Habib University
CS 451 - Computational Intelligence
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Optimisation and Swarm Intelligence
Assignment - 02

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Introduction

Ant Colony Optimization (ACO) is a well-known metaheuristic in which a colony of articial ants cooperates in exploring good solutions to a combinatorial optimisation problem. In this report, an ACO algorithim is presented for the graph coloring problem.

Problem Definition

Let $G = (V, E)$ be an undirected graph where V is the set of vertices and E is the set of edges. A $q-$coloring of G is a mapping $c\colon V \mapsto \{1, 2, 3, \cdots, q\}$ that assigns colors to vertices. The coloring is feasible if no two adjacent vertices have the same color i.e $\forall\{u, v\} \in E\colon c(u) \neq c(v)$, otherwise conflicts happen. An optimal coloring of G is a feasible coloring with smallest number of colors. This minimum number of colors q for which a feasible $q-$coloring exsists is called the chromatic number of G and is denoted by $\chi(G)$. Given a graph G,the graph coloring problem is to find an optimal coloring.

ANTCOL Algorithm for Graph colouring

In ANTCOL a colony of articial ants iteratively colors a specific graph, at each iteration, initially, ants produce feasible colorings by considering pheromone trails and heuristic information, and afterwards pheromone trails are updated according to the quality of colorings. The quality of colorings are measured using the following evaluation function[1],

$$f(s) = \frac{1}{q(s)} \tag{1}$$

where $q(s)$ denotes the number of colors are applied in coloring s.

Pheromone trails are related to pairs of nonadjacent vertices.Therefore, each pair of nonadjacent vertices $\{v_i, v_j\}$ has an associated pheromone trail τ_{ij} that represents the colony experience of colorings in which the two mentioned vertices have the same color i.e. belong to the same class of color.

There are multiple constructive methods that are used for graph coloring but here we are just focusing on ANTRLF. In ANTRLF,there exsits several stages, at stage k the artificial ant constructs color class C_k, and stage k also consists of several steps, at each step the artificial ant determines which uncolored vertices are to be added the color class C_k. Let W be the set of uncolored vertices that can be added to C_k, and B be the set of uncolored vertices which are not allowed to be added in C_k. In order to choose the uncolored vertex v_i,ANTRLF can use three diffrent peices of heuristic information as follows

$$\eta_{ik} = deg_B(v_i) \tag{2}$$

$$\eta_{ik} = deg_{B\cup W}(v_i) \tag{3}$$

$$\eta_{ik} = \mid W \mid - deg_w(v_i) \tag{4}$$

However,at the beginning of stage k,there are no vertices in B, so the following two strategies are applied to the first vertex to C_k

- Randomly selecting an uncolored vertex from W
- Selecting vertex v_i with maximum $deg_w(v_i)$

Pheromone trails are initially set to 1 and at the end of each iteration they are updated by considering the following rule,

$$\tau_{ij} = (1-\rho)\tau_{ij} + \sum_{s\in S_{ij}} \frac{1}{q(s)} \tag{5}$$

where $q(s)$ represents the number of colors applied to coloring s and ρ denontes the pheromone evaporation rate. S_{ij} is the subset of colorings in which the two vertices v_i and v_j belong to the same color class. In order to choose an uncolored vertex v_i to be added to the color class C_k , pheromone trail is defined as follows

$$\tau_{ik} = \frac{\sum_{j\in C_k} \tau_{ij}}{\mid C_k \mid} \tag{6}$$

[1]For presenting results in this report, f(s) = q(s).

τ_{ik} contains all the pheromone trails between the vertex v_i and so far added vertices in color class C_k. Consequently at each step of stage k, the probabilistic decision rule determines which uncolored vertex $v_i \in W$ is to be added to the color class C_k is defined as follows

$$p_{ik} = \begin{cases} \frac{\tau_{ij^\alpha} \eta_{ij^\beta}}{\sum_{j \in W} \tau_{ij^\alpha} \eta_{ij^\beta}} & v_i \in W \\ 0 & v_i \notin W \end{cases} \tag{7}$$

where p_{ik} is the probability of selecting vertex v_i. Stage k continues while W remains nonempty.

