

# Game Theoretic Modeling of Interactive Traffic for Verification and Validation of Autonomous Vehicle Control Systems

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## Background

In the near to medium term, autonomous vehicles will operate in traffic scenarios together with human-driven vehicles. One source of complexity in autonomous vehicle control is the need to deal with the extensive and constantly-occurring interactions among autonomous and human-driven vehicles.

To verify the correctness of autonomous vehicle control systems in terms of safety and performance in such interactive scenarios, traffic simulators used for virtual tests of these systems should represent the interactive behavior of vehicles with reasonable fidelity. This motivates our development of microscopic traffic models that can represent the interactions among vehicles.

Game theory is a suitable tool for modeling strategic interactions between rational decision-makers, such as drivers in traffic. Our vehicle interaction modeling approach is based on application of dynamic game theory.

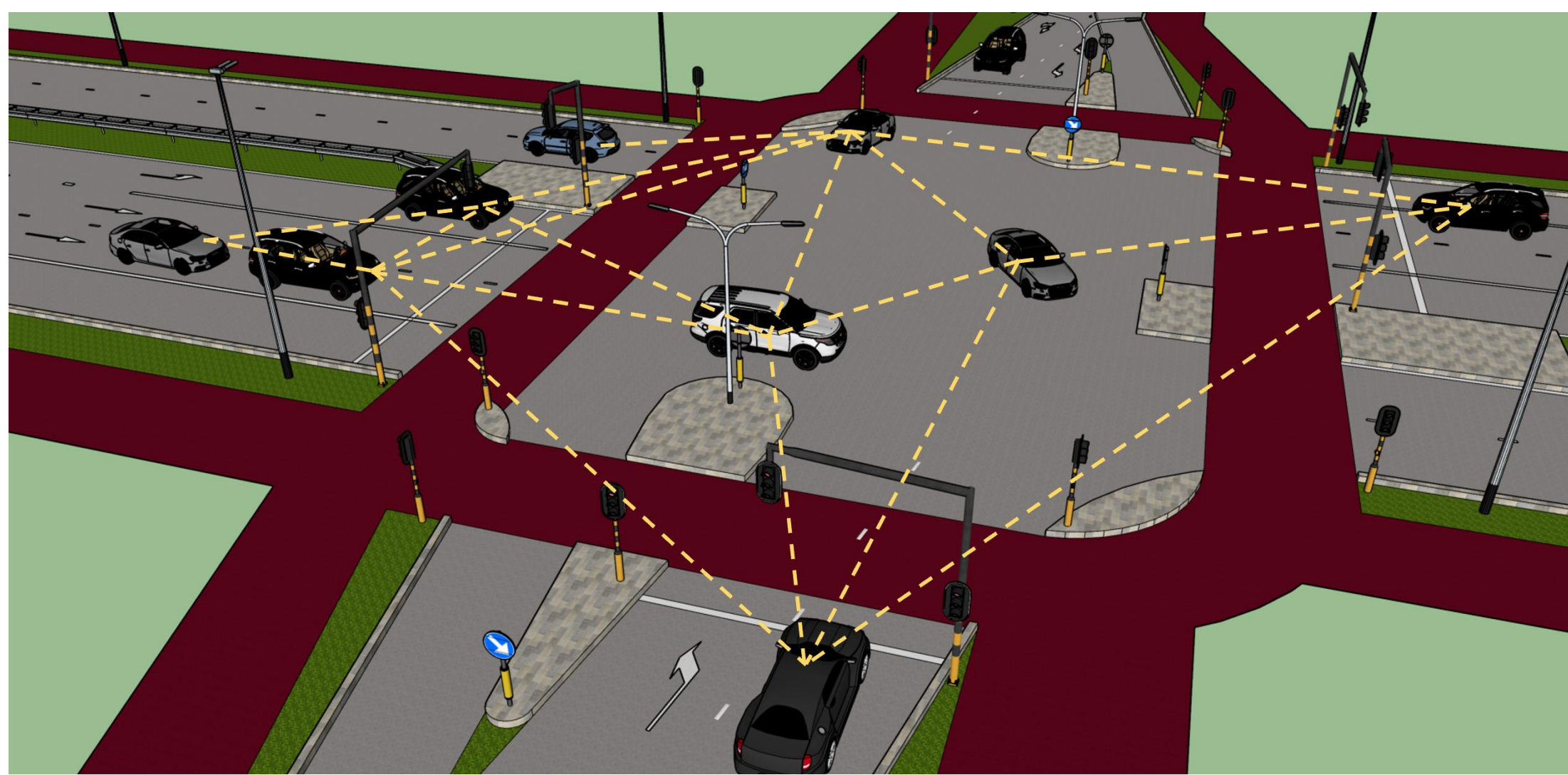


Fig. 1. Interactions between autonomous and human-driven vehicles in traffic.

## Main Developments

- A game-theoretic framework for modeling vehicle-to-vehicle interactions.
- An interactive highway traffic model, integration with higher-fidelity simulator, and validation using traffic data.
- An interactive intersection traffic model, and validation using data.
- Application of the models to evaluation and calibration of autonomous vehicle control systems.

