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MASTER'S IN AUTOMATION AND IT

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Assignment 2 : Modeling and Simulation of Vehicle Model

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Module : Modelling and simulation of continuous systems

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1 Problem Statement

Develop a simulation model for a vehicle (Quarter wheel Vehicle Model). Find relevant parameters such as mass, stiffness and damping of the suspension and the wheel in the model. Perform simulation studies to analyze the behaviour of the model. Also, analyze the model with the methods of linear system theory.

2 Introduction

The quarter-wheel vehicle model is the simplest representation of a vehicle used to study the dynamics of a vehicle's suspension. The model consists of the wheel/axle, the suspension system (damper and coil) and a quarter of the vehicle's body mass. This type of modelling considers various parts of the vehicle as lumped masses and thus considers only the essential degrees of freedom (DOFs) associated with the few lumped masses, such as the vehicle body and wheels, all in the vertical translational direction. Due to the two DOFs, a typical lumped-mass quarter vehicle model is basically capable of representing the body bounce and wheel hop modes of movement of a vehicle [2].

Block Diagram of the Model:

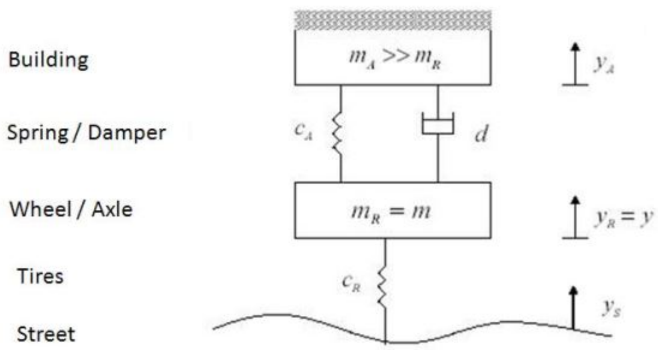

Figure 1: Block Diagram of Vehicle Model

where,

y_A - position of the car
y_R - position of the wheel
y_S - road profile change/ excitation by road
m_A – the mass of a quarter of the vehicle body
m_R – the mass of the wheel and suspension
c_A – spring constant (stiffness) of the suspension system
d – damping constant of the suspension system
c_R – spring constant (stiffness) of the wheel and tire

3 Modeling and Simulation

3.1 Mathematical Model:

In modelling mechanical systems, we shall see that all internal forces and torques at any point of the body must add up to zero. Also, it is true that the positions, velocities, and accelerations, both transnational and rotational must be same at any connecting point in the system [3].

Equation of Motion (Newton's Law):

Newton's law is: The sum of all forces exerted on a body equals the mass of the body multiplied by its acceleration, i.e:

$$m\ddot{y} = \sum forces \tag{1}$$

$$m\ddot{y} = -d\dot{y} - c_A y - c_R(y - y_s) \tag{2}$$

$$m\ddot{y} + d\dot{y} + \underbrace{(c_A + c_R)}_{\text{c}} y = \underbrace{c_R * y_s}_{\text{k(t)}} \tag{3}$$

State representation:

$$x_1 = y \qquad Path \tag{4}$$

$$x_2 = \dot{y} \qquad Speed \tag{5}$$

The resulting state equations are as follows:

$$\dot{x_1} = x_2 \tag{6}$$

$$\dot{x_2} = \frac{1}{m}(-dx_2 - cx_1 + c_R(y_s)) \tag{7}$$

$$\dot{x_2} = -\frac{d}{m} \cdot x_2 - \frac{c}{m} \cdot x_1 + \frac{c_R}{m} \cdot y_s(t) \tag{8}$$

Matrix Notation:

From eq 6 and 8 of the mathematical model, we will derive the state space model as:

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{c}{m} & -\frac{d}{m} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{c_R}{m} \end{bmatrix} y_s(t) \tag{9}$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \tag{10}$$

3.2 Normalization

Normalization is necessary for creating a simulation model in next step. To make an equation dimensionless, we need to normalize the variables in the equation so that they have the same units or are unitless. We need to choose suitable parameters for a dimensionless equation.