Computational Results

The parameter value for ANTLRF are $\alpha = 2, \beta = 2$ and $\rho = 0.5$. The colony size is set to 100 ants and termination condition is defined as the number of iterations exceeds 50. ANTRLF was applied on the gcol1 dataset and the optimal coloring s used 16 colors. There 6 diffrent strategies that are obtainaible in ANTRLF via combining heuristic information strategies and the strategy to the add the first vertex v_i to the color class C_k. The experimental results determined the selection of 3 and "Randomly selecting an uncolored vertex from W " as the best strategy. The results obtained are summarised in the table which are then followed by the graphs for each combination.

Combination	Number of Colors Used	Average Number of Colors used
1	16	17.923
2	16	17.481
3	16	17.974
4	18	19.978
5	18	19.365
6	17	19.32

From the results presented in the table, we can see that values obtained for average number of colors used were similar regardless of what strategy was choosen to add vertex v_i to the color class C_k. On the other hand, choosing which strategy to select when adding the first vertex v_i to the color class C_k had a significant impact on the results that were obtained. The strategy of randomly selected an uncolored vertex from W performed better than selecting vertex v_i which has the maximum degree in W. Intitutively, this makes sense as randomization allows the search to escape the local optima, thereby increasing the diversity of the solutions obtained.

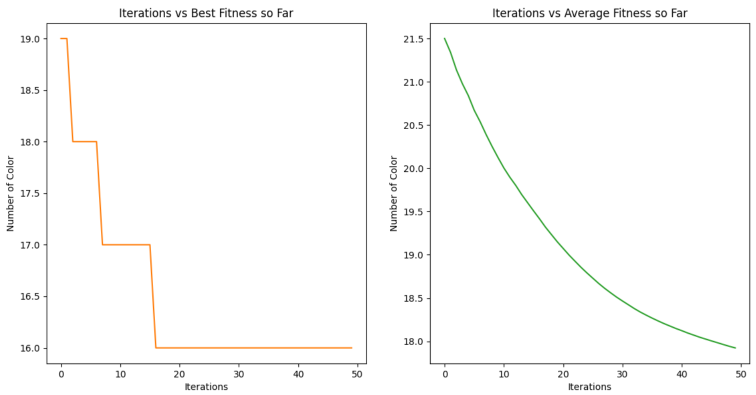

Figure 1: 2 and Randomly selecting an uncolored vertex from W.

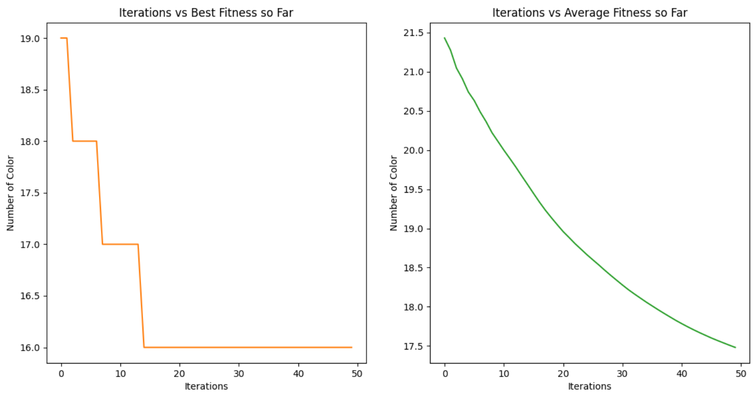

Figure 2: 3 and Randomly selecting an uncolored vertex from W.

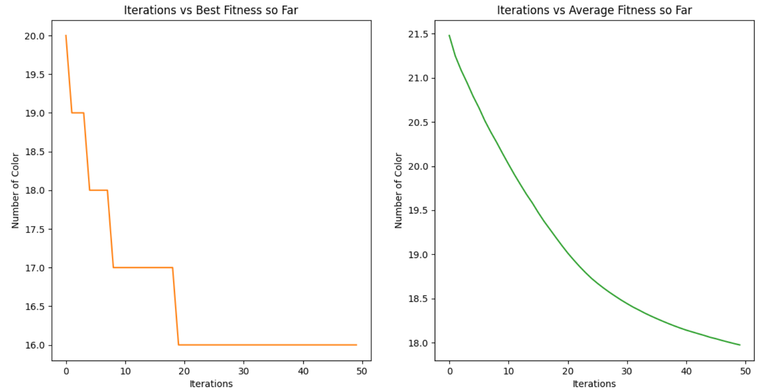

Figure 3: 4 and Randomly selecting an uncolored vertex from W.

