Technology Arts Sciences
TH Köln

Master's in Automation and IT

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Assignment 3 : NonLinear Gain Compensation

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1 Introduction

For this assignment, we have to work with a non-linear valve characteristic. For that, we have chosen an Equal percentage valve which has a non-linear characteristic. In an equal percentage valve, for equal increments of rated travel, will ideally give equal-percentage changes of the flow coefficient (Cv) from the existing Cv.The overall flow rate, however, reveals non-linear exponential behavior at the valve.

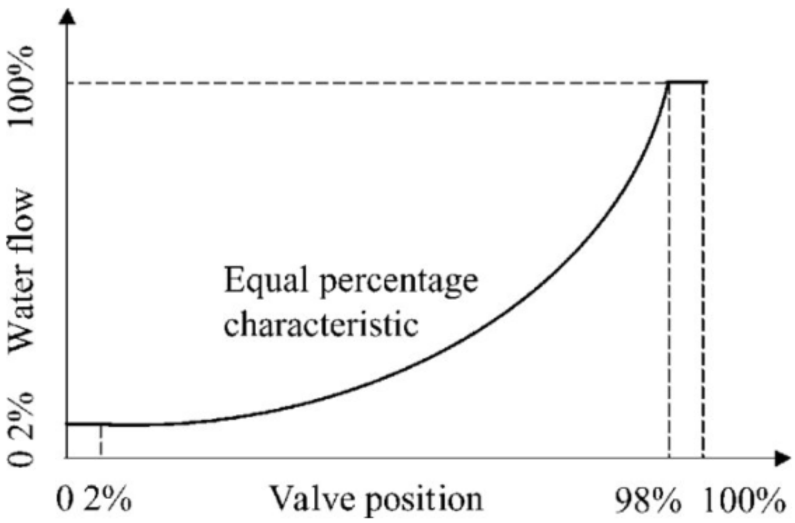


Figure 1: Non-linear or Equal Percentage Valve Characteristic

This characteristic results in a non-linearly growing plant gain, which must be compensated so that the system as a whole maintains linear behaviour.

2 Unisim Model Without Gain Compensation

In the first step, A simple system of plant represented by a valve controlled by PI controller is modeled in the UniSim.

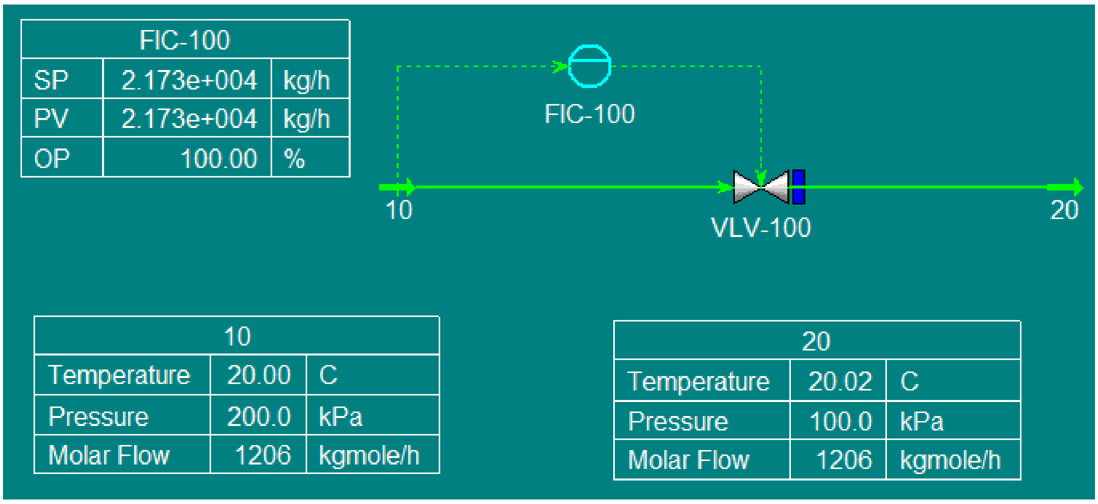


Figure 2: Unisim model simulation

In this model, we have used the non-linear equal percentage valve described as a first-order lag function which is controlled by the PI controller. The pressure at the inlet of the valve is 200 kPa and 100 kPa at the outlet.

2.1 Calculations

After simulating the model, the following calculations were performed:

2.1.1 Time Constant

To calculate the time constant, step-up and step-down steps were performed. The controller was set to manual mode and OP value was changed at 10%, 20%, 30%, 40%, 50%, 60%, 70%, 80% and 90%. The step up and step down of 5% were done at these mentioned points and data was logged and analyzed. The time taken for the 63.2% change of PV value after the step up/step down test is noted. In each step test the time constant was of 2 seconds which is also confirmed in the simulation in the setting of the plant.

	OP	PV	ΔPV	ΔOP	0.632 ΔPV	PV+0.632 ΔPV	Start Time	End time	Time Constant
	[%]	[kg/h]					Time at PV	Time at PV+0.632 ΔPV	
↑	95	18501,2	2750,4	5	1738,2528	17489,0528	2600,5	2602,5	2
•	90	15750,8							
↓	85	13409,3	-2341,5	-5	-1479,828	14270,972	2480,5	2482,5	2
↑	85	13409,3	1993,4	5	1259,8288	12675,7288	2060,5	2062,5	2
•	80	11415,9							
↓	75	9718,8	-1697,1	-5	-1072,567	10343,3328	2180,5	2182,5	2
↑	75	9718,8	1444,8	5	913,1136	9187,1136	1760,5	1762,5	2
•	70	8274							
↓	65	7043,99	-1230,01	-5	-777,3663	7496,63368	1880,5	1882,5	2
↑	65	7043,99	1047,16	5	661,80512	6658,63512	1460,5	1462,5	2
•	60	5996,83							
↓	55	5105,34	-891,49	-5	-563,4217	5433,40832	1580,5	1582,5	2
↑	55	5105,34	758,96	5	479,66272	4826,04272	1160,5	1162,5	2
•	50	4346,38							
↓	45	3700,25	-646,13	-5	-408,3542	3938,02584	1280,5	1282,5	2
↑	45	3700,25	550,08	5	347,65056	3497,82056	860,5	862,5	2
•	40	3150,17							
↓	35	2681,86	-468,31	-5	-295,9719	2854,19808	980,5	982,5	2
↑	35	2681,86	398,68	5	251,96576	2535,14576	560,5	562,5	2
•	30	2283,18							
↓	25	1943,76	-339,42	-5	-214,5134	2068,66656	680,5	682,5	2
↑	25	1943,76	288,96	5	182,62272	1837,42272	260,5	262,5	2
•	20	1654,8							
↓	15	1408,8	-246	-5	-155,472	1499,328	380,5	382,5	2
↑	15	1408,8	209,43	5	132,35976	1331,72976	10,5	12,5	2
•	10	1199,37							
↓	5	1021,07	-178,3	-5	-112,6856	1086,6844	130,5	132,5	2

