Lecture No. 9

Introduction To Statistics, Statistics And Probability

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Measures of Skewness and Kurtosis

Shape of the Distribution (Graphical Representation)

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In this lecture

- **Shape of the Distribution**
- **Measures of the Shape**
- **Symmetry and Skewness**
- **Tests of Skewness and Measures of Skewness**
- **Kurtosis and Measures of Kurtosis**

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Shape of the Distribution

- The shape of the distribution provides information about the central tendency and variability of measurements.
- Three common shapes of distributions are:

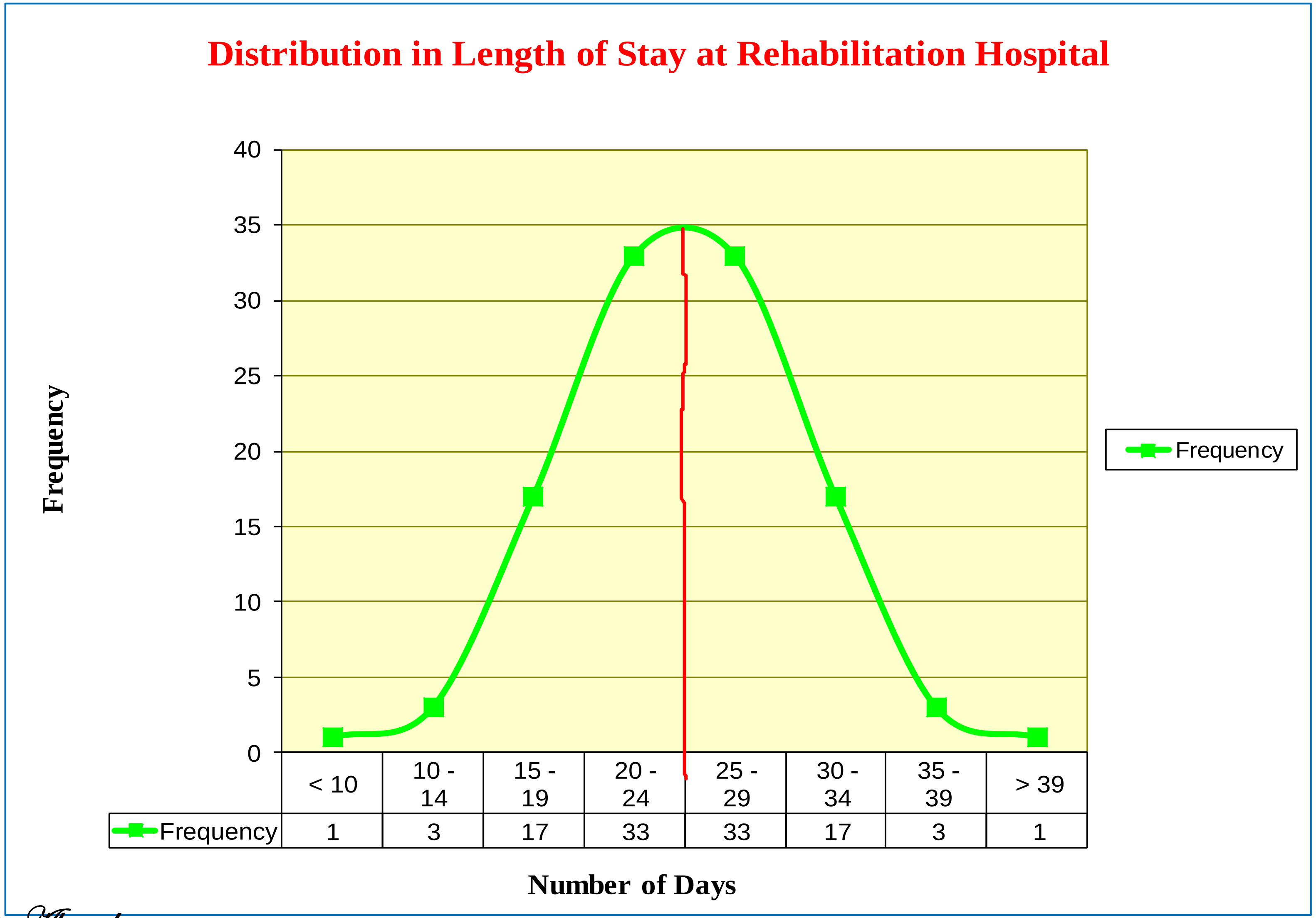
  Normal: bell-shaped curve; symmetrical

