A Survey of Probability Concepts

Chapter 5

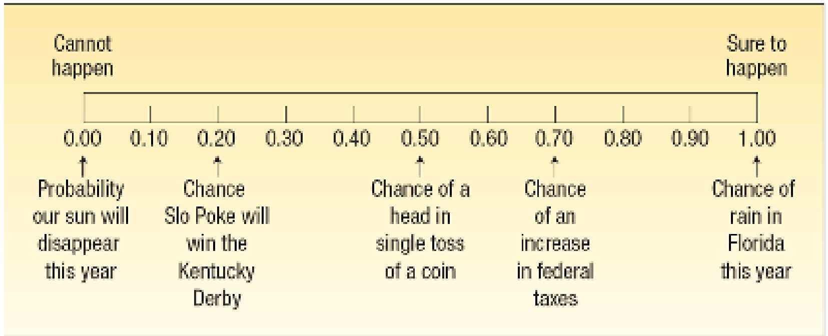
McGraw-Hill/Irwin

GOALS

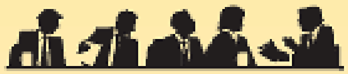
1. Define *probability*.
2. Describe the *classical, empirical*, and *subjective* approaches to probability.
3. Explain the terms *experiment, event, outcome*.
4. Define the terms *conditional probability* and *joint probability*.
5. Calculate probabilities using the *rules of addition* and *rules of multiplication*.
6. Apply a *tree diagram* to organize and compute probabilities.

Probability, Experiment, Outcome, Event: Defined

PROBABILITY A value between zero and one, inclusive, describing the relative possibility (chance or likelihood) an event will occur.

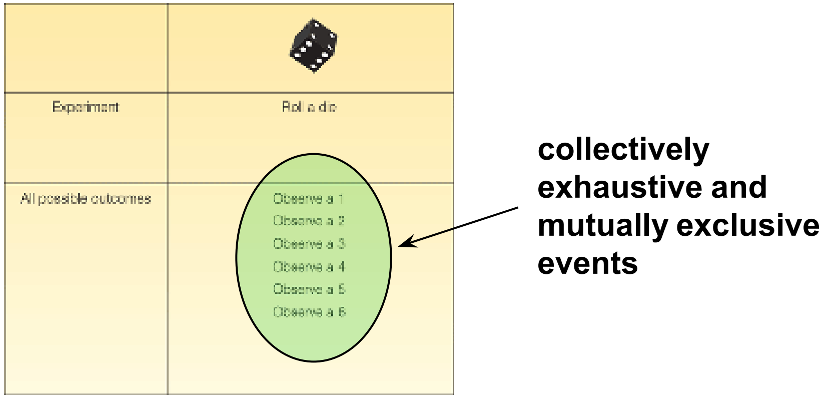


- An experiment is a process that leads to the occurrence of one and only one of several possible observations.
- An outcome is the particular result of an experiment.
- An event is the collection of one or more outcomes of an experiment.

		
Experiment	Roll a die	Count the number of members of the board of directors for Fortune 500 companies who are over 60 years of age
All possible outcomes	Observe a 1 Observe a 2 Observe a 3 Observe a 4 Observe a 5 Observe a 6	None are over 60 One is over 60 Two are over 60 ... 29 are over 60 ... ... 48 are over 60 ...
Some possible events	Observe an even number Observe a number greater than 4 Observe a number 3 or less	More than 13 are over 60 Fewer than 20 are over 60

Mutually Exclusive Events and Collectively Exhaustive Events

- Events are mutually exclusive if the occurrence of any one event means that none of the others can occur at the same time.
- Events are collectively exhaustive if at least one of the events must occur when an experiment is conducted.
- The sum of all collectively exhaustive and mutually exclusive events is 1.0 (or 100%)



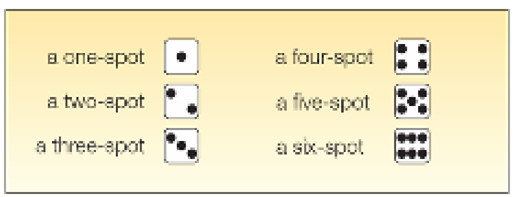
- Events are independent if the occurrence of one event does not affect the occurrence of another.

