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CS19-037

Section "A"

Euler's Method.

$e^y \frac{dy}{dx} + x^2y^2 = 2\sin(3x)$ $\quad y(0) = 5$

$h = 0.5 \quad y(10) = ?$

SOLUTION

$$f(x,y) = \frac{2\sin(3x) - x^2y^2}{e^y}$$

$x_0 = 0 \qquad y_0 = 5 \qquad h = 0.5$

1st iteration

$n = 0$

$$y_{n+1} = y_n + h[f(x_n, y_n)]$$

$$y_1 = y_0 + h[f(x_0, y_0)]$$

$$f(x_0, y_0) = \frac{2\sin(3x_0) - x_0^2y_0^2}{e^{y_0}}$$

$$f(x_0, y_0) = \frac{2\sin(3(0)) - (0)^2(5)^2}{e^5}$$

$$f(x_0, y_0) = \frac{2\sin 0^\circ - 0}{e^5} = \frac{0}{e^5} = 0$$

$$y_1 = y_0 + h(0)$$

$$y_1 = 5 + 0.5(0)$$
$$y_1 = 5 + 0$$
$$y_1 = 5$$

$$x_n = x_{n-1} + h$$
$$x_1 = x_0 + h = 0 + 0.5 = 0.5$$
$$x_1 = 0.5$$

$x_1 = 0.5$ $\qquad$ $y_1 = 5$

2nd iteration.

$n = 1$

$$y_2 = y_1 + h[f(x_1, y_1)]$$

$$f(x_1, y_1) = \frac{2\sin(3x_1) + x_1^2 y_1^2}{e^{y_1}}$$

$$f(x_1, y_1) = \frac{2\sin(3(0.5)) + (0.5)^2(5)^2}{e^5}$$

$$f(x_1, y_1) = \frac{2\sin(1.5) + (0.25)(25)}{e^5}$$

$$f(x_1, y_1) = \frac{2(0.026) + 31.25}{e^5} = \frac{-31.198}{(2.718)^5}$$

$$f(x_1, y_1) = -0.2102$$

$$y_2 = 5 + 0.5(-0.2102)$$
$$y_2 = 5 - 0.105 = 4.8948$$

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$x_2 = x_1 + h = 0.5 + 0.5 = 1$

