Electric current

Ques.- What is electric current ? Write its units and dimensions.

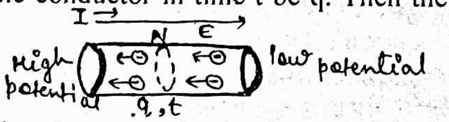
Ans.- When potential difference is applied across a conductor then an electric field comes into action inside the conductor from its higher potential end to its lower potential end due to which the free electrons present inside the conductor experience electric force in the direction opposite to electric field and they begin to move in the direction opposite to the electric field.

When potential difference is applied across a conductor then the amount of charge which flow through any cross-section of the conductor in a unit time is called electric current.

In other words, when potential difference is applied across a conductor then the rate of flow of charge through any cross-section of the conductor is called electric current.

If the amount of charge which flow through any cross-section of the conductor in time t be q. Then the electric current

$$\boxed{I = \frac{q}{t}}$$



The direction of electric current is taken in the direction of flow of positive charge or in the direction opposite to the of flow of negative charge.

Though, the electric current has both magnitude as well as direction yet it is considered to be a scalar quantity because it does not obey the laws of vector algebra.

Unit- From relation $I = \frac{q}{t}$

S.I. unit of $I = \frac{C}{s}$ = ampere (A)

C.G.S. esu of $I = \frac{statC}{s}$ = statampere (statA)

and C.G.S. emu of $I = \frac{abC}{s}$ = abampere (abA)

Dimensions- Electric current is a fundamental physical quantity therefore its dimensional formula is [A].

Note- (i) If N electrons flow through any cross-section of the conductor in time t then the charge which flow through any cross-section of the conductor in time t is given by $q = Ne$

∴ Electric current which flow through the conductor

$$I = \frac{q}{t}$$

or $\boxed{I = \frac{Ne}{t}}$

(ii) If I = 1A and t = 1sec

Then $I = \frac{Ne}{t}$

or $1 = \frac{N \times 1.6\times10^{-19}}{1}$

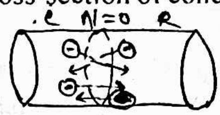
or $N = \frac{1}{1.6\times10^{-19}}$

or $N = 6.25\times10^{18}$

Thus, when 1A of electric current flows through any conductor then 6.25×10^{18} electrons pass through any cross-section of the conductor in 1second.

(ii) In the absence electric field the free electrons present inside a conductor move irregularly in all posible directions by virtue to thermal energy (thermal velocity 10^5 m/s). Due to this the number of electrons which pass through any cross-section of conductor from one side to another side in a given time interval is same as the number of electrons which passes through the same cross-section from other side to the first side in the same time interval. Therefore in the absence of electric field there is no net flow of electrons through any cross-section of conductor. i.e. N = 0

$$\therefore \quad I = \frac{Ne}{t} = \frac{0e}{t} = 0$$



Thus, in the absence of electric field the flow of electric current does not take place through any cross-section of conductor.

Definition of S.I and C.G.S. esu of electric current and relation between them

Ques.- Define of S.I and C.G.S. esu of electric current and establish relation between them.

Ans.- In relation $I = \frac{q}{t}$

If q = 1C and t = 1s

Then $I = \frac{1C}{1s}$

or $I = 1A$

Thus, if a charge of 1C flows through any cross-section of a conductor in 1s then the electric current flowing through the conductor is said to be 1 A.

Again if q = 1statC and t = 1s

Then $I = \frac{1statC}{1s}$

or $I = 1statA$

Thus, if if a charge of 1statC flows through any cross-section of a conductor in 1s then the electric current flowing through the conductor is said to be 1statA.

Relation between SI and CGS esu-

$$1A = \frac{1C}{1s}$$

$$= \frac{3\times10^9 statC}{1s}$$

$$= 3\times10^9 statA$$

$$\therefore \quad 1A = 3\times10^9 stA$$

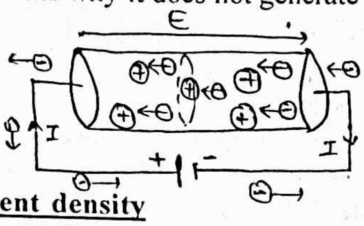
Note- \$ (i) Electirc current is of two types-

(a) Conventional electric current- The electric current whose direction is taken in the direction of flow of positive charge or opposite to the direction of flow of negative charge is called conventional electric current.

(b) Electronic current- The electric current whose direction is taken in the direction of flow of negative charge or opposite to the direction of flow of positive charge is called electronic current.

(ii) When potential difference is applied across a conductor then the number of electrons which enter into it through one of its end in a given time interval is same as the number of electrons which leaves the conductor through its other end in the same interval. Therefore, in a current carrying conductor the positive charge due to positive ionic cores is always equal and opposite to the negative charge due to free electrons as a result of which the current carrying conductor always remains electrically neutral.

That is why it does not generate electric field.



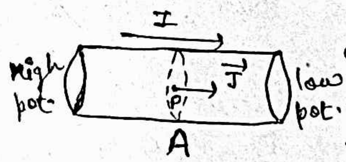
Current density

Ques.- Define current density. Write its units and dimensions.

Ans.- When potential difference is applied across a conductor then the electirc current flowing through a unit cross-sectional area chosen about a point of conductor is called current density at that point.

If the area of cross-section of a conductor be A and the electric current flowing through it be I then the current density at any point of that cross-section is given by $J = \frac{I}{A}$

It is a vector quantity and its direction at any point of conductor is same as the direction of electric current at that point.



Units- From relation $J = \frac{I}{A}$

S.I. unit of $J = \frac{A}{m^2} = Am^2$

C.G.S. esce of $J = \frac{statA}{cm^2} = statAcm^2$

Dimensions- From relation $J = \frac{I}{A}$

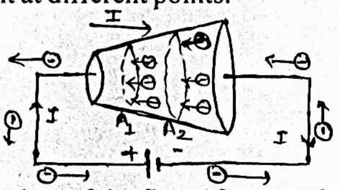
Dimensional formula of $J = \frac{[A]}{[L^2]}$
$= [AL^{-2}]$

Note- When potential difference is applied across a conductor of variable area of corss-section then the electric current which flow through different cross-section of conductor is same. Since the area of cross-section of conductor is different as different points therefore the current density will be different at different points.

$J = \frac{I}{A}$

$\because \quad A_1 < A_2$

$\therefore \quad J_1 > J_2$



Drift velocity

SQues.- Explain the mechanism of the flow of current in a conductor. Hence define the terms drift velocity and relaxation time. Obtain a relation between them.

Ans.- In the absence of electric field the free electrons present inside a conductor move irregularily in all possible directions with thermal velocities $v_{t1}, v_{t2}, v_{t3}, ...v_{tn}$, then the average thermal velocity of per electrons is given by

$$\vec{v}_{tav} = \frac{\vec{v}_{t1} + \vec{v}_{t2} + \vec{v}_{t3} + ...+\vec{v}_{tn}}{N}$$

or $\quad \vec{v}_{tav} = 0$

In the presence of electric field the free electrons present inside the conductor experience electric force in the direction opposite to the electirc field under whose effect electrons begin to accelerate in the direction opposite to the electric field.

Electric force experienced by free electrons

$$\vec{F}_e = -e\vec{E}$$

and acceleration of free electrons $\vec{a} = \frac{\vec{F}_e}{m}$

$$= \frac{-e\vec{E}}{m}$$

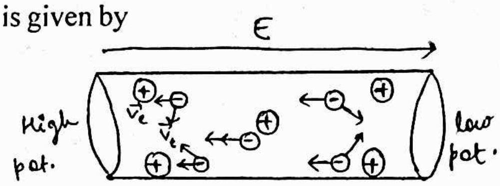
where m is the mass of electron.

If the time elapsed since the electrons suffered last collision with the poisitive ionic cores be t then the velocity acquired by the free electrons under the effect of electric field in time t is given by

$$\vec{v}_e = \vec{u}_e + \vec{a}t$$

or $\quad \vec{v}_e = 0 - \frac{e\vec{E}\,t}{m}$

or $\quad \vec{v}_e = -\frac{e\vec{E}\,t}{m}$



Thus, in the presence of electric field free electrons acquire a velocity in the opposite direction of electric field in addition to the thermal velocity.

The resultant velocity of free electrons in the presence of electric field is given by

$$\vec{v}_1 = \vec{v}_{t1} + \vec{v}_{e1}, \vec{v}_2 = \vec{v}_{t2} + \vec{v}_{e2}, \vec{v}_3 = \vec{v}_{t3} + \vec{v}_{e3}$$
$$..........., \vec{v}_N = \vec{v}_{tN} + \vec{v}_{eN}$$

Therefore, the average velocity of free electrons in the presence of electric field is given by

$$\vec{v}_{av} = \frac{\vec{v}_1 + \vec{v}_2 + \vec{v}_3 + ...+\vec{v}_N}{N}$$

or $\quad \vec{v}_{av} = \frac{(\vec{v}_{t1}+\vec{v}_{e1}) + (\vec{v}_{t2}+\vec{v}_{e2}) + (\vec{v}_{t3}+\vec{v}_{e3}) + ...+ (\vec{v}_{tN} + \vec{v}_{eN})}{N}$

or $\quad \vec{v}_{av} = \frac{\vec{v}_{t1}+\vec{v}_{t2}+\vec{v}_{t3}+...+\vec{v}_{tN}}{N} + \frac{\vec{v}_{e1}+\vec{v}_{e2}+\vec{v}_{e3}+..+\vec{v}_{eN}}{N}$

or $\quad \vec{v}_{av} = 0 + \frac{\vec{v}_{e1}+\vec{v}_{e2}+\vec{v}_{e3}+.....\vec{v}_{eN}}{N}$

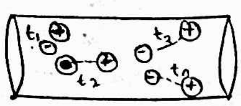
or $\quad \vec{v}_{av} = \frac{\vec{v}_{e1}+\vec{v}_{e2}+\vec{v}_{e3}+.....+\vec{v}_{eN}}{N}$

Thus, in the presence of electric field the average velocity of free electrons depends only upon the velocity acquired by the electrons under the effect of electric field.

$\because \quad \vec{v}_e = \frac{-e\vec{E}}{m}t$

$\therefore \quad \vec{v}_{av} = \frac{-\frac{e\vec{E}}{m}t_1 - \frac{e\vec{E}}{m}t_2 - \frac{e\vec{E}}{m}t_3 - ....-\frac{e\vec{E}}{m}t_N}{N}$

or $\quad \vec{v}_{av} = -\frac{e\vec{E}}{m}\left(\frac{t_1+ t_2+ t_3+ ....+t_N}{N}\right)$



or $\quad \vec{v}_{av} = -\frac{e\vec{E}}{m}\tau$

where τ is the average time elapsed since the electrons have suffered last collision with positive ionic cores and is called average relaxation time.

Thus, the average constant velocity acquired by free electrons in the direction opposite to the electric field is called drift velocity and it is represented by v_d.

$\therefore$ Drift velocity $\vec{v}_d = -\frac{e\vec{E}}{m}\tau$

Note- $(i) The path covered by free electrons between two successsive collisions with positive ionic cores is called free path and the mean of free path of all the electrons present in a conductor is called mean free path. It is represented by $\overline{\lambda}$ and it is of the order of 10^{-18}m.

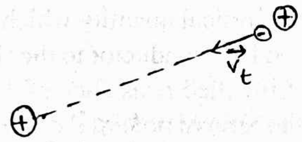
The time elapsed since the electrons suffer last. collision with positive ionic cores is called relaxation time and at any instant the mean of relaxation time of all the electrons present inside a conductor is called mean relaxation time. It is represented by τ and is of the order of 10^{-14} sec.

$\therefore$ Drift velocity of electrons = $\frac{\text{mean free path}}{\text{avg. relaxation time}}$

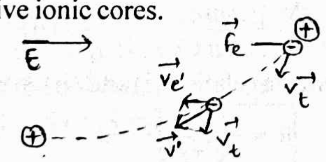
or $\quad v_d = \frac{\overline{\lambda}}{\tau} = \frac{10^{-18}}{10^{-14}} = 10^{-4}\ ms^{-1}$

or $\quad v_d = .1\ mms^{-1}$

(ii) In the absence of electric field free electrons possess thermal velocity therefore in the absence of electric field free electrons travel along straight path between two successive collisions with positive ionic cores.