Figure 3: Time Constant Calculation Table

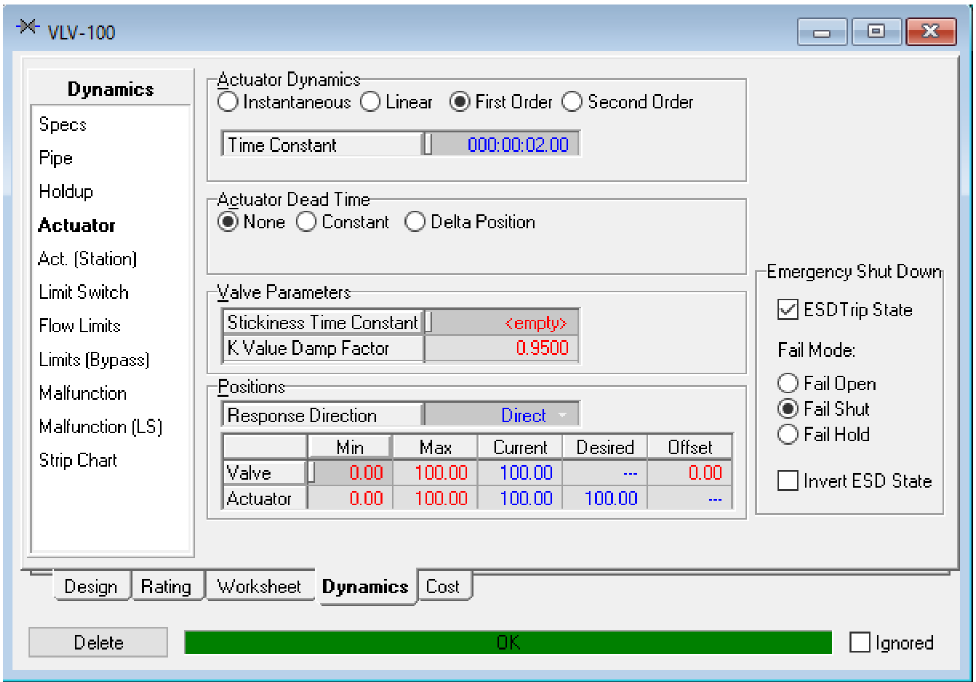


Figure 4: Time Constant Setting in UniSim

2.1.2 Plant Gain

For the calculation of the plant gain, again step-up and step-down steps were performed. The controller was set to manual mode and OP value was changed at 10%, 20%, 30%, 40%, 50%, 60%, 70%, 80% and 90%. The step up and step down of 5% were done at these mentioned points and data was logged, analyzed and calculated.

To calculate the Plant gain OP was scaled by dividing the OP value by max OP value which is 100 since the OP range is 0-100% and PV was also scaled by dividing it by its max value which was 21732 as its range was 0-21732. The Gain was calculated using the formula:

$$PlantGain = \frac{\Delta PV}{\Delta OP}$$

	OP [%]	PV [kg/h]	OP OP/Opmax	PV PV/Pvmax	ΔOP	ΔPV	Plant Gain
↑	95	18501,2	0,95	0,851338355	0,05	0,126560494	2,53
•	90	15750,8	0,9	0,724777861			
↓	85	13409,3	0,85	0,617033025	-0,05	-0,10774484	2,15
↑	85	13409,3	0,85	0,617033025	0,05	0,091726908	1,83
•	80	11415,9	0,8	0,525306117			
↓	75	9718,8	0,75	0,447213543	-0,05	-0,07809257	1,56
↑	75	9718,8	0,75	0,447213543	0,05	0,066482912	1,33
•	70	8274	0,7	0,380730631			
↓	65	7043,99	0,65	0,324131346	-0,05	-0,05659928	1,13
↑	65	7043,99	0,65	0,324131346	0,05	0,048185386	0,96
•	60	5996,83	0,6	0,27594596			
↓	55	5105,34	0,55	0,234923776	-0,05	-0,04102218	0,82
↑	55	5105,34	0,55	0,234923776	0,05	0,034923776	0,7
•	50	4346,38	0,5	0,2			
↓	45	3700,25	0,45	0,170268131	-0,05	-0,02973187	0,59
↑	45	3700,25	0,45	0,170268131	0,05	0,025312099	0,51
•	40	3150,17	0,4	0,144956032			
↓	35	2681,86	0,35	0,123406605	-0,05	-0,02154943	0,43
↑	35	2681,86	0,35	0,123406605	0,05	0,018345382	0,37
•	30	2283,18	0,3	0,105061223			
↓	25	1943,76	0,25	0,089442709	-0,05	-0,01561851	0,31
↑	25	1943,76	0,25	0,089442709	0,05	0,013296582	0,27
•	20	1654,8	0,2	0,076146126			
↓	15	1408,8	0,15	0,064826361	-0,05	-0,01131976	0,23
↑	15	1408,8	0,15	0,064826361	0,05	0,009636985	0,19
•	10	1199,37	0,1	0,055189376			
↓	5	1021,07	0,05	0,046984847	-0,05	-0,00820453	0,16

Figure 5: Palnt Gain Calculation Table

The measured Plant curve was plotted using the data from UniSim Model. It is observed that for the first 5% of the OP value the valve acts linearly having a constant gain. After 5% of the OP value non-linear behaviour can be seen as the valve is an equal percentage valve.

