Lecture
Basic Examples of Probability

Probability and Statistics

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Introduction to Probability

What is the Probability?

- Probability is the measure of how likely an event is.

Or

- Probability is the quantitative measure of uncertainty.

For example:

- Weather predictions are based on probability. The weatherman might say, "*There is an 80 percent chance of rain in the afternoon.*"

Introduction to Probability

- Generally the word probability is used in our day to day conversations by coming across following statements such as :
  - Probably it may rain today.
  - He may possibly join politics .
  - Pakistan Cricket Team has good chances of winning World –Cup.
- In this all statements, the terms *probably* , *possibly* & *chance* are used by me , which conveys the sense that there is uncertainty about what has happened and what is going to happen. Therefore , the term probability can substitute the word uncertainty.

Probability of an Event:

If an experiment can result in any one of N different equally likely outcomes, and if exactly n of these outcomes correspond to event A, then the probability of event A is denoted by $P(A)$, and is defined as:

Probability of an Event:

The ratio of the number of favourable cases to the number of all the cases, i.e.,

$$P(A) = \frac{no.\,of\ outcomes\ in\ favor\ of\ A}{no.\,of\ all\ possible\ outcomes\ of\ an\ experiment}$$

$$P(A) = \frac{n(A)}{n(S)}$$

Where,

- $n(A)$= no. of possible outcomes in favour of A, or ,the no. of ways to get Success.
- $n(S)$ = no. of all possible outcomes in S.

Steps to find probability:

1. Find total possible outcomes $n(S)$.

2. Find event of interest, say A.

3. Find no. of outcomes in favour of A, i.e., $n(A)$.

4. Use formula to find probability, i.e.,

$$P(A) = \frac{n(A)}{n(S)}$$

Example 1:

Question: Imagine you are rolling a die. Calculate the probability of rolling a "5."

Solution:

There are '6' possible outcomes in rolling a $die\ i.e., 1, 2, 3, 4, 5, 6$. Thus,

$$n(S) = 6$$

As there is only 1 '5'on the die so there is only one way to get success. So,

$$n(5) = 1$$

Thus,

$$P(5) = \frac{n(5)}{n(S)} = \frac{1}{6}$$

Axioms of Probability:

The probability of an event "A" must satisfy the following axioms:

- The probability is a non-negative real number and cannot exceed unity, i.e., for any event "A", $0 \leq P(A) \leq 1$.
- $P(S) = 1$, for the sure event S.
- Sum of the probability of success and failure of an outcome is always equal to one. i.e., $P(A) + P(\bar{A}) = 1$.
- $P(\emptyset) = 0$

Example 2:

Question: In an experiment of a rolling a die. What is the probability of rolling an even number.

Solution: There are '6' possible outcomes in rolling a $die\ i.e., 1, 2, 3, 4, 5, 6.$

Thus,

$$n(S) = 6$$

Let "*E*" denote rolling an even number, there are '3' outcomes in favour of an even roll, i.e., 2, 4, 6, i.e.,

$$n(E) = 3$$

Thus the probability of rolling an even number is:

$$P(E) = \frac{n(E)}{n(S)} = \frac{3}{6} = \frac{1}{2}$$

Example 3:

Question: A fair coin is tossed three times. What is the probability that at least one head appears?

Solution: The total no. of possible outcomes are:

$$n(S) = 2 \times 2 \times 2 = 8$$

the sample space for this experiment is:

$$S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$$

Let *A* denote the event that at least one head appears. Then

$$A = \{HHH, HHT, HTH, THH, HTT, THT, TTH\}$$

And therefore, $n(A) = 7$

Hence, $P(A) = \frac{n(A)}{n(S)} = \frac{7}{8}$

Example 4:

Question: If two fair dice are thrown, what is the probability of getting:

a. A double six?

b. A sum of 10 or more?

Solution: The total no. of possible outcomes are:

$$n(S) = 6 \times 6 = 36$$

The sample space S is represented as:

1st Dice

2nd Dice

	1	2	3	4	5	6
1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

Example 4(cont)

a) Let “A” represent the event that a double 6 occurs, then

$$A = \{(6, 6)\}$$

Thus,

$$n(A) = 1$$

Hence, the probability of a double “6” occurs is:

$$P(A) = \frac{n(A)}{n(S)} = \frac{1}{36}$$

Example 4(cont)

b) Let "*B*" represent the event that a sum of "*10 or more*" dots occurs.

