Electromagnetic Induction

Ques.- What is magnetic flux? Give its units and dimensions.

Ans.- The number of magnetic field lines which pass normally (perpendicularly) through a surface when placed in magnetic field is called magnetic flux linked with that surface. It is denoted by ϕ_B.

If a surface of area A be placed in uniform magnetic field B in such a manner that its area vector $\vec{A}$ makes an angle θ with the direction of magnetic field $\vec{B}$. Then the magnetic flux linked with it given by

$$\phi_B = BA\cos\theta$$

or $\phi_B = \vec{B}.\vec{A}$

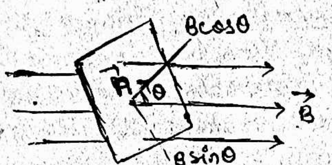
Units- S.I. unit of ϕ_B = Tm^2 = weber (Wb)

C.G.S. emu of ϕ_B = Gcm^2 = maxwell (Mx)

Dimensions- The dimensional formula of ϕ_B

$= [MT^{-2}A^{-1}][L^2]$

$= [ML^2T^{-2}A^{-1}]$

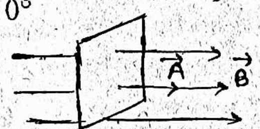


Particular cases- (i) When the surface is perpendicular to the magnetic field i.e. θ = 0°

then ϕ_B = BA cos0°

or ϕ_B = BA (1)

or ϕ_B = BA (max)

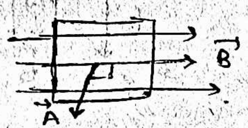


(ii) When the surface is parallel to the magnetic field i.e. θ = 90°

then ϕ_B = BA cos90°

or ϕ_B = BA (0)

or ϕ_B = 0 (minimum)



Note-(i) Relation between S.I. and C.G.S.emu-

1 Wb = 1 Tm^2

= $10^4G \times 10^4cm^2$

= 10^8 Gcm^2

= 10^8 Mx

∴ 1 Wb = 10^8 Mx

(ii) If the magnetic field be non-uniform and the surface is open then the magnetic flux linked with an area element

$$d\phi_B = \vec{B}.d\vec{A}$$

Magnetic flux linked with the whole surface

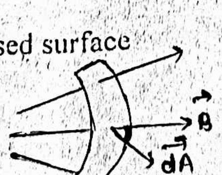
$$\phi_B = \int_S \vec{B}.d\vec{A}$$

(iii) If the magnetic field be non-uniform and the surface is closed then the magnetic flux linked with an area element

$$d\phi_B = \vec{B}.d\vec{A}$$

Magnetic flux linked with the whole closed surface

$$\phi_B = \oint_S \vec{B}.d\vec{A}$$

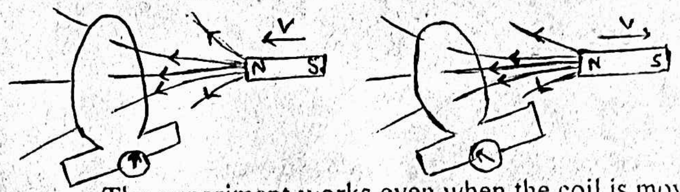


Experiments of Faraday and Henry

Ques.-Describe the various experiments performed by Faraday and Henry which led to the discovery of the phenomenon of electromagnetic inductiion

Ans.-Experiment I- When a bar magnet is pushed towards a coil, the galvanometer connected to it gives deflection. This shows that current has been set up in the coil. The deflection stops when the magnet is made stationary and reverses while the magnet is pushed away from the coil.

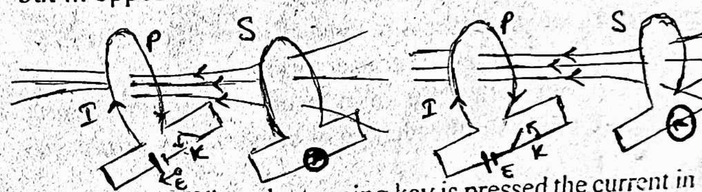
If one uses the south pole of the magnet instead of north, the results are same except that the galvanometer **deflections** are reversed.



The experiment works even when the coil is moved towards and away from the magnet.

Explaination- As the magnetic field lines are crowded near the poles of a magnet therefore when a magnet is pushed towards a coil the magnetic flux linked with the coil increases and when the magnet is pulled away from the coil the magnetic flux linked with the coil decreases. Due to this induced current is produced in the coil and the galvanometer gives deflection. When the magnet is made stationary, the magnetic flux linked with the coil remains same and hence galvanometer does not give deflection.

Experiment II- Consider two coils a battery and a tapping key k are connected to the first coil and a galvanometer G is connected to the second coil. When the tapping key k in the first coil pressed, the galvanometer in the second coil gives deflection for small time and when the tapping key is released the galvanometer again gives deflection for small time but in opposite direction.



