Lecture No. 4

Introduction to Statistics
Statistics and Probability

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Measures of Central Tendency

Concept of CT, Arithmetic Mean, Geometric Mean, Harmonic Mean, Relationship among AM, GM and HM

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In this lecture

- Measures of Central Tendency (Objective and Requirements)
- Objective of Measures of Central Tendency
- Requirement of a Good Measures of Central Tendency
- Arithmetic Mean (Merits and Demerits)
- Geometric Mean (Merits and Demerits)
- Harmonic Mean (Merits and Demerits)
- Relationship among AM, GM and HM

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What is Measures of Central Tendency

A measure of central tendency is a single value that attempts to describe a set of data by identifying the central position within that set of data. As such, measures of central tendency are sometimes called measures of central location. They are also classed as summary statistics. It is also defined as a single value that is used to describe the "**centre**" of the data.

Objective of Measures of Central Tendency

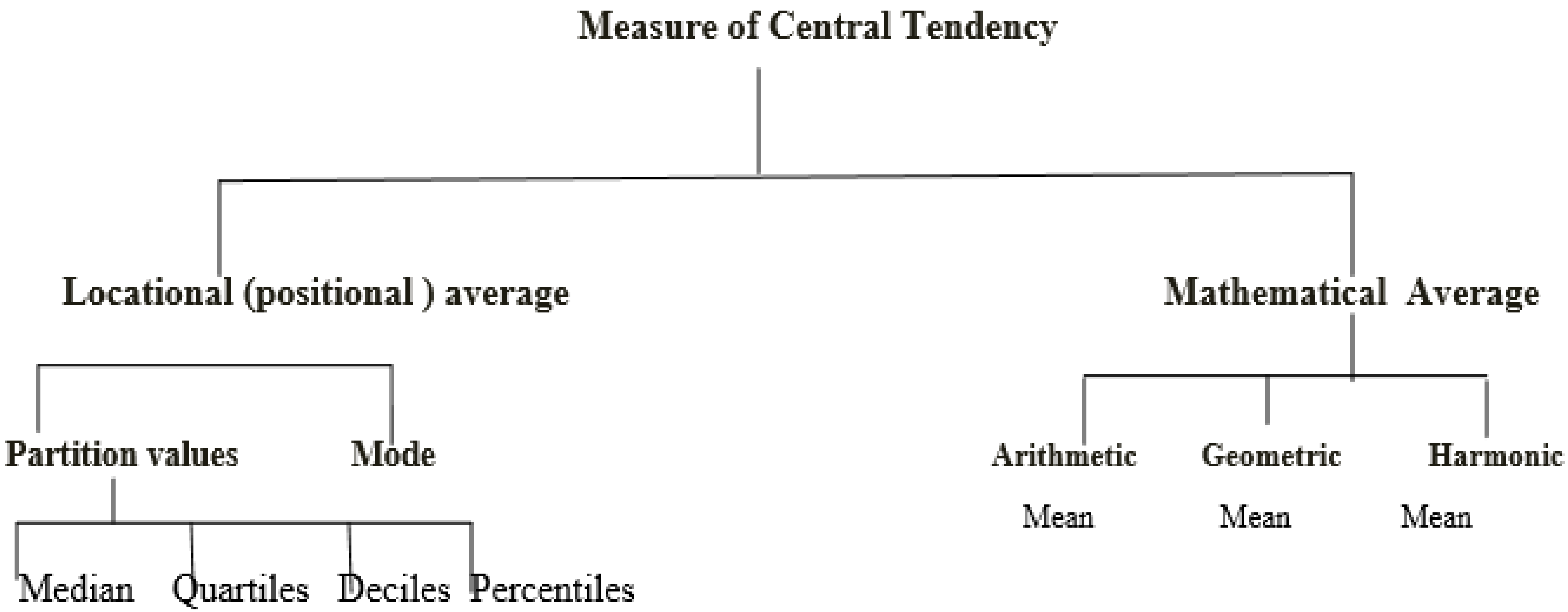
The main objectives of Measure of Central Tendency are To condense data in a single value. To facilitate comparisons between data.

Importance of Measures of Central Tendency

The measures of central tendency allow researchers to determine the typical numerical point in a set of data. The data points of any sample are distributed on a range from lowest value to the highest value. Measures of central tendency tell researchers where the center value lies in the distribution of data.