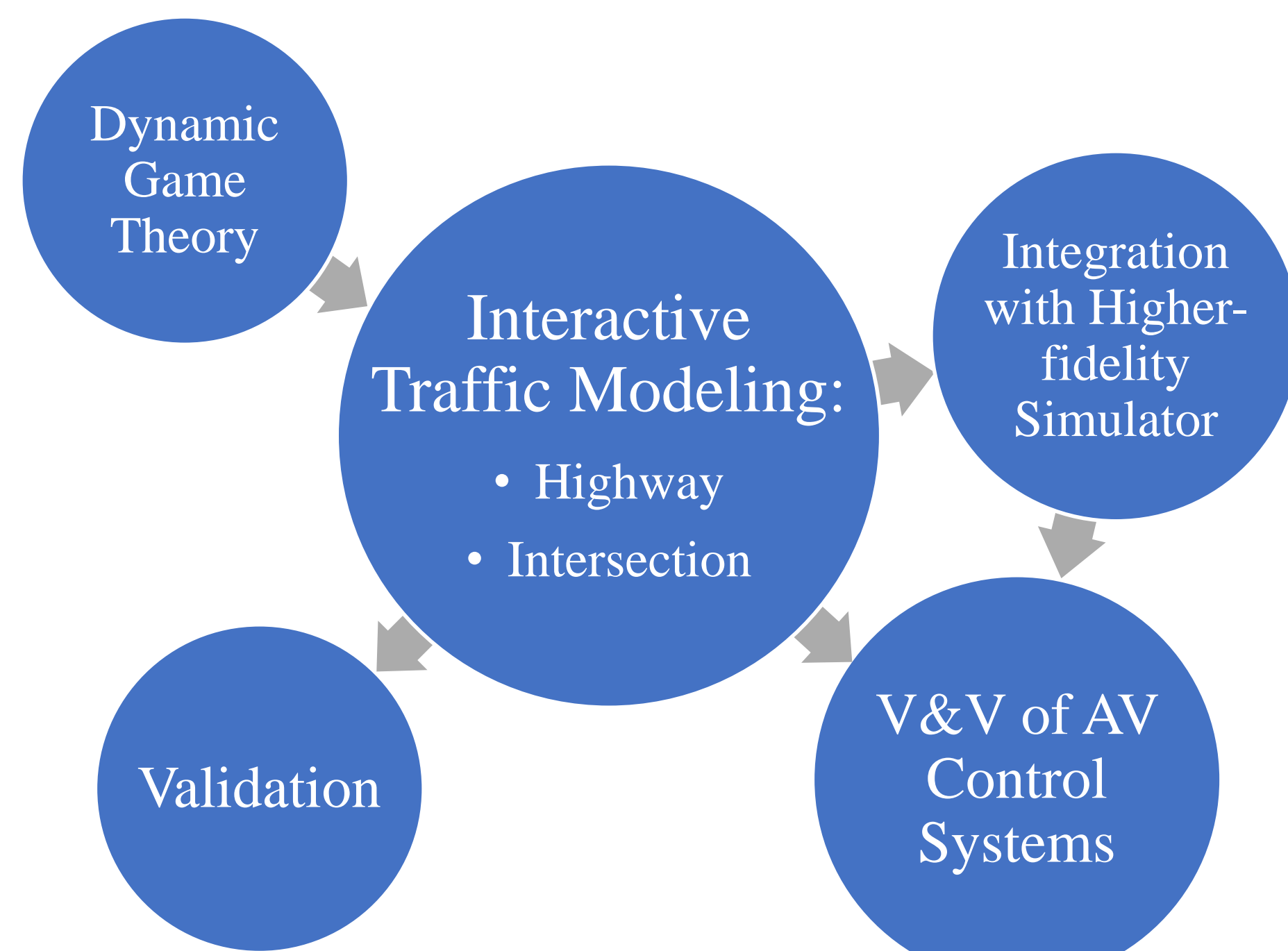


Fig. 2. Our developments and their architecture.

## Interactive Highway Traffic Model

Our approach to modeling the interactive behavior of vehicles in highway traffic is based on cognitive hierarchy theory, in particular, the level- $k$  framework.

Key assumptions:

- Humans make decisions through a finite number of reasoning steps, called “levels.”
- Humans have different reasoning levels in their interactions.

Level- $k$  decision-making:

- Level- $k$  driver optimally responds to level- $(k-1)$  drivers.
- Driving policies are determined using learning.