Explanation- When the tapping key is pressed the current in the coil P increases due to which the magnetic field and hence the magnetic flux linked with the coil S also increases. This causes induction of current in coil S. When the tapping key is released the current in the coil P decreases due to which the magnetic field and hence the magnetic flux linked with the coil S decreases. This again causes induction of current in S. But when the tapping key is kept pressed the current in the coil P becomes steady after same time due to which the magnetic field and hence the magnetic flux linked with the coil S becomes constant and no current is induced in coil S.

Faraday's law of electromagnetic induction

Ques.- State Faraday's law of electromagnetic induction. Express these laws mathematically.

Ans.- On the basis of results of his experiments Faraday enunciated two laws which are known as Faraday's law of electromagnetic induction. These laws are as follows-

I law- Whenever the magnetic flux linked with a circuit changes an emf is induced in the circuit and the induced emf lasts so long as the change in magnetic flux continues.

II law- The magnitude of induced emf in a circuit is directly proportional to the time rate of change of magnetic flux linked with the circuit.

If the change in magnetic flux linked with a coil in time interval Δt be Δϕ. Then, the rate of change of magnenetic flux

$$= \frac{\Delta\phi}{\Delta t}$$

According to the faraday's second law the emf induced in the coil

$$e \propto \frac{\Delta\phi}{\Delta t}$$

or $$e = \frac{-k\Delta\phi}{\Delta t}$$

where k is a constant of proportionality and negative shows that the induced emf always opposes the change in magnetic flux.

In SI and CGS electromagnetic system k = 1

$$\therefore \quad e = \frac{-\Delta\phi}{\Delta t} \quad \text{-------(i)}$$

If $d\phi$ be the small change in magnetic flux in small time dt then the above equation can be written as

$$e = \frac{-d\phi}{dt} \quad \text{-------(ii)}$$

If there are N turns in the coil and the magnetic flux linked with each turn of the coil be ϕ then the total magnetic flux linked with the coil will be $N\phi$.

$\therefore$ The emf induced in the coil $e = \frac{-d}{dt} N\phi$

or $$e = -N\frac{d\phi}{dt}$$

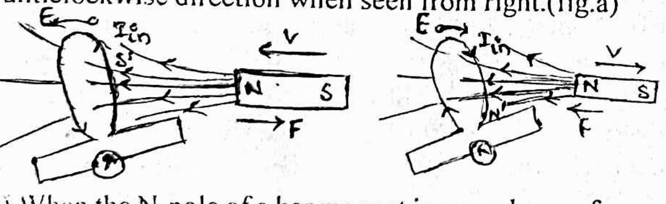
Lenz's law

Ques.- State and illustrate Lenz's law.

Ans.- Lenz's law gives the direction of induced current in a circuit. It is based on law of conservation of energy.

It states that the direction of induced current in a circuit is such that it opposes the change or cause which produces it.

Illustration-(i) When the N-pole of a bar magnet is moved towards a coil then the current induced in the coil should oppose the motion of the bar magnet. This will happen if the face of the coil towards the approaching magnet acquires N-polarity. Therefore the induced current in the coil will flow in anticlockwise direction when seen from right.(fig.a)

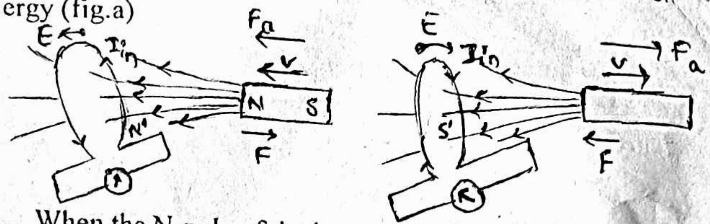


(ii) When the N-pole of a bar magnet is moved away from a coil then the current induced in the coil should oppose the motion of the bar magnet. This will happen if the face of the coil towards the receding magnet acquires S-polarity. Therefore the induced current in the coil will flow in clockwise direction when seen from right.(fig.b)

Lenz's law and law of conservation of energy

Ques.- Prove that Lenz's law is in accordance with law of conservation of energy.

Ans.- When the N-pole of a bar magnet is moved towards a coil, the direction of current induced in the coil is such that the face of the coil towards the approaching magnet acquires N-polarity. Due to which there is force of repulsion between the two N-poles and in order to move the magnet against the repulsive force some mechanical energy has to be spent. This mechanical energy gets converted into electrical energy which is in accordance with law of conservation of energy (fig.a)

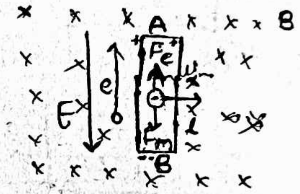


When the N-pole of the bar magnet is moved away from the coil the direction of induced current is such that the face of the coil towards the receding magnet acquires S-polarity. Due to which there is force of attraction between the south pole of the coil and the N-pole of the magnet and in order to move the magnet against the attractive force some mechanical energy has to be spent. This mechanical energy gets converted into electrical energy which is in accordance with law of conservation of energy (fig.b).