$x_2 = 1 \qquad y_2 = 4.8948$

3rd iteration

$n = 2$

$y_3 = y_2 + h\left[f(x_2, y_2)\right]$

$f(x_2, y_2) = \dfrac{2\sin(3x_2) - x_2^2 y_2^2}{e^{y_2}}$

$f(x_2, y_2) = \dfrac{2\sin(3(1)) - (1)^2 (4.8948)^2}{e^{4.8948}}$

$f(x_2, y_2) = \dfrac{2\sin 3 - 1(23.96)}{133.63}$

$f(x_2, y_2) = \dfrac{3(0.0523) - 23.96}{133.63}$

$F(x_2, y_2) = -0.17812$

$y_3 = 4.8948 + 0.5(-0.17812)$

$y_3 = 4.8948 - 0.0890$

$y_3 = 4.8057$

$x_3 = x_2 + h = 1 + 0.5 = 1.5$

$x_3 = 1.5 \qquad y_2 = 4.8057$

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4th iteration

$n = 3$

$$y_4 = y_3 + h[f(x_3, y_3)]$$

$$f(x_3, y_3) = \frac{2\sin(3x_3) - x_3^2 y_3^2}{e^{y_3}}$$

$$f(x_3, y_3) = \frac{2\cdot\sin(3(1.5)) - (1.5)^2(4.8057)^2}{e^{4.8057}}$$

$$f(x_3, y_3) = \frac{2(\sin 4.5) - (2.25)(23.04)}{122.205}$$

$$f(x_3, y_3) = -0.4229$$

$$y_4 = 4.8057 + [0.5\ (-0.4229)]$$

$$y_4 = 4.5942$$

$$x_4 = x_3 + h = 1.5 + 0.5 = 2$$

$$x_4 = 2 \qquad y_4 = 4.5942$$

5th iteration

$n = 4$

$$y_5 = y_4 + h[f(x_4, y_4)]$$

$$f(x_4, y_4) = \frac{2\sin(3x_4) - x_4^2 y_4^2}{e^{y_4}}$$

$$f(x_4, y_4) = \frac{2\sin[3(2)] - (2)^2(4.5942)^2}{e^{4.5942}}$$

$$F(x_4, y_4) = \frac{2\sin 6 - (4)(21.107)}{e^{4.5942}}$$

$$f(x_4, y_4) = \frac{2(0.1045) - 84.428}{98.908}$$

$$F(x_4, y_4) = \frac{0.209 - 84.428}{98.908}$$

$$F(x_4, y_4) = -0.8514$$

$$y_5 = 4.5942 + 0.5(-0.8514)$$

$$y_5 = 4.1684$$

$$x_5 = x_4 + n = 2 + 0.5 = 2.5$$

$$x_5 = 2.5 \qquad y_5 = 4.1684$$

6th iteration

$$n = 5$$

$$y_6 = y_5 + n\,[F(x_5, y_5)]$$

$$y_6 = 4.1684 + 0.5\left[\frac{2(\sin 3(2.5)) - (2.5)^2(4.1684)^2}{e^{4.1684}}\right]$$

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$$y_6 = 4.1684 + 0.5\left[\frac{2\sin 7.5 - (6.25)(17.375)}{64.611}\right]$$

$$y_6 = 3.3300$$

$$x_6 = x_5 + h = 2.5 + 0.5 = 3.0$$

$$x_6 = 3.0 \qquad y_6 = 3.3300$$

7th iteration

$$n = 6$$

$$y_7 = y_6 + h[f(x_6, y_6)]$$

$$y_7 = 3.3300 + 0.5\left[\frac{2\sin(3(3)) - 3^2(3.3300)^2}{e^{3.3300}}\right]$$

$$y_7 = 1.549$$

$$x_7 = x_6 + h = 3 + 0.5 = 3.5$$

$$x_7 = 3.5 \qquad y_7 = 1.549$$

$$x_7 = 3.5$$

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8th iteration

$$y_8 = y_7 + h[f(x_7, y_7)] \quad n=7$$

$$y_8 = 1.549 + 0.5\left[\frac{2\sin(3x_7) - x_7^2 y_7^2}{e^{y_7}}\right]$$

$$y_8 = 1.549 + 0.5\left[\frac{2\sin[3(3.5)] - (3.5)^2(1.549)^2}{e^{1.549}}\right]$$

$$y_8 = 1.549 + 0.5\left[\frac{2(\sin 10.5) - 29.39}{4.706}\right]$$

$$y_8 = -1.532$$

$$x_8 = x_7 + h = 3.5 + h = 3.5 + 0.5 = 4$$

$$x_8 = 4 \qquad y_8 = -1.532$$

9th iteration

$$y_9 = y_8 + h[f(x_8, y_8)] \quad n=8$$

$$y_9 = -1.532 + 0.5\left[\frac{2\sin(3(4)) - 4^2(-1.532)^2}{e^{1.532}}\right]$$

$$y_9 = -5.5441$$

$$x_9 = x_8 + h = 4 + 0.5 = 4.5$$

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10th iteration

$n = 9$

$$y_{10} = y_9 + h[f(x_9, y_9)]$$

$$y_{10} = -5.5446 + 0.5\left[\frac{2\sin 3(4.5) - (4.5)^2(-5.54)^2}{e^{5.5446}}\right]$$

$$y_{10} = -6.7582.$$

$$x_{10} = x_9 + h = 4.5 + 0.5 = 5$$

$x_{10} = 5$ $\quad$ $\boxed{y_{10} = -6.7582}$