

# Voting power in the Council of the European Union: A comprehensive sensitivity analysis

Dóra Gréta Petróczy\*

László Csató†

29th December 2023

*“Ideally, every responsible human being should have equal power in a world assembly. This situation, however, can only arise in an assembly of nations of equal sizes, each enfranchised to the same degree. To award nations voting powers proportional to their populations would give the spokesmen for large nations too much power in relation to the numbers of their sponsors.”<sup>1</sup>*

## Abstract

The Council of the European Union (EU) is one of the main decision-making bodies of the EU. Many decisions require a qualified majority: the support of 55% of the member states (currently 15) that represent at least 65% of the total population. We investigate how the power distribution, based on the Shapley-Shubik index, and the proportion of winning coalitions change if these criteria are modified within reasonable bounds. The influence of the two countries with about 4% of the total population each is found to be almost flat. The level of decisiveness decreases if the population criterion is above 68% or the states criterion is at least 17. The proportion of winning coalitions can be increased from 13.2% to 20.8% (30.1%) such that the maximal relative change in the Shapley–Shubik indices remains below 3.5% (5.5%). Our results are indispensable to evaluate any proposal for reforming the qualified majority voting system.

**Keywords:** cooperative game theory; European Union; qualified majority voting; power index; sensitivity analysis

**MSC class:** 91A80, 91B12

**JEL classification number:** C71, D72

---

\* Corresponding author E-mail: [petroczy.dora@nje.hu](mailto:petroczy.dora@nje.hu)

MNB Institute, John von Neumann University, Budapest, Hungary

†E-mail: [laszlo.csato@sztaki.hun-ren.hu](mailto:laszlo.csato@sztaki.hun-ren.hu)

HUN-REN Institute for Computer Science and Control (SZTAKI), Laboratory on Engineering and Management Intelligence, Research Group of Operations Research and Decision Systems, Budapest, Hungary

Corvinus University of Budapest (BCE), Institute of Operations and Decision Sciences, Department of Operations Research and Actuarial Sciences, Budapest, Hungary

<sup>1</sup> Source: [Penrose \(1946, p. 56\)](#).

# 1 Introduction

The Council of the European Union is one of two legislative bodies in the European Union (EU) besides the European Parliament. Since the EU Council is an explicitly intergovernmental institution where each country is represented by a national minister, power distribution within the Council has a fundamental impact on the member states: the power politics view can explain as much as 90% of the budget shares (Kauppi and Widgrén, 2004). Although the role of political power may be lower, it is certainly a significant driver of budgetary allocation (Zaporozhets et al., 2016).

The distribution of power in any institution depends on the voting rules. While some decisions require unanimity among the Council members, all major treaties have shifted further policy areas from unanimity to qualified majority. The definition of qualified majority was subject to many reforms in the past, especially because new members should have been provided with voting weights, which usually led to the modification of thresholds, too (Le Breton et al., 2012). Unsurprisingly, the qualified majority voting system of the European Union has been analysed by several studies both in the fields of operational research (Freixas, 2004; Algaba et al., 2007; Alonso-Meijide et al., 2009) and social choice theory (Brams and Affuso, 1976, 1985; Felsenthal and Machover, 2001).

Currently, Article 16 of the Treaty on European Union states the following conditions for a qualified majority:

- *Population criterion*: the supporting countries should represent at least 65% of the EU population;
- *States criterion*: the supporting countries should represent at least 55% of the EU member states.

In addition, any blocking minority needs to contain four member states. This system is effective from 1 November 2014.

Since any change in the set of member states is guaranteed to change voting powers, the effects of Brexit (Grech, 2021; Kóczy, 2021), as well as other possible entries (Kirsch, 2022) and exits (Petróczy et al., 2022) have been thoroughly analysed. However, we do not know any work that investigates how the power distribution will change if the majority rule of 65-55% is modified. The current paper aims to fill this research gap.

The issue is interesting not only for the academic community. The German government has recently taken several steps towards the wider use of qualified majority voting (OSW, 2023). In particular, a Franco-German working group made several recommendations for the institutional reform of the EU on 18 September 2023, including the adjustment of the thresholds to 60-60%. But the group of twelve experts has not examined the impact of their proposal—which will be presented in the following, among many other results.

A possible reason for the lack of sensitivity analyses in the extant literature is that measuring voting power in a weighted voting system where voters have different weights is far from trivial. For instance, consider a simple situation with two voters,  $A$  and  $B$ , having weights of 2 and 4, respectively. If the decision threshold is 3, the second voter  $B$  is a dictator and holds all power. On the other hand, if the decision threshold is 5, both voters are veto players and they have equal power.

Several power indices have been proposed to quantify the influence or power of a voter in similar situations. According to our knowledge, the first suggestion has been made by Penrose (1946). Nonetheless, his approach has gained more attention only after Banzhaf (1965) and Coleman (1971) have “rediscovered” it; thus, it is called the

Penrose–Banzhaf–Coleman, or, simply, Banzhaf index. The other popular power measure is the Shapley–Shubik index (Shapley and Shubik, 1954). In addition, since the power indices are usually defined via a weighted voting game, the influence of the voters can be quantified by essentially any solution concept of cooperative game theory such as the nucleolus (Schmeidler, 1969; Zaporozhets et al., 2016).

Our calculations are based on the Shapley–Shubik index, which can be considered as the expected relative share of the country in a fixed prize available to the winning coalition (Felsenthal and Machover, 2004). Because the computation of power indices is complex due to the exponential complexity of the algorithms (Bilbao et al., 2002), we provide numerical results on how power distribution in the EU Council depends on the population and states criteria. It is assumed that the proportion of total population required to accept a proposal varies between 51% and 80%, while the number of states required lies between 11 (40.7%) and 20 (74.1%).

The main findings of our sensitivity analysis can be summarised as follows:

- The voting power of a country comprising about 4% of the total population (the Netherlands and Romania) is almost insensitive to both thresholds of qualified majority (Section 4.1);
- The current proportion of winning coalitions (13.2%) will decline if the population criterion is above 68% or the states criterion is at least 17 (Section 4.2);
- Decisiveness can be improved only by lowering the states threshold if the level of inequality (the influence of large countries) is not allowed to decrease (Section 4.2);
- The recently proposed 60-60% rule, which requires the support of 17 member states, strongly favours small countries (the level of inequality is reduced by more than two-thirds) and substantially reduces the proportion of winning coalitions (Section 4.2);
- A rule with a higher decision probability than the current 13.2% is disadvantageous for either the majority of the countries or the majority of the population (Section 4.3);
- The decision probability can be increased to 20.8% (30.1%) such that the maximal relative change in the Shapley–Shubik indices remains below 3.5% (5.5%) (Section 4.3).

Clearly, these results are indispensable to evaluate any proposal for reforming the criteria of the qualified majority voting system used in the EU Council.

The paper has the following structure. Section 2 provides a concise overview of related papers. Section 3 introduces the basic concepts of our approach (the Shapley–Shubik index, the decision probability, and the Herfindahl–Hirschman index) that are used for the sensitivity analysis in Section 4. Finally, Section 5 ends with some concluding remarks.