Expression for motional EMF

Ques.- What is motional emf ? Deduce an expression for motional emf by considering the lorentz magnetic force acting on free charge carriers in a conductor moving in perpendicular direction to the magnetic feild.

Ans.- Consider a rod of length ℓ which is moved with constant velocity $\vec{v}$ in the perpendicular direction of uniform magnetic field $\vec{B}$ as shown in fig.



When the rod is moved then the free electrons present inside it experience a lorentz magnetic force in the downward direction (in the plane of paper). The lorentz magnetic force causes electrons to move from top to bottom. Due to which the end B of rod becomes negatively charged and the end A of rod becomes positively charged and an electric field comes into action inside the rod from end A to B. The electric field so produced exerts electric force on electrons from B to A which opposes lorentz magnetic force. As more and more electrons accumulate at end B, the force on electrons increases due to increase in electric field finally a stage comes when the electric force acting on electrons becomes equal and opposite to the lorentz magnetic force. If E be the electric field intensity at that stage then electric force on electrons

$$F_e = qE$$

and lorentz magnetic force acting on electrons

$$F_m = qvB\sin\theta$$

or $F_m = qvB\sin 90^\circ$

or $F_m = qvB\,(1)$

or $F_m = qvB$

In the state of equilibrium

$$F_e = F_m$$

or $qE = qvB$

or $E = vB$ ----(i)

If e be the potential difference (induced emf) between the two ends of the rod then $E = \frac{e}{\ell}$ ----(ii)

From relation (i) and (ii)

$$\frac{e}{\ell} = vB$$

or $\boxed{e = Bv\ell}$

Thus is the required expression of motional emf.

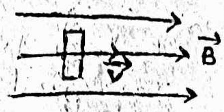
Note- (1) When the velocity of the rod is in the direction of magnetic field i.e. $\theta = 0°$

Lorentz magnetic force experienced by free electrons in the rod

$F_m = qvB \sin 0°$

or $F_m = qvB\ (0)$

or $F_m = 0$



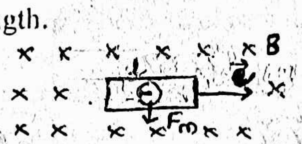
Therefore no emf will induce in the rod in this situation

(2) When the rod is moved in such a manner that its velocity is in the direction of its length.

$F_m = qvB \sin 90°$

or $F_m = qvB\ (1)$

or $F_m = qvB$



In this case the lorentz magnetic force in the perpendicular direction of rod. Therefore no emf will induce in the rod in this situation.

Conclusion- (i) When a rod moves in uniform magnetic field in such a manner that B. v and ℓ are mutually perpendicular each other than the induction of emf takes place in the rod which is given by $e = Bv\ell$

In this case it is said that the rod cut magnetic field lines.

(ii) When a rod moves in uniform magnetic field in such a manner that any two or all of three B.v and ℓ are parallel or antiparallel to one another then the induction of emf does not take place in the rod.

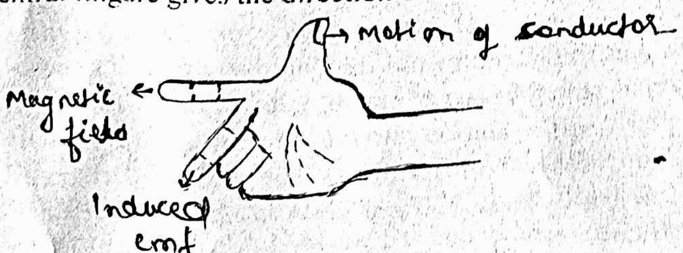
In this case it is said that the rod does not cut magnetic field lines.

Fleming's right hand rule (FRHR)

Ques.- State Fleming's right hand rule to find out the direction of induced current in motional emf.

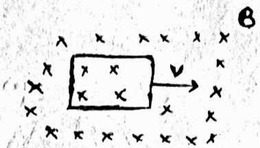
Ans.- When a conductor (rod) is moved in a uniform magnetic field then the direction of induced current inside the conductor can be find out by using FRHR.

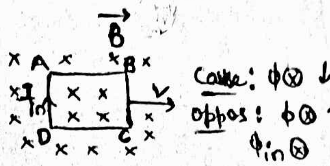
According to this rule when the central fingure, the fore fingure and the thumb of right hand are streched mutually perpendicular to one another in such a manner that the fore fingure is in the direction of magnetic field and the thumb is in the direction of motion of conductor then the central fingure gives the direction of induced current.



Since the direction of induced current is same as the direction of induced emf therefore by using FRHR the direction of induced emf can also be find out.

Note of Faraday's experiment- (i) When a coil is moved in magnetic field the magnetic flux linked with the area of the coil does not change. Due to this no emf is induced and no indued current flows in the coil and the galvanometer does not give any deflection.





But when the coil is moved out of the magnetic field the area of coil inside the magnetic field continuously decreases and the magnetic flux linked with the coil also decreases. In accordance with faraday's law of electromagnetic induction an emf will induce in the coil. The induced emf will cause the current to flow in such a direction that the decrease in flux be opposed. This will happen when the direction of induced current is clockwise which generated induced magnetic field and hence the induced magnetic flux in the downward direction.

