Foundations of Computational Math 2 Spring 2023

Programming assignment 3

General Task

The following methods should be implemented to work on scalar differential equations:

- Explicit Methods:
  1. Forward Euler:
  $$y_n = y_{n-1} + hf_{n-1}$$
  2. Adams Bashforth two-step (AB2)
  $$y_n = y_{n-1} + \frac{h}{2}(3f_{n-1} - f_{n-2})$$
- Implicit Methods:
  1. Backward Euler:
  $$y_n = y_{n-1} + hf_n$$
  2. Adams Moulton one-step (Trapezoidal Rule)
  $$y_n = y_{n-1} + \frac{h}{2}(f_n + f_{n-1})$$

The Family of Problems

Consider the parameterized family of initial value problems (IVP) given by:

$$\begin{aligned} f &= \lambda(y - F(t)) + F'(t) \\ y(0) &= y_0 \\ y(t) &= (y_0 - F(0))e^{\lambda t} + F(t) \end{aligned}$$

This family has parameters $\lambda \in \mathbb{R}$, $F : \mathbb{R} \to \mathbb{R}$, and $y_0 \in \mathbb{R}$ to define an initial value problem with solution $y : \mathbb{R} \to \mathbb{R}$ on $T_L \leq t \leq T_U$.

(Note that the integral curve is specificed by the choice of y_0 but all of integral curves contain an exponential and $F(t)$. So even if $y(0) = F(0)$ and $y(t) = F(t)$, integral curves with $y_0 \neq F(0)$ have an exponential component that can be seen by points in the numerical solution y_n since it contains points on many different integral curves of the system.)

The Tasks

0.1 General Comments

In this programming assignment you will explore the behavior of different methods on a finite interval $0 \leq t \leq 10$ with a fixed stepsize h for different choices of λ, y_0 and $F(t)$. You will compare the observed behaviors to those predicted by the theory of local error order, global error (convergence) order, and absolute stability.

For a given IVP, you will solve using a particular method and a series of stepsizes h. For each mesh you should quantify, at least, the error $|y(t_1) - y_1|$ which is the first step's error and therefore a local error, the final global error $|y(t_N) - y_N|$ where $t_N = 10$ and the maximum global error $\max_{0 \leq n \leq N} |y(t_n) - y_n|$. You should use these data to estimate the local error order and the global error (convergence) order of the methods.

You must organize your observations into a compact and clear presentation of evidence to support your conclusions.

0.2 Absolute Stability

Take $F(t) = 0$ and $y(0) = 1$.

- Probe the absolute stability properties of the methods for two $\lambda = \pm 1$
- Identify the intervals on the real axis where the methods are damping and where they are growing. Determine if this is consistent with the theory.

0.3 Accuracy and Stability

Take $F(t) = \sin(\omega t)$ and $y_0 = 0$.

- Test for $\omega = 0.01$ and $\omega = 10$
- Test for $\lambda = -1$ and $\lambda = -0.01$
- Discuss stability and accuracy.

Comment on Codes:

Note that any of the implicit LMS methods can be written as

$$y_n - h\beta_0 f(y_n, t_n) = S$$

where S is a known value that depends upon the method, stepsizes and past points. Substituting f into the expression and solving yields

$$y_n = [S + h\beta_0(F' - \lambda F)](1 - h\lambda\beta_0)^{-1}$$

which can be used to advance the implicit methods to y_n, t_n.

Submission of Results

Expected results comprise:

- A document describing your solutions as prescribed in the notes on writing up a programming solution posted on the Canvas.
- The source code, makefiles, and instructions on how to compile and execute your code including the Math Department machine used, if applicable.
- Code documentation should be included in each routine. (You don't need to paste your code in the writing report).
- All text files that do not contain code or makefiles must be PDF files. **Do not send Microsoft word files of any type.**

These results should be submitted by 11:59 PM on the due date. Submission of results is to be done via Canvas.