Lecture 1

Normal Distribution: Application and Properties

In this lecture, the general idea is

- **What is Normal Distribution**
- **Real Life Examples of Normal Distribution**
- **Properties of Normal Distribution**
- **Key Points about Normal Curve**

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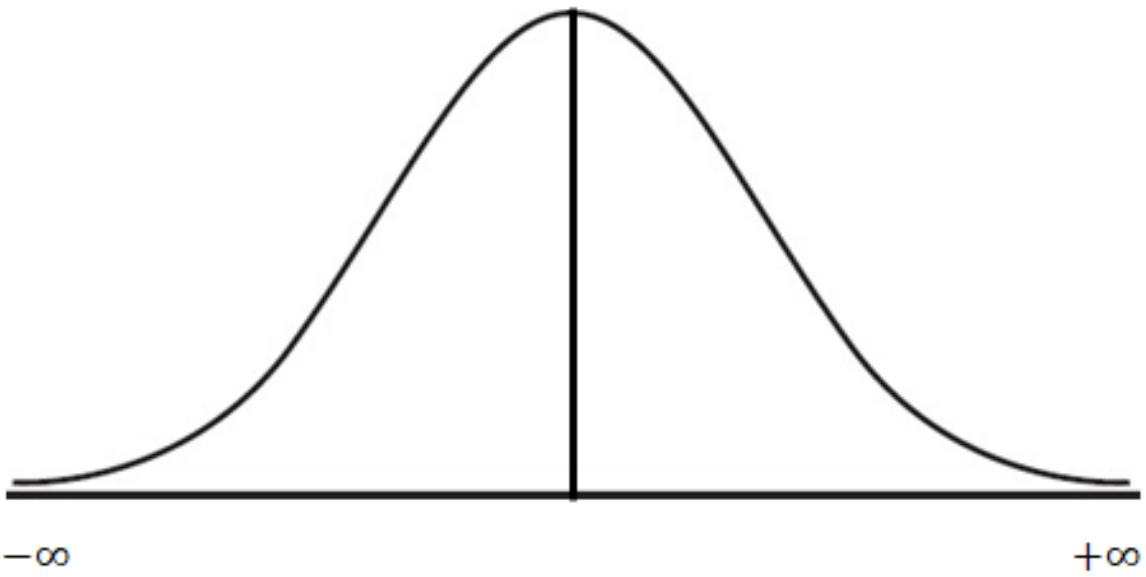
What is Normal Distribution?

Normal/Gaussian Distribution is a bell-shaped graph which encompasses two basic terms mean and standard deviation. It is a symmetrical arrangement of a data set in which most values cluster in the mean and the rest taper off symmetrically towards either extreme. Numerous genetic and environmental factors influence the trait.



The normal distribution is the most important probability distribution in statistics because it fits many natural phenomena. For example, heights, blood pressure, measurement error, and IQ scores follow the normal distribution.

It is also known as the Gaussian distribution and the bell curve. The normal distribution is a probability function that describes how the values of a variable are distributed.

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Parameters of the Normal Distribution

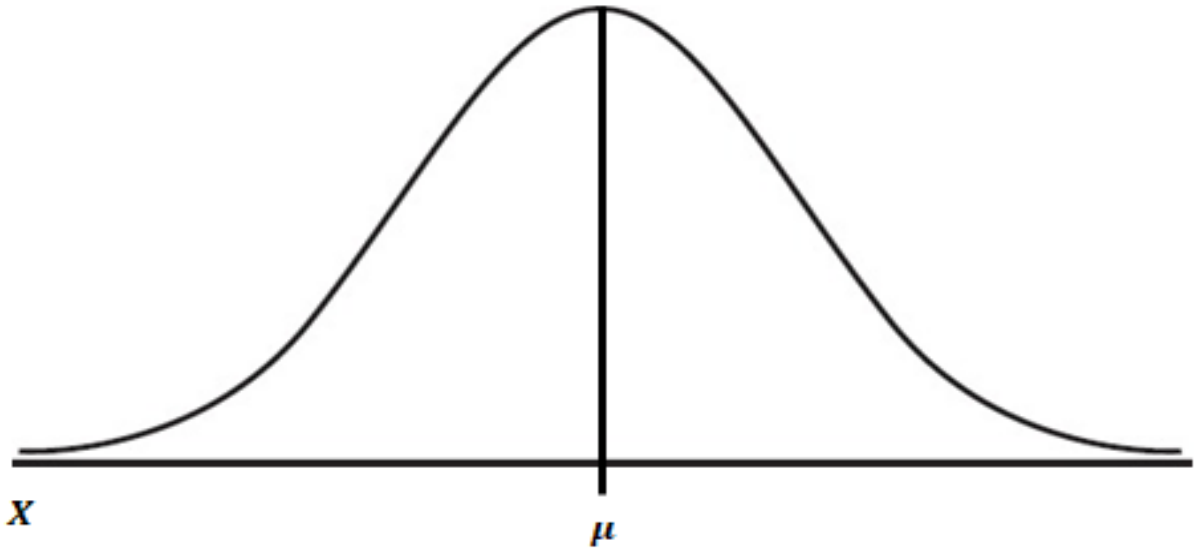
The parameters for the normal distribution define its shape and probabilities entirely. It has two parameters, the mean μ and standard deviation σ. The normal distribution has the following probability density function.

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-1/2\,[(x-\mu)^2/\sigma^2]} \qquad \forall\ -\infty < x < \infty$$

where, μ is mean, σ is the standard deviation and π = 3.1416.

Mean μ : The mean is the central tendency of the distribution. It defines the location of the peak for normal distributions. Most values cluster around the mean. On a graph, changing the mean shifts the entire curve left or right on the *X*-axis.

Standard deviation σ : The standard deviation is a measure of variability. It defines the width of the normal distribution. It determines how far away from the mean the values tend to fall.



