## Lecture No. 6

# Introduction To Statistics, Statistics And Probability


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# Measures of Dispersion

## Different Tools (Range, M.D, Coefficient

## of Range and MD.

## Merits and Demerits of

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# In this lecture

- What is Measure of Dispersion
- Why measures of dispersion?
- Characteristics of a good measure
- Types of Dispersion Measures
- Range, Coefficient of Range, Merits and Demerits
- Mean Deviation (M.D) , Merits and Demerits of M.D and Caff. of M.D

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# WHAT AND WHY MEASURE OF DISPERSION

- A quantity that measures the variability among the data, or how the data are dispersed about the average, known as Measures of dispersion, scatter, or variations.
- In statistics, dispersion (also called variability, scatter, or spread) is the extent to which a distribution is stretched or squeezed.
- A measure of statistical dispersion is a nonnegative real number that is zero if all the data are the same and increases as the data become more diverse.

- Serve as a basis for the control of the variability.
- To compare the variability of two or more series.
- Determine the reliability of an average.
- Facilitate the use of other statistical measures.

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# CHARACTERISTICS OF A GOOD MEASURE OF DISPERSION

An ideal measure of dispersion is expected to possess the following properties

1. It should be rigidly defined.
2. It should be based on all the items observations of the data.
3. It should not be unduly affected by extreme items.
4. Must be easily subjected to further mathematical operations.
5. It should be simple to understand and easy to calculate.

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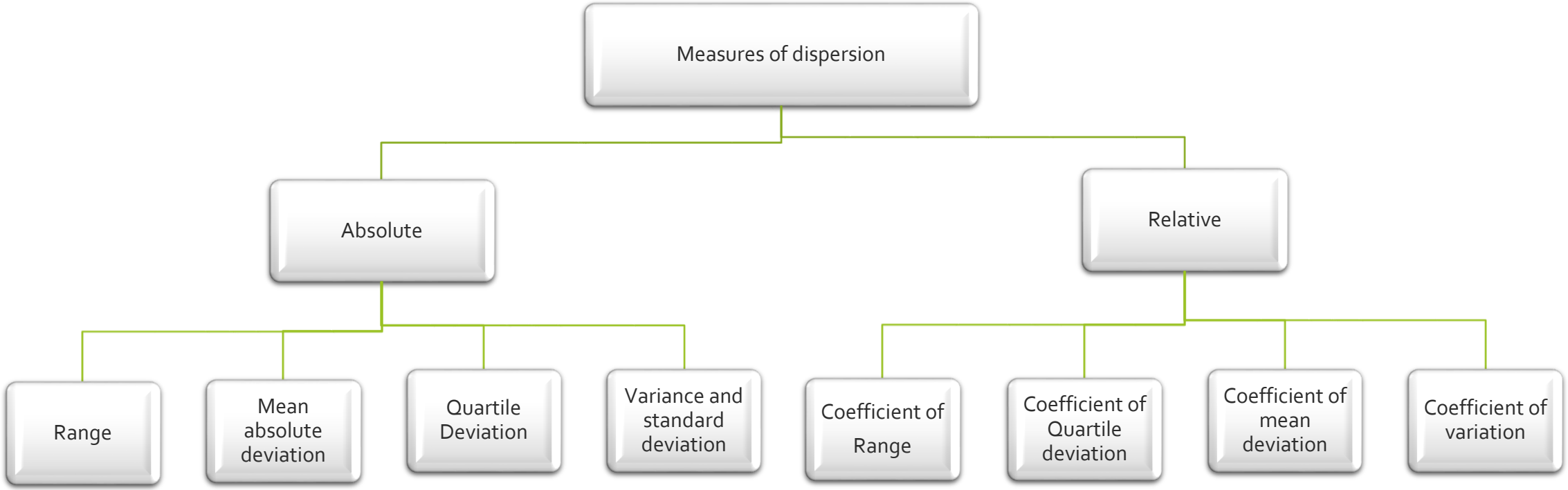
# TYPES OF DISPERSION MEASURES

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# RANGE

- This is the simplest possible measure of dispersion and is defined as the difference between the largest and smallest values of the variable. In symbols,

$$Range = X_{max} - X_{min}.$$

Where,

$X_{max} = Largest\ value$ and $X_{min} = Smallest\ value$

- In individual observations and discrete series, $X_{max}$ and $X_{min}$ are easily identified.
- In continuous series, the following two methods are followed.

