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Assignment: Modeling and Simulation of Discrete Systems

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Module: Modeling and Simulation of Discrete Systems

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1 Introduction

Since the development of what are called discrete processes, challenges related to time and efficiency, such as unexpected machine downtime, material shortages, and unpredictable demand were faced. These challenges can lead to increased waiting times and decreased overall efficiency.

In this report, a comprehensive analysis explaining how these challenges can be overcome with the application of **queuing theory**, is presented.

2 Applying Queuing Theory to Car Wash Systems

A real-life application of queuing theory is a car wash system, which involves studying the flow of vehicles arriving and exiting, the time they spend in the queues, and the time they spend getting serviced.

The car wash system considered for this report can be viewed as a combination of two queuing systems, an $M/M/1$ queue, in series with an $M/M/s$ queue, where $s = 4$ is assumed.

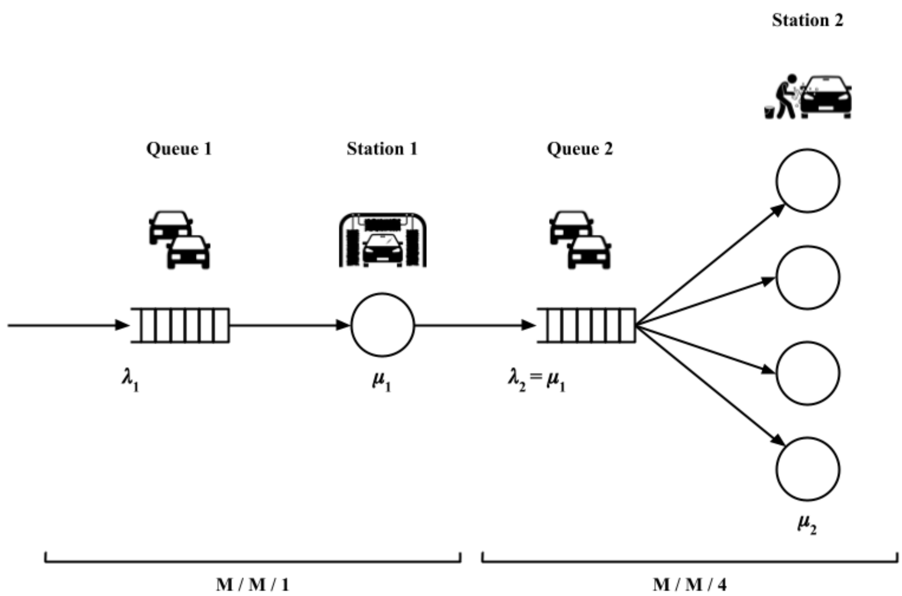

Figure 1: Car Wash System Queuing Model

The first $M/M/1$ model represents an automatic washing station with a single queue and a single server, with certain corresponding parameters like arrival rate λ_1 and service rate μ_1.

The second $M/M/4$ model represents the final cleaning stations where human helpers perform cleaning and drying services on the cars, with an arrival rate of λ_2, and a service rate of μ_2.

By using queuing theory, it is possible to analyze and optimize the performance of a car wash system by identifying bottlenecks, reducing waiting times, and improving overall efficiency. This modeling can help car wash owners and managers make data-driven decisions to improve their service.

2.1 Analysis of $M/M/1$ Queue

For the first $M/M/1$ queuing model of the process, an arrival rate $\boldsymbol{\lambda_1 = 8}$ **customers per hour**, and a service rate $\boldsymbol{\mu_1 = 12}$ **customers per hour**, is assumed.

Then the **utilization** of the first system (defined as the percentage of total time in which the station is working) is given as,

$$\rho_1 = \frac{\lambda_1}{\mu_1} = \frac{8}{12} = 0.6667 = 66.67\% \tag{1}$$

The **concurrency** or the average number of total customers present in the system at any instance of time is given as,

$$L = \frac{\rho_1}{1 - \rho_1} = \frac{0.666}{1 - 0.666} = 1.994 \approx 2 \text{ customers} \tag{2}$$

The total **waiting time in the system** is given as,

$$W = \frac{1}{\mu_1 - \lambda_1} = \frac{1}{12 - 8} = 0.25 \text{ hours} = 15 \text{ minutes} \tag{3}$$

The total number of **customers in the queue** at any instance of time, is given as,

$$L_q = \frac{{\rho_1}^2}{1 - \rho_1} = \frac{(0.666)^2}{1 - 0.666} = 1.3280 \approx 1 \text{ customers} \tag{4}$$

The total **waiting time in the queue** is given as,

$$W_q = \frac{\rho_1}{\mu_1(1 - \rho_1)} = \frac{0.666}{12(1 - 0.6667)} = 0.1666 \text{ hours} \approx 10 \text{ minutes} \tag{5}$$

2.2 Analysis of $M/M/4$ Queue

The $M/M/4$ queue model will have the same arrival rate as the exit rate of the previous model, which is same as the service rate of the first queuing model. And hence, $\boldsymbol{\lambda_2 = \mu_1 = 12}$ **customers per hour**. Along with that, each of the four servers has a service rate of $\boldsymbol{\mu_2 = 4}$ **customers per hour**.

