

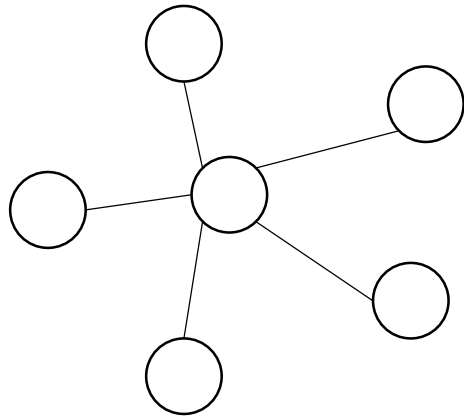
# Kruskal's Algorithm for Computing MSTs

Presented by: Raj Kumar Ranabhat  
M.E in Computer Engineering(I/I)  
Kathmandu University

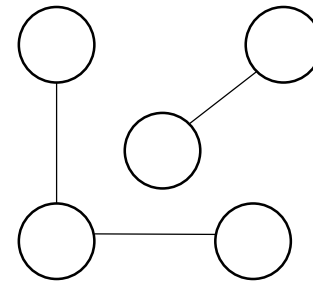
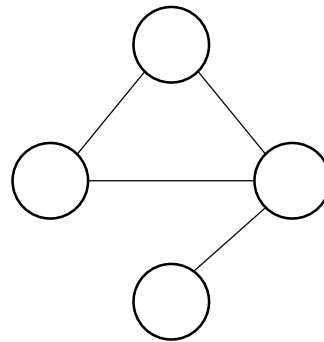
# Tree

A tree is a graph with the following properties:

- The graph is connected (can go from anywhere to anywhere)
- There are no cycles (acyclic)



Tree



Graphs that are not trees

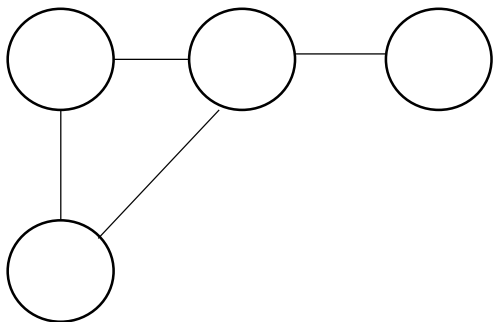
# Minimum Spanning Tree (MST)

Let  $G=(V,E)$  be an undirected connected graph.

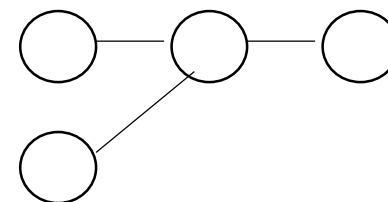
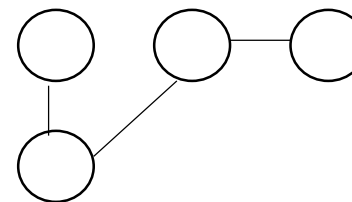
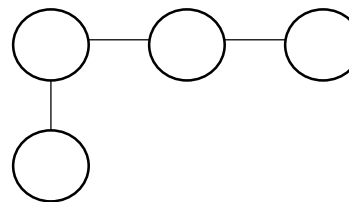
A sub graph  $T=(V,E')$  of  $G$  is a spanning tree of  $G$  if :-

- $T$  is a tree (i.e., it is acyclic)
- $T$  covers all the vertices  $V$ 
  - contains  $|V| - 1$  edges
- $T$  has minimum total weight
- A single graph can have many different spanning trees.

Connected undirected graph

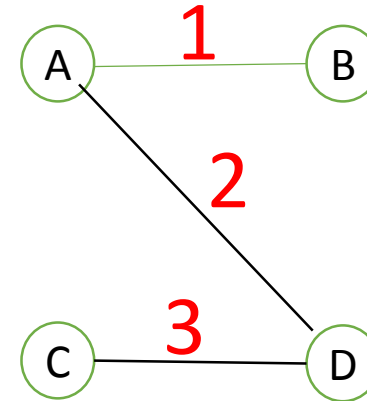
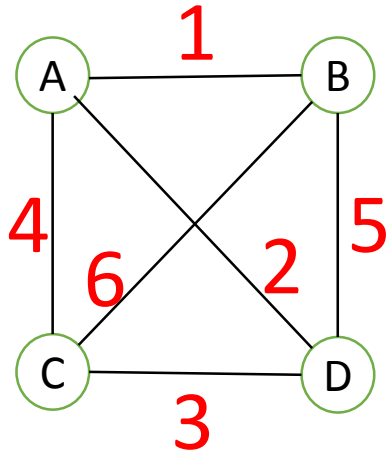


Spanning Tree



# Kruskal's Algorithm

- It is a algorithm used to find a minimum cost spanning tree for connected weighted undirected graph
- This algorithm first appeared in Proceedings of the American Mathematical Society in 1956, and was written by Joseph Kruskal



Connected Weighted

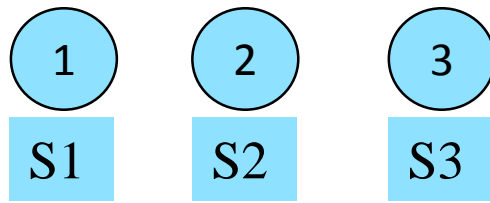
- It's a spanning tree because it connects all vertices without loops.
- Tree weight is minimum of all possibilities hence minimum cost spanning tree

# Disjoint Sets

- A disjoint set is a data structure which keeps track of all elements that are separated by a number of disjoint (not connected) subsets.
- It supports three useful operations
  - MAKE-SET( $x$ ): Make a new set with a single element  $x$
  - UNION ( $S_1, S_2$ ): Merge the set  $S_1$  and set  $S_2$
  - FIND-SET( $x$ ): Find the set containing the element  $x$

# Disjoint Sets

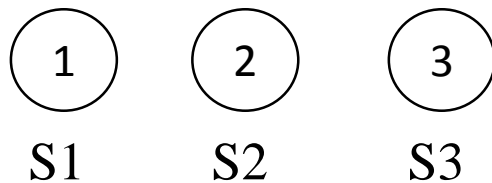
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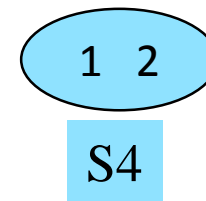
MAKE-SET(1), MAKE-SET(2), MAKE-SET(3) creates new set S1,S2,S3

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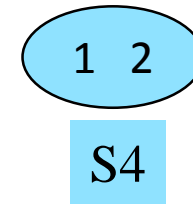
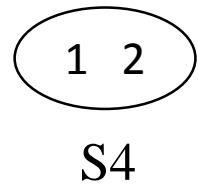
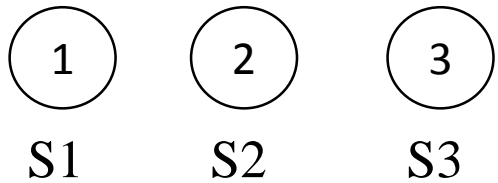


UNION (1,2) merge set S1 and set S2 to create set S4



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FIND-SET(2) returns set S4

# Kruskal's Algorithm

KRUSKAL(V,E):

$A = \emptyset$

**foreach**  $v \in V$ :

    MAKE-SET( $v$ )

Sort E by weight increasingly

**foreach**  $(v_1, v_2) \in E$ :

**if** FIND-SET( $v_1$ )  $\neq$  FIND-SET( $v_2$ ):

$A = A \cup \{(v_1, v_2)\}$

        UNION( $v_1, v_2$ )

**else**

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**return** A

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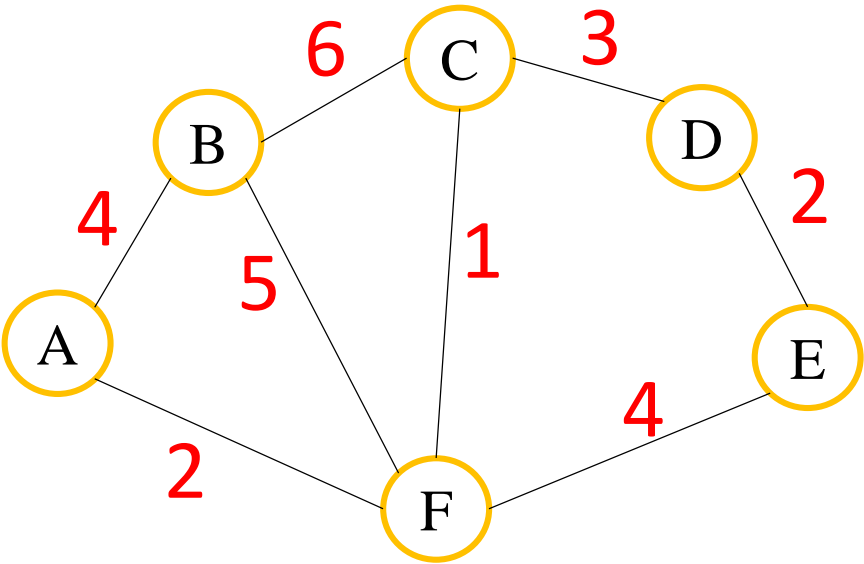
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Edges	Weight
AB	4
BC	6
CD	3
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CF	1

