Time Series Analysis

Time Series: The study of data according to its time of occurance is called Time Series Data.

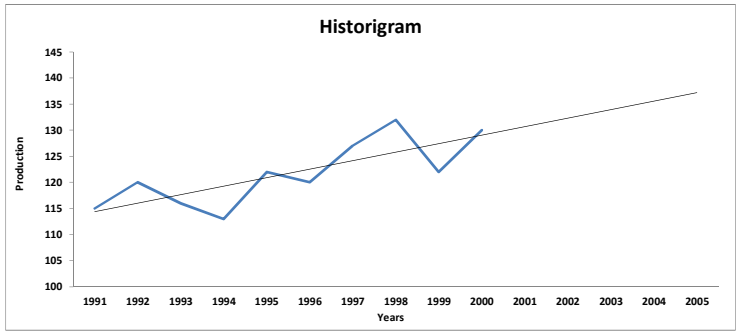
Every time series is characterized by the following four features called COMPONENTS of time series.

1. Seasonal Variation
2. Cyclical Flacuations
3. Irregular Variations
4. Trend or Secular Trend
   - i Method of free hand curve
   - ii Method of Semi Average
   - iii Method of Moving Average
   - iv Method of Least Square

i Method of free hand curve

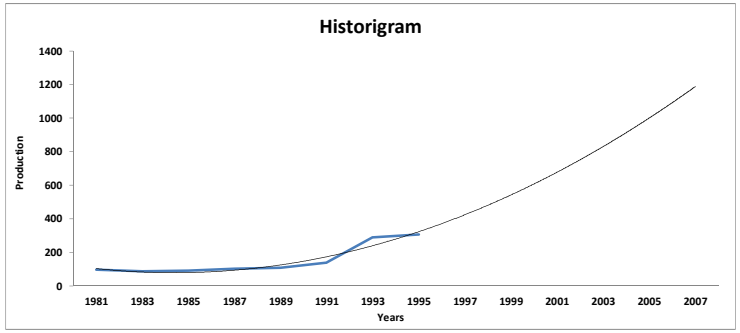
Question 1: Smoth the data with the help of free hand method and estimate production for 2001, beside other trend values.

Year	Produciton
1991	115
1992	120
1993	116
1994	113
1995	122
1996	120
1997	127
1998	132
1999	122
2000	130
2001	
2002	
2003	
2004	
2005	



Question 2: Smoth the data with the help of free hand method and find trend values.

Year	Produciton
1981	96
1983	87
1985	91
1987	102
1989	108
1991	139
1993	289
1995	307
1997	
1999	
2001	
2003	
2005	
2007	



Years	Y
1990	91
1991	87
1992	97
1993	55
1994	80
1995	93
1996	94
1997	86
1998	53
1999	58
2000	63
2001	69
2002	60
2003	85

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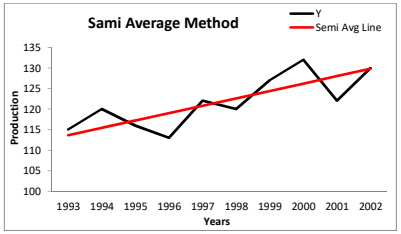
ii Method of Semi Average

Question 1: Compute Trend Values by Method of Semi Average

Case I

Year	Produciton Y	Semi Average	X	Trend Values $\hat{y} = 117.2 + 1.8x$
1993	115		-2	113.6
1994	120		-1	115.4
1995	116	$\bar{y}_1 =$ 117.2	$x_1 =$ 0	117.2
1996	113		1	119.0
1997	122		2	120.8
1998	120		3	122.6
1999	127		4	124.4
2000	132	$\bar{y}_2 =$ 126.2	$x_2 =$ 5	126.2
2001	122		6	128.0
2002	130		7	129.8

$$y - \bar{y}_1 = \frac{\bar{y}_2 - \bar{y}_1}{x_2 - x_1}(x - x_1)$$
$$y - 117.2 = \frac{126.2 - 117.2}{5 - 0}(x - 0)$$
$$\hat{y} = 117.2 + 1.8x$$



