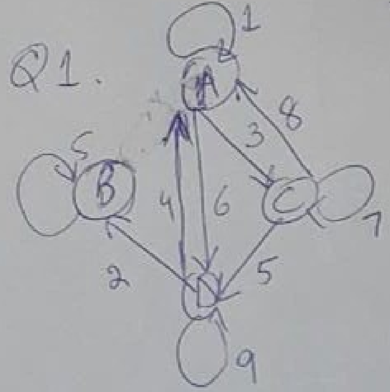
Assignment 2 Graph Solution
By Floyd (Modified warshall Algorithm)

Q1.



$$W = \begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{pmatrix} 1 & 0 & 3 & 6 \\ 0 & 5 & 0 & 0 \\ 8 & 0 & 7 & 5 \\ 4 & 2 & 0 & 9 \end{pmatrix} \end{array}$$

$$Q_K[i][j] = \min\left(Q_{K-1}[i][j],\ Q_{K-1}[i][K] + Q_{K-1}[K][j]\right)$$

$$Q_0 = \begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{pmatrix} 1 & \infty & 3 & 6 \\ \infty & 5 & \infty & \infty \\ 8 & \infty & 7 & 5 \\ 4 & 2 & \infty & 9 \end{pmatrix} \end{array} \qquad \begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{pmatrix} \overline{AA} & - & \overline{AC} & \overline{AD} \\ - & \overline{BB} & - & - \\ \overline{CA} & - & \overline{CC} & \overline{CD} \\ \overline{DA} & \overline{DB} & - & \overline{DD} \end{pmatrix} \end{array}$$

After including node A $(k=1)$

$$Q_1 = \begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{pmatrix} 1 & \infty & 3 & 6 \\ \infty & 5 & \infty & \infty \\ 8 & \infty & 7 & 5 \\ 4 & 2 & \boxed{7} & 9 \end{pmatrix} \end{array} \qquad \begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{pmatrix} \overline{AA} & - & \overline{AC} & \overline{AD} \\ - & \overline{BB} & - & - \\ \overline{CA} & - & \overline{CC} & \overline{CD} \\ \overline{DA} & \overline{DB} & \boxed{\overline{DAC}} & \overline{DD} \end{pmatrix} \end{array}$$

After including node B $(k=2)$

$$Q_2 = \begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{pmatrix} 1 & \infty & 3 & 6 \\ \infty & 5 & \infty & \infty \\ 8 & \infty & 7 & 5 \\ 4 & 2 & 7 & 9 \end{pmatrix} \end{array} \qquad \begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{pmatrix} 1 & - & \overline{AC} & \overline{AD} \\ - & \overline{BB} & - & - \\ \overline{CA} & - & \overline{CC} & \overline{CD} \\ \overline{DA} & \overline{DB} & \overline{DAC} & \overline{DD} \end{pmatrix} \end{array}$$

After including node C $(k=3)$

$$Q_3 = \begin{array}{c|cccc} & A & B & C & D \\ \hline A & 1 & \infty & 3 & 6 \\ B & \infty & 5 & \infty & \infty \\ C & 8 & \infty & 7 & 5 \\ D & 4 & 2 & 7 & 9 \end{array} \qquad \begin{array}{c|cccc} & A & B & C & D \\ \hline A & \overline{AA} & - & \overline{AC} & \overline{AD} \\ B & - & \overline{BB} & - & - \\ C & \overline{CA} & - & \overline{CC} & \overline{CD} \\ D & \overline{DA} & \overline{DB} & \overline{DAC} & \overline{DD} \end{array}$$

After including node D $(k=4)$

$$Q_4 = \begin{array}{c|cccc} & A & B & C & D \\ \hline A & 1 & \boxed{18} & 3 & 6 \\ B & \infty & 5 & \infty & \infty \\ C & 8 & \boxed{7} & 7 & 5 \\ D & 4 & 2 & 7 & 9 \end{array} \qquad \begin{array}{c|cccc} & A & B & C & D \\ \hline A & \overline{AA} & \boxed{\overline{ADB}} & \overline{AC} & \overline{AD} \\ B & - & \overline{BB} & - & - \\ C & \overline{CA} & \boxed{\overline{CDB}} & \overline{CC} & \overline{CD} \\ D & \overline{DA} & \overline{DB} & \overline{DAC} & \overline{DD} \end{array}$$

By Dijkstra's Algorithm (Taking A as source node).

Step 0: Initialization.

v	A	B	C	D
d[v]	0	∞	∞	∞
pred[v]	Nil	Nil	Nil	Nil
color[v]	W	W	W	W

PQ:

v	A	B	C	D
d[v]	0	∞	∞	∞

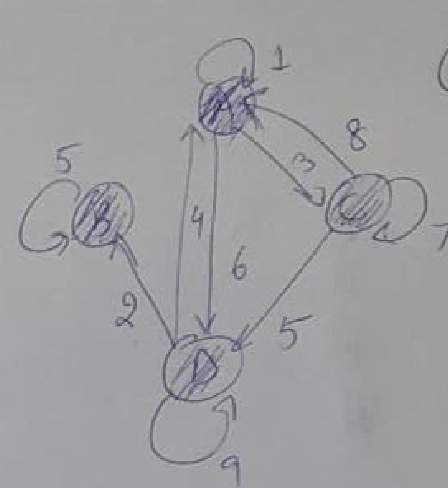
Step 1: As $Adj[A] = \{A, C, D\}$, work on A, C and D and update information.

v	A	B	C	D
d[v]	1	∞	3	6
pred[v]	A	Nil	A	A
color[v]	B	W	W	W

if $v.dist > u.dist + w(u,v)$
$v.d = u.d + w(u,v)$
$v.\pi = u$

PQ:

v	C	D	B
d[v]	3	6	∞



Step 2: After step 1, C has the minimum key in the PQ. As Adj[C] = {A, C, D}, work on A, C and D and update information.

v	A	B	C	D
d[v]	1	∞	3	6
pred[v]	A	Nil	A	A
color[v]	B	W	B	W

PQ:

v	D	B
d[v]	6	∞

Step 3: After step 2, D has the minimum key in the PQ. As Adj [D] = {A, B and D}, work on A, B and D and update information.

v	A	B	C	D
d[v]	1	8	3	6
pred[v]	A	D	A	A
color[v]	B	W	B	B

PQ:

v	B
d[v]	8

Step 4: After step 3, B has the minimum key in the PQ. As Adj [B] = {B}, work on B and update information.

v	A	B	C	D
d[v]	1	8	3	6
pred[v]	A	D	A	A
color[v]	B	B	B	B

PQ: = ∅

A → A = AA (1)

A → B = ADB (8)

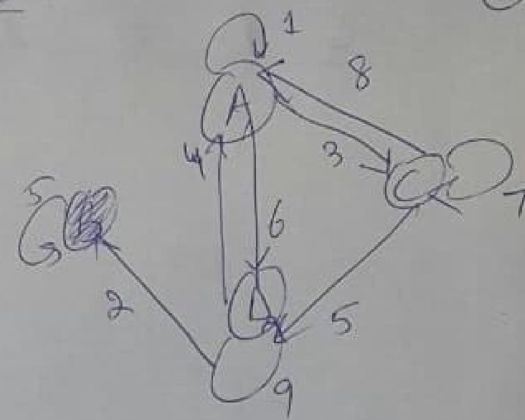
A → C = AC (3)

A → D = AD (6)

(verified the 1st row of Q4 obtained from Modified Warshall)

Now, taking B as source node

Step 0: Initialization



v	B	A	C	D
d[v]	0	∞	∞	∞
pred[v]	Nil	Nil	Nil	Nil
color[v]	W	W	W	W

PQ:

v	B	A	C	D
d[v]	0	∞	∞	∞

Step 1: As Adj [B] = {B}, work on B and update information.

v	B	A	C	D
d[v]	5	∞	∞	∞
pred[v]	B	Nil	Nil	Nil
color[v]	B	W	W	W

PQ:

v	A	C	D
d[v]	∞	∞	∞

Can not proceed further b/c all are infinity and we can not take the minimum value.