(ii) If the resistance of the circuit be R then the induced current in the circuit will be given by

$$I_m = \frac{\varepsilon_{in}}{R}$$

But $\varepsilon_{in} = \frac{-d\phi}{dt}$

$\therefore$ $I_{in} = \frac{-d\phi}{Rdt}$

If the time interval dt the amount of induced charge which flows through the circuit be dq_{in}.

Then $I_{in} = \frac{dq_{in}}{dt}$

or $dq_{in} = I_{in}dt$

or $dq_{in} = \frac{-d\phi}{Rdt} \times dt$

or $dq_{in} = \frac{-d\phi}{R}$

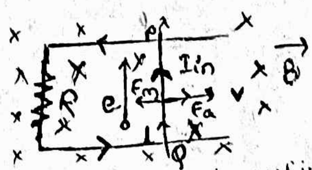
Thus, in the phenomenon of electromagnetic induction both the induced emf and the induced current depends upon the time interval (duration) in which the change in magnetic flux takes place but the induced charge is independent of the time interval in which the change in magnetic flux takes place.

Energy consideration in motional emf

Ques.- Deduce exressions for induced current, force required to pull the conductor, power delivered by the external source and power dissipated as joule loss in motional emf.

Ans.- Consider a rectangle or conductor having resistance R whose one arm PQ is free to move. It is placed in uniform magnetic field which is perpendicular to the plane of conductor when the arm PQ is moved toward right with uniform velocity v. Then a motional emf e will get induced in the arm PQ which is given by

$\boxed{e = Bv\ell}$



where ℓ is the length of the arm. Due to the emf induced in the arm PQ the induced current flowing the circuit.

$$I_{in} = \frac{e}{R} \quad \text{or} \quad \boxed{I_{in} = \frac{Bv\ell}{R}}$$

NOTE: Similarly a disk rotating in UMF can be assumed to be made of large no. of rods connected in parallel b/w its centre & peripheral (boundary)

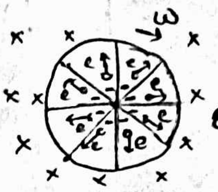
Because of the presence of magnetic field there will be lorentz magnetic force on arm PQ which will act in the opposite direction of its velocity in accordance with FLHR

$F_m = I_m \ell B \sin\theta$

or $F_m = \frac{Bv\ell}{R} \ell B \sin 90°$

or $F_m = \frac{B^2 v \ell^2}{R}$ (1)

or $\boxed{F_m = \frac{B^2 v \ell^2}{R}}$



In order to move the arm with constant velocity a force has to be applied on the arm PQ which should be equal in magnitude and opposite in direction to the lorentz magnetic force.

i.e. applied force $F_a = F_m$

or $F_a = \frac{B^2 v \ell^2}{R}$

The power required to move the arm with constant velocity

$P = F_a v$

or $P = \frac{B^2 v \ell^2}{R} . v$

or $\boxed{P = \frac{B^2 v^2 \ell^2}{R}}$

This mechanical power supplied by the external agency will get dissipated in the form of Joulean power (heat) which is also given by

$\boxed{P_J = \frac{B^2 v^2 \ell^2}{R}}$ [Joulean power- Thermal power]

EMF induced in a rotating rod [Derive the formula if given in exam].

Ques.- Derive the relation for EMF induced in a rotating rod in uniform magnetic feild.

Ans.- Consider a rod of length ℓ which is rotating in uniform magnetic field B with constant angular velocity ω such that its one end is at the centre and the other end is at the circumference of the circle.

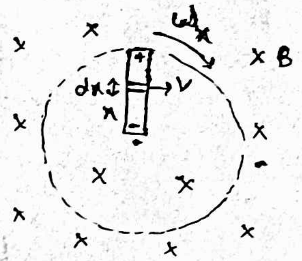
Consider a small length element dx of the rod at a distance x from the centre O of the circle. The linear velocity of the length element $v = \omega x$

EMF induced in the length element $de = Bvdx$

or $de = B\omega x \, dx$

By integrating the above relation with in proper limits the emf induced in the whole rod can be calculated as follows-

$$\int_0^e de = \int_0^{\ell} B\omega x \, dx$$



or $$\left[e\right]_0^e = B\omega \int_0^{\ell} x \, dx$$

or $$e - 0 = B\omega \left[\frac{x^2}{2}\right]_0^{\ell}$$

or $$e = B\omega \left[\frac{\ell^2}{2} - \frac{0^2}{2}\right]$$

or $$\boxed{e = \frac{1}{2} B\omega \ell^2}$$

This is required expression of emf induced in the rod.

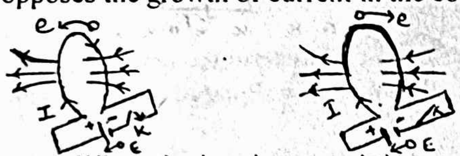
Note- When a bicycle's wheel is rotated in uniform magnetic field about its axle In this case its spokes can be assumed to be connected in parallel between its axle and rim Therefore the potential difference between its axle and rim will be equal is the emf induced across one of its spoke.