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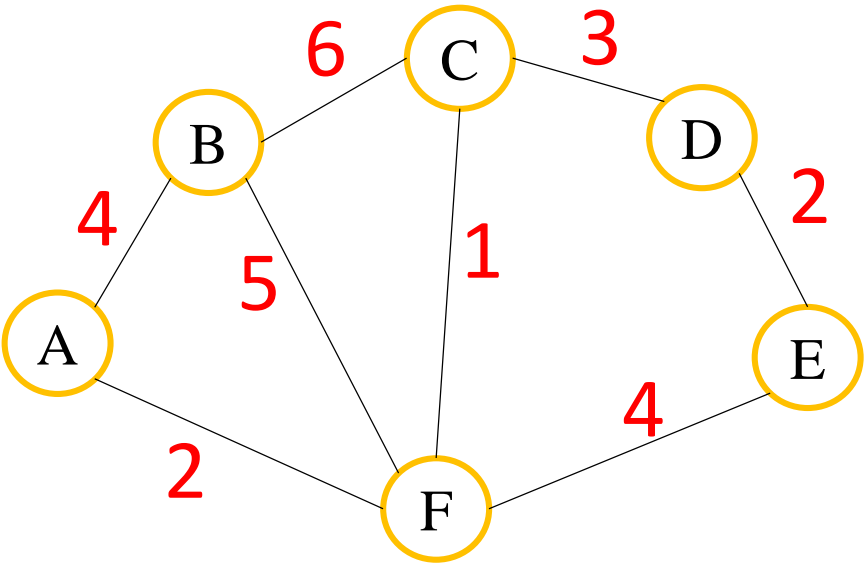
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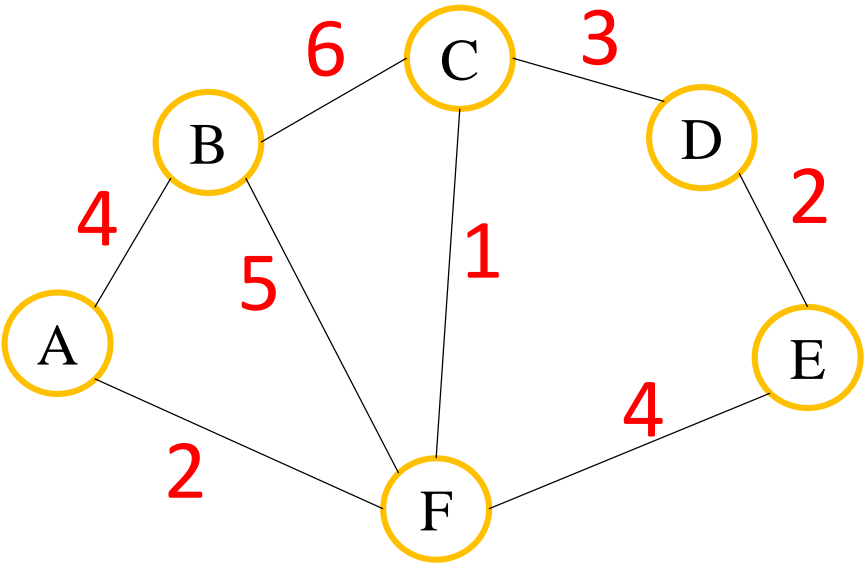
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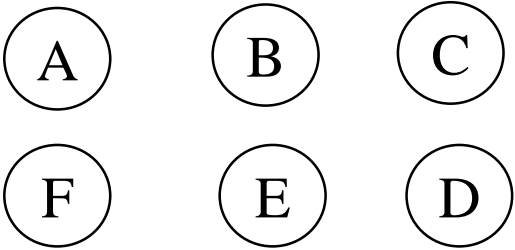
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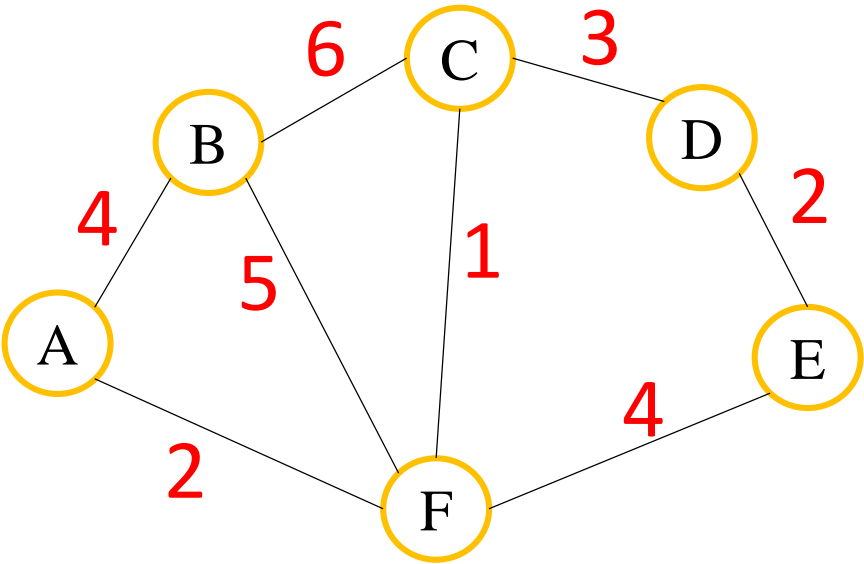
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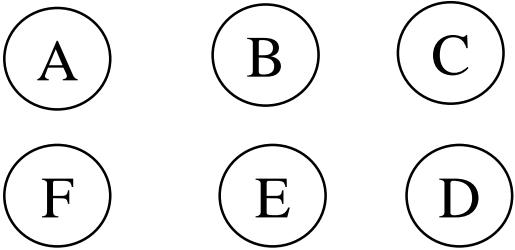
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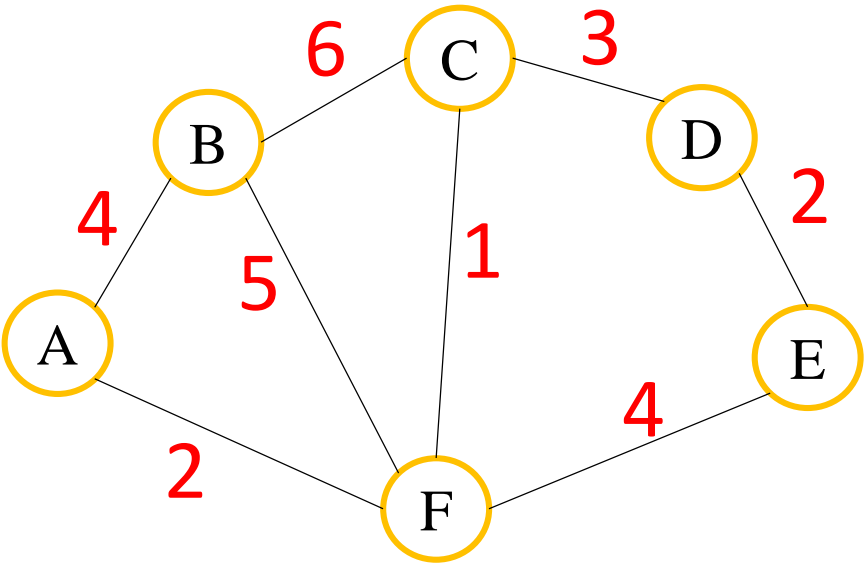
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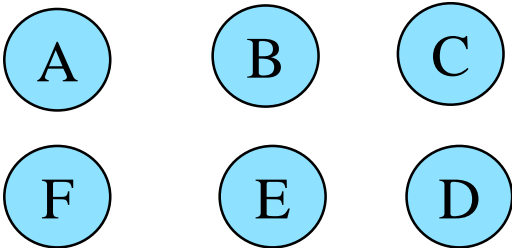
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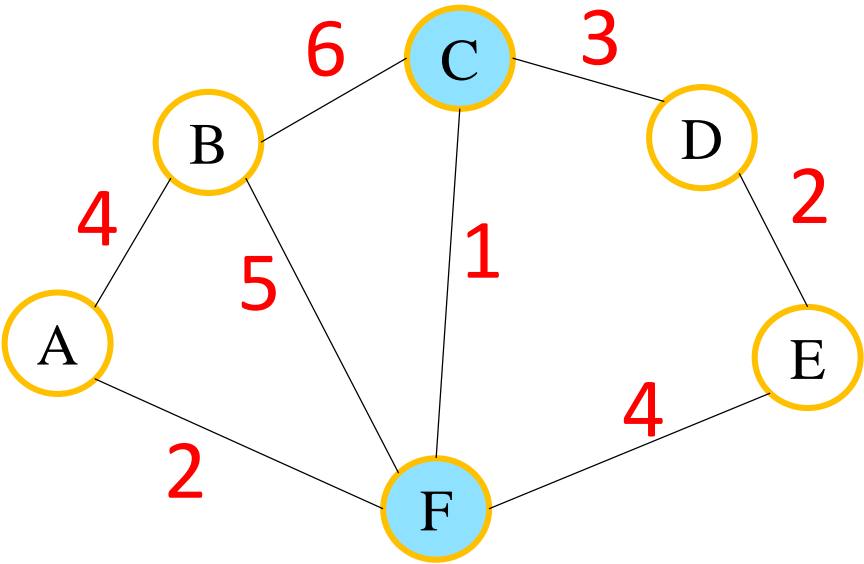
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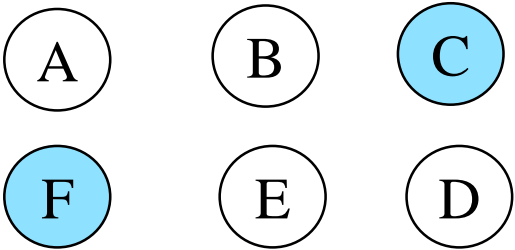
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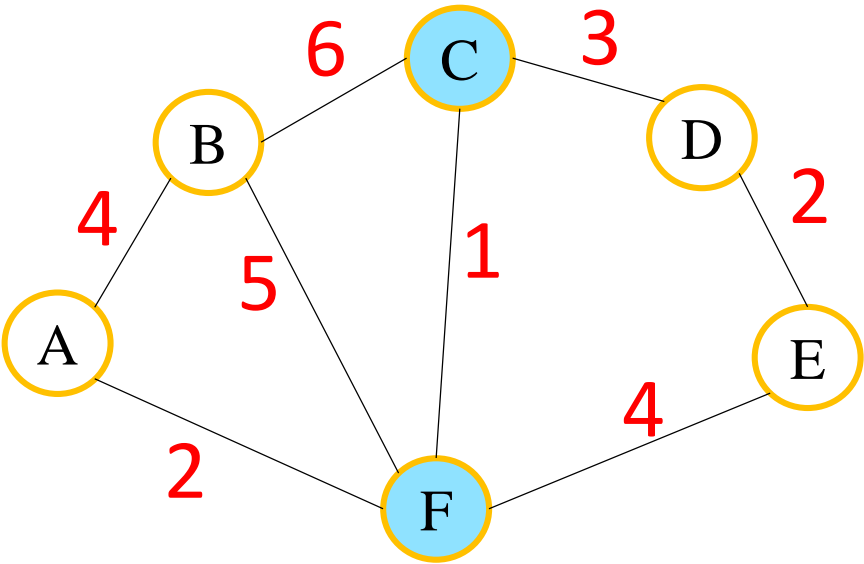
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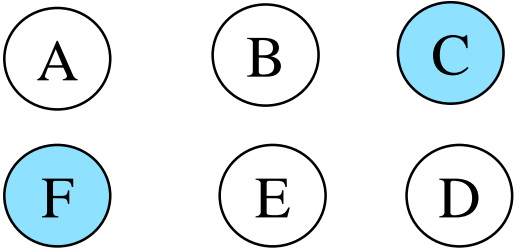
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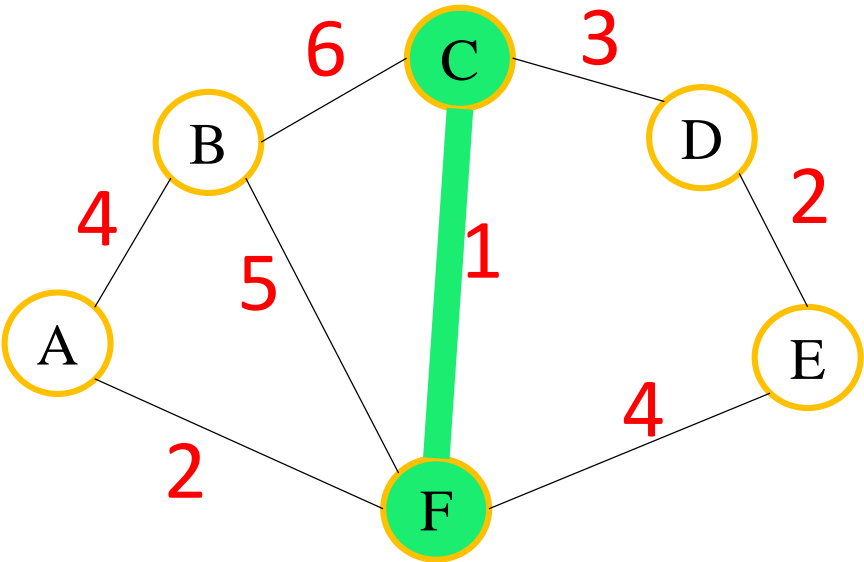
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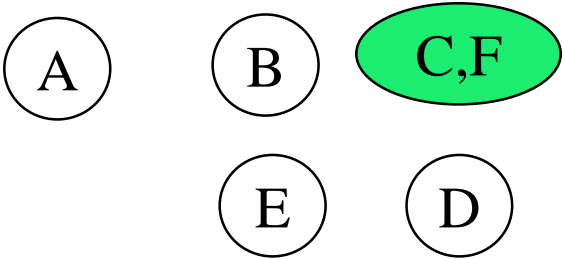
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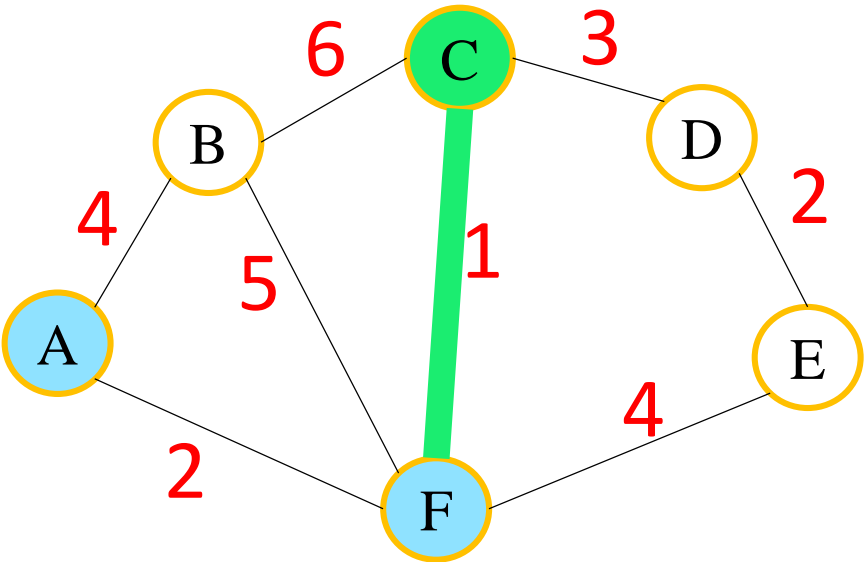