(verified the 2nd row of Q4 obtained from Modified warshall)

B→A (No path)
B→B (5)
B→C (No path)
B→D (No path

Now taking C as source node

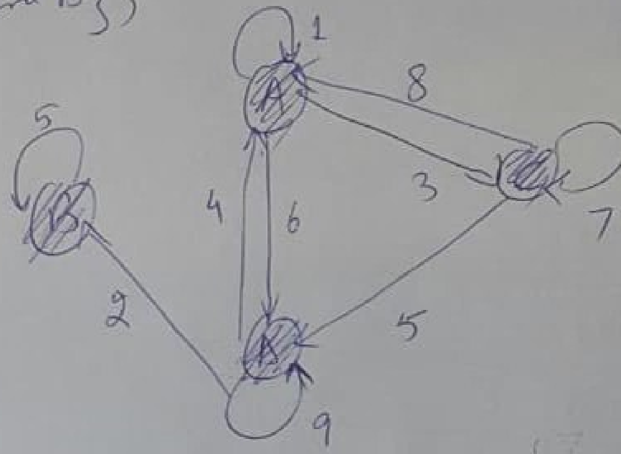
Step 0: Initialization

v	C	A	B	D
d[v]	0	∞	∞	∞
pred[v]	Nil	Nil	Nil	Nil
color[v]	W	W	W	W

PQ:

v	C	A	C	D
d[v]	0	∞	∞	∞

step 1 : As Adj[C] = {A, C and D}, work on A, C and D and update information.



V	C	A	B	D
d[v]	7	8	∞	5
pred[v]	C	C	Nil	C
color[v]	B	W	W	W

PQ:

v	D	A	B
d[v]	5	8	∞

step 2 : After step 1, D has the minimum key in the PQ. As Adj[D] = {A, B and D}, work on A, B and D and update information.

V	C	A	B	D
d[v]	7	8	7	5
pred[v]	C	C	D	C
color[v]	B	W	W	B

PQ:

v	B	A
d[v]	7	8

step 3 : After step 2, B has the minimum key in the PQ. As Adj[B] = {B}, work on B and update information.

V	C	A	B	D
d[v]	7	8	7	5
pred[v]	C	C	D	C
color[v]	B	W	B	B

PQ:

V	A
d[v]	8

Step 4: After step 3, A has the minimum key in the PQ. As Adj [A] = { C, D }, work on C and D and update information.

v	C	A	B	D
d[v]	7	8	7	5
pred[v]	C	C	D	C
color[v]	B	B	B	B

PQ : = ϕ

C → A = CA (8)
C → B = CDB (7)
C → C = CC (7)
C → D = CD (5)

(verified the 3rd row of Q_4 obtained from Modified Warshall)

Now taking D as source node

Step 0: Initialization

v	D	A	B	C
d[v]	9	∞	∞	∞
pred[v]	Nil	Nil	Nil	Nil
Color[v]	W	W	W	W

PQ:

v	D	A	B	C
d[v]	9	∞	∞	∞

Step 1: As Adj[D] = {A, B and D}, work on A, B and D and update information.

v	D	A	B	C
d[v]	9	4	2	∞
pred[v]	D	D	D	Nil
color[v]	B	W	W	W

PQ:

v	B	A	C
d[v]	2	4	∞

Step 2: After step 1, B has the minimum key in the PQ. As Adj [B] = {B}, work on B and update information.

v	D	A	B	C
d[v]	9	4	2	∞
pred[v]	D	D	D	Nil
Color[v]	B	W	B	W

PQ:

v	A	C
d[v]	4	∞

Step 3: After step 2, A has the minimum key in the PQ. As Adj[A] = {C, D}, work on C and D and update information.

v	D	A	B	C
d[v]	9	4	2	7
pred[v]	D	D	D	A
Color[v]	B	B	B	W

PQ:

v	C
d[v]	7

Step 4: After step 3, C has the minimum key in the PQ. As Adj[C] = {A, C and D}, work on A, C and D and update information.

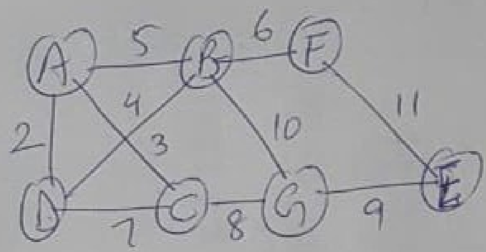
v	D	A	B	C
d[v]	9	4	2	7
pred[v]	D	D	D	A
Color[v]	B	B	B	B

PQ = ∅

D → A = DA (4)
D → B = DB (2)
D → C = DAC (7)
D → D = DD (9)

(verified the 4th row of Q4 obtained from Modified warshall)

Q2.



$$W = \begin{array}{c|ccccccc} & A & B & C & D & E & F & G \\ \hline A & 0 & 5 & 3 & 2 & 0 & 0 & 0 \\ B & 5 & 0 & 0 & 4 & 0 & 6 & 10 \\ C & 3 & 0 & 0 & 7 & 0 & 0 & 8 \\ D & 2 & 4 & 7 & 0 & 0 & 0 & 0 \\ E & 0 & 0 & 0 & 0 & 0 & 11 & 9 \\ F & 0 & 0 & 0 & 0 & 11 & 0 & 0 \\ G & 0 & 0 & 0 & 0 & 9 & 0 & 0 \end{array}$$

$$Q_k[i][j] = \min\left(Q_{k-1}[i][j],\ Q_{k-1}[i][k] + Q_{k-1}[k][j]\right)$$

$Q_0 =$

	A	B	C	D	E	F	G
A	∞	5	3	2	∞	∞	∞
B	5	∞	∞	4	∞	6	10
C	3	∞	∞	7	∞	∞	8
D	2	4	7	∞	∞	∞	∞
E	∞	∞	∞	∞	∞	11	9
F	∞	6	∞	∞	11	∞	∞
G	∞	10	8	∞	9	∞	∞

	A	B	C	D	E	F	G
A	-	$\overline{AB}$	$\overline{AC}$	$\overline{AD}$	-	-	-
B	$\overline{BA}$	-	-	$\overline{BD}$	-	$\overline{BF}$	$\overline{BG}$
C	$\overline{CA}$	-	-	$\overline{CD}$	-	-	$\overline{CG}$
D	$\overline{DA}$	$\overline{DB}$	$\overline{DC}$	-	-	-	-
E	-	-	-	-	-	$\overline{EF}$	$\overline{EG}$
F	-	$\overline{FB}$	-	-	$\overline{FE}$	-	-
G	-	$\overline{GB}$	$\overline{GC}$	-	$\overline{GE}$	-	-

