#REF!

Moments

The arithematic mean of kth power of deviation of data from mean, 0 or assumed value A are called moments about mean, 0 and A respectively.

Pure Moments / Momnets about Mean

$$m_k = \frac{\sum (X-\bar{X})^k}{n}, k = 1,2,3,4 \quad \text{(Un grouped data)}$$

$$m_k = \frac{\sum f(X-\bar{X})^k}{\sum f}, k = 1,2,3,4 \quad \text{(grouped data)}$$

Raw Moments About

Assumed value A

$$m'_k = \frac{\sum (X-A)^k}{n}, k = 1,2,3,4 \quad \text{(Un grouped data)}$$

$$m'_k = \frac{\sum f(X-A)^k}{\sum f}, k = 1,2,3,4 \quad \text{(grouped data)}$$

Zero or A=0

$$m'_k = \frac{\sum X^k}{n}, k = 1,2,3,4 \quad \text{(Un grouped data)}$$

$$m'_k = \frac{\sum fX^k}{\sum f}, k = 1,2,3,4 \quad \text{(grouped data)}$$

Z=3X	Z=X/3	T=3X+5		X	Y	X+Y	X-Y
0	0	5		15	16	31	-1
0	0	5		18	42	60	-24
0	0	5		23	51	74	-28
0	0	5		54	89	143	-35
0	0	5		23	53	76	-30
0	0	5		48	64	112	-16
0	0	5		10	21	31	-11
0	0	5		59	75	134	-16
0	0	5	Mean	31.25	51.375	82.625	-20.125
0.00	0.00	0.00	Sd	19.26	25.07	43.27	11.21
0.00	0.00	0.00	Var	370.79	628.27	1872.55	125.55
0	0	0			Mx+My	82.625	-20.125

Sum

Intervals (Marks)		f	x	fx	logX	flogX
1	9	6	5	30	0.69897	4.19382
10	18	12	14	168	1.146128	13.75354
19	27	8	23	184	1.361728	10.89382
28	36	13	32	416	1.50515	19.56695
37	45	8	41	328	1.612784	12.90227
46	54	3	50	150	1.69897	5.09691
		Sum f		Sum fx		
		50		1276		66.40731
					GM	1.328146

Note

$m_1 = 0$ (First moment about mean is ZERO)

$m'_1 =$ Mean (First moment about zero is MEAN)

$m_2 = S^2$ (Second moment about mean is Variance)

Moments about MEAN — UnGrouped Data

X	$X-\bar{X}$	$(X-\bar{X})^2$	$(X-\bar{X})^3$	$(X-\bar{X})^4$
15	-15	225	-3375	50625
8	-22	484	-10648	234256
23	-7	49	-343	2401
54	24	576	13824	331776
23	-7	49	-343	2401
48	18	324	5832	104976
10	-20	400	-8000	160000
59	29	841	24389	707281
240	0	2948	21336	1593716
Mean	m_1	m_2	m_3	m_4
30	0	368.5	2667	199214.5

Moments about value 'A' — UnGrouped Data

X	$X-A$	$(X-A)^2$	$(X-A)^3$	$(X-A)^4$
15	15	225	3375	50625
8	8	64	512	4096
23	23	529	12167	279841
54	54	2916	157464	8503056
23	23	529	12167	279841
48	48	2304	110592	5308416
10	10	100	1000	10000
59	59	3481	205379	12117361
240	240	10148	502656	26553236
A	m'_1	m'_2	m'_3	m'_4
0	30	1268.5	62832	3319154.5
	30			

Moments about MEAN — Grouped Data

f	$X-\bar{X}$	$f(X-\bar{X})$	$f(X-\bar{X})^2$	$f(X-\bar{X})^3$	$f(X-\bar{X})^4$
6	-20.5	-123.1	2526.4	-51842.2	1063801.7
12	-11.5	-138.2	1592.5	-18345.9	211344.6
8	-2.5	-20.2	50.8	-128.0	322.6
13	6.5	84.2	545.9	3537.3	22921.5
8	15.5	123.8	1917.0	29675.8	459381.8
3	24.5	73.4	1797.8	44010.4	1077375.0
Sum f			$\Sigma f(x-\bar{x})^2$		
50	11.88	0.00	8430.48	6907.42	2835147.30
	Mean	m_1	m_2	m_3	m_4
	25.52	0.00	168.61	138.15	56702.95

Moments about Assumed value — Grouped Data

$X-A$	$f(X-A)$	$f(X-A)^2$	$f(X-A)^3$	$f(X-A)^4$
0.0	0.0	0.0	0.0	0.0
9.0	108.0	972.0	8748.0	78732.0
18.0	144.0	2592.0	46656.0	839808.0
27.0	351.0	9477.0	255879.0	6908733.0
36.0	288.0	10368.0	373248.0	13436928.0
45.0	135.0	6075.0	273375.0	12301875.0
135.00	1026.00	29484.00	957906.00	33566076.00
A	m'_1	m'_2	m'_3	m'_4
5	20.52	589.68	19158.12	671321.52

Why we need to study moments ?

To check presence of Skewness and Kurtosis by using different formations of the ratios of pure moments.

$$\beta_1 = \frac{m_3}{\sqrt{m_2^3}}$$

$\beta_1 < 0$ (Negative Skewed)

$\beta_1 = 0$ (Symetric)

$\beta_1 > 0$ (Positive Skewed)

X	logX
15	1.176091
6	0.778151
23	1.361728
54	1.732394
	5.048364
18.28483	1.262091

X	1/X
15	0.066667
6	0.166667
23	0.043478
54	0.018519
	0.29533
HM	13.54417

13.54417

Vx+Vy

Cov XY

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