  Skewed: non-normal; non-symmetrical; can be positively or negatively skewed

  Multimodal: has more than one peak (mode)

- *Normal Distribution* is *symmetrical & bell-shaped*; often called "*bell-shaped curve*"
- When a variable's distribution is *non-symmetrical*, it is *skewed*. This means that the mean is not in the center of the distribution.

Normal Distribution



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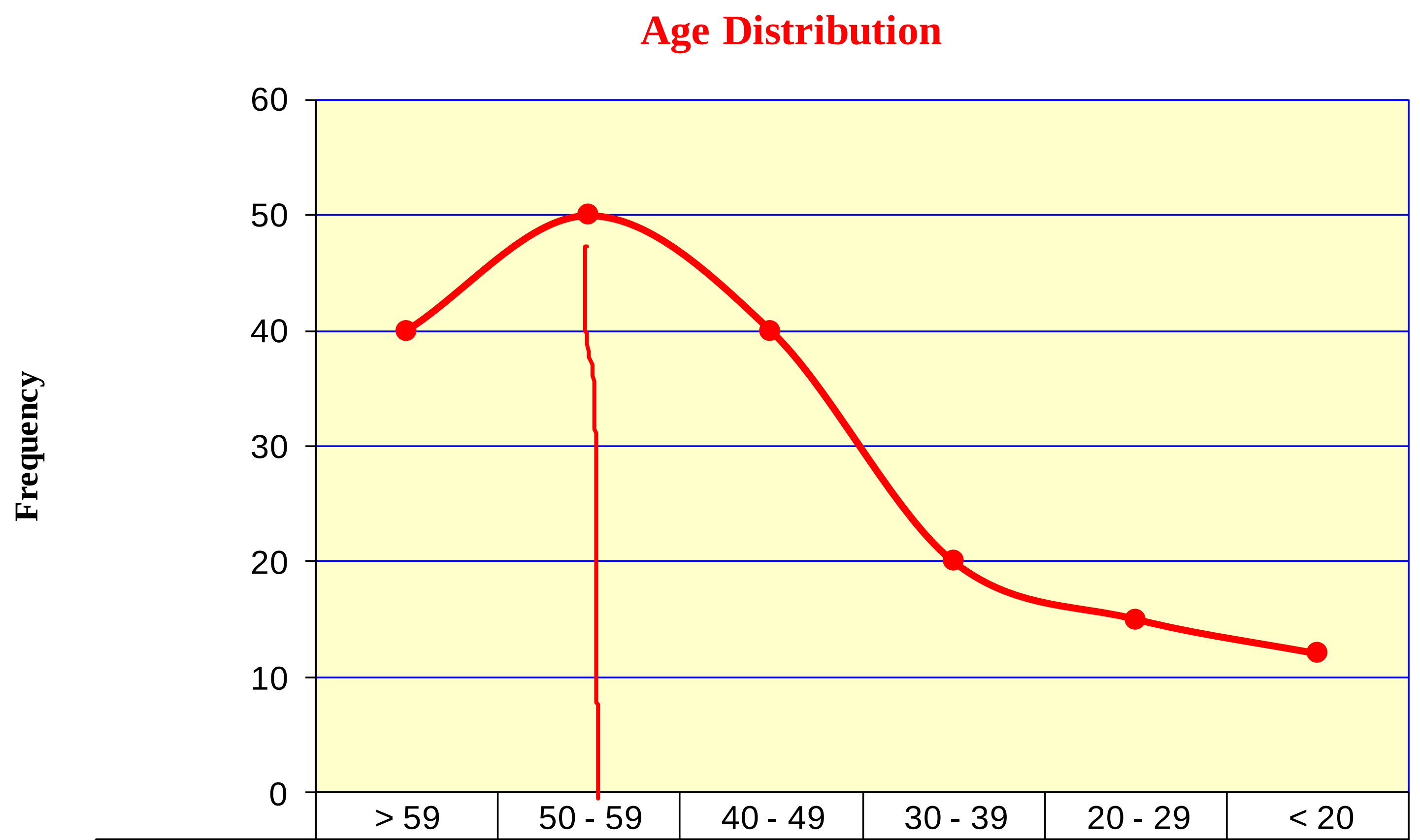
Positively Skewed Distribution:



	> 59	50 - 59	40 - 49	30 - 39	20 - 29	< 20
Frequency	40	50	40	20	15	12

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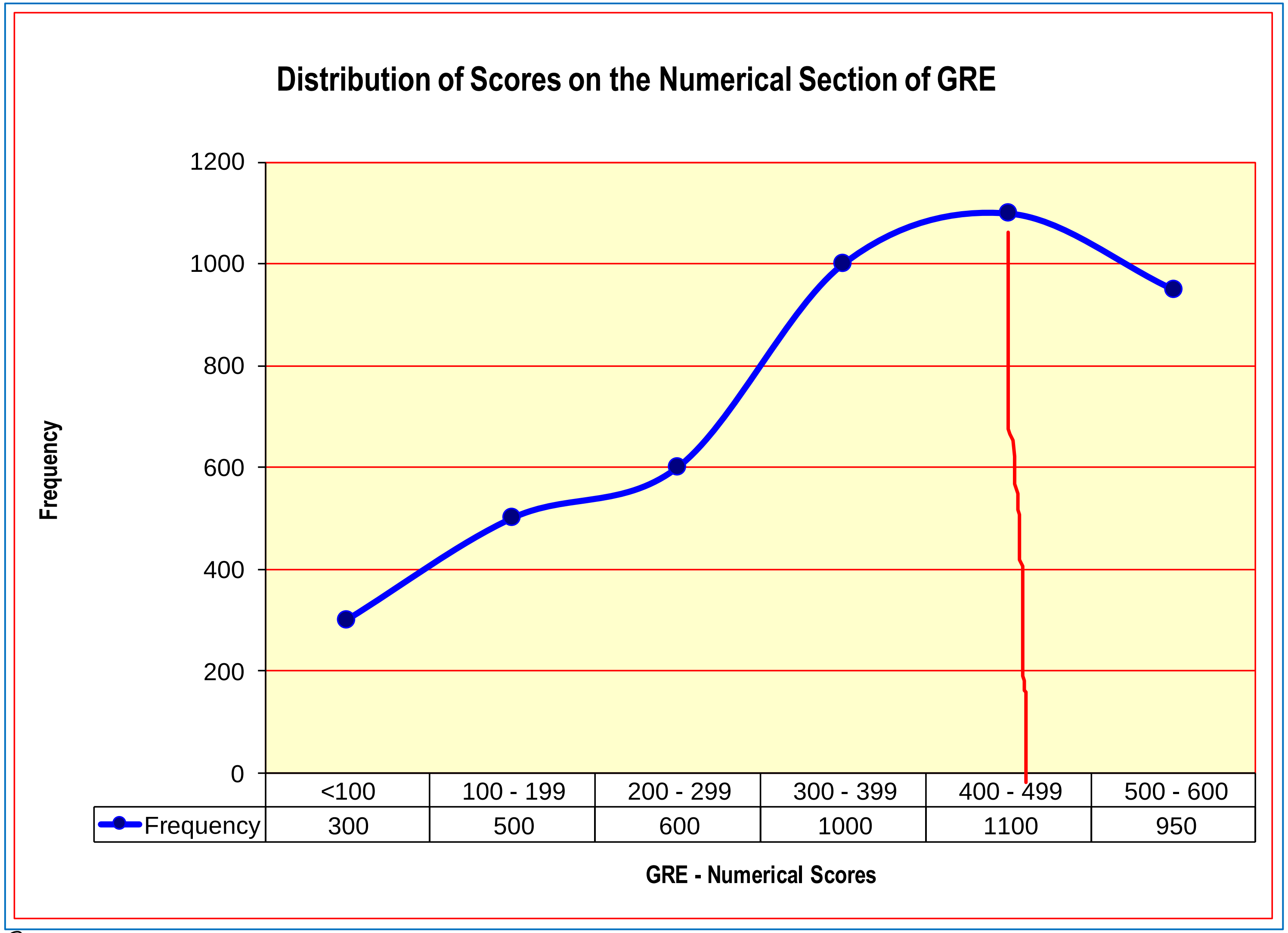
Negatively Skewed Distribution



