

Date: _____

FINAL PAPER

NUMERICAL ANALYSIS

CS19-037

MUNIB-UL-HASSAN

SECTION: "R"

DATE: JUNE 7, 2021

ANSWER Q1(a)

Bisection Method:

Roll no = 2019-CS-037

$$R = 37$$

$$f(x) = x - (R+10)^{-x} \quad [0, 1]$$

$$f(x) = x - (37+10)^{-x}$$

$$f(x) \Leftarrow x - (47)^{-x}$$

$$f(x) = x - \frac{1}{(47)^x}$$

For 1st iteration:

$$a = 0 \quad b = 1 \quad c = \frac{a+b}{2} = \frac{0+1}{2} = 0.5$$

$$f(c) = c - \frac{1}{(47)^c} = 0.5 - \frac{1}{(47)^{0.5}}$$

$$f(c) = 0.354135$$

$$f(c) > 0 \quad a = c$$

$$f(c) < 0 \quad b = c$$

$f(c) = 0$ iteration end.

Date: _____

i	a	b	c	$f(a)$	$f(b)$	$f(c)$
1	0	1	0.5	-1	0.978723	0.354135
2	0.5	1	0.75	0.354135	0.978723	0.694291
3	0.75	1	0.875	0.694291	0.978723	0.840572
4	0.875	1	0.9375	0.840572	0.978723	0.910435
5	0.9375	1	0.96875	0.910435	0.978723	0.944753

ANSWER 01 (b)

$$f(x) = x^3 - 4x^2 + x - 10 \leftarrow$$

$$x_0 = R - 10$$

$$R = 37$$

$$x_0 = 37 - 10 = 27$$

$$f'(x) = 3x^2 - 8x + 1$$

$$C = x_0 - \frac{f(x_0)}{f'(x_0)}$$

i	x_0	C
1	27	18.48884
2	18.48884	12.84201
3	12.84201	9.124493
4	9.124493	6.729468
5	6.729468	5.280018
6	5.280018	4.549633

ANSWER # 2

There are 4 Quadrature formula.

1- Trapezoidal

2- Simpson's $\frac{1}{3}$

3- Simpson's $\frac{3}{8}$

4- Boole's Rule.

$$1) \cos(x) [2+R, 3+R]$$

$$R = 37$$

$$f(x) = \cos x [39, 40]$$

$$n = 1$$

Trapezoidal

$$f(x) = \cos x$$

$$a = 39 \quad b = 40$$

$$h = \frac{b-a}{n} = \frac{40-39}{1} = 1$$

$$\int_a^b f(x) dx = \frac{f(a) + f(b)}{2h}$$

$$\int_{39}^{40} \cos x dx = \frac{\cos a + \cos b}{2(1)} = \frac{\cos 39 + \cos 40}{2}$$

$$\int_{39}^{40} \cos x dx = \frac{0.777146 + 0.766044}{2}$$

Date: _____

$$\int_{39}^{40} \cos x \, dx = 1.54319 \quad \# \text{ answer}$$

$$\boxed{\int_{39}^{40} \cos x \, dx = 0.771595}$$

- Simpson's $\frac{1}{3}$

$$\int_a^b f(x) \, dx = \frac{h}{3} [f(a) + f(b) + f(c)]$$

$$h = \frac{b-a}{2} = \frac{1}{2} \quad c = \frac{a+b}{2} = \frac{39+40}{2} = 39.5$$

$$\int_{39}^{40} \cos(x) \, dx = \frac{1}{3} (\cos a + \cos b + \cos c)$$

$$= \frac{1}{3} [\cos 39 + \cos 40 + \cos 39.5]$$

$$= \frac{1}{3} \left[0.777146 + 0.766044 + \cancel{0.796597} \right] \quad 0.771624$$

$$= 0.771604 \quad \frac{1}{6} (2.314814)$$

$$\boxed{\int_{39}^{40} \cos(x) \, dx = 0.771604 \quad 0.385802}$$

Date: _____

Simpson's 3/8:

$$\int_a^b f(x) dx = \frac{3h}{8} [f_0 + 3f_1 + 3f_2 + f_3]$$

It is also written as

$$\int_a^b f(x) dx = \frac{3h}{8} [f(a) + f(a+h) + f(a+2h) + f(a+3h)]$$

$$h = \frac{b-a}{n} = \frac{40-39}{3} = \frac{1}{3}$$

$$\int_{39}^{40} \cos x dx = \frac{3(\frac{1}{3})}{8} [f(39) + f(39 + \frac{1}{3}) + f(39 + \frac{2}{3}) + f(39 + \frac{3}{3})]$$

$$\int_{39}^{40} \cos x dx = \frac{1}{8} [f(39) + f(39.333) + f(39.666) + f(40)]$$

$$\int_{39}^{40} \cos x dx = \frac{1}{8} [\cos 39 + \cos 39.333 + \cos 39.666 + \cos 40]$$

$$\int_{39}^{40} \cos x dx = \frac{1}{8} [0.777146 + 0.773475 + 0.769778 + 0.766044]$$

$$\int_{39}^{40} \cos x dx = 0.385805$$

Date: _____

Bodle's Rule

$$\int_a^b f(x) dx = \frac{h}{45} [7f(a) + 32f(a+h) + 12f(a+2h) \\ + 32f(a+3h) + 7f(a+4h)]$$

$$h = \frac{b-a}{4} = \frac{40-39}{4} = \frac{1}{4}$$

$$\int_{39}^{40} \cos x dx = \frac{1}{4} [7f(39) + 32f(39+\frac{1}{4}) + \\ 12f\left[39 + 2\left(\frac{1}{4}\right)\right] + 32f\left(39 + 3\left(\frac{1}{4}\right)\right) + \\ 7f\left(39 + 4\left(\frac{1}{4}\right)\right)] \\ = \frac{1}{45} [7(0.777146) + 32f(39.25) + 12f(39.5) + \\ 32f(39.75) + 7f(40)]$$

$$= \frac{1}{90} [5.440022 + 32 \cos(39.25) + 12 \cos(39.5) + \\ 32 \cos(39.75) + 7 \cos(40)] \\ = \frac{1}{90} [5.440022 + 32(0.77439) + 12(0.771624) + \\ 32(0.768841) + 7(0.766044)]$$

$$= \frac{1}{90} [5.440022 + 24.78048 + 9.259488 + \\ 24.602912 + 5.362308]$$

Date: _____

$$\boxed{\int_{39}^{40} \cos(x) dx = 0.771613}$$

(ii)

$$\ln(x+2) \quad [1.1+R, 2+R]$$

$$R = 37$$

$$[38.1 \quad 39]$$

$$a = 38.1$$

$$b = 39$$

$$f(x) = \ln(x+2)$$

- Trapezoidal.

$$f(x) = \ln(x+2) \quad a = 38.1 \quad b = 39$$

$$h = \frac{b-a}{n} = \frac{39-38.1}{1}$$

$$h = 0.9$$

$$\int_a^b f(x) dx = \frac{f(a) + f(b)}{2h}$$

$$\int_{38.1}^{39} \ln(x+2) dx = \frac{f(38.1) + f(39)}{2(0.9)}$$

$$\int_{38.1}^{39} \ln(x+2) dx = \frac{\ln(38.1+2) + \ln(39+2)}{1.8}$$

$$\boxed{\int_{38.1}^{39} \ln(x+2) dx = 4.113860}$$

Date: _____

Simpson's 1/3

$$\int_a^b f(x) dx = \frac{h}{3} [f(a) + f(b) + 4f(c)]$$

$$h = \frac{b-a}{2} = \frac{39-38.1}{2} = \frac{0.9}{2} = 0.45$$

$$c = \frac{39+38.1}{2} = 38.55$$

$$\begin{aligned}\int_{38.1}^{39} \ln(x+2) dx &= \frac{0.45}{3} [f(38.1) + f(39) + f(38.55)] \\&= \frac{0.45}{3} [\ln(38.1+2) + \ln(39+2) + \ln(38.55+2)] \\&= \frac{0.45}{3} [\ln(40.1) + \ln(41) + \ln(40.55)] \\&= \frac{0.45}{3} [3.691376 + 3.713572 + 3.702535] \\&= 1.666122\end{aligned}$$

$$\int_a^b f(x) dx =$$

$$\boxed{\int_{38.1}^{39} \ln(x+2) dx = 1.666122}$$

Date: _____

Simpson's $\frac{3}{8}$.

