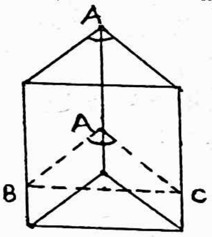
Dispersion of light

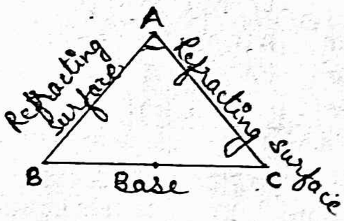
Prism

Ques.- What is a prism? What do you mean by refracting faces, refracting edge, base, angle of prism and principal section of prism?

Ans.- It is a transparent medium bounded by three rectangular and two triangular surfaces. The rectangular surfaces at which refraction takes place are called refracting surfaces, the common edge of two refracting surface is called refracting edge and the angle between the two refracting surfaces is called angle of prism. The rectangular surface which is opposite to the refracting edge is called base of the prism. The triangular cross-section of the prism which is perpendicular to its refracting edge is called principal section of prism.

A prism is always represented by its principal section.





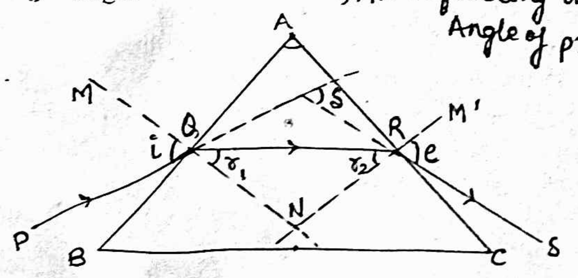
Refraction of light through prism

Ques.- Discuss the phenomenon of refraction through a prism?

Ans.- When a ray of light falls at the first refracting surface of a prism then it bends towards the normal drawn on the first refracting surface at the point of incidence because the refraction taken place from rarer to denser medium. When the refracted ray falls on the second refracting surface then it bends away from the normal drawn on the second refracting surface at the point of incidence because the refraction takes place from denser to rarer medium.

In this manner when a ray of light passes through a prism then it bends towards its base.

PQ = Incident ray, QR = refracted ray
RS = emergent ray, MN and M'N are normal
i or i_1 = angle of incidence at first surface
r_1 = angle of refraction at first surface
r_2 = angle of incidence at second surface
e or i_2 = angle of refraction at second surface
and δ = angle of deviation , A= Refracting angle / Angle of prism



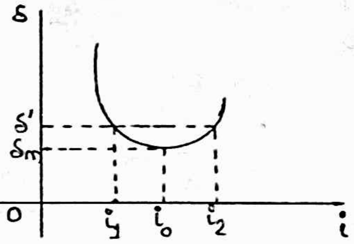
Angle of deviation

Ques.- Discuss the variation of theangle of deviation with that of the angle of incidence for a ray of light passing through a prism.

Ans.- The angle between the incident ray and the emergent ray is called angle of deviation. It measures the deviation suffered by a ray when it passes through a prism.

Experimentally it is found that with increase in the value of angle of the incidence initially the angle of deviation decreases then for a certain value of angle of incidence the angle deviation becomes minimum with the further increase in thevalue of angle of incidence the angle of deviation begin to increase.

From graph it is clear that corresponding to the angle of minimum deviation there is only one value of angle of incidence and for any other angle of deviation then are two angles of incidence. When the angle of deviation is minimum the prism is said to be in the state of minimum deviation.



Note- In the state of minimum deviation-
(i) Angle of incidence = Angle of emergence
i.e. $i = e$
(ii) Angle of refraction at first surface = Angle of incidence at second surface
i.e. $r_1 = r_2$
(iii) If the base angles are equal then the refracted ray is parallel to the base of prism.
(iv) the light ray passes symmetrically through the prism.

Formula for refractive index of prism (Relation between i, e, δ and A)

Ques.- Derive an expression for the refractive index of the material of prism in terms of angle of prism and the angle of minimum deviation.

Ans.- Let ABC be the principal section of a prism whose refractive index be μ. When the ray of light falls on the face AB of the prism then it ret refracted in the form of ray QR and emerges out as ray RS.

In fig.