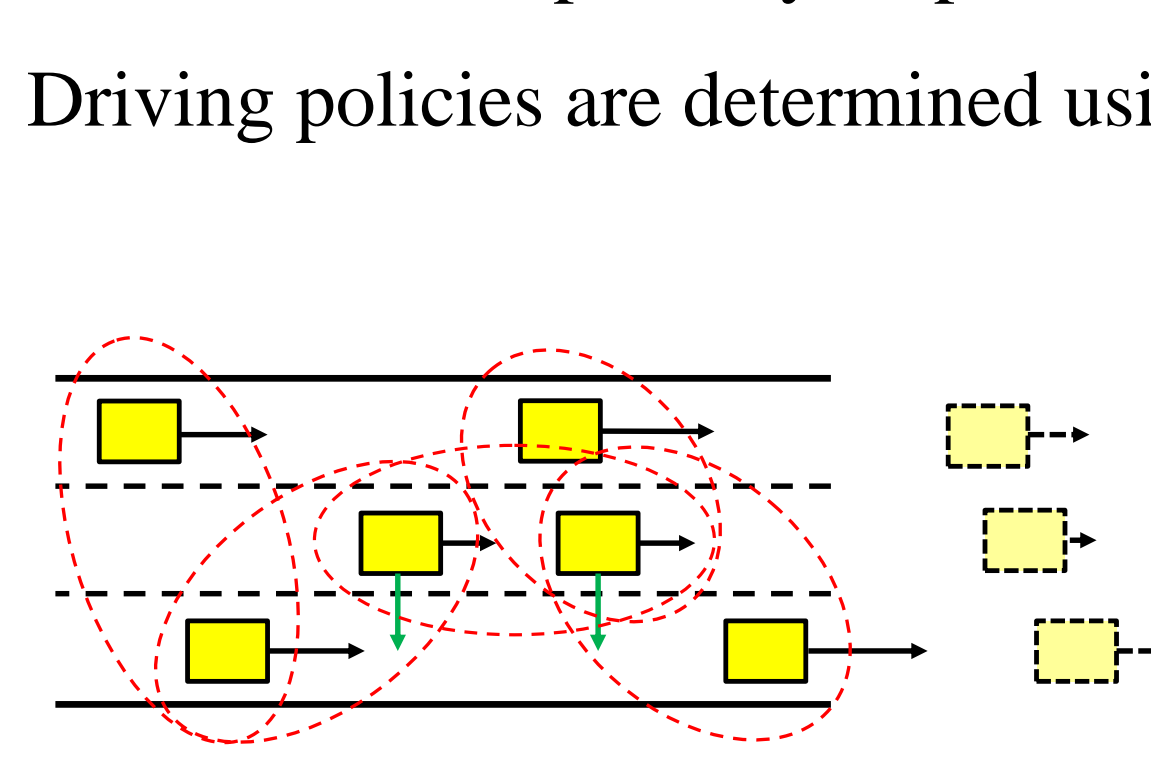


Fig. 3. Interaction topology modeled by our game-theoretic approach.

