Date:

FINAL PAPER

NUMERICAL ANALYSIS

CS19-037

MUNIB-UL-HASSAN

SECTION: "A"

DATE: JUNE 7, 2021

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ANSWER 01 (a)

Bisection Method:

Roll no = 2019-CS-037

$R = 37$

$f(x) = x - (R+10)^{-x}$ $[0, 1]$

$f(x) = x - (37+10)^{-x}$

$f(x) = x - (47)^{-x}$

$$f(x) = x - \frac{1}{(47)^x}$$

For 1^{st} iteration:

$a = 0$ $b = 1$ $c = \frac{a+b}{2} = \frac{0+1}{2} = 0.5$

$$f(c) = c - \frac{1}{(47)^c} = 0.5 - \frac{1}{(47)^{0.5}}$$

$f(c) = 0.354135$

$f(c) > 0$ $a = c$

$f(c) < 0$ $b = c$

$f(c) = 0$ iteration end.

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i	a	b	c	f(a)	f(b)	f(c)
1	0	1	0.5	-1	0.978723	0.354135
2	0.5	1	0.75	0.354135	0.978723	0.694291
3	0.75	1	0.875	0.694291	0.978723	0.840572
4	0.875	1	0.9375	0.840572	0.978723	0.910435
5	0.9375	1	0.96875	0.910435	0.978723	0.944753

ANSWER 01 (b)

$$f(x) = x^3 - 4x^2 + x - 10$$

$$x_0 = R - 10$$

$$R = 37$$

$$x_0 = 37 - 10 = 27$$

$$f'(x) = 3x^2 - 8x + 1$$

$$c = x_0 - \frac{f(x_0)}{f'(x_0)}$$

i	x_0	c
1	27	18.48884
2	18.48884	12.84201
3	12.84201	9.124493
4	9.124493	6.729468
5	6.729468	5.280018
6	5.280018	4.549633

ANSWER # 2

There are 4 Quadrature formula.

1- Trapezoidal

2- Simpson's $\frac{1}{3}$

3- Simpson's $\frac{3}{8}$

4- Boole's Rule.

i) $\cos(x)$ $[2+R, 3+R]$

$R = 37$

$f(x) = \cos x$ $[39, 40]$

$n = 1$

Trapezoidal

$f(x) = \cos x$ $a = 39$ $b = 40$

$h = \frac{b-a}{n} = \frac{40-39}{1} = 1$

$$\int_a^b f(x)\,dx = \frac{f(a) + f(b)}{2h}$$

$$\int_{39}^{40} \cos x\,dx = \frac{\cos a + \cos b}{2(1)} = \cancel{0.7771} \frac{\cos 39 + \cos 40}{2}$$

$$\int_{39}^{40} \cos x\,dx = \frac{0.777146 + 0.766044}{2}$$

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$$\int_{39}^{40} \cos x \, dx = \frac{1.54319}{2}$$

$$\boxed{\int_{39}^{40} \cos x \, dx = 0.771595}$$

- Simpson's $\frac{1}{3}$

$$\int_a^b f(x)\,dx = \frac{h}{3}\left[f(a) + f(b) + f(c)\right]$$

$$h = \frac{b-a}{2} = \frac{1}{2} \qquad c = \frac{a+b}{2} = \frac{39+40}{2} = 39.5$$

$$\int_{39}^{40} \cos(x)\,dx = \frac{1}{3}\left(\cos a + \cos b + \cos c\right)$$

$$= \frac{1/2}{3}\left[\cos 39 + \cos 40 + \cos 39.5\right]$$

$$= \frac{1/2}{3}\left[0.777146 + 0.766044 + \cancel{1.5496597}\right] \quad 0.771624$$

$$= \cancel{0.771604} \quad \frac{1}{6}(2.314814)$$

$$\boxed{\int_{39}^{40} \cos(x)\,dx = \cancel{0.771604} \; 0.385802}$$

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Simpson's $\frac{3}{8}$:

$$\int_a^b f(x)\,dx = \frac{3h}{8}\left[f_0 + 3f_1 + 3f_2 + f_3\right)$$

It is also written as

$$\int_a^b f(x)\,dx = \frac{3h}{8}\left[f(a) + f(a+h) + f(a+2h) + f(a+3h)\right]$$