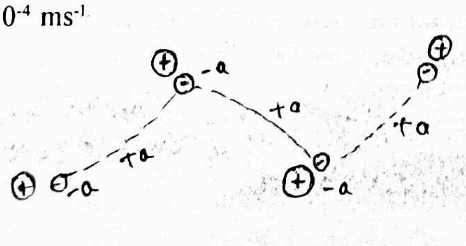


(iii) In the presence of electric field the free electrons acquire a velocity in the direction opposite to the electric field in addition to the thermal velocity. As the velocity acquired by free electrons under the effect of electric field continuously increases with time. Therefore, both the magnitude and direction of resultant velocity of free electrons changes continuously with time as a result of which free electrons follow parabolic path between two successive collisions with positive ionic cores.



(iv) In the presence of electric field the acceleration of electrons takes place between two successive collisions with positive ionic cores but when electrons suffer collision with positive ionic cores then their retardation takes place and they instantaneously come to rest. After collision the acceleration of electrons again take place. Due to the repetition of this process the average acceleration of electrons becomes zero and they move with a constant average velocity in the direction opposite to the electric field which is called drift velocity (v_d).

The value of drift velocity is of the order of .1 mms^{-1} or 10^{-4} ms^{-1}



Relation between electric current and drift velocity

Ques.- Derive relation between electric current and drift velocity.

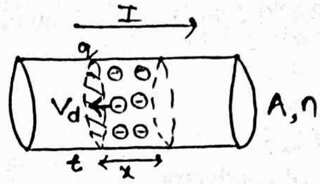
Ans.- Consider a conductor of length ℓ and area of crosssection A. On applying a potential difference across the ends of the conductor the drift velocity acquired by the free electrons present inside the conductor be v_d.

The distance covered by the free electrons present inside the conductor in time t is given by

$$x = v_d t \qquad \left[\because v_d = \frac{d}{t}\right]$$

If the number density (number of electrons present in a unit volume) of free electrons be n then the number of electrons which pass through any cross-section of conductor in time t is given by

N = Number of electrons which lie in distance x
= Number of electrons which lie in volume Ax
= nAx



$\therefore$ Amount of charge which flow through any cross-section of conductor in time t.

$$q = Ne = nAxe$$

By definition, the electric current flowing through any cross-section of conductor

$$I = \frac{q}{t}$$

or $\quad I = \frac{nAxe}{t}$

or $\quad I = \frac{nAv_d te}{t} \quad [\because x = v_d t]$

or $\quad \boxed{I = nAv_d e}$

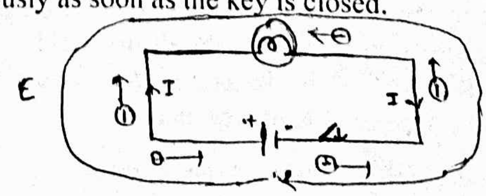
$\because J = \frac{I}{A}$

$J = \frac{nAV_d e}{A}$

$\boxed{J = nV_d e}$

This is the required relation.

Note- (i) As soon as the key is closed in the adjoining circuit, the electric field gets established through out the circuit with the speed of light and electric force begin to act in the direction opposite to the electric field due to which flow of electrons takes place in the direction opposite to the electric field and the flow of electric current takes place in the direction of electric field. Since the flow of electric current in an electric circuit is due to all the free electrons present in the circuit and not due to single electrons therefore as soon as key is closed the electric field gets established with the speed of light and hence the electric current is set up in the circuit with the speed of light. As a result of which the bulb glows instantaneously as soon as the key is closed.



(ii) Even though the values of A, v_d and e are very small yet a large current can flow through the conductor because of very large value of n (number density of free electrons).

$10^{29}\,m^{-3}$

$I = nAv_d e$ $\quad 10^{-19}$ C

$10^{-4}\,m^2 \quad 10^{-4}\,ms^{-1}$

Ohm's law

Ques.- State Ohm's law. Write the conditions for the validity of Ohm's law.

Ans.- In the year 1828 George Simon Ohm established a relation between the potential difference applied across a conductor and the electric current flowing through it which in his honour is called Ohm's law.

It states that if all the physical conditions (like length, area, temperature, pressure, density etc) remain unchanged then the potential difference applied across a conductor is directly proportional to the electric current flowing through it.

If on applying a potential difference V across a conductor the electric current flowing through it be I then in accordance with Ohm's law

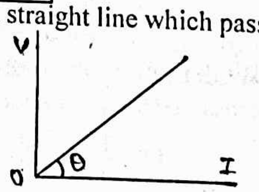
Ohm's law ← $V \propto I$

or $\quad V = RI$

where R is a constant of proportionality and is called resistance of conductor.

$\therefore$ Resistance of conductor $\boxed{R = \frac{V}{I}}$

Graph between V and I- It is a straight line which passes through the origin.



Slope of V-I graph

$= \tan\theta$

$= \frac{V}{I}$

$= R$ (resistance of the conductor)

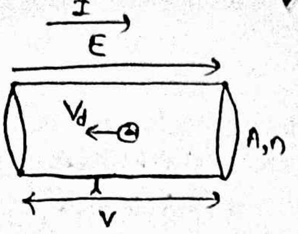
Conditions for the validity of Ohm's law- (i) The length and area (dimensions) of the conductor should remain unchanged.

(ii) Temperature, pressure, density etc. should also remain unchanged.

Derivation of Ohm's law on the basis of electronic theory of charge

Ques.- Derive Ohm's law on the basis of electronic theory of charge.

Ans.- Consider a conductor of length ℓ and area of cross-section A. When a potential difference V is applied across a conductor then the electric current flowing through it be I.



Electric field present inside the conductor is given by

$$E = \frac{V}{\ell} \qquad [\because V = E\ell]$$

Under the effect of electric field the electric force experienced by the free electrons present inside the conductor in the direction opposite to the electric field.

$$F_e = eE$$

By NSLM, the acceleration of free electrons

$$a = \frac{F_e}{m} = \frac{eE}{m}$$

If the average relaxation time of free electrons be τ then the average velocity (drift velocity) acquired by free electrons during the average relaxation time is given by

$$v = u + at$$

or $\quad v_{av} = 0 + \frac{eE}{m}\tau$

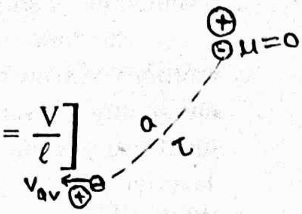
or $\quad v_d = \frac{eE}{m}\tau$

$\because \quad I = nAv_d e$

or $\quad v_d = \frac{I}{nAe}$ ----(i)

where n is the number density of free electrons.

From relation (i) and (ii)



$$\frac{I}{nAe} = \frac{eE\tau}{m}$$

or $\quad \frac{I}{nAe} = \frac{eV\tau}{\ell m} \qquad \left[\because E = \frac{V}{\ell}\right]$

or $\quad V = \frac{m\ell}{ne^2\tau A} I$

For a given conductor at constant temperature

$\frac{m\ell}{ne^2\tau}$ is constant.

$\therefore \quad V = $ constant I

or $\quad V \propto I$

This is the required Ohm's law.

Resistance

Ques.- Define resistance. On what factors does it depends. Write its units & dimensions.

Ans.- The physical quantity which measures the opposition offered by a conductor to the flow of electric current through it is called resistance of that conductor. It is defined as the ratio of potential difference applied across a conductor and the electric current flowing through it.

If on applying a potential difference V across a conductor the electric current flowing through it be I then its resistance is given by

$\boxed{R = \frac{V}{I}}$ ---(i)

$\because \quad V = \frac{m\ell}{ne^2\tau A} I$

$\therefore \quad \frac{V}{I} = \frac{m\ell}{ne^2\tau A}$ ----(ii)

Comparing relation (i) and (ii), we get

$$\boxed{R = \frac{m\ell}{ne^2\tau A}}$$

Factors on which the resistance of a conductor depends-

(i) It is directly proportional to the length of conductor.

i.e. $\quad R \propto \ell$

(ii) It is inversely proportional to the area of cross-section of conductor.

i.e. $\quad R \propto \frac{1}{A}$

(iii) It is inversely proportional to the number density of free electrons present inside the conductor.

i.e. $\quad R \propto \frac{1}{n}$

Since the number density of free electrons depends upon the nature of material of conductor. Therefore, the resistance of a conductor also depends upon nature of its material.

(iv) It is inversely proportional to the average relaxation time of free electrons present inside the conductor..

i.e. $R \propto \frac{1}{\tau}$

Since the average relaxation time of free electrons depends upon temperature. Therefore, the resistance of a conductor also depends upon its temperature.

Units - From relation

$$R = \frac{V}{I}$$

S.I unit of $R = \frac{V}{A}$ = ohm (Ω)

& C.G.S. esu of $R = \frac{\text{statV}}{\text{statA}}$ = statohm (statΩ)

Dimensions - From relation

$$R = \frac{V}{I}$$

Dimensional formula of $R = \frac{[ML^2T^{-3}A^{-1}]}{[A]}$

$= [ML^2T^{-3}A^{-2}]$

Definition of S.I and C.G.S esu of resistance

Ques.- Define of S.I and C.G.S. esu of resistance and establish relation between them.

Ans.- In relation

$$R = \frac{V}{I}$$

If V = 1V and I = 1A

Then $R = \frac{1V}{1A}$

$= 1\Omega$

Thus, if on applying a potential difference of 1V across a conductor the electric current flowing through it be 1A then the resistance of the conductor is said to be 1ohm.

Again, in relation $R = \frac{V}{I}$

If V = 1statV and I = 1statA

Then, $R = \frac{1\text{statV}}{1\text{statA}}$

$= 1\text{stat}\Omega$

Thus, if on applying a potential difference of 1statV across a conductor the electric current flowing through it be 1statA then the resistance of the conductor is said to be 1statohm.

Relation between SI and CGS esu-

$$1\Omega = \frac{1V}{1A}$$

$$= \frac{\frac{1}{300}\text{ statV}}{3\times10^9 \text{ statA}}$$

$$= \frac{1}{9\times10^{11}} \text{stat}\Omega$$

$$\therefore \quad 1\Omega = \frac{1}{9\times10^{11}} \text{stat}\Omega$$

Resistivity or specific resistance

Ques.- Define resistivity or specific resistance. On what factors does it depends. Write its units & dimensions.

Ans.- It is that physical quantity which measure the opposition by the material of conductor to the flow of electric current through it.

Since, the resistance of a conductor is directly proportional to its length and inversely proportional to its area of cross-section.

i.e. $R \propto \ell$

and. $R \propto \frac{1}{A}$

Combining the above relations

$$R \propto \frac{\ell}{A}$$

or $R = \rho\frac{\ell}{A}$ -----(i)

where ρ is a constant of proportionality and is called resistivity of material of conductor.

$\therefore$ Resisticity of material of conductor $\boxed{\rho = \frac{RA}{\ell}}$

If $\ell = 1$ and $A = 1$

then $\rho = R\frac{1}{1}$

or $\rho = R$

Thus, the resistivity of material of a conductor is numerically equal to its resistance if its length and area of cross-section be unity.

$\because \quad R = \frac{m}{ne^2\tau}\frac{\ell}{A}$ ---(ii)

From relation (i) and (ii)

$$\boxed{\rho = \frac{m}{ne^2\tau}}$$

Factors on which the resistivity of material of a conductor depends- (i) It is inversely proportional to the number density of free electrons present inside the conductor.

i.e. $\rho \propto \frac{1}{n}$

Since the number density of free electrons depends upon the nature of material of conductor therefore the resistivity also depends upon the nature of material of conductor.

(ii) It is inversely proportional to the average relaxation time of free electrons present inside the conductor.

i.e. $\rho \propto \frac{1}{\tau}$

Since the average relaxation time of free electrons depends upon temperature of conductor therefore the resistivity also depends upon the temperature of the conductor.

Units - From relation

$$\rho = \frac{RA}{\ell}$$

S.I. unit of $\rho = \frac{\Omega m^2}{m} = \Omega m$

C.G.S. esu of $\rho = \frac{\text{stat}\Omega \text{cm}^2}{\text{cm}}$ = statΩcm

Dimensions- From relation

$$\rho = \frac{RA}{\ell}$$

Dimensional formula of $\rho = \frac{[ML^2T^{-3}A^{-2}][L^2]}{[L]}$

$= [ML^3T^{-3}A^{-2}]$

Note- (i) Resistance is defined for conductor where as resistivity is defined for the material of conductor.

(ii) Comparision of resistivity of different metals (at the same temperature)

$$\rho_{Ag} < \rho_{Cu} < \rho_{Au} < \rho_{Al} < \rho_{Fe}$$

(iii) When a conductor is streched, hamnred or remoulded into another shape then its volume remains constant.