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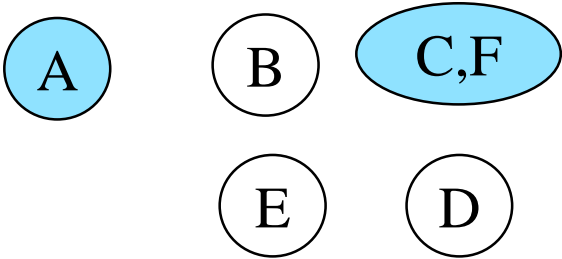
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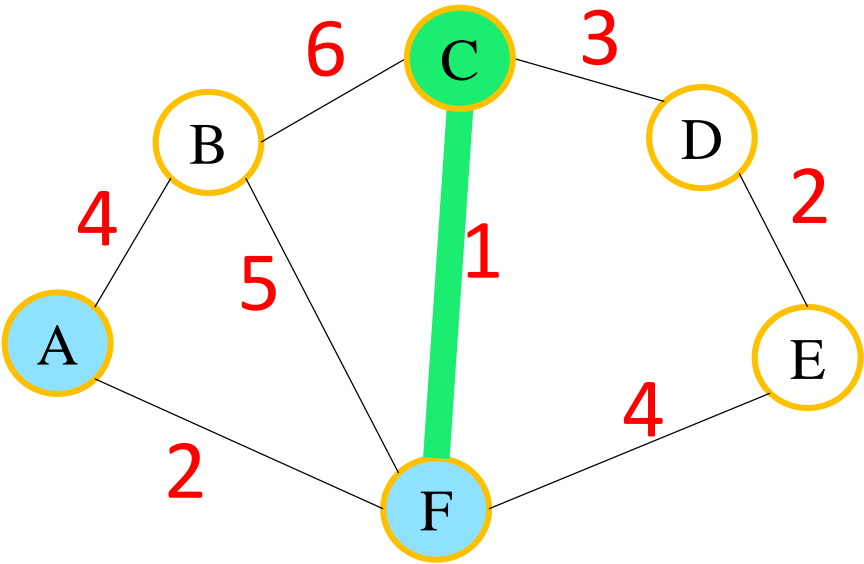
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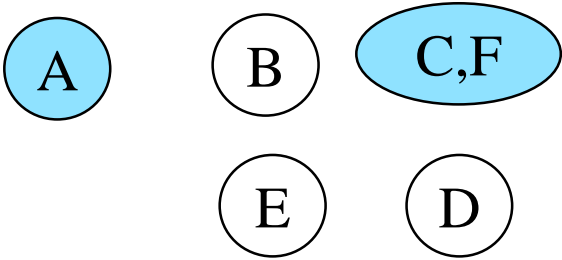
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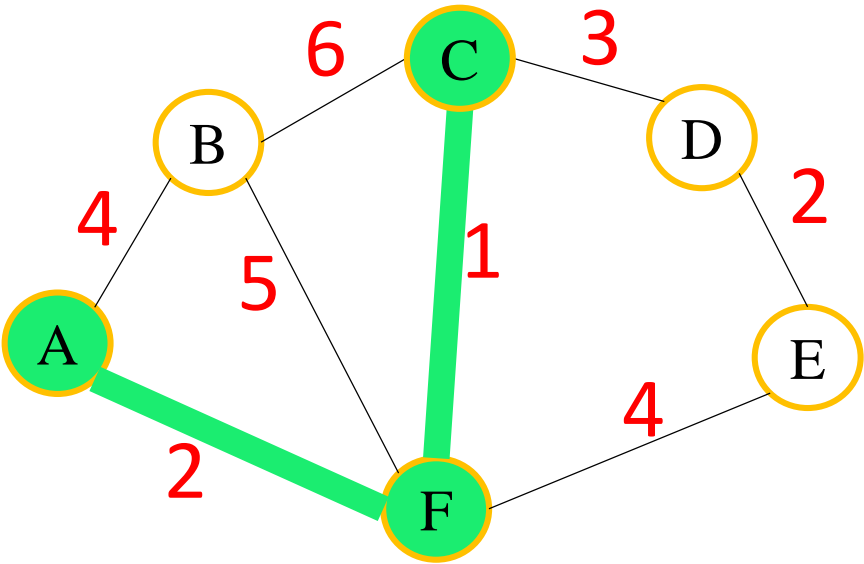
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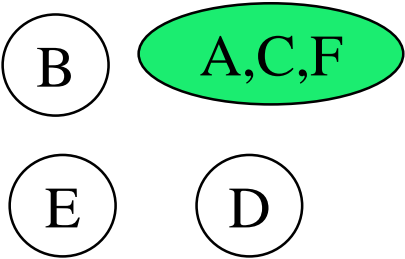
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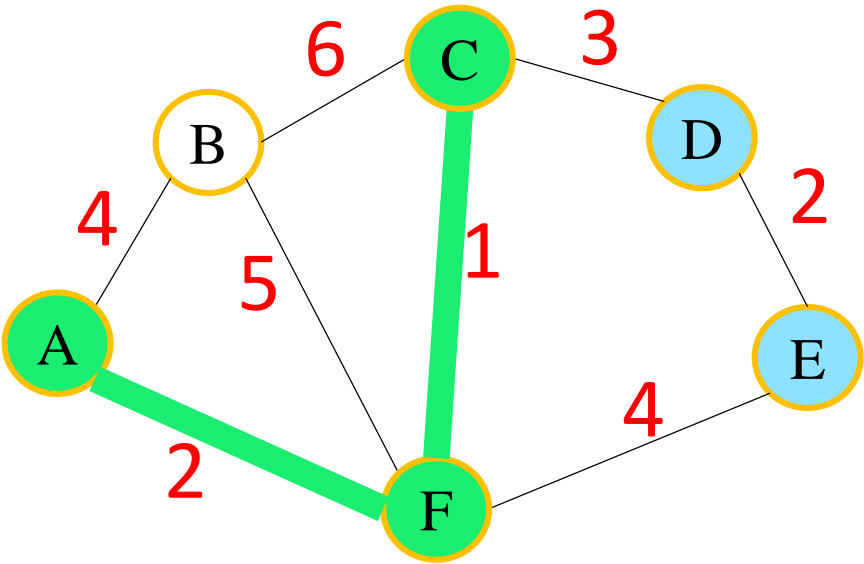
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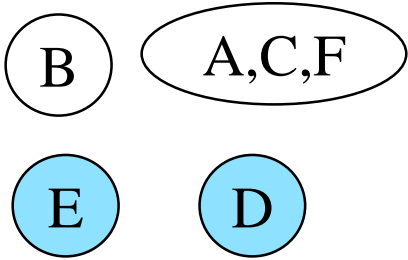
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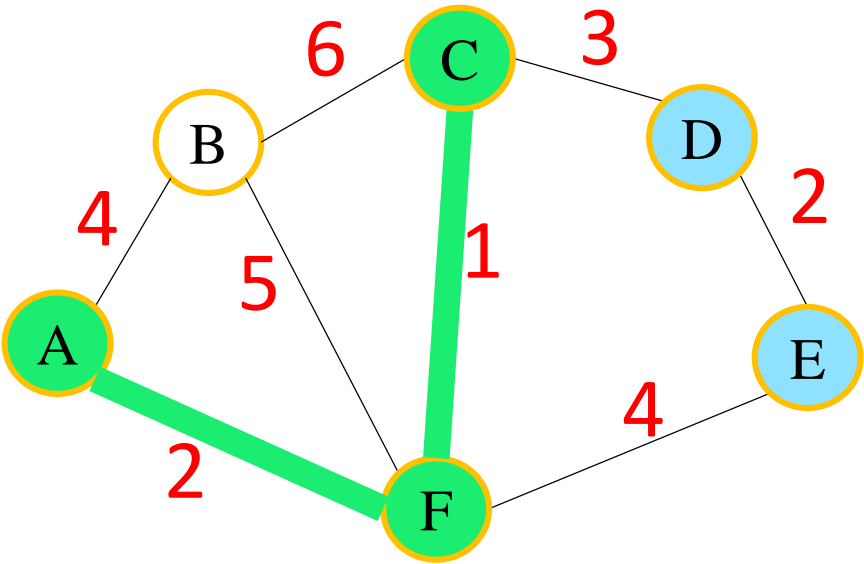
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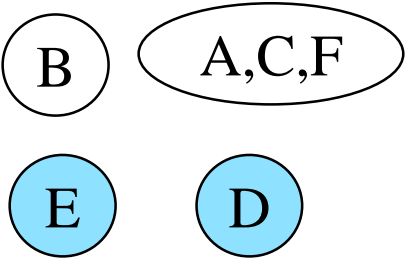
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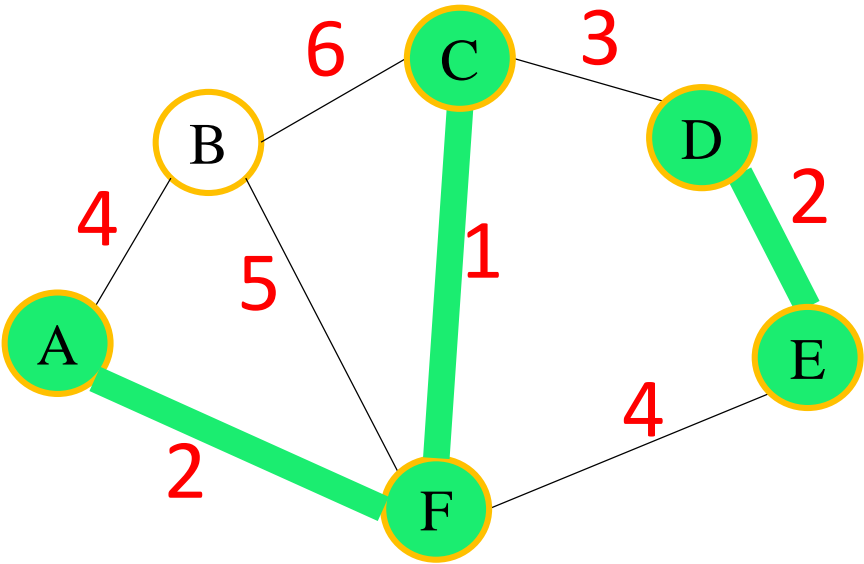
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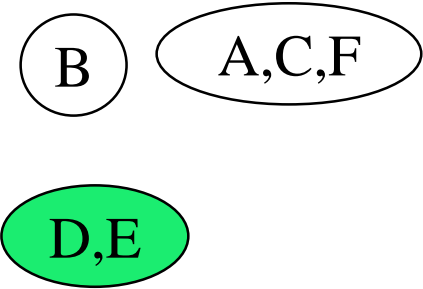
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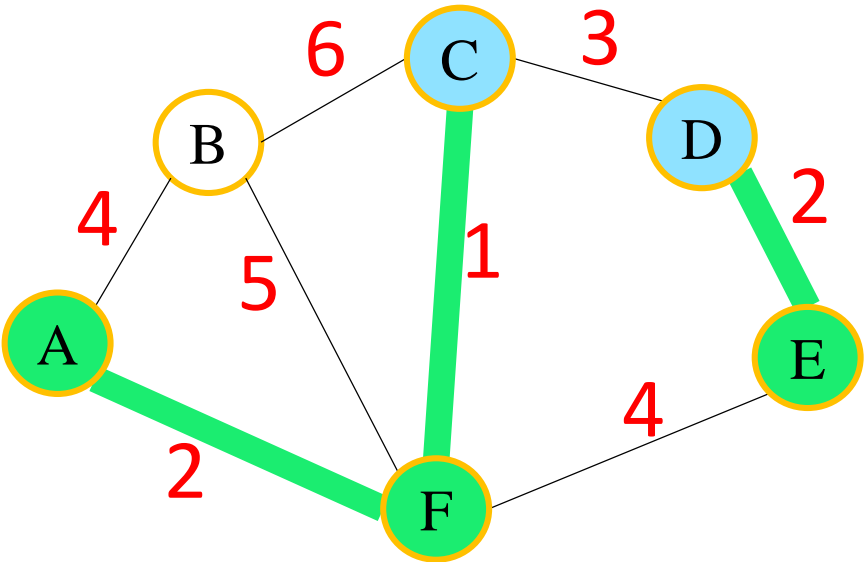
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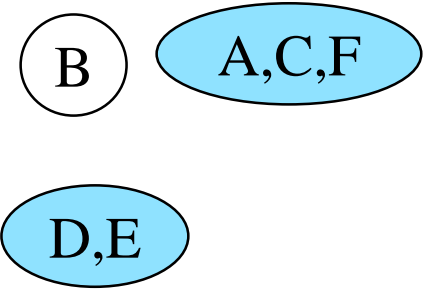
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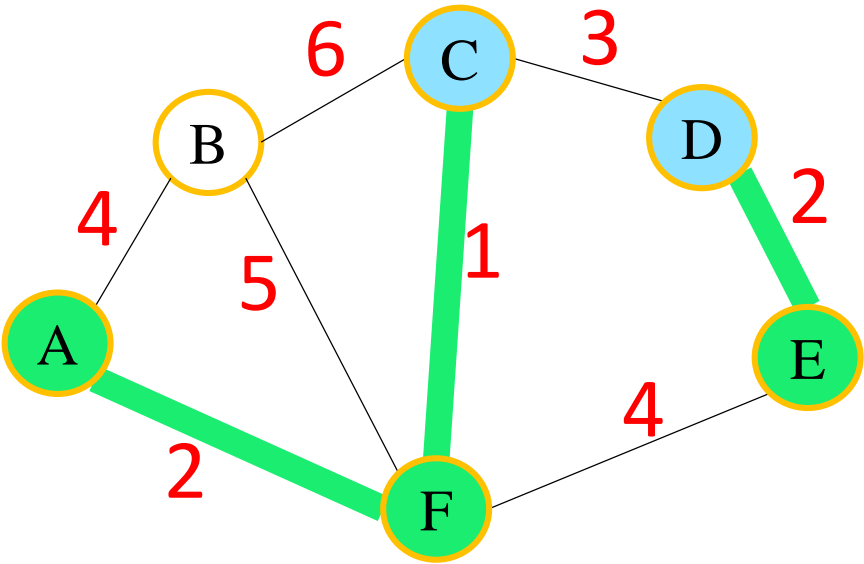
A = A ∪ {(v<sub>1</sub>,v<sub>2</sub>)}

