Ex: 7.2 $\quad f(x) = 1 + e^{-x}\sin(4x)$

fixed interval $= [0, 1]$

For trapezoidal Rule

$$h = x_1 - x_0 = 1 - 0 = 1$$

$$\int_{x_0}^{x_1} f(x)\,dx \simeq \frac{h}{2}(f_0 + f_1).$$

$$\int_0^1 f(x)\,dx \simeq \frac{1}{2}\left[1 + 0.721588\right] = 0.86079$$

For Simpson's Rule (3 nodes) $\underset{x_0}{0} - \underset{x_1}{0.5} - \underset{x_2}{1}$.

$$h = x_1 - x_0 = 0.5 - 0 = 0.5$$

$$h = x_2 - x_1 = 1 - 0.5 = 0.5. \qquad h = 1/2.$$

$$\int_0^1 f(x)\,dx \simeq \frac{h}{3}(f_0 + 4f_1 + f_2)$$

$$\simeq \frac{1/2}{3}\left[1 + 4(1.55152) + (0.72159)\right]$$

$$\simeq \frac{1}{6}\left[1 + 6.20608 + 0.72159\right].$$

$$= 1.32128.$$

Simpson's 3/8 rule:- $h = 1/3$

$x_0 - x_1 - x_2 - x_3$

$0 \quad \frac{1}{3} \quad \frac{2}{3} \quad 1$

$h = x_1 - x_0 = 1/3 - 0 = 1/3$

$h = 2/3 - 1/3 = 1/3$

$h = 1 - 2/3 = 1/3$.

$$\int_{x_0}^{x_3} f(x)\,dx \simeq \frac{3h}{8}\left[f_0 + 3f_1 + 3f_2 + f_3\right].$$

$$\int_0^1 f(x)\,dx = \frac{3(1/3)}{8}\left[f(0) + 3f(1/3) + 3f(2/3) + f(1)\right]$$

$$= \frac{1}{8}\left[1 + 3(1.69642) + 3(1.23447) + 0.72153\right]$$

$$= 1.31440.$$

Boole's Rule:- $h = 1/4$

$x_0 - x_1 - x_2 - x_3 - x_4$

$0 \quad \frac{1}{4} \quad \frac{2}{4} \quad \frac{3}{4} \quad 1$

$0 \quad 1/4 \quad 1/2 \quad 3/4 \quad 1$

$h_1 = 1/4 - 0 = 1/4$

$h_2 = 1/2 - 1/4 = 1/4.$

$h_3 = 3/4 - 1/2 = 1/4$

$h_4 = 1 - 3/4 = 1/4.$

$$\int_{x_0}^{x_4} f(x)\,dx \simeq \frac{2h}{45}\left(7f_0 + 32f_1 + 12f_2 + 32f_3 + 7f_4\right)$$

$$\int_0^1 f(x)\,dx \simeq \frac{2(1/4)}{45}\left[7f(0) + 32f(1/4) + 12f(1/2) + 32f(3/4) + 7f(1)\right]$$

$$= \frac{1}{90}\left[7(1) + 32(1.65534) + 12(1.55152) + 32(1.066666) + 7(0.72159)\right]$$

$$= 1.30859.$$

Scanned with CamScanner