Classical and Empirical Probability

CLASSICAL PROBABILITY $$\text{Probability of an event} = \frac{\text{Number of favorable outcomes}}{\text{Total number of possible outcomes}} \quad [5\text{-}1]$$

Consider an experiment of rolling a six-sided die. What is the probability of the event "an even number of spots appear face up"?

The possible outcomes are:



There are three "favorable" outcomes (a two, a four, and a six) in the collection of six equally likely possible outcomes.

EMPIRICAL PROBABILITY The probability of an event happening is the fraction of the time similar events happened in the past.

The empirical approach to probability is based on what is called the law of large numbers. The key to establishing probabilities empirically is that more observations will provide a more accurate estimate of the probability.

EXAMPLE:

On February 1, 2003, the Space Shuttle Columbia exploded. This was the second disaster in 113 space missions for NASA. On the basis of this information, what is the probability that a future mission is successfully completed?

$$\text{Probability of a successful flight} = \frac{\text{Number of successful flights}}{\text{Total number of flights}}$$

$$= \frac{111}{113} = 0.98$$

Subjective Probability - Example

SUBJECTIVE CONCEPT OF PROBABILITY The likelihood (probability) of a particular event happening that is assigned by an individual based on whatever information is available.

- If there is little or no past experience or information on which to base a probability, it may be arrived at subjectively.
- Illustrations of subjective probability are:
  1. Estimating the likelihood the New England Patriots will play in the Super Bowl next year.
  2. Estimating the likelihood you will be married before the age of 30.
  3. Estimating the likelihood the U.S. budget deficit will be reduced by half in the next 10 years.

Subjective Probability

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Subjective Probabilities in Household Surveys

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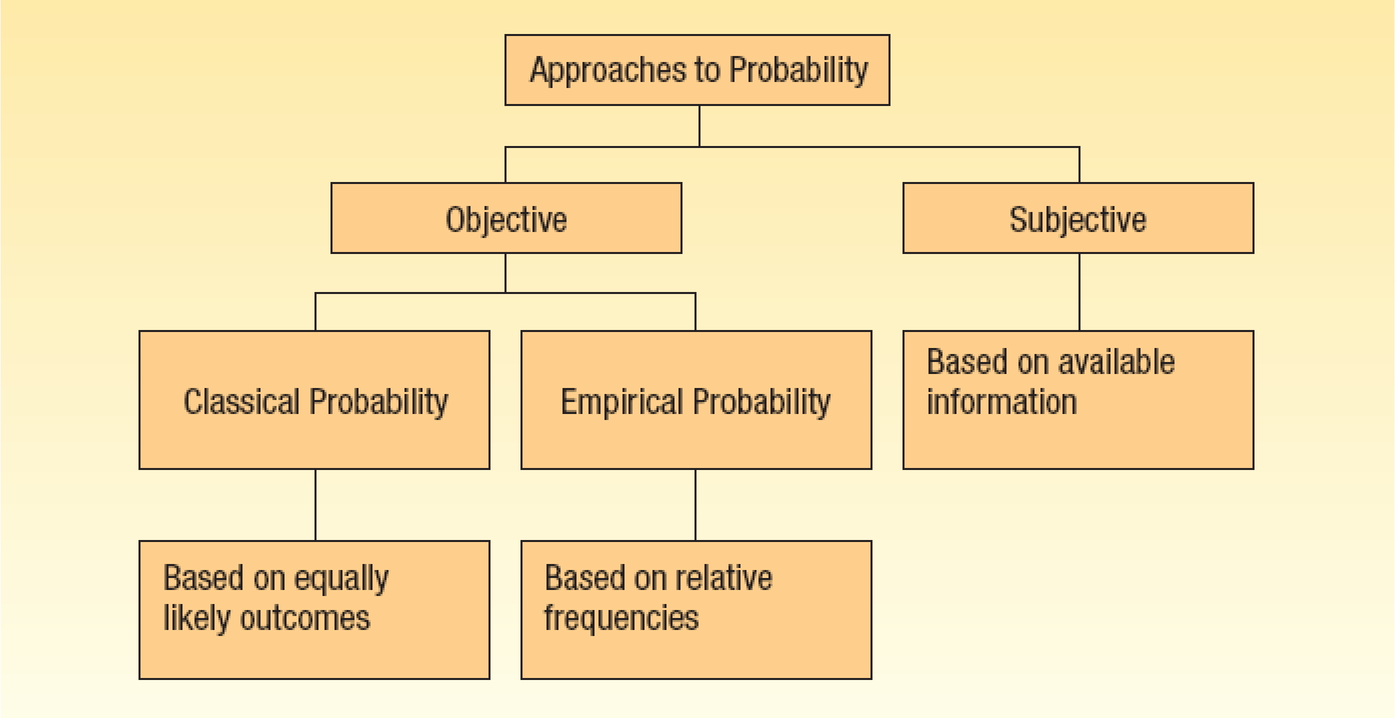
Abstract