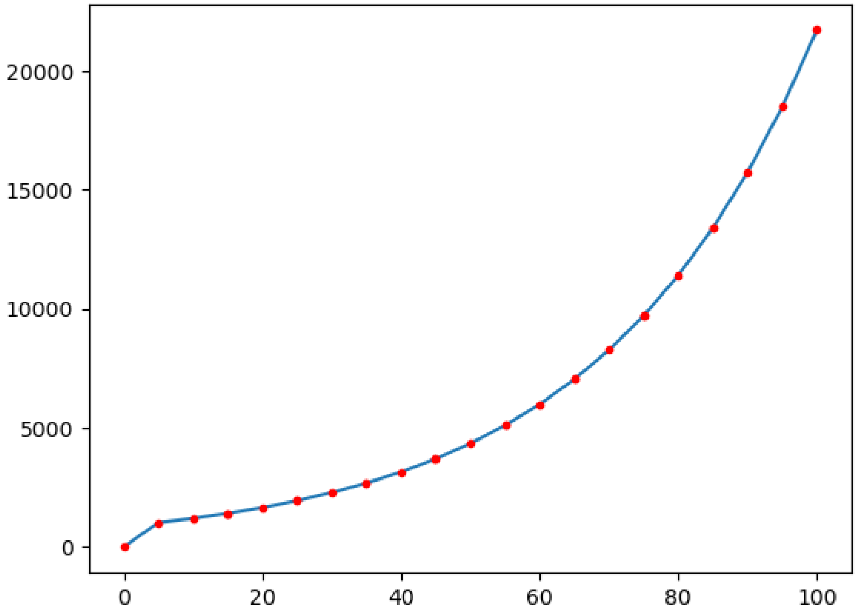


Figure 6: Measured Plant Curve

2.2 Transfer Function

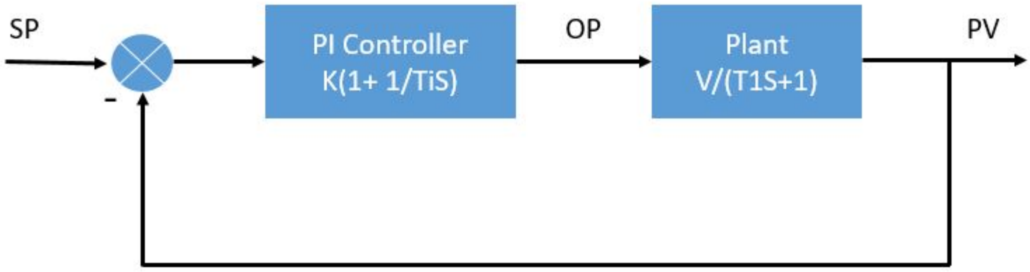
In the UniSim model, we have a PI controller and the transfer function of the PI controller is:

$$C = K(1 + \frac{1}{T_i S})$$

We have used an equal percentage characteristic valve in our UniSim model and is represented by a first-order lag transfer function which is:

$$\frac{PV}{OP} = \frac{V}{T_1 S + 1}$$

The transfer function of the close loop system in our model is calculated as:



$$TF = \frac{CP}{1 + CP}$$

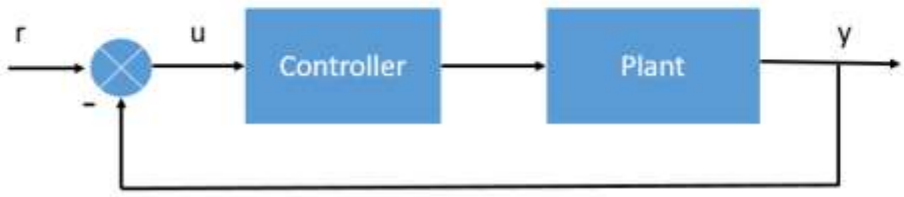
$$\frac{PV(S)}{SP(S)} = \frac{K(1 + \frac{1}{T_i S})(\frac{V}{T_1 S+1})}{1 + K(1 + \frac{1}{T_i S})(\frac{V}{T_1 S+1})}$$

$$\frac{PV(S)}{SP(S)} = \frac{(K + \frac{K}{T_i S})(\frac{V}{T_1 S+1})}{1 + (K + \frac{K}{T_i S})(\frac{V}{T_1 S+1})}$$

$$\frac{PV(S)}{SP(S)} = \frac{\frac{VKT_i S+VK}{T_i T_1 S^2+T_i S}}{\frac{T_i T_1 S^2+(VKT_i+T_i)S+VK}{T_i T_1 S^2+T_i S}}$$

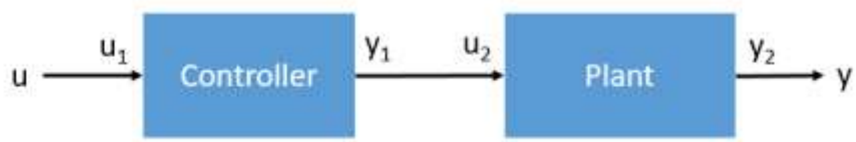
$$\frac{PV(S)}{SP(S)} = \frac{VKT_i S + VK}{T_i T_1 S^2 + (VKT_i + T_i)S + VK}$$

2.3 STATE SPACE MODEL



This represents the close loop of our system. To find the space state model of our loop we will first solve the serial connection between the Controller and the Plant and then we will solve it with the feedback loop.

1. SERIAL CONNECTION:



Here the controller is System 1 and the Plant is System 2. The equations defining the two systems are:

System 1 : $\dot{x}_1 = A_1x_1 + B_1u_1$

$y_1 = C_1x_1 + D_1u_1$

System 2 : $\dot{x}_2 = A_2x_2 + B_2u_2$

$y_2 = C_2x_2 + D_2u_2$

Common State Vector: $x = \begin{bmatrix} x1 \\ x2 \end{bmatrix}$

Since the Output of the first system is the input of the second system, therefore;