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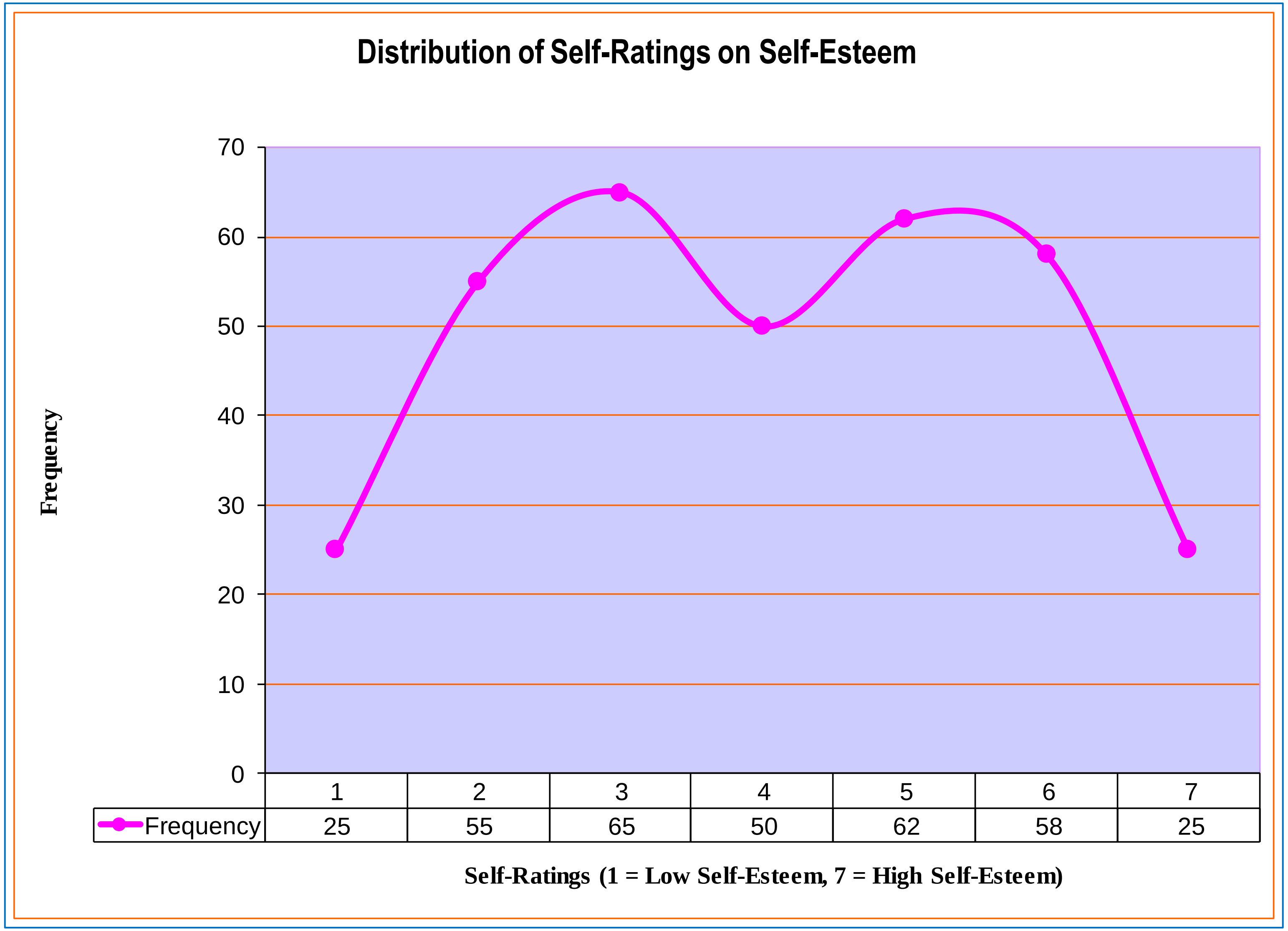
Bimodal Distribution