$$\int_a^b f(n) dx = \frac{3h}{8} [f(a) + 3f(a+h) + 3f(a+2h) + f(a+3h)]$$

$$h = \frac{b-a}{3} = \frac{39-38.1}{3} = \frac{0.9}{3} = 0.3.$$

$$\int_{38.1}^{39} \ln(n+2) dx = \frac{3}{8} (0.3) [f(38.1) + 3f(38.1+0.3) + 3f(38.1+2(0.3)) + f(38.1+3(0.3))]$$

$$= \frac{0.9}{8} [f(38.1) + 3f(38.4) + 3f(38.7) + f(39)]$$

$$= \frac{0.9}{8} [\ln(38.1+2) + 3(\ln(38.4+2)) + 3(\ln(38.7+2)) +$$

$$= \frac{0.9}{8} [\ln 40.1 + 3(\ln 40.4) + 3(\ln 40.7) + \ln 41]$$

$$= \frac{0.9}{8} [3.691376 + 3(3.698829) + 3(3.706228) + 3.713572]$$

$$\boxed{\int_{38.1}^{39} \ln(n+2) dx = 3.271513}$$

Date: _____

Boole's Rule.

$$\int_a^b f(x) dx = \frac{2h}{45} [7f(a) + 32f(a+h) + 12f(a+2h) + 32f(a+3h) + 7f(a+4h)]$$

$$h = \frac{b-a}{4} = \frac{39-38.1}{4} = 0.9 = 0.225$$

$$\begin{aligned} \int_{38.1}^{39} \ln(x+2) dx &= \frac{3(0.225)}{45} [7(\ln(38.1)) + 32\ln(38.1+0.225) \\ &\quad + 12\ln(38.1+2(0.225)) + 32\ln(38.1+3(0.225)) \\ &\quad + 7\ln(38.1+4(0.225))] \\ &= \frac{0.675}{45} [7\ln(38.1) + 32\ln(38.325) + 12\ln(38.55) \\ &\quad + 32\ln(38.775) + 7\ln(39)] \\ &= \frac{0.675}{45} [7\ln(38.1+2) + 32(\ln(38.325+2)) + \\ &\quad 12(\ln(38.55+2)) + 32(\ln(38.775+2)) + \\ &\quad 7(\ln(39+2))] \end{aligned}$$

$$\begin{aligned} &= 0.015 [7(\ln(40.1)) + 32(\ln 40.325) + 12(\ln 40.55) \\ &\quad + 32(\ln 40.775) + 7(\ln 41)] \end{aligned}$$

Date: _____

$$= 0.015 [7(3.691376) + 32(3.696971) + 12(3.7025) \\ + 32(3.708069) + 7(3.7135)]$$

$$= 0.015 [25.839632 + 118.303072 + 44.43 + \\ 118.658208 + 25.9945]$$

$$= 0.015 [333.225412]$$

$$= \underline{4.998381} \quad 4.998381$$

$$\int_{38.1}^{39} \ln(n+2) dn = \boxed{4.998381}$$

Date: _____

ANSWER # 03(a)

$$y = \sqrt{x} \quad x = R, R+1, R+2.$$

$$R = 37$$

$$x = 37, 38, 39$$

$$x = 37 \quad 38 \quad 39$$

$$y \quad 6.0827 \quad 6.1644 \quad 6.2449$$

Newton's difference table

x	y	Δ	Δ^2
x_0 37	6.0827 a_0	$6.1644 - 6.0827$	
x_1 38	6.1644	$38 - 37$ $= 0.0817 a_1$	$0.0805 - 0.0817$
x_2 39	6.2449	$6.2449 - 6.1644$ $39 - 38$ $= 0.0805$	$39 - 37$ $= -0.0006$ a_2