$$m = \widetilde{m}\, kg, \qquad d = \widetilde{d}\, \frac{N.s}{m}, \qquad c = \widetilde{c}\, \frac{N}{m}, \qquad c_R = \widetilde{c_R} \frac{N}{m}$$

$$x_1 = \widetilde{x}_1.m, \qquad x_2 = \widetilde{x}_2.\frac{m}{s}, \qquad y_s = \widetilde{y}_s.m, \qquad t = \widetilde{t}.s$$

$$\frac{d\widetilde{x}_1}{d\widetilde{t}}.\frac{m}{s} = \widetilde{x}_2.\frac{m}{s} \tag{11}$$

$$\frac{d\widetilde{x}_1}{d\widetilde{t}} = \widetilde{x}_2 \tag{12}$$

$$\frac{d\widetilde{x}_2}{d\widetilde{t}}.\frac{m}{s^2} = \frac{1}{m}(-d\widetilde{x}_2.\frac{m}{s} - c\widetilde{x}_1.m + c_R\widetilde{y}_s.m) \tag{13}$$

By Substituting values for m, c, d and c_R with units

$$\frac{d\widetilde{x}_2}{d\widetilde{t}}.\frac{m}{s^2} = \frac{1}{\widetilde{m}\, kg}(\widetilde{d}\, \widetilde{x}_2.\frac{N.s}{m} \cdot \frac{m}{s} - \widetilde{c} \cdot \widetilde{x}_1.\frac{N}{m} \cdot m + \widetilde{c_R} \cdot \widetilde{y}_s.\frac{N}{m} \cdot m) \tag{14}$$

$1N = 1kg.\frac{m}{s^2}$

$$\frac{d\widetilde{x}_2}{d\widetilde{t}}.\frac{m}{s^2} = \frac{1}{\widetilde{m}\, .kg}(\widetilde{d}\, \widetilde{x}_2.\frac{kg\frac{m}{s^2}.s}{m} \cdot \frac{m}{s} - \widetilde{c} \cdot \widetilde{x}_1.\frac{kg\frac{m}{s^2}}{m} \cdot m + \widetilde{c_R} \cdot \widetilde{y}_s.\frac{kg\frac{m}{s^2}}{m} \cdot m) \tag{15}$$

$$\frac{d\widetilde{x}_2}{d\widetilde{t}} = \frac{1}{80}(-\widetilde{m}\, \widetilde{x}_2 - \widetilde{c}\, \widetilde{x}_1 + \widetilde{c_R}\, \widetilde{y}_s) \tag{16}$$

Substituting following *eq. 16*,

$$\widetilde{x}_1 = x_1, \qquad \widetilde{x}_2 = x_2, \qquad \widetilde{y}_s = y_s, \qquad \widetilde{t} = t$$

$$\widetilde{m} = m, \qquad \widetilde{d} = d, \qquad \widetilde{c} = c, \qquad \widetilde{c_R} = c_R$$

we get,

$$\frac{dx_2}{dt} = \frac{1}{m}(-dx_2 - cx_1 + c_R y_s) \tag{17}$$

eq. 17 gives us the normalized dimensionless equation for vehicle model which can be used to find most suitable parameters by observing the response upon excitation by road.

3.3 Simulation Model

The road profile is considered to be the input into the system.

On a rough surface, an automobile may undergo pitch, bounce, and roll motions. A 2-DOF vehicle model with pitch and bounce motion, as presented in Figure 1, can provide a preliminary suspension model. [4]

We are considering following parameter values in *eq. 17* for simulating the model:

$$m = 80kg, \qquad d = 1500\frac{N.s}{m} \qquad , c = 40000\frac{N}{m} \qquad , c_R = 36000\frac{N}{m} \tag{18}$$

4 Analysis

4.1 With Step Input as Excitation of Road

The purpose of the study is to analyze the system's response (outputs) for a step input of the road profile, which can be regarded as the wheel going above a steep rigid object on the road (rock, brick, etc.)