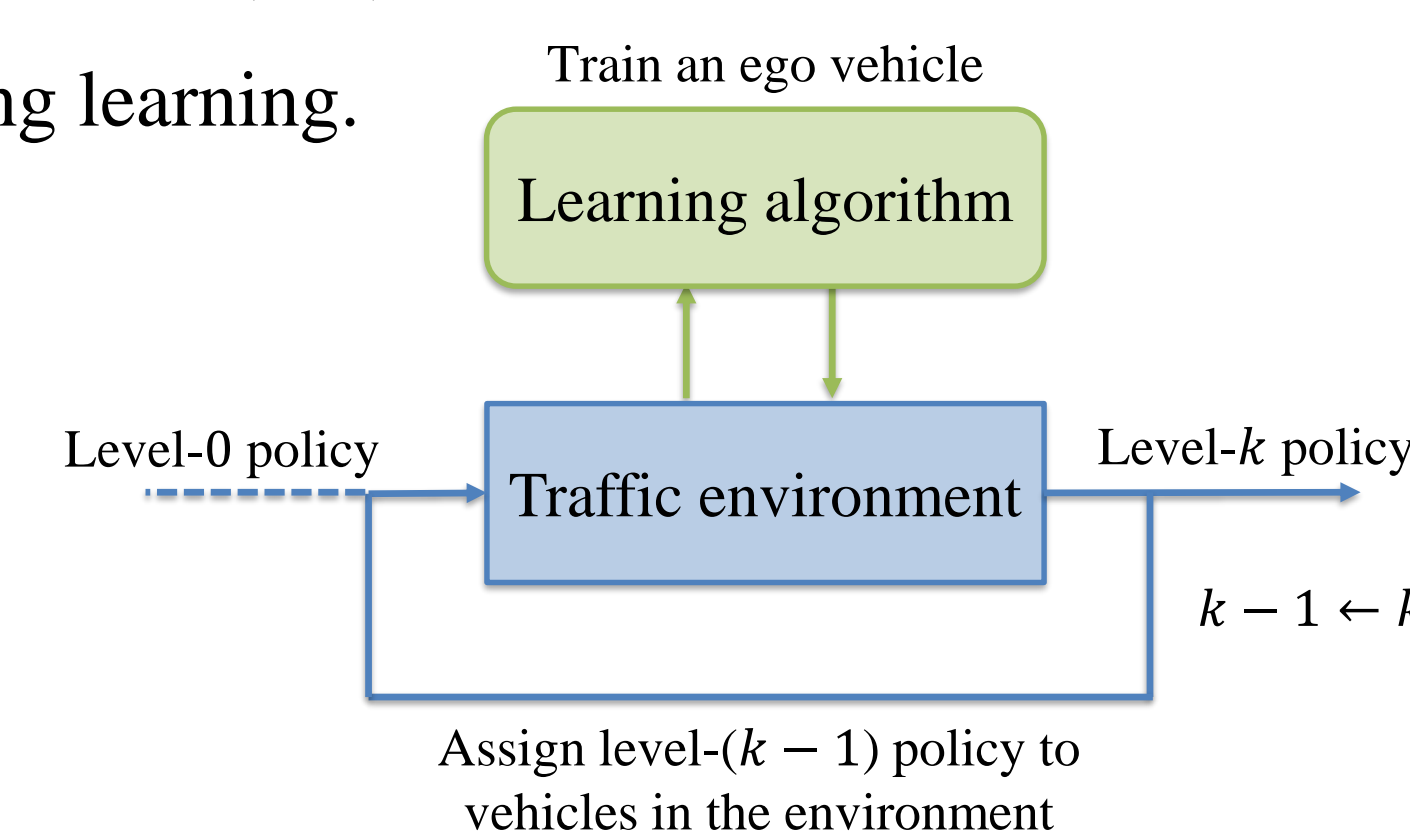


Fig. 4. Iterative reinforcement learning algorithm to obtain level- $k$  driver models.

- Once level- $k$  policies have been obtained, a traffic model with heterogeneous and interactive drivers is constructed using a mixture of level- $k$  drivers.
- The model can be integrated with existing traffic/driving simulators with higher-fidelity in terms of modeling vehicle dynamics and environmental factors.

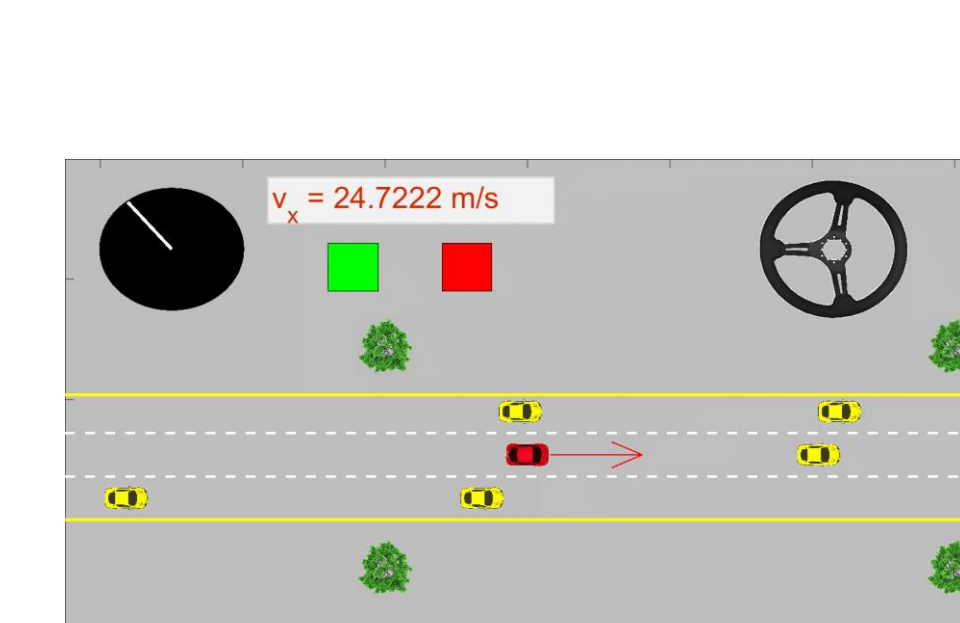


Fig. 5. Traffic model with a mixture of level- $k$  drivers (10% level-0, 60% level-1, and 30% level-2).