$U_2 = y_1$

Also,

$U_1 = u$ and $y_2 = y$

The equations of the first system can be re-written as:

$$\dot{x}_1 = A_1x_1 + B_1u$$

$$y_1 = C_1x_1 + D_1u$$

Now for the second system, Since,

$$U_2 = y_1$$

$$U_2 = C_1x_1 + D_1u$$

The system equations for the second system are reformulated as:

$$\dot{x}_2 = A_2x_2 + B_2 C_1x_1 + B_2D_1u$$

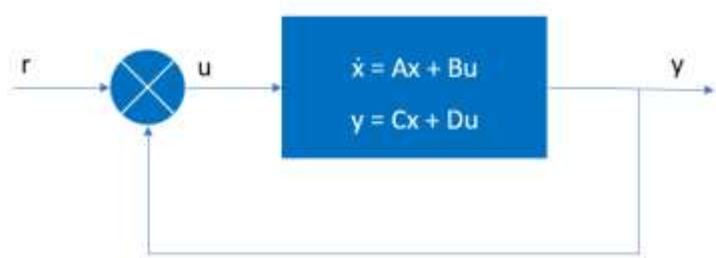
$$y_2 = C_2x_2 + D_2 C_1x_1 + D_2D_1u$$

The state equations for the total system of serial connection can be formulated as:

$$\begin{bmatrix} \dot{x}1 \\ \dot{x}2 \end{bmatrix} = \begin{bmatrix} A1 & 0 \\ B2\,C1 & A2 \end{bmatrix} \begin{bmatrix} x1 \\ x2 \end{bmatrix} + \begin{bmatrix} B1 \\ B2D1 \end{bmatrix} u$$

$$y = \begin{bmatrix} D2\,C1 & C2 \end{bmatrix} \begin{bmatrix} x1 \\ x2 \end{bmatrix} + [D2\ D1]u$$

2. FEEDBACK LOOP:



Now we will solve the feedback loop, the equation defining our system is

$$\dot{x} = Ax + Bu$$

$$y = Cx + Du$$

here we have:

$$u = r - y$$
$$u = r - (Cx + Du)$$
$$u = r - Cx - Du$$
$$u + Du = r - Cx$$

$$u = (I - D)^{-1}(r - Cx)$$

The equation of our system can be re-written as:

$$\dot{x} = Ax + Bu$$

$$\dot{x} = Ax + B(I - D)^{-1}(r - Cx)$$

$$\dot{x} = (A - B(I + D)^{-1}C)x + B(1+D)^{-1}r$$

$$y = Cx + Du$$

$$y = Cx + D(I - D)^{-1}(r - Cx)$$

$$y = (C - CD(1+D)^{-1})x + D(1+D)^{-1}r$$

After substituting and considering D2=0 to avoid an algebraic loop, the state equations for the total system can be formulated as:

$$\begin{bmatrix} \dot{x}1 \\ \dot{x}2 \end{bmatrix} = \begin{bmatrix} A1 & -B1C2 \\ B2\,C1 & A2 - B2D1C2 \end{bmatrix} \begin{bmatrix} x1 \\ x2 \end{bmatrix} + \begin{bmatrix} B1 \\ B2D1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & C2 \end{bmatrix} \begin{bmatrix} x1 \\ x2 \end{bmatrix}$$

Now solving for our system, with the help of octave and using the transfer function of the controller $K\left(1 + \frac{1}{Tis}\right)$ and plant $\frac{V}{1+T1s}$, here we have:

$A_1 = 0$ $B_1=1$ $C_1 = K/Ti$ $D_1 = K$

$A_2 = -\frac{1}{T1}$ $B_2=1$ $C_2 = V/T_1$ $D_2 = 0$

Now substituting the values,

$$\begin{bmatrix} \dot{x}1 \\ \dot{x}2 \end{bmatrix} = \begin{bmatrix} 0 & -V/T1 \\ K/Ti & -(KV+1)/T1 \end{bmatrix} \begin{bmatrix} x1 \\ x2 \end{bmatrix} + \begin{bmatrix} 1 \\ K \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & V/T1 \end{bmatrix} \begin{bmatrix} x1 \\ x2 \end{bmatrix}$$

The values of A, B, C, and D are

$A = \begin{bmatrix} 0 & -V/T1 \\ K/Ti & -(KV+1)/T1 \end{bmatrix}$ $B = \begin{bmatrix} 1 \\ K \end{bmatrix}$

$C = \begin{bmatrix} 0 & \frac{V}{T1} \end{bmatrix}$ $D = 0$

3 PI Controller with non-linear Gain compensation

To design a PI controller which dynamically compensates for the non-linear behaviour of the Plant, the Controller gain should be the inverse of the Plant Gain for the overall system gain to be constant and 1.

$$GainofPIController = \frac{1}{GainofPlant}$$

3.1 Calculation of Controller Gain

To calculate the controller gain, we first need to find the plant gain at every point and we don't have enough data so we will perform regression with the available data to find the approximate function defining the plant curve and then we will calculate the controller gain by taking the inverse of the plant gain.
Instead of getting a single high-order polynomial defining the whole plant curve, we have divided the whole plant curve into three parts on the basis of OP ranges to get three different low-order polynomials since high order polynomials are sensitive to the degree of the polynomial and lead to noisy estimates.

