# Lecture 2

# Normal Distribution: Direct use of Area Table

# (X→ Z → Get Area as Answer)

## Review of Previous Lecture

- **What is Normal Distribution**
- **Application of Normal Distribution in Daily Life.**
- **Properties of Normal Distribution and Normal Curve**
- **Key points for Normal Curve**

## In this lecture, the general idea is

- **Direct use of Area Table: Find Probability under Normal Distribution.**
- **Z → Area (Area is Answer)**
- **(X → Z → Area (Area is Answer)**

**Exercise Questions:** 9.19 to 9.37 (page 390 to 392)



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# Direct Use of Area Table (Z → Area)

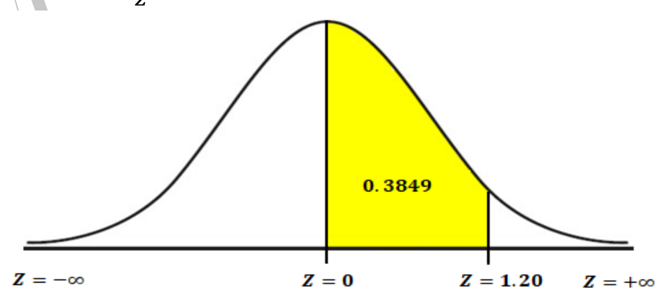
**Example 9.6 (page. 366):** Let the random variable $Z$ have standard normal distribution. Find the following probabilities.

i. $P(0 \leq Z \leq 1.20)$
ii. $P(-1.65 \leq Z \leq 0)$
iii. $P(0.6 \leq Z \leq 1.67)$
iv. $P(-1.30 \leq Z \leq 2.18)$
v. $P(-1.96 \leq Z \leq -0.84)$
vi. $P(Z \geq 1.96)$
vii. $P(Z \leq -2.15)$



**Solution:** As given $Z \sim N(0, 1)$ so $\mu_z = 0$ and $\sigma_z = 1$

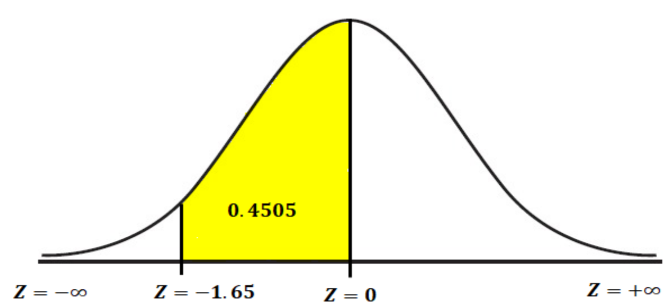
**i.** $\boldsymbol{P(0 \leq Z \leq 1.20)}$**=?**




See in Area Table ($Z = 1.2$ at 0.00)

$P(0 \leq Z \leq 1.20) = 0.3849$

**ii.** $\boldsymbol{P(-1.65 \leq Z \leq 0)}$**=?**




See in Area Table ($Z = 1.6$ at 0.05)

$P(-1.65 \leq Z \leq 0) = 0.4505$



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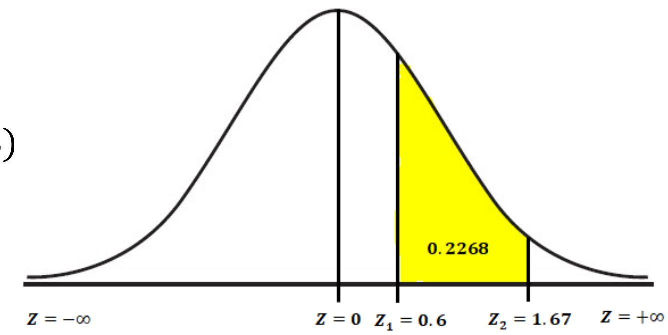
iii. $\boldsymbol{P(0.6 \leq Z \leq 1.67)}$**=?**