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Measures of the Shape

There are three measures of shape:

- **Skewness**
  - Absence of symmetry
  - Extreme values in one side of a distribution
- **Kurtosis**
  - Peakedness of a distribution
- **Box and Whisker Plots**
  - Graphic display of a distribution
  - Reveals skewness

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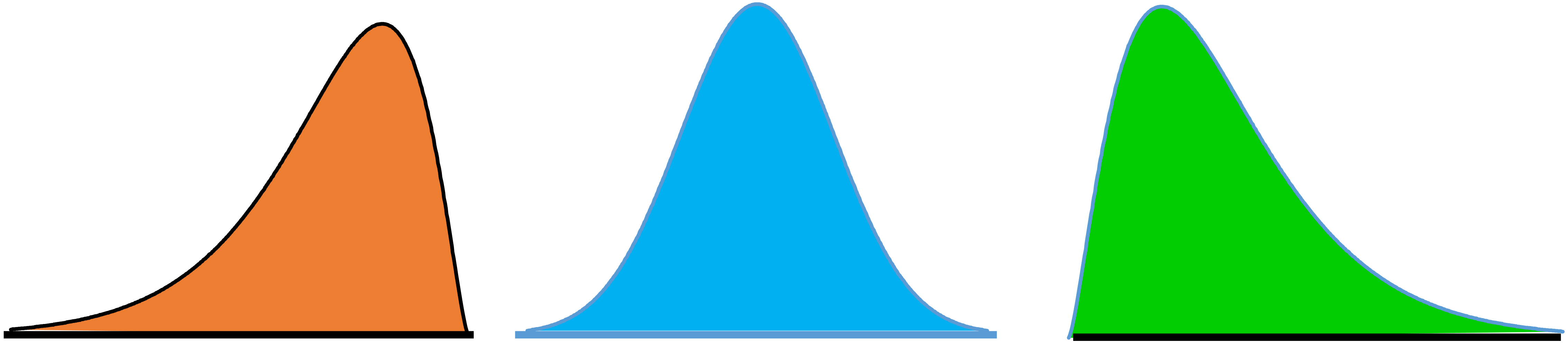
Symmetry and Skewness

- A frequency distribution is said to be symmetrical if the frequencies are equally distributed on both the sides of central value. In a symmetrical distribution, there is only one mode and the values of mean, median and mode are equal. This is called *symmetry*.
- *Skewness* is the *lack of symmetry* in a distribution around some central value (mean, median or mode). It is thus the degree of asymmetry.
- Skewness is the measure of the shape of a nonsymmetrical distribution.
- Two sets of data can have the same mean & SD but different skewness.
- Two types of skewness are:

  Positive skewness

  Negative skewness

Skewness



Negatively Skewed

Symmetric (Not Skewed)