$\angle MQP = i$ (angle of incidence at face AB)
$\angle RQN = r_1$ (angle of refraction at face AB)
$\angle QRN = r_2$ (angle of incidence at face AC)
and $\angle M'RS = e$ (angle of emergence)

The angle obtained between PQ produced in forward direction and RS produced in backward direction is angle of deviation

i.e. $\angle UTR = \delta$

Vertically opposite angles are equal

$\angle MQP = \angle TQN$
or $i = \angle 1 + r_1$
or $\angle 1 = i - r_1$ ----(i)
and $\angle M'RS = \angle TRN$
or $e = \angle 2 + r_2$ ----(ii)

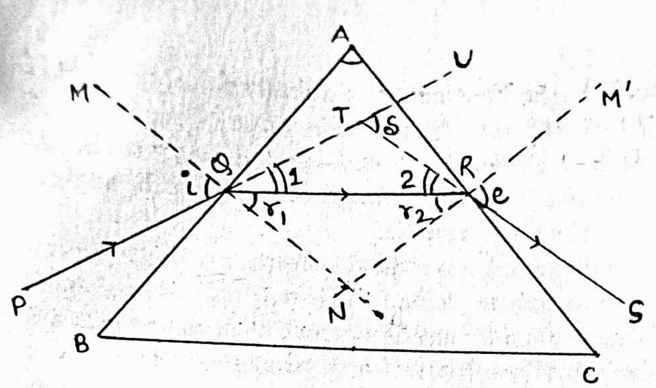
The sum of exterior angles of Δ is equal to sum of interior opposite angles

$\therefore \angle UTR = \angle TQR + \angle TRQ$
or $\delta = \angle 1 + \angle 2$ ----(iii)

Substituting tha value of ∠1 and ∠2 in relation (iii)

$\delta = (i - r_1) + (e - r_1)$
or $\delta = (i + e) - (r_1 + r_2)$ ----(iv)

In quadrilateral AQNR.



$$\angle AQN = \angle ARN = 90^\circ$$

$\therefore \quad A + \angle QNR = 180^\circ$ ----(v)

Also in Δ QNR

$r_1 + \angle QNR + r_2 = 180^\circ$ ----(vi)

From relation (v) and (vi)

$$A + \angle QNR = r_1 + r_2 + \angle QNR$$

or $\quad A = r_1 + r_2$ ------(vii)

From equation (iv)

$$\delta = i + e - (r_1 + r_2)$$

or $\quad \delta = i + e - A$

or $\quad \delta + A = i + e$

or $\quad i + e = \delta + A$ —(viii)

Now in the state of minimum daviation

$$i = e$$

$$r_1 = r_2 = r \text{ (say)}$$

and $\quad \delta = \delta_m$

From relation (vii)

$$A = r + r$$

or $\quad A = 2r$

or $\quad \boxed{r = \dfrac{A}{2}}$

From relation (viii)

$$i + i = \delta_m + A$$

or $\quad 2i = \delta_m + A$

or $\quad \boxed{i = \dfrac{\delta_m + A}{2}}$

Now the refractive index of material of prism

$$\mu = \frac{\sin i}{\sin r}$$

or $\quad \boxed{\mu = \dfrac{\sin\left(\dfrac{\delta_m + A}{2}\right)}{\sin \dfrac{A}{2}}}$

This is the required relation for refrative index of material of prism.

Relation between δ and A for small angled prism

Ques.- Derive an expression for the angle of deviation of a thin prism in terms of refractive index and angle of prism.

Ans.- When the angle of prism is small then all of its angles are also small. For refraction at surface AB

$$_r\mu_d = \frac{\sin i}{\sin r}$$

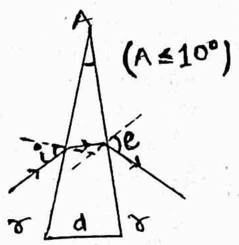
or $\quad _r\mu_d = \dfrac{\sin i}{\sin r_1}$

