

# ASSIGNMENT 2 (NUMERICAL ANALYSIS)

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SEC "A"

Question 1-

i) Compare the two solutions.

$$x = g(x)$$

$$x = 1 + x - \frac{x^2}{4}$$

Rearranging the above equation to find the fixed point

$$x^2 = 4$$

$$x = \pm 2$$

Hence the fixed points are  $+2, -2$

(ii) Write the property that the fixed point iteration process will converge to a fixed point.

To converge the iteration method we consider an interval  $[a, b]$  in which the root lies. The iteration method converges if  $|g'(x)| < 1$  whenever  $x \in [a, b]$ .

(iii) what will be happen when  $g'(x) \neq 1$

if  $g'(x) = 1$  then the given iteration method cannot converge in general. Because the error term may vanish or may not vanish increasing the number of iteration.

iv) From the table discuss converging/diverging behaviours of iteration for both cases.

For Case I

$$g'(x) = 1 - x$$

when  $-3 \leq x \leq -1$   $\frac{3}{2} < |g'(x)| < \frac{5}{2}$  and

so  $|g'(x)| > 1$  and so the method is not convergent.

For Case II

when  $1 < x < 2$   $|g'(x)|$  varies from 0 to  $\frac{1}{2}$  and so  $|g'(x)| < 1$  and hence the iteration converge to 2



Q2)

1- Compute Specific heat Capacity  $C_p$  at  $T = 1300$ .

By Newton's Forward Difference table

$T$	$H$	$\Delta H$	$\Delta^2 H$	$\Delta^3 H$
800	1305	.	.	.
1000	1460	$1460 - 1305 =$ 155	.	.
1200	1585	$1585 - 1460 =$ 125	$155 - 125 =$ -30	.
1400	1705	$1705 - 1585 =$ 120	$120 - 125 =$ -5	$-5 - (-30) =$ 25
1600	1825	$1825 - 1705 =$ 120	$120 - 120 = 0$	$0 - (-5) =$ 5

$\Delta^3 H$
.
.
.
.
$5 - 25 = -20$



Apply Newton's Forward Difference Table.

$$f(\bar{x}) = H_0 + \frac{1}{h} (\bar{x} - \bar{x}_0) \Delta H_0 + \frac{1}{2!h^2} (\bar{x} - \bar{x}_0)(\bar{x} - \bar{x}_1) \Delta^2 H_0 + \frac{1}{3!h^3} (\bar{x} - \bar{x}_0)(\bar{x} - \bar{x}_1)(\bar{x} - \bar{x}_2) \Delta^3 H_0 + \frac{1}{4!h^4} (\bar{x} - \bar{x}_0)(\bar{x} - \bar{x}_1)(\bar{x} - \bar{x}_2)(\bar{x} - \bar{x}_3) \Delta^4 H_0$$

$$h = x_1 - x_0 = x_2 - x_1 = \dots = x_n - x_{n-1}$$

$$h = 1000 - 800 = 200$$

Put values from table

$$F(1300) = 1305 + \frac{1}{200} (1300 - 800) 155 + \frac{1}{2(200)^2} (1300 - 800)(1300 - 1000) (-30) + \frac{1}{6(200)^3} (1300 - 800)(1300 - 1000)(1300 - 1200) (25) + \frac{1}{24(200)^4} (1300 - 800)(1300 - 1000)(1300 - 1200)(1300 - 1400) (-20)$$

$$F(1300) = 1305 + \frac{500}{200} (155) + \frac{1}{2(40000)} (500)(300)(-30) + \frac{1}{6(8 \times 10^6)} (500)(300)(100)(25) + \frac{1}{24(1.6 \times 10^9)} (500)(300)(100)(-100)(-20)$$

$$F(1300) = 1305 + 387.5 + 112.5 + 7.8125 + 0.78125$$

$$\boxed{F(1300) = 1588.59375}$$