$P(0.6 \leq Z \leq 1.67)$

$= P(0 \leq Z \leq 1.67) - P(0 \leq Z \leq 0.6)$

$= 0.4525 - 0.2257$

$P(0.6 \leq Z \leq 1.67) = 0.2268$ ✓



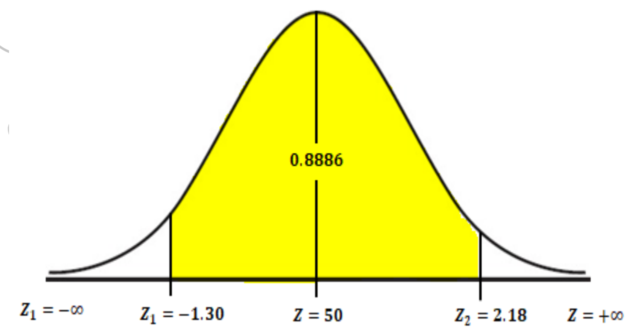

iv. $\boldsymbol{P(-1.30 \leq Z \leq 2.18)}$**=?**

$P(-1.30 \leq Z \leq 2.18)$

$= P(1.30 \leq Z \leq 0) + P(0 \leq Z \leq 2.18)$

$= 0.4032 + 0.4854$

$P(-1.30 \leq Z \leq 2.18) = 0.8886$ ✓



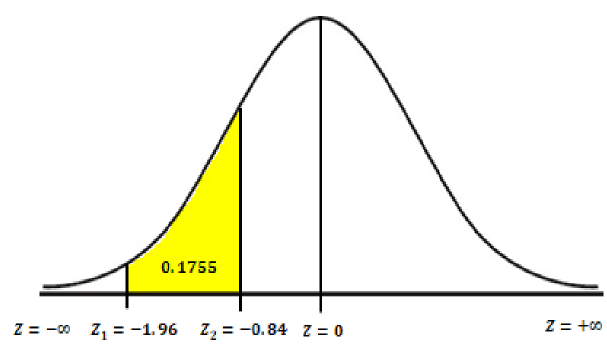

v. $\boldsymbol{P(-1.96 \leq Z \leq -0.84)}$**=?**

$P(-1.96 \leq Z \leq -0.84)$

$= P(-1.96 \leq Z \leq 0) - P(-0.84 \leq Z \leq 0)$

$= 0.4750 - 0.2995$

$P(-1.96 \leq Z \leq -0.84) = 0.1755$ ✓





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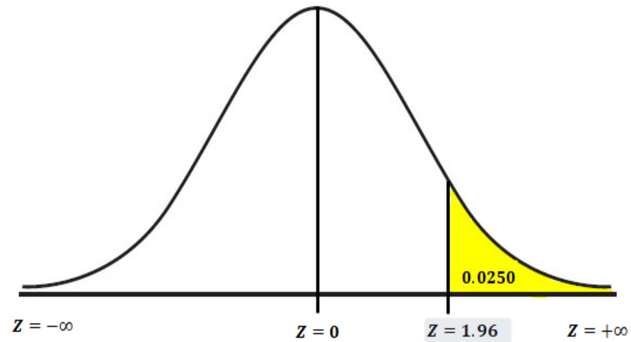
**vi.** $\boldsymbol{P(Z \geq 1.96)}$**=?**



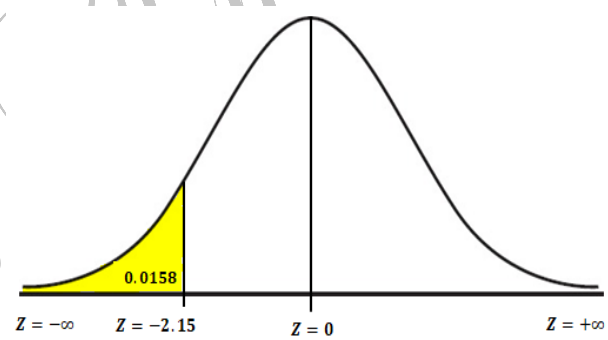

$P(Z \geq 1.96)$

$= P(0 \leq Z \leq +\infty) - P(0 \leq Z \leq 1.96)$

$= 0.50 - 0.4750$

$P(Z \geq 1.96) = 0.0250$

**vii.** $\boldsymbol{P(Z \leq -2.15)}$**=?**




$P(Z \leq -2.15)$

$= P(-\infty \leq Z \leq 0) - P(-2.15 \leq Z \leq 0)$

$= 0.50 - 0.4842$

$P(Z \leq -2.15) = 0.0158$





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# Direct Use of Area Table (X → Z → Area)

**Example 9.7 (page. 367):** A random variable X is normally distributed with $\mu = 50$ and $\sigma^2 = 25$. Find the probability that will be fall (i) between 0 and 40; (ii) between 55 and 100; (iii) larger than 54; (iv) smaller than 57.

