Lecture

Probability and Statistics (STS-202)

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General Addition Rule
General Multiplication Rule

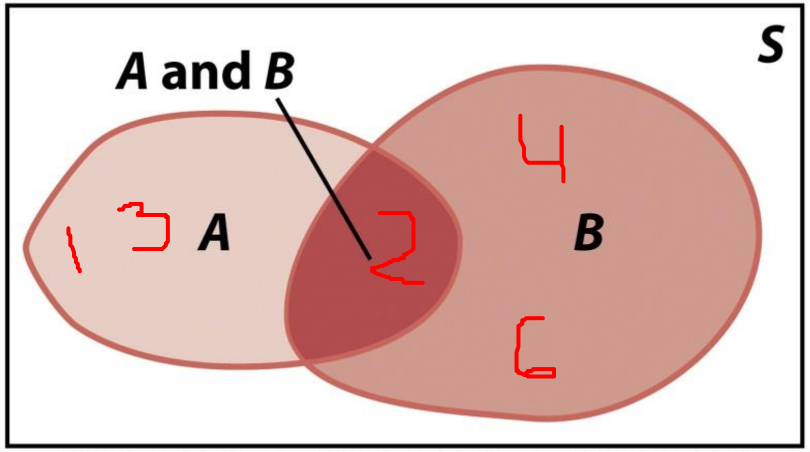
Learning Objectives

By the end of this lecture, you should be able to:

- **Apply the general addition rule and the general multiplication rule.**
- **Describe what is meant by the term ‘general’ in the general addition rule and general multiplication rule.**

General addition rule

$$P(A \overset{\cup}{\text{or}}\ B) = P(A) + P(B) - P(A \overset{\cap}{\text{and}}\ B)$$



$A = \{1\ 2\ 3\}$

$B = \{2, 4, 6\}$

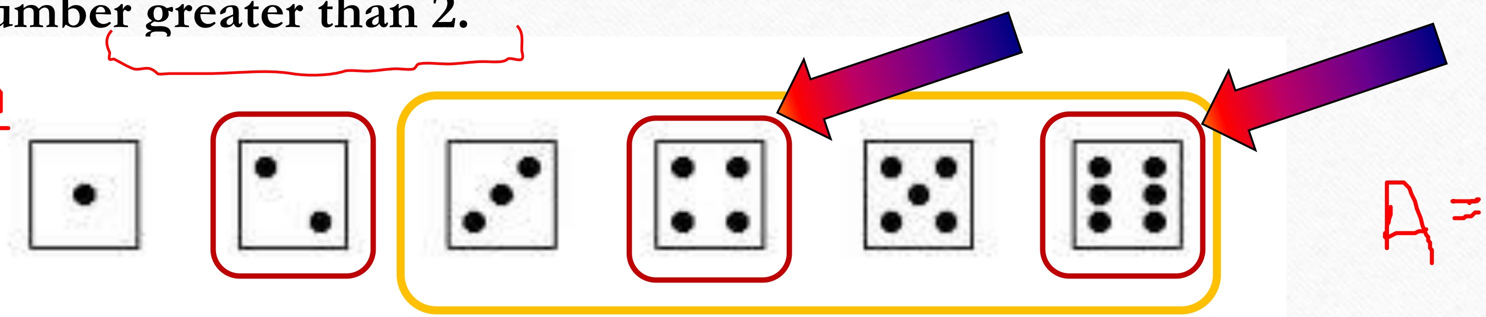
$A \cap B = \{2\}$

Why is it called the "general" rule?

"General" means can be used on BOTH joint <u>and</u> disjoint events!

Example: If rolling a single die, determine the probability of rolling an even number, or a number greater than 2.

$P(A) = \frac{n(A)}{n(S)} = \frac{3}{6}$



P(Even or >2) = ?

$A = \{2, 4, 6\}$

$B = \{3, 4, 5, 6\}$

P(Outcome is an Even) = 3/6

P(Outcome is a number >2) = 4/6

P(Outcome is an Even AND >2) = 2/6

$A \cap B = \{4, 6\}$

Applying General Addition Rule:

P(A or B) = P(A) + P(B) – P(A and B)

$\cup$

= P(Even) + P(>2) – P(Even and >2)

= 3/6 + 4/6 – 2/6

= <u>5/6</u>

Why do we call it the "general" addition rule?