Then,

$$B = \{(4, 6)\ ,(5, 5)\ (5, 6)\ ,(6, 4)\ (6, 5)\ (6, 6)\ \}$$

Thus,

$$n(B) = 6$$

Hence, the probability of a "*10 or more*" occurs is:

$$P(B) = \frac{n(B)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

Example 5:

Question: In an experiment of throwing of two dice together, what is the probability of the same dots occur?

Solution: Let "*E*" represent the event that "*same*" dots occurs. Then,

$$E = \{(6, 6) , (5, 5) (4, 4) , (3, 3) (2, 2) (1, 1) \}$$

Thus,

$$n(E) = 6$$

Hence, the probability of "*same dots*" occurs is:

$$P(E) = \frac{n(E)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

Similarly, the probability of getting "*different dots*" occurs is: $1 - \frac{1}{6} = \frac{5}{6}$

Example 6:

Question: If a card is drawn from an ordinary deck of 52 playing cards, find the probability that:

a. The card is red,

b. The card is diamond,

c. The card is 10,

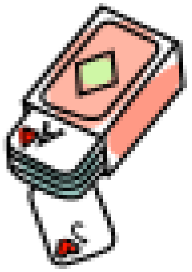
d. The card is face.

Solution: The total number of possible outcomes is 52, thus

$$n(S) = 52$$

Summary of an Ordinary Deck of 52 Playing Cards:

A little note about a deck of cards



A deck of cards = 52 cards

Each deck has four parts (suits) with 13 cards in them.

Each suit has 3 face cards.

52 cards = 1 deck

13 spades	13 hearts	13 clubs	13 diamonds
♠	♥	♣	♦

No. of Red Cards= 26, No. of Black Cards=26, No. of Face Cards=12

(4)

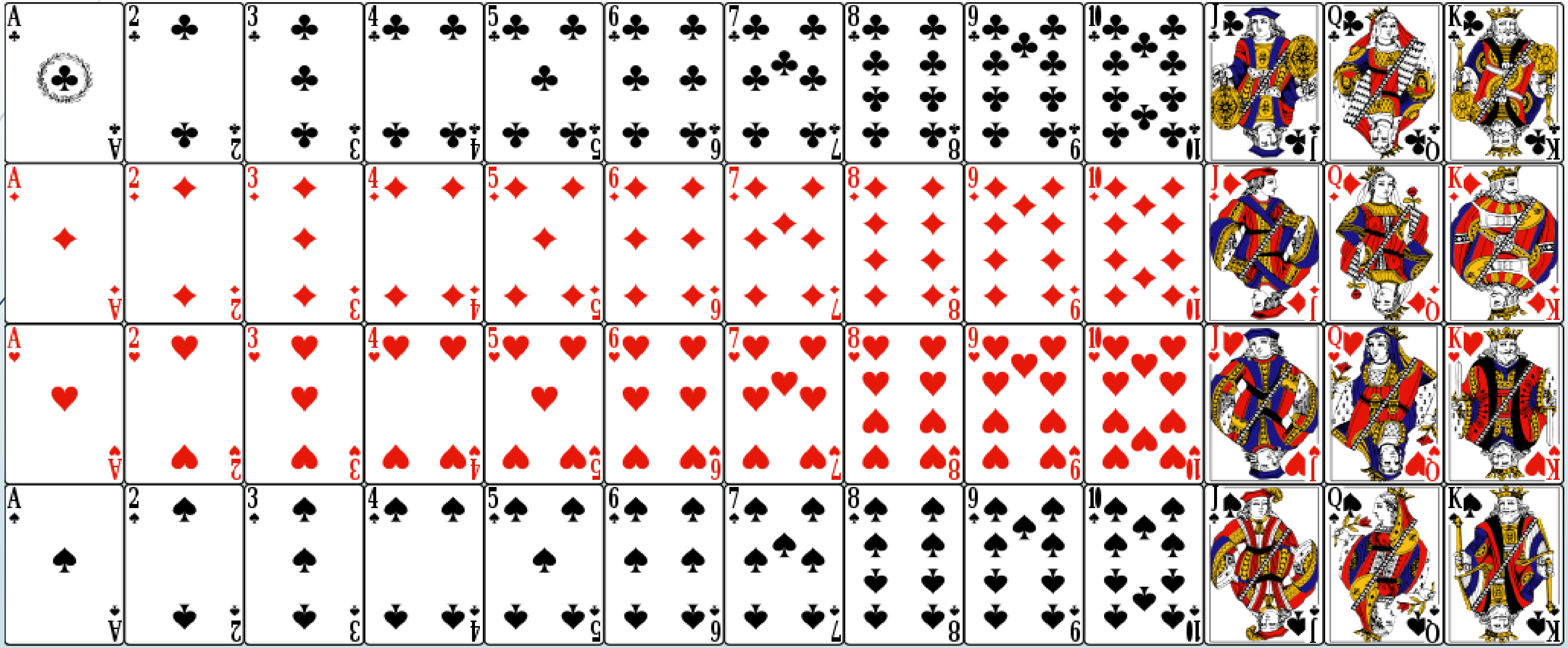
Aces 2's 3's 4's 5's 6's 7's 8's 9's 10's Jacks Queens Kings