$\boxed{\text{After including node } A\ (k=1)}$

$Q_1 =$

	A	B	C	D	E	F	G
A	∞	5	3	2	∞	∞	∞
B	5	$\underline{10}$	$\underline{8}$	4	∞	6	10
C	3	$\underline{8}$	$\underline{6}$	$\underline{5}$	∞	∞	8
D	2	4	$\underline{5}$	$\underline{4}$	∞	∞	∞
E	∞	∞	∞	∞	∞	11	9
F	∞	6	∞	∞	11	∞	∞
G	∞	10	8	∞	9	∞	∞

	A	B	C	D	E	F	G
A	-	$\overline{AB}$	$\overline{AC}$	$\overline{AD}$	-	-	-
B	$\overline{BA}$	$\overline{BAB}$	$\overline{BAC}$	$\overline{BD}$	-	$\overline{BF}$	$\overline{BG}$
C	$\overline{CA}$	$\overline{CAB}$	$\overline{CAC}$	$\overline{CAD}$	-	-	$\overline{CG}$
D	$\overline{DA}$	$\overline{DB}$	$\overline{DAC}$	$\overline{DAD}$	-	-	-
E	-	-	-	-	-	$\overline{EF}$	$\overline{EG}$
F	-	$\overline{FB}$	-	-	$\overline{FE}$	-	-
G	-	$\overline{GB}$	$\overline{GC}$	-	$\overline{GE}$	-	-

After including node B (k=2)

$$
\begin{array}{c}
\\ A \\ B \\ Q_2 = C \\ D \\ E \\ F \\ G
\end{array}
\begin{array}{c}
\begin{array}{ccccccc} A & B & C & D & E & F & G \end{array} \\
\left[\begin{array}{ccccccc}
\underline{10} & 5 & 3 & 2 & \infty & \underline{11} & \underline{15} \\
5 & 10 & 8 & 4 & \infty & 6 & 10 \\
3 & 8 & 6 & 5 & \infty & \underline{14} & 8 \\
2 & 4 & 5 & 4 & \infty & \underline{10} & \underline{14} \\
\infty & \infty & \infty & \infty & \infty & 11 & 9 \\
\underline{11} & 6 & \underline{14} & \underline{10} & 11 & \underline{12} & \underline{16} \\
\underline{15} & 10 & 8 & \underline{14} & 9 & \underline{16} & \underline{20}
\end{array}\right]
\end{array}
$$

$$
\begin{array}{c}
\\ A \\ B \\ C \\ D \\ E \\ F \\ G
\end{array}
\begin{array}{c}
\begin{array}{ccccccc} A & B & C & D & E & F & G \end{array} \\
\left[\begin{array}{ccccccc}
\overline{ABA} & \overline{AB} & \overline{AC} & \overline{AD} & - & \overline{ABF} & \overline{ABG} \\
\overline{BA} & \overline{BAB} & \overline{BAC} & \overline{BD} & - & \overline{BF} & \overline{BG} \\
\overline{CA} & \overline{CAB} & \overline{CAC} & \overline{CAD} & - & \overline{CABF} & \overline{CAG} \\
\overline{DA} & \overline{DB} & \overline{DAC} & \overline{DAD} & - & \overline{DBF} & \overline{DBG} \\
- & - & - & - & - & \overline{EF} & \overline{EG} \\
\overline{FBA} & \overline{FB} & \overline{FBAC} & \overline{FBD} & \overline{FE} & \overline{FBF} & \overline{FBG} \\
\overline{GBA} & \overline{GB} & \overline{GC} & \overline{GBD} & \overline{GE} & \overline{GBF} & \overline{GBG}
\end{array}\right]
\end{array}
$$

After including node C (k=3)

$$
\begin{array}{c}
\\ A \\ B \\ Q_3 = C \\ D \\ E \\ F \\ G
\end{array}
\begin{array}{c}
\begin{array}{ccccccc} A & B & C & D & E & F & G \end{array} \\
\left[\begin{array}{ccccccc}
\underline{6} & 5 & 3 & 2 & \infty & 11 & \underline{11} \\
5 & 10 & 8 & 4 & \infty & 6 & 10 \\
3 & 8 & 6 & 5 & \infty & 14 & 8 \\
2 & 4 & 5 & 4 & \infty & 10 & \underline{13} \\
\infty & \infty & \infty & \infty & \infty & 11 & 9 \\
11 & 6 & 14 & 10 & 11 & 12 & 16 \\
\underline{11} & 10 & 8 & \underline{13} & 9 & 16 & 20
\end{array}\right]
\end{array}
$$

$$
\begin{array}{c}
\\ A \\ B \\ C \\ D \\ E \\ F \\ G
\end{array}
\begin{array}{c}
\begin{array}{ccccccc} A & B & C & D & E & F & G \end{array} \\
\left[\begin{array}{ccccccc}
\overline{ACA} & \overline{AB} & \overline{AC} & \overline{AD} & - & \overline{ABF} & \overline{ACG} \\
\overline{BA} & \overline{BAB} & \overline{BAC} & \overline{BD} & - & \overline{BF} & \overline{BG} \\
\overline{CA} & \overline{CAB} & \overline{CAC} & \overline{CAD} & - & \overline{CABF} & \overline{CG} \\
\overline{DA} & \overline{DB} & \overline{DAC} & \overline{DAD} & - & \overline{DBF} & \overline{DACG} \\
- & - & - & - & - & \overline{EF} & \overline{EG} \\
\overline{FBA} & \overline{FB} & \overline{FBAC} & \overline{FBD} & \overline{FE} & \overline{FBF} & \overline{FBG} \\
\overline{GCA} & \overline{GB} & \overline{GC} & \overline{GCAD} & \overline{GE} & \overline{GBF} & \overline{GBG}
\end{array}\right]
\end{array}
$$

After including node D (k=4)

$$
\begin{array}{c}
\\ A \\ B \\ Q_4 = C \\ D \\ E \\ F \\ G
\end{array}
\begin{array}{c}
\begin{array}{ccccccc} A & B & C & D & E & F & G \end{array} \\
\left[\begin{array}{ccccccc}
\underline{4} & 5 & 3 & 2 & \infty & 11 & 11 \\
5 & \underline{8} & 8 & 4 & \infty & 6 & 10 \\
3 & 8 & 6 & 5 & \infty & 14 & 8 \\
2 & 4 & 5 & 4 & \infty & 10 & 13 \\
\infty & \infty & \infty & \infty & \infty & 11 & 9 \\
11 & 6 & 14 & 10 & 11 & 12 & 16 \\
11 & 10 & 8 & 13 & 9 & 16 & 20
\end{array}\right]
\end{array}
$$

$$
\begin{array}{c}
\\ A \\ B \\ C \\ D \\ E \\ F \\ G
\end{array}
\begin{array}{c}
\begin{array}{ccccccc} A & B & C & D & E & F & G \end{array} \\
\left[\begin{array}{ccccccc}
\overline{ADA} & \overline{AB} & \overline{AC} & \overline{AD} & - & \overline{ABF} & \overline{ACG} \\
\overline{BA} & \overline{BDB} & \overline{BAC} & \overline{BD} & - & \overline{BF} & \overline{BG} \\
\overline{CA} & \overline{CAB} & \overline{CAC} & \overline{CAD} & - & \overline{CABF} & \overline{CG} \\
\overline{DA} & \overline{DB} & \overline{DAC} & \overline{DAD} & - & \overline{DBF} & \overline{DACG} \\
- & - & - & - & - & \overline{EF} & \overline{EG} \\
\overline{FBA} & \overline{FB} & \overline{FBAC} & \overline{FBD} & \overline{FE} & \overline{FBF} & \overline{FBG} \\
\overline{GCA} & \overline{GB} & \overline{GC} & \overline{GCAD} & \overline{GE} & \overline{GBF} & \overline{GBG}
\end{array}\right]
\end{array}
$$