## 2 Related literature

The paper by Brams and Affuso (1976) is probably the first academic demonstration that Luxembourg was a null player in the Council of Ministers in the predecessor of the European Union. Between 1958 and 1973, the weight of France, Germany, and Italy

was 4, Belgium and the Netherlands had a weight of 2, while Luxembourg had 1. The qualified majority required 12 out of the 17 votes, hence, Luxembourg exercised absolutely no influence on decisions. [Brams and Affuso \(1976\)](#) have used the same example of the Council to uncover a real-life occurrence of the new member paradox, too: Luxembourg gained more voting power with the joining of Denmark, Ireland, and the United Kingdom in 1973. Similarly, the relative weight of Luxembourg continued to decline after Greece's accession in 1981, but its voting power increased ([Brams and Affuso, 1985](#)). In addition, Denmark, Ireland, and Luxembourg had the same voting power.

[Widgrén \(1994\)](#) calculates the effect of two possible enlargements, the potential joining of Sweden, Austria, Finland, and Norway (EC16), as well as Switzerland, Iceland, and Liechtenstein (EC19). Although these enlargements have never happened, the study considers not just the Banzhaf and Shapley–Shubik indices but takes into account possible a priori coalitions (France–German axis, Benelux countries, Mediterranean countries, Nordic countries). [Felsenthal and Machover \(1997\)](#) analyse the impact of four historical enlargements (1973, 1981, 1985, 1995), and determine a responsiveness index of the voting rule, which is a measure of decision ability.

[Laruelle and Widgrén \(1998\)](#) examine some concepts of fairness in the EU voting system. If the Union is a loose federation of independent states, then the “one state, one vote” principle should be satisfied. However, if it is treated as one large country, then the “one man, one vote” principle should be met: the weight of each member state needs to be proportional to its population. Finally, if the EU is a federation between states, then a fair solution must lie somewhere between these two extremities. The weighted qualified majority voting system used between 1995–2004 is shown to be unfair in all three senses.

First, the Treaty of Nice of 2001 took the populations of the member states into account. A proposal could be adopted if the supporting countries had at least 74% of the weights and 62% of the population. According to [Felsenthal and Machover \(2001\)](#), these quotas were fair as each voter had about the same influence in the Council, but they were too high and almost paralysed decision-making. [Freixas \(2004\)](#) verified that the dimension of this voting system was 3, which is a potential measure of complexity.

[Leech \(2002\)](#) essentially reproduced the work of [Felsenthal and Machover \(2001\)](#), and proposed a fair quota system before the 2004 EU enlargement, too, in which all citizens have roughly equal influence on decisions, regardless of the number of member states. The suggested algorithm adjusts the weights until the relative voting power and the population share are sufficiently close. [Słomczyński and Życzkowski \(2006\)](#) investigate a system based on the law of Penrose, where the weight of each country is proportional to the square root of its population.

[Kóczy \(2012\)](#) looks at the immediate impact of the Lisbon Treaty reform in 2014, as well as at the long-term effects of the different demographic trends in the countries. The reform is found to hurt medium sized countries, especially Central Eastern European states with declining populations.

[Casajus and Huettner \(2019\)](#) give an axiomatic characterisation of a new power index, called the Coleman–Shapley index. This decomposes the Banzhaf index into a voter's direct power as such and its impact on the power of the other voters by threatening to block any proposal. In contrast to the Shapley–Shubik index, the voters are not ordered by their agreement with a potential bill, but by their vested interest in it. The index is applied to the Council of the European Union and the UN Security Council. In particular, the differences between large and small countries are revealed to be much smaller according to the Coleman–Shapley index than according to the Shapley–Shubik index.

Several studies have shown independently that the Brexit has mainly benefited the large countries (Gábor, 2020; Grech, 2021; Göllner, 2017; Kirsch, 2016; Kirsch et al., 2018; Kóczy, 2021; Mercik and Ramsey, 2017; Szczypińska, 2018). On the other hand, a further exit would harm the large and benefit the smaller countries due to retaining the states criterion at 15 (Petróczy et al., 2022). Kirsch et al. (2018) use the normal approximation of the Banzhaf index in double-majority games to uncover that such non-monotonicity is in most cases inherent in a double-majority system, however, it is strongly exacerbated by the peculiarities of the population vector in the European Union.

Kirsch (2022) investigates the impact of the accession of Montenegro, Turkey, and Ukraine on the current 27 members. In this case, the 55% states threshold will be raised from 15 to 16. Consequently, the power of small countries would increase, and the power of large countries would decrease. Because of its significant size, the accession of Turkey would tip the balance of power: France, Poland, Poland, Germany, Italy, and Spain would lose almost 20% of their voting power.

### 3 Methodology

This section overviews our tools to be used for the sensitivity analysis: the Shapley–Shubik index is introduced in Section 3.1, while an aggregated measure of decisiveness and inequality, respectively, is overviewed in Section 3.2. Finally, Section 3.3 illustrates all these methods.

#### 3.1 A measure of voting power: the Shapley–Shubik index

Voting situations are usually modelled by a cooperative game with transferable utility where the voters are the players, and the value of any coalition is maximal if it can accept a proposal and minimal otherwise.

Let  $N$  denote the set of players and  $S \subseteq N$  be a coalition. The cardinal of a set is denoted by the corresponding small letter, namely, the number of players in coalition  $S$  is  $|S| = s$ , and the number of players is  $|N| = n$ . The value of any coalition is given by the characteristic function  $v : 2^N \rightarrow \mathbb{R}$ .

**Definition 3.1.** *Simple voting game:* A game  $(N, v)$  is called a simple voting game if

$$v(S) \in \{0, 1\} \text{ for all } S \subseteq N.$$

Let vector  $\mathbf{w} \in \mathbb{R}^n$  denote the weights of the players and  $q \in [0, 1]$  denote the decision threshold.

**Definition 3.2.** *Weighted voting game:* A game  $(N, v, \mathbf{w}, q)$  is called a weighted voting game if for any coalition  $S \subseteq N$ :

$$v(S) = \begin{cases} 1 & \text{if } \sum_{j \in S} w_j \geq q \\ 0 & \text{otherwise.} \end{cases}$$

The influence of the players can be quantified by power indices. As we have already seen, one popular measure for this purpose is the Shapley–Shubik index.

**Definition 3.3.** *Shapley–Shubik index:* Let  $(N, v, \mathbf{w}, q)$  be a weighted voting game. The Shapley–Shubik index of player  $i$  is

$$\varphi_i(N, v, \mathbf{w}, q) = \sum_{S \subseteq N \setminus \{i\}} \frac{s!(n-s-1)!}{n!} [v(S \cup \{i\}) - v(S)].$$

Consider a random order of the players. The Shapley–Shubik index of a player is its marginal contribution to the coalition formed by the preceding players, averaged over the set of all the possible  $n!$  orders. Consequently, the Shapley–Shubik index of a player in a weighted voting game is the probability that it becomes pivotal if the sequence of the players is chosen randomly.

In our sensitivity analysis, the Shapley–Shubik indices will be calculated for all the 27 current EU member states. The population criterion will be investigated between 51% and 80%. 51% is a natural lower bound, while it does not make sense to choose a quota over 80% since Germany accounts for 18.59% of the total population: in the absence of the specific blocking minority rule, the largest country would be a veto player for a threshold of at least 82%. The number of member states required for a qualified majority is assumed to be between 11 (which is 40.7% of the number of EU countries) and 20 (74.1%). A lower limit would be harmful for the small states, and a higher limit would make it challenging to accept any proposal.

The population shares of the countries, which determine their weights, are updated every year. We use the weights from the Council Decision (EU, Euratom) 2022/2518 of 13 December 2022 amending the Council’s Rules of Procedure ([Council of the European Union, 2022](#), Annex III); they are presented in Table 1 together with the Shapley–Shubik indices. The latter are calculated by the python package *powerindices* of Frank Huettner ([Huettner, 2023](#)). These are exact values, not approximations by simulations.