Date: _____

newton's polynomial

$$f(x) = a_0 + a_1(x - x_0) + a_2(x - x_0)(x - x_1)$$

$$f(x) = 6.0827 + 0.0817(x - 37) + 0.0006(x - 37)(x - 38)$$

$$f(x) = 6.0827 + 0.0817(x - 37) + 0.0006(x - 37)(x - 38)$$

ANSWER # 3 (b)

x	0	1	2	3
y	6	5+R	12+R	15+R

$$f(1.5) = ?$$

Sol:

$$R = 37$$

x	0	1	2	3
y	0	42	49	52

$$L(n) = \frac{(n-n_1)(n-n_2)(n-n_3)(n-n_4)}{(x_1-x_2)(x_1-x_3)(x_1-x_4)} y_1 +$$

$$\frac{(n-n_1)(n-n_2)(n-n_3)(n-n_4)}{(x_2-x_1)(x_2-x_3)(x_2-x_4)} y_2 + \frac{(n-n_1)(n-n_2)(n-n_3)(n-n_4)}{(x_3-x_1)(x_3-x_2)(x_3-x_4)} y_3$$

$$y_3 + \frac{(n-n_1)(n-n_2)(n-n_3)(n-n_4)}{(x_4-x_1)(x_4-x_2)(x_4-x_3)} y_4$$

$$L(n) = \frac{(n-1)(n-2)(n-3)(0)}{(0-1)(0-2)(0-3)} y_0 + \frac{(n-0)(n-2)(n-3)}{(1-0)(1-2)(1-3)} y_2$$

$$+ \frac{(n-0)(n-1)(n-3)}{(2-0)(2-1)(2-3)} y_9 +$$

$$\frac{(n-0)(n-1)(n-2)}{(3-0)(3-1)(3-2)} y_5$$

$$L(n) = 0 + \frac{n(n-1)(n-2)(n-3)}{1(-1)(-2)} y_0 + \frac{n(n-1)(n-3)}{2(1)(-1)} y_2$$

$$+ \frac{n(n-1)(n-2)}{3(2)(1)} y_5$$

$$L(n) = \frac{n(n-1)(n-2)(n-3)}{2} y_0 + \frac{n(n-1)(n-3)}{2} y_2$$

$$+ \frac{n(n-1)(n-2)}{3} y_5$$

$$L(n) = \frac{n(n-1)(n-2)(n-3)(21)}{2} - \frac{(n-1)(n-3)}{2} n y_0$$

$$+ \frac{n(n-1)(n-2)}{3} y_5$$

Date: _____

For $f(1.5)$

(10) N # 931130 A

$$L(1.5) = 1.5(1.5-2)(1.5-3) \cancel{21} - \frac{(1.5-1)(1.5-3)(1.5)}{2}$$
$$(49) + \frac{1.5(1.5-1)(1.5-2)}{3} \cancel{26}$$

$$L(1.5) = \left\{ 1.5(-0.5)(-1.5) \cancel{21} \right\} - \left\{ (0.5)(1.5)(1.5) \cancel{49} \right\}$$
$$+ \left[\frac{1.5(0.5)(-0.5)(26)}{3} \right]$$

$$L(1.5) = 23.625 - (-55.125/26) + \left(-\frac{9.75}{3} \right)$$

$$L(1.5) = 23.625 + 2.12019 - 3.25$$

$$\boxed{L(1.5) = 22.49519}$$

ANSWER 4(a)

$$y' = n + y$$

$$y(0) = 1$$

$$h = \frac{R+S}{10}$$

Sol:

Euler's Method.

$$y_{n+1} = y_n + h f(x_n, y_n)$$

$$x_0 = 0 \quad y_0 = 1 \quad h = \frac{37+5}{16} = 4.2$$

$$y_1 = y_0 + h [f(x_0, y_0)]$$

$$y_1 = 1 + 4.2 [f(0, 1)]$$

$$y_1 = 1 + 4.2 (0+1)$$

$$y_1 = 1 + 4.2 (1)$$

$$y_1 = 1 + 4.2$$

$$y_1 = \cancel{4.2} 5.2$$

$$x_1 = x_0 + h = 0 + 4.2 = 4.2$$

$$y_2 = y_1 + h[f(x_1, y_1)]$$

$$y_2 = 5.2 + 4.2 [f(4.2, 5.2)]$$
$$y_2 = 5.2 + 4.2 (4.2 + 5.2)$$

$$y_2 = 5.2 + 4.2 (9.4)$$

$$y_2 = 44.68$$

$$\boxed{y_1 = 5.2}$$

$$\boxed{y_2 = 44.68}$$

~~H~~evn's Formula.

