

Habib University  
Artificial Intelligence  
Fall 2020  
Assignment 3

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**Question 1.1**

$$e_{ij}^2 = (r_{ij} - \hat{r}_{ij})^2 = (r_{ij} - \sum_k p_{ik} q_{kj})^2$$

$$\frac{\partial}{\partial p_{ik}} e_{ij}^2 = -2(r_{ij} - \hat{r}_{ij}) q_{kj} = -2e_{ij} q_{kj}$$

$$\frac{\partial}{\partial q_{kj}} e_{ij}^2 = -2(r_{ij} - \hat{r}_{ij}) p_{ik} = -2e_{ij} p_{ik}$$

$$p'_{ik} = p_{ik} - \alpha \frac{\partial}{\partial p_{ik}} e_{ij}^2 = p_{ik} + 2\alpha e_{ij} q_{kj}$$

$$q'_{kj} = q_{kj} - \alpha \frac{\partial}{\partial q_{kj}} e_{ij}^2 = q_{kj} + 2\alpha e_{ij} p_{ik}$$

**Question 1.2**

$$\hat{r}_{ij} = bu_i + bi_j + \sum_k p_{ik} q_{kj}$$

$$e_{ij}^2 = (r_{ij} - \hat{r}_{ij})^2 = (r_{ij} - \sum_k p_{ik} q_{kj} - bu_i - bi_j)^2$$

$$\frac{\partial}{\partial p_{ik}} e_{ij}^2 = -2(r_{ij} - \hat{r}_{ij}) q_{kj} = -2e_{ij} q_{kj}$$

$$\frac{\partial}{\partial q_{kj}} e_{ij}^2 = -2(r_{ij} - \hat{r}_{ij}) p_{ik} = -2e_{ij} p_{ik}$$

$$\frac{\partial}{\partial bu_i} e_{ij}^2 = -2(r_{ij} - \hat{r}_{ij}) = -2e_{ij}$$

$$\frac{\partial}{\partial b_{ij}} e_{ij}^2 = -2(r_{ij} - \hat{r}_{ij}) = -2e_{ij}$$

$$p'_{ik} = p_{ik} - \alpha \frac{\partial}{\partial p_{ik}} e_{ij}^2 = p_{ik} + 2\alpha e_{ij} q_{kj}$$

$$q'_{kj} = q_{kj} - \alpha \frac{\partial}{\partial q_{kj}} e_{ij}^2 = q_{kj} + 2\alpha e_{ij} p_{ik}$$

$$bu'_i = bu_i - \alpha \frac{\partial}{\partial bu_i} e_{ij}^2 = bu_i + 2\alpha e_{ij}$$

$$bi'_j = bi_j - \alpha \frac{\partial}{\partial bi_j} e_{ij}^2 = bi_j + 2\alpha e_{ij}$$