Subjective probabilities are now collected on a number of large household surveys with the objective of providing data to better understand inter-temporal decision making. Comparison of subjective probabilities with actual outcomes shows that the probabilities have considerable predictive power in situations where individuals have considerable private information such as survival and retirement. In contrast the subjective probability of a stock market gain varies greatly across individuals even though no one has private information and the outcome is the same for everyone. An explanation is that there is considerable variation in accessing and processing information. Further, the subjective probability of a stock market gain is considerably lower than historical averages, providing an explanation for the relatively low frequency of stock holding. An important research objective will be to understand how individuals form their subjective probabilities.

Summary of Types of Probability



Rules of Addition

Rules of Addition

- Special Rule of Addition - If two events *A* and *B* are mutually exclusive, the probability of one *or* the other event's occurring equals the sum of their probabilities.

  P(A or B) = P(A) + P(B)

- The General Rule of Addition - If A and B are two events that are not mutually exclusive, then P(A or B) is given by the following formula:

  P(A or B) = P(A) + P(B) - P(A and B)

EXAMPLE:

An automatic Shaw machine fills plastic bags with a mixture of beans, broccoli, and other vegetables. Most of the bags contain the correct weight, but because of the variation in the size of the beans and other vegetables, a package might be underweight or overweight. A check of 4,000 packages filled in the past month revealed:

Weight	Event	Number of Packages	Probability of Occurrence	
Underweight	*A*	100	.025	← $\frac{100}{4,000}$
Satisfactory	*B*	3,600	.900	
Overweight	*C*	300	.075	
		4,000	1.000	

What is the probability that a particular package will be either underweight or overweight?

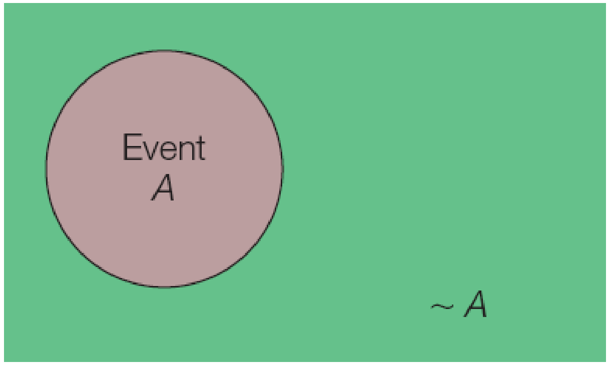
P(A or C) = P(A) + P(C) = .025 + .075 = .10

The Complement Rule

The complement rule is used to determine the probability of an event occurring by subtracting the probability of the event *not* occurring from 1.

$P(A) + P(\sim A) = 1$

or $P(A) = 1 - P(\sim A)$.