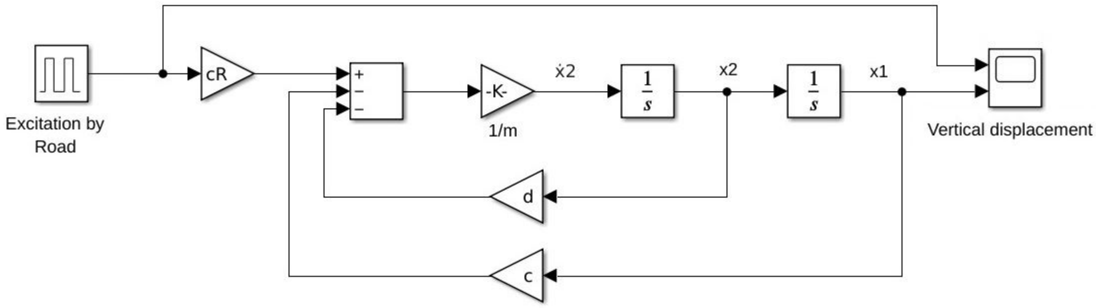

Figure 2: Simulation Model of Vehicle

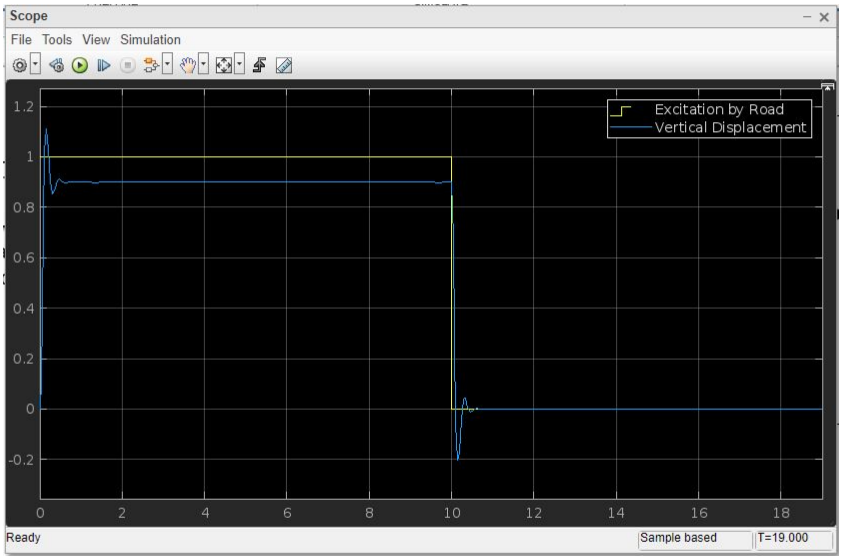

Figure 3: Simulation Model of Vehicle

4.2 With Sine Input as Excitation of Road

In order to do the analysis of the system sinusoidal input was given and it's response was observed at different frequencies.

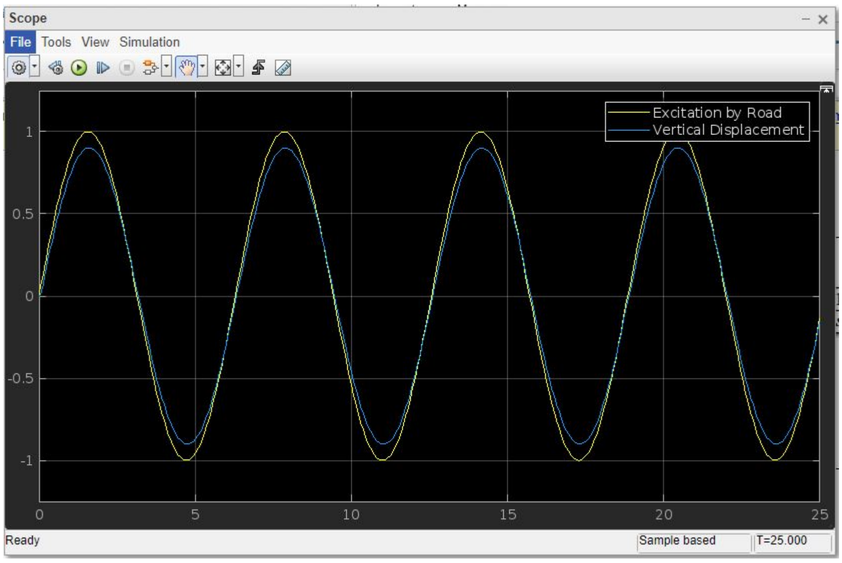

Figure 4: Response at input excitation frequency = 1rad/s

The analysis was done with low frequency of *1rad/sec*, for this the response was as depicted in *Fig. 4*. We can observe that the response of the model is following the input excitation by road closely with minor damping by damper system.