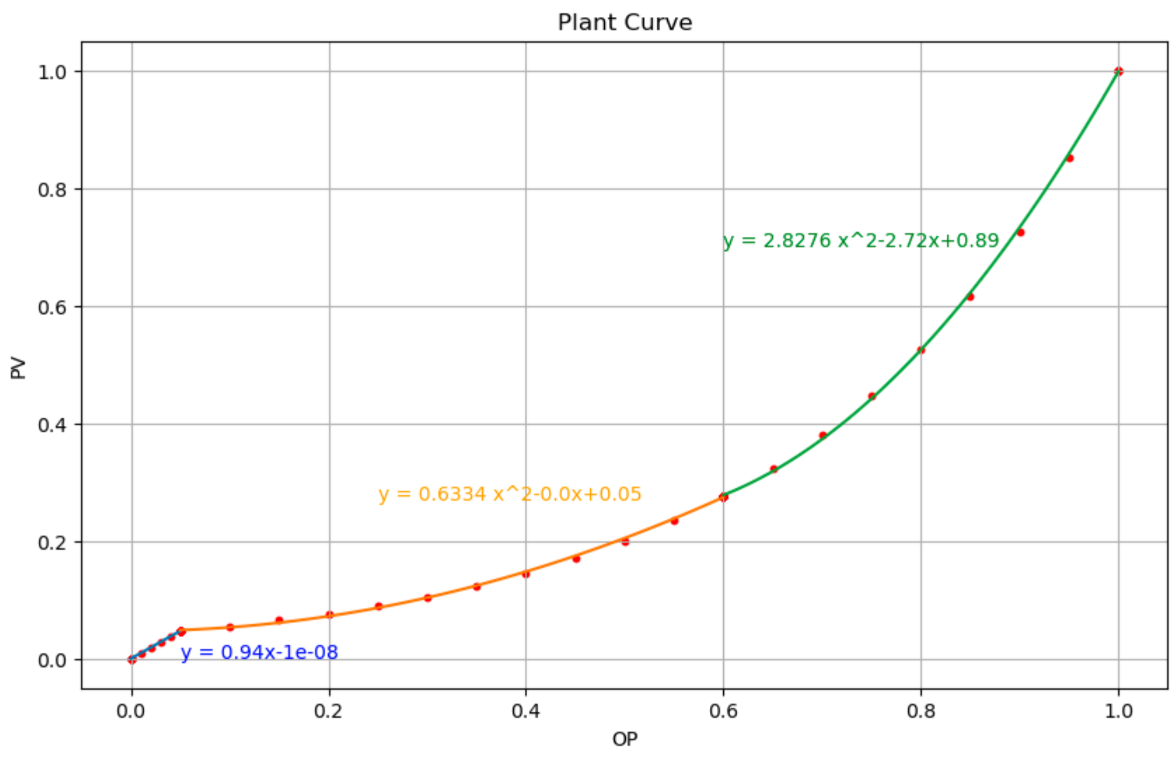


Figure 7: Plant Curve

For OP range 0% to 5%:

$$PV = 0.939696737OP - 8.06878306e^{-09}$$

$$PlantGain = \frac{dPV}{dOP} = 0.939696737$$

For OP range 5% to 60%:

$$PV = 0.63344238OP^2 - 0.00103153OP + 0.0468872$$

$$PlantGain = \frac{dPV}{dOP} = 1.26688476OP - 0.00103153$$

For OP range 60% to 100%:

$$PV = 2.82763752OP^2 - 2.72415453OP + 0.89441626$$

$$PlantGain = \frac{dPV}{dOP} = 5.65527504OP - 2.72415453$$

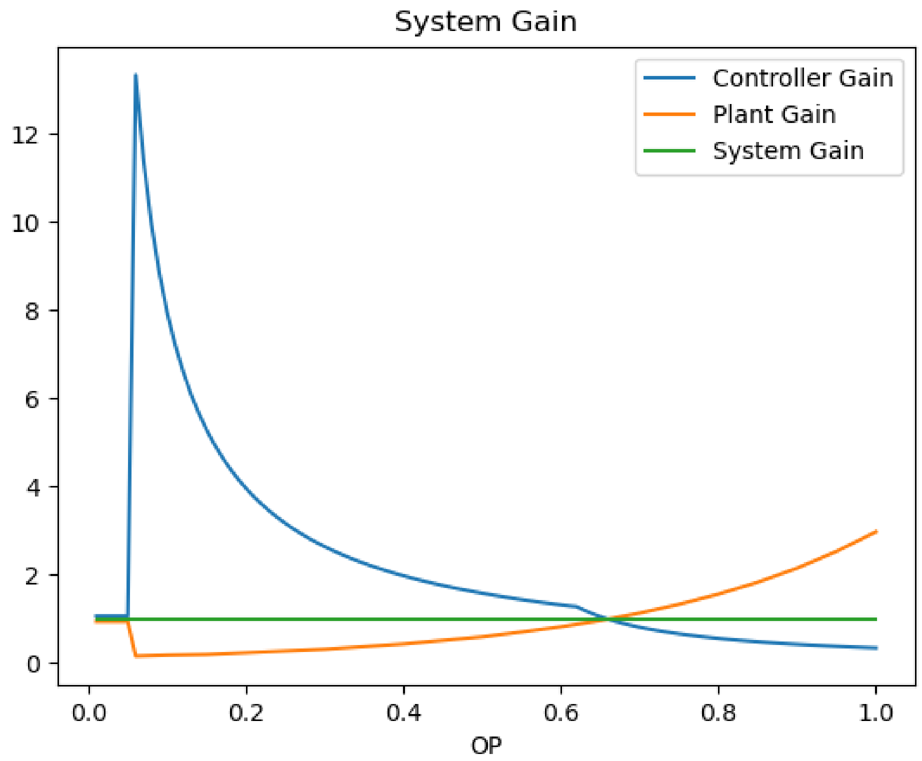


Figure 8: Plant Gain Compensation

3.2 Implementation in UniSim

The compensated model is now implemented in UniSim.

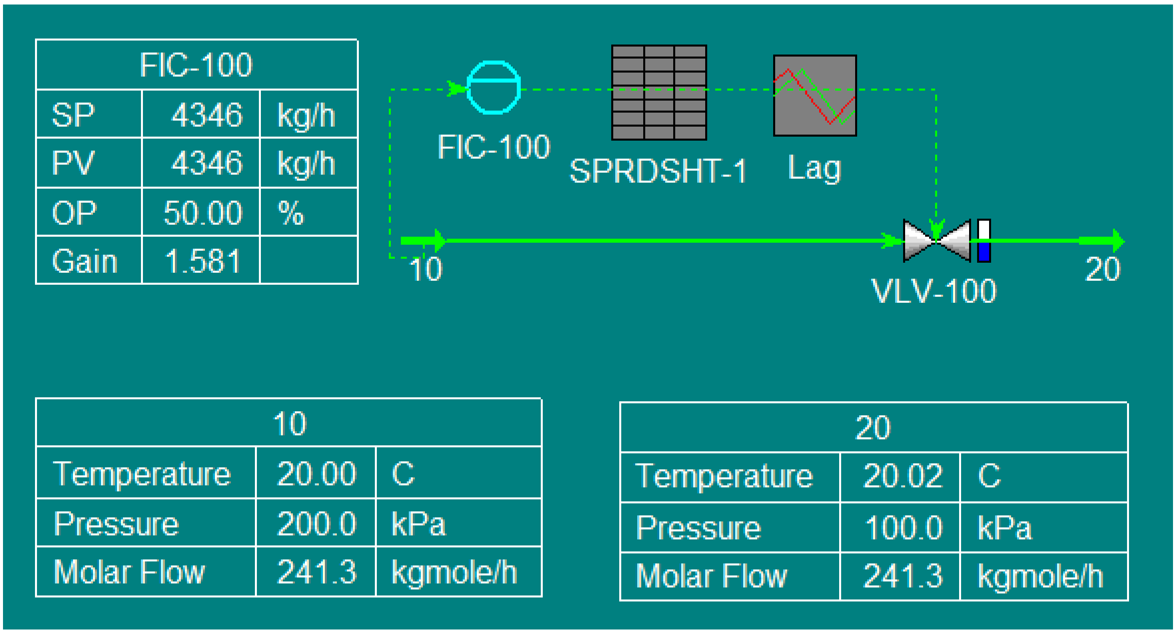


Figure 9: Non linear Gain Compensated UniSim Model

To implement the model spreadsheet is introduced in the model. The OP value is imported into the spreadsheet in cell A1 and then scaled into cell A2.