**Method 1:**

$X_{max}$ = Upper C.B of the highest class and $X_{min}$ = Lower C.B of the lowest class.

**Method 2:**

$X_{max}$ = Mid value of the highest class. and $X_{min}$ = Mid value of the lowest class.

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**Example 1:**

The yields (kg per plot) of a cotton variety from five plots are 8, 9, 8, 10 and 11. Find the range.

**Solution:**

As, $X_{max} = 11$ and $X_{min} = 8$

$$\text{Range} = X_{max} - X_{min} = 11 - 8 = 3$$

**Example 2:**

Calculate range from the following distribution.

| Size: | $60-63$, | $63-66$, | $66-69$, | $69-72$, | $72-75$ |
|---|---|---|---|---|---|
| f: | 5, | 18, | 42, | 27, | 8 |

Solution:

$X_{max}$ = Upper C.B of the highest class= 75, $X_{min}$ = Lower C.B of the lowest class= 60

$$\text{Range} = X_{max} - X_{min}$$

$$\text{Range} = 75 - 60 = 15$$

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# Coefficient of Range

The relative measure of dispersion based on range is the coefficient of range also known as range coefficient of dispersion.

**Formula:**

$$Coefficient\ of\ Range = \frac{X_{max} - X_{min}}{X_{max} + X_{min}}$$

**Example:**

*The yields (kg per plot) of a cotton variety from five plots are 8, 9, 8, 10 and 11. Find the coefficient of range.*

**Solution:**

As, $X_{max} = 11 \quad \text{and} \quad X_{min} = 8$

$$Coefficient\ of\ Range = \frac{X_{max} - X_{min}}{X_{max} + X_{min}} = \frac{11 - 8}{11 + 8} = \frac{3}{19} = 0.1578$$

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# MERITS AND DEMERITS OF RANGE

## Merits

1. It is simple to understand.
2. It is easy to calculate.
3. In certain types of problems like quality control, weather forecasts, share price analysis, etc., range is most widely used.

## Demerits

1. It is very much affected by the extreme items.
2. It is based on only two extreme observations.
3. It cannot be calculated from open-end class intervals.
4. It is not suitable for mathematical treatment.
5. It is a very rarely used measure.

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# MEAN DEVIATION (M.D)

- A **deviation** is the difference between individual observation with average (Mean, Median, Mode) of the data, i.e. $(X_i - Average)$.
- The average of the absolute values of deviation from the mean (median or mode) is called **mean deviation** or **mean absolute deviation**.

**Formula:** $$M.D = \frac{\sum|X_i - average|}{n}$$

**M.D from mean for sample data:** $$M.D = \frac{\sum|X_i - \bar{X}|}{n}$$

**M.D from mean for population data:** $$M.D = \frac{\sum|X_i - \mu|}{N}$$

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# FORMULAS OF M.D

## Ungrouped data

**M.D from Mean:**

$$M.D = \frac{\sum |X_i - \bar{X}|}{n}$$

**M.D from Median:**

$$M.D = \frac{\sum |X_i - median|}{n}$$

**M.D from Mode:**

$$M.D = \frac{\sum |X_i - mode|}{n}$$

## Grouped data

**M.D from Mean:**

$$M.D = \frac{\sum f|X_i - \bar{X}|}{\sum f}$$

**M.D from Median:**

$$M.D = \frac{\sum f|X_i - median|}{\sum f}$$

**M.D from Mode:**

$$M.D = \frac{\sum f|X_i - mode|}{\sum f}$$

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**Example 1:** Let the ages of 5 boys in a class is 12, 14, 14, 15, 18. Calculate the M.D from mean.