UNION(v<sub>1</sub>,v<sub>2</sub>)

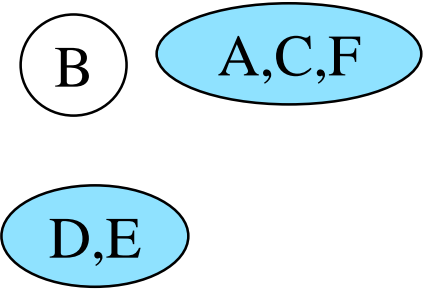
else

Remove edge (v<sub>1</sub>,v<sub>2</sub>)

return A



A = {(C,F),(A,F),(D,E),  
(C,D)}



Edges	Weight
CF	1
AF	2
DE	2
CD	3
AB	4
FE	4
BF	5
BC	6

KRUSKAL(V,E):

A = ∅

foreach v ∈ V:

MAKE-SET(v)

Sort E by weight increasingly

foreach (v<sub>1</sub>,v<sub>2</sub>) ∈ E:

if FIND-SET(v<sub>1</sub>) ≠ FIND-SET(v<sub>2</sub>):

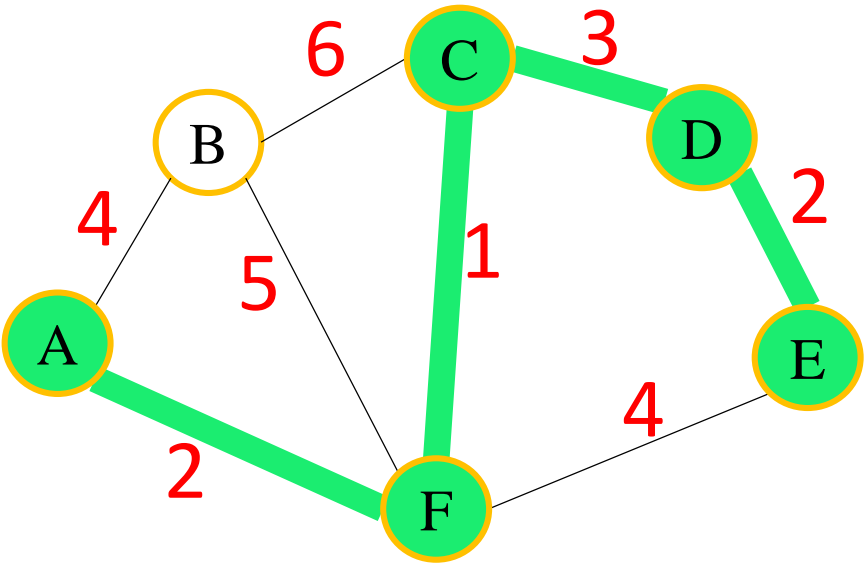
A = A ∪ {(v<sub>1</sub>,v<sub>2</sub>)}

UNION(v<sub>1</sub>,v<sub>2</sub>)

else

Remove edge (v<sub>1</sub>,v<sub>2</sub>)

return A



A = { (C,F), (A,F), (D,E),  
(C,D) }



Edges	Weight
CF	1
AF	2
DE	2
CD	3
AB	4
FE	4
BF	5
BC	6

KRUSKAL(V,E):

A = ∅

foreach v ∈ V:

MAKE-SET(v)

Sort E by weight increasingly

foreach (v<sub>1</sub>,v<sub>2</sub>) ∈ E:

if FIND-SET(v<sub>1</sub>) ≠ FIND-SET(v<sub>2</sub>):

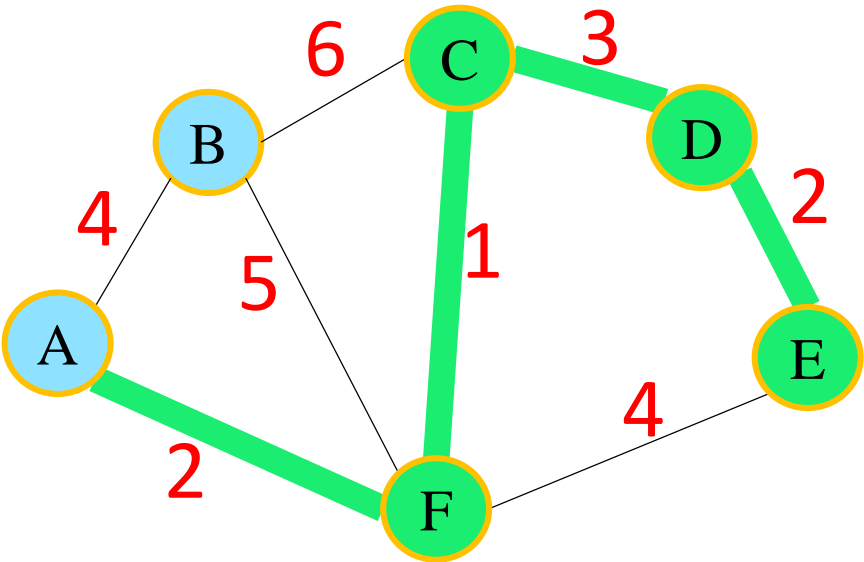
A = A ∪ {(v<sub>1</sub>,v<sub>2</sub>)}

UNION(v<sub>1</sub>,v<sub>2</sub>)

else

Remove edge (v<sub>1</sub>,v<sub>2</sub>)

return A



$A = \{ (C,F), (A,F), (D,E), (C,D) \}$



Edges	Weight
CF	1
AF	2
DE	2
CD	3
AB	4
FE	4
BF	5
BC	6

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MAKE-SET(v)