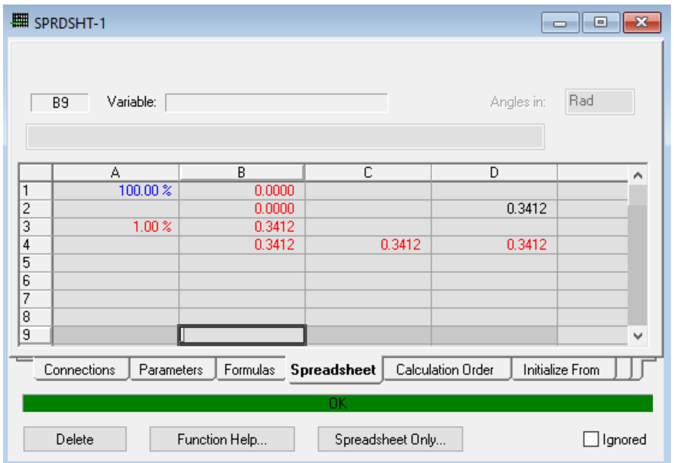


Figure 10: Spreadsheet-UniSim Model

In B column, real-time controller gain is calculated using three different polynomials of different OP ranges with the help of IF statement and summed into B4. The controller gain is then exported to Lag function from cell C4 and imported back to D2 after the lag function. From D4 The gain is finally exported to controller gain.

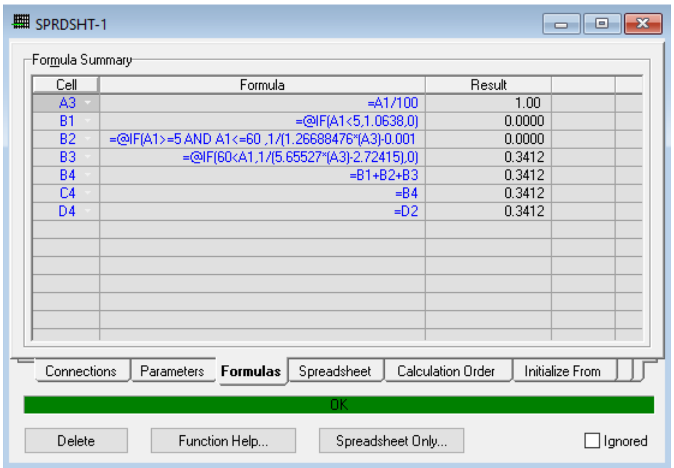


Figure 11: Spreadsheet-UniSim Model

3.3 Transfer Function

For the non linear gain compensation of the plant, we have assumed that the gain of the controller (K) is equal to the inverse of the plant gain (V).

$$GainofPIController(K) = \frac{1}{GainofPlant(V)}$$

Therefore the transfer function of the controller is:

$$C = \frac{1}{V}.(1 + \frac{1}{T_i S})$$

We have already established the transfer function of our close loop system before compensation in Section 2.2 of this document. The transfer function is:

$$\frac{PV(S)}{SP(S)} = \frac{VKT_i S + VK}{T_i T_1 S^2 + (VKT_i + T_i)S + VK}$$

After Substitution of $K = \frac{1}{V}$,

$$\frac{Y(S)}{R(S)} = \frac{T_i S + 1}{T_i T_1 S^2 + 2T_i S + 1}$$

3.4 State Space Form

$$\frac{Y(S)}{R(S)} = \frac{T_iS+1}{T_iT_1S^2+2T_iS+1}$$

$$\frac{Y(S)}{R(S)} = \frac{T_iT_1(\frac{1}{T_1}TS+\frac{1}{T_iT_1})}{T_iT_1(S^2+\frac{2}{T_1}S+\frac{1}{T_iT_1})}$$

$$\frac{Y(S)}{R(S)} = \frac{\frac{1}{T_1}TS+\frac{1}{T_iT_1}}{S^2+\frac{2}{T_1}S+\frac{1}{T_iT_1}}$$

Let,

$$A_1 = \frac{1}{T_1}, A_2 = \frac{2}{T_1}, A_3 = \frac{1}{T_iT_1}$$

$$\frac{Y(S)}{R(S)} = \frac{A_1S+A_3}{S^2+A_2S+A_3}$$

$$\frac{Y(S)}{R(S)} = \frac{Y(S)}{X(S)}.\frac{X(S)}{R(S)} = (A_1S+A_3)x\frac{1}{S^2+A_2S+A_3}$$

First considering,

$$\frac{X(S)}{R(S)} = \frac{1}{S^2+A_2S+A_3}$$

$$R(S) = S^2X(S)+A_2SX(S)+A_3X(S)$$

Taking Inverse Laplace,

$$r(t) = \frac{d^2}{dt^2}x(t)+A_2\frac{d}{dt}x(t)+A_3x(t)............(1)$$

Let,

$$x_1 = x(t)$$

$$x_2 = \dot{x}_1 = \frac{d}{dt}x(t)$$

$$\dot{x}_2 = \ddot{x}_1 = \frac{d^2}{dt^2}x(t)$$

By substituting the values in equation 1,

$$r(t) = \ddot{x}_1+A_2\dot{x}_1+A_3x_1$$

$$r(t) = \dot{x}_2+A_2x_2+A_3x_1$$

$$\dot{x}_2 = -A_2x_2-A_3x_1-r(t)$$

And we know,

$$\dot{x}_1 = x_2$$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ -A_3 & -A_2\end{bmatrix}.\begin{bmatrix}x_1\\ x_2\end{bmatrix}+\begin{bmatrix}0\\ 1\end{bmatrix}r(t)$$