**Solution:** first calculating the mean age:

$$\bar{X} = \frac{12 + 14 + 14 + 15 + 18}{5} = 14.6$$

Now using formula of M.D: $\boldsymbol{M.D = \frac{\sum|X_i - \bar{X}|}{n}}$

$$\text{M. D} = \frac{|12 - 14.6| + |14 - 14.6| + |14 - 14.6| + |15 - 14.6| + |18 - 14.6|}{5}$$

$$\text{M. D} = \frac{|-2.6| + |-0.6| + |-0.6| + |0.4| + |3.4|}{5}$$

$$\text{M. D} = \frac{2.6 + 0.6 + 0.6 + 0.4 + 3.4}{5}$$

$$\text{M. D} = \frac{7.6}{5} = 1.52$$

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**Example 2:** Calculate the M.D from mean for the following frequency distribution:

| Classes | $x$ | $f$ | $fx$ | $X_i - \bar{X}$ $X_i$- 156.1667 | $\|X_i$ | $f\|X_i - 156.166\|$ |
|---|---|---|---|---|---|---|
| 110 – 109 | 114.5 | 1 | 114.5 | −41.6667 | 41.66667 | 41.66667 |
| 120 – 129 | 124.5 | 4 | 498 | −31.6667 | 31.66667 | 126.6667 |
| 130 – 139 | 134.5 | 17 | 2286.5 | −21.6667 | 21.66667 | 368.3333 |
| 140 – 149 | 144.5 | 28 | 4046 | −11.6667 | 11.66667 | 326.6667 |
| 150 – 159 | 154.5 | 25 | 3862.5 | −1.66667 | 1.666667 | 41.66667 |
| 160 – 169 | 164.5 | 18 | 2961 | 8.333333 | 8.333333 | 150 |
| 170 – 179 | 174.5 | 13 | 2268.5 | 18.33333 | 18.33333 | 238.3333 |
| 180 – 189 | 184.5 | 6 | 1107 | 28.33333 | 28.33333 | 170 |
| 190 – 199 | 194.5 | 5 | 972.5 | 38.33333 | 38.33333 | 191.6667 |
| 200 – 209 | 204.5 | 2 | 409 | 48.33333 | 48.33333 | 96.66667 |
| 210 – 219 | 214.5 | 1 | 214.5 | 58.33333 | 58.33333 | 58.33333 |
| ***Total*** | | **120** | **18740** | | | **1810** |

**Solution:**

$$\bar{X} = \frac{\sum fx}{\sum f} = \frac{18740}{120} =$$

156.1667

Now using formula of M.D:

$$M.D = \frac{\sum f|X_i - \bar{X}|}{\sum f}$$

Putting values from table:

$$M.D = \frac{1810}{120} = 15.0833$$

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# MERITS AND DEMERITS OF M.D

**Merits**

1. Simplifies calculations.
2. Can be calculated by mean, median and mode.
3. Is not affected by extreme measures.
4. Used to make healthy comparisons.

**Demerits**

1. Mathematically illogical to assume all negatives as positives.
2. Not reliable.
3. Not suitable for comparing series.

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# COEFFICIENT OF MEAN DEVIATION

The relative measure of dispersion we get by dividing Mean Deviation by Mean or Median, is called Coefficient of Mean Deviation.

**Formula:** $$Coefficient\ of\ M.D = \frac{M.D}{mean\ or\ median}$$

**Example:** Let the ages of 5 boys in a class is 12, 14, 14, 15, 18. Calculate the coefficient of M.D from mean.

**Solution:**

first calculating the mean age: $\overline{X} = 14.6$

Now using formula of M.D: $M.D = 1.52$

$$\text{Coefficient of } M.D = \frac{M.D}{\overline{X}} = \frac{1.52}{14.6} = 0.1041$$

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# MERITS & DEMERITS OF COEFFICIENT OF M.D

**Merits**

1. Better Result than Range & Quartile Coefficient.
2. Least sampling fluctuation.
3. Rigidly defined.

**Demerits:**

1. Fractional Average.
2. Cannot be used for sociological studies
3. Less reliable than Coefficient of Variation

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**Question 1:** The following sample data of the number of communications are taken from logs of communication with Distance Education students: 5, 9, 5, 23, 27, 55, 34, 7, 30, 15, 22, 60, 14, 52, 297, 8, 51, 15, 51, 35, 15, 39, 137, 43, 38, 14, 93, 7. Calculate mean deviation from mean.

**Question 2: The following is the age distribution of 1000 persons working in an organization**

| Age Group | Number of Persons |
|---|---|
| 20-25 | 30 |
| 25-30 | 160 |
| 30-35 | 210 |
| 35-40 | 180 |
| 40-45 | 145 |
| 45-50 | 105 |
| 50-55 | 70 |
| 55-60 | 60 |
| 60-65 | 40 |

**Calculate mean deviation from mean and mean deviation from median.**

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# ANY QUESTION

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