$\boxed{e = \frac{1}{2} B\omega R^2}$ [Here $\ell = R$]

Self induction

Ques.- Define self induction. Describe an experiment to demonstrate the phenomenon of self induction. Also write its physical significance.

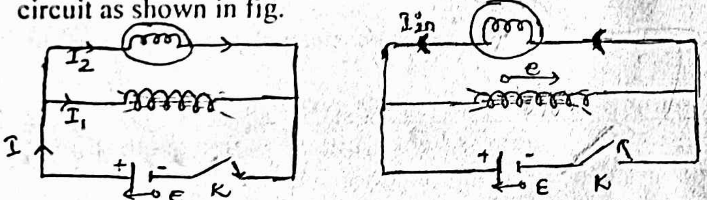
Ans.- Consider a coil which is connected to a battary through key k. When the key is closed the current in the coil increases from zero to maximum and the magnetic flux linked with it also increases from zero to max. In accordance with faraday's laws of EMI an emf is induced in the coil which opposes the growth of current in the coil.



When the key is opened the current in the coil decreases from max to zero and the magnetic flux linked with it also decreases from maximum to zero. In accordance with faraday's law of EMI an EMF is induced in the coil which opposes the decay of current in the coil.

The phenomenon of induction of an opposing emf in a coil due to the change in magnetic flux linked with the coil is called self induction.

Experimental demonstration of self induction- Consider the circuit as shown in fig.



When the key k is closed the illumination of bulb B takes place. But when the key is opened the decay of electric current and hence the decrease in magnetic flux takes place in the solenoid. Due to which is accordance with faraday's law of EMI, induction of an emf takes place in the solenoid with opposes the decay of current. This induced emf keeps the current flowing through the bulb for some time even after the key is opened. As a result of which the bulb glows some time even after the key is opened.

Physical significance of self induction- Just as the mass measures the inertia of transtational motion i.e. it opposes any change in the state of rest or uniform motion of a body. Similarly self induction measures the inertia of electricity i.e. it opposes both the growth and the decay of electric current in a coil. Thus, self induction is called the inertia of electricity.

Cofficient of self induction

Ques.- Define cofficient of self induction or self inductance. Give its units and dimensions.

Ans.- It is found that the magnetic flux linked with a coil at any instant is directly proportional to the current flowing through the coil at that instant.

i.e. $\phi_{tot} \propto I$

$\phi_{tot} = LI$

when L is a constant proportionlity and is called cofficient of self induction or self inductance. Its value depends upon the no. of turns in the coil, the dimensions of the coil and the nature of material present in its core.

When the electric current flowing in a coil changes the magnetic flux linked with the coil also changes and an emf is induced in the coil in accordance with Faraday's laws of EMI.

If the rate of change of magnetic flux and linked with a coil at any instant be $\frac{d\phi_{tot}}{dt}$ then the emf induced in the coil at that instant is given by

$$\varepsilon_{in} = -\frac{d\phi_{tot}}{dt}$$

or $$\varepsilon_{in} = -\frac{d}{dt}LI$$

or $$\boxed{\varepsilon_{in} = -\frac{LdI}{dt}}$$

Definition of cofficient of self induction- From relation

$$\varepsilon_{in} = -\frac{LdI}{dt}$$, we have

or $$L = \frac{\varepsilon_{in}}{dI/dt} = \frac{\varepsilon_{in}}{dI/dt} \text{ (nemerically)}$$

If $\frac{dI}{dt} = 1$ then $L = \varepsilon_{in}$

Thus, the cofficient of self induction of a coil is numerically equal to the emf induced in the coil when the rate of change of electric current in the coil is unity.

Units and dimensions of cofficient of self induction- From relation $\varepsilon_{in} = -\frac{LdI}{dt}$, we have

or $$L = \frac{\varepsilon_{in}}{dI/dt} \text{ (nemerically)}$$

$$\text{S.I. unit of } L = \frac{V}{A/s} = \frac{Vs}{A} = \text{henry (H)}$$

$$\text{C.G.S. emu of } L = \frac{abV}{abA/s} = \frac{abVs}{abA} = \text{abhenry (abH)}$$

$$\text{Dimensions of } L = \frac{[ML^2T^{-3}A^{-1}]}{[AT^{-1}]} = [ML^2T^{-2}A^{-2}]$$

Note- Definition of S.I unit of cofficient of self induction-

From relation $\varepsilon_{in} = -\frac{LdI}{dt}$, we have

or $$L = \frac{\varepsilon_{in}}{dI/dt} \text{ (numerically)}$$

If $\varepsilon_{in} = 1V$ & $\frac{dI}{dt} = \frac{1A}{s}$

Then $$L = \frac{1V}{1A/s} = \frac{1Vs}{A} = 1H$$

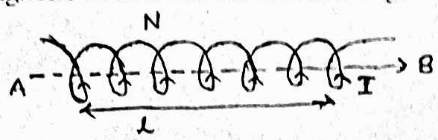
Thus, the cofficient of self induction of a coil is said to be 1 henry if on changing the electric current flowing in the coil, at a rate of 1 ampere per second the emf induced in it be 1 volt.