Now Considering,

$$\frac{Y(S)}{X(S)} = (A_1S + A_3)$$
$$Y(S) = A_1SX(S) + A_3X(S)$$

By Taking Inverse Laplace Transform,

$$y(t) = A_1\frac{d)}{dt}x(t) + A_3x(t)............(2)$$

Let,

$$x_1 = x(t)$$
$$x_2 = \dot{x}_1 = \frac{d}{dt}x(t)$$

By substituting the values in equation 2,

$$y(t) = A_1\dot{x}_1 + A_3x_1$$
$$y(t) = A_1\dot{x}_2 + A_3x_1$$

$$y(t) = \begin{bmatrix} A_3 & A_1 \end{bmatrix} . \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Now substituting the values,

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{1}{T_iT_1} & -\frac{2}{T_1} \end{bmatrix} . \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r(t)$$

$$y(t) = \begin{bmatrix} \frac{1}{T_iT_1} & \frac{1}{T_1} \end{bmatrix} . \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

So The values of A,B, C, D are,

$$A = \begin{bmatrix} 0 & 1 \\ -\frac{1}{T_iT_1} & -\frac{2}{T_1} \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} r(t)$$

$$C = \begin{bmatrix} \frac{1}{T_iT_1} & \frac{1}{T_1} \end{bmatrix}, D = \begin{bmatrix} 0 \end{bmatrix}$$

4 Control Parameters Selection

After gain compensation, our system transfer function is dependent on the values of two variables Ti and T1. To choose their appropriate values, since our valve has a Time constant of 2 sec meaning T1=2, we will change Ti and observe the behaviour of our system.

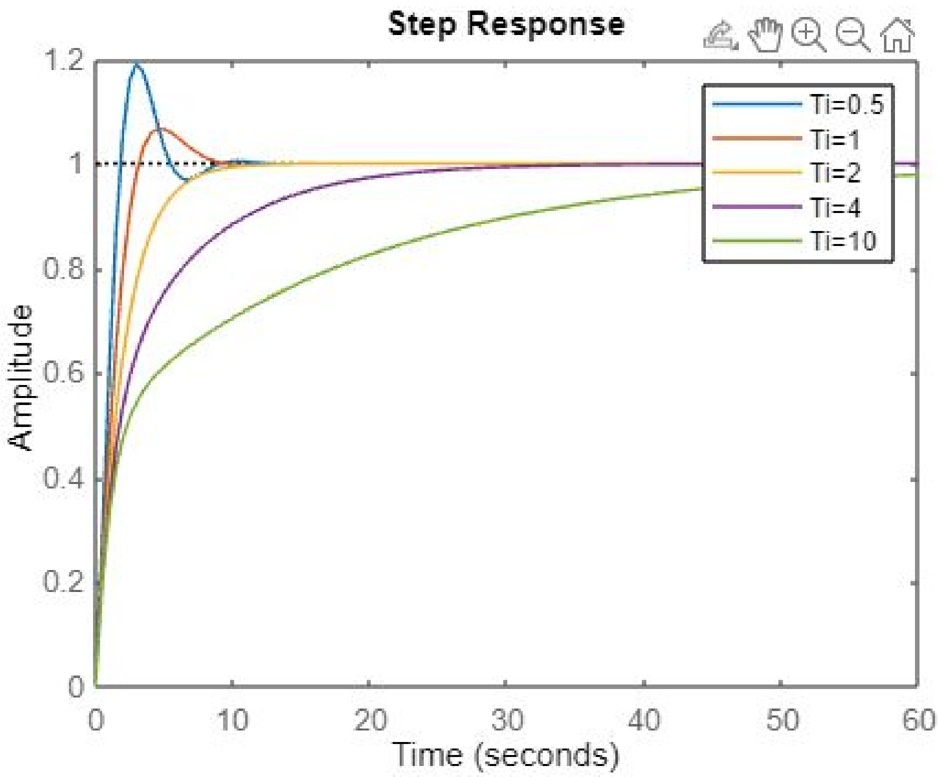


Figure 12: Step Response of Compensated system

It was observed that when Ti i.e.0.5,1 is smaller than T1=2, the system has an over-damped behaviour and has oscillations, Similarly when Ti is larger than T1 it has an under damped behaviour. But when Ti=T1=2 it is critically damped. Therefore we are choosing these parameters for our analysis.

The Transfer Function with these parameters will be:

$$TF = \frac{0.5S + 0.25}{S^2 + S + 0.25}$$

and the state space model is:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -0.25 & -1 \end{bmatrix} . \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r(t)$$

$$y(t) = \begin{bmatrix} 0.25 & 0.5 \end{bmatrix} . \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

5 Analysis

5.1 EigenValues

Eigen Values of a stats space system are given by:

$$\left|\lambda I - A\right| = 0$$

The eigen values after the calculations are:

$$\lambda_1, \lambda_2 = -0.5$$

After calculation, the eigen values of the system are negative and real means the system is stable.

5.2 Controllability

The system is said to be controllable if the determinant of C_o is not equal to zero and the rank of the matrix C_o is equal to the order of matrix A.

$$C_o = \begin{vmatrix} B & AB \end{vmatrix} \neq 0$$

$$C_o = \begin{vmatrix} 0 & 1 \\ 1 & -1 \end{vmatrix} = -1$$

Since

$$\left|C_o\right| \neq 0$$

and rank of C_o is 2 same as the order of matrix A, the system is Controllable.

5.3 Observability

The system is said to be observable if the determinant of O is not equal to zero and the rank of the matrix O is equal to the order of matrix A.

$$O = \begin{vmatrix} C \\ CA \end{vmatrix} \neq 0$$

$$O = \begin{vmatrix} 0.25 & 0.5 \\ -0.125 & -0.25 \end{vmatrix} = 0$$

Since

$$\left|O\right| = 0$$

and rank of C_o is 1 not same as the order of matrix A, the system is not Observable.

6 Final Simulation

After the Gain Compensation in our system and calculating the suitable parameters, we performed the final simulation and observed the system behaviour and compared it with the system without gain compensation. During the simulation, step up and step down were performed by changing the setpoint.

For the compensated system, the first Ti was 2 sec as calculated that it gives the best response of the system.