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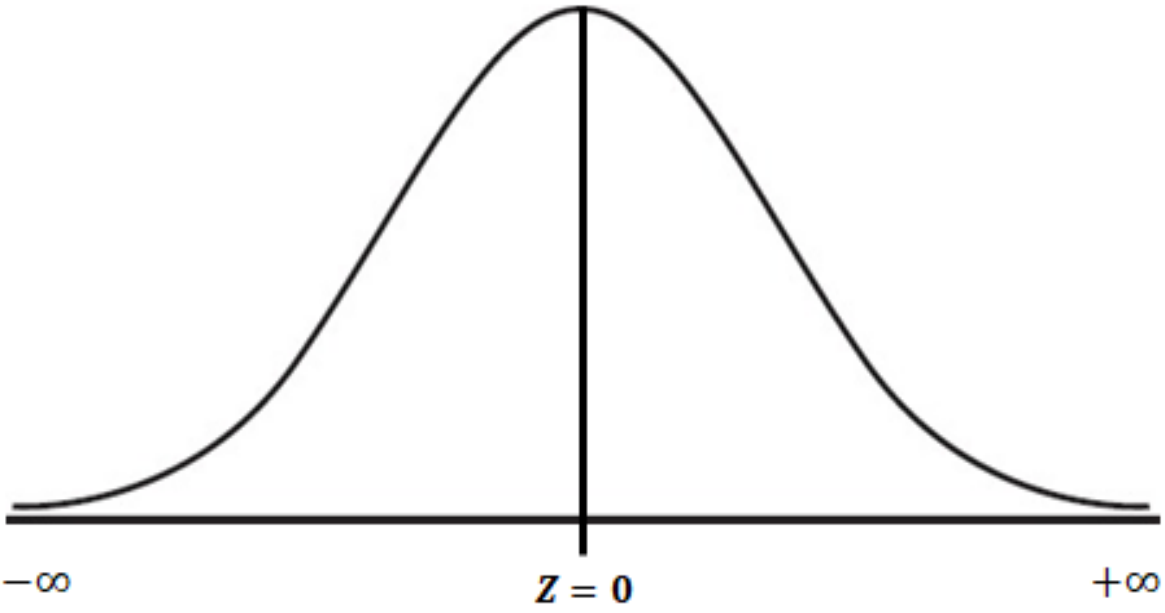
Standard Normal Distribution and Standard Scores

The normal distribution has many different shapes depending on the parameter values mean μ and standard deviation σ. However, the standard normal distribution is a special case of the normal distribution where the mean is zero and the standard deviation is 1. This distribution is also known as the Z-distribution.



A value on the standard normal distribution is known as a standard score or a Z-score. A standard score represents the number of standard deviations above or below the mean that a specific observation falls. For example, a standard score of 1.5 indicates that the observation is 1.5 standard deviations above the mean. On the other hand, a negative score represents a value below the average. The mean has a Z-score of 0.

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Real Life Examples of Normal Distribution

The normal distribution is widely used in understanding distributions of factors in the population. The normal distribution approximates many natural phenomena. Let's understand the daily life examples of Normal Distribution.

Tossing a Coin

Flipping a coin is one of the oldest methods for settling disputes. We all have flipped a coin before a match or game. The perceived fairness in flipping a coin lies in the fact that it has equal chances to come up with either result. The chances of getting head are 1/2, and the same is for tails. When we add both, it equals to one. If we toss coins multiple times, the sum of the probability of getting heads and tails will always remain 1.

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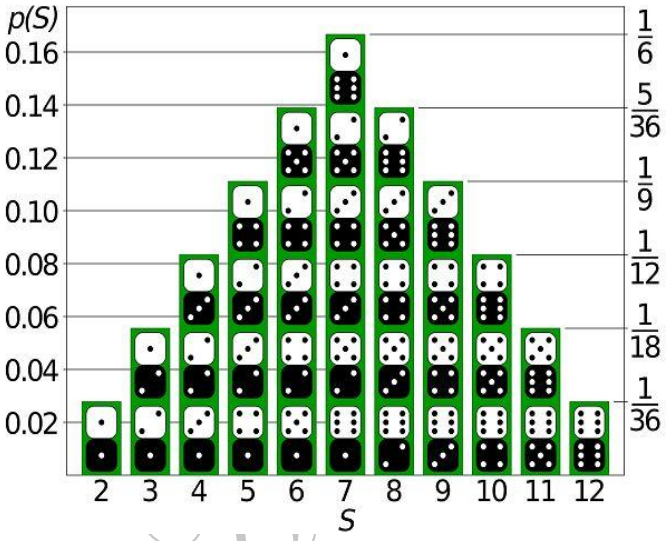
Rolling a Dice



A fair rolling of dice is also a good example of normal distribution. In an experiment, it has been found that when a dice is rolled 100 times, chances to get '1' are 15-18% and if we roll the dice 1000 times, the chances to get '1' are, again, the same, which averages to 16.7% (1/6). If we roll two dices simultaneously, there are 36 possible combinations. The probability of rolling '1' (with six possible combinations) again averages to around 16.7%, i.e., (6/36). More the number of dices more elaborate will be the normal distribution graph.

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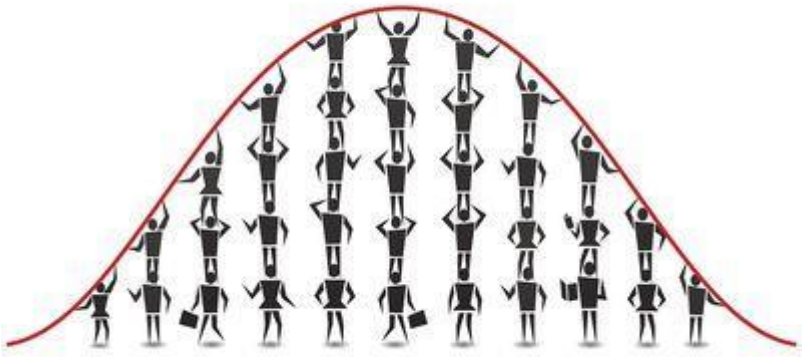
Height



Height of the population is the example of normal distribution. Most of the people in a specific population are of average height. The number of people taller and shorter than the average height people is almost equal, and a very small number of people are either extremely tall or extremely short. However, height is not a single characteristic; several genetic and environmental factors influence height. Therefore, it follows the normal distribution.

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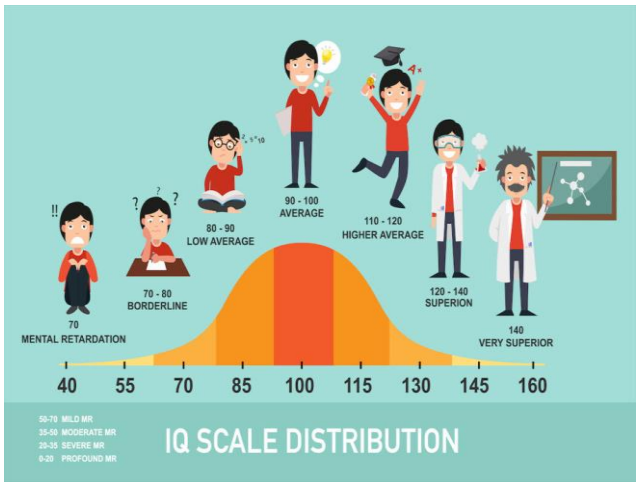
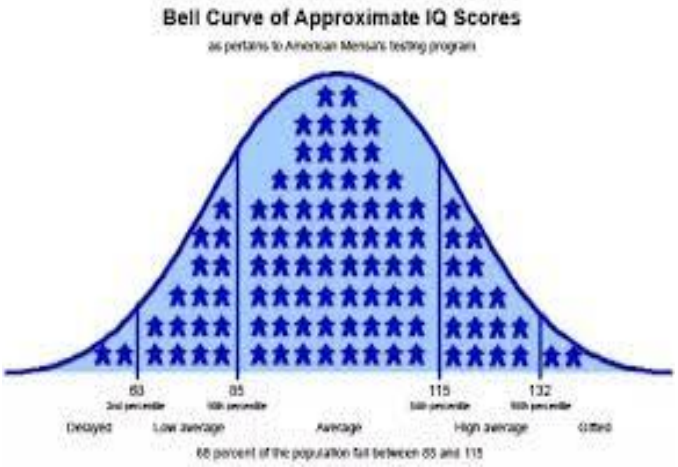
IQ of Test