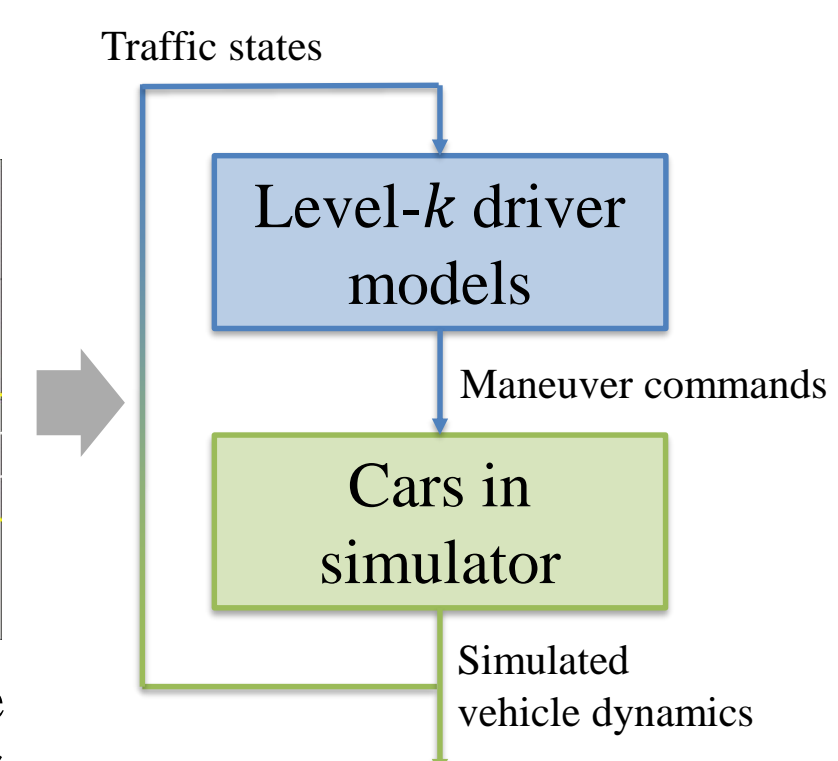


Fig. 6. Integration architecture.



Fig. 7. Integration with *The Open Racing Car Simulator*.

## Interactive Intersection Traffic Model

Our approach to modeling the interactive behavior of vehicles at intersections is based on a formulation of leader-follower game, which accounts for the “right-of-way” traffic rules via assigning a leader and a follower to each pair of interacting vehicles.

Leader-follower decision-making:

- A follower applies a “maximin” conservative strategy, and a leader makes rational decisions taking into account its awareness of the follower’s conservative strategy.

$$\begin{aligned} Q_l(s, \gamma_l) &= R_l(s, \gamma_l, \gamma_f^*), \\ Q_f(s, \gamma_f) &= \min_{\gamma_l \in \Gamma_l} R_f(s, \gamma_l, \gamma_f), \\ \gamma_l^* &= \arg \max_{\gamma_l \in \Gamma_l} Q_l(s, \gamma_l), \\ \gamma_f^* &= \arg \max_{\gamma_f \in \Gamma_f} Q_f(s, \gamma_f). \end{aligned}$$

Eqn. 1. Leader-follower decision-making. Eqn. 2. Generalization to multi-vehicle interactions.

$$\begin{aligned} Q_i(s_{\text{traffic}}, \gamma_i) &:= \min_{j \in \{1, \dots, n\}, j \neq i} Q_{i,j}(s_{i,j}, \gamma_i), \\ Q_{i,j}(s_{i,j}, \gamma_i) &:= \begin{cases} Q_l(s_{i,j}, \gamma_i) & \text{if } i < j, \\ Q_f(s_{i,j}, \gamma_i) & \text{if } i \geq j, \end{cases} \\ \gamma_i^* &\in \arg \max_{\gamma_i \in \Gamma_i} Q_i(s_{\text{traffic}}, \gamma_i). \end{aligned}$$

- The closed-loop operation of each vehicle is based on receding-horizon optimization.

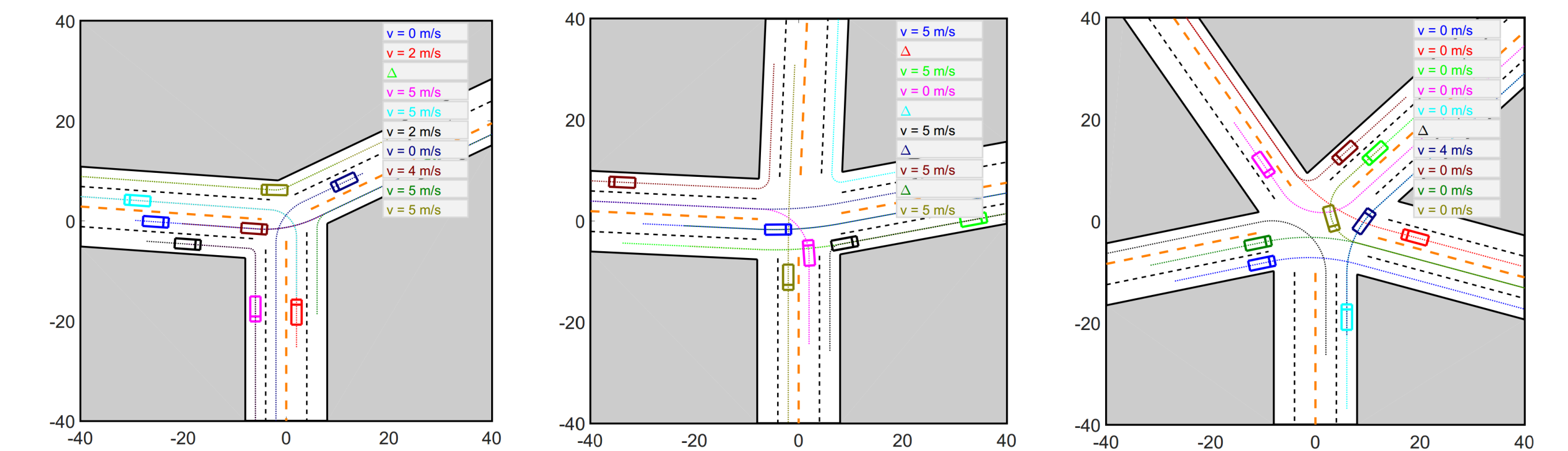


Fig. 8. Modeled traffic scenarios at three-way, four-way, and five way intersections.