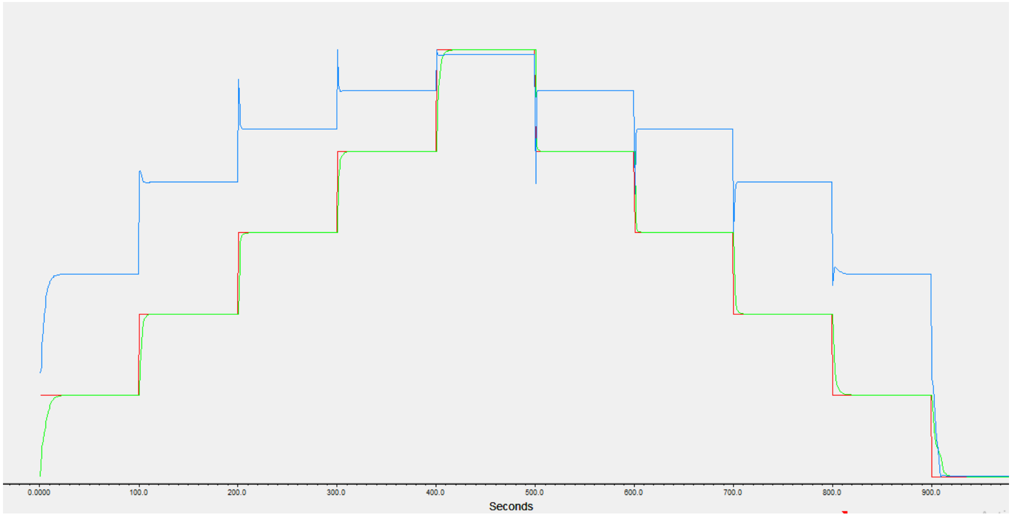


Figure 13: Compensated system when Ti=2sec

After that simulation with Ti= 6 sec was performed to observe the system behaviour.

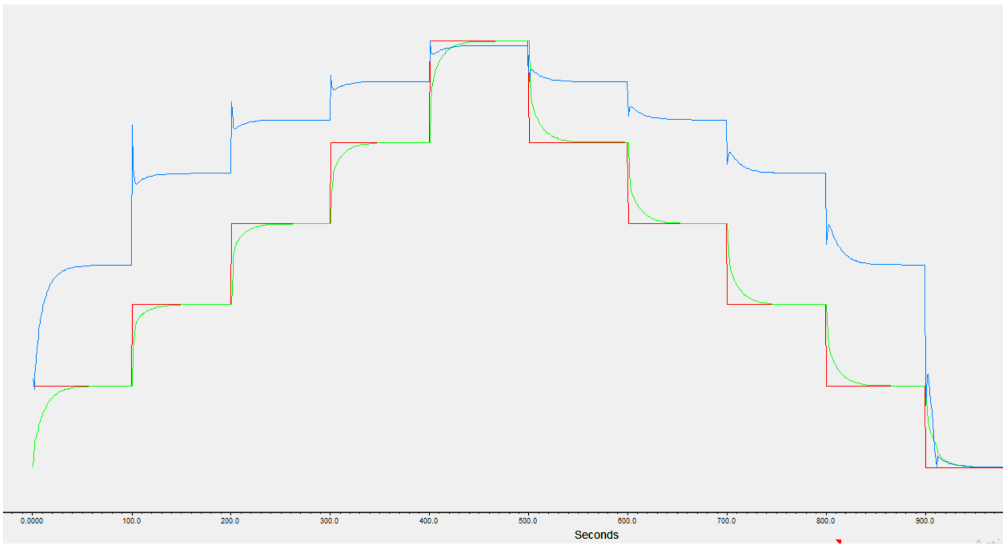


Figure 14: Compensated system when Ti=6sec

And the behaviour was then compared with the model of the system without gain compensation.

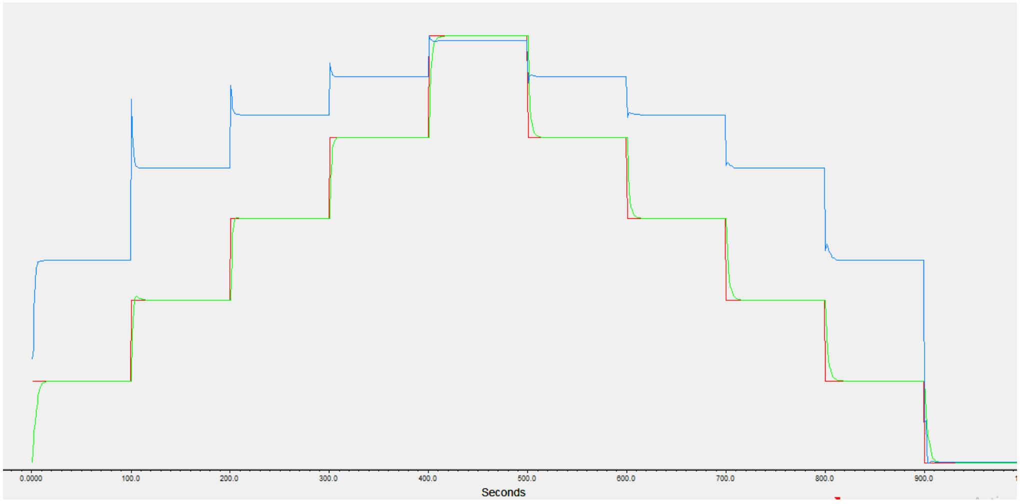


Figure 15: Non Compensated system when Ti=2sec

After the analysis the following observations were made: The gain-compensated system has no undershoots as compared to the uncompensated system.
The rise time is also constant throughout the increase of the OP range expect the beginning as it varies there whereas the rise time in the uncompensated system seems to be varying throughout and decrease with the increasing SP.
Also if we change Ti in the compensated system the system has an underdamped and overdamped response. As in our case when Ti=6s the rise time increases and OP behaviour is not smooth.
Interestingly, the compensated system has much higher overshoots in the start of the SP range.

7 Conclusion

In this assignment, we have analysed and implemented one of the methods to linearize the non-linear behaviour of the plant. We have done this with gain compensation by making the controller gain inverse of the plant gain dynamically.

After non-linear gain compensation, it was analyzed from our system transfer function and the new state space model that our model is now independent of Plant Gain V and PI controller Gain K and totally depends on time constants Ti and T1. It was also observed that to have a critically damped response Ti and T1 should be equal. If Ti is greater or smaller then T1 then the response of the system is under damped or over damped.

The system has negative and same eigenvalues means the system is stable. The system is also Controllable but not Observable.

After gain compensation, the system has a constant rise time throughout the SP range and no undershoots.

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