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cup B) = = P(A) + P(B)$

- Because it applies to any addition events. That is, you can use it for both joint events and disjoint events.
- Why does it also work for disjoint events?
  - Recall that if 2 events are disjoint, this means that the two events are mutually exclusive. In other words, if one of the two events or occurs, the other event will not occur.
  - Therefore, P(A <u>and</u> B), i.e. the probability of <u>both</u> events being true will always equal <u>0</u>.

So: P(A or B) = P(A) + P(B) - P(A and B)
However, if the events are disjoint, then P(A and B) is 0,
Therefore: P(A or B) = P(A) + P(B) <u>- 0</u> (i.e. This is our addition rule for disjoint events)

Let's look at an example of applying the general rule to a disjoint events:

Example: What is the probability of randomly drawing either an Ace or a 7 from a deck of 52 playing cards?

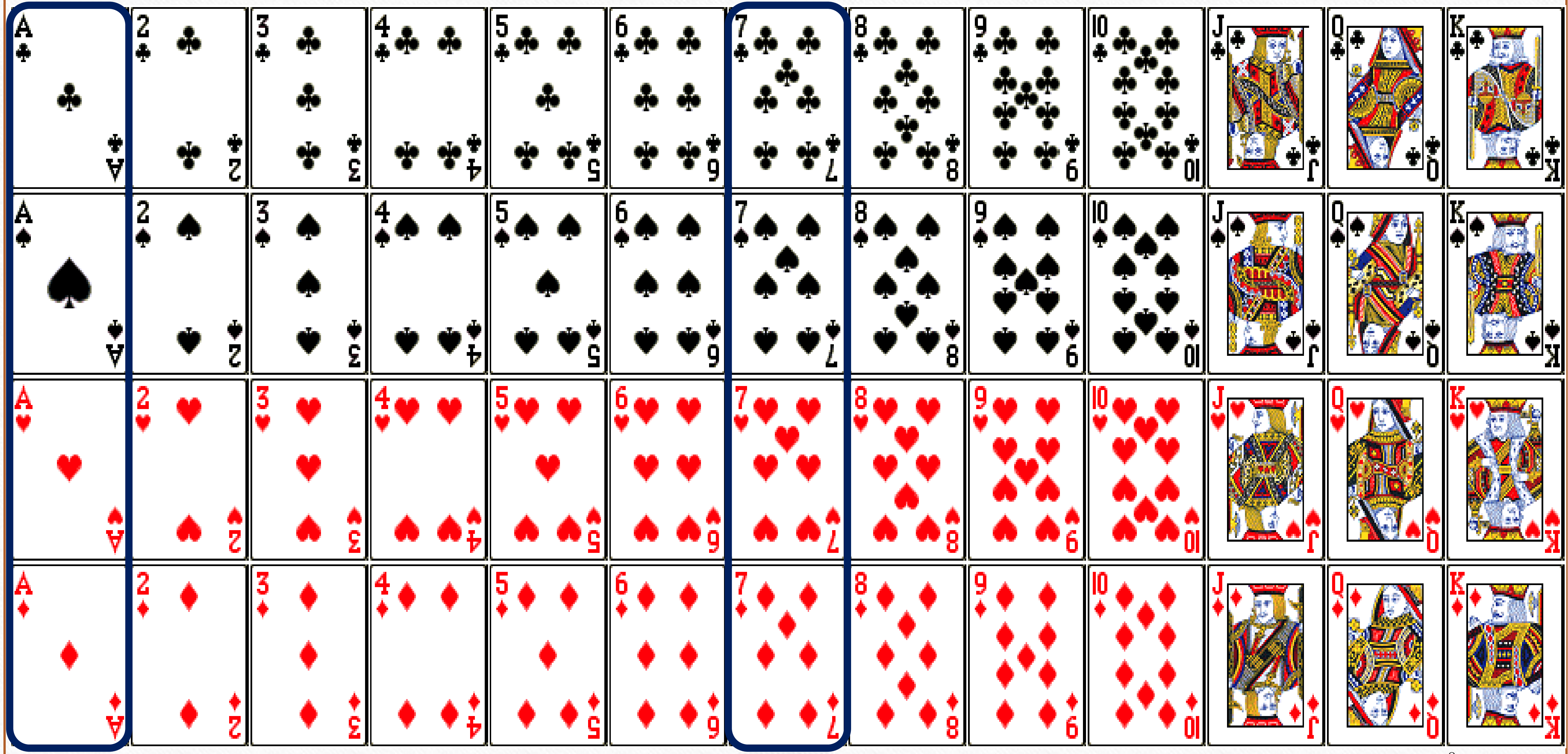
$P(A \cup B) = P(A) + P(B)$

- P(Card is an Ace) → 4/52
- P(Card is a 7) → 4/52
- P(Card is an Ace AND a 7) → 0

P(Draw an Ace OR Draw a 7) ?

= P(Ace) + P(7) – P(Ace and 7)