In this scenario of increasing competition, most parents, as well as children, want to analyze the Intelligent Quotient level. Well, the IQ of a particular population is a normal distribution curve; where IQ of a majority of the people in the population lies in the normal range whereas the IQ of the rest of the population lies in the deviated range.

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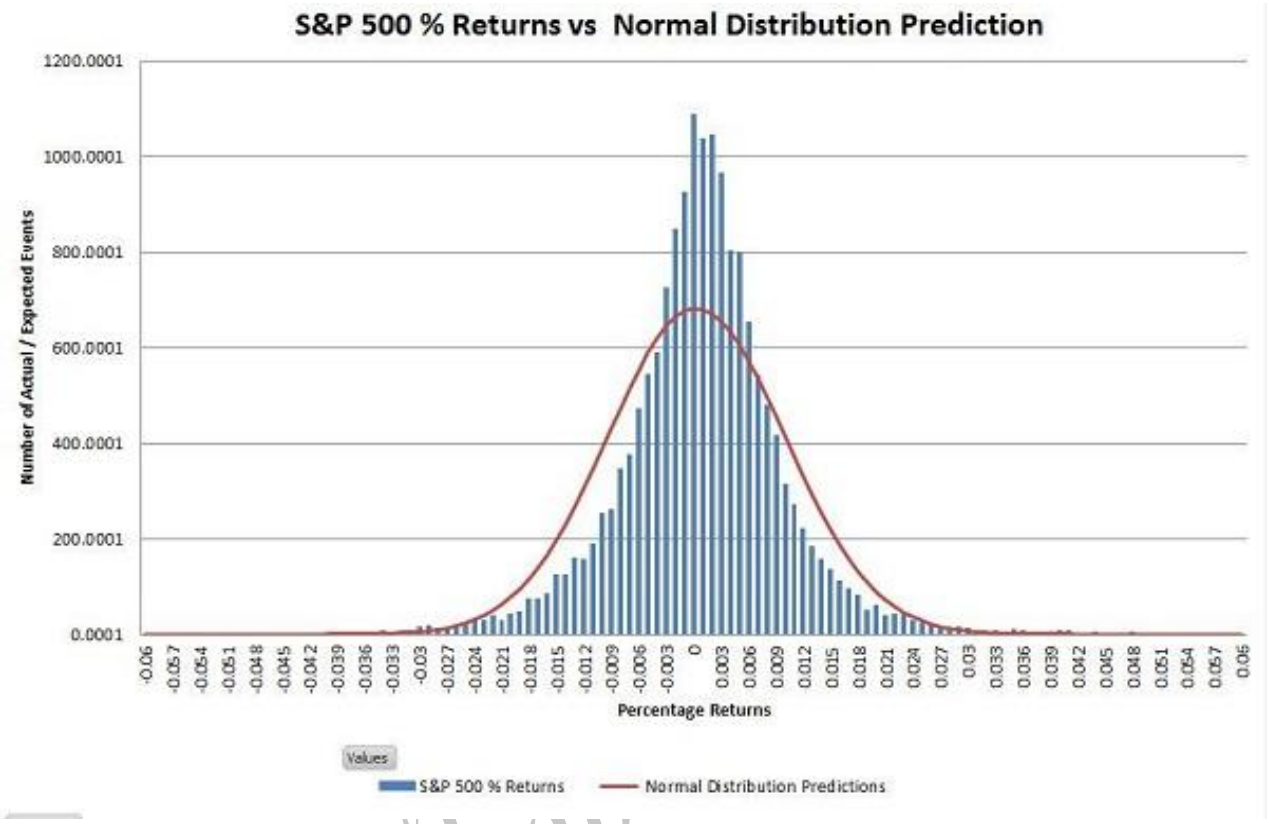
Technical Stock Market



Most of us have heard about the rise and fall in the prices of the shares in the stock market. These changes in the log values of Forex rates, price indices, and stock prices return often form a bell-shaped curve. For stock returns, the standard deviation is often called volatility. If returns are normally distributed, more than 99 percent of the returns are expected to fall within the deviations of the mean value. Such characteristics of the bell-shaped normal distribution allow analysts and investors to make statistical inferences about the expected return and risk of stocks.

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Income Distribution in Economy

The income of a country lies in the hands of enduring politics and government. It depends upon them how they distribute the income among the rich and poor community. We all are well aware of the fact that the middle-class population is a bit higher than the rich and poor population. So, the wages of the middle-class population makes the mean in the normal distribution curve.

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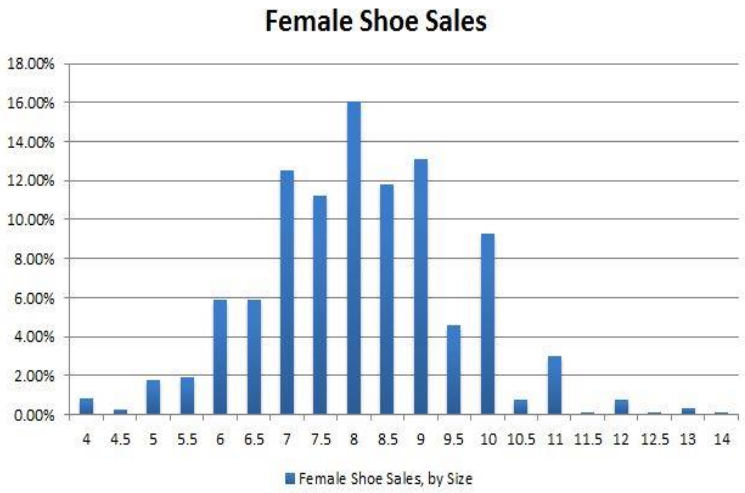
Shoe Size



Have you wondered what would have happened if the glass slipper left by Cinderella at the prince's house fitted another woman's feet? He would have ended up marrying another woman. It has been one of the amusing assumptions we all have ever come across. As per the data collected in the US, female shoe sales by size are normally distributed because the physical makeup of most women is almost the same.