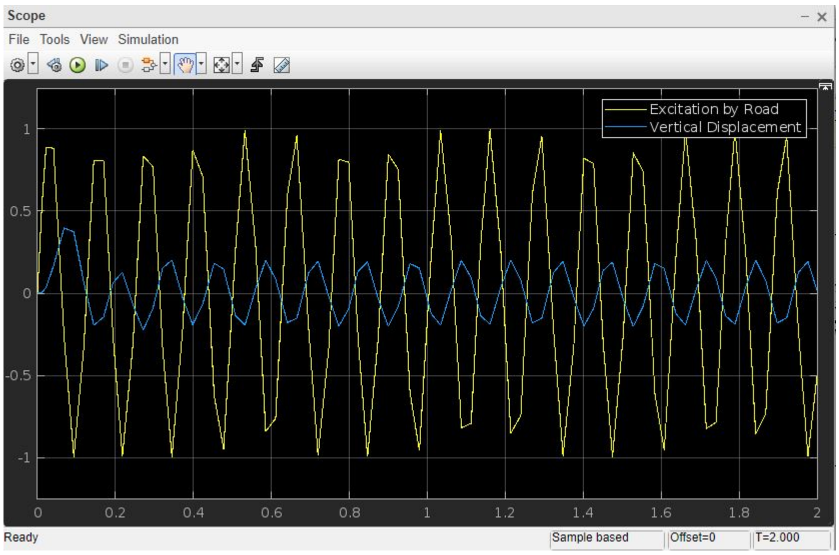

Figure 5: Response at input excitation frequency = 50rad/s

Next frequency chosen for analysis was *50 rad/sec*, for which the response was as recorded in *Fig. 5* .The frequency was increased meaning the irregularities are more frequent in the road. Due to the damping effect of the system the vehicle is only experiencing the jerk in the beginning but after that there is little displacement meaning the damper system is doing a good job at absorbing the shocks by road.

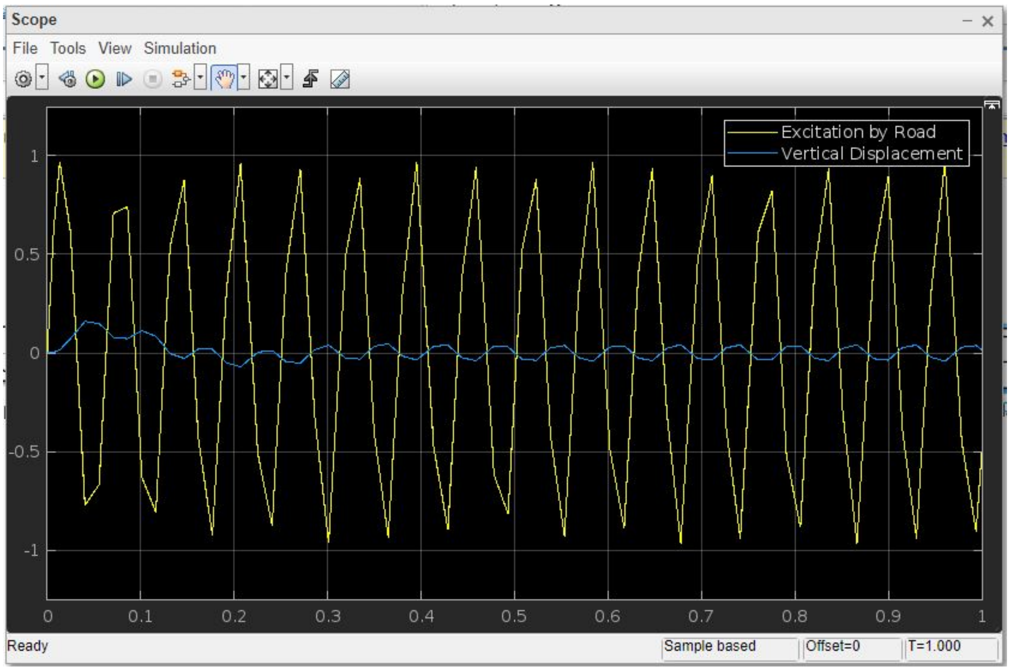

Figure 6: Response at input excitation frequency = 100rad/s

As we increase the frequency even further to *100 rad/sec*, this absorbtion effect becomes more prominent due to increased damping and the displacement decreases even further.

