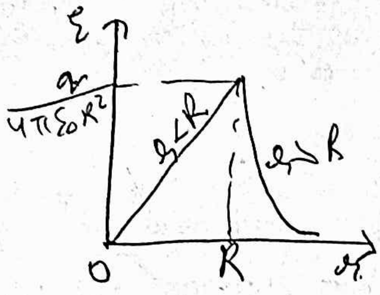
When r = R

then $E = \dfrac{q}{4\pi\varepsilon_0 R^2}$

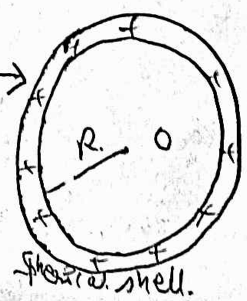
When r < R

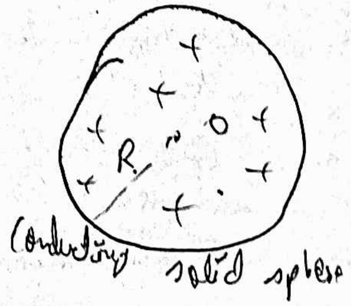
then $E = \dfrac{qr}{4\pi\varepsilon_0 R^3}$

$\therefore \quad E \propto r$



(ii) When charge is given to a conductor then due to mutual repulsion it gets distributed on the surface of conductor that is why a spherical conductor can be treated as spherical shell for the calculation of EFI because the charge distribution is same for both of them.





Derivation of Coulomb's inverse square law from Gauss theorem

Ques.- Derive Coulomb's inverse square law from Gauss theorem .

Ans.- Consider a point charge q at point O. By taking O as centre and r as radius draw a sphere. This sphere will behave like Gaussian surface for the point charge.

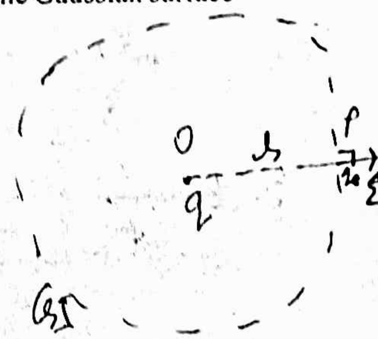
Net charge enclosed by the Gaussian surface = q

As the electric field due to the point charge is directed radially outwards and from symmetry the magnitude of EFI will be the same at every point of Gaussian surface. Therefore, the electric field will be perpendicular to the Gaussian surface.

Total electric flux linked with the Gaussian surface

$\phi_E = ES$

$= E4\pi r^2$ ---(i)



By Gauss theorem

$\phi_E = \dfrac{1}{\varepsilon_0} q$ -----(ii)

From relation (i) and (ii) we get

$E4\pi r^2 = \dfrac{1}{\varepsilon_0} q$

or $E = \dfrac{q}{4\pi\varepsilon_0 r^2}$

If another charge q_0 be placed at point P then electric force acting on it.

$F = q_0 E$

or $F = \dfrac{q_0 q}{4\pi\varepsilon_0 r^2}$

or $F = \dfrac{1}{4\pi\varepsilon_0} \cdot \dfrac{qq_0}{r^2}$

This is the required mathematical expression of Coulomb's inverse square law.