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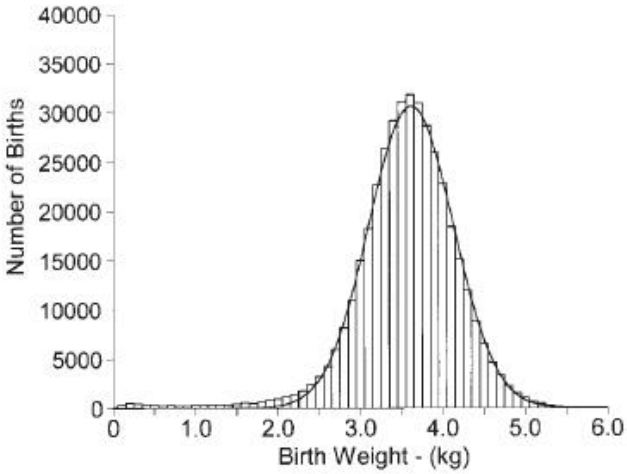
Birth Weight



The normal birth weight of a new born range from 2.5 to 3.5 kg. The majority of newborns have normal birth weight whereas only a few percentages of newborns have a weight higher or lower than the normal. Hence, birth weight also follows the normal distribution curve.

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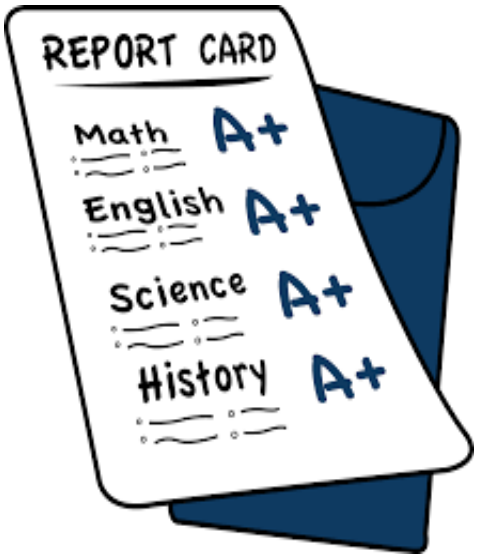
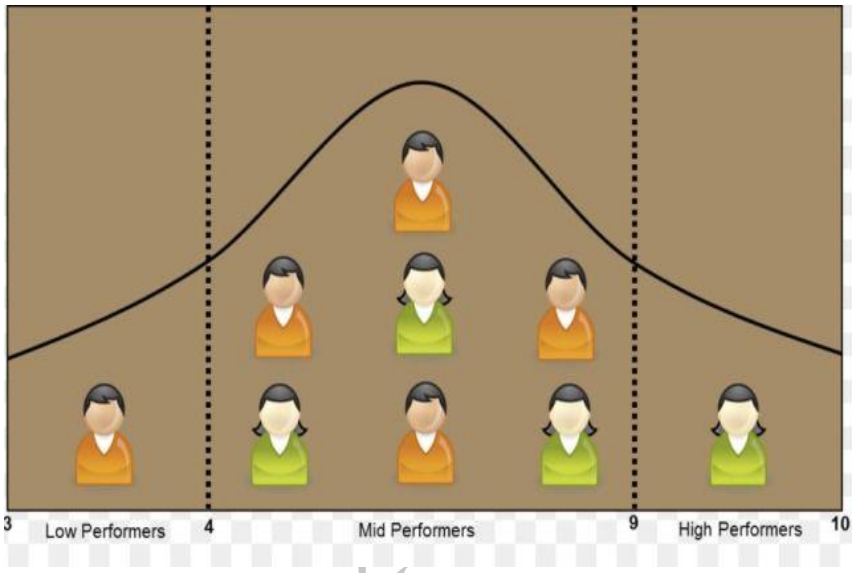
Student's Average Report





Nowadays, schools are advertising their performances on social media and TV. They present the average result of their school and allure parents to get their child enrolled in that school. School authorities find the average academic performance of all the students, and in most cases, it follows the normal distribution curve. The number of average intelligent student is higher than most other students.

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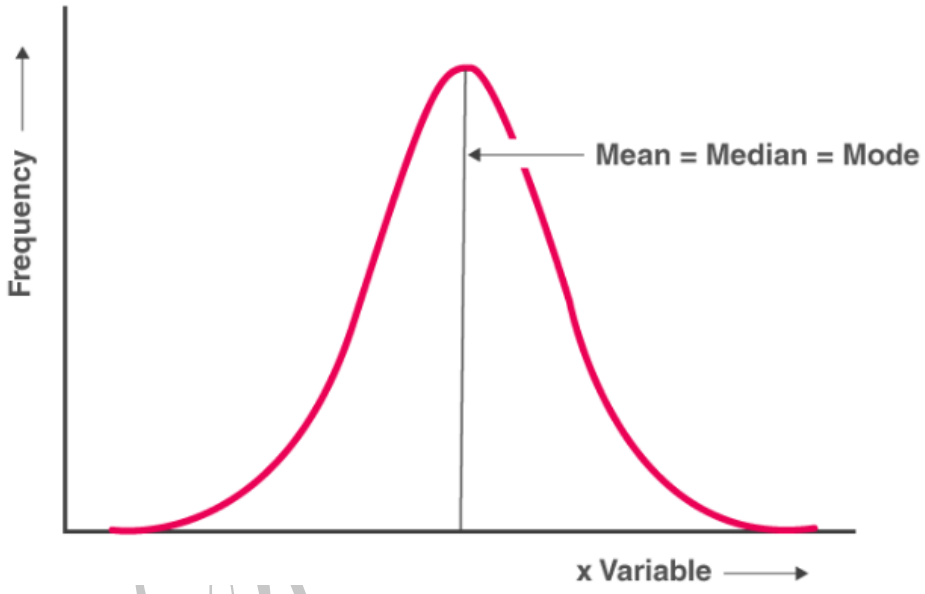
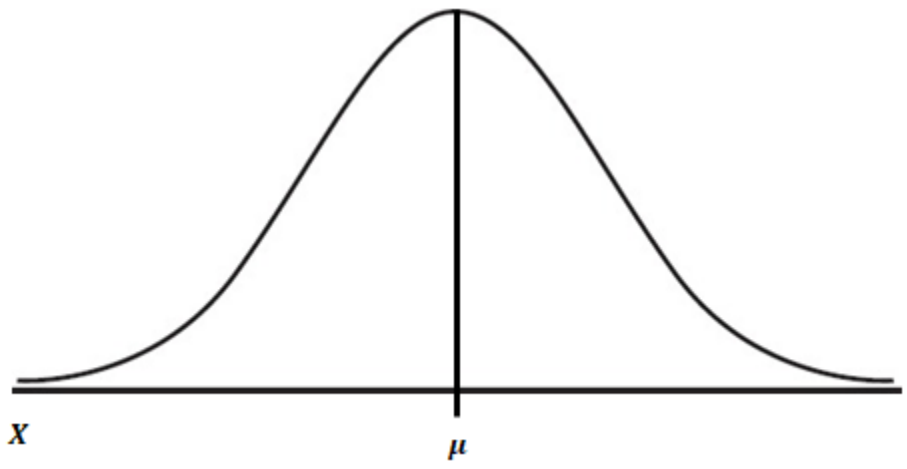
Properties of Normal Distribution

A normal distribution is a continuous probability distribution for a random variable x. The graph of a normal distribution is called the normal curve. A normal distribution has the following properties.