Self inductance of a long solenoid

Ques.- Derive relation for the self inductance of a long solenoid. State the factors on which the self inductance of a long solenoid depends.

Ans.- Consider a solenoid of length ℓ and area of cross-section A having N number of turns. The number of turns The no. of turns in a unit length of solenoid $n = N/\ell$, when I amount of electric current is allowed to flow through the solenoid then the magnetic field inside the solenoid is given by

$$B = \mu_0 nI$$



or $$B = \frac{\mu_0 NI}{\ell}$$

Magnetic flux linked with a single turn of solenoid

$$\phi = BA$$

or $$\phi = \frac{\mu_0 NIA}{\ell}$$

Total magnetic flux linked with the N turns of the solenoid

$$\phi_{tot} = N\phi$$

or $$\phi_{tot} = \frac{N\mu_0 NIA}{\ell}$$

or $$\phi_{tot} = \frac{\mu_0 N^2 AI}{\ell} \quad \text{-----(i)}$$

Also, the total magnetic flux linked with the whole solenoid is given by

$$\phi_{tot} = LI \quad \text{-----(ii)}$$

where L is the self inductance of the coil.

From relation (i) and (ii) we get,

$$LI = \frac{\mu_0 N^2 AI}{\ell}$$

or $$L = \frac{\mu_0 N^2 A}{\ell}$$

If a substance of relative magnetic permeability μ_r be present in the core of solenoid then (ii)

$$\boxed{L = \mu_r \frac{\mu_0 N^2 A}{\ell}}$$

This is the required expression of self inductance of solenoid.

Note- Multiplying both numerator and denominator of RHS of above relation by ℓ we get

$$L = \frac{\mu_r \mu_0 N^2 A\ell}{\ell.\ell}$$

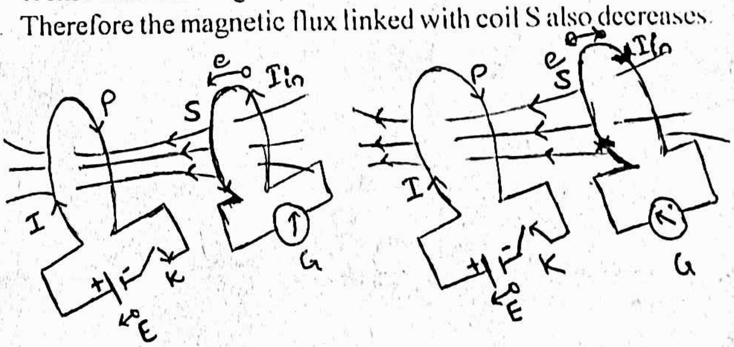
or $$\boxed{L = \mu_r \mu_0 n^2 A\ell}$$

Mutual induction

Ques.- Define mutual induction.

Ans.-.- Consider two coils P and S which are placed close to earth other. A battery and a key are connected to the P coil which is called primary coil and a galvanometer is connected to the S coil which is called secondary coil. When key in P coil is closed, the current in the P coil increases and the magnetic flux linked with it also increase. As the coil S is placed close to coil P therefore the magnetic flux linked with S coil also increases. In accordance with faraday's law of EMI and EMF is induced is the coil S which opposes the grow the of current in coil P.

When the key in coil P is opened the current in coil P decrease and the magnetic flux linked with it also decreases. Therefore the magnetic flux linked with coil S also decreases.



In accordance ~~as~~ with faraday's law of EMI, and emf is induced in the coil S which opposes the decay of current in coil P.

The phenomenon of induction of an opposing emf in a coil due to the change in magnetic flux linked with a neighbouring coil is called mutual induction.

Cofficient of mutual induction

Ques.- Define coefficient of mutual induction or mutual inductance. Give its units and dimensions.

Ans.- Cofficient of mutual induction- It is found that the magnetic flux ϕ_S linked with the secondary coil at any instant is directly proportional to the current following through the primary coil at that instant.

i.e. $\phi_{S_{tot}} \propto I_P$

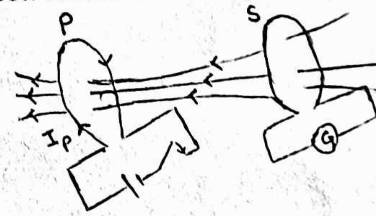
or $\phi_{S_{tot}} = MI_P$

where M is a constant of proportionality and is called cofficient of mutual induction or mutual inductance. It value depends upon the no. of turns in the two coils, the dimensions of two coils and the nature of ~~material~~ material/medium present in their core.

When the electric current flowing in the primary coil changes the magnetic flux linked with the secondary coil changes and an ~~emf~~ emf is induced in the secondary coil in accordance with faraday's laws of EMI.