After including node E ($k=5$)

$$Q5 = \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \\ F \\ G \end{array} \begin{array}{c} \begin{array}{ccccccc} A & B & C & D & E & F & G \end{array} \\ \begin{bmatrix} 4 & 5 & 3 & 2 & \infty & 11 & 11 \\ 5 & 8 & 8 & 4 & \infty & 6 & 10 \\ 3 & 8 & 6 & 5 & \infty & 14 & 8 \\ 2 & 4 & 5 & 4 & \infty & 10 & 13 \\ \infty & \infty & \infty & \infty & \infty & 11 & 9 \\ 11 & 6 & 14 & 10 & 11 & 12 & 16 \\ 11 & 10 & 8 & 13 & 9 & 16 & 20 \end{bmatrix} \end{array}$$

	A	B	C	D	E	F	G
A	Same as Q4.						
B							
C							
D							
E							
F							
G							

After including node F ($k=6$)

$$Q6 = \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \\ F \\ G \end{array} \begin{array}{c} \begin{array}{ccccccc} A & B & C & D & E & F & G \end{array} \\ \begin{bmatrix} 4 & 5 & 3 & 2 & \underline{20} & 11 & 11 \\ 5 & 8 & 8 & 4 & \underline{17} & 6 & 10 \\ 3 & 8 & 6 & 5 & \underline{25} & 14 & 8 \\ 2 & 4 & 5 & 4 & \underline{21} & 10 & 13 \\ \underline{22} & \underline{17} & \underline{25} & \underline{21} & \underline{22} & 11 & 9 \\ 11 & 6 & 14 & 10 & 11 & 12 & 16 \\ 11 & 10 & 8 & 13 & 9 & 16 & 20 \end{bmatrix} \end{array}$$

	A	B	C	D	E	F	G
A					$\overline{ABFE}$		
B					$\overline{BFE}$		
C					$\overline{CABFE}$		
D					$\overline{DBFE}$		
E	$\overline{EFBA}$	$\overline{EFB}$	$\overline{EFBAC}$	$\overline{EFBD}$	$\overline{EFE}$		
F							
G							

After including node G ($k=7$)

$$Q7 = \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \\ F \\ G \end{array} \begin{array}{c} \begin{array}{ccccccc} A & B & C & D & E & F & G \end{array} \\ \begin{bmatrix} 4 & 5 & 3 & 2 & 20 & 11 & 11 \\ 5 & 8 & 8 & 4 & 17 & 6 & 10 \\ 3 & 8 & 6 & 5 & \underline{17} & 14 & 8 \\ 2 & 4 & 5 & 4 & 21 & 10 & 13 \\ \underline{20} & 17 & \underline{17} & 21 & \underline{18} & 11 & 9 \\ 11 & 6 & 14 & 10 & 11 & 12 & 16 \\ 11 & 10 & 8 & 13 & 9 & 16 & 20 \end{bmatrix} \end{array}$$

	A	B	C	D	E	F	G
A	$\overline{ADA}$	$\overline{AB}$	$\overline{AC}$	$\overline{AD}$	$\overline{ACGE}$	$\overline{ABF}$	$\overline{ACG}$
B	$\overline{BA}$	$\overline{BDB}$	$\overline{BAC}$	$\overline{BD}$	$\overline{BFE}$	$\overline{BF}$	$\overline{BG}$
C	$\overline{CA}$	$\overline{CAB}$	$\overline{CAC}$	$\overline{CAD}$	$\overline{CGE}$	$\overline{CABF}$	$\overline{CG}$
D	$\overline{DA}$	$\overline{DB}$	$\overline{DAC}$	$\overline{DAD}$	$\overline{DBFE}$	$\overline{DBF}$	$\overline{DACG}$
E	$\overline{EGCA}$	$\overline{EFB}$	$\overline{EGC}$	$\overline{EFBD}$	$\overline{EGE}$	$\overline{EF}$	$\overline{EG}$
F	$\overline{FBA}$	$\overline{FB}$	$\overline{FABC}$	$\overline{FBD}$	$\overline{FE}$	$\overline{FBF}$	$\overline{FBG}$
G	$\overline{GCA}$	$\overline{GB}$	$\overline{GC}$	$\overline{GCAD}$	$\overline{GE}$	$\overline{GBF}$	$\overline{GBG}$

By Dijkstra's Algorithm
(Taking A as source node)

Step 0: Initialization.

V	A	B	C	D	E	F	G
~~pred~~ d[v]	0	∞	∞	∞	∞	∞	∞
pred[v]	Nil	N	N	N	N	N	N
color[v]	W	W	W	W	W	W	W

PQ:

V	A	B	C	D	E	F	G
d[v]	0	∞	∞	∞	∞	∞	∞

Step 1: As Adj[A] = {B, C, D}, so:

V	A	B	C	D	E	F	G
d[v]	0	5	3	2	∞	∞	∞
pred[v]	N	A	A	A	N	N	N
color[v]	B	W	W	W	W	W	W

if v.d > u.d + w(u,v)
update v.d

PQ:

V	D	C	B	E	F	G
d[v]	2	3	5	∞	∞	∞

Step 2: As Adj[D] = {A, B, C}, so:

V	A	B	C	D	E	F	G
d[v]	0	5	3	2	∞	∞	∞
pred[v]	N	A	A	A	N	N	N
color[v]	B	W	W	B	W	W	W

PQ:

V	C	B	E	F	G
d[v]	3	5	∞	∞	∞

Step 3: As Adj[C] = {A, D, G}, so:

v	A	B	C	D	E	F	G
d[v]	0	5	3	2	∞	∞	11
pred[v]	N	A	A	A	N	N	C
Color[v]	B	W	B	B	W	W	W

PQ:

v	B	G	E	F
d[v]	5	11	∞	∞

Step 4: As Adj[B] = {A, D, F, G}, so:

v	A	B	C	D	E	F	G
d[v]	0	5	3	2	∞	11	11
pred[v]	N	A	A	A	N	B	C
color[v]	B	B	B	B	W	W	W

PQ:

v	F	G	E
d[v]	11	11	∞

Step 5: As Adj[F] = {B, E}, so:

v	A	B	C	D	E	F	G
d[v]	0	5	3	2	22	11	11
pred[v]	N	A	A	A	F	B	C
color[v]	B	B	B	B	W	B	W

PQ:

v	G	E
d[v]	11	22

Step 6: As Adj[G] = {B, C, E}, so:

v	A	B	C	D	E	F	G
d[v]	0	5	3	2	20	11	11
pred[v]	N	A	A	A	G	B	C
color[v]	B	B	B	B	W	B	B

PQ:

v	E
d[v]	20

Step 7: As Adj[E] = {F, G}, so:

v	A	B	C	D	E	F	G
d[v]	0	5	3	2	20	11	11
pred[v]	N	A	A	A	G	B	C
color[v]	B	B	B	B	B	B	B

PQ: $\emptyset$

$A \to B = \overline{AB}$ (5)
$A \to C = \overline{AC}$ (3)
$A \to D = \overline{AD}$ (2)
$A \to E = \overline{ACGE}$ (20)
$A \to F = \overline{ABF}$ (11)
$A \to G = \overline{ACG}$ (11)