- Our model can reproduce scenarios extracted from real-world traffic data, and exhibits reasonable performance in resolving traffic conflicts.

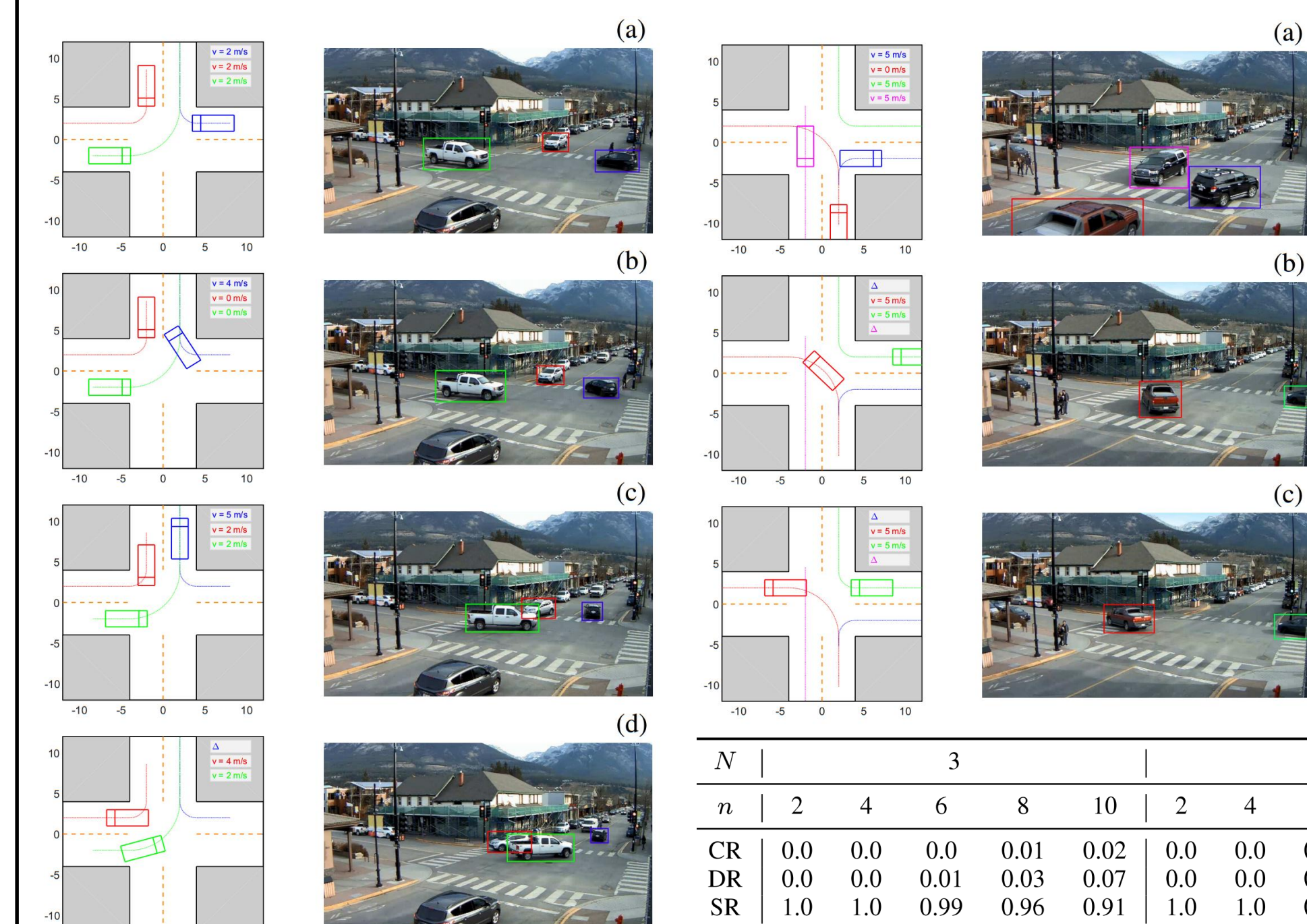


Fig. 9. Our model can easily reproduce traffic scenarios extracted from the video dataset at [https://berkeleyautomation.github.io/Traffic\\_Camera\\_Pipeline/](https://berkeleyautomation.github.io/Traffic_Camera_Pipeline/).

Tab. 1. Our model has high rates of success (SR), and low rates of collision (CR) and deadlock (DR), at intersection traffic scenarios with different numbers of road arms ( $N$ ) and cars ( $n$ ).

	3					4					5				
	2	4	6	8	10	2	4	6	8	10	2	4	6	8	10
CR	0.0	0.0	0.0	0.0	0.01	0.02	0.0	0.0	0.01	0.02	0.05	0.0	0.0	0.01	0.03
DR	0.0	0.0	0.01	0.03	0.07	0.0	0.0	0.02	0.08	0.04	0.0	0.02	0.06	0.10	0.12
SR	1.0	1.0	0.99	0.96	0.91	1.0	1.0	0.97	0.90	0.90	1.0	0.98	0.93	0.87	0.84

## For Autonomous Vehicle Control System Verification and Validation

- A significant feature of simulation-based virtual test using our interactive traffic model is that the simulation is closed-loop, i.e., not only does the AV under test responds to the traffic, but the traffic also responds correspondingly to the AV behavior.