**Solution:** As given $X \sim N(50, 5)$ so $\mu = 50$ and $\sigma = 5$.

i. $\boldsymbol{P(0 \le X \le 40) = ?}$

At $X_1 = 0$, we have $Z_1 = \frac{X_1 - \mu}{\sigma} = \frac{0-50}{5} = -10$

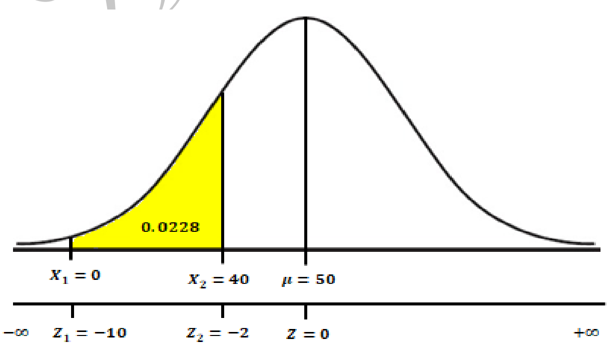
At $X_2 = 40$, we have $Z_2 = \frac{X_2 - \mu}{\sigma} = \frac{40-50}{5} = -2$

$P(0 \le X \le 40) = P(-10 \le Z \le -2)$

$= P(-10 \le Z \le 0) - P(-2 \le Z \le 0)$

$= 0.50 - 0.4772$

$P(0 \le X \le 40) = 0.0228$ ✓




ii. $\boldsymbol{P(55 \le X \le 100) = ?}$

At $X_1 = 55$, we have $Z_1 = \frac{X_1 - \mu}{\sigma} = \frac{55-50}{5} = 1$

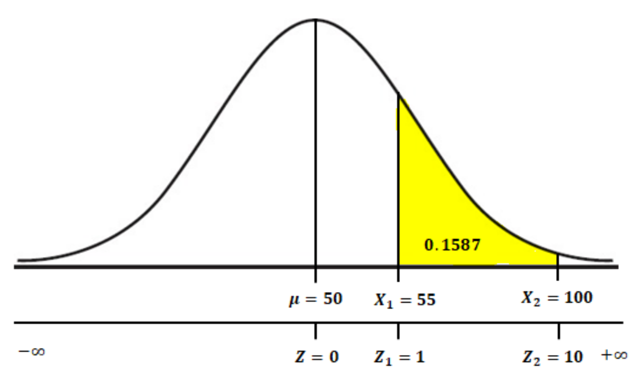
At $X_2 = 100$, we have $Z_2 = \frac{X_2 - \mu}{\sigma} = \frac{100-50}{5} = 10$

$P(55 \le X \le 100) = P(1 \le Z \le 10)$

$= P(0 \le Z \le 10) - P(0 \le Z \le 1)$

$= 0.50 - 0.3413$

$P(55 \le X \le 100) = 0.1587$ ✓



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**iii.** $\boldsymbol{P(X > 54)}$**=?**

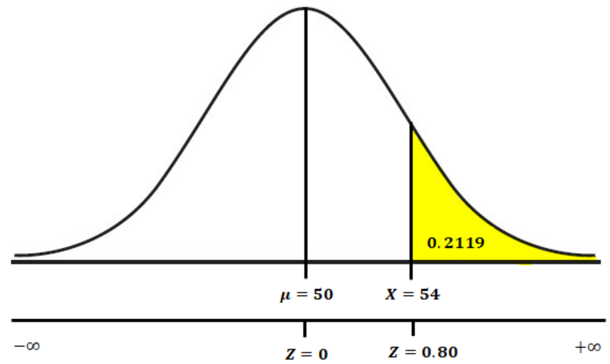
At $X = 54$, we have $Z = \frac{X-\mu}{\sigma} = \frac{54-50}{5} = 0.80$