1. The mean, median, and mode are equal.



2. The normal curve is bell shaped and is symmetric about the mean.



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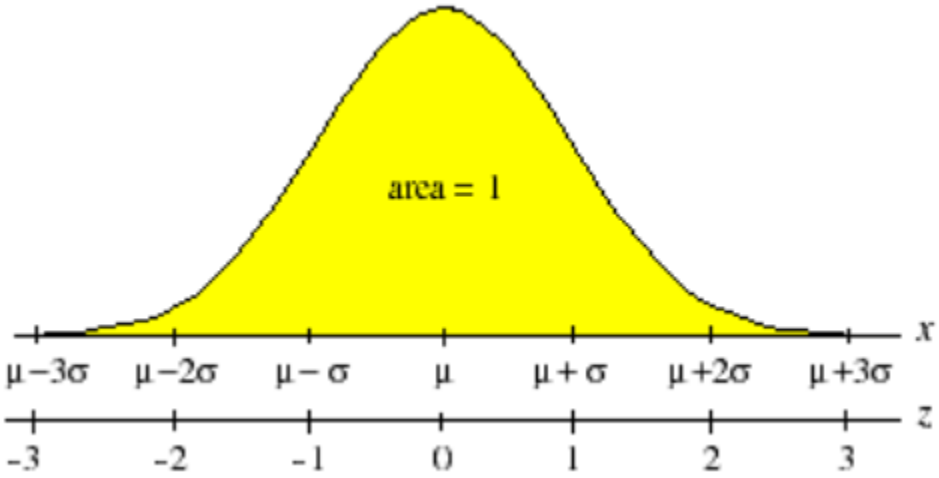
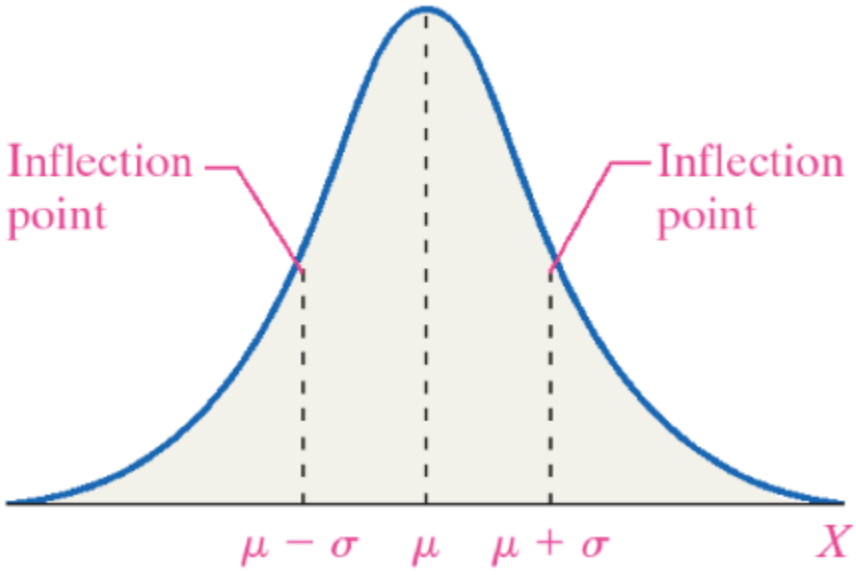
3. The total are under the normal curve is equal to one.



4. The normal is asymptotic to x-axis: The curve approaches, but never touches, the x-axis as it extends farther and farther away from the mean.

5. Between $\mu - \sigma$ and $\mu + \sigma$ (in the center of curve) the graph curves downward. The graph curves upward to the left $\mu - \sigma$ of and to the right of $\mu + \sigma$.

6. The points at which the curve changes from curving upward to curving downward are called inflection points.



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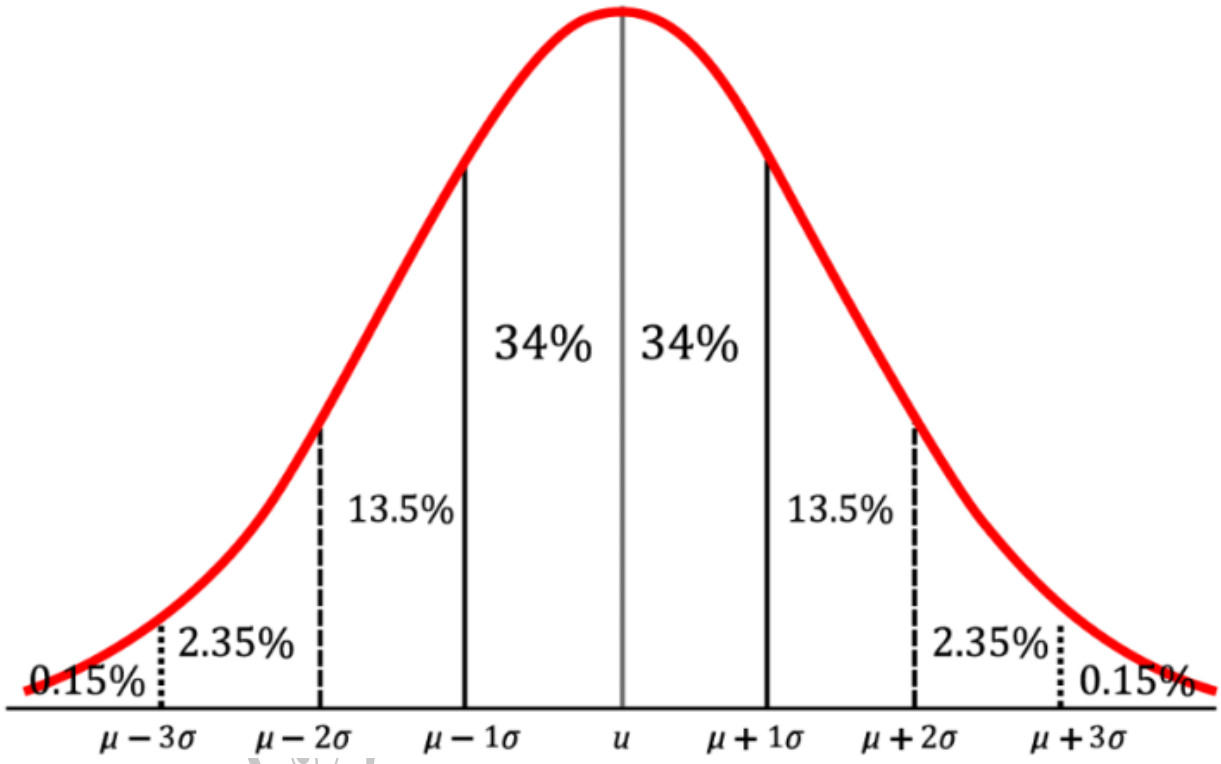
7. **The Empirical Rule:** Approximately 68% of the area under the normal curve is between $\mu - \sigma$ and $\mu + \sigma$. Approximately 95% of the area under the normal curve is between $\mu - 2\sigma$ and $\mu + 2\sigma$. Approximately 99.7% of the area under the normal curve is between $\mu - 3\sigma$ and $\mu + 3\sigma$



