Weekly Assignment 2

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Algorithm Design and Analysis

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Solution 2:

Given in the textbook, the BinSearch computes

$$mid = \frac{lo + hi}{2}$$

at every iteration and checks if the item i in in the array A. We have to prove that the algorithm terminates. If we can show that in every iteration the search interval decreases until it no longer satisfies the loop condition $lo \leq hi$, that is, at some points we will have $lo > hi$, then we will have the proof.
Let's the values of lo and hi in two successive iterations be $[lo_1, hi_1]$ and $[lo_2, hi_2]$. We need to show, $hi_2 - lo_2 < hi_1 - lo_1$.

- **Case-1:** $A[mid] = i$
  The element i is found and the algorithm returns **True**, so the **while** loop terminates.

- **Case 2:** $A[mid] < i$
  i is in the right half of the array, so $lo_2 = mid+1$ and $hi_2 = hi_1$. The size of the interval becomes, $hi_2 - lo_2 = hi_1 - (mid + 1)$. We can write, $mid \geq \frac{lo_1+hi_1}{2}$ and substituting that in the new interval size

  $$hi_2 - lo_2 = \frac{hi_1 - lo_1}{2}$$

  we get exactly half of the previous interval size, so we have $hi_2 - lo_2 < hi_1 - lo_1$

- **Case 3:** $A[mid] > i$
  In this case, i is in the left half of the array, the new size becomes, $hi_2 = mid - 1$ and $lo_2 = lo_1$. Using $mid \leq \frac{lo_1+hi_1}{2}$ we get

  $$hi_2 - lo_2 = \frac{hi_1 - lo_1}{2} - 1,$$

  which shows that the new interval size is smaller than the half of the interval. So, we again can say that $hi_2 - lo_2 < hi_1 - lo_1$

In all the cases, the size of the search interval $hi - lo$ decreases by at least half in each iteration. Thus, the algorithm will eventually terminate.