(Verified the 1st row of S_7 matrix obtained from Modified Warshall Algorithm)

Taking B as source node

Step 0: Initialization

v	B	A	C	D	E	F	G
d[v]	0	∞	∞	∞	∞	∞	∞
pred[v]	N	N	N	N	N	N	N
color[v]	W	W	W	W	W	W	W

PQ:

v	B	A	C	D	E	F	G
d[v]	0	∞	∞	∞	∞	∞	∞

Step 1: As Adj[B] = {A, D, F, G}, so:

v	B	A	C	D	E	F	G
d[v]	0	5	∞	4	∞	6	10
pred[v]	N	B	N	B	N	B	B
color[v]	B	W	W	W	W	W	W

PQ:

v	D	A	F	G	C	E
d[v]	4	5	6	10	∞	∞

Step 2: As Adj[D] = {A, B, C}, so:

v	B	A	C	D	E	F	G
d[v]	0	5	11	4	∞	6	10
pred[v]	N	B	D	B	N	B	B
color[v]	B	W	W	B	W	W	W

PQ:

v	A	F	G	C	E
d[v]	5	6	10	11	∞

Step 3: As Adj[A] = {B, C, D}, so:

v	B	A	C	D	E	F	G
d[v]	0	5	8	4	∞	6	10
pred[v]	N	B	A	B	N	B	B
color[v]	B	B	W	B	W	W	W

PQ:

v	F	C	G	E
d[v]	6	8	10	∞

step 4: As Adj[F] = {B, E}, so:

v	B	A	C	D	E	F	G
d[v]	0	5	8	4	17	6	10
pred[v]	N	B	A	B	F	B	B
color[v]	B	B	W	B	W	B	W

PQ:

v	C	G	E
d[v]	8	10	17

step 5: As Adj[C] = {A, D, G}, so:

v	B	A	C	D	E	F	G
d[v]	0	5	8	4	17	6	10
pred[v]	N	B	A	B	F	B	B
color[v]	B	B	B	B	W	B	W

PQ:

v	G	E
d[v]	10	17

step 6: As Adj[G] = {B, C, E}, so:

v	B	A	C	D	E	F	G
d[v]	0	5	8	4	17	6	10
pred[v]	N	B	A	B	F	B	B
color[v]	B	B	B	B	W	B	B

PQ:

v	E
d[v]	17

Step 7: As Adj[E] = {F, G}, so:

V	B	A	C	D	E	F	G
d[v]	0	5	8	4	17	6	10
pred[v]	N	B	A	B	F	B	B
Color[v]	B	B	B	B	B	B	B

PQ = ϕ

(verified the 2nd row of matrix Q_4 obtained from Modified Warshall Algorithm)

$B \to A = \overline{BA}$ (5)
$B \to C = \overline{BAC}$ (8)
$B \to D = \overline{BD}$ (4)
$B \to E = \overline{BFE}$ (17)
$B \to F = \overline{BF}$ (6)
$B \to G = \overline{BG}$ (10)

Taking C as source node

Step 0: Initialization.

V	C	A	B	D	E	F	G
d[v]	0	∞	∞	∞	∞	∞	∞
pred[v]	N	N	N	N	N	N	N
Color[v]	W	W	W	W	W	W	W

PQ:

V	C	A	B	D	E	F	G
d[v]	0	∞	∞	∞	∞	∞	∞

Step 1: As Adj[C] = {A, D, G}, so:

V	C	A	B	D	E	F	G
d[v]	0	3	∞	8	∞	∞	8
pred[v]	N	C	N	C	N	N	C
Color[v]	B	W	W	W	W	W	W

PQ:

V	A	D	G	B	E	F
d[v]	3	8	8	∞	∞	∞

Step 2: As Adj[A] = {B, C, D}, so:

V	C	A	B	D	E	F	G
d[v]	0	3	8	5	∞	∞	8
pred[v]	N	C	A	A	N	N	C
color[v]	B	B	W	W	W	W	W

PQ:

v	D	B	G	E	F
d[v]	5	8	8	∞	∞

Step 3: As Adj[D] = {A, B, C}, so:

V	C	A	B	D	E	F	G
d[v]	0	3.	8	5	∞	∞	8
pred[v]	N	C	A	A	N	N	C
Color[v]	B	B	W	B	W	W	W

PQ:

v	B	G	E	F
d[v]	8	8	∞	∞

Step 4: As Adj[B] = {A, F, G}, so:

V	C	A	B	D	E	F	G
d[v]	0	3	8	5	∞	14	8
pred[v]	N	C	A	A	N	B	G
color[v]	B	B	B	B	W	W	W

PQ:

v	G	F	E
d[v]	8	13	∞

Step 5: As Adj[G] = {B, E}, so:

V	C	A	B	D	E	F	G
d[v]	0	3	8	5	17	14	8
pred[v]	N	C	A	A	G	B	G
color[v]	B	B	B	B	W	W	B

PQ:

V	F	E
d[v]	13	17

Step 6: As Adj[F] = {B and E}, so:

V	C	A	B	D	E	F	G
d[v]	0	3	8	5	17	14	8
pred[v]	N	C	A	A	G	B	G
color[v]	B	B	B	B	W	B	B

PQ:

V	E
d[v]	17

Step 7: As Adj[E] = {F, G}, so:

V	C	A	B	D	E	F	G
d[v]	0	3	8	5	17	14	8
pred[v]	N	C	A	A	G	B	G
color[v]	B	B	B	B	B	B	B

PQ: $\emptyset$

$C \to A = \overline{CA}$ (3)
$C \to B = \overline{CAB}$ (8)
$C \to D = \overline{CAD}$ (5)
$C \to E = \overline{CGE}$ (17)
$C \to F = \overline{CABF}$ (14)
$C \to G = \overline{CG}$ (8)

(Verified the 3rd row of Q7 matrix obtained from Modified Warshall Algorithm)

Taking D as source node

Step 0: Initialization.

V	D	A	B	C	E	F	G
d[v]	0	∞	∞	∞	∞	∞	∞
p[v]	N	N	N	N	N	N	N
c[v]	W	W	W	W	W	W	W

PQ:

V	D	A	B	C	E	F	G
d[v]	0	∞	∞	∞	∞	∞	∞

Step 1: As Adj[D] = {A, B, C}, so:

v	D	A	B	C	E	F	G
d[v]	0	2	4	7	∞	∞	∞
p[v]	N	D	D	D	N	N	N
c[v]	B	W	W	W	W	W	W

PQ:

v	A	B	C	E	F	G
d[v]	2	4	7	∞	∞	∞

Step 2: As Adj[A] = {B, C, D}, so:

v	D	A	B	C	E	F	G
d[v]	0	2	4	5	∞	∞	∞
p[v]	N	D	D	A	N	N	N
c[v]	B	B	W	W	W	W	W

PQ:

v	B	C	E	F	G
d[v]	4	5	∞	∞	∞

Step 3: As Adj[B] = {A, D, F, G}, so:

v	D	A	B	C	E	F	G
d[v]	0	2	4	5	∞	10	14
p[v]	N	D	D	A	N	B	B
c[v]	B	B	B	W	W	W	W

PQ:

v	C	F	G	E
d[v]	5	10	14	∞

Step 4: As Adj[C] = {A, D, G}, so:

v	D	A	B	C	E	F	G
d[v]	0	2	4	5	∞	10	13
p[v]	N	D	D	A	N	B	C
c[v]	B	B	B	B	W	W	W

PQ:

v	F	G	E
d[v]	10	13	∞

Step 5: As Adj[F] = {B, and E}, so:

v	D	A	B	C	E	F	G
d[v]	0	2	4	5	21	10	13
p[v]	N	D	D	A	F	B	C
c[v]	B	B	B	B	W	B	W

PQ:

v	G	E
d[v]	13	21

Step 6: As Adj[G] = {B, C, E}, so:

v	D	A	B	C	E	F	G
d[v]	0	2	4	5	21	10	13
p[v]	N	D	D	A	F	B	C
c[v]	B	B	B	B	B	B	B

PQ:

v	E
d[v]	21

Step 7: As Adj[E] = {F, G}, so:

v	D	A	B	C	E	F	G
d[v]	0	2	4	5	21	10	13
p[v]	N	D	D	A	F	B	C
c[v]	B	B	B	B	B	B	B

PQ = ∅

D→A = $\overline{DA}$ (2)
D→B = $\overline{DB}$ (4)
D→C = $\overline{DAC}$ (5)
D→E = $\overline{DBFE}$ (21)
D→F = $\overline{DBF}$ (10)
D→G = $\overline{DACG}$ (13)

(verified the 4th row of Q_7 matrix)

Taking E as source node

Step 0: Initialization:

v	E	A	B	C	D	F	G
d[v]	0	∞	∞	∞	∞	∞	∞
p[v]	N	N	N	N	N	N	N
c[v]	W	W	W	W	W	W	W

PQ:

v	E	A	B	C	D	F	G
d[v]	0	∞	∞	∞	∞	∞	∞

Step 1: As Adj[E] = {F, G}, so:

v	E	A	B	C	D	F	G
d[v]	0	∞	∞	∞	∞	11	9
p[v]	N	N	N	N	N	E	E
c[v]	B	W	W	W	W	W	W

PQ:

v	G	F	A	B	C	D
d[v]	9	11	∞	∞	∞	∞

Step 2: As Adj⁻[G] = {B, C, E}, so:

V	E	A	B	C	D	F	G
d[V]	0	∞	19	17	∞	11	9
P[V]	N	N	G	G	N	E	E
c[V]	B	W	W	W	W	W	B

PQ:

V	F	C	B	A	D
d[V]	11	17	19	∞	∞

Step 3: As Adj⁻[F] = {B, E}, so:

V	E	A	B	C	D	F	G
d[V]	0	∞	17	17	∞	11	9
P[V]	N	N	F	G	N	E	E
c[V]	B	W	W	W	W	B	B

PQ:

V	B	C	A	D
d[V]	17	17	∞	∞

Step 4: As Adj⁻[B] = {A, D, F}, so:

V	E	A	B	C	D	F	G
d[V]	0	22	17	17	21	11	9
P[V]	N	B	F	G	B	E	E
c[V]	B	W	B	W	W	B	B

PQ:

V	C	D	A
d[V]	17	21	22

Step 5: As Adj⁻[C] = {A, D, G}, so:

V	E	A	B	C	D	F	G
d[V]	0	20	17	17	21	11	9
P[V]	N	C	F	G	B	E	E
c[V]	B	W	B	B	W	B	B

PQ:

V	A	D
d[V]	20	21

Step 6: As Adj⁻[A] = {B, C, D}, so:

V	E	A	B	C	D	F	G
d[V]	0	20	17	17	21	11	9
P[V]	N	C	F	G	B	E	E
c[V]	B	B	B	B	W	B	B

PQ:

V	D
d[V]	21

Step 7: As Adj[D] = {A, B, C}, so:

v	E	A	B	C	D	F	G
d[v]	0	20	17	17	21	11	9
p[v]	N	C	F	G	B	E	E
c[v]	B	B	B	B	B	B	B

PQ: ∅

$E \to A = \overline{EGCA}$ (20)
$E \to B = \overline{EFB}$ (17)
$E \to C = \overline{EGC}$ (17)
$E \to D = \overline{EFBD}$ (21)
$E \to F = \overline{EF}$ (11)
$E \to G = \overline{EG}$ (11)

(verified the 5th row of Q7 matrix)

Taking F as source node

Step 0: Initialization:

v	F	A	B	C	D	E	G
d[v]	0	∞	∞	∞	∞	∞	∞
p[v]	N	N	N	N	N	N	N
c[v]	W	W	W	W	W	W	W

PQ:

v	F	A	B	C	D	E	G
d[v]	0	∞	∞	∞	∞	∞	∞

Step 1: As Adj[F] = {B, E}, so:

v	F	A	B	C	D	E	G
d[v]	0	∞	6	∞	∞	11	∞
p[v]	N	N	F	N	N	F	N
c[v]	B	W	W	W	W		W

PQ:

v	B	E	A	C	D	G
d[v]	6	11	∞	∞	∞	∞

Step 2: As Adj[B] = {A, D, F, G}, so:

v	F	A	B	C	D	E	G
d[v]	0	11	6	∞	10	11	16
p[v]	N	B	F	N	B	F	B
c[v]	B	W	B	W	W	W	W

PQ:

v	D	A	E	G	C
d[v]	10	11	11	16	∞

Step 3: As Adj⁻[D] = {A, B, C}, so:

v	F	A	B	C	D	E	G
d[v]	0	11	6	17	10	11	16
p[v]	N	B	F	D	B	F	B
c[v]	B	W	B	W	B	W	W

PQ:

v	A	E	G	C
d[v]	11	11	16	17

Step 4: As Adj⁻[A] = {B, C, D}, so:

v	F	A	B	C	D	E	G
d[v]	0	11	6	14	10	11	16
p[v]	N	B	F	A	B	F	B
c[v]	B	B	B	W	B	W	W

PQ:

v	E	C	G
d[v]	11	14	16

Step 5: As Adj⁻[E] = {F, G}, so:

v	F	A	B	C	D	E	G
d[v]	0	11	6	14	10	11	16
p[v]	N	B	F	A	B	F	B
c[v]	B	B	B	W	B	B	W

PQ:

v	C	G
d[v]	14	16

Step 6: As Adj⁻[C] = {A, D, G}, so:

v	F	A	B	C	D	E	G
d[v]	0	11	6	14	10	11	16
p[v]	N	B	F	A	B	F	B
c[v]	B	B	B	B	B	B	W

PQ:

v	G
d[v]	16

Step 7: As Adj⁻[G] = {B, C, E}, so:

PQ = ϕ

v	F	A	B	C	D	E	G
d[v]	0	11	6	14	10	11	16
p[v]	N	B	F	A	B	F	B
c[v]	B	B	B	B	B	B	B

(Verified the 6th row of Q7 matrix)

$F \to A = \overline{FBA}$ (11)
$F \to B = \overline{FB}$ (6)
$F \to C = \overline{FABC}$ (14)
$F \to D = \overline{FBD}$ (10)
$F \to E = \overline{FE}$ (11)
$F \to G = \overline{FBG}$ (16)