The **utilization** of the system is given as,

$$\rho_2 = \frac{\lambda_2}{\mu_2 \cdot s} = \frac{12}{4 \cdot 4} = 0.75 = 75\% \tag{6}$$

The **probability of having 0 customers** in the system can be calculated as,

$$P_0 = \left[\left(\sum_{i=0}^{s-1} \frac{(s \cdot p)^i}{i!}\right) + \left((sp)^s \cdot \frac{1}{s!} \cdot \frac{1}{1-\rho_2}\right)\right]^{-1}$$

$$P_0 = \left[\left(\sum_{i=0}^{s=1} \frac{(3.0)^i}{i!}\right) + \left((3.0)^4 \cdot \frac{1}{4!} \cdot \frac{1}{1-0.75}\right)\right]^{-1}$$

$$P_0 = \left[\left(\frac{(3.0)^0}{0!} + \frac{(3.0)^1}{1!} + \frac{(3.0)^2}{2!} + \frac{(3.0)^3}{3!}\right) + (13.5)\right]^{-1}$$

$$P_0 = \{[1.0 + 3.0 + 4.5 + 4.5] + [13.5]\}^{-1}$$

$$P_0 = \{[13.0] + [13.5]\}^{-1} = (26.5)^{-1} = 0.037736 \tag{7}$$

The **average number of customers in the queue** is given as,

$$L_q = \left[\frac{1}{(s-1)!} \cdot (\frac{\lambda_2}{\mu_2})^2 \cdot \frac{\lambda_2 \cdot \mu_2}{(s\mu_2 - \lambda_2)^2}\right] \cdot P_0$$

$$L_q = \left[\frac{1}{(4-1)!} \cdot (\frac{12}{4})^2 \cdot \frac{12 \cdot 4}{(4 \cdot 4 - 12)^2}\right] \cdot 0.037736$$

$$L_q = 1.5283 \approx 2 \text{ customers} \tag{8}$$

The **concurrency** can be given as,

$$L = L_q + \frac{\lambda_2}{\mu_2} = 1.5283 + 3.0 = 4.528 \approx 5 \text{ customers} \tag{9}$$

The **waiting time of the queue** is,

$$W_q = \frac{L_q}{\lambda_2} = \frac{1.5283}{12} = 0.1273 \tag{10}$$

The **system waiting time** is,

$$W_s = W_q + \frac{1}{\mu_2} = \frac{4.5283}{12} = 0.37735 \tag{11}$$

The **probability of all servers being busy** can be calculated as,

$$P(n >= s) = \left[\frac{1}{(s)!} \cdot (\frac{\lambda_2}{\mu_2})^s \cdot \frac{s \cdot \mu_2}{(s\mu_2 - \lambda_2)^2}\right] \cdot P_0 \tag{12}$$

$$P(n >= s) = \left[\frac{1}{(4)!} \cdot (\frac{12}{4})^4 \cdot \frac{4 \cdot 4}{(4 \cdot 4 - 12)^2}\right] \cdot 0.037736 = 0.12735 \tag{13}$$

2.3 Simulation Model

Modelling and simulation of the queuing models were performed using the **Tecnomatix Plant Simulation** software. The model is built with one source to represent the arrival of customers, buffers to represent the waiting queues, stations to represent the servers, and corresponding drains to signify the exits.

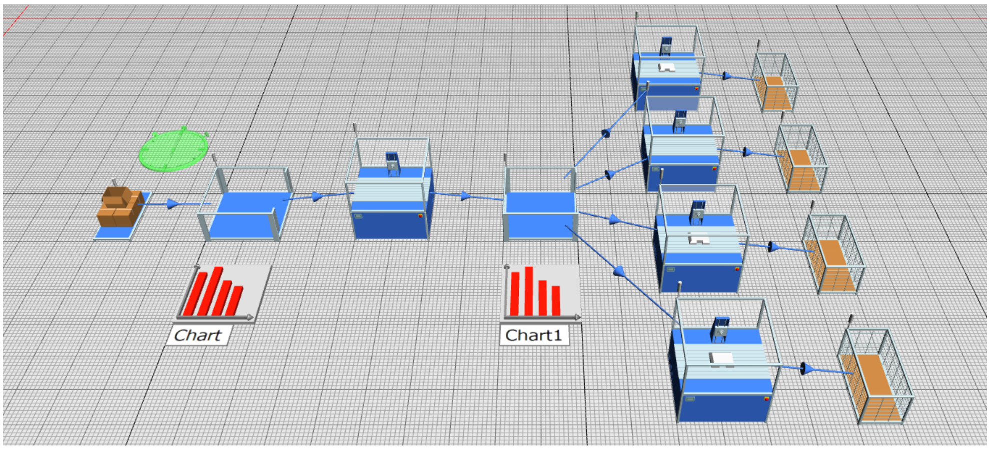

Figure 2: Car Wash System Simulation Model

For the first $M/M/1$ queue, the source is configured to have an arrival interval of **7 minutes and 30 seconds** with a **negative exponential** behaviour to signify $\lambda_1 = 8$ customers per hour. The first buffer is configured to have a capacity $= -1$ signifying infinite holding capacity. The first station has a processing time of **5 minutes**, also with a **negative exponential** behaviour, signifying the service rate of $\mu_1 = 12$ customers per hour.

