

Maximum Subarray

Given an integer array `nums`, find the contiguous subarray (containing at least one number) which has the largest sum and return *its sum*.

A **subarray** is a **contiguous** part of an array.

Input: `nums = [-2,1,-3,4,-1,2,1,-5,4]`

Output: 6

Explanation: `[4,-1,2,1]` has the largest sum = 6.

The naive method is to run two loops. The outer loop picks the beginning element, the inner loop finds the maximum possible sum with first element picked by outer loop and compares this maximum with the overall maximum. Finally, return the overall maximum. The time complexity of the Naive method is $O(n^2)$.

Using **Divide and Conquer** approach, we can find the maximum subarray sum in $O(n\log n)$ time. Following is the Divide and Conquer algorithm.

1. Divide the given array in two halves
2. Return the maximum of following three
 - Maximum subarray sum in left half (Make a recursive call)
 - Maximum subarray sum in right half (Make a recursive call)
 - Maximum subarray sum such that the subarray crosses the midpoint

```
3. static int maxCrossingSum(int arr[], int l, int m,
4.                          int h)
5. {
6.     // Include elements on left of mid.
7.     int sum = 0;
8.     int left_sum = Integer.MIN_VALUE;
9.     for (int i = m; i >= l; i--) {
10.         sum = sum + arr[i];
11.         if (sum > left_sum)
12.             left_sum = sum;
13.     }
14.
15.     // Include elements on right of mid
16.     sum = 0;
17.     int right_sum = Integer.MIN_VALUE;
18.     for (int i = m + 1; i <= h; i++) {
19.         sum = sum + arr[i];
20.         if (sum > right_sum)
21.             right_sum = sum;
22.     }
23.
```

```

24.         // Return sum of elements on left
25.         // and right of mid
26.         // returning only left_sum + right_sum will fail for
27.         // [-2, 1]
28.         return Math.max(left_sum + right_sum,
29.                         Math.max(left_sum, right_sum));
30.     }
31.
32.     // Returns sum of maximum sum subarray
33.     // in aa[l..h]
34.     static int maxSubArraySum(int arr[], int l, int h)
35.     {
36.         // Base Case: Only one element
37.         if (l == h)
38.             return arr[l];
39.
40.         // Find middle point
41.         int m = (l + h) / 2;
42.
43.         /* Return maximum of following three
44.         possible cases:
45.         a) Maximum subarray sum in left half
46.         b) Maximum subarray sum in right half
47.         c) Maximum subarray sum such that the
48.         subarray crosses the midpoint */
49.         return Math.max(
50.             Math.max(maxSubArraySum(arr, l, m),
51.                     maxSubArraySum(arr, m + 1, h)),
52.             maxCrossingSum(arr, l, m, h));
53.     }
54.

```

Kadane's Algorithm:

Initialize:

```
max_so_far = INT_MIN
```

```
max_ending_here = 0
```

Loop for each element of the array

```
(a) max_ending_here = max_ending_here + a[i]
```

```
(b) if(max_so_far < max_ending_here)
```

```
    max_so_far = max_ending_here
```

(c) if(max_ending_here < 0)

max_ending_here = 0

return max_so_far

```
public int maxSubArray(int[] nums) {  
    int maxsum=Integer.MIN_VALUE;  
    int sum=0;  
    int n=nums.length;  
    for(int i=0;i<n;i++)  
    {  
        if(nums[i]>sum+nums[i])  
        {  
            sum=nums[i];  
        }  
        else  
        {  
            sum+=nums[i];  
        }  
        maxsum=Math.max(maxsum,sum);  
    }  
    return maxsum;  
}
```