Taking G as source node

Step 0: Initialization

v	G	A	B	C	D	E	F
d[v]	0	∞	∞	∞	∞	∞	∞
p[v]	N	N	N	N	N	N	N
c[v]	W	W	W	W	W	W	W

PQ:

v	G	A	B	C	D	E	F
d[v]	0	∞	∞	∞	∞	∞	∞

Step 1: As Adj[G] = {B, C, E}, so:

v	G	A	B	C	D	E	F
d[v]	0	∞	10	8	∞	9	∞
p[v]	N	N	G	G	N	G	N
c[v]	B	W	W	W	W	W	W

PQ:

v	C	E	B	A	D	F
d[v]	8	9	10	∞	∞	∞

Step 2: As Adj[C] = {A, D, G}, so:

v	G	A	B	C	D	E	F
d[v]	0	11	10	8	17	9	∞
p[v]	N	C	G	G	C	G	N
c[v]	B	W	W	B	W	W	W

PQ:

v	E	B	A	D	F
d[v]	9	10	11	17	∞

Step 3: As Adj[E] = {F, G}, so:

v	G	A	B	C	D	E	F
d[v]	0	11	10	8	17	9	20
p[v]	N	C	G	G	C	G	E
c[v]	B	W	W	B	W	B	W

PQ:

v	B	A	D	F
d[v]	10	11	17	20

Step 4: As Adj[B] = {A, D, F, G}, so:

v	G	A	B	C	D	E	F
d[v]	0	11	10	8	14	9	16
p[v]	N	C	G	G	B	G	B
c[v]	B	W	B	B	W	B	W

PQ:

v	A	D	F
d[v]	11	14	16

Step 5: As Adj[A] = {B, C, D}, so:

v	G	A	B	C	D	E	F
d[v]	0	11	10	8	13	9	16
p[v]	N	C	G	G	A	G	B
c[v]	B	B	B	B	W	B	W

PQ:

v	D	F
d[v]	13	16

Step 6: As Adj[D] = {A, B, C}, so:

v	G	A	B	C	D	E	F
d[v]	0	11	10	8	13	9	16
p[v]	N	C	G	G	A	G	B
c[v]	B	B	B	B	B	B	W

PQ:

v	F
d[v]	16

Step 7: As Adj[F] = {B, E}, so:

v	G	A	B	C	D	E	F
d[v]	0	11	10	8	13	9	16
p[v]	N	C	G	G	A	G	B
c[v]	B	B	B	B	B	B	B

PQ: ϕ

$G \to A = \overline{GCA}$ (11)

$G \to B = \overline{GB}$ (10)

$G \to C = \overline{GC}$ (8)

$G \to D = \overline{GCAD}$ (13)

$G \to E = \overline{GE}$ (9)

$G \to F = \overline{GBF}$ (16)

(verified the 7th row of Q7 matrix)

Q3. i) Shortest path from A → E.

v	A	B	C	D	E	F	G
A	$\boxed{0_A}$	5_A	3_A	2_A	∞	∞	∞
D		5_A	3_A	$\boxed{2_A}$	∞	∞	∞
C		5_A	$\boxed{3_A}$		∞	∞	11_C
B		$\boxed{5_A}$			∞	11_B	11_C
F					22_F	$\boxed{11_B}$	11_C
G					20_G		$\boxed{11_C}$
E					$\boxed{20_G}$		

Shortest path from A → E = $\overline{ACGE}$ (20) (same as obtained in Q7 and by Dijkstra algorithm).

ii) Shortest path from E → D.

v	E	A	B	C	D	F	G
E	$\boxed{0_E}$	∞	∞	∞	∞	11_E	9_E
G		∞	19_G	17_G	∞	11_E	$\boxed{9_E}$
F		∞	17_F	17_G	∞	$\boxed{11_E}$	
B		22_B	$\boxed{17_F}$	17_G	21_B		
C		20_C		$\boxed{17_G}$	21_B		
A		$\boxed{20_C}$			21_B		
D					$\boxed{21_B}$		

Shortest path from E → D = $\overline{EFBD}$ (21) (same as in Q7 & Dijkstra)

iii) Shortest path from C → F.

V	C	A	B	D	E	F	G
C	$\boxed{0_C}$	3_C	∞	7_C	∞	∞	8_C
A		$\boxed{3_C}$	8_A	5_A	∞	∞	8_C
D			8_A	$\boxed{5_A}$	∞	∞	8_C
B			$\boxed{8_A}$		∞	14_B	8_C
G					17_G	14_B	$\boxed{8_C}$
F					17_G	$\boxed{14_B}$	

Shortest path from C → F = $\overline{CABF}$ (14)
(Same as obtained in Q7 and Dijkstra algorithm)

(iv) Shortest path from G → D.

V	G	A	B	C	D	E	F
G	$\boxed{0_G}$	∞	10_G	8_G	∞	9_G	∞
C		11_C	10_G	$\boxed{8_G}$	15_C	9_G	∞
E		11_C	10_G		15_C	$\boxed{9_G}$	20_E
B		11_C	$\boxed{10_G}$		15_C		16_B
A		$\boxed{11_C}$			13_A		16_B
D					$\boxed{13_A}$		

Shortest path from G → D = $\overline{GCAD}$ (13)
(Same as obtained in Q7 and Dijkstra algorithm)

Q4. MST from PRIM's Algorithm

Initialization

u	A	B	C	D	E	F	G
Key[u]	∞	∞	∞	∞	∞	∞	∞
π[u]	N	N	N	N	N	N	N

Q	A B C D E F G
A Minimum (Spanning Tree)	Empty

1. (u = A)

u	A	B	C	D	E	F	G
Key[u]	0	5	3	2	∞	∞	∞
π[u]	N	A	A	A	N	N	N

Q	B C D E F G
A	A

2. (u = D)

u	A	B	C	D	E	F	G
Key[u]	0	4	3	2	∞	∞	∞
π[u]	N	D	A	A	N	N	N

Q	B C E F G
A	A D

3. (u = C)

u	A	B	C	D	E	F	G
Key[u]	0	4	3	2	∞	∞	8
π[u]	N	D	A	A	N	N	C

Q	B E F G
A	A D C

4. (u = B)

u	A	B	C	D	E	F	G
Key[u]	0	4	3	2	∞	6	8
π[u]	N	D	A	A	N	B	C

Q	E F G
A	A D C B

5. (u = F)

u	A	B	C	D	E	F	G
Key[u]	0	4	3	2	11	6	8
π[u]	N	D	A	A	F	B	C

Q	E G
A	A D C B F

6. (u = G)

u	A	B	C	D	E	F	G
key[u]	0	4	3	2	9	6	8
π[u]	N	D	A	A	G	B	C

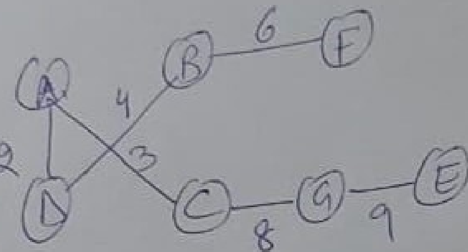
Q	E
A	A D C B F G

7. (u = E)

u	A	B	C	D	E	F	G
key[u]	0	4	3	2	9	6	8
	N	D	A	A	G	B	C

Q	empty
A	A D C B F G E

MST



$2+3+4+6+8+9$
$= 31$

By Kruskal's Algorithm

Sorting Edges in ascending order

A B C D E F G

Edges	weights
AD	2
AC	3
BD	4
AB	5
BF	6
CG	8
GE	9
BG	10
EF	11

✓ 1. Edge $\overline{AD}$ is taken, A and B are in separate sets.
$A = \{(A,D)\}$ AD B C E F G