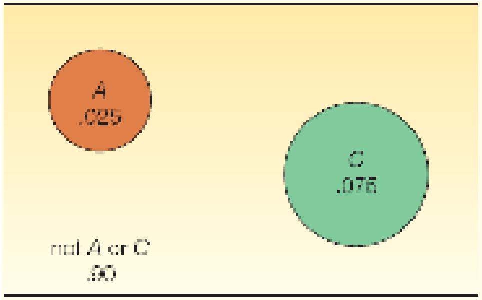


EXAMPLE

An automatic Shaw machine fills plastic bags with a mixture of beans, broccoli, and other vegetables. Most of the bags contain the correct weight, but because of the variation in the size of the beans and other vegetables, a package might be underweight or overweight. Use the complement rule to show the probability of a satisfactory bag is .900

Weight	Event	Number of Packages	Probability of Occurrence
Underweight	*A*	100	.025
Satisfactory	*B*	3,600	.900
Overweight	*C*	300	.075
		4,000	1.000

$$\begin{aligned} P(B) &= 1 - P(\sim B) \\ &= 1 - P(A \text{ or } C) \\ &= 1 - [P(A) + P(C)] \\ &= 1 - [.025 + .075] \\ &= 1 - .10 \\ &= .90 \end{aligned}$$

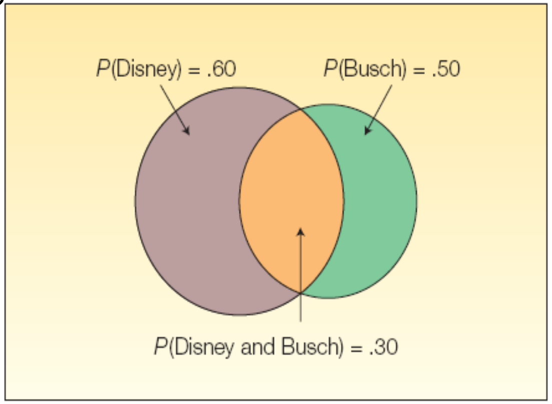


The General Rule of Addition and Joint Probability

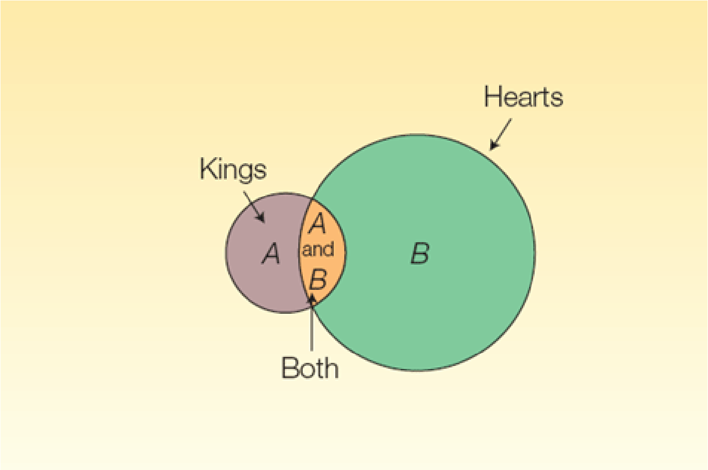
GENERAL RULE OF ADDITION $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ **[5–4]**

The Venn Diagram shows the result of a survey of 200 tourists who visited Florida during the year. The survey revealed that 120 went to Disney World, 100 went to Busch Gardens and 60 visited both.

What is the probability a selected person visited either Disney World or Busch Gardens?



JOINT PROBABILITY A probability that measures the likelihood two or more events will happen concurrently.



P(Disney or Busch) = P(Disney) + P(Busch) - P(both Disney and Busch)
= 120/200 + 100/200 – 60/200
= .60 + .50 – .80

Special and General Rules of Multiplication

- The special rule of multiplication requires that two events *A* and *B* are *independent*.
- Two events *A* and *B* are independent if the occurrence of one has no effect on the probability of the occurrence of the other.
- This rule is written: $P(A \text{ and } B) = P(A)P(B)$

EXAMPLE

A survey by the American Automobile association (AAA) revealed 60 percent of its members made airline reservations last year. Two members are selected at random. Since the number of AAA members is very large, we can assume that *R1 and R2 are independent.* What is the probability *both* made airline reservations last year?