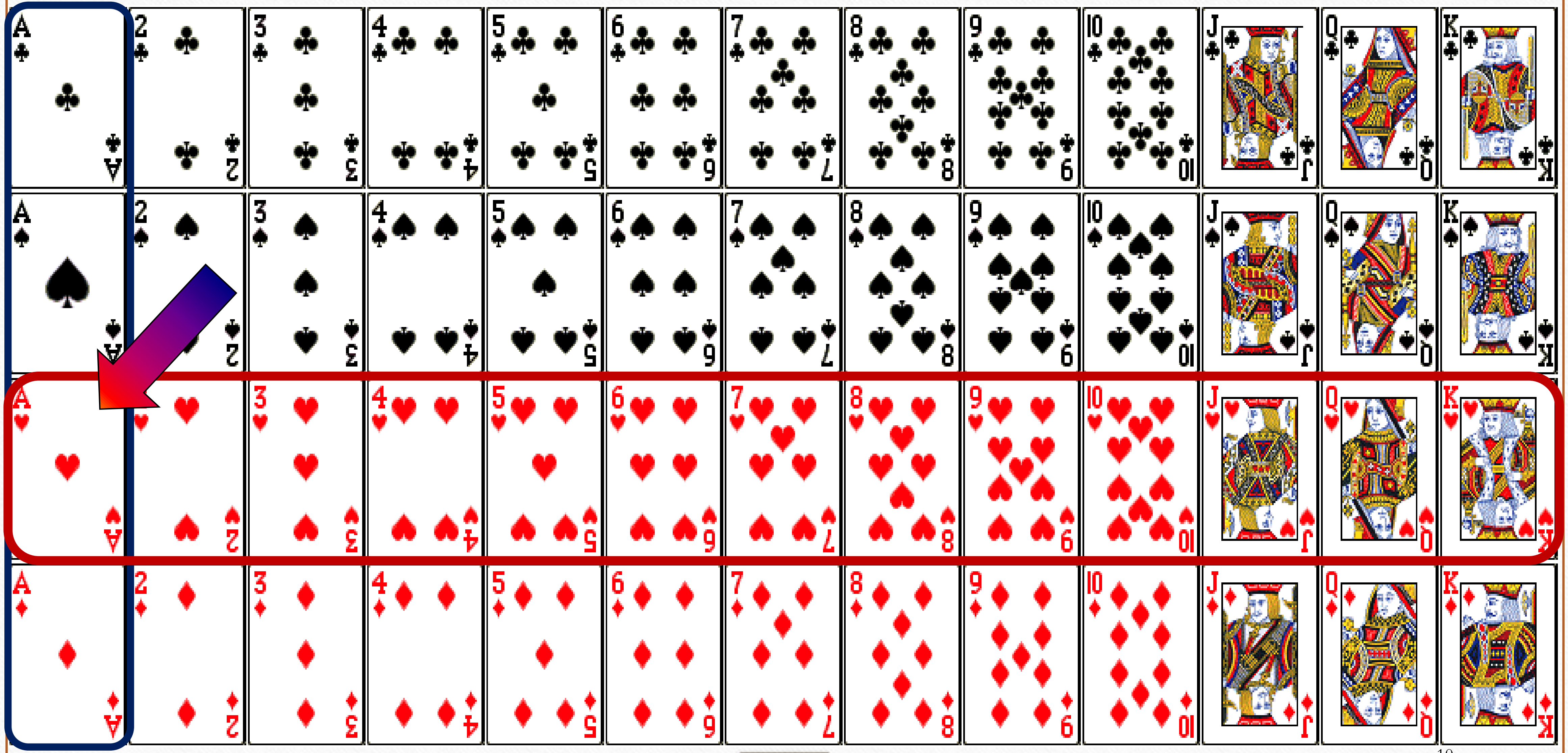
= 4/52 + 4/52 – 0/52) = 8/52



Example: What is the probability of randomly drawing either an ace or a heart from a deck of 52 playing cards?

$P(A \cup H) = P(A) + P(H) - P(A \cap H)$

- P(Ace) → 4/52
- P(Heart) → 13/52
- P(Ace and Heart) → 1/52
- There is one NON-disjoint event present. Notice how the Ace of Hearts has been counted twice. Therefore we must subtract this doubled item. So the *correct* answer is: (4/52 + 13/52 – 1/52) = <u>16/52</u>.



Question: What is the probability of randomly drawing either an ace or a heart from a deck of 52 playing cards?

Answer: There are 4 aces in the pack and 13 hearts. However, 1 card is both an ace and a heart. If you simply added the two probabilities separately, you would end up counting that same card twice.

The general addition rule tells us that if some of the outcomes are non-disjoint, then we will over count those non-disjoint outcomes – an additional time for each outcome.

Therefore, we need to subtract those overlaps. In this problem, there is exactly one disjoint event.

Thus: *P*(ace or heart) = *P*(ace) + *P*(heart) – *P*(ace and heart)

= 4/52 (the 4 aces) + 13/52 (the 13 hearts) - 1/52 (the Ace of Hearts)

= 16/52

P

Example: What is the probability that a card from a deck is either a King or a Queen or a Diamond?

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$

P(King) + P(Queen) + P(Diamond) is NOT correct since there are non-disjoint events that will be over counted.