$$h = \frac{b-a}{n} = \frac{40-39}{3} = \frac{1}{3}$$

$$\int_{39}^{40} \cos x\,dx = \frac{3\left(\frac{1}{3}\right)}{8}\left[f(39) + f\left(39+\frac{1}{3}\right) + f\left(39+\frac{2}{3}\right) + f\left(39+3/3\right)\right]$$

$$\int_{39}^{40} \cos x\,dx = \frac{1}{8}\left[f(39) + f(39.333) + f(39.666) + f(40)\right]$$

$$\int_{39}^{40} \cos x\,dx = \frac{1}{8}\left[\cos 39 + \cos 39.333 + \cos 39.666 + \cos 40\right]$$

$$\int_{39}^{40} \cos x\,dx = \frac{1}{8}\left[0.777146 + 0.773475 + 0.769778 + 0.766044\right]$$

$$\boxed{\int_{39}^{40} \cos x\,dx = 0.385805}$$

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Boole's Rule

$$\int_a^b f(x)\,dx = \frac{2h}{45}\left[7f(a) + 32f(a+h) + 12f(a+2h) + 32[f(a+3h)] + 7f(a+4h)\right]$$

$$h = \frac{b-a}{4} = \frac{40-39}{4} = \frac{1}{4}$$

$$\int_{39}^{40} \cos x\,dx = \frac{2\left(\frac{1}{4}\right)}{45}\; 7f(39) + 32f\left(39+\frac{1}{4}\right) + 12f\left[39+2\left(\frac{1}{4}\right)\right] + 32f\left(39+3\left(\frac{1}{4}\right)\right) + 7f\left(39+4\left(\frac{1}{4}\right)\right)\Big]$$

$$= \frac{\frac{1}{2}}{45}\left[7(0.777146) + 32f(39.25) + 12f(39.5) + 32f(39.75) + 7f(40)\right]$$

$$= \frac{1}{90}\left[5.440022 + 32\cos(39.25) + 12\cos(39.5) + 32\cos(39.75) + 7\cos(40)\right]$$

$$= \frac{1}{90}\left[5.440022 + 32(0.77439) + 12(0.771624) + 32(0.768841) + 7(0.766044)\right]$$

$$= \frac{1}{90}\left[5.440022 + 24.78048 + 9.259488 + 24.602912 + 5.362308\right]$$

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$$\boxed{\int_{39}^{40} \cos(x)\,dx = 0.771613}$$

(ii)

$\ln(x+2)$ $[1.1+R, 2+R]$

$R = 37$

$[38.1 \quad 39]$

$a = 38.1$ $b = 39$

$f(x) = \ln(x+2)$

- Trapezoidal.

$f(x) = \ln(x+2)$ $a = 38.1$ $b = 39$

$$h = \frac{b-a}{n} = \frac{39-38.1}{1}$$

$h = 0.9$

$$\int_a^b f(x)\,dx = \frac{f(a)+f(b)}{2h}$$

$$\int_{38.1}^{39} \ln(x+2)\,dx = \frac{f(38.1)+f(39)}{2(0.9)}$$

$$\int_{38.1}^{39} \ln(x+2)\,dx = \frac{\ln(38.1+2)+\ln(39+2)}{1.8}$$

$$\boxed{\int_{38.1}^{39} \ln(x+2)\,dx = 4.113860}$$

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Simpson's 1/3

$$\int_a^b f(x)\,dx = \frac{h}{3}\left[f(a) + f(b) + f(c)\right]$$

$$h = \frac{b-a}{2} = \frac{39-38.1}{2} = \frac{0.9}{2} = 0.45$$

$$c = \frac{39+38.1}{2} = 38.55$$

$$\int_{38.1}^{39} \ln(x+2)\,dx = \frac{0.45}{3}\left[f(38.1) + f(39) + f(38.55)\right]$$