However, the blocking minority rule that requires at least four countries is disregarded. It has a marginal role as the number of blocking coalitions always remains above 50% (see Section 4.2), that is, at least  $2^{27}/2 = 67,108,864$  blocking coalitions exist, from which only 21 are excluded. Furthermore, we mainly focus on the differences compared to the current power indices. Ignoring the blocking minority rule is standard in the literature, see, for example, [Kirsch \(2022, p. 404\)](#): “The effect of the aforementioned “additional rule,” i.e., forbidding blocking decisions by only three states, is negligible in the present context. For the current EU, there are only 22 coalitions of 24 states which do not represent 65% of the total population of the EU. This number is of (almost) no consequence for the power indices.” Note that the number of coalitions affected by the blocking minority rule depends on population shares of the countries; while there are 21 such coalitions under the 2022 data, this increases to 22 under the 2023 data, as well as under the data used in [Kirsch \(2022\)](#).

## 3.2 Quantifying decisiveness and inequality

For any qualified majority criteria, the decision ability of the European Union is quantified by the proportion of winning coalitions, similar to [Kirsch \(2022\)](#).

**Definition 3.4.** *Decision probability:* Let  $(N, v, \mathbf{w}, q)$  be a weighted voting game. The decision probability is the number of winning coalitions divided by the number of possible coalitions  $2^n$ .

Since the 27 member states have 27 different power indices, it is difficult to assess their changes. Therefore, a widely used measure of market concentration, the *Herfindahl–Hirschman index* ([Herfindahl, 1950](#); [Hirschman, 1945, 1964](#)), is used to measure the inequality of the power distribution.

**Definition 3.5.** *Herfindahl–Hirschman index (HHI):* Let  $(N, v, \mathbf{w}, q)$  be a weighted voting game,  $|N| = n$  be the number of players, and  $\varphi = [\varphi_i]$  be the vector Shapley–Shubik



Table 1: Populations and voting powers in the European Union

| Country        | Abbreviation | Population share | Shapley–Shubik index |
|----------------|--------------|------------------|----------------------|
| Germany        | DE           | 18.59%           | 17.87%               |
| France         | FR           | 15.16%           | 13.60%               |
| Italy          | IT           | 13.32%           | 11.69%               |
| Spain          | ES           | 10.60%           | 9.15%                |
| Poland         | PL           | 8.41%            | 6.84%                |
| Romania        | RO           | 4.25%            | 3.83%                |
| Netherlands    | NL           | 3.96%            | 3.61%                |
| Belgium        | BE           | 2.60%            | 2.62%                |
| Greece         | EL           | 2.37%            | 2.45%                |
| Czech Republic | CZ           | 2.36%            | 2.44%                |
| Sweden         | SE           | 2.33%            | 2.43%                |
| Portugal       | PT           | 2.31%            | 2.41%                |
| Hungary        | HU           | 2.16%            | 2.31%                |
| Austria        | AT           | 2.00%            | 2.19%                |
| Bulgaria       | BG           | 1.53%            | 1.85%                |
| Denmark        | DK           | 1.31%            | 1.70%                |
| Finland        | FI           | 1.24%            | 1.64%                |
| Slovakia       | SK           | 1.21%            | 1.62%                |
| Ireland        | IE           | 1.13%            | 1.56%                |
| Croatia        | HR           | 0.86%            | 1.37%                |
| Lithuania      | LT           | 0.63%            | 1.19%                |
| Slovenia       | SI           | 0.47%            | 1.08%                |
| Latvia         | LV           | 0.42%            | 1.05%                |
| Estonia        | EE           | 0.30%            | 0.96%                |
| Cyprus         | CY           | 0.20%            | 0.89%                |
| Luxembourg     | LU           | 0.14%            | 0.85%                |
| Malta          | MT           | 0.12%            | 0.83%                |

Notes: Abbreviations are ISO 3166-1 alpha-2 codes.

Population shares come from the Council Decision (EU, Euratom) 2022/2518 ([Council of the European Union, 2022](#)), Shapley–Shubik indices are own calculations.

indices. The Herfindahl–Hirschman index is

$$HHI(\varphi) = \sum_{i=1}^N \varphi_i^2.$$

The maximum of  $HHI$  is 1, reached if and only if a dictator exists in the game. The minimum of  $HHI$  is  $1/n$ , reached if and only if all players have the same power. Since the lower bound of  $HHI$  depends on the number of players, it is usual to consider its normalised version that lies between 0 and 1 ([Petróczy and Csató, 2021](#)).

**Definition 3.6.** *Normalised Herfindahl–Hirschman index ( $HHI^*$ ):* Let  $(N, v, \mathbf{w}, q)$  be a weighted voting game,  $|N| = n$  be the number of players, and  $\varphi = [\varphi_i]$  be the vector Shapley–Shubik indices. The normalised Herfindahl–Hirschman index is

$$HHI^*(\varphi) = \frac{HHI(\varphi) - 1/n}{1 - 1/n}.$$

Table 2: The sensitivity analysis of Example 1 with respect to the quota  $q$

| Threshold ( $q$ ) | $A$ | Voter<br>$B$ | $C$ | Decision<br>probability | $HHI$ | $HHI^*$ |
|-------------------|-----|--------------|-----|-------------------------|-------|---------|
| 3                 | 2/3 | 1/6          | 1/6 | 62.5%                   | 1/2   | 1/4     |
| 4                 | 1   | 0            | 0   | 50%                     | 1     | 1       |
| 5                 | 2/3 | 1/6          | 1/6 | 37.5%                   | 1/2   | 1/4     |
| 6                 | 1/2 | 1/2          | 0   | 25%                     | 1/2   | 1/4     |
| 7                 | 1/3 | 1/3          | 1/3 | 12.5%                   | 1/3   | 0       |

The second, third, and fourth columns show the Shapley–Shubik indices for the corresponding voters.

### 3.3 An illustrative example

Take the following simple qualified majority voting system.

**Example 1.** A committee consists of three members  $A$ ,  $B$ , and  $C$  who have the voting weights 4, 2, and 1, respectively.

If the decision threshold in Example 1 is  $q = 4$ , then voter  $A$  is a dictator and has the maximal power of 1. Even though the ratio of the voting weights is 4:2:1, the power distribution is 1:0:0.

**Example 2.** Consider Example 1. The Shapley–Shubik indices of the voters are presented in Table 2 as a function of the majority threshold  $q$ .

We have already seen that voter  $A$  is a dictator and has a maximal influence if  $q = 4$ . Voter  $C$  has no power if  $q$  is odd, however, the powers of voters  $B$  and  $C$  are equal if  $q$  is even. Interestingly, the Shapley–Shubik index is not a monotonic function of the criterion  $q$  for any voter.

**Example 3.** Consider Example 1. There are three voters, hence, the number of coalitions is  $2^3 = 8$ . If the threshold is  $q = 4$ , then four winning coalitions exist ( $A$ ,  $A \cup B$ ,  $A \cup C$ ,  $A \cup B \cup C$ ), and the decision probability is  $4/8 = 50\%$ . Table 2 shows the proportion of winning coalitions for other values of  $q$ .

Since Example 1 contains only one majority threshold, decisiveness is a monotonically decreasing function of this parameter as expected.