Sort E by weight increasingly

foreach (v<sub>1</sub>,v<sub>2</sub>) ∈ E:

if FIND-SET(v<sub>1</sub>) ≠ FIND-SET(v<sub>2</sub>):

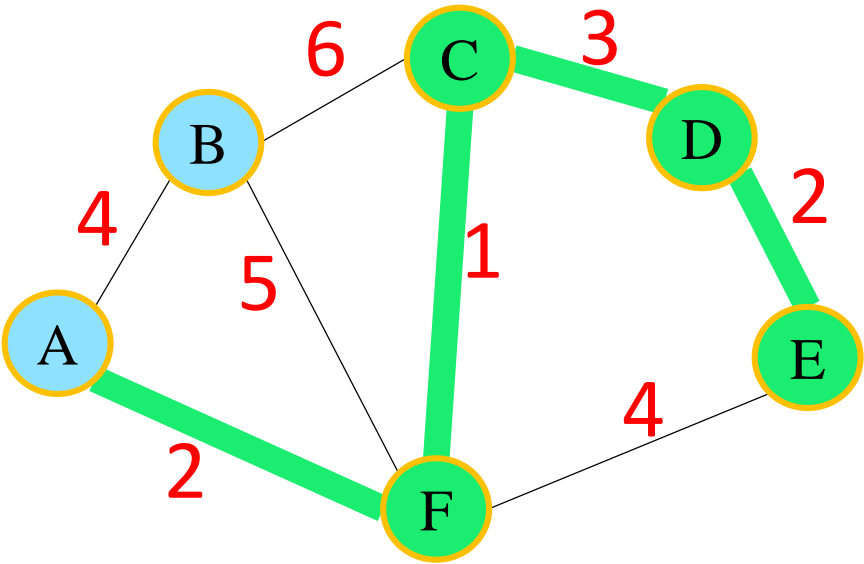
A = A ∪ {(v<sub>1</sub>,v<sub>2</sub>)}

UNION(v<sub>1</sub>,v<sub>2</sub>)

else

Remove edge (v<sub>1</sub>,v<sub>2</sub>)

return A



A = { (C,F), (A,F), (D,E),  
(C,D), (A,B) }



Edges	Weight
CF	1
AF	2
DE	2
CD	3
AB	4
FE	4
BF	5
BC	6

KRUSKAL(V,E):

$A = \emptyset$

**foreach**  $v \in V$ :

    MAKE-SET( $v$ )

Sort E by weight increasingly

**foreach**  $(v_1,v_2) \in E$ :

**if** FIND-SET( $v_1$ )  $\neq$  FIND-SET( $v_2$ ):

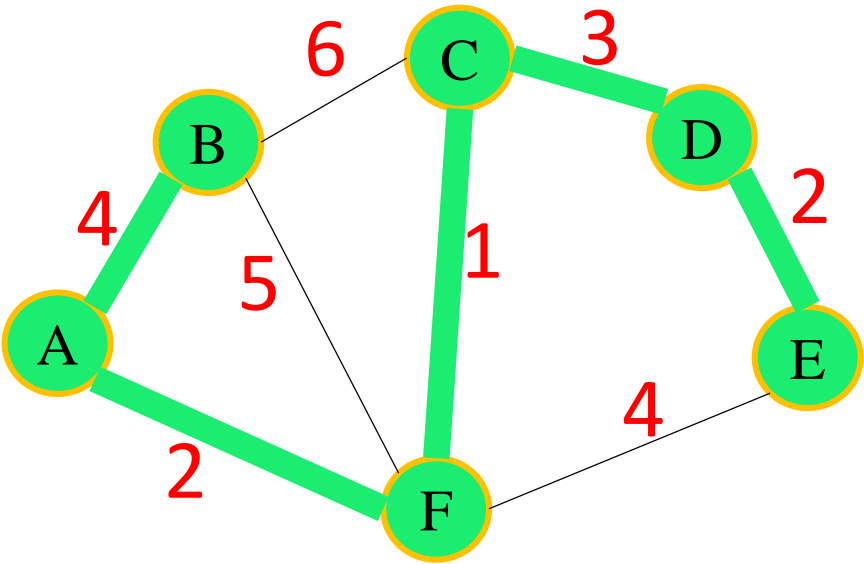
$A = A \cup \{(v_1,v_2)\}$

        UNION( $v_1,v_2$ )

**else**

        Remove edge  $(v_1,v_2)$

**return** A



$A = \{ (C,F), (A,F), (D,E), (C,D), (A,B) \}$

A,B,C,D,E,F

Edges	Weight
CF	1
AF	2
DE	2
CD	3
AB	4
FE	4
BF	5
BC	6

KRUSKAL(V,E):

A = ∅

foreach v ∈ V:

MAKE-SET(v)

Sort E by weight increasingly

foreach (v<sub>1</sub>,v<sub>2</sub>) ∈ E:

if FIND-SET(v<sub>1</sub>) ≠ FIND-SET(v<sub>2</sub>):

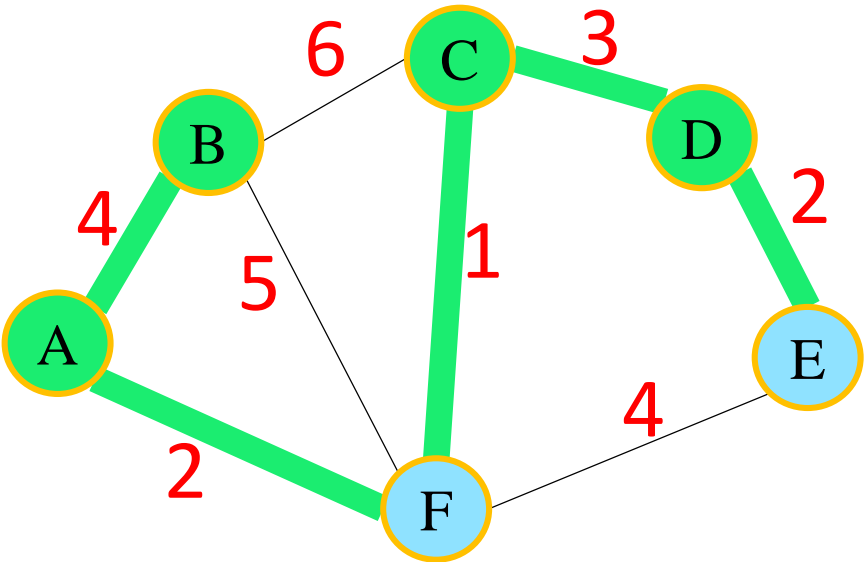
A = A ∪ {(v<sub>1</sub>,v<sub>2</sub>)}

UNION(v<sub>1</sub>,v<sub>2</sub>)

else

Remove edge (v<sub>1</sub>,v<sub>2</sub>)

return A



A = { (C,F), (A,F), (D,E),  
(C,D), (A,B) }

A,B,C,D,E,F

Edges	Weight
CF	1
AF	2
DE	2
CD	3
AB	4
FE	4
BF	5
BC	6

KRUSKAL(V,E):

$A = \emptyset$

**foreach**  $v \in V$ :

MAKE-SET( $v$ )

Sort E by weight increasingly

**foreach**  $(v_1,v_2) \in E$ :

**if** FIND-SET( $v_1$ )  $\neq$  FIND-SET( $v_2$ ):

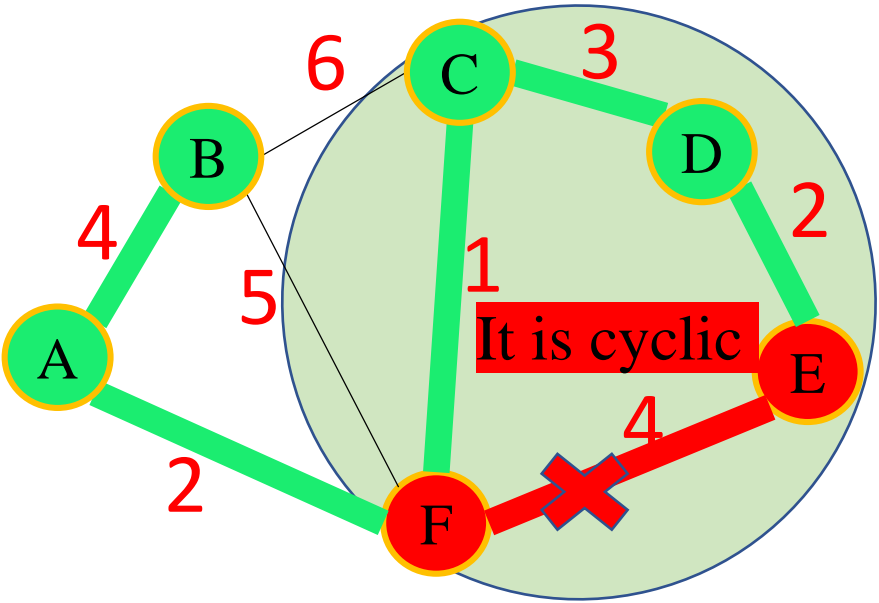
$A = A \cup \{(v_1,v_2)\}$

UNION( $v_1,v_2$ )

**else**

Remove edge  $(v_1,v_2)$

**return** A



$A = \{ (C,F), (A,F), (D,E), (C,D), (A,B) \}$

A,B,C,D,E,F  
Is in a same set.