$$y_{n+1} = y_n + \frac{h}{2} [f(x_n, y_n) + f(x_n + h, y_n + h f(x_n, y_n))]$$

$$y_1 = y_0 + \frac{h}{2} [f(x_0, y_0) + f(x_0 + h, y_0 + h f(x_0, y_0))]$$

$$f(x_0, y_0) = f(0, 1) = 0 + 1 = 1$$

$$y_1 = 1 + \frac{4.2}{2} [1 + f(0 + 4.2, 1 + 4.2(1))]$$

$$y_1 = 1 + 2.1 [1 + f(4.2, 5.2)]$$

$$y_1 = 1 + 2.1 [1 + (4.2 + 5.2)]$$

$$y_1 = 22.84$$

$$x_n = x_{n-1} + h$$

$$x_1 = x_0 + h$$

$$x_1 = 0 + 4 \cdot 2 = 4 \cdot 2$$

$$y_2 = y_1 + \frac{h}{2} [f(x_1, y_1) + f(x_1 + h, y_1 + h)] f(x_1, y_1)$$

$$f(x_1, y_1) = f(4 \cdot 2, 22.84)$$

$$= 4 \cdot 2 + 22.84$$

$$f(x_1, y_1) = 27.04$$

$$y_2 = 22.84 + \frac{4 \cdot 2}{2} [27.04 + f(4 \cdot 2 + 4 \cdot 2, 22.84 + 4 \cdot 2(27.04))]$$

$$y_2 = 22.84 + 2 \cdot 1 [27.04 + f(8 \cdot 4, 22.84 + 113.568)]$$

$$y_2 = 22.84 + 2 \cdot 1 [27.04 + f(8 \cdot 4, 136.408)]$$

$$y_2 = 22.84 + 2 \cdot 1 [27.04 + (8 \cdot 4 + 136.408)]$$

$$y_2 = 383.7208$$

$$y_1 = 22.84$$

$$y_2 = 383.7208$$

Date: _____

ANSWER u(b)

$$y' = y - xy \quad y(1) = 1 \quad y_1 = ?$$

$$h = \frac{R}{100}$$

Sol.

$$R = 37$$

$$h = \frac{37}{100} = 0.37$$

$$x_0 = 1$$

$$y_0 = 1$$

$$y_1 = y_0 + \frac{1}{6} [k_1 + 2k_2 + 2k_3 + k_4] h$$

$$k_1 = f(x_0, y_0) = y_0 - x_0 y_0$$

$$k_1 = 1 - 1(1) = 1 - 1$$

$$k_1 = 0$$

$$k_2 = f\left(x_0 + \frac{1}{2}h, y_0 + \frac{1}{2}k_1 h\right)$$

$$k_2 = f\left(1 + \frac{1}{2}(0.37), 1 + \frac{1}{2}0(0.37)\right)$$

$$k_2 = f(1.185, 1)$$

$$k_2 = 1 - (1.185)(1)$$

$$k_2 = 1 - 1.185$$

$$k_2 = -0.185$$

Date: _____

$$k_3 = f \left(x_0 + \frac{1}{2}h, y_0 + \frac{1}{2}k_2 h \right)$$

$$k_3 = f \left(1 + \frac{1}{2}(0.37), 1 + \frac{1}{2}(0.185)(0.37) \right)$$

$$k_3 = f (1 + 0.185, 1 + (-0.034225))$$

$$k_3 = f (1.185, -0.034225)$$

$$k_3 = f (1.185, -0.965775)$$

$$k_3 = 0.965775 - (1.185)(-0.965775)$$

$$k_3 = -0.17867$$

$$k_4 = f (x_0 + h, y_0 + k_3 h)$$

$$k_4 = f (1 + 0.37, 1 + (-0.17867)(0.37))$$

$$k_4 = f (1.37, 0.933893)$$

$$k_4 = 0.933893 - (1.37)(0.933893)$$

$$k_4 = -0.34554$$

Date: _____

$$y_1 = 1 + \frac{1}{6} [0 + 2(-0.185) + 2(-0.17867) + (-0.34554)] 0.37$$

$$y_1 = 1 + \frac{1}{6} [-0.37 - 0.35734 - 0.34554] 0.37$$

$$y_1 = 1 + \frac{1}{6} [-1.67288] (0.37)$$

$$y_1 = 1 - 0.06616$$

$$\boxed{y_1 = 0.93384}$$