$P(Z > 0.80)$

$= P(0 \leq Z \leq +\infty) - P(0 \leq Z \leq 0.80)$

$= 0.50 - 0.2881$

$P(X > 54) = 0.2119$ ✓




**iv.** $\boldsymbol{P(X < 57)}$**=?**

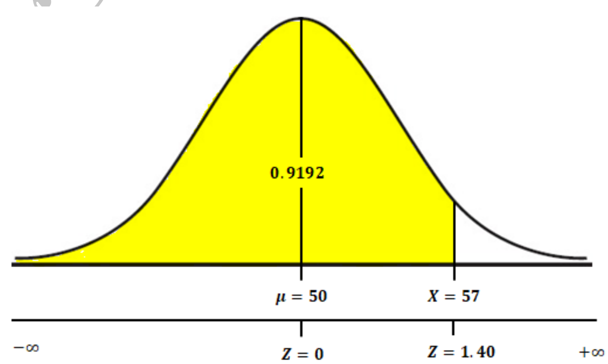
At $X = 57$, we have $Z = \frac{X-\mu}{\sigma} = \frac{57-50}{5} = 1.40$

$P(Z < 1.40)$

$= P(-\infty \leq Z \leq 0) + P(0 \leq Z \leq 1.40)$

$= 0.50 + 0.4192$

$P(X < 57) = 0.9192$ ✓





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**Example 9.8 (page. 368):** The length of life for an automatic dishwasher is approximately normally distributed with mean of 3.5 years and a standard deviation of 1.0 years. If this type of dishwasher is guaranteed for 12 month, what fraction of the sales will require replacement?

**Solution:** As given $X \sim N(3.5,\ 1)$ so $\mu = 3.5$ and $\sigma = 1$.

The fraction of sales requiring replacement is equal to the area under the normal curve for $X \leq 1$ year (12 months), the guaranteed period.

$\boldsymbol{P(X < 1)}$**=?**

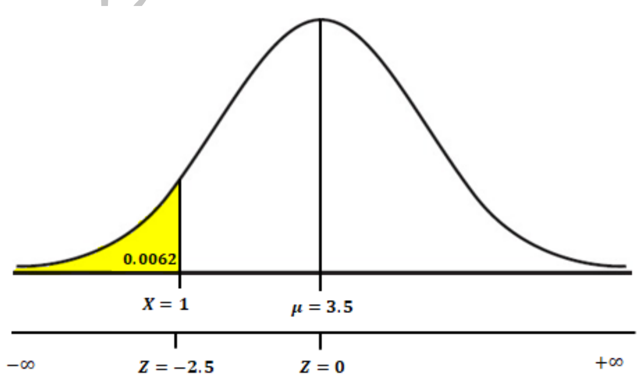
At $X = 1$, we have $Z = \frac{X-\mu}{\sigma} = \frac{1-3.5}{1} = -2.5$

$P(Z < -2.5)$

$= P(-\infty \leq Z \leq 0) - P(-2.5 \leq Z \leq 0)$

$= 0.50 - 0.4938$

$P(X < 1) = 0.0062$




About 0.62% of sales need replacing before 1 year (12 months).



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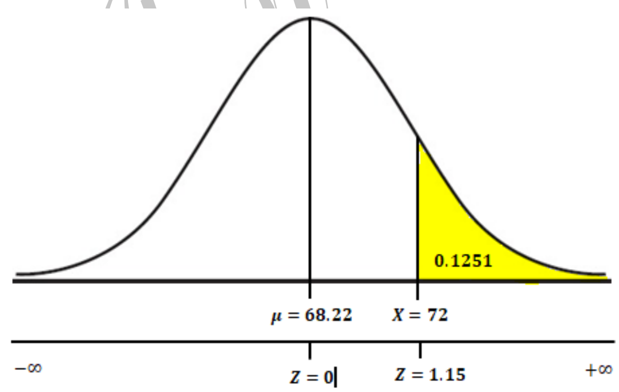
**Example 9.9 (page. 368):** The mean height of soldiers is 68.22 inches with a variance of $10.8(\text{in.})^2$. Assuming the distribution of heights to be normal, how many soldiers in a regiment of 1000 would you expect to be over 6 feet (72 inches) tall?