Question 2: Compute Trend Values by Method of Semi Average

Case II

Year	Produciton Y	Semi Average	X	Trend Values $\hat{y} = 116.33 + 2.5x$
1981	120		-1	113.8
1982	116	$\bar{y}_1 =$ 116.33	$x_1 =$ 0	116.3
1983	113		1	118.8
1984	122		2	121.3
1985	120		3	123.8
1986	127	$\bar{y}_2 =$ 126.33	$x_2 =$ 4	126.3
1987	132		5	128.8

$$y - \bar{y}_1 = \frac{\bar{y}_2 - \bar{y}_1}{x_2 - x_1}(x - x_1)$$
$$y - 116.33 = \frac{126.33 - 116.33}{4 - 0}(x - 0)$$
$$\hat{y} = 116.33 + 2.5x$$

Question 3: Compute Trend Values by Method of Semi Average

Case III

Year	Produciton	Semi Average	X	Trend Values $\hat{y} = 94 + 14.60x$
1981	96		-3	50.2
			-2	
1982	87		-1	79.4
		$\bar{y}_1 =$ 94	$x_1 =$ 0	
1983	91		1	108.6
			2	
1984	102		3	137.8
			4	
1985	108		5	167
			6	
1986	139		7	196.2
		$\bar{y}_2 =$ 210.75	$x_2 =$ 8	
1987	289		9	225.4
			10	
1988	307		11	254.6
			12	

$$y - \bar{y}_1 = \frac{\bar{y}_2 - \bar{y}_1}{x_2 - x_1}(x - x_1)$$
$$y - 94 = \frac{210.75 - 94}{8 - 0}(x - 0)$$
$$\hat{y} = 94 + 14.60x$$

Data

Year	Y
1981	96
1983	87
1985	91
1987	102
1989	108
1991	139
1993	289
1995	307

Question 4: Compute Trend Values by Method of Semi Average

Case IV

Year	Produciton	Semi Average	X	Trend Values $\hat{y} = 94 + 19.225x$
1981	96		-3	36.325
			-2	
1982	87		-1	74.775
		$\bar{y}_1 =$ 94	$x_1 =$ 0	
1983	91		1	113.225
			2	
1984	102		3	151.675
			4	
1985	108		5	190.125
			6	
1986	139		7	228.575
			8	
1987	289		9	267.025
		$\bar{y}_2 =$ 286.25	$x_2 =$ 10	
1988	307		11	305.475
			12	
1989	410		13	343.925

$$y - \bar{y}_1 = \frac{\bar{y}_2 - \bar{y}_1}{x_2 - x_1}(x - x_1)$$
$$y - 94 = \frac{1286.28 - 94}{10 - 0}(x - 0)$$
$$\hat{y} = 94 + 19.225x$$

Data

Year	Y
1981	96
1983	87
1985	91
1987	102
1989	108
1991	139
1993	289
1995	307
1997	410

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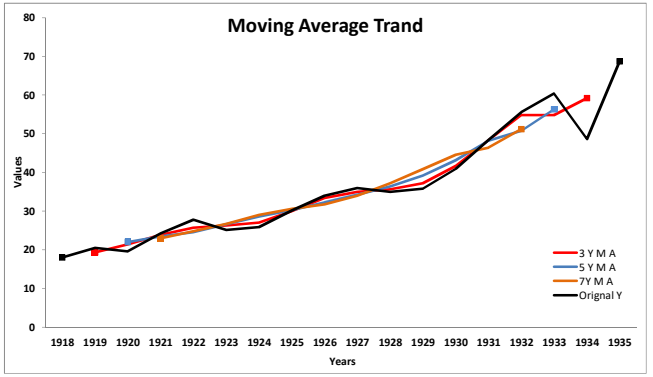
iii Method of Moving Average

Question 1: Determin the trend line by moving average method using 3 years, 5 years and 7 years moving averages.