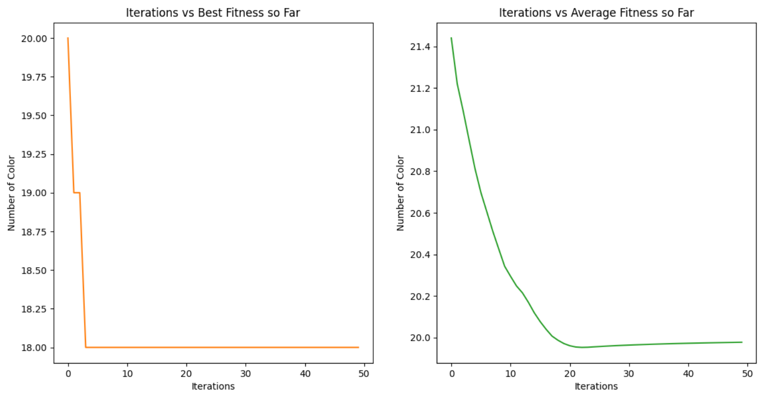

Figure 4: 2 and selecting uncolored vertex from W with maximum degree

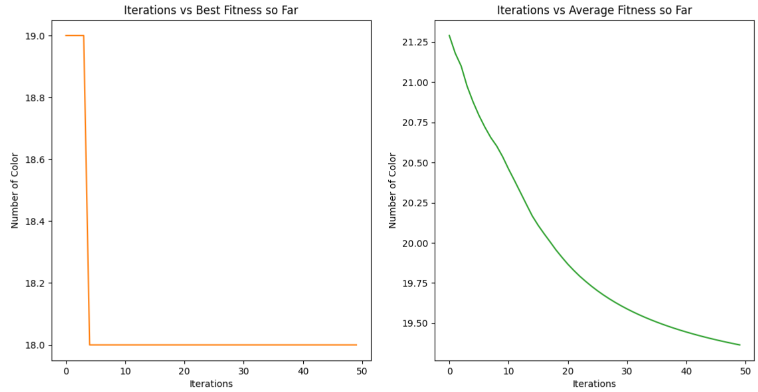

Figure 5: 3 and selecting uncolored vertex from W with maximum degree

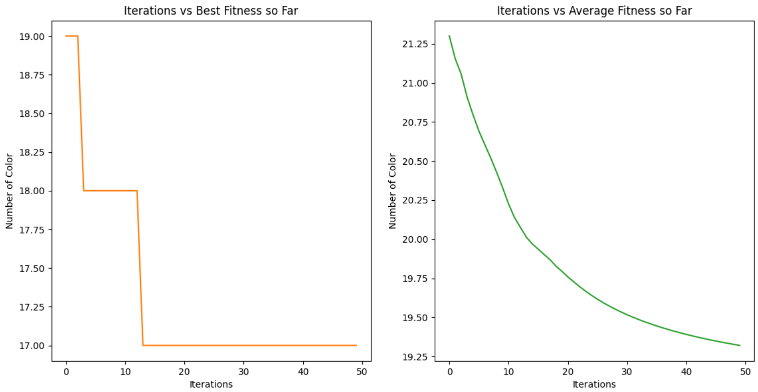

Figure 6: 4 and Randomly selecting an uncolored vertex from W.

Adjusment of the Parameters

Various test were performed in order to obtain appropriate values of the parameters governing the search in the ANTLRF algorithim. The results obtained were compared with graph in 2 to determine the effacy of the changes made. When adjusting the values of diffrent parameters, the value of other parameters was kept constant. The results are summarised in the following table. [2]

[2]Since this a minimisation problem, we want the value of fitness to decrease.

Parameter	Value	Best fitness so far	Average fitness so far	Change in Best Fitness so Far.	Change in Average Fitness so Far
α	1	18	20.735	2	3.254
β	2	16	17.632	0	0.151
ρ	0.1	16	19.402	0	1.921
N_{cycles}	100	15	16.718	-1	-0.763
N_{ants}	300	16	17.101	0	-0.38
Q	10	16	17.938	0	0.457