When the volume of a conductor remains constant.

Case (I)- Dependence of R on L-

$$R = \rho\frac{\ell}{A}$$

or $R = \rho\frac{\ell}{A}\times\frac{\ell}{\ell}$

or $R = \rho\frac{\ell^2}{A\ell}$

or $R = \frac{\rho\ell^2}{V}$ $[\because A\ell = V]$

$\because$ $\frac{\rho}{V}$ is constant

$\therefore$ $\boxed{R \propto \ell^2}$

i.e. $\frac{R_1}{R_2} = \frac{\ell_1^2}{\ell_2^2}$

Case (II)- Dependence of R on A-

$$R = \rho\frac{\ell}{A}$$

or $R = \rho\frac{\ell}{A}\times\frac{A}{A}$

or $R = \rho\frac{\ell A}{A^2}$

or $R = \frac{\rho V}{A^2}$ $[\because \ell A = V]$

$\therefore$ ρV is a constant

$\therefore$ $\boxed{R \propto \frac{1}{A^2}}$

i.e. $\frac{R_1}{R_2} = \frac{A_2^2}{A_1^2}$

$Case (III)- Dependence of R on r-

$$R = \rho\frac{\ell}{A}$$

or $R = \rho\frac{\ell}{A}\times\frac{A}{A}$

or $R = \rho\frac{\ell A}{A^2} = \frac{\rho V}{A^2}$

$\because$ ρV = constant

$\therefore$ $R \propto \frac{1}{A^2}$

or $R \propto \frac{1}{(\pi r^2)^2}$ $[\because A = \pi r^2]$

or $R \propto \frac{1}{\pi^2 r^4}$

$\therefore$ $\boxed{R \propto \frac{1}{r^4}}$

i.e. $\frac{R_1}{R_2} = \frac{r_2^4}{r_1^4}$

$\because$ $r = \frac{d}{2}$ (d - diameter)

$\therefore$ $R \propto \frac{1}{(d/2)^4}$

or $R \propto \frac{1}{d^4/16}$

or $R \propto \frac{16}{d^4}$

$\boxed{R \propto \frac{1}{d^4}}$

i.e. $\frac{R_1}{R_2} = \frac{d_2^4}{d_1^4}$

Conductance

Ques.- Define conductance. Write its units & dimensions.

Ans.- Conductance a conductor is defined as the reciprocal of resistance of conductor.

i.e. Conductance $= \frac{1}{\text{Resistance}}$

or $G = \frac{1}{R}$

$\because$ $R = \frac{V}{I}$

$\therefore$ $G = \frac{1}{V/I}$

or $\boxed{G = \frac{I}{V}}$

Units- From relation $G = \frac{1}{R}$

S.I. unit of $G = \frac{1}{\text{ohm}} = (\text{ohm})^{-1} = \Omega^{-1}$

= mho

= siemens (S)

Dimensions- From relation $G = \frac{1}{R}$

Dimensional formula of $G = \frac{1}{[ML^2T^{-3}A^{-2}]}$

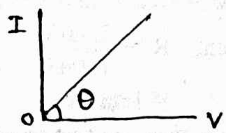
$= [M^{-1}L^{-2}T^3A^2]$

Note-(i) It is that physical quantity which measures the ability of a conductor to allow the flow of electric current through it.

(ii) Graph between I and V- By ohm's law

$V \propto I$ $\Rightarrow I \propto V$

$\therefore$ The graph between I and V is a straight line which will pass through the origin.



Slope of I-V graph = $\tan\theta$

$= \frac{I}{V}$

= G (Conductance of conductor)

Conductivity or specific conductance

Ques.- Define conductivity or specific conductance. Write its units & dimensions.

Ans.-It is defined as the reciprocal of resistivity of material of conductor.

i.e. Conductivity $= \frac{1}{\text{Resistivity}}$

or $\sigma = \frac{1}{\rho}$

$\because$ $\rho = \frac{m}{ne^2\tau}$

$\therefore$ $\sigma = \frac{ne^2\tau}{m}$

Note- $\because$ $\rho = \frac{RA}{\ell}$

$\therefore$ $\sigma = \frac{1}{\rho} = \frac{\ell}{RA}$

Units- From relation $\sigma = \frac{1}{\rho}$

S.I. unit of $\sigma = \frac{1}{\Omega m} = \Omega^{-1}m^{-1}$

If the av. value of α is given for a temp. range $(t_2 - t_1)$, then $\alpha_{av} = \frac{R_2 - R_1}{R_1(t_2 - t_1)}$.
~~Its nature also~~ It's value depends on temp.

= mho m^{-1} = Sm^{-1}

Dimensions- From relation $\sigma = \frac{1}{\rho}$

Dimensional formula of $\sigma = \frac{1}{[ML^3T^{-3}A^{-2}]}$
$= [M^{-1}L^{-3}T^3A^2]$

Note- It is that physical property which measures the ability of material of a conductor to allow the flow of electric current through it.

Relation between current density and conductivity

Ques.- Derive relation between current density and conductivity.

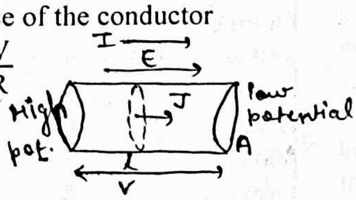
Ans.- Consider a conductor of length ℓ and area of cross-section A. On applying a potential difference V across its ends the electric current flowing through it be I.

Then, the current density at any point inside the conductor is given by

$$J = \frac{I}{A} \quad \text{-----(i)}$$

By ohm's law the resistance of the conductor

$$R = \frac{V}{I} \quad \text{or} \quad I = \frac{V}{R}$$



From relation (i)

$$J = \frac{V/R}{A}$$

or $$J = \frac{V}{RA}$$

In vector form $\vec{J} = \sigma\vec{E}$

$\because$ $$R = \rho\frac{\ell}{A} \quad \& \quad V = E\ell$$

$\therefore$ $$J = \frac{E\ell}{\rho\frac{\ell}{A}A}$$

$\because$ for a given conductor, σ is constant
$\therefore$ $\vec{J} \propto \vec{E}$ [Vector form of ohm's law]

or $$J = \frac{E}{\rho}$$

Thus, current density at any pt. of a conductor is directly proportional to EFI at that pt. and its direction is same as that of EFI.

$\because$ $\frac{1}{\rho} = \sigma$, conductivity of material of conductor.

$\therefore$ $\boxed{J = \sigma E}$

i.e. Current density = Conductivity × Electric field intensity

Temperature cofficient of resistance

Ques.- Define temperature cofficient of resistance. Write its practical unit.

Ans.- Consider a conductor whose resistance at 0°C and t°C be R_0 and R respectively. Then, the resistance of conductor at t°C can be expressed as

$$R = R_0 (1 + \alpha t)$$

where α is called temperature cofficient of resistance.

$$R = R_0 + R_0\alpha t$$

or $$R - R_0 = R_0\alpha t$$

or $$\boxed{\alpha = \frac{R - R_0}{R_0 t}}$$

If $R_0 = 1\Omega$ and $t = 1°C$

Then $$\alpha = \frac{R - R_0}{1.1}$$

or $$\alpha = R - R_0$$

Thus, if the resistance of a conductor at 0°C be 1Ω then on increasing its temperature through 1°C increase in its resistance is numerically equal to its temperature cofficient of resistance.

Its value depends upon the nature of material of conductor and its practical unit is $°C^{-1}$.

Note- If the length and area of cross-section of a conductor be ℓ and A respectively. Then, the resistance of the conductor at 0°C.

$$R_0 = \rho_0\frac{\ell}{A}$$

where ρ_0 is the resistivity of material of conductor at 0°C. and the resistance of conductor at t°C.

$$R = \rho\frac{\ell}{A}$$

where ρ is the resistivity of material of conductor at t°C.

$\because$ $$R = R_0 (1 + \alpha t)$$

or $$\rho\frac{\ell}{A} = \rho_0\frac{\ell}{A}(1 + \alpha t)$$

or $$\rho = \rho_0 (1 + \alpha t)$$

or $$\alpha = \frac{\rho - \rho_0}{\rho_0 t}$$

Therefore, α is also called the temperature cofficient of resistivity.

Effect of temperature on the resistance and resistivity of a conductor

Ques.- How the resistance and resistivity of a conductor varies with temperature? Explain.

Ans.- With the increase in temperature the thermal velocity of free electrons increases due to which the resultant velocity of free electrons also increase. Because of the above reason with the increase in temperature the average relaxation time of free electrons decreases

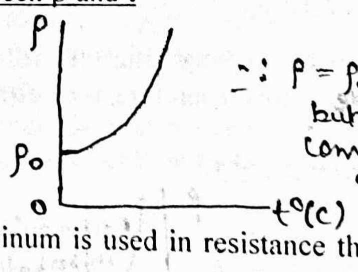
From relation $\rho = \frac{m}{ne^2\tau}$

It can be concluded that with the increase in temperature due to decrease in the average relaxation time of free electrons the resistivity of material of conductor increases.

From relatoin $R = \rho\frac{\ell}{A}$ it can be concluded that with the increase in temperature due to increase in resistivity the resistance of conductor also increases.

That is why the value of temperature cofficient of resistance for conductors is positive.

Graph between ρ and t-



$\because$ $\rho = \rho_0(1 + \alpha t + \beta t^2)$
but β is small in comparision with α, so, ignoring βt^2.

Note- Platinum is used in resistance thermometer because-

(a) Melting point of platinum is very high therefore it can be used for measuring very high temperature.

(b) Its temperature cofficient of resistance very large therefore even when the change in temperature is very small its resistance increases by considerable amount due to which even a small change in temperature could be measured by using platinum resistance thermometer.

Effect of temperature on the resistance and resistivity of a alloys (metallic)

Ques.- How the resistance and resistivity of alloys varies

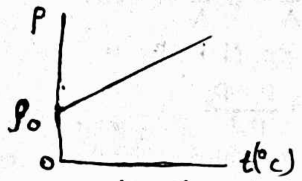
(*) Standard resistance - Resistance whose value doesnt change on change in temp.

with temperature? Explain.

Ans.- At the same temperature the resistivity of an alloy is greater than the resistivity of its component metals but with the increase in temperature the resistivity of alloys increases but the increase in their resistivity is less than thus of component metals.(Therefore,the temperature cofficient of resistivity of alloys is less than the temperature cofficient of resistivity of its component metals.)

From relatoin $R = \rho\frac{\ell}{A}$ it can be concluded that with the increase in temperature the resistance of alloys also increases. Therefore the value of α is positive for alloys.

Graph between ρ and t-



Note- Alloys like manganin and constantan (eureka) are used for making standard resistance coil and are used in electrical appliances like meterbridge, potentiometer, resistance box etc.

Reason- The resistivity of alloys like manganin and constantan is very high and their temperature cofficient of resistance is very small. That is why with the change in temperature their resistance nearly remains unchanged.

Effect of temperature on the resistance and resistivity of an electrolyte

Ques.- How the resistance and resistivity of an electrolyte varies with temperature? Explain.

Ans.- With the increase in temperature

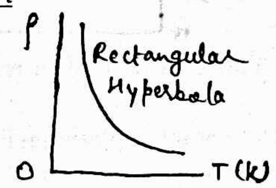
(a) The viscosity of electrolyte decreases as a result of which the space available for the free motion of ions increases.

(b) The rate of ionization of electrolyte molecules increases as a result of which the number density of charge carries increases.

Because of the above two reasons with the increase in temperature the resistivity of an electrolyte decreases.

From relatoin $R = \rho\frac{\ell}{A}$ it can be concluded that with the increase in temperature the resistance of electrolyte also decreases. Therefore, for electrolyte α is negative.

Graph between ρ and t-



Note- (i) Effect of temperature on the resistivity on semi-conductors- With the increase in temperature the resistivity and hence the resistance of semiconductors decreases.

Therefore, for semiconductors α is negative.

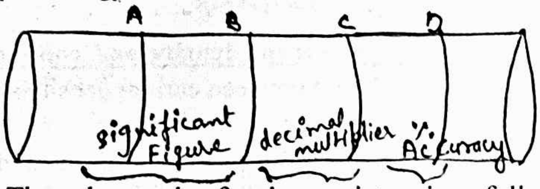
(ii) Effect of temperature on the resistivity of insulator- With the increase in temperature the resistivity and hence the resistance of insulators decreases.