Clubs (13)

Diamonds (13)

Hearts (13)

Spades (13)



Example 6(cont)

a) Let "A" represent the event that the card drawn is a red card. Then the number of outcomes in favour of "A" is 26, i.e., $n(A) = 26$, Since there are 26 red cards.

Hence,

$$P(A) = \frac{n(A)}{n(S)} = \frac{26}{52} = \frac{1}{2}$$

Example 6(cont)

b) Let "B" represent the event that the card drawn is a diamond. Then the number of outcomes in favour of "B" is 13, i.e., $n(B) = 13$, Since there are 13 diamond cards.

Hence,

$$P(B) = \frac{n(B)}{n(S)} = \frac{13}{52} = \frac{1}{4}$$

Example 6(cont)

c) Let "C" represent the event that the card drawn is a 10. Then the number of outcomes in favour of "C" is 4, i.e., $n(C) = 4$, Since there are 4 ten cards.

Hence,

$$P(C) = \frac{n(C)}{n(S)} = \frac{4}{52} = \frac{1}{13}$$

Example 6(cont)

d) Let "*D*" represent the event that the card drawn is a face card.

Then the number of outcomes in favour of "*D*" is *12*, i.e., $n(D) = 3 * 4 = 12$, Since there are 12 face cards (3 in each suit).

Hence,

$$P(D) = \frac{n(D)}{n(S)} = \frac{12}{52}$$

Example 7:

Question: Six white balls and four black balls, which are indistinguishable apart from colour, are placed in a bag. If six balls are taken from the bag, find the probability of their being three white and three black.

Solution:

The total number of possible outcomes in *S* is:

$$n(S) = \binom{10}{6} = \frac{10!}{6!\,(10-6)!} = 210$$

Example 7(cont)

Let A represent the event that three white and three black balls are taken. Then the number of outcomes correspond to A is:

$$n(A) = \binom{6}{3}\binom{4}{3} = 80$$

Therefore,

$$P(A) = \frac{n(A)}{n(S)} = \frac{80}{210} = \frac{8}{21}$$

Example 8:

Four items are taken at random from a box of 12 items and inspected. The box is rejected if more than 1 item is found to be faulty. If there are 3 faulty items in the box, find the probability that the box is accepted.

Solution:

The total number of possible outcomes in S is:

$$n(S) = \binom{12}{4} = \frac{12!}{4!\,(12-4)!} = 495$$

Example 8(cont)

The box contains 3 *faulty* and 9 *good* items. The box is accepted if there is:

i. No faulty item, or ii. One faulty item in the sample of four items selected.

Let "*A*" denote the event the number of faulty items chosen is *0 or 1*.

$$n(A) = \binom{3}{0}\binom{9}{4} + \binom{3}{1}\binom{9}{3}$$

$$n(A) = 126 + 252 = 378\ sample\ points.$$

Hence,

$$P(A) = \frac{n(A)}{n(S)} = \frac{378}{495} = 0.76$$

Hence the probability that the box is accepted is 0.76.

Practice Questions:

Question 1: A deck of playing cards is composed of 52 cards, with 13 cards in each of four suits called clubs, diamonds, hearts, and spades. In each suit there is one card each of the following thirteen denominations: ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, jack (J), queen (Q), king (K).What is the probability of selecting at random:

a. A diamond from the deck of 52 cards?
b. A king from the deck?
c. A diamond or a king?
d. An ace or a jack?
e. A king, queen or a jack?

Question 2: What is the probability of getting **(i)** a total of 7 and **(ii)** a sum of 11 when a Pair of fair dice are tossed?

Question 3: If the probabilities are, respectively, 0.09, 0.15, 0.21 and 0.23 that a person purchasing a new automobile will choose the colour green, white, red, or blue. What is the probability that a given buyer will purchase a new automobile that comes in one of those colours?

ANY QUESTION