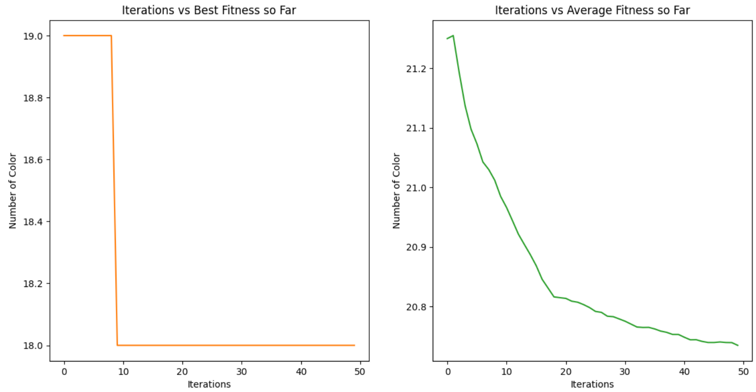

Figure 7: Influence of $\alpha = 1$ on the perfomance of ANTCOL algorithim

When we decrease the value of $\alpha = 1$, the quality of the optimal solution also decreased. Ants follow the path which has a higher pheromone concentration. In turn, the pheromone concentration increases after the increasing number of ants. Overtime, the solution converges to the shortest path. However, decreasing the value of α means there is a less chance of picking a route with high pheremone concentration. The solution obtained does not converge to the shortest path and the solution that is obtained is of a lower quality.

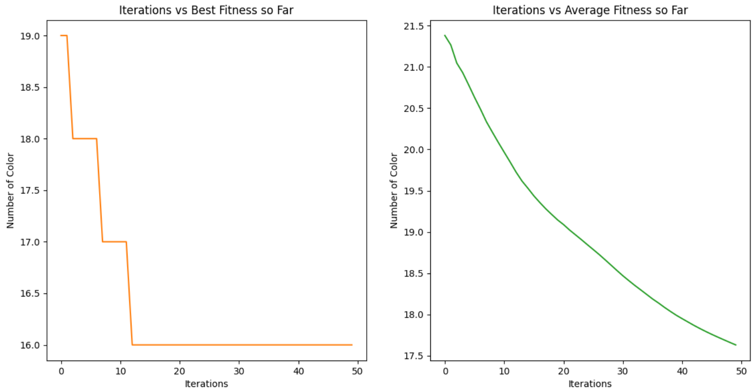

Figure 8: Influence of $\beta = 2$ on the perfomance of ANTCOL algorithim

Decreasing the value of $\beta = 2$ did not have a significant impact on the quality of the coloring that was obtained. This is because the probability of choosing a solution with a higher pheromone concentration is controlled by determined by α while β determines the desirability of a particular solution.

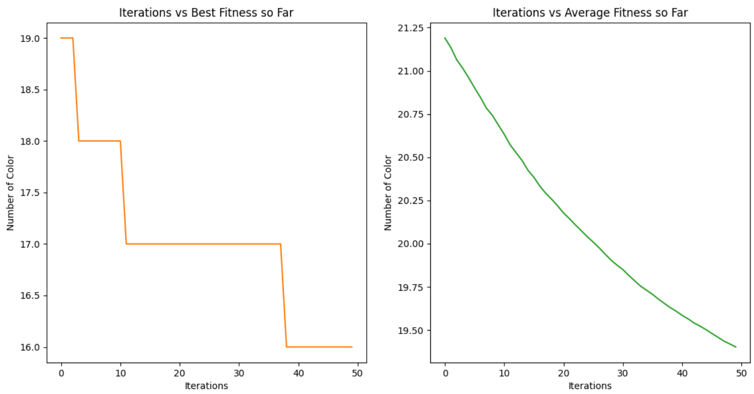

Figure 9: Influence of $\rho = 0.1$ on the perfomance of ANTCOL algorithim

Decreasing the value of $\rho = 0.1$ had a negeligble impact on the quality of the solutions that were obtained but the average quality of the solutions that were obtained greatly decreased. This because a higher concentration of pheremone can lead to risk of being stuck in the local optima. Decreasing the evaporation rate, means that pheremone concentration is builds up at a faster rate and ants follow the router which has a higher concentration of pheremone.

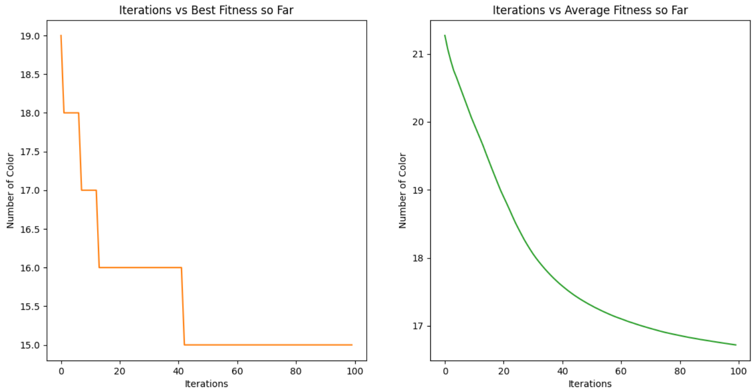

Figure 10: Influence of $N_{cycles} = 100$ on the perfomance of ANTCOL algorithim

Increasing the value of $N_{cycles} = 100$ improved the quality of the optimal solution greatly. This because we are using randomization to add the first vertex v_i to the color class C_k and that can result in ineffecient exploring of the search space. A suitable number of iterations would allow the search to escape the local optima and converge to the optimal solution.