Positively Skewed

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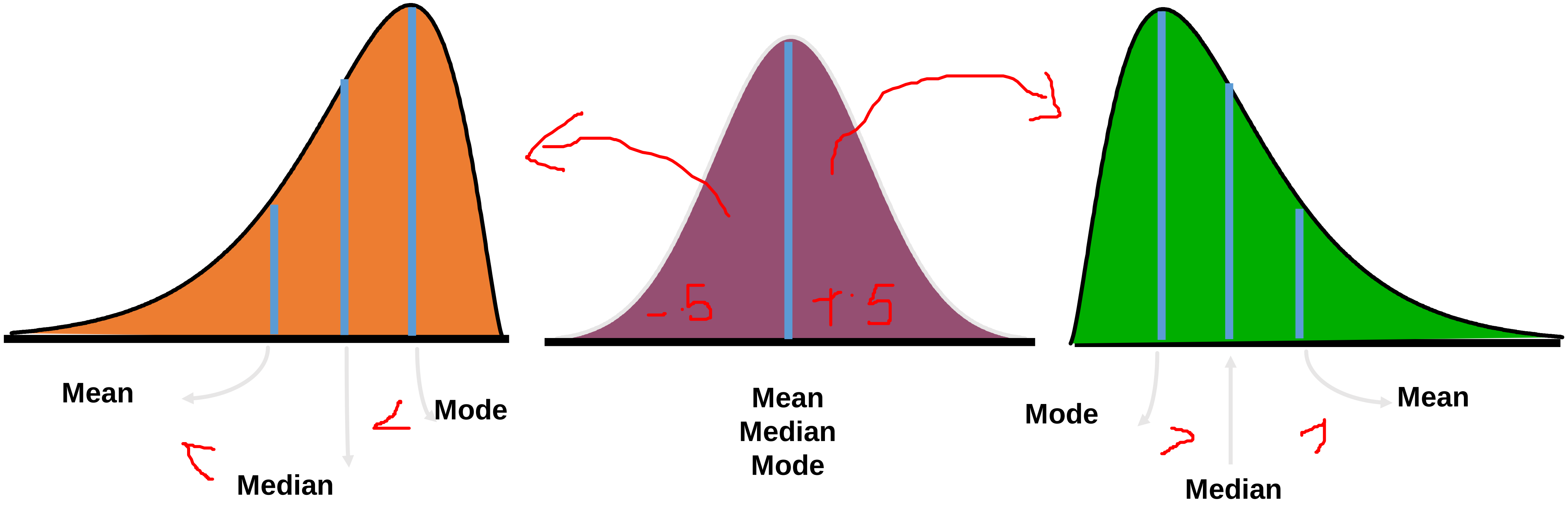
Skewness

(Relative Locations for Measures of Central Tendency)



Negatively Skewed

Symmetric (Not Skewed)

Positively Skewed

Tests of Skewness

In order to ascertain whether a distribution is skewed or not the following tests may be applied. Skewness is present if:

- The values of mean, median and mode do not coincide.
- When the data are plotted on a graph they do not give the normal bell shaped form i.e. when cut along a vertical line through the center the two halves are not equal.

Measures of Skewness

1.Karl Pearson coefficient of skewness

$$\text{Coefficient of skewness} = \frac{Mean - Mode}{Standard\ Deviation}$$

OR

$$\text{Coefficient of skewness} = \frac{3(Mean - Median)}{Standard\ Deviation}$$

2. Bowley's quartiles coefficient of skewness

$$\text{Quartile Coefficient of skewness} = \frac{Q_1 + Q_3 - 2Q_2}{Q_3 - Q_1}$$

3. Pearson's moment's coefficient of skewness

$$\text{Moment Coefficient of skewness} = \frac{m_3^2}{m_2^3}$$

First moment ratio also denoted by b_1

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Key to Interpret Skewness:

The coefficient of skewness give positive result for positively skewed distribution and negative result for negatively skewed.

This measure is always zero for a symmetrical distribution.

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Example 1. Find the Karl Pearson coefficient of skewness in given data 3, 7, 7, 7, 7, 8, 8, 8, 18.
Solution.

X	$(X-\bar{X})^2$
3	26.11
7	1.23
7	1.23
7	1.23
7	1.23
8	0.012
8	0.012
8	0.012
18	97.81
$\sum X = 73$	$\sum (X-\bar{X})^2 = 128.87$