**Example 4.** Consider Example 1. The values of the Herfindahl–Hirschman index, as well as its normalised version are summarised in Table 2 for various values of  $q$ . The power distribution has the maximal level of inequality if  $q = 4$ , however, all voters have equal power if  $q = 7$ .

Note that the value of  $HHI^*$  (and  $HHI$ ) remains the same if the threshold is increased from 5 to 6, even though the power of each voter changes.

## 4 Results

The effects of changing the qualified majority thresholds are analysed in three parts: Section 4.1 discusses Shapley–Shubik indices at the level of countries, Section 4.2 focuses on how the reform affects the whole European Union, and Section 4.3 compares different population and states quota pairs with respect to their plausibility.



## 4.1 Country-level changes in voting power

In the current double majority system, the voting power of any country depends exclusively on its population. Therefore, if two member states have approximately the same size, the dynamics of their Shapley–Shubik indices is similar. Six different patterns can be identified that are plotted in Figure 1.

Germany, the largest state, almost always benefits from a higher population criterion and a lower states criterion. Its current voting power could not decrease with a states quota of 11, but could not increase with a states quota of 20. Although its influence is now slightly below its share of the total population, this proportion is not impossible to exceed, especially for a lower states threshold.

Poland is also a large state, however, its current voting power remains substantially below its population share. The Shapley–Shubik index presents an unexpected function of the population quota: (1) it is flat, then increasing, then decreasing if 11 states are required for a majority; (2) it is increasing, then flat, then increasing, then decreasing if 15 or 17 states are required for a majority. The influence of Poland is higher for a lower states criterion, however, it rarely achieves its population share.

The Shapley–Shubik index of Romania is remarkably robust with respect to both decision thresholds. Its voting power can be slightly higher than its population share only for a high population criterion.

Belgium is a country whose voting power and population share almost coincide at the moment. Its Shapley–Shubik index is slightly decreasing as the population threshold grows, except if only 11 states are required for a qualified majority. Its voting power is lower for a lower states criterion.

Croatia benefits from a lower population limit and a higher states threshold. Voting power is intensively decreasing as a function of the population quota. However, the influence of Croatia is almost guaranteed to be higher than its share of the total population.

Finally, Malta is the smallest member state, hence, increasing the states criterion is strongly favourable for it. Nonetheless, the shapes of the functions are essentially analogous to the case of Croatia, although the gap between the voting power and the population share is naturally higher.

The Shapley–Shubik indices of the further 21 member states are not presented since their pattern are not fundamentally different from one of the six presented above.

## 4.2 Aggregated changes: decision probability and inequality

Figure 2 shows how the proportion of winning coalitions depends on the two decision thresholds. The general relationship is obvious: a higher population or states criterion reduces the number of coalitions which are able to accept a proposal. On the other hand, it is far from trivial that retaining the current level of decisiveness excludes a population quota over 68%, as well as a states quota over 16. The recently suggested 60-60% rule decreases the proportion of winning coalitions to 8.4%.

Figure 3 presents the connection between the proportion of winning coalitions and the level of inequality in power distribution, as a function of the two criteria of majority. For each sequence of dots, the 80% population quota can be found at the top left corner, which becomes lower by moving to the right. Under a fixed states criterion, a higher level of decisiveness usually implies less inequality, except for a high population criterion and a low states criterion. Consequently, if the large countries want to maintain the current *HHI*, the decisiveness of the EU can be improved only by choosing a lower threshold in

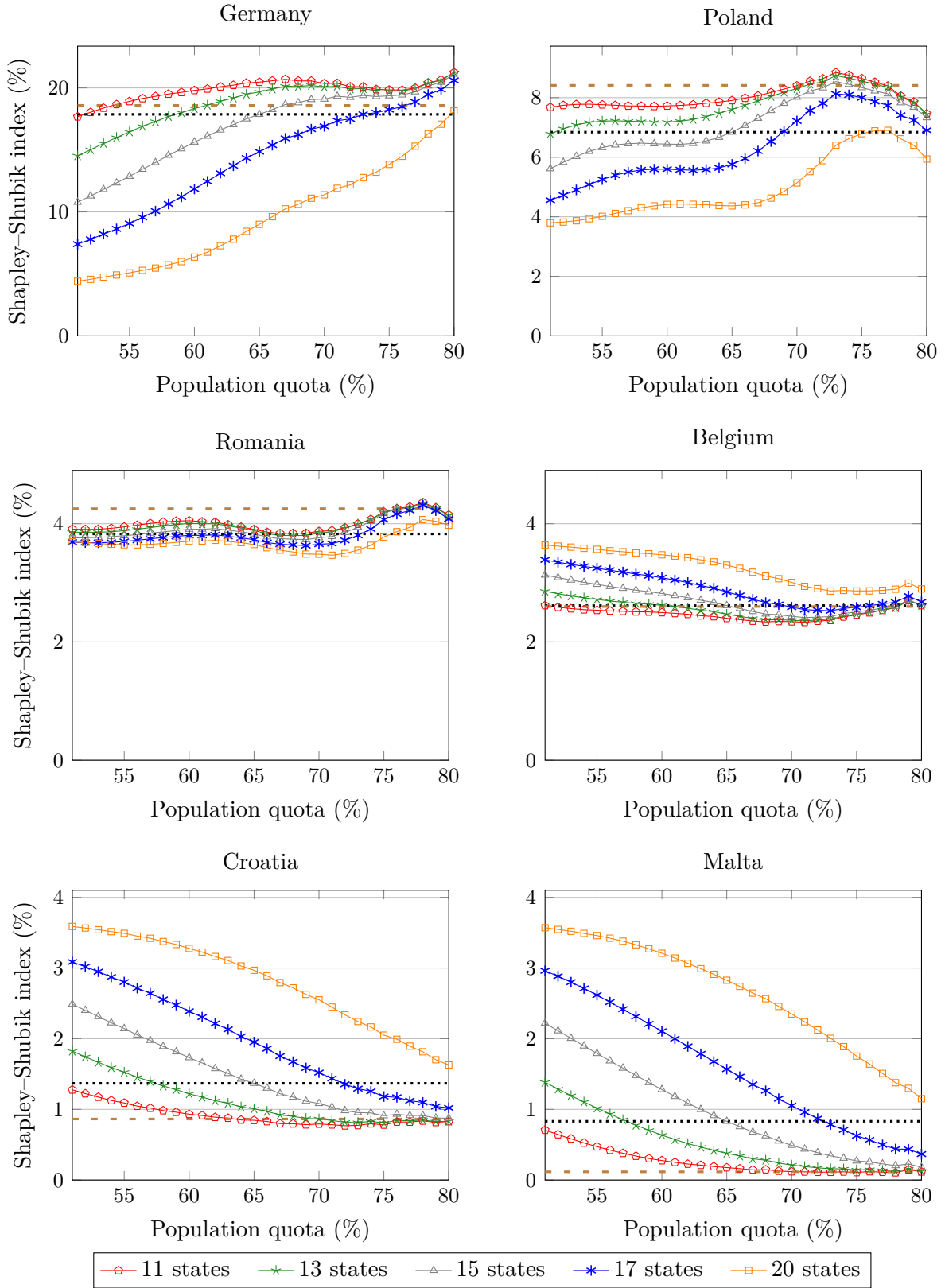


Figure 1: The voting power of six characteristic member states as a function the population and states criteria

*Notes:* The dotted black line shows the current voting power. The loosely dashed brown line shows the share of the EU population.