$$= \frac{0.45}{3}\left[\ln(38.1+2) + \ln(39+2) + \ln(38.55+2)\right]$$

$$= \frac{0.45}{3}\left[\ln(40.1) + \ln(41) + \ln(40.55)\right]$$

$$= \frac{0.45}{3}\left[3.691376 + 3.713572 + 3.702535\right]$$

$$= 1.666122$$

$$\int_a^b f(x)\,dx =$$

$$\boxed{\int_{38.1}^{39} \ln(x+2)\,dx = 1.666122}$$

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Simpson's $\frac{3}{8}$:

$$\int_a^b f(x)\,dx = \frac{3h}{8}\left[f(a)+3f(a+h)+3f(a+2h)+f(a+3h)\right]$$

$$h = \frac{b-a}{3} = \frac{39-38.1}{3} = \frac{0.9}{3} = 0.3$$

$$\int_{38.1}^{39} \ln(x+2)\,dx = \frac{3(0.3)}{8}\left[f(38.1)+3f(38.1+0.3)+3f(38.1+2(0.3))+f(38.1+3(0.3))\right]$$

$$= \frac{0.9}{8}\left[f(38.1)+3f(38.4)+3f(38.7)+f(39)\right]$$

$$= \frac{0.9}{8}\left[\ln(38.1+2)+3(\ln(38.4+2))+3(\ln(38.7+2))+\ln(39+2)\right]$$

$$= \frac{0.9}{8}\left[\ln 40.1 + 3(\ln 40.4)+3(\ln 40.7)+\ln 41\right]$$

$$= \frac{0.9}{8}\left[3.691376+3(3.698829)+3(3.706228)+3.713572\right]$$

$$\boxed{\int_{38.1}^{39} \ln(x+2)\,dx = 3.271513}$$

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Boole's Rule.

$$\int_a^b f(x)\,dx = \frac{2h}{45}\left[7f(a) + 32f(a+h) + 12f(a+2h) + 32f(a+3h) + 7f(a+4h)\right]$$

$$h = \frac{b-a}{4} = \frac{39 - 38.1}{4} = \frac{0.9}{4} = 0.225$$

$$\int_{38.1}^{39} \ln(x+2)\,dx = \frac{3(0.225)}{45}\left[7(f(38.1)) + 32f(38.1+0.225) + 12f(38.1+2(0.225)) + 32f(38.1+3(0.225)) + 7f(38.1+4(0.225))\right]$$

$$= \frac{0.675}{45}\left[7f(38.1) + 32f(38.325) + 12f(38.55) + 32f(38.775) + 7(f(39))\right]$$

$$= \frac{0.675}{45}\left[7\ln(38.1+2) + 32(\ln(38.325+2)) + 12(\ln(38.55+2)) + 32(\ln(38.775+2)) + 7(\ln(39+2))\right]$$

$$= 0.015\left[7(\ln(40.1)) + 32(\ln 40.325) + 12(\ln 40.55) + 32(\ln 40.775) + 7(\ln 41)\right]$$

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$= 0.015 [7(3.691376) + 32(3.696971) + 12(3.7025)$

$+ 32(3.708069) + 7(3.7135)]$

$= 0.015 [25.839632 + 118.303072 + 44.43 +$

$118.658208 + 25.9945]$

$= 0.015 [333.225412]$

$= \cancel{4.9983838} \quad 4.998381$

$$\int_{38.1}^{39} \ln(x+2)\,dx = \boxed{4.998381}$$

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ANSWER # 03(a)

$y = \sqrt{x}$ $x = R, R+1, R+2$.

$R = 37$

$x = 37, 38, 39$.

$x =$	37	38	39
y	6.0827	6.1644	6.2449

Newton's difference table

	X	Y	Δ	Δ^2
x_0	37	a_0 6.0827		
			$\frac{6.1644-6.0827}{38-37}$	
x_1	38	6.1644	$= 0.0817$ a_1	$\frac{0.0805-0.0817}{39-37}$
x_2	39	6.2449	$\frac{6.2449-6.1644}{39-38}$	$= -0.0006$
			$= 0.0805$	a_2