Therefore, for insulators α is negative.

Colour code of carbon resistors

Ques.- Describe the colour code used for carbon resistors.

Ans.- In carbon resistors there are four co-axial rings out of which the colour of first two rings give significant figures, the colour of third ring gives us decimal multiplier and the colour of fourth ring gives percentage accuracy or percentage tolerance.



The colour code of carbon resistors is as follows-

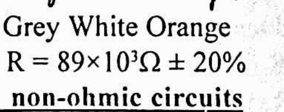
Colour	Significant figure	Decimal multiplier	% Accuracy
Black	0	10^0	-
Brown	1	10^1	-
Red	2	10^2	-
Orange	3	10^3	-
Yellow	4	10^4	-
Green	5	10^5	-
Blue	6	10^6	-
Violet	7	10^7	-
Grey	8	10^8	-
White	9	10^9	-
Gold	-	10^{-1}	5%
Silver	-	10^{-2}	10%
No colour	-	-	20%

The sequence of colours in colour code of carbon resistors can be remenbred with the help of following statement.

"B.B.ROY of Great Britain has Very Good Wife".

The capital letters of above statement gives us the initials of colours in the sequence in which they appear in the colour code of carbon resistances.

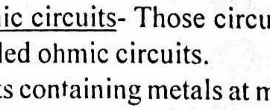
e.g.- (i)



Red Blue Green Gold

$R = 26\times10^{5}\Omega \pm 5\%$ ⇒ $R = 26\times10^{5}\,\Omega \pm 5\%$

(ii)



Grey White Orange

$R = 89\times10^{3}\Omega \pm 20\%$

Ohmic and non-ohmic circuits

Ques.- What are ohmic & non-ohmic circuits ? Give examples.

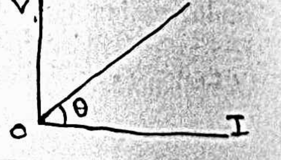
Ans.- Ohmic circuits- Those circuits which obey Ohm's law are called ohmic circuits.

e.g.- circuits containing metals at moderate temperatures.

Since ohmic circuits obey ohm's law therefore V-I graph for them is a straight line which passes through the origin.

Slope of V-I graph $= \tan\theta = R$

Thus, the slope of V-I graph gives the resistance of ohmic

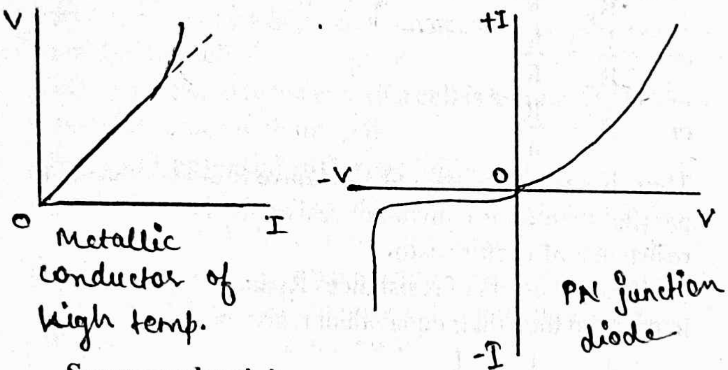


Moderate temp : -100°C to 100°C

circuits.

Non-ohmic circuits- Those circuits which do not obey ohm's law are called non-ohmic circuits.
e.g.- Circuits containing metals at high temperature, PN junction diode, diode valve, triode valve, electrolyte, thyristor etc.

For non-ohmic circuits V-I graph may have one or more of the following characteristics-
(i) The graph is non-linear.
(ii) The value of I depends upon the sign of V.
(iii) The value of I is not unique.

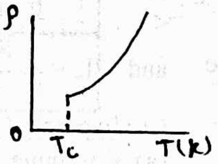


Superconductivity

Ques.- What is superconductivity ? What is its cause? Write its advantages.

Ans.-In 1911, H. Kammerling Onnes found that resistivity of some metals suddenly becomes zero at low temperatures. This property of certain metals by virtue of which their resistivity suddenly becomes zero at low temperature is called superconductivity. The metals which exhibit this property are called superconductors. The temperature at which their resistivity becomes zero is called critical temperature.

Metals	critical temperature
Hg	4.2 K
Sn	3.7 K
Pb	7.2 K



Cause- At low temperatures cohrence is developed between free electrons present inside a metal as a result of which they begin to follow one another due to which the collision of free electrons does not take place with the positive ionic cores and the resistivity of super conductors become zero at low temperatures.

Advantages- (i) If the super conductivity be acheived at room temperature then the loss of electrical energy in the form of heat during transmission of electricity can be avoided.

(ii) In super conducting loops large amount of electric current can flow. Therefore such loops behave like strong electromagnets. In this manner super conductivity can be used for generating strong magnetic feilds.

Series combination of resistors

Ques.- What do you mean by series combination of resistors ? Obtain the expression of equivalent resistance of three resistors connected in series ? What is its importance?

Ans.- When two or more resistors are connected between the terminals of a cell in such a manner that the first end of first resistor is connected to the positive terminal of the cell and the other end of first resistor is connected to the first end of third resistor and in this manner the second end of last resistor is connected to the negative terminal of the cell, then such a combination of retsistors is said to be series combination of resistors.

Consisder there resistors of resistance R_1, R_2 and R_3 which are connected in series across the terminals of a cell whose terminal potential difference be V.

If the electric current flowing through the three resistor be I then the potential difference across the first resistor $V_1 = IR_1$

the potential difference across the second resistor

$$V_2 = IR_2.$$

and the potential difference across the third resistor

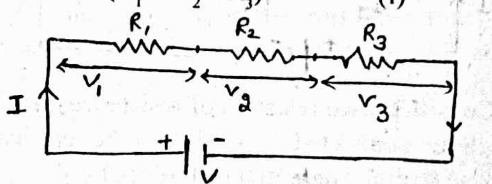
$$V_3 = IR_3$$

Since the total potential difference across the series combination of resistors is equal to the terminal potential difference of the cell therefore

$$V = V_1 + V_2 + V_3$$

or $V = IR_1 + IR_2 + IR_3$

or $V = I(R_1 + R_2 + R_3)$ -----(i)



If the equivalent resistance of series combination of resistor be R then

$$V = IR \qquad \text{-----(ii)}$$

From relation (i) and (ii)

$$IR = I(R_1 + R_2 + R_3)$$

or $R = R_1 + R_2 + R_3$

Thus, the equivalent resistance of series combination of resistors equal to the sum of resistances involved in series combination.

Importance- As in series combination the equivalent resistance is equal to the sum of resistances involved in series combination therefore when the desired resistance is greater than the given resistances then they sould be connected in series.

Note- (i)Genalization- If n resistors of $R_1, R_2, R_3, \ldots R_n$ are connected in series then their equivalent resistance is given by

$$R_s = R_1 + R_2 + R_3 + \ldots + R_n$$

Particular case- If $R_1 = R_2 = R_3 = \ldots = R_n = R$ (say)

Then $R_s = R + R + R + \ldots + R$

$$= nR$$

Thus, if n resistors each of resistance R are connected in series then their equivalent resistance is n times the value of each resistance.

(ii) Potential division formula In series combination

$$V \propto R$$

$$\therefore \quad \boxed{V_1 = \frac{R_1}{R_1 + R_2} V}$$

and $\boxed{V_2 = \frac{R_2}{R_1 + R_2} V}$

(iii) In series combination.

(a) I is same.

(b) $V \propto R$

(c) $V = V_1 + V_2 + V_3$

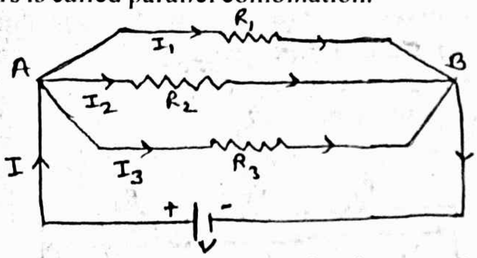
(d) $R = R_1 + R_2 + R_3$

(e) $R > R_1, R > R_2, R > R_3$.

Parallel combination of resistors

Ques.- What do you mean by parallel combination of resistors ? Obtain the expression of equivalent resistance of three resistors connected in parallel ? What is its importance.

Ans.- When two or more resistors are connected in such a manner that the one end of all the resistors are connected to one point which is in contact with the positive terminal of a cell and the other end of all the resistors are connected to another point which is in contact with the negative terminal of the cell then such a combination of resistors is called parallel combination.



Consider three resistors of resistances R_1, R_2, and R_3 which are connected in parallel to the terminals of a cell whose terminal potential difference be V.

As the three resistors are connected in parallel across the terminals of the cell. Therefore, the potential difference across them will be the same which is equal to the terminal potential difference of the cell V.

By ohm's law the electric current flowing through the first resistance $I_1 = \frac{V}{R_1}$

the electric current flowing through the second resistance

$$I_2 = \frac{V}{R_2}$$

and the electric current flowing through the third resistance $I_3 = \frac{V}{R_3}$

Therefore, the total electric current flowing through the parallel combination of resistors is given by

$$I = I_1 + I_2 + I_3$$

or $$I = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} \quad \text{----(i)}$$

If the equivalent resistance of the parallel combination of resistors be R then

or $$I = \frac{V}{R} \quad \text{----(ii)}$$

From relation (i) and (ii)

$$\frac{V}{R} = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

or $$\boxed{\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

Thus, the reciprocal of equivalent resistance of parallel combination of resistors is equal to the sum of reciprocals of resistances of resistors involved in parallel combination.

Importance- As the equivalent resistance of parallel combination of resistors is less than the resistance of resistors involved in parallel combination. Therefore, when the desired resistance is less then the resistance of given resistors then they sould be connected in parallel.

Note-(i) Generalization- If n resistors of resistances R_1, R_2, R_3.... R_n are connected in parallel then the equivalent resistance is given by.

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots\dots\dots + \frac{1}{R_n}$$

Particular case- If $R_1 = R_2 = R_3 = \dots\dots = R_n = R$ (say)

Then $$\frac{1}{R_p} = \frac{1}{R} + \frac{1}{R} + \frac{1}{R} + \frac{1}{R} + \dots\dots\dots + \frac{1}{R}$$

or $$\frac{1}{R_p} = \frac{n}{R}$$

or $$R_p = \frac{R}{n}$$

Thus, if n resistors each of resistance R are connected in parallel then their equivalent resistance is 1/n times the resistance of each resistor.

(ii) If two resistors of resistances R_1 and R_2 are connected in paralled then their equavalent resistance,

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

or $$\frac{1}{R} = \frac{R_2 + R_1}{R_1 R_2}$$

or $$\boxed{R = \frac{R_1 R_2}{R_2 + R_1}}$$

(iii) Current division formula in parallel combination

$$I \propto \frac{1}{R}$$

$\therefore$ $$\boxed{I_1 = \frac{R_2}{R_1 + R_2} \cdot I}$$

and $$\boxed{I_2 = \frac{R_1}{R_1 + R_2} . I}$$

(iv) In parallel combination of resistors-

(a) V is same.

(b) $I \propto \frac{1}{R}$

(c) $I = I_1 + I_2 + I_3$

(d) $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$

(e) $R < R_1, R < R_2, R < R_3$.

Internal resistance of a cell

Ques.- What is internal resistance of a cell? On what factors does it depends?

Ans.- The resistance offered by the electrolyte present between the electrodes of a cell to the flow of electric current through it is called internal resistance of the cell. It is represented by r.

The internal resistance of a cell depends upon the following factors-

(a) It is directly proportional to the distance between the two electrodes.

i.e. $r \propto l$.

(b) It is inversely proportional to the area of electrodes dipped inside the electrolyte.

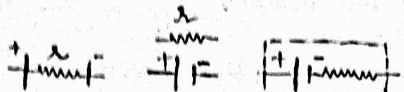
i.e. $r \propto \frac{1}{A}$

(c) It is directly proportional to the concentration of electrolyte.

i.e. $r \propto C$

(d) It is inversely proportional to the temperature of electrolyte..

i.e. $r \propto \frac{1}{T}$



Note-(i) Internal resistance of a cell can be represented in any one of the following manner.

(ii) Outside a cell flow of electric current takes place from positive to negative terminal and inside the cell the flow of electric current takes place from negative to positive terminal of the cell.

(iii) The internal resistance of a cell is assumed to be connected in series with the cell.

Terminal potential difference of a cell

Ques.- Define terminal potential difference of a cell.