$\because$ i and r_1 are small

$\Rightarrow \sin i \simeq i$

and $\sin r \simeq r$

$\therefore \quad \mu = \dfrac{i}{r_1}$



or $\quad i = \mu r_1$ —(i)

and for refraction at surface AC, $_d\mu_r = \dfrac{\sin r_2}{\sin e}$

or $\quad \dfrac{1}{_d\mu_r} = \dfrac{\sin e}{\sin r_2}$

or $\quad _r\mu_d = \dfrac{\sin e}{\sin r_2}$ (Principle of reversibility of light)

or $\quad \mu = \dfrac{\sin e}{\sin r_2}$

$\because$ e and r_2 are small

$\Rightarrow \sin e \simeq e$ and $\sin r_2 \simeq r_2$

$\therefore \quad \mu = \dfrac{e}{r_2}$

or $\quad e = \mu r_2$

For any prism $\; i + e = \delta + A$ and $r_1 + r_2 = A$

$$\mu r_1 + \mu r_2 = \delta + A$$

or $\quad \mu(r_1 + r_2) = \delta + A$

or $\quad \mu A = \delta + A$

or $\quad \boxed{\delta = (\mu - 1)A}$

This is the required expression for thin prism.

Dependence of angle of deviation on various factors

Ques.- State the factors on which the angle of deviation depends.

Ans.- The angle of deviation depends on following factors-

(i) Angle of prism- The larger the angle of prism the greater will be the angle of deviation and vice-versa.

(ii) Nature of material of prism- The larger the refractive index of material of prism the greater will be the angle of deviation and vice-versa.

e.g.- As the refractive index of flint glass is greater than the refrective index of crown glass therefore for the same angle of prism the angle of deviation of flint glass prism in greater then the angle of deviation of crown glass prism.

(iii) Colour of light- The refractive index of material of prism is different for light of different colours therefore angles of deviation of prism will also be different for light of different colours.

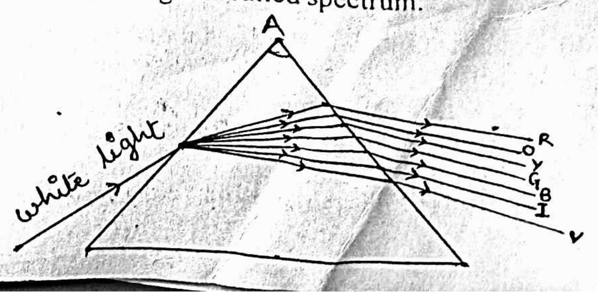
e.g.-In visible light the refractive index of material of prism is maximum for violet light and minimum for red light therefore the angle of deviation of prism will also be maximum for violet light and minimum for red light.

Dispersion of light

Ques.- What is dispersion of light? Explain with the help of ray diagram. Also explain the cause of dispersion.

Ans.- In the year 1666 Newton found that when white light is allowed to pass through a prism then in the emergent light a band of seven colours namely violet, indigo, blue, green, yellow, orange and red are obtained.

This phenomena of splitting of white light into its constituent colours when it is allowed to pass through a prism is called dispersion and the band of seven colours obtained in the emergent light is called spectrum.



Cause of dispersion- In accordance with Cauchy's relation $\mu = A + \frac{B}{\lambda^2}$

the refractive index of material of prism is different for different colours (wavelength) of light. As the white light is made up of seven colours therefore the refractive index of prism will be different for different colours and in accordance with the relation $\delta = (\mu - 1)A$ the deviation suffered by different colours will also be different i.e. is why different colours emerge from different points of the prism and the formation of spectrum takes place. This is the cause of dispersion.

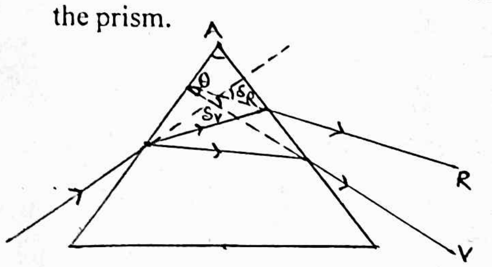
In visible light the refractive index of prism in minimum for red light and maximum for violet light therefore the deviation suffered by the red light is minimum and the deviation suffered by the violet light is maximum that is why red colour is obtained at the top and violet colour to obtained at the bottom of the spectrum.