Non disjoint events: King of Diamonds and Queen of Diamonds

To solve this question, we count all the outcomes, and then subtract all outcomes that have overlapped. I.e. All non-disjoint outcomes.

= P(King) + P(Queen) + P(Diamond) – P(King and Diamond) – P(Queen and Diamond)

$= \quad 4/52 \quad + \quad 4/52 \quad + \quad 13/52 \quad - \quad 1/52 \quad - \quad 1/52 \quad - \quad 0$

− P(King and Queen)

0

= 19 /52

General multiplication rule ("And")

When dealing with events that are dependent, we need to look at our 'conditional' event and account for the possible change in probability.

- Recall that if A and B are independent, then $P(\text{A and B}) = P(\text{A}) * P(\text{B})$
  - However, if P(B) changes based on whether or not A has occurred, then we are saying that the events are dependent.
  - Therefore, rather than simply saying P(B), we must adjust it to say P(B given that A has occurred).
  - There is a special notation for this: P(B | A).

$$P(\text{A and B}) = P(\text{A}) * P(\text{B}|\text{A})$$

This is called the general multiplication rule. That is, this is a version of the multiplication rule that is not limited to independent events.

P(A and B) = *P*(A) * *P*(B | A)

Example: What is the probability of randomly drawing a card from the deck that is an Ace AND a Heart?

P(ace and heart) = *P*(ace) * *P*(heart **| ace**)

P(Ace) = (4/52)

P(Heart **| Ace**) = (1/4)

Heart A A A A

→ Take a moment and think about this! We are limiting the situation to Aces only!!