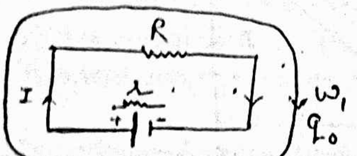
Ans.- The work done in moving a unit positive charge from positive to negative electrode through the external circuit of a cell is called terminal potential difference of that cell. It is represented by V.

If the work done in moving a test charge q_0 from positive electrode to the negative electrode of the cell through the external circuit be W_1 then the terminal potential difference of the cell is given by

$$\boxed{V = \frac{W_1}{q_0}}$$

If the external resistance be R and the electric current flowing through it be I then by Ohm's law the terminal potential difference of the cell is given by

$$V = IR$$



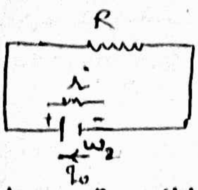
Potential drop inside a cell

Ques.- Define potential drop inside a cell.

Ans.- The work done in moving a unit positive charge from the negative electrode to the positive electrode o through the internal circuit of a cell is called potential inside that cell.

If the work done in moving a test charge q_0 from negative electrode to the positive electrode through the internal circuit be W_2 then the potential drop inside the cell is given by

$$\boxed{V' = \frac{W_2}{q_0}}$$



If the internal resistance of a cell be r and the electric current flowing through it be I then in accordance with Ohm's law, the potential drop inside the cell is given by

$$V' = Ir$$

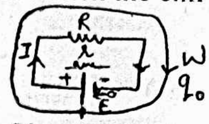
Electromotive force (emf)

Ques.- Define electromotive force (emf) of a cell.

Ans.- The work done in moving a unit positive charge throughout the circuit (external as well as internal) of a cell is called emf of that cell. It is represented by E and its direction is always taken from negative terminal to the positive terminal of the cell.

If the work done in moving a test charge q_0 throughout the circuit of a cell be W. Then the emf of the cell is given by

$$\boxed{E = \frac{W}{q_0}}$$



Note- (i) The nature of emf is same as that of potential difference therefore its units and dimensions are same as that of potential difference.

(ii) EMF of a cell depends upon the nature of electrolyte and electrodes of a cell and is independent of the internal resistance of cell and concentration of its electrolyte.

Relation between emf terminal potential difference and internal resistance of a cell

Ques.- Derive relation between emf terminal potential difference and internal resistance of a cell.

Ans.- Consider a cell of emf E and internal resistance r. When an external resistance R is connected across the terminals of the cell then the electric current flowing through it be I.

By Ohm's law the electric current flowing through the cell $= \frac{\text{Total potential difference}}{\text{Total resistance}}$

$= \frac{\text{EMF of the cell}}{\text{External resistance + Internal resistance}}$

or $\boxed{I = \frac{E}{R + r}}$ ----(i)

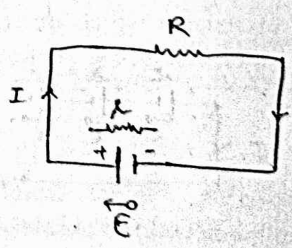
This is called circuit equation.

As the electric current flowing the external resistance R is I. Therefore the terminal potential difference is given by

$$V = IR$$

or $I = \frac{V}{R}$ ----(ii)

From relation (i) and (ii)



$\frac{V}{R} = \frac{E}{R+r}$

or $VR + Vr = ER$

or $Vr = ER - VR$

or $Vr = (E-V)R$

or $r = \frac{(E-V)R}{V}$ --(iii)

or $\boxed{r = \left(\frac{E}{V} - 1\right) R}$

$\because V = IR$

From relation (iii)

$r = \frac{(E-V)}{IR} R$

or $r = \frac{E - V}{I}$

or $Ir = E - V$

or $\boxed{V = E - Ir}$

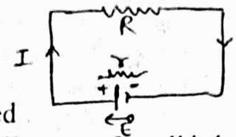
This is the required relation for terminal potential difference.

Note- (i) When the circuit is closed then

$I \neq 0$

$\therefore \quad V = E - Ir$

or $\quad V < E$



Thus, when a circuit is closed then the terminal potential difference of a cell is less than its emf.

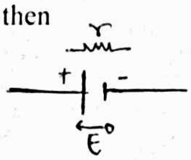
(ii) When the circuit is open then

$I = 0$

$\therefore \quad V = E - Ir$

or $\quad V = E - 0(r)$

or $\quad V = E$

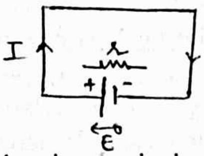


Thus, when a circuit is open then the terminal potential difference of a cell is equal to its emf.

(iii) When a cell is short circuited i.e. when the terminals of a cell are brought in electrical contact without external resistance.

$$I = \frac{E}{R+r} = \frac{E}{0+r} = \frac{E}{r}$$

$$\therefore \quad V = E - Ir = E - \frac{E}{r}r = 0$$



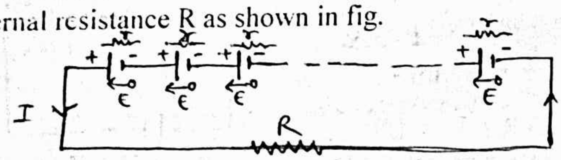
Thus, when a cell is short circuited then its terminal potential difference becomes zero.

Series combination of cells

Ques.- What do you mean by series combination of cells ? Derive the condition of maximum current through an external resistance connected across a series combination of cells.

Ans.- When two or more cells are connected across the ends of a resistance in such a manner that the positive terminal of first cell is connected to one end of the resistance, the negative terminal of the first cell is connected to positive terminal of the second, the negative terminal of second cell is connected to the positive terminal of third cell and in this manner the negative terminal of last cell is connected to the other end of the resistance then the combination of cells is said to be series combination.

Consider n identical cells each of emf E and internal resistance r which are connected in series across the external resistance R as shown in fig.



Then the net emf of the series combination of cells

$$E_{net} = E + E + E + ...+ E$$

or $\quad E_{net} = nE$

and the net internal resistance of the series combination of cells

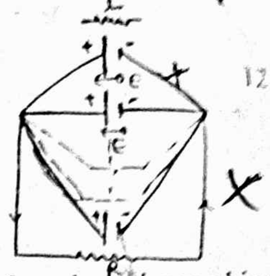
$$r_{net} = r + r + r + ...+ r$$

or $\quad r_{net} = nr$

The total electric current flowing through in the series combination of cells

$$I = \frac{E_{net}}{R + r_{net}}$$

or $\quad \boxed{I = \frac{nE}{R + nr}}$



Particular cases- (i) If $R >> nr$

Then $\quad I = \frac{nE}{R} = n \times \frac{E}{R}$

In this case the current obtained from the series combination of cells is equal to n times the current obtained from single cell. Therefore, the cells should be connected in series in this situation.

(ii) If $R << nr$

Then $\quad I = \frac{nE}{nr} = \frac{E}{r}$

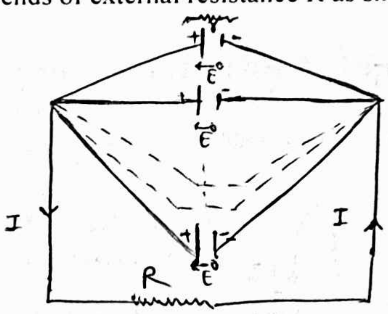
In this case the current obtained from the series combination of cells is equal to the current obtained from single cell. Therefore, there is no advantage of connecting the cells in series in this situation.

Parallel combination of cells

Ques.- What do you mean by parallel combination of cells ? Derive the condition of maximum current through an external resistance connected across a parallel combination of cells.

Ans.- When two or more cells are connected across the ends of a resistance in such a manner that the positive terminal of all the cells are connected to the one end of resistance and the negative terminals of all the cells are connected to the other end of resistance then the combination of cells is said to be parallel combination.

Consider n identical cells each of emf E and internal resistance r which are connected in parallel across the ends of external resistance R as shown in fig.



Thus, the net emf of parallel combination of cells

$$E_{net} = E$$

and net internal resistance of cells

$$\frac{1}{r_{net}} = \frac{1}{r} + \frac{1}{r} + \frac{1}{r} + ...+ \frac{1}{r}$$

or $\quad \frac{1}{r_{net}} = \frac{n}{r}$

or $\quad r_{net} = \frac{r}{n}$

$\therefore$ The total electric current flowing through the parallel combination

$$I = \frac{E_{net}}{R + r_{net}}$$

or $\quad I = \frac{E}{R + \frac{r}{n}}$

or $\quad I = \frac{nE}{nR + r}$

Particular cases- (i) If $R >> \frac{r}{n}$

Note: If nothing is given about the grouping of cells, then the cells should b[e] assumed to be connected in mixed grouping

then $I = \frac{E}{R}$

In this case, the current obtained from the parallel combination of cells is equal to the current obtained from single cell. Therefore there is no advantage of connecting cells in parallel in this situation.

(ii) If $R << \frac{r}{n}$

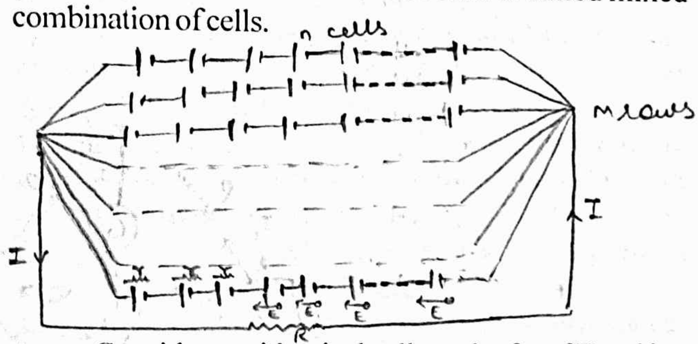
Then $I = \frac{E}{r/n} = \frac{nE}{r} = n \times \frac{E}{r}$

In this case the current obtained from the parallel combination of cells is equal to n times the current obtained from single cell. Therefore, the cells should be connected in parallel in this situation.

Mixed combination of cells

Ques.- What do you mean by mixed combination of cells? Derive the condition of maximum current through an external resistance connected across mixed combination of cells.

Ans.- When a fixed number of cells are connected in series to from a certain number of rows and all such rows are connected in parallel across the ends of an resistance then such a combination of cells is called mixed combination of cells.



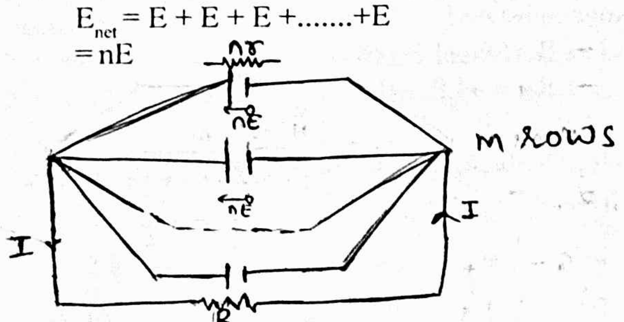
Consider mn identical cells each of emf E and internal resistance and which are connected in m rows such that each row contain n cells. These m rows are connected in parallel across an external resistance R as shown in fig.

Then, the net internal resistance of one row of cells.

$r_{net} = r + r + r + r + ... + r$
$= nr$

The net internal emf of one row of cells

$E_{net} = E + E + E + ... + E$
$= nE$



Net internal resistance of combination of cells

$$\frac{1}{r_{net}} = \frac{1}{nr} + \frac{1}{nr} + \frac{1}{nr} + ... + \frac{1}{nr}$$

or $\frac{1}{r_{net}} = \frac{m}{nr}$

or $r_{net} = \frac{nr}{m}$

Electric current flowing through the combination of cells

$$I = \frac{E_{net}}{R + r_{net}}$$

or $I = \frac{nE}{R + nr/m}$

or $I = \frac{mnE}{mR + nr}$

or $I = \frac{mnE}{(\sqrt{mR})^2 + (\sqrt{nr})^2 - 2\sqrt{mRnr} + 2\sqrt{mRnr}}$

or $I = \frac{mnE}{(\sqrt{mR} - \sqrt{nr})^2 + \sqrt{mRnr}}$

The value of I will be maximum when

$(\sqrt{mR} - \sqrt{nr})^2 + 2\sqrt{mRnr}$ is minimum

when $(\sqrt{mR} - \sqrt{nr})^2 = 0$

or $\sqrt{mR} - \sqrt{nr} = 0$

or $\sqrt{mR} = \sqrt{nr}$

Squaring both sides

$mR = nr$

or $R = \frac{nr}{m}$

Thus when the external resistance is equal to the net internal resistance then the current obtained from the mixed combination of cells is maximum.

