Homework 1: Regular Languages: Automata and Expressions

CS 212 Nature of Computation
Habib University

Fall 2025

1. 10 points

Definition 1 (Power on a language)**.** *Given a language L, we define L^n, for $n \in \mathbb{N}$ recursively as follows:*

1. $L^0 = \{\varepsilon\}$
2. $L^1 = L$
3. $L^{k+1} = \{w_1 w_2 \mid w_1 \in L^k \wedge w_2 \in L\}$

Definition 2 (Kleene closure)**.** *With this definition of L^n, we can define Kleene closure of L, L^* as follows:*

$$L^* = \bigcup_{i \in \mathbb{N}} L^i$$

Definition 3 (Positive closure)**.** *With this definition of L^n, we can define Positive closure of L, L^+ as follows:*

$$L^+ = \bigcup_{i \in \mathbb{Z}^+} L^i$$

Let $L \subseteq \Sigma^*$ be a language. Prove or disprove the following claim: $(L^+)^* = (L^*)^+$

Note: $\mathbb{N}$ here denotes the set of natural numbers which includes 0. See von Neumann ordinals for the construction of $\mathbb{N}$. $\mathbb{Z}^+$ denotes the set of positive integers.

2. 10 points

Definition 4 (Shuffle Operator)**.** *Let $L \subseteq \{0,1\}^*$ be a some language. The operator* SHUFFLE *is defined as,* $\textsc{shuffle}(L) = \{w \in \{0,1\}^* \mid \exists x \in L \text{ s.t. } w \text{ is obtained by shuffling the characters of } x\}$. *More formally,* $\textsc{shuffle}(L) = \{w = w_1 w_2 \ldots w_n \in \{0,1\}^* \mid \exists x = x_1 x_2 \ldots x_n \in L, \text{ s.t. } \forall w_i ! \exists x_j : w_i = x_j\}$.

Prove or disprove the following claim: If L is a regular language then $\textsc{shuffle}(L)$ is also regular. (You can not assume the pumping lemma in this problem.)

3. After failing to make it big with your CS degree from Habib university, you tried to try your luck with gambling. You notice that at poker tables to avoid cheating decks are "cut" before the round starts. The traditional method for cutting a deck of playing cards, the deck is arbitrarily split two parts, which are exchanged before reassembling the deck. But you can

never be too careful with card cheats, for this you see people using a more complex method for cutting the deck in which the deck is broken into three parts and the middle part in placed first in the reassembly. But your brain is broken after taking CS212 in fall 2025, and all you can think about are regular languages. You wonder if regular languages can be cut like that. With the mathematical skills you obtained from the course you were able to come up with the following formalism for the two types of cuts.

$$\textsc{cut}(L) = \{w_2w_1 \in \Sigma^* \mid w_1w_2 \in L, \text{ where each } w_i \text{ is some string in } \Sigma^*\}$$

$$\textsc{complex-cut}(L) = \{w_2w_1w_3 \in \Sigma^* \mid w_1w_2w_3 \in L, \text{ where each } w_i \text{ is some string in } \Sigma^*\}$$

Where L is some language. Now realizing that you have already lost all your life savings, you shift your focus on more important matters and start to wonder if regular languages are closed under these operators that you defined.

(a) [10 points] Prove or disprove the following claim: The class of regular language is closed under CUT operation.

(b) [10 points] Prove or disprove the following claim: The class of regular language is closed under COMPLEX-CUT operation.