Requirement of a Good Measures of Central Tendency

1. It should be rigidly defined.
2. It should be simple to understand & easy to calculate.
3. It should be based upon all values of given data.
4. It should be capable of further mathematical treatment.
5. It should have sampling stability.



Partition values: The points which divide the data in to equal parts are called Partition values.

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Arithmetic Mean

- This is what people usually intend when they say "average"
- Suppose that $x_1, x_2, \dots, x_n$ are the observations in a sample. The most important measure of central tendency in the sample is the sample average (or simply mean),

$$\overline{\boldsymbol{x}} = \frac{Sum\ of\ observations}{Number\ of\ Observations} = \frac{\boldsymbol{x_1+x_2+\cdots+x_n}}{\boldsymbol{n}} = \frac{\sum_{i=1}^{n} \boldsymbol{x_i}}{\boldsymbol{n}}$$

- Where n is total number of sample information.
- $\sum$ notation is use for sum.

Example 1

Example: Additional irrigation utilization from major and medium schemes at the end of various plans is **97**, **110**, **130**, **152**, **168**, **187**, **22**, **266**, **226**. Find the average additional irrigation during this period.

Solution:

$$\boldsymbol{\bar{x}} = \frac{\boldsymbol{x_1 + x_2 + \cdots + x_n}}{\boldsymbol{n}}$$

$$\bar{x} = \frac{97+110+130+152+168+187+22+266+226}{9} = \frac{1358}{9} = 150.8889$$

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Mean for Grouped Data

The arithmetic mean for grouped data is obtained by using formula:

$$\bar{X} = \frac{\sum fx}{\sum f}$$

Where,

$\sum \boldsymbol{fx}$= Sum of the product of mid points (x) with respective class frequency, may be obtained as "first take the product of x and f then add the results"

$\sum \mathbf{f} = \boldsymbol{N}$= Sum of all the frequency

Example 2

Question: Calculate the mean weight of 60 apples (weight nearest to grams), from the data given in the following frequency distribution:

Calculation: $\bar{X} = \dfrac{\sum fx}{\sum f}$

where, $\sum fx$=7350 and $\sum f$=60

$$\bar{X} = \frac{7350}{60} = 122.5 \text{ grams}$$

Solution:

Weights	f	Mid points (x)	fx
65-84	9	74.5	670.5
85-104	10	94.5	945.0
105-124	17	114.5	1946.5
125-144	10	134.5	1345.0
145-164	5	154.5	722.5
165-184	4	174.5	698.0
185-204	5	194.5	972.5
Total	60		**7350**