✓ 2. Edge $\overline{AC} = 3$ is taken, A and C are in separate sets.
$A = \{(A,D),(A,C)\}$ ACD B E F G

✓ 3. Edge $\overline{BD} = 4$ is taken, B and D are in separate sets
$A = \{(A,D),(A,C),(B,D)\}$ ABCD E F G

X 4. Edge $\overline{AB} = 5$ is taken, A and B are in the same set
$A = \{(A,D),(A,C),(B,D)\}$ ABCD E F G

5. Edge $\overline{BF}=6$ is taken, B and F are in separate sets.

$A = \{(A,D),(A,C),(B,D),(B,F)\}$ ABCDF E G

6. Edge $\overline{CG}=8$ is taken, C and G are in separate sets.

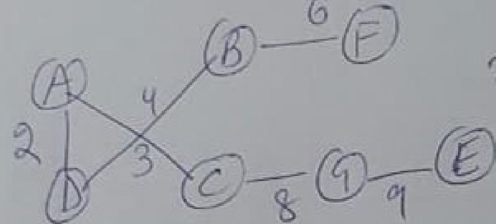
$A = \{(A,D),(A,C),(B,D),(B,F),(C,G)\}$ ABCDFG E

7. Edge $\overline{GE}=9$ is taken, G and E are in separate sets.

$A = \{(A,D),(A,C),(B,D),(B,F),(C,G),(G,E)\}$ ABCDEFG

No need to see further edges b/c can not be included.

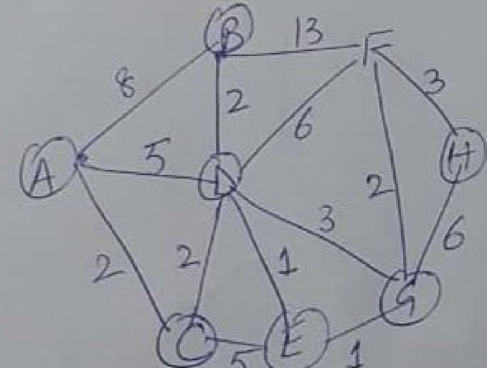
MST



$2+3+4+6+8+9=31$

verified the MST obtained from PRIM'S Algorithm

Q5.



By PRIM'S Algorithm

Initialization

u	A	B	C	D	E	F	G	H
key[u]	∞	∞	∞	∞	∞	∞	∞	∞
π[u]	N	N	N	N	N	N	N	N

Q	A B C D E F G H
A	Empty

1. (u = A)

u	A	B	C	D	E	F	G	H
key[u]	0	8	2	5	∞	∞	∞	∞
π[u]	N	A	A	A	N	N	N	N

Q	B C D E F G H
A (MST)	A

2. (u = C)

u	A	B	C	D	E	F	G	H
key[u]	0	8	2	2	5	∞	∞	∞
π[u]	N	A	A	C	C	N	N	N

Q	B D E F G H
A	A C

3. (u = D)

u	A	B	C	D	E	F	G	H
key[u]	0	2	2	2	1	6	3	∞
	N	D	A	C	D	D	D	N

Q	B E F G H
A	A C D

4. (u = E)

u	A	B	C	D	E	F	G	H
key[u]	0	2	2	2	1	6	1	∞
π[u]	N	D	A	C	D	D	E	N

⇓

[D will not be updated although ED=1
b/c D is not available in Queue]

Q	B F G H
A	A C D E

5. (u = G)

u	A	B	C	D	E	F	G	H
key[u]	0	2	2	2	1	2	1	6
π[u]	N	D	A	C	D	G	E	G

Q	B F H
A	A C D E G

6. (u = B)

u	A	B	C	D	E	F	G	H
key[u]	0	2	2	2	1	2	1	6
π[u]	N	D	A	C	D	G	E	G

Q	F H
A	A B C D E G

7. (u = F)

u	A	B	C	D	E	F	G	H
key[u]	0	2	2	2	1	2	1	3
π[u]	N	D	A	C	D	G	E	F

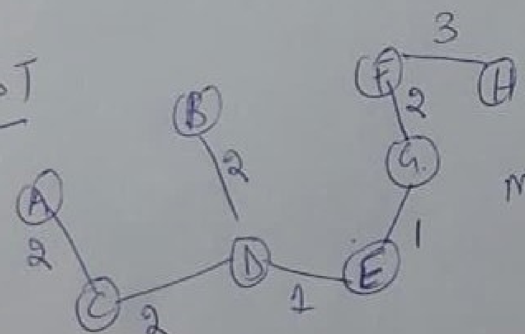
Q	H
A	A B C D E G F

8. (u = H)

u	A	B	C	D	E	F	G	H
key[u]	0	2	2	2	1	2	1	3
π[u]	N	D	A	C	D	G	E	F

Q	empty
A	A B C D E G F H

MST



MST = 1 + 1 + 2 + 2 + 2 + 2 + 3 = 13

By Kruskal's Algorithm
Sorting edges in ascending order

Edges	DE	EG	AC	CD	BD	FG	FH	DG	AD	CE	GH	DF	AB	BF
Weights	1	1	2	2	2	2	3	3	5	5	6	6	8	13

A B C D E F G H

1. Edge $\overline{DE} = 1$ is taken, D and E are in separate sets.

$A = \{(D,E)\}$ A B C DE F G H

2. Edge $\overline{EG} = 1$ is taken, E and G are in separate sets.

$A = \{(D,E), (E,G)\}$ A B C DEG F H

3. Edge $\overline{AC} = 2$ is taken, A and C are in separate sets.

$A = \{(D,E), (E,G), (A,C)\}$ AC B DEG F H

4. Edge $\overline{CD} = 2$ is taken, A and D are in separate sets.

$A = \{(D,E), (E,G), (A,C), (C,D)\}$ ACDEG B F H

5. Edge $\overline{BD} = 2$ is taken, B and D are in separate sets.

$A = \{(D,E), (E,G), (A,C), (C,D), (B,D)\}$ ABCDEG F H

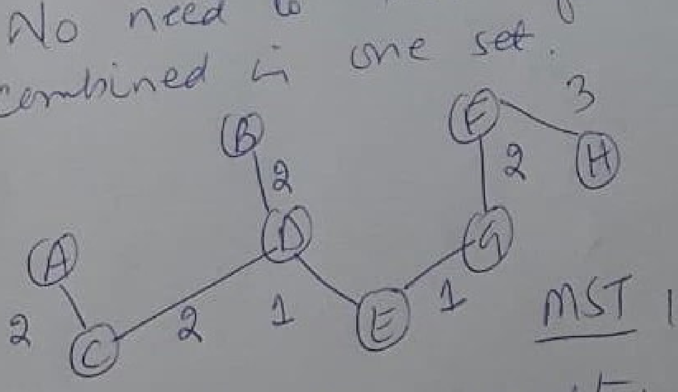
6. Edge $FG = 2$ is taken, F and G are in separate sets.

$A = \{(D,E), (E,G), (A,C), (C,D), (B,D), (F,G)\}$ ABCDEFG H

7. Edge $\overline{FH} = 3$ is taken, F and H are in separate sets.

$A = \{(D,E), (E,G), (A,C), (C,D), (B,D), (F,G), (F,H)\}$ ABCDEFGH

No need to take further edges b/c all are combined in one set.



MST $1+1+2+2+2+2+3 = 13$

(Verified the MST obtained from PRIM's Algorithm)