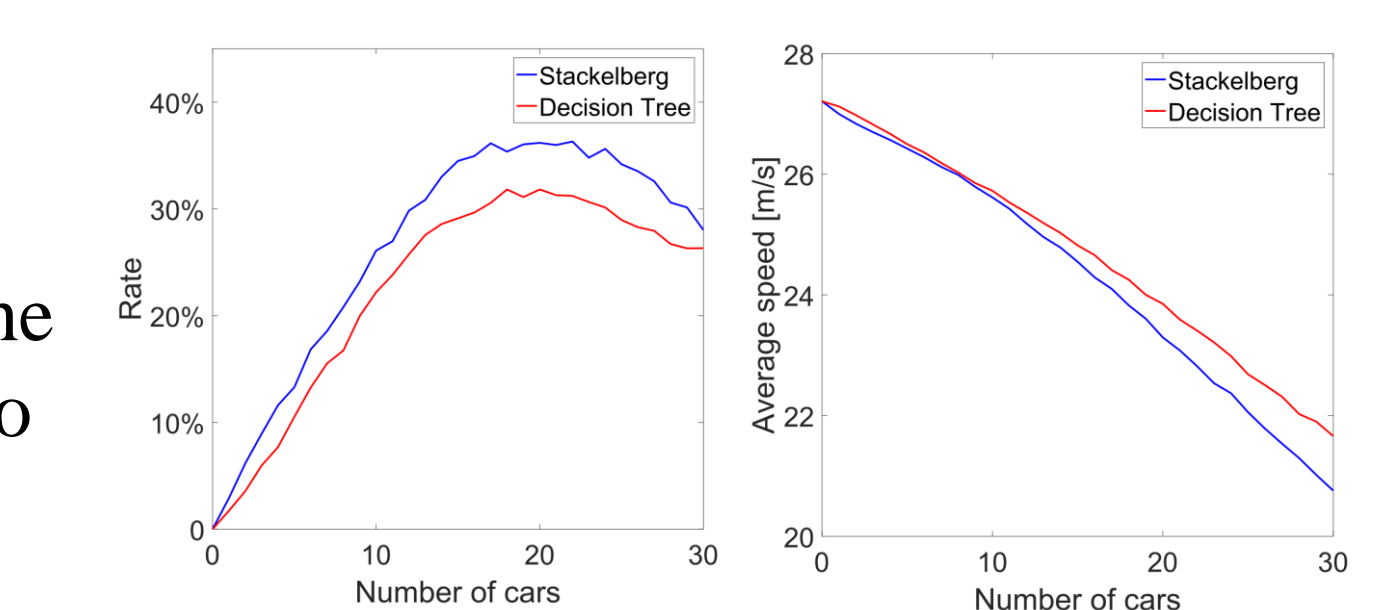


Fig. 11. Evaluation and comparison of two autonomous driving systems using our model

