Probability and Computer Science

LECTURE 1: INTRODUCTION TO PROBABILITY

Logistic details

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- Time: 7-8 weeks

- Textbook:
  - A first course in probability (Sheldon Ross)
  - Probability and Computing – Randomized Algorithm and Probabilistic Analysis (Mitzenmacher and Upfal)

E-books for Both can be found @ gigapedia.com

Introduction to Probability

- Mathematical tools to deal with uncertain events.
- Applications include:
  - Web search engine: Markov chain theory
  - Data Mining, Machine Learning: Data mining, Machine learning: Stochastic gradient, Markov chain Monte Carlo,
  - Image processing: Markov random fields,
  - Design of wireless communication systems: random matrix theory,
  - Optimization of engineering processes: simulated annealing, genetic algorithms,
  - Finance (option pricing, volatility models): Monte Carlo, dynamic models, Design of atomic bomb (Los Alamos): Markov chain Monte Carlo.

Plan of the course

- Combinatorial analysis; i.e counting
- Axioms of probability
- Conditional probability and inference
- Discrete & continuous random variables
- Multivariate random variables
- Properties of expectation, generating functions
- Additional topics:
  - Poisson and Markov processes
  - Simulation and Monte Carlo methods

- Applications

Combinatorial (Counting)

- Many basic probability problems are counting problems.
- Example: Assume there are 1 man and 2 women in a room. You pick a person randomly.
  - What is the probability P1 that this is a man?
  - If you pick two persons randomly, what is the probability P2 that these are a man and woman
- Answer: …

- Both problems consists of counting the number of different ways that a certain event can occur.

Basic Principle of Counting

- Basic Principle of Counting:
  - Suppose that two experiments are to be performed.
  - Experiment 1 can result in any one of n_1 possible outcomes
  - For each outcome of experiment 1, there are n_2 possible outcomes of experiment 2,
  - Then there are $n_1 \cdot n_2$ possible outcomes of the two experiments.
- Example:
  - A football tournament consists of 14 teams, each of which has 11 players. If one team and one of its players are to be selected as team and player of the year, how many different choices are possible?
  - Answer: $14 \cdot 11 = 154$

Generalized Principle of Counting

- Generalized Principle of Counting:
  - If r experiments that are to be performed are such that the 1st one may result in any of n1 possible outcomes;
  - and if, for each of these n1 possible outcomes, there are n2 possible outcomes of the 2nd experiment;
  - and if, for each of the n1 n2 possible outcomes of the first two experiments, there are n3 possible outcomes of the 3rd experiment; and if...,
  - then there is a total of n1 x n2 x nr possible outcomes of the r experiments.

- Example: A university committee consists of 4 undergrads, 5 grads, 7 profs and 2 non-university persons. A sub-committee of 4, consisting of 1 person from each category, is to be chosen. How many different subcommittees are possible?

  Answer: 4 . 5 . 7 . 2 = 280

More examples

- Example: How many different 6-place license plates are possible if the first 3 places are to be occupied by letters and the final 3 by numbers.
- Example: How many different 6-place license plates are possible if the first 3 places are to be occupied by letters, the final 3 by numbers and if
  - repetition among letters were prohibited?
  - repetition among numbers were prohibited
  - repetition among both letters and numbers were prohibited?

Permutations

- Example: Consider the acronym UBC. How many different ordered arrangements of the letters U, B and C are possible?
  - Answer: We have (B,C,U), (B,U,C), (C,B,U), (C,U,B), (U,B,C) and (U,C,B); i.e. 6 possible arrangement. Each arrangement is known as a permutation.
- General Result. Suppose you have n objects. The number of permutations of these n objects is given by $n (n - 1) (n - 2) \dots 3 . 2 . 1 = n!$
- Remember the convention $0! = 1$.
- Example: Assume we have an horse race with 12 horses. How many possible rankings are (theoretically) possible?

Permutations: Examples

- Example: A class in "Introduction to Probability" consists of 40 men and 30 women. An examination is given and the students are ranked according to their performance. Assume that no two students obtain the same score.
  - How many different rankings are possible?
  - If the men are ranked among themselves and the women among themselves, how many different rankings are possible?

More examples

- Example: You have 10 textbooks that you want to order on your bookshelf: 3 mathematics books, 3 physics books, 2 chemistry books and 2 biology books. You want to arrange them so that all the books dealing with the same subject are together on the shelf. How many different arrangements are possible?

- Solution:
  - For each ordering of the subject, say M/P/C/B or P/B/C/M, there are $3!3!2!2! = 144$ arrangements.
  - As there are $4!$ ordering of the subjects, then you have $144 \cdot 4! = 3456$ possible arrangements.

More examples

- Example: How many different letter arrangements can be formed from the letters EEPPPR?
- Solution: There are 6! possible permutations of letters E1E2P1P2P3R but the letters are not labeled so we cannot distinguish E1 and E2 and P1, P2 and P3; e.g. E1P1E2P2P3R cannot be distinguished from E2P1E1P2P3R and E2P2E1P1P3R. That is if we permuted the E.s and the P.s among themselves then we still have EPEPPR. We have 2!3! permutations of the labeled letters of the form EPEPPR.
- Hence there are 6! / (2!3!) = 60 possible arrangements of the letters EEPPPR.
- General Result. Suppose you have n objects. The number of different permutations of these n objects of which n1 are alike, n2 are alike,..., nr are alike is given by: n! / (n1! n2! ... nr!)

More examples

- Example: A speed skating tournament has 4 competitors from South Korea, 3 from Canada, 3 from China, 2 from the USA and 1 from France. If the tournament result lists just the nationalities of the players in the order in which they placed, how many outcomes are possible?

Combinations

- We want to determine the number of different groups of r objects that could be formed from a total of n objects.
- Example: How many different groups of 3 could be selected from A,B,C,D and E?
- Answer: There are 5 ways to select the 1st letter, 4 to select the 2^{nd} and 3 to select the 3rd so 5 . 4 . 3 = 60 ways to select WHEN the order in which the items are selected is relevant.
- When it is not relevant, then say the group BCE is the same as BEC, CEB, CBE EBC, ECB; there are 3! = 6 permutations. So when the order is irrelevant, we have 60/6 = 10 different possible groups.
- General result:
  - When the order of selection is relevant, there are: $n!/(n-r)!$ possible groups.
  - When the order of selection is irrelevant, there are $n! / (n-r)!r!$ (Binomial coefficient)

Combinations: Examples

- Example: Assume we have an horse race with 12 horses. What is the possible number of combinations of 3 horses when the order matters and when it does not?
- Answer:
  - When it matters, we have 12! (12-3)! = 12 11 10 = 1320 and
  - when it does not matter, we have 12! / (12-3)!3! = 220.
- Example: From a group of 5 women and 7 men, how many different committers consisting of 2 women and 3 men can be form?
  - What is 2 of the men are feuding and refuse to be serve on the committee together?
- Answer:
  - We have 5 choose 2 = 10 possible W groups and 7 choose 3 = 35 possible M groups, so 10 35 = 350 groups. In the 35 groups, we
  - We have 5 = (2 choose 2) x (5 choose 1) groups where the 2 feuding men can be so there are 10 30 = 300 possible committees.

More examples

- Example: Assume we have a set of n antennas of which m are defective. All the defectives and all the functional are indistinguishable. How many linear orderings are there in which no two defectives are consecutives?

- Answer: (n-m+1 choose m)