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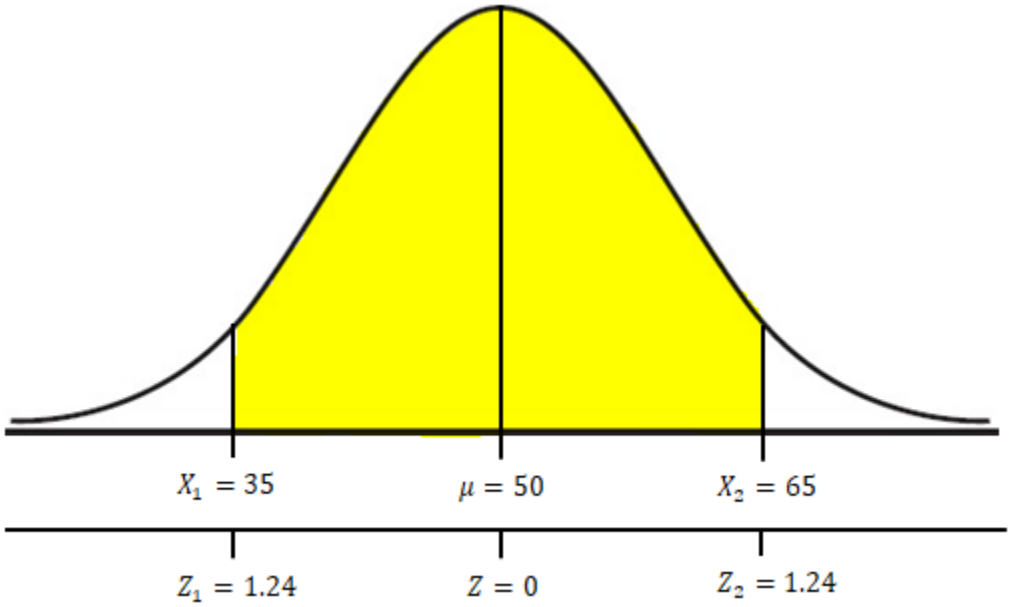
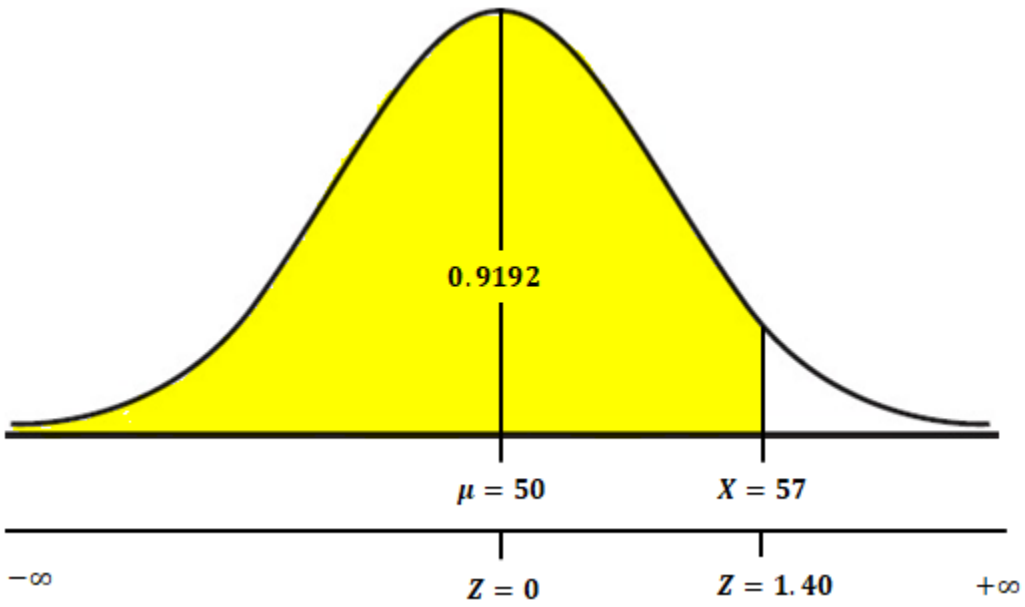
Key Points about Normal Curve

Addition of Probability: If central line lies within shaded area then add probabilities.





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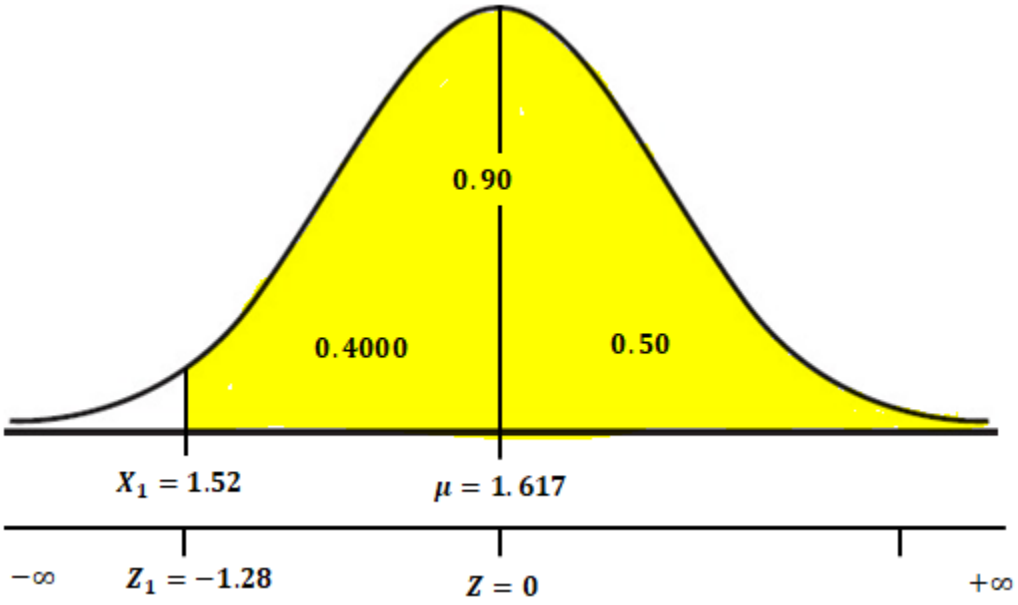