4.3 State space analysis

The system is represented by following state equations as stated in section 3.1.

$$\dot{x_1} = x_2 \tag{19}$$

$$\dot{x_2} = -\frac{d}{m} \cdot x_2 - \frac{c}{m} \cdot x_1 + \frac{c_R}{m} \cdot y_s(t) \tag{20}$$

Also, the path

$$y = x_1 \tag{21}$$

Comparing these equations with the general state space form,

$$\dot{X} = AX + BU \tag{22}$$

$$\dot{Y} = CX + DU \tag{23}$$

We get following matrices:

$$A = \begin{bmatrix} 0 & 1 \\ \frac{-c}{m} & \frac{-d}{m} \end{bmatrix}, B = \begin{bmatrix} 0 \\ \frac{c_R}{m} \end{bmatrix}$$

$$C = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, D = [0]$$

4.3.1 Eigen Values

Eigen Values of a state space system are given by:

$$\left|\lambda I - A\right| = 0$$

$$\begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ \frac{-c}{m} & \frac{-d}{m} \end{bmatrix} = 0$$

$$\begin{bmatrix} \lambda & -1 \\ \frac{c}{m} & \lambda + \frac{d}{m} \end{bmatrix} = 0$$

$$\lambda(\lambda + \frac{d}{m}) + \frac{c}{m} = 0$$

$$\lambda^2 + \lambda(\frac{d}{m}) + \frac{c}{m} = 0$$

The eigen values are found after solving above quadratic equation are:

$$\lambda_1, \lambda_2 = \frac{\frac{-d}{m} \pm \sqrt{(\frac{d}{m})^2 \; - 4(\frac{c}{m})}}{2}$$

Since damping constant of suspension system, d and mass, m can never be negative therefore for a system to be stable following condition must be satisfied:

$$(\frac{d}{m})^2 - 4(\frac{c}{m}) < 0 \tag{24}$$

OR

$$(\frac{d}{m})^2 \; - 4(\frac{c}{m}) < \frac{d}{m} \tag{25}$$

From various sources found online it was observed that the value of c»>d, due to this, condition stated in *eq. 23* will always be satisfied. As a result, eigen values will lie in negative half of s-plane making the system stable. We need to be concerned about system stability and choosing other parameters carefully only if this condition is violated.

After substituting the values of *d, m, c* from *eq. 18* in *eq.* , we get:

$$\lambda_1, \lambda_2 = -9.375 \pm 20.30i$$

Since eigen values lie in the negative half of the s plane indicating that the system is stable.

4.3.2 Observability

A system is said to be observable when all states can be known from system output. The observability matrix for the system is defined as:

$$O(A,C) = \begin{bmatrix} C \\ CA \end{bmatrix} \neq 0$$

$\because$

$$CA = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ \frac{-c}{m} & \frac{-d}{m} \end{bmatrix}$$

$$CA = \begin{bmatrix} 0 & 1 \end{bmatrix}$$

$$O = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \neq 0$$

Since the Matrix O has a full column rank i.e. 2, the system is completely observable.

4.3.3 Controllability

The system is said to be controllable if the determinant of C_o is not equal to zero and the rank of the matrix C_o is equal to the order of matrix A.

$$C_o = \begin{vmatrix} B & AB \end{vmatrix} \neq 0$$

$\because$

$$AB = \begin{bmatrix} 0 & 1 \\ \frac{-c}{m} & \frac{-d}{m} \end{bmatrix} \begin{bmatrix} 0 \\ \frac{c_R}{m} \end{bmatrix}$$

$$AB = \begin{bmatrix} \frac{c_R}{m} \\ \frac{dc_R}{m^2} \end{bmatrix}$$

$$C_o = \begin{vmatrix} 0 & \frac{c_R}{m} \\ \frac{c_R}{m} & \frac{dc_R}{m^2} \end{vmatrix} \neq 0$$

Since the Matrix Co has a full column rank i.e. 2, the system is fully controllable.

5 Conclusion

Ultimately we can conclude that the model created has suitable parameters so as to provide a good response as evident from the above analysis.

References

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