Case I Odd Year Moving Average

Year	Orignal Y	3 Y M A		5 Y M A		7Y M A	
		Total	Average	Total	Average	Total	Average
1918	18						
1919	20.5	58.1	**19.4**				
1920	19.6	64.3	**21.4**	110.1	**22.0**		
1921	24.2	71.6	**23.9**	117.2	**23.4**	161.1	23.0
1922	27.8	77.1	**25.7**	122.6	**24.5**	173.3	24.8
1923	25.1	78.8	**26.3**	133.2	**26.6**	186.8	26.7
1924	25.9	81.2	**27.1**	143.0	**28.6**	203.2	29.0
1925	30.2	90.1	**30.0**	151.2	**30.2**	214.0	30.6
1926	34	100.2	**33.4**	161.1	**32.2**	222.0	31.7
1927	36	105.0	**35.0**	171.0	**34.2**	237.8	34.0
1928	35	106.8	**35.6**	181.7	**36.3**	260.3	37.2
1929	35.8	111.7	**37.2**	196.1	**39.2**	285.7	40.8
1930	40.9	125.1	**41.7**	215.7	**43.1**	312.1	44.6
1931	48.4	144.9	**48.3**	241.1	**48.2**	324.7	46.4
1932	55.6	164.4	**54.8**	253.9	**50.8**	358.4	51.2
1933	60.4	164.6	**54.9**	281.7	**56.3**		
1934	48.6	177.7	**59.2**				
1935	68.7						



Question 2 Determin the trend line by moving average method using 4 years Centered and 6 years Centered moving averages.

Case II Even Year Centered Moving Average

Year	Orignal Y	4 Y C M A			6 Y C M A		
		4 Y M T	8 Y M T	4 Y C M A	6 Y M T	12 Y M T	6 Y C M A
1918	**18**						
1919	**20.5**						
		82.3					
1920	**19.6**		174.4	21.8			
		92.1			135.2		
1921	**24.2**		188.8	23.6		278.3	23.2
		96.7			143.1		
1922	27.8		199.7	25.0		295.9	24.7
		103.0			152.8		
1923	25.1		212.0	26.5		320.0	26.7
		109.0			167.2		
1924	25.9		224.2	28.0		346.2	28.9
		115.2			179.0		
1925	30.2		241.3	30.2		365.2	30.4
		126.1			186.2		
1926	34		261.3	32.7		383.1	31.9
		135.2			196.9		
1927	36		276.0	34.5			
		140.8					
1928	35						
1929	35.8						

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iv Method of Least Squares

Question 1: Determin the trend line by method of least squares and plot trend values on graph.

Case I	Year	Y	X	XY	X2	Y2	Trend Values $\bar{Y}$	
Odd number of Values	1918	18	-8	-144	64	324	**15.46**	-15
	1919	20.5	-7	-143.5	49	420.25	**17.84**	-14
	1920	19.6	-6	-117.6	36	384.16	**20.21**	-13
	1921	24.2	-5	-121	25	585.64	**22.59**	-12
	1922	27.8	-4	-111.2	16	772.84	**24.96**	-11
	1923	25.1	-3	-75.3	9	630.01	**27.34**	-10
	1924	25.9	-2	-51.8	4	670.81	**29.72**	-9
	1925	30.2	-1	-30.2	1	912.04	**32.09**	-8
	1926	34	**0**	0	0	1156	**34.47**	-7
	1927	36	1	36	1	1296	**36.85**	-6
	1928	35	2	70	4	1225	**39.22**	-5
	1929	35.8	3	107.4	9	1281.64	**41.60**	-4
	1930	40.9	4	163.6	16	1672.81	**43.98**	-3
	1931	48.4	5	242	25	2342.56	**46.35**	-2
	1932	55.6	6	333.6	36	3091.36	**48.73**	-1
	1933	60.4	7	422.8	49	3648.16	**51.11**	0
	1934	48.6	8	388.8	64	2361.96	**53.48**	1
		586	**0**	**969.6**	**408**	**22775.24**		
		$\sum Y$	$\sum X$	$\sum XY$	$\sum X^2$	$\sum Y^2$		