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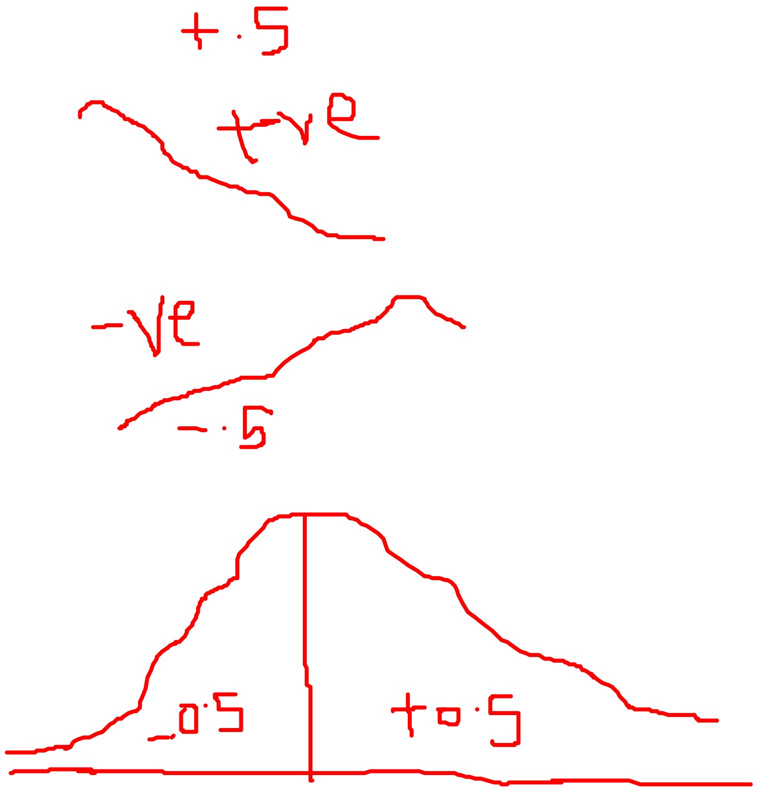

$$Mean = \bar{X} = \frac{\sum X}{n} = \frac{73}{9}$$

$$\boldsymbol{Mean = \bar{X} = 8.11}$$

Mode is the most repeated value in data

$$\boldsymbol{Mode = 7}$$

$$S.D = \sqrt{\frac{\sum(X-\bar{X})^2}{n}} = \sqrt{\frac{128.87}{9}}$$

$$\boldsymbol{S.D = 3.78}$$

$$\text{Coefficient of skewness} = \frac{Mean - Mode}{Standard\ Deviation}$$

$$\text{Coefficient of skewness} = \frac{8.11 - 7}{3.78} = \frac{1.11}{3.78}$$

$$\text{Coefficient of skewness} = 0.29365$$

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Example 2: let suppose we calculate the first quartile is 15, second quartile is 52 and third quartiles is 80, by Bowley quartiles coefficient of skewness calculate the coefficient of Skewness.

Solution.

$$\text{Coefficient of skewness} = \frac{Q_1 + Q_3 - 2Q_2}{Q_3 - Q_1} = \frac{15 + 80 - 2(52)}{80 - 15}$$

$$\text{Coefficient of skewness} = \frac{-9}{65}$$

$$\text{Coefficient of skewness} = -0.14$$

As the value of skewness is less than zero, the data is negatively skewed.

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Example 3: Lecture 18 and 19, example 1, we calculate the first four moments about mean such as $\boldsymbol{m_1 = 0}$, $\boldsymbol{m_2 = 4.67}$, $\boldsymbol{m_3 = 0}$, $\boldsymbol{m_4 = 32.67}$, using Pearson's moment's coefficient of skewness to find the coefficient of skewness.

Solution.

$$Coefficient\ of\ skewness = b_1 = \frac{m_3^2}{m_2^3} = \frac{(0)^2}{(4.67)^2}$$