Solution:

The probability the first member made an airline reservation last year is .60, written as $P(R_1) = .60$

The probability that the second member selected made a reservation is also .60, so $P(R_2) = .60$.

Since the number of AAA members is very large, you may assume that R_1 and R_2 are independent.

$$P(R_1 \text{ and } R_2) = P(R_1)P(R_2) = (.60)(.60) = .36$$

The general rule of multiplication is used to find the joint probability that two *independent* events will occur.

GENERAL RULE OF MULTIPLICATION $P(A \text{ and } B) = P(A)P(B|A)$

EXAMPLE

A golfer has 12 golf shirts in his closet. Suppose 9 of these shirts are white and the others blue. He gets dressed in the dark, so he just grabs a shirt and puts it on. He plays golf two days in a row and does not do laundry.

What is the likelihood both shirts selected are white?

The event that the first shirt selected is white is W_1. The probability is $P(W_1) = 9/12$

The event that the second shirt (W_2) selected is also white. The conditional probability that the second shirt selected is white, given that the first shirt selected is also white, is $P(W_2 | W_1) = 8/11$.

To determine the probability of 2 white shirts being selected we use formula: $P(AB) = P(A)\ P(B|A)$

$$P(W_1 \text{ and } W_2) = P(W_1)P(W_2|W_1) = (9/12)(8/11) = 0.55$$

Contingency Tables

A CONTINGENCY TABLE is a table used to classify sample observations according to two or more identifiable characteristic

EXAMPLE:

A sample of executives were surveyed about their loyalty to their company. One of the questions was, "If you were given an offer by another company equal to or slightly better than your present position, would you remain with the company or take the other position?" The responses of the 200 executives in the survey were cross-classified with their length of service with the company. What is the probability of randomly selecting an executive who is loyal to the company (would remain) and who has more than 10 years of service?

	Length of Service				
Loyalty	Less than 1 Year, B_1	1–5 Years, B_2	6–10 Years, B_3	More than 10 Years, B_4	Total
Would remain, A_1	10	30	5	75	120
Would not remain, A_2	25	15	10	30	80
	35	45	15	105	200

Event A_1 happens if a randomly selected executive will remain with the company despite an equal or slightly better offer from another company. Since there are 120 executives out of the 200 in the survey who would remain with the company

$P(A_1) = 120/200$, or .60.

Event B_4 happens if a randomly selected executive has more than 10 years of service with the company. Thus, $P(B_4| A_1)$ is the conditional probability that an executive with more than 10 years of service would remain with the company. Of the 120 executives who would remain 75 have more than 10 years of service, so