**Solution:** As given $X \sim N(68.22,\ 3.29)$ so $\mu = 68.22$ and $\sigma = 3.29$.

$\boldsymbol{P(X > 72)}$**=?**

At $X = 72$, we have $Z = \frac{X-\mu}{\sigma} = \frac{72-68.22}{3.29} = 1.15$




$P(Z > 1.15)$

$= P(0 \leq Z \leq +\infty) - P(0 \leq Z \leq 1.15)$

$= 0.50 - 0.3749$

$P(X > 72) = 0.1251$

As there are 1000 soldiers in the regiment, the number expected to be over 6 feet (or 72 inches) is $1000 \times 0.1251 = 125$



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**Example:** Let $X$ be a continuous random variable that has a normal distribution with mean 50 and standard deviation 8. Find the probability $P(30 \leq X \leq 39)$.

**Solution:** As given $X \sim N(50,\ 8)$ so $\mu = 50$ and $\sigma = 8$.

$\boldsymbol{P(30 \leq X \leq 39)}$**=?**

At $X_1 = 30$, we have $Z_1 = \frac{X_1 - \mu}{\sigma} = \frac{30-50}{5} = -2.5$

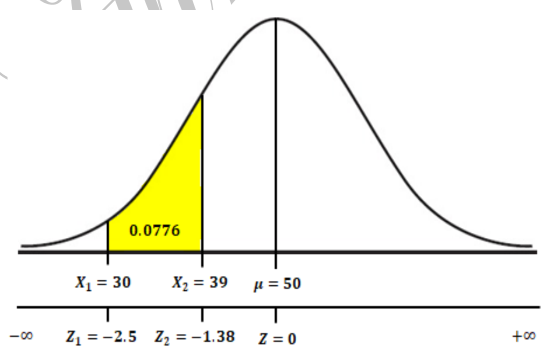
At $X_2 = 39$, we have $Z_2 = \frac{X_2 - \mu}{\sigma} = \frac{39-50}{5} = -1.38$

$P(30 \leq X \leq 39) = P(-2.5 \leq Z \leq -1.38)$

$= P(-2.5 \leq Z \leq 0) - P(-1.38 \leq Z \leq 0)$

$= 0.0838 - 0.0062$

$P(30 \leq X \leq 39) = 0.0776$





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**Example:** According to a Sallie Mae survey and credit bureau data, in 2008, college students carried an average of \$3173 debt on their credit cards (USA TODAY, April 13, 2009). Suppose that current credit card debts for all college students have a normal distribution with a mean of \$3173 and a standard deviation of \$800. Find the probability that credit card debt for a randomly selected college student is between \$2109 and \$3605.

**Solution:** As given $X \sim N(3173,\ 800)$ so $\mu = 3173$ and $\sigma = 800$.

$\boldsymbol{P(2109 \leq X \leq 3605) = ?}$

At $X_1 = 2109$, we have $Z_1 = \frac{X_1 - \mu}{\sigma} = \frac{2109 - 3173}{800} = -1.33$

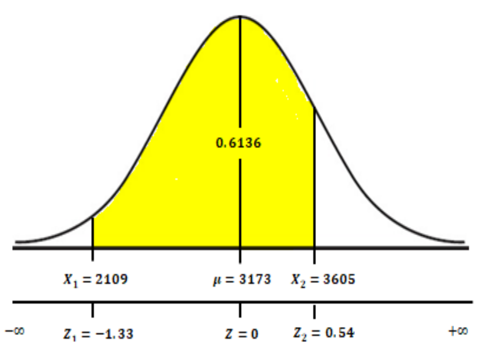
At $X_2 = 3605$, we have $Z_2 = \frac{X_2 - \mu}{\sigma} = \frac{3605 - 3173}{800} = 0.54$

$P(2109 \leq X \leq 3605) = P(-1.33 \leq Z \leq 0.54)$

$= P(-1.33 \leq Z \leq 0) + P(0 \leq Z \leq 0.54)$

$= 0.4082 + 0.2054$

$P(2109 \leq X \leq 3605) = 0.6136$ ✓






# In next Lecture

- **Normal Distribution: Inverse use of Area Table**

**THE END ☺**

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