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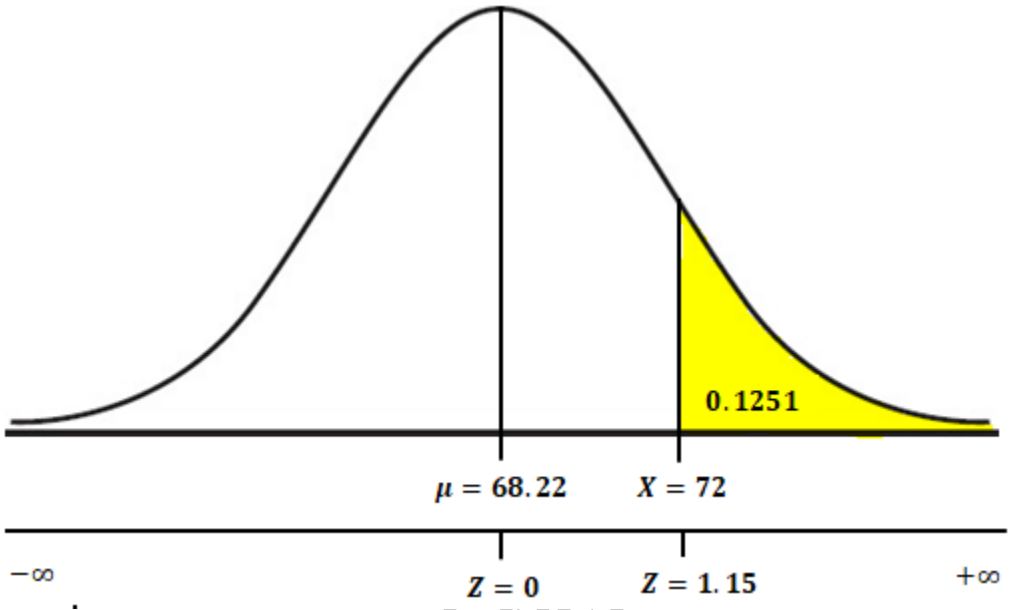
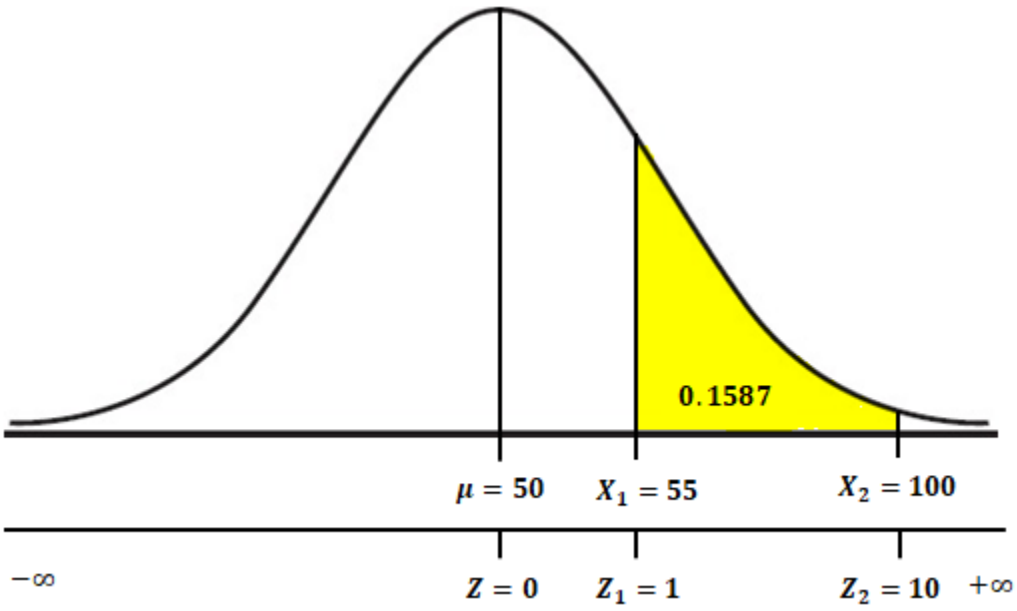
Subtraction of Probability: If central line lies outside of shaded area then subtract probabilities.





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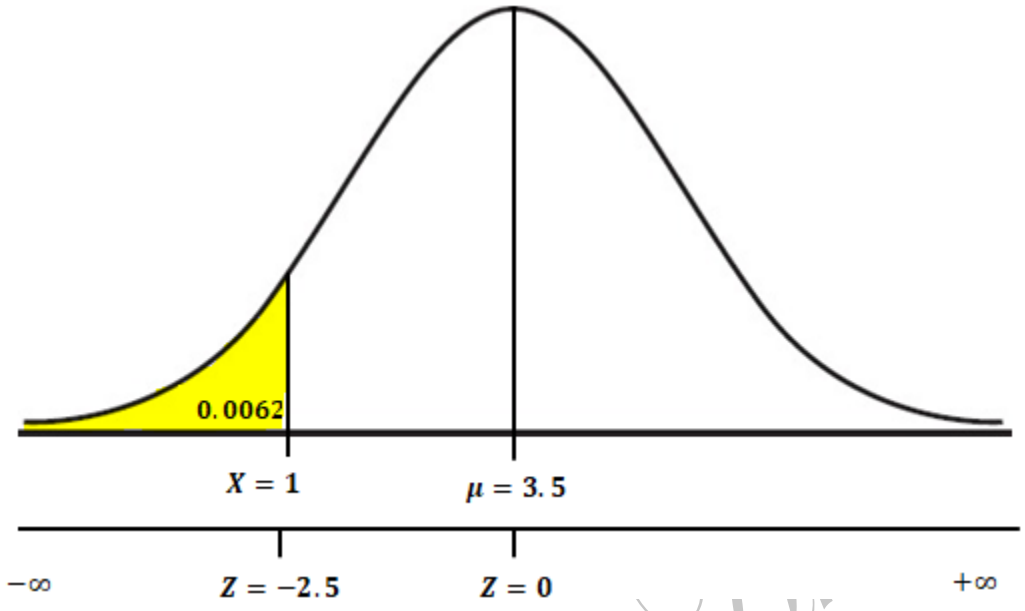