Angular dispersion

Ques.- Define angular dispersion. Write its expression in terms of refractive index.

Ans.- The angle between the rays of any two colours which emerge out of the prism (when produced in backward direction) is called angular dispersion of prism for those two colours. It is represented by θ.

It is also equal to the difference of angles of deviation of rays of those two colours when they pass through the prism.



If the angle of deviation suffered by the ray of red and voilet colour by a prism be δ_r and δ_v respectively then the angular dispersion of prism for red and violet colour will be given by

$$\theta = \delta_V - \delta_R$$

$\because \delta_V = (\mu_V - 1) A$ and $\delta_R = (\mu_R - 1) A$

$\therefore \quad \theta = (\mu_V - 1) A - (\mu_R - 1) A$

or $\quad \theta = \mu_V A - A - \mu_R A + A$

or $\quad \theta = (\mu_V - \mu_R) A$

Thus, angular dipersion depends on two factors-

(i) Angle of prism- The larger the angle of prism the greater will be the angular dispersion and vice-versa.

(ii) Nature of material of prism- The larger the refractive index of material of prism, the more will be angular dispersiom and vice-versa. As the refractive index of flint glass is greater than crown glass therefore for the same angle of a prism angular dispersion for flint glass is greater then crown glass

Note- Angle of dispersion measures the angular spread for any two colours which emerge out of the prism.

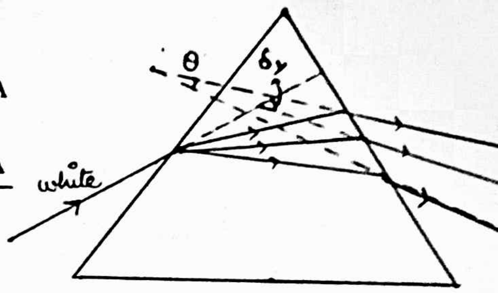
Dispersive power

Ques.- Define dispersive power. Write its expression in terms of refractive index.

Ans.- Dispersive power of material of prism for any two colours is defined as the ratio of angular dispersion of prism for those two colours and the angle of deviation suffered by the mean ray of those two colours. It is denoted by ω.

i.e. $\quad \omega = \frac{\theta}{\delta_{mean}}$

If the refractive index of material of prism for red, violet and yellow (mean ray) light be μ_v, μ_r and μ_y respectively then the dipersive power of material of prism for red and violet colour is given by



$$\omega = \frac{\theta_{RV}}{\delta_Y}$$

$\because \quad \theta_{RV} = (\mu_V - \mu_R) A$

and $\quad \delta_Y = (\mu_Y - 1) A$

$\therefore \quad \omega = \frac{(\mu_V - \mu_R) A}{(\mu_Y - 1) A}$

or $\quad \boxed{\omega = \frac{\mu_V - \mu_R}{\mu_Y - 1}}$

The dispersive power of material of prism is a unitless and dimensionsless physical quantity and its value depends only upon the nature of material of prism.

Note- Dispersive power of flint glass of prism is greater than crown glass prism.

@ **Combination of prisms** (Not in CBSE)

(I) Deviation without desperion

Ques.- What do you mean by deviation without dispersion? Draw a diagram of the combination of prisms for it and establish a relationship between their refracting angles. Also calculate the resultant deviation.

Ans.- In order to obtain deviation without disperion two prisms of different materials and different angles are combined to obtain deviation only but no dispersion. Both the prism produces equal and opposite angular dispersion as a result of whith the resultant angular dispersion becomes zero.

Let C be a crown glass and F be a flint glass prism whose refracting angles are A and A' respectively. If the refractive index of crown glass prism for violet, red and yellow light be μ_V, μ_R and μ_Y respectively then angular dispersion for crown glass prism.

$$\theta = (\mu_V - \mu_R) A$$

and angle of deviation suffered by the mean ray when it passes through crown glass prism.

$$\delta = (\mu_Y - 1) A$$

and if the refractive index of flint glass prism form violet, red and yellow light be μ_V', μ_R' and μ_Y' respectively then angular dispersion for flint glass prism.

$$\theta' = (\mu_V' - \mu_R') A'$$

and angle of deviation suffered by the mean ray when it passes through flint glass prism

$$\delta' = (\mu_Y' - 1) A'$$

Condition for deviation without dispersion- Since the resultant dispersion for the combination of prism is zero.