Edges	Weight
CF	1
AF	2
DE	2
CD	3
AB	4
FE	4
BF	5
BC	6



KRUSKAL(V,E):

$A = \emptyset$

**foreach**  $v \in V$ :

    MAKE-SET( $v$ )

Sort E by weight increasingly

**foreach**  $(v_1,v_2) \in E$ :

**if** FIND-SET( $v_1$ )  $\neq$  FIND-SET( $v_2$ ):

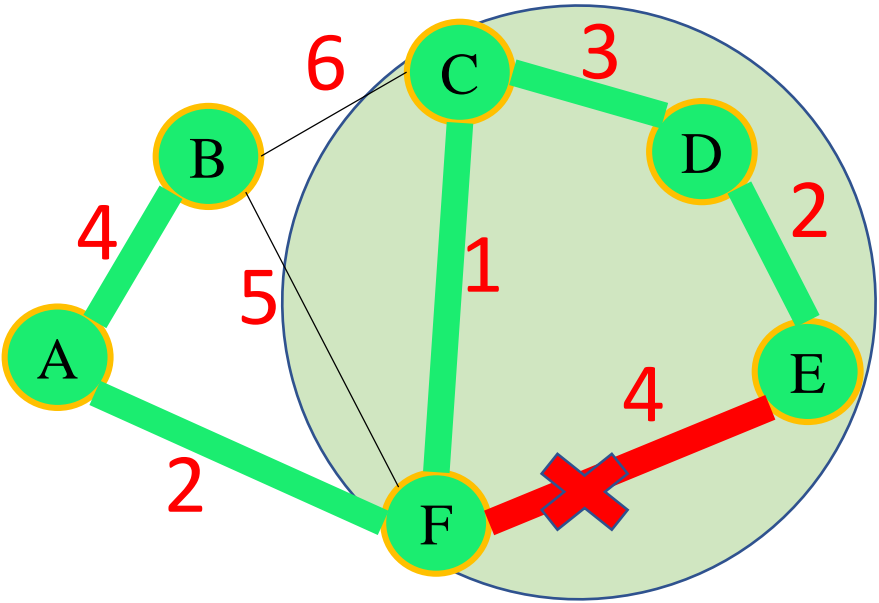
$A = A \cup \{(v_1,v_2)\}$

        UNION( $v_1,v_2$ )

**else**

        Remove edge  $(v_1,v_2)$

**return** A



$A = \{ (C,F), (A,F), (D,E), (C,D), (A,B) \}$

A,B,C,D,E,F

Edges	Weight
CF	1
AF	2
DE	2
CD	3
AB	4
FE	4
BF	5
BC	6

KRUSKAL(V,E):

A = ∅

foreach v ∈ V:

MAKE-SET(v)

Sort E by weight increasingly

foreach (v<sub>1</sub>,v<sub>2</sub>) ∈ E:

if FIND-SET(v<sub>1</sub>) ≠ FIND-SET(v<sub>2</sub>):

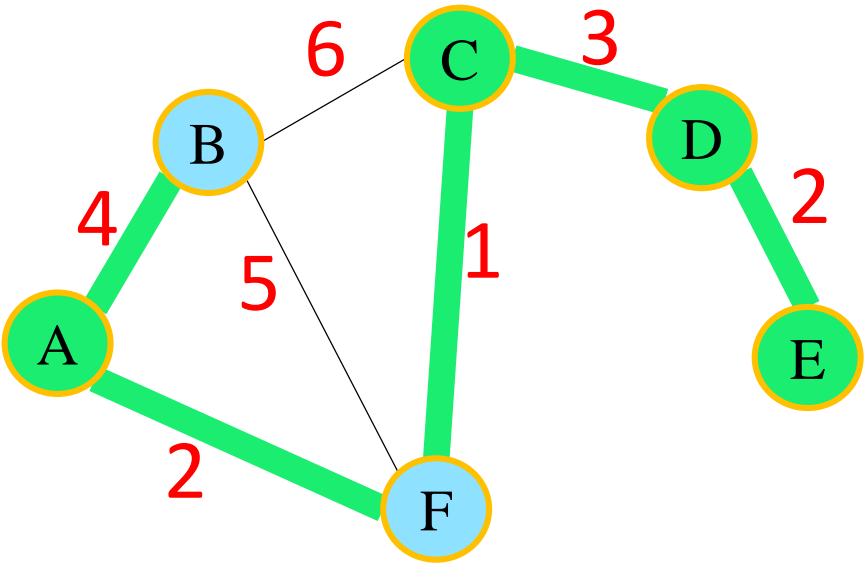
A = A ∪ {(v<sub>1</sub>,v<sub>2</sub>)}

UNION(v<sub>1</sub>,v<sub>2</sub>)

else

Remove edge (v<sub>1</sub>,v<sub>2</sub>)

return A



$A = \{ (C,F), (A,F), (D,E), (C,D), (A,B) \}$

A,B,C,D,E,F

Edges	Weight
CF	1
AF	2
DE	2
CD	3
AB	4
FE	4
BF	5
BC	6

KRUSKAL(V,E):

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MAKE-SET(v)

Sort E by weight increasingly

foreach (v<sub>1</sub>,v<sub>2</sub>) ∈ E:

if FIND-SET(v<sub>1</sub>) ≠ FIND-SET(v<sub>2</sub>):

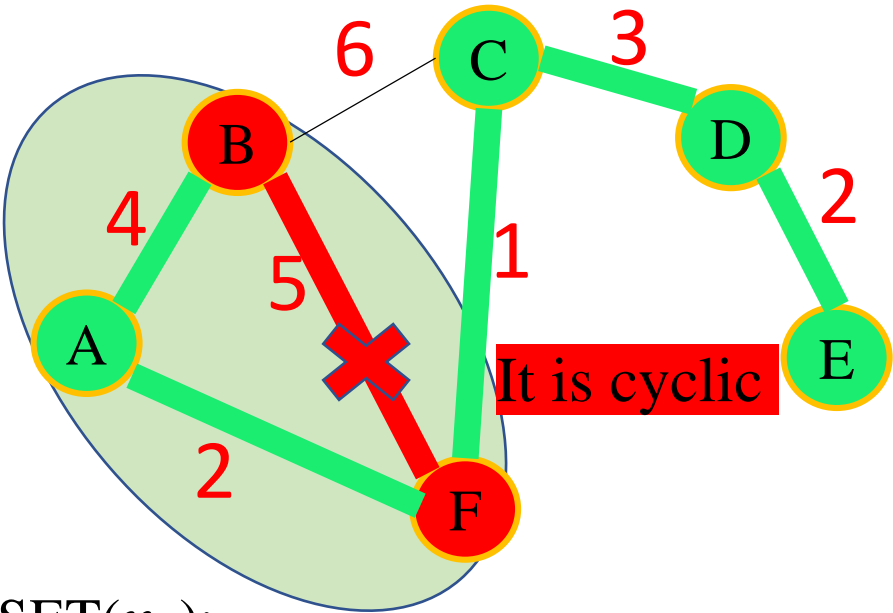
A = A ∪ {(v<sub>1</sub>,v<sub>2</sub>)}

UNION(v<sub>1</sub>,v<sub>2</sub>)

else

Remove edge (v<sub>1</sub>,v<sub>2</sub>)

return A



A = { (C,F),(A,F),(D,E),  
(C,D),(A,B) }

A,B,C,D,E,F  
Is in a same set.

Edges	Weight
CF	1
AF	2
DE	2
CD	3
AB	4
FE	4
BF	5
BC	6

KRUSKAL(V,E):

A = ∅

foreach v ∈ V:

MAKE-SET(v)

Sort E by weight increasingly

foreach (v<sub>1</sub>,v<sub>2</sub>) ∈ E:

if FIND-SET(v<sub>1</sub>) ≠ FIND-SET(v<sub>2</sub>):

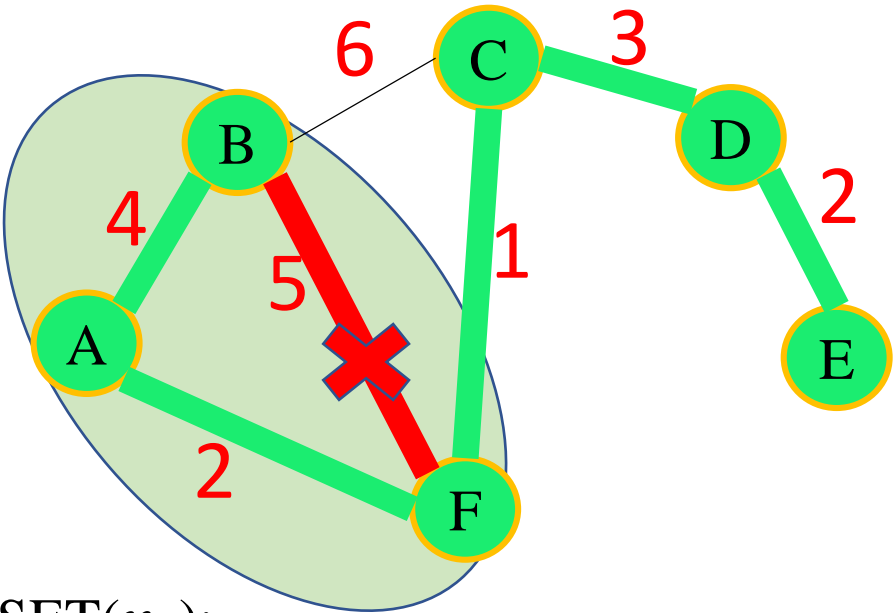
A = A ∪ {(v<sub>1</sub>,v<sub>2</sub>)}

UNION(v<sub>1</sub>,v<sub>2</sub>)

else

Remove edge (v<sub>1</sub>,v<sub>2</sub>)

return A



$A = \{ (C,F), (A,F), (D,E), (C,D), (A,B) \}$

A,B,C,D,E,F

Edges	Weight
CF	1
AF	2
DE	2
CD	3
AB	4
FE	4
BF	5
BC	6

KRUSKAL(V,E):

A = ∅

foreach v ∈ V:

MAKE-SET(v)

Sort E by weight increasingly

foreach (v<sub>1</sub>,v<sub>2</sub>) ∈ E:

if FIND-SET(v<sub>1</sub>) ≠ FIND-SET(v<sub>2</sub>):

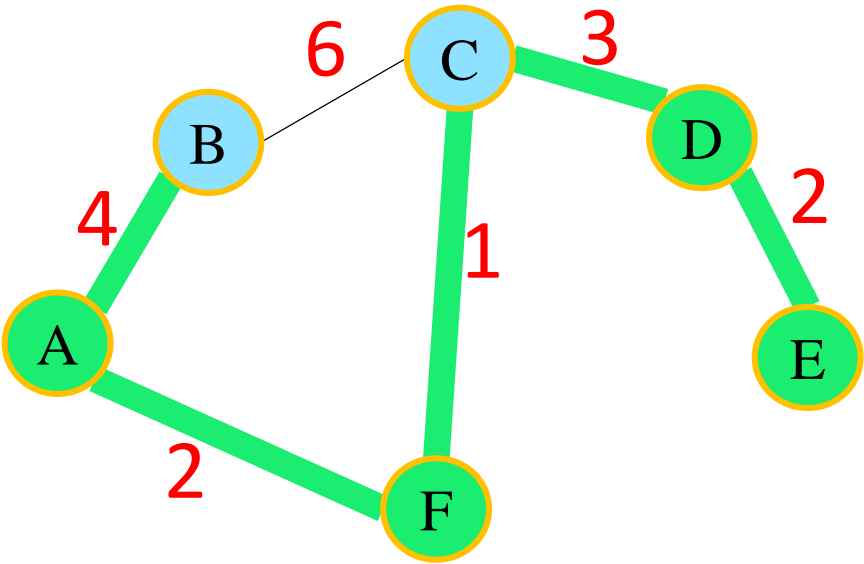
A = A ∪ {(v<sub>1</sub>,v<sub>2</sub>)}

UNION(v<sub>1</sub>,v<sub>2</sub>)

else

Remove edge (v<sub>1</sub>,v<sub>2</sub>)

return A



A = { (C,F), (A,F), (D,E),  
(C,D), (A,B) }

A,B,C,D,E,F

Edges	Weight
CF	1
AF	2
DE	2
CD	3
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FE	4
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MAKE-SET(v)