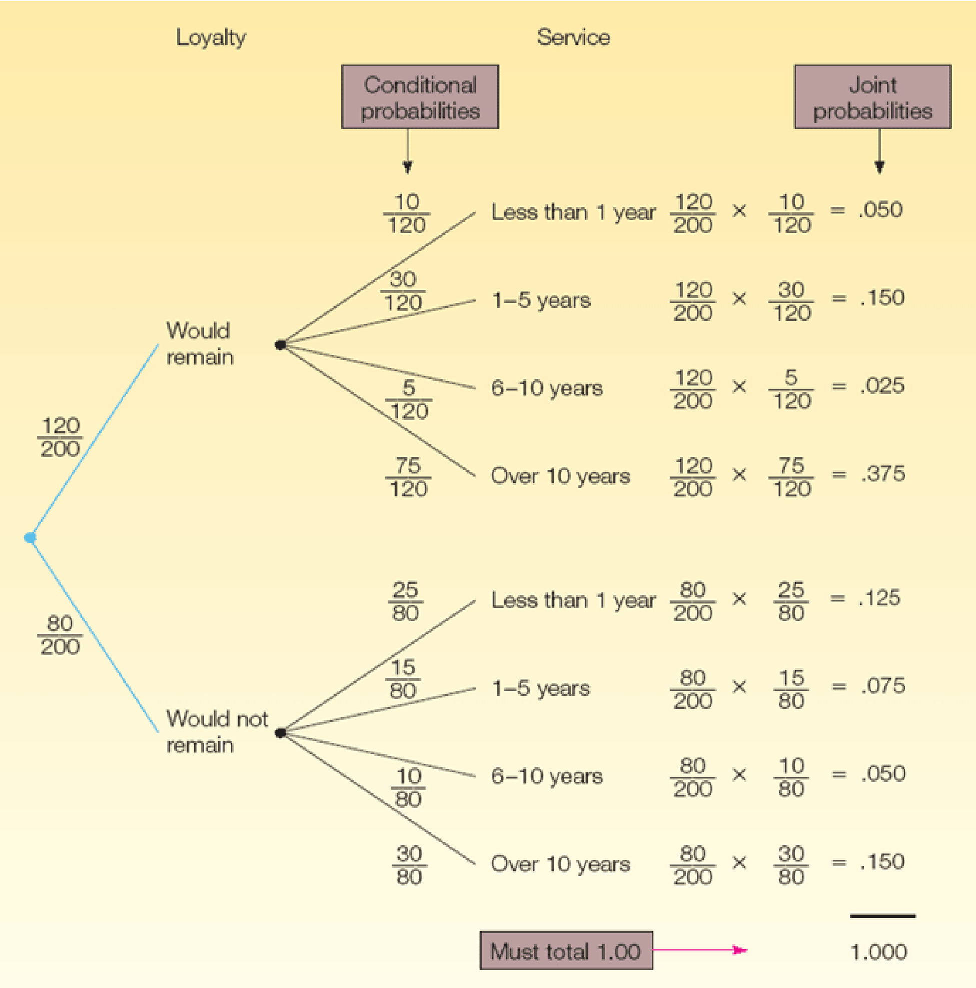
$P(B_4| A_1) = 75/120$.

$$P(A_1 \text{ and } B_4) = P(A_1)P(B_4|A_1) = \left(\frac{120}{200}\right)\left(\frac{75}{120}\right) = \frac{9{,}000}{24{,}000} = .375$$

Tree Diagrams

A tree diagram is useful for portraying conditional and joint probabilities. It is particularly useful for analyzing business decisions involving several stages.

A tree diagram is a graph that is helpful in organizing calculations that involve several stages. Each segment in the tree is one stage of the problem. The branches of a tree diagram are weighted by probabilities.



Home-work

23-26. Suppose a field may contain OIL. There is a seismic test which is useful in helping to judge whether oil is present. By "+" or "-" we mean that the test comes back positive or negative respectively. So P(+ | OIL) is the conditional probability for OIL if the test comes back positive. A reasonably good test should have P(OIL | +) > P(OIL), meaning that a positive test increases the chance above what it was before the test came back positive. Suppose

P(OIL) = 0.2 (probability of no oil is 0.8)

P(+ | OIL) = 0.9 (false negative probability of 0.1)

P(+ | no OIL) = 0.3 (false positive probability of 0.3)

Requirements

23. Fill out the tree diagram.

P(OIL) =

P(+ | OIL) = P(OIL +) =

P(- | OIL) = P(OIL -) =

conditional total = 1 (law of total probability) ⇒ total = P(OIL)

P(no OIL) =

P(+ | no OIL) = P(no OIL +) =

P(- | no OIL) = P(no OIL -) =

conditional total = 1 (law of total probability) ⇒ total = P(no OIL)

grand total = 1

24. Determine P(+) = P(OIL+) + P(OIL-) by the law of total probability.

25. Determine P(OIL | +) = $\frac{P(\text{OIL}+)}{P(+)}$. This is our revised probability for OIL if the test comes back positive.

26. Determine P(OIL | -). This is our revised probability for OIL if the test comes back negative.