$\therefore \quad \theta - \theta' = 0$

or $\quad (\mu_V - \mu_R) A + (\mu_V' - \mu_R') A' = 0$

or $\quad (\mu_V' - \mu_R') A' = -(\mu_V - \mu_R) A$

or $\quad \frac{A'}{A} = \frac{-(\mu_V - \mu_R)}{(\mu_V' - \mu_R')}$

or $A' = \dfrac{-(\mu_v - \mu_R)}{(\mu_v' - \mu_R')} A$

This is the required condition of deviation without dispersion.

Here negative sign shows that the angle of prisms are oppositely directed.

Resultant deviation for combination of prism- The resultant deviation for the combination of prism is

$$\delta_{net} = \delta + \delta'$$

or $\delta_{net} = (\mu_Y - 1) A + (\mu_Y' - 1) A'$

or $\delta_{net} = (\mu_Y - 1) A - (\mu_Y' - 1) \dfrac{(\mu_V - \mu_R)}{(\mu_V' - \mu_R')} A$

or $\delta_{net} = (\mu_Y - 1) A \left[1 - \dfrac{(\mu_y' - 1)(\mu_v - \mu_R)}{(\mu_y - 1)(\mu_v' - \mu_R')}\right]$

$\because$ $\dfrac{\mu_V - \mu_R}{\mu_Y - 1} = \omega$ dispersive power of crown glass prism

and $\dfrac{\mu_V' - \mu_R'}{\mu_Y' - 1} = \omega'$ dispersive power of flint glass prism

$\therefore$ $\boxed{\delta_{net} = \delta\left(1 - \dfrac{\omega}{\omega'}\right)}$

As ω and ω' can never equal therefore resultant deviation will never be zero.

(II) Dispersion without deviation

Ques.- What do you mean by dispersion without deviation? Draw a diagram of the combination of prisms for it and establish a relationship between their refracting angles. Also calculate the resultant dispersion.

Ans.- In order to obtain dispersion without deviation two prisms of different materials and different angles are combined to produce dispersion only but no deviation. Both the prism produces equal and opposite deviation in the path of mean ray. As a result of which the resultant deviation of mean ray becomes zero.

Let C be crown glass and F be flint glass prism, whose refracting angles are A and A' respectively. If the refractive index of crown glass prism for violet, red and yellow light be μ_V, μ_R and μ_Y respectively then angular dispersion of prism of is given by

$$\theta = (\mu_v - \mu_R) A$$

and the angle of deviation sufferd by mean ray is given by

$$\delta_Y = (\mu_y - 1) A$$

and if the refractive index of flint glass prism for violet, red and yellow light be μ_V', μ_R' and μ_Y' respectively then angular dispersion for flint glass prism is given by

$$\theta' = (\mu_V' - \mu_R') A'$$

and the angle of deviation suffered by mean ray is given by

$$\delta_Y' = (\mu_Y' - 1) A'$$

Condition for dispersion without deviation- As the resultant deviation of mean ray by the combination is zero.

$\therefore$ $\delta_{net} = \delta + \delta' = 0$

or $0 = (\mu_Y - 1) A + (\mu_Y' - 1) A'$

or $(\mu_Y' - 1) A' = -(\mu_Y - 1) A$

or $\dfrac{A'}{A} = \dfrac{-(\mu_Y - 1)}{(\mu_Y' - 1)}$

or $A' = \dfrac{-(\mu_Y - 1)}{(\mu_Y' - 1)} A$

Resultant dispersion for combination of prisms- The resultant dispersion produced by the combination of prisms is given by

$$\theta_{net} = \theta + \theta'$$

or $\theta_{net} = (\mu_V - \mu_R) A + (\mu_V' - \mu_R') A'$

or $\theta_{net} = (\mu_V - \mu_R) A - (\mu_V' - \mu_R') \dfrac{(\mu_Y - 1)}{(\mu_Y' - 1)} A$

or $\theta_{net} = (\mu_V - \mu_R) A \left[1 - \dfrac{(\mu_V' - \mu_R')(\mu_Y - 1)}{(\mu_V - \mu_R)(\mu_Y' - 1)}\right]$