Sort E by weight increasingly

foreach (v<sub>1</sub>,v<sub>2</sub>) ∈ E:

if FIND-SET(v<sub>1</sub>) ≠ FIND-SET(v<sub>2</sub>):

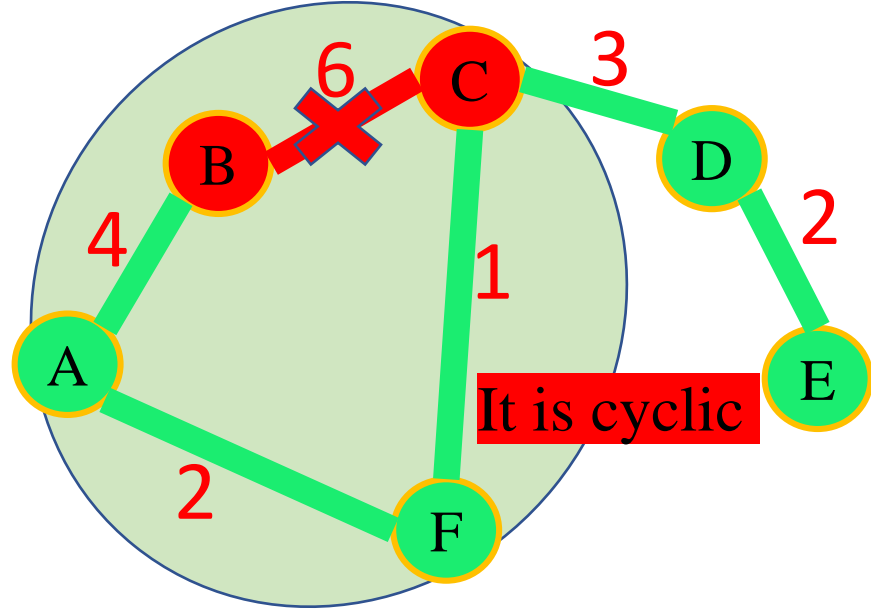
A = A ∪ {(v<sub>1</sub>,v<sub>2</sub>)}

UNION(v<sub>1</sub>,v<sub>2</sub>)

else

Remove edge (v<sub>1</sub>,v<sub>2</sub>)

return A



A = { (C,F), (A,F), (D,E),  
(C,D), (A,B) }

A, ~~B~~, ~~C~~, D, E, F

Is in a same set.

Edges	Weight
CF	1
AF	2
DE	2
CD	3
AB	4
FE	4
BF	5
BC	6

KRUSKAL(V,E):

$A = \emptyset$

**foreach**  $v \in V$ :

    MAKE-SET( $v$ )

Sort E by weight increasingly

**foreach**  $(v_1,v_2) \in E$ :

**if** FIND-SET( $v_1$ )  $\neq$  FIND-SET( $v_2$ ):

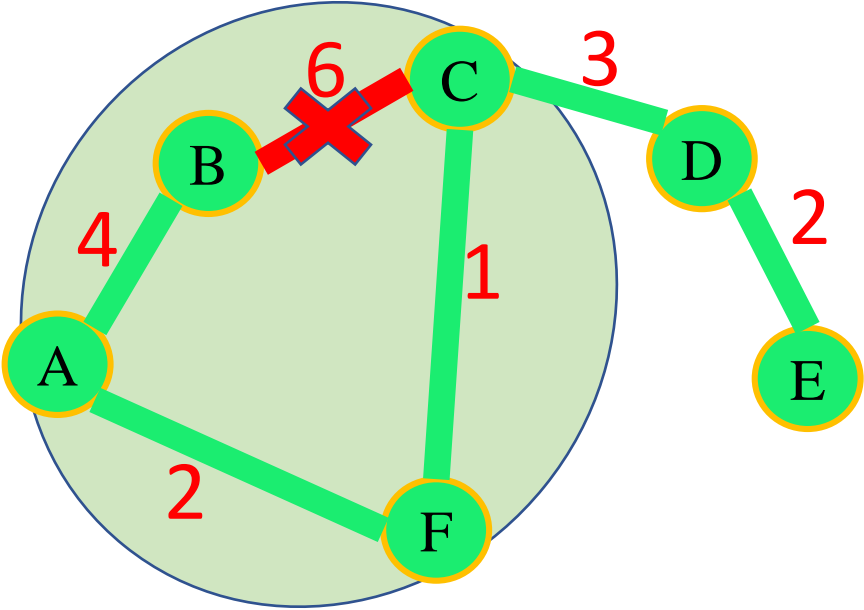
$A = A \cup \{(v_1,v_2)\}$

        UNION( $v_1,v_2$ )

**else**

        Remove edge  $(v_1,v_2)$

**return** A



$A = \{ (C,F), (A,F), (D,E), (C,D), (A,B) \}$

A,B,C,D,E,F

Edges	Weight
CF	1
AF	2
DE	2
CD	3
AB	4
FE	4
BF	5
BC	6

KRUSKAL(V,E):

A = ∅

foreach v ∈ V:

MAKE-SET(v)

Sort E by weight increasingly

foreach (v<sub>1</sub>,v<sub>2</sub>) ∈ E:

if FIND-SET(v<sub>1</sub>) ≠ FIND-SET(v<sub>2</sub>):

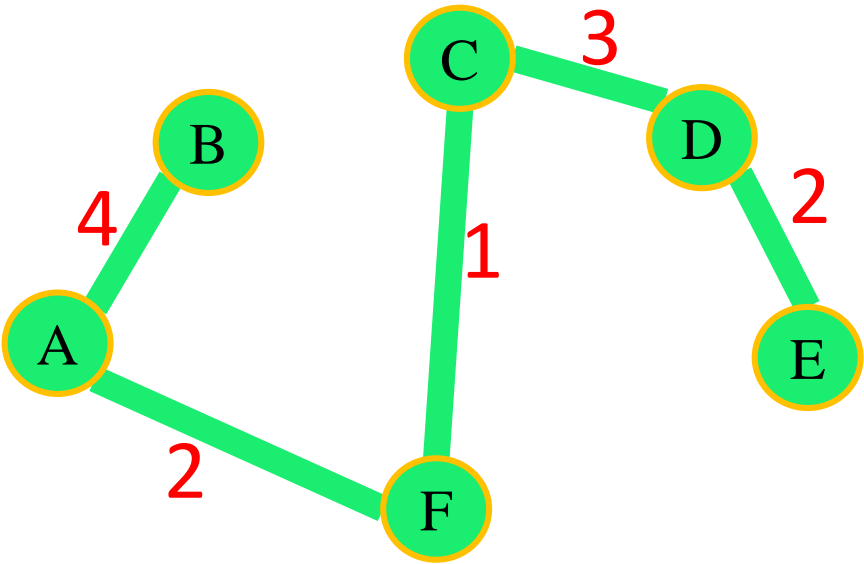
A = A ∪ {(v<sub>1</sub>,v<sub>2</sub>)}

UNION(v<sub>1</sub>,v<sub>2</sub>)

else

Remove edge (v<sub>1</sub>,v<sub>2</sub>)

return A



A = { (C,F), (A,F), (D,E),  
(C,D), (A,B) }

Edges	Weight
CF	1
AF	2
DE	2
CD	3
AB	4
FE	4
BF	5
BC	6



KRUSKAL(V,E):

$A = \emptyset$

**foreach**  $v \in V$ :

    MAKE-SET( $v$ )

Sort E by weight increasingly

**foreach**  $(v_1,v_2) \in E$ :

**if** FIND-SET( $v_1$ )  $\neq$  FIND-SET( $v_2$ ):

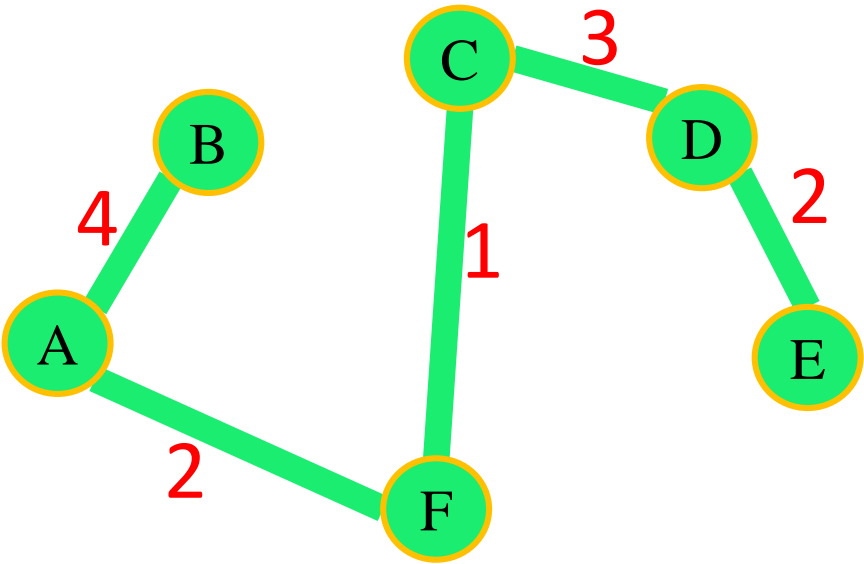
$A = A \cup \{(v_1,v_2)\}$

        UNION( $v_1,v_2$ )

**else**

        Remove edge  $(v_1,v_2)$

**return** A







$A = \{(C,F), (A,F), (D,E), (C,D), (A,B)\}$

Total Weight :-    1   +   2   +   2   +   3   +   4  
                         = 12

Edges	Weight
CF	1
AF	2
DE	2
CD	3
AB	4
FE	4
BF	5
BC	6


# Time Complexity

KRUSKAL(V,E):

O(1)		<b>A</b> = $\emptyset$
O(V)		<b>foreach</b> $v \in V$ : <b>MAKE-SET</b> ( $v$ )
O(E log E)		Sort E by weight increasingly
O(E log V)		<b>foreach</b> $(v_1, v_2) \in E$ : <b>if</b> <b>FIND-SET</b> ( $v_1$ ) $\neq$ <b>FIND-SET</b> ( $v_2$ ): <b>A</b> = <b>A</b> $\cup$ $\{(v_1, v_2)\}$ <b>UNION</b> ( $v_1, v_2$ ) <b>else</b> Remove edge $(v_1, v_2)$
		<b>return</b> <b>A</b>

# Time Complexity

KRUSKAL(V,E):

O(1)		<b>A</b> = $\emptyset$
O(V)		<b>foreach</b> $v \in V$ : <b>MAKE-SET</b> ( $v$ )
O(E log E)		Sort E by weight increasingly
O(E log V)		<b>foreach</b> $(v_1, v_2) \in E$ : <b>if</b> <b>FIND-SET</b> ( $v_1$ ) $\neq$ <b>FIND-SET</b> ( $v_2$ ): $A = A \cup \{(v_1, v_2)\}$ <b>UNION</b> ( $v_1, v_2$ ) <b>else</b> Remove edge $(v_1, v_2)$ <b>return</b> A