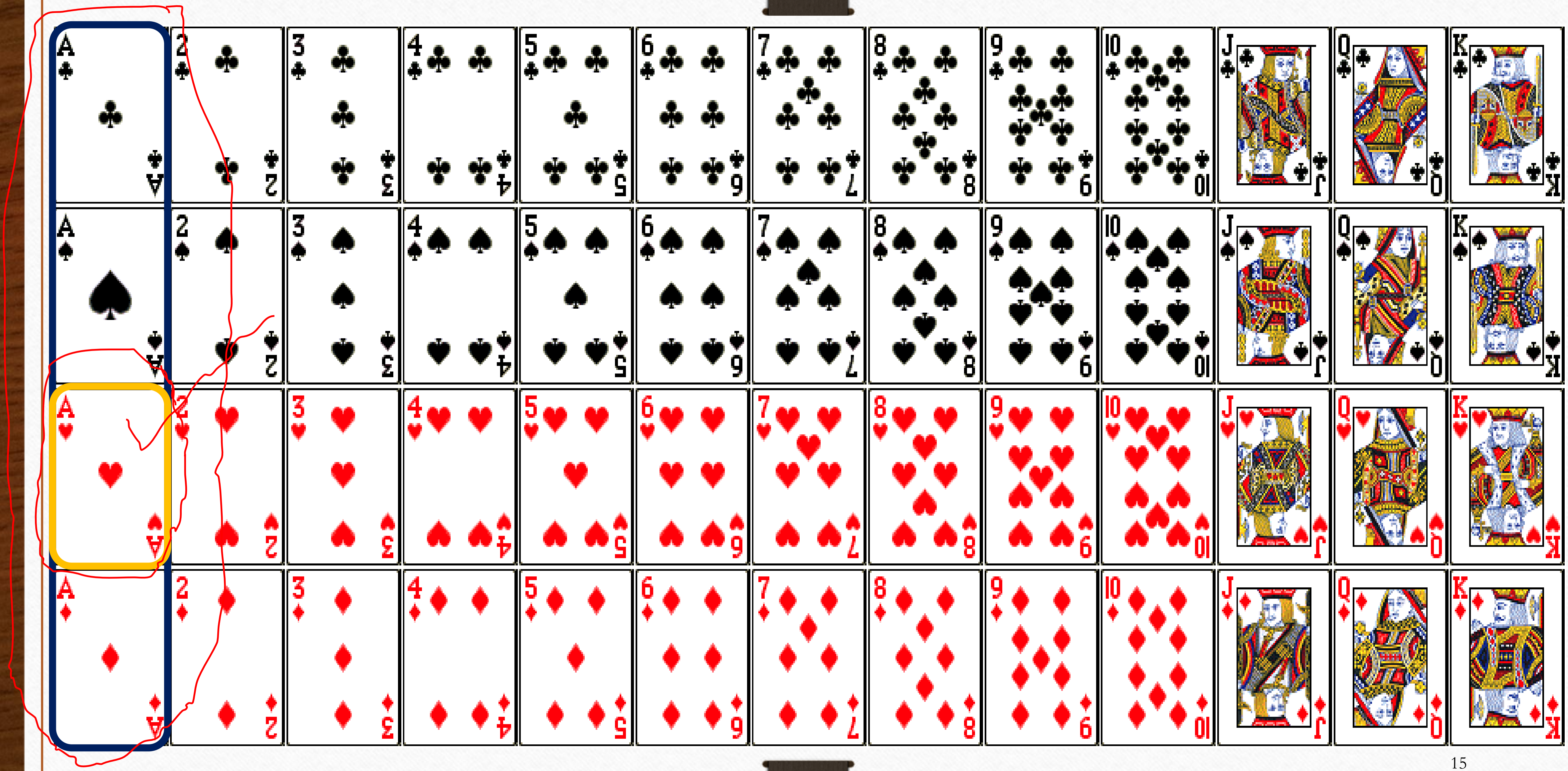
→ Probability of a Heart GIVEN that we are looking at Aces = 1/4

Answer: P(ace) * P(heart | ace)

= (4/52) * (1/4)

= 1/52

$P(A \cap H) = \frac{1}{52}$



Why do we call it the "general" multiplication rule?

- Same story as with the "general" addition rule. That is, this rule applies to ANY multiplication events – BOTH independent and non-independent.

Why does it also work for independent events?

– Recall that if two events are independent, this means that P(B) is NOT affected by P(A).

– That is, $P(B \mid A) = P(B)$.

– Our general rule states: $P(A \text{ and } B) = P(A) * P(B \mid A)$

- If our events are independent, then $P(B \mid A) = P(B)$
- So: $P(A \text{ and } B) = P(A) * P(B)$

ANY QUESTION