Note-(i). When the external resistance is much greater than the net internal resistance then in order to obtain the maximum current the cells should be conncted in series.

(ii). When the external resistance is less than the net internal resistance then in order to obtain maximum current the cells should be connected in parallel.

(iii). When the external resistance is equal to the net resistance then in order to obtain maximum current the cells should be connected in mixed combination.

Thermistors

Ques.- What do you mean by thermistors? Write some important applications of thermistors?

Ans.- Thermistor is a heat sensitive device whose resistance changes rapidly with temperature. Its temperature coefficient of resistance is very high which may be positive or negative..

Thermistors are made of oxides of metals (like nickel, cobalt, copper etc.). The thermistors are generally in the form of discs or rods which are enclosed in a small glass bulb with platinum pins at its two ends.

Applications- 1. It is used for the detection and measurement of very small temperatures.

2. It is used for voltage stablisation and current control.

3. It is used as relay switch in electrical appliances like refrigerator, television etc.

Electric Measurement

Kirchoff's law

Ques.- State kirchoff's laws for electrical circuits. State the sign conventions used. On which conservation laws they depend?

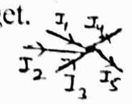
Ans.- In the year 1842 Kirchoff enunciated two laws in order to analyse typical electric circuit. In his honour these laws are known as Kirchoff law.

1. Kirchoff's I law or law of current or junction rule- It states that the algebraic sum of all the electric currents which meet at a junction of an electirc circuit is always equal to zero.

i.e. $\Sigma I = 0$

While applying junction rule all the electric currents which approaches towards a junction are taken +ve. and all the electric currents which recedes away from that junction are taken -ve.

Applying Kirchoff's I law at junction, we get.



$$+ I_1 + I_2 + I_3 - I_4 - I_5 = 0$$

or $$I_1 + I_2 + I_3 = I_4 + I_5$$

Thus the algebraic sum of all the electric currents which approaches towards the junction is equal to the algebraic sum of all the electric currents which recedes away from that junction.

This law is based on the law of conservation of charge.

X 2. Kirchoff's II law or law of voltage or mesh rule- It states that the algebraic sum of the product of all the resistances present in a closed mesh and electric currents flowing through them is equal to the algebraic sum of emf's of all the cells present in that loop or mesh.

i.e. $\Sigma IR = \Sigma E$

While applying mesh rule when we go through a resistance in the direction of electric current then the current should be taken +ve and when we go through a resistance in direction opposite to that of electric current then the current should be taken -ve. Similarly when we go through a cell from its -ve to +ve terminal then its emf is taken +ve and when we go through a cell from its +ve to -ve terminal then its emf is taken -ve.

Applying Kirchoff's II law in mesh ABEFA we get

$$- I_2R_2 + I_1R_1 = -E_2 + E_1$$

Again applying Kirchoff's II law in mesh BCDEB we get

$$+ I_3R_3 - I_2R_2 = E_2$$

Kirchoff's II law is based on the law of conservation of energy.

Wheatstone bridge

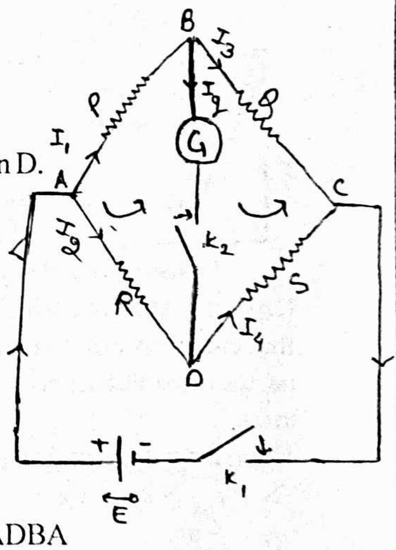
Ques.- What is Wheatstone bridge? Apply Kirchoff's laws to derive the balanced condition of Wheatstone bridge.

Ans.- In the year 1842, British physicist Wheatstone arranged four resistors in a difinite manner with the help of which an unknown resistance can be determined. This arrangement of resistances is known as Wheatstone bridge. In WSB. four resistors are connected along the arms of a rectangle. A cell is connected across one diagonal of the rectangle through a key and a galvanometer is connected across the other diagonal through another key. In WSB the values of resistances are adjusted in such a manner that the deflection of galvanometer becomes zero. In this situation the WSB is said to be in balanced state.

In the balanced state of WSB. the ratio of resistances of any two adjacent arms is equal to the ratio of resistances of other two adjacent arms.

i.e. $$\frac{P}{Q} = \frac{R}{S}$$

Derivation- When the keys K_1 and K_2 are closed then the flow of EC takes place in different parts of W.S.B. as shown in fig. Applying KIL at junction B



$$+I_1 - I_g - I_3 = 0$$

In balanced state $I_g = 0$

$\therefore$ $+I_1 - 0 - I_3 = 0$

or $I_1 = I_3$ ----(i)

Again applying KIL at junction D.

$$+I_2 + I_g - I_4 = 0$$

In balanced state $I_g = 0$

$\therefore$ $+I_2 + 0 - I_4 = 0$

or $I_2 = I_4$ -----(ii)

~~Dividing rel'n (i) by (ii)~~

$$\cancel{\frac{I_1}{I_2} = \frac{I_3}{I_4}}$$

or $\cancel{\frac{I_1}{I_3} = \frac{I_2}{I_4}}$ ---~~(iii)~~

Now applying KIIL in mesh ADBA

$$- I_2R + I_gG + I_1P = 0$$

In balanced state $I_g = 0$

$\therefore$ $-I_2R + 0G + I_1P = 0$

or $-I_2R + 0 + I_1P = 0$

or $I_2R = I_1P$ ----(iii)

Again applying KIIL in mesh BDCB

$$-I_gG - I_4S + I_3Q = 0$$

In balanced state $I_g = 0$

$\therefore$ $-0G - I_4S + I_3Q = 0$

or $-0 - I_4S + I_3Q = 0$

or $I_4S = I_3Q$ ----(iv)

Dividing rel'n (iii) and (iv)

or $\frac{I_2R}{I_4S} = \frac{I_1P}{I_3Q}$ ---(v)

From rel'n (ii) and (v), (i)

$$\frac{I_4 R}{I_4 S} = \frac{I_3 P}{I_3 Q}$$

$$\frac{R}{S} = \frac{P}{Q}$$

or $$\boxed{\frac{P}{Q} = \frac{R}{S}}$$

When the resistance of all the four resistor of WSB are of same order then the sensitivity of WSB is maximum.

Thus is the reqd. working formula of WSB.

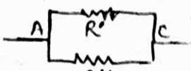
Note- 1. P and Q are called ratio arms, R is called known arm and S is called unknown arm of WSB.

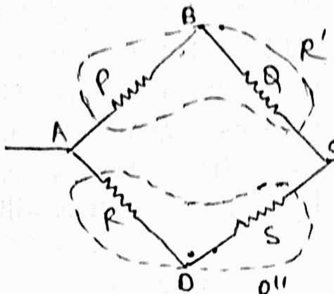
2. AC and BD are called conjugate arms of WSB because on interchanging the positions of cell and galvanometer in these arms the balanced state of WSB remains unaffected.

3. In the balanced state the equivalent resistance of WSB can be calculated as follows

$$R_{eq} = \frac{R'R''}{R'+R''}$$

$$= \frac{(P+Q)(R+S)}{(P+Q)+(R+S)}$$





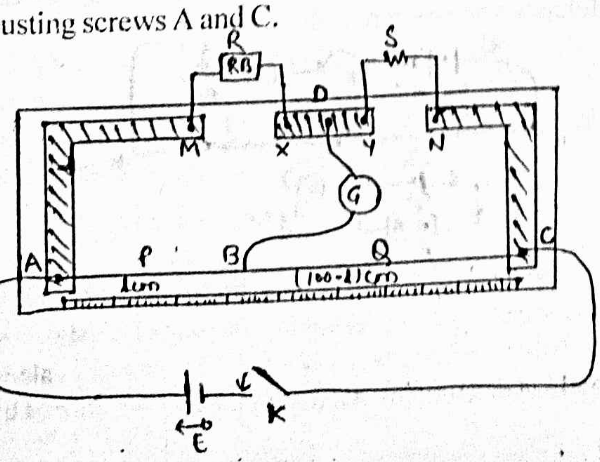
Meter bridge or slide wire bridge

Ques.- What is meter bridge? With the help of circuit diagram, explain how it can be used to find an unknown resistance.

Ans.-Meter bridge is an electrical device based on the principle of WSB with the help of meter bridge an unknown resistance can be find out and it is also used for comparing two unknown resistances.

Construction- In meter bridge there is a manganin or constantan wire which is streched between two L-shaped thick brass strips AM and CN fitted on a wooden planck A meter scale is also fitted on the wooden planck parallel to the wire. Between the two L-shaped strips another thick brass strip XY is fitted such that two empty spaces MX and NY are left between the three strips. At the ends of all three strips and at the midpoint D of third strip XY adjusting screws are present.

To determine the unknown resistance with the help of meter bridge the wire whose resistance S is unknown is connected in the gap YN and resistance box is connected in the gap MX. A galvanometer G is connected between the adjusing screws D and the sliding jockey B and a cell E is connected through a key K between the adjusting screws A and C.



First of all a known resistance R is taken but from the resistance box then after closing the key K the sliding jockey B is moved along the wire until the deflection of galvanometer a becomes zero. In this situation meter bridge is said to be in balanced state and the point B where the sliding jockey touches the wire is called balanced point whose distance from the end A of the wire is called balancing length.

If the resistance of parts AB and BC of the wire are P and Q respectively then the circuit of meter bridge will be equivalent to W.S.B. as shown in fig.

Let the distance of balanced point B from the end A of wire be ℓ cm.

i.e. AB = ℓ cm

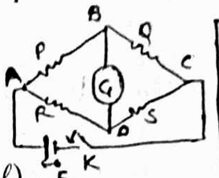
then BC = $(100-\ell)$ cm

If the resistance per unit length of wire be x then

$P = x\ell$ and $Q = x(100-\ell)$ $\because \left[P = \frac{\rho l}{A} = xl \ \& \ Q = \frac{\rho(100-l)}{A} = x(100-l)\right]$

From the principle of W.S.B., in the balanced state

$$\frac{P}{Q} = \frac{R}{S}$$



$\because R = \rho \frac{l}{A}$

$\Rightarrow \frac{R}{l} = \frac{\rho}{A}$

x = Resistance per unit length

or $$S = \frac{Q}{P} R$$

or $$S = \frac{x(100-\ell)}{x\ell} R$$

or $$\boxed{S = \frac{(100-\ell)}{\ell} R}$$

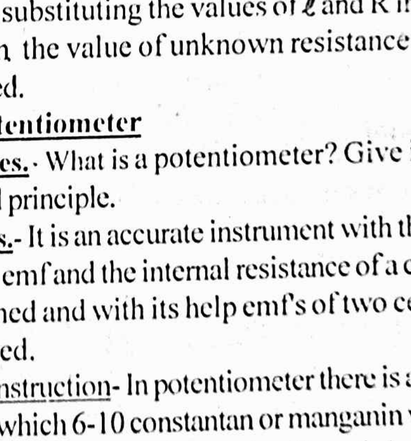
By substituting the values of ℓ and R in the above relation the value of unknown resistance S can be calculated.

Potentiometer

Ques.- What is a potentiometer? Give its construction and principle.

Ans.- It is an accurate instrument with the help of which the emf and the internal resistance of a cell can be determined and with its help emf's of two cells can be compared.

Construction- In potentiometer there is a wooden planck on which 6-10 constantan or manganin wires of uniform area of cross-section each of length 1 m are connected in series with the help of Cu strips. At the end of wires there are two adjusting screws A and B. There is a sliding jockey J wihch contains a large number of pressing keys. The position of sliding jockey can be noted with the help of meter scale which is fitted parallel to the wires.



Principle- Let AB the potentiometer wire whose length be ℓ_{AB}. An accumulator cell C', key K', rheostat Rh and ammeter A are connected in series to potentiometer wire.

