Course on Theory of Partial Differential Equations

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1 Prerequisites:

- Advanced Calculus
- Advanced Mathematical Analysis
- Functional Analysis
- Measure Theory

2 Course Content

- Review of fundamentals of functional analysis and facts from measure theory: Banach and Hilbert spaces, Dual spaces, Reisz-Representation theorem, Lax-Milgram theorem, weak and weak-star convergence, Lebesgue measure and integral.
- Function spaces, L^p spaces, convolution and mollifers, and useful functional inequalities in analysis of PDEs.
- Sobolev spaces and weak derivative calculus, Poincare inequality, and some embedding theorems.
- Theory of second order linear elliptic equations: existence and uniqueness of weak solutions, elliptic regularity (smoothness of solutions), and eigenfunction expansions. Laplace and Poisson equations will be core examples.
- Fourier series, the Galerkin Method, and weak solutions of linear parabolic equations. The heat equation will be a core example.
- The space of distributions, fundamental solutions, Fourier Transform of distributions and L^2 functions, and its applications to PDEs. The wave and heat kernels will be core examples.
- Theory of existence and uniqueness of the solutions of Navier-Stokes equations in 2D and 3D.

- Discussion on the Leray's celebrated paper on Navier-Stokes i.e, *"Sur le mouvement d'un liquide visqueux emplissant l'espace."* A modern view of this paper will be discussed.

3 Textbooks:

- Partial Differential equations by **Lawrence Evans.**
- Introduction to Partial Differential Equations by **David Borthwick.**
- Infinite Dimensional Dynamical Systems by **James Robinson.**