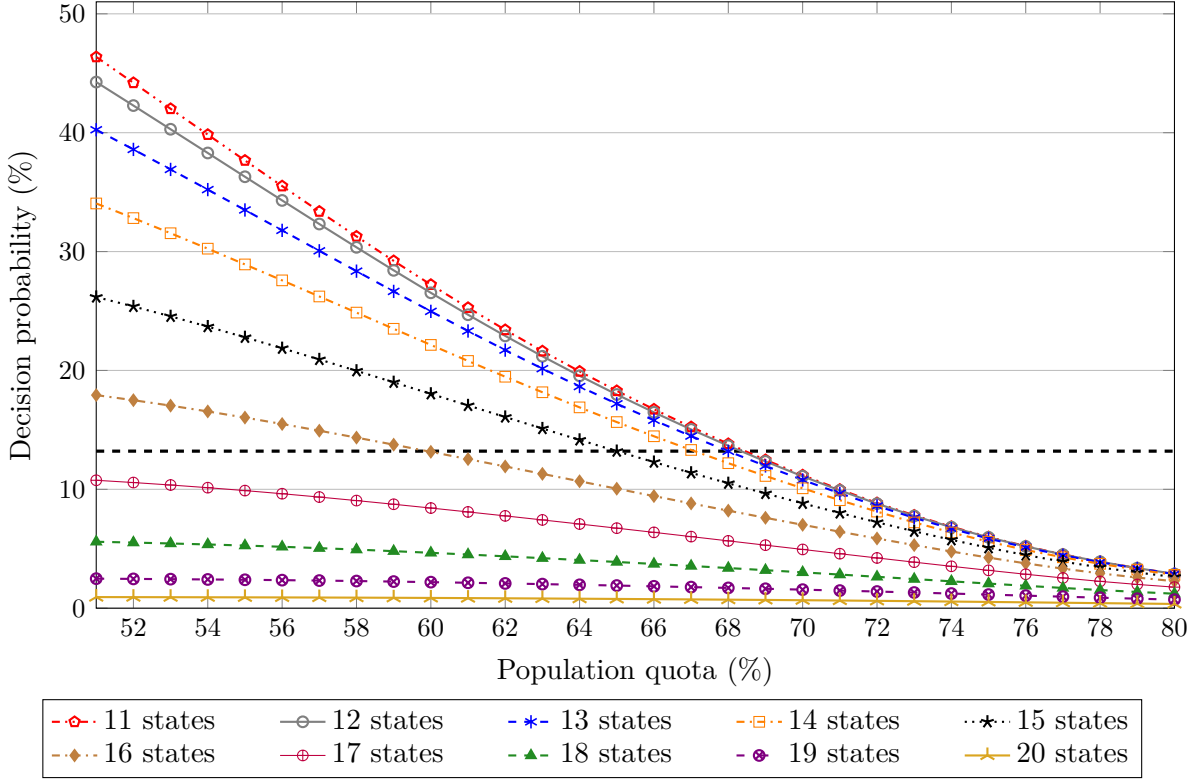


Figure 2: Decisiveness as a function of population and states criteria

*Note:* The dashed black line shows the current decision probability.

the number of states. Interestingly, if the states criterion is 11 or 12, the normalised Herfindahl–Hirschman index reaches its maximum approximately at the current decision probability of 13.2%. The recently suggested 60-60% rule reduces the value of  $HHI^*$  by more than two-thirds compared to the status quo (65-55%) as it decreases the population threshold and increases the states criterion to 17.

### 4.3 The optimal qualified majority criteria

Any change in the two qualified majority thresholds will reallocate voting powers among the member states, thus, it will produce some winners and losers as well. Here we focus on this aspect of the problem to understand which reforms can enjoy more support from the countries.

Figure 4 uncovers the proportion of winners: the number of states with a higher voting power (Figure 4.a) and their population shares (Figure 4.b). The number of winners varies between 4 and 23, while their proportion of total population remains between 20.03% and 84.66%. Unsurprisingly, it is almost impossible to favour both the majority of citizens and the majority of countries, although two extreme sets of quotas (79% of the population and 19 member states; 80% of the population and 20 member states) are beneficial for 23 countries representing 51.8% of the population. Generally, any change is good for either the numerous small member states without a majority of the total population, or for the few large countries where more than half of the people live.

However, some pairs of quota clearly seem to be dominated. For example, the qualified majority rule (63%, 14#) favours 7 states representing 75% of the population, while the neighbouring rule (62%, 14#) is advantageous for 12 countries representing 78.54% of the

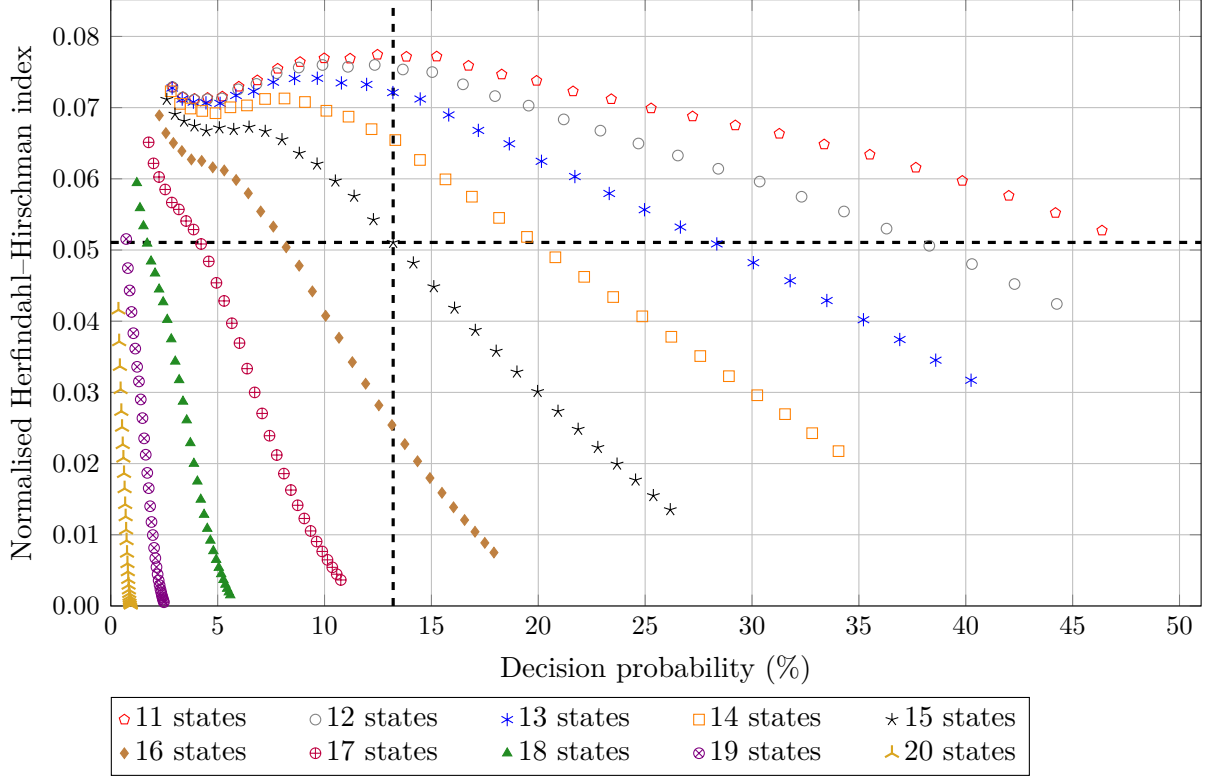


Figure 3: The relationship between decisiveness and inequality

*Note:* The vertical (horizontal) dashed black line shows the current decision probability (normalised Herfindal–Hirschman index).