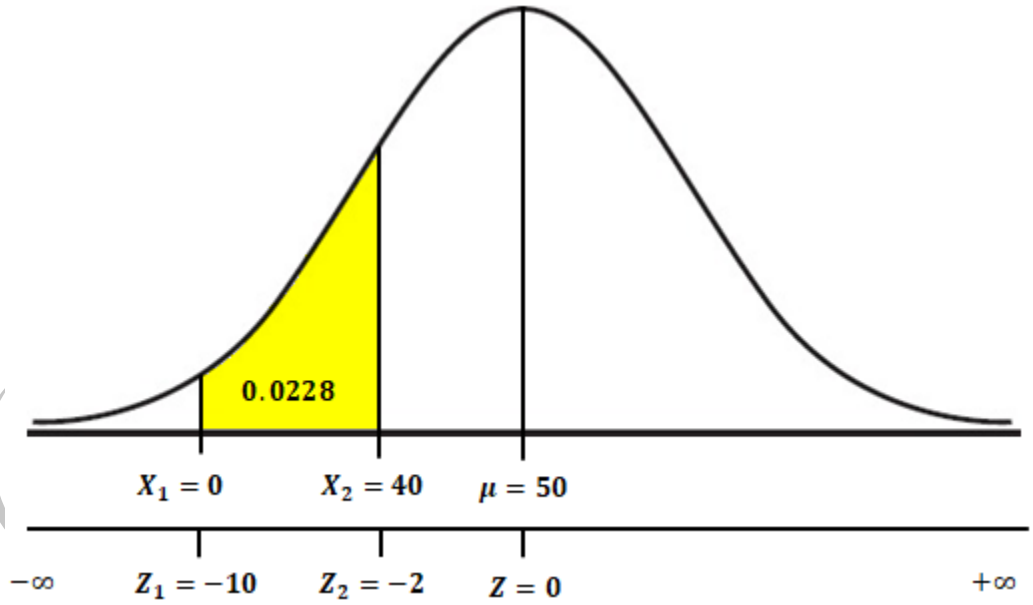

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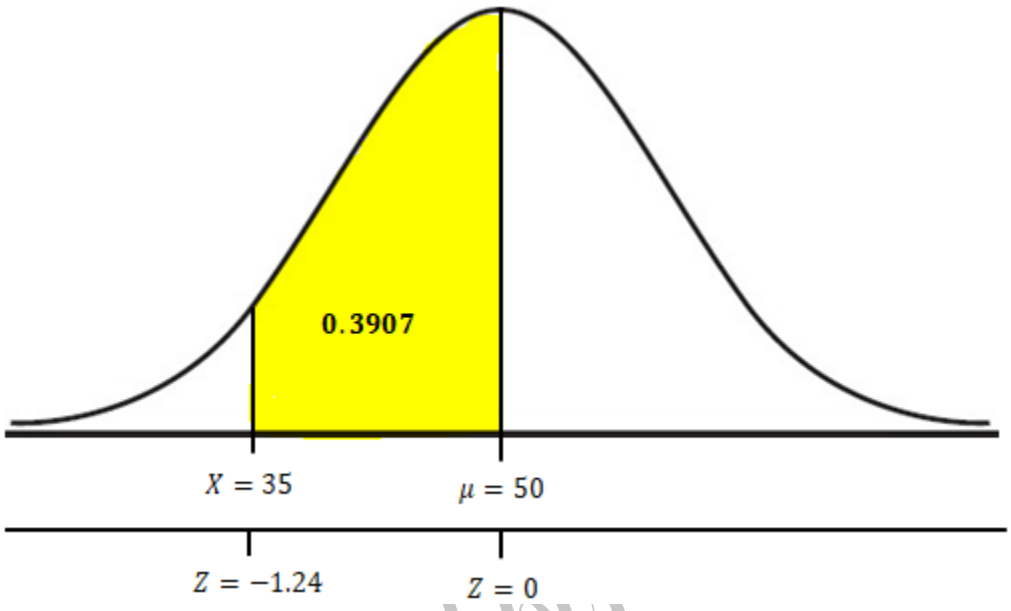
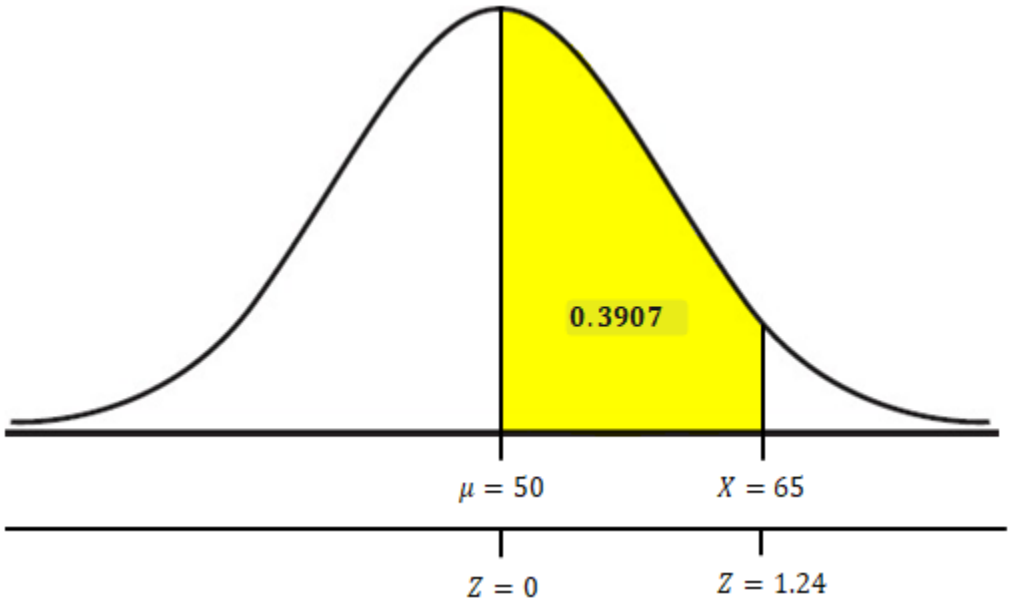
Neither Add nor Subtraction: If central line lies at start or end of shaded area then neither add nor subtraction.





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In next Lecture

- **Normal Distribution: Direct use of Area Table**

  (X→ Z → Get Area as Answer)

THE END ☺

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