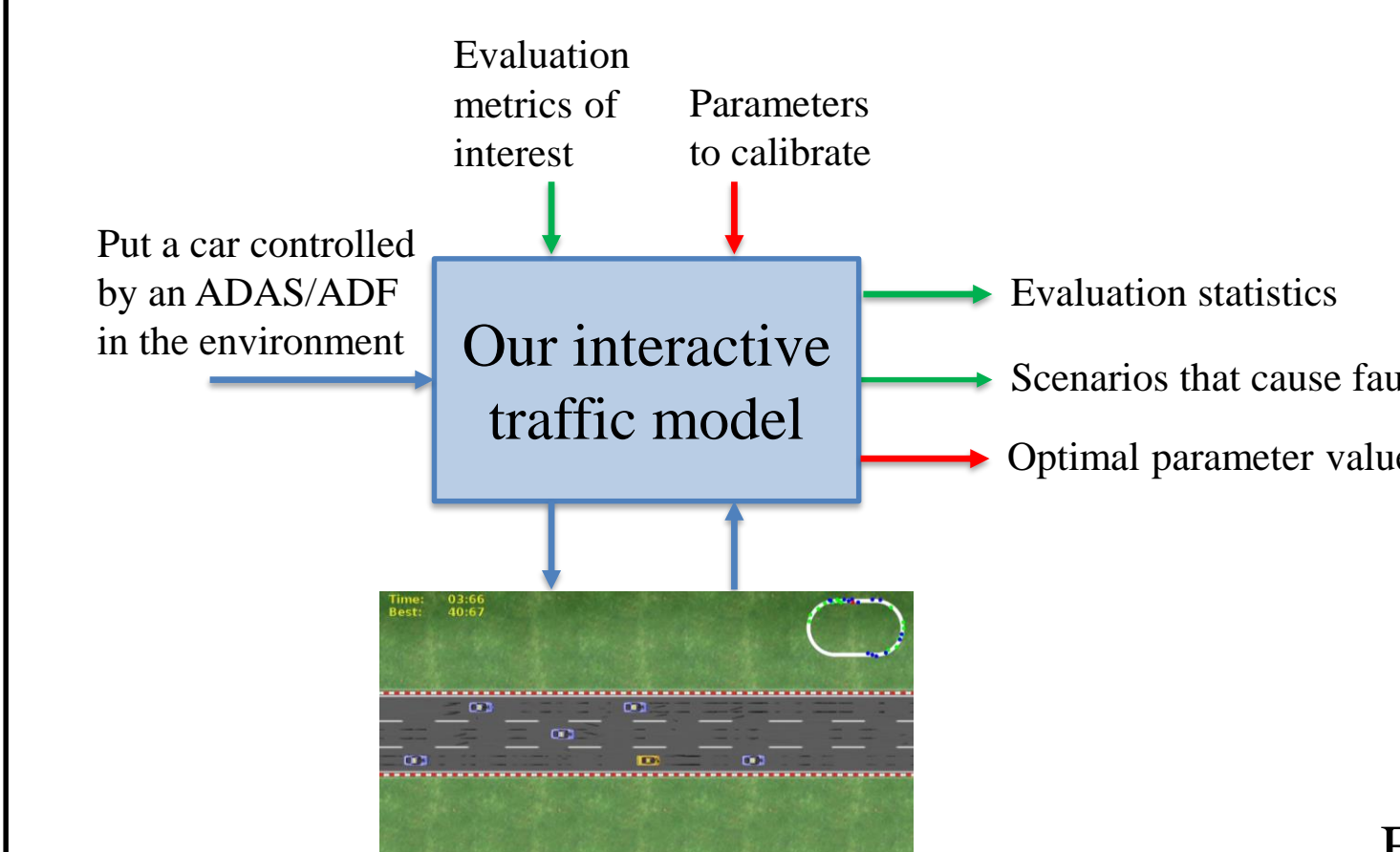


Fig. 10. Monte Carlo simulation-based evaluation and a given objective function, e.g., representing a weighted combination of safety and performance concerns.

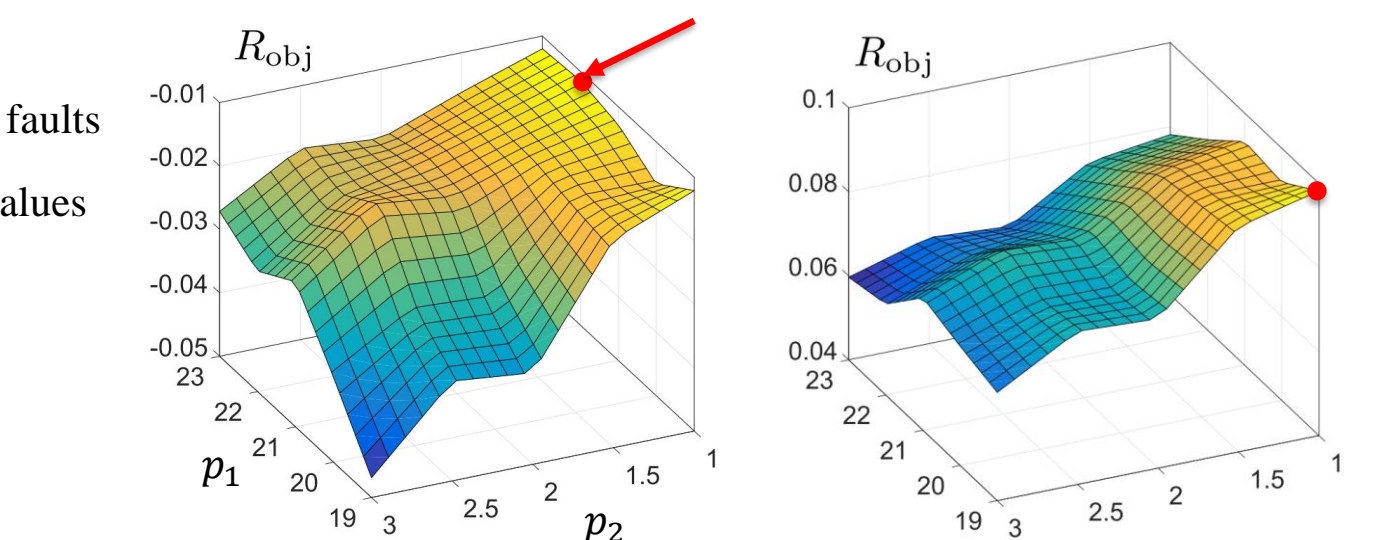


Fig. 12. Optimal design of a parameter pair ( $p_1, p_2$ ) for

## References

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- [3] Li, N., Yao, Y., Kolmanovsky, I., Atkins, E., & Girard, A. (2019). Game-Theoretic Modeling of Multi-Vehicle Interactions at Uncontrolled Intersections. *arXiv preprint arXiv:1904.05423*.