Table 3: Maximal relative losses in the Shapley–Shubik indices compared to the current qualified majority rule (65%, 15#) with a decision probability of 13.2%

| Majority criteria |                  | Maximal loss |         | Decision probability |
|-------------------|------------------|--------------|---------|----------------------|
| Population        | Number of states | Value        | Country |                      |
| 61%               | 14               | 1.98%        | Spain   | 20.8%                |
| 64%               | 15               | 2.54%        | Poland  | 14.2%                |
| 72%               | 17               | 3.19%        | Austria | 4.2%                 |
| 57%               | 13               | 3.20%        | Germany | 30.1%                |
| 67%               | 16               | 3.21%        | Romania | 8.8%                 |

population. Therefore, the rule (62%, 14#) is more likely to be accepted instead of (63%, 14#). Analogously, more countries (11 versus 8) and people (84.66% instead of 77.6%) benefit from the rule (59%, 13#) than from the rule (60%, 13#). Consequently, even though a rule with a higher decision probability is disadvantageous for either the majority of the countries or the population, an analysis of the beneficiaries reveals the advantage of some qualified majority thresholds.

Last but not least, even if many member states with a substantial share of total population are favoured by a change, a country will strongly oppose this reform if it loses a lot of voting power. Thus, examining the maximal relative loss is also important because it might reflect the level of resistance against the change. Figure 5 presents these values such that the size of the dots is proportional to the maximal relative loss. Quota pairs along the diagonal generate the smallest maximal losses; the five minimal values are reported in

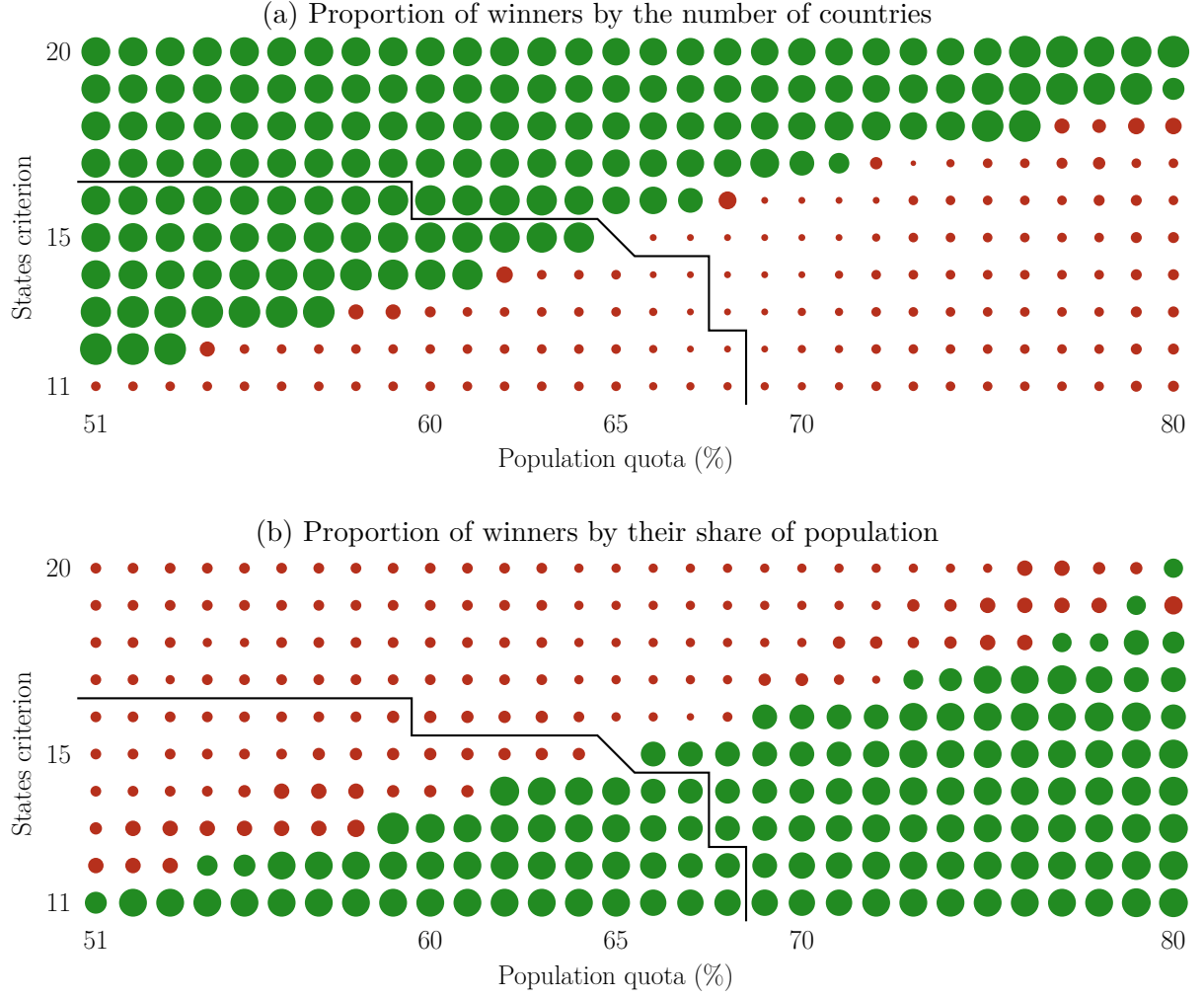


Figure 4: Proportion of winners in voting power as a function of the two criteria

*Notes:* The size of the dots is proportional to the proportion of winners in voting power.

A green (red) dot indicates that the proportion of winners does (not) achieve the majority (50%).

The thick black line shows the boundary corresponding to the current decision probability; all points below left (above right) to this line have a higher (lower) number of winning coalitions compared to the current situation.

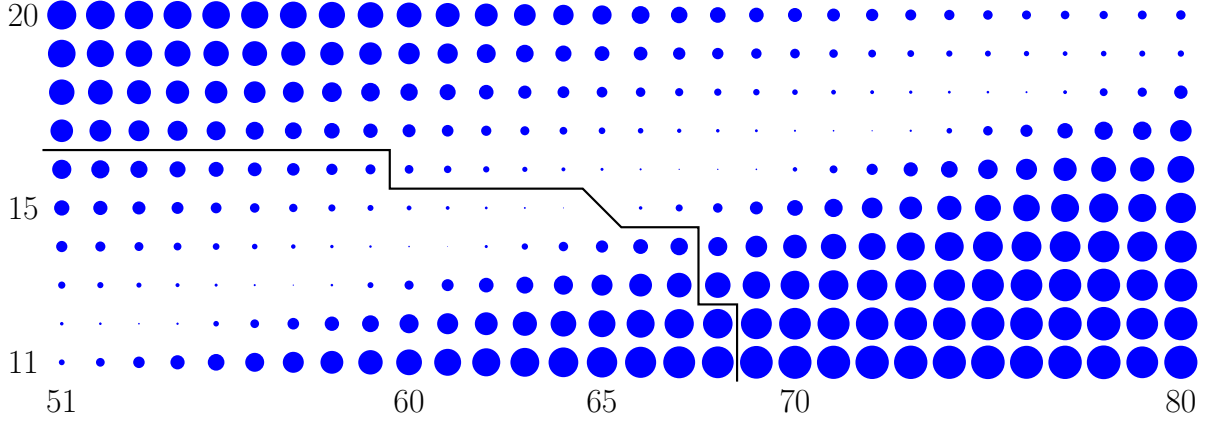


Figure 5: Maximal relative loss in voting power as a function of the two criteria

*Notes:* The size of the dots is proportional to the maximal relative loss in voting power.