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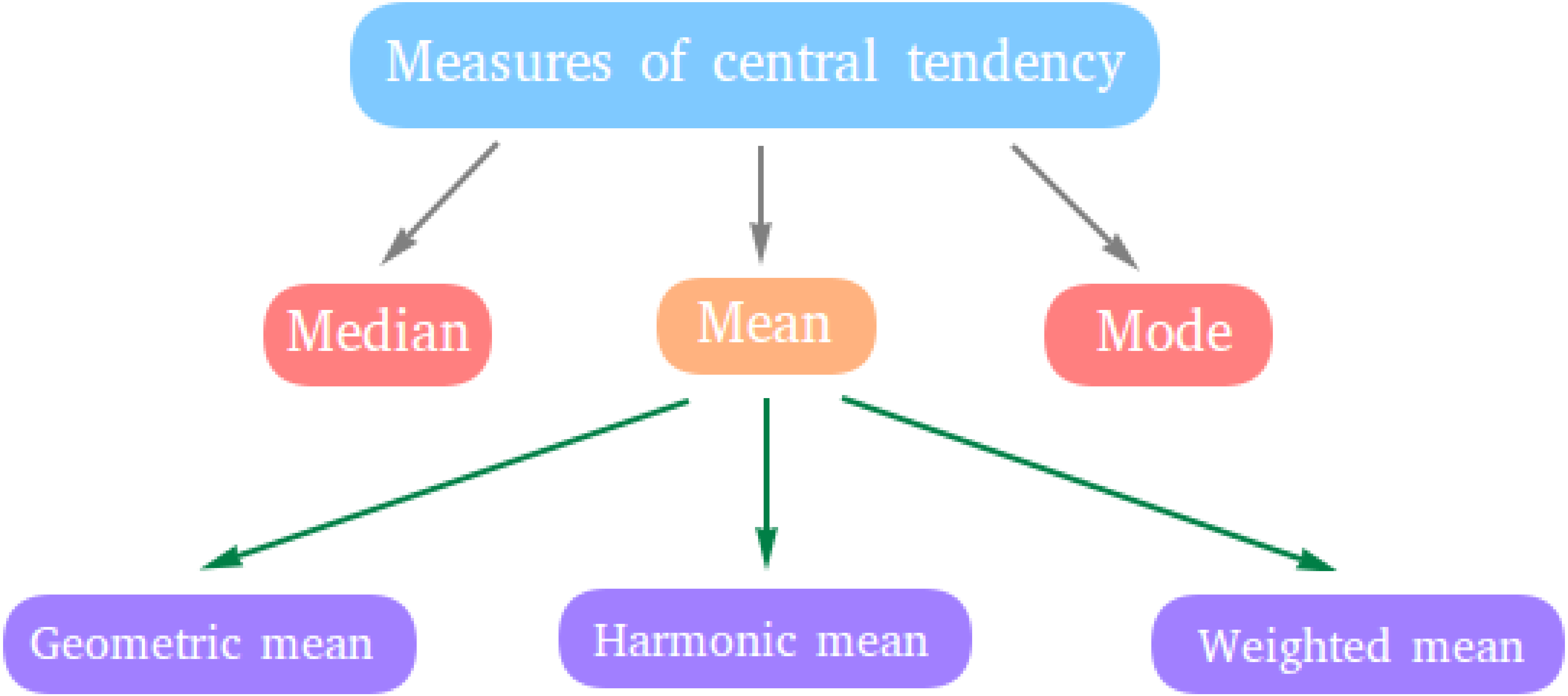
Merits and Demerits of Mean

Merits

i. It is based on all observation
ii. It is well defined by mathematical formulae
iii. It is best estimator of unknown population parameter when data is symmetrical
iv. It can be used for further algebraic mathematical treatment
v. It provides good a good basis for comparison

Demerits

i. It is affected by extreme values
ii. It gives worst estimate when the data is not symmetrical
iii. It cannot be located by inspection of the data graphically



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Geometric Mean

- Geometric mean is a kind of average of a set of numbers that is different from the arithmetic average.
- The geometric mean is well defined only for sets of positive real numbers. This is calculated by multiplying all the numbers (call the number of numbers n), and taking the nth root of the total.
- A common example of where the geometric mean is the correct choice is when averaging growth rates.
- The geometric mean is NOT the arithmetic mean and it is NOT a simple average.
- Mathematical definition: The nth root of the product of n numbers.

Formula of G.M

G.M. is the nth positive root of product of non-zero and none-negative values $x_1, x_2, \ldots, x_n$.

$$\mathbf{G.M.} = \sqrt[n]{\boldsymbol{x_1 \times x_2 \times \cdots \times x_n}}$$

For ungrouped data:

$$\mathbf{G = antilog}\left(\frac{\sum_{\mathrm{i=1}}^{n} \mathbf{log} x_i}{n}\right)$$

Where, $\sum_{i=1}^{n} log x_i$ = Sum of the log of the observations

n =no. of observations in a data set

For grouped data:

$$\mathbf{G = antilog}\left(\frac{\sum_{\mathrm{i=1}}^{n} f_i \mathbf{log} x_i}{\sum_{\mathrm{i=1}}^{n} f}\right)$$

Where, $\sum_{\mathrm{i=1}}^{n} f_i \mathbf{log} x_i$= Sum of the product of log of midpoint (x) with respective class frequency (f). $\sum_{\mathrm{i=1}}^{n} f_i$= sum of all frequencies

Example

Question: Calculate the geometric mean of the following frequency distribution:

Classes	x	f	log(x)	flog(x)
110-119	114.5	1	2.058805	2.058805
120-129	124.5	4	2.095169	8.380677
130-139	134.5	17	2.128722	36.18828
140-149	144.5	28	2.159868	60.4763
150-159	154.5	25	2.188928	54.72321
160-169	164.5	18	2.216166	39.89099
170-179	174.5	13	2.241795	29.14334
180-189	184.5	6	2.265996	13.59598
190-199	194.5	5	2.28892	11.4446
200-209	204.5	2	2.310693	4.621387
210-219	214.5	1	2.331427	2.331427
Total		**120**		**262.855**