This is called primary circuit or main circuit of potentiometer. When key K' is closed then flow of electric current takes place in the wire AB. As the end A of potentiometer wire is in contact with the +ve terminal and the end B of the wire in contact with -ve terminal of the cell accumalator C'. Therefore, the potential of point A of the wire will be greater than the potential of point B. Thus, the potential continuously decreases from point A to B along the wire AB. The rate of change of electric potential with distance along the potentiometer wire is called potential gradient.

If the potential differnce across the wire AB be V_{AB} then the potential gradient along the wire is given by

$$k = \frac{V_{AB}}{\ell_{AB}}$$

Its value ($k = I\rho/A$) remains constant throughout the wire and it is expressed in Vcm^{-1}.

Determination of emf of a cell- An experimental cell C of emf E and internal resistance r is connected between the end A of potentiometer wire and the sliding jockey J through a galvanometer G such that its +ve terminal is in contact with the end A of potentiometer wire. This is called secondary or auxillary circuit of potentiometer. Now by sliding the jockey along the potentiometer wire a point J is obtained in such a manner that on pressing the key the galvanometer gives no deflection. In this situtaion the flow of electric current does not takes through the galvanometer. This point J is called balanced point or null point and its distance from the end A of wire is called balancing length. In the balanced state of potentiometer the ~~point~~ potential difference between point A and J of potentiometer wire is equal to the emf of experimental cell.

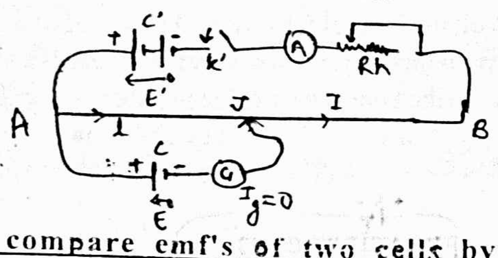
i.e. $E = V_{AJ}$.

If the balancing length be ℓ then

$$V_{AJ} = k\ell$$

$$\therefore E = k\ell$$

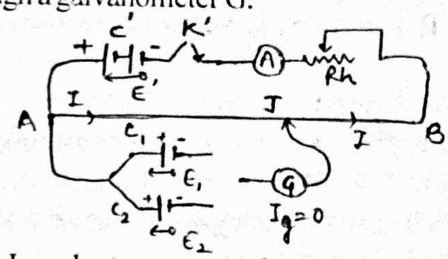
By substituting the values of k and ℓ in the above rel'n the emf of the cell can be determined.



To compare emf's of two cells by using potentiometer

Ques.- With the help of circuit diagram, explain how can a potentiometer be used to compare the emf's of two cells?.

Ans.- Two cells of emf E_1 and E_2 are connected to the primary circuit of potentiomenter in such a manner that +ve terminals of both the cells are connected to the end A of potentiometer wire and -ve terminals of both the cells are connected to terminal 1 and 2 of a two way key whose third terminal is connected to a sliding jockey J through a galvanometer G.



In order to compare the emf's of two cells a current is passed through the potentiometer wire by closing the key K' due to which a potential difference appears across the potentiometer wire.

When the plug is put in the gap between the terminal 1 and 3 of two way key then the cell of emf E_1 will come in the circuit. After this the cell of emf E_1 of the cell is balanced on the potentiometer wire by sliding the jockey. If the balancing length corresponding to the emf E_1 of the potentiometer will be ℓ then

$$E_1 = k\ell_1 \quad \text{---(i)}$$

where k is potential gradient along the potentiometer wire.

When the plug is put in the gap between the terminal 2 and 3 of the two way key the cell of emf E_2 will come in the circuit. After this the emf E_2 of the second cell is balanced on the balancing length corresponding to the emf E_2 of the second cell on potentiometer wire be ℓ_2 then

$$E_2 = k\ell_2 \quad \text{---(ii)}$$

Dividing rel'n (i) by (ii) we get

$$\frac{E_1}{E_2} = \frac{k\ell_1}{k\ell_2}$$

or

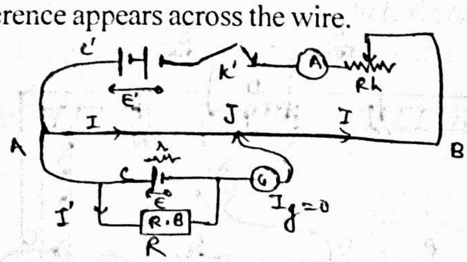
$$\frac{E_1}{E_2} = \frac{\ell_1}{\ell_2}$$

By substituting the values of ℓ_1 and ℓ_2 in the above rel'n the emf of the two cells can be compared.

To determine the internal resistance of a cell by using potentiometer

Ques.- With the help of circuit diagram, explain how can a potentiometer be used to measure internal resistance of a cell?.

Ans.- Let E be the emf and r be internal resistance of the experimental cell. By closing the key K' current is passed through the potentiometer wire due to which a potential difference appears across the wire.



First of all, the resistance box is taken out from the electrical circuit by removing the plug of resistance ∞ Ω from the resistance box then by sliding the jockey along the potentiometer wire the emf of the cell is balanced by the potential difference between the points A and J_1 of the wire.

If the balancing length corresponding to the emf of the cell be ℓ_1 then

$$E = k\ell_1 \quad \text{---(i)}$$

where k is potential gradient along the potentiometer wire.

Now by taking a plug of known resistance R, the resistance box is taken into the circuit then by sliding the jockey along the potentiometer wire the terminal potential difference of the cell is balanced by the potential difference between the points A and J_2 of the potentiometer wire.

If the balancing length corresponding to the terminal potential difference of the cell be ℓ_2 then

$$V = k\ell_2$$

∵The internal resistance of cell is given by

$$r = R\left(\frac{E}{V} - 1\right)$$

∴

$$r = R\left(\frac{k\ell_1}{k\ell_2} - 1\right)$$

or

$$r = R\left(\frac{\ell_1}{\ell_2} - 1\right)$$

By substituting the values of ℓ_1, ℓ_2 and R in the above rel'n the internal resistance of cell can be calculated.

Note- 1. While using potentiometer the following two things should be taken into consideration.

(i). The emf of the accumulator cell connected in the primary circuit and should be greater than the emf of experimental cell connected in the secondary circuit otherwise the experimental cell cannot be balanced by the potential difference between the two points on the potentiometer wire.

(ii) The +ve terminal of accumulator cell and the experimental cell should be connected to the same end of potentiometer wire otherwise the experimental cell cannot be balanced by the potential difference between the two points on the potentiometer wire.

In both the above situations the galvalnometer will give deflection in only one direction.

2.A potentiometer is said to be sensitive if it can measure very small potential difference.

The sensitivity of a potentiometer can be increased by decreasing its potential gradient. This can be acheived by-

(a) increasing the length of potentiometer wire.

(b) decrasing the current in the potentiometer wire.

3. Potentiometer behaves like an ideal voltmeter (explanation- magnetic effect of current.)

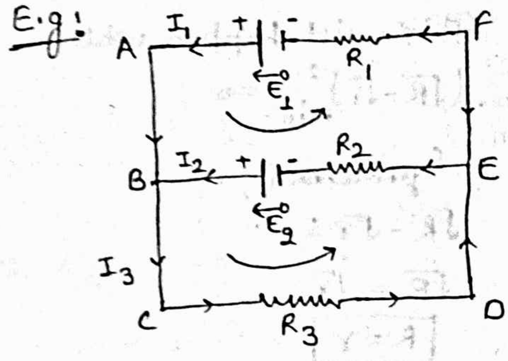
★ Kirchoff's II law or voltage law or mesh rule ★

It states that the algebric sum of all the voltages (changes in potential) in a closed loop of an electric circuit is always zero.

i.e; $\Sigma IR + \Sigma E = 0$

while applying mesh rule when we go around in a closed loop, then the increase in potential is taken +ve, and decrease in potential is taken -ve.

E.g:



Applying K II L in mesh ABEFA

$$-E_2 + I_2R_2 - I_1R_1 + E_1 = 0$$

Applying K II L in mesh BCDEB

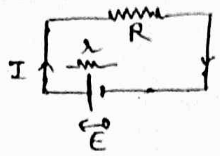
$$-I_3R_3 - I_2R_2 + E_2 = 0$$

★ Kirchoff's second law is based on law of conservation of energy.

⇒ While applying Kirchoff IInd law, when we pass through a resistor in the direction of current the the p.d across it is taken -ve and when we pass through a resister in the opposite direction current then p.d across it is taken +ve.

Similarly, when we pass through a cell in direction of its e.m.f, then its e.m.f is taken +ve and when we pass through a cell in the opposite direction of its e.m.f. then its e.m.f is taken -ve.

★★ Maximum power theorem (for output power in external resistor)



$$I = \frac{E}{R+r}$$

$$I = \frac{E}{(\sqrt{R})^2 + (\sqrt{r})^2}$$

$$I = \frac{E}{(\sqrt{R})^2 + (\sqrt{r})^2 - 2\sqrt{Rr} + 2\sqrt{Rr}}$$

$$I = \frac{E}{(\sqrt{R} - \sqrt{r})^2 + 2\sqrt{Rr}}$$

$I \to$ Maximum

$(\sqrt{R} - \sqrt{r})^2 + 2\sqrt{Rr} \longrightarrow$ minimum

This will happen when,

$(\sqrt{R} - \sqrt{r})^2_{min} = 0$

Squareroot,

$\sqrt{R} - \sqrt{r} = 0$

$\sqrt{R} = \sqrt{r}$

$\boxed{R = r}$

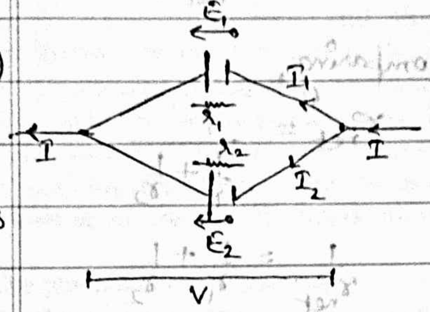
$$I_{max} = \frac{E}{R+R} = \frac{E}{r+r}$$

when I is max. then P will also be max.

$$\therefore P_{max} = I_{max}^2 R$$

*) Equivalent emf and internal resistance of two cells:

i) Without kirchoff's rule



let equi. internal resistance be r_{net} and equi. emf be $\mathcal{E}_{net}$.

$$\frac{1}{r_{net}} = \frac{1}{r_1} + \frac{1}{r_2}$$

$$r_{net} = \frac{r_1 r_2}{r_1 + r_2}$$

Here, $V = \mathcal{E}_1 - I_1 r_1$

$$\Rightarrow I_1 = \frac{\mathcal{E}_1 - V}{r_1}$$

and $V = \mathcal{E}_2 - I_2 r_2$

$$I_2 = \frac{\mathcal{E}_2 - V}{r_2}$$

Since, $I = I_1 + I_2$

$$I = \frac{\mathcal{E}_1 - V}{r_1} + \frac{\mathcal{E}_2 - V}{r_2}$$

$$I = \frac{\mathcal{E}_1}{r_1} + \frac{\mathcal{E}_2}{r_2} - \frac{V}{r_1} - \frac{V}{r_2}$$

$$\frac{V}{r_1} + \frac{V}{r_2} = \frac{\mathcal{E}_1}{r_1} + \frac{\mathcal{E}_2}{r_2} - I$$

$$V\left(\frac{1}{r_1} + \frac{1}{r_2}\right) = \frac{\mathcal{E}_1}{r_1} + \frac{\mathcal{E}_2}{r_2} - I$$

$$V = \frac{\frac{\mathcal{E}_1}{r_1} + \frac{\mathcal{E}_2}{r_2} - I}{\frac{r_1 + r_2}{r_1 r_2}} \quad \text{—i)}$$

for combination :-

$V = \mathcal{E}_{net} - I r_{net}$ —ii)

from (i) & (ii)

$$\frac{\frac{\mathcal{E}_1}{r_1} + \frac{\mathcal{E}_2}{r_2} - I}{\frac{r_1 + r_2}{r_1 r_2}} = \mathcal{E}_{net} - I\left(\frac{r_1 r_2}{r_1 + r_2}\right)$$