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newton's polynomial

$$f(x) = a_0 + a_1(x - x_0) + a_2(x - x_0)(x - x_1)$$

$$f(x) = 6.0827 + 0.0817(x-37) + 0.0006(x-37)(x-38)$$

$$\boxed{f(x) = 6.0827 + 0.0817(x-37) + 0.0006(x-37)(x-38)}$$

ANSWER # 3 (b)

x	0	1	2	3
y	6	5+R	12+R	15+R

$$f(1.5) = ?$$

Sol:

$R = 37$

x	0	1	2	3
y	0	42	49	52

$$L(x) = \frac{(x-x_2)(x-x_3)(x-x_4)}{(x_1-x_2)(x_1-x_3)(x_1-x_4)} y_1 + \frac{(x-x_1)(x-x_3)(x-x_4)}{(x_2-x_1)(x_2-x_3)(x_2-x_4)} y_2 + \frac{(x-x_1)(x-x_2)(x-x_4)}{(x_3-1)(x_3-x_2)(x_3-x_4)} y_3 + \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_4-x_1)(x_4-x_2)(x_4-x_3)} y_4$$

$$L(x) = \frac{(x-1)(x-2)(x-3)}{(0-1)(0-2)(0-3)}(0) + \frac{(x-0)(x-2)(x-3)}{(1-0)(1-2)(1-3)} 42 + \frac{(x-0)(x-1)(x-3)}{(2-0)(2-1)(2-3)} 49 + \frac{(x-0)(x-1)(x-2)}{(3-0)(3-1)(3-2)} 52$$

$$L(x) = 0 + \frac{x(x-2)(x-3)}{1(-1)(-2)} 42 + \frac{x(x-1)(x-3)}{2(1)(-1)} 49 + \frac{x(x-1)(x-2)}{3(2)(1)} 52$$

$$L(x) = \frac{x(x-2)(x-3)}{2} 42 + \frac{x(x-1)(x-3)}{-2} 49 + \frac{x(x-1)(x-2)}{\cancel{6}\ 3} \cancel{52}\ 26$$

$$L(x) = x(x-2)(x-3)(21) - \frac{(x-1)(x-3)x\,49}{2} + \frac{x(x-1)(x-2)\,26}{3}$$

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For $f(1.5)$

$$L(1.5) = 1.5(1.5-2)(1.5-3)\,21 - \frac{(1.5-1)(1.5-3)(1.5)}{2}$$

$$(49) + \frac{1.5(1.5-1)(1.5-2)}{3}\,26$$

$$L(1.5) = \{1.5(-0.5)(-1.5)\,21\} - \left\{\frac{(0.5)(-1.5)(1.5)(49)}{26}\right\}$$

$$+ \left\{\frac{1.5(0.5)(-0.5)(26)}{3}\right\}$$

$$L(1.5) = 23.625 - (-55.125/26) + \left(\frac{-9.75}{3}\right)$$

$$L(1.5) = 23.625 + 2.12019 - 3.25$$

$$\boxed{L(1.5) = 22.49519}$$

ANSWER 4(a)

$y' = x + y$ $\qquad\qquad$ $y(0) = 1$

$h = \dfrac{R+5}{10}$

Sol:

Euler's Method

$$y_{n+1} = y_n + h\, f(x_n, y_n)$$

$x_0 = 0$ $\qquad$ $y_0 = 1$ $\qquad$ $h = \dfrac{37+5}{10} = 4.2$

$y_1 = y_0 + h\,[f(x_0, y_0)]$

$y_1 = 1 + 4.2\,[f(0, 1)]$

$y_1 = 1 + 4.2\,(0+1)$

$y_1 = 1 + 4.2(1)$

$y_1 = 1 + 4.2$

$y_1 = \cancel{4}\cancel{3}5.2$

$x_1 = x_0 + h = 0 + 4.2 = 4.2$

$y_2 = y_1 + h[f(x_1, y_1)]$

$y_2 = 5.2 + 4.2\,[f(4.2, 5.2)]$

$y_2 \quad 5.2 + 4.2\,(4.2 + 5.2)$

$y_2 = 5.2 + 4.2\,(9.4)$

$y_2 = 44.68$

$\boxed{y_1 = 5.2}$ $\qquad$ $\boxed{y_2 = 44.68}$

~~HUE~~ Heun's Formula

$$y_{n+1} = y_n + \frac{h}{2}\left[f(x_n, y_n) + f(x_n + h, y_n + h f(x_n, y_n))\right]$$

$$y_1 = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_0 + h, y_0 + h f(x_0, y_0))\right]$$