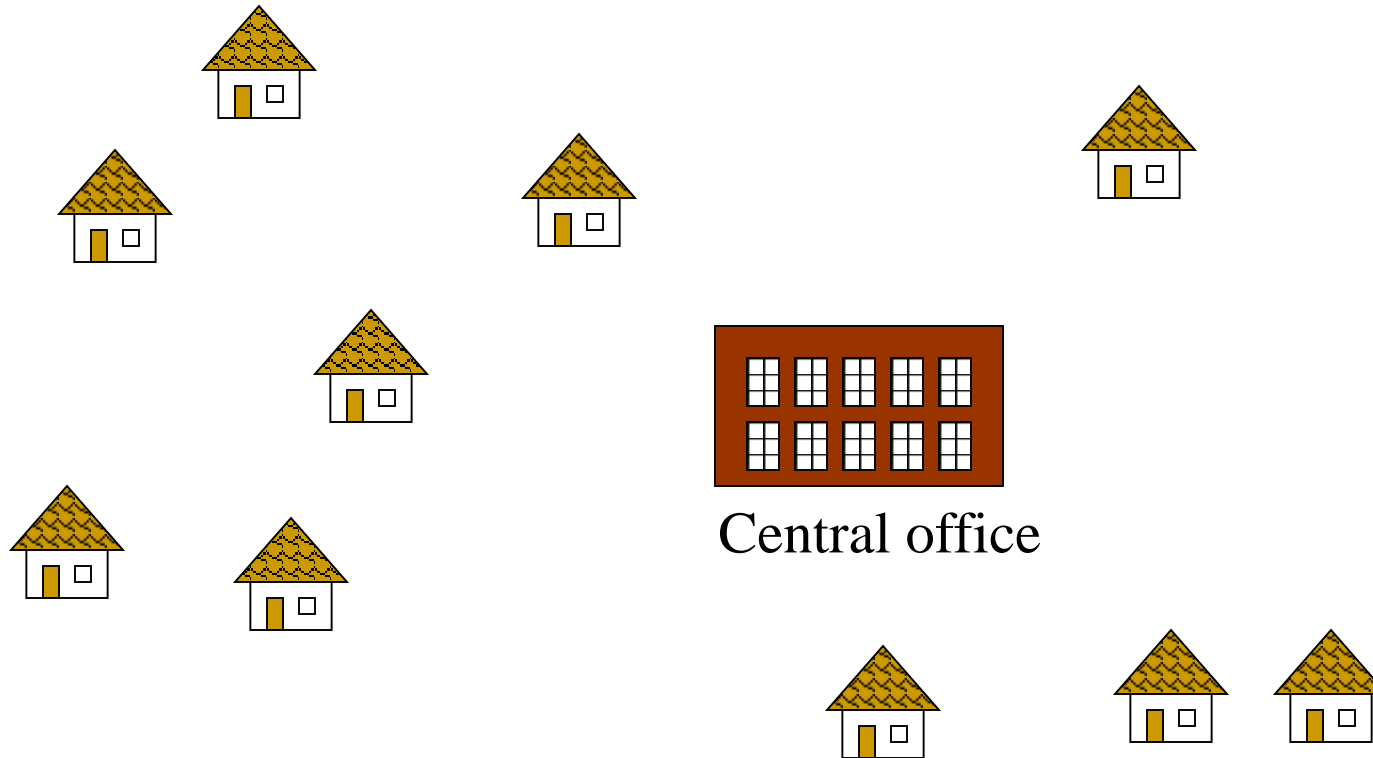
$$\begin{aligned}\text{Time Complexity} &= O(1) + O(V) + O(E \log E) + O(E \log V) \\ &= O(E \log E) + O(E \log V)\end{aligned}$$

$$\begin{aligned}\text{Since, } |E| \leq |V|^2 &\Rightarrow \log |E| = O(2 \log V) = O(\log V). \\ &= O(E \log V) + O(E \log V) \\ &= O(E \log V)\end{aligned}$$

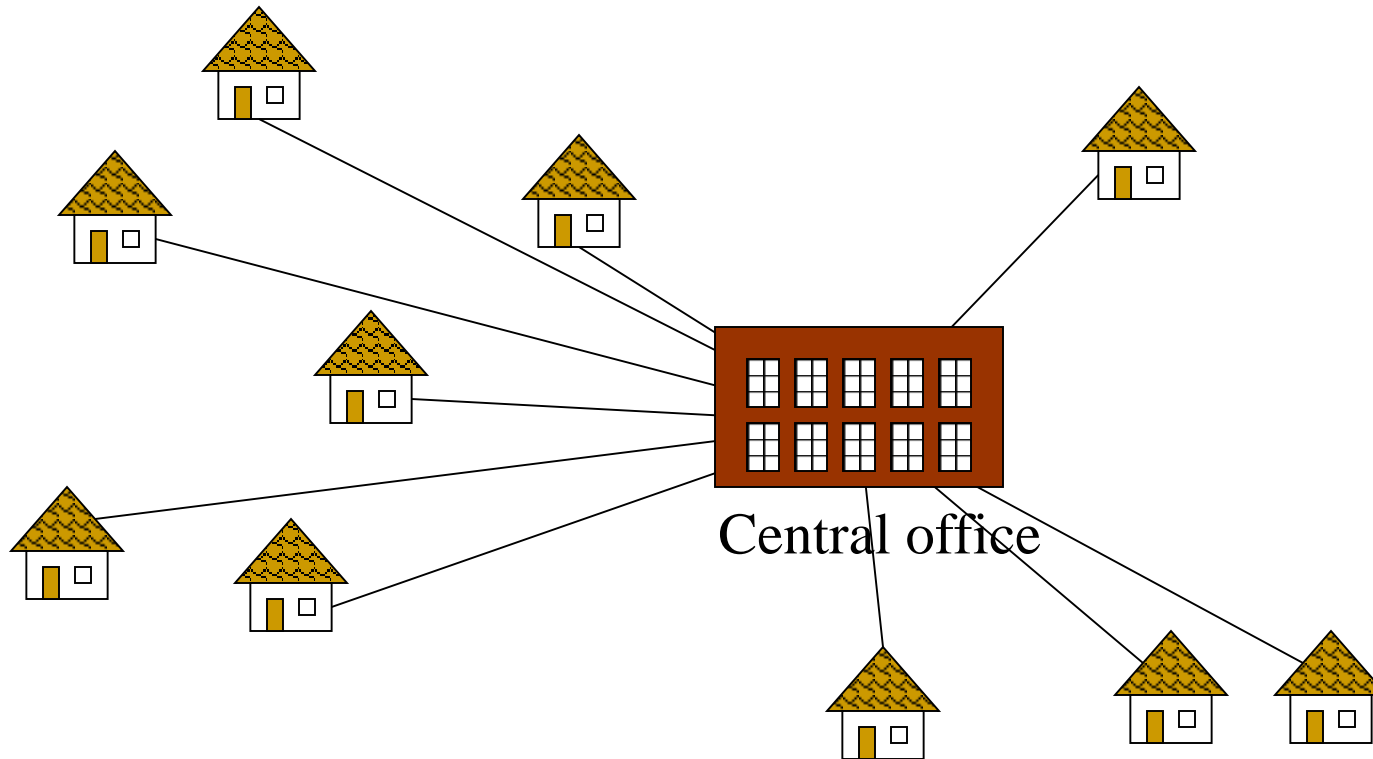
# Real-life applications of Kruskal's algorithm

- Landing Cables
- TV Network
- Tour Operations
- Computer networking
- Study of Molecular bonds in Chemistry
- Routing Algorithms

# Problem: Laying Telephone Wire

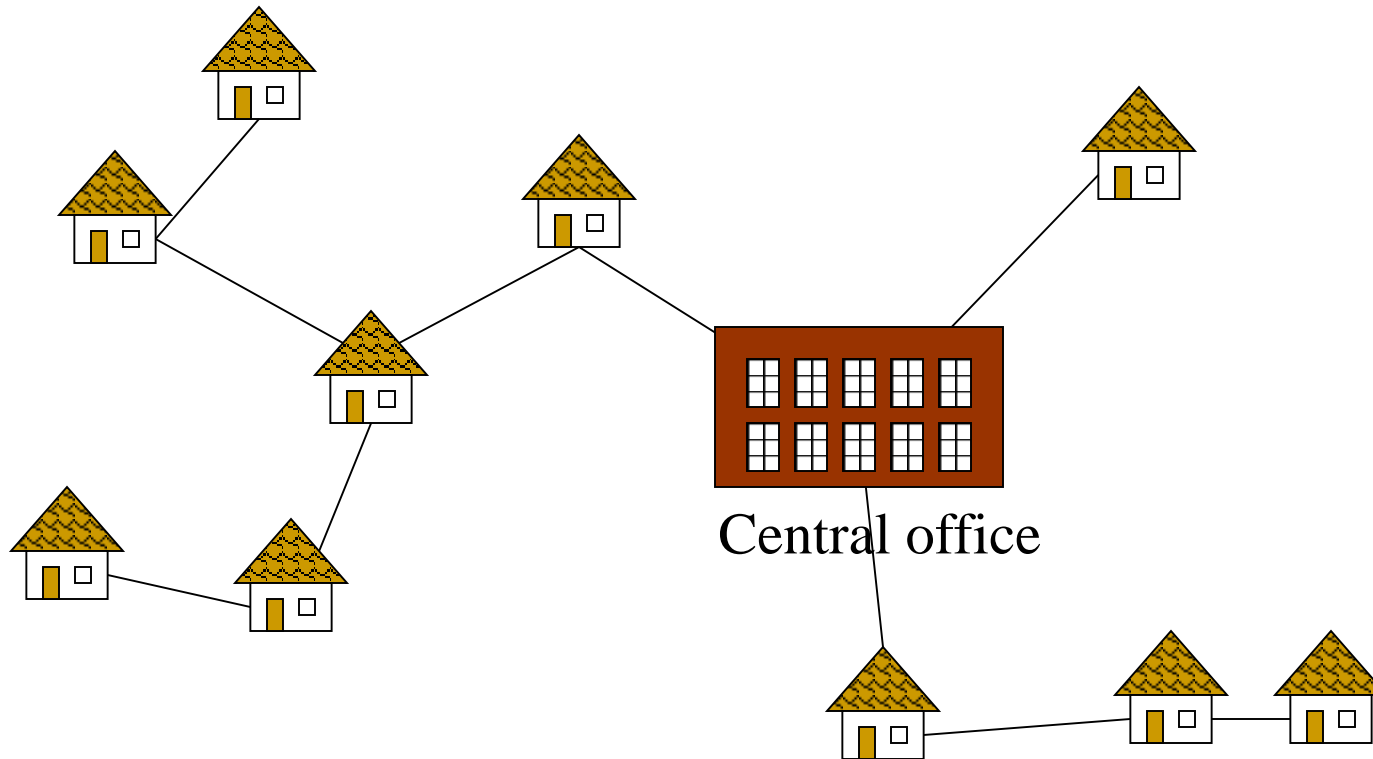


# Wiring: Naïve Approach



**Expensive!**

# Wiring: Better Approach



Minimize the total length of wire connecting the customers