Similarly, for the $M/M/4$ queue, the buffer is configured to have infinite capacity, and each station has a processing time of **15 minutes** ($\mu_2 = 4$).

The simulation was performed for a period of **24 hours**, and the occupancy probability charts were produced for both the queuing models. The charts represent the fraction of total time (y-axis) a given number of customers (x-axis) occupied a certain part of the system.

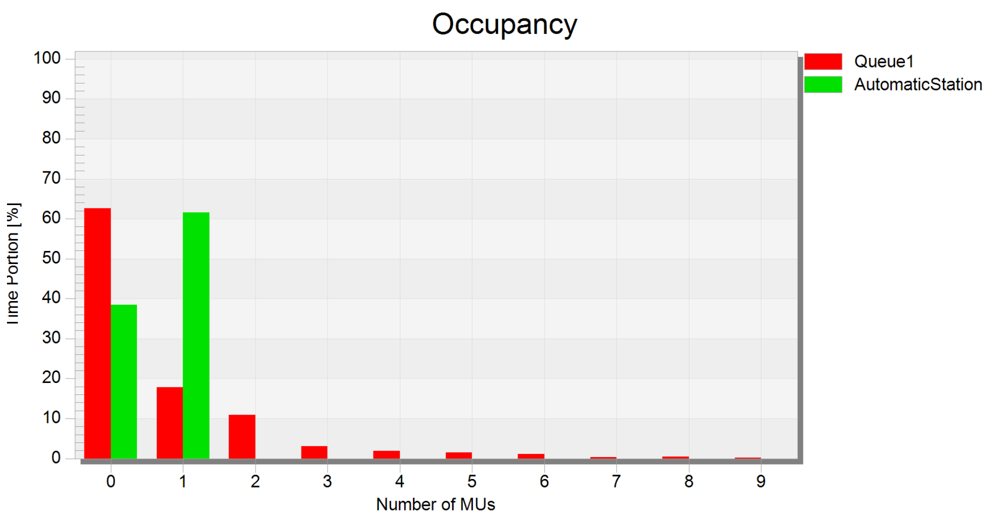

Figure 3: Occupancy chart for M/M/1

It is evident that the system has the capacity to cover the customer demand, and the queue occupancy behaviour could be described with a negative exponential distribution. The probability of having 0 customers in the queue is about 63%, which decreases exponentially as the number of customers increase. The automatic cleaning station (server), is busy (with a maximum of 1 customer) for approximately 60% of the total time, and idle for the remaining 40%.

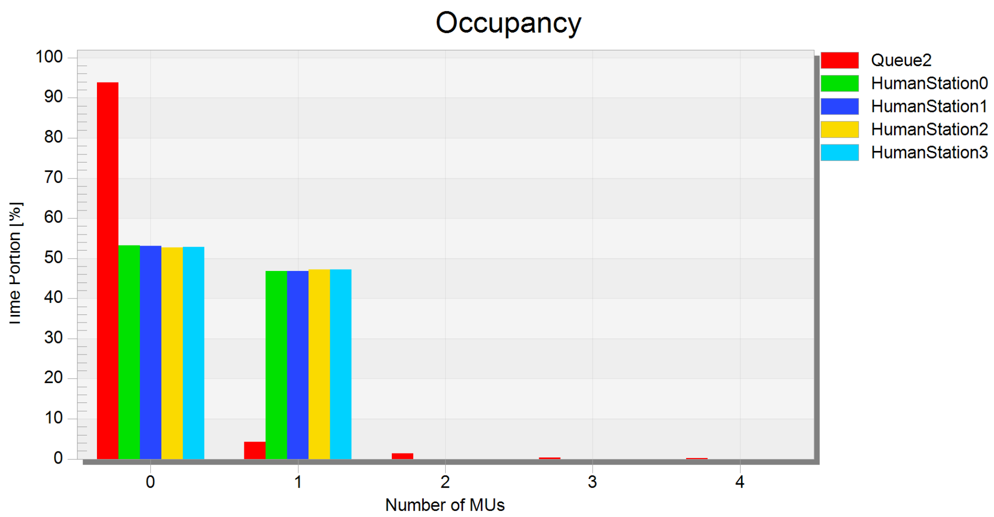

Figure 4: Occupancy chart for M/M/4

In a similar way, it can be interpreted that as the number of parallel servers increase, the system demand can be handled efficiently. Each of the four parallel servers are busy for about 50% of the total working time. The behaviour of the second queue can also be visualized to be decreasing exponentially.

3 Conclusion

Queuing theory is a branch of mathematics that handles the study of queues or waiting lines. It has various applications in real-world scenarios, and its concepts can be seen in daily life. As the most significant application of queuing theory is in managing service systems, the example developed above allowed us to understand how the optimization of the allocation of resources, and reduction of customer waiting times are performed.