$\bar{Y} =$ 34.47 $\bar{X} =$ 0.00

$$\bar{Y} = a + b\bar{X}$$

$$b = \frac{n\sum XY - \sum X \sum Y}{n\sum X^2 - (\sum X)^2} = \frac{16483.2}{6936} \quad b = 2.38$$

$$a = \bar{Y} - b\bar{X} = 34.47$$

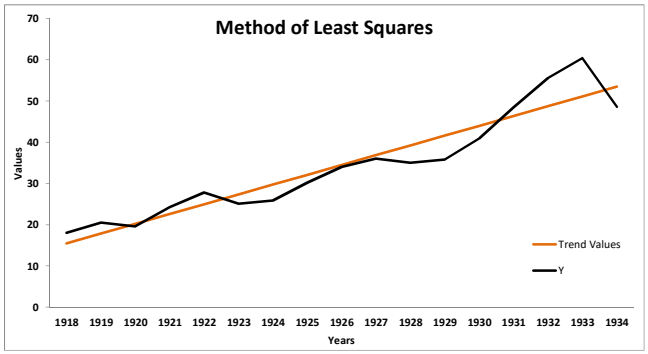
Y = 34.47 + 2.38 X

Shifing of Orgion

$X =$ -8

Y = 34.47 + 2.38 [X + (-8)]

Y = 15.46 + 2.38 X



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Question 2: Determin the trend line by method of least squares and plot trend values on graph.

Case II	Year	Y	X	XY	X2	Y2	Trend Values $\bar{Y}$
Even number of Values	1918	18	-17	-306	289	324	**14.26**
	1919	20.5	-15	-307.5	225	420.25	**16.86**
	1920	19.6	-13	-254.8	169	384.16	**19.46**
	1921	24.2	-11	-266.2	121	585.64	**22.06**
	1922	27.8	-9	-250.2	81	772.84	**24.66**
	1923	25.1	-7	-175.7	49	630.01	**27.27**
	1924	25.9	-5	-129.5	25	670.81	**29.87**
	1925	30.2	-3	-90.6	9	912.04	**32.47**
	1926	34	-1	-34	1	1156	**35.07**
	1927	36	1	36	1	1296	**37.67**
	1928	35	3	105	9	1225	**40.27**
	1929	35.8	5	179	25	1281.64	**42.88**
	1930	40.9	7	286.3	49	1672.81	**45.48**
	1931	48.4	9	435.6	81	2342.56	**48.08**
	1932	55.6	11	611.6	121	3091.36	**50.68**
	1933	60.4	13	785.2	169	3648.16	**53.28**
	1934	48.6	15	729	225	2361.96	**55.89**
	1935	68.7	17	1167.9	289	4719.69	**58.49**
		654.7	**0**	**2521.1**	**1938**	**27494.93**	
		$\sum Y$	$\sum X$	$\sum XY$	$\sum X^2$	$\sum Y^2$	

$\bar{Y} =$ 36.37 $\bar{X} =$ 0.00

$$\bar{Y} = a + b\bar{X}$$

$$b = \frac{n\sum XY - \sum X \sum Y}{n\sum X^2 - (\sum X)^2} = \frac{42858.7}{32946} \quad b = 1.30$$

$$a = \bar{Y} - b\bar{X} = 36.37$$

Shifing of Orgion

$X =$ -7

Y = 36.37 + 1.3 [X + (-7)]

Y = 27.27 + 1.3 X