If the rate of change of magentic flux linked with the primary coil at any instant be $\frac{d\phi_P}{dt}$ then the emf induced in the secondary coil at that instant is given by

$$\varepsilon_S = -\frac{d\phi_{S_{tot}}}{dt}$$



or $$\varepsilon_S = -\frac{d\,MI_P}{dt}$$

or $$\boxed{\varepsilon_S = -\frac{MdI_P}{dt}}$$

Definition of cofficient of mutual induction- From relation

$$\varepsilon_S = -\frac{MdI_P}{dt} \quad \text{we have}$$

$$M = \frac{\varepsilon_S}{dI_P/dt} \quad \text{(nemerically)}$$

If $\frac{dI_P}{dt} = 1$ then $M = \varepsilon_S$

Thus, the cofficient of mutual induction of two coils is numerically equal to the emf induced in the secondary coil when the rate of change of electric current in the primary coil is unity.

Units and dimensions of cofficient of mutual induction-

From relation $\varepsilon_S = -\frac{MdI_P}{dt}$ We have

$$M = \frac{\varepsilon_S}{dI_P/dt} \quad \text{(numerically)}$$

S.I. unit of $M = \frac{V}{A/s} = \frac{Vs}{A}$ = henry (H)

C.G.S. emu of $M = \frac{abV}{abA/s} = \frac{abVs}{abA}$ = ab henry (ab H)

Dimensions of $M = \frac{[ML^2T^{-3}A^{-1}]}{[AT^{-1}]} = [ML^2T^{-2}A^{-2}]$

Note-Definition of S.I unit of cofficient of mutual induction- From relation

$$\varepsilon_S = -\frac{MdI_P}{dt} \quad \text{We have}$$

$$M = \frac{\varepsilon_S}{dI_P/dt} \quad \text{(numerically)}$$

If $\varepsilon_S = 1V$ & $\frac{dI_P}{dt} = \frac{1A}{s}$

then $M = \frac{1V}{1A/s} = \frac{1Vs}{A} = 1H$

Thus, the cofficient of mutual induction of turn coils is said to be 1 henery if on changing the electric current flowing in the primary coil at a rate of 1 ampere per second the emf induced in the secondary coil be 1 volt.

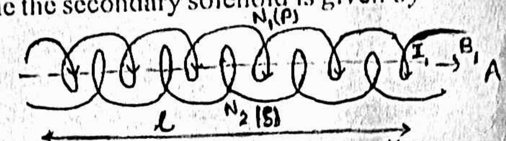
Mutual inductance of two long solenoids [derive the formula if given in paper]

Ques.- Derive relation for the mutual inductance of two long solenoids. State the factors on which the mutual inductance of two long solenoids depends.

Ans.-Consider two long solenoids of length ℓ and area of cross-section A having N_1 and N_2 no. of turns. The no. of turns in a unit length of two solenoids will be $n = \frac{N}{\ell}$. When I amount of electric current is allowed is flow through the primary solenoid then the magnetic field inside the secondary solenoid is given by

$$B_1 = \mu_0 n_1 I$$

or $$B_1 = \frac{\mu_0 N_1 I}{\ell}$$



Magnetic flux linked with a single turn of secondary coil

$$\phi_2 = B_1 A$$

or $$\phi_2 = \frac{\mu_0 N_1 IA}{\ell}$$

Total magnetic flux linked with the N_2 turns of the secondary solenoid

$$\phi_{2\,tot} = N_2\phi_2$$

or $$\phi_{2\,tot} = \frac{N_2\mu_0 N_1 IA}{\ell}$$

or $$\phi_{2\,tot} = \frac{\mu_0 N_1 N_2 AI}{\ell} \quad \text{----(i)}$$

Also the total magnetic flux linked with the secondary coil is given by

$$\phi_{2\,tot} = MI \quad \text{----(ii)}$$

where M is the mutual induction of two coils

From relation (i) and (ii) we get,

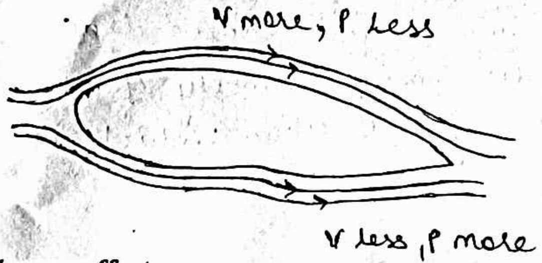
$$M = \frac{\mu_0 N_1 N_2 A}{\ell}$$

If a substance of relative magnetic permealibity μ_r be present in the core of two solenoids then

$$\boxed{M = \frac{\mu_r \mu_0 N_1 N_2 A}{\ell}}$$

This is the required expression of mutual inductance of two long solenoids.