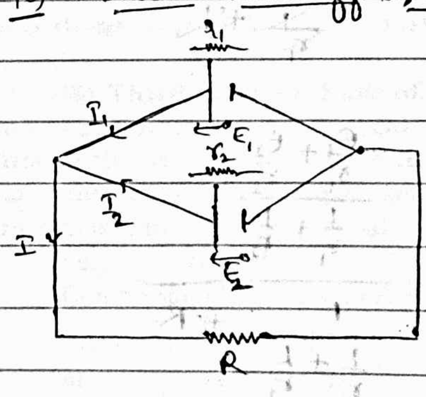
$$\frac{\frac{\mathcal{E}_1}{r_1} + \frac{\mathcal{E}_2}{r_2}}{\frac{r_1 + r_2}{r_1 r_2}} - \frac{I r_1 r_2}{r_1 + r_2} = \mathcal{E}_{net} - I\left(\frac{r_1 r_2}{r_1 + r_2}\right)$$

$$\therefore \mathcal{E}_{net} = \frac{\frac{\mathcal{E}_1}{r_1} + \frac{\mathcal{E}_2}{r_2}}{\frac{r_1 + r_2}{r_1 r_2}}$$

$$\mathcal{E}_{net} = \frac{\mathcal{E}_1 r_2 + \mathcal{E}_2 r_1}{r_1 + r_2}$$

$$\Rightarrow \mathcal{E}_{net} = \frac{\frac{\mathcal{E}_1}{r_1} + \frac{\mathcal{E}_2}{r_2}}{\frac{1}{r_1} + \frac{1}{r_2}}$$

ii) with kirchoff's rule :-



$$-IR - I_1 r_1 + \mathcal{E}_1 = 0$$

$$I_1 = \frac{\mathcal{E}_1 - IR}{r_1}$$

$$-IR - I_2 r_2 + \varepsilon_2 = 0$$

$$I_2 = \frac{\varepsilon_2 - IR}{r_2}$$

$$\because I = I_1 + I_2$$

$$I = \frac{\varepsilon_1 - IR}{r_1} + \frac{\varepsilon_2 - IR}{r_2}$$

$$I = \frac{\varepsilon_1}{r_1} - \frac{IR}{r_1} + \frac{\varepsilon_2}{r_2} - \frac{IR}{r_2}$$

$$I + \frac{IR}{r_1} + \frac{IR}{r_2} = \frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2}$$

$$I\left(1 + \frac{R}{r_1} + \frac{R}{r_2}\right) = \frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2}$$

$$I = \frac{\frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2}}{1 + \frac{R}{r_1} + \frac{R}{r_2}}$$

$$I = \frac{\frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2}}{1 + R\left(\frac{1}{r_1} + \frac{1}{r_2}\right)}$$

$$\cancel{I = \frac{\frac{\frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2}}{\frac{1}{r_1} + \frac{1}{r_2}}}{\frac{1}{r_1} + \frac{1}{r_2}}}$$

$$I = \frac{\frac{\frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2}}{\frac{1}{r_1} + \frac{1}{r_2}}}{\frac{1}{\frac{1}{r_1} + \frac{1}{r_2}} + R}$$

for equivalent cell,

$$I = \frac{\varepsilon_{net}}{r_{net} + R}$$

Comparing,

$$r_{net} = \frac{1}{\frac{1}{r_1} + \frac{1}{r_2}}$$

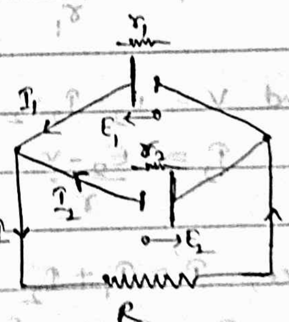
$$\frac{1}{r_{net}} = \frac{1}{r_1} + \frac{1}{r_2}$$

$$\Rightarrow r_{net} = \frac{r_1 r_2}{r_1 + r_2}$$

$$\text{\&} \quad \varepsilon_{net} = \frac{\frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2}}{\frac{1}{r_1} + \frac{1}{r_2}}$$

$$\varepsilon_{net} = \frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{r_1 + r_2}$$

Note:



$(\varepsilon_1 > \varepsilon_2)$

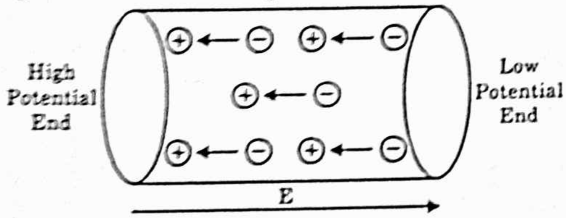
$$r_{net} = \frac{r_1 r_2}{r_1 + r_2}$$

$$\varepsilon_{net} = \frac{\varepsilon_1 r_2 - \varepsilon_2 r_1}{r_1 + r_2}$$

Heating Effect of Current

When a potential difference is maintained across the ends of a conductor then an electric field comes into action inside the conductor from its higher potential and to its lower potential end. Under the effect of electric field the free electrons present inside the conductor experiences electric force in the direction opposite to the electric field and they begin to accelerate in the direction opposite to the electric field. Thus, the free electrons acquire speed and hence K.E. due to the work done by the electric force. During their motion the collision of free electrons takes place with the +ve ionic loses present inside the conductor and transformation of their K.E. takes place into heat energy.

The phenomenon of generation of heat energy is the conductor when electric current is allowed to from through it is called heating effect of current.



Expression of Heat Generated

Consider a conductor of resistance R in which flow of electric current I takes place when a potential difference V is applied across is it.

Then, the charge which flow through any cross-section of conductor in time t is given by

$$q = It$$

By def. the work done in moving a unit positive charge from one end of the conductor to the other end is equal to the potential difference across the conductor.

Therefore, the work done is moving q amount of charge from one end of the conductor to the other end is given by

$$W = qV$$

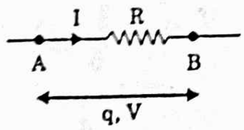
or $$W = ItV \qquad [\because q = It]$$

or $$W = VIt$$

$\because$ By Ohm's law $$V = IR$$

$\therefore$ $$W = (IR)It$$

or $$W = I^2Rt$$



Also $$I = \frac{V}{R}$$

$\therefore$ $$W = \left(\frac{V}{R}\right)^2 Rt$$

or $$W = \frac{V^2}{R}t$$

As the amount of heat generated is directly proportional to the work done by the electric field (or by the source of e.m.f.)

i.e. $$W \propto H$$

or $$W = JH$$

where J is Joule's mechanical equivalent of heat and its value depends upon the units of W & H.

$\therefore$ $$H = \frac{W}{J}$$

or $$H = \frac{VIt}{J} = \frac{I^2Rt}{J} = \frac{V^2}{R}\frac{t}{J}$$

If W is Joule and H is in calorie then

$$J = 4.186\frac{J}{cal} = 4.2\frac{J}{cal}$$

$\therefore$ $$H = \frac{VIt}{4.2} = \frac{I^2Rt}{4.2} = \frac{V^2t}{4.2R}$$

If W is in *erg* and H is in caloric then

$$J = 4.186\times10^7\frac{erg}{cal} = 4.2\times10^7\frac{erg}{cal}$$

$\therefore$ $$H = \frac{VIt}{4.2\times10^7} = \frac{I^2Rt}{4.2\times10^7} = \frac{V^2t}{4.2\times10^7}$$

and if W and H both are in the same units then

$$J = I$$

$\therefore$ $$H = VIt = I^2Rt = \frac{V^2}{R}t$$

Note : Appliances like electric bulb, electric iron, electric heater etc., works on the principle of heating effect of current.

Joule's Law of Heating Effect of Current

In the year 1841 on the basis of his experiments related to heating effect of current, Joule enunciated three laws which are known as Joule's law of heating.

(1) First Law or Law of Currents : When different amounts of electric current are allowed to flow through the same resistor for a given time interval then the amount of heat generated is directly proportional to the square of electric current.

i.e. $$H \propto I^2 \qquad [R \text{ and } t \text{ constant}]$$

(2) Second Law or Law of Resistance : When the same amount of electric current is allowed to flow through different resistances for a given time interval than the amount of heat generated is directly proportional to the resistance of resistor.

i.e. $$H \propto R \qquad [I \text{ and } t \text{ constant}]$$

(3) Third Law or Law of Time : When the same amount of electric current is allowed to flow through the same resistor for different time intervals than the amount of heat generated is directly proportional to the time interval.

i.e. $$H \propto t \qquad [I \text{ and } R \text{ constant}]$$

Combining all the above three laws

$$H = I^2Rt$$

or $$H = KI^2Rt$$

where $K = \frac{1}{J}$ is a constant of proportionality

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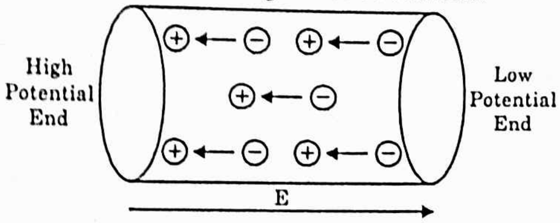

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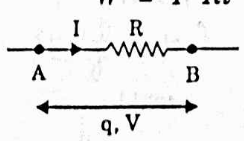

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$$J = I$$

$\therefore$ $H = VIt = I^2Rt = \dfrac{V^2}{R}t$

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Combining all the above three laws

$$H = I^2Rt$$

or $H = KI^2Rt$

where $K = \dfrac{1}{J}$ is a constant of proportionality

$$\therefore \quad H = \frac{I^2Rt}{J}$$

where J is Joule's mechanical equivalent of heat and its value depends upon the units in which H, I, R and t are taken.

If H is in calorie, I is in ampere, R is in ohm and t is in second.

Then, $J = 4.2\frac{J}{cal}$

$$\therefore \quad H = \frac{I^2Rt}{4.2}$$

Electric Power

The rate of which work is done by the source of e.m.f. in order to maintain the flow of electric current through an electric circuit is called electric power of that source.

In other words, the rate at which the electrical energy is supplied by the source of e.m.f. in order to maintain the flow of electric current in an electric circuit is called electric power of that source.

If the work done by the source of e.m.f. in order to maintain the flow of current in an electric circuit for time t be W. Then, the electric power of source of e.m.f. is given by

$$P = \frac{W}{t}$$

$$\therefore \quad W = VIt = I^2Rt = \frac{V^2}{R}t$$

or $$P = VI = I^2R = \frac{V^2}{R}$$

It is scalar quantity. Its SI unit is watt, CGS unit is $erg\,s^{-1}$ and dimensional formula is $[ML^2T^{-3}]$.

Note : Electric power is also defined for electric circuit in the following manner :

The rate at which work is done in an electric circuit.

Or

The rate at which electrical energy is supplied to the electric circuit.

Or

The rate at which dissipation of heat takes in an electric circuit.

Definition of SI Unit of Power

In relation, $P = VI$

If $V = 1V$ and $I = 1A$

Then $P = 1V \cdot 1A = 1W$

Thus, if on applying a potential difference of $1V$ across an electric circuit the electric current flowing through it be $1A$ then its electric power is said to the $1W$.

Note : Other units of electric power

$1kW = 10^3W, 1MW = 10^6W$

and $1HP = 746W$

Definition of kWhr and its Relation with Joule

From relation $P = \frac{W}{t}$, we have

$$W = Pt$$

If $P = 1kW$ and $t = 1hr$

then $W = 1kW \times 1h$

$= 1kWh$

Thus, $1kWh$ is the electrical energy consumed in $1hr$ in a electric circuit whose power is $1kW$.

It is the commercial unit of electrical energy and it is also called unit.

Relation Between kWh and Joule

$$1kWh = 1kW \times 1h = 10^3W \times 3600s = 3.6 \times 10^6 J$$

Calculation of No. of Units (kWh)

If an applying a potential difference V across a conductor the electric current flowing through it be I then the electrical energy consumed in it in time t is given by

$$W = VIt$$

It V is in volt, I is in ampere and t is in hour, then

$W = V(volt) \times I(amp) \times t(hr)$

or $W = VI\,(volt\text{-}amp) \times t\,(hr)$

or $W = P\,(watt) \times t\,(hr)$

or $W = \frac{P(watt) \times t(hr)}{1000}$

(in kWh)

$\therefore$ No. of units $= \frac{P(watt) \times t(hr)}{1000}$

* Rating of electric applicance : The value specified on an electric appliance is called its rating or specified value.

e.g.: 100W - 220 V
(P_0) (V_0)
Rating (specified) Power — Rating (specified) voltage

(a) when applied voltage, $V = V_0$ then power $P = P_0$

(b) when applied voltage, $V > V_0$, then the appliance get permanently damaged.

(c) when, $V < V_0$, then, $P = \frac{V^2}{R}$ (where $R = \frac{V_0^2}{P_0}$).

Note: whenever the reading of an electric appliance is given, the first of all its resistance should be calculated by using $P_0 = \frac{V_0^2}{R}$.