The thick black line shows the boundary corresponding to the current decision probability; all points below left (above right) to this line have a higher (lower) number of winning coalitions compared to the current situation.

Table 3.

To summarise, if the level of decisiveness is not allowed to decrease, the rule (61%, 14#) seems to be the best option, followed by (57%, 13#), since (64%, 15#) is close to the status quo. The implied changes in the influence of each country are plotted in Figure 6. Crucially, the current decision probability of 13.2% can be increased to 20.8% such that the maximal change in the Shapley–Shubik indices remains below 3.5%, or even to 30.1% such that the maximal relative change in the Shapley–Shubik indices remains below 5.5%.

## 5 Conclusions

The Council of the European Union applies a complex rule of qualified majority since 2014: a decision requires the support of at least 55% of the member states (currently 15 countries) that represent 65% of the total population. We have investigated the effects of modifying these arbitrary thresholds with respect to the power of individual countries, the (in)equality of the power distribution, and the decisive ability of the EU. According to our findings, even though any reform will be harmful to the majority of either the countries or the population, policy-makers can choose some quota pairs that minimise resistance and substantially decrease/increase the decision ability of the EU.

Because the complicated voting system of the Council of the European Union is able to produce unexpected and unwanted changes, similar analyses of power indices are important to understand and, possibly, to improve it. Hopefully, the current study will inspire further research along this line. In particular, it would be instructive to look at another widely used power measure, the Banzhaf index: although the Shapley–Shubik and Banzhaf indices have fairly similar values in many cases, they can behave quite differently in certain settings (Felsenthal and Machover, 1998, p. 277–278).

## Acknowledgements

We are grateful to *Frank Huettnner* for developing the python package *powerindices*.

The research was supported by the National Research, Development and Innovation Office



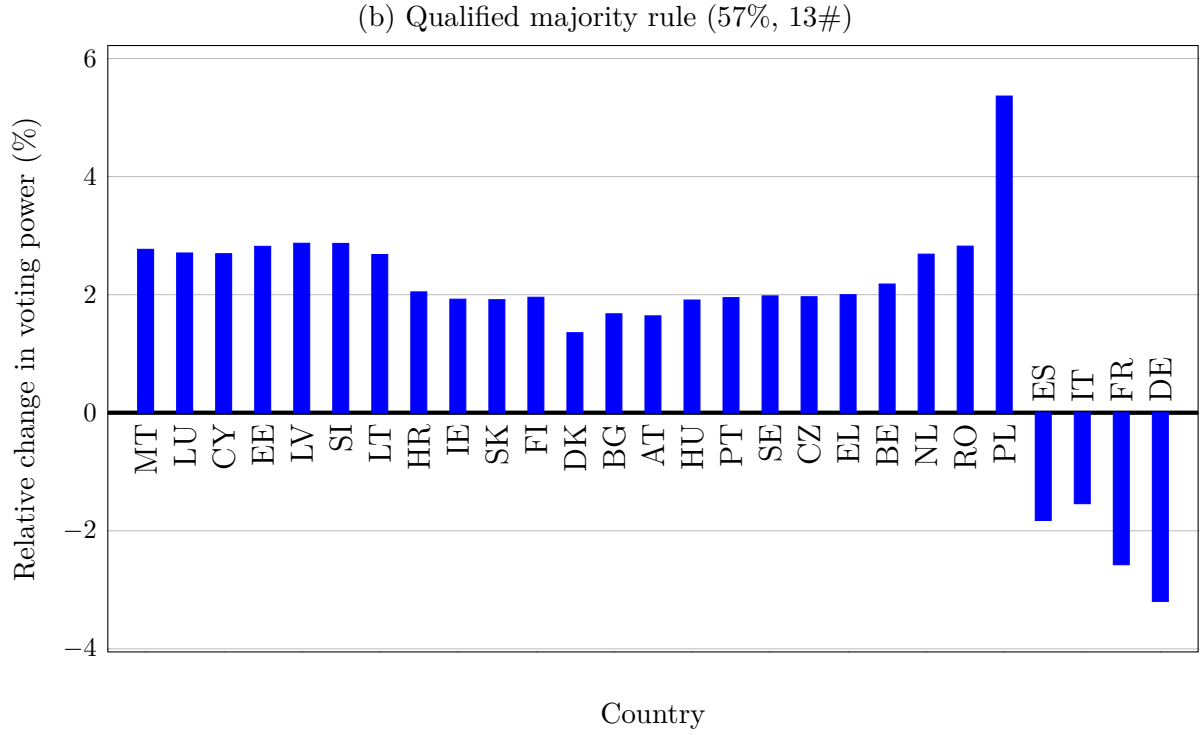
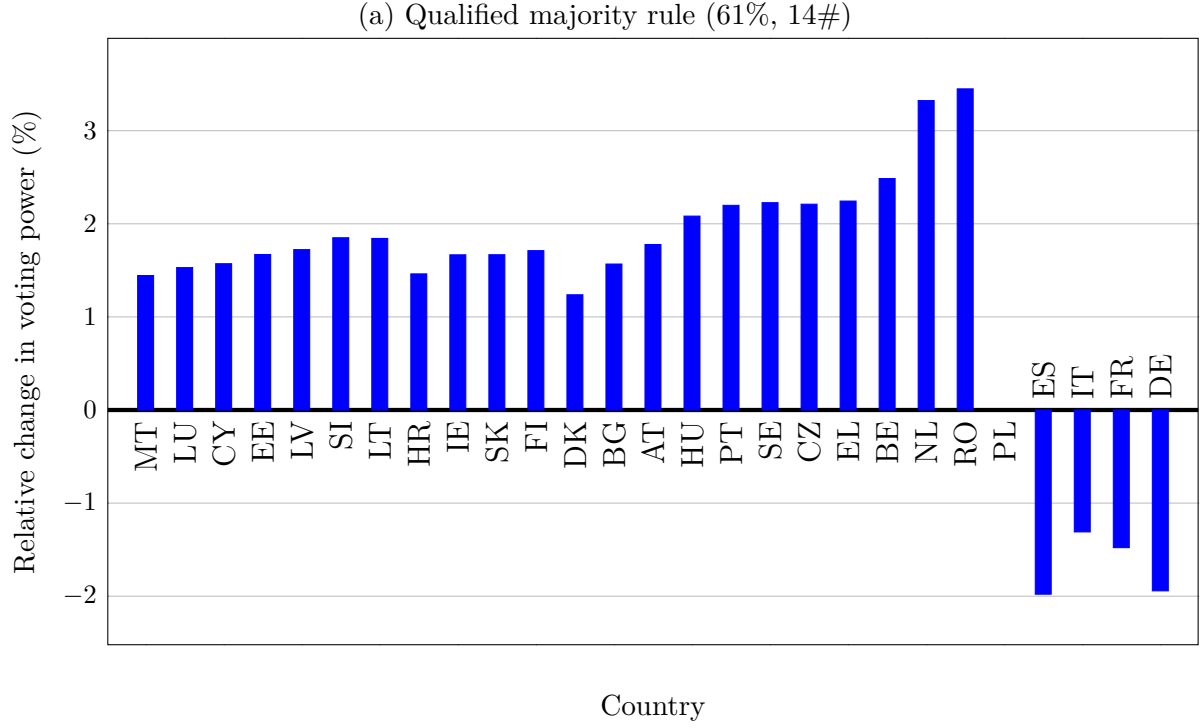


Figure 6: Relative changes in the Shapley–Shubik indices compared to the current qualified majority rule (65%, 15#)  
*Note:* See Table 1 for the abbreviations of country names.

under Grants FK 145838 and PD 146055.