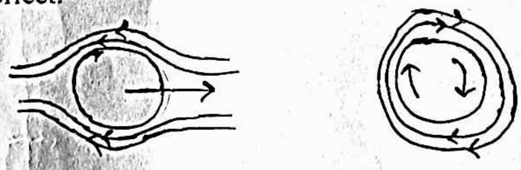
Note-(i) Mutual inductance of two coils and solenoids remains the same wheather the flow of electric current flows in the first solenoid or second solenoid. (so long as the dimensions of two coil or solenoid remain same)

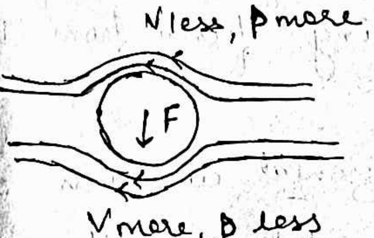


Magnus effect

Ques- What is Magnus effect ?

Ans.- If a ball spins as well as move linearly the stream-lines at the top of ball due to two types of motion opposed each other and those below are in the same direction. As a resuølt of it, the velocity of air flow is greater below than above the ball. According to Bernoulli's theorem the pressure on the upper side of the ball becomes less than the pressure the lower side of the ball. Due to which a resultant force acts on the ball at right angle to the linear motion in the downward direction. This force provides the necessary centripital force required to move the ball along a curved path. This effect is called Magnus effect.





★ Toricelli's theorem:- It states that

Statement: velocity of efflux of liquid from a orifice which is made into a tank is equal to velocity acquired by a body in falling freely from the surface of liquid to the orifice.

Proof: Consider a tank filled with liquid to height H when a fine hole (orifice) is made into tank at a depth h below the surface of liquid then the liquid comes out of orifice with velocity v, which is called velocity of efflux.

Since the tank is broad & hole is fine therefore velocity of free surface of liquid will be approximately equal to zero.

Total energy per unit volume at the free surface of liquid equal to

$= P_A + \frac{1}{2}\rho v_A^2 + \rho g h_A$

$= P_0 + \frac{1}{2}\rho (0)^2 + \rho g H$

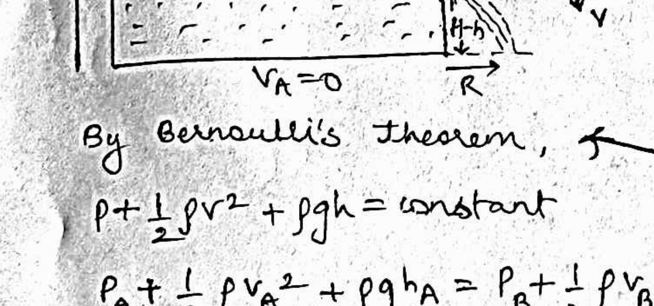
$= P_0 + \rho g H$. $\left[\begin{array}{l} P_0 = \text{atm. pressure} \\ \rho = \text{density} \end{array} \right]$.

· Total energy per unit volume at orifice $= P_B + \frac{1}{2} v_B^2 + \rho g h_B$

$= P_0 + \frac{1}{2}\rho v^2 + \rho g (H-h)$

★ Torricelli's Theorem★

Proof:



By Bernoulli's theorem,

$P + \frac{1}{2}\rho v^2 + \rho g h = \text{constant}$

$P_A + \frac{1}{2}\rho v_A^2 + \rho g h_A = P_B + \frac{1}{2}\rho v_B^2 + \rho g h_B$

$\cancel{P_0} + \frac{1}{2}\rho (0)^2 + \rho g H = \cancel{P_0} + \frac{1}{2}\rho v^2 + \rho g (H-h)$

$0 + \cancel{\rho g H} = \frac{1}{2}\rho v^2 + \cancel{\rho g H} - \rho g h$

$\cancel{\rho}\frac{1}{2}v^2 = \cancel{\rho} g h \Rightarrow v = \sqrt{2gh}$

** Two streamlines can never intersect each other because if they intersect each other then at their point of intersection two tangents can be drawn on streamline which will give us two different directions of velocity of that particle of liquid which lies at the point of intersection which is physically impossible.

NOTE: i) If a body is released from the free surface of liquid then the velocity acquired by it in reaching the orifice can be calculated by using III rd eq. of motion,

$$v^2 - u^2 = 2as$$
$$v^2 - (0)^2 = 2gh$$
$$v^2 = 2gh$$
$$v = \sqrt{2gh}$$

Thus, the velocity of efflux of liquid from the orifice is equal to the velocity acquired by body in falling freely from the surface of liquid to the orifice.

ii) Since, the velocity of efflux is in horizontal direction therefore motion of liquid will be same as horizontal projectile motion.

for horizontal projectile motion,

$$y = \frac{1}{2} g \frac{x^2}{u^2}$$

$$H - h = \frac{1}{2} g \frac{R^2}{v^2}$$

where, R is horizontal Range of liquid.

$$H - h = \frac{1}{2} g \frac{R^2}{(\sqrt{2gh})^2}$$

$$H - h = \frac{1}{2} \cancel{g} \frac{R^2}{\cancel{2}\, 2gh}$$

$$4h(H - h) = R^2$$

$$R = 2\sqrt{h(H-h)}$$