$\because$ $\dfrac{\mu_V - \mu_R}{\mu_Y - 1} = \omega$ dispersive power of crown glass prism

and $\dfrac{\mu_V' - \mu_R'}{\mu_Y' - 1} = \omega'$ dispersive power of flint glass prism

$\therefore$ $\boxed{\theta_{net} = \theta\left(1 - \dfrac{\omega'}{\omega}\right)}$

As ω and ω' can never be equal therefore resultant dispersion can never be zero.

@Real and virtual spectrum

Ques.- What do you mean by real and virtual spectrum?

Ans.-Real spectrum- The spectrum which can be obtained on the screen is called real spectrum.

In real spectrum, red colour lies at the top and violet colour lies at the bottom and this spectrum can be observed from all directions. It is obtained on the other side of the source.

Virtual spectrum- The spectrum which cannot be obtained on screen is called virtual spectrum.

In virtual spectrum violet colour lies at the top and red colour lies at the bottom and this spectrum can only be observed in the direction of emergent light only. It is obtained on the same side of the source.

@Pure and impure spectrum

Ques.- What do you mean by pure and impure spectrum?

Ans.-Pure spectrum- In pure spectrum there is no overlapping of different colours and the boundaries of different colours are distinctly visible.

Impure spectrum- In impure spectrum there is overlapping of different colours and the boundaries of different colours are not distinctly visible.

@Conditions for obtaining pure spectrum

Ques.- Write the conditions for obtaining pure spectrum.

Ans.- (i) The slit should be narrow.

(ii) The incident ray should be made parallel by using an achromatic convex lens.

(iii) The prism should be in the position of minimum deviation.

(iv) The emergent rays of different colours should be focussed by using an achromatic convex lens.

Note- The achromatic convex lens makes the spectrum clear and the prism in the state of minimum deviation makes the spectrum intense.

Rayleigh's law of scattering

Ques.- State Rayleigh's law of scattering.

Ans.- When sunlight passes through the earth's atmosphere then the scattering of light takes place by air molecules whose size is of the order of wavelength of light.

According to Rayleigh's law of scattering the intensity of scattered light is inversely proportional to the fourth power of wavelength of light.

If the intensity of scattered light be I and the wavelength of light be λ then according to Rayleigh's law

$$I \propto \frac{1}{\lambda^4}$$

In the sunlight wavelength of violet light is minimum and red light is maximum therefore the scattering of violet light is maximum and red light is minimum.

Phenomena of daily life based on scattering of light

Ques.- Explain different phenomena of daily life based on scattering of light.

Ans.-(i) Sky appears blue- As sunlight passes through earth's atmosphere, it gets scattered by air molecules. In accordance with Rayleigh's law light of shorter wavelengths scattered much more than the light of longer wavelengths. As blue colour has shorter wavelength therefore it scattered more strongly that is why sky appears blue.(In fact as the wavelength of violet colour is shorter than blue therefore it gets scattered even more than blue. But our eyes are more sesitive to blue than violet that is why sky is seen blue.)

(ii) Sun appears red at the time of sunrise and sunset - During sunrise and sunset sun rays have to cover larger distance in earth's atmosphere. Therefore, in accordance with Rayleigh's law all the colours of sunlight gets scattered except the red which reaches to our eyes. That is why sun appears red at the time of sunrise and sunset.

(iii) Sky appears black in the absence of atmosphere- In the absence of atmosphere the scattering of light does not take place due to which sky appears black. That is why to the passengers of aeroplane flying at higher altitudes sky appears black.

For the same reason the sky appears black at moon because moon has no atmosphere.

(iv) Clouds appear white- Clouds contain large particles like water drops, dust particles etc. which scatter all the colours of sunlight equally i.e. why clouds appear white.

(v) Danger signals are made of red colour- In accordance with Rayleigh's law the red colour can travel maximum distance without being scattered as compared to other colours. That is why danger signals of red colour can be observed from maximum distance.