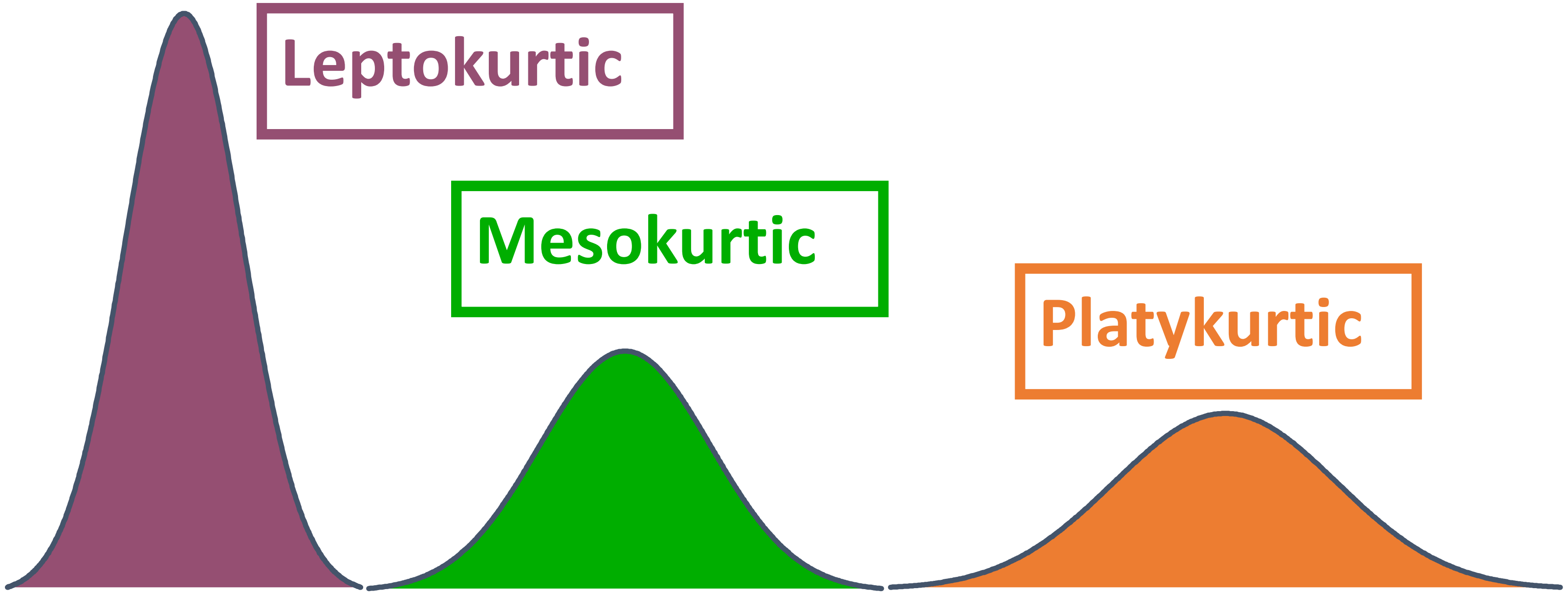
$$Coefficient\ of\ skewness = b_1 = 0$$

As the skewness is equal to zero, the distribution is symmetrical.

Kurtosis

Kurtosis is the degree of peakedness of a distribution usually taken as relative to normal distribution.

- A distribution having a relatively high peak is called leptokurtic.
- A distribution having which is flat-topped is called platykurtic.
- The normal distribution which is neither very peak nor very flat-topped is called mesokurtic.



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Measures of Kurtosis

❖ Measure of kurtosis based on the fourth moments about mean and second moments about mean.

$$Moments\ coefficient\ of\ kurtosis = b_2 = \frac{m_4}{m_2^2}$$

- ❑ $if\ b_2 > 3, the\ distribution\ is$ leptokurtic
- ❑ $if\ b_2 < 3, the\ distribution\ is$ platykurtic
- ❑ $if\ b_2 = 3, the\ distribution\ is$ mesokurtic

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Example 1. Lecture 20, example 1, we calculate the first four moments about mean such as $\boldsymbol{m_1 = 0}$, $\boldsymbol{m_2 = 4.6, m_3 = 0, m_4 = 32.67}$, find the kurtosis.

Solution.

$$Moments\ coefficient\ of\ kurtosis = b_2 = \frac{m_4}{m_2^2} = \frac{32.67}{(4.67)^2}$$

$$Moments\ coefficient\ of\ kurtosis = b_2 = 1.50$$

As b_2<3, the distribution is platykurtic.

ANY QUESTION