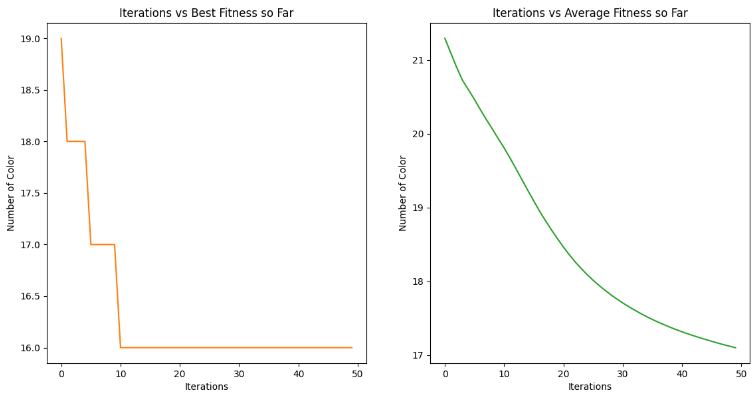

Figure 11: Influence of $N_{ants} = 300$ on the perfomance of ANTCOL algorithim

Although increasing the $N_{ants} = 300$ did not improve the optimal coloring that was obtained the average quality of the solutions that were obtained improved. This is because as increasing number of ants travel through a route which has high pheremone concentration, the obtained solution will converge to the optimal solution faster i.e 10 iterations instead of about 15 iterations. One downside of increasing the number of ants is that the time taken to execute increases linearly with the size of ant colony.

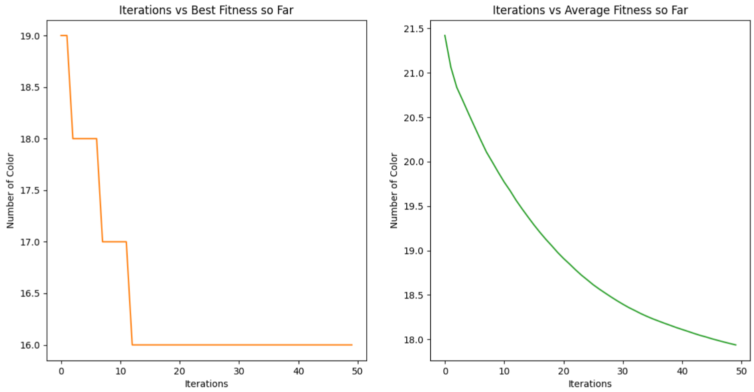

Figure 12: Influence of $Q = 10$ on the perfomance of ANTCOL algorithim

Increasing the value of $Q = 10$ didn't have a significant affect on the quality of the colorings that were obtained. This is because Q is a constant and acts as a scaling constant of the pheromone trail values.

Conclusion

Based on the results, we saw that using a random strategy to choose the first vertex led to diverse solutions and prevented the search from being stuck in a local optima. Furthermore, the trail factor plays an essential role in the quality of the results that were obtained. Moreover, we saw that a larger colony of ants meant a tradeoff between the rate of convergence to a optimal solution and the running time of the algorithim. Another factor that played a huge role in improving the quality of solutions was the number of cycles. Fewer cycles could lead to the solution not convreging an on the other hand larger number of cycles could lead to uneccessary exploration in the search space. The value of $\rho = 0.5$ is appropriate as decreasing the evaporation rate can lead to premature convergence.

References

[1] E. Salari and K. Eshghi, "An ACO algorithm for graph coloring problem," *2005 ICSC Congress on Computational Intelligence Methods and Applications*, Istanbul, Turkey, 2005, pp. 5 pp.-, doi: 10.1109/CIMA.2005.1662331.

[2] D. Costa and A. Hertz, "Ants can colour graphs," *The Journal of the Operational Research Society*, vol. 48, no. 3, p. 295, 1997.