## References

- Algaba, E., Bilbao, J. M., and Fernández, J. R. (2007). The distribution of power in the European Constitution. *European Journal of Operational Research*, 176(3):1752–1766.
- Alonso-Meijide, J. M., Bilbao, J. M., Casas-Méndez, B., and Fernández, J. R. (2009). Weighted multiple majority games with unions: Generating functions and applications to the European Union. *European Journal of Operational Research*, 198(2):530–544.
- Banzhaf, J. F. (1964-1965). Weighted voting doesn't work: A mathematical analysis. *Rutgers Law Review*, 19(2):317–343.
- Bilbao, J. M., Fernandez, J. R., Jiménez, N., and Lopez, J. J. (2002). Voting power in the European Union enlargement. *European Journal of Operational Research*, 143(1):181–196.
- Brams, S. J. and Affuso, P. J. (1976). Power and size: A new paradox. *Theory and Decision*, 7(1-2):29–56.
- Brams, S. J. and Affuso, P. J. (1985). New paradoxes of voting power on the EC Council of Ministers. *Electoral Studies*, 4(2):135–139.
- Casajus, A. and Huettner, F. (2019). The Coleman–Shapley index: being decisive within the coalition of the interested. *Public Choice*, 181(3-4):275–289.
- Coleman, J. S. (1971). Control of collectivities and the power of a collectivity to act. In Lieberman, B., editor, *Social Choice*, pages 269–300. Gordon and Breach, New York, New York, USA.
- Council of the European Union (2022). Council Decision (EU, Euratom) 2022/2518 of 13 December 2022 amending the Council's Rules of Procedure. <https://eur-lex.europa.eu/eli/dec/2022/2518/oj>.
- Felsenthal, D. S. and Machover, M. (1997). The weighted voting rule in the EU's Council of Ministers, 1958–1995: Intentions and outcomes. *Electoral Studies*, 16(1):33–47.
- Felsenthal, D. S. and Machover, M. (1998). *The Measurement of Voting Power*. Edward Elgar Publishing, Cheltenham, United Kingdom.
- Felsenthal, D. S. and Machover, M. (2001). The Treaty of Nice and qualified majority voting. *Social Choice and Welfare*, 18(3):431–464.
- Felsenthal, D. S. and Machover, M. (2004). A priori voting power: What is it all about? *Political Studies Review*, 2(1):1–23.
- Freixas, J. (2004). The dimension for the European Union Council under the Nice rules. *European Journal of Operational Research*, 156(2):415–419.
- Gábor, J. (2020). Impact of Brexit on voting power in Council of the European Union. *Open Political Science*, 3(1):192–197.
- Göllner, R. (2017). The Visegrád Group – A rising star post-Brexit? Changing distribution of power in the European Council. *Open Political Science*, 1(1):1–6.

- Grech, P. D. (2021). Power in the Council of the EU: organizing theory, a new index, and brexit. *Social Choice and Welfare*, 56(2):223–258.
- Herfindahl, O. C. (1950). *Concentration in the Steel Industry*. PhD thesis, Columbia University, New York.
- Hirschman, A. O. (1945). *National Power and the Structure of Foreign Trade*. University of California Press, Berkeley and Los Angeles, California, USA.
- Hirschman, A. O. (1964). The paternity of an index. *The American Economic Review*, 54(5):761–762.
- Huettner, F. (2023). Powerindices. <https://github.com/frankhuettner/powerindices>.
- Kauppi, H. and Widgrén, M. (2004). What determines EU decision making? Needs, power or both? *Economic Policy*, 19(39):222–266.
- Kirsch, W. (2016). Brexit and the distribution of power in the Council of the EU. 26 December. <https://www.ceps.eu/publications/brexit-and-distribution-power-council-eu>.
- Kirsch, W. (2022). The distribution of power within the EU: perspectives on a Ukrainian accession and a Turkish accession. *International Economics and Economic Policy*, 19(2):401–409.
- Kirsch, W., Słomczyński, W., Stolicki, D., and Życzkowski, K. (2018). Double majority and generalized Brexit: Explaining counterintuitive results. Manuscript. DOI: [10.48550/arxiv.1812.07048](https://doi.org/10.48550/arxiv.1812.07048).
- Kóczy, L. Á. (2012). Beyond Lisbon: Demographic trends and voting power in the European Union Council of Ministers. *Mathematical Social Sciences*, 63(2):152–158.
- Kóczy, L. Á. (2021). Brexit and power in the Council of the European Union. *Games*, 12(2):51.
- Laruelle, A. and Widgrén, M. (1998). Is the allocation of voting power among EU states fair? *Public Choice*, 94(3):317–339.
- Le Breton, M., Montero, M., and Zaporozhets, V. (2012). Voting power in the EU Council of Ministers and fair decision making in distributive politics. *Mathematical Social Sciences*, 63(2):159–173.
- Leech, D. (2002). Designing the voting system for the Council of the European Union. *Public Choice*, 113(3-4):437–464.
- Mercik, J. and Ramsey, D. M. (2017). The effect of Brexit on the balance of power in the European Union Council: an approach based on pre-coalitions. In Nguyen, N., Kowalczyk, R., and Mercik, J., editors, *Transactions on Computational Collective Intelligence XXVII*, pages 87–107. Springer, Berlin, Heidelberg, Germany.
- OSW (2023). The EU debate on qualified majority voting in the Common Foreign and Security Policy. Reform and enlargement. Ośrodek Studiów Wschodnich (Centre for Eastern Studies). 12 October. <https://www.osw.waw.pl/en/publikacje/osw-commentary/2023-10-12/eu-debate-qualified-majority-voting-common-foreign-and>.

- Penrose, L. S. (1946). The elementary statistics of majority voting. *Journal of the Royal Statistical Society*, 109(1):53–57.
- Petróczy, D. G. and Csató, L. (2021). Revenue allocation in Formula One: A pairwise comparison approach. *International Journal of General Systems*, 50(3):243–261.
- Petróczy, D. G., Rogers, M. F., and Kóczy, Á. L. (2022). Exits from the European Union and their effect on power distribution in the Council. *Games*, 13(1):18.
- Schmeidler, D. (1969). The nucleolus of a characteristic function game. *SIAM Journal on Applied Mathematics*, 17(6):1163–1170.
- Shapley, L. S. and Shubik, M. (1954). A method for evaluating the distribution of power in a committee system. *American Political Science Review*, 48(3):787–792.
- Słomczyński, W. and Życzkowski, K. (2006). Penrose voting system and optimal quota. *Acta Physica Polonica B*, 37(11):3133–3143.
- Szczypińska, A. (2018). Who gains more power in the EU after Brexit? *Finance a Uver*, 68(1):18–33.
- Widgrén, M. (1994). Voting power in the EC decision making and the consequences of two different enlargements. *European Economic Review*, 38(5):1153–1170.
- Zaporozhets, V., García-Valiñas, M., and Kurz, S. (2016). Key drivers of EU budget allocation: Does power matter? *European Journal of Political Economy*, 43:57–70.