$f(x_0, y_0) = f(0, 1) = 0 + 1 = 1$

$$y_1 = 1 + \frac{4.2}{2}\left[1 + f(0 + 4.2, 1 + 4.2(1))\right]$$

$y_1 = 1 + 2.1\,[1 + f(4.2, 5.2)]$

$y_1 = 1 + 2.1\,[1 + (4.2 + 5.2)]$

$y_1 = 22.84$

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$$x_n = x_{n-1} + h$$

$$x_1 = x_0 + h$$

$$x_1 = 0 + 4.2 = 4.2$$

$$y_2 = y_1 + \frac{h}{2}\left[f(x_1, y_1) + f(x_1 + h, y_1 + h\, f(x_1, y_1))\right]$$

$$f(x_1, y_1) = f(4.2, 22.84)$$

$$= 4.2 + 22.84$$

$$f(x_1, y_1) = 27.04$$

$$y_2 = 22.84 + \frac{4.2}{2}\left[27.04 + f(4.2 + 4.2, 22.84 + 4.2(27.04))\right]$$

$$y_2 = 22.84 + 2.1\left[27.04 + f(8.4, 22.84 + 113.568)\right]$$

$$y_2 = 22.84 + 2.1\left[27.04 + f(8.4, 136.408)\right]$$

$$y_2 = 22.84 + 2.1\left[27.04 + (8.4 + 136.408)\right]$$

$$y_2 = 383.7208$$

$$\boxed{y_1 = 22.84} \qquad \boxed{y_2 = 383.7208}$$

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ANSWER 4(b)

$y' = y - xy \qquad y(1) = 1 \qquad y_1 = ?$

$h = \frac{R}{100}$

sol.

$R = 37$

$h = \frac{37}{100} = 0.37 \qquad x_0 = 1$

$y_0 = 1$

$$y_1 = y_0 + \frac{1}{6}\left[K_1 + 2K_2 + 2K_3 + K_4\right]h$$

$K_1 = f(x_0, y_0) = y_0 - x_0 y_0$

$K_1 = 1 - 1(1) = 1 - 1$

$K_1 = 0$

$K_2 = f\left(x_0 + \frac{1}{2}h, y_0 + \frac{1}{2}K_1 h\right)$

$K_2 = f\left(1 + \frac{1}{2}(0.37), 1 + \frac{1}{2}0(0.37)\right)$

$K_2 = f(1.185, 1)$

$K_2 = \cancel{1.185}\ 1 - (1.185)(1)$

$K_2 = 1 - 1.185$

$K_2 = -0.185$

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$$K_3 = f\left(x_0 + \frac{1}{2}h, y_0 + \frac{1}{2}k_2 h\right)$$

$$K_3 = f\left(1 + \frac{1}{2}(0.37), 1 + \frac{1}{2}(-0.185)(0.37)\right)$$

$$K_3 = f(1 + 0.185, 1 + (-0.034225))$$

$$K_3 = f(1.185, 1 - 0.034225)$$

$$K_3 = f(1.185, 0.965775)$$

$$K_3 = 0.965775 - (1.185)(0.965775)$$

$$K_3 = -0.17867$$

$$K_4 = f(x_0 + h, y_0 + K_3 h)$$

$$K_4 = f(1 + 0.37, 1 + (-0.17867)(0.37))$$

$$K_4 = f(1.37, 0.933893)$$

$$K_4 = 0.933893 - (1.37)(0.933893)$$

$$K_4 = -0.34554$$

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$$y_1 = 1 + \frac{1}{6}\left[0 + 2(-0.185) + 2(-0.17867) + (-0.34554)\right] 0.37$$

$$y_1 = 1 + \frac{1}{6}\left[-0.37 - 0.35734 - 0.34554\right] 0.37$$

$$y_1 = 1 + \frac{1}{6}\left[-1.07288\right](0.37)$$

$$y_1 = 1 - 0.06616$$

$$\boxed{y_1 = 0.93384}$$