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Calculations:

$$G = antilog\left(\frac{\sum_{i=1}^{n} f_i log x_i}{\sum_{i=1}^{n} f}\right)$$

$$G = antilog\left(\frac{262.855}{120}\right)$$

$$G = antilog(2.19045)$$

$$G = 155.0452$$

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Merits and Demerits of GM

Merits

i. It is based on all observation
ii. It is well defined by mathematical formulae
iii. It can be used for further algebraic mathematical treatment
iv. It is suitable average for averaging ratios and percentages and the rates of increase and decrease
v. It is not much affected by extreme values

Demerits

i. It cannot be located by inspection of the data graphically.
ii. It cannot be computed if the value in the data is zero or negative.
iii. It gives great importance to the small values

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Harmonic Mean

Harmonic mean is "sum of the reciprocals of the given values".

A simple way to define a harmonic mean is to call it the reciprocal of the arithmetic mean of the reciprocals of the observations. The most important criteria for it is that none of the observations should be zero.

A harmonic mean is used in averaging of ratios. The most common examples of ratios are that of speed and time, cost and unit of material, work and time etc.

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Formula of H.M.

Harmonic mean is the reciprocal of the A.M. of the reciprocal of none-zero values $x_1, x_2, \ldots, x_n$

$$\mathrm{H.M.} = \text{Reciprocal of } \frac{\frac{1}{x_1}+\frac{1}{x_2}+\cdots+\frac{1}{x_n}}{n} = \frac{n}{\sum_{i=1}^{n}\frac{1}{x_i}}$$

In case of *frequency distribution* (grouped data) the harmonic mean is given by:

$$\mathrm{H.M.} = \text{Reciprocal of } \frac{\frac{f1}{x_1}+\frac{f2}{x_2}+\cdots+\frac{fn}{x_n}}{\sum f} = \frac{\sum f}{\sum_{i=1}^{n}\frac{f}{x_i}}$$

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Example for Ungrouped

Question: Calculate the harmonic mean of the numbers: 15, 20, 25.

Solution:

The harmonic mean is calculated as below: $H.M.=\frac{n}{\sum\frac{1}{x}}$

$$H.M.=\frac{3}{\frac{1}{15}+\frac{1}{20}+\frac{1}{25}}=\frac{3}{0.06667+0.5000+0.04000}$$

$$H.M=\frac{3}{0.15667}=19.15$$

Example for Grouped Data

Question: Calculate the harmonic mean of the following frequency distribution

Classes	x	f	f/x
110-119	114.5	1	0.008734
120-129	124.5	4	0.032129
130-139	134.5	17	0.126394
140-149	144.5	28	0.193772
150-159	154.5	25	0.161812
160-169	164.5	18	0.109422
170-179	174.5	13	0.074499
180-189	184.5	6	0.03252
190-199	194.5	5	0.025707
200-209	204.5	2	0.00978
210-219	214.5	1	0.004662
Total		120	0.77943

Calculations

Using formula for grouped data:

$$\mathrm{H.M.} = \frac{\sum f}{\sum_{i=1}^{n} \frac{f}{x_i}}$$

$$\mathrm{H.M.} = \frac{120}{0.77943}$$

$$\mathrm{H.M.} = 153.9586$$

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Merits and Demerits of HM

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Relation Between Three Averages

The relation between three averages is that the arithmetic mean is always greater than or equal to geometric mean.

$$A.M \geq G.M \geq H.M$$

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Questions for Practice

Question 1: The following sample data of the number of communications are taken from logs of communication with Distance Education students: 5, 9, 5, 23, 27, 55, 34, 7, 30, 15, 22, 60, 14, 52, 297, 8, 51, 15, 51, 35, 15, 39, 137, 43, 38, 14, 93, 7. Calculate the arithmetic mean, geometric mn and harmonic mean and show the relationship $A.M \geq G.M \geq H.M$.

Question 2: The following is the age distribution of 1000 persons working in an organization:

Groups	20-25	25-30	30-35	35-40	40-45	45-50	50-55	55-60	60-65
Frequency	30	160	210	180	145	105	70	60	40

Calculate the arithmetic mean, geometric mean and harmonic mean; prove the relationship $A.M \geq G.M \geq H.M$.

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ANY QUESTION